A NAVIGATION AND AUTOMATIC COLLISION AVOIDANCE SYSTEM FOR MARINE VEHICLES

Keith McGowan Miller

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A NAVIGATION AND AUTOMATIC COLLISION AVOIDANCE SYSTEM

FOR MARINE VEHICLES

Keith McGowan Miller BSc.

This thesis is submitted to the Council for National Academic Awards in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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Sponsoring Establishment: Polytechnic South West
Drake Circus
Plymouth

Collaborating Establishment: J&S Marine Ltd.

February 1990
DECLARATION

No part of this thesis has been submitted for any award or degree at any other institute.

While registered as a candidate for the degree of Doctor of Philosophy the author has not been a registered candidate for another award of the CNAA or of a university.
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ABSTRACT

A Navigation and Automatic Collision Avoidance System for Marine Vehicles

Keith M. Miller BSc.

Collisions and groundings at sea still occur, and can result in financial loss, loss of life, and damage to the environment. Due to the size and capacity of modern vessels, damage can be extensive. Statistics indicate that the primary cause of accidents at sea is human error, which is often attributed to misinterpretation of the information presented to the mariner. Until recently, data collected from sensors about the vessel were displayed on the bridge individually, leaving the mariner to assimilate the material, make decisions and alter the vessel's controls as appropriate. With the advent of the microprocessor a small amount of integration has taken place, but not to the extent that it has in other industries, for example the aerospace industry. This thesis presents a practical method of integrating all the navigation sensors. Through the use of Kalman filtering, an estimate of the state of the vessel is obtained using all the data available. Previous research in this field has not been implemented due to the complexity of the ship modelling process required, this is overcome by incorporating a system identification procedure into the filter. The system further reduces the demands on the mariner by applying optimal control theory to guide the vessel on a predetermined track. Hazards such as other vessels are not incorporated into this work but they are specified in further research. Further development work is also required to reduce computation time.
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CHAPTER 1

DEVELOPMENTS IN MARINE NAVIGATION

1.1 Introduction

In spite of modern electronic aids to navigation, accidents still occur at sea. These usually take the form of collisions between vessels or groundings and 90% of them occur in coastal waters. Cockcroft (1984) has established that in the ten years between 1973 and 1982, 276 trading ships were lost due to collisions, 57.5% of these occurred in restricted visibility, a vast improvement on the figure of 75% prior to 1973. It should be noted that marine traffic separation zones were introduced in some coastal areas at that time. Cockcroft goes on to suggest some reasons for the errors which caused the latest collisions. Of those which occurred in clear visibility, 75% were during the hours of darkness, and many in areas of low traffic density, suggesting that the predominant cause is a poor visual look-out. He goes on to state that in several instances the collision took place immediately before a change of watch. In restricted visibility the time of day is shown to have little or no effect on the likelihood of a collision, and over 85% take place between vessels travelling in opposite, or nearly opposite directions. Such occurrences have been reduced by the implementation of traffic separation zones, but due to the local Geographic factors involved, it is not practical to establish these schemes in all areas. In all conditions human failure is almost invariably the cause, usually from negligence, ignorance of the International Regulations for Prevention of Collisions at Sea or improper use of equipment. Trouse (1978) defends the navigator and suggests that his job is being made increasingly more complicated in two ways. These are firstly, by all the extra equipment being installed on the bridge, both to assist him with the task of navigating and to monitor the engine room and cargo, and secondly, by the shortening of
his decision time with the increase in ship speed and traffic density. Paffett (1981) and Paymans (1977) have independently produced schemes to solve the problem of confusion arising from the density of information being presented to the navigator, and on which he has to base his decision. They have suggested a serious study into the ergonomics of ship's bridge design. This would incorporate an investigation into engineering, systems analysis, psychology and physiology to define basic characteristics of shipborne operators and their requirements. Paffett goes on to suggest a paged computer system with pages for general information, engine room, navigation equipment and so on, bringing the information from all sensors to computer terminals, placed about the ship as required. One system developed along these guidelines is the MANAV integrated navigation system by Racal-Decca. Millar and Hansford (1983) have described the design process and final product, which consists of an automatic plotting table with standard paper charts, a computer terminal for user interface and display together with an advanced radar plotting aid (ARPA). Ship's officers who have used the system have described it as useful, effective and easy to use. Some criticisms have also been made, mainly in the lack of ability to interface the system to some of the common navigation aids. It is a small step from this approach automatically to monitor and predict the movement of other vessels. This can be achieved by processing data acquired from the ARPA, which is already carried on vessels of 10000 GRT or more by requirement of the International Maritime Organisation (IMO), and advise a course of action to be taken. This would help to alleviate the second of the problems suggested by Trousse (1978). The requirements of such a system would be precise location and state of the vessel on which it is installed, projected tracks of other vessels in the vicinity, knowledge of the local topology and the International Regulations for the Prevention of Collisions at Sea forming a rule base on which to make decisions, to avoid collisions and grounding. The information required is available from existing navigation aids but problems occur in putting this information to use in evaluating the.
most probable state of own ship, and in the decision making process. The system which, perhaps at present, comes closest to meeting these requirements is produced by Sperry Marine, an article entitled "All alone at the helm" in the September 1989 issue of Marine Engineers Review describes this system. Central to the integrated system is the 'touchscreen' controlled ARPA. This is interconnected to the autopilot and a Voyage Management Station, all integrated by Sperry Marine's own Seanet Token Ring Data Network, which, in the event of a malfunction of one processor, does not make the whole system inoperative. Kelvin Hughes, a competing radar manufacturer, have also launched a new product. Their "Tactical radar" was designed specifically for military use but many of the features such as the ability to digitise coastlines into the system do have direct applications in the commercial environment and will doubtless be included in a wider range of future designs.

1.2 Survey of Marine Electronic Navigation Systems

The development of modern electronic navigation dates from the period 1939-1945, to meet the exacting demands of the armed forces during World War II. Developments of this period form the basis of many of the systems in use today. Beck (1971) outlines the history of some of the systems which have evolved, including the Decca Navigator. This was developed by United Kingdom naval scientists and was in use for the Normandy (D-Day) landings of June 1944, whilst Loran, an acronym for "Long Range Navigation" was developed at the Radiation Laboratory of the Massachusetts Institute of Technology during this period. Prior to the development of frequency standards the direct measurement of range was impractical, and hence these early systems relied on the measurement of phase difference between two radio signals so that position fixes were related to hyperbolic position lines. The two systems discussed still rely on these basic hyperbolic principles, but improvements have been made to give an unambiguous fix, and to the information displayed on the mobile receiver.
A number of individual systems are now available to the commercial operator, each having its advantages and disadvantages. Loran C coverage, for example, is limited almost entirely to the northern hemisphere, and whilst Omega provides world-wide coverage, it is insufficiently accurate for inshore navigation. Decca is sufficiently accurate for coastal navigation, but accuracy falls off with range and each chain covers only a small area. The Naval Navigation Satellite System (NNSS or Transit) is also sufficiently accurate, but the long time between satellite passes makes it unsuitable for coastal navigation. A typical fit in a British merchant ship would thus comprise a gyro compass, with autopilot, electromagnetic and/or Doppler log, Decca Navigator, Loran C together with Omega and/or Transit. This would give the navigator world-wide coverage and sufficient accuracy. Radar or ARPA and radio direction finder would also be fitted.

The accuracy which will be available from the Navstar or Global Positioning System (GPS) has yet to be established. Cook (1983) suggested that positional errors of less than 20 metres will be achieved. Henderson and Strada (1980) give details of a small scale sea trial in which a mean distance of 25.3 metres was claimed for passages in and out of San Diego Naval Base, California. More recently Napier and Ashkenazi (1987) have suggested that the codes used will be downgraded to meet the original commercial design specification giving an instantaneous position to between 100 and 250 metres. Current policy dictates 100 metre spheroidal root mean square (srms) accuracy to reduce the threat to security. They also state that Omega and Loran C are due to be phased out shortly after the full introduction of GPS. Jorgensen (1987) backs up their views and points out that the specified 100 metre accuracy is the 2drms (twice the horizontal root mean squared error) value which corresponds to a navigational error in the vicinity of 150 to 200 metres. The system can be operated in differential mode, which uses two receivers, one fixed at a previously surveyed location, the other sited where the position is required. Nolan and Carpenter (1984)
have shown that over a period of 70 seconds a position accurate to between 4 and 5 metres is available. Grover-Brown and Hwang (1983) have improved the technology to obtain centimetre accuracy, but the position takes 10 minutes to calculate on a large mainframe computer. It is unlikely that differential mode of operation will be used at sea for purposes other than surveying, but the navigator will have instantaneous world-wide coverage with accuracy between 100 and 250 metres. It is interesting to note that in daytime, the accuracy of the Decca Navigator is better than 100 metres within approximately 300 kilometres of the master transmitting station in a typical array, and considerably better (25m) close to the centre of the chain.

With the advent of GPS, questions arise about the future of terrestrial based systems currently in use. Loran is likely to be maintained, while the prospects for Omega are in doubt. A decision has yet to be made on Decca. With ageing transmitter stations requiring excessive maintenance and manufacturers other than Racal Marine Electronics now being able to build and sell receivers, who will continue to finance the systems? This question is not only in the minds of owners of trading vessels, but also is of concern to small boat owners. Dahl (1988) estimated that by this year, 100,000 Decca sets will be installed on leisure craft. He goes on to suggest that the leisure users requirements for navigational aids are often more demanding than those of the professional who, in general, has better equipment and is better trained for its use. For obvious safety reasons, it is important that the leisure user is provided with adequate navigational aids, however the presentation of too much information from excess equipment can distract attention from the real issue of vessel safety, and thus itself create a hazard. Considering all the navigation aids available, or likely to be available within the foreseeable future, only Decca will give a fix accuracy of better than 100m, and this will only be attained during daytime and in areas close to the transmitters. The shipowner has therefore been left with a difficult choice when choosing suitable navigation aids. To further
complicate the problem the selection of instrumentation installed on vessels has often been governed by political and financial considerations rather than by sound technological judgements. No single navigation aid is capable of satisfying the requirements of collision avoidance systems and advanced track-keeping autopilots currently in the research and development stage. However, integration of a combination of aids is likely to give the desired result.

1.3 Integration of Navigational Data

Single system deficiencies have led to the development of integrated systems for world-wide use. Systems such as MANAV discussed earlier and Racal's MNS2000 are capable of receiving data from a number of aids, but only give a position update from the selected source. The accuracy of the position can be improved by combining the data received from a number of navigation aids. Firstly we may consider the actions a ship's officer might take to fix the position of his vessel. He may plot a number of fixes obtained from various sources; for example, from log, compass and knowledge of set and drift of the current he can derive an estimated position from a previous fix; another fix may be obtained from the radar, and finally one from an electronic aid such as Decca. Using his knowledge of the likely random errors in all three fixes, he can take a weighted mean to establish the most probable position of his vessel. One of the requirements of such an integrated system, is to minimise the random errors in some way. The Decca Navigator Co. (1973) suggested that a Gaussian distribution gave the best fit for the spread of random errors in radio navigation aids. The problem can be stated in terms of minimising the variance and leads to the use of Kalman-Bucy filters. These have been developed extensively for aerospace, and latterly for marine, navigation since the publication of the original works by Kalman (1960), and Kalman and Bucy (1961).
The Kalman filter has found many useful applications in the aircraft world, where sudden environmental changes do not often occur. However in marine operations where disturbances can change quickly, applications do not appear to have been so successful, although this is not true of the offshore oil industry, where modern dynamic positioning systems employ Kalman filters extensively. Tysso (1981), although discussing modelling and parameter estimation of a ships boiler, states that a successful application of the extended Kalman filter (EKF) method is strongly dependent on a reasonable choice of design parameters, the noise co-variance matrices. He goes on to suggest that choice of initial estimates is critical. At sea then if the initial values of current are incorrect, or change suddenly, the best estimates of position and velocity may diverge. A general analysis of the convergent properties of the EKF method is given by Ljung (1971).

In marine navigation, Kalman filtering can be applied to two independant navigation aids, or to a range of sensors, and provided the statistical information required is correct will give a result which minimises the variance of the random errors. McPherson (1979) describes a system designed, built and in use in large oil tankers. However, two independant aids are not always available in all operational areas for trading vessels, unlike other maritime applications such as hydrographic survey, where it is common practice to monitor position from two aids, although the data received from the back up system is often discarded. Bennett (1980) gives an algorithm to combine the data received using Kalman filtering and obtain a more accurate position of the vessel. On a trading vessel it is always possible to obtain an estimate of position from DR. This can be combined with any other data available to update the positional information. For precise navigation in the approaches to a port, when the vessel may be undertaking various manoeuvres, the traditional DR update from log and compass alone is inadmissable. Cross (1986) has devised a method for improving the estimate using additional velocity data obtainable from dual
axis Doppler log. The positional information is now improved, but further state parameters may be required in control applications. For example, in the automatic guidance system described by Burns and Dove (1986) eight states of the vessel are required to pass to the optimal control algorithm, which maintains a vessel on track at the desired speed. In the use of this technique it is necessary to develop a full non-linear mathematical model of the vessel and to obtain transition matrices for it at discrete sampling intervals. This has a further advantage of enabling estimates of states to be made even if measurements are not taken. Preliminary investigations undertaken as part of this work have shown the modelling and filtering process to be a cumbersome task involving extensive sea trials initially and time consuming calculations while running the system on-line. It is perhaps for these reasons that few of the integrated systems available today employ Kalman filtering, and instead other filtering techniques are used. The lack of recently published papers describing the use of Kalman filters in marine navigation does suggest that perhaps there has not been the progress hoped for in this area.

The requirement for further work in high accuracy position fixing is highlighted in a UK Department of Transport report (1984) of the collision between the car ferries European Gateway and Speedlink Vanguard off Harwich, UK in December 1982, in which each master believed that the other would alter course to let him pass. The report states, "It is our belief that this collision occurred because of a degree of over-complacency, on the bridge of both vessels, in the performance of what may have appeared routine and unexacting navigation". If the European Gateway had been in the correct position in the deep-water channel such a collision might not have happened. The importance of automatic collision avoidance is illustrated by another example, reported by Yudovich (1988), concerning a collision on the night of 31st August 1986 between the bulker Petr Vasev (18604grt), entering the port of Novorossiysk, and a passenger vessel, the Admiral Nakhimov.
(17053gt), leaving the port. Both were owned by the Black Sea Shipping company and captained by experienced qualified mariners. They agreed, while about 7 miles apart, and through a shore based radio station that Vasev would give way, although it was Nakhimov's duty to do so. With about 2 miles between them, the Vasev, carrying ARPA, considered the situation safe in spite of the frequent communications from Nakhimov requesting assurance that they would be giving way. Finally the master of the Nakhimov could wait no longer and altered course, applying 20° rudder in small steps over some 4 minutes, contrary to regulations which state "a succession of small alterations to course and/or speed should be avoided". The master of Vasev remained transfixed to his radar, making his contemplated alteration to half speed at about the time that Nakhimov applied helm. He did not see Nakhimov alter course until too late. Both masters were at fault, initially for ignoring regulations. When Nakhimov finally applied helm it should have been one large alteration, also the master of Vasev did not combine what he saw on his radar display with reality.

The author believes that further work in the use of Kalman filtering will lead to improved accuracy of navigation systems to ensure that each vessel is in the correct position at the right time. While in the field of modern marine operations there is a need to bring together the disciplines of computing, telecommunications, navigation, control engineering and artificial intelligence. Kalman filter theory has a role to play within this development area.

1.4 Objectives

This project has been concerned primarily with an investigation into accurate navigation. It is an integral part of the research programme being undertaken by the Ship Control Group at Polytechnic South West. The
The overall aim of this research is to develop a fully integrated navigation system capable of providing the master of the vessel with accurate information on the status of his own ship and other vessels in the vicinity. The system is also required to give advise on a suitable course of action to take to avoid collisions at sea.

The specific objectives of the investigation are:

(i) To give an outline specification for a shipboard navigation and advisory collision avoidance system to improve safety at sea;

(ii) To examine accuracy specifications of marine navigation aids, and perform an investigation into the practical application of Kalman filters in the marine environment. This will be based on the theoretical work of Dove (1984);

(iii) To conduct further investigations into filtering to overcome problems highlighted in the preliminary investigation and provide a navigation algorithm suitable for use in the overall system;

(iv) To link the filter to an optimal control algorithm to provide further insight into potential problems which may arise when advisory collision avoidance is included into the system.

This thesis comprises of segments of work from a wide range of disciplines including computing, mathematical modelling and signal processing, with further discussions on system identification and optimal control. It is necessary to understand fixed and random errors in current navigation equipment. These are described briefly in chapter 2 through the theory of operation. Chapter 3 describes the test vessel and instrumentation installed for data collection and trials on computer algorithms for filtering and
control. Kalman filter theory employs a mathematical model of the dynamic system, and hence ship modelling techniques are presented in chapter 4. Kalman filter theory is introduced in chapter 5, and chapter 6 presents a practical investigation into its operation through a simple filter for position fixing. Possible solutions to problems encountered with initial trials are also given. The filter developed is not adequate for control and a more sophisticated model employing further states is presented in chapter 7, although modifications are made to overcome problems encountered. Solutions to these problems, particularly those of modelling errors are discussed in chapter 8, where a method for system identification is implemented. The state estimate given by the filter is then passed to an optimal control algorithm defined in chapter 9 where results of controller trials are also presented. Finally, chapter 10 reviews the work and puts forward suggestions for further research in this area.

A schematic diagram of the full integrated navigation system is shown in figure 1.1. The system will comprise two computers linked together by a parallel interface. Both machines will be interfaced to ship's equipment and will run graphical displays. One machine will run navigation programs which
RADAR

read target data

read own ship data

send model parameters

ACAS heuristics

is manoeuvre needed?

yes

divert track

no

format target data, own ship data, control advice,

track control algorithm

Figure 1.2. Flow Diagram for Computer 1.

BACKGROUND

Interact with user via keyboard for input of desired track, ship parameters, etc.

Detect errors in ship’s instrumentation and use ship status to adjust model parameters.

send model parameters

ACAS heuristics

is manoeuvre needed?

yes

divert track

no

format target data, own ship data, control advice,

track control algorithm

Figure 1.3. Flow Diagram for Computer 2.
must be cycled quickly. It will interface to the ship's navigation aids, perform the mathematical model and filter computations and display an electronic chart with ship's status, desired track and information on target vessels. Meanwhile the second machine will interface to the radar, run heuristics for collision avoidance (ACAS) and ship operations, making modifications to the mathematical model if needed. It will also present track information to the mariner and be used as the man-machine interface for the system. Flow diagrams for the software in both computers are shown in figures 1.2 and 1.3. The machines, which are described fully in chapter 3, will be multi-tasking. This facility, through a time-sharing process, enables the processor to work on more than one task at a time. For example, while communicating with a peripheral, which is usually a slower process than the speed at which the processor can operate, spare time can be used for computations. So the software has been divided into primary and background tasks.
2.1 Introduction

A combination of deficiencies in individual items of navigation equipment and the emergence of the microprocessor has led to the development of integrated systems to aid navigation. Some such systems do no more than present the mariner with information received from a variety of sensors on a compact display, while others perform computations on data received from the sources to provide him with the most probable estimates of the information required. Optimal estimates are usually obtained from least squares and Kalman filtering techniques. On those occasions where the computation time required to perform such calculations prohibits their use, or system errors cannot be accurately established, suboptimal methods are used, as discussed by Gelb (1988). When considering the problem of filtering it is essential to have an understanding of the operation of the various sensors and the likelihood of errors in the output. Essential measurements for navigation are considered to be position, heading and speed, while additional information on depth and other traffic movements are required for the vessel's safety.

2.2 Electronic Position Fixing

Electronic position fixing systems for navigation at sea can be considered in two categories:

(i) Land based systems, primarily Omega, Loran and Decca, which are described fully by Tetley and Calcutt (1988), and which measure either phase or time differences between signals transmitted simultaneously from pairs of stations;
Satellite systems, such as GPS and Transit, also treated by Tetley and Calcutt (1988), measure ranges from satellites and range differences from satellites respectively.

Omega has worldwide coverage but low accuracy, whereas Loran has better accuracy but covers only the majority of the Northern hemisphere. Decca improves accuracy further but coverage is particularly restricted to specific areas around the world, primarily North-West Europe. While the accuracy of satellite systems is comparable to that of Decca, fixes available from Transit can be intermittent and GPS is not yet fully operational. The operation and errors of each of these systems will now be considered in more detail.

2.3 Decca Navigator

Measurement of phase difference between two signals transmitted simultaneously from sources at known locations gives a position line which is a hyperbola on a map of the earth's surface. Ambiguity arises as the pattern is repetitive; this is overcome by interrupting the standard transmission at regular intervals and all stations in turn transmitting a multipulse lane identification signal. The resultant periodic waveform has a wavelength equivalent to one lane, which is the minimum ambiguity available. In order to derive a position two intersecting position lines are obtained from one common master transmitting station and two slaves. Usually three slaves are combined with a master to constitute a chain and the fix is obtained by using the two position lines which give the best angle of cut. It is possible to obtain a fix from two position lines obtained from different chains, this is known as cross chain mode. Figure 2.1 shows the areas covered by Decca navigator worldwide.

Fixed or systematic errors within the Decca navigator are due to variation in velocity along the line of propagation and refraction due to land masses. Fixed error corrections to be applied to the position lines obtained are
tabulated with respect to geographical location and published by Decca Marine Systems in the Decca Navigator Operating Instructions and Marine Data Sheets. Variable errors due largely to local variations in the atmosphere are entirely random and increase with range. Random errors likely to be obtained in the finally computed position fix for the South West British chain are shown in figure 2.2. As the atmospheric variations leading to variable errors are likely to be affected themselves by the time of day, weather etc. so will the error probability in the position fix. A further set of charts show the 95% probability level of random error in metres. These indicate the likely accuracy of the fix with geographical location and are given in figures 2.3 and 2.4. Skywave total internal refractions interfering with the desired ground wave signal effectively limits the range of the system to 240 nautical miles by night in Northern Europe but 220 nautical miles nearer the equator.
Figure 2.2. Random Errors of South West British Chain in Full Daylight

Figure 2.3. Decca Random Error Contours - South West British Chain
Like the Decca navigator, Loran is a hyperbolic system. In this instance the hyperbolae are generated by measuring the time difference in two wave pulses appearing at the receiver. The fixed master station, at a known location, initiates the cycle by transmitting a pulse which is received by both the mobile receiver and the fixed slave, also at a known location. The slave waits for a given period before transmitting an identical pulse, this is then used by the receiver to compute an unambiguous line of position. The advantage of using a pulse system instead of phase comparison is that interference due to skywave reflections is now minimised. This latter signal will have taken a longer path and will arrive about 35µsecs after the ground wave signal at 1000 nautical miles, the maximum range of the Loran system.
The transmitted pulse length is 250μsecs, a tracking point 30μsecs after the start of the pulse is used for measurements to minimise skywave interference. The range of the system can be increased by using a 70μsec tracking point, this point represents maximum signal amplitude, however accuracy is greatly reduced in this mode of operation as measurements are likely to be made using the skywave which will extend coverage as it suffers less attenuation. Accuracy figures beyond the 1000 mile ground wave coverage area are not given but tables giving corrections to be applied to measurements made using skywaves are available. Figure 2.5 shows theoretical accuracy contours for the Loran Norwegian Sea chain, while figures 2.6 and 2.7 shows Loran coverage worldwide by ground and skywaves.

Figure 2.5. Loran Random Errors - Norwegian Sea Chain
Figure 2.6. Loran Coverage Worldwide
Figure 2.7. Loran Coverage Worldwide
2.5 Omega

Like the Decca Navigator, Omega is a phase comparison hyperbolic system. The low frequency (10-14kHz), long wavelength (30 km) transmissions from the eight stations give worldwide coverage. All stations can be considered to comprise a single chain and each station can be paired with any other to give a position line. This is achieved by all stations transmitting each of the three frequencies used in navigation for about 1.1 seconds during the 10 second cycle, transmissions at other frequencies throughout the remainder of the cycle are mainly used for communications. The system is configured such that all stations are transmitting different frequencies during the eight time slots.

The use of long radio waves leads to poor resolution. Furthermore they are totally refracted internally by the ionosphere, therefore the waves move up and down while they travel from the transmitter to the receiver. The path covered and, as a consequence, the phase of the received waves are affected by variations of the height of the ionosphere, diurnal variations, the earth's magnetic field and other conditions. Tables are available to give some corrections but accuracy remains in the order of 0.75 nautical miles during the day and 1.5 nautical miles at night. In spite of these claims a Western Pacific Omega validation study during 1976 and 1977 estimated the 95% accuracy to be between 6.3 and 7.3 nautical miles. In coastal waters the problem of poor absolute accuracy can be overcome by using the systems good relative accuracy and operating in differential mode. A local shore based receiver station at known coordinates calculates corrections and transmits these to local shipping. As yet few shore stations capable of performing this function have been set up.

2.6 Transit (Navy Navigation Satellite System)

In October 1989, six Transit satellites were in operation, in polar orbits approximately 1075km above the earth's surface travelling at 7.3kms\(^{-1}\). A fix
is available from a satellite visible to the receiver for more than two minutes. Each satellite is tracked from the earth and its predicted orbit for the next 16 hours transmitted to it. This prediction accounts for precession and irregularities to the circular orbit which will be slightly elliptical and subject to gravitational forces of the geoid. Corrections to the elliptical orbit are then retransmitted by the satellite at two minute intervals, phase modulated on to carrier frequencies of 399.968kHz and 149.988kHz. A receiver measures the Doppler shift of these signals and computes a hyperboloid, the intersection of this with the earth's surface gives a line of position. Repetition of the computation two minutes later gives a second position line, and hence an estimate of position. To account for any discrepancy between transmitted frequency and comparative frequency generated in the receiver a third measurement is made, thus this source of error is removed. Other error sources and their effect on the final fix are:

(i) Anomalies of signal propagation. The purpose of using two frequencies is to eliminate errors due to refraction in the ionosphere; the two frequencies will be affected differently and the receiver can compute the correction. Refraction due to the troposphere however cannot be evaluated (5m);

(ii) Errors in the orbit predictions (10 - 20m);

(iii) Incorrectly estimated forces acting on the satellite orbit (15 - 25m);

(iv) Satellite position rounding error (5m);

(v) Inaccuracies in the receiver and its computational restrictions (5m);

(vi) Error in the antenna height entered into the receiver (10m);

(vii) Errors in speed and heading entered into the receiver. A vessel travelling at twenty knots will move about 2500metres during reception of the three signals required to calculate position. Speed and heading input is used to evaluate a dead reckoning position between receptions. A speed inaccuracy of 1 knot can lead to an error in the longitude of 90m if the vessel is heading east or west, whereas an error of 400m could result if it were heading north or south;

(vii) If two or more satellites are received simultaneously, they can cause mutual signal interference. Occasionally satellites are temporarily
switched off to overcome this problem. Close to the poles however, where satellite passes are concentrated, this problem can still lead to a total loss of useful information.

The main disadvantage with Transit is that it does not provide 24 hour global coverage. The original design specification intended for each point on the earth to be covered twice by each of the six satellites in a 24 hour period, giving a maximum of two hours between fixes with least coverage on the equator. Due to precession of the five useful satellites this figure deteriorated to ten hours in some parts of the world.

2.7 Global Positioning System (GPS)

GPS is intended to overcome the primary problem of Transit to give complete global coverage to compute a position in three dimensions 24 hours a day. The final GPS system is to comprise of twenty one satellites in six circular orbits 20200km above the surface of the earth, with each satellite having a power source to maintain it in orbit. Limited coverage is already available with full operation in 1992. It will then take time for manufacturers to produce low cost receiver units and perhaps by the mid 1990's GPS will be a common piece of navigation equipment on vessels.

Each satellite transmits signals at given times in accordance with an on board time source. This is updated every 24 hours by a caesium atomic clock on the ground. A less accurate clock in the receiver unit anticipates reception of the signals, the time delay to actual reception enables satellite ranges to be observed. Observation of a single range gives a position line in the form of a circle on the map of the earth's surface. Ranges from two satellites give two position line intersections, an approximate estimate of position will lead to the correct solution. The clock in the receiver unit may be in error, giving a third variable, thus a third satellite position line is required to solve the problem in two dimensions. Reception from four
satellites enables computation of a position fix in three dimensions. Furthermore measurement of the Doppler shift of the signals enables velocity of the receiver unit to be calculated.

The satellite data required to compute position is coded, and the receiver unit must contain the necessary information to decode the messages. Coded messages from satellites are modulated on to two carrier frequencies, 1575.42MHz (L1) and 1227.60MHz (L2). Course and acquisition (C/A) code is transmitted on the L1 carrier only, whereas precision code (P) is transmitted on both frequencies. Each signal will be refracted by a different amount as it passes through the ionosphere, thus, by making measurements on both frequencies this source of error can be reduced. While commercial receivers will be able to make measurements on both frequencies, they will only have access to the C/A code and hence, in theory, accuracy of the position fix obtained will be reduced from 5m to 100m. Methods of improving this error have however been devised by commercial system manufacturers and it is now possible that the C/A codes will be further downgraded in order that the original design specification of 100m will once again apply.

The most common method used to improve accuracy is Kalman filtering, Napier (1989). An eight state filter is implemented to estimate position (3 components), velocity (3 components), and clock errors (clock bias and clock rate). Essentially the filter is used to reduce the random noise found in the radio signals from the satellites. A schematic diagram is shown in figure 2.8 where the code tracking logic obtains the pseudo-ranges, ranges which include clock error, and range rate from four satellites. The Kalman filter combines these measurements with predictions obtained from a model based on vehicle kinematics. A constant velocity model disturbed by a constant acceleration between updates is used. The constant acceleration is modelled as an unbiased, normally distributed random forcing function. Filtered position and velocity is then output with the velocity also being fed back into the code tracking logic.
Differential GPS, another method of improving accuracy, is often used in applications such as land and nearshore hydrographic survey where it is practical to employ a second stationary receiver unit. The stationary receiver, sited at a known location computes an error, which is assumed constant in the vicinity. The position obtained by the mobile is then corrected. To obtain even greater accuracy the mobile is positioned at the required location while several ranges are made from each of the available satellites. Analysis of the results reduces random error.

2.8 Positioning Errors

All of the systems mentioned above rely on the intersection of lines of position obtained from some measurement arising from the transmission of electromagnetic waves. Each line of position obtained contains some random error due to atmospheric, solar, magnetic and other effects. The central limit theorem of statistics, Gelb(1988), states that a superposition of independent random variables always tend towards normality, regardless of the distribution of the individual random variables contributing to the sum. On this basis it is fair to state that the random error of each line of position obtained will be normally distributed with some standard deviation $\sigma$. Figure 2.9 shows the variation in one lane of a Decca Navigator for a night sample period. For a one dimensional error standard deviation and root mean square (RMS) error are identical. RMS error is defined as the sum of the squares of the deviations from the mean or average values divided by the number of measurements. Numerically 68.3% of the readings
fall within the standard deviation, and 95.4% fall within twice the standard deviation for an assumed Gaussian distribution.

Figure 2.9. Measured Decca Errors on One Line of Position

Figure 2.10. Error Ellipse and DRMS radius
The position fix is computed in two dimensions from the intersection of two or more lines of position and therefore the error is given by a combination of error sources and is described as a bivariate error distribution. Consider two lines of position intersecting at some angle $\beta$, as shown in figure 2.10. The standard deviation of each LOP is indicated by lines drawn parallel to it, there is a 68.3% chance of the measured LOP falling between these lines. The area enclosed by the lines of standard error forms a diamond which would contain $0.683^2$ or 46.6% of the fixes taken at the point of intersection of the LOP’s. The ellipse inscribed in the diamond would contain 39.35% of the fixes. It is however more usual to quote the DRMS value, which is the radius of the circle about the intersection which would contain 68.3% of the fixes. The DRMS value is given by:

$$\text{DRMS} = \sqrt{a^2 + b^2}$$

where $a$ and $b$ are the semi-major and semi-minor axis of the error ellipse.

Cross (1983) gives computational procedures for evaluating these parameters from the angle of cut and the standard deviation for each position line.

A fault of many papers quoting errors in position fixing systems is that they rarely state which value they have given. Where this is the case it is assumed that the DRMS value is quoted.

2.9 Ship’s Compass

Primarily there are three types of compass, namely the magnetic compass, the fluxgate compass and the gyro compass. Compasses combining the various modes of operation have also been built, for example magnetic compasses with fluxgate to drive repeaters and fluxgate compensated gyros.
The magnetic compass relies on the magnetic properties of ferrous materials and the earth's magnetic field. A permanent magnet swings to point North. It therefore comprises one moving part. Due to the simplicity of its design, the magnetic compass is extremely cheap.

