

Analytical investigation of hydrodynamic performance of a dual pontoon WEC-type breakwater

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Abstract

Based on the linear potential flow theory and matching eigen-function expansion technique, an analytical model is developed to investigate the hydrodynamics of two-dimensional dual-pontoon floating breakwaters that also work as oscillating buoy wave energy converters (referred to as the integrated system hereafter). The pontoons are constrained to heave motion independently and the linear power take-off damping is used to calculate the absorbed power. The proposed model is verified by using the energy conservation principle. The effects of the geometrical parameters on the hydrodynamic properties of the integrated system, including the reflection and transmission coefficients and CWR (capture width ratio, which is defined as the ratio of absorbed wave power to the incident wave power in the device width). It is found that the natural frequency of the heave motion and the spacing of the two pontoons are the critical factors affecting the performance of the integrated system. The comparison between the results of the dual-pontoon breakwater and those of the single-pontoon breakwater shows that the effective frequency range (for condition of transmission coefficient $K_T < 0.5$ and the total capture width ratio $\eta_{\text{total}} > 20\%$) of the dual-pontoon system is broader than that of the single-pontoon system with the same total volume.

Key words: *linear potential flow theory; floating breakwaters; wave energy extraction; effective frequency range.*

1. Introduction

Extracting energy from ocean waves has become an important research focusing in ocean

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29 engineering in recent years. To date, a wide variety of wave energy converters (WECs) have been
30 developed, such as oscillating water column (OWC), oscillating buoy and overtopping wave energy
31 converters (Falcão, 2010). However, the high construction cost of these energy conversion devices is
32 still a big challenge (Ferro, 2006). Therefore, reducing the cost of wave energy devices through the
33 improved design has become utmost important.

34 One solution to reduce the cost is the concept of embedding WECs into other offshore structures
35 (Vicinanza et al., 2014). Combining the wave energy devices with breakwaters has drawn increasing
36 attention during the past years. Takahashi et al. (1992), Boccotti (2007) and Arena et al. (2013)
37 proposed the concept of building wave energy devices into caisson breakwaters. The wave energy
38 devices were integrated into the pile-supported breakwater, the rubble mound breakwaters and the
39 submerged plate-type breakwater by He and Huang (2014), Vicinanza et al. (2014) and Orer and
40 Ozdamar (2007), respectively. It is understood that the floating breakwaters are preferable due to
41 their relatively low costs, independence on subsea geological conditions, low environmental impact,
42 aesthetic considerations and flexibility (McCartney, 1985). He et al. (2012, 2013), Michailides and
43 Angelides (2012), Ning et al. (2016), Martinelli et al. (2016) and Chen et al. (2016) investigated the
44 performance of hybrid systems consisting of a floating breakwater and a wave energy extraction
45 device. In addition, some research has been conducted to study the coastal protection of wave farms
46 (Zanuttigh and Angelelli, 2013; Mendoza et al., 2014). From the literature, the advantages of the
47 integrated systems can be concluded as follows: (1) the cost sharing between wave energy devices
48 and breakwaters can be achieved; (2) the additional ocean space is unneeded for the wave energy
49 device; (3) the multi-purpose use of WECs may be achieved.

50 This study follows the study by Ning et al. (2016). They conducted laboratory experiments to
51 evaluate the performance of an integrated system consisting of an oscillating buoy WECs and a
52 pile-restrained floating breakwater. In this study, an analytical method is developed to calculate the
53 performance of an integrated system, which allows the parametric studies in a wide range of wave
54 and structural parameters. The transmission and reflection coefficients are the important factors to
55 evaluate the performance of a breakwater and the capture width ratio is often used to quantify the
56 performance of the WECs. It is understood that breakwaters are often considered as operating
57 satisfactory when $K_T < 0.5$ (K_T denotes the transmission coefficient) and the effective capture width
58 ratio CWR for a wave energy converter shall be greater than 20% (Koutandos et al., 2005; Babarit et

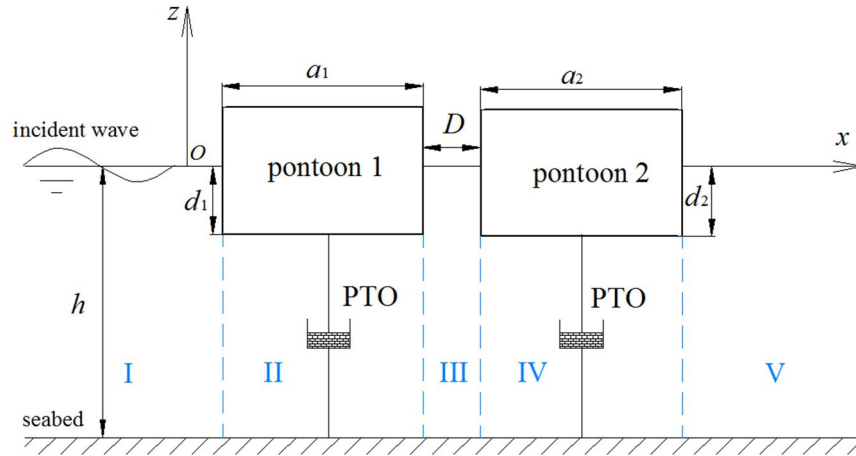
59 al., 2012; Ning et al., 2016). In this paper, the frequency range corresponding to $K_T < 0.5$ and $CWR >$
60 20% is named as effective frequency bandwidth. For a system with a single pontoon, the qualified
61 transmission coefficient and the effective CWR can be achieved only for a narrow frequency range
62 (Ning et al. (2016)). Additionally, the theoretical maximum energy conversion efficiency is only 50%
63 and the effective frequency range of energy conversion is narrow for a two-dimensional symmetrical
64 device with heave motion (Falnes, 2002; Arena et al., 2013). The present study aims at broadening
65 the effective frequency bandwidth of the integrated systems by introducing an improved arrangement
66 of dual-pontoon breakwaters. A power take-off (PTO) system is installed on each pontoon to harvest
67 the energy of its heave motion. Two pontoons are arranged in tandem and work independently. The
68 schematic sketch of the improved arrangement is shown in Fig. 1. From the point view of the
69 engineering costs, the total volume of the two breakwaters shall be smaller than that of the case with
70 a single pontoon.

71 The hydrodynamics of offshore structures consisting of dual pontoons have been studied by
72 many researchers using analytical (Liu and Li, 2014; Zheng and Zhang, 2016), numerical (Weng and
73 Chou, 2007; Williams and Abul-Azm, 1997; Williams et al., 2000) and experimental methods
74 (Koutandos et al., 2005). For structures with regular shapes, analytical methods with high
75 computational efficiency are often used to predict the wave-structure interaction (Li and Teng, 2015).
76 In this study, the analytical method based on the linear potential theory is used to calculate the
77 diffraction and radiation problems of the two-pontoon system. The exciting wave force and
78 hydrodynamic coefficients in the heave mode are computed based on the analytical model for 2-D
79 wave-structure interaction developed by Zheng and Zhang (2016). The reflection and transmission
80 coefficients and the CWR are calculated for a wide parametric range. The rest of the paper is
81 organized as follows. In Section 2, the formulas are described. In Section 3, the validation, the results
82 and the discussions are presented. In Section 4, the conclusions are given.

