



UNIVERSITY OF  
PLYMOUTH



School of Engineering, Computing and Mathematics  
Faculty of Science and Engineering

2019-03-01

## A new guidance law for trajectory tracking of an underactuated unmanned surface vehicle with parameter perturbations

Haibin Huang

Mian Gong

Yufei Zhuang

Sanjay Sharma *School of Engineering, Computing and Mathematics*

Dianguo Xu

*Let us know how access to this document benefits you*

### General rights

All content in PEARL is protected by copyright law. Author manuscripts are made available in accordance with publisher policies. Please cite only the published version using the details provided on the item record or document. In the absence of an open licence (e.g. Creative Commons), permissions for further reuse of content should be sought from the publisher or author.

### Take down policy

If you believe that this document breaches copyright please [contact the library](#) providing details, and we will remove access to the work immediately and investigate your claim.

Follow this and additional works at: <https://pearl.plymouth.ac.uk/secam-research>

---

### Recommended Citation

Huang, H., Gong, M., Zhuang, Y., Sharma, S., & Xu, D. (2019) 'A new guidance law for trajectory tracking of an underactuated unmanned surface vehicle with parameter perturbations', *Ocean Engineering*, . Available at: <https://doi.org/10.1016/j.oceaneng.2019.02.042>

This Article is brought to you for free and open access by the Faculty of Science and Engineering at PEARL. It has been accepted for inclusion in School of Engineering, Computing and Mathematics by an authorized administrator of PEARL. For more information, please contact [openresearch@plymouth.ac.uk](mailto:openresearch@plymouth.ac.uk).

# A New Guidance Law for Trajectory Tracking of an Underactuated Unmanned Surface Vehicle with Parameter Perturbations

Haibin Huang<sup>a</sup>, Mian Gong<sup>a,b</sup>, Yufei Zhuang<sup>a,\*</sup>, Sanjay Sharma<sup>c</sup>, and Dianguo Xu<sup>b</sup>

<sup>a</sup> Harbin Institute of Technology, Weihai, PR China

<sup>b</sup> Harbin Institute of Technology, Harbin, PR China

<sup>c</sup> University of Plymouth, Plymouth, United Kingdom

**Abstract:** This paper proposes a novel trajectory tracking controller for an underactuated unmanned surface vessel (USV) with multiple uncertainties and input constraints. Unlike previous dynamic surface control (DSC) methods that utilize a first-order filter to obtain derivatives and avoid “explosion of complexity”, we introduce nonlinear tracking differentiators (NTDs) that achieve satisfactory differential performance and fast tracking response and control the vessel to converge to the pseudo-yaw angle and surge. First, a new guidance law for yaw angle and surge is constructed. Second, inner and outer disturbances caused by uncertainties can be observed by reduced-order extended state observers. With the new guidance law, the design process of the controller is simplified and easy to implement. The simulation results show the trajectory tracking error can be stabilized to a certain extent under the parameter perturbations and other uncertainties, which verified the effectiveness of the proposed algorithm.

*Keywords:* Unmanned surface vehicle, Trajectory tracking, Guidance law, Active disturbance rejection control, Input saturation

## 1. Introduction

A great amount of research in the field of unmanned surface vessel motion control has been witnessed in recent decades. Unlike the path-following problem, which only involves a spatial constraint, the aim of trajectory tracking is forcing the vehicle to follow a time-varying trajectory. Moreover, these vessels are usually underactuated, which means that the direct force for sway control is unavailable. All of these factors increase the difficulty of controller design for USV. The plane motion of unmanned underactuated underwater vehicles is similar to that of USVs. In previous trajectory

tracking research, exponential asymptotical stability could be achieved with a backstepping design in Godhavn *et al.* (1998) and Pettersen and Nijmeijer (1999). In Lefeber (2000) and Lefeber, *et al.* (2003), the tracking control strategies proposed were based on a cascading approach. In Pettersen and Nijmeijer (2001), the ship mode was transformed into a chained form system, and related experiments were carried out. In Zhong-Ping Jiang (2002), the inherent cascade structure of ship dynamics was analysed, and the tracking controller was designed using Lyapunov's direct method. Based on Barbalat's lemma and backstepping techniques, the controller can achieve global asymptotical and exponential tracking in Dong and Guo (2005). The early studies of trajectory tracking based on the cascade and back-stepping approaches require a persistent excitation (PE) condition, i.e., a straight-line reference trajectory cannot be tracked. To resolve the PE problem, Do *et al.* (2015) and Do *et al.* (2002) relaxed the restricted conditions using Lyapunov's direct method.

Generally, there are two types of trajectory tracking methods: direct and indirect. Direct methods consider the problem as a stabilization problem for tracking error equations and apply adaptive, back-stepping or sliding mode control directly to determine the control law, while indirect methods divide the problem into a two-step process. First, the guidance laws, i.e., the desired surge, sway or yaw angle, are obtained. Then, the actual inputs are designed so that the surge, sway or yaw angle follow the desired pseudo-variables. Most of the literature on this topic adopts direct methods. The control laws obtained using this approach often result in complex algorithms and a large amount of calculations. In Ashrafiuon *et al.* (2008), the desired surge and sway velocities are designed as guidance laws. Then, sliding mode controllers are applied to guarantee asymptotic convergence to the desired values. However, the desired values proposed in Ashrafiuon *et al.* (2008) are special cases. For this, Yu *et al.* (2012) suggests that the desired surge and sway should relate to the position errors and be modified. Motivated by Yu *et al.* (2012), the desired surge and sway velocities are improved using hyperbolic tangent functions in Elmokadem *et al.* (2016). Cui *et al.* (2017) proposed an integral sliding mode controller based on multiple-input and multiple-output extended state observer to estimate the linear and angular velocities and unknown external disturbances. Terminal sliding mode control is adopted to increase the robustness of the system. Since the desired sway velocity is not forced by the control input directly, the correct heading of the USV is difficult to guarantee. In Chwa, D. (2011), a guidance law is proposed to solve the trajectory tracking problem. However, the reference trajectory is generated by a virtual ship that depends on exact model parameters. For

fluctuating parameters, the lateral velocity and reference will deviate because control in the sway direction is not available.

To obtain a simpler control structure and ensure the correct heading, we must develop a guidance law for the surge and yaw angle. Furthermore, the law should track various reference trajectories without restrictions on the initial conditions. Finally, since the actual USV has to meet control input constraints, the influence of input saturation must be considered.

This paper is organized as follows. Section 2 states the underactuated USV mode and trajectory tracking control objective. Section 3 proposes a trajectory tracking guidance law based on surge and yaw angle. In section 4, controllers are proposed based on dynamic surface control (DSC) and active disturbance rejection control (ADRC). Section 5 provides a simulation to illustrate the proposed methods. Finally, conclusions are summarized in section 6.

## 2. Problem formulation

This section presents the kinematic and dynamic models of an underactuated USV with three degrees of freedom.

To study an underactuated surface vessel moving in the horizontal plane, the kinematic and dynamic models can be described with 3 degrees of freedom. Detailed derivation procedures can be found in Fossen (2002). The kinematic equations for an USV can be described as:

$$\begin{cases} \dot{x} = u \cos \psi - v \sin \psi \\ \dot{y} = u \sin \psi + v \cos \psi \\ \dot{\psi} = r \end{cases} \quad (1)$$

where  $x$  and  $y$  represent the inertial coordinates of the USV.  $\psi$  is the yaw angle.  $u$  and  $v$  denote the surge and sway velocities, respectively, and  $r$  is the yaw velocity.

Considering the internal parameters perturbations, unmodeled dynamics and external environmental disturbances, the dynamic equations can be modified as follows:

$$\begin{aligned}
\dot{u} &= \frac{m_{22} + \Delta m_{22}}{m_{11} + \Delta m_{11}} vr - \frac{d_{11} + \Delta d_{11}}{m_{11} + \Delta m_{11}} u + \frac{\tau_u + \Delta \tau_u}{m_{11} + \Delta m_{11}} + \Delta_u \\
&= \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{\tau_u}{m_{11}} + F_u \\
\dot{v} &= -\frac{m_{11} + \Delta m_{11}}{m_{22} + \Delta m_{22}} ur - \frac{d_{22} + \Delta d_{22}}{m_{22} + \Delta m_{22}} v + \frac{\Delta \tau_v}{m_{22} + \Delta m_{22}} \Delta_v \\
&= -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v + F_v \\
\dot{r} &= \frac{(m_{11} + \Delta m_{11}) - (m_{22} + \Delta m_{22})}{m_{33} + \Delta m_{33}} uv - \frac{d_{33} + \Delta d_{33}}{m_{33} + \Delta m_{33}} r + \frac{\tau_r + \Delta \tau_r}{m_{33} + \Delta m_{33}} + \Delta_r \\
&= \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_r}{m_{33}} + F_r
\end{aligned} \tag{2}$$

Where  $\tau_u$  and  $\tau_r$  denote the control inputs. Obviously, there is not a control force in the sway direction, therefore, the USV model is an underactuated system. Positive constant parameters  $m_{ii}$  and  $d_{ii}$  are the ship inertia including the added mass and hydrodynamic damping terms, respectively.  $\Delta m_{ii}$  and  $\Delta d_{ii}$  represent the parameter perturbations in the vessel model.  $\Delta \tau_u$ ,  $\Delta \tau_v$  and  $\Delta \tau_r$  are the unknown external environmental disturbances, and  $\Delta_u$ ,  $\Delta_v$  and  $\Delta_r$  denote other unmodeled dynamics.  $F_u$ ,  $F_v$ , and  $F_r$  are the lumped uncertainties.

**Assumption 1.** For the lumped uncertainties  $F = [F_u \ F_v \ F_r]^T$  in (2), there are

positive constants  $\bar{F}_u$ ,  $\bar{F}_v$ ,  $\bar{F}_r$ , such that  $F_u$ ,  $F_v$ ,  $F_r$ , satisfy  $\left| \frac{d^k F_i}{dt^k} \right| \leq \bar{F}_i$ ,  $i = u, v, r$ ,

$k = 0, 1$ .

