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# Modelling machine induced noise and vibration in a ship structure

Wang, Weihui

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**MODELLING MACHINE INDUCED NOISE AND VIBRATION  
IN A SHIP STRUCTURE**

by

**WEI-HUI WANG**

A thesis submitted to the University of Plymouth  
In partial fulfilment for a degree of

**DOCTOR OF PHILOSOPHY**

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In collaboration with

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# **ABSTRACT**

## **Modelling Machine Induced Noise and Vibration in a Ship Structure**

by

**Wei-Hui Wang**

Most high speed vessels are fitted with powerful high speed engines which are installed in confined spaces and, as a consequence, cause an extremely high level of noise and vibration. Often structure-borne sound power is transmitted to a sound carrying structure from a source via a number of contact points. In turn, the noise and vibration are propagated in the structure and could possibly cause an undesired noise radiation.

In this study, a model for predicting power flow based on the mobility theory has been addressed. The unique parts of the study include the establishment of the relationship of mobility functions with respect to four-pole parameters and the dynamic stiffness coefficients of a coupled machine/mount/foundation system. Also expressions to represent the sound input power, the output power and the transmitted power in relation to mobility functions are clarified.

From a detailed analysis of relevant literature, it is shown that no validated models for predicting the propagation of structure-borne noise within the intermediate frequency range of 125 Hz to 1kHz exist. As a consequence, a new numerical stress wave model has been developed to bridge this knowledge gap. This innovative approach extends the earlier works of Cremer, Heckl and Ungar in the field of stress wave propagation.

Finally, a novel holistic model has been developed to line up the transmission, propagation and radiation predictions of a machine induced noise and vibration in ship's structure to take in account the fluid-structure interaction effect. A number of experiment measurements have been performed to validate the established models. From the comparisons, the prediction models are shown to be credible with an accuracy higher than 95 per cent.

The established models are of a generic nature and can be applicable to diverse engineering fields regarding to the predictions of structure-borne noise and vibration transmission, propagation and radiation. Applications of these models to characterize the vibration reduction countermeasures, as in the case of resilient mounts and squeeze-film damping plates, from a machine are also discussed.

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## NOMENCLATURE

$A_{ijr}$	Modal constant of $r^{\text{th}}$ mode
$B$	Bending rigidity of beam
$B'$	Bending stiffness per unit length
$c$	Sound speed in surrounding fluid
$c_B$	Bending wave speed
$c_{Lb}$	Longitudinal wave speed in beam
$c_{Lp}$	Longitudinal wave speed in plate
$c_T$	Torsional wave speed in beam
$c_{eq}$	Equivalent viscous damping
$E$	Young's modulus
$F_1$	Force at input side
$F_2$	Force at output side
$F_i^m$	Force at point $m$ in direction $i$
$\hat{F}$	Force amplitude
$\{F_k\}$	Known force vector
$\{F_u\}$	Unknown force vector
$F_y$	Shear force
$f$	Exciting frequency in Hertz
$f_n$	Natural frequency in Hertz
$G$	Shear modulus
$G(\vec{r}, \vec{r}_s)$	Green's function
$G_{ff}$	Force spectrum density function
$G_{fv}$	Cross power spectrum density function
$G_{vv}$	Velocity spectrum density function
$H_0^{(2)}$	Hankel function
$H_{if}$	Transfer or frequency response function for force excitation
$H_{iv}$	Transfer or frequency response function for velocity excitation
$h$	Hysteresis damping coefficient; Thickness of plate/beam
$h_{ij}$	Hysteresis damping coefficient
$j$	Unit imaginary number
$K$	Shear rigidity
$k$	Stiffness coefficient
$k_{ij}$	Stiffness coefficient
$n(\vec{r}_s)$	Normal direction vector at $\vec{r}_s$
$M$	Mobility
$M_{ij}$	Point mobility

$M_{ij}$	Mobility matrix element; Transfer mobility
$M_z$	Bending moment
$M_{ij}^{mk}$	Mobility matrix element
$m$	Mass
$m'$	Mass per unit length
$m''$	Mass per unit area
$m_{ij}$	Mass coefficient
$P$	Power
$P(\vec{r})$	Underwater acoustic pressure at $\vec{r}$
$p$	Sound pressure
$Q$	Shear force in plate
$R$	Receptance
$R_{ij}$	Receptance coefficient
$r$	Distance from excitation point
$\vec{r}$	Position vector in water
$\vec{r}_{ij}$	Displacement vector from nodal point $i$ to nodal point $j$
$\vec{r}_s$	Position vector on immersed hull surface
$S$	Cross-sectional area
$S_s$	Immersed hull surface area
$S_{ij}$	Dynamic stiffness function
$T$	Torsional rigidity
$t$	Time
$V_1$	Velocity at input side
$V_2$	Velocity at output side
$V_+$	Amplitude of progressive wave velocity along positive $x$ axis
$V_-$	Amplitude of progressive wave velocity along negative $x$ axis
$V_j$	Amplitude of decayed near-field wave velocity along negative $x$ axis
$V_{-j}$	Amplitude of decayed near-field wave velocity along positive $x$ axis
$V_j^k$	Velocity at point $k$ in direction $j$
$\{V_k\}$	Known velocity vector
$\{V_u\}$	Unknown velocity vector
$\hat{X}$	Response amplitude
$x$	vibration response
$x_i$	Displacement of interior structure as well as tangential displacement on the water contacting surface
$x_n$	Normal displacement of surface contacted with ambient water
$w_z$	Angular velocity about $z$ axis
$\{Y\}$	Modal displacement vector
$Z_{ii}$	Point impedance coefficient

$Z_{ij}$	Transfer impedance coefficient
$v_y$	Velocity along y axis
$\alpha_{ij}$	Four-pole parameters
$[\Phi]$	Normalized modal matrix
$\phi$	Phase angle
$\mathcal{I}'_z$	Rotary mass moment of inertia about z axis
$\kappa$	Shear distribution factor
$k$	Wave number
$\rho_p$	Density of plate material
$\eta$	Structural damping loss factor; Lateral displacement
$\eta_r$	Structural damping loss factor of rth mode
$\sigma$	Radiation efficiency
$\mu$	Poisson's ratio
$\lambda_B$	Bending wavelength
$\lambda_r$	rth damped eigenvalue
$\omega$	Exciting frequency in radian per second
$\omega_n$	Natural frequency in radian per second
$\omega_r$	Undamped natural frequency of rth mode

## GLOSSARY

BE	Boundary Element
BEM	Boundary Element Method
CLF	Coupling Loss Factor
DAA	Doubly Asymptotic Approximation
FE	Finite Element
FEM	Finite Element Method
FEA	Finite Element Analysis
FFT	Fast Fourier Transform
MDOF	Multi-Degrees of Freedom
NR	Noise Reduction
rpm	revolutions per minute
SDOF	Single Degree of Freedom
SEA	Statistical Energy Analysis
SPL	Sound Pressure Level
TDOF	Two Degrees of Freedom

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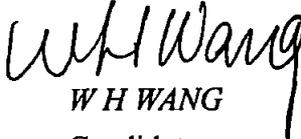
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## AUTHOR'S DECLARATION

I declare that the work submitted is the result of my own investigations and has not been submitted in support of a Candidature for any other degree of this or any other University

  
*W H WANG*  
Candidate

# CHAPTER 1

## INTRODUCTION

### 1.1 The Structure-Borne Noise Source

Noise in a ship can be a problem. For reasons of comfort and habitability, the control of structure-borne noise is of paramount importance in the design of high performance ships. Effective noise control has been one of the most difficult objectives to accomplish in ship building. In essence, noise is acoustic energy in air radiated from vibratory solid bodies. Therefore, to control noise is to reduce vibration. Vibrations are generated from periodical mechanical forces such as impact, torsion, shearing, bending, etc., produced by various machinery. They propagate via connecting junctions to beams, plates, shells, etc., to all structural parts of a ship. During propagation, vibration waves transmit in two principal modes, the longitudinal mode along the propagation orientation, and the transverse mode in all directions normal to the propagation orientation. In a ship structure, these two modes constantly transform in fraction into each other at the perpendicular joint of two structural members.

For instance, consider two beams A and B perpendicularly cross jointed. A longitudinal wave propagation in beam A reaches the junction. There a major portion of the incoming wave continues its journey in beam A, and a fraction of the wave is scattered into beam B, with the vibration mode changing to transversal one. The magnitude of the scattered fraction depends on the frequency of the wave, the nature of the connecting joint, the fixtures and materials of either beams, etc. To complicate the matter further, as the vibration waves encounter the material discontinuities in the structure, some reflections will occur. In summary, there are vibrations with various frequencies, amplitudes,

oscillation orientations, and phases existing in a ship structure. As those vibrations transmit throughout the ship structure, the transversal modes radiate acoustic energy into the air in direct contact with the structural solid surfaces. Thus, in a noisy compartment, for example, it is often difficult to identify the specific noise sources.

As ships adopt lighter weight structure to improve their speed performance, this causes stronger structural vibrations and louder noises. Especially for those fast patrol boats, where the structure is super light, with comparatively more powerful propulsion engines confined in relatively smaller spaces, the vibrations and noises become unbearable. The noise sources generated by the mechanical forces or the forces acting directly on structural parts of a machine are called structure-borne noise sources. From an onboard measurement survey of structure-borne noise level of sources, such as the main diesel engine, the reduction gear, the generator and the exhaust pipe etc, onboard a fast vessel, it has been found that the main diesel engine is often the most dominant one, Wang (1996).

Naturally, it is desirable to contain the vibrations at the origin. To this day, resilient mounts are utilized between the engine block and the base structure as the vibration isolator. But in reality, there is no resilient material in the world capable of isolating the entire vibration power by its molecular friction mechanism alone. Thus, a large portion of the vibration power does pass through the mounts into the structure. Then every portion of the structure is a noise source. To analyse and describe the whole picture is a mission next to impossible.

However, with sufficient knowledge of the vibration transmission mechanisms in the structure, a simplified model may be constructed and an approximate solution may be obtained for this noise control problem.

## **1.2 Noise Source Transmission in Structure**

Structure-borne sound power is predominantly transmitted to a sound carrying structure from a source via a number of contact points. In turn, the noise and vibrations are propagated in the structure possibly causing sensitive equipment to vibrate or to cause

undesired radiated noise. In principle, this may be avoided by measures at source, in transmission, during propagation or at radiation. It is, of course, preferable to cope with the problems at the generation sites and thereby avoid more comprehensive and expensive measures later in the chain. To this end, the resilient mounts are utilized as an effective countermeasure to reduce noise and vibration power transmitted from the source to sound receiver.

In reviewing noise transmission paths, it is noted that from the machinery room compartment to other areas of a ship, there are three parallel paths : air, water, and structure, the latter being the most important. While structure-borne noise is familiar, it takes an uncommon form in ship hulls : compared to industrial environments, the hull is relatively homogeneous and long wave-guide and whose transmission characteristics are affected by spatially periodic stiffening frames. Furthermore, the waterborne path is intimately coupled to the structure-borne path, partially short-circuiting structural impedance discontinuities such as bulkheads, which, in the atmosphere, would constitute effective barriers to structure-borne noise propagation. Another peculiarity of a hull is that, over much of the relevant frequency range, radiation loading is associated with inertia forces, exerted by the entrained mass of water, which modify the hull plating response. Because of the high sound velocity of water, the coincidence frequency of surface ship hull plating lies in the ultrasonic range. Flexural waves therefore radiate from areas where wave number conversion occurs, viz. drive points and impedance discontinuities. Once launched, the waterborne acoustic signal suffers propagation losses which are small compared to those encountered in the atmosphere.

Noise propagation in ship structures as well as the choice of appropriate steps for control/reduction are intimately related. Figure 1.1 shows the relation between each step on structure-borne noise transmission in ships.

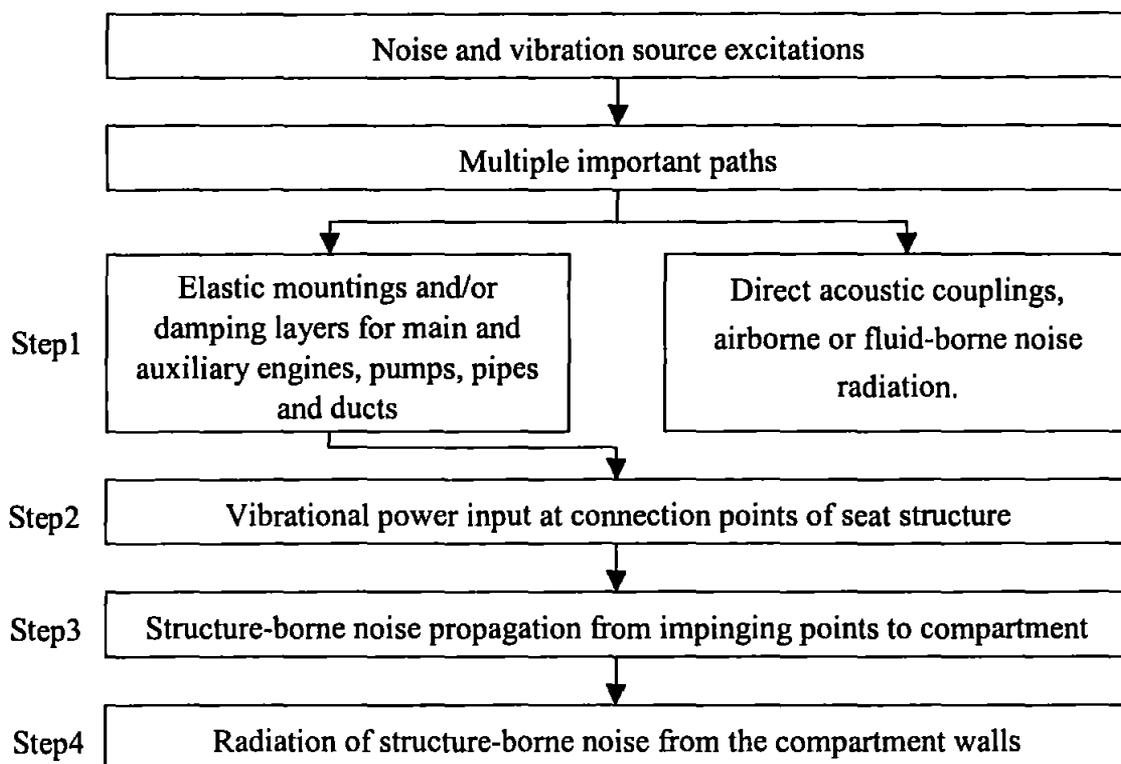


Figure 1.1 Relation between each step on structure-borne noise transmission

Machines fitted to a ship structure generate unwanted vibrations, therefore, they are often mounted on the structures via vibration isolators as a vibration control measure. In practice, machines are mounted on large flexible structures (referred to as the seating) which may have many modes of vibration, therefore, it is not convenient to consider each mode separately. For this situation, the dispersion properties of elastic waves in rib-stiffened plate systems need to be determined for application with geometries typical of a ship's structure.

### 1.3 Background and Objective of the Study

Operational motions of engine and other machinery start vibrations, and the resilient mounts under the machinery seatings initiate the noise control reduction efforts. The effectiveness of the mounts can be defined in terms of the force or velocity at the machine seating. But the vibration force and the velocity may not be in phase. Besides, a large response in a structure does not always lead to a high level radiated noise. Here a more appropriate method to describe the vibration condition is adopted for each machine. The

effectiveness of isolation is defined as the measure of the power transmitted to the structure. This single quantity includes both the vibration parameters of force and velocity. An important reason for using the concept of power is that it provides a means of comparing the effects of both translational and rotational excitations. Minimizing the power transmitted through an isolator is also consistent with the first step in any vibration control exercise, ie, to deal with the problem at the source. Normally, the resilient mounts are composed of elastic and viscoelastic materials and designed in various configurations.

The vibration power fed into a structure for a multipoint mounting system can be calculated by utilizing the concept of effective mobility of the source /mount /receiver system and the source characterization as a force or a velocity. Furthermore, if the relations between the mobility function pertaining to the coupled and decoupled source/ mount/ receiver systems can be derived, then the effectiveness and performance of the mount can be identified. To achieve a successful design and an appropriate selection of the mount, sensitivity analyses of the mobility function value with respect to the dynamic properties should be undertaken.

Owing to the complexity of a ship structure, the effectiveness of the structure-borne noise isolation measure should be tightly dependent on the propagation wave patterns.

Besides considering the resilient engine mounts, a model of structure-borne noise propagation in the foundation structure has to be developed. In most cases, however, the foundation is formed in branch structure, which means the interactions between the contact points may be reduced due to intermediate stiffeners and/or energy transmission to adjacent structural parts, Cremer et al (1988). The main reason of this phenomenon has been found to be the standing wave effect in the complex branch structure. Basically, a structure which is excited, e.g. by means of a point force, will disperse motion. To describe the dynamic behaviour of the structure, the transfer mobility function matrix approach is used in the study.

Both in the predictions of the effect of design modifications for vibration isolation and in predictions of structure-borne noise propagation through complex systems, the structural characteristics of the source and the receiver are required. The quantity primarily sought is the active sound power transmitted to the supporting structure. The aim of this research investigation is to establish a new holistic model to line up the transmission, propagation and radiation predictions of a machine induced noise and vibration in a ship's structure that takes into account the fluid-structure interaction effect.

To achieve the above, the research objectives are:

- (a) Engine or machine characterization as a velocity source coupled with mobility data in order to obtain convenient measurements and power calculations;
- (b) Develop models for transfer mobilities and transmissibilities from engine to foundation;
- (c) Numerical analysis and validation of the results by experimental measurements;
- (d) Establish a model of propagation of structure-borne noise and vibration from the engine or machine foundation;
- (e) Numerical analysis based on the stress wave model for the sound and vibration distribution in the ship's structure by using a computer package and self-generated software;
- (f) Develop a combined finite element and boundary element method in dealing with the prediction of underwater sound radiation.
- (g) Validation of the analysis accuracy by undertaking experiment measurements.

It is intended that the developed model will be suitable for use in the design stage of such, or similar, marine craft and to assist in the reduction of vibratory noise in existing ones.

#### **1.4 Contributions of this Research Work**

The contributions to knowledge that have been made as a result of this research study are as follows:

- (a) Establishment of the relations of the mobility functions with respect to the four-pole

parameters and the dynamic stiffness coefficients of a coupled machine/mount /foundation system.

- (b) Clarification of the expressions to represent the sound input power, the output power and the transmitted power in relation to mobility functions of a coupled machine/mount /foundation system.
- (c) Development of a new numerical stress wave model to predict the propagation of structure-borne noise within the intermediate frequency range of 125 Hz to 1 kHz where existed a knowledge gap in the pervious literature.
- (d) Accomplishment of a holistic model to line up the transmission, propagation and radiation predictions of a machine induced noise and vibration in ship's structure to take in account the fluid-structure interaction effect.

### **1.5 Outline of the Report**

In more detail, the works carried out in this study are summarized in Table 1.1. In which the stress wave finite element method (FEM) is developed by continuing the previous work by Cremer et al (1998). An interface program to link the finite element and boundary element software is also developed. In the transfer mobility prediction model, the establishment of the relations of the mobility functions with respect to the four-pole parameters and the dynamic stiffness coefficients is a unique contribution.

Table 1.1 Summary of the works carried out in this study

Application Model	Mobility and Transmission					Structure-Borne Sound Propagation			Underwater Acoustic Radiation	
	Beam	Plate	Test Rig (I)	Test Rig (II)	Patrol vessel	Test Rig (I)	Test Rig (II)	Patrol vessel	Test Rig(II)	Spherical shell
FEM (ANSYS)	√	√	√	√	√			√		
FEM/BEM			√	√		√	√		√	√
Stress wave FEM	√	√								
Analytical	√									√
Experimental Measurement	√	√	√	√					√	

A literature survey in those areas where vibrational power transmitted from machines into various ship structures and prediction models for propagation of structure-borne noise in ships is given in Chapter 2. The survey clearly reveals the fact that there are not enough validated models for predicting the propagation of structure-borne noise within the medium frequency range, i.e. from 125Hz to 1kHz, present in the published literature. Thus this gap in knowledge needs to be bridged.

The mobility model to calculate the transmitted power flow to the foundation structure from a machine is introduced and developed in Chapter 3. A mobility theory is offered in this Chapter for evaluating the point and transfer mobilities of various machine-isolator-receiving structure systems. It is the predominant and important aspect to cope with the machine induced structure-borne noise and vibration problems.

In order to evaluate the mobility reduction behaviour of structure-borne sound transfer from a marine engine via resilient mounts, the concept of mechanical four-pole parameters, which represent the physical state at a junction, is adopted to explain the sound attenuation

theory of a resilient mount.

A coupling model, combining finite element method (FEM) and boundary element method (BEM), is developed in Chapter 4 for the purpose of predicting the sound propagation in a ship structures to take into account for the fluid-structure interaction effect. In the derivations of the transfer matrices of a finite beam and a finite rectangular plate element, a stress wave model has derived. A FORTRAN program is developed based on this stress wave model. This program is especially suitable for analysing structure-borne noise and vibration within medium frequency range in a more efficient way. The validations of the developed stress wave model and the coupled FEM/BEM model are given in this Chapter. Also, the developed coupled FEM/BEM model has applied to study the machine induced underwater radiation problem.

In Chapter 5, the experimental model studies for measuring point and transfer mobilities from a machine to foundation are described. Validation of the four-pole parameters model is made by comparing the measurement results on two different scale engine room models. Also, the investigations of the effect of the resilient mounts in the attenuation of structure-borne noise power transmission are discussed.

A mobility study on the improved engine room structure by using squeeze film damping plates in a patrol vessel is also exhibited in Chapter 5. The improved radiated cabins' noise levels from the machines were discussed by measurements.

In Chapter 6, an application of the coupled FEM/BEM model to predict a machine induced underwater acoustic radiation is performed on the scale engine room model. Experimental measurement for validation was also given.

The discussion and the conclusions of this work are presented in Chapter 7. Recommendations for further work are also given.

Some of the chapters have appendices related to them. In connection with Chapter 5,

Appendix A details derivations of the transfer function matrix of finite rectangular element for the elastic bending wave. Furthermore, in Chapter 4, the discretization model of the Helmholtz equation by boundary element method is given in Appendix B. Also, there is a requirement to consider the Green's function for the solution of Helmholtz equation, this is discussed in Appendix C.

Appendix D are the technical papers published during the progress of this study.

Finally, shown in Appendix E is the FORTRAN program written for analysing the structure-borne noise propagation based on the stress wave model described in Chapter 4.

## **CHAPTER 2.**

### **OVERVIEW OF THE FIELD OF RESEARCH**

#### **2.1 General Background**

The interest in acoustics and structural dynamics has increased in Naval Architecture Engineering during the last decade. The possibilities of looking into the details of sound and vibration transmission, propagation and radiation to, in and from the solid structures have also been enhanced due to the rapid progress of advanced electronic devices, such as the piezoelectric accelerometer, the impedance head, the hammer kit, the hydrophone, the digital sound level meter and the fast Fourier transformer. Owing to the increased complexity with respect to both technical equipment and constructions as well as to the environment demand sophisticated methods are required to predict the consequences of planned alterations and to validate by measurements.

Many noise problems do not surface until the ship is in operation. Simple as well as inexpensive methods to rectify the undesirable situations must be sought. Otherwise, it may be extremely difficult to get a solution. In addition, the assembly of different subsystems can lead to unintended sound and vibration problems despite the fact that each subsystem works extremely well separately. Therefore, all parts of the sound and vibration chain are important and should be equivalently treated.

The following summary is a short version of the research works done on the problem, providing a general background of this study. The references are arranged more or less in accordance with the steps outlined in Fig1.1.

The works of Middleton (1976), Kihlman (1978), and Plunt (1982) have discussed the possible ways in which the structure-borne noise source strength could be defined and measured. They concluded that the practical methods, available for the noise level prediction in buildings and ships, were those which define source strength of machinery by reference to the response of a typical foundation to the force source. Machinery impedance methods might appear to offer some assistance in the derivation of the vibration source strength of a prime mover or other item of machinery. Classical impedance theories treat vibration generators as ideal elements such as force sources or velocity sources. Real machines tend not to fall neatly into these categories because the machine itself is a structure. Large machines, such as main engines, appreciably affect the structure of the ship. The source strength of an engine could be quantified by measuring the velocity or acceleration response of the bed plate in the test house when the engine is undergoing land trial. However, the information would only be useful if the characteristics of the bed plate could be defined and compared with the characteristics of the ship's hull.

The transmission in turn, is completely determined by the source strength together with the dynamic characteristics of the source, of the receiver and of the coupling elements in between the two subsystems. A series of research works, such as Pinnington (1980, 1987, 1988), Pinnington and White (1981), Pinnington and Pearce (1990), Dobson et al. (1993); Janssen and Buiten (1973), Buiten (1976), Ohlrich (1979, 1980), Petersson (1980, 1983), Petersson and Plunt (1980, 1981, 1982a, 1982b), Plunt (1983), Plunt and Jensen (1983), have been undertaken in United Kingdom and Nordic Co-operative Organization respectively since 1980. Such works concentrated on investigating the parameters controlling power transmission from a vibrating machine to an extensive receiving structure, such as a ship hull, via spring-like vibration isolators. Part of the work was an investigation looking into convenient ways of representing a machine as a source of vibration power. Two simple models of a machine were developed, namely a mass and a finite beam. For the first one, a mass was used to represent a machine at low frequencies, when it moves as a rigid body. This model was used by Pinnington and White (1981), which was applied to an experimental investigation of power transmission from a mass

excited by a force, the mass being coupled to a long finite beam by a rubber spring. The rubber spring represented a machine isolator, and the long finite beam, represented an extensive receiving structure. At low frequencies a machine can be considered as a rigid mass, but at higher frequencies it vibrates in its natural modes of vibration, therefore impossible to accurately model it with a simple element. Thus a free-free beam has been chosen, at this first instance, to characterise a resonant machine source. This choice is simple enough to permit mathematical analysis, yet it has the necessary feature to provide an understanding of power transmission from a resonant machine source. Furthermore, the theory of Pinnington (1980), Pinnington and White (1981) considered the power input to, and transmitted from, a short finite beam connected to a semi-infinite beam by a spring-like isolator. The semi-infinite beam was the simplest way in which to represent the receiving structure. For the second one, the theory considered power transmitted from a short finite beam connected to a long finite beam by a spring-like isolator. In this case the long finite beam, having many resonances, represents an extensive but finite receiving structure. Experiments were then carried out with physical models of the above two configurations, and the measurements of peak and frequency average power were compared with the predictions. Most experimental measurements in these research works were undertaken in a laboratory. While in the case of a ship structure, the characterization of power flow of structure-borne noise source by measurements will be much more complicated.

Heckl (1976) pointed out that the behaviour of elastic mounts at low frequencies was fairly well understood and it was possible to predict their performance provided the stiffness of the mount and the impedances of the structures on both sides of the mount were known. For higher frequencies, elastic mounts must be considered as wave-guides having higher order resonances. It was shown that the first higher order resonance depends mainly on the mass of the resilient element. It was also shown that for large resiliently mounted engines at medium and higher frequencies (100-500 Hz) the sound transmission through the surrounding air can be more prominent than that through a good resilient mount.

Kihlman (1982) developed a simple model to calculate the vibrational power fed into a structure for a multipoint mounting system based on the concept of effective overall mobility of the source and the foundation and the effective transfer mobility of the vibration isolators. However, the vibration source was considered to be under rigid body motion only.

Horner (1990), Horner and White (1990, 1991) have developed the models to determine the wave type, either flexural or longitudinal wave, which carry most vibrational power through bends or joints in beam-like structures. By establishing the wave type which predominantly carries power transmission, it is then possible to apply the most suitable vibration control measure. Gibbs and Tattersall (1987) considered all modes of vibration, including compression, torsion and bending, both for the incident and generated waves of vibrational energy transmission at a corner-junction of square section rods.

At the beginning of the 1980's, Goyder and White's works (1980a, 1980b, 1980c) studied the phenomena of wave propagation and power flow in the far field of an infinite plate with a single infinite line-beam. Such a structure is excited by forces or torques applied to the beam at the driving point. In the far field, power transmitted by flexural waves in the beam is radiated into the plate but power transmitted by torsional waves in the beam is not radiated. The general conclusion from their analysis is that the power flowing into the system is controlled by the beam while the subsequent flow of power throughout the structure is governed by the plate. Also the plate carries a cylindrical wave with a strong directivity. However, their method is hardly applicable to the vibrational power flow analysis of real ship structure, as the latter is composed of plate-stiffener combination with limited plate extent and multiple stiffeners.

Regarding the development of the measurement technique of power flow, Pavic (1976) examined the signal propagation characteristics in both nondispersive and dispersive media based on the signal coherence at two different points along propagation paths. In propagation along nondispersive paths, coherence was achieved by characteristic time lag

shifting and crosscorrelation analysis was used. For dispersive paths in which the coherence of the traveling signal was degraded along the path and conventional crosscorrelation ceased to be useful as a method of analysis, Pavic showed that the coherence between the signals can be restored by applying some transformation to the spectra of the signals. The resultant simplified power flow measurement methods and the conceptual design of analog electronic measuring devices could be applicable in some special cases of flexural vibrations in both beams and plates. The finite difference approach was chosen for obtaining spatial derivatives of kinematical quantities relevant to power flow determination.

The two processes, i.e., generation and transmission of the noise in ship structure are not independent. Knowledge of the mechanisms governing the generation as well as the dynamic characteristics of the source allows for an approximate description of the source strength. Verheij (1976) discussed an experimental method in which the internal excitation of a machine was replaced by an equivalent external force. Application of this method to three diesel engines for the octave bands with central frequencies from 125 to 2kHz showed that the real excitation may be replaced by an external equivalent force, which generates within 3dB the same response as that of the running engine, for various receiver positions, various sound transmission paths, various ways of mounting and various environments. There are some limitations of such a method: (1) the vibrational energy should be distributed over the respective transmission paths in the same ratio; (2) the equivalent forces measured with the aid of a loudspeaker should be easily recognizable by their narrow band signature of the open circuit voltage of a reciprocal transducer in the mechano-electrical two-port system, whereas for broad band sources this will be less easy; (3) the transmission system would be reciprocal.

The works of Cremer et al (1988) have dealt with analysis of the propagation of structure-borne noise in vehicles and buildings, especially concentrating on the propagation wave pattern for the fundamental structure such as simple infinite (or semi-infinite) beam, thin plate or shell.

Nilsson (1976, 1977, 1981) investigated the propagation of the structure-borne noise in the superstructures for typical medium-sized tankers. It was found that the attenuation of structure-borne noise was sufficiently well described by a simple flexural wave model for a frequency range of 31.5-4kHz. After Sawley (1969) and Lyon (1975) developed a framework of the study of the energy flow of structural vibration systems called Statistical Energy Analysis (SEA), Jensen (1976) discussed the possibility of calculating expected vibration amplitude distribution throughout a ship's structure with a higher degree of accuracy using a power flow model based on SEA. The comparison between measured and calculated values of the insertion loss on a ship model section has shown that the agreement was reasonable; somewhere acceptable and elsewhere poor. The experimental investigation on the ship structure model has shown that it is hardly appropriate to limit the SEA model of a ship section to include only flexural vibration modes in the structure. At last, when the intention is to carry out a complete analysis of the structure-borne noise propagation in complicated steel plate construction (composed of combinations of plates joined in either T-junction, L-junctions or cruciform junctions), it is necessary to consider the flexural wave combined with longitudinal and shear wave transmission through the hull plate sections simultaneously. The main problems involved in the application of SEA for prediction of noise level concerning structure-borne noise propagation, as pointed out by Sawley (1969), Kihman and Plunt (1976) and Irie and Takagi (1978), was in getting proper input data to the calculation.

Sound radiation as a result of structure-borne vibration has been treated by Gophey (1967), Ginsberg and Rosenkilde (1986), Cremer et al. (1988), Chen and Ginsberg (1988) and Fahy (1989). Fahy explained that sound radiation from vibrating surfaces could be expressed in terms of the distribution of normal surface velocity. Analysis of sound radiation from planar surfaces by means of far-field evaluation of the Rayleigh integral and synthesis of travelling wave Fourier components were presented in such a way that the equivalence of these dual approaches could be appreciated. Gophey derived a regular integral equation from the classical Helmholtz integral and a numerical quadrature technique, successfully solved the integral equation to obtain the distributions of pressure

and velocity over a vibrating surface submerged in an unbounded acoustic medium. The acoustic radiation from an elastic plate driven by an underwater shock (Ginsberg and Rosenkilde, 1986) or by a central point force (Chen and Ginsberg, 1988) was numerically solved by the Doubly Asymptotic Approximation (DAA) model which took the fluid-structure interaction into consideration. The results showed that the pressure distribution was not in good accord with the displacement. The main reason is that in the formulation of DAA model the numerical error for displacement is of the first order whilst that for pressure, which is proportional to the velocity, is of the second order.

## **2.2 Special Topics**

There are some subjects of particular interest as this study is concerned. The relevant references are assembled accordingly.

### **2.2.1 Multi-Point Transmission**

The classical approach for calculating structure-borne noise and vibration transmission is based on the assumption of a velocity source performing rigid body motions. Thus a multi-point coupling between the source and the receiving structure may be considered as a combination of separate poles according to Pinnington and Pearce in 1990. For a machine mounted at four sites there will be four possible modes of motion; the monopole motion with all sites moving in phase, two dipoles with opposite pairs moving in anti-phase (rocking about two perpendicular axes), and a quadrupole in which diagonal pairs move in anti-phase.

Although it was possible to handle mathematically all six degrees of freedom in motion as well as the potential coupling between them, the unwieldiness of the classical approach grows rapidly as the number of degrees of freedom included increases (Timoshenko and Young, 1955). It is therefore customary to simplify the transmission problem to only one degree of freedom. The rigid body motion treatment underlying this approach is often questionable. Instead it is usually necessary to treat both the source and the receiving structure as continuous systems for low frequencies. Such a description of linear source

and receiver systems is conveniently obtained by means of the concept of mobility which, most often, is more preferable than the mechanical impedance in dealing with structure-borne sound and vibration transmission problems.

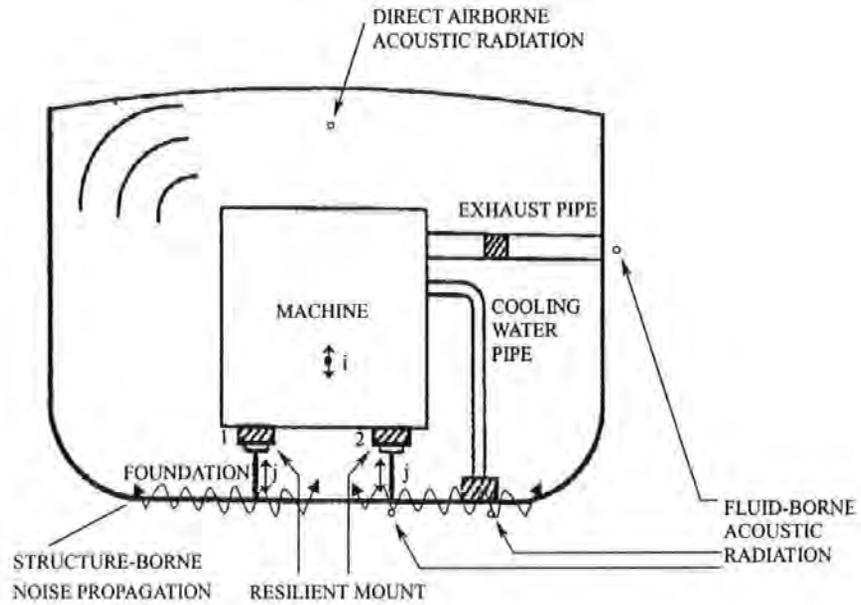


Figure 2.1 Possible acoustic transmission paths

The source and receiver structures are commonly coupled to each other at several discrete points as shown in Figure 2.1. The word "point" herein is used in a somewhat extended meaning, denoting an area with its dimensions smaller than a fraction of the governing wavelength. The physical coupling of the structural subsystems is often constituted by relatively small elastic elements ( chocks, isolators, etc) called the transmission elements. For a general system, the subsystems are thus consisting the source (including possible feet), the transmission elements and the receiver. If no transmission elements are present, the interface between the source and the receiver is located below the feet. Of course, there are other possible assembly variations, but the one described facilitates a unified treatment of rigid and resilient installations.

In the general case, the interaction between the different contact points as well as between the different components of motion must be taken into account. For the wide frequency band of interest, the handling of the general matrix formulation results in an excessive task

since not only the contact points between the source and the receiver can be quite numerous, but also the six components of motion relative to Cartesian axes and excitation at each connection must be taken into account.

A large number of works concerning developing the methodology and describing experimental methods for the determination of structure-borne noise and vibration transmission have been reported. It is also shown that in the experimental situation the benefits of the mobility representation are apparent.

Ohlrich (1979, 1980) compared the dynamic characteristics of four types of bedplates used for engine mounting in ships and revealed that the characteristic of the bedplates has a local effect to the structure-borne noise control in the frequency range from 20Hz to 20kHz of interest for shipboard acoustics. The bedplate structure is composed of two parallel platestrips supported by girders and frames, which may form an integral part of the double-bottom structure in a ship. The results presented were derived from experiments on models of 5-frame bottom sections of ships. Also the vibration transmission from the bedplate to the hull structure by applying a harmonic point force excitation normal to the bedplate in a single point was discussed. This relationship is an important factor for the prediction of engine-generated noise in remote areas.

In 1980 to 1983, Petersson(1980, 1983), Petersson and Plunt (1981, 1982a, 1982b), Plunt and Jensen (1983) and Plunt (1983) applied the effective mobility concept to the analysis of a structure-borne sound power transmission between coupled structures via multi-paths from a machine. Also the methods for determining the effective mobilities in the transmission of vibrations between complex structures connected at several points were developed. The general mobility matrix of multi-point, coupled structures can be formulated into a  $6N \times 6N$  matrix, where  $N$  is the number of contact points. The effective mobility represents the ratio of the total velocity due to all applied forces to the force acting at a contact point to be considered. Thus it is possible to neglect the influence of some element in the mobility matrix and to rearrange the general mobility matrix into some

corresponding effective mobilities. Some results from full scale measurements of point and transfer mobilities on the feet of shipboard diesel engine and the foundations were presented and some possible simplifications of the mobility description of these structures were discussed. The measurements were performed for the frequency range 50-3200Hz. The examples of measured point and transfer mobilities for the structures in their studies showed that the point mobilities were generally greater than the transfer mobilities because of the inhomogeneity of the structures. Also, the point mobilities were found to be mainly determined by the local dynamic properties of the structure. Accordingly, the effective point mobilities of complex, inhomogeneous structures can be approximated by the corresponding ordinary point mobilities in many cases. For homogeneous structure and structures where there are no strong discontinuities between the contact points, at which the source and the receiving structure are connected, the interaction between the points must be taken into account. However, the effective point mobility can be approximated in such cases by the ordinary point mobility multiplied by different correction factors. The agreements between the measurement mobility results and the results from the estimation procedures were very good. Besides, in the utilization of the effective mobilities as the descriptive parameters to evaluate the structure-included acoustic energy input on the combination of diesel engine---coupling elements---foundation structure, the prediction models of structure-borne noise power transmission still need the appropriate source velocity data.

Verheij (1986) made some significant contributions to the methodology for analysing the multi-path sound transfer in resilient mounting systems in ships, and for improving such systems in an economical way by using the experimental methods of reciprocal transfer function measurement for quantifying the sound transfer path. The concept of this approach for the path analysis is that, the measurements were performed with the engine stopped, whereas the machinery noise is simulated by excitation upon the outside of the engine with a number of incoherent exciters. It is assumed that for the frequency range where the engine no longer vibrates as a rigid body, the internal excitation may be modelled with sufficient accuracy by this artificial excitation. Only by using this artificial

excitation does it become practicable to interrupt sound transfer path, e.g. by removing a flexible pipe coupling. If then for constant excitation the sound transfer to a distant position decreases, it is certainly a quantitative indication for the relative importance of the interrupt path. An essential step simplifying these experiments is to perform the transfer measurements reciprocally. The great advantage of this approach might be that the predominant transfer paths can be detected with relatively simple experiments. Moreover, a quantitative insight is obtained as to what extent a certain resilient mounting system can be improved without modifying the paths through the mounts and through the air.

The sound source under investigation was 4MW medium-speed propulsion diesel engine onboard a twin propeller passenger and car-ferry. Measurements had been performed both at sea during a normal service trip and at the yard before the ship went into service. During the measurements at sea, the two outer of the four main engines were in service. They were running deliberately at different speeds to facilitate source identification. The engine close to the receiver location was running at 512 rpm, the more distant engine at 480 rpm. The receiver location for determining the sound transfer to the accommodation, was on the hull at approximately 10m above the tanktop of the double bottom in the engine room. To characterise the total sound transfer to this location, the open circuit voltage of a reciprocal electrodynamic vibration exciter was used. It's response at sea was completely determined by the nearest main engine for the frequency range of the path analysis, i.e., the 63Hz-1kHz octave bands.

More recently, Koh (1992) recognised that for the study of machine-induced vibration problems, considering resonance responses of the coupled systems as well as the interactions among governing degrees-of-freedom, a more detailed model is necessary. The unifying concept of time-averaging vibrational power was utilized to examine the vibration transmission from a machine source to flexible beam and plate-like seating structures via the translational and rotational motions as well as the coupling between these motions, for the case when the seating structures were subjected to co-located simultaneously acting sinusoidal force and moment excitations. For linear structures subjected to simultaneously

acting force and moment excitations, the driving point coupling mobility function always exist, except for the special case when the excitation point coincides with a symmetrical point of the mode shape. These coupling mobility functions contribute to vibrational power components transmitting to the structures as important as the direct force and / or moment mobility functions. Thus the cancellation among the transmitted vibrational power components to the seat structure is possible, which constitutes a special vibration control technique. Koh proposed this vibration control technique to reduce the unwanted machine-induced vibration levels on the seat structure, at a specific frequency, by means of controlling the rate of the applied moment to the applied force via suitably designed force and moment seatings attached to the mounting point. This control technique is most suitable for the case of a low to medium constant speed machine mounted on a flexible seating structure.

### **2.2.2 Mobility Prediction**

The mechanical point mobility at a point of a structure is defined as the complex ratio between the velocity and the force acting at the same point. The moment mobility may analogously be defined as the ratio between the angular velocity and the moment. Generally the magnitude and the phase of both the velocity (angular velocity) and the force (moment) are required. The force and moment mobility functions are completely determined by the dynamic characteristics of the actual structure, such as the mass (inertia), the stiffness and the losses (internal losses as well as coupling or radiation losses).

In case where the subsystems (machinery, foundation) are connected at several points, as shown in Figure 2.1, the so-called transfer mobilities must also be defined. These transfer mobilities are defined as the ratio of the velocity component at point  $k$  to the force component acting at point  $m$ . Generally, there are six independent components of motion at a point, three translations and three rotations. Accordingly there must be transfer mobilities defined between all six components, obviously including point mobilities. Thus O'Hara (1967) generally defined the elements in a total mobility matrix as:

$$M_{ij}^{mk} = \frac{V_j^k}{F_i^m}, \quad (m,k=1,2,\dots,N; \quad i,j=1,2,\dots,6) \quad (2.1)$$

Where  $F_i^m$  is the force at point  $m$  in the direction  $i$  and  $V_j^k$  the velocity at point  $k$  in the direction  $j$  due to this force. For the linear systems, the reciprocity principle should be valid, which means that the indices could be transposed but with the restriction that both the point indices and the direction indices are transposed simultaneously, i.e.,

$$M_{ij}^{mk} = M_{ji}^{km} \quad (2.2)$$

For continuous systems where the number of degrees of freedom are infinite, the mobility can be deduced from the wave equation. Such a deduction can be rather complicated, especially for finite structures in resonances. Generally, the mobility can only be expressed in a closed form for infinite system. However, there are special cases often of great practical importance for first order calculations and estimations. Figure 2.2 presents a typical mobility plot. The mobility of an infinite elementary structure is called the characteristic mobility. Whilst finite structures possess more or less recurrent resonances. The expressions of the mobility are quite complicated, since they are mainly determined by these resonances. In many cases, it is not even possible to find practically useful expressions unless special methods for the determination of the eigen-functions are used.

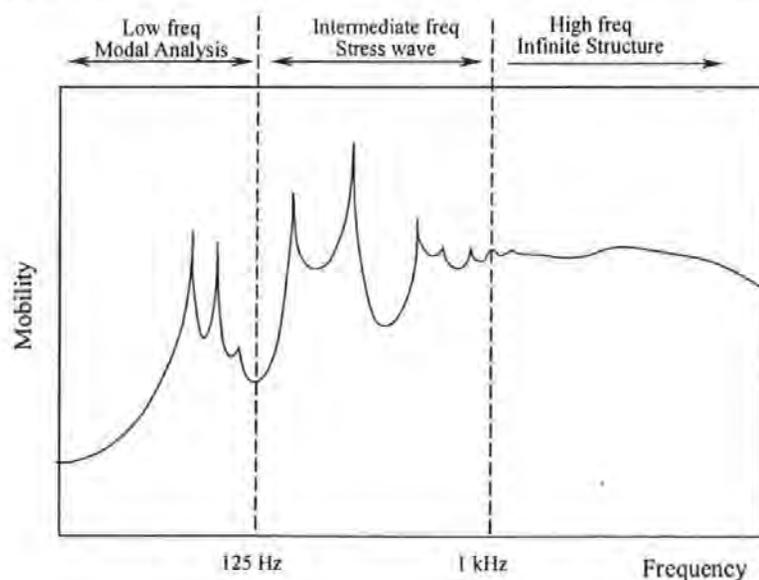


Figure 2.2 Typical mobility plot

The mobility (or the inverse quantity of mechanical impedance) has been derived for a number of systems that are either of infinite or semi-infinite in extent. Two compilations of expressions were given by Cremer et al. (1988) and Pinnington (1988). For a limited number of finite systems, closed form expressions representing the mobility have been established, such as by Snowdon (1968). However, in the general case of a finite system, the mobility is not obtainable in closed form.

For the low frequency region the mobility may be estimated using a series solution approach (Cremer et al., 1988), or a finite element analysis (Hale and Meirovitch, 1980). But, to consider a moderate number of eigen-functions (modes) contributing to the dynamic behaviour of the system, the numerical work required is very time-consuming. Moreover, the fineness of the finite element mesh needed is related to the maximum frequency of interest, which in sound and vibration problems can be relatively high. Naturally, the physical size of the system under consideration also put a limit to this approach.

Damping is the removal of energy from a vibratory system. The energy loss is either transmitted away from the system by some mechanism of radiation or dissipated within the system. Most measurements of damping are performed under conditions of cyclic or near cyclic oscillation. Usually, the decay of free oscillation is observed directly, or by measurements of steady-state forced vibration at (or in the vicinity of) resonance. In both cases the total energy removed in a cycle can be inferred, but the results are seldom precise enough to provide a detailed picture of how the instantaneous rate of energy removal fluctuates within a single cycle. Apart from the correct data input problems, the handling of amplitude and frequency dependent losses can be rather intricate (Crandall, 1970). Surveys and discussions of the different mechanisms governing the damping are found in the works of Crandall (1970), Unger (1973), Bert (1973), Nashif et al. (1985) and Sun and Lu (1995).

Another approach which has become increasingly attractive is the synthesis of experimental and numerical methods owing to the development of the Fast Fourier

Transform (FFT)-technique. The modal parameters were extracted from a few transfer function measurements and a complete mobility matrix was found numerically by Ewins (1980) and Goyder (1980). Unfortunately, it is found that such an approach is critically dependent on the measurement technique and the subsequent analysis of the results, and misleading conclusions can be reached (Ewins and Griffin, 1981). The discrepancies between measurement and analysis are due to localised errors (loss of stiffness at joints, etc), or to more general factors (such as incorrect values of elastic modulus or material density etc), or to the case that the location of the response points used in the modal test does not coincide exactly with any of the mesh of grid points used in the analytical model. In addition, the extraction of the modal parameters can be very cumbersome for heavily damped systems or for cases of high modal density with strong coupling (Looser, 1982).

For the high frequency region of the non-resonant system, i.e., where the modal density of built-up structures is generally high, Skudrzyk (1980) and Cremer et al. (1988) discussed the validity that the mobility of the actual finite system can be approximated by the mobility of the corresponding infinite system. It was demonstrated for some structure configurations for which expressions giving the frequency for the onset of the infinite system behaviour were proposed by Goyder and White (1980a). Moreover, as frequency increases, the mobility is determined better by the dynamic characteristics of a small substructure surrounding the point under consideration. This means that the infinite system behaviour and thereby the mobility may be estimated for the structural element within the nearest discontinuities (Bishop and Mahalingam, 1981).

Consequently, the prediction of the mobility is most difficult in the mid-frequency range. However, if details of the vibrations are not of interest but rather the average dynamic behaviour of the system is under consideration, the mobility can be derived from that of the corresponding infinite system (Skudrzyk, 1980).

The inadequate understanding of the numerous mechanisms governing the losses of a structural system, both with respect to the dissipation and to the coupling or radiation to

external systems, makes the prediction of the mobility more difficult. This in turn, is essential for the determination of the structure-borne noise and vibration power transmission. Thus, in general, such predictions still, to a great extent, must be based on experience and empirically acquired data.

The problems in predicting the real part or mobility, i.e., that part associated with the power transmission, were discussed by Goyder and White (1980a, 1980b, 1980c) and Pinnington (1980). For the low and mid-frequency regions, the real part of the mobility of the finite system is normally, on the average, less than predicted on the assumption of an infinite behaviour, and is approximately equal to that value at high frequencies.

### **2.2.3 Prediction Method for Propagation of Structure-Borne Noise in Ship**

The majority of prediction methods for structure-borne noise propagation are basically empirical and written in computer software programs which are easy and inexpensive to operate. This approach views previous ships as full-scale models for future similar ones, meanwhile trying to find rules for arranging experimental findings and for translating them into reliable predictions. The accuracy of these programs is satisfactory for standard ship designs. In principle, this approach comes down to asking what is the difference in acoustical effect between a new ship and ships that could be investigated. However, parameter studies and prediction for non-standard ship design cannot be accommodated using these methods. Among these, probably the most well-known is the method proposed by Janssen and Buiten (1973). Also further developments of this method, where more detailed data for the reductions in velocity levels per deck and per frame were taken into account by Buiten (1976) and Plunt (1980), are available. Common to all these empirical methods is the requirement that a velocity level for the foundation of the source is used as the input data.

Other prediction methods are, in general, based on the Statistical Energy Analysis (SEA) method (Lyon, 1975) or the so-called waveguide methods (Nilsson, 1978a). Such methods, which describe the propagation of structure-borne noise in a ship structure, have certain

limitations in the low frequency region, i.e., below the first few plate resonances (Nilsson, 1980). The first resonance frequency can be fairly high for a plate element in the tanktop construction or in the engine foundation. This naturally causes uncertainties for the predicated noise levels in the octave bands with centre frequencies at and below 125 Hz. However, Nilsson (1976, 1978b, 1981) has developed a method for extending the waveguide model for calculating the attenuation of structure-borne sound in the junction between a plate panel and a boundary stiffener in a ship structure.

The SEA provides a means for high-frequency vibration prediction by estimating the equilibrium energies of a network of subsystems which are subjected to an assumed distribution of external sources of time-stationary vibrational power input. It offers a good tool to analyse and predict acoustic and vibration transmissions in coupled systems. From the practical point of view, the capital element for applying SEA is the determination of the coupling loss factors (CLF). Theoretical estimations of CLF has been obtained in simple cases from wave propagation in infinite coupled beams by Gibbs and Tattersall (1987). The SEA approach for analysing complicated structures with many resonant modes in every frequency band considered has been developed by Lyon (1975). It was applied firstly to predict a ship's noise level by Plunt (1980). Other studies by Sawley (1969), Jensen (1976), Kihlman and Plunt (1976), and Irie and Takagi (1978) also reported positive results from the use of the method on marine vessels. They use the transmitted structure-borne noise power, which is not directly measurable, as the quantity to describe the source strength. However, knowing the mobilities of the foundation and the machinery as well as the velocity level of the contact points of the freely suspended machinery, it is possible to calculate the input power.

The waveguide method or the grillage method has been developed by Nilsson (1977, 1978 a&b, 1980, 1981) for the calculation of the attenuation of the structure-borne noise in a ship structure, which is modelled by a grillage consisting of the plates between two successive frames. The moments and rotation at the plate junctions are calculated. In this method the input power to the plates is given by the product of the moment and the angular

velocity. Accordingly, the primary quantity for the description of the source is the fore-and-aft directed moment, transmitted from the engine to the foundation. Direct measurements of these moments are very complicated to perform. Nilsson investigated various methods to decrease the noise levels in the accommodation spaces in superstructure by model tests. The attenuation of structure-borne sound in the propagation path between source and receiver is increased by means of damping layers, resilient mounts and changed boundary conditions between main deck and superstructure. Damping layers were found to have only a local effect. Resilient mounts between superstructure and main deck can reduce the noise levels in the superstructure by the order of 10 dB(A).

More recently, Cabos and Jokat (1998) proposed a Noise Finite Element Method (Noise-FEM) to predict the propagation of structure-borne noise in complex structure by using the concept of “sound temperature” and “sound conductivities”. The sound temperature is defined as the mean energy per mode of a subsystem that is capable of vibration. The sound temperature can determine the direction of sound energy flux. The sound conductivities represent the coefficients of energy flux in different directions of a subsystem considering the losses at discontinuities and the changed coupling loss factors. To derive the conductivities relation, methods combining the acoustical waveguide model, the power flow finite element analysis and the SEA are used. Once the sound conductivities are determined, a power balance equation can be solved with the aid of the method of finite elements. The result is the sound temperature distribution and hence the energy distribution based on the given power fed into the system. Owing to the sound temperature just represents a mean energy per mode, thus when applying the Noise-FEM to analyse the structure-borne sound of a container ship ECOBOX 42, larger deviations of the noise level generally occur in the lower frequency range (below 160 Hz) compared with measurements.

