LIGHT-FRONT QUANTUM CHROMODYNAMICS
A framework for the analysis of hadron physics
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Abstract

An outstanding goal of physics is to find solutions that describe hadrons in the theory of strong interactions, Quantum Chromodynamics (QCD). For this goal, the light-front Hamiltonian formulation of QCD (LFQCD) is a complementary approach to the well-established lattice gauge method. LFQCD offers access to the hadrons’ nonperturbative quark and gluon amplitudes, which are directly testable in experiments at existing and future facilities. We present an overview of the promises and challenges of LFQCD in the context of unsolved issues in QCD that require broadened and accelerated investigation. We identify specific goals of this approach and address its quantifiable uncertainties.

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I. INTRODUCTION

Quantum Chromodynamics (QCD), the theory of strong interactions, is a part of the Standard Model of elementary particles that also includes, besides QCD, the theory of electro-weak (EW) interactions. In view of the difference in strength of these interactions, one may treat the EW interactions as a perturbation in systems consisting of hadrons, the composite particles that respond to the strong interactions. Perturbation theory has its place in QCD also, but only at large values of the transferred energy or momentum where it exhibits the property of asymptotic freedom. The field of perturbative QCD is well developed and many phenomena have been described using it, such as factorization, parton distributions, single-spin asymmetries, and jets. However, at low values of the energy and momentum transfer, the strong interaction must be treated in a nonperturbative manner, since the interaction strength becomes large and the confinement of quarks and gluons, as the partonic components of the hadrons, cannot be ignored. There is a wealth of data in this strong interaction regime that is waiting for explanation in terms of calculations proceeding directly from the underlying theory. As one prominent application of an \textit{ab initio} approach to QCD, we mention that many extensive experimental programs either measure directly, or depend upon the knowledge of, the probability distributions of the quark and gluon components of the hadrons.

Three approaches have produced considerable success in the strong-coupling area up to the present. First, hadronic models have been formulated and applied successfully, such as in Refs. \cite{1-8}. This success comes sometimes at the price of introducing parameters that need to be identified quantitatively. For example, the Relativistic String Hamiltonian developed by Simonov et al. \cite{9-16} depends on the current quark masses, the string tension, and a parameter corresponding to $\Lambda_{QCD}$. The second method, lattice QCD \cite{17-21}, is an \textit{ab initio} approach directly linked to the Lagrangian of QCD. Based on a Euclidean formulation, lattice QCD provides an estimate of the QCD path integral and opens access to low-energy hadronic properties such as masses. Although lattice QCD can estimate some observables directly, it does not provide the wave functions (WF) that are needed for the description of the structure and dynamics of hadrons. Third is the Dyson–Schwinger approach \cite{22-24}. It is also formulated in Euclidean space-time and employs models for vertex functions.

Light-Front QCD (LFQCD) is an alternative \textit{ab initio} approach to strongly interacting systems \cite{19, 25-48}. It is, like perturbative and lattice QCD, directly connected to the QCD Lagrangian, but it is a Hamiltonian method, formulated in Minkowski space rather than Euclidean space. The essential ingredient is Dirac’s front form of Hamiltonian dynamics \cite{49-51}, where one quantizes the theory at fixed light-cone time $\tau = t + z/c$ rather than ordinary time $t$. An interpolation between the instant form and the front form of the relativistic Hamiltonian dynamics is discussed in Ref. \cite{52-54}. Thus, initial conditions for a WF are set not at a single time $t$, but on the space-time hyperplane swept by the front of a plane wave of light. The solutions will be exact mass spectra and light-front wave functions (LFWFs) capable of describing a wide range of experiments in a relativistically covariant manner. For example, one obtains the probability distributions of the quark and gluon components of the hadrons from the squared modulae of the LFWFs. Hence, LFQCD exhibits the promise of accessing a much wider range of experimental situations than previously addressed.

The light-front framework has many attractive features. On the technical side, LFQCD provides the largest number of kinematic (interaction-independent) generators of the Poincaré transformations in relativistic Hamiltonian dynamics, i.e., seven instead of only
six in other frameworks. The eigenvalues of the LFQCD Hamiltonian are the discrete masses and continuous invariant-mass hadronic spectra, instead of the frame-dependent energies. The method yields the boost-invariant and process-independent LFWFs needed for form factors, scattering amplitudes, correlations, spin effects, decay rates, momentum space distributions, and other hadronic observables.