The control objective of trajectory tracking is to design control laws for the surge force  $\tau_u$  and yaw torque  $\tau_r$ , and ensure the USV tracks a desired, time-varying, smooth trajectory.

### 3. Trajectory tracking guidance law

The along-track error  $x_e$  and the cross-track error  $y_e$  can be defined as follows:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} \cos \psi_d & -\sin \psi_d \\ \sin \psi_d & \cos \psi_d \end{bmatrix}^T \begin{bmatrix} x - x_d(t) \\ y - y_d(t) \end{bmatrix} \tag{3}$$

where  $\psi_d(\omega) = \text{atan2}(y'_d(t), x'_d(t)) \in [-\pi, \pi]$  denotes the tangent angle of the trajectory.

Taking the time derivative of  $x_e$ , we have

$$\begin{aligned}\dot{x}_e &= (\dot{x} - \dot{x}_d) \cos \psi_d - \dot{\psi}_d (x - x_d) \sin \psi_d + (\dot{y} - \dot{y}_d) \sin \psi_d + \dot{\psi}_d (y - y_d) \cos \psi_d \\ &= \dot{x} \cos \psi_d + \dot{y} \sin \psi_d - \underbrace{(\dot{x}_d \cos \psi_d + \dot{y}_d \sin \psi_d)}_{U_d} + \dot{\psi}_d \underbrace{[-(x - x_d) \sin \psi_d + (y - y_d) \cos \psi_d]}_{y_e}\end{aligned}\quad (4)$$

where  $U_d = \dot{x}_d \cos \psi_d + \dot{y}_d \sin \psi_d = \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \cos(\psi_d + \alpha) = \sqrt{\dot{x}_d^2 + \dot{y}_d^2}$ . Note that

$\alpha = -\psi_d$ . Substituting kinematic equations (1) into (4), we have

$$\begin{aligned}\dot{x}_e &= u \cos \psi \cos \psi_d - v \sin \psi \cos \psi_d + u \sin \psi \sin \psi_d + v \cos \psi \sin \psi_d - U_d + y_e \dot{\psi}_d \\ &= u \cos(\psi - \psi_d) - v \sin(\psi - \psi_d) - U_d + y_e \dot{\psi}_d \\ &= U \cos(\psi - \psi_d + \beta) - U_d + y_e \dot{\psi}_d\end{aligned}\quad (5)$$

where  $U = \sqrt{u^2 + v^2}$  represents total speed of the USV.  $\beta = \text{atan2}(v, u)$  represents the sideslip angle between the vessel heading and the orientation of the velocity vector.

Similarly, taking the time derivative of  $y_e$ , and substituting kinematic equations (1) into it, we have

$$\begin{aligned}\dot{y}_e &= U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d + (\dot{x}_d \sin \psi_d - \dot{y}_d \cos \psi_d) \\ &= U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d\end{aligned}\quad (6)$$

Finally, the time derivative of (3) becomes

$$\begin{cases} \dot{x}_e = U \cos(\psi - \psi_d + \beta) - U_d + y_e \dot{\psi}_d \\ \dot{y}_e = U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d \end{cases}\quad (7)$$

Up to this point, pseudo-variables  $\psi_{\text{pseudo}}$  and  $u_{\text{pseudo}}$  are designed to make the tracking errors  $x_e$  and  $y_e$  converge to zero.

**Proposition 1** Let the desired yaw angle and surge be such that

$$\begin{cases} \psi_{\text{pseudo}} = \psi_d + \arctan\left(-\frac{y_e}{\Delta}\right) - \beta \\ u_{\text{pseudo}} = \sqrt{U_{\text{pseudo}}^2 - v^2} \end{cases}\quad (8)$$

where  $U_{\text{pseudo}} = \frac{(U_d - kx_e) \sqrt{y_e^2 + \Delta^2}}{\Delta}$ ,  $\Delta$  is the look-ahead distance.  $k > 0$  is the

controller gain.

If the following error signals

$$E_\psi = \psi - \psi_{\text{pseudo}} \quad (9)$$

$$E_u = u - u_{\text{pseudo}} \quad (10)$$

converge to zero, the convergence of the position error is guaranteed.

**Proof** Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 \quad (11)$$

Differentiating (11), and substituting (7) into it, yields

$$\begin{aligned} \dot{V}_1 &= x_e \dot{x}_e + y_e \dot{y}_e \\ &= x_e [U \cos(\psi - \psi_d + \beta) - U_d + y_e \dot{\psi}_d] + y_e [U \sin(\psi - \psi_d + \beta) - x_e \dot{\psi}_d] \\ &= -kx_e^2 - \frac{Uy_e^2}{\sqrt{y_e^2 + \Delta^2}} \end{aligned} \quad (12)$$

Since  $k > 0$ , it is obvious that  $\dot{V}_1 \leq 0$ . Therefore, it can be concluded that  $x_e$  and  $y_e$  converge to zero.

**Remark** Different from the existing trajectory tracking guidance laws in Ashrafiuon *et al.* (2008), Yu *et al.* (2012) and Elmokadem *et al.* (2016), the proposed guidance law has the following advantages.  $\psi$  and  $u$  can be controlled directly by the control inputs  $\tau_u$  and  $\tau_r$ , as shown in dynamic equations (2). Therefore, the complexity of the control law can be reduced. Additionally, the correct yaw angle can be guaranteed. In other words, if  $u$  and  $v$  are selected as the desired values, the heading angle could be in the wrong direction. In Chwa, D. (2011), a guidance law is proposed for  $\psi$  and  $u$ . The main difference between our paper and Chwa, D. (2011) is the reference trajectory generation, with Chwa, D. (2011) relying on the model parameters of a virtual ship, as discussed in the introduction section. However, considering the limited performance of USV, the reference trajectory in this paper cannot be generated randomly.

#### 4. Trajectory tracking control law

The reduced-order extended state observer (ESO) proposed in Huang and Xue (2014) has been studied extensively in Shao and Wang (2015) and Liu *et al.* (2017). Compared with the traditional ESO, the reduced-order ESO yields faster response with the same bandwidth as that in Shao and Wang (2015). More importantly, the peaking phenomenon during the initial phase may result in performance deterioration and even lead to system instability. This issue can be totally eliminated using first-order reduced-order ESO. Therefore, in this paper, we use the reduced-order ESO to estimate the

lumped uncertainties in dynamic equation (2).

$$\begin{cases} \dot{p}_1 = -\omega_1 p_1 - \omega_1^2 u - \omega_1 \left( \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{\tau_u}{m_{11}} \right) \\ \hat{F}_u = \omega_1 u + p_1 \end{cases} \quad (13)$$

$$\begin{cases} \dot{p}_2 = -\omega_2 p_2 - \omega_2^2 r - \omega_2 \left( \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_r}{m_{33}} \right) \\ \hat{F}_r = \omega_2 r + p_2 \end{cases} \quad (14)$$

where  $\omega_1 > 0$  and  $\omega_2 > 0$  are the **observer bandwidths** to be determined.

**Lemma 1** Consider the proposed reduced-order ESO in (13) and (14), if Assumption 1 is satisfied, then

$$|\tilde{F}_x(t)| \leq \zeta_k \frac{1}{\omega_k}, \text{ where } t > t_0 + \max \left\{ 0, \frac{\ln \omega_k}{\omega_k} \right\}, \tilde{F}_x = \hat{F}_x - F_x \quad \zeta_k > 0 \text{ is a positive}$$

constant, ( $x = u, r; k = 0, 1$ ).

Lemma 1 implies that properly increasing the observer gains  $\omega_1$  and  $\omega_2$  can reduce the estimation error  $\tilde{F}_x$ . However, the observer gains cannot be too large. The increase in gains is subject to noise effects. Ultimately, this approach is a reasonable compromise based on estimation quality and the noise effect, see Liu *et al.* (2017).

To facilitate controller design, the trajectory tracking guidance law was established in the previous section. In this section, our motivation is very clear: design  $\tau_u$  and  $\tau_r$ , and make the USV yaw angle  $\psi$  and surge velocity  $u$  converge to pseudo-variables  $\psi_{\text{pseudo}}$  and  $u_{\text{pseudo}}$ , respectively. The guidance law is designed according to control inputs  $\tau_u$  and  $\tau_r$ , so many control approaches such as adaptive control, sliding mode control, and PID, can be employed easily. In this paper, to avoid direct derivation of intermediate variable values and ease implementation, the DSC technique is applied to the dynamic level.

It is well known that discrete systems are widely applied in actual engineering practice. To obtain high-quality tracking and differential signals from virtual control commands, nonlinear tracking differentiators (NTDs) were chosen to implement the functions of a first-order filter for DSC. NTD is a time-optimal solution in a discrete form proposed in Han (2009) and is given in the following form:

$$\begin{cases} fh = fhan(x_1(k) - v(k), \dot{x}_1(k), R, h_0) \\ x_1(k+1) = x_1(k) + h \cdot \dot{x}_1(k) \\ \dot{x}_1(k+1) = \dot{x}_1(k) + h \cdot fh \end{cases} \quad (15)$$



where  $h$  is the sampling period,  $R$  is an acceleration factor,  $h_0$  can be adjusted individually according to the noise signal.  $\text{fhan}(x_1(k) - v(k), \dot{x}_1(k), R, h_0)$  is

$$\left\{ \begin{array}{l} d = Rh \\ d_0 = hd \\ y = x_1 + h\dot{x}_1 \\ a_0 = \sqrt{d^2 + 8R|y|} \\ a = \begin{cases} \dot{x}_1 + \frac{a_0 - d}{2} \text{sign}(y), & |y| > d_0 \\ \dot{x}_1 + \frac{y}{h}, & |y| \leq d_0 \end{cases} \\ \text{fhan} = \begin{cases} R\text{sign}(a), & |y| \leq d_0 \\ R\frac{a}{d}, & |y| > d_0 \end{cases} \end{array} \right. \quad (16)$$

More detailed discussion and study of NTD can be found in Guo and Zhao. (2011).