### **2.3 Summary of the Field of Previous Research**

A number of the methods for previous activities regarding the prediction and control of structure-borne noise induced by a onboard machine have been proposed. These research

works can be categorised as calculation methods about structure-borne sound propagation in the ship's structure, the methodologies for determining structure-borne sound transmission from engine to foundation, and the measures for reducing the structure-borne sound transmission. Comparisons of what each process does and how these methods have been applied are summarized in Table 2.1. The review clearly highlights the fact that there are very few validated models for predicting the propagation of structure-borne noise within the medium frequency range, i.e., from 125 Hz to 1 kHz, and thus served as the impetus for this research work reported herein.

**Table 2.1 Comparison of the methods for previous activities concerning the prediction and control of structure-borne noise**

<b>Calculation of structure-borne sound propagation in the ship structure</b>	
<b>Calculation methods</b>	<b>Description</b>
1. Empirical methods (Janssen and Butien, 1973; Plunt, 1980; Buiten, 1976)	Statistical approach of previous ships as full-scale models to find the propagation rules of vibration velocity different of sound transmission through different parts of the hull for frequency range 32Hz-8kHz. Input data is the shaft speed, shaft power, main structure characteristics and the velocity levels of engine and foundation.
2. Statistical energy analysis (Jensen, 1976; Sawley, 1969; Kihlman and Plunt, 1976; Irie and Takagi, 1978)	High-frequency vibration prediction by estimating the equilibrium energies of a network of subsystems. Input data is the transmitted power ( velocity levels and effective mobility of foundation ) from sources.
3. Grillage method (Nilsson, 1977, 1978b, 1981)	The calculation of the attenuation of the structure-borne noise power flow, represented by the velocity squared, in the vertical direction in a ship structure is determined by the propagation of flexural waves in the plate elements between two successive frames. Based on the prediction model it is found that the velocity level of a deck is a function of the input data at the source, wave numbers, masses, losses and dimensions of the plate elements in the structure. The input data is the transmitted moments ( angular velocity and moment mobility of foundation).
4. Noise Finite Element Method (Cabos and Jokat, 1998)	Applied the concept of sound temperature and sound conductivities and combined the acoustical waveguide method, the power flow finite element method and the statistical energy analysis to calculate the structure-borne noise propagation in complex structures. The input data is the transmitted power fed into the structure system.
<b>Determination of structure-borne sound transmission from engine to foundation</b>	
<b>Methods</b>	<b>Description</b>
1. Petersson and Plunt's method (1981, 1982a, 1982b)	Applied the effective mobility concept to analyse structure-borne sound power transmission between coupled structures via multi-paths from a machine. Besides the full scale measurements of point and transfer mobilities on the feet of shipboard diesel engine and foundation, the source velocity is needed for transmitted power calculation.
2. Verheij's method (1986)	Facilitate source and path identification by using the experimental method of reciprocal transfer function measurement. Linear structural system is assumed.
3. Koh's method (1992)	The unifying concept of time-averaged vibrational power was used to examine the vibration transmission from a machine source to flexible beam and plate-like seating structures via the translational and rotational motions as well as the coupling between these motions.
<b>Reduction of structure-borne sound power transmission by changes in design of foundations</b>	
<b>Methods</b>	<b>Description</b>
1. Empirical methods (Middleton, 1976; Steenhoek, 1976; Besio and Loredan, 1993)	Undertook model and /or full-scale experiments to measure the velocity levels at various foundations and the hull.
2. Partly theoretical methods (Heckl, 1976; Petersson, 1983)	Predicting the behaviour of elastic mounts at low frequencies (<100 Hz) fairly well provided the stiffness of the mount and the impedances of structures on both sides of the mount were known. For higher frequencies (>500Hz) elastic mounts must be considered as wave-guides having higher order resonances. The first higher order resonance depends mainly on the mass of resilient element. At medium frequencies (100-500Hz) the sound transmission through air cavity beneath the engine can be more important than that through a good resilient mount.

**CHAPTER 3**  
**THEORY OF MOBILITY**  
**AND**  
**POWER FLOW TRANSMISSION**

The mobility of a structure is the ratio between velocity and force for simple harmonic vibration in the range of linear behaviour. The term point mobility is used where velocity and force are at the same position and in the same direction. The term transfer mobility is used when velocity and force are at different positions and/or in different directions.

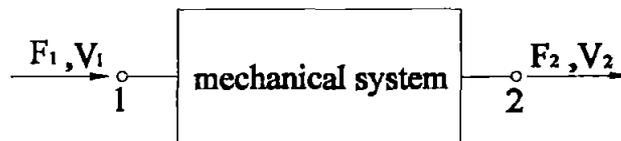


Figure 3.1 Input and output of a mechanical system

Assumed that a mechanical system as shown in Figure 3.1, with instantaneous forces  $F_1$  and  $F_2$  applied at the input and output respectively resulting in corresponding instantaneous velocities  $V_1$  and  $V_2$ . The ratio  $\frac{V_1}{F_1} = M_{11}$  is the point mobility at 1 and

$\frac{V_2}{F_1} = M_{21}$  is the transfer mobility at 2 induced by force acting at 1. For linear system the

transfer mobilities  $M_{21}$  and  $M_{12}$  are reciprocal, i.e.  $M_{21} = M_{12}$ . The velocity vector

$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}$  is related to the force vector  $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$  by the full mobility matrix, i.e.

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (3.1)$$

In this Chapter consideration is given to developing the theory of mobility prediction and power flow transmission.

### 3.1 Mobility of Single Degree of Freedom (SDOF) Vibration Model

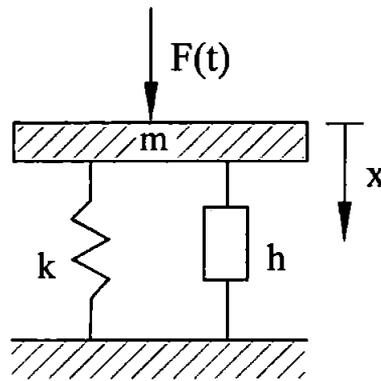


Figure 3.2 Single degree of freedom spring-mass-hysteresis damping system

Consider a single degree of freedom spring-mass-hysteresis damping system, as shown in Figure 3.2, with mass, damping and stiffness coefficients denoted by  $m$ ,  $h$ , and  $k$ , respectively. If the system is subjected to a harmonic exciting force, i.e.,  $F(t) = F_0 e^{j\omega t}$ , then the equation of motion for the response  $x(t)$  can be expressed as

$$m\ddot{x} + \frac{h}{\omega} \dot{x} + kx = F(t) \quad (3.2)$$

Thus for the harmonic excitation, where  $\dot{x} = j\omega x$ , equation (3.2) becomes

$$m\ddot{x} + k(1 + j\eta)x = F_0 e^{j\omega t} \quad (3.3)$$

where  $\eta = \frac{h}{k}$  is called the structural damping loss factor. The quantity  $k(1 + j\eta)$  is called the complex stiffness.

The mobility of the system is

$$M(\omega) = \frac{\dot{x}(t)}{F(t)} = j\omega R(\omega) = \frac{j\omega}{[(k - \omega^2 m) + j\eta k]} = \frac{j\omega}{m[(\omega_n^2 - \omega^2) + j\eta\omega_n]} \quad (3.4)$$

where

$$R(\omega) = \frac{x(t)}{F(t)} = \frac{1}{[(k - \omega^2 m) + j\eta k]} \quad : \text{receptance}$$

$$\omega_n = \left(\frac{k}{m}\right)^{\frac{1}{2}} = 2\pi f_n \quad : \text{natural frequency}$$

The mobility of such a damped system is always a complex quantity and is frequency dependent. Evidently, the mobility is influenced by the system parameters, namely mass, damping and stiffness. If the damping ratio is small then the mobility has the following characteristics:

1. As  $\omega \gg \omega_n$ ,

$$M(\omega) \approx -\frac{j}{m\omega} \quad (3.5)$$

In this situation, the mobility is said to be mass dependent only.

2. As  $\omega = \omega_n$ ,

$$M(\omega) = \frac{1}{\eta} \quad (3.6)$$

At resonance, the mobility is said to be damping dependent only.

3. As  $\omega \ll \omega_n$ ,

$$M(\omega) \approx \frac{j\omega}{k} \quad (3.7)$$

In this case, the mobility is stiffness dependent only.

Taking logarithms and absolute values on both sides of equations (3.5) to (3.7), the following relations is obtained:

$$\log|M(\omega)| = \log\left(\frac{1}{\omega m}\right) = -\log(m) - \log(\omega), \quad \text{for } \omega \gg \omega_n \quad (3.8)$$

$$\log|M(\omega)| = \log\left(\frac{1}{\eta}\right), \quad \text{for } \omega = \omega_n \quad (3.9)$$

$$\log|M(\omega)| = \log\left(\frac{\omega}{k}\right) = \log(\omega) - \log(k), \quad \text{for } \omega \ll \omega_n \quad (3.10)$$

Equations (3.8) to (3.10) show that providing  $M(\omega)$  and  $\omega$  expressed in logarithmic coordinates, they have linear relations in the very high and very low frequency ranges as shown in Figure3.3.

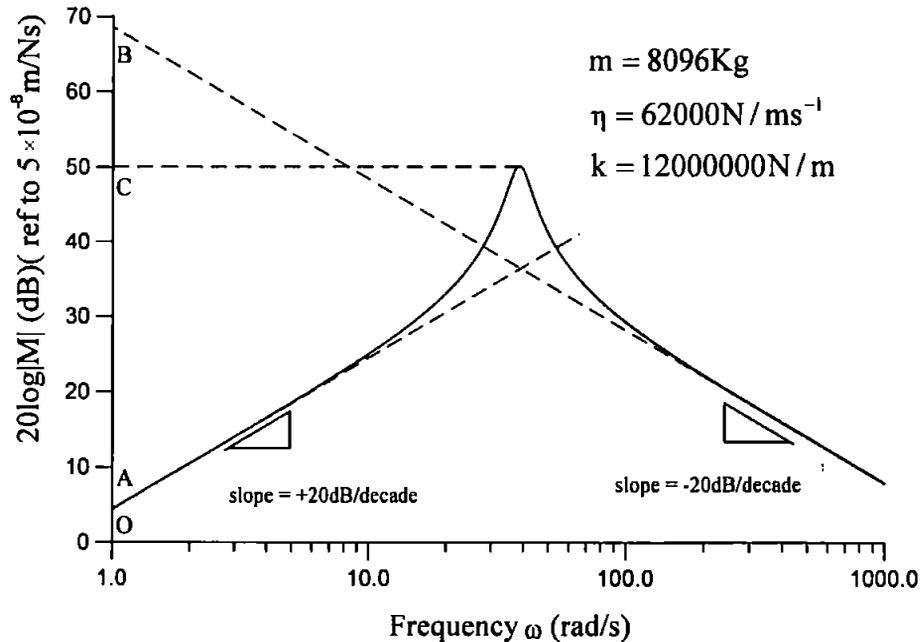


Figure 3.3 Characteristics of mobility spectrum for a SDOF system

These correspond to “mass-like” and “spring-like” behaviour as would be expected. Figure 3.3 represents the mobility spectrum of a SDOF system with  $m=8,096 \text{ kg}$ ,  $\eta=62,000$

Ns/m,  $k=12,000,000$  N/m and  $\omega_n = 38.5$  rad/s.

In Figure 3.3, it is seen firstly that the magnitude of mobility spectrum curve approaches a straight line with slope equaling +20 dB/decade in the lower frequency region. The intersection distance  $\overline{OA}$  along the ordinate axis is,

$$\overline{OA} = \log|M(\omega)|_{\log(\omega)=0} = -\log(k) \quad (3.11)$$

or

$$k = \left[ |M(\omega)|_{\log(\omega)=0} \right]^{-1} \quad (3.12)$$

Secondly, in the mobility spectrum curve has a peak value  $\overline{OC}$  at resonance, that is,

$$\overline{OC} = \log|M(\omega_n)| = \log\left(\frac{1}{\eta}\right) \quad (3.13)$$

or

$$\eta = \left[ |M(\omega_n)| \right]^{-1} \quad (3.14)$$

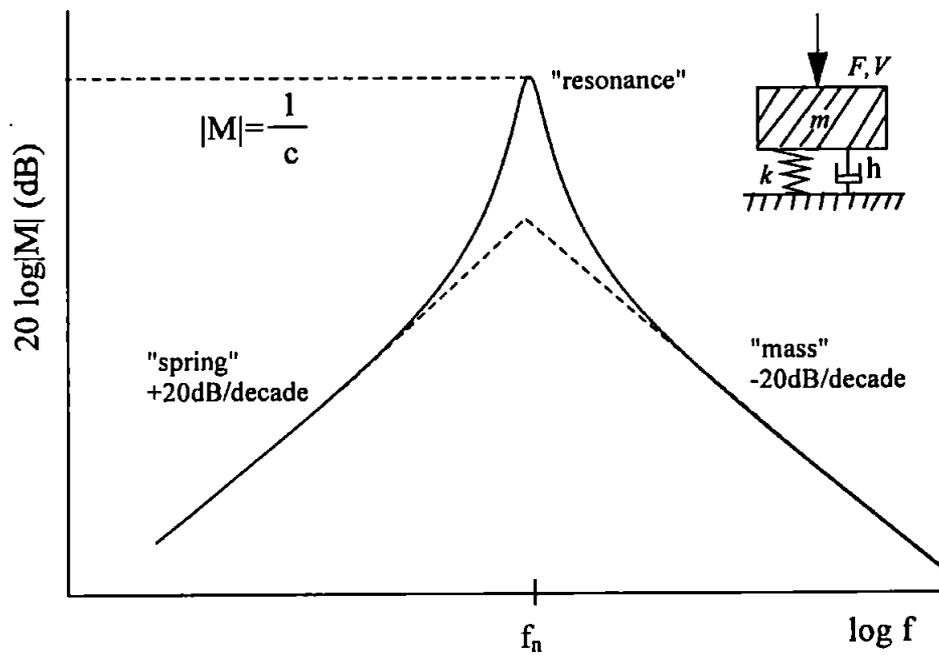
Finally, the mobility spectrum curve approaches another straight line with slope equaling -20 dB/decade in the higher frequency region. The intersection distance  $\overline{OB}$  along the ordinate axis is

$$\overline{OB} = \log|M(\omega)|_{\log(\omega)=0} = -\log(m) \quad (3.15)$$

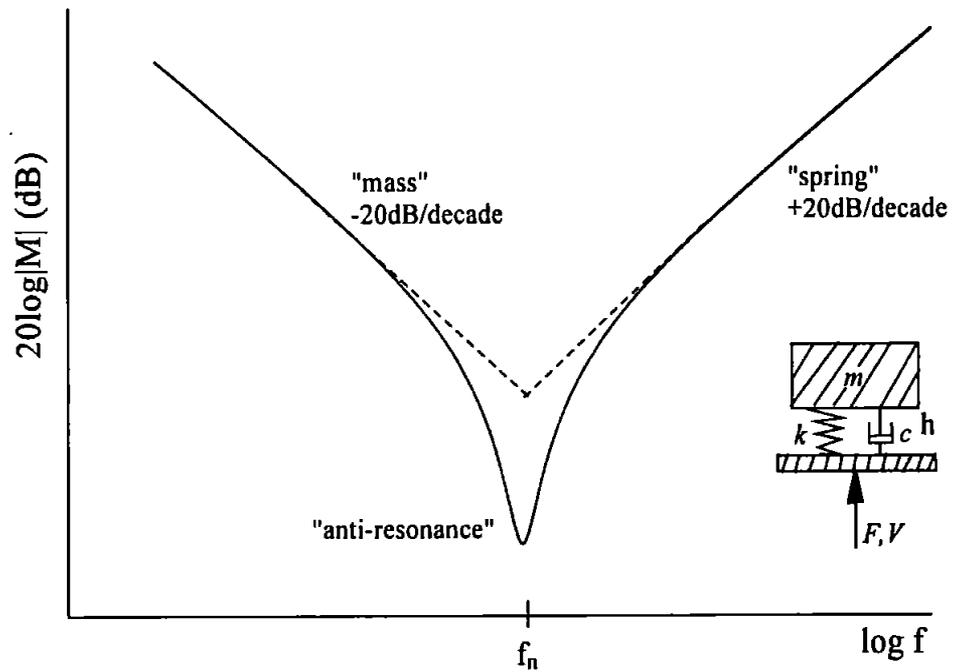
or

$$m = \left[ |M(\omega)|_{\log(\omega)=0} \right]^{-1} \quad (3.16)$$

Equations (3.11), (3.13) and (3.15) constitute the bases for the data checking methods of the mobility measurements.



(a) Excited on mass (rigid foundation)



(b) Excited on spring

Figure 3.4 Mobility of general SDOF system

For generality, the mobility curve of SDOF spring-mass-dashpot system can be shown as Figure 3.4. Figure 3.4(a) represents the driving point mobility for excitation on the mass. The height of the maximum in the mobility curve is determined by the damping. The lower the damping, the higher the  $|M|_{\max}$ . For  $f \gg f_n$  the curve shows “mass-behaviour”, i.e.,

$|M| \approx 1/\omega m$  (slope -20dB/decade) and for  $f \ll f_n$  the curve shows “spring-behaviour”, i.e.,  $|M|_{\max} \approx \omega/k$  (Slope +20dB/decade). The mobilities in the ranges of “spring-behaviour” and “mass-behaviour” are symmetrical about the axis  $f=f_n$  with equal magnitude but opposite slopes.

Figure 3.4 (b) shows the driving point mobility for excitation on the lower side of the spring. There is now an “anti-resonance” at  $f_n$ , giving a minimum mobility. Now the mass-behaviour is seen for  $f \ll f_n$ . For the “tuning” frequency such a spring-mass system may apply a high dynamical load using a relatively small mass. This set-up can be used for vibration suppression, especially on resonant structures.

### 3.2 Mobility of Two Degrees of Freedom (TDOF) Vibration Model

A two degrees of freedom vibration system, as shown in Figure 3.5, is represented by two discrete lumped mass  $m_1$  and  $m_2$  and linked by two pairs of massless spring and hysteresis damper in series. The spring constants are  $k_1$  and  $k_2$  and structural damping loss coefficients are  $h_1$  and  $h_2$  respectively.

When the exciting forces  $F_1$  and  $F_2$  are applied at mass 1 and mass 2 respectively, then the equations of motion for mass 1 and 2 are:

$$[m]\{\ddot{x}\} + \frac{1}{\omega}[h]\{\dot{x}\} + [k]\{x\} = \{F(t)\} \quad (3.17)$$

where  $[m]$ ,  $[h]$  and  $[k]$  are the  $2 \times 2$  mass, damping and stiffness matrices, respectively.

$\{\ddot{x}\}$ ,  $\{\dot{x}\}$  and  $\{x\}$  are the acceleration, velocity and displacement vector, respectively.

$\{F(t)\}$  is the exciting force vector. Their expanded forms are:

$$[m] = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \quad [h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}, \quad [k] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix},$$

$$\{\ddot{x}\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}, \quad \{\dot{x}\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}, \quad \{x\} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix},$$

and

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

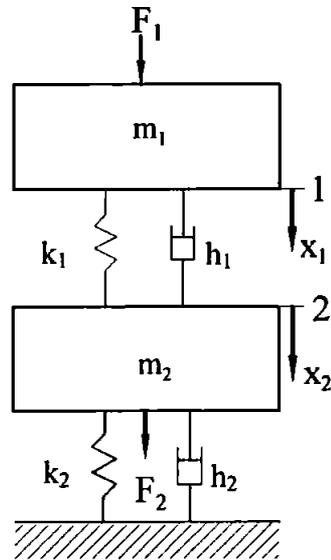


Figure 3.5 General two degrees of freedom system

For discrete lumped mass system,

$$m_{11} = m_1, \quad m_{12} = m_{21} = 0, \quad m_{22} = m_2.$$

By equilibrium relations, \$h\_{ij}\$ and \$k\_{ij}\$ are found to be:

$$h_{11} = h_1, \quad h_{12} = h_{21} = -h_1, \quad h_{22} = h_1 + h_2$$

and

$$k_{11} = k_1, \quad k_{12} = k_{21} = -k_1, \quad k_{22} = k_1 + k_2$$

The expanded form of equation (3.18) is

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} \quad (3.18)$$

When harmonic forcing functions are applied, i.e.,

$$F_i(t) = \hat{F}_i e^{j\omega t}, \quad i = 1, 2. \quad (3.19)$$

then the vibration displacements can be represented as

$$x_i(t) = \hat{X}_i e^{j\omega t}, \quad i = 1, 2. \quad (3.20)$$

in which  $\hat{X}_1$ ,  $\hat{X}_2$  are called complex response amplitudes. Substituting equation (3.19), (3.20) into (3.18), and defining the dynamic stiffness functions  $S_{ij}(\omega)$  of the system as

$$S_{ij}(\omega) = -\omega^2 m_{ij} + jh_{ij} + k_{ij}, \quad i, j = 1, 2 \quad (3.21)$$

then, equation (3.16) becomes

$$S_{ij}(\omega) \hat{X}_j = \hat{F}_i, \quad i, j = 1, 2 \quad (3.22)$$

The response amplitudes can be solved from equation (3.22) as

$$\hat{X}_j = S_{ij}^{-1}(\omega) \hat{F}_i, \quad i, j = 1, 2. \quad (3.23)$$

or

$$\hat{X}_1(\omega) = \frac{S_{22}(\omega) \hat{F}_1 - S_{12}(\omega) \hat{F}_2}{S_{11}(\omega) S_{22}(\omega) - S_{12}(\omega) S_{21}(\omega)} \quad (3.24a)$$

$$\hat{X}_2(\omega) = \frac{-S_{21}(\omega) \hat{F}_1 + S_{11}(\omega) \hat{F}_2}{S_{11}(\omega) S_{22}(\omega) - S_{12}(\omega) S_{21}(\omega)} \quad (3.24b)$$

The vibration velocity vector is

$$V_i = \dot{X}_i(t) = j\omega \hat{X}_i e^{j\omega t} = \hat{V}_i e^{j\omega t}, \quad i = 1, 2. \quad (3.25)$$

where

$$\hat{V}_1(\omega) = j\omega \hat{X}_1 = j\omega \frac{S_{22}(\omega) \hat{F}_1 - S_{12}(\omega) \hat{F}_2}{S_{11}(\omega) S_{22}(\omega) - S_{12}(\omega) S_{21}(\omega)} \quad (3.26a)$$

$$\hat{V}_2(\omega) = j\omega \hat{X}_2 = j\omega \frac{-S_{21}(\omega) \hat{F}_1 + S_{11}(\omega) \hat{F}_2}{S_{11}(\omega) S_{22}(\omega) - S_{12}(\omega) S_{21}(\omega)} \quad (3.26b)$$

Then the mobility matrix of a TDOF system is related by:

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (3.27)$$

where the mobility functions  $M_{ij}$  ( $i=1,2$  and  $j=1,2$ ) can be expressed in terms of the dynamic stiffness functions:

$$M_{11} = \frac{j\omega S_{22}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (3.28a)$$

$$M_{12} = \frac{-j\omega S_{12}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (3.28b)$$

$$M_{21} = \frac{-j\omega S_{21}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (3.28c)$$

$$M_{22} = \frac{j\omega S_{11}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (3.28d)$$

Expanding equation (3.27) and rearranging gives:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (3.29)$$

where  $\alpha_{ij}$  are the four-pole parameters and can be expressed in terms of mobility functions as:

$$\alpha_{11} = \frac{M_{11}}{M_{12}}, \quad \alpha_{12} = \frac{M_{12}M_{21} - M_{11}M_{22}}{M_{21}}, \quad \alpha_{21} = \frac{1}{M_{12}}, \quad \alpha_{22} = -\frac{M_{22}}{M_{12}} \quad (3.30)$$

For the linear system with proportional damping, there exists the symmetry properties for  $m_{ij}$ ,  $h_{ij}$  and  $k_{ij}$ , therefore, the dynamic stiffness  $S_{ij}$  and the transfer mobility  $M_{ij}$  are also symmetrical.

### 3.3 Mobility of Multidegrees of Freedom (MDOF) Vibration Model

Consider an  $N$  degrees of freedom system with structural or hysteresis damping. The equation of motion can be expressed as:

$$[m_{ij}]_{NXN} \{\ddot{X}\}_{NX1} + j[h_{ij}]_{NXN} \{\dot{X}\}_{NX1} + [k_{ij}]_{NXN} \{X\}_{NX1} = \{F\}_{NX1} \quad (3.31)$$

where the mass matrix  $[m_{ij}]$ , the damping matrix  $[h_{ij}]$  and the stiffness matrix  $[k_{ij}]$  are  $N \times N$  matrices.

When the exciting force vector  $\{F\}$  equals zero, equation (3.31) becomes the equation of motion for the free vibration of a linear dynamic system. There exists the mass normalized modal matrix  $[\Phi]$ , in which the  $r$ th damped eigenvalue is

$$\lambda_r^2 = \omega_r^2 (1 + j\eta_r) \quad (3.32)$$

where

$\omega_r$  : the undamped natural frequency of the  $r$ th mode;

$\eta_r$  : the structural damping loss factor of the  $r$ th mode.

For a linear dynamic system, the mass normalized modal matrix  $[\Phi]$  possesses the following orthogonality properties,

$$[\Phi]^T [m] [\Phi] = [I] = \begin{bmatrix} M_1 & & \\ & \ddots & \\ & & M_r = 1 \end{bmatrix} \quad (3.33a)$$

$$[\Phi]^T [k_{ij}] [\Phi] = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_r^2 \end{bmatrix} \quad (3.33b)$$

$$[\Phi]^T [h_{ij}] [\Phi] = \begin{bmatrix} \eta_1 \omega_1^2 & & \\ & \ddots & \\ & & \eta_r \omega_r^2 \end{bmatrix} \quad (3.33c)$$

The response  $\{X\}$  can be expressed as a linear combination of modal shapes, i.e.,

$$\{X\}=[\Phi]\{Y\} \quad (3.34)$$

where  $\{Y\}$  is called modal displacement vector. Substitute equation (3.34) into equation (3.31) then multiply both sides by  $[\Phi]^T$ , equation (3.31) becomes:

$$[\Phi]^T [m_{ij}] [\Phi] \{\ddot{Y}\} + j[\Phi]^T [h_{ij}] [\Phi] \{\dot{Y}\} + [\Phi]^T [k_{ij}] [\Phi] \{Y\} = [\Phi]^T \{F\} \quad (3.35)$$

By utilizing the orthogonality relations, equation (3.35) can be simplified and decoupled as:

$$\{Y\} + j[\eta_r \omega_r^2] \{Y\} + [\omega_r^2] \{Y\} = [\Phi]^T \{F\} = [F_{eq}] \quad (3.36)$$

If there is a single force  $F_j$  applied at the  $j$ th degree of freedom, then the external modal force is

$$\{F_{eq}\} = \begin{bmatrix} \Phi_{11} & \Phi_{21} & \cdots & \Phi_{j1} & \cdots & \Phi_{N1} \\ \Phi_{12} & \Phi_{22} & \cdots & \Phi_{j2} & \cdots & \Phi_{N2} \\ \vdots & \vdots & & \vdots & & \vdots \\ \Phi_{1r} & \Phi_{2r} & \cdots & \Phi_{jr} & \cdots & \Phi_{Nr} \\ \vdots & \vdots & & \vdots & & \vdots \\ \Phi_{1N} & \Phi_{2N} & \cdots & \Phi_{jN} & \cdots & \Phi_{NN} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ F_j \\ \vdots \\ 0 \end{Bmatrix} = \begin{Bmatrix} \phi_{j1} F_j \\ \phi_{j2} F_j \\ \vdots \\ \phi_{jr} F_j \\ \vdots \\ \phi_{jN} F_j \end{Bmatrix} \quad (3.37)$$

The solution of the  $r$ th modal displacement in equation (3.36) is obtained as:

$$Y_r = \frac{\Phi_{jr} F_j}{(\omega_r^2 - \omega^2 + j\eta_r \omega_r^2)} \quad (3.38)$$

where  $\omega$  is the excitation frequency of the external force  $F_j$ .

By equation (3.34) the displacement of the  $i$ th degree of freedom is:

$$X_i = \Phi_{i1} Y_1 + \Phi_{i2} Y_2 + \cdots + \Phi_{ir} Y_r + \cdots + \Phi_{iN} Y_N \quad (3.39)$$

Substituting equation (3.38) into equation (3.39), the receptance coefficient can be expressed as:

$$R_{ij}(\omega) = \frac{X_i}{F_j} = \sum_{r=1}^N \frac{A_{ijr}}{\omega_r^2 - \omega^2 + j\eta_r \omega_r^2} \quad (3.40)$$

where

$$A_{ijr} = \Phi_{ir} \Phi_{jr} \quad : \text{ the modal constant.}$$

Therefore the mobility coefficient can then be written as:

$$M_{ij}(\omega) = \sum_{r=1}^N \frac{j\omega A_{ijr}}{(\omega_r^2 - \omega^2) + j\eta_r \omega_r} \quad (3.41)$$

From equation (3.41), it is seen that the mobility coefficient of an N degrees of freedom system can be expressed as the sum of the N modal mobilities.

In reality, a mechanical structure system has an infinite number of degrees of freedom. It can be approximated by a N degrees of freedom system, with approximations being made to the higher modes. For the mobility of the real system, a residual term  $C+jD$  has to be added to equation (3.41),

$$M_{ij}(\omega) = \sum_{r=1}^N \frac{j\omega A_{ijr}}{(\omega_r^2 - \omega^2) + j\eta_r \omega_r} + (C + jD) \quad (3.42)$$

### 3.4 Mobility of Infinite Beam and Plate Model

Not only for beams, but also for structures like plates, cylinders, etc., the asymptotic approximation obtained by averaging over frequency is nearly equal to the mobility of the corresponding infinitely extended system, as shown by Skudrzyd (1980) and Cremer et al. (1988). Such infinite system mobilities are often applied in machinery acoustics calculations. The reason is that, for many problems a detailed frequency analysis is not relevant. Often the noise spectrum is broadband and/or varying with rotation speed. If such an excitation generates structural responses in the frequency range far above the lowest eigenfrequency, a change of structural design parameters would affect many relevant eigenfrequencies simultaneously. Averaging over frequency bands is then profitable in analysis procedures.

The driving point mobilities for infinite beams and plates are respectively given by Cremer et al. (1988):

$$M_{b\infty} = \frac{0.19(1-j)}{\rho_b S \sqrt{c_{Lb}} h f} \quad (3.43)$$

$$M_{p\infty} = \frac{0.125}{\sqrt{B' m''}} \approx \frac{0.453}{\rho_p c_{Lp} h^2} \approx \frac{5.41}{\omega \rho_p \lambda_B^2 h} \quad (3.44)$$

where  $m''$  : mass per unit area;  $B'$  : bending stiffness per unit width;  $\rho_b$  : density of beam;  $\rho_p$  : density of plate material;  $c_{Lp}, c_{Lb}$  : longitudinal wave speed in plate and beam respectively;  $h$  : thickness of plate/beam;  $\lambda_B$  : bending wavelength.

Figure 3.6 shows the frequency behaviour of  $20 \log |M|$  for a mass, a spring, an infinite beam and an infinite plate. Note that, for  $|M|_{p\infty}$  to be approximately valid not only averaging over frequencies is required, but also the driving point has to be at least a distance of  $\lambda_B/2$  away from plate boundaries.

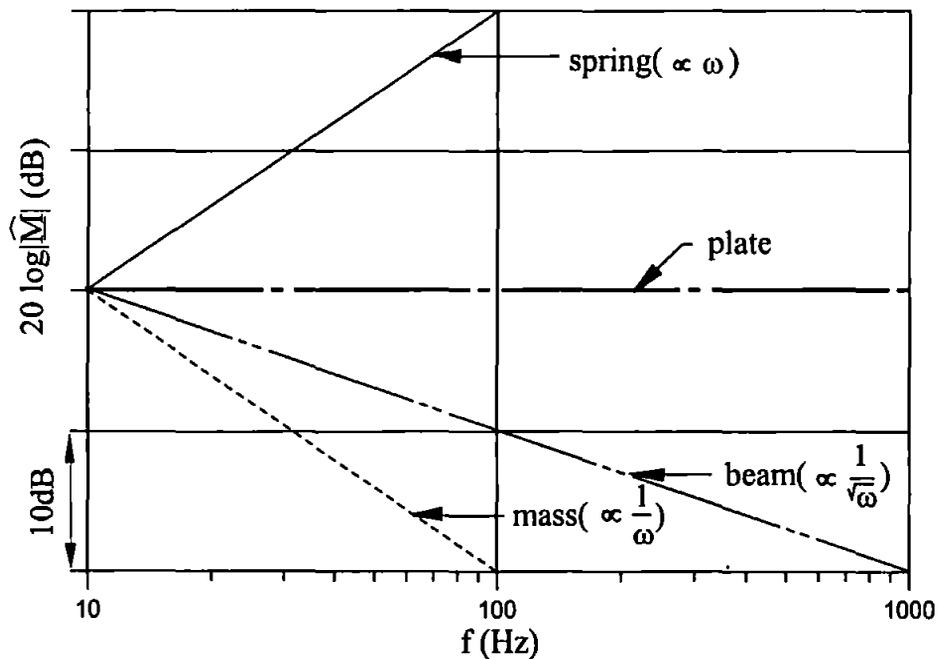


Figure 3.6 Mobility from idealized structural elements and infinite beam and plate

### 3.5 Mobility of Multipole Coupling System

Structure-borne noise excitation caused by mechanical coupling between an engine and a hull structure via a resilient mount can be represented as shown in Figure 3.7. By assuming that it can be simplified to a 1-direction coupling then it may be modelled as an isolator suspension system.

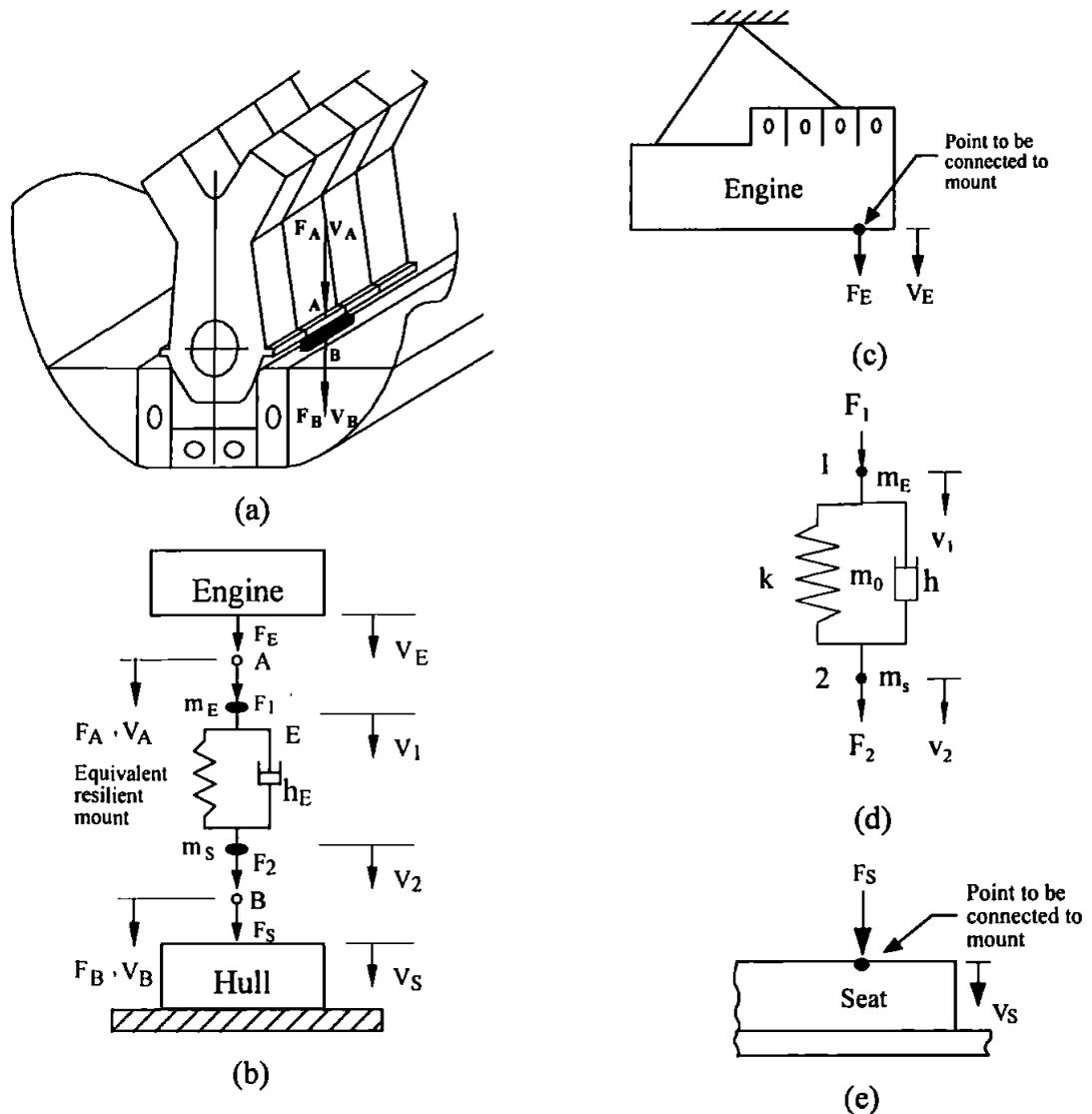


Figure 3.7 Modelling the engine-mount-hull coupling as a 1-D isolator suspension system

The dynamic behaviour of this coupled system can be expressed in terms of frequency response characteristics. If the responses are defined as velocities  $V_A$ ,  $V_B$ , the frequency response function will take the form of the mobility (or impedance) relationships. Thus, similar to equation (3.27), it is written as

$$\begin{Bmatrix} V_A \\ V_B \end{Bmatrix} = \begin{bmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{bmatrix} \begin{Bmatrix} F_A \\ F_B \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} F_A \\ F_B \end{Bmatrix} = \begin{bmatrix} Z_{AA} & Z_{AB} \\ Z_{BA} & Z_{BB} \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \end{Bmatrix} \quad (3.45)$$

where the mobility matrix  $[M] = \begin{bmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{bmatrix}$  is the inverse of the impedance matrix

$$[Z] = \begin{bmatrix} Z_{AA} & Z_{AB} \\ Z_{BA} & Z_{BB} \end{bmatrix}, \quad \text{i.e.,}$$

$$[M] = [Z]^{-1} \quad (3.46)$$

From equation (3.46), the driving point impedance  $Z_{AA}$ , the receiving point impedance  $Z_{BB}$  and the transfer impedance  $Z_{AB}$  or  $Z_{BA}$  can be expressed as,

$$Z_{AA} = \frac{M_{BB}}{M_{AA}M_{BB} - M_{AB}M_{BA}} \quad (3.47a)$$

$$Z_{BB} = \frac{M_{AA}}{M_{AA}M_{BB} - M_{AB}M_{BA}} \quad (3.47b)$$

$$Z_{AB} = Z_{BA} = \frac{-M_{AB}}{M_{AA}M_{BB} - M_{AB}M_{BA}} \quad (3.47c)$$

where

$$M_{AA} = \frac{V_A}{F_A}, \quad M_{BB} = \frac{V_B}{F_B}, \quad M_{AB} = \frac{V_A}{F_B}, \quad M_{BA} = \frac{V_B}{F_A}. \quad (3.48)$$

$F_A$ ,  $F_B$  and  $V_A$ ,  $V_B$  are the forces and velocities at the engine side and seating side of the equivalent resilient mount respectively.  $M_{AA}$ ,  $M_{BB}$  are the point mobilities and  $M_{AB}$ ,  $M_{BA}$  are the transfer mobilities of the engine-mount-hull coupled suspension system. According to the relations in (3.48) all of these mobility spectra can be measured by accelerometer transducers mounted on point A and B with an exciting force, such as an impulse, applied at point A and B.

Through the relations of velocity consistency and force equilibrium at the junctions, the mobility functions  $M_{AA}$ ,  $M_{BB}$ ,  $M_{AB}$ ,  $M_{BA}$  can be analyzed and combined from the individual mobility characteristics of the separate subsystems, such as engine, resilient mount and seating structure, as shown in Figure 3.7 (c) (d) (e).

Considering the individual hoisted engine system shown in Figure 3.7 (c), if  $F_E$  and  $V_E$  are respectively the exciting force and velocity response at the site on the engine bed which is connected to the mount, then the relation between  $F_E$  and  $V_E$  is,

$$V_E = M_E F_E \quad (3.49)$$

The point mobility  $M_E$  of the engine can be expressed as,

$$M_E = \frac{j\omega}{[(k_E - \omega^2 m_E) + jh_E]} \quad (3.50)$$

where  $m_E$ ,  $k_E$  and  $h_E$  are the effective mass, stiffness and damping coefficients of the engine respectively, at the point to be linked to the mounts.

For the individual resilient mount subsystem shown in Figure 3.7 (d), this can constitute a two degrees of freedom system and the relation of equation (3.27) holds as well. Equations (3.28a) to (3.28d) can be utilized to attain the mobility functions of such a separate resilient mount. While in this case the added effective masses of the engine and the seating structure at the junction points have to be considered. That is:

$$M_{11} = \frac{j\omega S'_{22}(\omega)}{S'_{11}(\omega)S'_{22}(\omega) - S'_{12}(\omega)S'_{21}(\omega)} \quad (3.51a)$$

$$M_{12} = \frac{-j\omega S'_{12}(\omega)}{S'_{11}(\omega)S'_{22}(\omega) - S'_{12}(\omega)S'_{21}(\omega)} \quad (3.51b)$$

$$M_{21} = \frac{-j\omega S'_{21}(\omega)}{S'_{11}(\omega)S'_{22}(\omega) - S'_{12}(\omega)S'_{21}(\omega)} \quad (3.51c)$$

$$M_{22} = \frac{j\omega S'_{11}(\omega)}{S'_{11}(\omega)S'_{22}(\omega) - S'_{12}(\omega)S'_{21}(\omega)} \quad (3.51d)$$

where

$$S'_{ij} = -\omega^2(m_{ij} + m_E) + jh_{ij} + k_{ij} \quad , \quad j = 1, 2 \quad (3.52a)$$

$$S'_{i2} = -\omega^2(m_{i2} + m_S) + jh_{i2} + k_{i2} \quad , \quad i = 1, 2 \quad (3.52b)$$

By a similar consideration, the separate individual seating structure, as Figure 3.7 (e), has the relation:

$$V_S = M_S F_S \quad (3.53)$$

and the point mobility function of the seating structure will be:

$$M_S = \frac{j\omega}{[(k_S - \omega^2 m_S) + j\omega h_S]} \quad (3.54)$$

By connecting the engine, mount, and seat structure together in series, equation (3.27) holds. Considering the relations of force equilibrium and velocity compatibility at the junction point A between the engine and the mount:

$$V_E = V_1 = V_A \quad (3.55)$$

$$F_A = \frac{V_A}{M_{AA}} = \frac{V_E}{M_E} + \frac{V_1}{M_{11}} \quad (3.56)$$

it yields the relation

$$M_{AA} = \frac{M_E M_{11}}{M_E + M_{11}} \quad (3.57)$$

Similarly, by the relations at the junction point B between the mount and the seat :

$$V_S = V_2 = V_B \quad (3.58)$$

$$F_B = \frac{V_B}{M_{BB}} = \frac{V_S}{M_S} + \frac{V_2}{M_{22}} \quad (3.59)$$

one has

$$M_{BB} = \frac{M_S M_{22}}{M_S + M_{22}} \quad (3.60)$$

Then, the transfer mobilities  $M_{AB}$  and  $M_{BA}$  are found, by equations (3.51b) and (3.51c), to be

$$M_{AB} = \frac{-j\omega S'_{12}(\omega)}{S'_{11}(\omega)S'_{22}(\omega) - S'_{12}(\omega)S'_{21}(\omega)} \quad (3.61a)$$

$$M_{BA} = \frac{-j\omega S'_{21}(\omega)}{S'_{11}(\omega)S'_{22}(\omega) - S'_{12}(\omega)S'_{21}(\omega)} \quad (3.61b)$$

The comparison of the results calculated from equations (3.57), (3.60), (3.61a) and (3.61b) with those by measurements following the relations of equation (3.48) will be carried out to check the validity of the current model.

### 3.6 Vibrational Power Input to a Receiving Structure from Engine

The instantaneous vibration power input ( $P_{inst}$ ) to a structure is defined as the product of the excitation force ( $F$ ) and the associated velocity ( $V$ ). For a harmonic excitation  $|F| \sin \omega t$  applying at a point on a structure of mobility,  $M = |M| e^{j\phi}$ , it causes a velocity  $|V| \sin(\omega t + \phi)$  at that point, where  $\phi$  is the phase angle between the driving excitation force and the associated velocity. The instantaneous vibration power input will be:

$$(P_{inst}) = F V^* = |F| |V| \sin \omega t \sin(\omega t + \phi) \quad (3.62)$$

where  $V^*$  is the conjugate of complex velocity  $V$ .

In general, it is of greater interest to know the time averaged power input (P). It is calculated by integrating equation (3.62) over one period of vibration to yield:

$$(P) = \frac{1}{T} \int_0^T |F| |V| \sin \omega t \sin (\omega t + \phi) dt = \frac{1}{2} |F| |V| \cos \phi \quad (3.63)$$

The averaged power input is related to the phase angle  $\phi$ . If  $\phi = \frac{\pi}{2}$  i.e., undamped, then the averaged power (P)=0.

At any point on a structure, the velocity and force is related via frequency response mobility (or impedance) functions, so that equation (3.63) may be rewritten in the following forms:

$$(P) = \frac{1}{2} |F|^2 \operatorname{Re}[M] \quad (3.64)$$

or

$$(P) = \frac{1}{2} |V|^2 \operatorname{Re}[Z] \quad (3.65)$$

where  $\operatorname{Re}[M]$  and  $\operatorname{Re}[Z]$  are the real part of the point mobility function and the point impedance at the excitation site, respectively. The choice of which equation to apply in a practical situation is controlled by the nature of the vibration source and the ability to measure either the force or velocity. In many cases, equation (3.65) can be applied if a machine is unaffected by the method of attachment, such that it behaves on site as if it were freely suspended. For a machine mounted on an isolation system, this condition is usually fulfilled velocity source.

The above equations are applicable only to a harmonically varying point force. In general, machinery is supported at a number of sites and each site will have some contact area over which both direct forces and moments may be transmitted. This will lead to a very

complicated set of inter-related expressions for the total input power. Suppose that there are  $N$  coordinates involved in the coupling of the machine to the sub-structure, then it will be necessary to define an  $N \times N$  matrix of mobility functions. Equations (3.64) and (3.65) are rewritten in matrix form as:

$$(P) = \frac{1}{2} \{F\}^T \text{Re}[M] \{F\} \quad (3.66)$$

or

$$(P) = \frac{1}{2} \{V\}^T \text{Re}[Z] \{V\} \quad (3.67)$$

Prediction, or measurement, of  $(P)$  value via these full matrix relationships can be extremely difficult. This is particularly true in situations where moment excitations represent a major source of the resultant vibrational power.

From expressions (3.66) and (3.67), it is clear that the power injected into a structure by the machine is dictated by the excitation force or velocity exerted by a machine and the mobility or impedance characteristics of the supporting structure. The characterization of an engine as a “velocity source” will now be considered .

There are many techniques available for modelling engineering structures in order to predict the mobility frequency response functions. These include finite element analysis, experimental modal analysis and analytical techniques based upon the use of infinite structures. Finite element analysis and experimental modal analysis are limited in their application to frequencies whose behaviour can be described as “moda”. For a finite element model, this implies that the use of many degrees of freedom may lead to difficulties and can be a serious concern. It has been shown (Pinnington,1980) that relatively simple models of “patches” around mounting sites can be used to predict the

higher frequency performance of a seating structure. The introduction of realistic damping ( which is essential if one is seeking to estimate the mathematically real component of the mobility function) into the preliminary modelling phase of a finite element idealization also poses a problem in many instances. However, it is possible to introduce a form of modal damping, extracted from experimental observations, into the calculations of frequency domain data based upon a finite element model.

In practical measurements by utilizing a FFT analyser, the power can be found by using the force spectrum density function  $G_{ff}$  , or the vibration velocity spectrum density function  $G_{vv}$ , or the cross power spectrum density function  $G_{fv}$  , which can be expressed as:

$$P(\omega) = G_{ff} \operatorname{Re}[M] = \operatorname{Re}[G_{fv}] = G_{vv} \operatorname{Re}[Z] \quad (3.68)$$

The driving point impedance matrix, the receiving point impedance matrix and the transfer impedance matrix for well-installed system can be obtained in the same way as equations (3.47a) to (3.47c), i.e.,

$$[Z_{\text{input}}] = [Z_{AA}] = \frac{[M_{BB}]}{[M_{AA}][M_{BB}] - [M_{AB}][M_{BA}]} \quad (3.69a)$$

$$[Z_{\text{output}}] = [Z_{BB}] = \frac{[M_{AA}]}{[M_{AA}][M_{BB}] - [M_{AB}][M_{BA}]} \quad (3.69b)$$

$$[Z_{\text{transmit}}] = [Z_{BA}] = [Z_{AB}] = \frac{-[M_{AB}]}{[M_{AA}][M_{BB}] - [M_{AB}][M_{BA}]} \quad (3.69c)$$

Substituting  $[Z_{\text{input}}]$ ,  $[Z_{\text{output}}]$  and  $[Z_{\text{transmit}}]$  into equation (3.68), the input power, output power and transmitted power can be calculated respectively, i.e.,

$$(P_{\text{input}}) = G_{v_A v_A} \operatorname{Re}[Z_{\text{input}}] \quad (3.70a)$$

$$(P_{\text{output}}) = G_{v_B v_B} \operatorname{Re}[Z_{\text{output}}] \quad (3.70b)$$

$$(P_{\text{transmit}}) = G_{v_A v_A} \text{Re}[Z_{\text{transmit}}] \quad (3.70c)$$

Thus the general methodology for obtaining the power fed into a receiving structure from the excitation sources of an engine via the contact points is derived. More details concerning the transfer mobility reduction of the structure-borne sound transmitted from a marine engine will be discussed later.

### 3.7 Logic of Structure-Borne Sound Reduction

The low noise design problem for a simple engine room onboard a vessel is schematically shown in Figure 3.8. The primary noise generating mechanism is the main engine that injects vibrations into the structure via the mounts and foundations. These vibrations will be transmitted throughout the structure and eventually radiate sound to the surroundings.

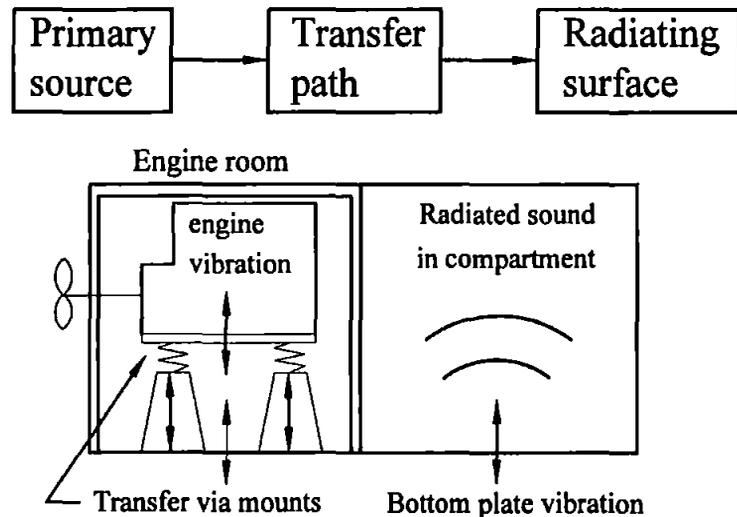


Figure 3.8 Schematic of machinery noise analysis for low noise design

To simulate the vibration reduction behaviour of the resilient mounts of a machine, a model can be established by utilizing simple input/output transfer mechanisms, which may be represented in terms of mobility functions. Figure 3.9 shows the analysis models which

can be used to model the structure-borne sound transfer between the primary source and the radiating surface. For a uni-directional simple harmonic excitation force  $F_1$ , the engine structure vibrates with a simple harmonic velocity  $V_1$  at the driving point. Structural waves propagate from the excitation point to the adjacent bottom plate, where bending waves generate a velocity field  $V_2(x,y)$ . This vibration field radiates a sound field  $P(x,y,z)$ .

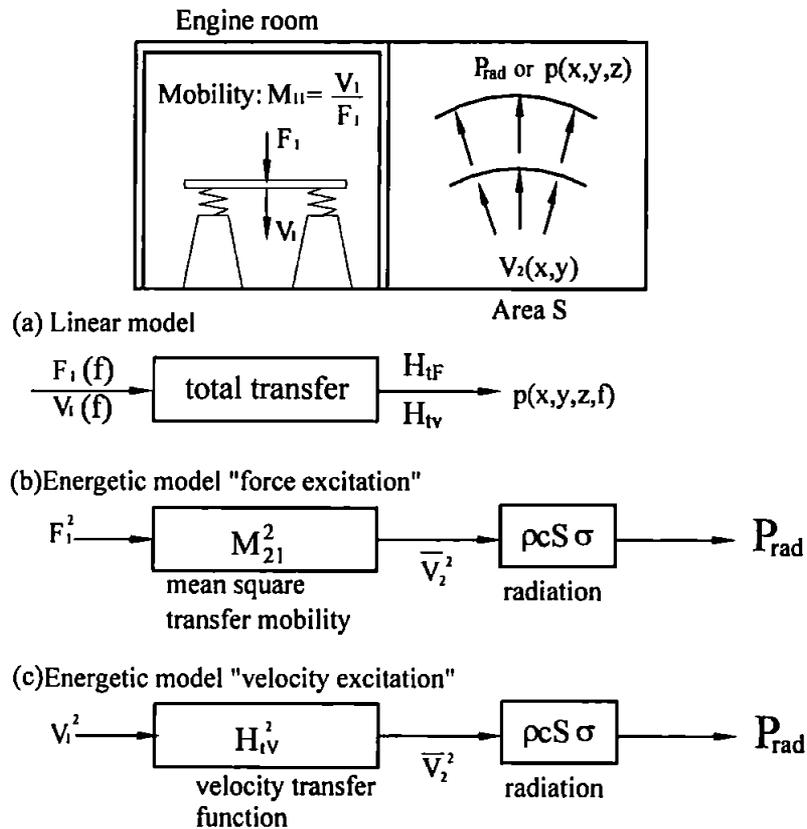


Figure 3.9 Schematic for the analysis of structure-borne noise transfer

The linear equations which relate the radiated sound pressure with the force and the velocity at the driving point are (Verheij, 1995):

$$p(x, y, z, f) = H_{iF} F_1(f) \quad (3.71)$$

$$p(x, y, z, f) = H_{iV} V_1(f) \quad (3.72)$$

here  $H_{iF}$  and  $H_{iV}$  are so-called frequency response functions, describing the sound

transfer for force and velocity excitation respectively. Basically, these frequency response functions have a complex value at each frequency, indicating the magnitude and the phase of the vibration.