The fluxgate compass relies on the properties of electro-magnetic induction. The earth's magnetic field cutting through a coil of wire induces a current into the wire. Two coils arranged perpendicularly produce voltages proportional to the sine and cosine of the angle of cut, thus removing the problem of ambiguity of the zero and giving a more accurate reading than a single coil. Such a system is known as a two phase fluxgate compass. Occasionally three phases, that is, three coils mutually at 120°, are used for further improvement of accuracy. Such a system has no moving parts and produces signals which are easily resolved by modern electronics. Fluxgate compasses are also inexpensive.

Both the magnetic and fluxgate compasses produce a bearing relative to magnetic North. They are also affected by masses of ferrous metals in the vicinity and any form of electrical appliances nearby. These errors are relative; therefore with careful selection of siting and by calibration they can be minimised. Remaining errors are from a variety of sources with various probability density functions, so by the central limit theorem of statistics the resultant random error tends towards a normal distribution.

The gyrocompass does not rely on the earth's magnetic properties, instead a spinning wheel is set in motion such that its spin axis is maintained parallel to the spin axis of the earth. Thus the gyrocompass measures a bearing relative to true and not magnetic North. The wheel is suspended in theoretically frictionless bearings such that it is allowed to move in all three degrees of rotation. It is set spinning at an accurately controlled rate and the entire system is then damped to obtain stability. External forces acting on the inertial mass are then the earth's rotation and gravity. Negative feedback is applied to overcome centrepetal acceleration of the
earth, while a mass is suspended to amplify the gravitational force and the gyro becomes true North seeking. With the gyro set on the centre line of the vessel remaining errors are due to the vessel's acceleration. This is greatest in rolling motion and is reduced by suspending the gyro in a viscous fluid to provide further damping. Remaining motions of the vessel give rise to a random heading error of approximately $1^\circ$. It should however be noted that the damping leads to a slow response time for the gyro which would never be noticed in a large vessel with an even slower response. When installed in smaller vessels, which respond quickly to the helm, the gyro requires time to respond to the vessel's movement.

2.10 Speed Measurement

Currently all speed measurement devices are mounted underwater. With the introduction of GPS this may change, but commercial manufacturers may not consider it financially viable to incorporate the Doppler shift measurement process into all satellite receivers. As yet accuracy figures for this additional measurement are not available.

Electromagnetic and Pitot tube pressure logs both measure speed through the water to accuracies of $0.1\%$ and $0.75\%$ respectively of the range in use. The electromagnetic type is also frequently capable of measuring lateral velocity also, although with a degraded accuracy of about $2\%$.

The patented acoustic correlation log tracks an acoustic signal reflected off the sea bed to depths of 200 metres; in deeper water it switches to tracking the water column about 12 metres below the transducer. In shallow water below 200 metres, speed over the ground is measured to an accuracy of 0.1 knots. In deeper water the relative error of water speed 12 metres below the transducer is also measured. A second transducer is required if lateral velocity is also to be obtained.
Acoustic signals returned from the sea bed are also used in Doppler logs. The velocity of the receiver unit is established by comparing the frequency of the received wave with that transmitted. In coastal waters, that is where the depth is less than 400 metres, the sea bed acts as the layer for reflecting the signal, so that speed over the ground is measured. In deeper water, some arbitrary boundary layer in the medium, 10 to 30 metres below the transducer, reflects the signal and the speed measurement is made relative to the water velocity of this layer. An accuracy of 0.2% of the measured value or 0.1 knot, whichever is greater is available using the Doppler system.

2.11 Radar

Radar measures the two-way travel time of microwave signals to reflective objects, using a rotating antenna to give range and bearing of the object. It is therefore used to inform the mariner of navigational hazards which, due to range and visibility restrictions, cannot be seen by eye. Automatic Radar Plotting Aids (ARPA) incorporate a microprocessor and software capable of computing the course and speed of mobile hazards or targets. Further calculations are then undertaken to predict the track of the target and compute its closest point of approach (CPA). This information is then displayed on the ARPA digitally or by plotting vectors which indicate course and speed of the target and may indicate CPA. The mariner thus has more information available sooner and he may be able to take evasive action at an earlier time, Chudley and Dove (1989) The disadvantage of such systems is that navigators are placing too much trust in them. In situations where he may have taken precautionary action without radar, the mariner may now read off the CPA and decide that no action is necessary. As a consequence vessels are passing closer than they would do without such advice.

Advanced radar is not yet perfected; some equipments track noise or clutter too easily, while others lose weak, intermittent targets. This is one aspect
of research into navigation systems which is currently being undertaken by the Ship Control Group at Polytechnic South West, where further investigations involve a system which will give advice to the navigator on what evasive action should be taken.

2.12 Integration

Integration of a number of the sensors described above leads to improvement in accuracy and hence safety of navigation at sea. One method of improving the accuracy of a position fix involves measuring position from two independent sources. A knowledge of errors of the individual systems allows weights to be assigned to each of the fixes. A weighted mean can be taken manually, or use of a microprocessor gives rise to practical application of a least squares algorithm. Other integration techniques have evolved, for example in the Transit receiver, where a fix is available from satellite data only; using data from three consecutive transmissions from one satellite, although the fix error can be up to 450 metres per knot of ship speed, depending on heading and satellite elevation. Additional data from ship's log and compass to compute dead reckoning between reception of the three satellite signals, reduces this error considerably. It is therefore necessary to integrate the Transit receiver by interfacing to log and compass, although this information can be entered manually into receivers used in the marine leisure industry. Furthermore, due to infrequency of satellite passes, fixes can be sparse and the dead reckoning update can be used within the receiver to estimate position between satellite passes. A new starting point is reset whenever a satellite is available. A further improvement is to integrate this system with Omega, which has a large relative error. Whenever a satellite is available, a fix is obtained and the Omega relative error calculated. This error is then applied to all subsequent Omega fixes until the next satellite pass. One such system in commercial use is the Magnavox MX 1105 satellite/Omega navigator.
Other commercial systems are employing log and compass inputs to reduce positional errors, for example the Mk53 Decca Navigator receiver. This device utilises a filtering technique to improve accuracy. A further facility exists to programme the receiver with a number of waypoints on route, details of distance off track and course error can then be displayed while underway. The ultimate aim must be to feed these signals into an autopilot to maintain the vessel not only on course, but also on track. The technology to perform this function is in existance, although more than just positional information would be required by the autopilot. Velocity feedback in the two dimensions and rate of turn are necessary to stabilise the system.

Dove (1984) has shown the validity of Kalman filtering as applied to marine navigation. The process he used combined state estimates from measurements with those from a mathematical model of the vessel to produce an optimal estimate. Trials conducted in simulation showed promising results. A fundamental comment on the work is that the model used to represent the vessel is also used within the filter. Any deviation between the two, or inaccuracies in estimating external forcing functions, may lead the optimal estimate to stray from the true value with time.

2.13 Further Developments

Development of a production system for fully automatic ship control is well into the future. Although the technology exists there are other considerations governing the instrumentation installed on a vessel. Regulations dictate the personnel required to man a ship, and while shipowners have to pay crew they avoid expenditure on a computer system. Instrumentation manufacturers are also reluctant to start manufacturing equipment which automatically controls the vessel. The computer assisted collision has not yet happened, but when it does no manufacturer would wish to be subject to the court case test. There is however an immediate requirement for improving the accuracy of marine navigation systems and
the way in which information is presented to the man on the bridge, assisting him with the decision making process and giving advice. For example, examination into the competitive business of radar manufacture shows rapid progress being made in this area. Kelvin Hughes, a leading UK manufacturer, have recently launched a new tactical radar aimed at the coastal defence market. This is a rasterscan ARPA with additional functions including the ability to digitise a chart onto the radar display. Problems identified include the need to stabilise the superimposed chart against random movement and the need to improve multiple target tracking. Future enhancements include interfacing to existing ship's equipment and combining the facilities on the radar with those of integrated navigation and collision avoidance, to produce optional additional displays which would give a product well ahead of those offered by competitors. Further enhancements could involve the development of an expert system to aid ship operations. This would entail additional mathematical modelling at low speeds, where extreme non-linearity can lead to large inaccuracies. This work would include stopping in narrow channels, emergency procedures, anchoring, manoeuvring in narrow channels and berthing.
CHAPTER 3

INSTRUMENTATION

3.1 Introduction

As indicated in chapter 2, Dove (1984) has undertaken computer simulation studies to show the potential of Kalman filtering applied to marine navigation. There are many other examples of theoretical and simulation studies, for example Cross (1987). This work did not consider how noise functions with constant offsets, such as wind and tide would be measured and applied to the system process. Much of the work detailed in this thesis was undertaken afloat in Plymouth Sound; a map showing reference points used is shown in figure 3.1 and a brief description of the test vessel with the on board instrumentation follows.

3.2 The Test Vessel

The vessel chosen for trials was the Polytechnic's training vessel 'CATFISH', an 11m, twin engined catamaran built on a Prout Snowgoose 37 hull.

| Number of Screws | 2 |
| Number of Rudders | 2 |
| Individual Rudder Area | 0.359 m² |
| Displacement | 8.5 tonnes |
| Overall Length (including rudders) | 11.17 m |
| Overall Width | 4.3 m |
| Hull Centre-Line Separation | 3.7 m |
| Main Engines | Volvo Panther |
| Power @ 2500 rpm | 25 kW |
| Engine/Screw Gear Reduction | 1.66 : 1 |
| Forward Speed @ 2200 rpm | 8.9 knots |
| Forward Speed @ 1100 rpm | 4.2 knots |
Figure 3.1. Plymouth Sound
Figure 3.2. Instrumentation on Catfish

**CONTROLS**
- Rudder (Hydraulic Pump)
- Engines (Electric RAM)

**MEASUREMENTS**
- Rudder (LDVT)
- Engines (Tachometers)
- Position (DECCA Mk 53)
- Glop
- Plop
- Speed (Pulse Log)
- Heading (2 Phase Fluxgate)

**EUROBEEB**
- D.A.C., A.D.C.
- SERIO
- 6502 Processor
- Acorn Cambridge Workstation
- VDU Printer
- Plotter
Figure 3.3. Catfish Equipment Installation
Because the hull was originally designed as a yacht, it has a large rudder area, and consequently a small amount of helm movement turns the vessel sharply. Also the drive shafts on both engines rotate in the same direction which applies both lateral thrust and a moment on the vessel.

3.3 Computer Instrumentation

One of the advantages of using a Kalman-Bucy filter is that not all parameters in the state vector require measurement; those which are not measured are predicted from the model alone. The instrumentation installed on the test vessel has been restricted where possible to the minimum expected on a commercial vessel. Only the control equipment for the engines and computer systems for interfacing and solving the guidance algorithms being added. The complete system is shown schematically in figure 3.2. Figure 3.3 shows the locations of the equipment installed and figure 3.4 gives details of the wiring; the 26 pin connector is used as the computer interface for analogue signals. The filter does, however, require statistical information on each of the measurements in the form of a covariance matrix.

For afloat trials an Acorn Cambridge Workstation (ACW) was used for real time control and guidance of the vessel. The ACW uses a 32016 (32 bit) main processor with a 6502 (8 bit) processor for I/O. It has 1 Mbyte of memory with a 20 Mbyte Winchester disc and 5 inch floppy disc drive built in. All software has been written in FORTRAN 77 as this is compatable with the FORTRAN used previously in simulations. Software packages purchased to enhance the system include a graphics facility, and a terminal emulator package which also enables files to be transferred between the ACW and the Polytechnics mainframe, or any other computer which has the package installed. In addition a microprocessor (CUBE) has been installed to act as in interface between the ACW and the measurement and control
systems. This consists of a Eurocard rack with three cards, namely:

(i) A processor card which also has 48kbytes of memory divided into 32k ROM and 16k battery backed RAM; a serial port which is used for communication with the ACW and two 8 bit digital interfaces which are not used,

(ii) A serial input/output (SERIO) card with four serial ports, two of which are used to receive positional information from the Decca Navigator and Trisponder,

(iii) An eight channel, 12 bit, analogue to digital converter (ADC) card is used to receive rudder angle and engine revolution data. This also has four digital to analogue (DAC) channels used to transmit control signals.

All software within the cube has been written in 6502 assembly language.

Work in the laboratory was transferred to one of the target machines for the prototype system. Two Acorn Archimedes RISC machines with high resolution colour monitors are being used. These machines, which have 4Mbytes of RAM, 20Mbytes Winchester disc and a 3½ inch floppy disc drive built in, were selected mainly for their processing power. The reduced instruction set and floating point co-processor enable calculations in floating point arithmetic to be performed faster than on any other computer in the price range. Furthermore, excellent documentation of the operating system gives direct access to resident machine code routines for graphics and interfacing from high level languages. All high level programming in these machines was written in ANSI C as this runs much faster than any other high level language available for the Archimedes and gives easier access to the computer memory and machine code routines. Two machines allow the running of the processors in parallel, linked with an IEEE488 interface connecting the two data highways. Further expansion cards, also communicating directly with the data bus, make the cube system redundant, improving the speed of communications with the vessel's equipment.
3.4 Vessel Instrumentation

Rudder angle ($\delta$) is measured using a Linear Variable Differential Transformer (LVDT), Mansfield (1973), mounted alongside the hydraulic ram used to actuate the rudders. The LVDT, when activated with 12v DC, energised a pair of fixed coils with an AC current. A core connected to the displacement mechanism induces the current into a third fixed coil. The phase of the induced current is proportional to the amount of core in the two primary windings. A phase shift demodulator and amplifier unit convert the phase shift into an analogue voltage between +5 and -5 volts. This is directly proportional to displacement, and is measured by the cube and transmitted to the ACW where a linear calibration converts voltage to rudder angle.

Engine revolutions (n) are derived from a pulse tachometer. A card was built to count pulses and convert them into an analog voltage proportional to the count. The voltage is measured by the cube and sent to the ACW.

Positional information (x and y) is derived from a Decca Navigator System (DNS) Mk53 receiver which receives signals from the Decca hyperbolic phase comparison positioning system described in chapter 2. The trials area of Plymouth Sound is covered by the South West British chain giving very small random errors; during daylight hours a DRMS value of 10 metres is available. In Plymouth Sound it is also possible to receive signals from the English chain, in which case the random errors are much larger, having a DRMS value of 100 metres. This enabled trials to be undertaken on the filter under both extremities of Decca coverage. The hyperbolic position lines as received are transmitted to the cube via a serial link. This is then passed on to the ACW which calculates a position fix in geographical coordinates. The solution is given in Appendix A. The fix is then mapped on
to the Ordnance Survey Great Britain 1936 grid (OSGB36), which is fully defined in Appendix C, and finally to ship coordinates with the aid of further information on the ship's heading.

Forward speed (u) is measured by a pulse log which is interfaced to the DNS. As the vessel moves through the water a small vane rotates generating electrical pulses. These are counted by the DNS MkS3 which uses knowledge of pulses per mile to calculate speed which is transmitted to the cube with the positional information. The log measures speed through the water only and hence set and drift of the current are added to obtain speed over the ground. This measurement is subject to a large amount of random noise, which is partly due to the poor discrimination available within the DNS. The standard deviation of the log is of the order of 1 knot.

Due to the slow response time of a gyro compass compared with that of the turn rate of the vessel, it was decided to install a 2 phase fluxgate compass to measure heading (ψ). This is also interfaced to the DNS MkS3 which contains the calibration constants required to resolve the two signals obtained into a magnetic heading. The compass has further been 'swung' and a table drawn up to account for variation and deviation. Rather than use look up table techniques, as the navigator would, a sine function has been fitted to the errors and programmed into the ACW. A graph showing the function and measured errors is shown in figure 3.5. After correction for these fixed errors the outstanding random errors are considered to be less than 4°.

It has already been indicated that lateral velocity (v) and yaw rate (r) are not measured directly. Values in the measured state vector z are obtained using a numerical method to differentiate lateral displacement and heading respectively. This technique has proved poor in deriving lateral velocity due
COMPASS VARIATION AND DEVIATION

ERROR = 8.85 × SIN(H - 44) - 5.4

+ MEASURED ERRORS

Figure 3.5. Compass Variation and Deviation.
to the large errors obtained for displacement using Decca. Heading, as measured by the compass, has little random noise and therefore the process gives a more reasonable result for yaw rate, but this is dependant on the number of previous measurements taken into consideration and on the cycle time of the measurement process. Taking just three terms, with a cycle time of 2 seconds the result will be accurate to within $5^\circ$/sec.

Errors in measurements from log and compass were evaluated by making a run in calm water with constant engine revs and steering on a transit, thus maintaining the vessel in a steady state, while continually logging data.

From the results of this run the variances of measurements have been evaluated as:

- Variance of $u = 0.5 \ (m/s)^2$
- Variance of $\psi = 0.0054 \ (rad)^2$

In the cases of lateral and angular velocity, which are not measured directly but are derived from other measurements, the variances are:

- Variance of $v = 2.0 \ (m/s)^2$
- Variance of $r = 0.02 \ (rad/s)^2$

These were obtained from their relationships with the measurements from which they are derived. Variances for rudder angle and engine revolutions were ignored because these parameters are not estimated by the model but are used to drive it.

When conducting trials on the filter software the fix obtained from Trisponder has been used as a reference position. This is a range measuring hydrographic survey system giving an accuracy to within 3 metres, Del Norte (1986). Trisponder measures the two way travel time of microwave signals between the ship-based master station and a number of fixed remote stations, whose locations must be known. Corrections are made for a delay,
due to signal turn around time in the remotes, and for remote station
heights before the true horizontal distances are transmitted through a serial
interface. The position fix obtained from two or more ranges is more
accurate than that of the DNS system, but Trisponder has a much shorter
range and is restricted to line of sight operation. This limits its use as a
navigation aid, but it is frequently used for surveying. There are four
permanent remote Trisponder stations situated around Plymouth Sound so a
least squares algorithm, given in Appendix B, has been employed to find the
most probable position of the vessel from the incoming ranges. The four
remote stations are not all visible across the whole trials area and
occasionally a false signal will arrive at the receiver. This usually results in
an overlarge range as the most common cause is reflection from or
refraction around land masses. Checking for these gross errors has not
been implemented as the Trisponder is only being used as a reference
during filter trials and large errors are obvious to the eye on the computer
plots.

Figure 3.6 shows that a measurement cycle within the cube is triggered on
reception of the ASCII character 0 from the ACW. It then reads data from
Trisponder, Decca and finally the analog voltages proportional to rudder
angle and engine speed. The frequency at which the Trisponder transmits
data is adjustable but has a minimum of one measurement set per second.
Similarly the Decca Mk53 transmits data along the serial computer interface
at nominally 1.56 second intervals but this may be as long as 6 seconds. To
reduce the probability of a long delay all peripheral functions available on
the Mk53 receiver should not be used. These consume processor time within
the DNS which may be put to better use for the desired I/O operations.

The Decca Mk53 receiver transmits a large quantity of data through the
serial interface, as described in the Decca Navigator Receiver Mk53
Installation Handbook. Only the messages containing position lines, compass
and log are required, all other information is discarded by the cube. In order to locate the desired data in the string, identification characters are transmitted amongst the data set. Memory locations 1E00, 1E07 within the cube must contain the ASCII values for DLI, VHW, respectively, as these are the identifying characters the software within the cube searches for amongst the DNS data string.

Several runs were made while plotting position fixes from Decca, ship model, filter and Trisponder using a digitised chart of the area on the ACW computer screen. The ship model is plotted as a continual update from its own state and not updated by the optimal estimate as it is in the filter algorithm. This is done to highlight any inaccuracies within the model, which will show as a cumulative error. All measurement data was stored on disc so that the runs could be repeated in the laboratory.

3.5 Control Instrumentation

As the test vessel was not already fitted with an autopilot, it was decided to install systems to control both engine revolutions and rudder angle for controller trials, although this would not be a requirement in the final advisory system. For operational safety the automatic operation had to be easily overridden by the helmsman. Required values, as calculated by the optimal controller within the ACW, are sent to the cube via the serial link where software, again written in assembly language, performs the required control functions by switching output voltages from digital to analog (DAC) channels between 0 and 10 volts.

In manual operation the rudders are driven from the ships wheel through a hydraulic system consisting of a vane pump, two pipes and a ram, forming a loop. Turning the wheel forces fluid around the system one way and moves the ram; reversing the motion of the wheel, forces fluid in the opposite
direction reversing the motion of the ram. The vane pump is equipped with valves which restrict direction of flow to motion of wheel, so automatic control was achieved by placing an electrical reversing pump in parallel with the vane pump. A signal of 10v, 5mA from one channel of the DAC card is amplified by a power amplifier to switch a small relay which in turn switches a 20A relay to drive the pump and finally the rudders. A signal from a different channel of the DAC card drives a similar system which
reverses the polarity of the power to the pump reversing rudder movement. While the pump is being driven the output from the LVDT is monitored by the cube so that when the required value is achieved the DAC channel in operation is switched to off, thus halting rudder movement.

The engines are controlled by a 24v electric ram which is connected mechanically to the throttle levers. The ram is driven from the cube through a power amplifier and relay, and different channels are used for different directions of movement. Due to the large time constant of the engines, it is impractical to move the throttles and monitor the tachometers until the desired value is reached, which would have created a long delay in the operation of the main program. So instead the throttles are moved by an amount which varies as a step function of error, in the appropriate direction on each cycle of the main process until the desired value is achieved.

Figure 3.6 shows that a control cycle is initiated on reception of an ASCII character I, the cube will then await further data from the ACW before issuing control signals. In the case of throttle control, a signal is transmitted for a short, but significant, time measured by the cube. For rudder control the cube compares the voltage from the LVDT with required voltage and sends signals until the result is within a threshold about zero. The rudders take time to move, and a large alteration of rudder angle will need a longer control cycle within the cube. During the whole of this process the cube is committed and unavailable for a measurement cycle. A future modification will remove the voltage comparison from within the control cycle. A hardware comparator (integrated circuit) will then perform this function previously undertaken by the cube and output the signal to drive the rudders.
4.1 Introduction

All moving rigid bodies have six degrees of freedom. For the purposes of this investigation the ship can be adequately described as a rigid body with three degrees of freedom, namely surge, sway and yaw. Ship motions in heave, pitch and roll are considered small enough to be neglected. The motion is described in terms of a moving system of axes coincident with the mass centre of the hull as shown in figure 4.1.

\[ X \text{ and } Y \text{ are the forces of surge and sway and } N \text{ is the yaw moment, } x, y \text{ and } \psi \text{ are the corresponding displacements of the vessel and } u, v \text{ and } r \text{ are their derivatives. The rudder angle is denoted by } \delta, \text{ the propeller revolutions by } n \text{ and the vessel moves on a grid } (x_0, y_0) \text{ defined on the earth's surface, where } x_0 \text{ is the direction of true North, giving a reference from which heading } (\psi) \text{ is measured.} \]
This gives rise to a Eulerian set of equations of motion which may be written in the form:

\[ m\ddot{u} - mrv = X \]  \hspace{1cm} (4.1)

\[ m\ddot{v} + mur = Y \]  \hspace{1cm} (4.2)

\[ I_1 \ddot{r} = N \]  \hspace{1cm} (4.3)

The usual method employed to obtain the \( X \) and \( Y \) forces and yaw moment, as originally given by Abkowitz (1964), is to consider the forces and moments acting on the vessel as functions of:

(i) Properties of the ship e.g. length and hull geometry.
(ii) Properties of motion e.g. velocity.
(iii) Properties of the fluid e.g. density of sea water.

For a given ship the general function can then be expressed as:

\[ f(x, y, \psi, u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta, \dot{\delta}, n) \]

This function can be reduced to useful mathematical form by the use of Taylor series expansion for a function of several variables. It has been shown by Dove (1984), that taking linear terms from the expansion is insufficient to define the ship accurately. Non-linear terms that give rise to more than 10\% of the global force or moment as evaluated by Burns (1984) were considered of major importance, and included in the equation set. So the surge equation becomes:

\[ m\ddot{u} - mrv = X_{u} \ddot{u} + X_{w}(u + u_{w}) + \bar{X}_{w}u^{2} + \bar{X}_{w}u^{3} + \bar{X}_{w}v^{2} + \bar{X}_{w}r^{2} + \bar{X}_{SS} \delta^{2} + \bar{X}_{sw}u_{w} + \bar{X}_{u}u_{w} + \bar{X}_{u}u_{w} \]  \hspace{1cm} (4.4)

and the sway equation becomes:

\[ m\ddot{v} + mur = Y_{v} \ddot{v} + Y_{w}(v + v_{w}) + Y_{r} \ddot{r} + Y_{r}r + Y_{S} \delta + Y_{SS} \delta^{2} + Y_{SS} \delta^{3} + Y_{SS} \delta^{3} + Y_{SS} \delta^{3} + Y_{w}n_{w} + Y_{v}v^{3} + \bar{Y}_{w}w_{w} + \bar{Y}_{w}w_{w} + \bar{Y}_{w}w_{w} + \bar{Y}_{w}w_{w} \]  \hspace{1cm} (4.5)
whilst the yaw equation is:

\[ I \ddot{r} = N_\varepsilon \dot{v} + N_v (v + v_s) + N_r \dot{r} + N_\delta \dot{\delta} + N_{\delta \delta} \delta^2 + N_{\delta \delta \delta} \delta^3 + N_m n_A + N_{v\varepsilon} v^3 + \]
\[ N_{v\varepsilon} \dot{v}^2 + N_{\delta \varepsilon} \delta \varepsilon v^2 + N_{v\varepsilon} v_s \]

(4.6)

In the above a shorthand notation has been adopted, for instance:

\[ X_u = \frac{\partial X}{\partial u} , \quad X_{uu} = \frac{1}{2} \frac{\partial^2 X}{\partial u^2} , \quad X_{uuu} = \frac{1}{6} \frac{\partial^3 X}{\partial u^3} \]

and the dimensionalised hydrodynamic coefficients are obtained from the non-dimensional values in the usual manner, for example

\[ X_u = (\frac{1}{2} \rho L^2 u) X_u \]

where \( \rho \) is water density and \( L \) and \( u \) are the vessels length and velocity.

The steering gear and main engines can both be modelled by first order linear differential equations.

\[ \delta_s = \frac{\delta_0}{T_s} - \frac{\delta_A}{T_s} \]

(4.7)

\[ n_A = \frac{n_0}{T_m} - \frac{n_A}{T_m} \]

(4.8)

One form of equations which can be used to represent the vessel are presented in equations 4.4 to 4.6, and although other techniques exist, Chudley et al (1989), this form will be used to represent the hull throughout the remainder of the project. Other aspects of the vessel are poorly modelled however, and consideration needs to be given to the expert system for operations. This part of the software decides which components of the vessel's dynamics are contributing most to its behaviour under the prevailing operating conditions and make modifications as necessary under machinery failures or emergencies. To suit these requirements a modular format developed by Ogawa and Kasai (1978) is used.
4.2 The Modular Model

The model used in this investigation is based upon the modular approach presented by Tapp (1989) for use in marine simulation. Forces and moments are decomposed into contributions associated with the system elements, for example hull, propeller, rudder and disturbance terms, and expressed as follows with the corresponding subscripts:

\[
X = X_h + X_p + X_r + X_d
\]

\[
Y = Y_h + Y_p + Y_r + Y_d
\]

\[
N = N_h + N_p + N_r + N_d
\] (4.9)

This has reduced equations 4.4 to 4.6 into their constituent components. Further modules can also be included thus enabling the expert system to decide which modules should be used under the given conditions and values for coefficients of the modules. At the same time elements such as propeller thrust and rudder drag are modelled more fully while maintaining the existing state space format.

4.3 The State Equation

For the purposes of this research the vessel is modelled using equations 4.9 with the constituent components being obtained from equations 4.4 to 4.8. Rearrangement of equations 4.9 into matrix form such that \( \dot{u}, \dot{v} \) and \( \dot{r} \) are expressed in their canonical form yields 4.10.

Expressions for the constants, which were obtained from the vessel dimensions, state, and hydrodynamic coefficients are given in Appendix D.
The state variables chosen to represent the vessel are rudder angle, engine revolutions, forward displacement, forward velocity, lateral displacement, lateral velocity, heading and yaw rate, so the state vector is defined as:

\[
x^T = (\delta_A, n_A, x, u, y, v, \psi, r)
\]

This state is affected by the forcing vector:

\[
u^T = (\delta_0, n_0, u_e, v_e, u_s, v_s)
\]

From these eight states the set of first order differential equations 4.10 used to define the ship can be written:

\[
\dot{x}(t) = F(t)x(t) + G(t)u(t)
\]

(4.11)

It is convenient to partition the \(G\) matrix into the control forcing functions \(G_c\) and the disturbance forcing functions \(G_d\):

\[
\dot{x}(t) = F(t)x(t) + G_c(t)u(t) + G_d(t)w(t)
\]

(4.12)
4.4 Solution of the State Equation

Integration of equation 4.12 yields the corresponding discrete solution:

\[ x(k+1) = A(k,k+1)x(k) + B(k,k+1)u(k) + C(k,k+1)w(k) \quad (4.13) \]

where the matrices \( A, B \) and \( C \) can be obtained from

\[ A(k,k+1) = e^{F_{(t)T}} \quad (4.14) \]
\[ B(k,k+1) = (e^{F_{(t)T}} - I)F(t)G_c(t) \quad (4.15) \]
\[ C(k,k+1) = (e^{F_{(t)T}} - I)F(t)G_o(t) \quad (4.16) \]

There are several methods for evaluating \( A, B \) and \( C \) in the above formulae, one involves summing the Maclaurin series for \( e^{F_{(t)T}} \) which is given by

\[ \sum_{k=0}^{\infty} \frac{F_{(t)T}^k}{k!} \]

and is convergent for all \( F \).

While, from the programming aspect, this is the simplest approach it is very slow. Tests conducted on the target computer showed that for this particular application convergence of the series requires a minimum of 50 terms, although to obtain consistent results 70 must be used. While the time constraints imposed by the number of terms needed may be adequate for the purposes of simulation, it would slow the computations of the modelling process down below the requirements for on-line use in real time. There are alternative methods and these are considered below:

(i) The eigenvalue method.

Performing a similarity transformation on \( e^{FT} \) to the form closest to a diagonal, the Jordan form yields
\[ e^{FT} = Te^{JT}T^{-1} \]

where \( e^{JT} \) can be found from the eigenvalues of \( F \) and \( T \) and the corresponding eigenvectors. Computation of the eigenvectors does however, slow down the calculation process.

(ii) The Cayley-Hamilton method.

The Cayley-Hamilton theorem allows any function of a matrix to be expressed as a polynomial of the matrix in the following way.

\[
e^{FT} = \gamma_1 F^{n-1} + \gamma_2 F^{n-2} + \ldots + \gamma_n F + \gamma_n I
\]

The scalars \( \gamma_i \) can be found by solving

\[
e^{JT} = \gamma_1 J^{n-1} + \gamma_2 J^{n-2} + \ldots + \gamma_n J + \gamma_n I
\]

This method is similar to the eigenvalue method in that Jordan forms are required. These can be found through the eigenvalues; however solution of the Jordan polynomial and evaluation of the original polynomial tends to take a little longer than the computation of the eigenvectors. As a consequence this method is seldom used.

(iii) The Resolvent Matrix method.

This constitutes a mapping of the original function on to the s plane. \( e^{FT} \) can then be found by taking inverse Laplace transforms of the resolvent matrix.

\[
e^{Ft} = L^{-1}(R(s)) \quad (4.17)
\]

where the resolvent matrix \( R(s) = (sl - F)^{-1} \) can be evaluated directly using Leverrier's algorithm.
Defining \( n \times n \) real matrices \( A_1, A_2, \ldots, A_n \) and scalars \( \theta_1, \theta_2, \ldots, \theta_n \):\[
A_1 = \mathbf{I} \quad \quad \quad \theta_1 = -\text{trace } FA_1 \\
A_2 = FA_1 + \theta_1 \mathbf{I} \quad \theta_2 = -\text{trace } FA_2 / 2 \\
\ldots \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \quad \ldots \ldots \\
A_n = FA_{n-1} + \theta_{n-1} \mathbf{I} \quad \theta_n = -\text{trace } FA_n / n \quad (4.18)
\]

then
\[
(sI - F)^{-1} = \frac{s^{n-1}A_1 + s^{n-2}A_2 + \ldots + sA_{n-1} + A_n}{s^n + \theta_1 s^{n-1} + \ldots + \theta_{n-1} s + \theta_n} \quad (4.19)
\]

A flow diagram of the computer program, written in ANSII C, to solve the state equation 4.13 using the resolvent matrix method is given in figure 4.2. The program commences by computing matrices \( A \) and scalars \( \theta \) and continues by computing the poles of the system. Using root-squaring for initial approximations, these are then improved using the Newton-Raphson formulae. Both of these numerical methods are treated fully by Pennington (1970). In practice poles of up to third order are obtained for the ship model. Marsden (1970) shows that inverse Laplace transforms, can be evaluated for each pole, or eigenvalue \( \lambda \), of order \( m \). By calculating residues \( R \), where:
\[
R_i = \lim_{s \to \lambda_i} \left( \frac{d^{m-1}}{ds^{m-1}} [(s-\lambda_i)^m R(s)] \right) \quad (4.20)
\]

In the case of multiple poles of order \( n \), a value for \( R \) should be obtained for each order i.e. equation 4.20 must be used for \( m=1 \) to \( n \). Then the complimentary function \( e^{FT} \) is finally obtained by the summation of the inverse transforms:
\[
e^{FT} = e^{\lambda_1 T} R_1 + e^{\lambda_2 T} R_2 + \ldots + e^{\lambda_n T} R_n \quad (4.21)
\]

for distinct roots. In the case of repeated roots additional terms in the summation are given by:
The particular integral, which gives the discrete control matrix, is then given by:

\[
\frac{1}{(m-1)!} \sum_{j} e^{\lambda_j T} R_j
\]  

(4.22)

Multiplications on previous results, and on residues calculated from this new pole lead to the complete solution.

\[
B = L^{-1} \left( A^T G \right)
\]  

(4.23)

Figure 4.2. Procedure for Solving the State Equation
In practice it is found that this algorithm improves the speed of the solution to the state equation by a factor of 3.5 when compared with the Maclaurin series.