Choose equation (10) as the error surface and let  $u_{\text{pseudo}}$  pass through NTD to obtain the command signal  $u_{\text{cmd}}$  and its derivative  $\dot{u}_{\text{cmd}}$ :

$$\left\{ \begin{array}{l} \text{fh} = \text{fhan}(u_{\text{cmd}}(k) - u_{\text{pseudo}}(k), \dot{u}_{\text{cmd}}(k), R, h_0) \\ u_{\text{cmd}}(k+1) = u_{\text{cmd}}(k) + h \cdot \dot{u}_{\text{cmd}}(k) \\ \dot{u}_{\text{cmd}}(k+1) = \dot{u}_{\text{cmd}}(k) + h \cdot \text{fh} \end{array} \right. \quad (17)$$

Take the derivative of

$$\bar{E}_u = u - u_{\text{cmd}} \quad (18)$$

We have

$$\dot{\bar{E}}_u = \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{\tau_u}{m_{11}} + F_u - \dot{u}_{\text{cmd}} \quad (19)$$

Therefore,  $\tau_u$  can be designed as

$$\tau_u = m_{11} \left( -\frac{m_{22}}{m_{11}} vr + \frac{d_{11}}{m_{11}} u - \bar{F}_u + \bar{\tau}_u \right) \quad (20)$$

where  $\bar{\tau}_u = -k_u \bar{E}_u + \dot{u}_{\text{cmd}}$ ,  $k_u > 0$ .

Choose equation (9) as the error surface and let  $\psi_{\text{pseudo}}$  pass through NTD.

Based on Chwa, D. (2011), let us choose

$$r_{\text{pseudo}} = k_\psi \sin(\bar{E}_\psi / 2) + \dot{\psi}_{\text{cmd}} \quad (21)$$

where  $k_\psi > 0$ ; then, let  $r_{\text{pseudo}}$  pass through NTD.

$$\begin{cases} \text{fh} = \text{fhan}(r_{\text{cmd}}(k) - r_{\text{pseudo}}(k), \dot{r}_{\text{cmd}}(k), R, h_0) \\ r_{\text{cmd}}(k+1) = r_{\text{cmd}}(k) + h \cdot \dot{r}_{\text{cmd}}(k) \\ \dot{r}_{\text{cmd}}(k+1) = \dot{r}_{\text{cmd}}(k) + h \cdot \text{fh} \end{cases} \quad (22)$$

Take the derivative of

$$\bar{E}_r = r - r_{\text{cmd}} \quad (23)$$

Then, we have

$$\dot{\bar{E}}_r = \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{\tau_r}{m_{33}} + F_r - \dot{r}_{\text{cmd}} \quad (24)$$

Therefore,  $\tau_r$  can be designed as

$$\tau_r = m_{33} \left( -\frac{m_{11} - m_{22}}{m_{33}} uv + \frac{d_{33}}{m_{33}} r - \bar{F}_r + \bar{\tau}_r \right) \quad (25)$$

where  $\bar{\tau}_r = -2\xi k_r \bar{E}_r - k_r^2 \sin(\bar{E}_r / 2) + \dot{r}_{\text{cmd}}$ ,  $\xi > 0$ ,  $k_r > 0$ .

## 5. Simulation results

This section presents the simulation results to validate the effectiveness and robustness of the proposed control strategies. CyberShip II, developed by Norwegian University of Science and Technology, is selected as the simulation USV, with the following parameters:  $m_{11} = 25.8$ ,  $m_{22} = 33.8$ ,  $m_{33} = 6.2$ ,  $d_{11} = 12$ ,  $d_{22} = 17$ , and  $d_{33} = 0.5$ . The internal parameter uncertainties are generated as follows:

$$\Delta m_{11} = 0.2m_{11} \sin(0.1t), \quad \Delta m_{22} = 0.2m_{22} \sin(0.2t + \pi/4),$$

$$\Delta m_{33} = 0.2m_{33} \sin(0.1t + \pi/6), \quad \Delta d_{11} = 0.2d_{11} \sin(0.1t + \pi/3),$$

$$\Delta d_{22} = 0.2d_{22} \sin(0.2t + \pi/2), \quad \text{and} \quad \Delta d_{33} = 0.2d_{33} \sin(0.1t + \pi/5). \quad \text{The external}$$

disturbances are assumed to be 
$$\begin{cases} \Delta \tau_u = \sin(0.2t) + \cos(0.2t + \pi/4) + \sin(0.2t + \pi/6) \\ \Delta \tau_v = \sin(0.2t) + \cos(0.2t + \pi/4) + \sin(0.2t + \pi/6) \\ \Delta \tau_r = \sin(0.2t) + \cos(0.2t + \pi/4) + \sin(0.2t + \pi/6) \end{cases}$$

The control parameters are  $h = 0.001$ ,  $h_0 = 10h$ ,  $R = 150$ ,  $k_u = 0.1$ ,  $k_\psi = 5$ ,  $k_r = 5$ , and  $\xi = 0.8$ . In practicality, the saturation problem frequently occurs in engineering systems. In this paper, the bounds of control inputs  $\tau_u$  and  $\tau_r$  are set as  $0 < \tau_u < \tau_{u\text{max}}$ ,  $|\tau_r| < \tau_{r\text{max}}$ , where  $\tau_{u\text{max}} = 10$ ,  $\tau_{r\text{max}} = 10$ . Input saturation can be found in many industrial processes, which may destroy the stability of the closed-loop

systems. To make this problem tractable, the desired pseudo-variable proposed in (8) can be limited. In this paper, we choose

$$u_{\text{pseudo}} = \begin{cases} u_{\text{pseudo}}, & u_{\text{pseudo}} \leq 1 \\ 1 & u_{\text{pseudo}} > 1 \end{cases} \quad (26)$$

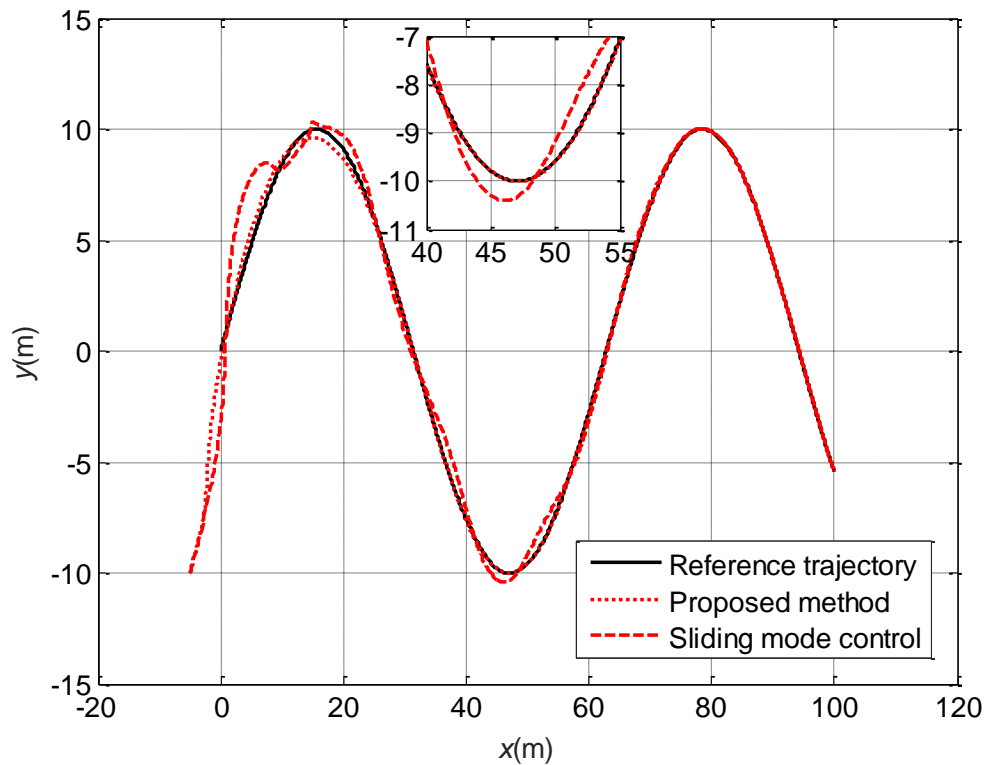
To demonstrate the advantage of the proposed guidance law, the previous method using  $u$  and  $v$  as the desired values presented in Elmokadem *et al.* (2016) is selected for comparison.