The force and velocity at the driving point are related as

$$M_{11} = \frac{V_1}{F_1} \quad (3.73)$$

here  $M_{11}$  is called driving point mobility. It is a measure of the vibration velocity for a given force  $F_1$ .

In many practical situations instead of linear equations, equations in terms of mean square quantities or so-called “energetic” quantities are used. In principle, these formulations are exact, if derived directly from equations (3.71) and (3.72). However, for practical purposes, approximations of transfer functions are used. Those functions relate e.g. 1/3-octave band levels of  $F_1^2$  and  $V_1^2$  with the radiated sound power  $P_{\text{rad}}$  (Verheij,1995). In the scheme of Figures 3.9(b) and 3.9(c), the “energetic” models are presented. The transmission chain is divided into two blocks. The first block represents the structure and gives the ratio between the spatially averaged mean square velocity  $\overline{V_2^2}$  of the bottom surface and  $F_1^2$  or  $V_1^2$ :

$$M_{21}^2 = \frac{\overline{V_2^2}}{F_1^2}, \quad H_{IV}^2 = \frac{\overline{V_2^2}}{V_1^2} \quad (3.74)$$

The function  $M_{21}^2$  is called the mean square transfer mobility and the function  $H_{IV}^2$  is called the velocity transfer function.

The second block represents the sound radiation and gives the ratio between the radiated sound power and  $\overline{V_2^2}$ :

$$\frac{P_{\text{rad}}}{V_2^2} = \rho c S \sigma \quad (3.75)$$

The right side of equation (3.75) contains the characteristic impedance  $\rho c$  of the surrounding fluid (air, water, etc), the area  $S$  of the radiating surface, and the radiation efficiency  $\sigma$ .  $\sigma$  is a measure of the efficiency with which a structure converts vibrations into sound.

The factors which determine the sound transfer is influenced by design as seen in Figure 3.9 and from the following equations:

$$\frac{P_{\text{rad}}}{F_1^2} = (M_{21}^2 S \sigma) \rho c = (M_{11}^2 H_{1V}^2 S \sigma) \rho c \quad (3.76)$$

$$\frac{P_{\text{rad}}}{V_1^2} = (H_{1V}^2 S \sigma) \rho c \quad (3.77)$$

If the nature of the excitation is such that for different engine design  $F_1$  is unaffected, equation (3.76) implies that sound reduction is obtained by:

- (a) decrease of driving point mobility ;
- (b) reduction of radiating surface area;
- (c) decrease of velocity transfer function;
- (d) decrease of radiation efficiency.

If the nature of the excitation is such that for different engine designs  $V_1$  is unaffected, equation (3.77) implies that sound reduction is obtained by:

- (a) reduction of radiating surface area;
- (b) decrease of velocity transfer function;
- (c) decrease of radiation efficiency.

A design measures to decrease  $S$  significantly, have very limited applications. However measures to decrease  $M_{11}$  and are  $H_{iv}^2$  are very important and will be discussed extensively in this study.

### **3.8 Velocity Transfer Function and Transfer Mobility**

The velocity transfer function  $H_{iv}^2$ , defined in equation (3.74), depends strongly on the size and nature of the structure. For noise reduction purposes, the main interest is in the eigenfrequency range. Here again the discussion will be limited to the elementary case of a finite plate. In principle,  $H_{iv}^2$  can be calculated with the aid of an eigenfunction model by the finite element method. Again it can be said that, if in the intermediate frequency range (from 100 Hz to 1 kHz) of interest a large number of eigenfrequencies is involved, the calculations become time consuming. Moreover, from the view point of the designer it is important to look at some smoothed data and to understand the relation between the design parameters and these data.

To derive a simple expression for  $H_{iv}^2$  on a plate, the following assumptions are made:

- (a) the plate length and width are large compared with  $\lambda_B$  ;
- (b) many weakly damped resonances occur simultaneously ;
- (c) results are for frequency bands that contain at least 3-5 eigenfrequencies.

In that case the velocity distribution over the plate is approximately homogeneous (so called diffuse field).

At equilibrium, the power injected into the plate is equal to the power dissipated by the plate.

$$(P_{in})=(P_{diss}) \quad (3.78)$$

From equations (3.62) and (3.63), the power injected by a point source into the plate at a single frequency is given by :

$$(P_{in}) = \frac{1}{2} \text{Re} \{F_1 V_1^*\} = F_{1,rms}^2 \text{Re} \{M_{11}\} = V_{1,rms}^2 \text{Re} \left\{ \frac{1}{M_{11}} \right\} \quad (3.79)$$

where  $F_{1,rms}$  is the root-mean-square of  $F_1$ . For averaging over frequencies,  $M_{11}$  may be replaced by  $M_{p,\infty}$ . Using equations (3.44) and (3.79) this leads to :

$$(P_{in}) = \frac{0.453 F_{1,\Delta f}^2}{\rho_p c_{LP} h^2} = 2.2 \rho_p c_{LP} h^2 V_{1,\Delta f}^2 \quad (3.80)$$

where  $c_{LP}$  is the longitudinal wave speed of the plate;  $h$  is the thickness of the plate;  $\rho_p$  is the density of the plate;  $V_{1,\Delta f}^2$  is the driving point velocity within the frequency band  $\Delta f$ .

The power dissipation from the plate is caused by material damping, by energy transport across the plate boundaries into the support, and by sound radiation from the plate. The well known parameter which includes all these damping mechanisms is the (apparent) loss factor  $\eta$ . For each frequency or frequency band it was defined by Verheij (1995) as:

$$\eta(f) = \frac{P_{diss}}{\omega m'' S V_2^2} = \frac{\text{energy dissipation per vibration period}}{2\pi(\text{reversible}) \text{ mechanical energy}} \quad (3.81)$$

where  $m''$  is the mass per unit area of the plate. In equation (3.81) it is assumed that the total mechanical energy (i.e. the sum of kinetic and potential energy) is twice the kinetic energy, i.e.

$$E = m'' S \overline{V_2^2} \quad (3.82)$$

Therefore, the dissipated power may be written as:

$$P_{\text{diss}} = \eta \omega m'' S \overline{V_2^2} \quad (3.83)$$

Using equations (3.78), (3.80) and (3.83), the second equation of (3.74) becomes

$$H_{iv}^2(f) = \frac{0.35 c_{LP} h}{f S \eta(f)} \quad (3.84)$$

Figure 3.10 shows the effect on  $H_{iv}^2$  of different loss factor. It is seen that at low frequencies  $H_{iv}^2 \gg 1$ . The frequency at which  $H_{iv}^2 = 1$  decreases with increasing  $\eta$  and decreasing plate thickness.

The square transfer mobility  $M_{21}^2$ , defined by the first equation of (3.74), in the eigenfrequency range follows from equations (3.78), (3.79), (3.83) and putting

$\text{Re}\{M_{11}\} = M_{p\infty}$ :

$$M_{21}^2 = \frac{\overline{V_{2,\Delta f}^2}}{F_{1,\Delta f}^2} = \frac{M_{p\infty}}{\eta \omega m'' S} \approx \frac{2.85}{\eta f \rho_p^2 c_{LP} h^3 S} \quad (3.85)$$

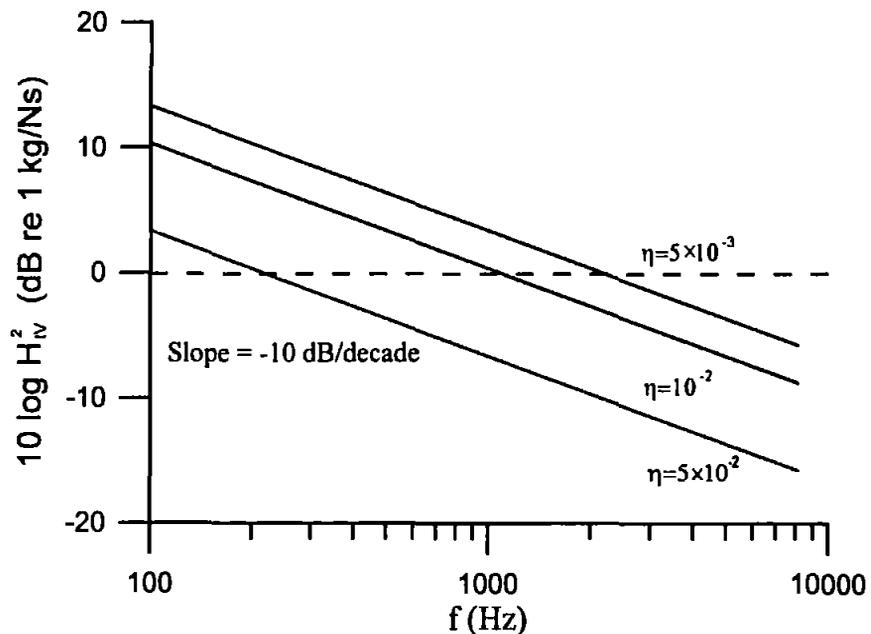


Figure 3.10 Velocity transfer function of a steel plate (480mm×340mm×5mm) of different loss factor

Comparing equation (3.84) and (3.85) reveals that the influence of damping upon  $H_{iv}^2$  and  $M_{21}^2$  is the same, but the influence of plate thickness variation is quite different. Therefore, it is of vital importance for a designer to distinguish the source (or component) type which drives the structure and to determine either equation (3.76) or equation (3.77) to apply.

### 3.9 Mechanical Four-Pole Parameters Method in System Mobility Evaluation

In 1970's, a powerful method used to calculate the transmission and isolation of structure-borne vibration and noise has been successfully developed. This is the so-called mechanical four-pole parameters method (Cremer et.al.,1988). It has been applied in the vibration and noise control field by Snowdon (1971).

#### 3.9.1 Basic Theory of the Four-Pole Parameters Method

The mechanism of high frequency noise transmission through a mechanical system such as an elastic mount can be characterized in terms of the parameters at each end as shown in Figure 3.1. This allows physical insight into how high frequency noise generated by the input end transmits through the elastic mount, and hence how it may be suppressed at the output end. To keep the analysis simple in the first instance, only two parameters at each end are considered. These are the complex frequency functions of force and velocity. The matrix equation relating velocities and forces to point and transfer mobilities for a general system is equation (3.27), repeated here for clarity,

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (3.86)$$

The main advantage of the mobility method is that system components can be coupled by way of these functions that are defined at the interfaces between components. This technique is used to investigate the effects of fitting a simple compliant element between

the engine and seat.

Expanding equation (3.86) and rearranging gives:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (3.87)$$

The coefficients  $\alpha_{11}$ ,  $\alpha_{12}$ ,  $\alpha_{21}$ ,  $\alpha_{22}$ , in equation (3.87) are called the four-pole parameters (Cremer et al., 1988). Their definitions are:

$$\alpha_{11} = \left. \frac{V_1}{V_2} \right|_{F_2=0}, \quad \alpha_{12} = \left. \frac{V_1}{F_2} \right|_{V_2=0}, \quad \alpha_{21} = \left. \frac{F_1}{V_2} \right|_{F_2=0}, \quad \alpha_{22} = \left. \frac{F_1}{F_2} \right|_{V_2=0} \quad (3.88)$$

$V_2=0$  means that the output end is fixed, while  $F_2=0$  means that the output end is free. From equation (3.88), it can be seen that  $\alpha_{11}$  and  $\alpha_{22}$  are dimensionless,  $\alpha_{12}$  has the dimension of mobility and  $\alpha_{21}$  has the dimension of impedance.

In general, all four-pole parameters are frequency dependent complex quantities. They are relevant to the characteristics of the mechanical system itself. Expanding equation (3.87), the driving point mobility can be obtained as:

$$M_{11} = \frac{V_1}{F_1} = \frac{\alpha_{11}V_2 + \alpha_{12}F_2}{\alpha_{21}V_2 + \alpha_{22}F_2} = \frac{\alpha_{11}M_{22} + \alpha_{12}}{\alpha_{21}M_{22} + \alpha_{22}} \quad (3.89)$$

where  $M_{22} = V_2/F_2$  is the receiving point mobility. Reciprocally, the driving point impedance is:

$$Z_{11} = \frac{F_1}{V_1} = \frac{\alpha_{21}V_2 + \alpha_{22}F_2}{\alpha_{11}V_2 + \alpha_{12}F_2} = \frac{\alpha_{21} + \alpha_{22}Z_{22}}{\alpha_{11} + \alpha_{12}Z_{22}} \quad (3.90)$$

where  $Z_{22} = \frac{F_2}{V_2}$  is the receiving point impedance. Also, the transfer mobilities and impedances can be derived as:

$$M_{12} = \frac{V_1}{F_2} = \alpha_{11}M_{22} + \alpha_{12} \quad (3.91)$$

$$M_{21} = \frac{V_2}{F_1} = \frac{M_{22}}{\alpha_{21}M_{22} + \alpha_{22}} \quad (3.92)$$

and

$$Z_{12} = \frac{F_1}{V_2} = \alpha_{21} + \alpha_{22}Z_{22} \quad (3.93)$$

$$Z_{21} = \frac{F_2}{V_1} = \frac{Z_{22}}{\alpha_{11} + \alpha_{12}Z_{22}} \quad (3.94)$$

The force and velocity transmissibilities are defined and expressed as:

$$T_{F_2} = \left| \frac{F_2}{F_1} \right| = \left| \frac{1}{\alpha_{21}M_{22} + \alpha_{22}} \right| = \left| \frac{Z_{22}}{\alpha_{21} + \alpha_{22}Z_{22}} \right| \quad (3.95)$$

$$T_{V_2} = \left| \frac{V_2}{V_1} \right| = \left| \frac{M_{22}}{\alpha_{11}M_{22} + \alpha_{12}} \right| = \left| \frac{1}{\alpha_{11} + \alpha_{12}Z_{22}} \right| \quad (3.96)$$

By equations (3.95) and (3.96), the system transmissibilities can be found provided the four-pole parameters and the receiving point mobility or impedance are known.

### 3.9.2 Connection of Mechanical Four-Pole Systems

Analogous to an electric circuit system, there are two basic types of connection for the mechanical four-pole systems, namely the series and parallel connections. If the output end of one system is rigidly joined to the input end of another system, it is said to be in series connection. While the input (output) ends of two or more mechanical systems are rigidly joined together and move with the same velocity, then it is said to be in parallel connection.

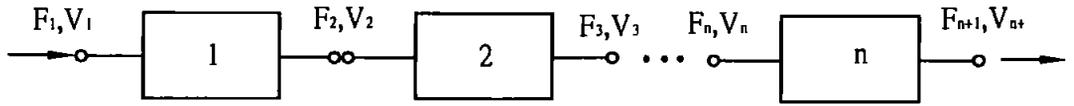


Figure 3.11 Four-pole subsystems in series connection

As the four-pole mechanical subsystems are connected in series, as shown in Figure 3.11, then the output of one subsystem is just the input of the subsequently adjacent subsystem.

For each subsystem, the matrix form expression in terms of the four-pole parameters is:

$$\begin{aligned} \begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} &= \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \\ \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} &= \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \begin{Bmatrix} V_3 \\ F_3 \end{Bmatrix} \\ &\vdots \\ \begin{Bmatrix} V_n \\ F_n \end{Bmatrix} &= \begin{bmatrix} \alpha^n_{11} & \alpha^n_{12} \\ \alpha^n_{21} & \alpha^n_{22} \end{bmatrix} \begin{Bmatrix} V_{n+1} \\ F_{n+1} \end{Bmatrix} \end{aligned}$$

Therefore,

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \dots \begin{bmatrix} \alpha^n_{11} & \alpha^n_{12} \\ \alpha^n_{21} & \alpha^n_{22} \end{bmatrix} \begin{Bmatrix} V_{n+1} \\ F_{n+1} \end{Bmatrix} \quad (3.97)$$

For example, if there only two subsystems connected in series, then

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{Bmatrix} V_3 \\ F_3 \end{Bmatrix} \quad (3.98)$$

in which

$$\begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} = \begin{bmatrix} \alpha'_{11}\alpha''_{11} + \alpha'_{12}\alpha''_{21} & \alpha'_{11}\alpha''_{12} + \alpha'_{12}\alpha''_{22} \\ \alpha'_{21}\alpha''_{11} + \alpha'_{22}\alpha''_{21} & \alpha'_{21}\alpha''_{12} + \alpha'_{22}\alpha''_{22} \end{bmatrix} \quad (3.99)$$

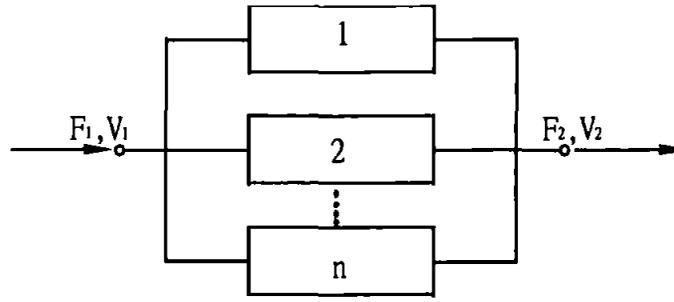


Figure 3.12 Four-pole subsystems in parallel connection

Figure 3.12 shows the  $n$  subsystems in parallel connection, in which the terminal input or output forces of the system is equal to the sum of the input and output forces of each individual subsystems. Then

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (3.100)$$

where

$$\beta_{11} = \frac{A}{B} \quad , \quad \beta_{12} = \frac{1}{B} \quad , \quad \beta_{21} = \frac{AC - B^2}{B} \quad , \quad \beta_{22} = \frac{C}{B} \quad (3.101)$$

and

$$A = \sum_{i=1}^n \frac{\alpha_{11}^i}{\alpha_{12}^i} \quad , \quad B = \sum_{i=1}^n \frac{1}{\alpha_{12}^i} \quad , \quad C = \sum_{i=1}^n \frac{\alpha_{22}^i}{\alpha_{12}^i} \quad (3.102)$$

For example, if the system is composed of two subsystems in parallel connection, then

$$A = \frac{\alpha'_{11}\alpha''_{12} + \alpha'_{12}\alpha''_{11}}{\hat{\alpha}'_{12}\hat{\alpha}''_{12}} = \frac{\varphi}{\varepsilon} \quad (3.103a)$$

$$B = \frac{\alpha'_{12} + \alpha''_{12}}{\alpha'_{12}\alpha''_{12}} = \frac{\theta}{\varepsilon} \quad (3.103b)$$

$$C = \frac{\alpha'_{22}\alpha'_{12} + \alpha'_{12}\alpha''_{22}}{\alpha'_{12}\alpha''_{22}} = \frac{\lambda}{\varepsilon} \quad (3.103c)$$

Thus

$$\begin{aligned} \beta_{11} &= \frac{A}{B} = \frac{\varphi}{\theta}, & \beta_{12} &= \frac{AC - B^2}{B} = \frac{\varphi\lambda - \theta^2}{\varepsilon\theta} \\ \beta_{21} &= \frac{1}{B} = \frac{\varepsilon}{\theta}, & \beta_{22} &= \frac{C}{B} = \frac{\lambda}{\theta} \end{aligned} \quad (3.104)$$

where

$$\varphi = \alpha'_{22}\alpha''_{12} + \alpha'_{12}\alpha''_{22} \quad (3.105a)$$

$$\theta = \alpha'_{12} + \alpha''_{12} \quad (3.105b)$$

$$\varepsilon = \alpha'_{12}\alpha''_{12} \quad (3.105c)$$

$$\lambda = \alpha'_{11}\alpha''_{12} + \alpha'_{12}\alpha''_{11} \quad (3.105d)$$

### 3.10 Explanation of Reduction in Transfer Mobility by Inserting an Isolator

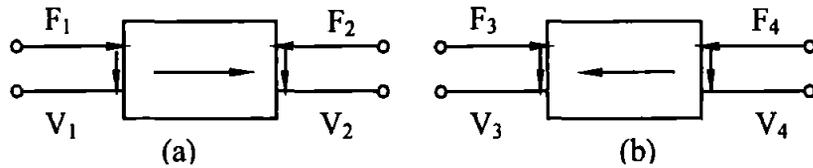


Figure 3.13 Mobility of intergrated system combined by mobility matrices of the subsystems

Suppose the two ends of the subsystems a, b are 1,2 and 3,4 respectively, as shown in Figure 3.13, where F represents the force and V represents the velocity. When the subsystems are linked together, then  $V_3=V_2$  and  $F_3 = -F_2$ , equation (3.97) the becomes:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \alpha_{33} & \alpha_{34} \\ -\alpha_{43} & -\alpha_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{14} \\ \alpha'_{41} & \alpha'_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} \quad (3.106)$$

It requires that  $F_4=0$  since 4 is the receiving point and 1 is the driving point. Thus,

$$\frac{1}{\alpha'_{41}} = M_{41} = \frac{V_4}{F_4} \quad (3.107)$$

From equation (3.106)

$$\alpha'_{41} = \alpha_{21}\alpha_{33} - \alpha_{22}\alpha_{43} = \frac{M_{22} + M_{33}}{M_{12}M_{34}}$$

or

$$M_{41} = \frac{M_{12}M_{34}}{M_{22} + M_{33}} \quad (3.108)$$

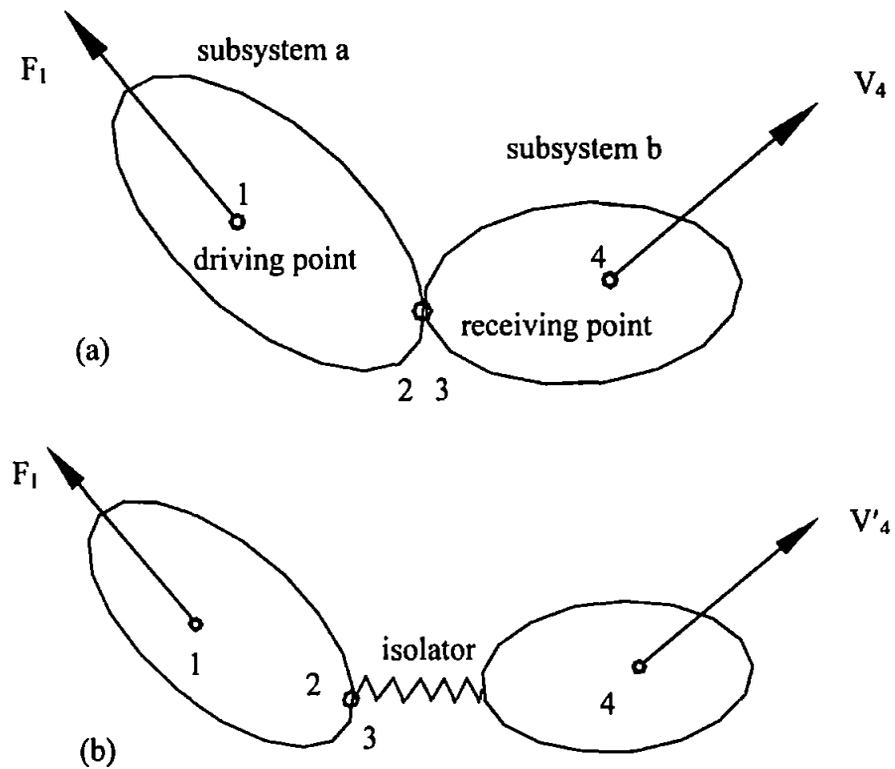


Figure 3.14 System before and after installation of isolator

From equation (3.108), it can be seen that the transfer mobility of the combined system ( $M_{41}$ ) is not a simple additive combination of the transfer mobilities ( $M_{12}$  and  $M_{34}$ ) of the two subsystems. The linking point mobilities ( $M_{22}$  and  $M_{33}$ ) of the subsystems play a vital role in determining the total transfer mobility. As an isolator is connected to the input end of the subsystem b, such as in Figure 3.14 (b) which represents the improved system of Figure 3.14 (a), the mobility  $M_{33}$  would increase significantly. It follows from equation

(3.108) that the total transfer mobility  $M_{41}$  is reduced. In other words, to reduce the total transfer mobility, it requires the introduction of a larger mobility than the original one. This is the reason of the basic principle for a good structure-borne noise attenuation result that, the structure on both sides of a resilient mount must be properly designed with smaller mobility than that of the mount itself. Simply said, the desired mobility changes for the coupled machine---resilient mount---foundation structure in series system should be “heavy-compliant-heavy” and/or “stiff-compliant-stiff”.

Thus, the evaluation model and control principle (equation (3.108)) for structure-borne sound transmission from engine to the receiving structure have been established. Besides this, numerical model for the analysis of the transfer function of a complicated stiffened-plate structure like a ship will describe in the next Chapter.

**CHAPTER 4**  
**STRUCTURE-BORNE VIBRATION AND NOISE**  
**PROPAGATION IN A SHIP STRUCTURE**

In most cases the engine foundation and hull structure are composed of beams and plates. Once the vibrational power flow feeding in the receiving structure from an engine via the contact points, the structure-borne sound and vibration can propagate in the structure. Structure-borne noise implies elastic waves in the sound frequency range. Besides the longitudinal waves, there are many transverse wave patterns existing in the stiffened-plate structure during the noise propagation, such as bending waves, shear waves and torsional waves etc. Structure-borne noise propagation is more complicated than sound propagating in fluid. To analyse the structure-borne noise propagation, the FEM for the lower frequency range and the SEA model for the higher frequency range can be applied. While in the medium frequency range there is a knowledge gap. If the FEM is used to solve this medium frequency propagation problem, it can be assumed as a rule of thumb that, in the discretization process, about ten elements are needed per wavelength. In the case of a container ship of length 250 m in the range of frequencies around 1 kHz, it leads to FE models with over  $10^6$  degrees of freedom, which is pointed out by Cabos and Jokat (1998). It sounds to be an impractical model of computation. Thus an innovative numerical model based on the stress wave concept is developed as follows.

## 4.1 Wave Propagation in a Finite Beam

### 4.1.1 Elastic Bending Wave in a Finite Beam

For the radiation of airborne noise or hydroacoustics, the elastic bending wave in a stiffened plate structure is the most important wave pattern owing to its larger lateral displacement. The one dimensional bending wave equation in a beam of finite length can be expressed as (Cremer et al, 1988):

$$\frac{B}{m'} \frac{\partial^4 v_y}{\partial x^4} + \frac{\partial^2 v_y}{\partial t^2} - \left[ \frac{\mathcal{I}'_z}{m'} + \frac{B}{K} \right] \frac{\partial^4 v_y}{\partial x^2 \partial t^2} + \frac{\mathcal{I}'_z}{K} \frac{\partial^4 v_y}{\partial t^4} = 0 \quad (4.1)$$

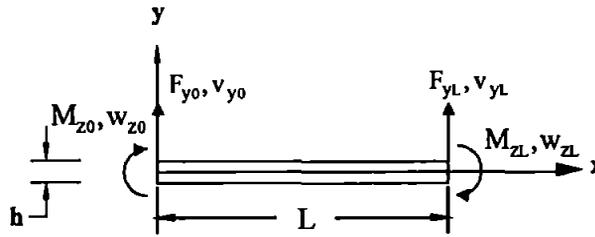


Figure 4.1 Coordinate system of a beam

where the coordinate system is shown in Figure 4.1, and the symbols are:

$$B = \text{bending rigidity of a rectangular beam} = EI = \frac{1}{12} Ebh^3 \text{ (N-m}^2\text{)}$$

$$m' = \text{mass per unit length} = \rho S \text{ ( kg / m )}$$

$$v_y = \text{velocity along y axis ( m / sec )}$$

$$w_z = \text{angular velocity about z axis ( rad / s )}$$

$$F_y = \text{force along y axis ( N )}$$

$$M_z = \text{moment about z axis ( Nm )}$$

$$\mathcal{I}'_z = \int \rho dx = \text{rotary mass moment of inertia of } dx \text{ about z ( kg-m )}$$

$$K = \frac{Gs}{\kappa} = \text{the shear rigidity ( N - m )}$$

- $E$  = Young's modulus ( N / m<sup>2</sup> )  
 $S$  = cross-sectional area ( m<sup>2</sup> )  
 $h$  = thickness of plate or web height of beam ( m )  
 $\kappa$  = shear distribution factor ( m<sup>-1</sup> )  
 $\mu$  = Poisson's ratio  
 $t$  = time ( sec )

The first two terms of equation ( 4.1 ) correspond to the differential equation for pure bending waves. The other three terms represent the corrections. The first of these, with the coefficient  $\vartheta'_z / m'$  (which is equal to the square of the radius of gyration), would occur by itself if only rotational inertia were considered, and the second would occur by itself if only shear deformations were taken into account. For homogeneous structures, these two parts are not independent of each other. First of all,

$$\frac{\vartheta'_z}{m'} = \frac{I}{S}$$

and secondly,

$$\frac{B}{K} = \frac{IE\kappa}{SG}$$

In the case of a steel beam, if the bending wavelength to the web height ratio is less than 6 then the last three terms in equation ( 4.1 ) should be maintained. Otherwise, Cremer et al, pointed out that there exists an error of around 10% to the solution of phase velocity. The phase velocity  $C_B$  and the elastic wavelength  $\lambda_B$  can be expressed as:

$$C_B = \left(\frac{B}{m'}\right)^{1/4} (\omega)^{1/2} \quad (4.2)$$

$$\lambda_B = \frac{2\pi C_B}{\omega} \quad (4.3)$$

where  $\omega$  is the circular frequency of the bending wave.

For a 3% error of phase velocity, the ratio of  $\lambda_B/h$  should be greater than 10, thus according to equation (4.3) the frequency range has to be less than 3.2 kHz for a steel beam of the scantling for ship structure type (Wang, 1996). In these cases equation (4.1) can be reduced to the pure bending wave equation:

$$B \frac{\partial^4 v}{\partial x^4} + m' \frac{\partial^2 v}{\partial t^2} = 0 \quad (4.4)$$

In an actual vibration carrying structure, there always exists the boundaries. The boundaries may change the wave direction, phase and amplitude, furthermore, they may even change the wave pattern. To consider the progressive wave, the reflection wave and exponentially decayed near-field wave due to the boundaries, the solution of equation (4.4) with a circular frequency  $\omega$  is:

$$v = v_+ e^{-jkx} + v_- e^{jkx} + v_{-j} e^{-kx} + v_j e^{kx} \quad (4.5)$$

where

$v = v(t)$  has the form of harmonic function of time

$$k = \text{bending wave number} = \frac{2\pi}{\lambda_B} = \frac{\omega}{C_B} = (\omega^2 m' / B)^{1/4} \quad (4.6)$$

$v_+$  = amplitude of the progressive wave along the positive x direction

$v_-$  = amplitude of the progressive wave along the negative x direction

$v_{-j}$  = amplitude of the decayed near-field wave along the positive x direction

$v_j$  = amplitude of the decayed near-field wave along the negative x direction

From equation (4.5) and the fundamental theory of plate, the angular velocity  $w$ , the bending moment  $M_z$  and the shear force  $F_y$  can be obtained from the following relations:

$$w = \frac{\partial v}{\partial x}, \quad M_z = -\frac{B}{j\omega} \frac{\partial w}{\partial x} = -\frac{B}{j\omega} \frac{\partial^2 v}{\partial x^2}, \quad F_y = -\frac{\partial M_z}{\partial x}, \quad \frac{\partial F_y}{\partial x} = -j\omega m'v \quad (4.7)$$

If the state parameters of both ends of a beam element of length  $L$  are  $\langle v_{y_0}, w_{z_0}, F_{y_0}, M_{z_0} \rangle$  and  $\langle v_{y_L}, w_{z_L}, F_{y_L}, M_{z_L} \rangle$ , then the transfer relationships between the end parameters can be represented by:

$$\begin{Bmatrix} v_{y_0} \\ w_{z_0} \\ v_{y_L} \\ w_{z_L} \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{Bmatrix} F_{y_0} \\ M_{z_0} \\ F_{y_L} \\ M_{z_L} \end{Bmatrix} \quad (4.8a)$$

or

$$\{v_{NP}\} = [M_{ij}] \{F_{NP}\} \quad (4.8b)$$

where  $[M_{ij}]$ , ( $i, j = 1, 2, 3, 4$ ), is called the complex transfer matrix or mobility matrix,

$$\{v_{NP}\} = \begin{Bmatrix} v_{y_0} \\ w_{z_0} \\ v_{y_L} \\ w_{y_L} \end{Bmatrix} \text{ is the nodal velocity vector, and } \{F_{NP}\} = \begin{Bmatrix} F_{y_0} \\ M_{z_0} \\ F_{y_L} \\ M_{y_L} \end{Bmatrix} \text{ is the nodal force vector}$$

Setting  $v = v_{y_0}$  and  $w = w_{z_0}$  at  $x=0$ ;  $v = v_{y_L}$  and  $w = w_{z_L}$  at  $x = L$ , from these four

conditions it can be collected as a matrix equation :

$$\begin{Bmatrix} v_{y_0} \\ w_{z_0} \\ v_{y_L} \\ w_{z_L} \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -jk & jk & -k & k \\ e^{-jkL} & -e^{jkL} & e^{-kL} & e^{kL} \\ -jke^{-jkL} & jke^{jkL} & -ke^{-kL} & ke^{kL} \end{bmatrix} \begin{Bmatrix} v_+ \\ v_- \\ v_{-j} \\ v_j \end{Bmatrix} \quad (4.9a)$$

$$\text{or } \{v_{NP}\} = [A] \{v_{coeff}\} \quad (4.9b)$$

where

$$\{v_{\text{coeff}}\} = \begin{Bmatrix} v_+ \\ v_- \\ v_{-j} \\ v_j \end{Bmatrix}$$

$$[A] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -jk & jk & -k & k \\ e^{-jkL} & -e^{jkL} & e^{-kL} & e^{kL} \\ -jke^{-jkL} & jke^{jkL} & -ke^{-kL} & ke^{kL} \end{bmatrix}$$

then

$$\{v_{\text{coeff}}\} = [A]^{-1} \{v_{\text{NP}}\} \quad (4.10)$$

From equation (4.8b), (4.9b) and (4.10)

$$\{F_{\text{NP}}\} = [M_{ij}]^{-1} \{v_{\text{NP}}\} = [M_{ij}]^{-1} [A] \{v_{\text{coeff}}\}$$

$$\text{or } \{F_{\text{NP}}\} = [\beta] \{v_{\text{coeff}}\} \quad (4.11)$$

$$\text{Where } [\beta] = [M_{ij}]^{-1} [A]$$

$$\text{or } [M_{ij}] = [A] [\beta]^{-1} \quad (4.12)$$

Using the relationships in equation (4.7) and setting  $F_{y_0} = 1$ ,  $M_{z_0} = 0$ ,  $F_{y_L} = 0$  and  $M_{z_L} = 0$  into equation (4.8a),  $B_{ij}$  can be derived.

Similarly, by setting  $F_{y_0} = 0$ ,  $M_{z_0} = 1$ ,  $F_{y_L} = 0$ ,  $M_{z_L} = 0$ ;  $F_{y_0} = 0$ ,  $M_{z_0} = 0$ ,  $F_{y_L} = 1$ ,  $M_{z_L} = 0$ ; and  $F_{y_0} = 0$ ,  $M_{z_0} = 0$ ,  $F_{y_L} = 0$ ,  $M_{z_L} = 1$  respectively, the transfer functions  $\beta_{2j}$ ,  $\beta_{3j}$  and  $\beta_{4j}$  can be obtained. The complete relation between the nodal force vector and the velocity coefficient vector is listed in matrix form as:

$$\begin{Bmatrix} F_{y_0} \\ M_{z_0} \\ F_{y_L} \\ M_{z_L} \end{Bmatrix} = \begin{bmatrix} \frac{Bk^3}{\omega} & -\frac{Bk^3}{\omega} & -\frac{Bk^3}{j\omega} & \frac{Bk^3}{j\omega} \\ \frac{Bk^2}{j\omega} & \frac{Bk^2}{j\omega} & -\frac{Bk^2}{j\omega} & -\frac{Bk^2}{j\omega} \\ \frac{Bk^3 e^{-jkL}}{\omega} & -\frac{Bk^3 e^{-jkL}}{\omega} & -\frac{Bk^3 e^{-kL}}{j\omega} & \frac{Bk^3 e^{xL}}{j\omega} \\ \frac{Bk^2 e^{-jkL}}{j\omega} & \frac{Bk^2 e^{-jkL}}{j\omega} & -\frac{Bk^2 e^{-kL}}{j\omega} & -\frac{Bk^2 e^{-xL}}{j\omega} \end{bmatrix} \begin{Bmatrix} v_+ \\ v_- \\ v_{-j} \\ v_j \end{Bmatrix} \quad (4.13)$$

From equation (4.12), the final form of  $[M_{ij}]$  can be derived as:

$$[M_{ij}] = \frac{j\omega}{Bk^2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -jk & jk & -k & k \\ e^{-jkL} & -e^{jkL} & e^{-kL} & e^{kL} \\ -jke^{-jkL} & jke^{jkL} & -ke^{-kL} & ke^{kL} \end{bmatrix} \begin{bmatrix} jk & -jk & -k & k \\ 1 & 1 & -1 & 1 \\ jke^{-jkL} & -jke^{jkL} & e^{-kL} & e^{kL} \\ e^{-jkL} & e^{jkL} & -e^{-kL} & -e^{kL} \end{bmatrix} \quad (4.14)$$

This forms the mobility matrix of a beam element of finite length for pure bending wave propagation.

#### 4.1.2 Longitudinal Stress Wave in a Finite Beam

If two straight beams are joined at right angles, the bending waves propagating in one beam may be partly changed to the longitudinal waves in another beam. Thus, in a grillage structure, the longitudinal wave and torsional wave are capable of coupling with the bending wave in the beam elements joined at right angle. For the longitudinal stress wave propagation in a finie beam, the governing equation of propagating velocity  $v_x$  can be expressed as (Cremer et al, 1988):

$$E \frac{\partial^2 v_x}{\partial x^2} = \rho \frac{\partial^2 v_x}{\partial t^2} \quad (4.15)$$

If the longitudinal wave velocity has the form of harmonic function of time, then the solution of equation (4.15) can be express as:

$$v_x(x) = v_+ e^{-jkx} + v_- e^{jkx} \quad (4.16)$$

where

$$k = \text{the longitudinal wave number} = \frac{2\pi}{\lambda_L} = \frac{\omega}{C_L} = \omega(E/\rho)^{-1/2} \quad (4.17)$$

and the coordinate system, the longitudinal nodal forces and velocities of a finite beam element are shown in Figure 4.2

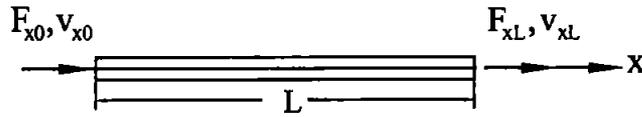


Figure 4.2 Longitudinal nodal force and velocity of bar element

In equation (4.16), the representation has considered the wave field on a rod to be composed of forward and backward propagating waves, where  $v_+$  and  $v_-$  are unknown for the present.

Taking into account the relations of nodal velocities,

$$\begin{Bmatrix} v_{x0} \\ v_{xL} \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ e^{-jkL} & e^{jkL} \end{bmatrix} \begin{Bmatrix} v_+ \\ v_- \end{Bmatrix} \quad (4.18a)$$

or

$$\{ v_{NP} \} = [A] \{ v_{coeff} \} \quad (4.18b)$$

Define the mobility matrix  $[M_{ij}]$ , which relates to the longitudinal wave propagation in a finite rod as :

$$\begin{Bmatrix} v_{x0} \\ v_{xL} \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} F_{x0} \\ F_{xL} \end{Bmatrix} \quad (4.19a)$$

$$\{ v_{NP} \} = [M_{ij}] \{ F_{NP} \} \quad (4.19b)$$

The corresponding force that acts in the beam is given by :

$$F(x) = Z(v_+ e^{-jkx} - v_- e^{jkx}) \quad (4.20)$$

where  $Z$  is the driving point impedance at free end and

$$Z = C_L \rho S = \sqrt{E \rho} S \quad (4.21)$$

From equation (4.20)

$$F_{x0} = Z(v_+ - v_-) \quad (4.22a)$$

$$F_{xL} = Z(v_+ e^{-jkL} - v_- e^{jkL}) \quad (4.22b)$$

Express equations (4.22a) (4.22b) in matrix form, it becomes

$$\begin{Bmatrix} F_{x0} \\ F_{xL} \end{Bmatrix} = \begin{bmatrix} Z & -Z \\ Ze^{-jkL} & -Ze^{jkL} \end{bmatrix} \begin{Bmatrix} v_+ \\ v_- \end{Bmatrix} \quad (4.23)$$

or

$$\{ F_{NP} \} = [\beta] \{ v_{coeff} \} = [\beta][A]^{-1} \{ v_{NP} \} \quad (4.24)$$

Compare equation (4.19b) and (4.24), the mobility matrix regarding to the longitudinal wave propagation in a finite beam can be obtained as:

$$[M_{ij}] = [A][\beta]^{-1} = \begin{bmatrix} 1 & 1 \\ e^{-jkL} & e^{jkL} \end{bmatrix} \begin{bmatrix} Z & -Z \\ Ze^{-jkL} & -Ze^{jkL} \end{bmatrix}^{-1} \quad (4.25)$$

### 4.1.3 Torsional wave in a Finite Beam

The torsional wave equation of a rectangular bar can be expressed by (Cremer et al,1988):

$$T \frac{\partial^2 w_x}{\partial x^2} = \mathcal{G} \frac{\partial^2 w_x}{\partial t^2} \quad (4.26)$$

where  $w_x$  is the angular velocity about the x-axis,  $\mathcal{G}'$  represents the mass moment of inertia per unit length of the bar,  $T$  is the torsional rigidity. For a rectangular bar of height  $h$  and width  $b$ ,

$$\mathcal{G}' = \frac{1}{12} \rho (bh^3 + hb^3) = \frac{1}{12} \rho S^2 \left( \frac{h}{b} + \frac{b}{h} \right) \quad (4.27)$$

$$T = G\kappa b^3 h \quad (4.28)$$

where  $G$  is the shear modulus.  $\kappa$  is a factor depending on the aspect ratio  $h/b$ , as shown in Table 4.1 from theory of elasticity.

Table 4.1  $\kappa$  value of rectangles with various ratios of  $h/b$

$h/b$	1.0	1.5	2.0	2.5	3.0	5.0	6.0	10	$\infty$
$\kappa$	0.141	0.196	0.229	0.249	0.263	0.291	0.298	0.312	0.333

Analogous to equation (4.15), the solution of equation (4.26) can be obtained by:

$$w_x = w_+ e^{-j\kappa x} + w_- e^{j\kappa x} \quad (4.29)$$

where

$$\kappa = \frac{2\pi}{\lambda} = \frac{\omega}{C_T} = \omega \left[ \frac{12\kappa G S b}{\rho(h^2 + b^2)h} \right]^{-\frac{1}{2}} \quad (4.30)$$

$$C_T = \left( \frac{T}{\mathcal{G}'} \right)^{1/2} = \left( \frac{12\kappa G S b}{\rho(h^2 + b^2)h} \right)^{1/2} \quad (4.31)$$

By the similar procedure, the mobility matrix for the torsional wave propagation in a finite bar is derived by:

$$[M_{ij}] = \begin{bmatrix} 1 & 1 \\ e^{-jkL} & e^{jkL} \end{bmatrix} \begin{bmatrix} Z & -Z \\ Ze^{-jkL} & -Ze^{jkL} \end{bmatrix}^{-1} \quad (4.32)$$

where

$$Z = \mathcal{G}'C_T \quad (4.33)$$

## 4.2 Wave Propagation in a Finite Rectangular Plate

### 4.2.1 Two Dimension Elastic Bending Wave in a Finite Rectangular Plate

The two dimensional bending wave equation of a finite thin plate is (Cremer et al, 1988):

$$\Delta\Delta\eta - \kappa^4\eta = 0 \quad (4.34a)$$

or

$$\Delta\Delta v - \kappa^4 v = 0 \quad (4.34b)$$

where  $\Delta$  is the Laplace operator,  $\eta$  and  $v$  are the lateral displacement and velocity of the plate particle in  $z$  direction respectively.

$$\Delta = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \quad : \quad \text{Laplace operator}$$

$$\kappa^4 = \omega^2 m'' / B' \quad : \quad \text{wave number}$$

$$m'' = \rho h \quad : \quad \text{mass per unit area of plate (kg/m}^2\text{)}$$

$$\rho \quad : \quad \text{density of plate}$$

$$h \quad : \quad \text{plate thickness (m)}$$

$$B' = \frac{Eh^3}{12(1-\mu^2)} \quad : \quad \text{bending rigidity of plate (N-m}^2\text{)}$$

For deriving the mobility function of a plate in a bending wave, it consists of solving the

differential equation (4.34a), subject to the following boundary condition:

1. There is no rotation of the plate particle at the site of excitation about x and y axes, in other words, this implies that the vibration is small and the first derivatives of the particle displacement with respect to x and y are equal to zero.
2. The excitation force is equal to the sum of the shear force  $Q_r$  in the plate around the excitation site.
3. Far away from the excitation point, a decayed wave field occurs.
4. The displacements must be axially symmetric.

Equation (4.34a) may then be replaced by two differential equations of second order, namely

$$\Delta\eta + \kappa^2\eta = 0 \quad (4.35a)$$

$$\Delta\eta - \kappa^2\eta = 0 \quad (4.35b)$$

Equation (4.35a) may be recognized as the usual wave equation for nondispersive media (e.g., for sound wave in air); its axially symmetric solution in two dimensions, as is well known, consists of zero-order cylindrical functions. If one takes account of the boundary condition (3), one finds that Hankel function of the second kind is the only type of these functions that can serve as a solution. Thus,

$$\eta_1 = C_1 H_0^{(2)}(\kappa r) \quad (4.36)$$

where  $r$  represents the distance from the excitation point. For the remainder of the discussion, one may use the asymptotic expression for the Hankel function,

$$H_0^{(2)}(\kappa r) \approx -\frac{2j}{\pi} \ln \frac{\kappa r}{2}, \quad \text{for } |\kappa r| \ll 1 \quad (4.37a)$$

$$H_0^{(2)}(\kappa r) \approx \sqrt{\frac{2}{\pi \kappa r}} e^{-j(\kappa r - \pi/4)}, \quad \text{for } |\kappa r| \gg 1 \quad (4.37b)$$

instead of a more exact representation. Detail derivations of equations (4.37a) and (4.37b) are shown in Appendix A.

The solution of equation (4.35b) that satisfies the previously mentioned boundary conditions simply by replacing  $\kappa r$  in equation (4.36) by  $-j\kappa r$ , thus,

$$\eta_2 = C_2 H_0^{(2)}(-j\kappa r) \quad (4.38)$$

Using equation (4.37b) for large values of  $\kappa r$ , one finds

$$H_0^{(2)}(-j\kappa r) \approx \sqrt{\frac{2}{-j\pi\kappa r}} e^{j\pi/4} e^{-\kappa r} \quad (4.39)$$

Thus equation (4.38) represents a near exponentially decay field.

The general solution, obtained by combining equations (4.36) and (4.38), may be expressed as:

$$\eta = C_1 H_0^{(2)}(\kappa r) + C_2 H_0^{(2)}(-j\kappa r) \quad (4.40)$$

The boundary condition (1) requires that

$$\frac{\partial \eta}{\partial r} = C_1 \kappa \left[ -\frac{2j}{\pi\kappa r} + \dots \right] - jC_2 \kappa \left[ \frac{2}{\pi\kappa r} + \dots \right] = \frac{2j}{\pi r} (-C_1 - C_2) + \dots = 0$$

Because the terms in  $\kappa r$  and in higher power of  $\kappa r$  reduce to zero at  $r \rightarrow 0$ , only the first term remains for consideration. Hence

$$C_1 = -C_2$$

Thus

$$\eta = C_1 [H_0^{(2)}(\kappa r) - H_0^{(2)}(-j\kappa r)] \quad (4.41)$$

The value of equation (4.41) at  $r = 0$  again may be found from the asymptotic expansion of the Hankel functions:

$$\begin{aligned}\eta(0) = \eta_0 &= C_1 \left[ -\frac{2j}{\pi} \ln \frac{\kappa r}{2} + \dots + \frac{2j}{\pi} \left( \ln \frac{\kappa r}{2} + \ln(-j) + \dots \right) \right] \\ &= C_1 \frac{2j}{\pi} \ln(-j) = C_1\end{aligned}$$

The constant  $C_1$  here is equal to the displacement amplitude at the excitation point, thus

$$\eta = \eta_0 [H_0^{(2)}(\kappa r) - H_0^{(2)}(-j\kappa r)] = \eta_0 \Pi(\kappa r) \quad (4.42)$$

where the propagation function  $\Pi(\kappa r)$  is defined by:

$$\Pi(\kappa r) = H_0^{(2)}(\kappa r) - H_0^{(2)}(-j\kappa r) \quad (4.43)$$

The remaining unknown  $\eta_0$  may be related to the exciting force on the basis of boundary condition (2). For this purpose, one may consider the excitation point to be at the center of a small circle of radius  $r_0$ , and one may take the external exciting force  $F_0$  to act on the area enclosed by that circle. Let the shear force that acts on the circumference of this circle, per unit length, be represented by  $Q_r$ . From mechanics of elasticity

$$Q_r = B' \frac{\partial \Delta \eta}{\partial r} \quad (4.44)$$

Substituting equation (4.42) into equation (4.44), it becomes

$$Q_r = -B' \kappa^3 \eta_0 \left[ \frac{dH_0^{(2)}(\kappa r)}{d(\kappa r)} + \frac{dH_0^{(2)}(-j\kappa r)}{d(\kappa r)} \right]$$

For  $\kappa r = \kappa r_0$  and by using of the small-argument expansion of the Hankel functions, then

$$Q_{r_0} = Q_r \Big|_{r=r_0} = \frac{4jB'\kappa^2}{\pi r_0} \eta_0 \quad (4.45)$$

Thus

$$F_0 = 2\pi r_0 Q_{r_0} = 8jB'\kappa^2 \eta_0 \quad (4.46)$$

The work of Cremer et al (1988) stopped here. The following derivation extends the Cremers' work to an innovative numerical model to cope with the analysis of structure-

borne noise propagation. If  $F_0$  is applied to one corner of a rectangular plate, then  $F_0$  can be considered as applied in a quarter of a small circle of radius  $r_0$  and with center at the corner. In this case,

$$F_0 = 2jB'\kappa^2\eta_0 \quad (4.47)$$

Now, equation (4.42) becomes:

$$\eta = \frac{F_0}{2jB'\kappa^2} [H_0^{(2)}(\kappa r) - H_0^{(2)}(-j\kappa r)] \quad (4.48)$$

Considering that  $v_0 = j\omega\eta_0$ , the driving point force impedance  $Z_{F_0}$  at one corner of a rectangular plate regarding to the bending wave propagation is :

$$Z_{F_0} = \frac{F_0}{\hat{v}_0} = \frac{2B'\kappa^2}{\omega} = \frac{2\omega m''}{\kappa^2} \quad (4.49)$$

Thus the lateral velocity distribution of the plate particle is:

$$v(x, y) = \frac{F_0\omega}{2B'\kappa^2} [H_0^{(2)}(\kappa r) - H_0^{(2)}(-j\kappa r)] = \frac{F_0}{Z_{F_0}} \Pi(\kappa r) \quad (4.50)$$

The rotational speed  $w_x$  and  $w_y$  about x and y axis can be expressed as :

$$w_x(x, y) = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial r} \cdot \frac{\partial r}{\partial x} = \frac{F_0\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r)}{\partial r} \right] \cdot \frac{x}{r} \quad (4.51)$$

$$w_y(x, y) = \frac{\partial v}{\partial y} = \frac{\partial v}{\partial r} \cdot \frac{\partial r}{\partial y} = \frac{F_0\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r)}{\partial r} \right] \cdot \frac{y}{r} \quad (4.52)$$

For a finite rectangular plate element, as shown in Figure 4.3 with nodal force vector  $\langle F_{z1} \dots F_{z4} M_{x1} \dots M_{x4} M_{y1} \dots M_{y4} \rangle^T$  and nodal velocity vector  $\langle v_{z1} \dots v_{z4} w_{x1} \dots w_{x4} w_{y1} \dots w_{y4} \rangle$ , the transfer relationships between the nodal parameters can be related by:

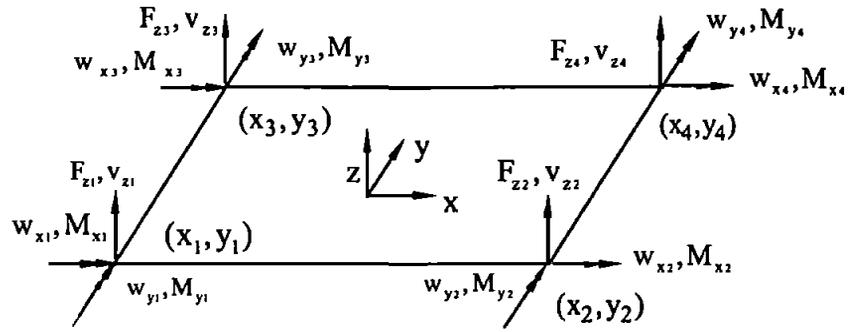


Figure 4.3 State nodal parameters of the bending wave pattern in a rectangular plate element

$$\begin{Bmatrix} v_{z1} \\ \vdots \\ v_{z4} \\ w_{x1} \\ \vdots \\ w_{x4} \\ w_{y1} \\ \vdots \\ w_{y4} \end{Bmatrix} = \begin{bmatrix} M_{1,1} & \cdots & M_{1,4} & M_{1,5} & \cdots & M_{1,8} & M_{1,9} & \cdots & M_{1,12} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ M_{4,1} & \cdots & M_{4,4} & M_{4,5} & \cdots & M_{4,8} & M_{4,9} & \cdots & M_{4,12} \\ M_{5,1} & \cdots & M_{5,4} & M_{5,5} & \cdots & M_{5,8} & M_{5,9} & \cdots & M_{5,12} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ M_{8,1} & \cdots & M_{8,4} & M_{8,5} & \cdots & M_{8,8} & M_{8,9} & \cdots & M_{8,12} \\ M_{9,1} & \cdots & M_{9,4} & M_{9,5} & \cdots & M_{9,8} & M_{9,9} & \cdots & M_{9,12} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ M_{12,1} & \cdots & M_{12,4} & M_{12,5} & \cdots & M_{12,8} & M_{12,9} & \cdots & M_{12,12} \end{bmatrix} \begin{Bmatrix} F_{z1} \\ \vdots \\ F_{z4} \\ M_{x1} \\ \vdots \\ M_{x4} \\ M_{y1} \\ \vdots \\ M_{y4} \end{Bmatrix} \quad (4.53)$$

By use of equations (4.50) , (4.51) and (4.52) and successively setting one nodal force  $F_0 = 1$  and all the other nodal forces and moments equal to zero, then  $M_{ij}$  ( $i=1,2,3,4$  and  $j=1,2,\dots,12$ ) can be obtained by equation (4.53).