4.5 Hydrodynamic Coefficients

In order to model the behavior of the hull it was first necessary to evaluate the derivatives used in equations 4.4 to 4.6. These hull constants are termed hydrodynamic coefficients and are usually obtained by conducting controlled tests on a scale model in a towing tank. Full-scale trials can be performed, but satisfactory control of the trials is difficult due to unknown forces such as wind, tide and current acting on the vessel and these results are more frequently used to validate the coefficients obtained from model tests. There are also some theoretical methods available for coefficient evaluation. Korvin-Kroukovsky (1957) developed a method whereby the vessel's length is divided into strips; each strip is then treated as a buoyant cylinder of equal area, a form which has been well researched and coefficients established. Integration along the length of the vessel then gives some of the coefficients, originally those in heave, pitch and roll. The technique has been extended by Clarke (1972) to include sway and yaw. This method is widely used by those concerned with ship handling, an application in which operators are not concerned with cycle time of the program, whereas one of the constraints of filtering is that the model must run in real time with a rapid update rate. This is particularly critical as the filtering theory used to date assumes that the system is linear between sampling intervals.

Evaluation of hull coefficients using towing tank experiments on a scale model is a costly process. Pourzanjani (1987) has adapted this technique to wind tunnel experiments where the problem of instrumentation is easier. Abkowitz (1980) suggests a series of manoeuvres on board the vessel for system identification. However all these techniques require expense and
delay before the model can be commissioned. Clarke (1982) has evaluated sway and yaw coefficients for a number of vessels using towing tank and rotating arm test data and then, by regression analysis, produced formulae for evaluation of the linear coefficients directly from vessel dimensions of length, beam and displacement. This analysis was performed on much larger vessels than the one being used for trials on this project and the poor results obtained using this technique suggest that extrapolation beyond the range of vessels tested by Clarke is not feasible. Surge coefficients, can be evaluated by fitting a curve to hull resistance against forward speed from data taken while the vessel is maintaining a straight course. This can be achieved using the vessels instrumentation while it is about its normal duties and is therefore cost efficient.

Although Clarke's numerical method for evaluating coefficients is not practical in this instance, it may be useful for evaluating initial estimates for hull coefficients in future trials on large mono-hulled vessels and the equations are given below (4.24). The prime denotes non-dimensionalisation using the Society of Naval Architects and Marine Engineers (SNAME) prime system, in which units for mass, length and time, respectively, are given by the mass of the ship, its length, and the time taken by the ship to cover the distance of its own length at its instantaneous speed. Tables 4.1 to 4.3 give dimensionalising factors for coefficients used to date.

\[
\begin{align*}
Y_s &= -\pi \left( \frac{T}{L} \right)^2 \left[ 1 + 0.16 C_s \frac{B}{T} - 5.1 \frac{B^2}{L^2} \right] \\
Y_r &= -\pi \left( \frac{T}{L} \right)^2 \left[ 0.67 \frac{B}{L} - 0.0033 \frac{B^2}{L^2} \right] \\
N_y &= -\pi \left( \frac{T}{L} \right)^3 \left[ 1.1 \frac{B}{L} - 0.041 \frac{B}{T} \right] \\
N_r &= -\pi \left( \frac{T}{L} \right)^3 \left[ 1 + 0.017 C_s \frac{B}{T} - 0.33 \frac{B}{L} \right] \\
Y_s &= -\pi \left( \frac{T}{L} \right)^2 \left[ 1 + 0.4 C_s \frac{B}{T} \right] \\
Y_r &= -\pi \left( \frac{T}{L} \right)^2 \left[ -0.5 + 2.2 \frac{B}{L} - 0.08 \frac{B}{T} \right] \\
N_y &= -\pi \left( \frac{T}{L} \right)^3 \left[ 0.5 + 2.4 \frac{B}{L} \right] \\
N_r &= -\pi \left( \frac{T}{L} \right)^3 \left[ -0.25 + 0.039 \frac{B}{T} - 0.56 \frac{B}{L} \right]
\end{align*}
\] (4.24)
where

\[ L = \text{length} \]
\[ B = \text{beam} \]
\[ T = \text{draught} \]
\[ C_b = \text{block coefficient} \]

### 4.6 Coefficients for the Test Vessel

In order to estimate a set of values to use in the mathematical model representing Catfish, a set of turning circles were performed while logging heading, rudder angle, forward speed and trisponder ranges. A run consisted of two 360° turns so that corrections could be made for disturbances as suggested by Douglas (1985). The results of a typical run are shown in Figure 4.3. Coefficients were estimated from those available for vessels of

![Figure 4.3. Turning Circle Trials On Catfish](image-url)
CATFISH MODEL EVALUATION

RUN 7 Rudder 5deg Start Speed 9.0kts

Figure 4.4a.

CATFISH MODEL EVALUATION

RUN 3 Rudder 10deg Start Speed 8.8kts

Figure 4.4b

Figure 4.4 Turning Circle Comparisons (Model and Vessel)
similar hull dimensions as applicable. Turning circles in opposite directions were of different diameters for equal, and opposite, rudder angles, due to rotation of both propellers in the same direction. This was allowed for in the mathematical model by adding a further coefficient in:

$$\bar{N} = \delta^2 \theta^2 d^2$$

The turning circles for the model were compared with those of the trials and the coefficients fine-tuned until the vessel and model turning circles matched. The final results are considered acceptable for rudder angles up to 15°, beyond this limit the model starts to deviate from the trials. However, with the optimal controller minimising rudder movement large rudder angles are not expected and this restriction is considered acceptable. Some results of vessel and model turning circles are shown in figures 4.4a and 4.4b.

The final values obtained for hydrodynamic coefficients are as follows. The term hydrodynamic coefficients is used loosely as the method of evaluation leads to a set of numbers which may represent the vessel adequately but are not necessarily the coefficients.

**Surge Coefficients:**

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Value</th>
<th>Dimensionalising Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_u$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$X_v$</td>
<td>-0.000624</td>
<td>0.5$\rho L^2$</td>
</tr>
<tr>
<td>$X_{uu}$</td>
<td>-269.84</td>
<td>0.5$\rho L U$</td>
</tr>
<tr>
<td>$X_{un}$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$X_{uv}$</td>
<td>0.0</td>
<td>0.5$\rho L U$</td>
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<tr>
<td>$X_{vv}$</td>
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<td>0.5$\rho L$</td>
</tr>
<tr>
<td>$X_{rr}$</td>
<td>0.0</td>
<td>0.5$\rho L^2$</td>
</tr>
<tr>
<td>$X_{ss}$</td>
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<td>0.5$\rho L^2$</td>
</tr>
<tr>
<td>$X_{nn}$</td>
<td>0.2772</td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.1*
Sway Coefficients:

<table>
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<tr>
<th>Derivative</th>
<th>Value</th>
<th>Dimensionalising Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y_s</td>
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<td>0.5pL^2</td>
</tr>
<tr>
<td>Y_{xx}</td>
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</tr>
<tr>
<td>Y_v</td>
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<td>0.5pL</td>
</tr>
<tr>
<td>Y_r</td>
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<td>0.5pL</td>
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<tr>
<td>Y_f</td>
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<td>0.5pL</td>
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<tr>
<td>Y_{xx}</td>
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</tr>
<tr>
<td>Y_{yy}</td>
<td>0.0</td>
<td>0.5pLU</td>
</tr>
<tr>
<td>Y_{rr}</td>
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<td>0.5pL/U</td>
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<tr>
<td>Y_{vv}</td>
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<td>0.5pL/U</td>
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<tr>
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<td>0.5pL/U</td>
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<tr>
<td>Y_{vv}</td>
<td>0.1818182</td>
<td>0.5pL/U</td>
</tr>
<tr>
<td>Y_{sv}</td>
<td>0.0</td>
<td>0.5pL</td>
</tr>
</tbody>
</table>

Table 4.2

Yaw Coefficients:

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<tr>
<th>Derivative</th>
<th>Value</th>
<th>Dimensionalising Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_s</td>
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<td>0.5pL^2</td>
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<td>N_{xx}</td>
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<tr>
<td>N_v</td>
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<td>N_{yy}</td>
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<td>N_{yy}</td>
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<td>0.5pL/U</td>
</tr>
<tr>
<td>N_{rr}</td>
<td>-0.05</td>
<td>0.5pL/U</td>
</tr>
<tr>
<td>N_{ss}</td>
<td>0.0116172</td>
<td>0.5pL/U</td>
</tr>
<tr>
<td>N_{sv}</td>
<td>-0.090909</td>
<td>0.5pL/U</td>
</tr>
<tr>
<td>N_{sv}</td>
<td>0.0</td>
<td>0.5pL</td>
</tr>
</tbody>
</table>

Table 4.3
CHAPTER 5

FILTERING

5.1 Introduction

The problem of accurately estimating the six parameters of planetary motion from telescopic measurements was investigated by Gauss in 1795. Due to inaccuracies in the measurements a direct computation gives an error in the result. Gauss used a redundancy in the number of observations for determination of unknown quantities and developed the method of least squares. He used an approximate knowledge of a planet's orbit in order to satisfy all observations in the most accurate manner possible, implying the use of dynamic modelling and linearization techniques. In order to obtain the most probable estimate Gauss minimised a function of the differences between estimated and observed values, known as the residuals. A maximum likelihood criteria was introduced by Fisher in 1912 and a probabilistic version of least squares was also chosen as the performance index by Wiener in 1942 when he developed the minimum mean squares estimate. This also differs from Gauss' approach in that the signal was no longer assumed constant, instead the signals obtained at each sampling interval were linked statistically. The recursive approach presented by Follin in 1955 laid the final foundation for Kalman (1960) to develop his filter and undertake further work with Bucy (1961). The Kalman filter can be described as an efficient computation of the least squares method. This is demonstrated by Cross (1983) who derives the filter equations directly from least squares. A less formal derivation is given in Appendix E.

5.2 The Linear Kalman Filter

Considering the discrete case, the random process can be modelled by:

\[ \hat{x}(k+1) = A(k+1,k)\hat{x}(k) + w(k) \]  

(5.1)
where \( \hat{x} \) denotes an estimate of the state vector and observations occur at discrete time intervals with the linear relationship:

\[
z(k) = H(k)\hat{x}(k) + v(k)
\]

where \( w \) and \( v \) are error vectors. These are uncorrelated white-noise sequences, that is random processes with a constant spectral density function, with known covariances, and \( w \) is uncorrelated with \( v \). \( H \) is the relationship between measurements and the state vector.

The covariance matrices are given by:

\[
E[w_i w_j] = \begin{cases} N(k+1) & j = i \\ 0 & j \neq i \end{cases}
\]

\[
E[v_i v_j] = \begin{cases} M(k+1) & j = i \\ 0 & j \neq i \end{cases}
\]

\[
E[w_i v_j] = 0 \quad \forall \ i, j
\]

where \( E \) is the expectation operator.

Given an initial state estimate \( \hat{x}(0/0) \) and an error covariance matrix \( P(1/0) \), the estimate at future sampling times can be obtained by a linear blending of the noisy measurements with the prior estimate:

\[
\hat{x}(k/k) = A(k,k-1)\hat{x}(k-1/k-1) + K(k)\left[z(k) - H(k)A(k,k-1)\hat{x}(k-1/k-1)\right]
\]

where \( A(k,k-1) \) denotes the transition matrix between time \( k-1 \) and time \( k \), and \( K \) is the blending factor which requires optimising in some way. In particular, the matrix \( K \) that minimizes the mean square estimation error is called the Kalman gain and is given by:

\[
K(k) = P(k/k-1)H^T(k)\left[H(k)P(k/k-1)H^T(k) + M(k)\right]^{-1}
\]
The matrix $P(k/k-1)$ is the covariance of the error in the estimate predicted at time $k$. It uses all the information obtained from time 0 to time $k-1$, and is given by:

$$P(k/k-1) = A(k/k-1)P(k-1/k-1)A^T(k/k-1) + N(k-1) \quad (5.6)$$

The covariance matrix associated with the optimal estimate can now be computed from:

$$P(k/k) = \left[ I - K(k)H(k) \right]P(k/k-1) \quad (5.7)$$

Equations (5.4), (5.5), (5.6) and (5.7) constitute the Kalman filter recursive loop and can be used as shown in figure 5.1.

5.3 The Non-Linear Kalman Filter

Section 5.2 gave a set of filtering equations based on the assumption that both the system and measurement processes are linear. Many applications exist where these processes are non-linear, for example the ship model described in section 4. There are two techniques generally used for dealing with non-linear systems:
(i) To linearize about some nominal trajectory in state space that does not depend on the measurement data. The result is referred to as a linearized Kalman filter.

(ii) To linearize about a trajectory that is continually updated with the state estimates resulting from the measurements. This filter is called an Extended Kalman Filter (EKF).

In circumstances where the mission time is short and measurements are sparse, or when the trajectory can be predetermined with reasonable accuracy, the linearized Kalman filter is usually preferred. When mission time is long it is preferable to use the EKF.

Both of these linearization techniques can lead to what is known as divergence of the filter, this occurs when the error covariance matrix $P(k/k)$, computed by the filter, becomes unjustifiably small compared with the actual error in the estimate. This causes the gain matrix to become small and puts too little weight on the measurements. It is shown by Kalman (1962) that the gain matrix remains stable under a wide range of conditions for linear systems. Thus divergence is due to a direct result of errors introduced by the linear approximation. Divergence is more likely to occur in the extended Kalman filter than in the linearised filter. The extended filter relies on the updated trajectory which is only better than the nominal one in a statistical sense. There is a chance that the updated trajectory may be poorer. Consider the situation where a large error occurs in the measurement process. There is then a small probability of such an occurrence on the Gaussian distribution, for example the probability of the measurement error exceeding four times the standard deviation is 0.006%, this number is finite and therefore some probability exists. The optimal estimate will incorporate this error, known as a gross error, and apply some weighting to it in the combination with the prediction. Thus the optimal estimate will itself be in error by some amount which is greater than the
expected value. This estimate is then used to obtain the next prediction and is incorporated into the recursive loop. In the linearised filter the prediction would have been computed prior to the event and is not affected by the measurement process. Therefore the extended Kalman filter is better on the average than the linearized filter, but it is more likely to diverge, particularly in situations where the initial uncertainty and measurement errors are large.

The extended Kalman filter is defined for a time-invariant continuous dynamic system with discrete measurements as follows:

The state estimate and its error covariance are propagated by the following non-linear differential equations:

\[
\begin{align*}
\dot{\hat{x}} &= f(\hat{x}) + w \\
\dot{P} &= F(\hat{x})P + PP^T(\hat{x}) + Q
\end{align*}
\]

where

\[
F(\hat{x}) = \frac{\partial f}{\partial x} \bigg|_{x=\hat{x}}
\]

Given initial values \( \hat{x}(0) \) and \( P(0) \), these equations can be integrated between measurements to obtain predictions \( \hat{x}(k+1,k) \) and \( P(k+1/k) \).

The discrete measurement model is given by:

\[
z(k) = h(x(k)) + v
\]

After the measurements, the estimates are corrected using the state estimate update equation:

\[
\hat{x}(k+1/k+1) = \hat{x}(k+1,k) + K(k+1)\left[ z(k+1) - h(\hat{x}(k+1,k)) \right]
\]

where the Kalman gain is given by:
\[ K(k+1) = P(k+1/k)H'(k+1) \left[ H(k+1)P(k+1/k)H'(k+1) + M(k+1) \right]^{-1} \]  
(5.12)

where \[ H(k+1) = \frac{\partial h}{\partial \hat{x}} \bigg|_{\hat{x}=\hat{x}} \]

The error estimate for the updated estimate can then be computed from:

\[ P(k+1/k+1) = \left[ I - K(k+1)H(k+1) \right]P(k+1/k) \]  
(5.13)

Equations (5.12), (5.11), (5.13) and the integral of (5.9) replace (5.5), (5.4), (5.7), and (5.6) respectively in the filter loop shown in figure 5.1.

The linearized Kalman filter follows a similar format with the nominal trajectory \( \bar{x} \) substituting for \( \hat{x} \). Also, the state estimate propagation equation (5.8) is reformulated using the Taylor series:

\[ \hat{x} = f(\bar{x}) + F(\bar{x})(\hat{x} - \bar{x}) \]  
(5.14)

giving a state estimate update:

\[ \hat{x}(k+1/k+1) = \hat{x}(k+1,k) + K(k+1) \left[ z(k+1) - h(\bar{x}(k+1,k)) - H(\bar{x})(\hat{x} - \bar{x}) \right] \]  
(5.15)

Higher order linearized filters can be obtained by taking more terms in the Taylor series.

5.4 Filters in Marine Position Fixing

The use of Kalman filters in GPS navigation receivers was discussed in section 1.7. In this instance a prediction obtained from a kinematic model is filtered with measurements of range and range rate to give displacement and velocity of a vessel. Integration with a heading sensor, such as a compass would enable computation of a similar state vector to that used to
represent the vessel in chapter 4, only rate of turn remains unmeasured.
Similar filters are used in inertial navigation systems where precise
positioning is required. In such situations measurements are often made
using accurate, but expensive, devices such as accelerometers and
gyroscopes. Signals are integrated electrically to give velocity. To obtain a
state estimate in three degrees of freedom, surge, sway and yaw, as
described by figure 4.1, two accelerometers and one gyroscope are required.
The system can also be described by a state equation which is obtained
from the simple kinematic relationship:

\[ x = \dot{x}t + \frac{1}{2}\ddot{x}t^2 \]

Writing this in matrix form to encompass sway and yaw yields (5.16), where
\( \Delta t \) indicates the update interval. The format presented here describes the
dynamics of the vessel only, dynamics of the measurement system, such as
gyro drift are not included. In practice the accelerations are not usually
known, they are included here for completeness as they do contribute to
the random noise process. Equation 5.16 can be used as the model to
generate the prediction to combine with measurements in the Kalman filter.
This model is a simple process compared with that described in section 4,
however it is only of value when accurate velocity measurements are
available from either an inertial system or dual axis log, a rare occurrence in
marine navigation due to expense.

\[
\begin{bmatrix}
    x \\
    \dot{x} \\
    \ddot{x} \\
    y \\
    \dot{y} \\
    \ddot{y} \\
    \psi \\
    \dot{\psi} \\
\end{bmatrix} = \begin{bmatrix}
    1 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & \Delta t & \frac{\Delta t^2}{2} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & \Delta t & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 & \Delta t & \frac{\Delta t^2}{2} & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t \\
\end{bmatrix} \begin{bmatrix}
    x \\
    \dot{x} \\
    \ddot{x} \\
    y \\
    \dot{y} \\
    \ddot{y} \\
    \psi \\
    \dot{\psi} \\
\end{bmatrix}
\]

(5.16)
In dynamic positioning where the required measurement data is often available, this form of model is still rarely used. Grimble (1980) and Balchen (1976) describe Kalman filter processes for DP. In both cases linear dynamic models of the vessel, as described in section 4, are used to represent the hull in the presence of a current, with an extension of the system process to incorporate wind loading. Both of these environmental forces can be taken as low frequency, that is they have slowly varying mean values with a random element superimposed. Under such circumstances Kalman filtering gives a good state estimate for automatic control of thrusters to maintain vessel state. There is, however a third natural phenomena in the form of a very much higher frequency wave motion also acting on the vessel. This force does not have an overall effect on displacing the vessel, it merely oscillates it about some mean position. Should the measurement process coincide with some harmonic of the wave motion, instability of the controller will result. It is therefore essential that the wave motion is included in the process. As with the aerospace industry, the high order of accuracy in the measurement process makes this particular application successful. Kalman gains computed in simulation for yaw being typically 0.1 for the low frequency motion and 0.2 for the high frequency.

Kalman filters have also been considered for use in hydrographic survey. For this application it is usual to monitor vessel position only; as no control loop is employed information on velocities is not normally required. The Kalman filter is employed in individual systems such as Microfix and Syledis. Microfix is a range microwave range measuring system similar to the Trisponder system described in Appendix A, while Syledis is capable of operating in either range or hyperbolic mode. The filter employed in the Microfix system filters all eight ranges independently. As the ranges are not measured simultaneously the measurements are subject to skew in that the vessel will move between the measurements made to compute position. A further function of the filter is to deskew the measurements.
In the cases of complete survey packages which take input from a number of position fixing systems and other devices, such as the echo sounder, the Kalman filter has found little favour. This is primarily attributed to the heavy demand on microprocessor resources required, and instead other methods are used. For example, the 950 survey system recently developed by Racal Marine Systems utilises a window filter. A prediction is made by fitting a straight line through several previous fixes and extrapolating. A rectangular window is placed around the prediction, if the next fix falls within the window it is accepted, if it falls outside it is rejected and the prediction is used in its place. There is no combination process, and as such leaves a certain amount of random noise in the track depending on the size of the window. This can either be set manually, or automatically. In the latter case the set window size is multiplied by a factor, or gain, which is computed from the recent history of differences between measured and predicted fix values. The effect is to produce a track plot which is representative of the random noise of the position fixing system in use, only gross errors being removed. For the purposes of marine navigation, where random errors are large in comparison to those of survey systems, this windowing process achieves little.

While the Decca 950 system monitors two position fixing systems, only one is selected to produce the ship's track, the other is used for back up in case of failure, and for quality control. In an equivalent system produced by Waverley, a subsidiary of Dowty, no filtering is employed. Instead, it is possible to compute a fix using up to ten position lines obtained from a variety of sources, with such redundancy in observations filtering is considered unnecessary.
6.1 Introduction

Prior to incorporating the ship model into a Kalman filter, it was decided to construct a simple filter for position fixing alone. This exercise was designed as a practical investigation into the effects of different gain values when combining measurements from an electronic positioning system with a simple linear prediction process. It goes on to investigate the Kalman gain and how variations in the processes can affect the optimum gain.

6.2 The Prediction Process

Employing the simple linear regression through the previous few optimal estimates of position and extrapolating gives a prediction process. The equation of the line is given by:

\[ y = ax + b \]  \hspace{1cm} (6.1)

where \( x \) and \( y \) are Eastings and Northings and \( a \) and \( b \) are the gradient and constant of the regression line through the last \( n \) fixes and are given by:

\[
a = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \hspace{1cm} (6.2)
\]

\[
b = \frac{\sum x}{n} - a \frac{\sum y}{n} \hspace{1cm} (6.3)
\]

A prediction for \( x \) is made by locating start and finish points on the line which correspond to the first and last of the \( n \) fixes. The distance along the \( x \) axis between these points is multiplied by \( \frac{1}{\sqrt{n}} \), this value is placed in equation 6.1 to obtain a prediction in \( y \). This process assumes that the
vessel is moving at constant velocity and thus constitutes a second order process. Any acceleration of the vessel will lead to errors.

The decision of variable, Eastings or Northings, on which to perform the regression line is made by computing the variance, the denominator of 6.1, for both variables. The variable with the minimum variance is chosen.

An alternative prediction process would be to fit a non-linear curve through the previous fixes. Extrapolation of such a curve can however give a poor prediction as the curve is only fitted through the defined points.

In order to investigate this algorithm further, a Kalman filter based on the linear process has been programmed into the Acorn Archimedes computer in ANSII C. Trials data was obtained using the Decca Navigator system, which was described fully in chapter 2.3, in Plymouth Sound.

6.3 Changing The Gain

The gain matrix (K) will be diagonal and the non-zero elements can take any value within the limits \( 0 \leq K_{ii} \leq 1 \). The gain matrix is used to produce a filtered output \( \hat{x} \) by combining predicted position \( x \) and measured fixes \( z \) using the formula:

\[
\hat{x} = x + K(z - x)
\]  

(6.4)

A value of 1 on the diagonal results in an output of measurement only for the corresponding element of the state vector, 0 gives prediction only, any value in between results in a combination.

Results showing plots of position for a short run in an Easterly direction conducted in Plymouth Sound are shown in figures 6.1a to 6.1k. The gain value changes in each plot from 1 down to 0 in steps of 0.1. In each case
the previous ten optimal estimates were used to compute the prediction. The initial ten fixes are unfiltered as these values are required to compute the first prediction.

In figure 6.1a the gain setting is 1, and thus shows raw measurements. Random error is clearly seen as is one gross error towards the end of the run. Effects of increasing the weighting applied to the prediction are seen in figures 6.1b to 6.1j; this last plot is heavily filtered and results in a smooth track, with the exception of the gross error which is still clearly visible. Finally figure 6.1k, which has a gain setting of 0, relies on prediction alone, thus any variation in the trend beyond the initial ten values will be ignored and the resulting track would continue in the straight line shown ad infinitum.

Figure 6.1a. Raw Decca Track
Figure 6.1b. Filtered Track

Figure 6.1c. Filtered Track
Figure 6.1d. Filtered Track

FILTER GAIN 0.7
--- decca
--- filter

Figure 6.1e. Filtered Track

FILTER GAIN 0.6
--- decca
--- filter
Figure 6.1f. Filtered Track

Figure 6.1g. Filtered Track
Figure 6.1h. Filtered Track

Figure 6.1i. Filtered Track
Figure 6.1j. Filtered Track

Figure 6.1k. Predicted Track
Figure 6.2a. Raw Decca Track
Figure 6.2b. Filtered Track
Figure 6.2c. Filtered Track
Figure 6.2d. Filtered Track
FILTER GAIN 0.6
decca filter

Figure 6.2e. Filtered Track
FILTER  GAIN  0.5
deca  filter

Figure 6.2f. Filtered Track
Figure 6.2g. Filtered Track
Figure 6.2h. Filtered Track
FILTER GAIN 0.2
decca filter

Figure 6.2i. Filtered Track
Figure 6.2j. Filtered Track
Figure 6.2k. Predicted Track
It can be seen in figures 6.1 that the linear process is most likely to fail in a turn, the heavily filtered output gives a smoother track but overshoots turns, this is because the natural tendency of the prediction is towards a straight line. Figures 6.2a to 6.2k show results from a longer run also undertaken in Plymouth Sound, with the start point in the South West corner. Several turns are made on the route into Plymouth. Figure 6.2k is seen to continue in a straight line as expected while figure 6.2j, with a gain of 0.1, overshoots the turns. Figure 6.2i, with a gain of 0.2, would appear to be closest to the optimal gain, the filtered track is smooth. Turns are negotiated although overshoot occurs, so the process appears optimal provided the vessel continues in a straight line. This algorithm will therefore have an application in the hydrographic industry where it is common practice to run straight lines at constant velocity.

6.4 The Optimal Gain

Computation of the optimal, or Kalman gain, as discussed in chapter 5, requires knowledge of the error covariance matrix for the measurements and the model of the prediction process. The DRMS value for the Decca Navigator in Plymouth Sound was given in chapter 2.3 as 10 metres under normal daylight operating conditions. The prediction process used in 6.2 above is in effect shifting the fix origin to the first of the ten, or an equivalent point on the line, then increasing both Eastings and Northings by \( \frac{1}{\sqrt{10}} \) of the distance moved in each direction during the last ten fixes. The prediction is made from filtered, not raw fixes, thus the filter is recursive. The prediction process can be described by:

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix}_{n} =
\begin{bmatrix}
  1.111 & 0 \\
  0 & 1.111
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}_{n-1}
\]

(6.5)

The optimum gain according to Kalman is the value of \( K \) which gives an output with minimum variance. When two measurement processes are
combined the optimum value for $K$ can be computed easily from the variances of the measurement processes. In this case, where the variance of the prediction is unknown, the optimum gain can be obtained by iterating around equations 5.5 to 5.7 until a constant value for $K$ is obtained to the desired accuracy. In cases such as this, where the transition is given by a diagonal matrix, the gain matrix will also be diagonal and can be computed independently for each variable. Furthermore, as variances and transitions are identical for both $x$ and $y$, both gains will be identical. The diagonal elements of the gain matrix will therefore be given by the iteration of:

$$P = A^2P$$  \hspace{1cm} (6.6)\

$$K = P(P + M)^{-1}$$  \hspace{1cm} (6.7)\

$$P = (1 - K)P$$  \hspace{1cm} (6.8)\

where

$P$ is the variance of the prediction,

$M$ is the variance of the measurement,

$A$ is the state transition matrix for the prediction.

The solution of equations 6.6 to 6.8 with $A=1.111$, which corresponds to the value used to obtain figures 6.1 and 6.2, and $M=100$, the variance of the Decca Navigator in Plymouth Sound, gives $K=0.1736$ after 70 cycles. This is close to the value of 0.2 estimated from the plots. With $K$ set to this optimum value, the corresponding value for $P$ is 17.36 metres$^2$.

The factor which will vary the gain is the number of points used to make the prediction, this is shown in table 6.1. The run used to produce figures 6.2 were rerun using the gain values given in the table. The resulting plots are shown in figures 6.3a to 6.3h. In this instance a more accurate track, as measured by Trisponder, has been plotted for comparison purposes.
Figure 6.3a. Variation of gain with n
FILTER GAIN 0.437
3 REGRESSION POINTS
decca filter trispo

Figure 6.3b. Variation of gain with n
FILTER GAIN 0.360
4 REGRESSION POINTS
decca filter trispo

Figure 6.3c. Variation of gain with n
FILTER GAIN 0.306
5 REGRESSION POINTS
decca filter trispo

Figure 6.3d. Variation of gain with n
Figure 6.3e. Variation of gain with $n$
FILTER GAIN 0.234
7 REGRESSION POINTS
decca filter trispo

Figure 6.3f. Variation of gain with n
FILTER GAIN 0.210
8 REGRESSION POINTS
decca filter trispo

Figure 6.3g. Variation of gain with $n$
FILTER GAIN 0.174
10 REGRESSION POINTS
decca filter trispo

Figure 6.3h. Variation of gain with n
Table 6.1. Variation in Gain with Number of Points

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>0.5556</td>
<td>0.4375</td>
<td>0.3600</td>
<td>0.3056</td>
<td>0.2653</td>
<td>0.2344</td>
<td>0.2099</td>
<td>0.1736</td>
</tr>
</tbody>
</table>

From figures 6.3, an optimal value for \( n \) can be estimated as 4, this value gives a smooth track and little overshoot on turns. In places the difference between the filtered and Trisponder tracks is still large at about 50 metres, however this is seen to be due to consistent deviation between the raw Decca and Trisponder during short periods. This may be due to local fixed errors within the Decca system arising from surrounding land masses. Decca would not normally be used in such confined waters where a buoyage navigation system operates. The use of larger values for \( n \) overcomes the effect of such errors, however without the use of Trisponder, or some other information such as heading perhaps, it is impossible to distinguish between a fixed error and a turn.

6.5 Variance of the Measurement Process

The Decca Navigator used to obtain the results above is primarily used for marine navigation. As suggested previously, the filtering process described is also applicable to hydrographic survey. For a brief investigation into this application the Trisponder, a typical range-range survey system with a standard deviation of 1 metre on each range, will be used. The solution for position is obtained by the least squares algorithm given in Appendix A, further computations enable the DRMS value of the measurement process to be evaluated, this can then be used in the computation of the Kalman gain. It is interesting to note, however that for the filter process described in this chapter the Kalman gain remains constant irrespective of the variance of the measurement process. This is because an improvement in the measurements improves the correlation of the straight line and hence the prediction process. This is confirmed by computations performed on equations 6.6 to 6.8 with \( n=5 \) and varying \( M \).
Figure 6.4a. Variation of gain with n
FILTER GAIN 0.437
3 REGRESSION POINTS
trispo filter

Figure 6.4b. Variation of gain with n
Figure 6.4c. Variation of gain with n
Figure 6.4d. Variation of gain with \( n \)
Results of variance of the prediction process are shown in table 6.2. Using the linear process described in this chapter, the only variable which affects the gain is the number of points used to make the prediction.