The desired reference trajectories of the three scenarios are as follows:

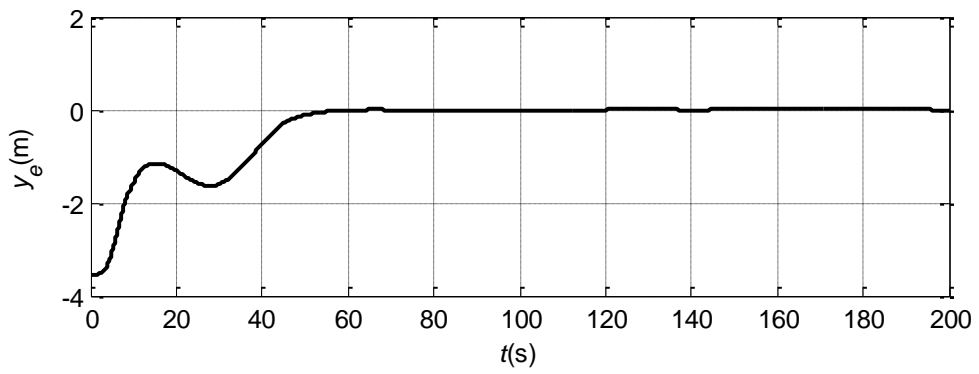
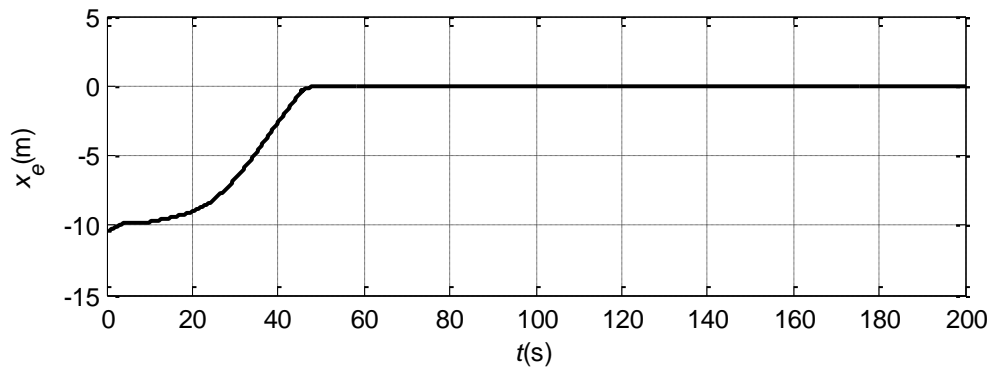
(1) Sinusoidal trajectory

$$\begin{cases} x_d(t) = 0.5t \\ y_d(t) = 10\sin(0.05t) \end{cases} \quad (27)$$

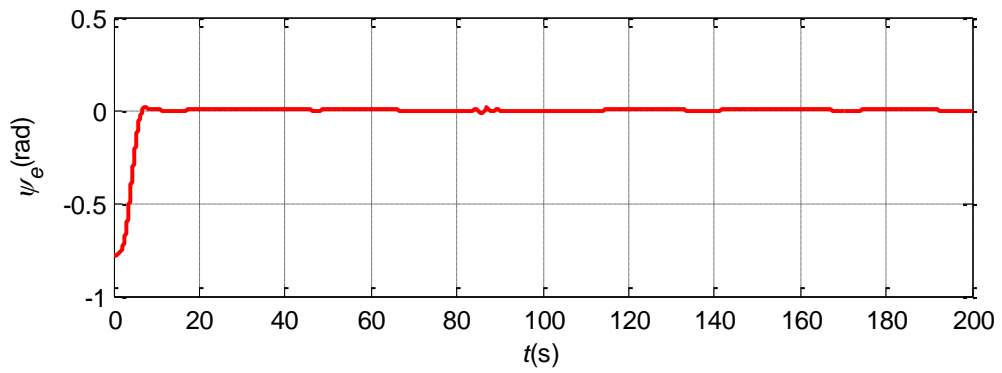
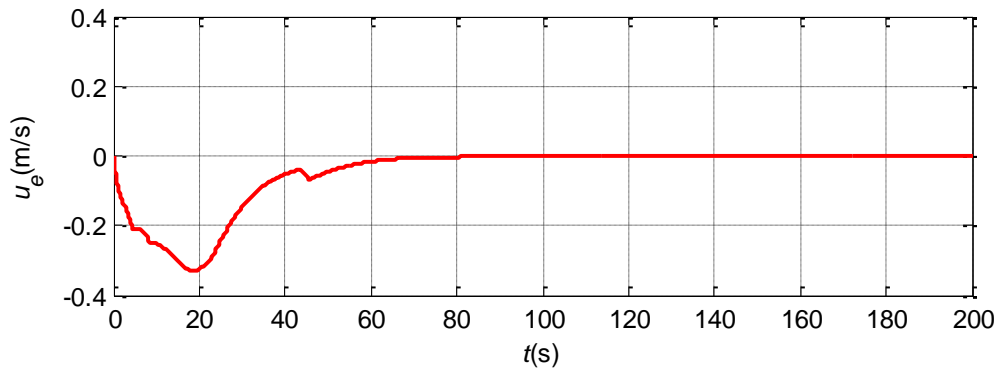
The initial velocities are  $[u(0) \ v(0) \ r(0)] = [1 \ 0 \ 0]$ . The initial position and yaw angle are  $[x(0) \ y(0) \ \psi(0)] = [-5 \ -10 \ \pi/4]$ .



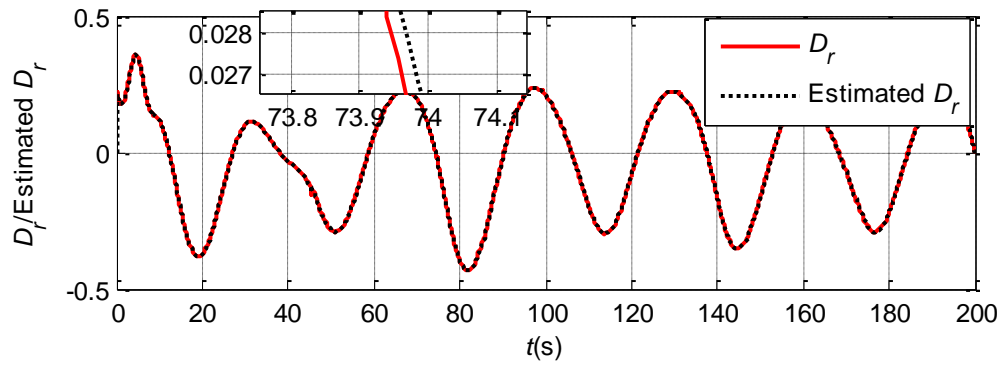
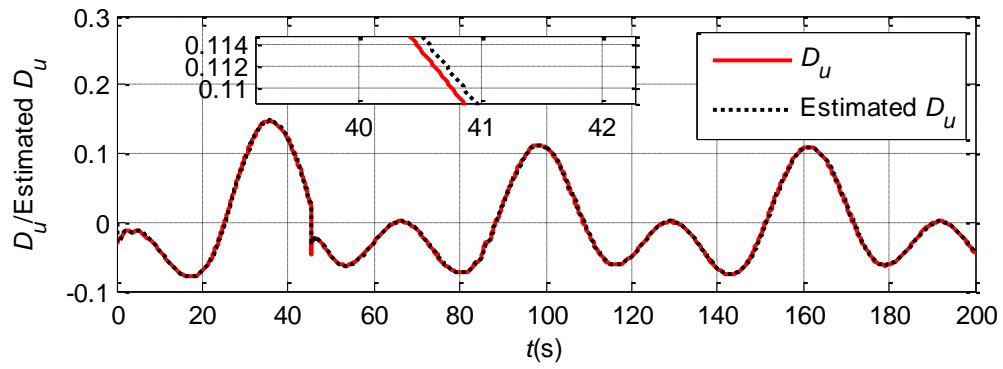
a) Sinusoidal trajectory tracking result



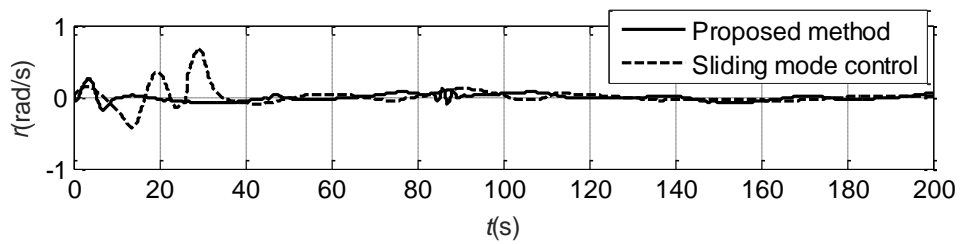
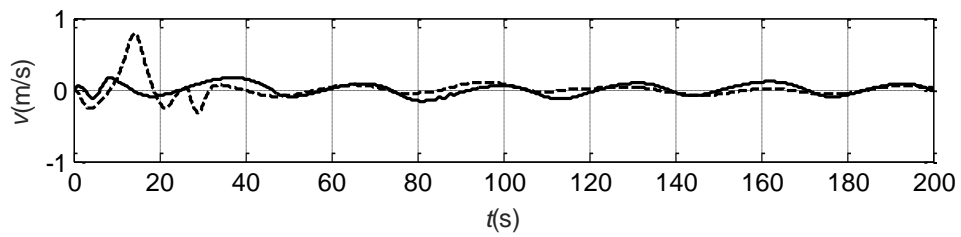
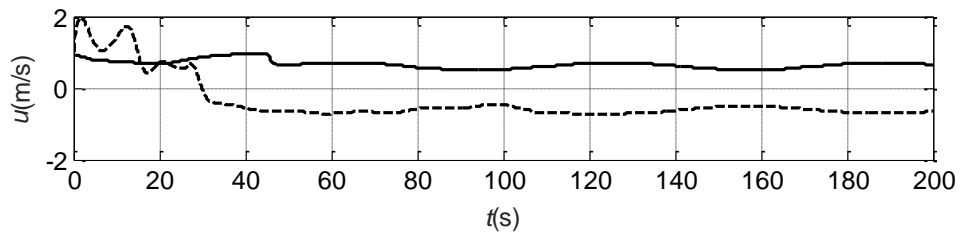
b) Tracking errors