Quantization in the light-front provides the field-theoretical realization of the intuitive ideas of the parton model which is formulated at fixed $t$ in the infinite-momentum frame. The same results are obtained in the front form for any frame; e.g., as already mentioned above, the structure functions and other probabilistic parton distributions measured in deep inelastic scattering are obtained from the squares of the boost invariant LFWFs, the eigensolution of the light-front Hamiltonian. In particular, the “handbag” contributions to the $E$ and $H$ generalized parton distributions for deeply virtual Compton scattering, which can be computed from the overlap of LFWFs, automatically satisfy the known sum rules. The LFWFs contain information about novel QCD features, such as color transparency, hidden color, intrinsic charm, sea-quark symmetries, dijet diffraction, direct hard processes, and hadronic spin dynamics. The familiar kinematic variable $x_{Bj}$ of deep inelastic lepton-hadron scattering becomes identified with the LF momentum fraction $x$ carried by the constituent in a hadron that is struck by the gauge boson emitted by the lepton. The BFKL Regge behavior of structure functions can be demonstrated from the behavior of LFWFs at small $x$. Hadronic matrix elements of currents can be obtained as overlaps of LFWFs as in the Drell-Yan-West formula. The gauge-invariant meson and baryon distribution amplitudes which control hard exclusive and direct reactions are the valence LFWFs integrated over transverse momentum at fixed $x$. The “ERBL” evolution of distribution amplitudes and the factorization theorems for hard exclusive processes can be derived most directly using LF methods.

One can also prove fundamental theorems for relativistic quantum field theories using the front form, including the cluster decomposition theorem and the vanishing of the anomalous gravitomagnetic moment for any Fock state of a hadron. One can show that a nonzero anomalous magnetic moment of a bound state requires nonzero angular momentum of the constituents. The cluster properties of LF time-ordered perturbation theory, together with $J^z$ conservation, can be used to derive the Parke-Taylor rules for multi-gluon scattering amplitudes. The counting-rule behavior of structure functions at large $x$ and Bloom-Gilman duality have also been derived in LFQCD. The existence of “lensing effects” at leading twist, such as the $T$-odd “Sivers effect” in spin-dependent semi-inclusive deep-inelastic scattering, was first demonstrated using LF methods.

LF quantization is thus the natural framework for description of the nonperturbative relativistic bound-state structure of hadrons using QCD. However, there exist subtle problems in LFQCD that require thorough investigation. For example, the complexities of the vacuum in the usual instant-time formulation, such as the Higgs mechanism and condensates in $\phi^4$ theory, have their counterparts in zero modes or, possibly, in additional terms in the LFQCD Hamiltonian that are allowed by power counting. LF considerations of the vacuum as well as the problem of achieving full covariance in LFQCD require close attention to the LF singularities and zero-mode contributions. The truncation of the light-front Fock-space calls for the introduction of effective quark and gluon degrees of freedom to overcome truncation effects, e.g., see Refs. Introduction of such effective degrees of freedom is what one desires in seeking the dynamical connection.
between canonical (or current) quarks and effective (or constituent) quarks that Melosh sought \cite{123}, and Gell-Mann advocated as a method for truncating QCD \cite{124}.

The LF Hamiltonian formulation thus opens access to QCD at the amplitude level and is poised to become the foundation for a common treatment of spectroscopy and the parton structure of hadrons in a single covariant formalism, providing a unifying connection between low-energy and high-energy experimental data that so far remain largely disconnected.

II. APPLICATIONS OF THE LIGHT-FRONT FORMALISM

A. Structure of Hadrons

Experiments that need a conceptually and mathematically precise theoretical description of hadrons at the amplitude level include investigations of: the structure of nucleons and mesons, heavy quark systems and exotics, hard processes involving quark and gluon distributions in hadrons, heavy ion collisions and many more. For example, LFQCD will offer the opportunity for an \textit{ab initio} understanding of the microscopic origins of the spin content of the proton and how the intrinsic and spatial angular momenta are distributed among the partonic components in terms of the WFs. This is an outstanding unsolved problem as experiments to date have not yet found the largest components of the proton spin. The components previously thought to be the leading carriers, the quarks, have been found to carry a small amount of the total spin. Generalized parton distributions (GPDs) were introduced to quantify each component of the spin content, and the interface between GPDs and experimental measurements in deeply virtual Compton scattering (DVCS) has been discussed in Ref. \cite{125–130}. As another example, LFQCD will reproduce or predict the masses, quantum numbers and widths of the already familiar hadrons or yet-to-be observed exotics such as glueballs and hybrids. Some preliminary analyses can be found in Refs. \cite{131–134}.