<table>
<thead>
<tr>
<th>M</th>
<th>1</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>7.36</td>
<td>1.736</td>
<td>4.338</td>
<td>8.677</td>
<td>17.36</td>
<td>34.71</td>
</tr>
</tbody>
</table>

Table 6.2. Variation in Predicted Error Variance with Measurement Variance

Figures 6.4a to 6.4d show a section of the Trisponder track used in figures 6.3 with and without filtering. In this instance, with more accurate fixing than Decca, it would appear that a prediction from the previous two points gives the best result without overshoot on the turn.

6.6 Removal of Gross Errors

Figure 6.1j shows a heavily filtered Decca track plot in which a gross measurement error is still obvious. In theory there is a small chance of such an error occurring, from the DRMS value for Decca in Plymouth Sound there is a 95% probability of the measurement being more than 20 metres from the true position. In practice such occurrences can be more frequent under certain circumstances. For example, when using a microwave system such as Trisponder for surveying, the transmitted signal can follow two paths. The direct path, which is the desired measurement, and a reflected path, a reflection off the water surface. If, on arrival at the receiver, both signals are out of phase by $180^\circ$ they cancel out and no measurement is made, a point where this may occur is known as a null point or range hole. If another path of the same signal from transmitter to receiver is another reflection off some other object such as a land mass or vessel, then this measurement may be available and will be measured. In the case of a ranging system such an occurrence will generate an overrange measurement. On the Trisponder tracks shown in figures 6.3 a similar set
Figure 6.5a. Filtered Track With Window To Detect Gross Errors

Figure 6.5b. Filtered Track With Window To Detect Gross Errors
of events have occurred, in this case Drakes Island has blocked the direct path to a remote sited across the Sound at Pier Cellars which is shown on the chart in figure 3.1.

Gross errors such as those described above can be removed by placing a window about the prediction and testing whether the measurement falls within the window. If it does then both estimates can be filtered, if not then the prediction must be accepted and the measurement ignored. There are two problems associated with this solution. Firstly, the window size which must be representative of the standard error of the measurement system in use and as such could be computed either theoretically from the DRMS value or practically from some statistic based on the recent history of events. Secondly, suppose the vessel makes a sharp turn and simultaneously a gross error occurs in the measurement. The subsequent measurement may be accurate but fail the window test as will all further measurements. As a consequence the computed track will continue on predictions alone. This problem can be overcome by opening the window on an update which follows a failure. The system should also test for several consecutive failures and restarted if this event occurs. As only gross errors are required to be removed, the window can be made very much larger than the DRMS value. Figure 6.5a is a rerun of the Decca data used to produce figures 6.1 under identical conditions to figure 6.1i, but with a 50 metre diameter window included. The resulting plot demonstrates the filter with a window size set too small and turns are overshot. Figure 6.5b shows a result with the window size at 100 metres. In this instance the majority of the plot is identical to that of figure 6.1i, with the exception of the region around the gross error. This has now been removed and as a result the filtered track immediately following this point is vastly improved. The data set used to produce figure 6.3c with 4 regression points used to make the prediction was rerun with a window size of 100 metres. Two points failed the test, these are indicated on figure 6.6 with crosses, and corresponds to 0.7% of the fixes. Unfortunately both of these fixes occur at critical points.
Figure 6.6. Filtered Decca Track With Window
on the track when the vessel is undergoing a turn and the filtered output is distorted.

As the modelling process used is linear, the filtering process described in this chapter performs best when the vessel is travelling in a straight line. It will manage turns purely because a small number of points have been used to make the prediction. One method of further improvement would be to increase the sampling rate, this is not possible on the system installed on the test vessel due to the restrictions imposed by the Decca Navigator Receiver.
CHAPTER 7

A FILTER FOR MARINE NAVIGATION AND CONTROL

7.1 Introduction

While the vessel's position is still of primary importance for marine navigation, other factors should also be considered. The state of the vessel is likely to be passed to a control algorithm for track keeping. Further requirements thus comprise accurate heading to maintain course, and velocity information for feedback, or damping, of the control loop. It can be seen that the system process for the vessel described in section 4 is tailored to suit these additional requirements, and that the following modifications are required to the filter algorithm presented in chapter 6 in order to suit this application.

(i) The system is not driven by white noise but by a deterministic control vector and noisy disturbances; the state update equation (5.8) therefore requires modification. Control is assumed to be stable and disturbances are taken as Gaussian processes with a non-zero mean. The integral of the system error covariance matrix (5.9) is therefore modified to include the control matrix C and becomes:

$$ P(k+1/k) = A(k+1,k)P(k,k)A^T(k+1,k) + C(k+1,k)N(k+1)C^T(k+1,k) $$  (7.1)

(ii) The measurement transformation matrix H assumes the identity matrix and is therefore omitted from original equations. Thus all measurements presented to the filter were required in state vector format. Forward and lateral displacements are obtained using the Decca navigator, a system which measures position on the earth's surface. This measurement is then transformed on to the moving axis with the aid of the compass measurement. This transformation constitutes a linear
mapping, and under this condition the measurement matrix $H$ reduces to the identity.

(iii) Improvements can be made to the speed of the filter algorithm by considering the manner in which the equations are used. Figure 7.1 shows an iterative loop which commences each cycle by taking a measurement, initiates the covariance to the identity, then computes the Kalman gain and its error covariance. The iterations are used to obtain convergence of the filter. An improvement on this technique can be made by changing the order of these operations. In practice the system error covariance and Kalman gain can be computed prior to performing the measurement process. During this time the computer may well be idling, while awaiting the Decca navigator to initiate the measurement cycle, so a saving may be made in computer time. Furthermore, by computing the initial error covariance less iterations may be required.

Enter measurement and disturbance covariance matrices $M$ and $N$, state transition matrices $A$, $B$ and $C$ and prior estimate $\hat{x}$. Use $A$, $B$ and $C$ to update estimate $\hat{x}(k+1/k)$.

- Delay -
- Take measurement $z(k+1)$ -

Find transition matrices $A$, $B$ and $C$ based on current estimate $\hat{x}(k/k)$.

- Initiate $P(k/k)$ to the identity matrix.
- Find predicted error covariance $P(k+1/k)$, the Kalman gain $K(k+1)$ and the final error covariance $P(k+1/k+1)$.
- Iterate around these equations to obtain convergence of gain.

Increment $k$

Use Kalman gain and measurement to update estimate to $\hat{x}(k+1/k+1)$.

Figure 7.1. The Kalman Filter Loop
It is now established that the filter applied to the marine navigation problem is an extended Kalman filter. That is the non-linear system process is linearized about the most recent optimal estimate, while the measurement process is linear and the errors are Gaussian. As a ship constitutes a non-linear system, when parameters such as large alterations of course and/or speed, shallow water effects, and trim are considered there must be some limitation to the technique. The linearization process assumes constant course and speed during each sample period. This is reasonable provided sample times are small when compared with such factors as ship time constants and time between waypoints. A block diagram of the filter is shown in figure 7.2.

Equations 5.11, 5.12, 5.13 and 5.17 are used recursively to obtain the state estimate at a future sampling time as shown in figure 7.1. Equations 7.1, 5.12 and 5.13 are iterated on each cycle to obtain convergence of the filter gain.
7.2 Results

Trials in Plymouth Sound were conducted on the Kalman Filter algorithm for marine navigation. Several runs were made while plotting position fixes from Decca, ship model, filter and Trisponder on a digitised chart of the area on the computer screen. Although the ship model is updated from the optimal state estimate, its position on the reference grid is updated from its own previous position. This is done to highlight any inaccuracies within the model which will show as a cumulative error. All measurement data were stored on disc so that the runs could be repeated in the laboratory. The results of estimated positions of the vessel for one run during which the vessel was guided along the standard approaches to the port of Plymouth are given in figure 7.3. The model is seen to deviate from the true track, and this is largely due to disturbances such as wind and tide and also errors in the hydrodynamic coefficients obtained for the vessel. The filtered estimate takes some weighted mean between the measured and model tracks as expected. Figures 7.4a and 7.4b show the control states which are used to drive the model. The various outputs are given in figures 7.5a to 7.5f. Forward displacement is cumulative and is plotted on such a small scale that little deviation is seen between the measured, model and hence filter outputs. Filtered forward velocity follows closely the smooth model, however it is affected by the noise in the measurement data. Lateral displacement is also a cumulative sum of the moving vessel axis on the reference grid, however it is plotted at a larger scale than forward displacement and measurement noise is evident. The filtered output follows the model closely suggesting a low gain setting. The measurement of lateral velocity, as obtained from the displacement data, is noisy and has been limited within the software to 2 knots, again the filtered output follows the predicted values and is unaffected by the very noisy measurement data indicating a very low gain setting. Filtered heading is seen to take a value between the measured and predicted values. Rate of turn is noisy for both model and measured data, thus a noisy filter output...
Figure 7.3. Estimated Positions With No Disturbance Input
**Figure 7.4a. Engine Control Input**

**Figure 7.4b Rudder Control Input**
Figure 7.5a. Displacement With No Disturbance Input

Figure 7.5b. Velocity With No Disturbance Input
Figure 7.5c. Displacement With No Disturbance Input

Figure 7.5d. Velocity With No Disturbance Input
Figure 7.5e. Displacement With No Disturbance Input

Figure 7.5f. Velocity With No Disturbance Input
Figure 7.6. Kalman Gains With No Disturbance Input

\[ K = \begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.19 & 1.08 & 10^{-3} & -10^{-3} & 1.33 & 0.01 \\
0.0 & 0.0 & 0.01 & 0.03 & 10^{-4} & -10^{-4} & -10^{-3} & -0.14 \\
0.0 & 0.0 & 10^{-3} & 0.08 & 0.08 & 0.02 & -12.0 & -10^{-4} \\
0.0 & 0.0 & -10^{-3} & -10^{-3} & 10^{-4} & 10^{-4} & -0.06 & 10^{-3} \\
0.0 & 0.0 & 10^{-4} & -10^{-4} & -10^{-4} & 0.44 & 10^{-3} & \\
0.0 & 0.0 & 10^{-5} & 10^{-4} & -10^{-3} & -10^{-7} & 0.01 & 10^{-4}
\end{bmatrix} \quad (7.2) \]

\[ P = \begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.96 & 10^{-3} & 18.85 & 0.62 & 2.30 & 0.70 & -0.64 & -0.22 \\
0.08 & 0.01 & 0.56 & 0.02 & 0.20 & 0.06 & -0.06 & -0.02 \\
-6.33 & -10^{-3} & -0.67 & -0.51 & -4.05 & -4.74 & 4.22 & 1.44 \\
0.70 & -10^{-4} & 0.10 & 0.06 & 1.42 & 0.53 & -0.48 & -0.16 \\
-0.37 & 10^{-4} & -0.06 & -0.03 & -0.80 & -0.28 & 0.25 & 0.08 \\
-0.22 & 10^{-6} & -0.04 & -0.02 & -0.44 & -0.17 & 0.15 & 0.05
\end{bmatrix} \quad (7.3) \]
is obtained. While the entire Kalman gain matrix used included the off-diagonal, or cross-coupling terms, it is of interest to investigate initially the uncoupled, diagonal terms. These are plotted in figure 7.6 which verifies the observations made above. Although the variance of lateral and forward displacements are identical, their respective gains are seen to take different values. This is due to the differences in the variance of their velocities, these parameters are fed back through the transition matrix to the displacements and will therefore influence the predicted error covariance and hence the Kalman gain. A typical gain matrix is given by 7.2 and the corresponding covariance of the prediction by 7.3. The gain matrix displays relatively large values on the diagonal, as expected. However some of the off diagonal terms are not small. In particular the seventh column shows some large terms indicating that the heading is a dominant variable.

7.3 Disturbance Estimation

It has been shown by Wills (1984) that the total disturbance effect can be estimated from the difference between position vectors obtained from smoothed Decca data and DR, where the DR, or dead reckoning position, is obtained by plotting only course and speed measurements from log and compass. Placing a window around a number of previous pairs of vectors and taking the mean value improves the estimate of the disturbance vector. This can then be used in conjunction with the next DR position to find the EP, or estimated position, of the vessel. Suppose that the filtered output suggests the vessel has moved distances $x_1$ and $y_1$ on the grid between fixes, whereas the model gives displacements $x_2$ and $y_2$. Any difference between $x_1$ and $x_2$ and $y_1$ and $y_2$ may be assumed to be due to external forces and errors in the model. The computer program was modified to compute this disturbance vector on each cycle. The previous twenty values being averaged to give combined values for $u$, and $u$, and $v$, and $v$, which were then applied to the model equations on the next cycle. Furthermore the variances are calculated and applied to the Kalman filter equations.
CATFISH FILTER TRIALS

RUN C210CTWD

# #
TRISPO

DECCA

FILTER

MODEL

Figure 77. Estimated Positions With Disturbance Input
Figure 7.8a. Displacement With Disturbance Input

Figure 7.8b. Velocity With Disturbance Input
Figure 7.8c. Displacement With Disturbance Input

Figure 7.8d. Velocity With Disturbance Input
Figure 7.8e. Displacement With Disturbance Input

Figure 7.8f. Velocity With Disturbance Input
Figure 7.9. Kalman Gains With Disturbance Input
Figure 7.10a. Computed Forward Disturbance

Figure 7.10b. Computed Lateral Disturbance
With this process added the data set used to obtain the results given in chapter 7.1 was rerun. Resulting plots of position are shown in figure 7.7, while figures 7.8a to 7.8f show vessel states and figure 7.9 gives diagonal gain elements. Figures 7.10a and 7.10b show the components obtained in the X and Y directions for disturbances on the moving ship's axis. This is seen to be noisy but the mean is not zero. Comparing positional error shown in figure 7.7 with that of the equivalent with no disturbances, figure 7.3, shows an improvement in the mathematical model and filtered output. It is interesting to note that the diagonal elements of the Kalman gains have increased, thus reducing the weighting applied to the improved model. Considering the off-diagonal terms, an increase is seen in general, however, this is most noticeable in sway and yaw. The heading terms are still seen to dominate the gain matrix, and it is therefore the heading which should be considered of primary importance when attempting to make further improvements.
Ideally a less noisy measurement device such as a gyro-compass should be used. Due to expense however this is not feasible and error sources within the existing system must be inspected. Heading errors may be present in both the measurement and model, and in these cases may be due to:

(i) A fixed error in the measurement device. The magnetic compass calibration, a sinusoidal curve fitted to the data as shown in figure 3.4, may be inadequate;

(ii) An error in the mathematical model. One or more of the hydrodynamic coefficients used may not truly represent the vessel.

Consider now figure 7.7 which shows the model deviating to the South on the initial Easterly run, then to the East on the Northerly section of the passage. This indicates that the heading produced by the model is larger than the actual value. The model is however driven by filtered input and it may be that an error in the measurement data is corrupting this input.

From the model equation 4.10 it can be seen that the heading is derived from yaw rate. Figure 7.8f shows that this state is above zero during the periods of straight runs. It would therefore appear that the model is in error, suggesting that one or more of the coefficients used in obtaining the \( N \) terms given in Appendix D is inaccurate. As final proof of a model error the data was rerun with the diagonal filter gain element for the heading set to 1.0. The resulting filtered track shown in figure 7.11 gives a more desirable result, with the filtered track following much more closely the true track as indicated by Trisponder.

The filter tests carried out afloat have verified the simulator work conducted by Dove (1984). They show that noisy measurement data have been improved by reducing random errors. However, the remaining problems
Figure 7.11. Estimated Positions With Disturbances And Fixed Heading Gain
established from these results will have to be overcome if a practical system is to be produced. The two outstanding problems are:

(i) For the extended Kalman filter to function satisfactorily it has been established that an accurate mathematical model is required. As explained in chapter 4 this is a difficult and costly process for marine vehicles. For the purposes of this thesis less accurate methods were used in this procedure and as a consequence the hydrodynamic coefficients found do not truly represent the vessel;

(ii) The time of about 5 seconds taken by the Acorn Archimedes to cycle the model, measurement and filter algorithms is in excess of the desired cycle time of about 1.5 seconds. Computation time will be improved by the incorporation of a floating point coprocessor unit in the computer when available from the manufacturer. This will reduce floating point calculation time by a factor of about 20. However, it is not expected to give an adequate reduction in cycle time.

### 7.4 Rearrangement of equations

Enter measurement and disturbance covariance matrices $M$ and $N$, prior estimate $x(0/0)$ and its error covariance inverted $P^{-1}(1/0)$. Set $k=1$.

- Compute Error Covariance for updated estimate from equation (7.6) and invert to get $P(k+1/k+1)$
- Find transition matrices $A(k/k-1)$ etc. Compute the covariance from (7.1) and invert for $P'(k+1/k)$
- Update estimate from (5.4) or for extended filter use (5.11)
- Increment $k$
- Compute Kalman gain from (7.7)

*Figure 7.11. Alternative Kalman Filter Recursive Loop*
The Kalman filters equations given previously can be manipulated and reformulated in many ways. One form given by Gelb (1988) enables the optimal estimate error covariance to be computed from:

\[ P^{-1}(k+1/k+1) = P^{-1}(k+1/k) + H^T(k+1)M^{-1}H(k+1) \]  

(7.6)

The Kalman gain is then given by:

\[ K(k+1) = P(k+1/k+1)H^T\]  

(7.7)

These equations are used in the recursive loop as shown in figure 7.11.

At first sight this alternative formulation appears more complex than the traditional formulae given previously, particularly when using iterations to obtain convergence of the gain. It can be seen that instead of cycling three equations 5.5, 5.6 and 5.7 involving one matrix inversion, now two will be involved in this loop, 5.6 and 7.1, and two matrix inversions are necessary. Matrix inversion is a particular problem as it is not easily computed, the solution is obtained for a matrix A by solving the equation \( A A^{-1} = I \) simultaneously. This is a time consuming process and can lead to errors if the system is ill-conditioned. This reformulation is however worth further consideration on three counts:

(i) Reformulation can lead to more rapid convergence. For example one method of solving systems of simultaneous equations is to reformulate and iterate, certain formulations will converge quicker than others, some may even diverge;

(ii) The number of additional multiplications involved in cycling three equations must be evaluated against cycling two equations with two matrix inversions;
(iii) Under certain conditions the matrix inversion can be simplified. This is discussed in chapter 10.

The alternative algorithm was programmed into the Acorn Archimedes with the original software. Both routines were run consecutively using identical data, both converged after the same number of iterations. The alternative algorithm took 2.7% more time than the original. Although it takes more time this form should not be dismissed as it may be useful in a reformatting of the whole procedure discussed under further research in chapter 10.
CHAPTER 8

SYSTEM IDENTIFICATION

8.1 Introduction

The assumption has been made throughout this thesis that the coefficients used in the $F$ matrix to model the vessel are known correctly, thus describing the system process accurately. It was established in chapter 7 that the mathematical model of Catfish is inaccurate. Furthermore, the vessel's dynamics may change over long periods of time. For example, as the underwater surface of the hull is fouled with growth the hydrodynamic derivatives used to model the hull and its resistance will require modification. Established techniques for parameter identification, based upon various optimization criteria, would require new algorithms to be included into the Kalman filter recursive loop. These methods are usually time consuming and consequently are unsuitable for real-time applications. Gelb (1988) proposes a method which can be easily implemented into the existing Kalman filter loop. This method was further investigated for the marine application, optimizing parameters on the minimum variance estimation algorithms already in use.

8.2 Augmenting the State Vector

Let the unknown parameters be denoted by a vector $a$, having dynamics defined by the differential equation:

$$\dot{a} = 0$$

(8.1)

with non-linear equations of motion, the system process can now be written:

$$\dot{x} = f(\hat{x}, a) + Gu$$

(8.2)

where both $a$ and $\hat{x}$ are to be estimated from the noisy measurement data. Combining $x$ and $a$ into a composite state vector denoted $x^*$ such that:
and applying this system process to the extended Kalman filter routine yields estimates for both states and unknown parameters.

8.3 The Modelling Process

Selecting the vector \( a \) to contain hydrodynamic coefficients for the vessel gives a large dimension augmented state vector and a largely populated transition matrix. This formulation would then lead to cumbersome computations, defeating one of the prime objectives of this research. Furthermore, due to cross coupling, some coefficients cannot be isolated and are therefore unidentifiable. An alternative method was suggested by Robbins (1982) who applied this method of parameter identification to aircraft, but used simplified mathematical models. To perform the identification process the system process was reduced to smaller components and controlled manoeuvres were performed. Then, assuming certain cross-coupling terms to be negligible under the control applied, for example when applying rudder to the aircraft surge and sway terms only are considered and the induced roll is neglected, a small number of parameters can be identified from each manoeuvre.

It is not economically viable to attempt to identify slowly varying parameters while on a voyage as the vessel may be delayed at great expense during identification manoeuvres. Furthermore, in order to keep the dimensions of the augmented state vector to minimum, maintain identifiability of parameters and to retain sparse population of the transition matrix it is necessary to identify the components of the latter directly. Initially only sway and yaw terms are considered, as these use the least accurate coefficients and were seen to give poorer results than the surge term. The augmented state vector can be written as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{a}
\end{bmatrix} =
\begin{bmatrix}
-f(a,x) + Gu \\
0
\end{bmatrix}
\]
where the augmented parameters are shown in matrix (4.10) and expressions for their initial evaluation are given in appendix D. These constants are dependent on the vessel states, and hence, assuming a slow transition time of the vessel in comparison to cycle time of the Kalman recursive loop, the parameters to be identified may be taken as having dynamics given by equation 8.1.

The Extended Kalman Filter estimated error covariance equation (5.9) requires evaluation of the partially differentiated state transition matrix. This is now given by:

$$ F^* = \frac{\partial f(x,u)}{\partial x^*} |_{x^* = z^*} $$

which is a matrix of dimensions 16 x 16:

$$ F^* = \begin{bmatrix}
\frac{\partial x}{\partial z_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial x}{\partial z_y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} $$

where $0_8$ is the zero matrix of dimensions 8 x 8.

Writing this as a partitioned matrix of 4 sub-matrices, each of 8x8 dimension:

$$ F^* = \begin{bmatrix}
F_1 & E \\
0 & 0 \\
\end{bmatrix} $$

(8.6)
The original 8 states of the system process are still described by equation 4.10. The discrete transition matrix is then given in partitioned form by:

\[ A^* = \begin{bmatrix} A & 0 \\ 0 & I_8 \end{bmatrix} \]  

(8.7)

where \( I_8 \) is the identity matrix of order 8, \( A \) is the discrete solution to \( \mathbf{F} \) and can be obtained using the algorithm given in section 4.4.

8.4 The Measurement Process

The original eight states of the measurement vector can be obtained as before but the augmented states are not measured. They can however be estimated from the previous optimal estimate. Rearrangement the equations 4.10 yields 8.8, thus \( Y_1, Y_2, Y_3, Y_4, N_1, N_2, N_3 \) and \( N_4 \) can be obtained from the previous optimal estimates. Solutions for the equations should be iterated to obtain convergence.

\[
\begin{align*}
Y_1 &= \frac{-Y_4u - Y_4v - Y_4r}{\delta} \\
Y_2 &= \frac{-Y_4u - Y_4v - Y_4r}{u} \\
Y_3 &= \frac{-Y_4u - Y_4v - Y_4r}{v} \\
Y_4 &= \frac{-Y_4u - Y_4v - Y_4r}{r} \\
N_1 &= \frac{-N_4u - N_4v - N_4r}{\delta} \\
N_2 &= \frac{-N_4u - N_4v - N_4r}{u} \\
N_3 &= \frac{-N_4u - N_4v - N_4r}{v} \\
N_4 &= \frac{-N_4u - N_4v - N_4r}{r}
\end{align*}
\]  

(8.8)
The measurement matrix can thus be written:

$$
\mathbf{h} = \begin{bmatrix}
\mathbf{X} & \mathbf{0} \\
\end{bmatrix}
$$

where $\mathbf{X}$ is an $8 \times 8$ diagonal matrix with diagonal elements equal to the corresponding elements of the state vector.

Then for the computation of the Kalman gain:

$$
\mathbf{H} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}^*} |_{\mathbf{x}^* = \mathbf{r}^*}
$$

### 8.5 Filtering

The filter currently in use must be modified to account for the additional terms included in the processes above. In order to estimate the prediction error covariance and hence the Kalman gain, $A^*$ should incorporate the entire matrix $F^*$:

$$
A^* = \begin{bmatrix}
A & D \\
0 & I_8 \\
\end{bmatrix}
$$

The matrix $D$ can then be obtained in a similar way to the discrete control and disturbance matrices defined in chapter 4 and is then given by:
The Kalman gain matrix, which now has dimensions $16 \times 16$, is computed by iterating around equations 5.12 to 5.13 and 7.1 as shown in figure 7.1.

8.6 Results

The data set used in chapter 7 was rerun using the filter algorithm with system identification, results are plotted in figures 8.1a to 8.1e and 8.2a to 8.2j. By comparison with figure 7.7, Figure 8.1a shows that position information from the model, and hence also the filter, is improved by the identification process. Figure 8.1b plots the track around the first turn at a larger scale; in this instance the Trisponder track has been filtered using the algorithm presented in chapter 6. Positional errors were computed from this track. Figure 8.1c plots absolute error, from which it is seen that the filter reduces noise. This error is then reduced into along-track and cross-track components in figures 8.1d and 8.1e. The cross-track error follows the trend set by the DNS but is much less noisy. It remains within 25 metres of the true position line compared with the 50 metres achieved by the DNS. The along-track error has a similar performance until reaching the northerly section of the passage where it is seen to have a mean value greater than that given by the DNS and the maximum value increases to about 40 metres. As a consequence the second turn is overshot causing the cross-track error to increase during the westerly passage. The initial along-track error suggests that the surge coefficients may be inaccurate; these parameters were not included in the identification procedure. The error source is confirmed by figure 8.2b which shows the modelled forward velocity constantly higher than the measured values.

The very low gain values obtained for lateral velocity cause the noisy measured values to be almost ignored and the model values to be used. The values obtained from the model are, however, realistic at less than 1 knot.
CATFISH
FILTER
TRIALS

RUN
C210CTSI

# TRISPO

* DECCA

FILTER

@ MODEL

Figure 8.1a. Position

- 142
Figure 8.1b. Position

Figure 8.1c. Absolute Position Error With Reference To Filtered Trisponder
Figure 8.1d. Along-Track Error With Reference To Filtered Trisponder

Figure 8.1e. Cross-Track Error With Reference To Filtered Trisponder
Figure 8.2a. Displacement

Figure 8.2b. Velocity
Figure 8.2c. Displacement

Figure 8.2d. Velocity
Figure 8.2c. Displacement

Figure 8.2f. Velocity
Figure 8.2g. Sway and Yaw System Coefficients

Figure 8.2h. Sway and Yaw System Coefficients
Figure 8.21. Kalman Gain Diagonal Elements
Figure 8.2j. Kalman Gain Diagonal Elements
In comparison with figure 7.8d, the corresponding diagram with no identification, the filtered output is much smoother and less erratic, giving what would appear a more reasonable estimation. While an improvement can also be seen in angular velocity in figure 8.2f, the filtered output still appears noisy. Although this may be a true representation, as the test vessel does turn rapidly when in waves, this noise is more likely to be an influence of the noisy measurements made of the rudder angle. Ideally, better measurements made by way of a gyro should be used to improve this parameter which is important when used in connection with a controller as discussed in chapter 9.

Figures 8.2g and 8.2h show the identified system coefficients. The rudder terms $Y_r$ and $N_r$, are seen to be noisy. These terms would be expected to reduce to zero when travelling in a straight line and increase during the use of rudders in a turn, which is seen to occur. Further noise is probably due to noisy rudder measurements. $Y_s$ and $N_s$, the surge terms are close to zero. These terms influence the turning characteristics with speed and over the 6 to 7 knot speed range used during the trial have little influence.

Diagonal elements of the filter gain matrix are plotted in figures 8.2i and 8.2j. Gains for the eight original states appear better than those given previously in figure 7.9. The lateral and angular velocity gains have been reduced below that of the forward velocity, as would be expected as the latter contains less noise. For the same reason, the lateral displacement has been reduced below that of the forward displacement. The heading gain is more consistent at 0.8, but the model has behaved satisfactorily so the problem of inaccurate coefficients leading to the model error discussed in chapter 7.3 has been overcome.
CHAPTER 9

CONTROL INTEGRATION

9.1 Introduction

Track keeping as an essential component for the safe navigation of the vessel, it is also included in the philosophy of the integrated navigation system as defined by Larsen (1989). In order to complete the project an automatic track keeping system was designed and installed in Catfish. This has been achieved by the following steps:

(i) prior to departure the mariner will enter a sequence of waypoints indicating the intended track;

(ii) while underway the computer will automatically control rudder angle and engine speed to maintain the vessel on track and arrive at the destination on time.

Optimal control theory utilises a mathematical model in conjunction with accurate knowledge of the vessel's state. Both of these requisites are available from the Kalman filter.

9.2 Optimal Control

The theory of an optimal multi-variable control system has been developed to control simultaneously position and velocity of the vessel, and tested in simulation by Burns (1984). Deviation from the desired values were corrected by operation of the rudders and main engines. An optimal system seeks to minimise a global parameter \( J \) called the cost function or performance index. This is based upon the summation of the weighted errors over some time interval, perhaps over one stage of the voyage. In addition to minimise
the errors in the output parameters, the optimal controller must also attempt to minimise the control effort, that is to minimise rudder and main engine activity. The cost function is normally stated in the following quadratic terms:

\[ J = \int_{t_0}^{t_1} (x-r)^T Q (x-r) + u Ru \, dt \]  \hspace{1cm} (9.1)

where \( r \) is the desired state vector and \( Q \) and \( R \) are usually diagonal matrices, with the values of the individual elements reflecting the importance of the parameters being controlled.

An initial requirement of the controller is the desired state \( r \) at each sample time. This is obtained by entering a series of waypoints into the computer, in practice waypoint position is entered through the keyboard using either a cursor on the chart display driven by the arrow keys, or by typing in co-ordinates using the alpha-numeric keys. The program assumes that each pair of waypoints alternately define a straight line followed by a curve. Thus taking the first waypoint as the starting point, the second is the "wheel over" position at the start of the arc required to reach waypoint three. On reaching this point, the vessel is required to be on a steady course to waypoint four which is the next "wheel over" position and so on. With each waypoint either a speed for that leg of the passage or an estimated time of arrival is entered, so the desired state of the vessel can be computed at each sample time prior to starting the voyage.

Values of the weighting parameters were selected to give an acceptable optimal closed-loop pole assignment but this is not a straightforward task. Starting with values obtained in the design of optimal controllers for previous vessels, the optimum control feedback matrix \( S \) can be calculated from

\[ S = -R^{-1}G^TW \]  \hspace{1cm} (9.2)
where $W$ is the solution to the matrix Riccati equations which, should $t_1$ be infinite, or far removed from $t_0$, converges to constant values, so that:

$$WF + F^tW + Q - WGR^{-1}G^tW = 0 \quad (9.3)$$

The control law is then given by:

$$u = -S(x - m) \quad (9.4)$$

where $m$ is the desired state of the vessel evaluated in reverse time so as to reduce transient errors which would normally occur when a change in the vessel state is required, primarily when the desired track changes direction. However, as the desired state vector $r$ is evaluated along a curve on these occasions, transient errors being assumed negligible and the control law is given by:

$$u = -S(x - r) \quad (9.5)$$

To assess the pole assignments of the system, equation (9.5) is combined with the state equations (4.11) to give:

$$x = (F - GS)x - GSr \quad (9.6)$$

where the term $(F - GS)$ may be identified as the closed-loop state matrix of the optimal system. The optimal closed-loop eigenvalues (poles) are then given by:

$$|S - (F - GS)| = 0 \quad (9.7)$$

It is apparent that when $F$ and $G$ are time-invariant the location of the optimal closed-loop poles depend upon the value of the $S$ matrix. There exists then, an infinite number of closed-loop poles, each being a measure of the optimality of the system as defined by relative weightings $Q$ and $R$. The matrices $Q$ and $R$ are diagonal and the values of $Q_{11}$ and $Q_{22}$ can be set to 0 as the rudder and main engines are control inputs and not controlled...
variables. Surge dynamics are affected by variations in $Q_{33}$ and $Q_{44}$ together with $R_{32}$. The sway and yaw weightings are $Q_{55}$, $Q_{66}$ and $Q_{77}$, $Q_{88}$ respectively together with $R_{n}$. Due to the coupling of sway and yaw, these elements closely interact, and changing any one value will affect all corresponding terms in the feedback matrix. Values of the weighting parameters must be selected to provide an acceptable optimal closed-loop pole assignment, so an iterative approach around the selection of $Q$ and $R$ and location of the poles is used to evaluate a gain matrix with an acceptable assignment.