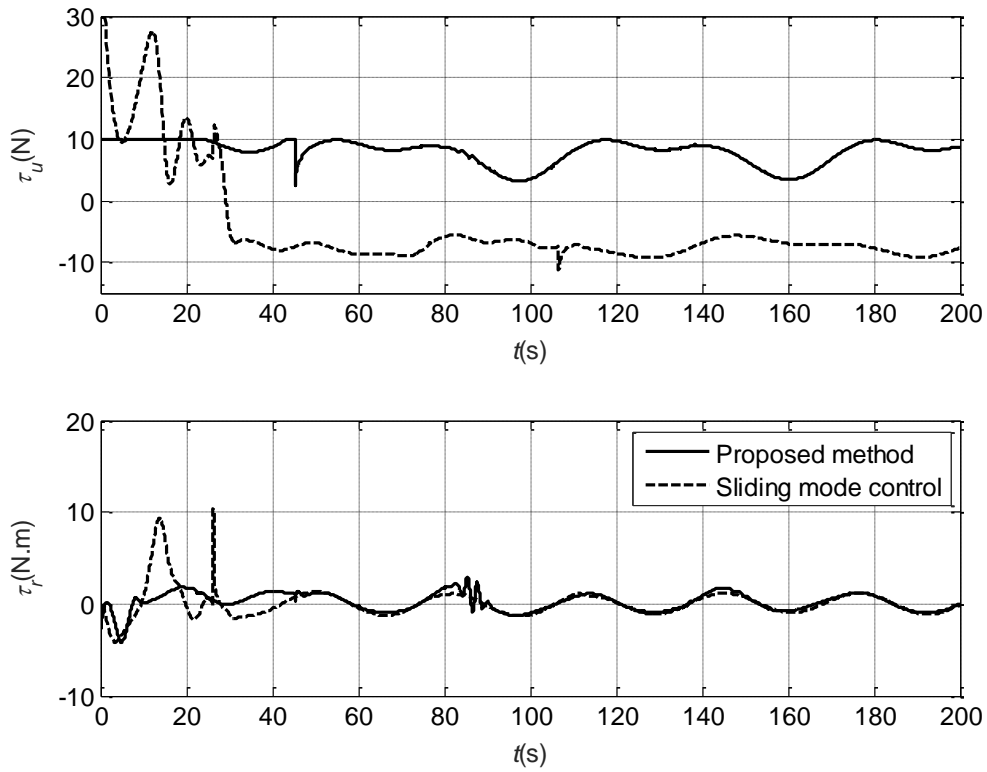
c) Surge and yaw angle errors



d) Observer errors



e) Velocities



f) Control inputs

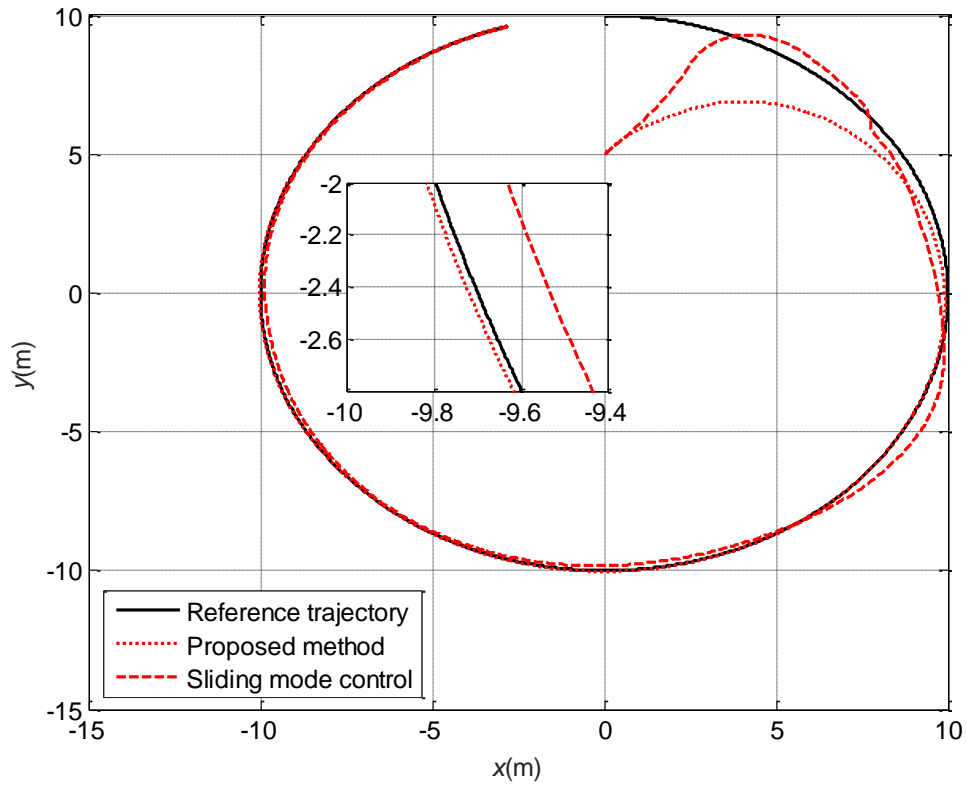
**Fig. 1.** Sinusoidal trajectory tracking performance

The sinusoidal trajectory can be tracked under the various disturbances and constraints by the proposed method, as shown in Fig. 1. Fig. 1 a) and b) indicate the errors between the reference and actual trajectory asymptotically decrease to approximately zero. The performance of yaw angle and surge velocity controllers is shown in Fig. 1 c). The observer performance for the reduced-order ESOs is shown in Fig. 1 d), which demonstrates that the designed observer can quickly and accurately estimate the lumped disturbance. The surge sway and yaw velocities are shown in Fig. 1 e). Note that the sway velocity is bounded. Although there is no actuation in the sway direction, the sway velocity  $v$  will remain bounded. This effect is due to the inputs and disturbance being bounded. Fig. 1 f) indicates that the proposed control system performs well despite the existence of limit bounds on the control inputs. The desired pseudo-variables and control input gains are adjusted until the constraint requirements are met. Note that the previous method did not show good performance for long times even though input saturation did not exist. Fig. 1 e) and f) show that the tracking control is not in the correct direction because the yaw angle of the ship cannot be controlled directly.

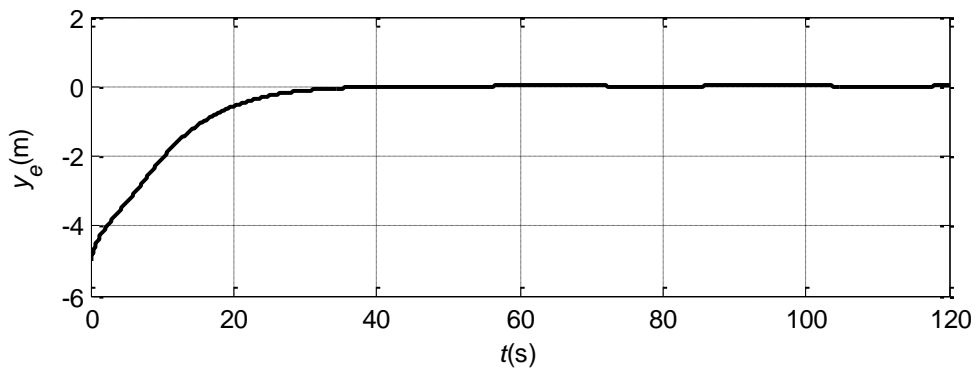
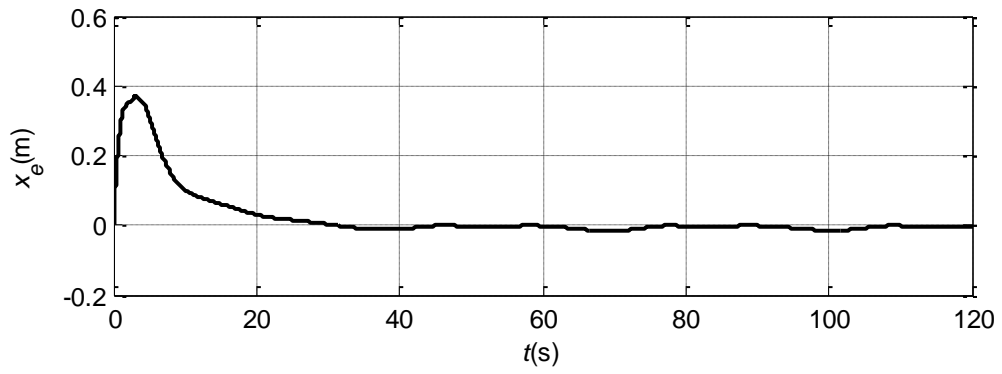
(2) Circular trajectory

$$\begin{cases} x_d(t) = 10\sin(0.05t) \\ y_d(t) = 10\cos(0.05t) \end{cases} \quad (28)$$

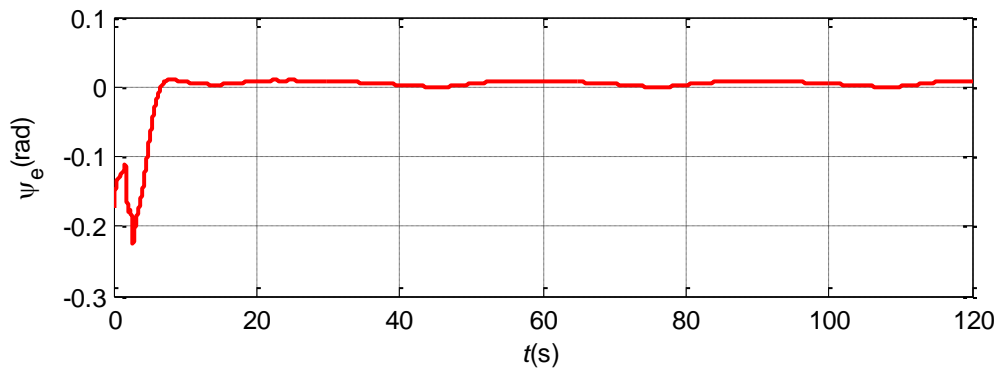
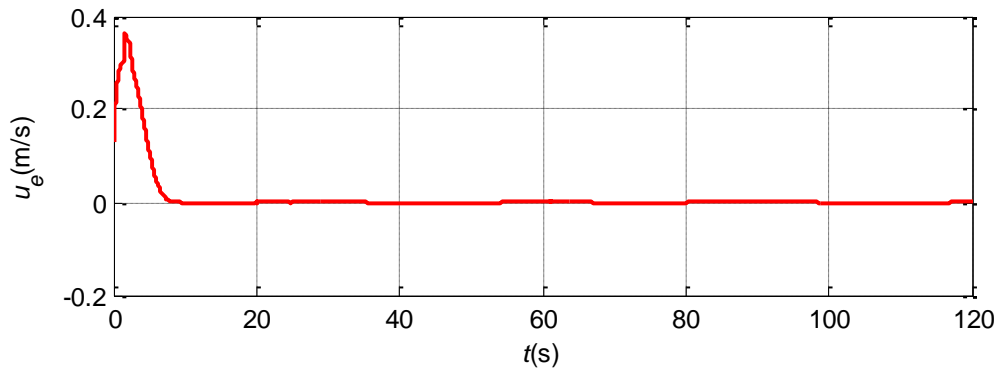
The initial velocities are  $[u(0) \ v(0) \ r(0)] = [1 \ 0 \ 0]$ . The initial position and yaw angle are  $[x(0) \ y(0) \ \psi(0)] = [0 \ 5 \ \pi/4]$ .