B. QCD at High Temperature and Density

There are major programs at accelerator facilities such as GSI-SIS, CERN-LHC, and BNL-RHIC to investigate the properties of a new state of matter, the quark-gluon plasma, and other features of the QCD phase diagram. In the early universe temperatures were high, while net baryon densities were low. In contrast, in compact stellar objects, temperatures are low and the baryon density is high. QCD describes both extremes. However, reliable perturbative calculations can only be performed at asymptotically large temperatures and densities, where the running coupling constant of QCD is small due to asymptotic freedom, and lattice QCD provides information only at very low chemical potential (baryon density). Thus, many frontier questions remain to be answered. What is the nature of the phase transitions? How does the matter behave in the vicinity of the phase boundaries? What are the observable signatures of the transition in transient heavy-ion collisions? LFQCD opens a new avenue for addressing these issues. In recent years a general formalism to directly compute the partition function in LF quantization has been developed and numerical methods are under development for evaluating this partition function in LFQCD \cite{135–137}. The goal is to establish a tool comparable in power to lattice QCD but extending the partition function to finite chemical potentials where experimental data are available.
C. Nuclear Reactions

There is a new appreciation that initial and final-state interaction physics, which is not intrinsic to the hadron or nuclear LFWFs, must be addressed in order to understand phenomena such as single-spin asymmetries, diffractive processes, and nuclear shadowing (see the report \[138\]). This motivates extending LFQCD to the theory of reactions and to investigate high-energy collisions of hadrons. Standard scattering theory in Hamiltonian frameworks can provide valuable guidance for developing a LFQCD-based analysis of high-energy reactions.

D. Intense Time-Dependent Fields

High-intensity laser facilities offer prospects for directly measuring previously unobserved processes in QED, such as vacuum birefringence \[139\], photon-photon scattering \[140\] and, still some way in the future, Schwinger pair production. Furthermore, ‘light-shining-through-walls’ experiments \[141\] can probe the low energy frontier of particle physics and search for beyond-standard-model particles \[142\]. These possibilities have led to great interest in the properties of quantum field theories, in particular QED, in background fields describing intense light sources \[143, 144\], and some of the fundamental predictions of the theory have been experimentally verified \[145\].

Despite the basic theory behind ‘strong-field QED’ having been developed over 40 years ago, there have remained until recent years several theoretical ambiguities that can in part be attributed to the use of the instant-form in a theory which, because of the laser background, naturally singles out light-like directions. Thus, light-front quantization is a natural approach to physics in intense laser fields. The use of the front-form in strong-field QED \[146\] has provided answers to several long standing questions, such as the nature of the effective mass in a laser pulse \[147\], the pole structure of the background-dressed propagator \[148\], and the origins of classical radiation reaction within QED \[149\].

Combined with non-perturbative approaches such as ‘time dependent basis light-front quantization’ \[150, 151\], which is specifically targeted at time-dependent problems in field theory, the front-form promises to provide a better understanding of QED in external fields. Such investigations will also provide groundwork for understanding QCD physics in strong magnetic fields at, for example, RHIC \[152\].

III. RELATIONSHIP WITH OTHER APPROACHES

A solution of the LFQCD Hamiltonian eigenvalue equation can utilize all available mathematical methods of quantum mechanics and contribute to the development of advanced computing techniques for large quantum systems, including nuclei. For example, in the Discretized Light Cone Quantization (DLCQ) \[43–48\], periodic conditions are introduced such that momenta are discretized and the size of the Fock space is limited without destroying Lorentz invariance. Solving a quantum field theory is then reduced to diagonalizing a large sparse Hermitian matrix. The DLCQ method has been successfully used to obtain the complete spectrum and LFWFs in numerous model quantum field theories such as QCD with one or two space dimensions for any number of flavors and quark masses. An extension of this method to supersymmetric theories, SDLCQ \[153\], takes advantage of the fact that the
LF Hamiltonian can be factorized as a product of raising and lowering ladder operators. SDLCQ has provided new insights into a number of supersymmetric theories including direct numerical evidence [154] for a supergravity/super-Yang–Mills duality conjectured by Maldacena [153].

One of the most interesting recent advances in hadron physics has been the application to QCD of a branch of string theory, Anti-de Sitter/Conformal Field Theory (AdS/CFT) [156]. Although QCD is not a conformally invariant field theory, one can use the mathematical representation of the conformal group in five-dimensional anti-de Sitter space to construct an analytic first approximation to the theory. The resulting model [157–165], called AdS/QCD, gives accurate predictions for hadron spectroscopy and a description of the quark structure of mesons and baryons which has scale invariance and dimensional counting at short distances, together with color confinement at large distances.