83 **2. Analytical formula**

84 As shown in Fig. 1, a breakwater comprises of dual floating pontoons that are installed in the
85 water with uniform depth h . The breadths of the pontoon 1 and pontoon 2 are defined as a_1 and a_2 ,
86 the drafts d_1 and d_2 , respectively, and the spacing between the two pontoons is D . To study the

87 interaction between the waves and the floating breakwater, a two-dimensional Cartesian coordinate
 88 ($O-xz$) system is employed with its origin located on the still water surface. Correspondingly, the
 89 mass and the stiffness of the n th pontoon in the heave mode can be expressed as M_n ($=$
 90 $\rho a_n d_n$) and K_n ($= \rho g a_n$), respectively, where ρ is the water density, g the gravitational acceleration
 91 and $n = 1$ or 2 . The structures are subjected to a train of regular waves travelling in the positive
 92 x -direction and are assumed to respond only in the heave mode only.
 93



94
 95 Figure 1 Sketch of the floating structures with the PTO systems
 96

97 As indicated in Fig. 1, the fluid domain is divided into five subdomains I, II, III, IV and V. The
 98 fluid motion in the whole domain can be described by the velocity potential

$$99 \quad \phi(x, z, t) = \text{Re}[\Phi(x, z)e^{-i\omega t}] \quad (1)$$

100 where t is the time, $i = \sqrt{-1}$, ω the angular frequency, Re denotes the real part of a complex, Φ is a
 101 complex velocity potential that satisfies the Laplace equation:

$$102 \quad \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad (2)$$

103 The velocity potential Φ can be divided into three components as:

$$104 \quad \Phi = \Phi_I + \Phi_D + \sum_{n=1}^2 \Phi_{R,n} \quad (3)$$

105 where Φ_I is the incident potential, Φ_D the diffraction potential and $\Phi_{R,n}$ the radiation potential due to
 106 the heave motion of the n th structure. The velocity potential for the incident waves can be written as
 107 follows

108

$$\Phi_1 = -\frac{igA \cosh k(z+h)}{\omega \cosh kh} e^{ikx} \quad (4)$$

109

where A is the wave amplitude, k the wave number, which satisfies the dispersion relation, i.e., $\omega^2 = gk \tanh(kh)$.

111

For the diffraction problem, the governing equation is Laplace equation and the boundary conditions can be written as follows:

112

113

$$\left\{ \begin{array}{l} \frac{\partial \Phi_D}{\partial z} - \frac{\omega^2}{g} \Phi_D = 0 \quad (z = 0, x < x_{l,1} \text{ or } x_{r,1} < x < x_{l,2} \text{ or } x > x_{r,2}) \\ \frac{\partial \Phi_D}{\partial z} = 0 \quad (z = -h) \\ \frac{\partial \Phi_D}{\partial z} = -\frac{\partial \Phi_1}{\partial z} \quad (z = -d_n, x_{l,n} < x < x_{r,n}, n = 1, 2) \\ \frac{\partial \Phi_D}{\partial x} = -\frac{\partial \Phi_1}{\partial x} \quad (-d_n < z < 0, x = x_{r,n} \text{ or } x = x_{l,n}, n = 1, 2) \\ \Phi_D \text{ outgoing: finite value, } |x| \rightarrow \infty \end{array} \right. \quad (5)$$

114

where $x_{l,n}$ denotes the coordinate of the left edge of the n th structure and $x_{r,n}$ denotes the coordinate of the right edge of the n th structure.

115

116

The radiation potential due to the heave motion of the n th pontoon with an amplitude $A_{R,n}$ and an angular frequency ω can be written as

117

118

$$\Phi_{R,n} = -i\omega A_{R,n} \varphi_{R,n}(x, z) \quad (6)$$

119

The complex spatial velocity potential $\varphi_{R,n}$ satisfies the Laplace equation and its boundary

120

conditions can be written as follows:

121

$$\left\{ \begin{array}{l} \frac{\partial \varphi_{R,n}}{\partial z} - \frac{\omega^2}{g} \varphi_{R,n} = 0 \quad (z = 0, x < x_{l,1} \text{ or } x_{r,1} < x < x_{l,2} \text{ or } x > x_{r,2}) \\ \frac{\partial \varphi_{R,n}}{\partial z} = 0 \quad (z = -h) \\ \frac{\partial \varphi_{R,n}}{\partial z} = \delta_{m,n} \quad (z = -d_m, x_{l,m} < x < x_{r,m}, m = 1, 2) \\ \frac{\partial \varphi_{R,n}}{\partial x} = 0 \quad (-d_m < z < 0, x = x_{r,m} \text{ or } x = x_{l,m}, m = 1, 2) \\ \varphi_{R,n} \text{ outgoing: finite value, } |x| \rightarrow \infty \end{array} \right. \quad (7)$$

122 where $\delta_{m,n}$ is the Kronecker delta.

123 The analytical expressions of the diffraction and radiation potentials (including the evanescent
 124 modes) in each domain can be obtained based on the method by Zheng and Zhang (2016). The
 125 equation sets can be formed by substituting the diffraction potentials into Eq. (5) and radiation
 126 potentials into Eq. (7) and using the orthogonality of the vertical eigen-function. Then the unknown
 127 coefficients of the diffraction and radiation potentials can be obtained. The potential in each domain
 128 can be further determined. Then the vertical exciting force $F_{z,n}$ on the n th structure can be calculated
 129 by

$$130 \quad F_{z,n} = -i\omega\rho \int_{S_n} (\Phi_I + \Phi_D) n_z ds \quad (8)$$

131 where S_n is the bottom surface of the n th structure and n_z is the unit normal vector in the negative
 132 z -direction.

133 The added mass μ_n^m and radiation damping λ_n^m on the n th structure in the heave motion
 134 subject to a unit forced motion of the m th structure can be written as:

$$135 \quad \mu_n^m = -\rho \int_{S_n} \text{Re}[\varphi_{R,m}] n_z ds \quad (9)$$

$$136 \quad \lambda_n^m = -\rho\omega \int_{S_n} \text{Im}[\varphi_{R,m}] n_z ds \quad (10)$$

137 where Im denotes the imaginary part of a complex and $m = 1, 2$.

138 Then the equation of motion can be written as:

$$139 \quad (-\omega^2(\mathbf{M} + \boldsymbol{\mu}) - i\omega(\boldsymbol{\lambda} + \boldsymbol{\lambda}_{\text{PTO}}) + \mathbf{K}) \mathbf{A}_R = \mathbf{F}_z \quad (11)$$

140 where \mathbf{M} and \mathbf{K} are the mass and stiffness matrices of the structures, respectively; $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ are the
 141 added mass and wave damping matrices of the structures, respectively; $\boldsymbol{\lambda}_{\text{PTO}}$ is the PTO damping
 142 matrix imposed on the structures. \mathbf{A}_R and \mathbf{F}_z denote heave response motion vector and heave
 143 excitation force vector, respectively.

144 The motion equation can be re-written as:
 145

$$146 \quad \left\{ -\omega^2 \left(\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} + \begin{bmatrix} \mu_1^1 & \mu_1^2 \\ \mu_2^1 & \mu_2^2 \end{bmatrix} \right) - i\omega \left(\begin{bmatrix} \lambda_1^1 & \lambda_1^2 \\ \lambda_2^1 & \lambda_2^2 \end{bmatrix} + \begin{bmatrix} \lambda_{\text{PTO},1}[1,1] & 0 \\ 0 & \lambda_{\text{PTO},2}[2,2] \end{bmatrix} \right) + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \right\} \begin{pmatrix} A_{R,1} \\ A_{R,2} \end{pmatrix} = \begin{pmatrix} F_{z,1} \\ F_{z,2} \end{pmatrix} \quad (12)$$

148 Note that, for a dual pontoon WEC-type floating breakwater, the nonzero elements of $\boldsymbol{\lambda}_{\text{PTO}}$ are

149 $\lambda_{PTO}[1, 1]$ and $\lambda_{PTO}[2, 2]$, which represent the PTO damping imposed on the first and second pontoon,
 150 respectively. In the present study, $\lambda_{PTO}[n, n]$ equals to the optimal PTO damping for an isolated single
 151 device, which can be expressed as $\lambda_{PTO}[n, n] = \sqrt{(K_n / \omega - \omega(M_n + \mu_n))^2 + \lambda_n^2}$, where μ_n and λ_n
 152 represent the added mass and damping coefficient of the n th structure in the isolated case (Falnes,
 153 2002; Wolgamot et al., 2016).