For an excitation moment  $M_0$  applied to one given corner, the moment can be replaced by a set of two coupling forces with a small distance  $2a$  apart each other as shown in Figure 4.4.

$$M_0 = 2aF \quad (4.54)$$

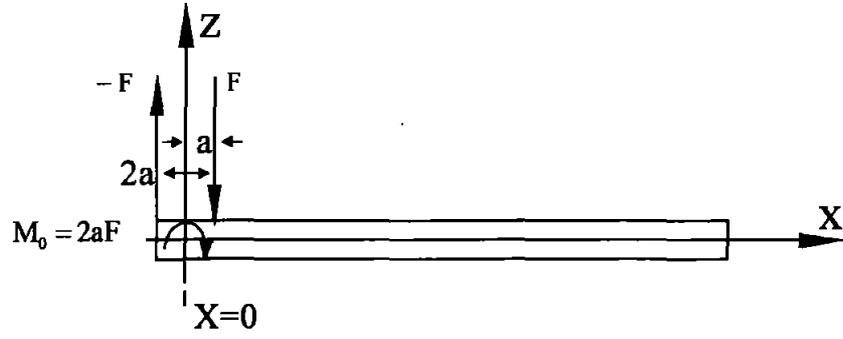


Figure 4.4 Moment excitation at a corner of a rectangular plate

From equation (4.50), it can be seen by superposition principle that the lateral velocity of the plate particle is :

$$v_z(x, y) = \frac{F\omega}{2B'\kappa^2} [\Pi(\kappa r_1) - \Pi(\kappa r_2)] \quad (4.55)$$

where  $r_1$ ,  $r_2$  are the distances from the excitation point  $x = a$  and  $x = -a$  to any point  $(x, y)$  in the plate respectively.

For the points along x axis, there are:

$$\begin{cases} \Pi(\kappa r_1) = H_0^{(2)}[k(x-a)] - H_0^{(2)}[-jk(x-a)] \\ \Pi(\kappa r_2) = H_0^{(2)}[k(x+a)] - H_0^{(2)}[-jk(x+a)] \end{cases} \quad (4.56)$$

Expanding the Hankel functions with respect to the small quantity of  $k(x-a)$  or  $k(x+a)$ , then

$$\begin{cases} \Pi(\kappa r_1) \approx 1 + 2k^2(y-a)^2 \left[ \alpha_1 + \alpha_2 \ln \frac{\gamma k(y-a)}{2} - \frac{1}{2} \alpha_2 \ln(-j) \right] \\ \Pi(\kappa r_2) \approx 1 + 2k^2(y+a)^2 \left[ \alpha_1 + \alpha_2 \ln \frac{\gamma k(y+a)}{2} - \frac{1}{2} \alpha_2 \ln(-j) \right] \end{cases}$$

where  $\alpha_1 = -\frac{1}{4} - \frac{j}{2\pi}$ ,  $\alpha_2 = \frac{j}{2\pi}$ ,  $\gamma = 1.781$ .

Thus, for  $a \rightarrow 0$ , the angular velocity at the excitation corner can be obtained from equation (4.55) as:

$$w_y(0) = \frac{\partial v_z}{\partial x} \Big|_{a \rightarrow 0} = \frac{F a \omega k^2}{2B'k^2} \left(1 - \frac{4j}{\pi} \ln \frac{\gamma ka}{2}\right) = \frac{M_0 \omega}{4B'} \left(1 - \frac{4j}{\pi} \ln \frac{\gamma ka}{2}\right) \quad (4.57)$$

Using equation (4.55) and the relations  $w_x = \partial v_z / \partial y$ ,  $w_y = \partial v_z / \partial x$ ; successively setting  $M_0 = 1$  at each node and substituting them by the coordinates of nodal points, then  $M_{ij}$ , ( $i = 5,6,7,8$ ;  $j = 1,2,\dots,12$ ), in equation (4.53) can be obtained.

A similar procedure can be followed to derived the transfer mobility function  $M_{ij}$ , ( $i = 9,\dots,12$ ;  $j = 1,2,\dots,12$ ), in equation (4.53) along  $y$  axis. Thus a full mobility matrix of a finite rectangular plate element in bending wave is derived as show in Table A.1 of Appendix A.

#### 4.2.2 Longitudinal Wave in a Rectangular Plate

Analogous to the longitudinal wave pattern in a finite rod, the longitudinal wave patterns in both  $x$  and  $y$  directions of a rectangular plate can be expressed as:

$$v_x(x, y) = v_{x+} e^{-jkx} + v_{x-} e^{jkx}$$

$$v_y(x, y) = v_{y+} e^{-jky} + v_{y-} e^{jky}$$

The transfer matrix between the nodal parameters, which includes the nodal forces and nodal velocities as shown in Figure 4.5, can be followed by:

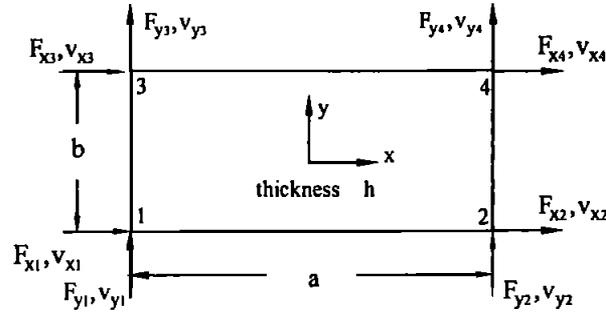


Figure 4.5 State nodal parameters of the longitudinal wave pattern in a rectangular plate element

$$\begin{Bmatrix} v_{x1} \\ v_{x2} \\ v_{x3} \\ v_{x4} \\ v_{x5} \\ v_{x6} \\ v_{x7} \\ v_{x8} \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & M_{18} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} & M_{27} & M_{28} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} & M_{37} & M_{38} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} & M_{47} & M_{48} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} & M_{57} & M_{58} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} & M_{67} & M_{68} \\ M_{71} & M_{72} & M_{73} & M_{74} & M_{75} & M_{76} & M_{77} & M_{78} \\ M_{81} & M_{82} & M_{83} & M_{84} & M_{85} & M_{86} & M_{87} & M_{88} \end{bmatrix} \begin{Bmatrix} F_{x1} \\ F_{x2} \\ F_{x3} \\ F_{x4} \\ F_{x5} \\ F_{x6} \\ F_{x7} \\ F_{x8} \end{Bmatrix} \quad (4.58)$$

where

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} M_{33} & M_{34} \\ M_{43} & M_{44} \end{bmatrix} = Z_1^{-1} \begin{bmatrix} 1 & 1 \\ e^{-jka} & e^{jka} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ e^{-jka} & -e^{jka} \end{bmatrix}^{-1} \quad (4.58a)$$

$$\begin{bmatrix} M_{55} & M_{57} \\ M_{75} & M_{77} \end{bmatrix} = \begin{bmatrix} M_{66} & M_{68} \\ M_{86} & M_{88} \end{bmatrix} = Z_2^{-1} \begin{bmatrix} 1 & 1 \\ e^{-jkb} & e^{jkb} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ e^{-jkb} & -e^{jkb} \end{bmatrix}^{-1} \quad (4.58b)$$

$M_{ij} = 0$  for all the elements not mentioned in equations (4.58a) and (4.58b). where

$$k = \frac{\omega}{\sqrt{D/\rho}}, \quad D = \frac{E}{1-\mu^2}, \quad Z_1 = \frac{1}{2} \sqrt{D\rho} bh, \quad Z_2 = \frac{1}{2} \sqrt{D\rho} ah$$

### 4.3 Modelling Noise Propagation in a Ship Structure

#### 4.3.1 Fluid-Structure Interaction

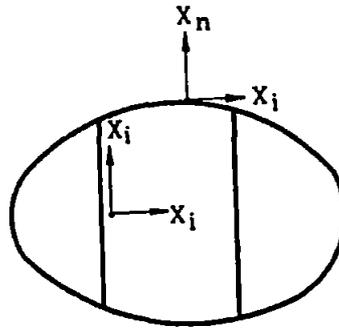


Figure 4.6 Definition of  $\{x_n\}$  and  $\{x_i\}$

When a ship is floating in water and subjected to machine induced vibrating forces  $\{f\}$ , then the equations of motion of the ship structure can be expressed in matrix form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} - \{p\} \quad (4.59)$$

where  $[M]$ ,  $[C]$  and  $[K]$  represent the mass, damping and stiffness matrices respectively.  $\{f\}$  is the acoustic loading transmitted from the machine mountings which is obtained by the mobility theory and characterising the vibrational sources as velocity sources. While  $\{p\}$  represents the hydrodynamic pressure which can be only applied to the surfaces of the structure in contact with the water and shall be analysed by the coupling algorithm which is a combination of finite element (FE) and boundary element (BE) methods.

If the internal structural damping effect can be neglected and the displacement vector  $\{x\}$  of the structure is divided into  $\{x_n\}$ , ie, the normal displacement of the surface contacted with the ambient water, and  $\{x_i\}$ , ie, the displacement of the interior structure as well as the tangential displacement on the water contacting surface as shown in Figure 4.6, then equation (4.59) becomes :

$$\begin{bmatrix} M_{nn} & M_{ni} \\ M_{in} & M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{x}_n \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} K_{nn} & K_{ni} \\ K_{in} & K_{ii} \end{bmatrix} \begin{Bmatrix} x_n \\ x_i \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_i \end{Bmatrix} - \begin{Bmatrix} p \\ 0 \end{Bmatrix} \quad (4.60)$$

### 4.3.2 Coupling of the FE and BE Methods

Consider a linear structure system and put  $\{x\} = \{X\} e^{i\omega t}$  into equation (4.60) then,

$$\begin{bmatrix} K_{nn} & K_{ni} \\ K_{in} & K_{ii} \end{bmatrix} \begin{Bmatrix} X_n \\ X_i \end{Bmatrix} - \omega^2 \begin{bmatrix} M_{nn} & M_{ni} \\ M_{in} & M_{ii} \end{bmatrix} \begin{Bmatrix} X_n \\ X_i \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_i \end{Bmatrix} - \begin{Bmatrix} p \\ 0_i \end{Bmatrix} \quad (4.61)$$

The second equation in (4.61) is:

$$([K_{in}] - \omega^2[M_{in}])\{X_n\} + ([K_{ii}] - \omega^2[M_{ii}])\{X_i\} = \{f_i\} \quad (4.62)$$

and  $\{X_i\}$  can be expressed in terms of  $\{X_n\}$  from equation (4.62) as :

$$\{X_i\} = ([K_{ii}] - \omega^2[M_{ii}])^{-1} (\{f_i\} - ([K_{in}] - \omega^2[M_{in}])\{X_n\}) \quad (4.63)$$

Substituting (4.63) into the first row column expansion of equation (4.61),

$$([K_{nn}] - \omega^2[M_{nn}])\{X_n\} + ([K_{ni}] - \omega^2[M_{ni}])\{X_i\} = \{f_n\} - \{p\}$$

or

$$\begin{aligned} & (([K_{nn}] - \omega^2[M_{nn}]) - ([K_{ni}] - \omega^2[M_{ni}])([K_{ii}] - \omega^2[M_{ii}])^{-1}([K_{in}] - \omega^2[M_{in}]))\{X_n\} \\ & = -([K_{ni}] - \omega^2[M_{ni}])([K_{ii}] - \omega^2[M_{ii}])^{-1}\{f_i\} + \{f_n\} - \{p\} \end{aligned} \quad (4.64)$$

Since the underwater acoustic pressure distribution radiated from a vibrating hull surface is governed by the Helmholtz equation:

$$\nabla^2 p(\vec{r}, \omega) + k^2 p(\vec{r}, \omega) = 0 \quad (4.65)$$

and  $p(\vec{r})$  can be expressed as in Appendix B :

$$P(\vec{r}) = \frac{1}{2\pi} \int_{S_s} [P(\vec{r}_s) \frac{\partial P(\vec{r})}{\partial n(\vec{r})} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r})}{\partial n(\vec{r})}] dS_s \quad (4.67)$$

where

- $\bar{r}$  = position vector in water;  
 $\bar{r}_s$  = position vector on immersed hull surface;  
 $n(\bar{r}_s)$  = normal direction vector at  $\bar{r}_s$ ;  
 $S_s$  = immersed hull surface area;  
 $P(\bar{r})$  = underwater acoustic pressure amplitude at  $\bar{r}$ ;  
 $G(\bar{r}, \bar{r}_s)$  = Green's function.

The relation between  $P(\bar{r})$  and the normal displacement amplitude  $X_n$  on the hull surface must satisfy :

$$\frac{\partial P(\bar{r}_s)}{\partial n(\bar{r}_s)} = -i\rho_0\omega v_n(\bar{r}_s) = \rho_0\omega^2 X_n \quad (4.68)$$

Utilizing the discretization technique with interpolating shape functions to the water contacting hull surface, equation (4.68) becomes :

$$[A] \{P\} = [G] \{X_n\} \quad (4.69)$$

Substituting equation (4.69) into (4.64), the governing equation for the combined FE and BE methods for the structure-borne noise transmission problem inboard ships can show to be:

$$\begin{aligned} & [([K_{nn}] - \omega^2[M_{nn}] - ([K_{ni}] - \omega^2[M_{ni}])([K_{ii}] - \omega^2[M_{ii}])^{-1}([K_{in}] - \omega^2[M_{in}]))\{X_n\} \\ & + [A]^{-1}[G]\{X_n\} = \{f_n\} - ([K_{ni}] - \omega^2[M_{ni}])([K_{ii}] - \omega^2[M_{ii}])^{-1}\{f_i\} \end{aligned} \quad (4.70)$$

#### 4.4 Flow Chart of the Computer Program

Based on the above mentioned formulation of mobility matrix regarding to the stress wave propagation in finite beam and rectangular plate elements, a computer program has been coded in Matlab software and FORTRAN language to analyse the structure-borne sound

propagation in a stiffened-plated structure. In principle , this program developed is somewhat analogous to the general existing finite element analysis program which created mainly for strength and vibration computations. The procedure is :

### 1. Assembly of element mobility matrix

Combining the mobility matrices of a beam for bending wave (equation (4.14)) , longitudinal wave (equation (4.25)) and torsional wave (equation (4.32)) and allocating each elements of  $H_{ij}$  to its corresponding position in the assembly mobility matrix with respect to the numbering of the degrees of freedom of the modal forces and nodal velocity , the complete mobility matrix of a beam element can be formed . Which has a dimension of  $12 \times 12$  . Similarly , the complete mobility matrix of a rectangular plate element can then be obtained by combining the matrix in equation (4.53), as shown in Table A.1 of Appendix A for bending wave, and the matrix in equation (4.58) for longitudinal wave .

### 2. Transfer the element mobility matrix to the global mobility matrix

Since the derivation of element mobility matrices is referred to the local coordinate system  $(x,y,z)$  , which is depending on the orientation of the element . While the coordinates of the locations of nodal points are referred to the unified global coordinate system  $(X,Y,Z)$  .

Thus the unit vectors ,  $\bar{n}_x$  ,  $\bar{n}_y$  ,  $\bar{n}_z$  , along  $x,y,z$  -axis are given by :

$$\bar{n}_x = \frac{1}{|\bar{r}_{12}|} \bar{r}_{12} \quad (4.95a)$$

$$\bar{n}_y = \bar{n}_z \times \bar{n}_x \quad (4.95b)$$

$$\bar{n}_z = \frac{\bar{r}_{12} \times \bar{r}_{13}}{|\bar{r}_{12} \times \bar{r}_{13}|} \quad (4.95c)$$

where  $\bar{r}_{ij}$  means the vector from nodal point  $i$  to nodal point  $j$  .

To calculate the direction cosines between the axes  $(x,y,z)$  and  $(X,Y,Z)$  , the rotation transformation matrix  $[T]$  between these two coordinate system can be attained . Thus the global mobility matrix for one element ,  $[M_G^e]$  , can be given by :

$$[M_G^e] = [T]^T [M^e] [T] \quad (4.60)$$

where  $[H^e]$  is the assembly element mobility matrix .

### 3. Form integrated structural mobility matrix

Recurring the process of step 2 for each element in the order of the numbering of the elements . The whole structural mobility matrix  $[M_G]$  can be attained .

### 4. Partition of $[M_G]$ and boundary conditions

For given kinematic boundary conditions at constrained supports and given source excitations the wave propagation equation , such as

$$\{v\} = [M_G] \{F\}$$

can be partitioned to the form as:

$$\begin{Bmatrix} v_k \\ v_u \end{Bmatrix} = \begin{bmatrix} M_{I I} & M_{I II} \\ M_{II I} & M_{II II} \end{bmatrix} \begin{Bmatrix} F_u \\ F_k \end{Bmatrix} \quad (4.61)$$

where  $\{v_k\}$  and  $\{F_k\}$  are the known velocities and forces ,  $\{v_u\}$  and  $\{F_u\}$  are the unknown velocities and force excitations .

## 5. Solution

Expanding equation (4.61) the solution of the unknown nodal velocities and nodal forces can be expressed in terms of the given velocities and forces as :

$$\begin{Bmatrix} F_u \\ v_u \end{Bmatrix} = \begin{bmatrix} M_{II} - M_{III} M_{III}^{-1} M_{III} & M_{III} M_{III}^{-1} \\ -M_{III}^{-1} M_{III} & M_{III}^{-1} \end{bmatrix}^{-1} \begin{Bmatrix} v_k \\ F_k \end{Bmatrix} \quad (4.62)$$

## 6. Backward transformation

The solved nodal velocities and nodal forces in global coordinate system should be transformed backward to the local coordinate system .

## 7. Power flow calculation

Detail of the calculation of the structure-borne sound power transmission in structure will be illustrated in the next section by example .

To follow this procedure, the FORTRAN program listed in Appendix E was written to perform the analysis of structure-borne sound propagation and Figure 4.7 illustrates a simplified flow chart of its development. This program allows the user to cope with the mobility distribution of a structure composed of rectangular plate elements and beam elements and is especially suitable for analysing structure-borne noise within the medium frequency range.

To validate the correctness of this program the comparison of the mobility spectra of simple structures obtained by experiment and conventional FEM solution are made. This will be discussed in the next section.

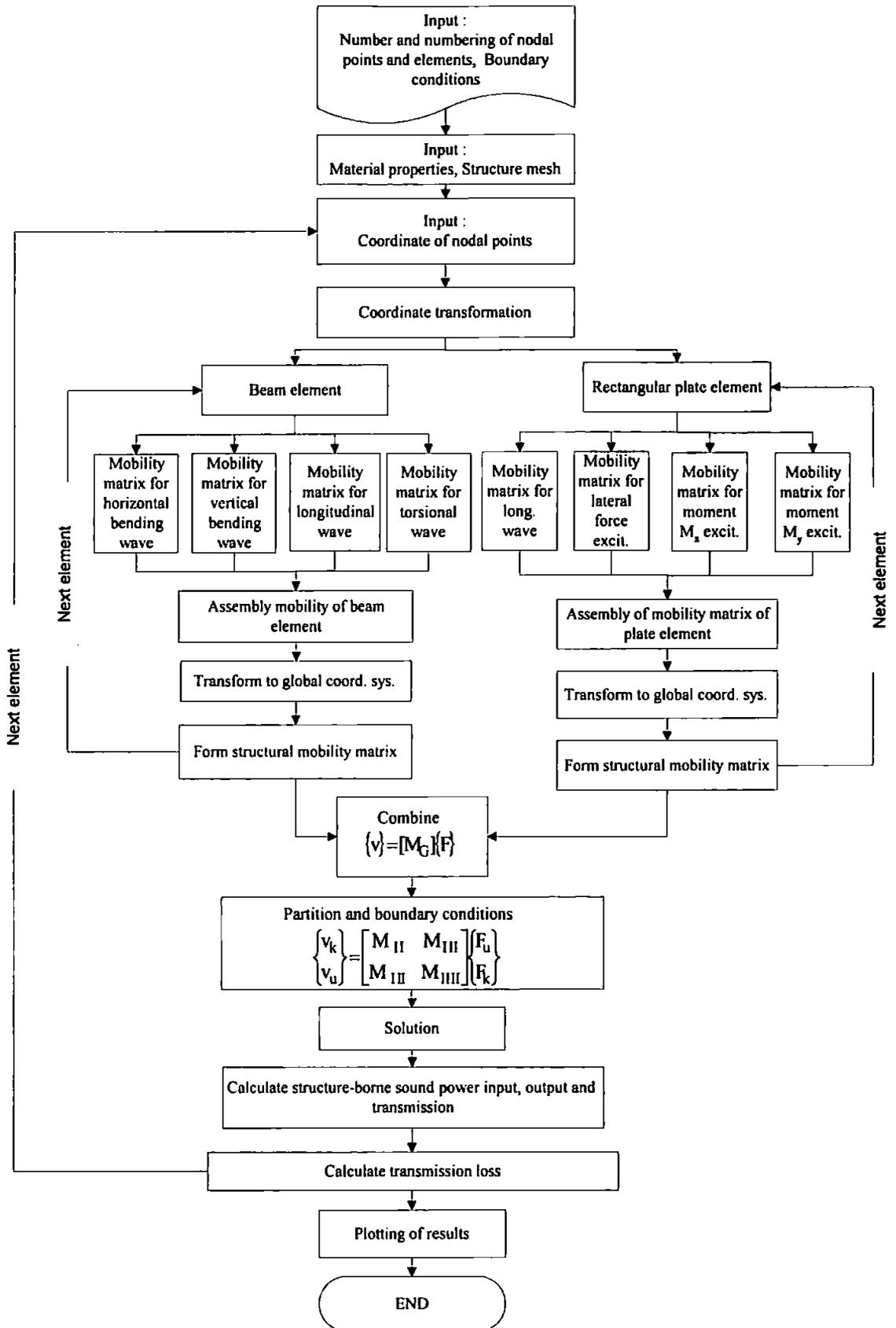


Figure 4.7 Flow chart of the program development for structure-borne sound propagation analysis

## 4.5 Validation of the Computer Code by Analysing the Structure-Borne Noise

### Propagation via Simple Structures

To validate the correctness of the developed software, the mobility analysis are carried out for simple structures such as a beam and a rectangular plate. Then compare the results to experiment measurements. Also the results are compared with that by FEM using the software ANSYS (De Salvo and Swanson,1983) for double check.

#### 4.5.1 Validation of Mobility of a Free-Free Beam

Consider a pipe beam of length 1m and diameter 0.0156 m. The beam was bolted to the shaker at midspan, so it acted as a free-free beam. The input data of the beam for FE analysis are:

pipe thickness (t)	:	1.0E-3 m
pipe diameter (d)	:	15.6E-3 m
cross section area (A)	:	4.585E-5 m <sup>2</sup>
moment of inertia (I)	:	1.227E-9 m <sup>4</sup>
Young's modulus (E)	:	2.1E+11 N/m <sup>2</sup>
density ( $\rho$ )	:	7850 kg/m <sup>3</sup>
Poisson's ratio ( $\mu$ )	:	0.3
damping ratio ( $\xi$ )	:	0.001
element length (L)	:	0.01 m
number of elements	:	100

The beam was divided into 100 equal elements as shown in Figure 4.8. Each beam element has a length 1cm. The origin of the coordinates put on the left end of the beam. Since the beam was rigidly linked to the shaker at midspan, hence the driving point of the beam has only the vertical force applied and without any moment and torque existed for such symmetrical arrangement. In this case only the flexural bending modes can be excited.

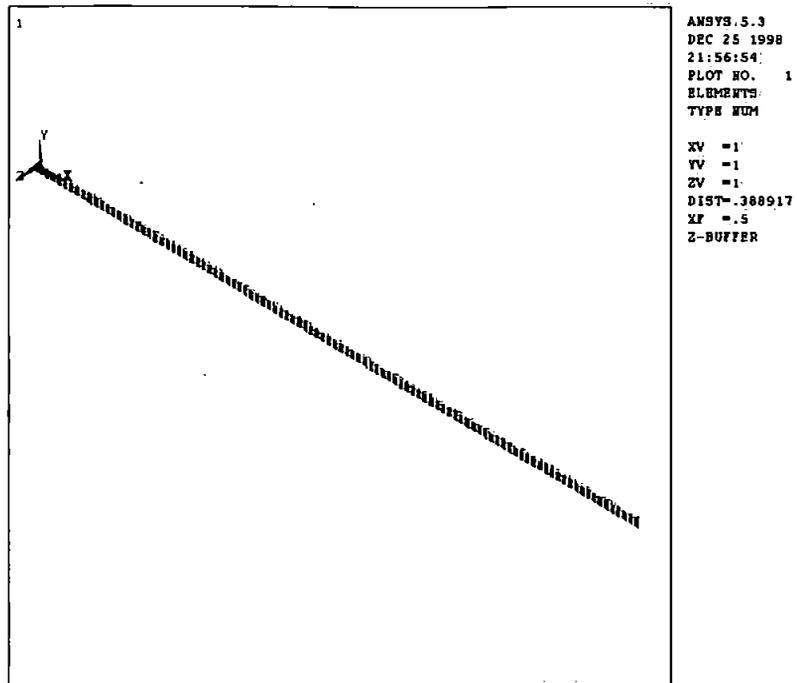


Figure 4.8 FE model of the pipe beam

In addition, experimental measurement of the mobilities at driving point and free end of the beam is conducted by utilizing the experiment instrument layout shown in Figure 4.9. In which an impedance head (B&K 8001) was put at A and an accelerometer (B&K 4374) put at B.

Pipe thickness	1.0E-3(m)	Material density	7850 (Kg/m <sup>3</sup> )
Cross section area	4.585E-5 (m <sup>2</sup> )	Poisson's ratio	0.3
Moment of inertia	1.227E-9 (m <sup>4</sup> )	Damping ratio	0.001
Young's modulus	2.1E+11 (N/m <sup>2</sup> )		

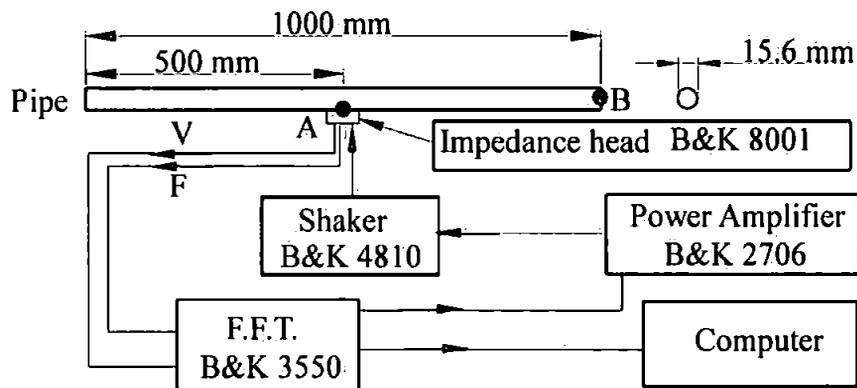


Figure 4.9 Experiment layout for the mobility measurement of the beam

The comparisons of the driving point mobility at A and the transfer mobility at B obtained by FE analysis and the measurement are shown in Figures 4.10 and 4.11 respectively. Good coincidence of the mobility spectra in the frequency below 800 Hz is displayed. To check the correctness of these two Figures by analytic solution of the model frequencies, the natural frequency of a beam in bending vibration, refer to Thomson (1993), can be expressed as:

$$\omega_n = C_n \left( \frac{EI}{mL^2} \right)^{1/2} \quad (\text{rad / s}) \quad (4.63)$$

Here

$$\omega_n = C_n \left( \frac{2.1 \times 10^{11} \times 1.22 \times 10^{-9}}{7850 \times 4.585 \times 10^5 \times 1} \right)^{1/2} = 26.68 C_n$$

Where  $C_n = 22.4, 121.0, 298.6, \dots$  for symmetrical modes. Thus, the natural frequencies of the first two symmetrical modes are 597.6 and 3228.3 rad/sec respectively, i.e., 95.1 Hz and 513.8 Hz. These are coinciding with the peak frequencies shown in Figures 4.10 and 4.11.

Also, the same driving point and transfer mobilities are analyzed by using the developed stress wave model with two equal beam elements only, each of length 0.5m, and compare to that by ANSYS. The results are shown in Figures 4.12 and 4.13. Very good coincidence is obtained.

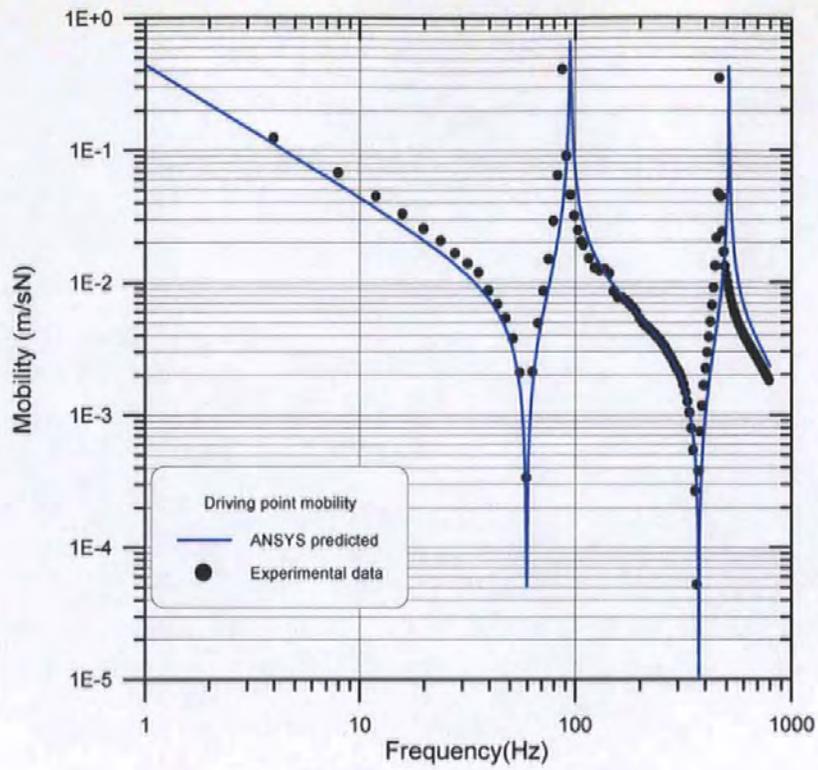


Figure 4.10 Comparison of the driving point mobility spectra of the beam by ANSYS and experiment.

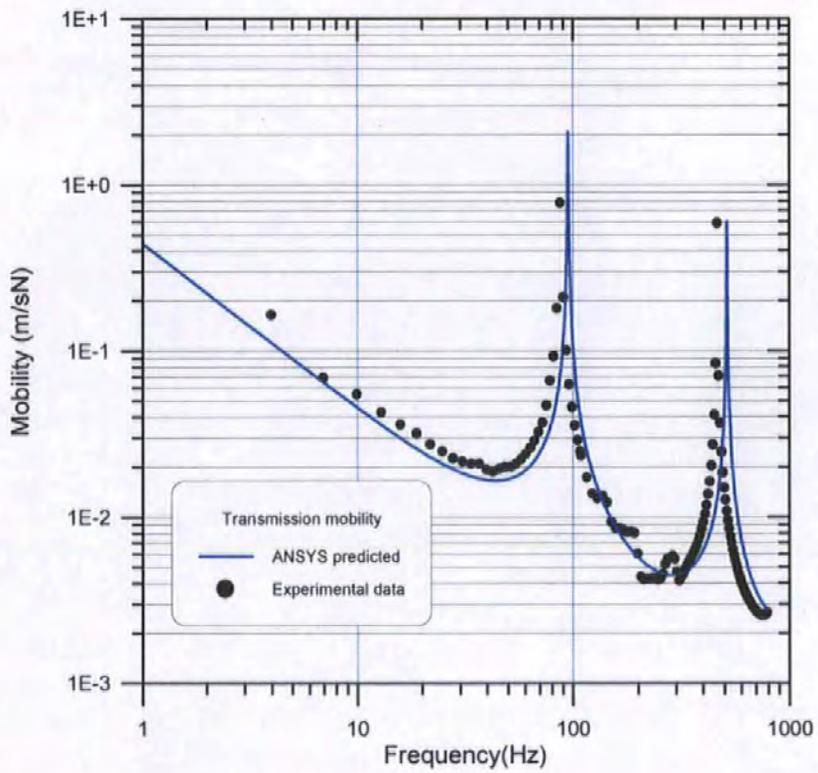


Figure 4.11 Comparison of the transfer mobility spectra of the beam by ANSYS and experiment

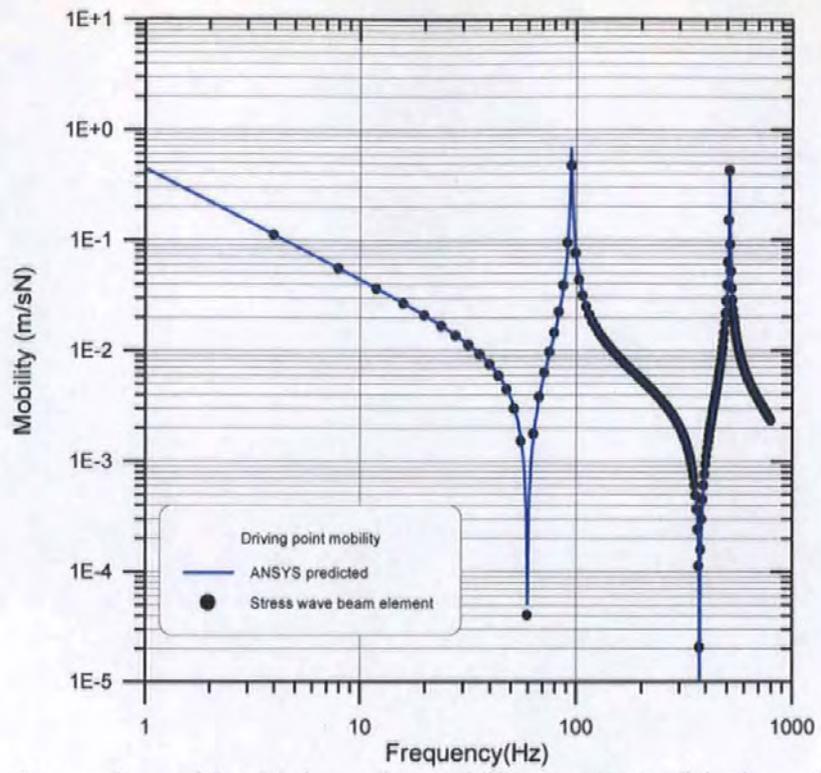


Figure 4.12 Comparison of the driving point mobility spectrum of the beam by stress wave model and ANSYS

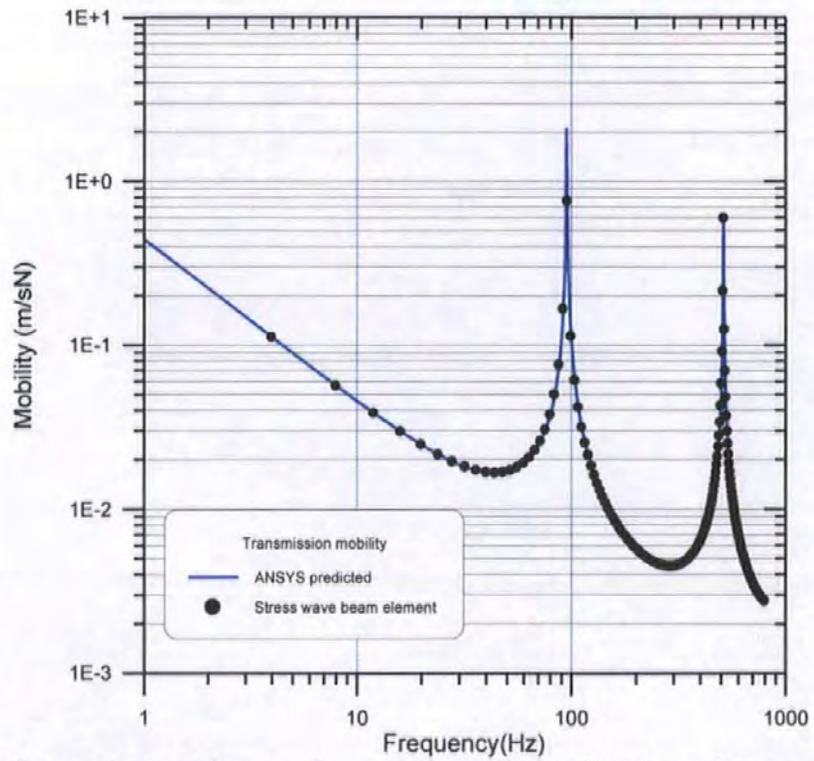


Figure 4.13 Comparison of the transfer mobility spectra of the beam by stress wave model and ANSYS

From these comparisons in Figures 4.10 to 4.13, it illustrates:

1. The mobility experiment is credible.
2. The stress wave model is very efficient in time saving to form the global mobility matrix by using fewer elements and accurate for mobility analysis by using only very few elements.
3. Even in higher frequency range up to 4000 Hz, the stress wave model for mobility analysis can maintain the precision by using very few number of elements, while the number of elements used for the FE analysis has to be increased as the frequency is increased. This is compared in Figure 4.14.

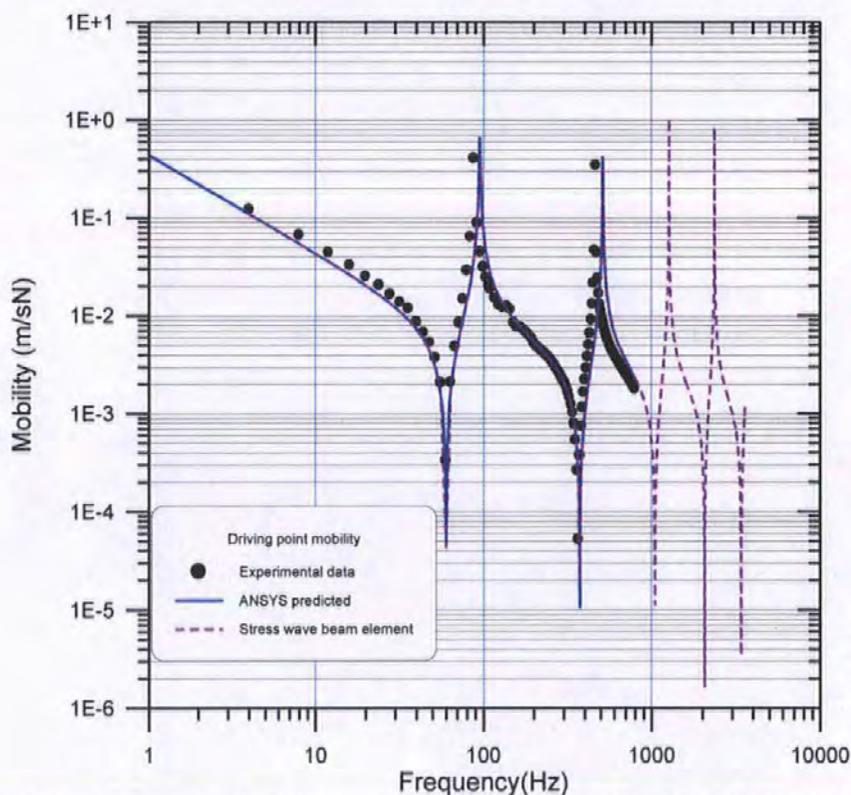


Figure 4.14 Comparison of the driving point mobility spectra of the beam in higher frequency range.

#### 4.5.2 Validation of Mobility of a Square Plate

Consider a square steel plate of dimension 1000mm×1000mm×2mm. All four edges of the plate are simply supported. Driving point of the excitation is located at the center of the

plate. The input data for the driving point mobility analysis by using the stress wave model and ANSYS are:

plate size	: 1000mm×1000mm×2mm
Young's modulus	: 2.1E+11 N/m <sup>2</sup>
density	: 7850 kg/m <sup>3</sup>
Poisson's ratio	: 0.25
damping ratio	: 0.001
element length	: 0.1m×0.1m for ANSYS and 0.5m×0.5m for stress wave model
number of elements	: 100 for ANSYS and 4 for stress wave model

The mesh generation for the FE mobility analysis is shown in Figure 4.15. Where 100 rectangular plate elements are used. While in the stress wave model only 4 stress wave plate elements are adopted.

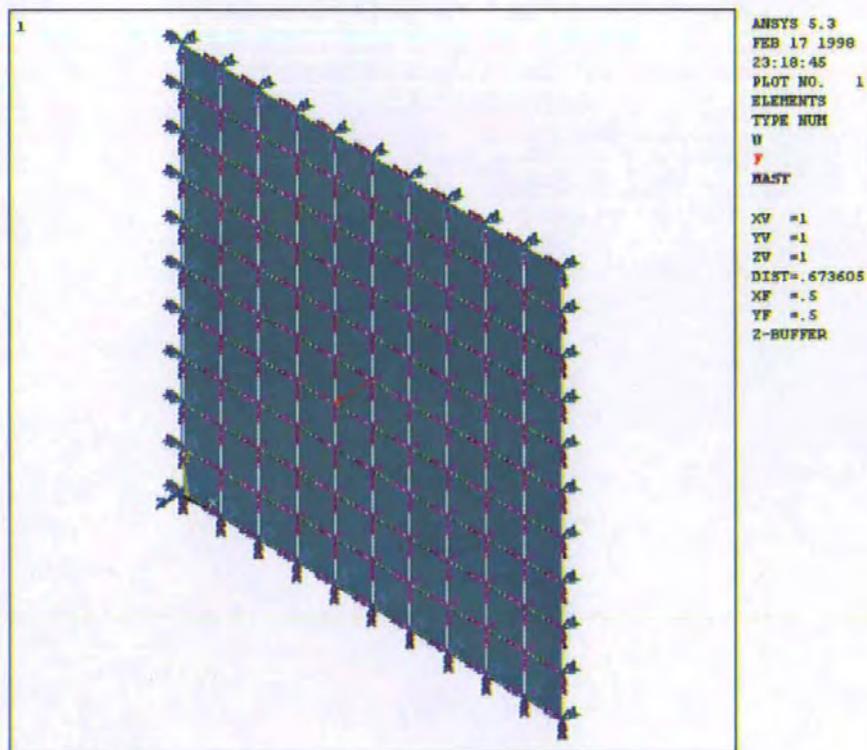


Figure 4.15 Mesh generation of the plate for FE mobility analysis.

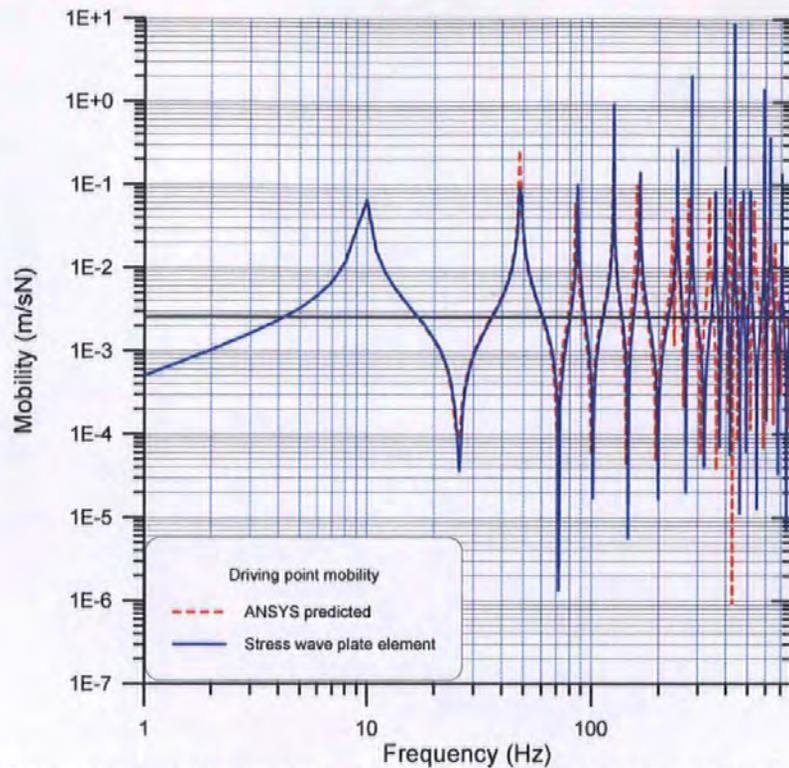


Figure 4.16 Comparison of the driving point mobility spectra of the plate by stress wave model and ANSYS.

The analysis results of the central driving point mobility spectra by the stress wave model and the FEM are compared in Figure 4.16. Also very good coincidence is shown in lower frequency range (to 120Hz ) only, while in higher frequency range the deviation of FEM becomes greater and greater. Once again it proves that the stress wave model for mobility analysis in plate is efficient by using fewer elements especially when applying in the higher frequency range.

#### 4.6 Validation of the Coupled FE and BE Model

Consider a submerged spherical shell of radius 1m and thickness 0.03m. When the shell is subjected to a harmonic internal pressure of magnitude 1 Pa, the analytical solution of the sound pressure and normal velocity on the surface of this spherical shell can be found in the studies of Everstine (1991) and Jeans and Mathews (1990). Figure 4.17 shows the coupled FEM/BEM model. Comparisons of the numerical solutions of the spectra of the surface pressure and normal velocity by using equations (4.61) to that by Everstine are

shown in Figures 4.18 and 4.19 respectively. Where the abscissa in the Figures adopts the wave number in air  $k_a$ , which is  $\omega/c_a$ ,  $c_a$  is the speed of sound in air. The coincidence of the curves validate the coupled FE and BE model being credible. Detail accuracy analysis of the application of the coupled FE/BE model for predicting the underwater sound radiation is discussed in section 7.1.3 of Chapter 7.

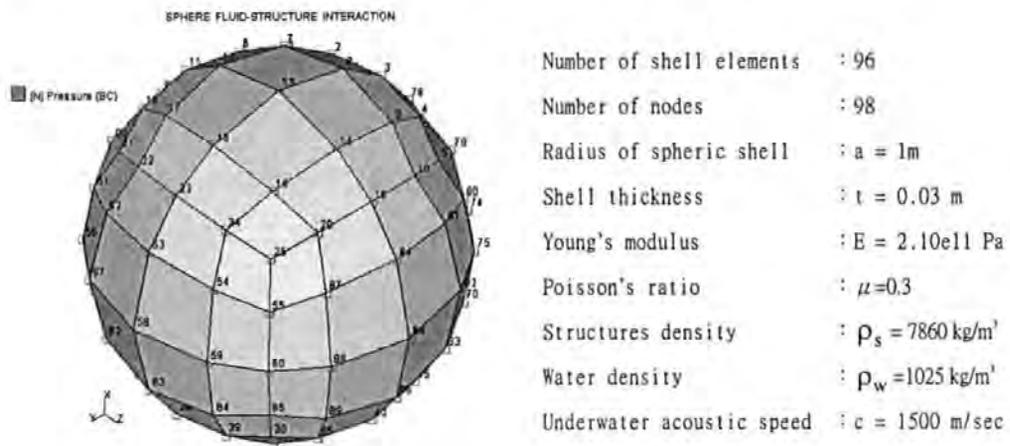


Figure 4.17 Coupled FEM/BEM model of a spherical shell

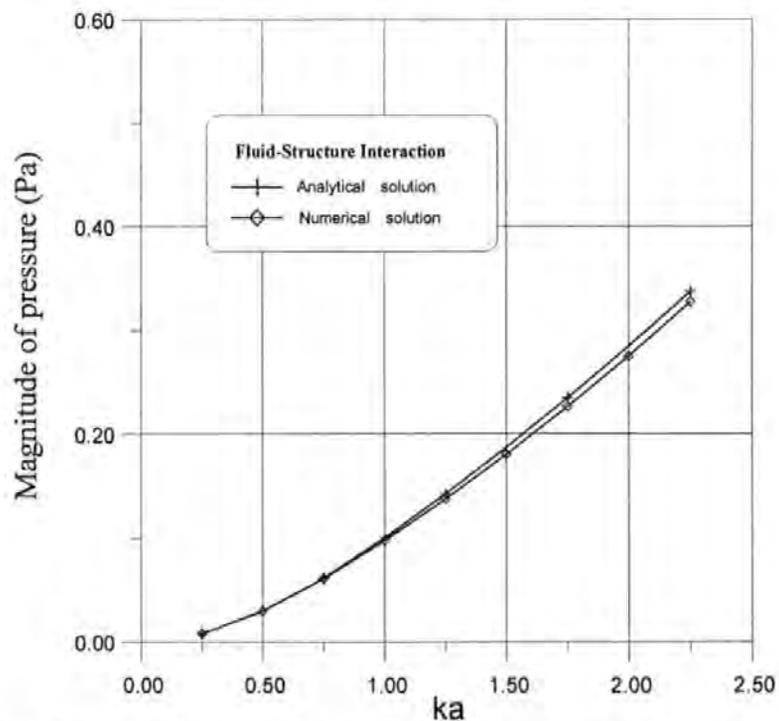


Figure 4.18 Surface pressure spectrum on a spherical shell under harmonic internal pressure excitation

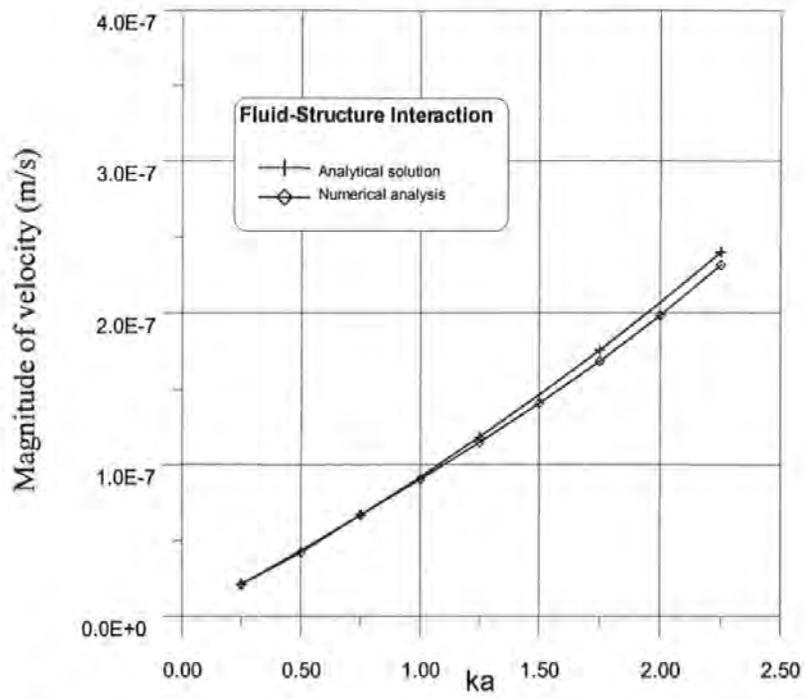


Figure 4.19 Surface normal velocity spectrum on a spherical shell under unit harmonic internal pressure excitation

**CHAPTER 5**

**EXPERIMENTAL MODELS FOR VALIDATING  
POINT AND TRANSFER MOBILITY OF  
A MACHINE TO FOUNDATION**

To validate the accuracy of applying the mobility analysis model established in Chapter 4 to the machine induced noise and vibration transmission, mobility measurement studies were undertaken on two scale models of engine room structure of two different ships. The larger model is 3m long and the smaller one is 0.585m long . Both models were performed the mobility measurements to validate the correctness of the four-pole parameters method described in Chapter 3 for ascertaining the mechanical vibration and structure-borne noise transmission. In addition, the effectiveness of the transmitted vibrational power reduction via the resilient mounts of a machine has also been characterized during the experiments. A resilient mount usually composed of an elastic element, such as a spring, and a viscoelastic element, such as a rubber piece, to absorb and dissipate the vibrational power. The characterization is performed by the comparison of the transfer mobilities, the vibrational velocity spectra and the transmitted power spectra when the scale model of engine room is installed resilient mounts with and without rubber pad respectively.

Secondly, the smaller model of the engine room structure also used to illustrate the validation of a machine induced underwater acoustic radiation prediction obtained by the coupled FEM/BEM model described in Chapter4 and experiment measurement in a water tank respectively . This part of the study will be described in the next Chapter. The reason for the adoption of a smaller scale model for the underwater acoustic radiation study is

owing to the consideration of reducing the reflection effect of the tank wall as far as possible.

