Transport of Fine Sediment with Mesoscale Currents in the Shelf–Slope Zone of the Sea

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Received December 28, 1998

Abstract—We developed a semianalytical model of the transport of fine suspended material with mesoscale currents in the near-bottom boundary layer in the shelf–slope zone of the ocean. With this model, the vertical profiles of the horizontal velocity and concentration of suspended material are calculated analytically in a quasi-one-dimensional approximation. The profiles obtained are used for calculations of the horizontal suspended matter fluxes. An integral advection–diffusion equation is deduced for describing the balance of suspended material in the near-bottom layer. The equation is solved numerically. The sedimentary material is assumed to be weakly consolidated. The model can be employed for the calculation of the transport of fine suspended material with various mesoscale currents, such as cyclonic and anticyclonic eddies, as well as by meandering longshore currents. It is shown that due to the strong nonlinearity of the problem, small variations in the intensity of a current result in significant variations in the rate of sedimentation and erosion. Because of the onset of the secondary circulation in the near-bottom Ekman’s layer, the direction of the suspension transport and the direction of the current in the water thickness are not the same. It is shown that mesoscale eddies moving along the continental slope form elongated zones of erosion and sedimentation in the direction of their propagation. This mechanism allows the matter to be transported both up and down to the continental slope.

INTRODUCTION

In recent years, the problem of predicting sediment and pollution transport in the coastal–shelf zone of the ocean has become increasingly urgent. This is primarily motivated by the need for ecological safety in this zone of the ocean when exploiting mineral deposits [1, 3]. Examples are the oil and gas deposit development project in the Barents Sea and the “Goluboi potok” gas pipeline construction project between Russia and Turkey on the bottom of the Black Sea.

At present, the problem of sediment transport is being studied quite thoroughly only in the coastal zone of the sea, where the prevailing factors are waves and currents induced by the wind. Beyond the coastal zone, at depths of 20 m, the effect of waves on the bottom is sufficiently weakened, and the most probable reasons for the transport of bottom sediments and pollution are mesoscale eddies, meanders of the longslope currents, internal waves, and storms that occur once every few decades [1, 3, 4, 7].

In the present paper, we propose a semi-analytical model for the estimation of bottom erosion, transport of sediments, and their redeposition under the action of mesoscale currents. We describe the model in detail, as well as present typical examples of calculations for the case when the bottom is affected by cyclonic and anticyclonic eddies, meandering currents, and the joint action of eddies and currents.

THEORY

A theoretical description of the processes of suspending, transporting, and settling of fine sedimentary matter by mesoscale currents is based on the hydrodynamical theory of the boundary layer. By mesoscale currents, we mean the currents, whose characteristic scales of spatial variability are on the order of the local Rossby deformation radius (in the near-coastal basins, this value is a few or a few tens of kilometers), and, whose characteristic time of variability exceeds the pendulum-related day. As will be shown below, the basic tenets of the theory are also valid for currents of a larger scale. In fact, the problem of the transport of suspended matter has two boundary layers. One of them, the dynamical boundary layer, is defined by the vertical structure of currents in the vicinity of the sea bottom, and its thickness is of the order of Ekman’s scale, $h_E$. The thickness of the second one, the diffusion boundary layer, $h_{dh}$, is defined by the balance between the entrainment of the solid particles by the near-bottom turbulence and their settling under the action of gravity. By the total thickness of the boundary layer, we mean the maximum value among these scales, $h_{BBL} = \max(h_E, h_d)$. In accordance with the boundary layer theory, the characteristic horizontal size is assumed to be significantly larger than the vertical one. It is also assumed that the density stratification does not strongly affect the distribution of the vertical structure of the currents within the near-bottom boundary layer.

The boundary layer theory allows one to split the initial three-dimensional problem into two more simple
subproblems: (i) evolution of integral parameters in the horizontal plane, and (ii) reconstruction of the vertical structure of the flow carrying suspension. In order to be able to calculate such characteristics as intensity of erosion and settling, as well as the horizontal flux of suspended material, it is only sufficient to solve the first problem. The effect of the details of the vertical structure on the integral (over the vertical) parameters is found to be weak. Therefore, in the present study, we primarily pay attention to the calculations of integral characteristics, whereas the vertical distribution of the parameters is described more approximately.