a) Circular trajectory tracking result

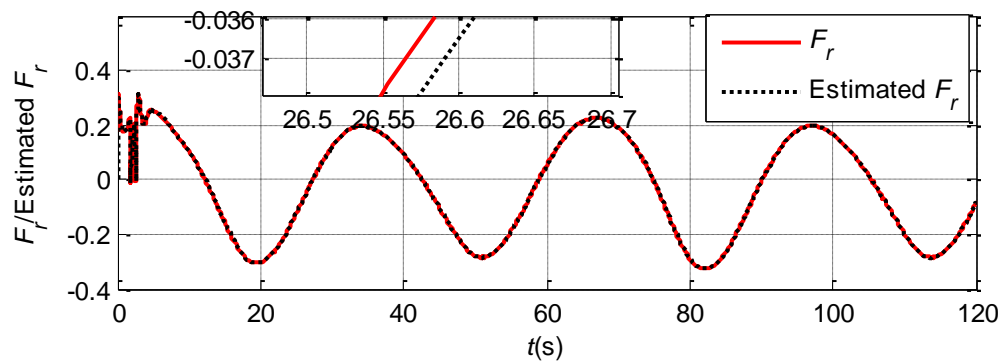
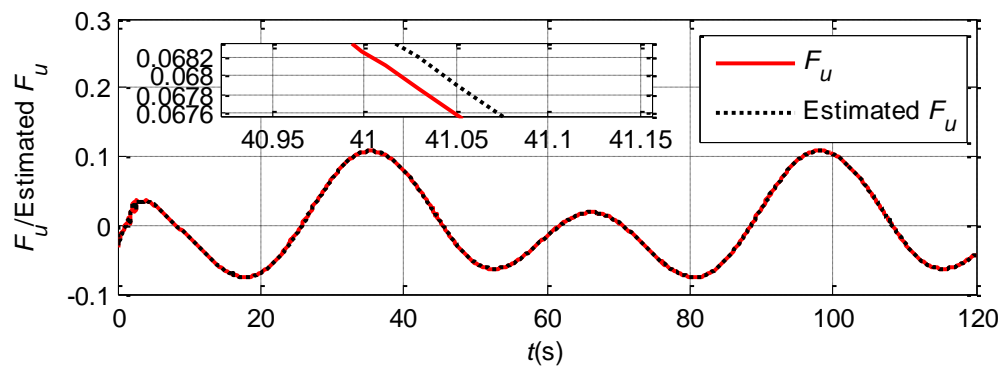


b) Tracking errors

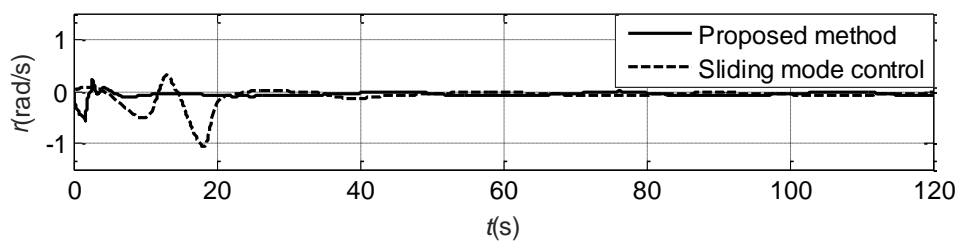
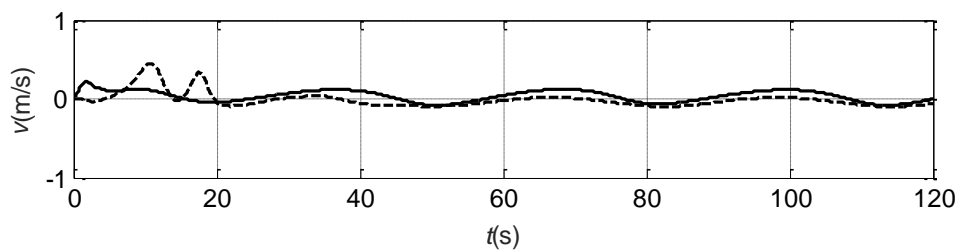
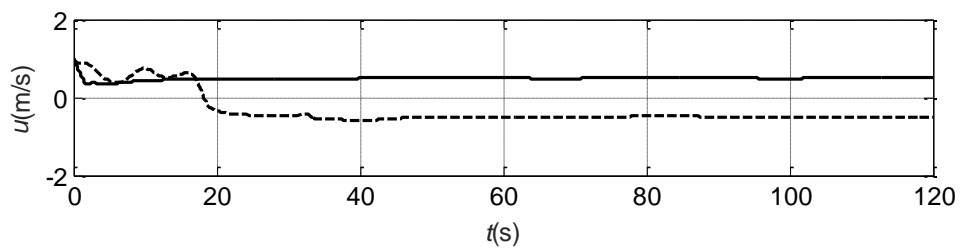


c) Surge and yaw angle errors

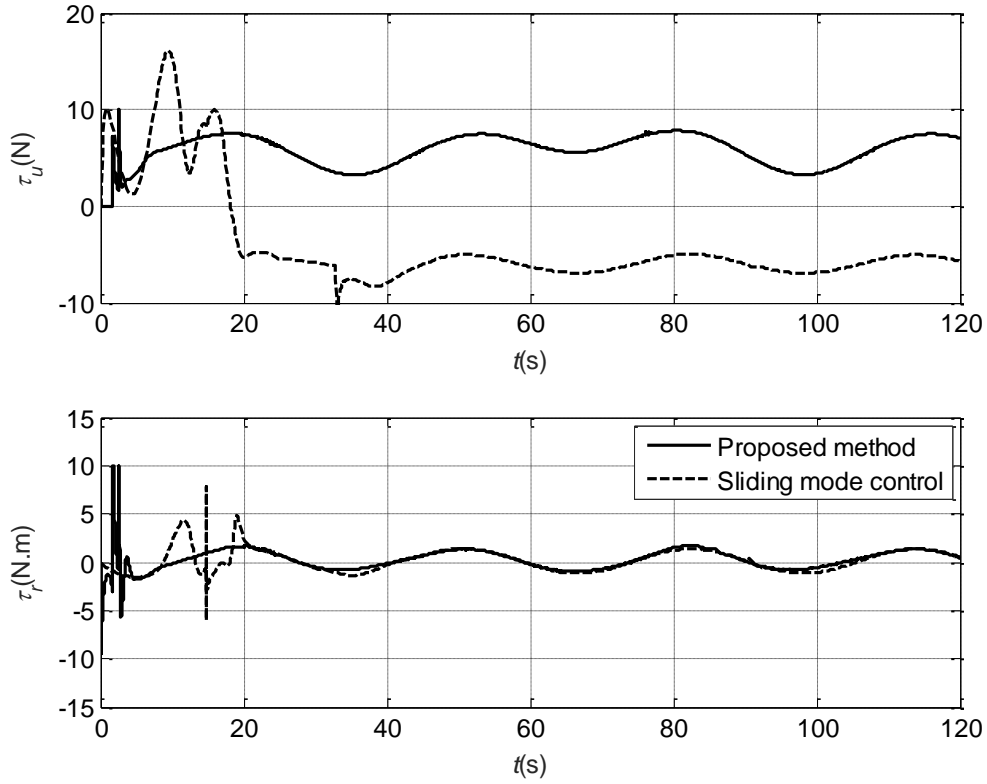




d) Observer errors



e) Velocities



f) Control inputs

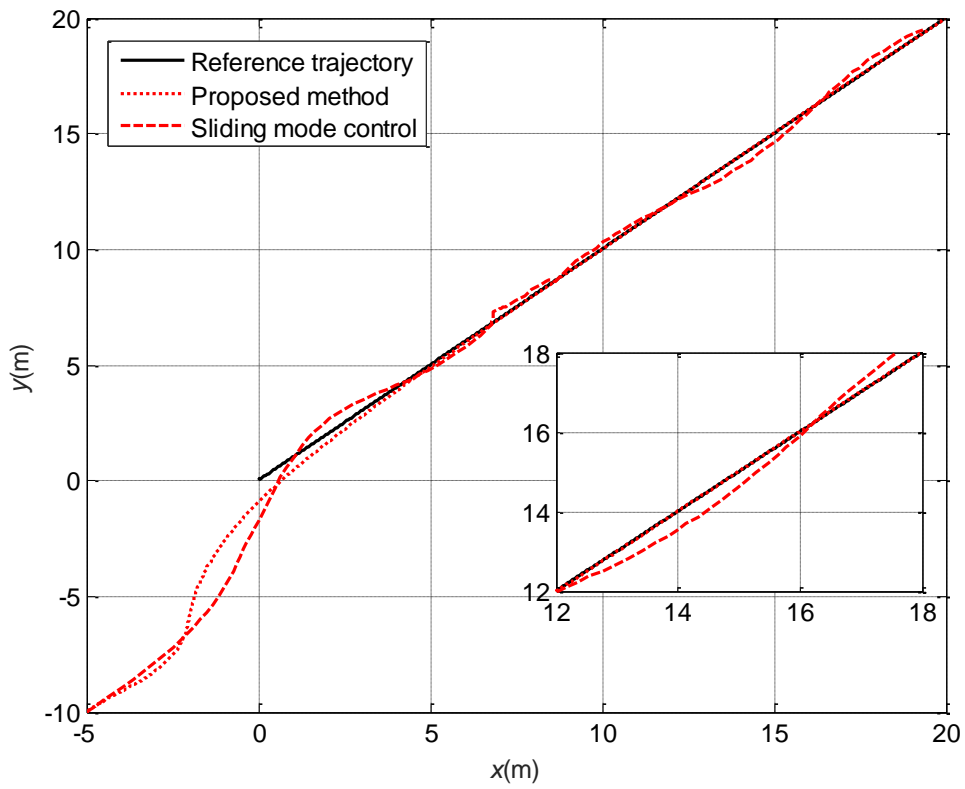
**Fig. 2.** Circular trajectory tracking performance

**Fig. 2** shows the tracking results for a circular trajectory. From Figs. 2 a) and b), using the proposed method, the tracking errors are sufficiently small and can be maintained even in the presence of parametric uncertainties and external disturbances. Figs. 2. a) shows that the previous method has higher convergence speed because the inputs are constrained. However, the previous method has low convergence accuracy. Figs. 2 e) and f) show that the tracking control cannot maintain the correct direction. The circular trajectory is generated with a constant velocity  $u$ ,  $v$  and  $r$ . The actual control inputs shown in Figs. 2 f) are time-varying to compensate for the time-varying disturbance.