Finally, a patrol vessel study was undertaken by applying the mobility theory and FEA to understand the structure-borne noise transmission from engine. For the purpose of reducing the noise level onboard, the effectiveness of squeeze-film technique was also assessed in the application. A squeeze film is an auxiliary plate attached parallel to the surface of a structure at corners, thereby trapping a thin layer of air. Relative vibration of the plates pump this air at high velocities, resulting in energy loss due to the viscosity of air. Once the loss factor of the squeeze film damping plate is determined, the mobility reduction can be analysed by FEA.

Basically, the measuring system common to the mobility measurements were arranged as shown in the block diagram of Figure 5.1. The specifications were:

- Accelerometer, B&K 4370, weight 54g , suitable for a frequency range of 0-5 kHz, 5% error for a range 0.2-3.5 kHz, and 10% error for a range of 0.1-4.8 kHz, maximum acceleration  $20\text{kms}^{-2}$ .
- Hammer-kit , B&K 8202, suitable for a frequency range of 0-7 kHz (which was defined as a 10dB force amplitude drop range) with steel tip, weight 10.3g, force range 500-5000N , exerted duration 0.25-2 ms; attached force transducer, B&K 8200, suitable for a frequency range of 0-10 kHz, weight 21g, force range 1000N (tension) - 5000N (compression) stiffness  $5 \times 10^8 \text{ Nm}^{-1}$ , resonant frequency 35 kHz, charge sensitivity 3.79 P C/N.
- Charge amplifier, B&K 2635\*8 sets, magnification factor range 0.1-1000 mv/unit.

- Tape recorder, TEAC RD-135T, 16 channels, reliable frequency band 0-5 kHz for 8 channels used simultaneously.
- Spectrum analyser, B&K 3550, 8 channels, highest effective analysis frequency range 26.5 kHz for each channel.
- Impedance head, B&K 8001, weight 31g , 5% error for range 1 Hz -6 kHz and 10% error for 1 Hz -10 kHz.

The accelerometers, charge amplifiers, and force transducer attached to the hammer-kit and the digital tape recorder were utilized together to measure the necessary data related to the mobility.

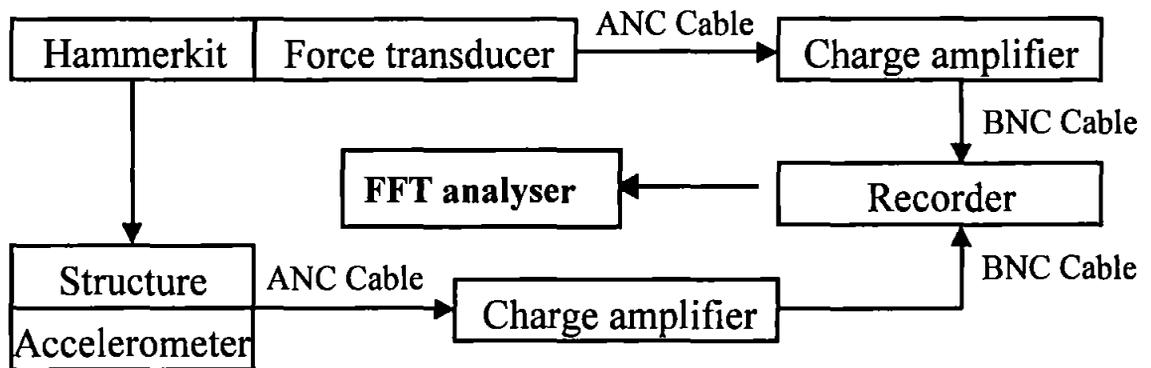


Figure 5.1 Instrument arrangement for mobility measurement

### 5.1 Mobility Study of Scale Engine Room Model (I)

Consider a scale model of an engine room structure as shown in Figure 5.2. This model with a length 3 m, consists of two equally spaced web frames, two end bulkheads, four girders on the bottom, one stringer on each side shell and two hatchside deck longitudinals. In the middle of the bottom a bedplate, 600mm×300mm, is installed and bolted to the bottom girders. On the top of the bedplate four resilient mounts were used to support a

steplees motor. On the shaft of the motor there is attached a deliberately designed eccentric mass to simulate a dynamical unbalance.

In order to recognize the structure-borne sound transmission via the resilient mount of the motor-resilient mount-foundation structure, also shown in Figure 5.2, all mobilities in equation (3.108) are necessary to be measured.

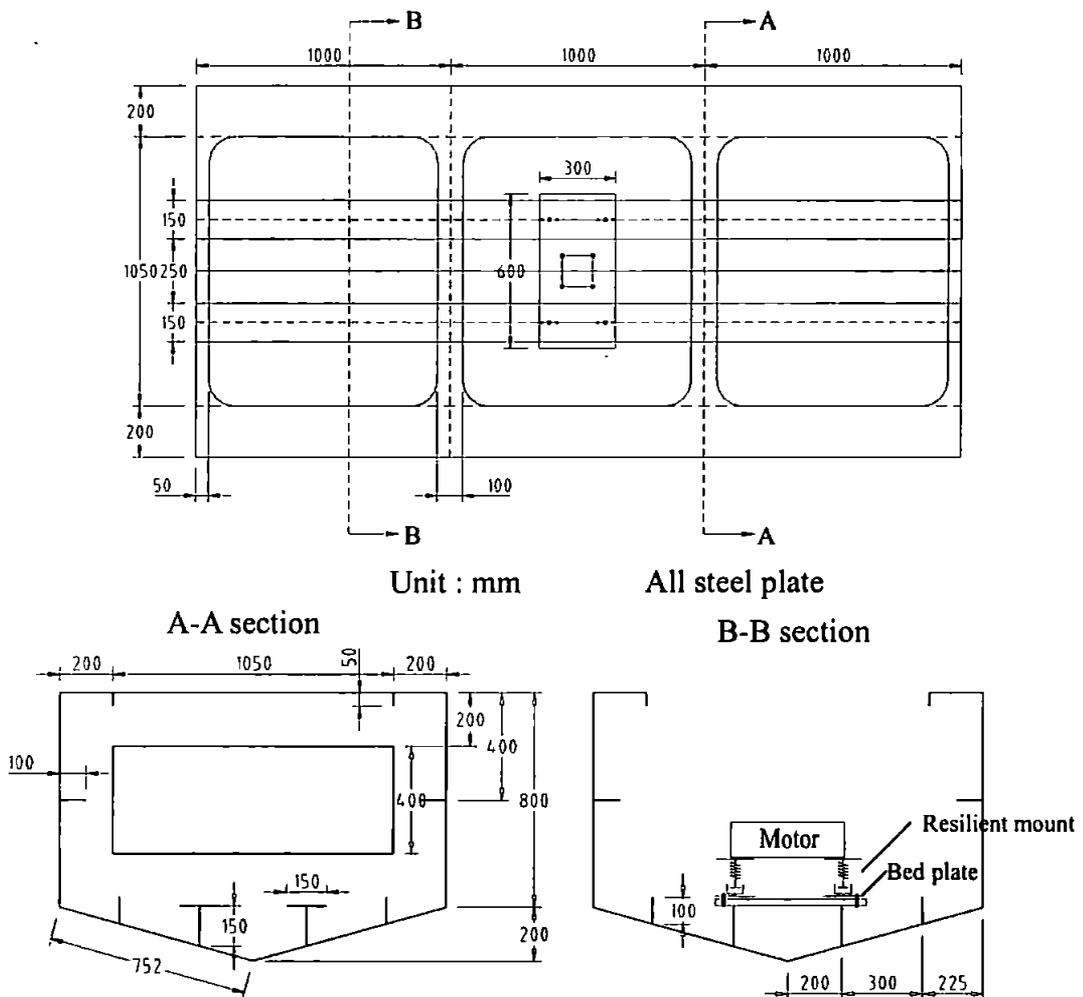


Figure 5.2 Scale engine room model (I) for mobility measurement

The locations of the mobility measuring and knocked points for the motor-resilient mount-foundation structure are shown in Figure 5.3. Where point ① was located at the centre of

gravity of the motor, points ② and ③ were located close to the bolt joint of the motor and the resilient mount while point ④ was close to the bolt joint of the bedplate and the bottom girder. The scale engine room model, had a mass of 500 kg, was hung in air and floating in a water tank respectively during the mobility measurements. The mobility measurements were undertaken by the conditions defined by :

- M<sub>12</sub>: hung and separated the subsystem A, knocked point ② by a hammer and measured the transfer mobility at point ①.
- M<sub>22</sub>: hung and separated the subsystem A, measured the point mobility at point ②.
- M<sub>33</sub>: put subsystem A on subsystem B together but did not fasten, measured the point mobility at point ③.
- M<sub>34</sub>: put subsystem A on subsystem B, knocked point ④ by a hammer and measured the transfer mobility at point ③.
- M<sub>41</sub>: linked and fastened the subsystems A and B, knocked point ① and measured the transfer mobility at point ④.

where subsystems A and B are coupled in series as shown in Figure 5.5.

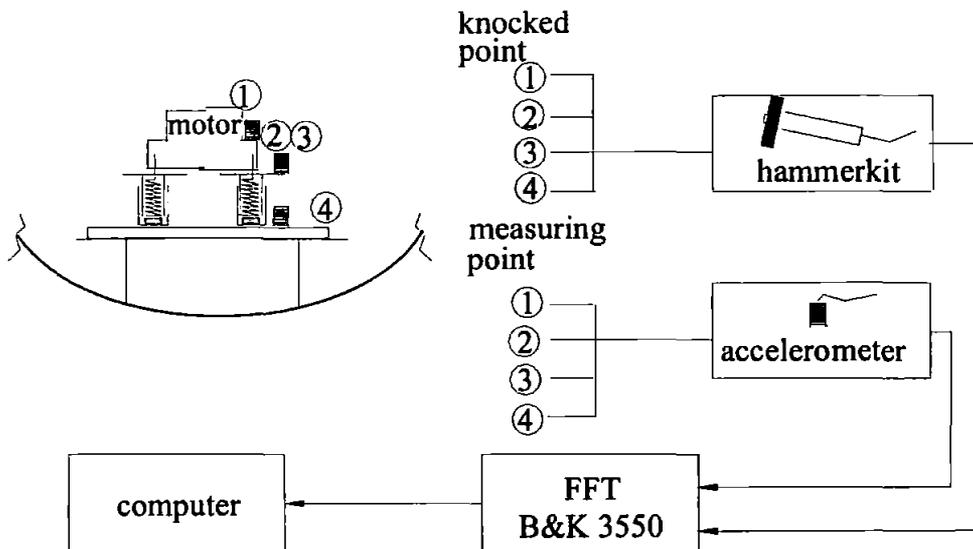


Figure 5.3 Mobility measuring points and experiment setup of the motor-resilient mount-foundation system

For the mobility testing of the mounts, which interposing the bedplate and the girders in

Figure 5.2, a sleeve type of resilient mount as shown in Figure 5.4 was selected to study. In which the spring and rubber piece put in series connection within a sleeve. Where the rubber piece can be replaced. Thus, the selection of this type of resilient mount is capable of studying and comparing the vibration isolation characteristics for different kind and thickness of rubber in a replaceable way.

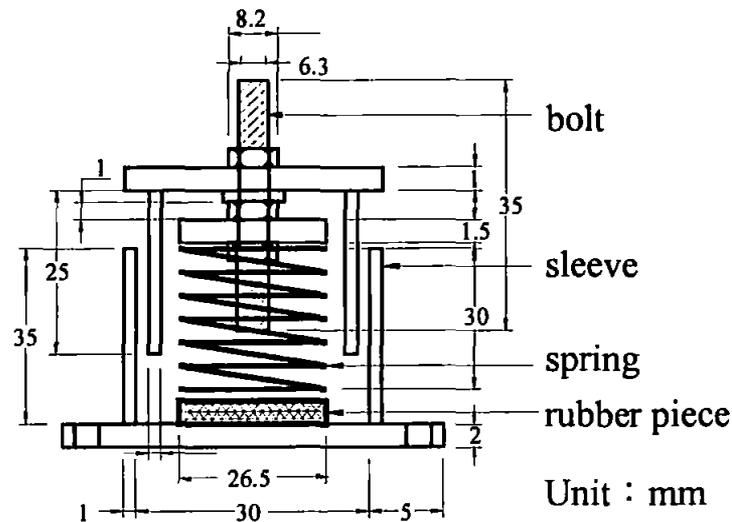


Figure 5.4 Configuration of the sleeve type of resilient mount

### 5.1.1 Validation of the Four-Pole Parameters Model

Consider the scale model as two subsystems A and B coupled in series as shown in Figure 5.5. Subsystem A consists of the motor and the upper part of the sleeve type mount, while subsystem B consists of the lower part of the sleeve type mount, bedplate and the hull structure. To validate the model of equation (3.108), the comparison of  $|M_{41}|$  by direct measurement and by calculation were made. The comparisons have carried out for four cases, i.e., the scale engine room model hung in air as shown in Figure 5.6, and floated in a water tank, of dimension 4.2m long, 3.6m wide and 2.4m deep, with and without inserting a Butyl rubber piece in the resilient mounts.

The comparison results are shown in Figures 5.7 to 5.10. Detail accuracy analysis of the

mobility model for evaluating vibration transmission via mounts is discussed in Section 7.1.1 of Chapter 7.

Figure 5.7 represents the comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model was hung in air with rubber pieces interposing in the mounts.

Figure 5.8 represents the comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model was hung in air without rubber pieces interposing in the mounts.

Figure 5.9 represents the comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model was floating in the tank with rubber pieces interposing in the mounts.

Figure 5.10 represents the comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model was floating in the tank without rubber pieces interposing in the mounts.

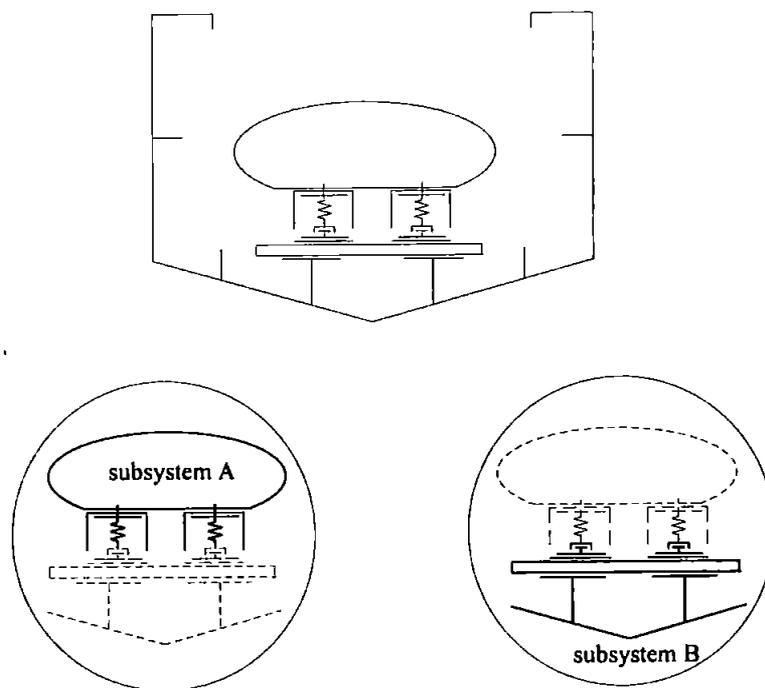


Figure 5.5 Coupled subsystems A and B of the scale engine room model (I)



Figure 5.6 The hanging arrangement of the scale engine room model (I)

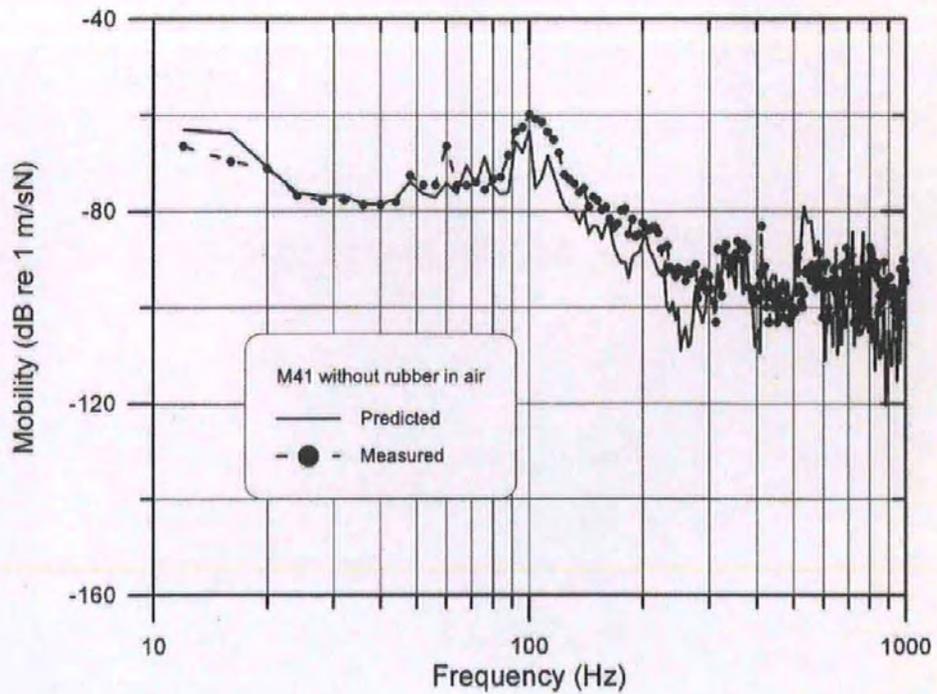


Figure 5.7 Comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model (I) was hung in air with rubber pieces interposing in the mounts

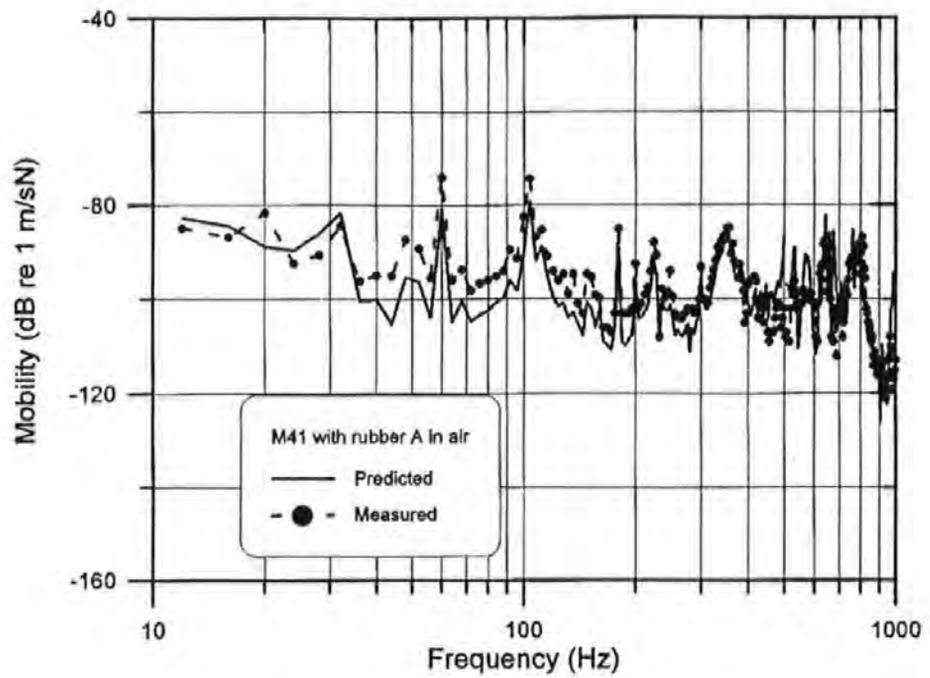


Figure 5.8 Comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model (I) was hung in air without rubber pieces interposing in the mounts

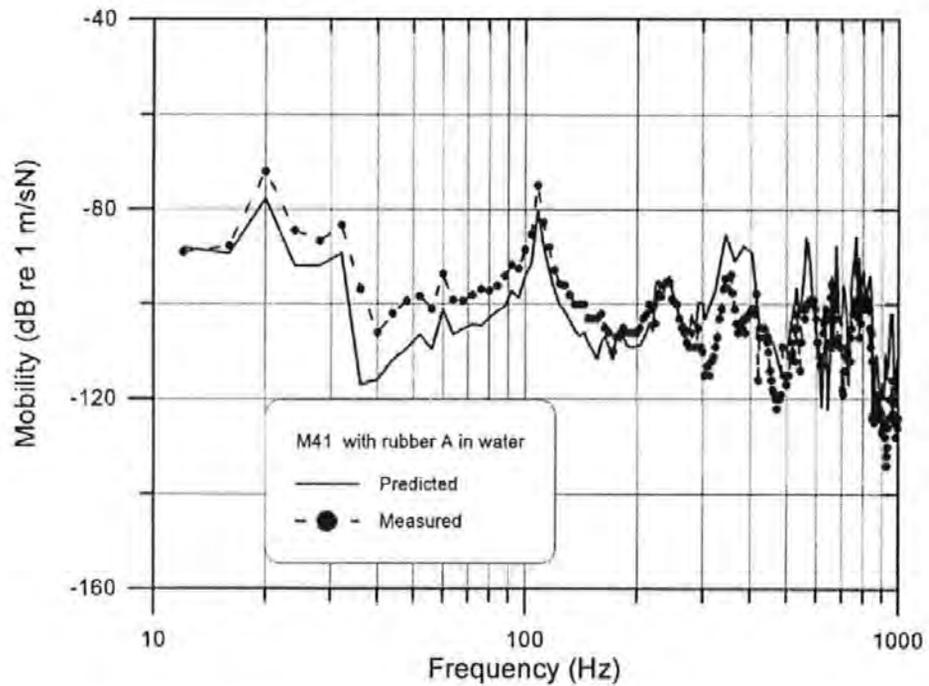


Figure 5.9 Comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model (I) was floating in the tank with rubber pieces interposing in the mounts

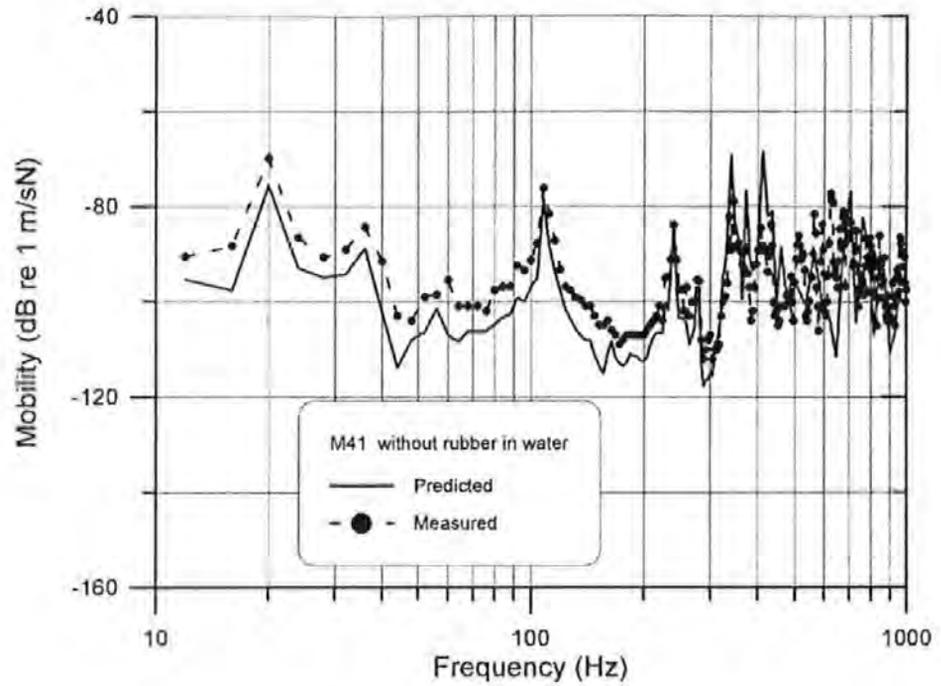


Figure 5.10 Comparison of the predicted and measured mobility spectrum  $M_{41}$  when the scale model (I) was floating in the tank without rubber pieces interposing in the mounts

From the measured transfer mobility spectra  $M_{41}$  shown in Figures 5.7 and 5.8, the effect of the rubber pad for the reduction of transfer mobility function from the motor to the bottom structure can be characterized and is shown in Figure 5.11. It can be seen that the rubber pad can reduce  $M_{41}$  by an average amount around 12dB at the peak frequencies below 200 Hz, while the reduction in higher frequency is not apparent.

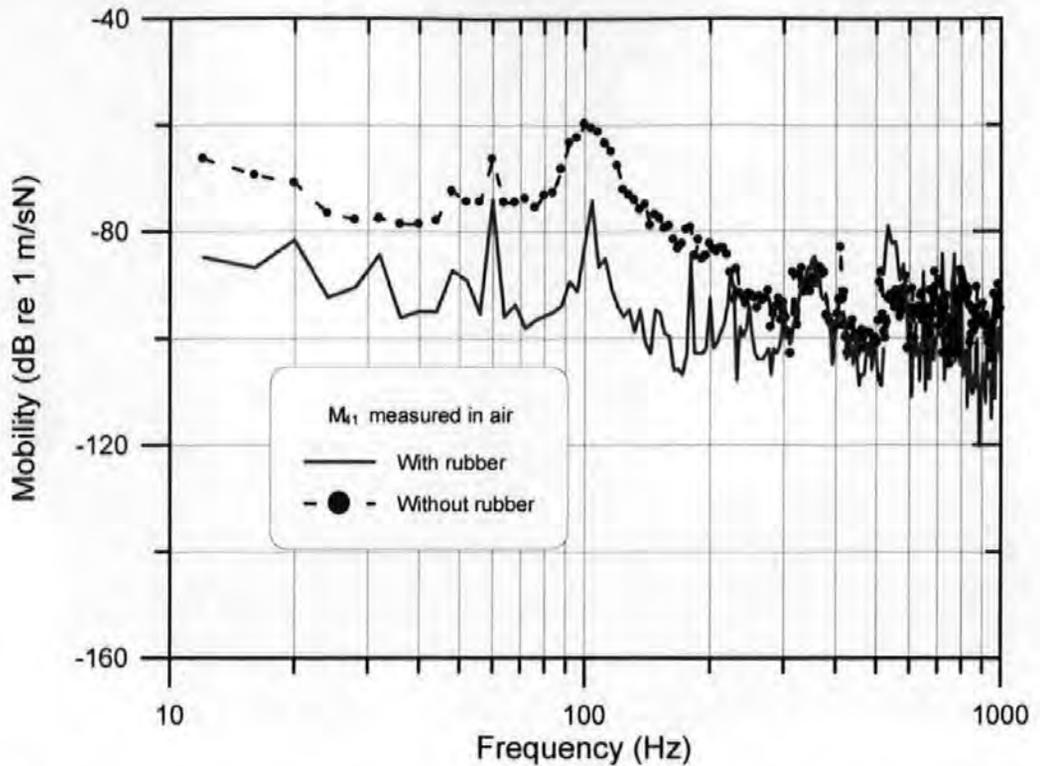


Figure 5.11 Effect of rubber pad in reducing the transfer mobility of the scale engine room model ( I ) hung in air.

### 5.1.2 Prediction of Vibrational Power Transmission via Resilient Mount from Motor

When the motor in the scale engine room model was rotating at a constant speed 1140 rpm, there was a vibration caused by the unbalanced mass attached to the shaft. This vibration can transmit to the foundation via the mounts. The vibrational power flow transmission can be predicted by using the velocity and impedance spectra as expressed in equations (3.70a) to (3.70c), in which the impedances are also related to the mobilities by equations (3.69a) to (3.69c). Thus the power flow calculation can be schemed as the flow chart shown in Figure 5.12.

The measured vibrational velocity spectrum density functions at motor side and girder side of the resilient mount with/without rubber pad are compared in Figures 5.13 and 5.14 . Combining the measured mobility spectra, the predicted vibration power at input of the

mount are calculated as shown in Figures 5.15 and 5.16. Comparison of the vibrational power reduction via the resilient mount with and without rubber pad is shown in Figure 5.17. It clearly shows that the Noise Reduction (NR) in power transmission via a resilient mount with rubber pad is better than that without rubber. Detail quantitative analysis of the effect of the rubber pad in reduction of vibration transmission is discussed in Section 7.1.4 of Chapter 7 about the results shown in Figures 5.11 to 5.16.

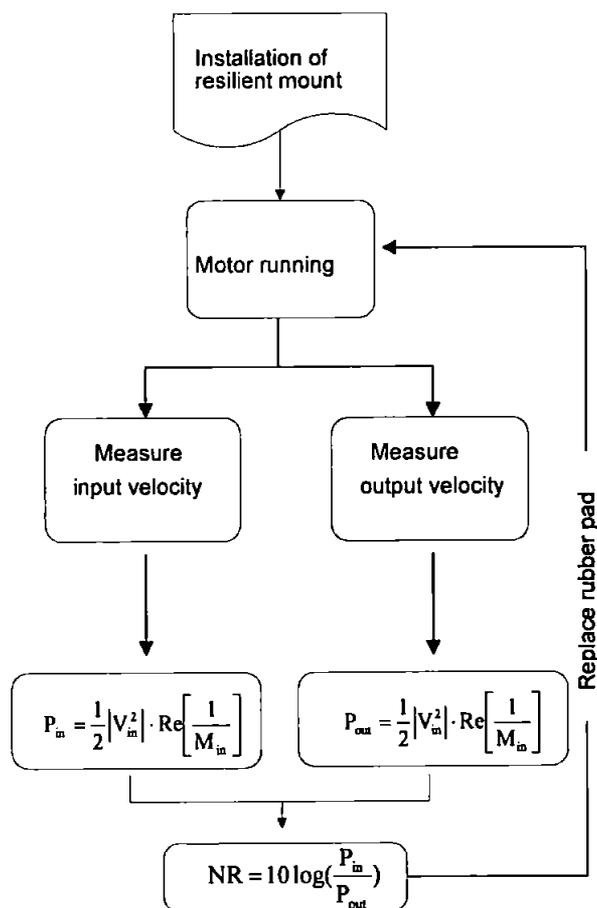


Figure 5.12 Flow chart for calculating the noise reduction in power flow transmission via resilient mount

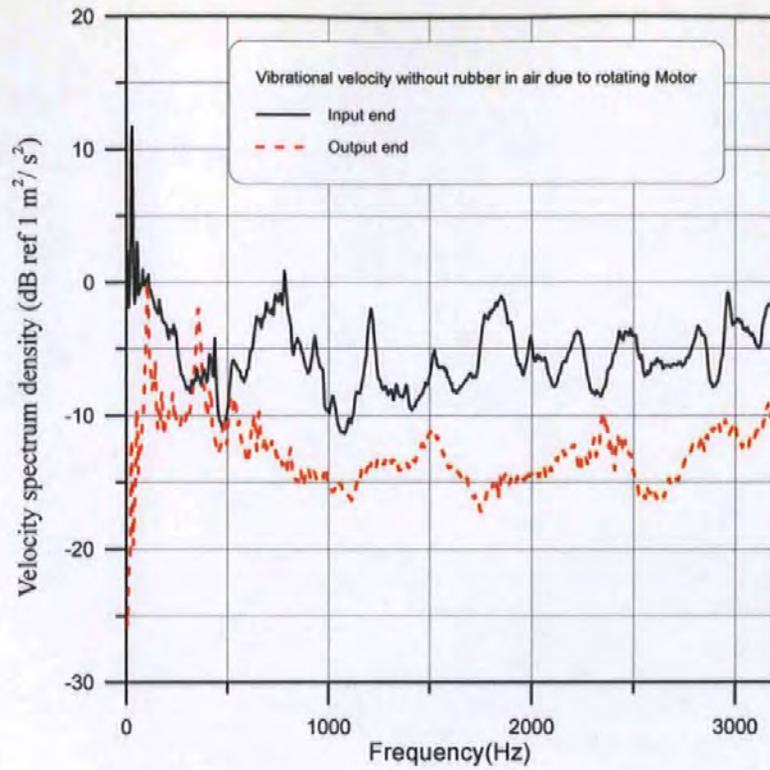


Figure 5.13 Velocity spectra density at input and output sides of the resilient mount without rubber pad.

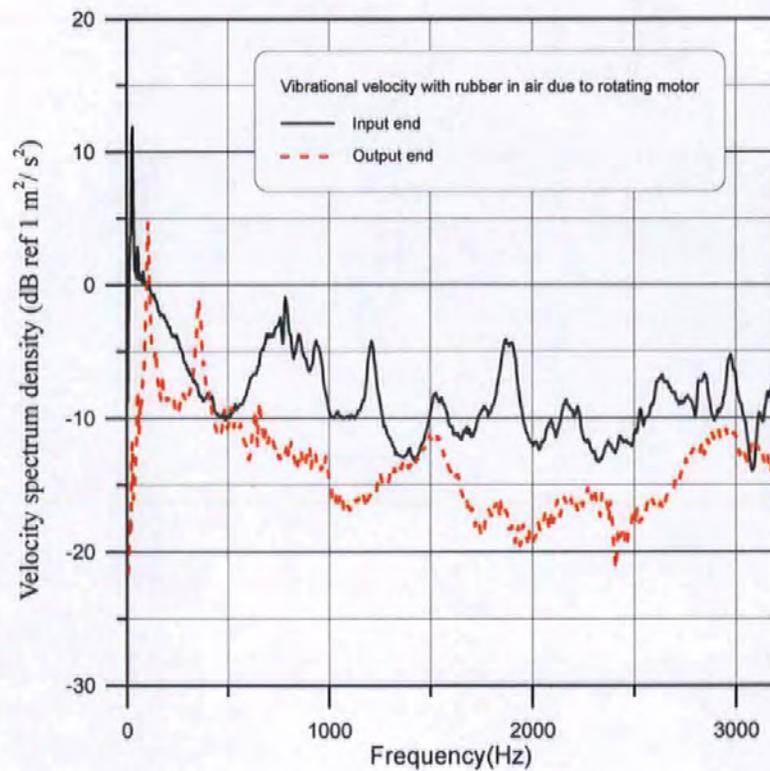


Figure 5.14 Velocity spectra density at input and output sides of the resilient mount with rubber pad.

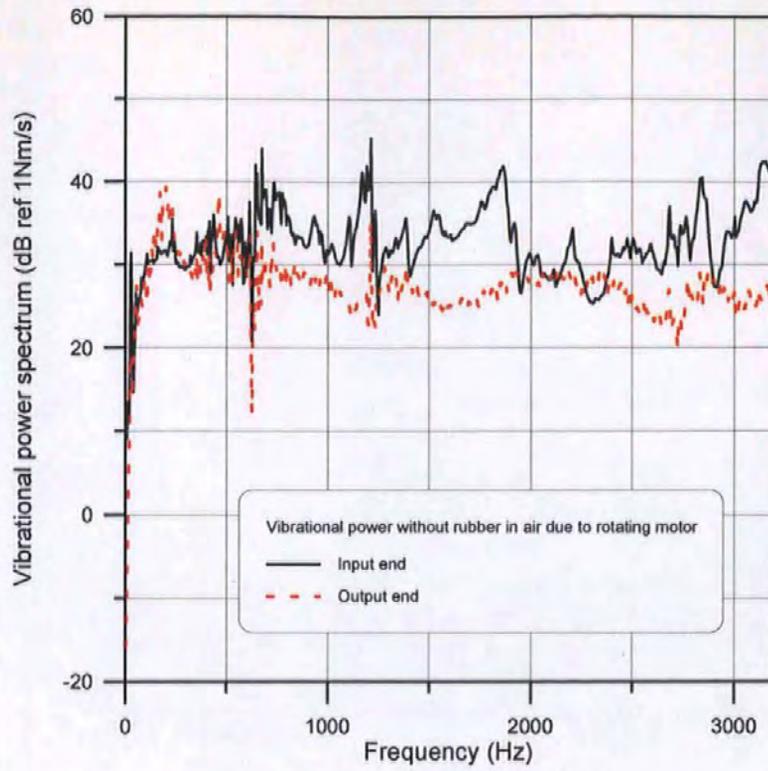


Figure 5.15 Vibrational power spectra at input and output sides of the resilient mount without rubber pad

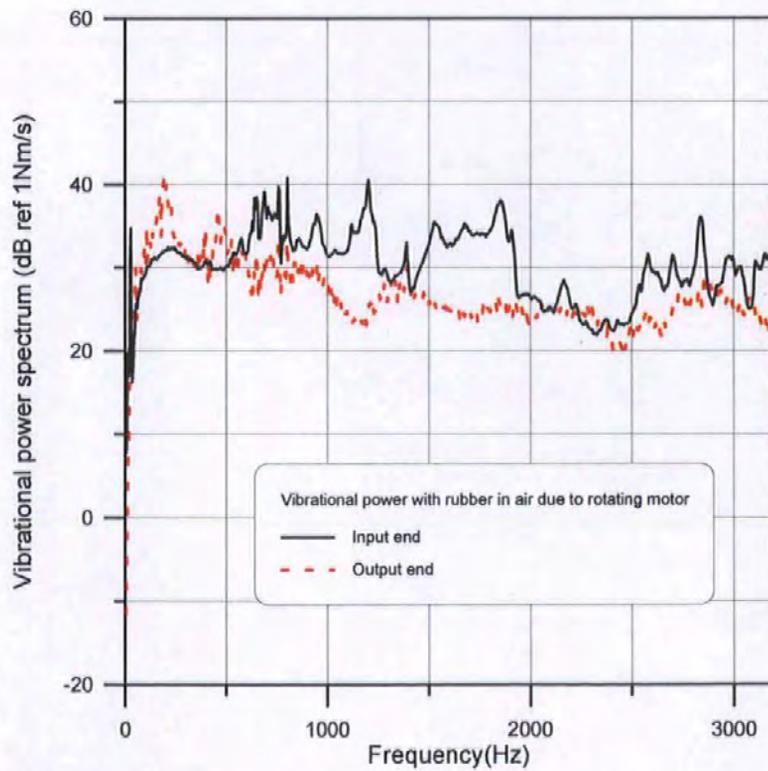


Figure 5.16 Vibrational power spectra at input and output sides of the resilient mount with rubber pad

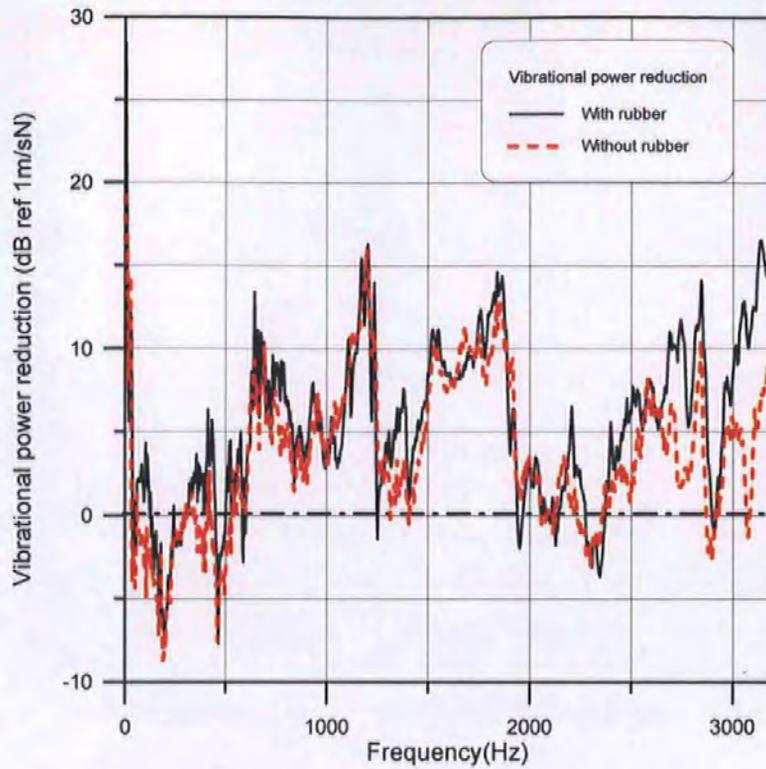


Figure 5.17 Comparison of Vibrational power reduction via mount with and without rubber pad

### 5.1.3 Validation of the FEM/BEM Model for Mobility Analysis

To validate the coupled FEM/BEM model described in Chapter 4, the driving point mobility of the separate subsystem B in Figure 5.5, of the scale engine room model (I) as shown in Figure 5.18 and 5.19, was analyzed by the software ANSYS/SYSNOISE and compared with the measured mobility. Detail accuracy evaluation of the mobility analysis by using the FEM/BEM model is discussed in Section 7.1.2 of Chapter 7.

Input data to the analysis are:

plate thickness	: 3 mm
density	: 7860 kg/m <sup>3</sup>
Young's modulus of steel	: $2.1 \times 10^{11}$ N/m <sup>2</sup>
Poisson's ratio	: 0.3
damping ratio	: 0.0014
Young's modulus of bungie rope	: $4.7 \times 10^6$ N/m <sup>2</sup>
linear stiffness of bungie rope	: 8.3 N/m
number of rectangular plate elements	: 8217
number of master degree of freedom	: 670
minimum element size	: 0.05 m

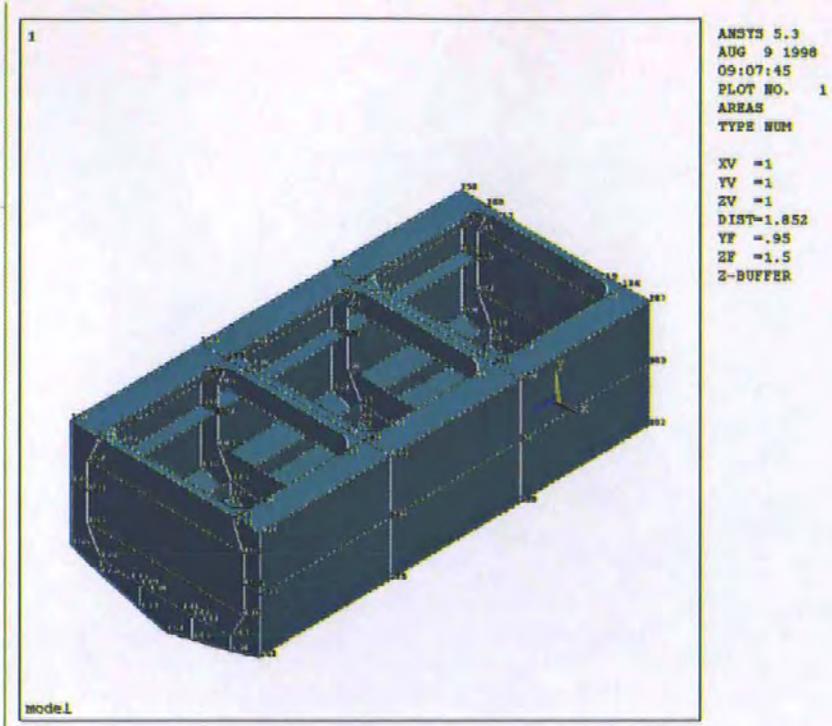


Figure 5.18 Geomentry of the experimental scale engine room model (I)

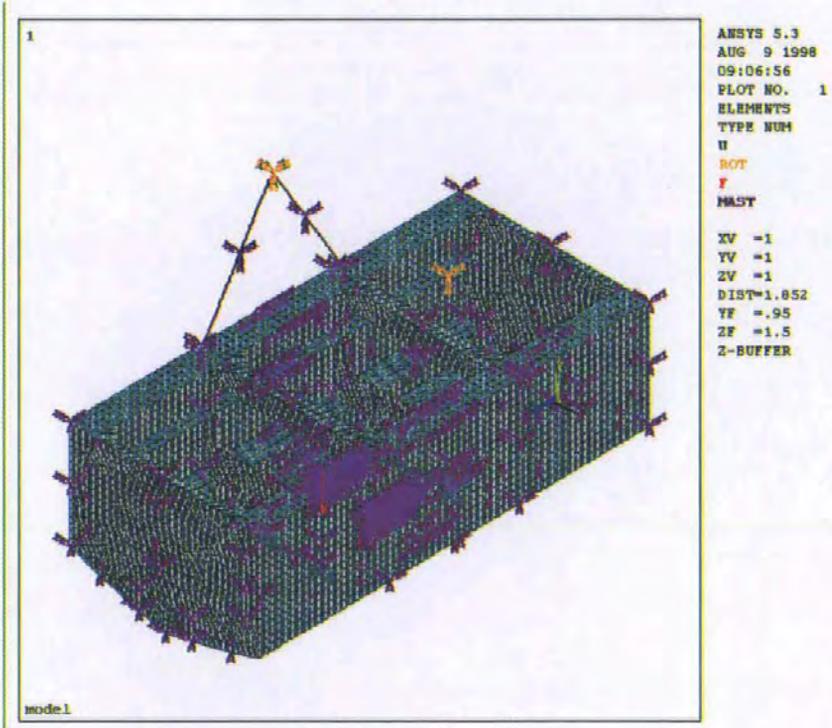


Figure 5.19 Mesh generation of the scale engine room mode (I).

The mobility experiment arrangement is shown in Figure 5.20. The driving point is located at the foot of the mount. The driving point mobility was measured when the scale model was floating in the water tank and hung by four bungee rope simultaneously.

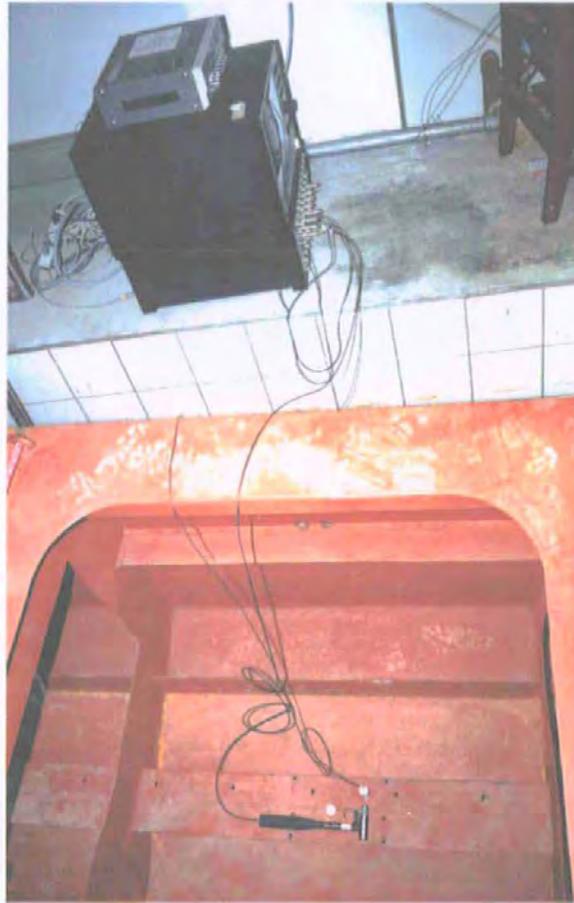


Figure 5.20 Experiment arrangement

The driving point mobilities predicted and measured in this condition is compared in Figure 5.21. The coincidence of the mobility spectra is fairly well below 550 Hz and deviates in higher frequency range.

Even in lower frequency range the measured mobility spectrum existed fluctuations deviated from the predicted spectrum curve. This illustrates the influence of the reflection

effect of the tank wall. Deviations in higher frequency range can be explained as that element size should be less than one tenth of the structure-borne wave length as a common practice, Cabos and Jokat (1998), i.e., the credible frequency range is below 450 Hz for the established FEA mesh model. Thus, it can be concluded that FEA is only suitable for mobility analysis in lower frequency range of the analysis and the one sixth rule is proved to be applicable.

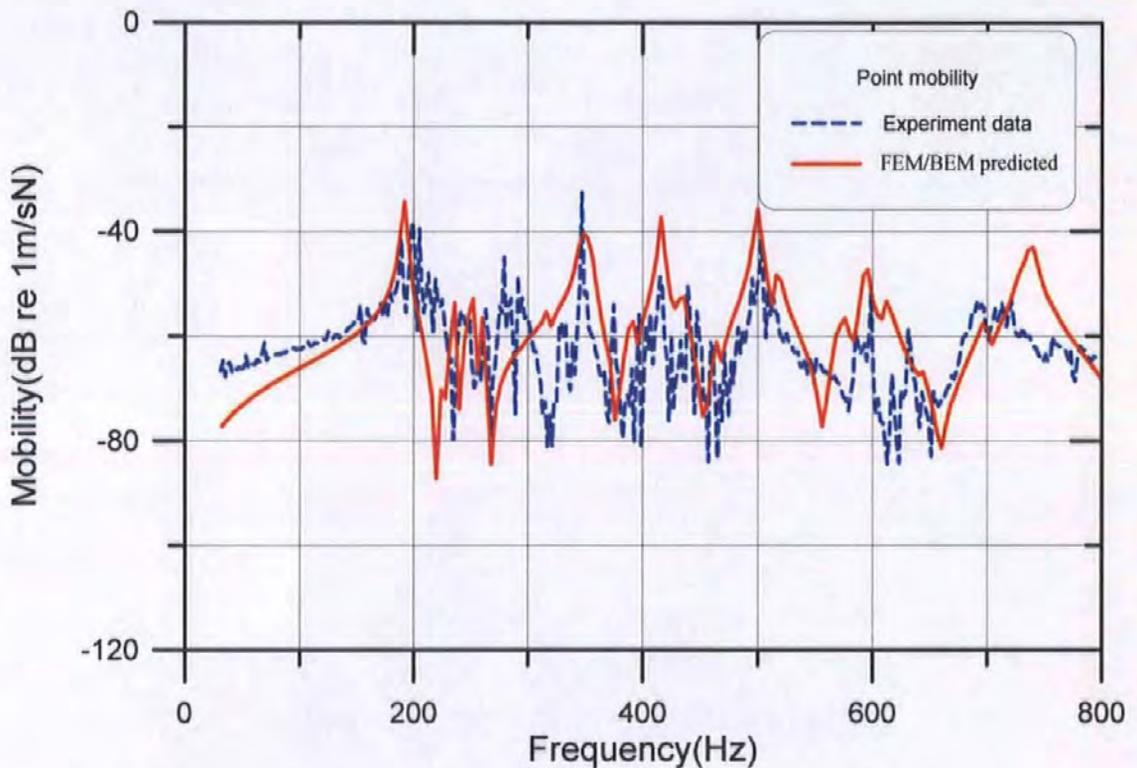


Figure 5.21 Comparison of the driving point mobility

## 5.2 Mobility Study of Scale Engine Room Model (II)

### 5.2.1 Description of the Experiment

In order to validate the accuracy of the mobility analysis and the FEM/BEM noise propagation models, experiments were undertaken on a smaller scale model of an engine room structure having a round bilge. This model with a length 808mm, is shown in Figure 5.22, consists of two end bulkheads, four girders on the bottom, one bilge tank on

each side. On the top of the girders there installed a bedplate and a stepless motor supported by four resilient mounts bolted on the bedplate. The motor was also designed with a deliberate eccentric mass on the shaft. The arrangement of mobility measurement is shown in Figure 5.23 and the configuration of the resilient mount used is the same as shown in Figure 5.4.

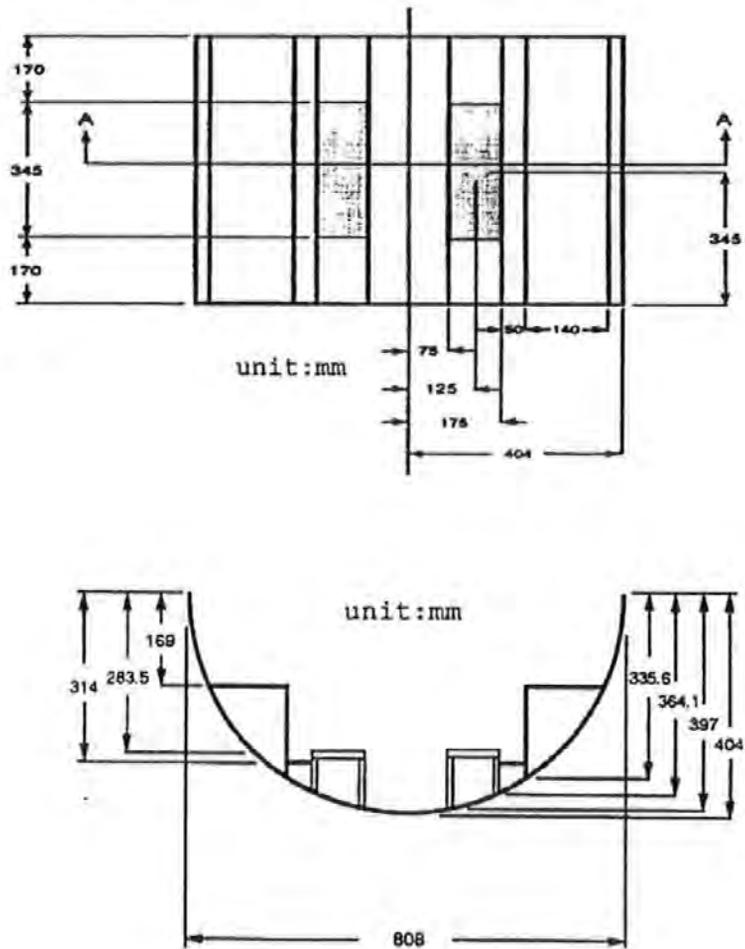


Figure 5.22 Scale engine room mode (II) for mobility measurement.

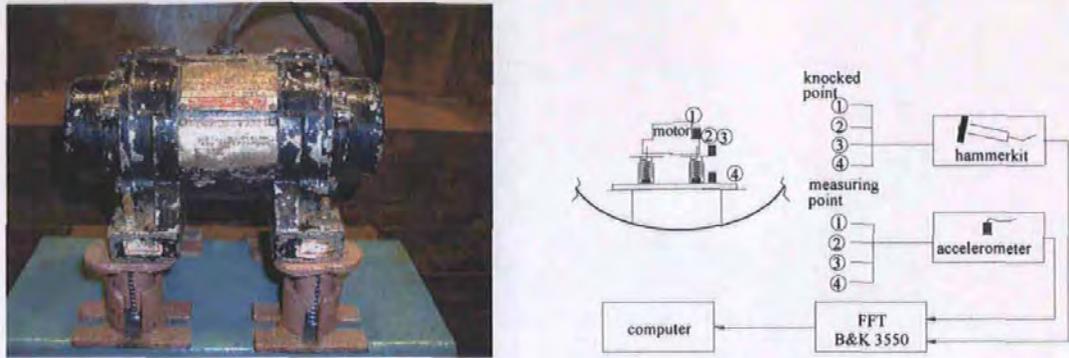


Figure 5.23 Instrument arrangement for mobility measurement

### 5.2.2 Comparisons of Mobility and Vibrational Power

The measured and predicted transfer mobility  $M_{41}$  when the model being put in water and in air with and without rubber pad are compared in Figure 5.24 to Figure 5.27. The resilient mount effect on measured transfer mobility of the same scale model (II) in water and in air are compared in Figures 5.28 and 5.29. When the motor ran at a speed 1800 rpm, the rubber pad effect on the exciting vibration velocity and the transfer power of the same scale model (II) at the input and output ends are shown in Figure 5.30 to Figure 5.37. From the results, it is evident that resilient mounts have the effects to reduce transfer mobility, vibratory velocity and vibrational power by an amount of 5-20dB, 2-5dB and 10-40% respectively. The entrained mass of water apparently reduced the transfer mobility by an amount 5-10dB in the lower frequency range.