154 The power P_n produced by the n th structure can be calculated by:

$$155 \quad P_n = \frac{1}{2} \omega^2 \lambda_{PTO}[n, n] |A_{R,n}|^2 \quad (13)$$

156 Then the total power absorbed is as follows:

$$157 \quad P_{total} = \sum_{n=1}^2 P_n \quad (14)$$

158 The incident wave power can be calculated as follows

$$159 \quad P_{incident} = \frac{1}{4} \frac{\rho g A^2 \omega}{k} \left(1 + \frac{2hk}{\sinh 2hk} \right) \quad (15)$$

160 The CWR is an important indicator to evaluate the hydrodynamic efficiency of WECs (Babarit,
 161 2015). The CWR (η_n) of the n th structure can be calculated as $\eta_n = P_n / P_{incident}$ and the total CWR
 162 as $\eta_{total} = P_{total} / P_{incident}$.

163 The performance of a breakwater can be evaluated by the reflection coefficients K_R and
 164 transmission coefficients K_T :

$$165 \quad K_R = \left| \frac{\Phi_D - i\omega \sum_{n=1}^2 A_{R,n} \varphi_{R,n}}{\Phi_I} \right|_{x=-\infty} \quad (16)$$

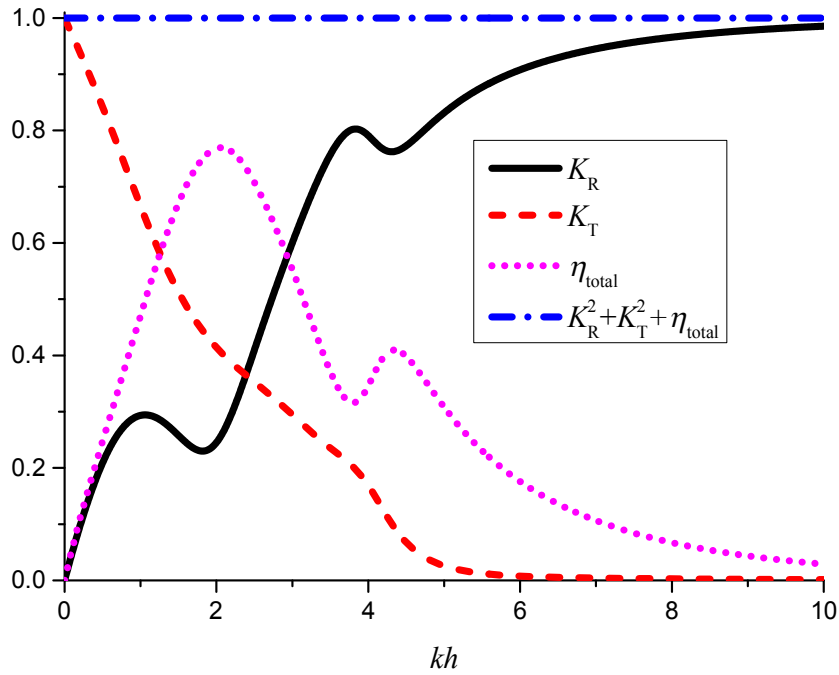
$$166 \quad K_T = \left| \frac{\Phi_I + \Phi_D - i\omega \sum_{n=1}^2 A_{R,n} \varphi_{R,n}}{\Phi_I} \right|_{x=+\infty} \quad (17)$$

167 3. Results and discussions

168 3.1 Validation

169 The present model is validated by using the energy conservation relationship of $K_R^2 + K_T^2 + \eta_{total}$

170 = 1. Fig. 2 shows the results of the reflection coefficient K_R , the transmission coefficient K_T , the total
 171 CWR η_{total} and the $K_R^2 + K_T^2 + \eta_{total}$ for geometrical parameters of $a_1 = a_2 = 6$ m, $d_1 = d_2 = 1.25$ m, D
 172 = 2 m and $h = 10$ m. The nonzero elements of the PTO damping matrix are chosen as $\lambda_{PTO}[1, 1]$ and
 173 $\lambda_{PTO}[2, 2]$. It can be seen that the relation of $K_R^2 + K_T^2 + \eta_{total} = 1$ is satisfied perfectly, which validates
 174 the present analytical model.



175
 176 Figure 2 Variations of reflection coefficient K_R , transmission coefficient K_T , the total CWR η_{total} and
 177 $K_R^2 + K_T^2 + \eta_{total}$ vs the dimensionless wavenumber kh .
 178

179 3.2 Parametric study

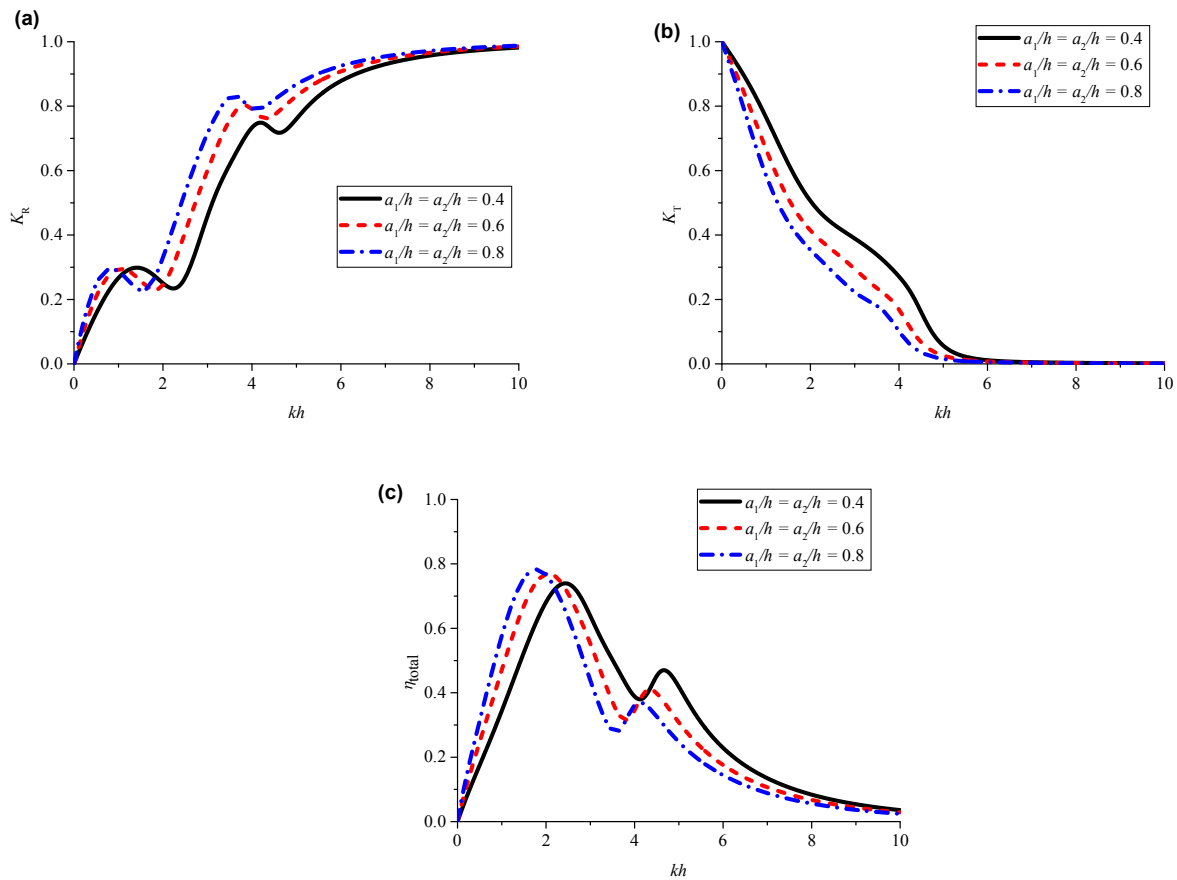
180 The performance of an integrated system combining a dual-pontoon floating breakwater and
 181 dual wave energy devices relies on several parameters, including the breadths, drafts, and spacing of
 182 the pontoons (a_n , d_n and D). A parametric study is conducted to investigate the sensitivity of the
 183 performance of an integrated system to various wave and geometrical parameters. The difference
 184 between the system with dual pontoons and that with a single pontoon are compared in Section 3.2.3.