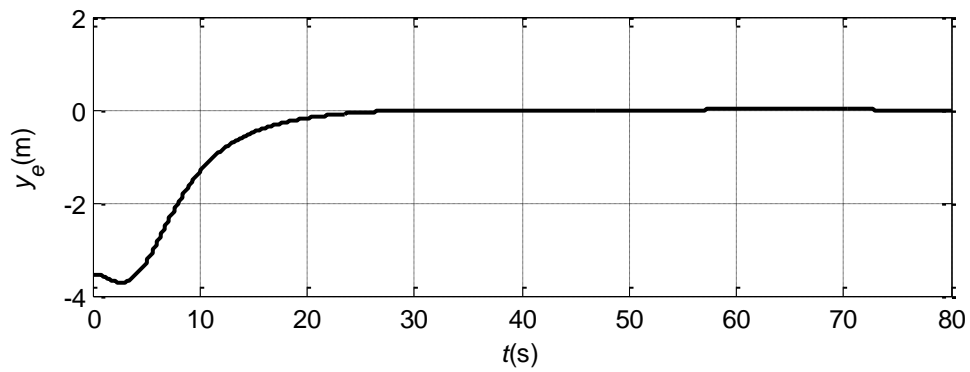
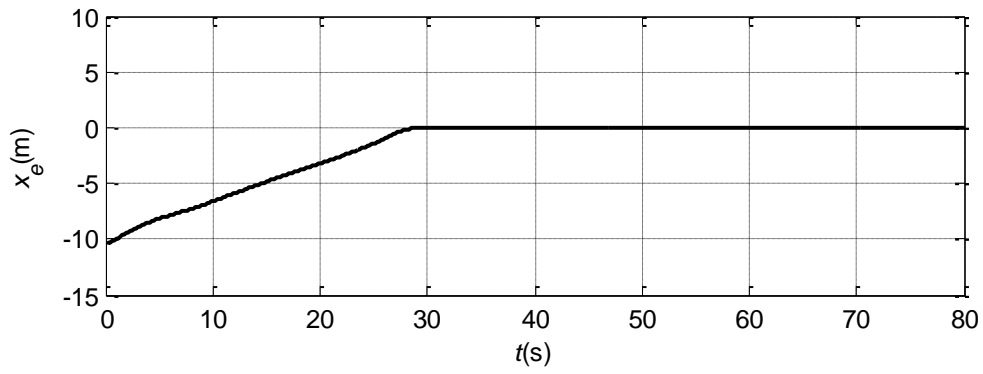
### (3) Straight trajectory

$$\begin{cases} x_d(t) = 0.25t \\ y_d(t) = 0.25t \end{cases} \quad (29)$$

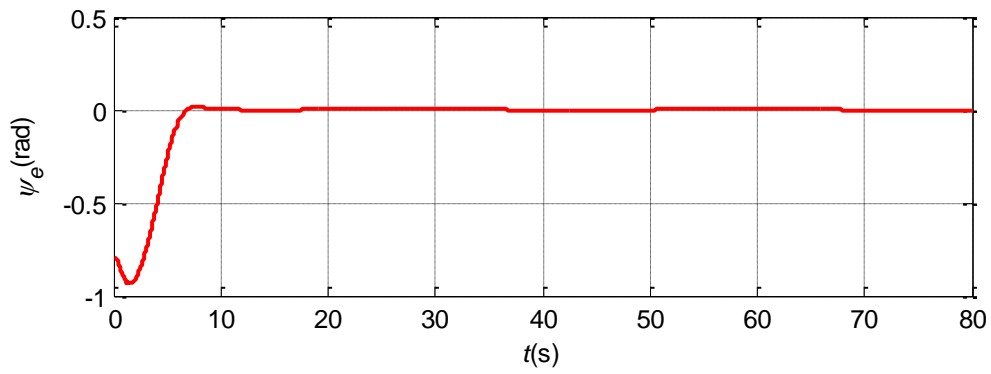
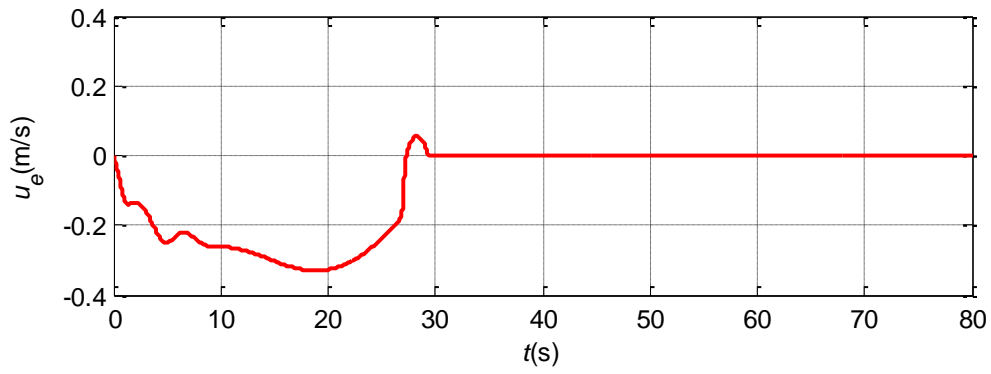
The initial velocities are  $[u(0) \ v(0) \ r(0)] = [1 \ 0 \ 0]$ . The initial position and yaw angle are  $[x(0) \ y(0) \ \psi(0)] = [-5 \ -10 \ \pi/4]$ .



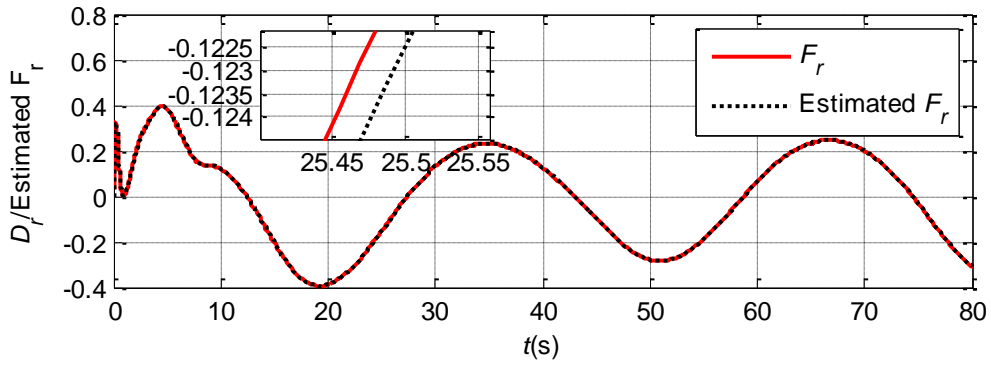
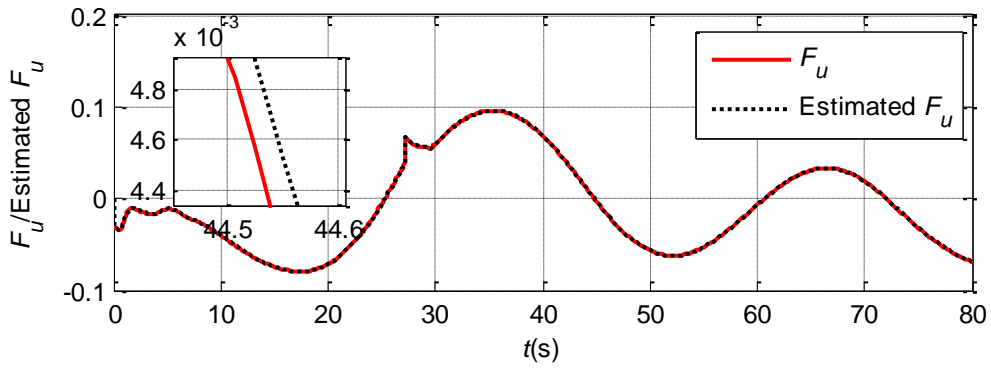
a) Trajectory tracking results