Detail discussions regarding the accuracy analysis of the mobility model applying to the scale model (II) and the quantitative analysis of the effect of the fluid-structure interaction on the structure-borne vibration propagation are given in Sections 7.1.1 and 7.1.5 of Chapter 7 respectively.

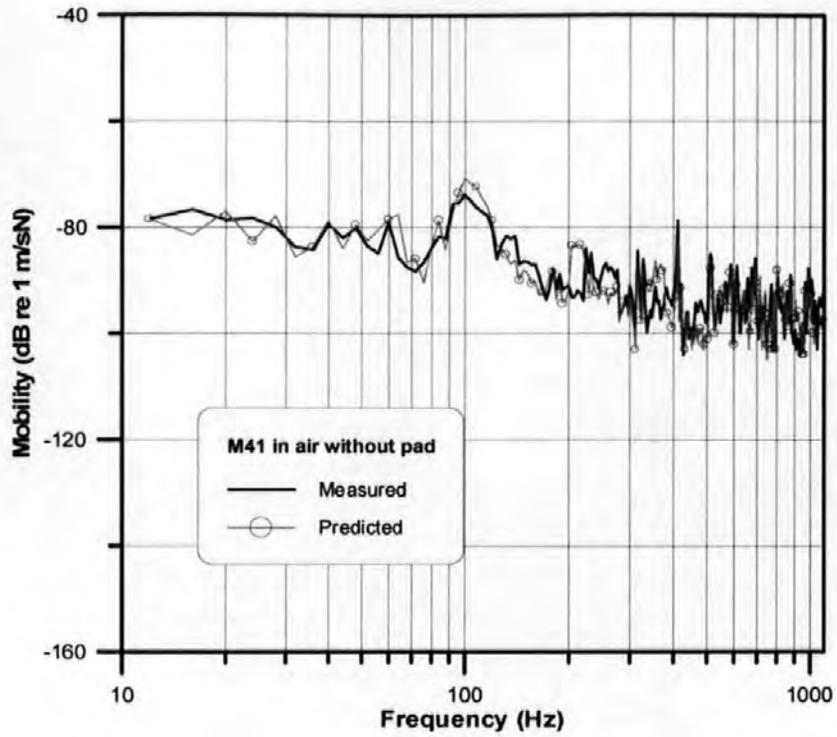


Figure 5.24 Comparison of transfer mobility  $M_{41}$  in air without rubber pad in resilient mount

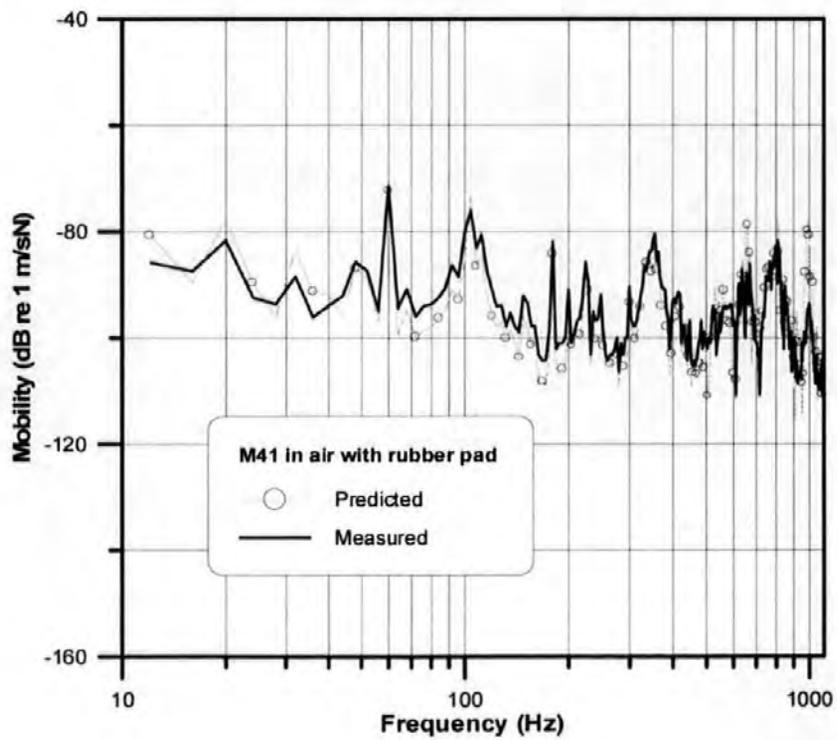


Figure 5.25 Comparison of transfer mobility  $M_{41}$  in air with rubber pad in resilient mount

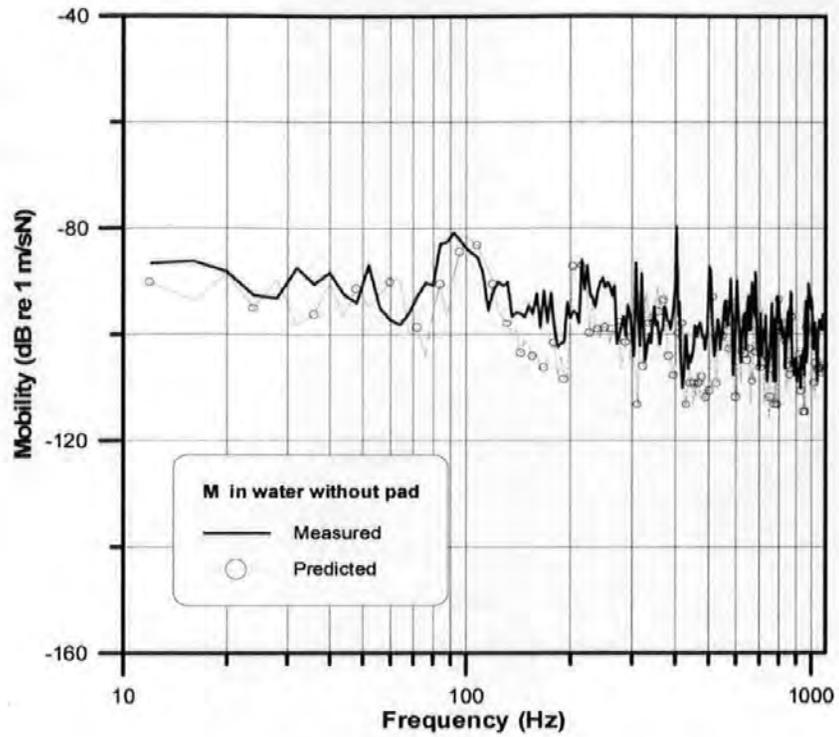


Figure 5.26 Comparison of transfer mobility  $M_{41}$  in water without rubber pad in resilient mount

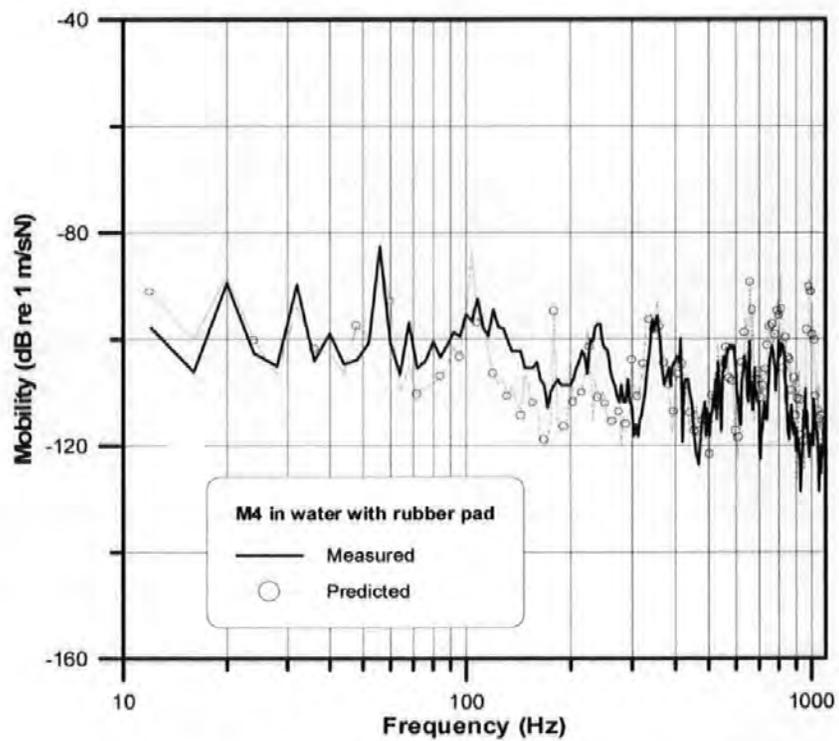


Figure 5.27 Comparison of transfer mobility  $M_{41}$  in water with rubber pad in resilient mount

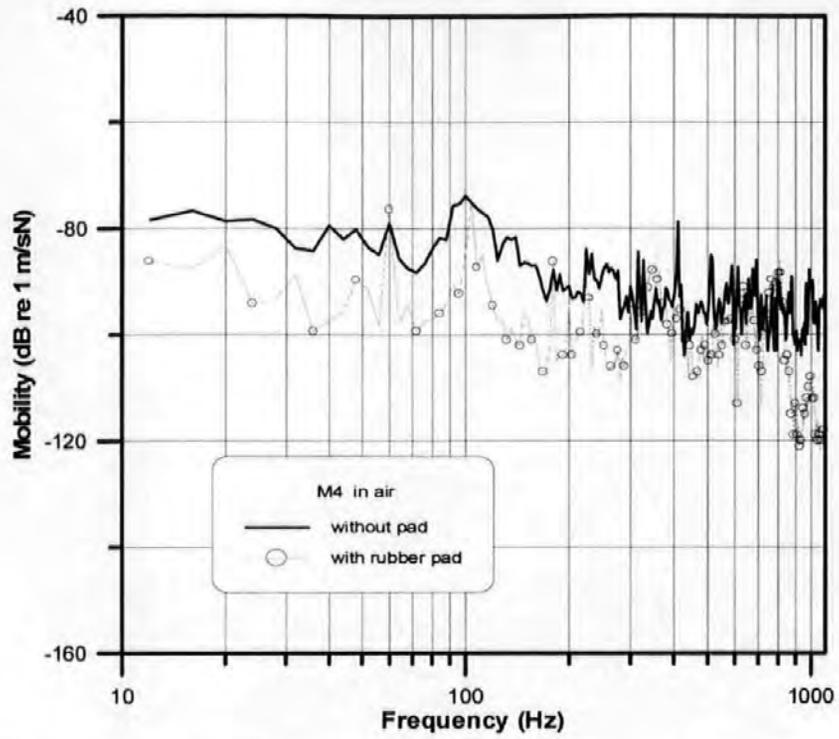


Figure 5.28 Effect of rubber pad in reducing transfer mobility of the scale model (II) hung in air.

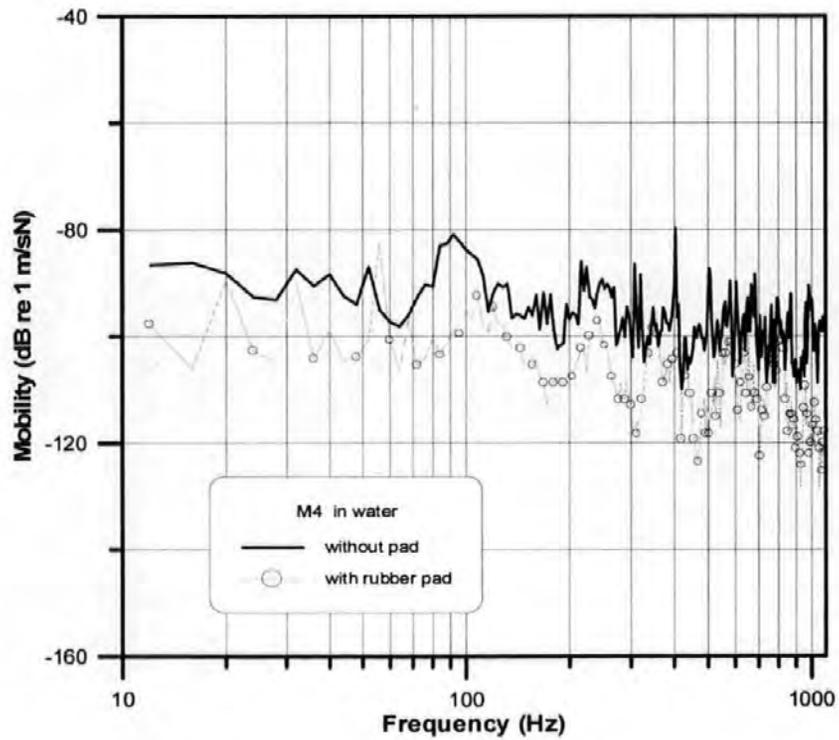


Figure 5.29 Effect of rubber pad in reducing transfer mobility of the scale model (II) floated in water tank

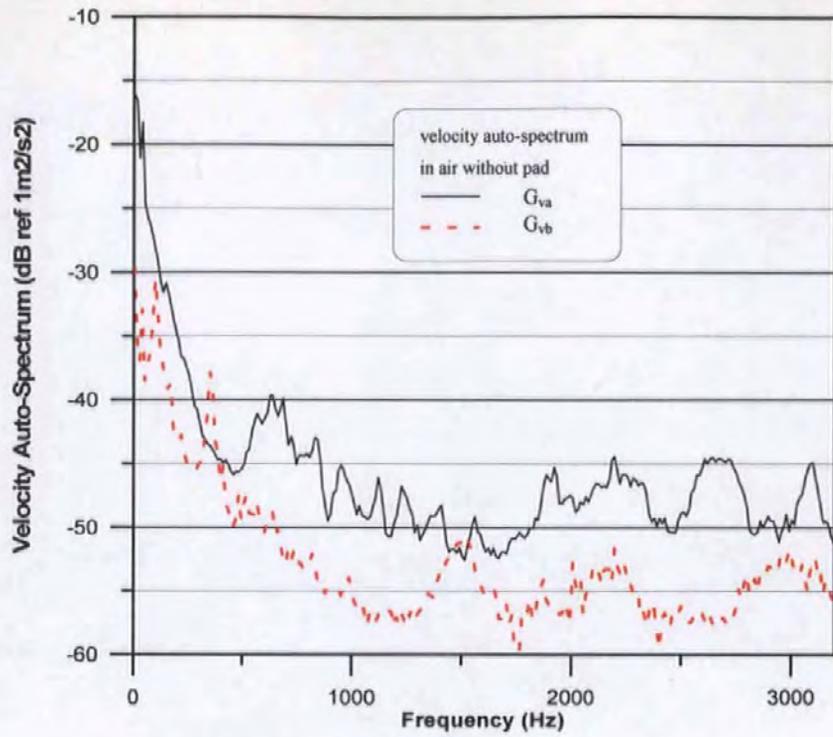


Figure 5.30 Velocity auto-spectra in air at input and out sides of the resilient mount without rubber pad

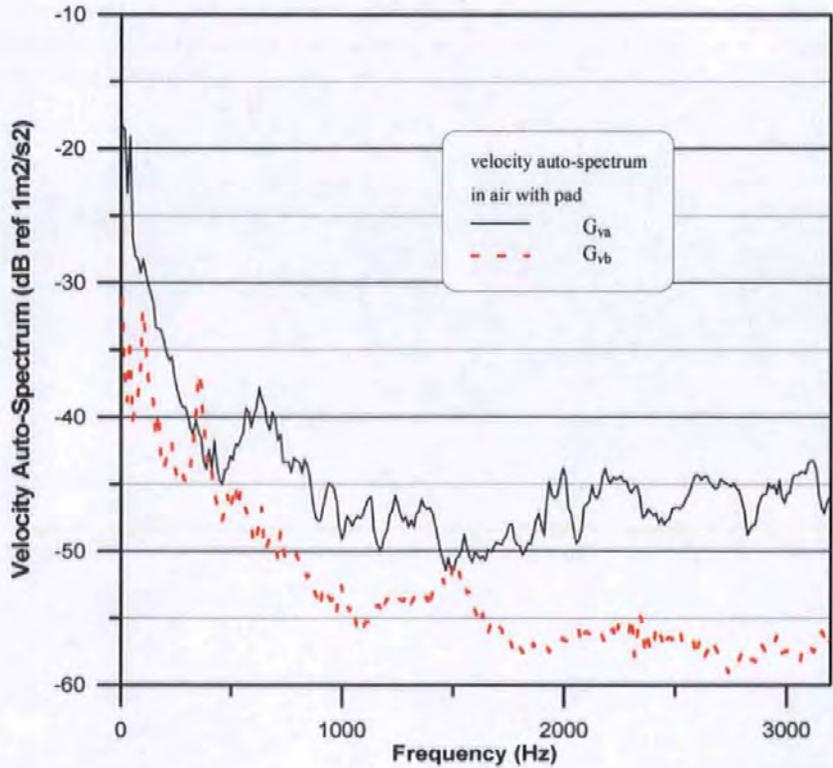


Figure 5.31 Velocity auto-spectra in air at input and out sites of the resilient mount with rubber pad

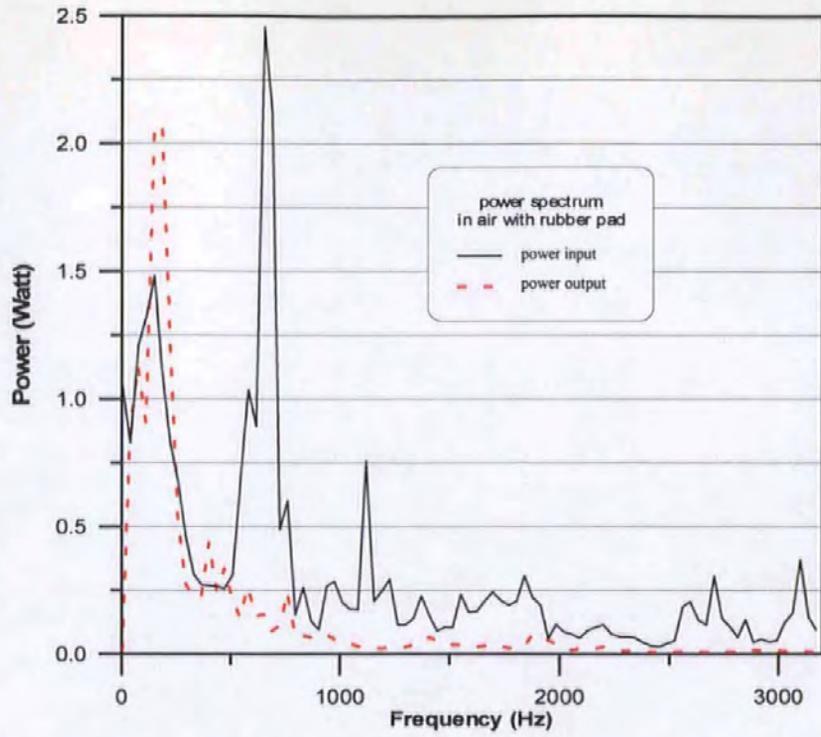


Figure 5.32 Vibrational power spectra in air at input and output sides of the resilient mount with rubber pad.

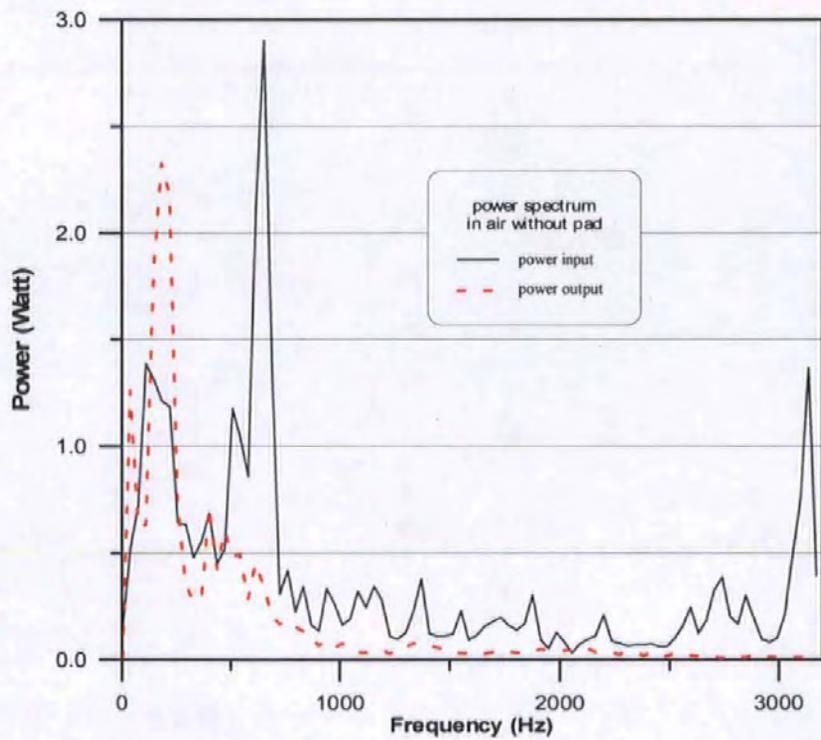


Figure 5.33 Vibrational power spectra in air at input and output sides of the resilient mount without rubber pad.

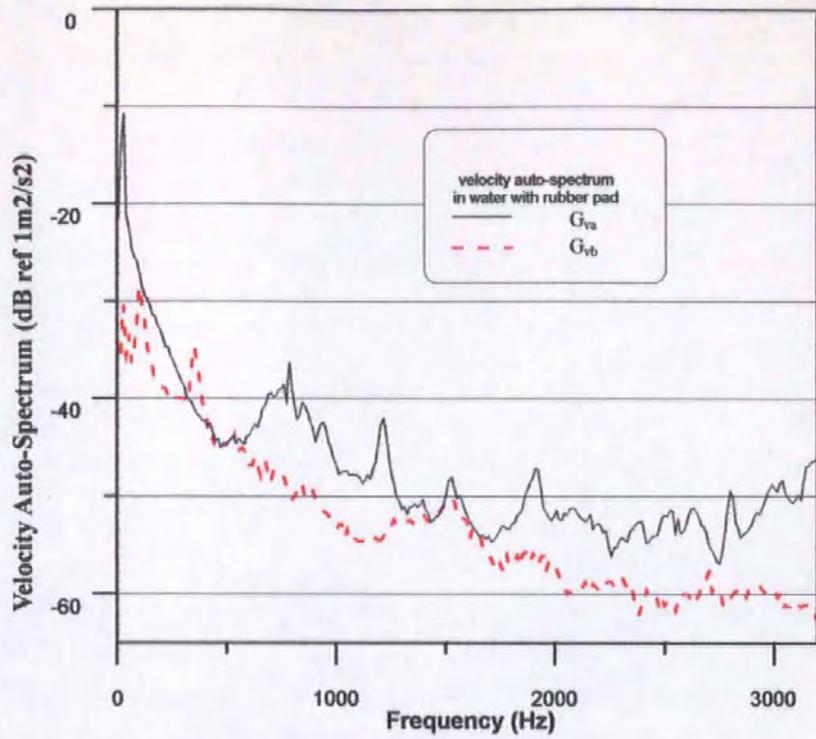


Figure 5.34 Velocity auto-spectra in water at input and output sides of the resilient mount with rubber pad

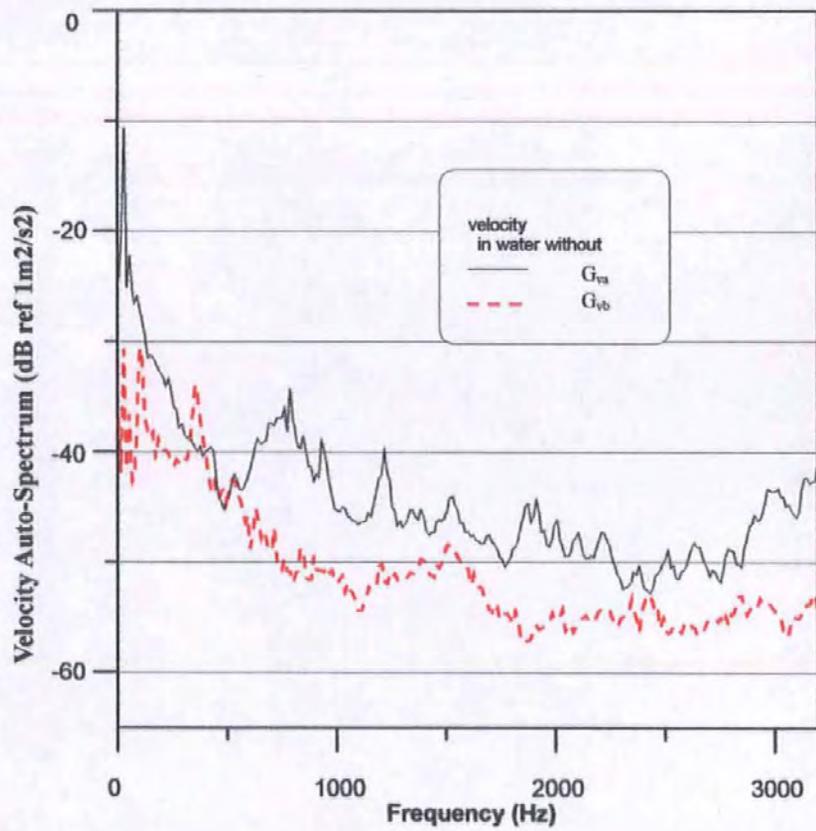


Figure 5.35 Velocity auto-spectra in water at input and output sides of the resilient mount without rubber pad

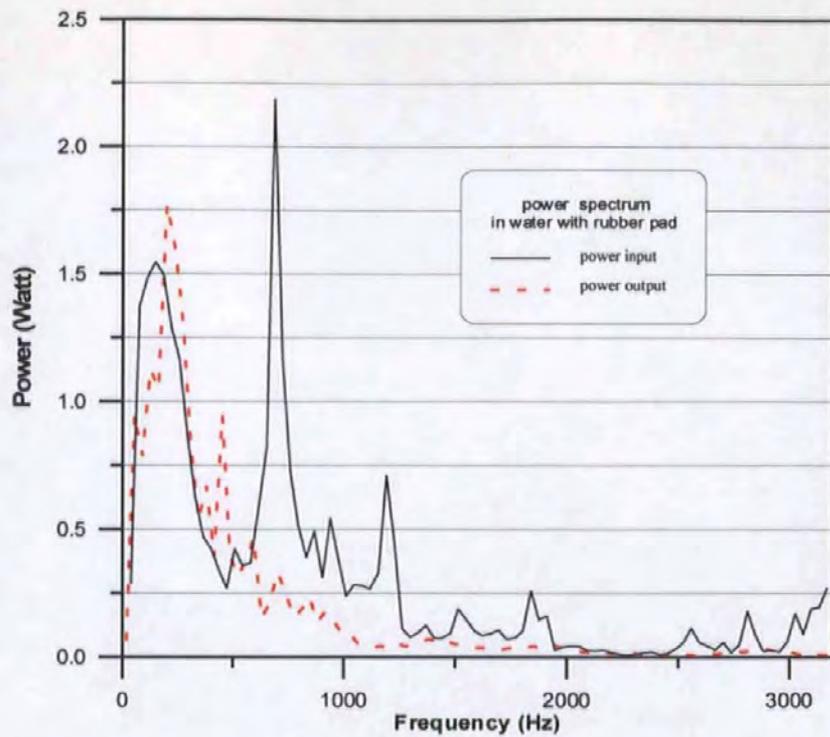


Figure 5.36 Vibrational power spectra in water at input and output sides of the resilient mount with rubber pad.

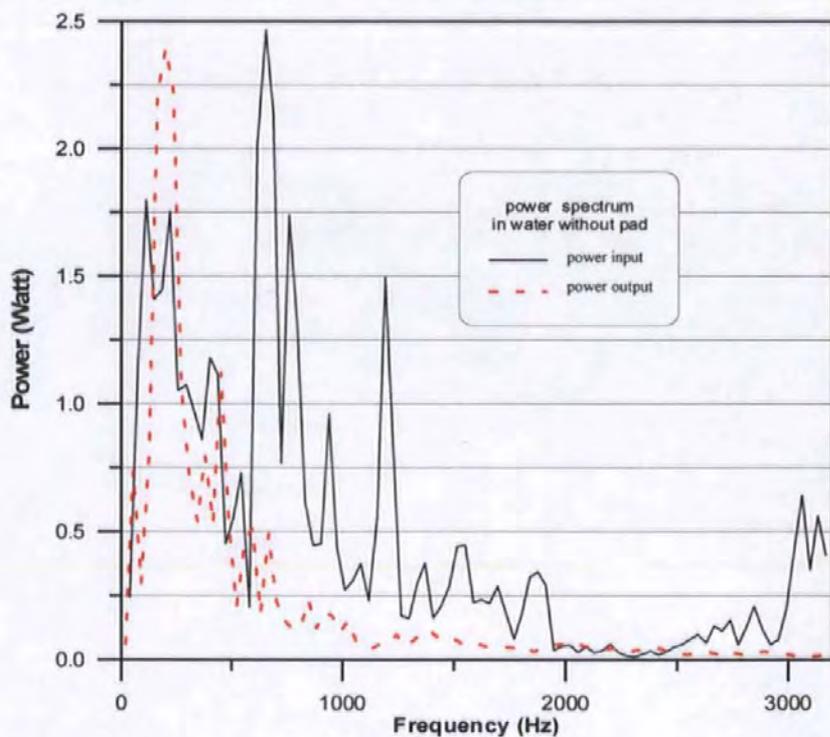


Figure 5.37 Vibrational power spectra in water at input and output sides of the resilient mount without rubber pad.

### 5.3 Patrol Vessel Study

The fast patrol vessel investigated in this study was of 100 gross tonnage, with a maximum continuous main diesel engine output of 3000 HP\*2 sets, with rpm=2100, gear ratio=2.7, and shaft rpm=780. It could achieve a maximum speed of approximately 31.55 knots.

The measured noise levels in cabins were:

engine room	115dB(A)
captain's room	90dB(A)
pilothouse	85dB(A)

The main structure-borne sound sources are from main engines, reduction gears, generators and exhaust pipes. A major part of the structure-borne noise onboard the vessel was identified to be the main engine vibration transmitted via the mounts (Wang, 1996). The principal particulars of the vessel are listed and the general arrangement of the vessel is shown as Figure 5.38.

length overall:	30.50 m;
breadth moulded:	6.80 m;
depth moulded:	3.55 m;
draft moulded:	1.70 m;
maximum speed:	31.55 knots;
cruising speed:	28 knots.

The specifications of the machinery in engine room were:

#### Main diesel engine

model:	MTU 16 V 396;
revolution speed:	2100 rpm
continuous rating:	3000 HP * 2 sets;
weight:	7.5 tons × 2 sets.

## Generator

revolution speed:	1900 rpm
output:	23 kW*2 sets.

## Reduction gear

reduction gear ratio:	2.7;
shaft speed:	780rpm;
type:	V drive.

In order to attenuate the noise level in the cabins of the vessel, one of the measures adopted to reduce the structure-borne noise transmission from the main diesel engines was to install squeeze plates to the web of the engine girders. The squeeze-film damping technique and the prediction of its loss factor were developed by Chow and Pinnington (1982 and 1985). Each squeeze plate used is a rectangular steel plate of dimensions 300 mm long, 200 mm wide and 1 mm thick and tack welded to the web, at its corners, beneath the engine mounts. Figure 5.39 shows the locations of these squeeze plates. When the squeeze plate and the web vibrate out-of-phase with each other or in different wave length, then it forms a air pumping effect, with the alternative action to squeeze the air in between the plates out and attract the outer air in, so as to dissipate the vibrational energy.

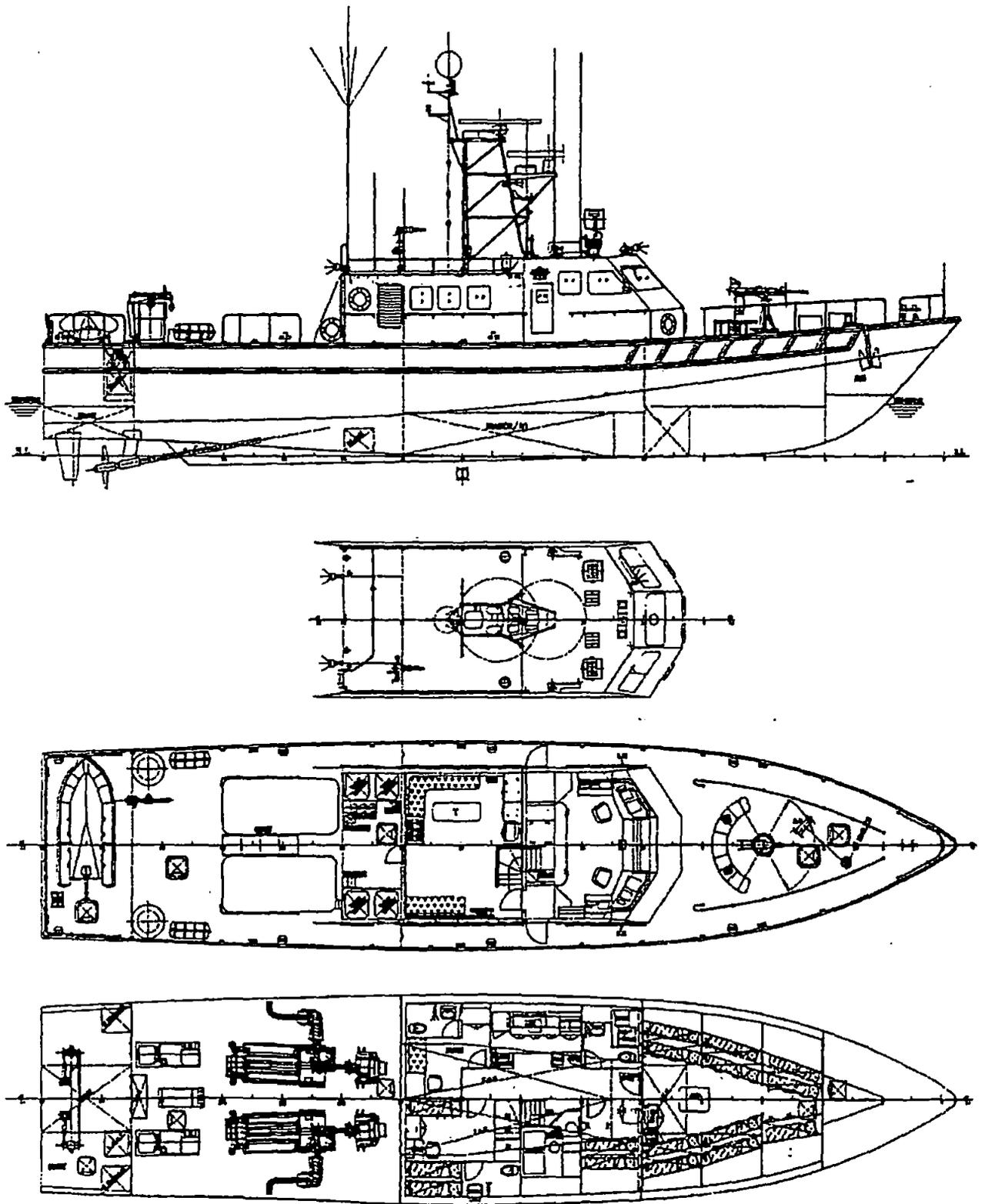


Figure 5.38 General arrangement of a fast vessel



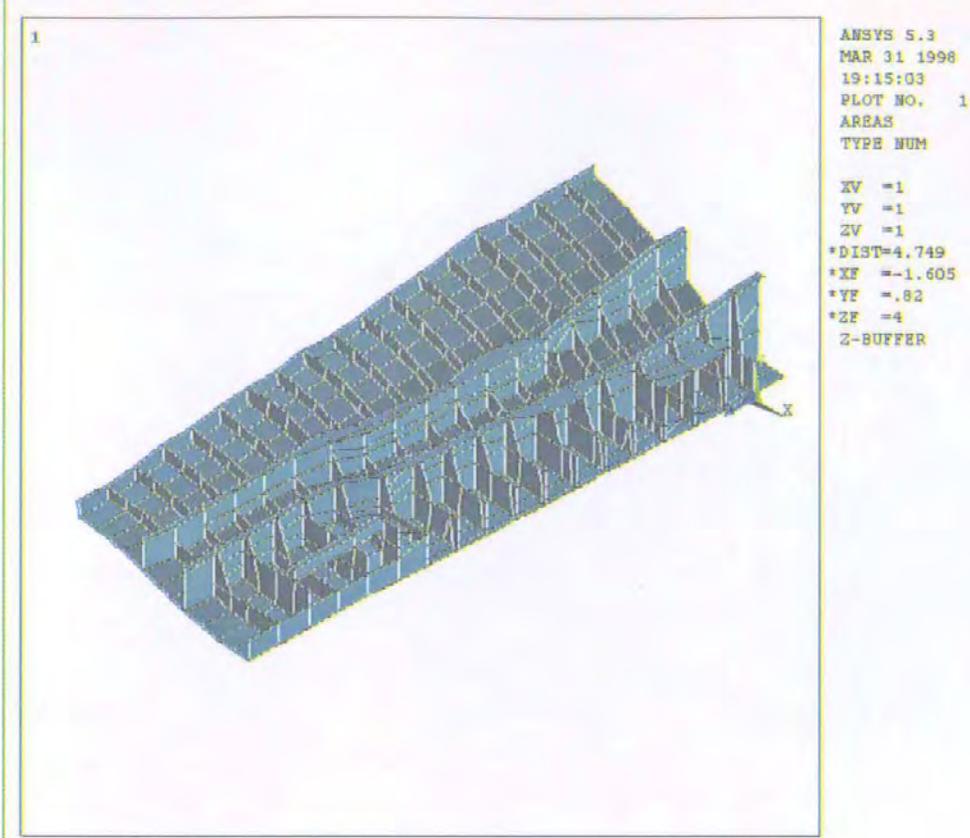


Figure 5.40 Bottom structure of the engine room of the patrol vessel

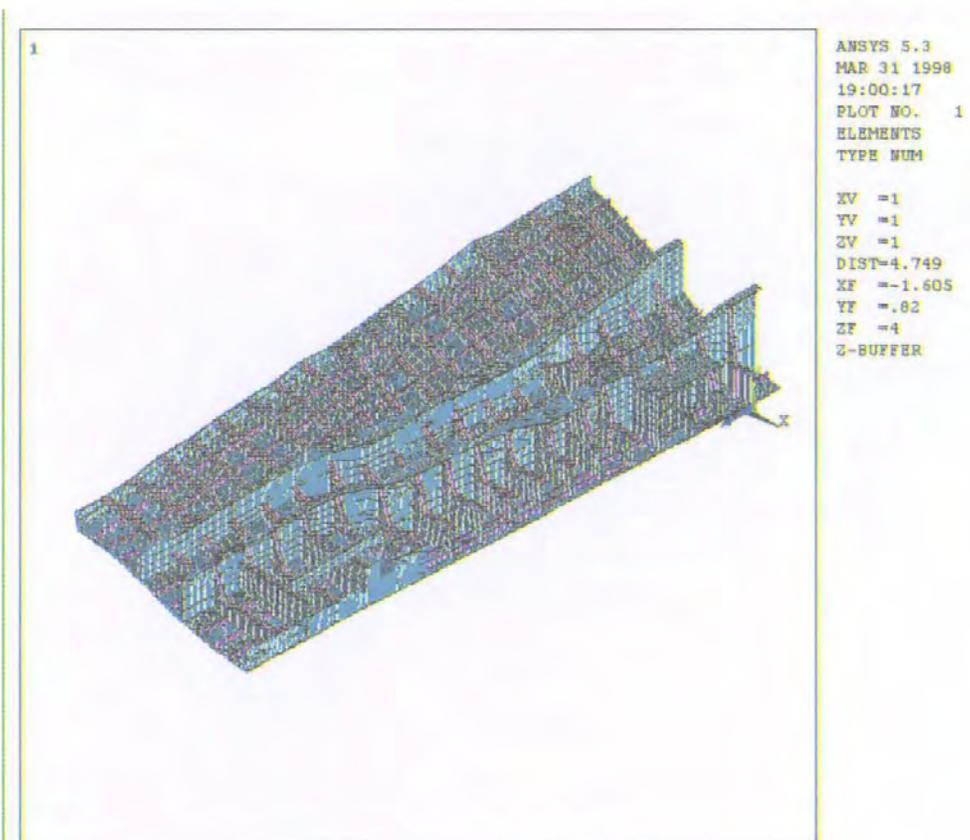


Figure 5.41 FEA model of the engine room bottom of the patrol vessel

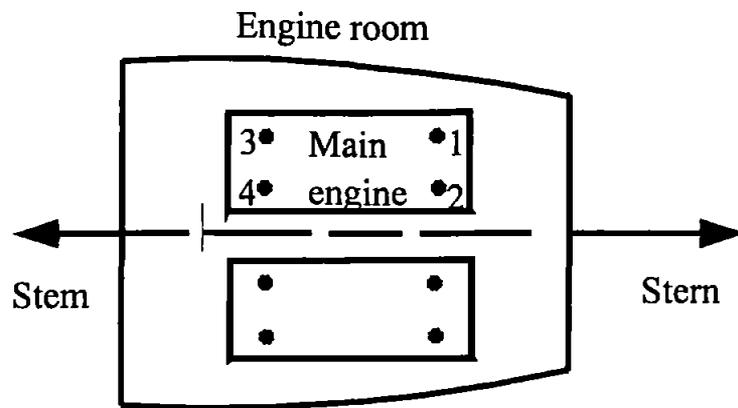


Figure 5.42 Number of the locations for mobility analysis

By this FEA model, the mobility's at the mount positions on the girder, which were numbered as in Figure 5.42, can be analysed for the conditions before and after the installation of the squeeze film damping plates. The boundary conditions and the locations of master degree of freedom of the FEA model are shown in Figures 5.43 and 5.44 respectively. The damping ratio at these portion of attached plates raised from 0.001 of the original structure to 0.01, corresponding to the loss factor 0.02 in the frequency range below 500 Hz as shown in Figure 5.45. This data was obtained by Wang and Yang (1999).

The comparsons of the analysed driving point mobilities at the mount positions are shown in Figure 5.46 to Figure 5.49.

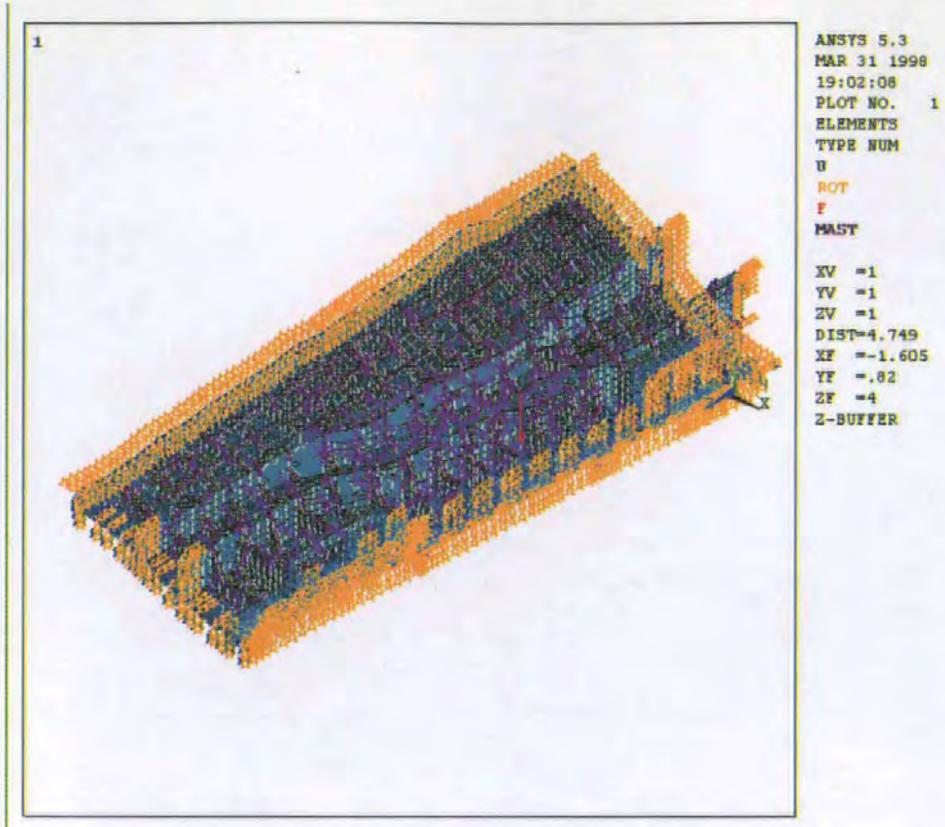


Figure 5.43 Boundary condition of the FEA model

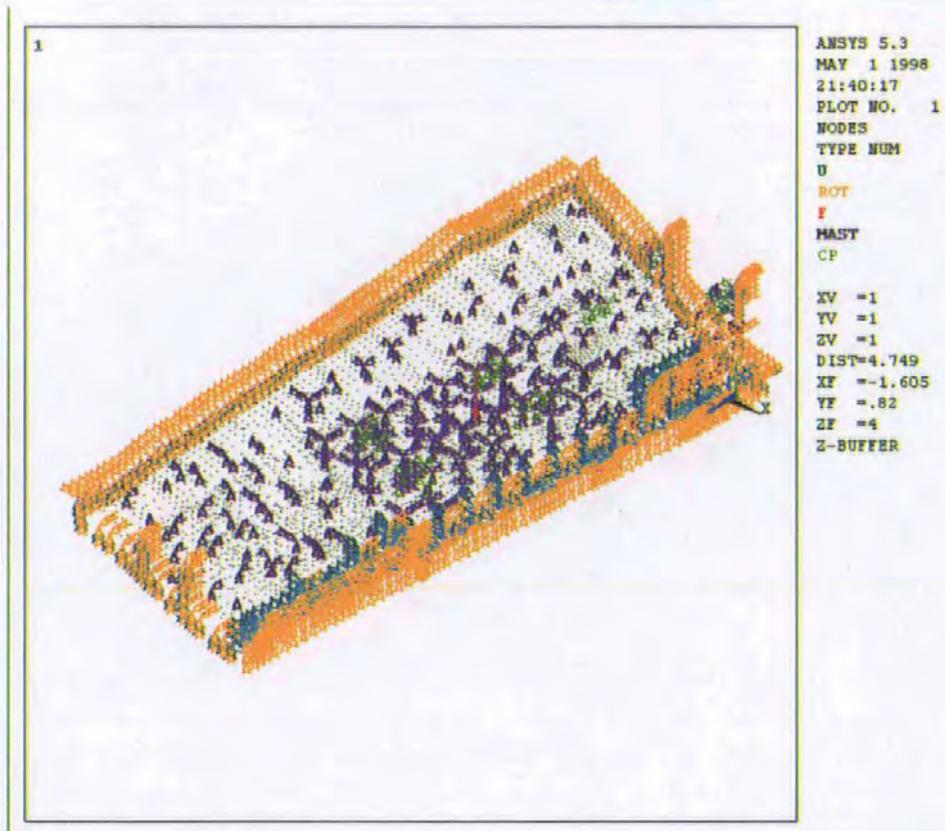


Figure 5.44 Locations of the master degree of freedom

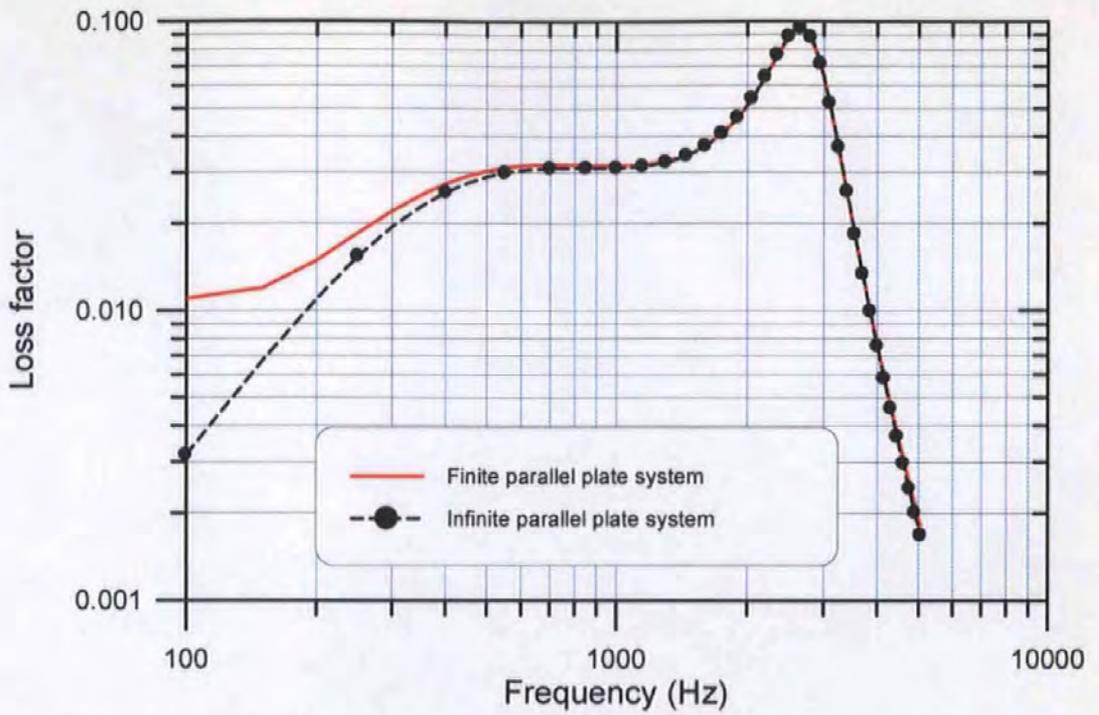


Figure 5.45 Comparison of loss factor by the SEA model between two finite parallel coupled plates and the compressive flow model between two infinite parallel coupled plates (Wang and Yang, 1998)

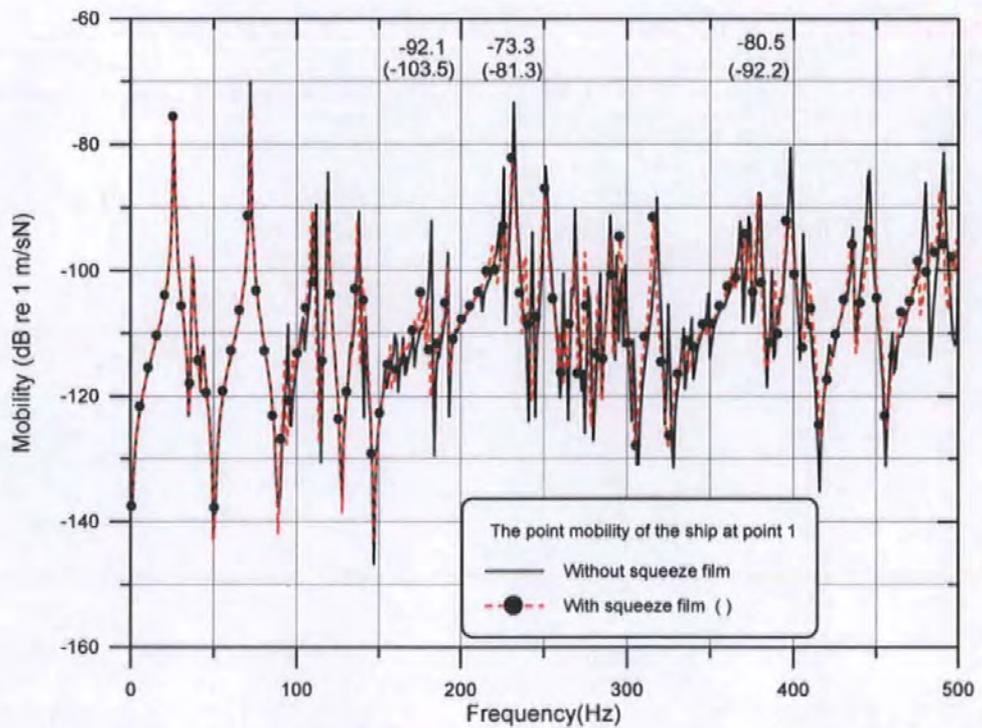


Figure 5.46 Comparison of mobility at mount 1 with and without squeeze film

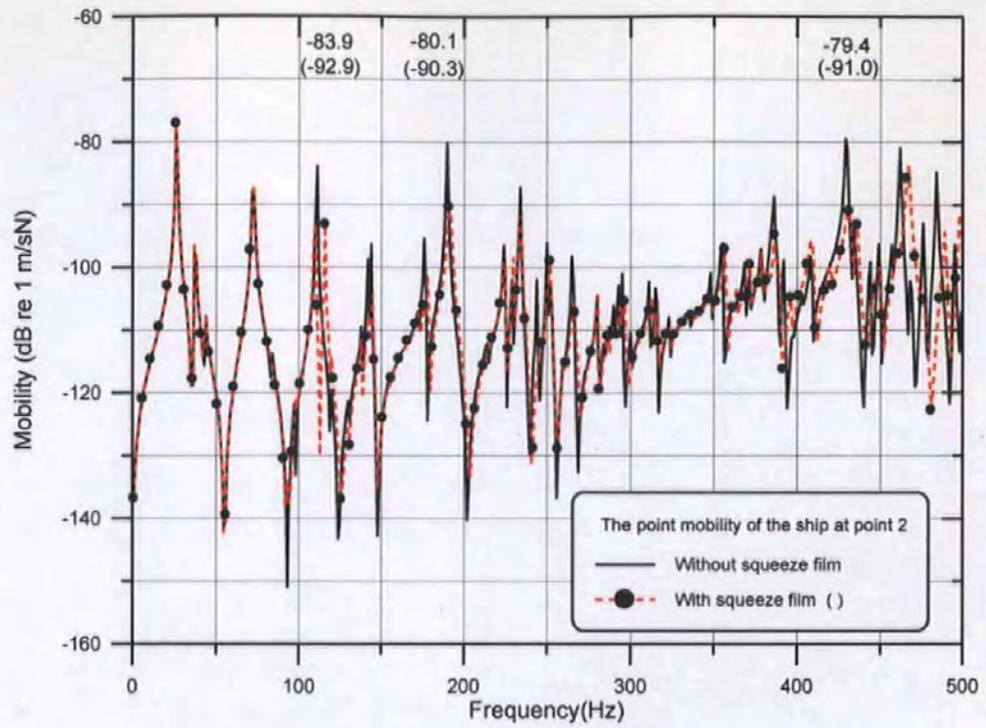


Figure 5.47 Comparison of mobility at mount 2 with and without squeeze film

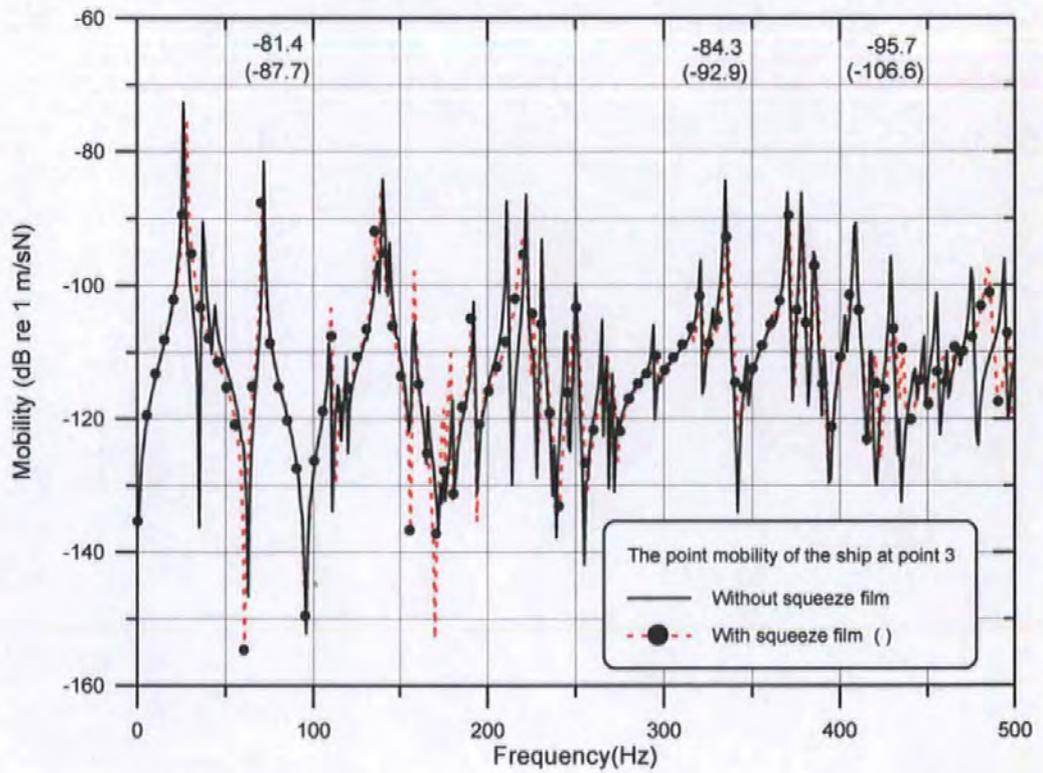


Figure 5.48 Comparison of mobility at mount 3 with and without squeeze film

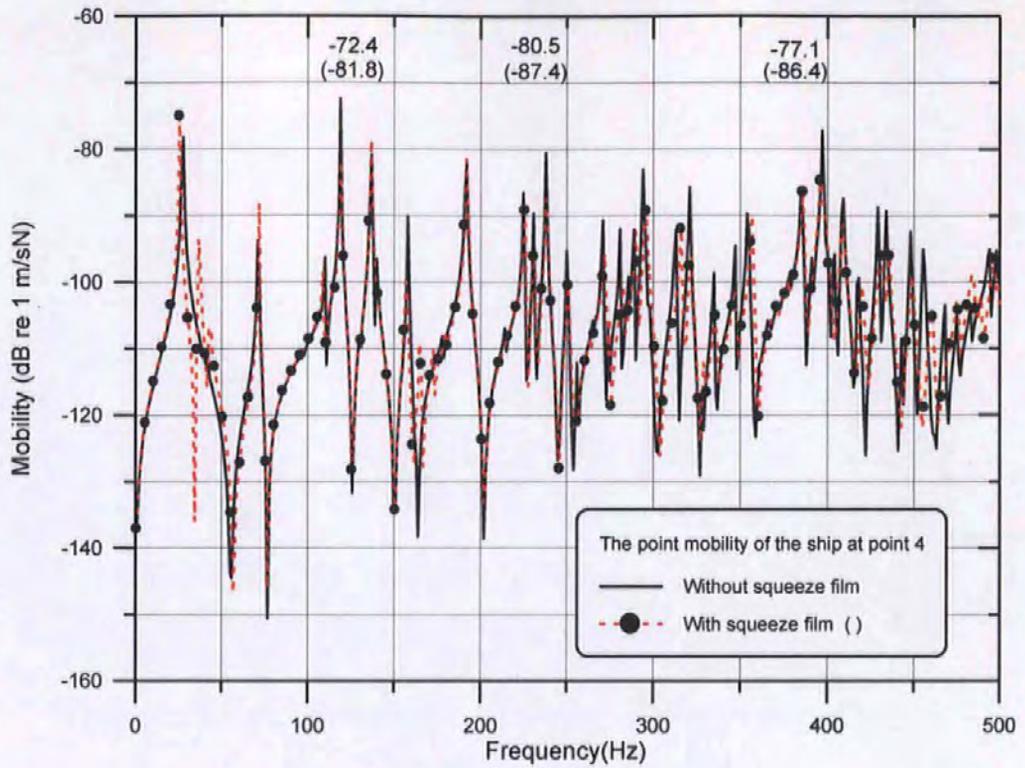


Figure 5.49 Comparison of mobility at mount 4 with and without squeeze film

It is apparent that the peak mobilities were reduced by 5~10 dB after the installation of the squeeze film damping plates. Also, the comparison of airborne noise level in the cabins were measured and the improvements can be seen in Table 5.1. There are also a 3~11 dB(A) in airborne noise reduction. Detail discussions of the quantitative effect of the squeeze-film damping on the reduction of structure-borne vibration transmission are given in Section 7.1.6 of Chapter 7.