185 3.2.1 Effect of the structure breadths (a_1 and a_2)

186 A breakwater with two identical pontoons is considered in Sections 3.2.1, 3.2.2 and 3.2.3.
 187 Firstly, the effect of the structure breadths is investigated. Figs. 3(a-c) show the variation of the
 188 reflection coefficient K_R , the transmission coefficient K_T and the total CWR η_{total} against the

189 dimensionless wavenumber kh for three breadths of $a_1/h = a_2/h = 0.4, 0.6$ and 0.8 . The other
 190 geometrical parameters are kept constant as $d_1/h = d_2/h = 0.125, D/h = 0.2$.

191 In each of Fig. 3 (a) – (c), the general trends for all three curves are the same. The reflection
 192 coefficient increases with increasing a_1/h . In particular, it is noted that the reflection coefficient
 193 exhibits small oscillations at some critical wave numbers among the generally upward curve and the
 194 wave numbers where the minimum and maximum values of K_R occur shift to the lower frequency
 195 region with the increase of the structure breadth. This may be due to the interference by the strong
 196 reflection. Similarly, oscillations can also be found for the curves of total CWR. However, for the
 197 transmission coefficient, the oscillation phenomenon is weak. A similar phenomenon was found by
 198 Garnaud and Mei (2009), who adopted the analytical multiple scales method under the framework of
 199 linear potential flow theory. The effective frequency ranges of different structure breadths are slightly
 200 different from each other. The effective frequency ranges for $a_1/h = a_2/h = 0.4, 0.6$ and 0.8 are $2.04 <$
 201 $kh < 6.25, 1.55 < kh < 5.75$ and $1.26 < kh < 5.37$, resulting in bandwidths of 4.21, 4.20 and 4.11,
 202 respectively.



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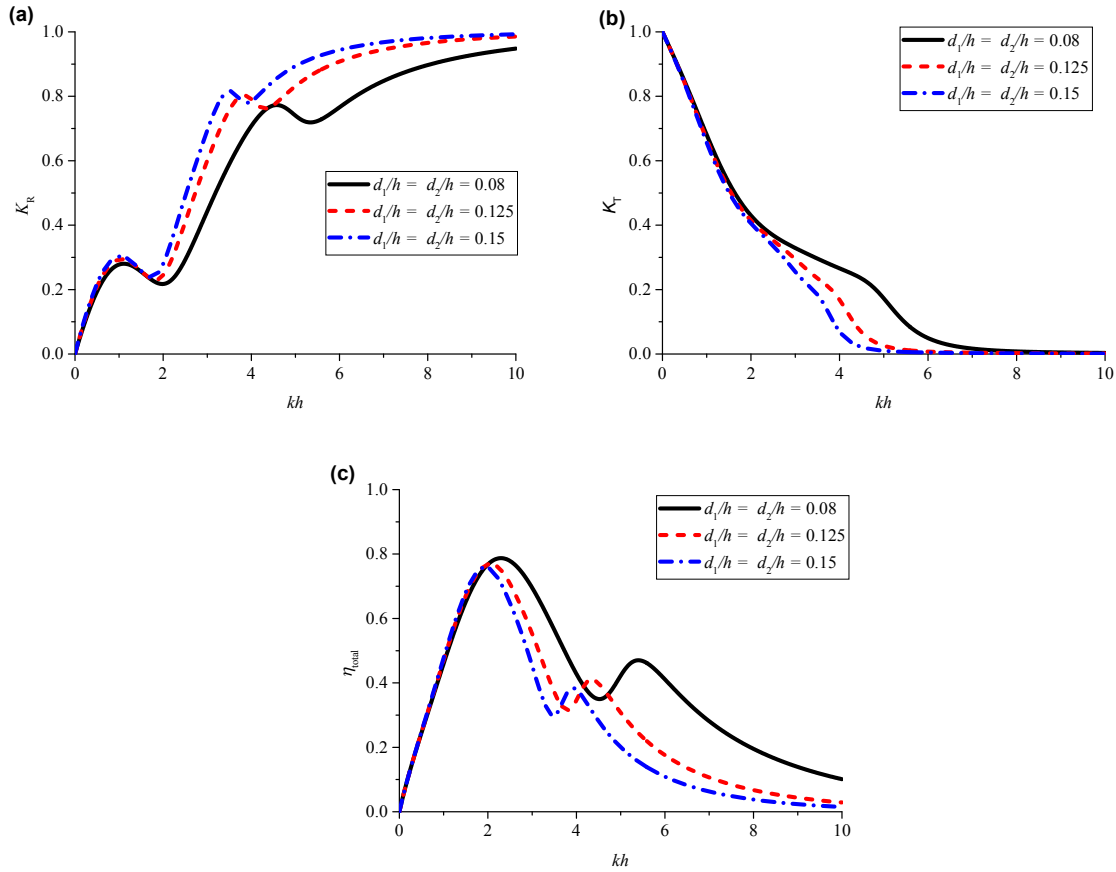
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205 Figure 3 Variations of reflection coefficient K_R , transmission coefficient K_T and CWR η_{total} vs the

206 dimensionless wavenumber kh for cases with different structure breadths ($d_1/h = d_2/h = 0.125$, $D/h =$
207 0.2).

208 3.2.2 Effect of the structure drafts (d_1 and d_2)

209 Figs. 4 (a-c) present the influence of the drafts on the performance of the integrated system.
210 Results are shown for three cases with drafts of $d_1/h = d_2/h = 0.08$, 0.125 and 0.15 . The other
211 parameters are kept constant as $a_1/h = a_2/h = 0.6$, $D/h = 0.2$. All the reflection coefficients increases
212 with increasing kh with some oscillations. The critical wave number corresponding to the maximum
213 values of K_R shifts towards the lower frequency with the increase of the draft. As expected, the
214 structures with larger draft provide the more effective wave barriers, and the effect of the draft on the
215 K_R is more obvious for $2 < kh < 7$ than the other kh . However, it appears that the effect of the draft on
216 η_{total} is in the apparently opposite trend to that on K_R as $kh > 2.5$. In addition, the wave numbers
217 where η_{total} reaches their maximum correspond to those where K_R values reaches its minimum. The
218 effective frequency range and the peak value of η_{total} decreases with the increase of the draft ratio.
219 The effective frequency ranges for $d_1/h = d_2/h = 0.08$, 0.125 and 0.15 are $1.65 < kh < 7.93$, $1.55 < kh$
220 < 5.76 and $1.50 < kh < 5.00$, and the corresponding effective bandwidths are 6.28 , 4.21 and 3.50 ,
221 respectively.