Figure 9.1. The Controller

The $S$ matrix can be applied to the desired state vector $r$, as obtained for each sampling time, before starting the passage, as shown in figure 9.1.

9.3 Integration

For the purposes of control and guidance of the vessel the track error is required in place of the lateral displacement. So at the start of the control algorithm the track error is calculated on the grid using:

$$ TE = \begin{bmatrix} \hat{y}_o - y_o & \hat{x}_o - x_o \end{bmatrix} \begin{bmatrix} \cos \psi_o \\ -\sin \psi_o \end{bmatrix} $$

(9.8)

where the $s$ suffix denotes demanded values.

This then replaces $y$ as $\hat{x}(5)$ throughout the control algorithm. Similarly $\hat{x}(6)$ is replaced by numerically differentiating the previous three values obtained for track error.
Disturbances \( w(k+1) \)

\[
\begin{align*}
\text{SHIP} & \quad u(k) \quad x(k+1) \\
\text{MODEL} & \quad W(k) \quad A(k+1,k) \quad B(k+1,k) \quad C(k+1,k) \\
\text{INC} & \quad k \\
\text{CONTROLLER} & \quad r(k+1) \\
\text{COORDINATE TRANSFORMATION} & \quad x_0 \\
\text{MEASUREMENT} & \quad z(k+1) \\
\text{KALMAN FILTER} & \quad \hat{x}(k+1/k+1)
\end{align*}
\]

Hydrodynamic Coefficients

\[ F = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0.004 & -0.138 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0.573 & 0 & 0 & 0 & -0.308 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-0.273 & 0 & 0 & 0 & -0.092 & 0 & -1.444
\end{bmatrix} \quad (9.9)
\]

\[ G = \begin{bmatrix}
1 \\
0 \\
0.5
\end{bmatrix} \quad (9.10)
\]

**Figure 9.2. The Complete Integrated Navigation and Control System**

The overall integrated navigation system is shown as a block diagram in figure 9.2, while figure 9.3 shows a flow diagram of the software developed. In practice the delay is generated automatically by the measurement process.

Prior to full scale trials a suitable gain matrix \( S \) was evaluated as described in section 9.2. For a forward speed of 4.5 ms\(^{-1}\), and using the hydrodynamic coefficients, the \( F \) and \( G \) matrices were computed:
Figure 9.3. Flow Diagram of Automatic Guidance Program
Using techniques established in the design of optimal controllers, and used by Burns (1985) for controller trials in simulation, the optimal control law for Catfish was evaluated as:

\[ Q = \text{diag}[0 \ 0 \ 0.05 \ 0 \ 0.004 \ 0 \ 1 \ 0] \]  

\[ R = \text{diag}[1 \ 1] \]

and computing the optimal controller feedback matrix:

\[ S = \begin{bmatrix} 0.177 & 0 & 0 & 0 & -0.0115 & 0.1299 & -0.708 & -0.404 \\ 0 & 0.0135 & 0.221 & 1.578 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

(9.12)

For the values of \( F \) and \( G \) given in 9.9 and 9.10, the closed-loop eigenvalues are:

\[ s = -1.352; -0.4098; -0.0819; -0.998; -0.500; -0.0069; -0.1380; 0.0 \]  

(9.13)

The gain matrix was tested in computer simulation with the vessel starting 50 metres off track; the resulting plot is shown in figure 9.4.

---

**Figure 9.4. Controller Simulation**
Full scale trials on the controller proceeded in three stages, firstly using course keeping alone. This was achieved using values for \( S_7 \) and \( S_1 \) only; these gains being evaluated using the transfer functions described and tested by Burns (1985). Secondly, following satisfactory results the track keeping terms \( S_n \) and \( S_{2n} \) were included. These cannot be tested independently, as it can be shown that a track keeping controller with velocity feedback is always unstable. The original course keeping terms require modification due to the cross coupling between yaw rate and lateral velocity clearly indicated in the hydrodynamic coefficients in equations (4.5) and (4.6). Finally the forward speed terms \( S_{23} \) and \( S_{24} \) were included. These are independent of any other terms in the \( S \) matrix.

Trials proceeded without Trisponder and with disturbances being entered at the start of the program. Track and course keeping terms in the \( S \) matrix were set as for the simulation, but the system proved to be unstable giving an oscillatory response, probably due to the long cycle time of about 6 seconds. To test the controller alone and reduce the cycle time of the computer program, the filter algorithm was removed and the gain fixed as a diagonal matrix given by:

\[
K = \text{diag}[1 \ 1 \ 0.1 \ 0.1 \ 0.05 \ 0.98 \ 0.05] \tag{9.14}
\]

Furthermore, the \( A, B \) and \( C \) matrices were not recalculated unless the driving forces of engine revolutions and rudder angle changed by more than 200rpm and 2° respectively. The gain settings (\( S \)) were adjusted further and expanded to include a term in angular acceleration \( S_9 \), the corresponding term in the state vector being evaluated by numerical differentiation. Finally, it was argued that the cross track velocity term \( S_5 \) was necessary as the cross coupling between yaw rate and lateral velocity allows the yaw rate term to act as the damping coefficient for both track and course errors. Fine-tuning of the controller parameters was performed on the vessel. In order to measure the performance of a particular gain setting a set of indices are defined as:
These may be considered as the constituent elements of the global index $J$ in equation (9.1). Results shown in tables 9.1 and 9.2 were obtained.

\[
J_x = \int (x - x_0)^2 dt \\
J_y = \int (y - y_0)^2 dt \\
J_\psi = \int (\psi - \psi_0)^2 dt \\
J_\delta = \int \delta^2 dt \\
J_* = \int \theta^2 dt
\]

(9.15)

Table 9.1. Performance Indices For Off-Track Control Gains

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Controller Gains</th>
<th>Performance Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_5$</td>
<td>$S_{15}$</td>
</tr>
<tr>
<td>1</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.006</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>0.006</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>0.008</td>
<td>-0.6</td>
</tr>
<tr>
<td>11</td>
<td>0.007</td>
<td>-0.5</td>
</tr>
<tr>
<td>12</td>
<td>0.007</td>
<td>-0.5</td>
</tr>
<tr>
<td>13</td>
<td>0.007</td>
<td>-0.5</td>
</tr>
<tr>
<td>14</td>
<td>0.007</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Table 9.2. Performance Indices For Along-Track Control Gains

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Controller Gains</th>
<th>Performance Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{23}$</td>
<td>$S_{24}$</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>0.9</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>18</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>19</td>
<td>0.22</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>0.44</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Run 20 was aborted due to Catfish being on a collision course. However, as indicated by the behaviour of the throttle levers, this set of gains was slightly oscillatory.

Considering the performance indices in tables 9.1 and 9.2, which are required to be a minimum, the optimal controller gains are established as:

\[
S = \begin{bmatrix}
0 & 0 & 0 & 0 & -0.007 & 0 & -0.5 & -0.08 & -3.5 \\
0 & 0 & 0.22 & 0.5 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

9.4 Results

As an example of the many runs undertaken, a short passage was undertaken up the river Tamar from Cremyll buoy to West Mud buoy, shown on the map in figure 4.1, followed by a turn to starboard with the final leg heading towards No.1 buoy but terminating before crossing the path of the Torpoint ferry. The first leg of the passage went extremely well, with the controller steering a straight course towards West Mud buoy. The vessel should have turned just before this target, but it took the turn slightly late, perhaps due to the accuracy with which points were taken from the chart, and turned immediately the buoy was past on the starboard side. The turn was smooth with no noticeable overshoot; a steady course was maintained on the final leg towards the target. The voyage was repeated in the reverse direction with an increased radius on the turn. To test the repeatability of the system both passages were rerun. The results of all four runs are plotted as figures 9.5a to 9.8d. The first plot in a sequence shows desired and actual track, this is followed by plots of displacement and velocity against time for the three degrees of freedom considered.

The tracks are slightly oscillatory, however, track error shows that the vessel is rarely more than 20 metres from the desired line (The large error
CATFISH CONTROL TRIALS

RUN CO4MAY02

DEMAND       ACTUAL

0 - - - - - - 500m

Figure 9.5a. Desired and Actual Track
Figure 9.5b. Across-track Control Variables

Figure 9.5c. Across-track Control Variables
Figure 9.5d. Along-track Control Variables
CATFISH CONTROL TRIALS
RUN c04MAY03
DEMAND ACTUAL

0 500m

Figure 9.6a. Desired and Actual Track
Figure 9.6b. Across-track Control Variables

Figure 9.6c. Across-track Control Variables
Figure 9.6d. Along-track Control Variables
Figure 9.7a. Desired and Actual Tracks
Figure 9.7b. Across-track Control Variables

Figure 9.7c. Across-track Control Variables
Figure 9.7d. Along-track Control Variables
Figure 9.8a. Desired and Actual Tracks
Figure 9.8b. Across-track Control Variables

Figure 9.8c. Across-track Control Variables
Figure 9.8d. Along-track Control Variables
seen at the start of figure 9.8a is due to the ship's master interacting with the system to avoid a collision). To reduce oscillation further, it will be necessary to reduce the sampling interval as the major hull time constant of 2.6 seconds is close to the sampling interval of 1.56 seconds (a constraint imposed by the DNS Mk53 receiver) which, as discussed in section 1.5 could be as long as 6 seconds. Under these conditions non-oscillatory control of the highly responsive hull is not possible. To implement the system on a larger monohulled vessel should be simpler as the hull time constants will be greatly increased and the response to commands reduced.

Control over forward displacement does not appear to be so good. For the first three passages the vessel was always within 50 metres of the desired value until the final leg is reached, where the error increases steadily from zero to 100 metres, with the vessel ahead of its desired location. In the final run the vessel was always behind its desired location along the track, sometimes by as much as 100 metres. The only areas where the forward track error is within acceptable limits are at the start of the voyage and at the end of the turn. Contrary to previous runs, at termination the vessel was still about 100 metres short of the desired finishing point. Large values obtained for forward track error are considered to be a combination of two effects. Firstly, incorrect hydrodynamic coefficients, as established in the results from filter trials, and secondly inaccuracies in disturbance estimates.
CHAPTER 10

CONCLUSIONS

10.1 Introduction

The aims and objectives specified in chapter one have been investigated and achieved. Specifications were given for a complete integrated collision avoidance system to control the vessel on a predetermined track, while avoiding other vessels. The work then concentrated on the filtering aspect of the system to produce an optimal estimate of the state of the own vessel at regular intervals. This was achieved through the use of Kalman filtering. The following developments were achieved by the investigation:

(i) a linear filtering process for positioning a vessel while maintaining constant course was developed;

(ii) non-linear filtering to obtain the vessel state in three degrees of freedom initially failed due to difficulties in establishing vessel dynamics and external forces acting on the vessel;

(iii) the filter was successfully integrated with an optimal control algorithm to maintain a vessel on track.

Whilst the filtering and control applications to ship manoeuvring have been investigated and optimal methods established, there are still outstanding problems, primarily in large computation time. Thus further research into the methods discussed is required.

10.2 Discussion Of Results

A linear filter for ship position

A method for filtering positional data was presented in chapter six. This consisted of combining the measurements from position fixing systems with a prediction obtained by extrapolating a straight line from previous fixes. The optimal gain, computed using a Kalman filter was shown to be
constant (0.17) for any system when the previous ten points were used to compute the regression line. Under this condition the variance of the optimal estimate reduces that of the measurement process by 17%. The gain, and hence the improvement in the variance of vessel position, does however depend on the number of regression points taken. Tests conducted on data acquired from the Decca Navigator System in a geographical location where the standard deviation is 100 metres\(^2\) gave best results when six regression points were used with a diagonal Kalman gain matrix with the non-zero elements equal to 0.26. Under these conditions, and provided the vessel followed a near straight track, the filtered track was seen to take a smooth line through the measurements. When the vessel's heading changed by 90° over a 2 minute period the filtered track overshot the turn and returned to the true track within 2 minutes. The overshoot is due to the linear prediction process used. Similar tests conducted on Trisponder data, a positioning system with a variance of 1 metre\(^2\), gave best results with just 3 regression points and a Kalman gain matrix with diagonal elements of 0.44. Under these conditions similar turns to those described above were also handled well. The system is however likely to fail on more rapid turns. This simple positioning filter is therefore of practical use in position fixing, for example in surveying where it is usual to run a survey in straight lines, but it does not fulfill all the objectives of this research. It does however, provide a suitable introduction to practical Kalman filtering, a method used throughout the remainder of the thesis. These preliminary results demonstrate the limitations of the linear Kalman filter.

The effects of gross measurement errors were also considered in chapter six. These were shown to have an effect on future positions, as would be expected from any recursive algorithm such as a Kalman filter. A simple method for the removal of such errors comprising of a window centred about the prediction was introduced. If the fix falls within the window the filter is employed as usual. Should the fix fall outside the window, the measurement is rejected and the prediction alone used as the optimal
position. The optimum window size for the Decca measurements was found by experiment to be 100 metres, the equivalent of the variance of the system.

A non-linear filter for ship control

A non-linear ship model was described fully in chapter four. Particular coefficients of the Taylor series expansion of the variables applying forces acting on the hull are taken to represent the hull mathematically. As some of the terms taken are non-linear, the modelling process is linearised about the most recent optimal estimate as evaluated by the filter. This model was incorporated into the filter for trials. Initial results presented in chapter seven show the model, and hence the filter, deviating from the true track; this is due primarily to external disturbances such as wind and tide acting on the vessel. These forces were then estimated from the filtered output and fed back into the modelling process to give improved performance. However the track output still drifts from the true track with time. This was shown to be due to errors in coefficients used to model the vessel. These coefficients were initially obtained by a series of trials manoeuvres, they could perhaps be obtained more accurately from trials in a more controlled environment such as a towing tank or wind tunnel. However such procedures are expensive and would be required for each hullform on which the system was installed. The work undertaken in this thesis short cut this procedure and attempted to evaluate the coefficients by sea trials. Manoeuvres undertaken afloat led directly to identification of some coefficients, however others are not directly identifiable due to cross-coupling. The manoeuvres undertaken afloat were performed in computer simulation using the terms identified and estimating those which were not available. By comparison with the trials results a set of coefficients which represented the vessel were established. In practise it would be too costly to delay a vessel for such manoeuvres prior to taking passage. This filtering procedure is therefore not viable for commercial applications in integrated systems for use in merchant vessels. In the case
of a new vessel however, when it is usual to conduct sea trials immediately after construction, coefficients may be established in this manner but two problems still exist. Firstly, it was established that the technique used did not evaluate the coefficients to the necessary accuracy; and secondly the coefficients of the hullform will change as the hull becomes fouled.

On-line coefficient identification
In order to overcome the difficulty in evaluating the model coefficients a new method of on-line system identification was investigated. This was carried out for sway and yaw only as these were considered to be the most inaccurate, but the method could also be applied to the surge terms. The method involves augmenting the state vector with the coefficients of the state transition matrix. Thus the Kalman filter estimates both the state of the vessel and the hull coefficients. For prediction purposes these are taken as constant, and for the measurement vector they are evaluated from the previous optimal estimate. The latter is a non-linear process and thus the measurement matrix must now be linearised about that estimate.

Results of filtered position showed the vessel remaining within 20 metres of the track as estimated by Trisponder, and where the difference reached this extreme value, the Decca Navigator System (DNS) was seen to have a constant offset from the Trisponder track. The filtering algorithm in use is intended to cope with random errors only, and fixed errors must be evaluated and removed prior to filtering, so the noise reduction achieved is a better assessment of performance. This was reduced from a DRMS value of 10 metres to 2 metres. The ability of the system to maintain a position independent of the noise of the DNS demonstrated that the filter was producing accurate outputs of displacement in the three degrees of freedom considered. It was shown previously that inaccuracy in one, or all of these outputs led to drift of the filter from the true track. As values for velocity are fed back into the modelling process, any inaccuracy in their estimates leads to cumulative errors in the displacement outputs. The turn rate does
however remain noisy and an improvement could be achieved by use of a gyro input.

Ship control

Finally, the filtered output was fed to a control algorithm. Optimal control theory was used to establish the control parameters to maintain the vessel on track, in both along-track and across-track directions. Ideally an optimal controller should compute new gains from the transition matrix on each cycle. In these tests, however, this proved to be a long and complex operation. So from a series of trials, a set of constant gains which best suited the vessel were established and the controller thus functioned in a proportional plus derivative form. In spite of the difficulty of controlling such a manoeuvrable vessel with short time constants, the results showed the vessel to remain within 20 metres of the desired track over a short duration. It was however less efficient along the track, always advancing too quickly. This was possibly due to the modelling process which was seen earlier to over-estimate the speed of the vessel. In order to overcome this difficulty the surge modelling coefficients should be included in the identification process.

10.3 Further Research

While the operation of the filter was satisfactory in removing noise, there are other factors which reduce the practicality of the system. The major problem is the time constraint and the computing limitation when using the Acorn Cambridge Workstation and later the Archimedes computers. Ideally the filter should cycle faster than 1.5 seconds, the rate at which the DNS produces output. The final solution, including system identification takes about 15 seconds. This can be improved by installation of a hardware arithmetic co-processor unit in the computer, but this will not completely solve the problem and this speed aspect in particular should be considered in further research.
New work should commence with extensive trials on alternative data sets. The results obtained for $Y_s$ and $N_s$ showed that these coefficients are almost zero and could perhaps be removed. However, as trials were conducted over such a small speed range, 6 to 7 knots, this evidence is not conclusive. Tapp (1989) shows that for a larger vessel capable of a wider range of speeds, turning characteristics may vary with speed. Therefore these coefficients require investigation over a wider speed range and on a variety of vessels. Other coefficients, namely $Y_0$, $Y_s$, $N_0$, and $N_s$, remain approximately constant at the values to which they were initially set. Further investigations are required to establish whether these are their true values. This can be established by initialising them to some other value and seeing if they correspond. If the values established in this thesis are correct under all conditions, then these coefficients can be fixed within the computer program and the dimensions of the state vector reduced, thus saving computer time. The erratic nature of the rudder coefficients, $Y_s$ and $N_s$, shown in plots 8.2e and 8.2f, suggest that the system may be susceptible to instability. Analysis should therefore be performed, and should any unstable situations be encountered, a heuristic approach can be incorporated to overcome difficulties. For example, upper and lower limits might be given to these coefficients. Further improvement to the filter process would be achieved by adding a window filter to remove gross errors. This would function in a similar way to the window described in chapter six. The window should also be used in the case of the state parameters, in particular for those of forward velocity in which gross measurement errors are obvious. It was shown in chapter six that such measurements are retained by the filter for future use and therefore probably give the noisy signal seen in the filtered forward velocity output, which may be removed by use of a window filter.

When working on the system coefficients, and their application to the model, it may be useful to consider the transfer functions which can be written from the model equations as:
assuming the velocity is constant. These give an alternative formulation for the mathematical model, and may prove cost efficient in computer time. They may also be of value for computing controller gains on-line.

Efficiency of the algorithm is a problem commonly encountered when applying Kalman filtering to real-time applications. The load on processor time has forced development in prospective applications to turn to sub-optimal filtering techniques, as for example the 950 hydrographic surveying system developed by Decca Marine Systems and discussed in section 5.4. Careful formulation and manipulation of the processes involved in the algorithm leads to a more efficient result as shown below.

If the measurement errors are uncorrelated then the covariance matrix M is diagonal. Furthermore, measurements can perhaps be grouped together so that uncorrelated blocks can be formed on the diagonal of the covariance matrix. This enables the expression for the updated error covariance using the alternative Kalman filter equations to take the following form, neglecting the recursive counter k and denoting correlated measurement transformation and covariance blocks with suffices a, b, ..., n,

\[
P^{t+} = P^{t} + \begin{bmatrix} H^t & 0 & \cdots & 0 \\ 0 & H^t & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H^t \end{bmatrix} \begin{bmatrix} M^t & 0 & \cdots & 0 \\ 0 & M^t & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M^t \end{bmatrix} \begin{bmatrix} H^t & 0 & \cdots & 0 \\ 0 & H^t & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H^t \end{bmatrix}
\]

\[
= P^{t} + \begin{bmatrix} H^t & M^t & H^t \\ 0 & H^t & M^t \\ 0 & 0 & H^t \end{bmatrix}
\]

Having simplified the matrix multiplication involved, perhaps to scalar
multiplication, simplification of the matrix inversion problem can also be achieved if certain blocks can be taken as \(0\). Even if they do have some value, processor time may still be used more efficiently by partitioning into four segments and performing the inversion process suggested by Fadeev and Fadeeva (1963);

writing

\[ p^{-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \]

where \(A\) and \(D\) are square matrices of order \(p\) and \(q\).

The inverse can be obtained in the form

\[ p = \begin{bmatrix} k & l \\ m & n \end{bmatrix} \]

where the blocks are given by:

\[ N = (D - CA^{-1}B)^{-1} \quad M = -NCA^{-1} \quad L = A^{-1}BN \quad K = A^{-1} - A^{-1}BM \]

This involves two matrix inversions, both of smaller dimensions than the original. The problem is further reduced if either of blocks \(A\) or \(C\) are zero.

Partitioning can therefore give more rapid cycling of the iterative loop used to obtain filter convergence. Computational aspects of the partitioned form for the Kalman filter offer distinct advantages in certain situations. Each application however, should be judged on its merits as many of the advantages are dependent on formulation of the original problem and on restrictions imposed by the implementation of the measurement process.

Suppose now that measurements are not available simultaneously. Using the partitioned scheme the system can be programmed to process data sequentially; processing measurements as they arrive and skipping those that are not available. Such a scheme would require complicated system organization but make most efficient use of data as it becomes available.
For the marine application, the sequential process will probably consist of sampling rudder angle and engine speed only between filtered updates when all other measurements, obtained via the Decca navigator, are available. This process, which is shown diagramatically in figure 10.1, offers the advantage of reducing the time scale over which the vessel's state appears constant.

10.4 System Development

The basic description of the filter for use in a complete integrated navigation and collision avoidance system has been discussed. The final objective is to combine this with the work on collision avoidance being undertaken by Blackwell (1989), who is using an expert system based on the International Regulations for the Prevention of Collisions at Sea. To date only single ship encounters have been considered in simulation. The computer software detects whether another vessel is likely to impinge on its domain, an area around the vessel the dimensions of which were evaluated by Colley (1986) using observations of genuine encounters. If the own vessel is the give way vessel, then appropriate control input is applied to avoid the collision and regain the original course. For the trials on the expert system, ship models and their response to control inputs have been
simplified. Further research in this area will incorporate multi-ship encounters, lane separation zones and shallow water areas into the system.

The complete integrated system will use the state vector obtained from the Kalman filter as the best estimate for the state of the own ship. A proportional derivative controller similar to that described in chapter nine will automatically control the vessel on some pre-determined track. Data taken from the ARPA set will indicate positions of other vessels, targets, within a selected range. Further software is required to estimate speed and direction of targets, and here again Kalman filtering was suggested by Castella and Dunnlecke (1974). With accurate target data available, it is envisaged that the expert system will alter the desired track of the own vessel used by the control unit, and continually update this until the danger is passed. At this point the original desired track can be regained for use by the controller.

10.5 Concluding Summary

The investigations undertaken in this research programme have demonstrated the capability of the Kalman filter to estimate the state of a vessel from noisy measurement data; used this estimate to control the vessel and given guidelines for the incorporation of collision avoidance. The questions of benefits over existing techniques and ability to cope with measurement failure must be considered.

It was clearly pointed out in chapter one that while assisting the mariner with information and advice, it must be presented clearly and one must be careful not to present too much at one time. In this respect there is a clear need for an ergonomic study of procedures on the bridge of a ship, to determine what information the mariner needs and when he needs it. Furthermore, in order not to confuse the mariner, data requires some standard form of presentation, which must be used by all manufacturers.
Without such standardisation, information is more likely to confuse, rather than assist the mariner, jeopardising the safety of the vessel rather than improving it. Vessel safety is the purpose of this and further research, so an ergonomic study of bridge layout, together with specifications for an integrated navigation system is considered necessary.

All the results obtained using the algorithms presented in this thesis assumed that appropriate measurements were available. Gross errors were considered and it was shown that in such cases better results are achieved with no measurement at all. Consider now a situation in which one of the measurement systems fails and no measurements, or false measurements, are made for an extended period. In the case of the positioning routine presented in chapter six, following a complete failure, predictions would continue to be the optimal estimate and the track would continue in a straight line. If just one Trisponder range, or one Decca lane were measured, then the optimal estimate could be adjusted to account for this information and the estimate improved. The possibility of partial system failure should also be considered for the complex filtering algorithm used in chapter eight. Analysis and trials should be conducted to establish the extent to which the system can cope with such failures, considering for what length of time the system will continue to give reliable results, and what degradation of results will occur with time.

When such a system as that described in this thesis is installed on a vessel and used in practise, it will be employed by the mariner in connection with other computer systems performing various tasks, such as voyage management, cargo handling and engine room monitoring. At this stage of development there will be a need for a central computer operations room on board the vessel. An integrated computer system could be utilised, performing all of these functions, with the added benefit of speed from the sharing of information. Voyage management, cargo handling and engine room installations already exist, although not within one unit, and the technology
to complete the integrated navigation system described in this thesis is awaited.

There are arguments offered, primarily by traditional mariners, against such developments, which may be summarised as:

(i) Computer systems are thought to be unreliable, so what would happen in the case of a machine or power failure?

(ii) The target detection capability of ARPA is not considered to be robust enough to pass reliable information to such a computer system;

(iii) What would a small craft without such a facility do on encountering a vessel controlled by computer with nobody on board, and needing to communicate by radio, with whom could he do so?

(iv) The duties of the modern mariner entail far more than just navigation and the vessel is not the only dynamic system he has to consider. The weather and voyage economics are constantly changing and these parameters must also be considered to route the vessel while underway. How could an automated system adjust to such variations?

These points may be countered by:

(i) Most of the equipment needed to run such a system has a very small power consumption. The greatest single power drain on a computer is for the display, and since rudders and engine speed are controlled by hydraulics, in the case of a complete power failure the total consumption of the system could easily be powered by batteries. If the displays were turned off under such circumstances, which could itself be done automatically, the power drain on the batteries would be minimal. In the case of component failure, a complete duplicate backup
system is envisaged, and in some cases perhaps a triplicate set of components. This is normal in the aerospace industry where the probability of all systems failing is almost nil, and there is no reason why this technology should not apply to the marine industry.

(ii) Similar reasoning applies to the argument against the ARPA set, in that certain vessels are already required to carry two sets by IMO regulations. Combining target data gathered from two or three sets must give more reliable results. Furthermore, the technology used in ARPA is constantly being improved and updated. For example, Kelvin Hughes, a leading manufacturer, are conducting extensive research and development on their products, continuously improving specifications of their ARPA equipment.

(iii) An automatically controlled vessel would have little need for communications. Indeed if all vessels were to obey the International Regulations for the Prevention of Collisions at Sea the requirement for radio communications would be reduced. For example, the collision between Admiral Nikhimov and the Vasev described in chapter 1 would almost certainly not have occurred if the masters of the vessels had not been in radio contact and certainly not if they had both adhered to the regulations. The only foreseeable use for radio contact is to acquire the movements of other vessels beyond the range of the radar set, for example in a confined waterway, when the radars' range is impaired by a land mass. Such areas are covered by communication with harbour authorities and if these establishments were to broadcast all movements in the vicinity over a telemetry link in a standard format, the automated systems installed would have access to this data. In this way port authorities could also be automated to a certain extent, again reducing the risk of collision due to human error. Although collisions due to this source are not common in the marine environment, similar mistakes made by air traffic control have been well publicised.
Voyage management systems are currently in use to advise the mariner of voyage economics, and research into weather routing is being undertaken by Calvert (1990). These systems obtain the required data from shore based transmitters by satellite communications. The output would simply be used by the automatic guidance system to guide the vessel. This reinforces the objective stated above for implementing a central operations room. For automatic guidance the control parameters would have to be modified for each stage of the voyage. For example, it is of little importance if the vessel is a mile off-track in mid-ocean while still heading in the right direction, but this is different to the requirements when operating in confined waters. Further research is necessary to define the various stages of a passage and identify the control criteria but the technology for such a system will be complete within the foreseeable future.
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APPENDIX A

THE DECCA SOLUTION ON THE SPHEROID

A Decca chain has a base frequency $f$, the master transmits $6f$, the red slave $8f$, green $9f$ and purple $5f$ giving comparison frequencies in the mobile receiver of red $24f$, green $18f$ and purple $30f$. The Decca chain local to Plymouth Sound is referenced as the South West British and denoted 1B, it operates on a base frequency $f = 14.0466\text{kHz}$. The Sound is best covered by the green and purple position lines. The transmitting stations which generate these are located at:

<table>
<thead>
<tr>
<th></th>
<th>master</th>
<th>green</th>
<th>purple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>50° 13' 56'' N</td>
<td>49° 55' 50'' N</td>
<td>51° 25' 42'' N</td>
</tr>
<tr>
<td>Longitude</td>
<td>3° 50' 13'' W</td>
<td>6° 18' 15'' W</td>
<td>3° 22' 41'' W</td>
</tr>
</tbody>
</table>

The receiver calculates the position lines and transmits them to the computer through the serial interface as zone, zone group and lane for each pattern. A routine is required in the computer to convert this data into a position on the earth's surface, this is achieved as follows:

For both LOPs calculate the distances along the baseline from the master transmitter as shown in figure 1A.

![Figure 1A. Distances Along Baseline](image-url)

- A1 -
\( T_1 \) and \( T_2 \) can be calculated using:

\[
T_i = \left( \left( \text{Lanes/zone} \times \text{Number of zones} \right) + \text{Lanes} \right) \times \frac{c}{2 \times \text{comparison frequency}}
\]

where \( c = \text{celerity} \approx 3 \times 10^4 \text{ms}^{-1} \)

Lanes/zone = 18 red
12 green
15 purple

Then for the hyperbola:

\[
P_i = X_i S - MS = MX_i - 2T_i, \quad i = 1, 2
\]

Figure 2A. Decca System on the Surface of a Spheroid

\( \theta \)'s are measured by the angles formed at the centre of the spheroid, \( \beta \)'s
and \( K \) are spherical angles.