The dynamics in AdS space in five dimensions is dual to a semiclassical approximation to Hamiltonian theory in physical 3 + 1 space-time quantized at fixed light-front time [166]. Remarkably, there is an exact correspondence between the fifth-dimension coordinate of AdS space and a specific impact variable \( \zeta^2 = b^2 \perp x (1 - x) \) which measures the physical separation of the quark constituents within the hadron at fixed light-cone time \( \tau \) and is conjugate to the invariant mass squared \( M_{q\bar q}^2 \). This connection allows one to compute the analytic form of the frame-independent simplified LFWFs for mesons and baryons that encode hadron properties and allow for the computation of exclusive scattering amplitudes.

The effective confining potential \( U(\zeta^2) \) in this frame-independent “light-front Schrödinger equation” systematically incorporates the effects of higher quark and gluon Fock states. The potential has a form of a harmonic oscillator potential if one requires that the chiral QCD action remains conformally invariant [167]. The result is a nonperturbative relativistic light-front quantum mechanical wave equation which incorporates color confinement and other essential spectroscopic and dynamical features of hadron physics.

These recent developments concerning AdS/CFT duality provide new insights about LFWFs which may form first approximations to the full solutions that one seeks in LFQCD, and be considered as a step in building a physically motivated Fock-space basis set to diagonalize the LFQCD Hamiltonian, as in the “basis light-front quantization” (BLFQ) method [168]. A complementary light-front interpretation of the duality and holography is found in Ref. [169].

IV. GOALS OF THE PROJECT

The purpose of the LFQCD program is to bring together experts in the field and attract new contributors who will together take advantage of the available theoretical and computational tools and develop them further in order to provide answers to the pertinent questions in an accelerated fashion. The central issue is the rigorous description of hadrons, nuclei, and systems thereof from first principles using QCD. We list the main goals of the required research.

1. Evaluation of masses and wave functions of hadrons using the light-front Hamiltonian of QCD.

2. The analysis of hadronic and nuclear phenomenology based on fundamental quark and gluon dynamics, taking advantage of the connections between quark-gluon and nuclear many-body methods.
3. Understanding of the properties of QCD at finite temperatures and densities, which is relevant for understanding the early universe as well as compact stellar objects.

4. Developing predictions for tests at the new and upgraded hadron experimental facilities – JLAB, LHC, J-PARC, GSI-FAIR.

5. Analyzing the physics of intense laser fields, including a nonperturbative approach to strong-field QED.

6. Providing bottom-up fitness tests for model theories as exemplified in the case of Standard Model [109].

To accomplish the nonperturbative analysis of QCD, we need to:

1. Continue testing the LF Hamiltonian approach in simple theories in order to improve our understanding of its peculiarities and treacherous points vis-à-vis manifestly-covariant quantization methods [111–120]. This will include work on theories such as Yukawa theory [170–176] and QED [177–182] and on theories with unbroken supersymmetry, in order to understand the strengths and limitations of different methods. Much progress has already been made along these lines.


3. Develop computer codes which implement the regularization and renormalization schemes [1]. Provide a platform-independent, well-documented core of routines that allow investigators to implement different numerical approximations to field-theoretic eigenvalue problems, including the light-front coupled-cluster method [201]. Consider various quadrature schemes and basis sets, including DLCQ, finite elements, function expansions [202], and the complete set of orthonormal wave functions obtained from AdS/QCD [203–205]. This will build on the Lanczos-based MPI code developed for nonrelativistic nuclear physics [206, 207] applications and similar codes for Yukawa theory and lower-dimensional supersymmetric Yang–Mills theories.

4. Address the problem of computing theoretical bounds on truncation errors and other ambiguities introduced by various simplifying assumptions, particularly for energy scales where QCD is strongly coupled. Understand the role of renormalization group methods [208–209], asymptotic freedom [210, 211] and spectral properties of $P^+$ in quantifying theoretical errors, as one could do in the case of model LF lattice dynamics [212] or in model studies of mathematical accuracy of the similarity renormalization

\[1\] An example of a related discussion is available at www.fuw.edu.pl/~lfqcd/inmemoriam/?part=20.
group procedure for Hamiltonians in Refs. [213, 214]. Such studies of theoretical accuracy are necessary for understanding and differentiating between inputs characterizing various approaches when estimating their predictive power and capability of falsifying theories.