222

223

224 Figure 4 Variations of the reflection coefficient K_R , transmission coefficient K_T and total capture
 225 width ratio η_{total} vs the dimensionless wavenumber kh for cases with different structure drafts ($a_1/h =$
 226 $a_2/h = 0.6$, $D/h = 0.2$).

227

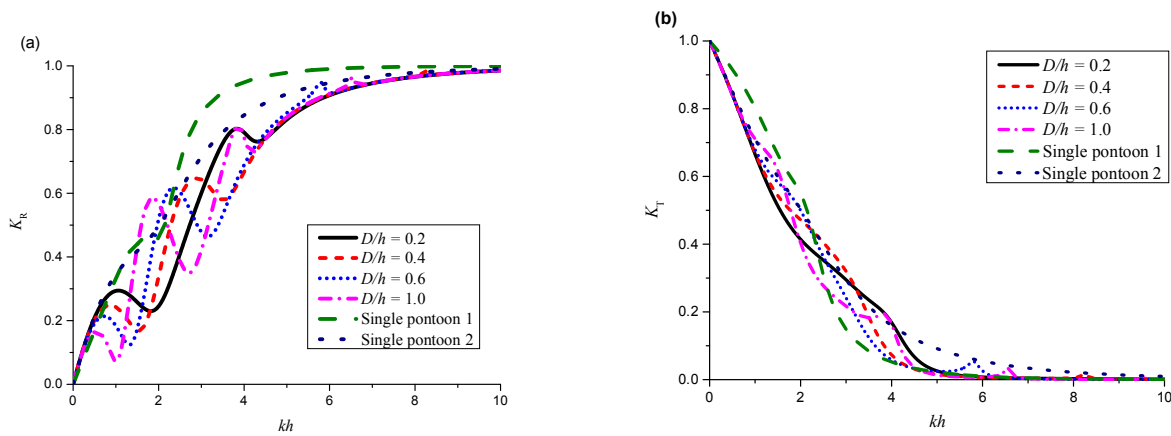
228 It can be understood that, for a two-dimensional pontoon, the natural frequency of the heave
 229 mode decreases with the increase of the structure breadth or draft, which can be identified by Figs. 3
 230 and 4. By combing the results in Figs. 3 and 4, it can be seen that breakwater performance of the
 231 system becomes better and the performance of the energy conversion becomes worse with the
 232 decrease of the natural frequency of the heave mode.

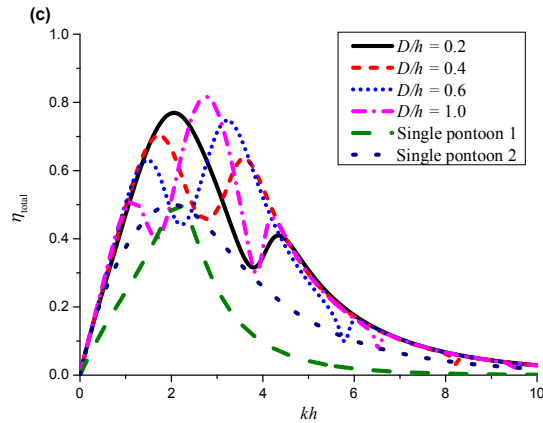
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234 3.2.3 Effect of the pontoon spacing (D)

235 The effect of pontoon spacing on the reflection coefficient K_R , transmission coefficient K_T and
 236 total CWR η_{total} is shown in Figs. 5(a-c) for four pontoons of $D/h = 0.2, 0.4, 0.6$ and 1.0 . Other
 237 geometrical parameters are kept constant as $a_1/h = a_2/h = 0.6$ and $d_1/h = d_2/h = 0.125$. For
 238 comparisons, the results of a single pontoon of $a/h = 0.6$ and $d/h = 0.25$ (referred to be single
 239 pontoon 1, where a is the breadth of the pontoon and d the draft) and $a/h = 1.2$ and $d/h = 0.125$

240 (referred to be single pontoon 2) are plotted in Fig. 5. Note that the volume of the pontoon of the
 241 single case equals to the total volume of the pontoons of the dual pontoon case. It can be seen that
 242 the reflection coefficient generally increases with increasing kh but with oscillations, whose
 243 amplitude appears to increase with increasing D/h . Similar phenomenon can be found for the total
 244 CWR η_{total} . The system with a smaller D/h gives a broader frequency range, for which the $K_T < 0.5$.
 245 For the four cases, the total CWR increases with increasing kh firstly and then decreases after it
 246 reaches the maximum. The frequency ranges corresponding to $\eta_{\text{total}} > 20\%$ are similar. In conclusion,
 247 the system with a smaller spacing may give a broader effective frequency range in terms of $K_T < 0.5$
 248 and $\eta_{\text{total}} > 20\%$. The effective frequency ranges are $1.55 < kh < 5.77$, $1.78 < kh < 5.71$, $2.02 < kh <$
 249 5.42 and $1.78 < kh < 5.67$ and the corresponding effective bandwidths are 4.22, 3.93, 3.40 and 3.89
 250 for $D/h = 0.2, 0.4, 0.6$ and 1.0 , respectively. The effective bandwidths of an isolated single pontoon 1
 251 and an isolated single pontoon 2 are bandwidth 1.09 and 2.53, respectively. It can be seen that
 252 effective frequency bandwidth of the two-pontoon system is broader than that of isolated single
 253 pontoons.