It is shown by Razin (1967) that:

\[
\cos \beta_i = \frac{u_1 u_i u_2 \overline{u_2^2} + u_2^2 - u_i^2}{u_1^2 + u_i^2}
\]

where

\[
\begin{align*}
u_i &= a_i \cos K - a_2, \\
u_2 &= a_i \sin K, \\
u_3 &= a_2 \frac{\sin P_i}{\sin \theta_{ux1}} - a_i \frac{\sin P_2}{\sin \theta_{ux2}}
\end{align*}
\]

and

\[
a_i = \frac{\cos P_i - \cos \theta_{ux1}}{\sin \theta_{ux1}}, \quad i = 1, 2
\]
Then the angle

\[ \theta_{w3} = \arctan \left( \frac{\cos P_1 - \cos \theta_{wx}}{\sin P_1 + \sin \theta_{wx} \cos \beta} \right) \]

and finally

\[ \theta_{x,i} = \theta_{w3} + P_i, \quad i = 1,2 \]

allows the position of the vessel to be given in terms of degrees of arc from the transmitters. Assuming the earth to be a sphere of radius \( R \) (for the purposes of Decca chains take \( R \) to be the radius at the mid-point of the coverage area) and using \( \varphi \) and \( \theta \) to represent the Latitude and Longitude of the receiver. Then the following equations must be satisfied:

\[
\begin{align*}
\cos \varphi_x \sin \theta_x, (\cos \varphi \sin \theta) + \cos \varphi_x \sin \theta_x, (\cos \varphi \sin \theta) + \sin \varphi_x \sin \theta_x (\sin \varphi) &= \cos \theta_{x1} \\
\cos \varphi_x \sin \theta_{x2} (\cos \varphi \sin \theta) + \cos \varphi_x \sin \theta_{x2} (\cos \varphi \sin \theta) + \sin \varphi_x \sin \theta_{x2} (\sin \varphi) &= \cos \theta_{x2} \\
\cos \varphi_u \sin \theta_u (\cos \varphi \sin \theta) + \cos \varphi_u \sin \theta_u (\cos \varphi \sin \theta) + \sin \varphi_u \sin \theta_u (\sin \varphi) &= \cos \theta_u
\end{align*}
\]

Putting

\[
\begin{align*}
g &= \cos \varphi \sin \theta \\
f &= \cos \varphi \sin \theta \\
h &= \sin \varphi
\end{align*}
\]

and treating these as 3 linear equations in 3 unknowns:

\[
\begin{align*}
f &= C_1 \cos \theta_{x1} + C_2 \cos \theta_u + C_3 \cos \theta_{x2} \\
g &= C_4 \cos \theta_{x1} + C_5 \cos \theta_u + C_6 \cos \theta_{x2} \\
h &= C_7 \cos \theta_{x1} + C_8 \cos \theta_u + C_9 \cos \theta_{x2}
\end{align*}
\]

where the \( C_i \)'s are given by:

\[
\begin{align*}
C_1 &= (\cos \varphi_u \sin \theta_u \sin \varphi_x - \sin \varphi_u \cos \varphi_x \sin \theta_x) / D \\
C_2 &= (\sin \varphi_x \cos \varphi_u \sin \varphi_x - \cos \varphi \sin \theta_x \sin \varphi_x) / D \\
C_3 &= (\cos \varphi_x \sin \theta_u \sin \varphi_u - \sin \varphi_x \cos \varphi_u \sin \theta_u) / D \\
C_4 &= (\sin \varphi_u \cos \varphi_x \cos \theta_x - \cos \varphi_u \sin \theta_x \sin \varphi_x) / D \\
C_5 &= (\cos \varphi_x \sin \theta_x \sin \varphi_u - \sin \varphi_x \cos \varphi_u \sin \theta_x) / D \\
C_6 &= (\sin \varphi_x \cos \varphi_u \cos \vartheta_u - \cos \varphi_x \sin \theta_u \sin \varphi_u) / D
\end{align*}
\]
Assuming the earth to be a spheroid with equatorial radius \( R \) and polar radius \( r \), its equation in spherical coordinates is:

\[
\left( \frac{\rho \cos \varphi}{R} \right)^3 + \left( \frac{\rho \sin \varphi}{r} \right)^3 = 1
\]

Taking \( P_0 \) as a point at approximately the centre of the service area of the Decca chain with coordinates \((\varphi_0, \theta_0)\) then the radius of curvature at \( P_0 \) is given by:

\[
r_e = \frac{R^2 \sin^2 \varphi_0 + r^2 \cos^2 \theta_0}{r}
\]

and the centre of the sphere osculating at \( P_0 \) is given in cartesian coordinates by:

\[
X = \left[ R - \sqrt{R^2 \sin^2 \varphi_0 + r^2 \cos^2 \theta_0} \right] \cos \varphi_0 \sin \theta_0
\]
\[
Y = \left[ R - \sqrt{R^2 \sin^2 \varphi_0 + r^2 \cos^2 \theta_0} \right] \cos \theta_0 \sin \varphi_0
\]
\[
Z = \left[ r - \frac{R}{r} \right] \left[ R^2 - \sqrt{R^2 \sin^2 \varphi_0 + r^2 \cos^2 \theta_0} \right] \sin \varphi_0
\]

If \( p \) is a point on the spheroid at \((\varphi, \theta)\) its cartesian coordinates are given by \((x, y, z)\) where \( x = R \cos \varphi \sin \theta \), \( y = R \cos \varphi \cos \theta \) and \( z = r \sin \theta \). If \( p' \) is the image of \( p \) on the osculating sphere, then the coordinates of \( p' \) are:

\[
x' = \frac{r_e (x - X)}{L}
\]
\[
y' = \frac{r_e (y - Y)}{L}
\]
\[
z' = \frac{r_e (z - Z)}{L}
\]

with \( L = \sqrt{(x - X)^2 + (y - Y)^2 + (z - Z)^2} \)
\((\phi', \theta')\) are then given by:

\[
\phi' = \sin^{-1}(z - \frac{Z}{L})
\]
\[
\theta' = \tan^{-1}\left(\frac{x - x'}{y - y'}\right)
\]

For the Decca system \(\phi'_w\) and \(\theta'_w\) should be found in order to evaluate the constants \(C_i\) through \(C_s\). The coordinates of the receiver on the osculating sphere can be computed from: \(\phi'_s = \sin^{-1}(h)\) \(\theta'_s = \tan^{-1}(g/f)\) To map these back onto the spheroid:

\[
\theta_s = \tan^{-1}\left(\frac{g + C_{10}}{h + C_{11}}\right)
\]
\[
\phi_s = \tan^{-1}\left(C_{10}\sin\theta_s + h + C_{11}\right)
\]

where \(C_{10} = \frac{X}{r_e} \quad C_{11} = \frac{Y}{r_e} \quad C_{12} = \frac{Z}{r_e} \quad C_{14} = \frac{R}{r_e}\)

\((\phi_s, \theta_s)\) is the required Decca position in geographical coordinates.
TRISPONDER AND LEAST SQUARES

The Trisponder system is manufactured by Del Norte (1986). It measures the two-way travel time of microwave signals between the mobile master station and up to 8 fixed remotes whose locations must be known. Corrections are made in the processor unit for a delay due to signal turn around time in the remotes, evaluated by calibration, and also for the remote station heights. The processor unit then displays the true horizontal ranges and transmits them through a serial interface.

![Diagram of Trisponder System]

*Figure B1. Trisponder System*

The manufacturers quote an accuracy of 1 metre on each range, but this is generally taken to be optimistic. With the processor unit taking out fixed errors by using a calibration only random errors remain, these fall into two categories. Small errors due mainly to refraction and change in velocity of propagation as the density of the medium through which the signal travels changes, the largest errors occurring if the signal has to travel over both land and sea. Gross errors are usually due to the receiver picking up a reflected, rather than the direct, signal and are often detectable and can be ignored provided there is a redundancy in the number of observations. It is
possible to find the location of the master unit by measuring just two ranges as in figure B1, with knowledge of $E_1, E_2, N_1,$ and $N_2$. In fact this leads to an ambiguity as there are two possible solutions, one on each side of the baseline, but we would hope to know which is the correct answer. With more than two remotes the ambiguity problem is cured (provided the remotes do not all lie on the same line) but we have redundant observations and can obtain $\sum_{i=1}^{n} v_i$ fixes where $n$ is the number of remotes. Figure B2 shows three remotes giving three fixes, we must assume that the true location of the master is somewhere in the cocked hat.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{cocked_hat}
\caption{The Cocked Hat From Three Ranges}
\end{figure}

To establish the most probable position of the master a least squares process is used which minimises the sum of squares of the residuals ($v$):

$$\sum_{i=1}^{n} v_i^2 = \text{minimum} \quad \text{where} \quad \bar{r}_i = r_i + v_i \quad (B1)$$

and $\bar{r}_i$ is the range to remote $i$ from the best estimate of the master's location.

$r_i$ is the measured range.

First establishing a model of the system using the observation equations:

$$\sqrt{(E - E_i)^2 + (N - N_i)^2} - \bar{r}_i = 0 \quad i = 1, n \quad (B2)$$

- B2 -
For the estimation of \( \bar{F} \) and \( \bar{N} \) it is necessary to linearise this basic model, this necessitates estimating provisional values of the quantities involved. We already have an approximation of \( \bar{r} \) since we know the observed values \( r \), we also require provisional values for \( \bar{x} = [\bar{E}, \bar{N}]^{\top} \), let these be \( \bar{x}^0 = [E^0, N^0]^{\top} \) and let them be related to \( \bar{x} \) by:

\[
\bar{x} = \bar{x}^0 + \bar{x}
\]  

(B3)

so it is now necessary to estimate the small quantities \( \bar{x} \), writing a general vector function:

\[
F(\bar{x}, \bar{r}) = F(\bar{x}^0 + \bar{x}, r + \bar{v}) = 0
\]  

(B4)

and applying a Taylor series expansion to first differentials we obtain:

\[
F(\bar{x}, \bar{r}) = F(\bar{x}^0, r) + \frac{\partial F}{\partial \bar{x}} \bar{x} + \frac{\partial F}{\partial \bar{r}} \bar{v} = 0
\]  

(B5)

where \( \frac{\partial F}{\partial \bar{x}} \) and \( \frac{\partial F}{\partial \bar{r}} \) are evaluated at the points \( \bar{x}^0 \) and \( r \) respectively, in this case \( \frac{\partial F}{\partial \bar{r}} = I_n \), the identity matrix of order \( n \).

\( F(\bar{x}^0, r) \) is a vector which contains the \( n \) values of \( F(\bar{x}, \bar{r}) \) computed at the known points \( (\bar{x}^0, r) \). Let this vector be \( -\bar{b} \).

\[
-\bar{b} = \begin{bmatrix}
    f_1(\bar{x}^0, r) \\
    f_2(\bar{x}^0, r) \\
    \vdots \\
    f_n(\bar{x}^0, r)
\end{bmatrix}
\]

\( \frac{\partial F}{\partial \bar{x}} \) is a matrix of order \( 2n \) and is denoted by the letter \( A \), the \( i \)th row will be the partial differential of \( f_i(\bar{x}, \bar{r}) \) with with respect to \( \bar{E} \) and \( \bar{N} \).

\[
A = \begin{bmatrix}
    \frac{\partial f_1}{\partial \bar{E}} & \frac{\partial f_1}{\partial \bar{N}} \\
    \frac{\partial f_2}{\partial \bar{E}} & \frac{\partial f_2}{\partial \bar{N}} \\
    \vdots & \vdots \\
    \frac{\partial f_n}{\partial \bar{E}} & \frac{\partial f_n}{\partial \bar{N}}
\end{bmatrix} = \begin{bmatrix}
    E^0 - E_1 & N^0 - N_1 \\
    E^0 - E_2 & N^0 - N_2 \\
    \vdots & \vdots \\
    E^0 - E_n & N^0 - N_n
\end{bmatrix}
\]
where \( r^o_1 = \sqrt{(E^o - E_i)^2 + (N^o - N_i)^2} \)

Sustituting \( A \) and \( b \) into (85):

\[
Ax = b + v \tag{86}
\]

It is shown by Cross (1983) that the least squares estimates (\( \hat{x} \) and \( \hat{v} \)) of \( x \) and \( v \) are given by:

\[
\hat{x} = (A^TWA)^{-1}A^TWb \tag{87}
\]

and

\[
\hat{v} = Ax - b \tag{88}
\]

where \( W \) is a weighting matrix which represents the covariances of the observations. In the trisponder application \( W \) will be diagonal as all observations are independent and the variance of measurements are taken as 1.0 as the figure given by the manufacturer is the best available. If an observation is not made it is given a weighting of 0.0. The new value obtained for \( \hat{x} \) is placed back into (83) and the process iterated until a result of the required accuracy is obtained.
A point \((p)\) on the earth's surface is usually described by a pair of coordinates \((\phi, \theta)\). The Longitude \((\theta)\) is the angle of arc about the centre of the spheroid between the central meridian (which runs through Greenwich) and \(p\) and has a range \(180^\circ E\) to \(180^\circ W\) \((-180^\circ < \theta < 180^\circ)\). Latitude \((\phi)\) is the angle of arc about the centre of the spheroid between the equator and \(p\) and has a range \(90^\circ S\) to \(90^\circ N\) \((-90^\circ < \phi < 90^\circ)\). The Ordnance Survey Great Britain 1936 (OSGB36) grid, fully defined in the Projection tables of Great Britain (1975), is a transverse mercator projection of the earth covering Great Britain and defined on the Airy spheroid. It has a true origin at latitude \(49^\circ\), longitude \(2^\circ W\), but points are described relative to a false origin \(400\text{km} \text{West} \text{and} \text{100km} \text{North}\) of the true origin giving all points positive values in Easterly and Northerly components.

Eastings\((E)\) and Northings\((N)\) can be computed from \((\phi, \theta)\) using the formula given by Clark as follows:

\[
E = v \Delta \lambda \cos \phi + v \frac{\Delta \lambda^3}{6} \cos^3 \phi \left( \frac{\nu}{\rho} - \tan^2 \phi \right)
\]

\[
N = M + v \sin \phi \frac{\Delta \lambda^2}{2} \cos \phi + v \sin \phi \frac{\Delta \lambda^4}{24} \cos^3 \phi \left( \left( \frac{2 \nu}{\rho} \right)^2 + \frac{\nu}{\rho} - \tan^2 \phi \right)
\]

where \(\Delta \lambda = \text{Longitude measured from origin (2°W) in radians}\)

\(M = \text{Meridian distance from the latitude of the origin}\)

\[
v = \frac{R}{(1 - e^2 \sin^2 \phi)^{1/2}}
\]

\[
\rho = \frac{R(1 - e^2)^{1/2}}{(1 - e^2 \sin^2 \phi)^{1/2}}
\]

\(R\) and \(e\) are radius and eccentricity of the spheroid.
APPENDIX D

STATE TRANSITION MATRIX COEFFICIENTS

Surge Coefficients:

\[ X_i = \frac{X_{68} \delta}{m - X_c} \]
\[ X_s = \frac{X_{68} n}{m - X_o} \]
\[ X_a = \frac{X_{68} u}{m - X_v} \]
\[ X_e = \frac{X_{68} v - mr}{m - X_v} \]

Sway Coefficients:

\[ Y_i = \frac{Y_{59} - Y_{68} \delta + Y_{68} \delta^2}{(m - Y_c)(1 - (Y_{68} N_{68}))} + \frac{Y_{68} (N_{59} + N_{68} \delta - N_{68} \delta^2)}{(l_s - N_s)(1 - (Y_{68} N_{68}))} \]
\[ Y_s = \frac{-mr}{1 - (Y_{68} N_{68})} \]
\[ Y_a = \frac{Y_{59} - Y_{68} r v + Y_{68} v^2}{(m - Y_c)(1 - (Y_{68} N_{68}))} + \frac{Y_{68} (N_{59} + N_{68} r v - N_{68} v^2)}{(l_s - N_s)(1 - (Y_{68} N_{68}))} \]
\[ Y_e = \frac{Y_r}{(m - Y_c)(1 - (Y_{68} N_{68}))} + \frac{Y_{68} N_r}{(l_s - N_s)(1 - (Y_{68} N_{68}))} \]

Yaw Coefficients:

\[ N_i = \frac{N_{59} - N_{68} \delta + N_{68} \delta^2}{(l_s - N_s)(1 - (Y_{68} N_{68}))} + \frac{N_{68} (Y_{59} + Y_{68} \delta - Y_{68} \delta^2)}{(m - Y_c)(1 - (Y_{68} N_{68}))} \]
\[ N_s = \frac{-N_{68} m r}{1 - (Y_{68} N_{68})} \]
\[ N_s = \frac{N_v - N_{ss} rv + N_{ss} v^2}{(l_s - N_v)(1 - (Y_{ss} N_{ss}))} + \frac{N_{ss} (Y_v + Y_{ss} rv - Y_{ss} v^2)}{(m - Y_v)(1 - (Y_{ss} N_{ss}))} \]

\[ N_{ss} = \frac{N_v}{(l_s - N_v)(1 - (Y_{ss} N_{ss}))} + \frac{N_{ss} v}{(m - Y_v)(1 - (Y_{ss} N_{ss}))} \]

where

\[ Y_{ss} = \frac{Y_v}{(m - Y_v)} \]

\[ N_{ss} = \frac{N_v}{(l_s - N_v)} \]

and \( m \) and \( l_s \) are the mass and moment of inertia of the vessel.

For Catfish: \( m = 8500 \text{kg} \)

\( l_s = 117469 \text{kgm} \)
Assuming that a value $x$ for a dimension is best estimated from the weighted mean of two independent measurements $x_1$ and $x_2$ then we can write:

$$\hat{x} = ux_1 + (1-u)x_2$$

(E1)

where $u$ is a weighting factor and $0 \leq u \leq 1$.

If $x_1$ and $x_2$ have variances $P_1$ and $P_2$ respectively we can deduce that

$$P = u^2 P_1 + (1-u)^2 P_2$$

(E2)

where $P$ is the minimum variance of $\hat{x}$.

It is desired to find the value of $u$ for which the variance $P$, and thus the random error, will be a minimum. This occurs when:

$$\frac{dP}{du} = 0$$

$$= 2\hat{u}P_1 - 2(1 - \hat{u})P_2 = 0$$

(E3)

so $\hat{u} = \frac{P_2}{P_1 + P_2}$

substituting into (E1):

$$\hat{x} = \frac{P_2}{P_2 + P_1} x_1 + \frac{P_1}{P_2 + P_1} x_2$$

(E4)

It can be shown that the variance $\hat{P}$ of the optimal combination of estimates $\hat{x}$ is less than that of both estimates individually.
Rewriting (E4) in predictor/corrector form:

\[ \hat{x} = x_1 - \left[ \frac{P_1}{P_1 + P_2} \right] (x_1 - x_2) \]  (E5)

and

\[ \hat{P} = P_1 - \left[ \frac{P_1}{P_1 + P_2} \right] P_1 \]  (E6)

Thus defining

\[ K = \frac{P_1}{P_1 + P_2} \]  (E7)

Equations (E5) and (E6) can be written:

\[ \hat{x} = x_1 - K (x_1 - x_2) \]  (E8)

\[ \hat{P} = P_1 - KP_1 \]  (E9)

Expanding into multidimensional form such that:

\[ x_1 = (x_{11}, x_{12}, \ldots, x_{1n}) \]

\[ x_2 = (x_{21}, x_{22}, \ldots, x_{2n}) \]

\( P_1, P_2 \) and \( K \) will be matrices of order \( n \) and

\[ K = P_1 (P_1 + P_2)^{-1} \]  (E10)

\[ \hat{x} = x_1 - K (x_1 - x_2) \]  (E11)

\[ \hat{P} = P_1 - KP_1 \]  (E12)

Now consider a system where estimates of \( x_1 \) and \( x_2 \) are made from two independant sources, say from a mathematical model and by measurements with random noise:
If the random noise $v(k+1)$ has a gaussian distribution with zero mean and variance $R$ then we can write:

$$z(k+1) = H_x(k+1) + v(k+1)$$

(E13)

and deduce that

$$E[z(k+1)] = H.E[x(k+1)]$$

(E14)

where $E$ is the expectation operator and $H$ is a scale factor.

Reverting to scalar quantities. $\frac{z}{H}$ is an independant estimate of $\hat{x}$ corresponding to $x_z$ in the above equations, and we can deduce directly from the definition of variance that

$$P_z = \frac{R}{H^2}$$

Substituting into equations (E5) and (E6):

$$\hat{x} = x_1 - \left[ \frac{P}{P + \frac{R}{H^2}} \right] (x_1 - \frac{z}{H})$$

(E15)
\[ \hat{p} = P - \left[ \frac{P}{P + \frac{R}{H}} \right] P \]  

Which may be written as equations (E10), (E11) and (E12):

\[ K = \left[ \frac{PH}{H^2 P + R} \right] \]

\[ \hat{x} = x_k - K(Hx_k - z_k) \]

\[ \hat{p} = P - KH \]

And finally in multidimensional form, with notation as for the system above:

\[ \hat{x}_{(k+1/k+1)} = \hat{x}_{(k+1/k)} - K \left[ H \hat{x}_{(k+1/k)} - z_{(k+1)} \right] \]  

\[ K_{(k+1)} = P_{(k+1/k)} H^T_{(k+1)} \left[ H_{(k+1)} P_{(k+1)} H^T_{(k+1)} + R_{(k+1)} \right] \]

\[ P_{(k+1/k+1)} = \left[ I - K_{(k+1)} H_{(k+1)} \right] P_{(k+1/k)} \]

These equations should be extended to include any further information available within the system. For example, when applied to modelling marine vehicles external forcing functions such as wind and tide will have to be catered for, any measurements made of these parameters will contain random noise, the variance of which will require inclusion in equations (E17), (E18) and (E19).
APPENDIX F

PUBLISHED PAPERS

The following papers have been published by the author in connection with the research described in this thesis.

Dove MJ  A Voyage Management System To Improve The Economy Of Ships. The 8th Biennial Seminar on The Economic Ship, Organised by The Institute of Marine Engineers and The Society of Naval Architects and Marine Engineers, Singapore, December 1987.

Burns RS  A Control and Guidance System For Ships In Port Approaches.


A VOYAGE MANAGEMENT SYSTEM TO IMPROVE THE ECONOMY OF OPERATION OF SHIPS

Michael J. Dove MSc PhD CEng MRINA MRIN

Roland S. Burns BSc MPhil PhD CEng MIMechE

Keith M. Miller BSc

Ship Control Group
Plymouth Polytechnic
Plymouth, United Kingdom

ABSTRACT

Working under contracts in conjunction with the United Kingdom Efficient Ship Programme the Ship Control Group is developing a Voyage Management System which will automatically guide a vessel from off the berth at the departure port to off the berth at the arrival port. To carry out this function the system will compute an accurate position fix and generate ship control commands in order to follow an optimum route. The system takes in to account predicted weather conditions and operates with the minimum of manual involvement to give an economic passage between ports.

The paper describes the extensive use of simulation and afloat facilities at Plymouth Polytechnic in the design of a Voyage Management System. Mathematical models developed in earlier research programmes have been used extensively, whilst Kalman filtering techniques, multivariable control processes, collision avoidance optimisation and weather routeing are being incorporated in the system.
NOMENCLATURE

a) Matrices and Vectors

A Discrete State Transition Matrix
B Discrete Control Matrix
C Discrete Disturbance Matrix
D Measurement Matrix
K Kalman Gain Matrix
Q State Error Weighting Matrix
R Control Weighting Matrix
r Desired State Vector
u Control Vector
v Noise Vector
w Disturbance Vector
x State Vector
x Best Estimate of State Vector
z Measurement Vector

b) Scalar Symbols

J Cost Function
k Integer Counters
t Time(s)

INTRODUCTION

A great deal has been written about the concept of the fully automated unmanned ship. Undoubtedly the technology exists to provide such a system and research and development is being undertaken in a number of centres around the world, with the objectives of eliminating or at least reducing the manning scales on merchant ships. Furthermore if one compares the maritime and aerospace industries it does appear that the aircraft industry is way ahead in the automation stakes. Is this necessarily the case, and if it is, then why? The answer to both of these questions is possibly bound up in the conservatism of the shipping industries. Proposals for integrated systems and automation appear qualified by statements which suppose that the industry is not yet ready for a system which removes the mariner from the command loop. Perhaps this is true, after all the aircraft pilot still has ultimate control over his aircraft and it would be reasonable to suppose that the travelling public would wish this to continue, in spite of the fact that statistics tell us automatic landing is safer than when the pilot is at the controls. Nevertheless recent accidents in the air and at sea continue to point to the fact that the majority of accidents have human error as the major factor. It has been suggested [1] that 85% of all marine collisions and groundings are due to human error. On these grounds alone there is a case for research and development into automating the control and guidance systems which are installed in ships. Furthermore this does not preclude relative levels of automation in the navigation and guidance process, neither does it preclude sub systems which advise the mariner of the action to take. For example, Automatic Radar Plotting Aids are now generally accepted as taking a great deal of tedious work away from the Officer of the Watch, whilst providing him with much more information than he could obtain manually. The work of the Ship Control Group is directed towards development of these concepts.

1-2.2
The Group's activities are divided into navigation, control and guidance, collision avoidance and weather routing. By the very nature of the research and development, mathematical modelling of ship systems is an all important part of the work. This paper describes the work undertaken to develop a voyage management system for automatic guidance of a ship through all phases of a passage, and sets out the case for considerable cost savings, without a decrease in safety, if more automatic systems are incorporated into the navigation guidance process.

THE SHIP GUIDANCE PROBLEM

In its research programmes the Ship Control Group has used stochastic optimal control theory [2]. An important part of this theory is the separation principle, which allows a given optimisation problem to be reduced to two other problems whose solutions are known, namely an optimal filter in cascade with a deterministic controller. Such a ship navigation and guidance problem is illustrated in Figure 1. The problem considered has thus been the design of an optimal multi-variable system to control simultaneously position and velocity of the vessel. The deviation from the desired values of these parameters can only be corrected, for most vessels, by operation of the rudder(s) and/or the main engine(s). A feature of an optimal system is that it will seek to maximise or minimise a global parameter, J, called the cost function or performance index. This is based upon the summation of the weighted errors over some time interval. Examples of the time interval might be the time to complete the pilotage phase of the voyage, the time to complete the oceanic phase of the voyage, or the total time from departure to arrival ports.

![Figure 1 The Automatic Navigation and Guidance Problem](image-url)
The pilotage, coastal and oceanic phases of the voyage may require different or modified cost functions. Whilst safety of the vessel, and hence collision avoidance, is paramount, the weighting of track keeping, position accuracy, course keeping, speed and other parameters will change during a passage. For example in the approaches to a port it is vitally important that a large vessel keeps to the buoied channel and hence automatic track keeping will have maximum weighting. In this instance the track is defined by the channel in to the port. However once in to open water the navigator has a wider choice available. There will still be limitations in coastal waters, but once clear of the coastline the minimum distance will involve a great circle track, possibly taking the vessel in to bad weather areas. Hence the shortest distance may not mean the minimum time at sea. The cost function will now change and as weather data is received the on-board computers will be required to update the best route in order to avoid the worst weather. Alternatively the cost function might now be required to minimise fuel consumption, entailing longer time at sea, but possibly reducing voyage costs. There is, after all, no point steaming at 20 knots, only to find that there is no berth available at the arrival port. So port knowledge (berths and bunkers) could provide limiting constraints which will affect the oceanic performance index.

In addition to minimising the errors in the output parameters, the optimal controller must also attempt to minimise the control effort, that is minimise rudder and main engine activity, so reducing wear and tear on such equipments as steering motors. The cost function is normally stated in the following quadratic terms;

\[ J = \int_{t_0}^{t_f} (x-r)^T Q (x-r) + u^T R u \, dt \]  

(1)

Q and R are usually diagonal matrices and the values of the individual elements reflect the importance of the parameters being controlled. For example in a track changing manoeuvre, large course errors will be incurred. If the track keeping elements in the Q matrix are weighted more heavily than the course keeping terms, then the majority of the control effort will be expended in reducing the track error, at the expense of course keeping. Similarly in the oceanic phase of the voyage if fuel consumption is more important than arrival time then the fuel element in the Q matrix can be suitably weighted. The R matrix gives weighting to the the components of the control vector \( u(k) \). For most shipping applications the controls are demanded rudder angle and main engine revolutions, although for specialist applications bow and stern thrusters might be incorporated.

Optimal filtering, using a Kalman-Bucy filter, is a stochastic technique which combines noise corrupted measurements of a dynamic system with other known information about the system, in order to obtain best estimates of the variables, or states, that govern the system. It should be noted, however, that the process assumes that the system is linear and the errors are gaussian. As a ship constitutes a non-linear system, when parameters such as large alterations of course and/or speed, shallow water effects, and trim are considered there must be some limitation to the technique. In the work undertaken by the Ship Control Group the problem has been overcome by assuming constant course and speed during each sample period [3]. This is reasonable providing sample times are small when compared with such factors as ship time constants and time between way points. In determining the value of the gain matrix consideration must be given to the control vector \( u(k) \) and
its associated control matrix $B(k, k+1)$. The overall filter equations are:

$$
\hat{x}(k+1) = A(k, k+1)\hat{x}(k) + B(k, k+1)u(k) + K(k+1)[z(k+1) - B(k+1)[A(k, k+1)\hat{x}(k) + B(k, k+1)u(k)]]
$$

(2)

Much attention was devoted to the choice of state variables and the states finally chosen were actual rudder angle, actual engine revolutions, $x$ coordinate of position, forward speed, $y$ coordinate of position, lateral speed, heading and yaw-rate. From these eight states a set of first order differential equations was used to define the ship, rudder and engine dynamics. These equations are expressed in discrete matrix form as:

$$
x(k+1) = A(k, k+1)x(k) + B(k, k+1)u(k) + C(k, k+1)w(k)
$$

(3)

Equations (1), (2) and (3) from the basis of the optimal filter and controller. The essential features of a ship automatic guidance system are shown in Figure 2.

---

Figure 2 The Ship Automatic Guidance System

With regard to position fixing at sea it is likely in the near future that the Global Positioning System (GPS) will become the main position fixing system, with accuracies acceptable during all phases of a voyage. However discussions are currently taking place on the future of the Decca Navigator and Loran. It does seem that Loran will emerge as an alternative or back-up navigation aid. There are still many political and technological problems in the way of complete Loran C coverage of European waters and the Mediterranean, so that it will probably be a long time before the Decca Chains disappear. These are prudent moves which will ensure that the maritime
nations are not entirely dependent upon a military satellite system, good though that system may be. It is also worth pointing out at this stage that, in addition to reducing the random errors associated with measurements of position and velocity, an optimal filter can provide estimates of states which are not measured, or where the navigation aid is temporarily out of action. In spite of GPS accuracy then, it is the contention of this Research Group that optimal filtering of position and velocity has a future [4].

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![Figure 3 Comparison of Simulated and Actual Turning Circles at 9 knots](image)

**Figure 3 Comparison of Simulated and Actual Turning Circles at 9 knots**

**AUTOMATIC GUIDANCE IN PORT APPROACHES**

Earlier work [5] has concentrated on the problem of guiding a vessel automatically along a predetermined track in the approaches to a port. This entailed a cost function where track keeping predominated, except during track changes, when course keeping becomes more important. After extensive computer simulation tests involving the Polytechnic's mainframe computer and ship simulation facilities, together with model trials on a reservoir, the development of the prototype navigation and guidance system was commenced in February 1987. The Group purchased two Acorn Work Stations (ACW) for software development. This microcomputer system was compatible with the mainframe facilities available, and was portable, so that development work could take place in the laboratory, using mainframe facilities whenever necessary. An ACW was then taken afloat for practical testing of the system.
In this way the design team were able to correct faults and make minor adjustments whilst a test programme was underway. The team has made use of the Polytechnic's hydrographic training vessel, an 11 metre twin-hulled vessel. Initially the requirement was to obtain a mathematical model for a multi-hull vessel. This was achieved by a series of turning circle and straight run manoeuvres in Plymouth Sound. Figure 3 shows a typical turning circle for the trials vessel against a turning circle for the model.

Figure 4 Passage in to the Port of Plymouth
On board the vessel, position data is derived from the Mark 53 Decca Navigator, whilst velocity is obtained from an electromagnetic log and magnetic compass. Rudder angle and engine revolutions are obtained from a displacement transducer and tachometer respectively, and thus six of the eight states mentioned in an earlier section of this paper are measured directly. The remaining two, lateral speed and yaw-rate are estimated by the filter. The measurement systems are interfaced to the ACW using a 6502 microprocessor in a Eurocard rack with A-D converter and serial interfaces. The best estimates of the states are then used in the optimal controller to produce demanded values of rudder angle and engine revolutions, and are therefore used to maintain the vessel on the correct path for entry into the port. Whilst the main objective is the safe passage of the ship along the navigable channel the cost function can be adjusted to ensure arrival off the berth at a predetermined time, so minimising the costs of associated port services such as tugs and personnel on the jetty. A typical test run is illustrated in Figure 4. Latitude and longitude is derived from the Mark 53 Decca Navigator. This, together with the filtered position fix is compared with the true position derived from the Trisponder. Trisponder is an accurate survey system, which obtains position by measuring ranges from fixed transmitting stations. It is unsuitable for marine navigation because of its range limitations. The researchers were, however, able to use it for reference as a chain exists within Plymouth Sound.

**Figure 5** Annotated Screen Display for Automatic Collision Avoidance
THE COASTAL PHASE

Once clear of the port the coastal phase of the voyage is undertaken using the same cost function as for the harbour phase. The vessel is automatically maintained on track using information from available navigation aids to minimise track and course error. Theoretical studies are underway to incorporate an Automatic Collision Avoidance System (ACAS). This project is being undertaken in two areas. One research programme is concerned with applications of Artificial Intelligence to the International Rule of the Road [6]. The work is at present in the early stages and computer simulations are being undertaken. An example of the type of computer simulation being undertaken is given in Figure 5. In another project researchers have been developing software to automatically guide the vessel when risk of collision or grounding exists [7]. Computer studies have shown that it is possible for the cost function to be modified when an approaching ship poses a threat to the safety of navigation. The on board computer is programmed to change the weighting to allow for a collision situation and for the vessel to take avoiding action. Trials will commence shortly to ensure that the automatic collision system can be integrated with the navigation and guidance systems.

Whilst the researchers are confident that the overall system is technically possible they are very aware of the many human and statutory difficulties which exist. What happens for example when an ACAS fitted ship is on collision course with a yacht and it is the duty of the smaller non ACAS vessel to give way? For these, and other reasons, the first stage of this development will be to advise the Officer of the Watch what action, if any, should be taken. As such this will be a development from the already successful automatic radar plotting aids which are available, but with the difference that the collision avoidance will be combined with navigation and guidance in a central display.