Let us introduce a Cartesian coordinate system and place its origin at some point on the bottom. Let the Z-axis be directed vertically upward. The topography of the bottom is assumed to be preset and defined by a function

\[ z_b = b(x, y), \]  

(1)

which describes the elevation of the bottom with respect to a given reference level.

Within the near-bottom boundary layer, the equations of the mesoscale dynamics can be written in a geostrophic approximation [2, 6]

\[ -f\nu = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( K_z \frac{\partial \nu}{\partial z} \right), \]  

(2)

\[ f\nu = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( K_z \frac{\partial \nu}{\partial z} \right), \]  

(3)

where \( u, \nu \) and \( w \) are the components of the current velocity; \( p \) is the pressure; \( f \) is the Coriolis parameter; \( \rho \) is the seawater density; and \( K_z \) is the coefficient of vertical viscosity. The equation of evolution of the suspended matter concentration has the common form

\[ \frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial \nu c}{\partial y} + \frac{\partial (w-w_j)c}{\partial z} = \frac{\partial}{\partial z} \left( K_z \frac{\partial c}{\partial z} \right), \]  

(4)

where \( c(x, y, z, t) \) is the suspension concentration, \( w_j \) is the velocity of precipitation of suspended particles averaged over the grain-size composition (i.e., fall velocity), and \( t \) is the time. The horizontal diffusion and horizontal viscosity are not taken into account, since, in the near-bottom boundary layer, the vertical gradients of velocity and concentration are significantly larger than the horizontal ones. From equation (4), it follows that the expression for vertical flow of suspended material has the form

\[ F_{z}(x, y, z, t) = (w-w_j)c - K_z \frac{\partial c}{\partial z}. \]  

(5)

The boundary conditions at the bottom can be written in the following form

\[ u = \nu = w = 0, \quad \tau = C_d(u_j^2 + \nu_j^2), \]  

\[ F_{z} = E - D, \quad c = c_b \quad \text{for} \quad z = b(x, y), \]  

(6)

where \( E \) is the intensity of erosion of the sedimentary material from the bottom, which is approximated by empirical formulas [10]

\[ E(x, y, t) = M(\tau(x, y, t) - \tau_0) \quad \text{for} \quad \tau > \tau_0, \]  

\[ E = 0 \quad \text{for} \quad \tau < \tau_0, \]  

(7)

where \( D(x, y, t) = \nu c_b \) is the intensity of precipitation of suspended particles from the water column onto the bottom, \( c_b \) is the near-bottom concentration of suspension, \( \tau = K_z \sqrt{\left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial \nu}{\partial z} \right)^2} \) is the near-bottom friction stress, \( \tau_0 \) the threshold value of the near-bottom friction stress, which determines the onset of erosion under the action of a near-bottom current, and \( M \) is a constant known from observations and depending on the physicochemical characteristics of sedimentary material.

The current, above the near-bottom boundary layer, is assumed to be known, so that its velocity \( u, \nu \) or geopotential \( \Phi(x, y, t) \) at some level \( z_p \) above the near-bottom boundary layer, is considered to be already given. The flux of suspended matter, above the near-bottom boundary layer, is assumed to equal zero. This means that the suspended matter supply from the surface and intermediate waters to the near-bottom boundary layer is assumed to be negligibly small

\[ F_{zc} = 0 \quad \text{for} \quad z \geq z_f. \]  

(8)

Equations (2) and (3) can be solved analytically for various depth dependences of the friction coefficient. In the present study, for the sake of simplicity, we assume that, within the near-bottom boundary layer, the turbulent friction coefficient and diffusivity are related as \( K_z = rK, \) \( r \) = const, depend only on the horizontal coordinates and time, and are independent of depth. Then, equations (2) and (3) have solutions in the form of Ekman's spiral

\[ u + iv = \frac{i(\Phi_x + i\Phi_y)}{f} \times \left[ 1 - \exp \left( \frac{(1+i)(z-b(x, y))}{h_E} \right) \right], \]  

(9)

where \( \Phi_x \) and \( \Phi_y \) are the horizontal derivatives of the geopotential, \( h_E(x, y, t) = \sqrt{\frac{2K_i}{f}} \) is Ekman’s scale, and \( i = \sqrt{-1} \). According to the second relation from (6), the coefficient of turbulent friction can be expressed in terms of the current velocity at the upper boundary of the viscous boundary layer

\[ K_z = \frac{C_d^2(u_j^2 + \nu_j^2)}{f}. \]  

(10)
Note that, in contrast to the classical Ekman's theory, equations (2) and (3) are nonlinear since the friction coefficient $K_e$ depends on the current velocity.