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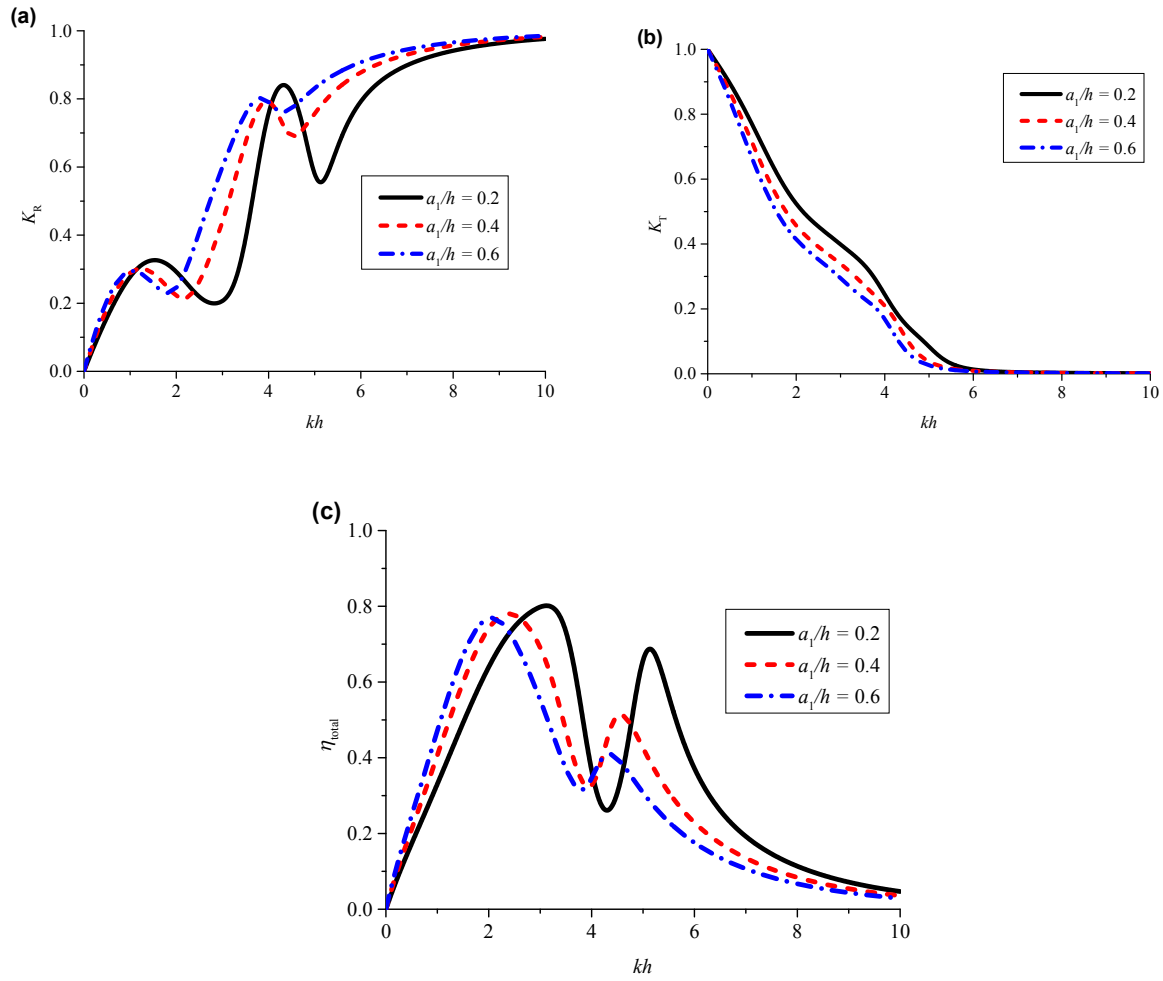
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257 Figure 5 Variations of the reflection coefficient K_R , transmission coefficient K_T and total capture
 258 width ratio η_{total} vs the dimensionless wavenumber kh for cases with different spacings ($a_1/h = a_2/h =$
 259 0.6 , $d_1/h = d_2/h = 0.125$). The size of single pontoon 1 is $a/h = 0.6$ and $d/h = 0.25$, and the size of
 260 single pontoon 2 is $a/h = 0.12$ and $d/h = 0.125$.

261

262 3.2.4 Effect of the breadth ratio (a_1/a_2)

263 It has been proved that, for a system consisting of two identical pontoons, the natural frequency
 264 in the heave mode of each pontoon and the spacing between the two pontoons are the critical factors
 265 for the performance of the dual pontoon system. From the point view of the engineering cost, the
 266 smaller the total volume of the two pontoons, the lower cost of the integrated system. The cost
 267 reduction should not compromise the performance of the system. It is understood that the reduction
 268 of the draft or the breadth can lead to a decrease in the volume of a pontoon. Therefore, the system
 269 consisting of two non-identical pontoons (i.e., the two pontoons with different dimensions and
 270 different natural periods) is of interest. In this section, the effect of breadth ratio of the two pontoons
 271 (a_1/a_2) is investigated. Two scenarios are considered: (1) a_2/h fixed as 0.6 with three breadths of the
 272 front pontoon of $a_1/h = 0.2, 0.4$ and 0.6 and (2) a_1/h fixed as 0.6 with three breadths of the rear
 273 pontoon of $a_2/h = 0.2, 0.4$ and 0.6 . The drafts and the distance are defined as $d_1/h = d_2/h = 0.125$ and
 274 $D/h = 0.2$, respectively. In the discussion, all the results are compared with the results of a reference
 275 case of $a_1/h = a_2/h = 0.6$, $d_1/h = d_2/h = 0.125$ and $D/h = 0.2$.



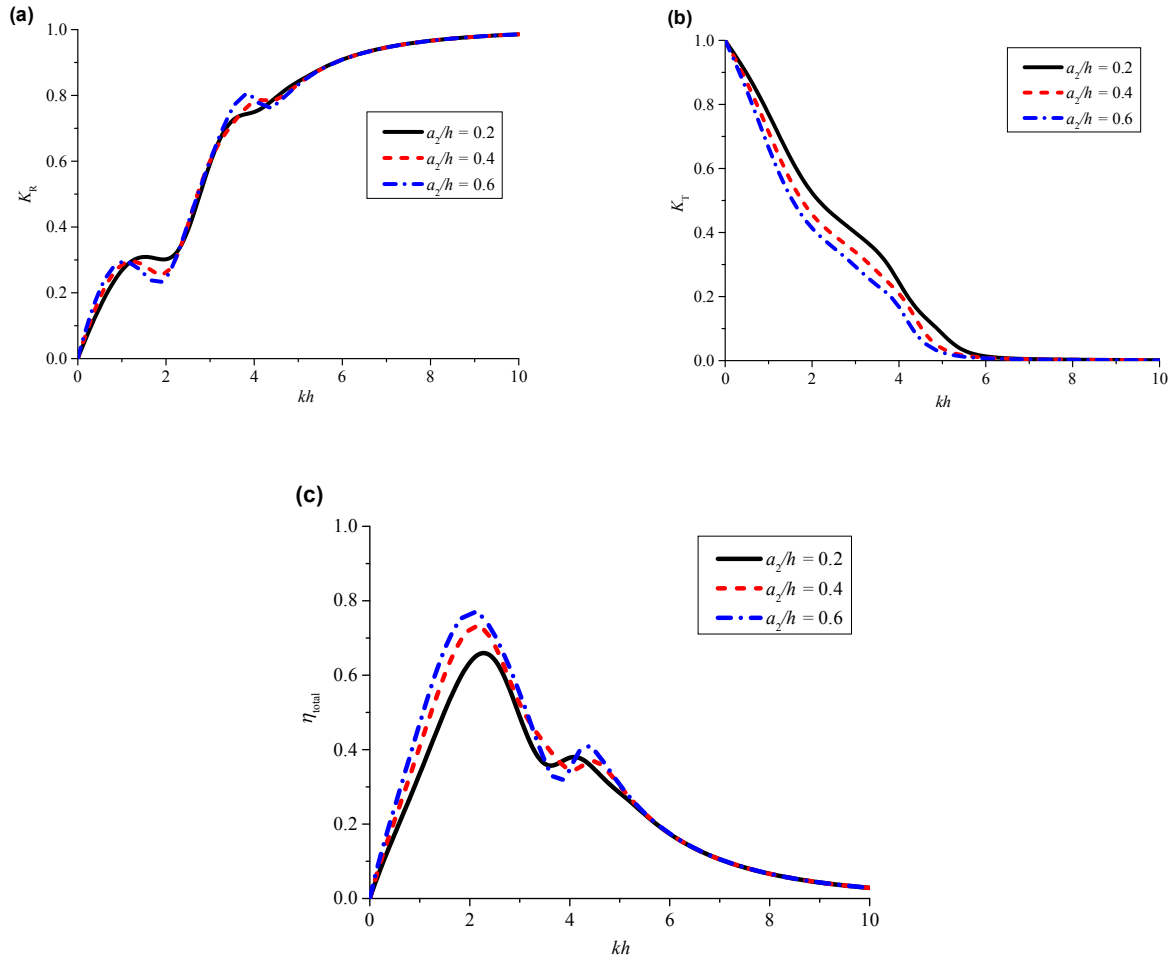
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278 Figure 6 Variations of the reflection coefficient K_R , transmission coefficient K_T and total capture
 279 width ratio η_{total} vs the dimensionless wavenumber kh for cases with different breadth ratios. ($a_2/h =$
 280 $0.6, d_1/h = d_2/h = 0.125, D/h = 0.2$)