5. Solve eigenvalue problems for hadronic masses and wave functions, cf. [205]. Use these wave functions to compute form factors, GPDs, scattering amplitudes, and decay rates. Compare with perturbation theory, lattice QCD, and model calculations, using insights from AdS/QCD, where possible. Study the transition to nuclear degrees of freedom, beginning with light nuclei.

6. Classify the spectrum with respect to total angular momentum. In equal-time quantization, the three generators of rotations are kinematic, and the analysis of total angular momentum is relatively simple. In light-front quantization, only the generator of rotations around the z-axis is kinematic; the other two, of rotations about the axes x and y, are dynamical. To solve the angular momentum classification problem, the eigenstates and spectra of the sum of squares of these generators must be constructed [215, 216]. This is the price to pay for having more kinematical generators than in equal-time quantization, where all three boosts are dynamical. In light-front quantization, the boost along z is kinematic, and this greatly simplifies the calculation of matrix elements that involve boosts, such as the ones needed to calculate form factors. The relation to covariant Bethe-Salpeter approaches projected on the LF [217, 224] may help in understanding the angular momentum issue and its relationship to the Fock-space truncation of the LF Hamiltonian. Model-independent constraints from the general angular condition [225, 227], which must be satisfied by the LF helicity amplitudes, should also be explored. The contribution from the zero mode appears necessary for the hadron form factors [228] to satisfy angular momentum conservation, as expressed by the angular condition [229, 230]. The relation to light-front quantum mechanics, where it is possible to exactly realize full rotational covariance and construct explicit representations of the dynamical rotation generators, should also be explored.

7. Explore the AdS$_5$/QCD correspondence and light-front holography [157, 165]. The approximate duality in the limit of massless quarks motivates few-body analyses of meson and baryon spectra based on a one-dimensional light-front Schrödinger equation in terms of the modified transverse coordinate $\zeta$. Models that extend the approach to massive quarks have been proposed, but a more fundamental understanding within QCD is needed. The nonzero quark masses introduce a non-trivial dependence on the longitudinal momentum, and thereby highlight the need to understand the representation of rotational symmetry within the formalism. Exploring AdS$_5$/QCD wave functions as part of a physically motivated Fock-space basis set to diagonalize the LFQCD Hamiltonian should shed light on both issues. The complementary Ehrenfest interpretation can be used to introduce effective degrees of freedom such as diquarks in baryons [163].

8. Develop numerical methods/computer codes to directly evaluate the partition function (viz. thermodynamic potential) as the basic thermodynamic quantity. Compare to lattice QCD, where applicable, and focus on a finite chemical potential, where reliable lattice QCD results are presently available only at very small (net) quark densities.
There is also an opportunity for use of LF AdS/QCD to explore non-equilibrium phenomena such as transport properties during the very early state of a heavy ion collision. LF AdS/QCD opens the possibility to investigate hadron formation in such a non-equilibrated strongly coupled quark-gluon plasma.

9. Develop a LF approach to the neutrino oscillation experiments that are possible at Fermilab and elsewhere, with the goal of reducing the energy spread of the neutrino-generating hadronic sources, so that the three-energy-slits interference picture (assuming there exist only three neutrinos) of the oscillation pattern [231,232] can be resolved and the front form of Hamiltonian dynamics utilized in providing the foundation for qualitatively new (treating the vacuum differently than it is treated in the instant form of dynamics) studies of neutrino mass generation mechanisms.

10. Take advantage of the possibility that, if the renormalization group procedure for effective particles (RGPEP) [233, 234] does allow one to study intrinsic charm, bottom, and glue in a renormalized and convergent LF Fock-space expansion, one might consider a host of new experimental studies of production processes using the intrinsic components that are not included in the calculations based on gluon and quark splitting functions.

V. CONCLUSION

As a theory and foundation for the phenomenology of processes involving hadrons, QCD faces challenges that by no means are resolved, neither directly nor at the current conceptual level of attempts to improve the standard model and seek a unified theory beyond it. A hadron eigenstate of the LFQCD Hamiltonian, calculated with modern tools of massive computing, can provide previously unavailable capabilities for in-depth exploration of the structure of the Fock-space wave functions. The discovery potential hidden in LFQCD for understanding basic theoretical issues in particle physics is as great as the utility of this approach as a tool, deeply rooted in theory, for the phenomenology of strong interactions.

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