On integrating the equation of the suspended matter transport over the vertical within the near-bottom boundary layer, we obtain
\[
\frac{\partial m}{\partial t} + \frac{\partial F_{ex}}{\partial x} + \frac{\partial F_{ey}}{\partial y} = M(\tau - \tau_0) - w_x c_b, \tag{11}
\]
where $m(x, y, t) = \int_0^z c(x, y, z, t)dz$ is the total mass of suspended matter above the bottom unit area, and $F_{ex}$, $F_{ey}$, and $c(x, y, z, t)$ are the horizontal fluxes of suspended matter integrated over the vertical within the near-bottom boundary layer.

To close the system of equations, it is necessary to express the vertical profile of the suspended matter concentration $c(x, y, z, t)$ in terms of its near-bottom concentration $c_0(x, y, t)$. This can be accomplished in different ways [8, 9, 11, 12]. Here, we use the fact that, under the action of currents, the balance of concentration of the suspended matter in the near-bottom boundary layer is mainly defined by the vertical motions, namely, the vertical turbulent diffusion and precipitation of the particles under the action of gravity [3]. Therefore, as a zero approximation, we can neglect the first three terms in equation (4). On integration of such a reduced equation, over the vertical under boundary conditions (6) and (8), and taking into account the widely used assumption that $w \ll w_x$, we obtain the well-known formula [3, 5]
\[
c = c_0 \exp \left[ -\frac{w_x(z - b)}{K_c} \right]. \tag{12}
\]
Substituting formulas (9) and (12) in equation (11), we obtain a closed advection–diffusion equation for a single unknown $m$
\[
\frac{\partial m}{\partial t} - \frac{\partial}{\partial x}(m R_1) + \frac{\partial}{\partial y}(m R_2) = E - w_x m h_d, \tag{13}
\]
where
\[
R_1 = A(\Phi_x + (2\beta + 1)\Phi_y)l/f,
R_2 = A(\Phi_x(2\beta + 1) - \Phi_y)l/f,
A = \frac{\beta}{[(\beta + 1)^2 + \beta^2]},
\]
and $h_d(x, y, t) = K_c/w_x$ is the thickness of the diffusive boundary layer. The intensity of erosion $E$ is given by formulas (7). The right-hand side of equation (10) represents the total vertical flux of suspended matter at the bottom. Its positive values correspond to the case when erosion prevails.

The problem is solved in the following manner. For the region selected, we give the values of the Coriolis parameter $f$, square friction coefficient $C_d$, matter constants $M$, $\tau_0$, and $w_x$, and other parameters, as well as the geopotential of the horizontal currents $\Phi(x, y, t)$ outside the Ekman's friction layer. The evolution of the total suspended matter mass $m$ is determined by numerically solving the base equation (13). The numerical scheme for solving equation (13) is similar to that described in [13], and is based on the use of the method of splitting with respect to directions and an improved Laks–Vendrøf scheme for the advective part of the transfer operator. After calculating the spatial distribution of the value of $m$ at every moment of time, we calculate the vertical profiles of suspended matter concentration from formula (12), components of vertical suspension flux at the bottom due to the sediment erosion from the bottom and gravitational precipitation of suspended matter from formulas (8), and then, we calculate the total vertical suspended matter flux at the bottom $F_{ex}(b)$ from the third formula (6). The total mass of the settled (or washed away) material is determined by integrating the value of the vertical flux $F_{ex}(b)$ over time.