281

282 Figs. 6(a-c) show the results of the reflection coefficient K_R , transmission coefficient K_T and
 283 total CWR η_{total} corresponding to the three values of a_1/h ($= 0.2, 0.4$ and 0.6) with a fixed $a_2/h=0.6$. It
 284 can be seen that the transmission coefficient of the system decreases and the frequency range for
 285 $\eta_{total} > 20\%$ is broadened slightly with the decrease of the a_1/a_2 . That is to say, the effective
 286 frequency range changes little by comparing with the case of the $a_1/h = a_2/h = 0.6, d_1/h = d_2/h =$
 287 0.125 and $D/h = 0.2$. The effective frequency ranges are $2.15 < kh < 6.93, 1.77 < kh < 6.24$ and 1.55
 288 $< kh < 5.76$ and the bandwidths are $4.78, 4.47$ and 4.21 for $a_1/h = 0.2, 0.4$ and 0.6 , respectively.



290

291 Figure 7 Variations of the reflection coefficient K_R , transmission coefficient K_T and total capture
 292 width ratio η_{total} vs the dimensionless wavenumber kh for cases with different breadth ratios. ($a_1/h =$
 293 0.6 , $d_1/h = d_2/h = 0.125$, $D/h = 0.2$)

294

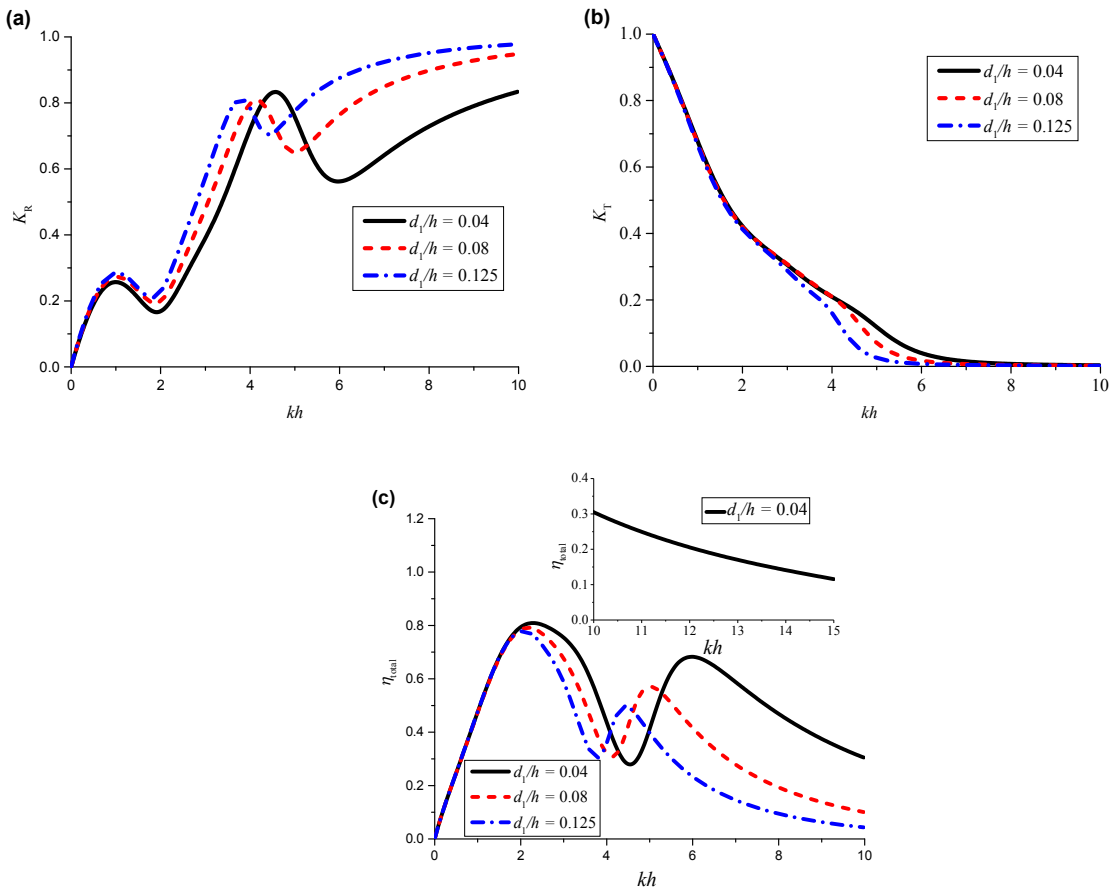
295 Figs. 7(a-c) show the results corresponding to the three values of a_2/h ($= 0.2, 0.4$ and 0.6) with a
 296 fixed $a_1/h=0.6$. It can be seen that the variations of K_R vs kh of the three cases are similar to each
 297 other. The transmission coefficient becomes smaller with the decrease of the a_1/a_2 at the low
 298 frequency region. The effective frequency ranges for $a_2/h = 0.2, 0.4$ and 0.6 are $2.14 < kh < 5.74$,
 299 $1.77 < kh < 5.74$ and $1.55 < kh < 5.74$ and the bandwidths are 3.60, 3.97 and 4.19, respectively. The
 300 effective frequency bandwidth becomes broader with the decrease of the a_1/a_2 . By comparing the
 301 results shown in Figs. 6 (b-c), it is found that the configuration with a smaller front pontoon and a
 302 bigger rear pontoon has wider effective bandwidth than the opposite arrangement of the pontoons.

303

304 **3.2.5 Effect of the draft ratio (d_1/d_2)**

305 The effect of the draft ratios (d_1/d_2) is investigated in this section. Firstly, the group with three
 306 drafts of $d_1/h = 0.04, 0.08$ and 0.125 are considered. The other parameters are fixed as $d_2/h = 0.125$,
 307 $a_1/h = a_2/h = 0.6$ and $D/h = 0.2$. Secondly, performance of the opposite configuration (i.e., $d_1/h =$
 308 0.125 $d_2/h = 0.04, 0.08$ and 0.125) is investigated. Similar to Section 3.2.4, the reference case is not
 309 changed.