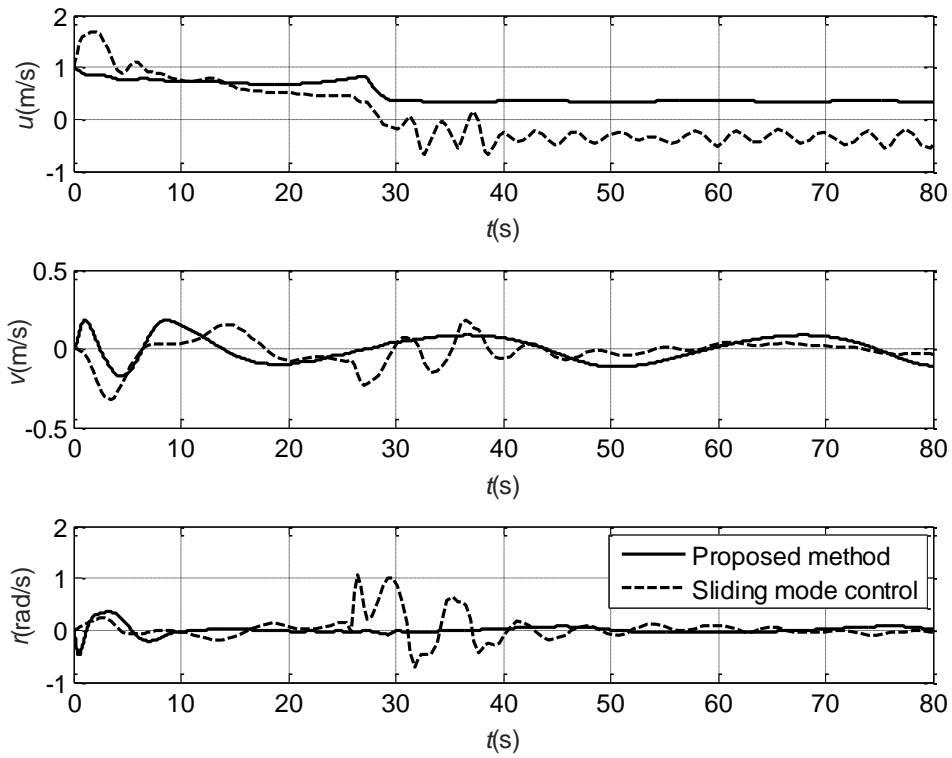
b) Tracking errors



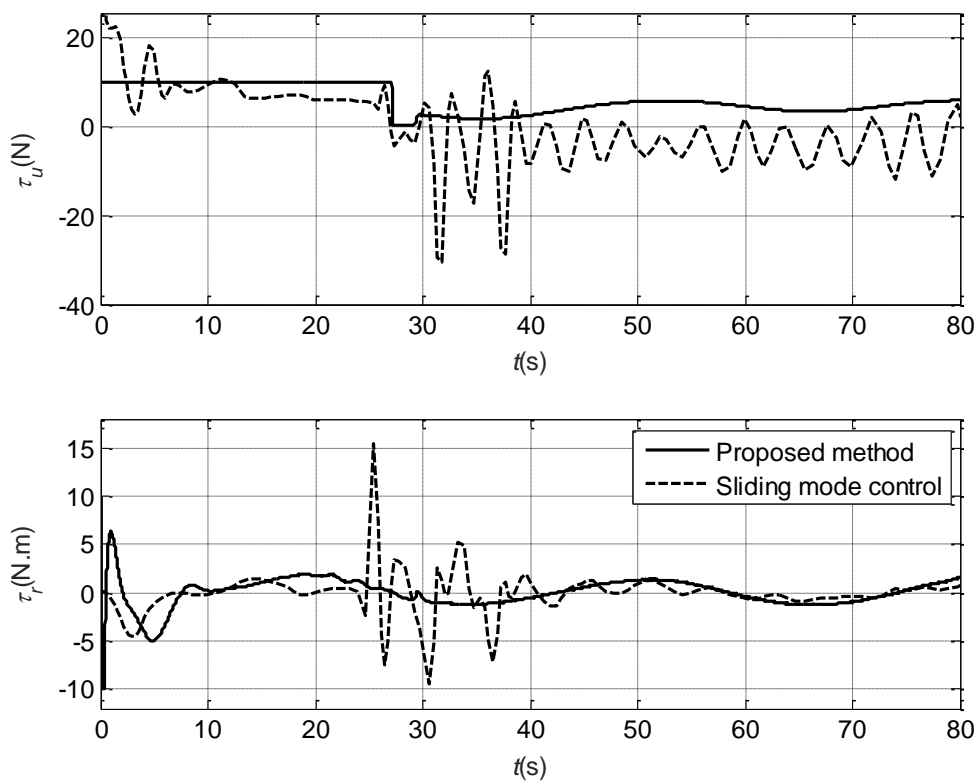
c) Surge and yaw angle errors



d) Observer errors



e) Velocities



f) Control inputs

**Fig. 3.** Straight trajectory tracking performance

Fig. 3 shows the tracking results for a straight line. As is shown in Fig. 3 a) and b), good performance is achieved even in the case of large initial errors and the abovementioned constraints. As is shown in Fig. 3c) and f), when thrust surplus appears, the controller adjusts quickly. However, the previous method cannot maintain a straight-line motion under the various disturbances. Additionally, it is difficult to maintain the correct yaw angle under the various disturbances.

## **6. Conclusion**

This paper addressed the problem of trajectory tracking control for an underactuated USV on the horizontal plane in the presence of parameter perturbation and environmental disturbances. Compared with the traditional trajectory tracking control strategy, the new approach proposed in this paper divides the problem into a guidance law and a control loop. This approach can effectively avoid complex manufacturing processes and difficulty in the actual application. We believe that the indirect method with a suitable guidance law provides a simpler solution to the trajectory tracking problem than that of the direct method. By introducing the ADRC technique, we can both reduce the dependence on USV mathematical models and achieve improvement of the DSC. According to the simulation results from the reference sinusoidal, circular, and straight trajectory, input constraints can be effectively handled by adjusting the desired pseudo-variables and control gains. Since the proposed control law does not require a persistent excitation condition and special initial states, more types of reference trajectories can be tracked. Future work will include sea trials to further validate the control algorithm.

## **Acknowledgments**

This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant no. HIT.KISTP.2014029, the Science and Technology Foundation for the Universities in Shandong Province under Grant no. J14LN93, the Discipline Construction Foundation in Harbin Institute of Technology, Weihai no. WH20160103, Education Research Project in Harbin Institute of Technology, Weihai no. BKQN201619.

## **Reference:**

- Adriaan Arie Johannes Lefeber. Tracking control of nonlinear mechanical systems. Universiteit Twente, 93, 213-244, 2000.
- Ashrafiuon H., Muske K. R., Mcninch L. C., and Soltan R. A. Sliding-mode tracking

- control of surface vessels. *IEEE Transactions on Industrial Electronics*, 55(11), 4004-4012, 2008.
- Chwa D. Global tracking control of underactuated ships with input and velocity constraints using dynamic surface control method. *IEEE Transactions on Control Systems Technology*, 19(6), 1357-1370, 2011.
- Do K. D., Jiang Z. P., and Pan, J. Underactuated ship global tracking under relaxed conditions. *IEEE Transactions on Automatic Control*, 47(9), 1529-1536, 2015.
- Do K. D., Jiang Z. P. and Pan, J. Universal controllers for stabilization and tracking of underactuated ships. *Systems & Control Letters*, 47(4), 299-317, 2002.
- Dong W., and Guo Y. Nonlinear tracking control of underactuated surface vessel. *American Control Conference, 2005. Proceedings of the (Vol.6, pp.4351-4356 vol. 6)*. IEEE, 2005.
- Elmokadem T., Zribi M., and Youcef-Toumi K. Terminal sliding mode control for the trajectory tracking of underactuated autonomous underwater vehicles. *Ocean Engineering*, 129, 2016.
- Elmokadem T., Zribi M. and Youcef-Toumi K. Trajectory tracking sliding mode control of underactuated AUVs. *Nonlinear Dynamics*, 84(2):1079-1091, 2016.
- Fossen T. I. *Marine Control Systems: Guidance, Navigation, and Control of Ships, Rigs and Underwater Vehicles*. Marine Control System, Guidance, Navigation and Control of Ships, Rigs and Underwater, 2002.
- Godhavn J. M., Fossen T. I. and Berge S. P. Non-linear and adaptive backstepping designs for tracking control of ships. *International Journal of Adaptive Control & Signal Processing*, 12(8), 649-670, 1998.
- Guo B. Z. and Zhao Z. L. On convergence of tracking differentiator. *International Journal of Control*, 84(4), 693-701, 2011.
- Han J. From pid to active disturbance rejection control. *IEEE Transactions on Industrial Electronics*, 56(3), 900-906, 2009.
- Huang Y. and Xue W. Active disturbance rejection control: methodology and theoretical analysis. *Isa Transactions*, 53(4), 963-976, 2014.
- Lefeber E., Pettersen, K. Y. and Nijmeijer H. Tracking control of an underactuated ship. *IEEE Transactions on Control Systems Technology*, 11(1), 52-61, 2003.
- Liu L., Wang D. and Peng Z. Eso-based line-of-sight guidance law for path following

- of underactuated marine surface vehicles with exact sideslip compensation. *IEEE Journal of Oceanic Engineering*, 42(2), 477-487, 2017.
- Pettersen, K. Y. and Nijmeijer, H. Global practical stabilization and tracking for an underactuated ship - a combined averaging and backstepping approach. *Modeling Identification & Control*, 20(4), 59-64, 1999.
- Pettersen K. Y. and Nijmeijer, H. Underactuated ship tracking control: theory and experiments. *International Journal of Control*, 74(14), 1435-1446, 2001.
- Cui R., Chen L., Yang C., and Chen M. Extended state observer-based integral sliding mode control for an underwater robot with unknown disturbances and uncertain nonlinearities. *IEEE Transactions on Industrial Electronics*, 64(8), 6785-6795, 2017.
- Shao X. and Wang H. Back-stepping active disturbance rejection control design for integrated missile guidance and control system via reduced-order ESO. *ISA Trans*, 57(4), 10-22, 2015.
- Yu R., Zhu Q., Xia G. and Liu, Z. Sliding mode tracking control of an underactuated surface vessel. *IET Control Theory & Applications*, 6(3), 461-466, 2012.
- Jiang Z. P., Global tracking control of underactuated ships by Lyapunov's direct method. *Automatica*, 38(2), 301-309, 2002.