Table 5.1 Comparison of airborne noise level in cabins of the patrol vessel

Compartment	Airborne noise level, dB(A)	
	Original	Improved
Engine room	115	112
Captain room	90	86
Lavatory of captain's room	—	91
Deputy captain's room	—	89
Lavatory of deputy captain's room	—	93
Cabin room	—	82
Dining room	—	84.5
Galley	—	85
Meeting room	—	80
Pilothouse	85	74

## CHAPTER 6

# APPLICATION OF THE COUPLED FEM/BEM MODEL TO PREDICT MACHINE INDUCED UNDERWATER ACOUSTIC RADIATION

The coupled FEM/BEM model can be applied for solving the fluid-structure interaction problem in structure-borne noise propagation and underwater acoustic radiation of ships. Accordingly, the estimation of the structure-borne noise characteristics of a ship is important in the design of a quieter ship structure. The hull vibration response and the pressure distribution on the wetted surface of a ship subjected to the excitation from the source like a machine can be analysed by utilizing the developed method. For this part of application and validation of the numerical model can be illustrated by taking the scale engine room model (II) into account.

### 6.1 Numerical Analysis Model of the Underwater Sound Radiation for Scale Engine Room Model (II)

The FEM and BEM meshes of the scale engine room model (II), as described in section 5.2, were established in Figures 6.1 and 6.2. Input data fed to the FEA are :

number of shell elements	: 711
number of nodes	: 654
plate thickness	: 0.0024 m
Young's modulus	: $2.1 \times 10^{11}$ Pa
Poisson's modulus	: 0.3
material density	: 7860 kg/m <sup>3</sup>

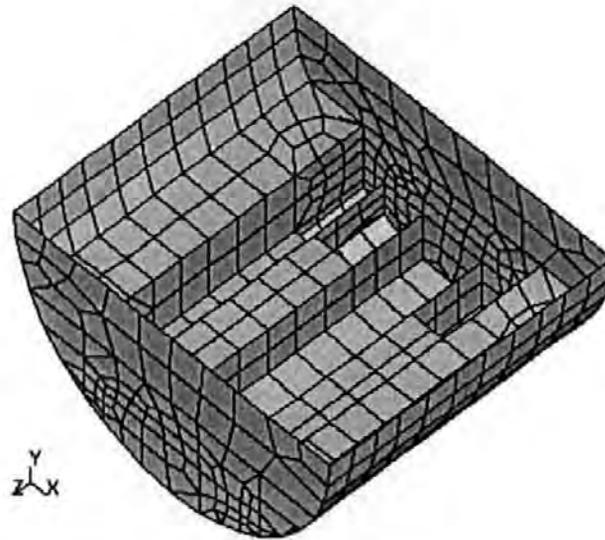


Figure 6.1 FEM mesh generation of the scale engine room model (II)

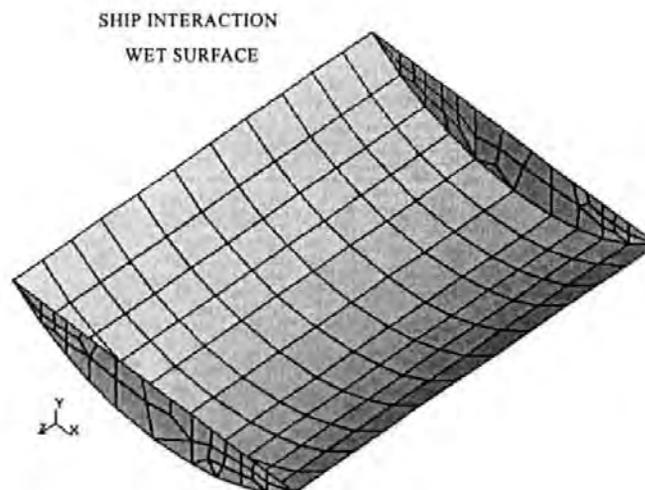


Figure 6.2 BEM grid generation of the wetted surface of the scale engine room model (II)

Input data fed to BEM are :

number of boundary elements	: 180
number of nodes	: 201
sound speed	: 1500 m/s
fluid density	: 998 kg/m <sup>3</sup>

When the scale model was floating at the center part of the free surface of the water tank,

with size 4.2 m×3.6 m×2.4 m (deep), and the motor ran at a speed 1800 rpm, the output power and vibrational velocity have been measured at the feet of resilient mounts as described in Section 5.2. Once these measured excitations input to the numerical model, the vibration velocity and pressure distribution on the wetted surface of the scale model (II) can be analysed by equations (4.62) and (4.63). The underwater radiated sound pressure can also be obtained by equation (4.67).

## 6.2 Experimental Arrangement for Underwater Acoustic Radiation Measurement

Put the scale engine room model (II) at the center part of the water surface and let the model floating freely but keeping it in the doubly symmetrical position relative to the tank with four bungee rope hung in assistance, as shown in Figure 6.3. Arrange four hydrophones also in the doubly symmetrical positions as shown in Figure 6.4. During the motor in the engine room was running at a speed 1800 rpm (30 Hz), the underwater radiated sound pressure from the surface of the bottom plate could be measured by these hydrophones.



Figure 6.3 Experimental arrangement for underwater acoustic radiation measurement

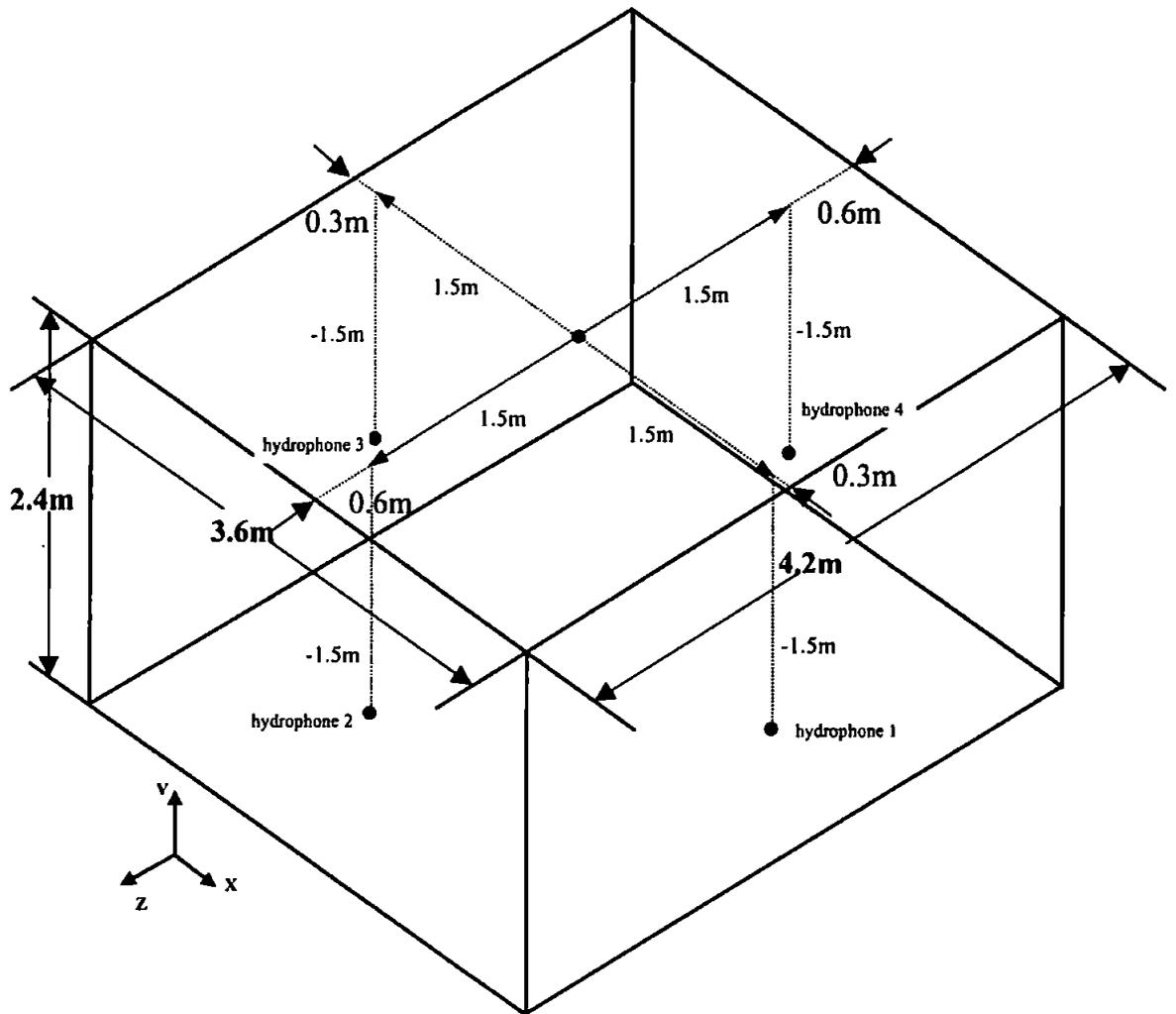


Figure 6.4 Locations of the hydrophones

### 6.3 Predicted and Measured Underwater Sound Pattern

When the scale model (II) was floating at the surface center of a rectangular water tank of length 4.2m, breadth 3.6m and water depth 2.4m, the motor ran at 1800 rpm. The distributions of sound pressure, normal velocity, and sound intensity on the wetted surface are predicted as shown in Figure 6.5 to Figure 6.7. The predicted underwater sound pressure on two symmetrical planes in the fluid domain, apart from the wall of a distance 0.3m for the location of hydrophones 1 and 3 and 0.6m for hydrophones 2 and 4, and a horizontal plane at a depth 1.5m are compared with the measured sound pressure level (SPL) by the four hydrophones in Figure 6.8 to Figure 6.10.

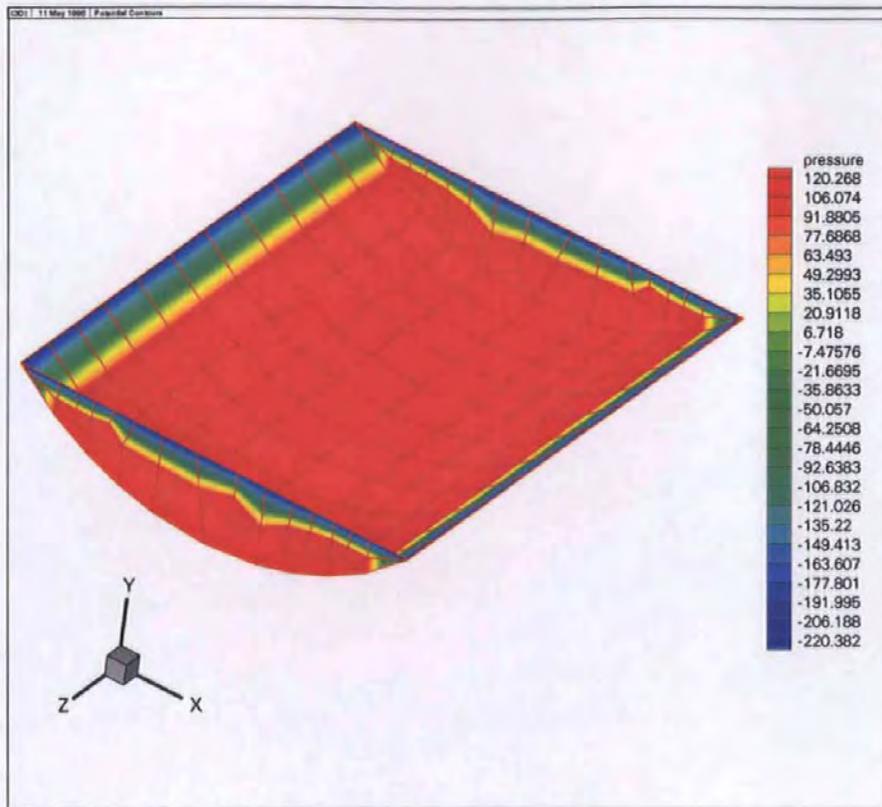


Figure 6.5 Predicted sound pressure distribution on the wetted surface

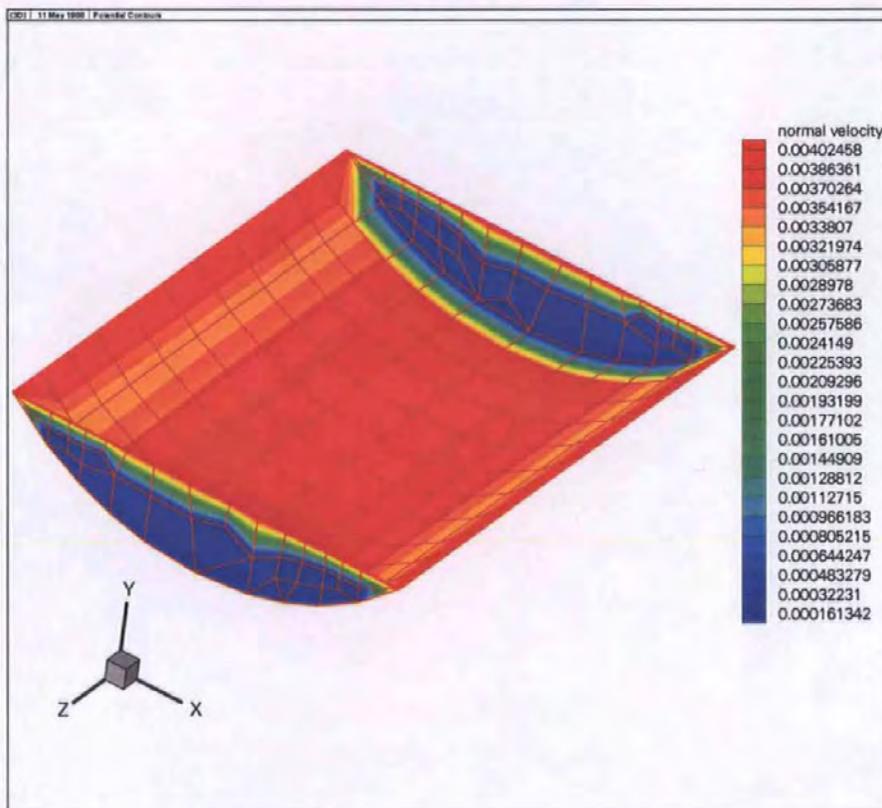


Figure 6.6 Predicted normal velocity distribution on the wetted surface

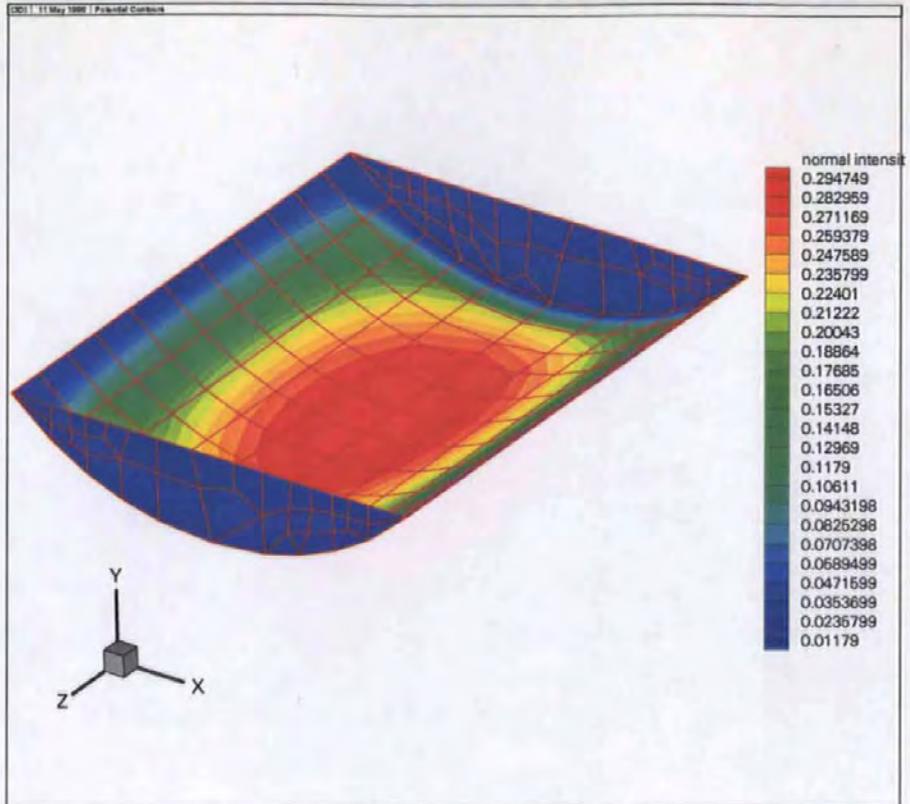


Figure 6.7 Predicted sound intensity distribution on the wetted surface

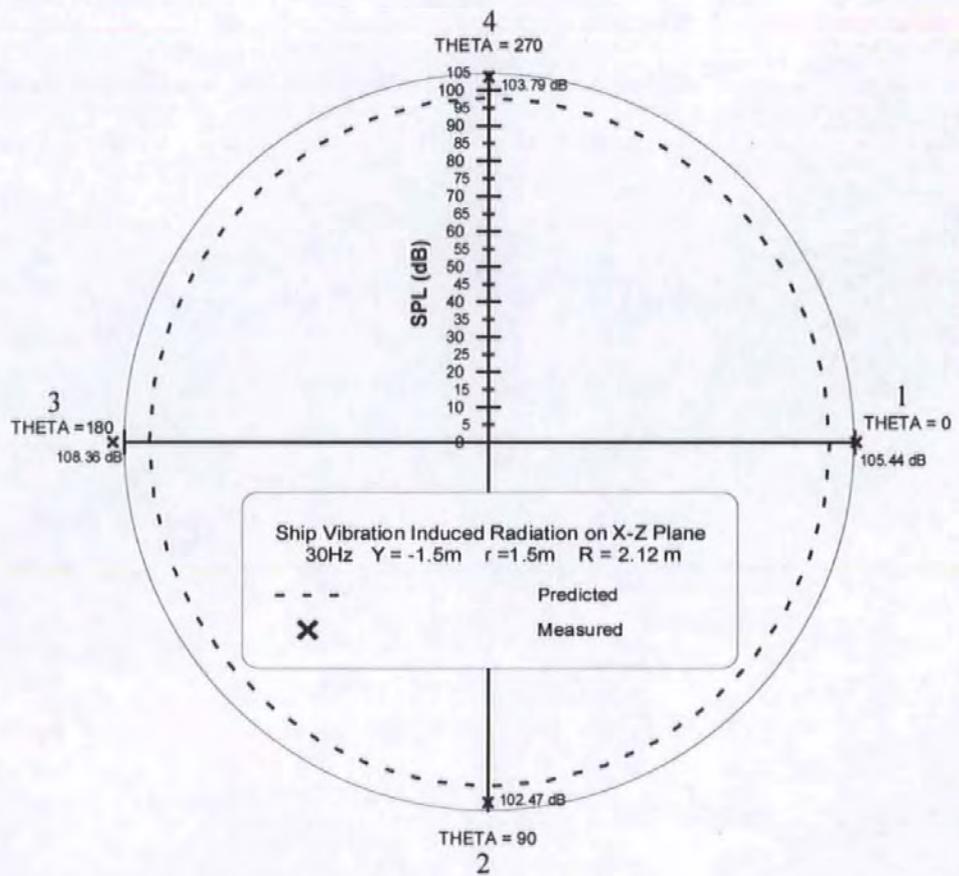


Figure 6.8 Machine vibration induced underwater sound radiation at 30Hz on X-Z plane

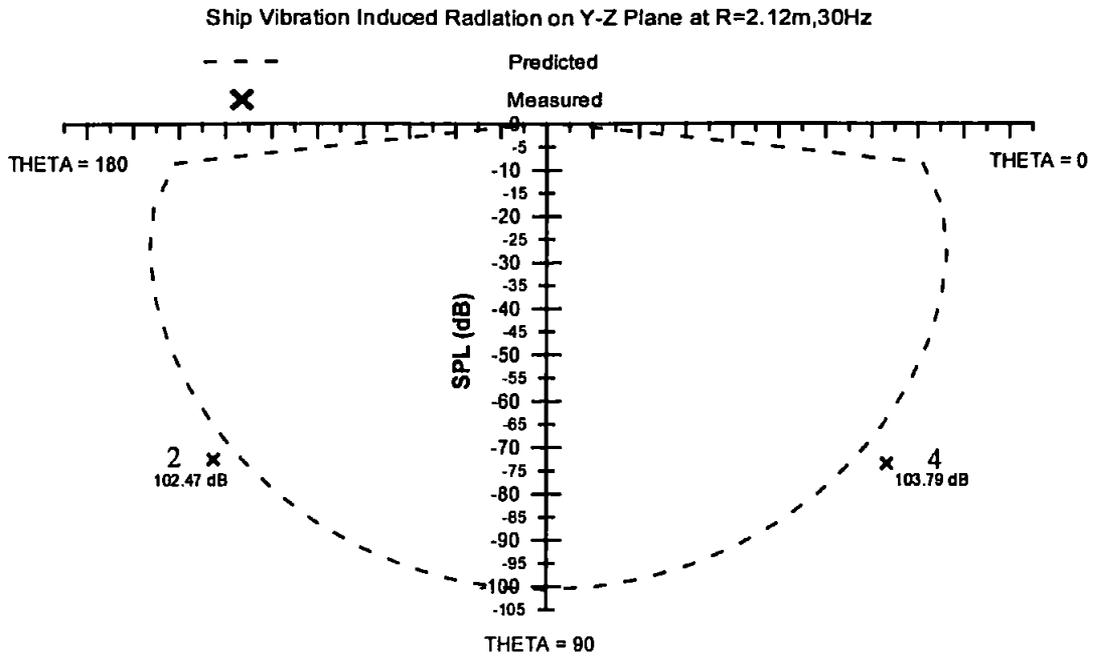


Figure 6.9 Machine vibration induced underwater sound radiation at 30 Hz on Y-Z plane

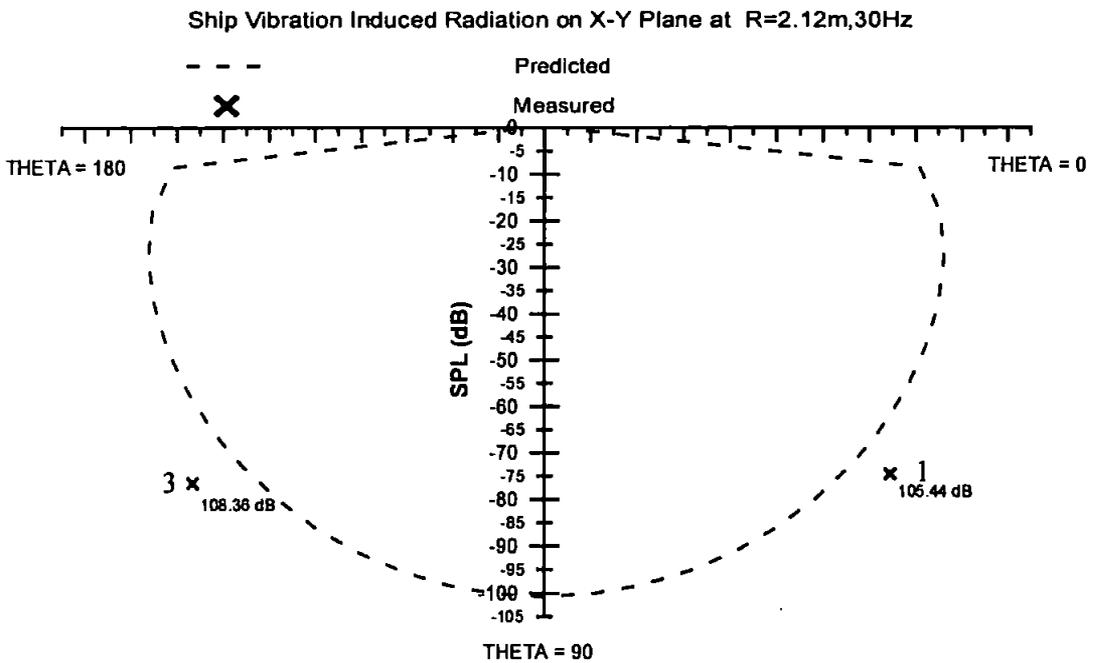


Figure 6.10 Machine vibration induced underwater sound radiation at 30Hz at X-Y plane

From the comparisons of the radiated SPL shown in Figure 6.8 to Figure 6.10, it can be found some deviations. By the error comparison shown in Table 6.1, points 1 and 3 have larger errors than points 2 and 4. This is owing to that the locations of points 1 and 3 are closer to the tank wall than points 2 and 4. Thus the influence of the sound reflection effect at points 1 and 3 is bigger than that at points 2 and 4. While the predicted model by using the FEM/BEM software only consider the free-field sound radiation. Indeed, the validation of the FEM/BEM model needs an underwater anechoic chamber to constitute a free field environment for underwater sound radiation measurement.

Table 6.1 Error analysis between the predicted and measured SPL

Measuring point	1	2	3	4
Predicted SPL (dB)	98.35	98.35	98.35	98.35
Measured SPL (dB)	105.44	102.47	108.36	103.79
Error (dB)	+7.09	+4.12	+10.01	+5.44

# CHAPTER 7

## CONCLUDING REMARKS

### 7.1 Discussion and Conclusions

In this study the machine (e.g. a motor or a diesel engine ) induced structure-borne sound propagation in the engine room model structures or the underwater acoustic sound radiation was analysed and measured. Basically the structure-borne sound power is predominantly transmitted through a sound carrying structure from a source via a number of contact points. In turn, the noise and vibrations are propagated in the structure possibly causing sensitive equipment to vibrate or to cause undesired radiated noise. In principle, this may be avoided by measures at sources, in transmission, during propagation or at radiation.

To this point, the resilient mounts were characterised as an effective countermeasure to reduce noise and vibrational power transmitted from the source to the sound receiver as discussed in Chapter 5. In addition, the effectiveness of the squeeze-film damping plate has also been evaluated in this study, shown in Figure 5.46 to Figure 5.49, as a measure for the attenuation of structure-borne noise propagation at the positions close to the excitation source of the receiving structure.

The quantitative evaluation of these countermeasures for attenuating the structure-borne noise and vibrations and the error analyses of the prediction models established in this study are discussed as follows.

### **7.1.1 Accuracy of the Mobility Model for Evaluating Vibration Transmission via Mounts**

The mobility theory discussed in Chapter 3 for the prediction of power flow transmission via a mount has been validated by a number of experimental measurement results on the scale engine room models (I) and (II) as shown in Chapter 5. The error analyses between the predicted and measured transfer mobilities, shown in Figure 5.7 to Figure 5.10, of the scale engine room model (I) are listed in Table 7.1. While that for the scale engine room model (II), shown in Figure 5.24 to Figure 5.27, are also analysed as appeared in Table 7.2. From Table 7.1, the mobility prediction model only has the average errors 2.8% and 4.0% different from that measured on the scale mode (I) in dry condition, with and without the rubber pad inserted in the resilient mounts, respectively. Whilst the mobility predictions of the scale model (I) have the corresponding errors 4.8% and 3.6% in water respectively. The greater errors are owing to the wave reflection effect of the tank wall in the measurements. This reflection effect does not have taken into consideration in the prediction model.

From Table 7.2, it is evident that the average mobility prediction errors in the case of the smaller scale engine room model (II) are obviously reduced, to 1.4% and 3.6% in air, and – 3.5% and 4.6% in water respectively. Since the smaller scale model subjected to less influence from the tank wall reflections.

Thus, it can be concluded that a credible mobility model for predicting the transfer mobility of engine vibrations via resilient mounts has been established with a considerable precision at least over 95% both in dry and wetted hull conditions.

Table 7.1 Mobility error analysis between prediction and measurement on the scale engine room model (I)

Condition		Frequency (Hz)																		Average error $\frac{1}{n} \sum_{i=1}^n  e_i , (%)$
		20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	
in air with rubber pad	predicted (dB)	-61	-78	-79	-72	-74	-73	-75	-64	-62	-83	-96	-105	-99	-100	-90	-92	-104	-92	2.8
	measured (dB)	-61	-79	-79	-74	-68	-75	-74	-62	-60	-82	-94	-99	-101	-102	-87	-93	-98	-90	
	prediction error $e_i$ (%)	0	+1.3	0	+2.7	-8.8	+2.7	-1.4	-3.2	-3.3	-1.2	-2.1	-6.1	+2.0	+2.0	-3.4	+1.1	-6.1	-2.2	
in air without rubber pad	predicted (dB)	-88	-82	-100	-95	-80	-102	-102	-99	-85	-97	-93	-101	-96	-112	-108	-95	-124	-102	4.0
	measured (dB)	-82	-84	-96	-89	-75	-98	-98	-93	-82	-92	-93	-102	-102	-110	-110	-96	-122	-110	
	prediction error $e_i$ (%)	-7.3	+2.4	-4.2	-6.7	-6.7	-4.1	-4.1	-6.5	-3.7	-5.4	0	+1.0	+5.9	-1.8	+1.9	+1.0	-1.4	+7.3	
in water with rubber pad	predicted (dB)	-79	-90	-115	-107	-101	-105	-102	-99	-107	-110	-102	-90	-117	-99	-102	-99	-120	-121	4.8
	measured (dB)	-73	-85	-106	-99	-94	-99	-98	-94	-105	-107	-110	-98	-116	-99	-108	-98	-123	-124	
	prediction error $e_i$ (%)	-8.2	-5.9	-8.5	-8.1	-7.4	-6.1	-4.1	-5.3	-1.9	-2.8	-7.3	-8.2	+0.9	0	+5.6	-1.0	+2.4	+2.4	
in water without rubber pad	predicted (dB)	-77	-95	-101	-105	-105	-104	-103	-100	-98	-112	-117	-92	-98	-102	-78	-90	-110	-97	3.6
	measured (dB)	-71	-92	-93	-102	-98	-101	-99	-98	-93	-107	-115	-92	-98	-101	-82	-88	-105	-98	
	prediction error $e_i$ (%)	-8.5	-3.3	-8.6	-2.9	-7.1	-3.0	-4.0	-2.0	-5.4	-4.7	-1.7	0	0	-1.0	+4.9	-2.3	-4.8	+1.0	

Table 7.2 Mobility error analysis between prediction and measurement on the scale engine room model (II)

Condition		Frequency (Hz)																	Average error $\frac{1}{n} \sum_{i=1}^n  e_i , (\%)$	
		20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900		1000
in air without rubber pad	predicted (dB)	-78	-82	-79	-82	-79	-86	-83	-80	-72	-96	-94	-92	-100	-102	-98	-101	-98	-90	1.4
	measured (dB)	-79	-82	-79	-83	-79	-87	-83	-80	-75	-96	-92	-90	-96	-102	-101	-101	-102	-92	
	prediction error $e_i$ (%)	+1.3	0	0	+1.2	0	+1.1	0	0	+4.0	0	-2.2	-2.2	-4.2	0	+3.0	0	+3.9	+2.2	
in air with rubber pad	predicted (dB)	-79	-90	-91	-87	-73	-98	-99	-96	-82	-92	-92	-104	-111	-110	-102	-117	-80	-110	3.6
	measured (dB)	-81	-91	-92	-86	-73	-96	-96	-94	-80	-100	-90	-102	-110	-102	-110	-110	-94	-110	
	prediction error $e_i$ (%)	+2.5	+1.1	+1.1	-1.2	0	-2.1	-3.1	-2.1	-2.5	+8.0	-2.2	-2.0	-0.9	-7.8	+7.3	-6.4	+14.9	0	
in water without rubber pad	predicted (dB)	-90	-94	-90	-91	-90	-99	-95	-92	-81	-98	-103	-85	-92	-112	-106	-114	-105	-99	3.5
	measured (dB)	-90	-90	-88	-90	-98	-95	-90	-82	-82	-98	-103	-80	-86	-108	-106	-110	-103	-100	
	prediction error $e_i$ (%)	0	-4.4	-2.3	-1.1	+8.2	-4.2	-5.6	-12.2	+1.2	0	0	-6.3	-7.0	-3.7	0	-3.6	-1.9	+1	
in water with rubber pad	predicted (dB)	-89	-100	-102	-99	-92	-107	-106	-102	-98	-107	-103	-110	-121	-119	-111	-95	-118	-110	4.6
	measured (dB)	-90	-98	-99	-102	-100	-102	-101	-100	-98	-109	-112	-103	-117	-110	-122	-100	-119	-120	
	prediction error $e_i$ (%)	+1.1	-2.0	-3.0	+2.9	+8.0	-4.9	-5.0	-2.0	0	+1.8	+8.0	-6.8	-3.4	-8.2	+9.0	-5.0	+0.8	+8.3	

### 7.1.2 Accuracy of the Coupled FE and BE Model for Mobility Prediction

When the machine induced vibrations fed into the bottom structure of a engine room via the contact points of mounts, this vibrations can propagate in the structure domain and radiate noise to ambient environment. To evaluate the vibration mobility spectral function of the structures, the coupled FE and BE model has been used to take the fluid-structure interaction effect into consideration. Applying this model the driving point mobility analysis of the engine scale room scale model (I) has been analysed and been compared with the measurements as described in Figure 5.21. Error analysis of the predicted mobility and the mean line of the fluctuating measured mobility spectrum is shown in Table 7.3. The mean error is exhibited to be only 4.0%. The fluctuations existing in the measured mobility spectral curve has been demonstrated to be the reflection effect of the tank wall.

Table 7.3 Error analysis of the mobility predicted by the coupled FE/BE model on scale engine room model (I) in water

Condition	Frequency (Hz)								Mean error $\frac{1}{n} \sum_{i=1}^n  e_i , (\%)$
	100	200	300	400	500	600	700	800	
predicted (dB)	-65	-52	-60	-59	-38	-55	-60	-66	4.0
measured (dB)	-62	-50	-61	-62	-40	-53	-58	-63	
Error $e_i$ (%)	-4.8	-4.0	+1.7	+4.8	+5.0	-3.8	-3.4	-4.8	

As described in Chapter 5, this FEM/BEM model is only suitable for mobility analysis of structure-borne propagation of vibrations in a lower frequency range, i.e., below 450 Hz. The mesh sizes of numerical model is limited by one-tenth of the structure-borne wave length, which is dependent on the frequency range to be considered, and can be considered as a common rule.

### 7.1.3 Accuracy of the Coupled FE/BE Model for Predicting Underwater Sound Radiation

The established coupled FEM and BEM model can also be applied to the underwater sound radiated from the vibrations of the wetted hull. In Chapter 4, a fully submerged spherical shell in infinite fluid domain has been studied. By using this numerical model the predicted pressure and velocity spectrum due to unit harmonic internal pressure has been compared with the analytical solutions by Everstine (1991) and Jeans and Mathews (1990) as shown in Figures 4.18 and 4.19. From the error analyses in Table 7.4 and 7.5, it can be seen that both mean errors within the range below the wave number under  $k_a=2.25$  are less than 3%. So this model can be concluded to be credible.

Again, application of this combined FE/BE model to predict the radiated underwater sound of the scale engine room model (II) in fluid domain has been described when it was excited by the operating motor. By the error analysis of the SPL shown in Table 6.1, there is a mean error of  $-6.3\%$  compared to the SPL measured at two pairs symmetrical positions. The greater error occurs owing to the developed FE/BE model was only considered the infinite and semi-infinite fluid domain conditions, thereby the error enlarged in the tank measurement.

Table 7.4 Error of the pressure spectrum on the shell surface by FEM/BEM prediction with respect to analytical solution

Condition	Wave number in air, $k_a$									Mean error $\frac{1}{n} \sum_{i=1}^n  e_i , (\%)$
	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	
Analytical (Pa)	0.01	0.03	0.05	0.087	0.13	0.17	0.23	0.27	0.33	2.93
FEM/BEM (Pa)	0.01	0.03	0.05	0.082	0.125	0.18	0.24	0.28	0.34	
Error $e_i$ (%)	0	0	0	-6.1	-4.0	-5.6	-4.2	-3.6	-2.9	

Table 7.5 Error of the velocity spectrum on the shell surface by FEM/BEM prediction with respect to analytical solution

Condition	Wave number in air, $k_a$									Mean error $\frac{1}{n} \sum_{i=1}^n  e_i $ , (%)
	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.00	2.25	
Analytical ( $\times 10^{-7}$ m/s)	0.20	0.40	0.65	0.80	1.20	1.30	1.76	2.00	2.30	2.63
FEM/BEM ( $\times 10^{-7}$ m/s)	0.20	0.40	0.65	0.82	1.19	1.40	1.70	2.10	2.40	
Error $e_i$ (%)	0	0	0	-2.4	-1.7	-7.1	-3.5	-4.8	-4.2	

#### 7.1.4 Quantitative Effect of Rubber Pad in the Reduction of Vibration Transmission

The mobility analyses and measurements have been discussed in Chapter 5. To quantify the effect of a rubber pad in the resilient mount to reduce the machine induced vibration transmission into the bottom structure, two scale engine room models (I) and (II) has been considered by interposing in the resilient mounts with or without rubber pads respectively during the motor was running.

In the case of the scale model (I) hung in air, the peaks of the transfer mobility spectrum were reduced by the amount summarized in Table 7.6 and the mean reduction of the effect of the rubber pad on the transfer mobility attained to be 6.42 dB. The comparisons of the vibrational velocity auto-spectra at the input and output sides of the resilient mount with or without rubber pad at spectral peaks are detailed in Table 7.7 and 7.8 respectively.

Table 7.6 Effect of the rubber pad on the transfer mobility reduction at the peak frequencies of the scale engine room model (I) hung in air

Mobility (dB)	Peak frequency (Hz)											
	20	32	48	60	101	140	160	180	200	360	420	540
with rubber	-81	-84	-87	-73	-77	-95	-95	-83	-92	-84	-96	-79
without rubber	-71	-78	-63	-65	-50	-78	-78	-79	-82	-84	-82	-90
reduction	10	6	14	8	14	17	17	4	10	0	-14	-11
mean reduction	6.42 (dB)											

Table 7.7 Effect of the rubber pad on the reduction of the vibrational velocity auto-spectrum reduction at the peak frequencies of the dry scale engine room model (I)

Velocity auto-spectrum (dB)		Peak frequency (kHz)															Mean (dB)	
		0.10	0.35	0.52	0.75	0.80	0.90	1.20	1.55	1.78	2.00	2.25	2.40	2.60	2.75	2.95		3.20
without rubber in mount	input side (A)	0.1	-2.5	-6.0	1.0	-4.0	-4.0	-2.5	-5.5	-1.0	-4.0	-3.5	-4.0	-6.0	-3.5	-1.0	-1.0	-2.96
	output side (B)	0.1	-7.0	-9.0	-15.0	-15.0	-14.5	-14.0	-12.0	-15.0	-14.5	-14.0	-12.0	-16.0	-12.0	-11.0	-10.0	-21.03
	reduction (C)=(A)-(B)	0	4.5	3.0	16.0	11.0	10.5	11.5	6.5	14.0	10.5	10.5	8.0	10.0	8.5	10.0	9.0	18.07
with rubber in mount	input side (D)	4.5	-2.0	-11.0	-1.0	-4.0	-4.5	-4.5	-8.0	-4.0	-12.0	-10.5	-12.0	-7.0	-7.0	-5.5	-7.0	-5.97
	output side (E)	0	-8.5	-10.0	-12.0	-13.5	-13.0	-15.0	-12.0	-17.0	-19.0	-17.5	-21.0	-16.5	-12.5	-11.0	-13.5	-13.25
	reduction (F)=(D)-(E)	0.1	6.5	-1.0	11.0	9.5	8.5	10.5	4.0	13.0	7.0	7.0	9.0	9.5	5.5	5.5	6.5	19.20
reduction effect of rubber	reduction in input side (G)=(A)-(D)	-4.5	-0.5	5.0	2.0	0	0.5	2.0	2.5	3.0	8.0	7.0	8.0	1.0	3.5	4.5	6.0	2.69
	reduction in output side (H)=(B)-(E)	-0.1	1.5	1.0	-3.0	-1.5	-1.5	1.0	0	5.0	4.5	3.5	9.0	0.5	0.5	0	3.5	1.49

Table 7.8 Effect of the rubber pad on the vibration power reduction at the peak frequencies of the dry scale engine room model (I) hung in air

Power spectrum (dB)		Peak frequency (kHz)																				Mean (dB)
		0.10	0.20	0.40	0.60	0.70	0.75	0.80	0.90	1.15	1.25	1.40	1.55	1.70	1.80	2.20	2.40	2.60	2.70	2.80	3.20	
Without rubber in mount	input side (A)	31	39.5	38	36	44	40	33	37.5	32	45	35	36	34	41	34	32	32	38	40.5	42	36.9
	output side (B)	26	37	37	31	32	29	29	28	28	25	29	25	25	29	29	28	24	24	29	28	28.6
	reduction (C)=(A)-(B)	5	2.5	1	5	12	11	4	9.5	4	20	16	11	9	12	5	4	8	14	11.5	14	8.3
with rubber in mount	input side (D)	34	40	30	33	39	40	33	36	31.5	40.5	32	35	35	34	28	23	32	32	37	31	33.8
	output side (E)	29	33	36	31	30	29	29.5	30	27	22.5	28	25	24	25	25	20	22	25	28	22	27.1
	reduction (F)=(D)-(E)	5	7	-6	2	9	11	4.5	6	4.5	18	4	10	11	9	3	3	10	7	9	9	6.7
reduction effect of rubber	reduction in input side (G)=(A)-(D)	-3	-0.5	8	3	5	0	0	1.5	0.5	4.5	3	1	-1	7	6	9	0	6	3.5	11	3.2
	reduction in output side (H)=(B)-(E)	-3	4	1	0	2	0	-0.5	-2	1	2.5	1	0	1	4	4	8	2	-1	1	6	1.6

From the quantitative analyses, it is found that the rubber pad in the resilient mount has the effect to reduce the vibration power flow transmission in three ways, i.e., (1) the reduction of the transfer mobility level at the peak frequencies in a wide band as summarized in Table 7.6, (2) the reduction of the input velocity auto-spectrum level at the peak frequencies in wide band as indicated in Table 7.7, and (3) the reduction of the output velocity auto-spectrum level at the peak frequencies also in wide band. While the velocity differences at input and output sides in both conditions of the resilient mount with or without rubber do not change apparently.

### 7.1.5 Quantitative Effect of the Fluid-Structure Interaction on Structure-Borne Vibration Propagation

To quantify the effect of the fluid-structure interaction on the structure-borne vibration propagation, the study of the scale engine room model (II) described in Chapter 5 can be illustrated as an example. Comparisons of the changes in the transfer mobilities of the model both in dry and wetted conditions are shown in Figures 5.28 and 5.29, the changes in the velocity auto-spectra are shown in Figures 5.30 and 5.34 and the changes in the vibrational power spectra are shown in Figures 5.32 and 5.36, respectively. Tables 7.9, 7.10 and 7.11 exhibit the individual comparison results for the condition concentrating on that the resilient mount interposed with rubber pad.

Table 7.9 Effect of the fluid-structure interaction on the transfer mobility of the scale engine room model (II)

Mobility spectrum (dB)	Peak frequency range (Hz)									
	20	32	40	55	105	180	230	350	600	700
			1	1					1	1
			50	60					650	800
Dry model	-83	-90	-91	-78	-78	-83	-90	-90	-90	-88
Wetted model	-90	-90	-100	-102	-92	-110	-97	-96	-103	-100
Reduction	7	0	9	24	14	27	7	6	13	12
Mean reduction	11.9 (dB)									

Table 7.10 Effect of the fluid-structure interaction on the velocity auto-spectrum of the scale model (II) at both input and output sides of mount

Velocity auto-spectrum (dB)		Peak frequency range (kHz)														Mean (dB)
		0.01   0.10	0.30   0.40	0.60   0.80	0.90   1.00	1.00   1.15	1.20   1.30	1.35   1.40	1.50   1.55	1.75   1.90	2.00   2.10	2.15   2.55	2.60   2.80	2.90   3.05	3.10   3.20	
in air with rubber in mount	input side (A)	-19	-40	-38	-45	-46	-46	-46.5	-48.5	-48	-44	-44	-44.5	-45.5	-43	-39.3
	output side (B)	-33	-37.5	-47	-52.5	-55	-53.5	-53.5	-52.5	-57	-56.5	-56.5	-57	-56	-55.5	-51.6
in water with rubber in mount	input side (C)	-11	-35	-36	-42	-44	-42	-50	-48	-48	-52	-54	-50	-48	-46	43.3
	output side (D)	-29	-42	-50	-51	-52	-54	-54	-51	-56	-60	-61	-60	-61	-63	-60.8
Reduction	input side (E)=(A)-(C)	-8	-5	-2	-3	-2	-4	3.5	-0.5	0	8	10	5.5	2.5	3	0.6
	output side (F)=(B)-(D)	-4	4.5	3	-1.5	-3	0.5	0.5	-1.5	-1	3.5	4.5	3	5	7.5	1.5

Table 7.11 Effect of the fluid-structure interaction on the vibrational power of the scale model (II) at both input and output sides of the mount

Vibrational power (Watt)		Peak frequency range (kHz)														Mean (Watt)	
		0.15	0.45	0.55	0.75	0.80	0.95	1.10	1.20	1.38	1.50	1.70	1.85	2.50	2.70		3.00
in air with rubber in mount	input side (A)	1.47	1.02	2.45	0.62	0.26	0.30	0.75	0.30	0.24	0.24	0.25	0.3	0.23	0.16	0.35	0.596
	output side (B)	1.10	0.40	0.30	0.24	0.10	0.10	0.05	0.02	0.10	0.05	0.05	0.08	0.01	0.01	0.01	0.181
in water with rubber in mount	input side (C)	1.55	0.40	2.20	0.50	0.55	0.30	0.70	0.15	0.20	0.10	0.25	0.15	0.15	0.12	0.2	0.489
	output side (D)	1.10	0.60	0.30	0.24	0.20	0.10	0.05	0.05	0.08	0.04	0.05	0.02	0.01	0.01	0.01	0.191
Reduction	input side (E)=(A)-(C)	-0.37	0.62	0.25	0.12	-0.19	0	0.05	0.15	0.04	0.14	0	0.15	0.08	0.04	0.15	0.082
	output side (F)=(B)-(D)	0	-0.2	0	0	-0.1	0	0	-0.03	0.02	0.01	0	0.06	0	0	0	-0.016

It is found that the mean reduction of the peak transfer mobilities with an amount 11.9 dB, as shown in Table 7.9, due to the fluid radiation impedance effect. When the motor on board was running, the mean velocity auto-spectrum level was reduced by 0.6 dB and 1.5 dB at input and output sides of the mount respectively by the fluid-structure interaction effect, as shown in Table 7.10. A special phenomenon has been found in Table 7.11 that the mean peak vibrational power at the output side of the mount was enhanced by 0.016 Watt due to the fluid-structure interactions. While in a usual condition the power at the input side of the mount should be reduced. This phenomenon could be explained by the reason that the vibrational power spectra at both sides of the mount changed hugely within the range between 450~550 Hz, as shown in Figures 5.32 and 5.36. Since the bigger the area between the power spectral curves the more the power flow was transmitted. Thus, in the wetted hull case, the input side of the mount absorbed more vibrational power from the output side owing to the change of vibration pattern within this frequency range compared with the dry hull condition.