AUTOMATIC WEATHER ROUTEING

Having cleared the land the navigator will now wish to follow the most economic route to the coastline at the other end of an oceanic passage. Weather routeing [8] is now an established process in reducing overall running costs. In computer simulations the researchers have shown that it is possible for the cost function to be changed so that weather data transmitted via a satellite link can be used to plan and execute the most cost effective route for the vessel to follow. An ACW is being used for this component of the overall voyage management system. Once laboratory tests are complete this will also be taken afloat and linked to the navigation system. An indication of the route which might be advised is given in Figure 6, which shows the great circle route between Cork and New York, with the suggested route, taking into consideration the expected weather conditions for the next few days. The simulated optimum route is very close to that advised for the 37000 dwt Dart Atlantic for a 25 knot passage in November 1986. At the time large depressions were forecast for the great circle route. It must be emphasised that the route planning is advisory in the initial stages of the research. However it is intended that this information will also be fed to the optimal controller so that the vessel can automatically follow the most cost effective route during the oceanic phase of the voyage.

1-2.9
CONCLUSIONS

The Ship Control Group's ultimate aim is to produce a Voyage Management System which takes navigational data from such navigation aids as are fitted on the vessel, collision avoidance data from the ship's radars, together with meteorological data from shore stations via a satellite link. After appropriate filtering, this data will be used as input to a multivariable optimal controller which will maintain the vessel on the correct track between ports, with due consideration to safety, efficiency and economy of operation. The Group is some way towards achieving this aim, as results discussed in this paper have demonstrated. The navigation and guidance system has been tested afloat, whilst simulation studies have demonstrated that the automatic collision avoidance and weather routeing systems can be incorporated in to the overall voyage management system. The next step is to develop a complete prototype system and undertake extensive trials in coastal and oceanic waters.

Operators of today's ocean-going and specialist ships have numerous electronic aids available. The traditional role of each navigation aid has been one of a stand-alone unit with the mariner using his experience and training to coordinate the data from all the available sources in order to optimize vessel performance. As casualty statistics indicate however, when under stress or at times of peak work load, he is liable to be a poor coordinator of the information available to him, particularly when that information is from a number of different sources and in differing forms. Even during the less stressful oceanic phase of the voyage the amount of data...
available may lead to decisions which are not cost effective in terms of passage time or even estimated time of arrival, whilst on the coastal phase the economies of operation may conflict with prudent navigational practice. Furthermore the rapid developments in the electronics industries at large have affected the traditional role of navigators in the air and at sea. The aircraft industries have led the way, perhaps because they are less inhibited by tradition, perhaps because their needs have been more paramount. Now the maritime world is starting to follow suit. The research programme described in this paper is, it is hoped making some small contribution to the developments in this area by applying modern control technology and computing techniques to the problem of automatically conducting a ship safely and economically from port to port, whilst at the same time helping to reduce some of the stress which affects mariners at all stages of a voyage.

REFERENCES


A Control and Guidance System for Ships in Port Approaches

R S Burns, BSc, MPhil, PhD, CEng MIMechE, M J Dove, MSc, PhD, CEng, MRINA, MRIN, and K M Miller, BSc
Ship Control Group, Plymouth Polytechnic

SYNOPSIS
As part of the United Kingdom Efficient Ship Programme, the Ship Control Group at Plymouth Polytechnic was awarded a contract to develop a voyage management system for marine vehicles. The role of the system is to automatically guide a vessel during port approaches at the beginning and during port approaches at the end of a voyage. This paper gives an account of the processes used in modelling the Polytechnic test vessel and goes on to describe the design and implementation of the guidance system. Results from the commissioning trials are presented, together with a discussion on the subsequent performance evaluation.

INTRODUCTION
The automatic pilot (autopilot) for ship steering has its origin near the beginning of this century, following the invention of the gyrocompass. Elmer Sperry discussed the problems of automatic steering in 1922 as an application of the gyrocompass and by 1932, 400 of Sperry's systems had been installed on merchant ships throughout the world. The autopilot of this era was a simple mechanical proportional controller and did not always prevent transient oscillation.

From about 1950, autopilots employing proportional integral and derivative (PID) control actions were introduced, using analogue electrical hardware. Such systems were manually tuned to take into account the ship's dynamics and the effect of disturbances. Self-adjusting, or adaptive autopilots came into being in the mid 1970's, a typical example being the S G Brown 3000 adaptive autopilot.

Most existing commercial ship autopilots fall under the category of single-input, single-output control systems. These systems are designed to minimise the error in a single variable such as heading or distance off track that has occurred due to changes in the desired value or to some disturbance effects. The control action is taken without regard to its effect on any of the other system parameters.

Multivariable control theory views the global problem and attempts to formulate a control policy that minimises the errors in all the state variables according to some predefined order of priority. Further, an optimal controller will seek to maximise the return from the system for a minimum cost. It is possible to construct a deterministic optimal controller with the assumption that the states are measurable and noise-free, and that the ship dynamics are linear and time-invariant.

Over a number of years, the Ship Control Group at Plymouth Polytechnic has conducted extensive theoretical studies into ship guidance and control, and has demonstrated, in both computer simulation and free-sailing trials using a car ferry, that stochastic optimal control theory may be applied to the ship guidance problem. This paper describes the design, implementation and evaluation of the first system of its kind aboard a full-size ship.

Dr R S Burns did his engineering apprenticeship at Lucas Aerospace, where he subsequently worked as a Development Engineer. In 1965 he moved to British Oxygen as a Project Engineer, and in 1968 he became the Chief Development Engineer for Evered and Co. In 1970, Dr Burns moved to Plymouth Polytechnic, where he is currently the Principal Lecturer for control and instrumentation in the Department of Mechanical Engineering. His research interests include optimal control, intelligent systems, system identification, state estimation, computer modelling and simulation and computer aided engineering.

Dr M J Dove started his career as a Deck Officer in the Merchant Navy and served a commission in the Royal Navy from 1956 to 1972. In 1972 he became a lecturer at London Polytechnic and moved to the College of Technology, Southampton in 1973. Dr Dove moved to Plymouth Polytechnic in 1975 and is currently Principal Lecturer in the Department of Marine Science and Technology. His research interests include control and guidance of marine vehicles, mathematical modelling of marine vehicles, marine simulators, computer simulation of marine traffic and improvement of the accuracy of electronic position fixing systems. From 1979 to 1984 Mr K M Miller worked for the National Maritime Institute, initially as an Assistant Scientific Officer and then as the Scientific Officer. From here, he moved to British Maritime Technology as a Development Engineer. Since 1986 Mr Miller has been a Research Assistant for the Ship Control Group at Plymouth Polytechnic. His research interests include position fixing and state estimation at sea, and computer modelling of marine vehicles and their environment.

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SHIP MATHEMATICAL MODELS

Prior to the design of any control system, is the identification of a mathematical model that adequately defines the dynamic performance of the plant, in this case the response of a vessel to changes in control inputs (rudder and engines) and disturbance inputs (wind and tide).

Equations of motion
The ship is considered to be a rigid body with three degrees of freedom, in surge, sway and yaw. Ship motions in the other three degrees of freedom, roll, pitch and heave are considered small enough to be neglected. This gives rise to an Eulerian set of equations of motion which may be written in the form:

\[ \begin{align*}
\dot{x} - m v &= X \\
\dot{v} + m u &= Y \\
\dot{r} + I_z &= N
\end{align*} \]

(1)

Transfer function model
The sway and yaw expressions in equation set (1) may be combined to give the Nomoto ship transfer function:

\[ \frac{\phi_A(s)}{\delta_A(s)} = \frac{K_\phi(1 + T_1 s)}{s(1 + T_1 s)(1 + T_2 s)} \]

(2)

The constants \( T_1, T_2, T_3 \) and \( K_\phi \) are computed using the ship's mass, moment of inertia and dimensionless hydrodynamic coefficients.

State-space model
The terms on the right-hand side of equation set (1) represent the total hydrodynamic and aerodynamic forces and moments acting on the hull. For an accurate description of the manoeuvring characteristics during tight turns, both linear and non-linear hydrodynamic coefficients must be employed, and should be re-dimensionalised at each sampling instant.

A multivariable model may be constructed using the state vector:

\[ x^T = (\delta_A \ n_A \ x \ u \ v \ \phi_A \ r) \]

(3)

This state is affected by the forcing function:

\[ u^T = (\delta_D \ n_D \ u \ c \ \varphi_c \ s) \]

(4)

Equation set (1) represents the state equations for the ship and is expressed by the state matrix vector differential equation:

\[ \dot{x}(t) = F(t) x(t) + G(t) u(t) \]

(5)

It is convenient to partition the \( G \) matrix in terms of the control forcing function \( \delta_c \) and \( n_c \) and the disturbance forcing functions \( u_c, v_c, u_h \) and \( v_h \) so that:

\[ \dot{x}(t) + F(t) x(t) + G_c(t) u(t) + G_D(t) w(t) \]

(6)

The corresponding discrete solution is:

\[ x((K + 1)T) = A(T, KT) x(KT) + B(T, KT) u(KT) + C(T, KT) w(KT) \]

(7)

Such a mathematical description accurately represents three degree of ship motion in tight manoeuvres and can be expanded to include roll, pitch and heave in a six degree of freedom model if required.

Mathematical model for test vessel
The Polytechnic hydrographic training craft ‘Catfish’, was used as the test vessel. The particulars of the vessel are given below:

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hulls</td>
<td>2</td>
</tr>
<tr>
<td>Number of screws</td>
<td>2</td>
</tr>
<tr>
<td>Number of rudders</td>
<td>2</td>
</tr>
<tr>
<td>Individual rudder area</td>
<td>0.395 m²</td>
</tr>
<tr>
<td>Displacement</td>
<td>8.5 tonnes</td>
</tr>
<tr>
<td>Overall length (including rudder)</td>
<td>11.17 m</td>
</tr>
<tr>
<td>Overall width</td>
<td>4.3 m</td>
</tr>
<tr>
<td>Hull centreline separation</td>
<td>3.7 m</td>
</tr>
<tr>
<td>Main engines</td>
<td>Volvo Panther</td>
</tr>
<tr>
<td>Power at 2500 rev/min</td>
<td>25 kw</td>
</tr>
<tr>
<td>Engine/screw gear reduction</td>
<td>1.66:1</td>
</tr>
<tr>
<td>Forward speed at 2200 rev/min</td>
<td>8.9 knots</td>
</tr>
<tr>
<td>Forward speed at 1100 rev/min</td>
<td>4.2 knots</td>
</tr>
</tbody>
</table>

The hydrodynamic coefficients were computed theoretically, and fine-tuned by comparing results from computer simulations with real test data. Turning circle manoeuvres provided information regarding yaw and sway coefficients, whilst acceleration and steady forward speed tests gave the surge coefficients. During the sea trials, data was logged every 15 seconds, measurements being taken of speed, heading and position. In order to achieve a high accuracy, position fixes were taken employing Trisponder, a microwave measurement system used for hydrographic surveying. Figure 1 shows a comparison between simulated and actual turning circles at 9 knots, 5° starboard rudder.

![Fig. 1: 'Catfish' mathematical model evaluation](image-url)
The Nomoto transfer function constants for the test vessel are:

\[ K_a = -0.3617 \, \text{s}^{-1} \]
\[ T_1 = 0.6038 \, \text{seconds} \]
\[ T_2 = 2.581 \, \text{seconds} \]
\[ T_3 = 1.658 \, \text{seconds} \] \hspace{1cm} (8)

These were computed for a forward speed of 4.455 m/s. The multivariable state equation set (5) for the above forward speed is given by:

\[
\begin{bmatrix}
\dot{\delta}_A \\
\dot{n}_A \\
\dot{x} \\
\dot{y} \\
\dot{\phi}_A \\
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.004 & 0 & -0.138 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.573 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.273 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\delta_A \\
n_A \\
x \\
y \\
\phi_A \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.05 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\delta_D \\
n_D \\
\end{bmatrix}
\]

The matrix equations (9) do not include disturbance inputs.

**THE SHIP GUIDANCE PROBLEM**

**Single-input single-output systems**

Most commercial autopilots control a single variable, normally heading. Figure 2 shows the block diagram of a typical system, the transfer functions \( G_1(s), G_2(s), G_3(s) \) and \( H_1(s) \) represent the dynamics of the controller, rudder, ship and gyrocompass respectively. These are the essential elements for course-keeping control. For track-keeping, the transformation function \( G_5(s) \) is required in the forward path and \( H_2(s) \) becomes the track measurement system.

**Multivariable systems**

In its research programmes the Ship Control Group has used stochastic optimal control theory. An important part of this theory is the separation principle, which allows a given optimisation problem to be reduced to two other problems whose solutions are known, namely an optimal filter in cascade with a deterministic controller. Such a ship navigation and guidance problem is illustrated in Fig. 3. The problem considered has thus been the design of an optimal multivariable system to simultaneously control position and velocity of the vessel. The deviation from the desired values of these parameters can only be corrected, for most vessels, by operation of the rudder(s) and/or the main engine(s). A feature of an optimal system is that it will seek to maximise or minimise a global parameter, \( J \), called the cost function or performance index. This is based upon the summation of the weighted errors over some time interval. Examples of the time interval might be the time to complete the piloting phase of the voyage, the time to complete the oceanic phase of the voyage, or the total time from departure to arrival ports.

In addition to minimising the errors in the output parameters, the optimal controller must also attempt to minimise the control effort, that is minimise rudder and main engine activity, so reducing wear and tear on such equipment as steering motors. The cost function is normally stated in the following quadratic terms:

\[
J = \int_0^T \left\{ \sum (x - r)^T Q (x - r) + u^T R u \right\} \, dt
\]  

(10)

\( Q \) and \( R \) are usually diagonal matrices and the values of the individual elements reflect the importance of the parameters being controlled. For example, in a track-changing manoeuvre, large course errors will be incurred. If the track-keeping elements in the \( Q \) matrix are weighted more heavily than the course-keeping terms, then the majority of the control effort will be expended in reducing the track error, at the expense of course-keeping. Similarly, in the oceanic phase of the voyage if fuel consumption is more important than arrival time then the fuel element in the \( Q \) matrix can be suitably weighted. The \( R \) matrix gives weighting to the components of the control vector \( u(k) \).
The control law for a deterministic optimal controller is:
\[ u_{np} = -R^{-1}G^TW(x - r) \]  
(11)
where \( W \) is the solution of the matrix Riccati equations.

Alternatively:
\[ u_{np} = -S(x - r) \]  
(12)
when
\[ S = -R^{-1}G^TW \]

Optimal filtering, using a Kalman-Bucy filter, is a stochastic technique, which combines noise corrupted measurements of a dynamic system with other known information about the system, in order to obtain the best estimates for the variables, or states, that govern the system. It should be noted, however, that the process assumes that the system is linear and the errors are gaussian. As a ship constitutes a non-linear system, when parameters such as large alterations of course and/or speed, shallow water effects, and trim are considered, there must be some limitation to the technique. In the work undertaken by the Ship Control Group the problem has been overcome by assuming constant course and speed during each sample period.

**CONTROLLER DESIGN**

**Single-input single-output controller**

The transfer functions shown in Fig. 2 for the test vessel are:

\[ G_2(s) = \frac{1}{s + 1} \]
\[ G_3(s) = \frac{0.3848(s + 0.603)}{s(1 + 1.656)(s + 0.3874)} \]
\[ G_4(s) = \frac{u}{s} \]
\[ H_4(s) = 1 \]  
(13)

A PD course-keeping control would have the following control action:
\[ \delta_D = K_1(\varphi_D - \varphi_A) - K_2\dot{\varphi}_A \]  
(14)

It is possible to demonstrate that PD control for track-keeping provides an unstable solution for all values of \( K_1 \). Under these conditions, it is necessary to introduce a cross-track acceleration term, or PDA control action of the form:
\[ \delta_D = K_1(y_D - y_A) - K_2\dot{y}_A - K_3\ddot{y}_A \]  
(15)
when
\[ K_2 = 2T_dK_1 \]
and
\[ K_3 = T_d^2K_1 \]

then
\[ \delta_D = K_1 \left\{ y_D - \left(1 + 2T_d\dot{s} + T_d^2\ddot{s}\right)y_A \right\} \]
or
\[ \delta_D = K_1 \left\{ y_D - \left(1 + T_d\dot{s} + (1 + T_d\dot{s})y_A \right) \right\} \]  
(16)

When \( l/T_d = 0.603 \), a triple zero occurs at \( s = -0.603 \). Figure 4 shows the root locus diagram and \( K_1 = 0.451 \) provides acceptable closed-loop poles.
Multivariable optimal control

If the optimal control law (12) is combined with the state equations (5) then:

\[ \dot{x} = Fx - GS(x - r) \]

or

\[ \dot{x} = (F - GS)x - GSr \]  \hspace{1cm} (17)

where the term \((F - GS)\) may be identified as the closed-loop state matrix of the optimal system. The optimal closed-loop eigenvalues (poles) are then given by:

\[ |sI - (F - GS)| = 0 \]  \hspace{1cm} (18)

It is apparent that when \(F\) and \(G\) are time-invariant the location of the optimal closed-loop poles depend upon the value of the feedback matrix \(S\), which in turn is dependent upon weighting matrices \(Q\) and \(R\). There exists then, an infinite number of closed-loop poles, each being a measure of the optimality of the system as defined by relative weightings of \(Q\) and \(R\).

The weighting matrices \(Q\) and \(R\) are diagonal and of the form:

\[ Q = \text{diag}(q_{11}, q_{22}, q_{33}, q_{44}, q_{55}, q_{66}, q_{77}, q_{88}) \]

\[ R = \text{diag}(r_{11}, r_{22}) \]  \hspace{1cm} (19)

Elements \(q_{11}\) and \(q_{22}\) are the rudder and main engines weightings and may be set at zero since the purpose of the control system is to employ the rudder and engines as control inputs, not as controlled variables. Surge dynamics are affected by variations in \(q_{12}\) and \(q_{15}\) together with \(r_{12}\). The sway and yaw weightings are \(q_{15}\), \(q_{16}\), \(q_{17}\), and \(q_{18}\) respectively. Due to the coupling of sway and yaw, these together with \(r_{15}\) closely interact, and changing any one value will affect all sway and yaw terms in the feedback matrix.

Values of the weighting parameters must be selected to provide an acceptable optimal closed-loop pole assignment. This is not a straightforward task. Using techniques established in the design of optimal controllers for previous vessels, the following control law was evaluated:

\[ Q = \text{diag}(0, 0, 0, 0, 0, 0, 0, 0) \]

\[ R = \text{diag}(1, 1) \]  \hspace{1cm} (20)

\[ S = \begin{bmatrix} 0.177 & 0 & 0 & 0 & -0.0115 & 0.1299 & -0.708 & -0.404 \\ 0 & 0.0135 & 0.221 & 1.578 & 0 & 0 & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (21)

For the values of \(F\) and \(G\), given in equation (9), the closed-loop eigenvalues become:

\[ s = -1.352 - 0.4098 - 0.0819 - 0.998 \]

\[ -0.500 - 0.0069 - 0.138 \]

\[ 0 \]  \hspace{1cm} (22)

Taking a simplistic view, feedback matrix \(S\) may be considered as the combination of a PD course controller \((s_{15}, s_{25})\), PD track controller \((s_{17}, s_{27})\) and PD forward position controller \((s_{35}, s_{37})\). If the course terms are removed, then, as previously explained, the track-keeping law becomes unstable. Thus, the course terms are an integral part of the multivariable optimal control policy.

Figure 5 shows a computer simulation of the test vessel in the approaches to Plymouth using the controller feedback matrix \(S\) given in equation (21). A desired state vector \(r\) is defined in terms of position, heading and speed. Initially, the test vessel has a large cross-track error, but the control system quickly pulls it onto track, but in doing so incurs an along-track error. To cater for this, the control system increases the engine speed, so when the first turn to port is reached, the vessel is in the correct position along the track. When Drake’s Island is passed, a sharp turn to port is demanded, resulting in both cross-track and along-track errors.
CONTROLLER IMPLEMENTATION

Upon completion of the theoretical evaluation, the system was implemented on the test vessel. The hardware problems to be overcome were:

1. Availability of suitable measurement systems.
2. Provision of a computer with sufficient power to perform the necessary optimal filter and optimal control computations in real time.
3. Installation of rudder and engine control servos.
4. Data communications interface.

All of this had to be achieved within a finite budget, and Fig. 6 is a schematic diagram of the system selected.

A Mk 53 Decca receiver was employed to monitor the vessel's position (Trisponder being used as a reference). The internal filter in the Mk 53 receiver was bypassed, only the raw hyperbolic data (GLOP, PLOP) being used. The pulse log speed measurement system and the 2 phase fluxgate compass were interfaced to the Mk 53 receiver.

A Eurocard rack system comprising of a 6502 processor, a serial I/O board and a DAC/ADC board was used to handle data communication. Position, speed and heading information was supplied on an RS 232 line to the serial I/O, and analogue rudder and engine speed signals to the ADC.

Processed information was relayed to an Acorn Cambridge Workstation (ACW) using an RS 232 serial interface. The optimal filter and control computations were performed by the 32 bit workstation processor.

Rudder and engine commands were relayed from the ACW along a serial line to the Eurocard system, and out through the DAC port as analogue demand signals to the rudder and engines, both of which contain feedback loops.

The rudder command signal was compared with the output from a linear variable differential transformer (LVDT) connected to the rudder, and the difference used to actuate a hydraulic pump and ram. The engine command signal was compared with the output from the engine tachometers, and the difference used to operate a ball and screw dc servo connected to the throttle linkages.

PERFORMANCE EVALUATION

In assessment of system performance it must be clearly understood that:

1. Any difference between filtered measurements and actual states of the vessel is a measure of performance of the optimal filter.
2. Any difference between filtered measurements and desired states is a measure of performance of the optimal controller.

The optimal filter parameters were obtained from knowledge of the known statistical errors of the measurement systems. Figure 7 shows data taken from a typical passage into the Port of Plymouth. It will be seen that there is good agreement between the filter output and measurements from the Trisponder.

When attempting to quantify the 'goodness' of a control system, it is convenient to consider an index of performance that is related to the errors that exist between desired and actual values over the control period. A set of performance indices may be defined:

\[ J_u = \int_0^T (u_D - u_A)^2 \, dt \]
\[ J_y = \int_0^T (y_D - y_A)^2 \, dt \]
\[ J_{\psi} = \int_0^T (\psi_D - \psi_A)^2 \, dt \]
\[ J_b = \int_0^T \delta_A \, dt \]
\[ J_n = \int_0^T \eta_A \, dt \]

The above indices may be considered as the constituent elements of the global index \( J \) given in equation (10).

The controller parameters were fine-tuned by starting with the \( S \) matrix given in equation (21) and performing small perturbations of each parameter about the theoretical value to achieve minimum values of the relevant performance indices. This was achieved by conducting a series of trial runs with identical desired state values and initial conditions, over the same control time period.

Table I shows results obtained whilst tuning the track-keeping controller gains and Table II provides results for the forward speed/position controller gains.

Figure 8 shows a set of results from a typical run. From the results presented in Tables I and II, the controller parameters in Run 12 (Table I) and Run 5 (Table II) provide a combination for a minimum for the global performance index \( J \).
Table II: Tuning forward speed position controller gains

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Controller gains</th>
<th>Performance indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{g3}$</td>
<td>$S_{g4}$</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.22</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>0.44</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Hence the optimal feedback matrix is:

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.007 & -0.5 & -0.8 & -3.5 \\ 0 & 0 & 0.22 & 0.5 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(34)}$$

CONCLUSIONS

The Ship Control Group's ultimate aim is to produce a control and guidance system which takes navigational data from such navigation aids as are fitted on the vessel, collision avoidance data from the ship's radar, together with meteorological data from shore stations via a satellite link. After appropriate filtering, this data will be used as input to a multivariable optimal controller which maintains the vessel on the correct track between ports, with due consideration to safety, efficiency and economy of operation. The group is some way towards achieving this aim, as results discussed in the paper have demonstrated.

It has been shown that a track-keeping control policy may be realised using a single-input, single-output transfer function approach. Such a control law, by its very nature, can only employ errors in the single variable, together with its time derivatives, i.e. cross-track error, velocity and acceleration.

These constraints are not imposed upon a multivariable control strategy, the control law being a function of all the selected state variables. But are the two solutions wildly different? Careful examination reveals that, perhaps as expected, they are not. For small heading errors, the cross-track velocity approximates to $u$ and the cross-track acceleration to $\Psi_A$. In other words, cross-track acceleration feedback and yaw-rate feedback have precisely the same effect.

The question also to be answered is why the finally selected feedback matrix $S$ in equation (24) differs from the theoretical prediction given in (21). These differences must be related to the accuracy of the model used for simulation purposes, and the assumptions made in the design analysis. It can be seen from Fig. 1 that the hydrodynamic modelling of the hull is quite accurate. What was discovered, however, after the theoretical analysis had taken place, was that the closed-loop time constants of the rudder and engines were significantly larger than had been assumed. In fact, the rudder did not behave in the manner of a first-order system at all, but had a constant velocity of 5° per second, irrespective of demand level or differences between demanded and actual rudder angle. Another problem not taken into account at the design stage was that the sampling time was controlled by the Mark 53 Decca receiver. This was nominally 1.56 seconds, but on occasions could be multiples of this. Hence, during a run, there were times when the sampling period was different to that used in simulation, and is the most probable reason for the differences between predicted and selected values used in the feedback matrix $S$.

In evaluating the optimal control parameters, it was assumed that the system was linear and time-invariant. It is well known that a ship is neither of these, and it might seem reasonable to argue that this assumption could lead to errors at the analysis stage. The mathematical model used in the simulation, however, was both non-linear and time-variant and was employed extensively to investigate the need for controller adaptation during (a) speed changes, and (b) tight manoeuvres. It was concluded that for the test vessel being used, over speeds of between 4 knots and 9 knots and for manouevres involving rudder angles of +20°, controller adaptation was not required.

REFERENCES


NOMENCLATURE

Matrices and vectors

A: Discrete state transition matrix.
B: Discrete control matrix.
C: Discrete disturbance matrix.
F: Continuous time system matrix.
G, G_e, G_d: Continuous time forcing matrices.
I: Identity matrix.
Q: State error weighting matrix.
R: Control weighting matrix.
S: Desired state vector.
T: Feedback gain matrix.
U: Control vector.
Noise vector.
Riccati matrix.
Disturbance vector.
Ship and earth-related state vector.
Best estimate of ship and earth related state vectors.

\( y, x_0, x \)  

\( q_{10}, q_{11} \)  

Elements of state error matrix.

\( r, r_{12} \)  

Yaw rate.

\( s \)  

Elements of control weighting matrix.

\( t \)  

Laplace operator.

\( T, T_1, T_2 \)  

Continuous time.

\( u, u, u' \)  

Sampling time interval.

\( v, v, v' \)  

Time constants.

\( x, y, z \)  

Forward velocity of ship, current and air.

\( X, Y, Z \)  

Lateral velocity of ship, current and air.

\( \alpha, \beta, \gamma \)  

Ship related cartesian coordinate system.

\( \delta, \delta' \)  

Total forces and moments in surge, sway and yaw.

\( \phi, \phi' \)  

Actual and demanded tracks.

\( \theta, \theta' \)  

Actual and demanded rudder angles.

\( \theta, \theta' \)  

Actual and demanded headings.
Kalman Filters in Navigation Systems

Michael J. Dove and Keith M. Miller

(Department of Marine Science and Technology, Polytechnic South West)

This paper puts forward a case for more precise navigation at sea. It commences by briefly outlining existing and forthcoming navigation aids. The integration of marine navigation systems, and in particular the use of Kalman filtering is then discussed. After outlining the developments in the aerospace industries, the authors suggest why marine developments have not been so rapid as those in the air. The idea of automatic collision avoidance is also raised and a method for tracking radar targets discussed. Finally, the authors survey marine applications of Kalman filtering techniques, including some references to their own work, and assess the need for further research and development.

1. INTRODUCTION. In spite of modern electronic aids to navigation, accidents still occur at sea. These usually take the form of collisions between vessels or groundings and 90 per cent occur in coastal waters. Cockcroft has established that in the 10 years between 1973 and 1982, 0.084 per cent of trading ships were lost due to collisions, 57.5 per cent of these occurred in restricted visibility, a vast improvement on the figure of 75 per cent prior to 1973. It should be noted that marine traffic separation zones were introduced in some coastal areas at that time. Cockcroft goes on to suggest some reasons for the errors which caused the latest collisions. Of those which occurred in clear visibility, 75 per cent were during the hours of darkness, and many in areas of low traffic density, suggesting the predominant cause is a poor visual look-out. He goes on to state that in several instances the collision took place immediately before a change of watch. In restricted visibility the time of day is shown to have little or no effect on the likelihood of a collision, and over 85 per cent take place between vessels travelling in opposite, or nearly opposite directions. Such occurrences have been reduced by the implementation of traffic separation zones, but it is not possible to establish these schemes in all areas. In all conditions human failure is almost invariably a factor, usually in the form of negligence, ignorance of the International Regulations for Prevention of Collisions at Sea or improper use of equipment. Trousse sympathizes with the navigator and suggests that his job is being made increasingly more complicated in two ways, first, with all the extra equipment being installed on the bridge, both to assist him with the task of navigating and to monitor the engine room and cargo, and secondly in the shortening of his decision time with the increase in ship speed and traffic mass. Paffett and Paymans have independently produced schemes to overcome the former problem of density of information being presented to the navigator on which he has to make a decision. They have suggested an in-depth study into the ergonomics of ships' bridge design. This would incorporate an investigation into engineering, systems analysis, anthropology, psychology and physiology to define basic characteristics of shipborne operators and their requirements. Paffett goes
on to suggest a paged computer system, bringing the information from all sensors to computer terminals, placed about the ship as required. One system developed along these guidelines is the MANAV integrated navigation system by Racal-Decca. Millar and Hansford have described the design process and final product, which consists of an automatic plotting table with standard paper charts, a computer terminal for user interface and display together with an automatic radar plotting aid (ARPA). Ship's officers who have used the system have described it as useful, effective and easy to use. Some criticisms have also been made, mainly in the lack of ability to interface the system to some of the common navigation aids. It is a small step from this approach to automatically monitor and predict the movement of other vessels, this can be achieved by processing data acquired from the ARPA and advise a course of action to be taken. This would help to alleviate the second of the problems suggested by Troussue. The requirements of such a system would be the precise location and state of the vessel on which it is installed, projected tracks of other vessels in the vicinity, knowledge of the local topology and a basis on which to make decisions to avoid collisions and grounding. The information required is available from existing navigation aids but the problems occur in putting this to use to evaluate the most probable state of own ship and in the decision-making process.

2. BRIEF SURVEY OF MARINE ELECTRONIC NAVIGATION SYSTEMS.

The development of modern electronic navigation began in the period 1939-45, to meet the exacting demands of the Second World War. Developments of this period form the basis of many of the systems in use today. Beck outlines the history of some of the systems which have evolved, including the Decca Navigator. This was developed in the United Kingdom and was in use for the D-Day landings of 1944, whilst Loran was developed in the USA and was in use prior to 1945. Before the development of suitable frequency standards the direct measurement of range was impractical, and hence these early systems relied on the measurement of phase or time difference between two radio signals so that position fixes were related to hyperbolic position lines. The two systems discussed still rely on these basic hyperbolic principles, but improvements have included the display of position in geographical coordinate form.

A number of individual systems are now available to the commercial operator and each has its advantages and disadvantages. Loran coverage, for example, is limited almost entirely to the northern hemisphere, and whilst Omega provides world-wide coverage, it is insufficiently accurate for inshore navigation. Decca is sufficiently accurate for coastal navigation, but accuracy falls off with range and each chain covers only a small area. The Transit Satellite System is also sufficiently accurate, but the time between satellite passes makes it unsuitable for coastal navigation. A typical fit in a British foreign-going merchant ship would thus comprise a gyro compass, with autopilot, electromagnetic and/or Doppler log, Decca Navigator, Loran C together with Omega and/or Transit. This would give the navigator world-wide coverage and sufficient accuracy. Radar or ARPA and a radio direction finder would also be fitted.

The accuracy which will be available from the Navstar or Global Positioning System (GPS) has yet to be established. Cook suggested that positional errors of
less than 20 metres will be achieved. Henderson and Strada\(^8\) give details of a small scale sea trial in which a mean distance of 25.3 metres was claimed for passages in and out of San Diego Naval Base in the USA. More recently Napier and Ashkenazi\(^9\) have suggested that the codes used will be downgraded to meet the original commercial design specification giving an instantaneous position to between 100 and 250 metres. Current policy dictates 100 metre spheroidal root mean square (s.r.m.s.) accuracy to reduce the threat to security. They also state that Omega and Loran are due to be phased out shortly after the full introduction of GPS. Jorgensen\(^10\) backs up their views and points out that the specified 100 metre accuracy is 2 d.r.m.s. (twice the root mean square) when using four satellites to obtain a three-dimensional fix and ignoring the vertical component. The 100 metres therefore corresponds to a 2 per cent probability of the navigational error on the horizontal plane exceeding 100 metres. The system can be operated in a differential mode, which uses two receivers, one fixed at a previously surveyed location, the other where the position is required. Nolan and Carpenter\(^11\) have shown that over a period of 70 seconds a position accurate to between 4 and 5 metres is available. Brown and Hwang\(^12\) have increased the technology to obtain centimetre accuracy, but the position takes 10 minutes to calculate on a computer. It is unlikely that differential mode of operation will be used at sea for purposes other than surveying, but the navigator will have instantaneous world-wide coverage with accuracy between 100 and 250 metres. It is interesting to note that in daytime, the accuracy of the Decca Navigator is better than 100 metres (d.r.m.s.) within approximately 300 kilometres of the master transmitting station in a typical array, and considerably better (25 m) close to the centre of the chain.