RESULTS OF NUMERICAL CALCULATIONS

We modeled several situations which differ mainly in the form of the given current: (i) stationary circular anticyclone, (ii) stationary circular cyclone, (iii) stationary elliptical anticyclone, (iv) meandering current, and (v) anticyclonic eddy transported by a longslope current. For natural fine-grained sediments, the value of $\tau_0$ and $M\tau_0$ lie within the ranges $\tau_0 = 1-16$ dyne/cm² and $M\tau_0 = 10^{-6}-5 \times 10^{-5}$ g/cm² s⁻¹, respectively. Since, as a rule, a thin surface layer of such sediments is weakly consolidated, for modeling, it is reasonable to choose the values $\tau_0$ and $M\tau_0$ close to the lower limit of these ranges. For most of the calculations, the values of the constants are the following: $C_D = 2.5 \times 10^{-3}$, $f = 1.0 \times 10^{-4}$, $\tau_0 = 1.1$ dyne cm⁻², $w_x = 5 \times 10^{-3}$ cm s⁻¹, $M = 5 \times 10^{-4}$ g cm⁻² s⁻¹ [14–16], which correspond to a clayey-silt sediment with a typical particle size of about 30 µm. The parameters of idealized currents given in the model were chosen characteristic of the mesoscale eddies, e.g., in the Black Sea [4].

(i) Stationary anticyclonic eddy. The geopotential of a free flow above the Ekman's layer has the Gaussian form, the maximum value of the orbital velocity being equal to 25.5 cm/s. The radius of the eddy is assumed to be equal to 20 km. The value of Ekman's scale and the scale of vertical mixing are determined in the calculations with the model. They depend on the distance from the center of the eddy and vary from zero at the periphery of the eddy up to 10 m (the Ekman's scale) and 39 m (the scale of the vertical mixing).

The eddy formation starts in a fluid with a zero suspended matter concentration; thus, the only reason for its appearance is the entrainment of suspended particles form the bottom. Erosion is most intensive at the highest velocity of current. Due to the secondary circulation
caused by friction, the particles of the fluid in the near-bottom Ekman's layer deflect to the left from the circular clockwise rotation. The horizontal suspended matter flux has a component directed outward, and suspension concentration reaches its maximum at larger radius. The suspended particles precipitate over the bottom in the outer region of the eddy and are not entrained again, since the actual shear stress at the bottom is less than the threshold value. At the beginning of the process, the total suspended matter amount increases until the erosion of the bottom material prevails. Then, precipitation becomes more intensive, and the equilibrium between erosion and precipitation is reached in \( t = 30 \text{ days} \).

Figure 1a shows the total vertical flux of the suspended matter at the bottom as a three-dimensional plot. Erosion prevails in the inner region of the eddy, whereas precipitation prevails at the periphery. (Positive values, as were mentioned above, correspond to the regions where erosion prevails). Figure 1b shows the total mass of suspended matter in the water column \( m \) above a unit area of the bottom upon reaching the steady-state distribution of suspended matter concentration. The maximum values are shifted to the periphery of the eddy. Figure 1c shows the total mass of the material washed away and settled on the bottom under the action of an anticyclone over about 30 days needed to reach steady-state conditions. As expected, the material washed away from the bottom in the central part of the anticyclone has settled down at its periphery. The maximum amount of the material washed away from a unit area is equal to 2.4 g/cm\(^2\); the similar value for the settled matter equals 1 g/cm\(^2\).

(ii) Figure 2 indicates the horizontal distribution of the masses washed away and settled on the bottom under the action of a stationary cyclonic eddy, whose parameters are the same as in the previous case except for the direction of rotation. In this case, the pattern is opposite, since the suspended matter flux has a component directed inward so that the region of precipitation lies in the vicinity of the center of the eddy, whereas the region of washing away is located at its periphery. The values of the maximum of washing and settling are on the order of 2 g/cm\(^2\).

(iii) Figure 3 shows a map of contour lines of the total (including erosion and settling) suspended matter

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**Fig. 1.** Patterns induced by a stationary anticyclonic eddy: (a) vertical near-bottom suspended matter flux \((E-D)\); (b) mass of the suspended matter \(m\) in the water column above a unit area; (c) total mass of the material washed away and settled on the bottom \((g/cm^2)\).