310



311

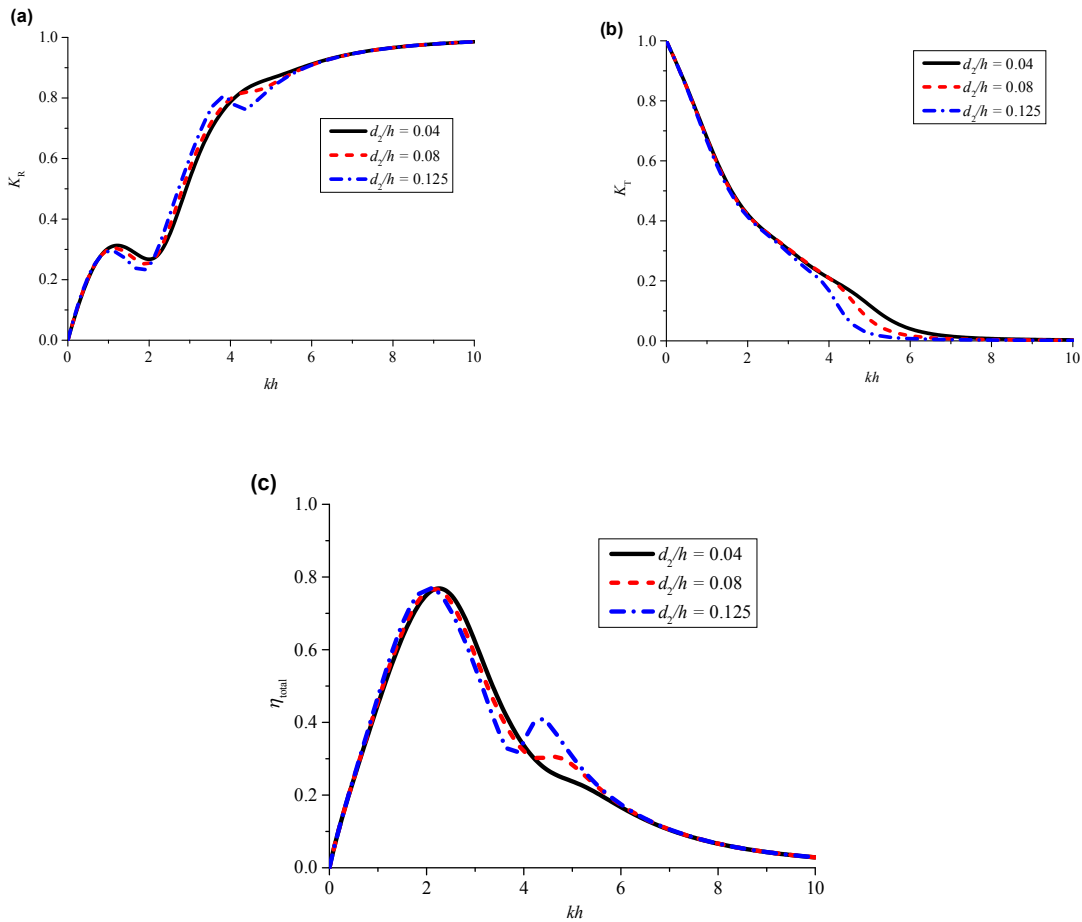
312 Figure 8 Variations of the reflection coefficient K_R , transmission coefficient K_T and total capture
 313 width ratio η_{total} for pontoons with different draft ratios ($d_2/h = 0.125, a_1/h = a_2/h = 0.6, D/h = 0.2$).

314

315

316 The influence of the draft ratio ($d_1/d_2 = 0.32, 0.64$ and 1) on the reflection coefficient K_R ,
 317 transmission coefficient K_T and total CWR η_{total} is shown in Figs. 8(a-c). It can be seen that the
 318 strongest oscillation occurs for the case with the smallest draft ratio. The transmission coefficient is
 319 found to be affected by d_1/h only at region of $3.5 < kh < 7.0$. The effective frequency range becomes
 320 broader with the decrease of the draft ratio. The K_T values for the three cases are the almost same as

321 $kh < 2$. That is to say, the rear pontoon with a large size determines the transmission coefficient of the
 322 integrated system in long waves. In terms of the total CWR, the integrated system gives a broader
 323 effective frequency range. The effective frequency ranges for $K_T < 0.5$ and $\eta_{total} > 20\%$ are $1.60 < kh$
 324 < 12.14 , $1.58 < kh < 7.9$ and $1.55 < kh < 6.32$ for $d_1/d_2 = 0.32$, 0.64 and 1 , respectively. Accordingly,
 325 the bandwidths are 10.56 , 6.32 and 4.77 . In a word, the integrated system with the smaller draft ratio
 326 performs better than that of the identical case in terms of the effective frequency bandwidth.
 327



328

329

330 Figure 9 Variations of the reflection coefficient K_R , transmission coefficient K_T and total capture
 331 width ratio η_{total} for pontoons with different draft ratios ($d_1/h = 0.125$, $a_1/h = a_2/h = 0.6$, $D/h = 0.2$).
 332

333 Figs. 9 (a-c) shows the results as the larger pontoon is located at the upstream side. The
 334 effective ranges are $1.60 < kh < 5.74$, $1.58 < kh < 5.73$ and $1.55 < kh < 5.57$ for $d_2/h = 0.04$, 0.08 and
 335 0.125 , respectively. Accordingly, effective frequency bandwidths are 4.14 , 4.15 and 4.02 ,
 336 respectively. It can be seen that the bandwidth changes little by decreasing the draft of the rear

337 pontoon. By comparing the results shown in Figs. 8 and 9, the configuration with the small-draft
338 front pontoon and large-draft rear pontoon performs better than the opposite configuration in terms
339 of the effective frequency bandwidth. But the transmission coefficient changes very little for the two
340 configurations with the same total volume.

341

342 **3.3 Discussions**

343 The hydrodynamic performance of an integrated breakwater system with two floating pontoons
344 is investigated. Results showed that the two-pontoon system results in a broader effective frequency
345 range in terms of the qualified transmission coefficient and acceptable total CWR than a
346 single-pontoon system. It is understood that the volume of the floating structures affects its cost.
347 Now we take the case with parameters of $d_1/h = 0.04$, $d_2/h = 0.125$, $a_1/h = a_2/h = 0.6$ and $D/h = 0.2$
348 as an example, the total volume of two pontoons is much smaller than that of the single pontoon with
349 the parameters of $a/h = 0.6$, $d/h = 0.25$ and $a/h = 1.2$, $d/h = 0.125$. Therefore, the two-pontoon system
350 is much more acceptable economically.

351 The comparisons of the performance with different configuration of two non-identical pontoons
352 are conducted. Interestingly, the configuration with a smaller front pontoon and a larger rear pontoon
353 performs better than the opposite configuration in terms of the effective frequency bandwidth. This is
354 because stronger reflection occurs at the high frequency region if the larger pontoon is in front of the
355 smaller pontoon, resulting in lower CWR. Thus, the effective frequency bandwidth is narrower. From
356 the point view of effective frequency bandwidth, the configuration with the smaller front pontoon
357 and the larger rear pontoon is suggested.

358 It is worthy to note that, since the viscous effect is not considered, the transmission coefficient
359 and the total CWR of the integrated system may be overestimated by using the potential flow theory
360 in the frequency domain. The future work will focus on the physical experiments by adopting the
361 general generator.

362

363 **4. Conclusions**

364 The hydrodynamic properties of a two-pontoon WEC-type breakwater have been investigated

365 analytically under the frame of linear potential flow theory. The matching eigen-function method is
366 used to solve the diffraction and radiation problems. The absorbed power is calculated by using the
367 linear PTO damping method. The numerical results for a range of configurations are presented to
368 illustrate the influence of the different wave and structural parameters on the performance of the
369 integrated system. The conclusions are summarized as follows.

- 370 (1) The reflection coefficient, transmission coefficient and the total CWR of a system with two
371 identical pontoons strongly depends on the natural frequency in heave mode and the spacing
372 between them;
- 373 (2) For a system with two non-identical pontoons but the total volume fixed, the broader effective
374 frequency bandwidth ($K_T < 0.5$ and $\eta_{total} > 20\%$) can be achieved for configuration with a front
375 pontoon with small-draft and a rear pontoon with large-draft;
- 376 (3) By comparing the transmission coefficient and the CWR of the system with those by the single
377 pontoon, the system with two small pontoons (i.e., the total volume of the two small pontoons is
378 less than that of the single pontoon) can give a better performance.

379
380

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