With the advent of GPS the future of the Decca Navigation System (DNS), with its ageing transmitter stations requiring excessive maintenance and manufacturers other than Racal Marine Electronics now being able to build and sell receivers, is also in doubt. This question is not only on the minds of owners of trading vessels, but also concerns small boat owners. Dahl\(^13\) estimates that by 1990, 100,000 Decca sets will be installed on leisure craft. He goes on to suggest that the leisure user’s requirements for navigational aids are often more demanding than those of the professional who, in general, has better equipment and is better trained for its use. For obvious safety reasons, it is important that the leisure user is provided with adequate navigational aids. Considering all the navigation aids available to, or likely to be available to, the navigator within the immediate future, only Decca will give a fix accuracy of better than 100 m, and this will only be attained during daytime and in areas close to the transmitters. The shipowner has therefore been left with a difficult decision when choosing suitable navigation aids. To further complicate the problem the selection of instrumentation installed on vessels has often been governed by political and financial considerations rather than by sound technological judgements. No single navigation aid is capable of satisfying the requirements of collision-avoidance systems and advanced track-keeping autopilots\(^14\) currently in the research and development stage. However, integration of a combination of aids is likely to give the desired result.
3. INTEGRATION OF NAVIGATIONAL DATA. Single system deficiencies have led to the development of integrated systems for world-wide use. Systems such as MANAV discussed earlier and Racal's MNS2000 are capable of receiving data from a number of aids, but only give a position update from the selected source. The accuracy of the position can be improved upon by combining the data received from a number of navigation aids. First consider the actions a ship's officer might take to fix the position of his vessel. He may plot a number of fixes obtained from various sources. For example, from log, compass and a knowledge of the set and drift of the current he can derive an estimated position from a previous fix, another fix may be obtained from radar information, and finally one from an electronic aid such as DNS. Using his knowledge of the likely random errors in all three fixes, he can take a weighted mean to establish the most probable position of his vessel. One of the requirements of an integrated system then, is to minimize in some way the random errors. The Decca Navigator Co. suggest that a Gaussian distribution gives the best fit for the spread of random errors in radio navigation aids and this suggests the problem can be stated in terms of minimizing the variance, which has led to the use of Kalman-Bucy filters. These have been developed extensively for aerospace, and latterly for marine, navigation since the publication of the original works by Kalman, and Kalman and Bucy.

A set of papers covering the development of the Kalman filter from the theory through to a vast range of applications is given by Sorenson, while Scovell et al. and Mattin are concerned with implementation. Following from the original works in 1960 and 1961, Kalman filters were in operational use in space flight in 1963. Further development saw their use in long-range missiles, later still in military aircraft and then in short-range missiles. The techniques have now been developed for commercial systems and are finding increasing use in signal processing. For example, in the high precision differential GPS application to geodosy given earlier by Brown and Hwang, banks of Kalman filters are used. In the marine environment applications have been suggested in specialist areas such as hydrographic survey and dynamic positioning. For general navigation and integrated navigation systems, Dove suggests the use of Kalman filter techniques. Gonthier and Ayers present two filter designs for such systems and perform comparison tests, while Liang and McMillan have integrated dead reckoning and Omega using several algorithms and tested each in simulation. Danson and Kibble and Dove et al. have been concerned with the precise navigation of a vessel in the pilotage and berthing stages of a voyage. Abbott and Gent, whilst acknowledging that Kalman filters have proved to be very successful in many applications, for example in space, where sudden changes to the environment do not occur, warn that in other applications their inability to adapt to such changes has led to many failures. This is borne out by the authors' recent experiences when undertaking trials in UK waters in two instances, both on different vessels and in different areas.

4. AEROSPACE APPLICATIONS. A brief history of integrated navigation systems within the aerospace industry has been given by Stokes and Smith in which they include applications of Kalman filtering and in particular discuss one
such system developed at the Royal Aircraft Establishment. The basic Kalman filter is designed for linear systems. However, real-world systems have non-linear dynamics or measurements. Alternatively when knowledge of the system is incomplete it may be necessary to estimate not only the time-varying states of the system, but also some of the parameters which govern the system dynamics. The Kalman filter can readily be modified to cope with such situations, thus making the filter applicable to a wide range of situations which are not of a simple linear form. The modified filter for non-linear systems is known as an Extended Kalman filter (EKF). Robbins has given details of the derivation of the EKF for use with non-linear systems, and describes a particular application in the estimation of aerodynamic derivatives from flight trials data. Ohlmeier has devised an EKF to estimate the state vector of bank-to-turn missiles, passing the estimate on to three types of optimal controllers to test the performance of the guidance laws used. An integrated system developed by Leach and Maskell combines VLF phase difference measurements with Doppler radar and magnetic compass using EKF. The system has further been used to navigate a research aircraft. Errors are known to occur within the VLF phase measurements and an adaptive approach has been used to detect these contingencies in flight, that is certain matrix elements of the filter are modified in order to maintain navigation accuracy.

The application of Kalman filtering to the alignment of inertial navigation systems was suggested by Weston. More recently Grewal et al. describe the development of dual extended Kalman filters for the calibration and alignment of complex inertial guidance systems, one for accelerometers and one for gyros and alignment. A technique for generating trajectories that will provide observability of instrument errors is also given. Richman et al. point out that applying similar technology to strapdown components is an effective way of reducing the avionics cost of precision guided weapons. They go on to suggest that this can also result in a performance which is superior to that obtained with more expensive equipment. Johnson has tackled the problem of alignment and calibration of such systems in vehicles launched from a carrier aircraft, and here also methods for calibrating unknown instrument coefficients have been developed.

The integration of inertial navigation systems with other aids has also been investigated. Griffiths et al. have used linear covariance analysis to predict the performance of such systems and cases where the associated Kalman filter is both matched and mismatched to the inertial system are considered. More recently, the possibilities of integrating GPS with inertial systems have been explored by Nielson et al. Integration allows in-flight alignment of the inertial navigation system, and in addition, feedback from the inertial system to the GPS receiver improves its ability to reacquire satellite signals after outages. Full scale tests have been conducted on the system which combines the accuracy of GPS with the jamming immunity of inertial navigation.

Kalman filtering techniques have also been applied to modern terrain aided navigation systems. Hostettler and Andreas and also Mealy and Wang Tang have used EKF to handle the highly non-linear terrain measurement function. In both cases the use of banks of reduced order filters has been investigated to
improve the systems convergence from large initial position uncertainties. Chen et al. mention the use of Kalman filters in the SITAN system, going on to explore the convergence region of the system. One method discussed is parallel filtering with adaptive sequential decision. The performance is described and compared with other methods.

5. MARITIME APPLICATIONS. Whilst the references given above for aerospace applications are far from exhaustive they do suggest that the Kalman filter has found many useful applications in the aircraft world, where sudden environmental changes do not occur. However, in marine operations where disturbances can change quickly, applications do not appear to have been so successful, although this is not true of the offshore oil industry where modern dynamic positioning (DP) systems employ Kalman filters extensively. Tyssø, although discussing modelling and parameter estimation of a ship’s boiler, states that a successful application of the EKF method is strongly dependent on a reasonable choice of design parameters and should include a precise knowledge of the noise covariance matrices. He goes on to suggest that choice of initial estimates is critical. At sea then if the initial values of a current, for instance, are incorrect, or change suddenly, the best estimates of position and velocity may diverge. A general analysis of the convergent properties of the EKF method is given by Ljung.

In marine navigation, Kalman filtering can be applied to two independent navigation aids, or to a range of sensors, and provided the statistical information required is correct will give a result which minimizes the variance of the random errors. McPherson describes a system designed, built and in use in large oil tankers. However, two independent position fixing systems are not available in all operational areas of trading vessels, unlike other maritime applications such as hydrographic survey, where it is common practice to monitor position from two different systems, although the data received from the back-up system is often discarded. Bennett gives an algorithm to combine the data received using Kalman filtering and thus to obtain a more accurate position of the vessel. On a trading vessel it is, however, always possible to obtain an estimate of position from DR. This can be combined with any other data available to update the positional information. For precise navigation in the approaches to a port, when the vessel may be undertaking various manoeuvres, the traditional DR update from log and compass alone is inadmissible. Cross has devised a scheme for improving the estimate using additional velocity data obtainable from a dual axis doppler log. The positional information is now improved, but further state parameters may be required in control applications. For example, in the automatic guidance system described by Burns and Dove eight states of the vessel are required to pass to the optimal control algorithm, which maintains a vessel on track at the desired speed. Therefore it is necessary to develop a full non-linear mathematical model of the vessel and to obtain transition matrices for it at discrete sampling intervals. This has a further advantage of enabling estimates of states to be made even if measurements are not taken. Work undertaken recently by the authors has shown the modelling and filtering process to be a cumbersome task involving extensive sea trials initially and time consuming.
calculations while running the system on-line. It is perhaps for these reasons that few of the integrated systems available today employ Kalman filtering, and instead other filtering techniques are used.

Returning to the offshore industry, Lockling\textsuperscript{51} describes a new generation of computerized dynamic positioning systems, based upon Kalman filtering, which have been successfully installed in different kinds of offshore vessels, while Witbeck\textsuperscript{52} gives a general explanation of the various systems, functions and applications of DP. Saelid \textit{et al.}\textsuperscript{53} are concerned with the Kalman filtering algorithms of the \textit{ALBATROSS} dynamic positioning system. A novel adaptive filtering technique, where the estimator includes both a self-tuning filter and a Kalman filter is given by Fung and Grimble,\textsuperscript{54} whilst Grimble\textsuperscript{55} has developed a new extended Kalman filter based on more direct modelling of the wave spectra energy. He suggests the use of towing tank and wind tunnels to formulate models of the particular vessel and partitions the Kalman filter gain matrix to reduce online calculations.

The lack of recently published papers describing the use of Kalman filters in marine navigation does suggest that perhaps there has not been the hoped for progress in this area. However, in the improvement of autopilot design there has been a steady development. From the original works of Sperry\textsuperscript{56} and Minorsky,\textsuperscript{57} who applied proportional control only, came the application of control theory, which in turn led to Proportional, Integral and Derivative (PID) controllers. Nowadays almost all ocean going vessels are equipped with this kind of autopilot system. With the advent of optimal control theory there has been a steady improvement in autopilot design. In the full description given by van Amerongen and Nauta Lemke,\textsuperscript{58} they report on recent work undertaken in adaptive steering, that is the development of an autopilot whose control parameters adapt themselves to alterations in the dynamics of the vessel or its environment. They go on to discuss rudder roll stabilization, with the inclusion of the roll-reduction system into the conventional controller. Under conventional techniques the two control loops can have a detrimental effect on each other, but Van Amerongen and van Nauta Lemke suggest that the effects of one can be used to minimize the errors in the other. Similarly track keeping systems have been developed whereby the controller not only maintains the vessel on course, but also aims to keep it on the desired track. Weightings must be applied to these control variables in order to evaluate the control parameters. Such systems require multivariable controllers and in order to estimate the input variables to such algorithms, Kalman filtering is suggested. This is particularly useful when no measurement of variables is available; for example in the case of the roll-reduction system referred to above roll is rarely measured.

6. TARGET TRACKING. The inclusion of collision avoidance into marine integrated navigation systems immediately involves the problem of monitoring and predicting the track of other vessels. Radar is the obvious sensor for this purpose, but as with all other instruments it has its limitations and inaccuracies leading to random errors in the range and bearing measurements made. Kalman tracking filters have been developed\textsuperscript{59} to determine the most probable speed and velocity of targets from these data. In such systems, when the target dynamics are
modelled in a rectangular coordinate system giving linear state equations, the measurements will of course be non-linear functions of the state variables. EKF is used to estimate the state variables and provide, via a prediction, linearization of the next measurement vector. Gholson and Moose\textsuperscript{60} suggest that such systems work well until the target makes a sudden change in its trajectory when the position and velocity estimates diverge from the true values. This can lead to bias errors or complete filter divergence. They go on to give two adaptive approaches to solve this tracking problem which is frequently applied to air targets for gun sighting. In such situations the sampling rate needs to be rapid and Baheti\textsuperscript{61} has devised a method to reduce the number of calculations in a Kalman tracking filter by a factor of four.

7. DISCUSSION. From the numerous references quoted it would appear that the linear Kalman filter theory alone has little practical use in marine navigation. This is largely due to the filter mathematical model. The model coefficients need to be evaluated independently for the particular vessel on which it is to be installed and ideally the model should be linear. In the real-world marine vehicles are very non-linear in their behaviour. Many authors overcome this problem by using EKF as described. Another approach\textsuperscript{62} is to assume the course and speed of the vessel are constant during each sample period. In both cases discrete transitions need to be recalculated frequently, and may therefore require a very powerful microprocessor for on-line use. Kalman theory also suggests that both disturbance and measurement noises are white with zero means. In practice, wind and tide may have random variations superimposed on changing values, suggesting that the exact values may not exactly model the disturbances. As far as measurement noise is concerned the system will require knowledge of the random errors, which are assumed Gaussian. These can either be calculated continuously, or the operator may wish to enter figures given in the manufacturer’s literature. In either case the error model may not be a realistic representation of the true errors.

Another critical factor in the design of an optimal filter is the modelling of the marine vehicle used, which can take the form given in Burns et al.\textsuperscript{83} Two methods have been suggested for obtaining the coefficients required, full-scale trials and model tests in both towing tanks and wind tunnels. These are expensive processes and may still not give accurate results for all situations, for example the coefficients will change with factors such as vessel speed, depth under the keel and draught of the vessel. Much work remains to be done in connection with the mathematical model either by direct measurement of a vessel’s parameters using system identification, as suggested by Abkowitz,\textsuperscript{64} or by a simplified series of open-loop tests that could be undertaken with a minimum disruption to a ship’s normal role. Also the disturbance conditions, as has already been mentioned, may change quite rapidly, for instance in the approaches to a port, when the highest degree of accuracy of position and velocity is expected.

It would seem then that although Kalman filtering is used extensively in the avionics industry, it is not easily adaptable to the marine industry, with the exception of specialized branches where perhaps accurate short range navigation aids or expensive inertial systems are available. With the advent of GPS we should
be looking forward to its integration with existing systems and combining the result with information from other sensors on the vessel and the mathematical model, but how many of the existing aids will be maintained once GPS is in full operation?

8. CONCLUSIONS. The ultimate aim must be to produce a system to take a ship to its destination automatically along a predetermined track avoiding both moving and stationary obstacles in its path. A recent report in a UK newspaper has suggested that in Japan progress is being made in this area with one vessel making a 2 day voyage unmanned. In the United Kingdom an optimal control theory for track keeping has been developed and tested, although some difficulty was found with establishing the optimal control coefficients. These can be derived from the coefficients obtained for the mathematical model employed within the filter, or by continuing work on an adaptive controller. With regard to collision avoidance, the detection and tracking of targets has been discussed and these are, for marine vehicles, currently displayed on their ARPA equipment. However, regulations need to be observed and an automatic system must ensure that the International Regulations for the Prevention of Collisions at Sea are complied with, so that the vessel is navigated safely avoiding all hazards. From original work by Davis et al., who conducted computer simulations of a collision avoidance system for up to five vessels and included land masses, Colley et al. have simulated the traffic flow over a 24-hour period in the Dover Strait. The simulation includes land masses and depth contouring in order that each vessel can route itself to its destination, while, at the same time, automatic collision avoidance is applied to every vessel. The result is compared with a typical plot over the same period taken from a land-based radar. Consideration must also be given to unusual circumstances such as the vessel which stands on when she should give way. This has led to consideration of the use of artificial intelligence techniques by Blackwell et al. and Smeaton et al. This suggests that there is scope for further work, not only on independent problems, but also in bringing together the areas of system identification, state estimation, optimal control and collision avoidance as applied to marine vehicles, with perhaps an increased awareness of modern control, and guidance methods by ship operators and navigators.

The requirement for further work in high accuracy position fixing is highlighted in a recent (1984) UK Department of Transport report of the collision between the car ferries European Gateway and Speedlink Vanguard off Harwich, UK in December 1982, when each master believed that the other would alter course to let him past. The report states, 'It is our belief that this collision occurred because of a degree of over-complacency, on the bridge of both vessels, in the performance of what may have appeared routine and unexacting navigation.' If the European Gateway had been in the correct position in the deep-water channel such a collision might not have happened. The authors believe that further work in the use of Kalman filter techniques will lead to improved accuracy of the navigation system to ensure that each and every vessel is in the correct position at the right time. The importance of automatic collision avoidance is illustrated by another example concerning a collision on the night
of 31 August 1986 between the bulk carrier *Petr Vasev* (18604 grt), entering the port of Novorossiysk, and passenger vessel *Admiral Nakhimov* (17053 grt), leaving the port. Both were owned by the Black Sea Shipping company and captained by experienced qualified mariners. They agreed, while about 7 miles apart, and through a shore-based radio station that the *Vasev* would give way, although it was the *Nakhimov*’s duty to do so. With about 2 miles between them, the *Vasev*, carrying ARPA, considered the situation safe in spite of the frequent communications from the *Nakhimov* requesting assurance that they would be giving way. Finally, the master of the *Nakhimov* could wait no longer and altered course, applying 20° rudder in small steps over some 4 minutes, contrary to regulations which state ‘a succession of small alterations to course and/or speed should be avoided’. The master of the *Vasev* remained glued to his radar, making his contemplated alteration to half speed at about the time that the *Nakhimov* applied helm. He did not see the other vessel alter course until it was too late. Both masters were held to be at fault, initially for ignoring regulations, and then, when the *Nakhimov* finally applied helm, it should have been one large alteration of course. The master of the *Vasev* could not combine what he saw on his radar display with reality.

The authors believe that further work in the use of Kalman filtering will lead to improved accuracy of navigation systems for both own-ship and in the tracking of target vessels. While in the field of modern marine operations there is a need to bring together the disciplines of computing, telecommunications, navigation, control engineering and artificial intelligence. Kalman filter theory has a role to play within this development area.

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Simulation of a marine guidance and expert collision avoidance system

R. S. Burns, G. R. Blackwell and K. M. Miller
Ship Control Group, Plymouth Polytechnic, Plymouth, Devon, U.K.

ABSTRACT

One of the major problems that stands in the way of the fully automated ship is that of safe guidance in and out of port. This paper addresses the problem of automatic guidance and collision avoidance, and draws upon the work undertaken over a number of years by the Ship Control Group at Plymouth Polytechnic, U.K., in terms of simulation, prototype model testing and full-scale system implementation.

Techniques to model and simulate the dynamic behaviour of a surface vessel are presented, together with simulated and actual results of an optimal ship guidance system. The problem of collision avoidance in port approaches is tackled using an expert system. Simulation results from a prototype system are given. Finally, the paper considers the need for an expert controller to provide integrated automatic guidance and collision avoidance.

INTRODUCTION

Although overall standards of safety at sea are very good, the approaches to a port, where traffic density is intense, may be considered a high risk area. It has been shown for example (Coldwell 1981) in the Humber Seaway, where there are 100 traffic movements per day, that there is at least one collision or grounding per week. In addition, there is the human element to consider. It has been highlighted (Panel on Human Error 1976) that 85 percent of all marine collisions and groundings are due to human error.

This suggests that there is a need for automatic guidance and collision avoidance systems for marine vehicles in confined waters. As electronic navigation aids become more sophisticated and onboard computer systems become more available, the concept of the fully automatic ship becomes a tangible reality, and it is predicted that above 50 percent of the world's shipping will be fully automated by the turn of the century.

The work presented in this paper is concerned with the simulation of a marine guidance and collision avoidance system. A ship is considered to be a multivariable system that may be controlled by formulating an optimal policy that seeks to maximise the return from the system for a minimum cost. The optimal guidance system therefore involves simultaneously the position, heading and speed of the vessel. In addition, a rule-based expert system is simulated to handle sensory data input, and to make collision avoidance decisions based upon the International Regulations for preventing Collisions at Sea (1981).

SHIP MATHEMATICAL MODEL

All moving rigid bodies contain six degrees of freedom. In this paper the ship is considered to be a rigid body with three degrees of freedom, namely surge, sway and yaw. Ship motions in heave, pitch and roll are considered small enough to be neglected. It is convenient to describe the motion in terms of a moving system of axes coincident with the mass centre of the hull, which gives rise to an Eulerian set of equations of motion.

It is necessary to obtain the hydrodynamic surge and sway forces together with the yaw moments acting on the hull. These may be considered as functions of:

(i) Properties of the ship e.g. length and hull geometry.
(ii) Properties of motion e.g. velocity.
(iii) Properties of the fluid e.g. density of sea water.

These may be reduced to a useful mathematical form by the use of Taylor's expansion for a function of several variables (Abkowitz 1964). It has been shown (Burns et al. 1985) that taking only the linear terms from the expansion is insufficient to define the ship accurately. Non-linear terms that give rise to more than 10% of the global force or moment were considered of major importance, and included in the equation set.

The state vector \( x(t) \) chosen to represent the vessel contains the following state variables:

1. Actual rudder angle
2. Actual engine speed
3. Forward position
4. Forward velocity
5. Lateral position
6. Lateral velocity
7. Heading
8. Yaw rate

The forcing vector \( u(t) \) contains the following variables:

a. Demanded rudder angle
b. Demanded engine speed
c. Component of current speed in surge direction
d. Component of current speed in sway direction
e. Component of wind speed in surge direction
f. Component of wind speed in sway direction
From these eight states a set of first-order differential equations can be used to define the ship.

\[ \dot{x}(t) = F(t)x(t) + G(t)u(t) \quad (1) \]

It is convenient to partition the control forcing functions \( G_c \) and the disturbance forcing functions \( G_d \).

\[ \dot{x}(t) = F(t)x(t) + G_c(t)u(t) + G_d(t)w(t) \quad (2) \]

The corresponding discrete time solution is:

\[ x(k+1) = A(k,k+1)x(k) + B(k,k+1)u(k) + C(k,k+1)w(k) \quad (3) \]

Equation (3) may be used in a recursive manner to simulate the passage of a vessel when the control vector \( u(k) \) and the disturbance vector \( w(k) \) are known.

Figure 1 shows a turning circle simulation of a fast cargo ship of displacement 17,100 tonnes. In the simulation the speed of approach is 15 knots and 20 degrees of starboard rudder has been applied.

\[ J = \int_{t_0}^{T} \{x-r\}^T Q(x-r) + u^T R u \, dt \quad (4) \]

The values of the individual elements reflect the importance of the parameter being controlled. For example, in a track changing manoeuvre, large course errors will be incurred. If the track-keeping elements in the \( Q \) matrix are weighted more heavily than the course-keeping terms, then the majority of the control effort will be expended in reducing the track error, at the expense of the course deviation.

Figure 3 shows a simulation of a full-size car ferry in the approaches to Plymouth. Here, the track-keeping is most heavily weighted, followed by the heading and forward speed. If the reduction in forward speed during a tight manoeuvre can be tolerated, the speed control loop may be dispensed with altogether.
Measurement and State Estimation

In Figure 1 it is assumed that all the state variables can be measured with complete accuracy. All navigational instruments contain measurement errors and a best estimate can be obtained by incorporating a minimum variance or Kalman-Bucy filter. These have been developed extensively for aerospace, and latterly marine navigation (Dove et al. 1985).

The Kalman filter is a recursive computational algorithm which works in a predictor-corrector manner. The current best estimate of the state vector $x(k)$ is used to drive the mathematical model of the ship in real time to predict the state of the vessel at time $(k+1)$. The predictions are compared with the measurements and multiplied by the Kalman gain matrix to obtain the best estimate $x(k+1)$.

In determining the value of the gain matrix, consideration has to be given to measurement errors. These are assumed to be random with a Gaussian distribution, and are stated in terms of a covariance matrix.

Figure 4 shows the simulation of a typical passage into the port under night-time conditions. The simulation assumes that positional data is being received from a Decca Navigator using a standard deviation of 200 m. It is seen that the true and filtered tracks are almost coincident.

The Collision Avoidance Problem

A rule-based approach is an essential prerequisite to the task of collision avoidance at sea, whether or not computers are involved in the process. At first sight, it might seem that a simple procedural application of a standard set of rules, with consequent actions, would cover all requirements; indeed, all mariners are expected to abide by just such a set of rules.

Such a procedural approach has been used most effectively in computer simulation of marine traffic flow and collision avoidance. (Colley et al. 1984) The effects of changes (e.g. in traffic volume) in high-density traffic lanes have been highlighted by such a model, and the technique has also been used to good effect in training mariners. However, this approach takes no account of the need for experience and common sense in applying these rules and, as such, is not a practical option in real-world situations.

The International Regulations for Preventing Collisions at Sea, although quite specific as far as they go, do not in themselves give rigorous definitions of preconditions and consequent actions. It is left to the mariner to decide such details as timing, clearances, and suitable course alterations; such discretion comes with years of accumulated experience and wisdom. One must also consider non-standard situations, particularly those involving more than two vessels which could not all be adequately covered by any reasonable reference guide.

Clearly, it is possible to provide 'rules of thumb' for specific types of situation, and to add supplementary rules for variations in these situations. Such a rule structure would be founded on the previously mentioned anti-collision regulations, would incorporate the accumulated wisdom of expert mariners and would, presumably, also be tailored to reflect the response characteristics of the system operating that rule structure - speed of response, breadth of information available to the system and possible consequence of misjudgement (confidence limits).

An expert system as described would require access to two types of information, static and dynamic. Static information relates to fixed characteristics of the vessel - length, beam, maximum speed, minimum turning circle, safe clearing distance, and a variety of technical data (some possibly specific to current voyage). Dynamic
Such an intelligent response system should be capable of evaluating an encounter between two or more vessels from the standpoint of an experienced mariner, and taking (or advising) appropriate action. On being provided with the information normally available on the bridge of a well-equipped ship, this system should be capable of:

(a) Recognising an encounter (potential hazard) situation in good time;

(b) Identifying the type of encounter and the status of one’s own vessel (own-ship) in that encounter, according to the International Regulations for Preventing Collisions at Sea - such status would normally be either ‘stand-on’ (having right of way) or ‘give-way’, according to one’s position relative to the other vessel (subject to types of vessel involved, fishing vessels and deep-draught vessels having right of way in many circumstances);

(c) Choosing a course of action which combines a sensible safety margin with the least practicable inconvenience (where own-ship is judged to be stand-on, this will - initially at least - mean no alteration of course);

(d) Maintaining a watching brief on the other vessel(s) in the encounter, and being ready to take avoiding action if necessary, should the situation change - it is not unknown for ‘rogue’ vessels to disregard the regulations and plough straight on when they should give way, or even to turn into danger.

**COLLISION AVOIDANCE SIMULATION**

The simulation package in current use is at the second prototype stage. The package comprises a simulator module, driving each of two vessels (own-ship and hazard vessel) independently on the knowledge bases for each of the two vessels. At 20 second intervals, ship time (simulated by 1/2 second intervals in real time) the following sequence is carried out:

(a) Certain minimal information, as would be available via instrumentation (speed, course, relative position) is communicated from the knowledge base of each ship to the knowledge base of the other;

(b) The expert system module considers the current state of each vessel independently, setting status indicators for consequent action, to be carried out by the simulator module;

(c) The simulator module updates the position, speed, and course information for each vessel. In turn, with due regard for the recommendations of the expert system with respect to that vessel;

(d) The screen display is updated to show the new current situation.

The first prototype [Blackwell et al. 1987] contained the decision-making logic for the expert system module in such a form that the rules were ‘hard-wired’ into the program code. Whist adequate for an initial test-bed, this approach was unsuited to a flexible, expanding rule base as envisaged at the outset of this paper. The second prototype incorporates a content-free ‘inference engine’ actioning a set of rules which form a ‘decision network’ - see Figure 5. These rules are held as a set of nodes, or objects, each comprising a simple boolean decision function, pointers to two other nodes, and six other relevant parameters defining consequent action. The most significant of these are the two flags, one for each of the ‘child nodes’, indicating whether that node is to be actioned immediately or at the next time-step (solid and dotted lines respectively in Figure 5). This form of structure allows for the inference/decision network to be expanded and enriched to any degree of sophistication. A simple extension will also permit ‘backtracking’ to explain the decision process at any stage, and parallel processing is a clear possibility without major changes.

**OPERATION OF EXPERT SYSTEM MODULES**

As outlined in the previous section, the prototype package drives two simulated vessels, each with its own operational parameters, and each (independently) subject to the same expert system module. As a consequence, both vessels act in accordance with the anti-collision regulations, and there is no likelihood of one vessel having to take evasive action at a late stage to deal with non-co-operation by the other. In the near future, it is intended to model the two ships on separate computers, interacting via a communications link; an additional feature of such a system would be the provision of a manual override on one of the ships - the hazard - to enable testing of an extension of the rule base to handle ‘rogue’ behaviour.
At the present phase of development, the rule base consists of rules for:

1. Identifying the presence of a potential hazard, and assessing the degree of threat in terms of expected time to infringement of the domain - this time factor is central to the decision on when to take avoiding action, if necessary.

2. Identifying the type of encounter, including several variants in some cases (the primary types of encounter being head-on, crossing and overtaking), and fixing the status of own-ship and the perceived status of hazard-ship in the encounter.

3. Negotiating the stages of the encounter, with due regard for appropriate safety margins.

SIMULATION RESULTS

Figure 6 shows a completed head-on encounter. Note that the status of own-ship is known in detail, but hazard ship is known only to be changing course.

Figure 7 shows an overtaking manoeuvre when own-ship is to starboard of hazard vessel. Here own ship turns starboard on the same heading, but parallel track of the hazard ship, and then turns to port on original course, passing ahead of hazard vessel.

The effectiveness of the present rule structure has been evaluated by a multiple run of 500 simulated encounters. The results of this exercise are shown in the bar chart in Figure 8.

The results of this multiple simulation illustrate that, in the large majority of cases, potential domain violations were avoided by the expert system invoking appropriate collision avoidance strategies; moreover, the manoeuvres involved did not, in the main, involve excessive course alterations - closest point of approach (CPA) separations for these manoeuvres are clearly bunched just outside the domain boundary. However, a small number of the encounters still show CPA's within the domain boundary - this is clearly unacceptable. The reasons for this are considered to be threefold:
1. The rule base in the initial expert system module was shown to have certain 'blind spots', in which each vessel assessed the other as the give-way vessel, and the required avoidance action was not taken.

2. In using a fairly simple model for generation of simulated vessels, some realism was lost - vessels with unlikely combinations of characteristics were involved in a number of the offending encounters.

3. A small number of these simulations highlighted the need for an increased level of sophistication in the strategy for recognising and handling encounter situations. Specifically, the fixed look-ahead, used to assess the time to initiate avoidance manoeuvres, appeared inadequate in certain cases; an ongoing look-ahead, extrapolating at regular intervals from current status, seemed indicated to ensure action at the optimum time. This is considered to be the most important development to arise from the multiple simulation exercise, and will form a central feature of future developments.

**AUTOMATIC GUIDANCE AND COLLISION AVOIDANCE**

At present, the collision avoidance system described in this paper acts as an expert advisor. The next stage of development is to link the guidance and collision avoidance systems together to form an expert controller. The advent of such systems should be viewed alongside recent advances in Vessel Traffic Systems (VTS). Discussions in the International Maritime Organisation (IMO) and the European Economic Community (EEC) have contributed to a global dialogue on the future aims and objectives of VTS.

The challenge facing the maritime world over the next decade will be the implementation of VTS in all major ports, using advanced surveillance and communication techniques, together with shipborne automatic guidance and collision avoidance systems. Such systems will increase the efficiency of port operations and at the same time improve safety levels, particularly in poor weather conditions.

**NOMENCLATURE**

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**Scalar Symbols**

| J                      | Cost Function or Performance Index |
| k                      | Integer Counter |
| t                      | Continuous Time |
| t<sup>0</sup>          | Initial and Final Times |
| u<sub>0</sub>·v<sub>d</sub> | Actual and Desired Forward Velocity |
| X<sub>0</sub>·y<sub>d</sub> | Earth Co-ordinate System |

| e                      | Course Error |

**REFERENCES**


International Marine Organisation. 1981. "International Regulations for Preventing Collisions at Sea."