**Fig. 2.** Total mass of the material washed away and settled on the bottom under the action of a stationary cyclone \((g/cm^2)\).
flux at the bottom for a stationary anticyclonic eddy of elliptical shape with small azimuth variations of velocity. The rest of the parameters are kept constant. Here, the nonlinearity of the process of suspended matter transport is well manifested, since a small variation in the velocity of the free flow, in comparison with case (a), results in a significant variation in the distribution of the suspended matter flux. Here, contrary to the axisymmetric distribution of the flow shown in Fig. 1a, the regions of erosion and settling form separated cells. Nevertheless, the erosion regions (positive values of the flux) are still closer to the center of the anticyclone, whereas the settling regions (negative values) are at the periphery.

(iv) Figure 4a indicates the horizontal distribution of the mass of the material washed away and settled on the bottom under the action of a meandering current at the initial stage of the process over about three days. The maximum velocity of the free flow, for this case, is rather high and equals 42 cm/s. As we assume that the motion begins in a fluid with a zero suspended matter concentration, the only source for its occurrence is the entrainment of the particles from the bottom. Therefore, at the beginning, the settling is virtually absent, and we only observe erosion, which is most intensive at the places where the current velocity reaches its maximum. The steady-state distribution of suspended matter concentration in seawater is reached after approximately 21 days. The distribution of the washed away and settled material over the bottom up to this moment is shown in Fig. 4b. The meandering current forms both cyclonic and anticyclonic loops. The regions of settling (surface elevation) are formed inside and outside the meander, respectively. Here, the amount of material washed away from a unit area of the bottom reaches significant values and, in the zone of the maximum erosion, is equal to 19 g/cm², whereas the amount of the settled material reaches 14 g/cm².

(v) Figure 5 shows a map of contour lines of the total amount of the material washed away and settled on the bottom under the action of an anticyclonic eddy transported by a longslope current. The parameters of the eddy are the same as in case (a). The calculation is performed for a more fine-grained sedimentary matter than in the previous case: its fall velocity is \( w_f = 5 \times 10^{-3} \text{ cm/s} \). The flow points to the northwest. The velocity of the center of the eddy is 5.6 cm/s. This case simulates the motion of near-coastal anticyclonic eddies in the Black Sea [4]. Dashed lines represent the regions of the material set-

Fig. 3. Total near-bottom vertical suspended matter flux \((E-D)\) for an anticyclonic eddy of elliptical shape \((g/cm^2 s)\).
ting, whereas solid lines denote the region of erosion. Thus, the anticyclonic eddy settles suspended particles both coastward and seaward.

**DISCUSSION**

Examples of the calculations shown in Figs. 1–5 demonstrate that, when the bottom is affected by mesoscale currents, the suspended matter transport is characterized by strong nonlinearity. This means that insignificant variations in the current velocity can result in significant variations of the parameters of the flow carrying suspension and the intensity of the sediment erosion or redeposition. The major reasons for such behavior are the existence of the threshold friction stress for particle erosion and precipitation, and nonlinear dependence of the bottom stress on the velocity of a free flow. The direction of suspended matter transport under mesoscale currents is not the same as the direction of velocity of a free flow due to the secondary circulation in the bottom boundary layer induced by the friction forces on the time scales of the order of a pendulum-related day.

One of the mechanisms that provide the suspended matter exchange between the shelf and the open ocean can be represented by the longslope currents carrying anticyclonic eddies. The results of the calculations for this case presented in Fig. 5 show that redeposition of the suspended matter on the bottom can occur both coastward and seaward from the axis of the eddy motion. Thus, this mechanism, even if weak, can provide bottom sediment transport from greater to smaller depths.

Another mechanism is associated with the current meandering. In this case, it was found that the bottom sediments are displaced only on the one side (to the left) of the axis of the current. The meanders of longslope currents result in the appearance of patches of erosion and settling of the sediment on the sea bottom. Asymmetrical (elongated over one of the horizontal axes) mesoscale eddies also give rise to a patchy structure.

**ACKNOWLEDGMENTS**

This work is supported by the Russian Foundation for Basic Research, project no. 97-05-65635, the Ministry of Science and Technology of the Russian Federation (project “Near-Shore Zone: Theoretical Investigation of the Physical Processes”) and the British Royal Society (“Cascading” project).

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