Thereby, it can be concluded that the fluid-structure interaction affects the structure-borne vibration propagation in many aspects. Normally, this interaction can reduce not only the mobilities and vibrational velocity levels, but also change the vibration pattern of the structure.

#### **7.1.6 Quantitative Effect of the Squeeze-Film Damping on the Reduction of Vibration Transmission**

By Figures 5.46 to 5.49, the driving mobilities at the four engine mounts supported by the bottom girders, which was improved by attaching the squeeze-film damping plates, have a mean reduction at the top twelve peak frequencies 6.3 dB at mount 1, 6.2 dB at mount 3; 5.3 dB at mount 2 and 5.6 dB at mount 4; as displayed in Table 7.12. It is seen that the reductions at mounts 1 and 3; and that at mounts 2 and 4 are analogical respectively. This is owing to the mounts 1 and 3; and the mounts 2 and 4 were respectively supported by the same individual girder as shown in Figure 5.39. The girders linked to the engine via the

Table 7.12 Effect of the squeeze-film damping in the reduction of the driving point mobility of the patrol vessel

Mount	Mobility (dB)	Top 12 peak frequencies (Hz)												Mean reduction (dB)
		25	72	125	140	232	255	320	380	398	445	480	490	
1	without squeeze-film	-75	-70	-85	-90	-73	-83	-89	-88	-80	-85	-87	-80	6.3
	with squeeze-film	-83	-83	-92	-92	-81	-87	-91	-89	-92	-93	-97	-88	
	reduction	8	7	7	2	8	4	2	1	11	8	10	8	
2	without squeeze-film	-78	-96	-88	-84	-96	-80	-88	-89	-80	-81	-93	-85	5.3
	with squeeze-film	-80	-98	-91	-93	-101	-90	-99	-95	-90	-84	-94	-92	
	reduction	2	2	3	4	5	10	11	6	10	3	1	7	
3	without squeeze-film	-73	-90	-81	-84	-88	-87	-93	-85	-87	-87	-91	-96	6.2
	with squeeze-film	-76	-105	-88	-91	-93	-93	-106	-92	-90	-89	-92	-111	
	reduction	3	15	7	7	5	6	13	7	3	2	1	5	
4	without squeeze-film	-75	-89	-73	-79	-81	-87	-80	-83	-86	-86	-78	-88	5.6
	with squeeze-film	-79	-94	-85	-82	-85	-90	-85	-89	-98	-89	-85	-92	
	reduction	4	5	12	3	4	3	5	6	12	3	7	4	

mounts disperse vibrations. Thus, once the driving point mobilities were reduced, the cabins' noise levels were attenuated accordingly.

### **7.1.7 Conclusions**

From the work of the study, it is concluded that the developed models for predicting the machine induced noise and vibration in a ship structure have been ascertained to be credible with an error less than 5%. These models were established in order to solve the transmission, propagation and radiation of noise and vibration problems starting from a machine source to the receiver in a logical and realistic ways.

Besides, the characterisation method of the resilient mount of a machine and the squeeze film damping plate used as the countermeasures for the structure-borne noise and vibration control has been preformed.

### **7.2 Recommendations for Further Work**

Based on the stress wave model of beam element and the rectangular plate element derived in Chapter 4, the stress wave shell element is necessary to derive for more diverse applications. Further, the interface program of the stress wave model should be edited to combine the BEM software to predict the structure-borne noise and vibration propagation in the intermediate and higher frequency range.

The optimization algorithm for determining the size, quantity and locations to apply the squeeze-film damping plates needs to be realized for practical use.

Once the underwater anechoic chamber at the author's Institution, the National Taiwan Ocean University, being completed, the underwater sound radiation experiment performed in this study shall be repeated by virtue of the free field environment and comparisons made.

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## **Appendix A**

### **Derivation of the Transfer Function Matrix of Finite Rectangular Plate For the Two Dimensional Elastic Bending Wave**

## Appendix A

### Derivation of the Transfer Function Matrix of Finite Rectangular Plate for the Two Dimensional Elastic Bending Wave

Wave equation of a plate in flexure (Cremer et al, 1988):

$$\Delta\Delta\eta - k^4\eta = 0 \quad (\text{A-1})$$

The solution of equation (A.1) can be expressed as:

$$\hat{\eta} = C_1 H_0^{(2)}(kr) + C_2 H_0^{(2)}(-jkr) \quad (\text{A-2})$$

where  $H_0^{(2)}(kr)$  and  $H_0^{(2)}(-jkr)$  are the Hankel functions of the second kind, The expanded form of  $H_0^{(2)}(kr)$  is:

$$H_0^{(2)}(kr) = J_0(kr) - jY_0(kr) \quad (\text{A-3})$$

where  $J_0(kr)$  is the Bessel function of the first kind of order zero and  $Y_0(kr)$  is the Bessel function of the second kind of order zero. They are :

$$J_0(kr) = 1 - \frac{(kr)^2}{2^2(1!)^2} + \frac{(kr)^4}{2^4(2!)^2} - \frac{(kr)^6}{2^6(3!)^2} + \dots \quad (\text{A-4})$$

$$Y_0(kr) = \frac{2}{\pi} \left[ J_0(kr) \left( \ln \frac{kr}{2} + \gamma \right) + \sum_{m=1}^{\infty} \frac{(-1)^{m-1} h_m}{2^{2m} (m!)^2} (kr)^{2m} \right] \quad (\text{A-5})$$

where

$\gamma \approx 0.5772$  is the Euler constant

$$h_m = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{m} \quad (\text{A-6})$$

Using an asymptotic expansion,  $H_0^{(2)}(kr)$  and  $H_0^{(2)}(-jkr)$  have the following characteristics :

$$H_0^{(2)}(kr) \approx \begin{cases} -\frac{2j}{\pi} \ln \frac{kr}{2} & \text{for } |kr| \ll 1 \\ \sqrt{\frac{2}{\pi kr}} e^{-j(kr - \frac{\pi}{4})} & \text{for } |kr| \gg 1 \end{cases} \quad (\text{A-7})$$

By using the boundary conditions explained in section 5.4, the solution of equation (A.1) has the form as:

$$\hat{\eta} = \frac{\hat{F}_0}{2jB'k^2} [H_0^{(2)}(kr) - H_0^{(2)}(-jkr)] \quad (\text{A-8})$$

for the force excitation  $F_{z1}$  acting at a corner, as shown in Figure A.1. Thus the lateral velocity and angular velocity distribution of the plate can be obtained by:

$$v_z(x, y) = j\omega\eta = \frac{F_z\omega}{2B'k^2} [H_0^{(2)}(kr) - H_0^{(2)}(-jkr)] = \frac{F_{z1}}{Z_{F_0}} \Pi(kr) \quad (\text{A-9})$$

$$W_x(x, y) = \frac{\partial v_z}{\partial y} = \frac{\partial v_z}{\partial r} \frac{\partial r}{\partial y} = \frac{F_z\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(kr)}{\partial r} - \frac{\partial H_0^{(2)}(-jkr)}{\partial r} \right] \frac{y}{r} \quad (\text{A-10})$$

$$W_y(x, y) = \frac{\partial v_z}{\partial x} = \frac{\partial v_z}{\partial r} \frac{\partial r}{\partial x} = \frac{F_z\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(kr)}{\partial r} - \frac{\partial H_0^{(2)}(-jkr)}{\partial r} \right] \frac{x}{r} \quad (\text{A-11})$$

where  $r = (x^2 + y^2)^{1/2}$

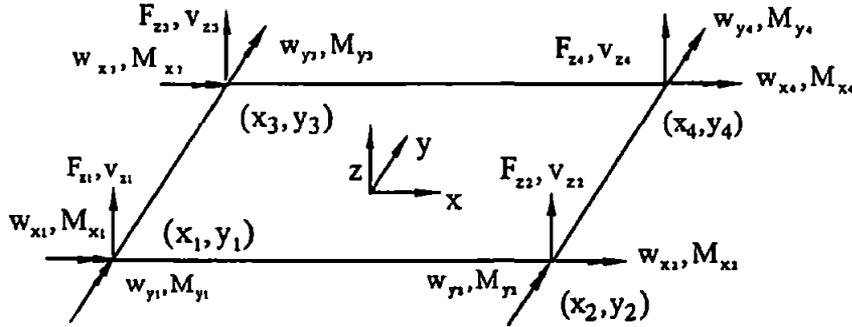


Figure A.1 Nodal forces and nodal velocities of a rectangular in bending waves

For the mobility transfer relations between nodal forces and nodal velocities, it can be written as:

$$\langle v_{z1} \cdots v_{z4}, w_{x1} \cdots w_{x4}, w_{y1} \cdots w_{y4} \rangle^T = [H_{ij}] \langle F_{z1} \cdots F_{z4}, M_{x1} \cdots M_{x4}, M_{y1} \cdots M_{y4} \rangle^T \quad (\text{A-12})$$

From equation (A.9) ~ (A.11), the relations between  $F_z$  and  $v_z$ ,  $w_x$  or  $w_y$  can be established respectively.

Setting  $F_{zj} = 1$  in equation (A.9) at node  $j$ , then

$$H_{ij} = v_{z1} = \frac{\kappa^2}{2\omega m'} = \frac{\omega}{2B'\kappa^2} \quad \text{for } i = j$$

$$H_{ij} = v_{z2} = \frac{\omega}{2B'\kappa^2} [H_0^{(2)}(\kappa r_{ij}) - H_0^{(2)}(-j\kappa r_{ij})] \quad \text{for } i \neq j$$

where  $r_{ij}$  is the distance between nodes  $i$  and  $j$ . Then  $H_{ij}$ ,  $i, j = 1, 2, 3, 4$  can be obtained.

By equations (A.10) and (A.11), the derivatives of Hankel functions with respect to  $\kappa r$  are:

$$\begin{aligned} \frac{\partial H_0^{(2)}(\kappa r)}{\partial r} &= -\frac{2k(\kappa r)}{2^2(1!)^2} + \frac{4k(\kappa r)^3}{2^4(2!)^2} - \frac{6k(\kappa r)^5}{2^6(3!)^2} + \dots - \frac{2j}{\pi} J_0(\kappa r) \frac{1}{r} - \frac{2j}{\pi} \left[ \left( \ln \frac{\kappa r}{2} + r \right) \right. \\ &\quad \left. \left( -\frac{2k(\kappa r)}{2^2(1!)^2} + \frac{4k(\kappa r)^3}{2^4(2!)^2} + \frac{4k(\kappa r)^5}{2^6(3!)^2} + \dots \right) + \sum_{m=1}^{\infty} \frac{(-1)^{m-1} h_m}{2^{2m}(m!)^2} 2mk(\kappa r)^{2m-1} \right] \end{aligned} \quad (\text{A.13})$$

and

$$\begin{aligned} \frac{\partial H_0^{(2)}(\kappa r)}{\partial r} &= -\frac{2k(\kappa r)}{2^2(1!)^2} + \frac{-4jk(-j\kappa r)^3}{2^4(2!)^2} + \frac{-6jk(-j\kappa r)^5}{2^6(3!)^2} + \dots \\ &\quad - \frac{2j}{\pi} J_0(-j\kappa r) \frac{1}{r} - \frac{2j}{\pi} \left[ \left( \ln \frac{-j\kappa r}{2} + r \right) \left( -\frac{-2jk(-j\kappa r)}{2^2(1!)^2} + \frac{-4jk(-j\kappa r)^3}{2^6(2!)^2} \right. \right. \\ &\quad \left. \left. - \frac{-6jk(-j\kappa r)^5}{2^6(3!)^2} + \dots \right) + \sum_{m=1}^{\infty} \frac{(-1)^m h_m}{2^{2m}(m!)^2} 2mj k(-j\kappa r)^{2m-1} \right] \end{aligned} \quad (\text{A.14})$$

Substituting the distance  $r = r_{ij}$  ( $i, j = 1, 2, 3, 4$ ), and the relative  $y$  distance  $y = y_{ij}$  for the mode  $i$  to the node  $j$  of force excitation  $F_{zj}$  into equations (A.10), then  $H_{ij}$  ( $i = 5, \dots, 8$ ;  $j = 1, \dots, 4$ ) can be found. Similarly, substituting the distance  $r_{ij}$  ( $i, j = 1, 2, 3, 4$ ) and the relative  $x$  distance  $x_{ij}$  for the node  $i$  to the force applied node  $j$  into equation (A.11),  $H_{ij}$  ( $i = 9, \dots, 12$ ;  $j = 1, \dots, 4$ ) can be found.

For the excitation couple  $M_{xy}$  applied at node  $j$  of a relative small arm  $2a$ , as shown in figure A2, the total velocity can be obtained as:

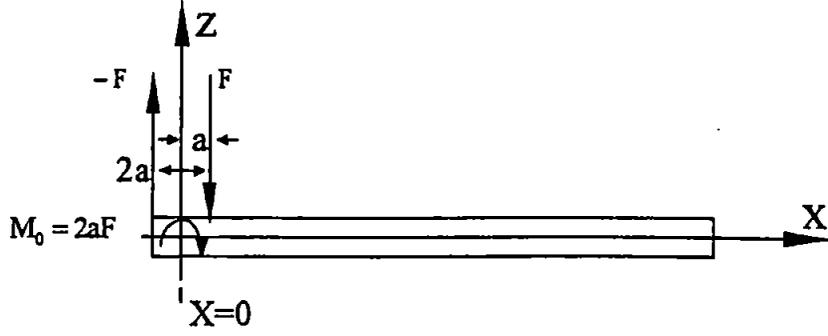


Figure A2 A couple applied at node j of a rectangular plate

$$V_2(x, y) = \frac{F_z w}{2B'k^2} \{ [H_0^{(2)}(kr_1) - H_0^{(2)}(-jkr_1)] - [H_0^{(2)}(kr_2) - H_0^{(2)}(-jkr_2)] \} \quad (A.16)$$

where

$$r_1 = [(y - a)^2 + x^2]^{\frac{1}{2}}$$

$$r_2 = [(y + a)^2 + x^2]^{\frac{1}{2}}$$

The angular velocities  $w_x$  and  $w_y$  are:

$$\begin{aligned} W_x(x, y) &= \frac{\partial V_z}{\partial y} = \frac{\partial V_z}{\partial r} \frac{\partial r}{\partial y} \\ &= \frac{M_{xj} \omega}{2B'k^2 a} \left\{ \left[ \frac{\partial H_0^{(2)}(kr_1)}{\partial r_1} - \frac{\partial H_0^{(2)}(-jkr_1)}{\partial r_1} \right] \frac{y-a}{r_1} \right. \\ &\quad \left. - \left[ \frac{\partial H_0^{(2)}(kr_2)}{\partial r_2} - \frac{\partial H_0^{(2)}(-jkr_2)}{\partial r_2} \right] \frac{y+a}{r_2} \right\} \end{aligned} \quad (A.17)$$

$$\begin{aligned} W_x(x, y) &= \frac{\partial \hat{V}}{\partial y} = \frac{\partial \hat{V}}{\partial r} \frac{\partial r}{\partial y} \\ &= \frac{M_{xj} \omega}{2B'k^2 a} \left\{ \left[ \frac{\partial H_0^{(2)}(kr_1)}{\partial r_1} - \frac{\partial H_0^{(2)}(-jkr_1)}{\partial r_1} \right] \frac{x}{r_1} \right. \\ &\quad \left. - \left[ \frac{\partial H_0^{(2)}(kr_2)}{\partial r_2} - \frac{\partial H_0^{(2)}(-jkr_2)}{\partial r_2} \right] \frac{x}{r_2} \right\} \end{aligned} \quad (A.18)$$

where " a " is set to be a small distance, namely  $a = 10^{-5}$  m. Expanding the Hankel functions  $H_0^{(2)}(kr_1)$  and  $H_0^{(2)}(kr_2)$  with respect to  $k(y+a)$  respectively then the propagation functions become:

$$\prod (kr_1) \approx 1 + 2k^2(y-a)^2 [\alpha_1 + \alpha_2 \ln \frac{\gamma k(y-a)}{2} - \frac{1}{2} \alpha_2 \ln(-j)] \quad (\text{A.19a})$$

$$\prod (kr_2) \approx 1 + 2k^2(y+a)^2 [\alpha_1 + \alpha_2 \ln \frac{\gamma k(y+a)}{2} - \frac{1}{2} \alpha_2 \ln(-j)] \quad (\text{A.19b})$$

where  $\alpha_1 = -\frac{1}{4} - \frac{j}{2\pi}$ ,  $\alpha_2 = \frac{j}{2\pi}$ ,  $\gamma = 1.781$ .

For

$$\frac{\partial r}{\partial y} = \frac{1}{2} \frac{2(y-a)}{r_1} = \frac{y-a}{r_1}, \quad \frac{\partial r_1}{\partial x} = \frac{x}{r_1}$$

$$\frac{\partial r_2}{\partial y} = \frac{y+a}{r_2}, \quad \frac{\partial r_2}{\partial x} = \frac{x}{r_2}$$

Successively substituting the x distance  $x = x_i$  ( $i = 1, \dots, 4$ ) and  $r_1 = (r_1)_{ij}$  of the distance from the corner node  $i$  to the moment excitation corner  $j$  into equations (A.16) to (A.18) respectively, then  $H_{ij}$  ( $i = 1, \dots, 12; j = 5, \dots, 8$ ) can be attained.

Similar procedure can be utilized to derive the mobility transfer functions  $H_{ij}$  ( $i = 1, \dots, 12; j = 9, \dots, 12$ ) for the moment excitations  $M_{yj}$  ( $j = 1, \dots, 4$ ) applied at corner  $j$  via the velocity propagation distributions as:

$$v_z(x, y) = \frac{M_{yj}\omega}{2B'k^2} \{ [H_0^{(2)}(kr_1) - H_0^{(2)}(-jkr_1)] - [H_0^{(2)}(kr_2) - H_0^{(2)}(-jkr_2)] \}$$

$$w_x = \frac{\partial v_z}{\partial y} = \frac{\partial \hat{v}}{\partial r} \frac{\partial r}{\partial y} = \frac{M_{yj}\omega}{2B'k^2 a} \left\{ \left[ \frac{\partial H_0^{(2)}(kr_1)}{\partial r_1} - \frac{\partial H_0^{(2)}(-jkr_1)}{\partial r_1} \right] \frac{x}{r_1} - \left[ \frac{\partial H_0^{(2)}(kr_2)}{\partial r_2} - \frac{\partial H_0^{(2)}(-jkr_2)}{\partial r_2} \right] \frac{x}{r_2} \right\}$$

$$w_y(x, y) = \frac{\partial v_z}{\partial x} = \frac{\partial \hat{v}}{\partial r} \frac{\partial r}{\partial x} = \frac{M_{yj}\omega}{2B'k^2 a} \left\{ \left[ \frac{\partial H_0^{(2)}(kr_1)}{\partial r_1} - \frac{\partial H_0^{(2)}(-jkr_1)}{\partial r_1} \right] \frac{x-a}{r_1} - \left[ \frac{\partial H_0^{(2)}(kr_2)}{\partial r_2} - \frac{\partial H_0^{(2)}(-jkr_2)}{\partial r_2} \right] \frac{x+a}{r_2} \right\}$$

Thus, to summarize the results of  $H_{ij}$   $i, j = 1, 2, \dots, 12$  and take  $a = 10^{-5}$  m, the full mobility matrix of a rectangular plate for bending wave propagation can be listed as table A1. Clearly,  $[H_{ij}]$  is symmetrical.

Table A.1 Full mobility matrix of finite rectangular plate in bending wave

	j=1	j=2	j=3	j=4
i=1	$\frac{k^2}{2\omega m^*} = \frac{\omega}{2B'k^2}$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{12}) - H_0^{(2)}(-jkr_{12})]$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{13}) - H_0^{(2)}(-jkr_{13})]$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{14}) - H_0^{(2)}(-jkr_{14})]$
i=2	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{21}) - H_0^{(2)}(-jkr_{21})]$	$\frac{k^2}{2\omega m^*} = \frac{\omega}{2B'k^2}$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{23}) - H_0^{(2)}(-jkr_{23})]$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{24}) - H_0^{(2)}(-jkr_{24})]$
i=3	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{31}) - H_0^{(2)}(-jkr_{31})]$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{32}) - H_0^{(2)}(-jkr_{32})]$	$\frac{k^2}{2\omega m^*} = \frac{\omega}{2B'k^2}$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{34}) - H_0^{(2)}(-jkr_{34})]$
i=4	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{41}) - H_0^{(2)}(-jkr_{41})]$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{42}) - H_0^{(2)}(-jkr_{42})]$	$\frac{\omega}{2B'k^2} [H_0^{(2)}(kr_{43}) - H_0^{(2)}(-jkr_{43})]$	$\frac{k^2}{2\omega m^*} = \frac{\omega}{2B'k^2}$
i=5	0	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{11})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{11})}{\partial r} \right] \cdot \frac{y_{11}}{r_{11}}$	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{13})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{13})}{\partial r} \right] \cdot \frac{y_{13}}{r_{13}}$	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{14})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{14})}{\partial r} \right] \cdot \frac{y_{14}}{r_{14}}$
i=6	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{21})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{21})}{\partial r} \right] \cdot \frac{y_{21}}{r_{21}}$	0	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{23})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{23})}{\partial r} \right] \cdot \frac{y_{23}}{r_{23}}$	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{24})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{24})}{\partial r} \right] \cdot \frac{y_{24}}{r_{24}}$
i=7	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{31})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{31})}{\partial r} \right] \cdot \frac{y_{31}}{r_{31}}$	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{32})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{32})}{\partial r} \right] \cdot \frac{y_{32}}{r_{32}}$	0	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{34})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{34})}{\partial r} \right] \cdot \frac{y_{34}}{r_{34}}$
i=8	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{41})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{41})}{\partial r} \right] \cdot \frac{y_{41}}{r_{41}}$	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{42})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{42})}{\partial r} \right] \cdot \frac{y_{42}}{r_{42}}$	$\frac{\omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(\alpha r_{43})}{\partial r} - \frac{\partial H_0^{(2)}(-j\alpha r_{43})}{\partial r} \right] \cdot \frac{y_{43}}{r_{43}}$	0

Table A.1 Full mobility matrix of finite rectangular plate in bending wave (continued)

	j=1	j=2	j=3	j=4
I=9	0	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_1)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_1)}{\partial r} \right] \cdot \frac{x_{11}}{r_{11}}$	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_2)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_2)}{\partial r} \right] \cdot \frac{x_{12}}{r_{12}}$	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_4)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_4)}{\partial r} \right] \cdot \frac{x_{14}}{r_{14}}$
I=10	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_1)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_1)}{\partial r} \right] \cdot \frac{x_{21}}{r_{21}}$	0	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_3)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_3)}{\partial r} \right] \cdot \frac{x_{23}}{r_{23}}$	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_4)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_4)}{\partial r} \right] \cdot \frac{x_{24}}{r_{24}}$
I=11	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_1)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_1)}{\partial r} \right] \cdot \frac{x_{31}}{r_{31}}$	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_2)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_2)}{\partial r} \right] \cdot \frac{x_{32}}{r_{32}}$	0	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_4)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_4)}{\partial r} \right] \cdot \frac{x_{34}}{r_{34}}$
i=12	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_1)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_1)}{\partial r} \right] \cdot \frac{x_{41}}{r_{41}}$	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_2)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_2)}{\partial r} \right] \cdot \frac{x_{42}}{r_{42}}$	$\frac{\omega}{2B'\kappa^2} \left[ \frac{\partial H_0^{(2)}(\kappa r_3)}{\partial r} - \frac{\partial H_0^{(2)}(-j\kappa r_3)}{\partial r} \right] \cdot \frac{x_{43}}{r_{43}}$	0

	j=5	j=6	j=7	j=8
i=1	0	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$
i=2	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	0	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$
i=3	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	0	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$
i=4	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	$\frac{0.5 \cdot 10^3 \omega}{2B'\kappa^2} \left\{ \begin{array}{l} [H_0^{(2)}(\kappa r_1) - H_0^{(2)}(-j\kappa r_1)] \\ - [H_0^{(2)}(\kappa r_2) - H_0^{(2)}(-j\kappa r_2)] \end{array} \right\}$	0







## **Appendix B**

### **Helmholtz Equation and Boundary Integral Method**

## Appendix B.

### Helmholtz Equation and Boundary Integral Method

#### B.1 Helmholtz Equation

If the underwater acoustic pressure can be expressed as a harmonic function of time with frequency  $\omega$ , then

$$p(\vec{r}, t) = P(\vec{r})e^{i\omega t} \quad (\text{B.1})$$

The governing wave equation for underwater sound propagation is,

$$\nabla^2 p(\vec{r}, t) = \frac{1}{c^2} \partial_{tt} p(\vec{r}) \quad (\text{B.2})$$

where  $c$  is the sound propagation speed in water and  $\partial_{tt} = \frac{\partial^2}{\partial t^2}$ . Take Fourier transform on both sides of equation (B.2).

$$\int_{-\infty}^{\infty} [\nabla^2 p(\vec{r}, t) - \frac{1}{c^2} \partial_{tt} p(\vec{r}, t)] e^{-i\omega t} dt = 0$$

or

$$\nabla^2 \left( \int_{-\infty}^{\infty} p(\vec{r}, t) e^{-i\omega t} dt \right) + \frac{\omega^2}{c^2} \left( \int_{-\infty}^{\infty} p(\vec{r}, t) e^{-i\omega t} dt \right) = 0 \quad (\text{B.3})$$

Define  $P(\vec{r}, \omega) = \int_{-\infty}^{\infty} p(\vec{r}, t) e^{-i\omega t} dt$  and the wave number  $k = \omega / c$ , then equation (B.3) becomes

$$\nabla^2 P(\vec{r}, \omega) + k^2 P(\vec{r}, \omega) = 0 \quad (\text{B.4})$$

Equation (B.4) is known as the Helmholtz equation.

The time domain pressure distribution can then be obtained by the inverse Fourier transform

$$p(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\vec{r}, \omega) e^{i\omega t} d\omega \quad (\text{B.5})$$

## B.2 Boundary Integral Method

### B.2.1 Basic Boundary Integral Theorem

Suppose there are two functions  $\Phi$  and  $\lambda$  which have continuous first and second order derivatives, then the following relation can be obtained from the Green's Second Identity :

$$\int_V (\Phi \nabla^2 \lambda - \lambda \nabla^2 \Phi) dV = \int_V \left( \Phi \frac{\partial \lambda}{\partial n} - \lambda \frac{\partial \Phi}{\partial n} \right) ds \quad (B.6)$$

where  $\frac{\partial}{\partial n}$  means the outward divergence on the surface of the control volume.

Consider the field point  $\vec{r} = \vec{r}(x, y, z)$  and the source point  $\vec{r}_s = \vec{r}_s(x_s, y_s, z_s)$  in the control volume, set  $r$  to be the distance between these two points, then the free space Green's function can be expressed as :

$$G = \frac{e^{ikr}}{r} \quad (B.7)$$

where

$$r = \sqrt{(x_s - x)^2 + (y_s - y)^2 + (z_s - z)^2}$$

If  $P$  represents the pressure function in  $V$ , then from the Green's theorem

$$\int_V (P \nabla^2 G - G \nabla^2 P) dV = \int_V \left( P \frac{\partial G}{\partial n} - G \frac{\partial P}{\partial n} \right) dS \quad (B.8)$$

Substituting the Helmholtz equation (B.4) and the Green's function (B.7) into equation (A.8), then it becomes

$$\nabla^2 G + k^2 G = 0 \quad (B.9)$$

From equations (B.4) and (B.9), it can be shown that within the control volume but excluding the points within the infinitesimal sphere around the field point there exists the relation :

$$\nabla(PVG - GVP) = 0 \quad (B.10)$$

or

$$PVG - GVP = 0 \quad (B.11)$$

### B.2.2 Boundary Integral Equation

As in the right side of equation (B.8) the boundary integral should consider the normal direction of the boundary surface. The propagation direction of sound can be characterised by the relative position of the source point and the field point. If within the control volume the sound field point is outside the sound source body, then it is an exterior propagation problem as shown in Figure B1. Otherwise, it is an interior propagation problem, shown in Figure B2.

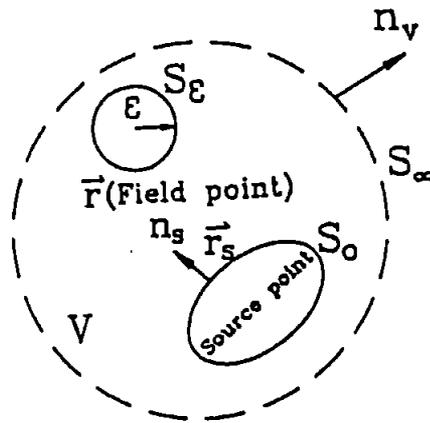


Figure B1. Exterior propagation problem

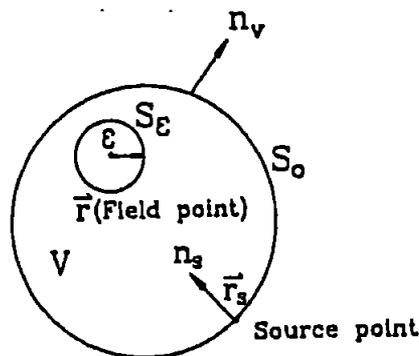


Figure B2. Interior propagation problem

Consider the exterior propagation problem described as in Figure B1, in which  $\vec{r}$  means the position vector of a sound field point,  $S_\epsilon$  is the spheric surface which encompassing the considered sound field point with a infinitesimal radius  $\epsilon$ ,  $S_\infty$  represents the surface area of the infinite control volume  $V$ ,  $n_\nu$  is outward normal vector of the surface  $S_\infty$ ,  $n_s$  is outward normal vector of the surface of source body. At the source point  $n_\nu$  and  $n_s$  have opposite directions, ie,  $\frac{\partial}{\partial n_s} = -\frac{\partial}{\partial n_\nu}$ .

From the Green's theorem, equation (B.8) becomes :

$$\int_{s_0+s_s+s_\infty} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_\nu} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n_\nu}] dS = \int_V (P \nabla^2 G - G \nabla^2 P) dV = 0 \quad (B.12)$$

in which

$$\int_{s_s} [P(\vec{r}) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_\nu} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r})}{\partial n_\nu}] dS(\vec{r}) = 4\pi P(\vec{r})$$

$$\int_{s_\infty} [P(\infty) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_\nu} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\infty)}{\partial n_\nu}] dS(\infty) = 0$$

so that

$$4\pi P(\vec{r}) + \int_{s_0} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_\nu} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n_\nu}] dS(\vec{r}_s) = 0 \quad (B.13)$$

or

$$4\pi P(\vec{r}) = \int_{s_0} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_\nu} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n_\nu}] dS(\vec{r}_s) \quad (B.14)$$

Equation (B.14) is named as the Exterior Helmholtz Integral Equation.

If both the sound source point and sound field point are sited on the boundary surface of the sound source body, then the field point  $\vec{r}$  can be only encompassed by an infinitesimal semi-spheric surface. Now equation (B.14) becomes :

$$2\pi P(\vec{r}) = \int_{s_0} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_\nu} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n_\nu}] dS(\vec{r}_s) \quad (B.15)$$

In the exterior propagation problem, if the field point is considered within the source body and is not pertaining to the interior region of the control volume  $V$ , then equation (B.13) should be taken account for deleting the surface integral domain  $S$ , ie,

$$\int_{s_0} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_v} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n_v}] dS(\vec{r}_s) = 0 \quad (B.16)$$

As for the interior propagation problem, the derivations are the same as the exterior propagation problem except to put  $n_s$  and  $n_v$  in the same directions.

After summarising various conditions of the  $\vec{r}$  positions, the Helmholtz equation can be expressed as :

$$C(\vec{r})P(\vec{r}) = \int_{s_0} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_v} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n_v}] dS(\vec{r}_s) \quad (B.17)$$

where the  $C(\vec{r})$  values are listed in Table B.1.

**Table B.1**  $C(\vec{r})$  values for various  $\vec{r}$  position

Position of $\vec{r}$	Exterior propagation	Interior propagation
$\vec{r}$ outside $S_0$	$C(\vec{r}) = 4\pi$	$C(\vec{r}) = 0$
$\vec{r}$ on $S_0$	$C(\vec{r}) = 2\pi$	$C(\vec{r}) = 2\pi$
$\vec{r}$ inside $S_0$	$C(\vec{r}) = 0$	$C(\vec{r}) = 4\pi$

As for other surface boundary like a corner or plane, then  $C(\vec{r})$  is taken to be :

$$C(\vec{r}) = 4\pi - \int_{s_0} \frac{\partial(\frac{1}{r(\vec{r}, \vec{r}_s)})}{\partial n} dS(\vec{r}_s) \quad (B.18)$$

### B.3 Fluid-Structure Interaction

As the density of water is much greater than air, the coupling effects of fluid-structure interaction have to be taken into consideration for the vibration problems of an immersed structure. As shown in equation (B.15),  $P(\vec{r})$  represents the relative acoustic pressure on the wetted surface of a submerged structure in vibration. On the wetted surface there exists the relation :

$$\frac{\partial P(\vec{r}_s)}{\partial n} = i\rho\omega v_n(\vec{r}_s) \quad (\text{B.19})$$

Now the Helmholtz Integral Equation of the submerged body surface can be expressed as :

$$C(\vec{r})P(\vec{r}) = \int_{S_s} [P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_s} + i\rho\omega G(\vec{r}, \vec{r}_s) v_n(\vec{r}_s)] dS(\vec{r}_s) \quad (\text{B.20})$$

Utilizing the boundary element discretization technique on the submerged body surface, equation (B.20) can be written as :

$$C(\vec{r})P(\vec{r}) = \sum_s \int_{S_s} [N(\vec{r}_s)P(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_s} - i\rho\omega G(\vec{r}, \vec{r}_s)N(\vec{r}_s)v_n(\vec{r}_s)] dS_s(\vec{r}_s) \quad (\text{B.21})$$

Equation (B.21) can be expressed in matrix form as :

$$[A] \{P\} = [B] \{V_n\} \quad (\text{B.22})$$

where

$$[A] = C(\vec{r}) - \sum_s \int_{S_s} [N(\vec{r}_s) \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n_s}] dS_s(\vec{r}_s) \quad (\text{B.23})$$

$$[B] = \sum_s \int_{S_s} i\rho\omega G(\vec{r}, \vec{r}_s)N(\vec{r}_s) dS_s(\vec{r}_s) \quad (\text{B.24})$$

By assuming that the modal point displacements of the boundary structure elements have the form :

$$\{x_n\} = \{X_n\}e^{i\omega t}$$

then

$$\{v_n\} = \frac{\partial \{x_n\}}{\partial t} = i\omega \{x_n\} \quad (\text{B.25})$$

Substituting into equation (B.22), it becomes

$$[A]\{P\} = i\omega[B]\{x_n\} \quad (\text{B.27})$$

or

$$[A]\{P\} = [G]\{x_n\} \quad (\text{B.28})$$

where

$$[G] = i\omega[B]$$

The fluid interacting forces applied to the nodal points of the wetted structure boundary surface are :

$$\{f_w\} = \{a\}^T \{P\} = \{a\}^T [A]^{-1} [B] i\omega \{x_n\} = \{a\}^T [A]^{-1} [G] \{x_n\} \quad (\text{B.28})$$

in which  $\{a\}$  represents the area parameters of each nodal point.

## **Appendix C**

### **Half-Space Green's Function for Helmholtz Equation**

## Appendix C.

### Half-Space Green's Function for Helmholtz Equation

The Green's function in equation (A.7) represents that of the submerged body situated within an unbounded water space. When the body nears or is located at the free surface or the seabed, there exists an infinite plane surface  $S_p$  as shown in Figure C1 and C2, the Green's function used in the boundary surface integral equation (B.8) should be changed. In this case the impedance boundary condition should be satisfied on both the submerged body surface  $S_0$  and the infinite plane surface  $S_p$ . The impedance boundary condition can be expressed as :

$$ikP + \frac{Z}{\rho_0 c} \frac{\partial P}{\partial n_0} = 0 \quad (C.1)$$

where  $Z$  represents the surface sound impedance and  $\rho_0 c$  is the characteristic value of the impedance in the medium. Now the boundary surface integral should be taken over the domains  $S_0$  and  $S_p$ . In order to overcome the difficulty of integrating over the unbounded plane surface  $S_p$ , it is intended to choose a Half-Space Green's Function  $G_h$  so as to make that

$$\int_{S_p} [P \frac{\partial G_h}{\partial n_0} + G_h \frac{\partial P}{\partial n_0}] dS = 0 \quad (C.2)$$

and simultaneously to satisfy the impedance boundary condition :

$$ikG_h + \frac{Z}{\rho_0 c} \frac{\partial G_h}{\partial n_0} = 0 \quad (C.3)$$

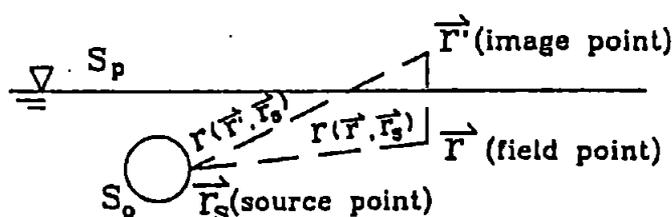


Figure C1 Submerged body nears half-space boundary

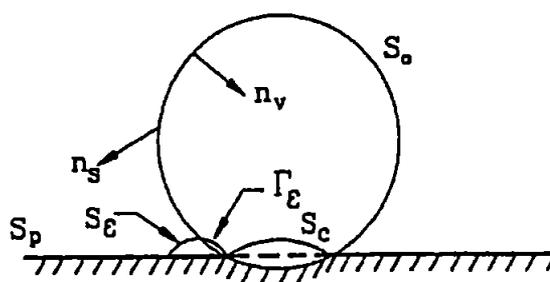


Figure C2 Submerged body situated in half-space

There are different boundary conditions which need to be considered for each type of the half-space propagation problem, and they are :

- (a) If the submerged body is near the free surface, then  $P = 0$  on  $S_p$ .
- (b) If the submerged body is situated on the seabed, then  $\frac{\partial P}{\partial n_0} = 0$  on  $S_p$ .

Choose the Half-Space Green's Function so considered as :

$$G_h = \frac{e^{ikr}}{r} + R_p \frac{e^{-ikr'}}{r'} \quad (C.4)$$

The above expression is just to increase an image point  $\vec{r}'$  to the field point  $\vec{r}$  with respect to the free surface. Where  $R_p$  represents the refraction ratio of the plane boundary surface  $S_p$ . When  $R_p = 1$ , ie, in the rigid body surface condition  $Z \rightarrow \infty$  and  $\frac{\partial G_h}{\partial n_0} = 0$  which means the velocity is zero as  $\vec{r}$ , located on  $S_p$ . When  $R_p = -1$ , ie, in the free surface conditions,  $Z=0$  and  $P=0$ . As  $\vec{r}$ , is on  $S_p$ ,  $G_h=0$ . Both the conditions  $R_p = 1$  and  $R_p = -1$  can satisfy equation (C.2).

## **Appendix D**

### **Publication Works**

The following technical papers related directly to the work of the study have been published during its progress.

1. W.H. Wang, R. Sutton and B. Dobson (1995) "Modelling Mobility and Transmissibility of Sound and Vibration from Machinery to Ship Structure", *Journal of the Society of Naval Architects and Marine Engineers, ROC*, Vol. 14, No.2, pp. 31-53.(This paper was awarded as to be the outstanding paper on the annual meeting of the Society of Naval Architects and Marine Engineers, ROC, 1995).
2. W.H. Wang, R. Sutton and B. Dobson (1997) "Structure-Borne Noise Control for a Marine Engine", *Proceedings of the 10th Symposium of the Acoustical Society, ROC, Taipei*, pp. 190-199.
3. W.H. Wang, R. Sutton and B. Dobson (1997) "Behaviour of Structure-Borne Noise Attenuation in a Resilient Mount", *Proceedings of TRAM'97 SINGAPORE*, pp. 241-251.
4. W.H. Wang, R. Sutton and B. Dobson (1998) "Behaviour of Structure-Borne Noise Attenuation in a Resilient Mount", *Journal the Society of Naval Architects and Marine Engineers, ROC*, Vol.17, No.2, pp. 49-62.
5. W.H. Wang, R.J. Shyu and J.R. Chang (1998) "Viscoelastic Passive Damping Technology on Ship's Vibration and Noise Control", *Proceedings of PRADS'98, Hague*, pp. 943-950.
6. W.H. Wang, and T.C. Yang (1999) "Mobility Analysis of the Improved Bottom Structure by Squeeze Film for Structure-Borne Noise Reduction", *Proceedings of Third International Students' Congress of the Asia-Pacific Region Countries, Part II*, pp. 7.1-7.18.(This paper was awarded as to be the outstanding paper on the field of ocean engineering and shipbuiding.)
7. W.H. Wang, J.H. Liou, R. Sutton and B. Dobson (2000) "Machine Vibration Induced Underwater Acoustic Radiation", *Journal of Marine Science and Technology*, Vol.8, No.1, pp. 1-11.

## MODELLING MOBILITY AND TRANSMISSIBILITY OF SOUND AND VIBRATION FROM MACHINERY TO SHIP STRUCTURE

W.H. Wang\* R. Sutton\*\* and B. Dobson\*\*

\* Department of Naval Architecture  
National Taiwan Ocean University, ROC

\*\* Institute of Marine Studies, University of Plymouth, UK

*Keywords: Point/Transfer Mobility, Impedance, Noise Source, Structure-Borne*

### ABSTRACT

The vibratory power of structure-borne noise sources in a vessel generated by the reduction gear, generator and reduction gear, all of which are located in the engine room and are fitted to the ship structure, have been investigated in this study. Calculation of the structure-borne vibratory power flow transmitted through the multi-paths between the above mentioned operating machineries and the connected structures has been determined by utilizing the measurement data of the mobilities and by characterizing the noise sources as velocity sources.

To ensure that the data measurements collected onboard a fast patrol boat were correct, details of the instrument calibration are given. Also valuable data checking methods are submitted and utilized in this work. The parallel measurement technique is validated and found to be applicable to the transfer mobility measurement of the elastic mountings.

Experimental data results and the calculations of the transmitted vibrational power flow show that the vertically transmitted power from the main engine to the seating structure is the most dominant structure-borne noise source in the engine room.

## 機械振動噪音傳入船體結構之動性及傳遞性模式

王偉輝\* R. Sutton\*\* B. Dobson\*\*

\* 國立海洋大學 造船工程系

\*\* 英國普利茅斯大學 海洋科技研究所

關鍵詞：點/傳動性，阻抗，音源，結構傳遞

### 摘 要

本文探討船舶機艙內之噪音源，如主機、發電機及減速機等，於運作時，其內之振動能量，如何藉由多管道(Multi-Path)傳遞至結構體之動性(Mobility)量測，而算出傳遞至結構域振動能量流。

對於量測儀器表規之校準方法作了詳盡的討論，以期於高速艇量測所得數據可信。對量測所得結果為證實可信，文中尚提出幾項頗具價值的核驗方法。

證實平行量測技術(PMT)應用在實船動性之量測為可行，因此提供一種簡化的近似動性量測法。

對於國內承造之高速艇實測結果知，主機垂向的振動能量傳遞為最大。該艇全速航行時，發現機艙內之機座，有些是將機械振動之能量傳遞至船底結構中；亦有一些機座是將船底結構之振動能量傳至轉運之機器內，船底之驅動振動能量是由軸系振動及螺旋轉動其四週流場之擾動水壓所造成。

## INTRODUCTION

### 1. Introduction to Structure-borne Noise Problem

Noise in a ship can be a problem. Depending on the characteristics of the noise, it can be classified into either being structure-borne or airborne. Airborne noise is usually radiated from sound sources such as a fan, ventilator and machinery etc. This kind of noise can be dealt with by using acoustic insulation enclosures or/and absorption material to efficiently reduce and improve the noise level. Basically, the airborne noise problem is pertinent to a more localized area.

However, structure-borne noise is caused by vibratory sources which can induce structural vibrations. Such structural vibrations can propagate throughout the ship's compartments or cabins, to form radiated noise. The frequency range for audible radiated noise is from 20 Hz to 20 kHz. In other words, if noise is present in some compartment due to structure-borne transmission, it is very hard to search for the noise source simply in area around the local compartment. Therefore, structure-borne noise analysis constitutes an integrated problem.

During the last decade, noise onboard ships has been regarded as a serious problem. Especially in a large number of fast patrol boats which are constructed of light structures and fitted with powerful propulsion engines that are located in confined spaces that, as a consequence, cause noise and vibration problems. The noise and vibration levels at or close to such a vibratory source can be extremely high and are, therefore, undesirable. Hence, the overall aim of this paper is to conduct an in depth investigation into the mechanical mobilities of diesel engines and the accompanying accessories for structure-borne noise and vibration transmission estimations.

In practice, machines are sometimes mounted on large flexible structures (referred

to as the seating) which have so many modes of vibration and, therefore, it is not convenient to consider each mode separately. For this situation the approach adopted in [1-4] has been to consider the seating as being of infinite extent in order to predict the frequency averaged power by using simple mobility formulae [5]. It is very useful to determine how to model simple finite seatings and thus to approximate it as an infinite seating by making frequency averaged power predictions like those of references [1-4]. Also it is beneficial to find the parameters which are required for a power transmission source from a structure of frequency dependent mobility.

### 2. Prediction Methods for Propagation of Structure-borne Noise Propagation in Ships

Comprehensive studies [6-16] with the aim of surveying noise transmission and developing prediction methods from diesel engines and various marine machineries to the seatings have been undertaken. The majority of prediction methods for the structure-borne noise propagation emphasized are basically empirical and are written in computer software programs which are easy to operate. The accuracy of these programs are satisfactory for some standard ship designs. However, parameter studies and prediction for non-standard ship design cannot be accommodated using these methods. Other prediction methods are in general based on the statistical energy analysis (SEA) method [17] or the so-called waveguide methods [18]. Such methods which describe the propagation of structure-borne noise in the ship structure have certain limitations in the low frequency region, i.e. below the first few plate resonances. The first resonance frequency can be fairly high for a plate element in the tanktop construction or in the engine foundation. This naturally causes uncertainties for the predicate noise levels in the octave bands with centre frequencies at and below 125 Hz. However, Nilson [19-21] has devel-

oped a method to extend the waveguide model for the calculation of the attenuation of structure-borne sound in the junction between plate panel and boundary stiffener in a ship structure.

A number of prediction methods based on measurement data of the structure-borne noise propagation have been proposed. Among these, probably the most well-known is the method by Janssen and Buiten [22]. Also further developments of this method [23,24], where more detailed data for the reductions in velocity levels

per deck and per frame are taken into account, are available. Common to all these empirical methods is the requirement that a velocity level for the foundation of the source is used as input data.

In Table 1 different input data requirements for the structure-borne sound sources are summarised for a couple of typical noise control activities. Also, the comparison of parameters which govern the structure-borne sound characteristics of different types of engines and foundation are given.

Table 1 Specification of the input data required for some activities concerning the reduction of structure-borne sound

Calculation of structure-borne sound propagation in the ship structure	
<u>Calculation method</u>	<u>Input data</u>
Empirical methods [22-24]	Velocity levels on foundations
Statistical energy analysis [25-28]	Transmitted power ( velocity levels and effective mobility of foundation )
Grillage method [19,21]	Transmitted moments ( angular velocity and moment mobility of foundation)
Comparison of structure-borne sound characteristics of different types of engines and foundations	
<u>Methods</u>	<u>Data</u>
Statistics of structure-borne sound levels at the contact points of engines [22,23]	Shaft speed, shaft power, main structure characteristics and velocity levels of engines and foundations
Comparison of structure - borne sound levels of one machinery on different foundations [7,29]	Mobility of foundations and mobility of machinery
Reduction of structure-borne sound power transmission by changes in design of machinery and / or foundations	
<u>Methods</u>	<u>Data</u>
Empirical methods (model and / or full-scale experiments) [30-32]	Velocity levels at the foundation and the hull
Partly theoretical methods [33,34]	Velocity levels and mobilities for the foundation and the hull

## MOBILITY AND POWER FLOW TRANSMISSION

### 1. Mobility of Single Degree of Freedom (SDOF) Vibration System

Consider a spring-mass-dashpot single degree of freedom system (SDOF) with mass, damping and stiffness coefficients being  $m$ ,  $c$  and  $k$  respectively. If the system is subjected to an exciting force  $f(t)$ , then the equation of motion for the vibration response  $x(t)$  can be expressed as,

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (1)$$

If the excitation is harmonic, i.e.,  $f(t) = Fe^{i\omega t}$ , then the steady-state solution of equation (1) will be  $x(t) = Xe^{i\omega t}$ , where  $X$  is the complex response amplitude and  $\omega$  is the excitation frequency. The dynamic flexibility or receptance  $R(\omega)$  is defined as,

$$R(\omega) = \frac{x(t)}{f(t)} = \frac{1}{[(k - \omega^2 m) + i\omega c]} \quad (2)$$

The mobility of the SDOF System is defined as,

$$M(\omega) = \frac{\dot{x}(t)}{f(t)} = i\omega R(\omega) = \frac{i\omega}{[(k - \omega^2 m) + i\omega c]} \quad (3)$$

Putting the natural frequency  $\omega_n = \sqrt{\frac{k}{m}}$  and the damping ratio  $\zeta = \frac{c}{2\omega_n m}$  into equation (3) then,

$$M(\omega) = \frac{i\omega}{m[(\omega_n^2 - \omega^2) + 2i\zeta\omega_n\omega]} \quad (4)$$

It is obvious that the mobility of a system is always a complex quantity and is frequency dependent. Rewrite equation (3) as,

$$M(\omega) = \frac{i\omega}{[m(\frac{k}{m} - \omega^2) + i\omega c]} \quad (5)$$

It can be seen that the mobility is influenced by the system parameters, which are mass, damping and stiffness, and related to the exciting frequency. If the damping ratio is small, then the mobility has the following characteristics.

1. As  $\omega \gg \omega_n$ ,

$$M(\omega) \approx -\frac{i}{m\omega} \quad (6)$$

In this condition, the mobility is said to be mass dependent only.

2. As  $\omega = \omega_n$ ,

$$M(\omega) = \frac{1}{c} \quad (7)$$

At resonance, the mobility is said to be damping dependent only.

3. As  $\omega \ll \omega_n$ ,

$$M(\omega) \approx -\frac{i\omega}{k} \quad (8)$$

In the case, the mobility is stiffness dependent only.

Taking logarithms and absolute values on both sides of the equations (6) to (8), then the following relations can be obtained.

$$\log |M(\omega)| = \log \left( \frac{1}{\omega m} \right) = -\log(m) - \log(\omega), \quad \text{for } \omega \gg \omega_n \quad (9)$$

$$\log |M(\omega)| = \log \left( \frac{1}{c} \right), \quad \text{for } \omega = \omega_n \quad (10)$$

$$\log |M(\omega)| = \log \left( \frac{\omega}{k} \right) = \log(\omega) - \log(k), \quad \text{for } \omega \ll \omega_n \quad (11)$$

All the equations (9) to (11) show that  $M(\omega)$  and  $\omega$ , which in case are expressed in logarithmic coordinates, have linear relations in the very high and very low frequency ranges as shown in Figure 1.

In Figure 1, the mobility spectrum curve approaches to a straight line with slope equals to 1 at far lower frequency region. The intersection distance  $\overline{OA}$  along the ordinate axis will be,

$$\overline{OA} = \log |M(\omega)|_{\log \omega = 0} = -\log(k) \quad (12)$$

or

$$k = [|M(\omega)|_{\log(\omega)=0}]^{-1} \quad (13)$$

Secondary, the mobility spectrum curve appears peak value,  $\overline{OC}$ , which means,

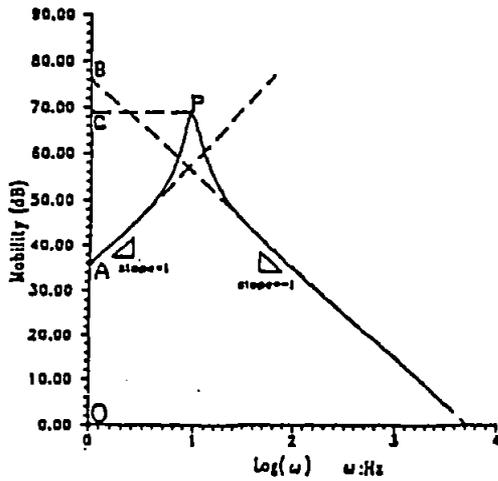


Fig.1 Characteristics of mobility spectrum for a SDOF system

$$\overline{OC} = \log |M(\omega_n)| = -\log \left( \frac{1}{c} \right) \quad (14)$$

or

$$c = [|M(\omega_n)|]^{-1} \quad (15)$$

Finally, the mobility spectrum curve approaches to another straight line with slope equals to -1 at higher frequency region. The intersection distance  $\overline{OB}$  along the ordinate axis will be,

$$\overline{OB} = \log |M(\omega)|_{\log \omega = 0} = -\log(m) \quad (16)$$

or

$$m = [|M(\omega)|_{\log(\omega)=0}]^{-1} \quad (17)$$

Equations (13), (15) and (17) constitute the basis for some valuable correct data checking methods for the mobility measurements undertaken onboard the fast patrol boats used in this research project.

## 2. Mobility of Multidegree of Freedom (MDOF) Vibration System

Consider an  $N$  degrees of freedom undamped system, both the mass matrix  $[m_{ij}]$  and the stiffness matrix  $[k_{ij}]$  are  $N \times N$  matrices. Similar to the SDOF system, the receptance matrix  $R(\omega)$  can be expressed as,

$$R(\omega) = [r_{ij}(\omega)] = ([k_{ij}] - \omega^2[m_{ij}])^{-1} \quad (18)$$

For a linear dynamic system, the normalized modal matrix  $\Phi$  possesses the following orthogonality properties,

$$\Phi^T [m_{ij}] \Phi = I = \begin{bmatrix} \ddots & & & \\ & m_1 = 1 & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \quad (19)$$

and

$$\Phi^T [k_{ij}] \Phi = I = \begin{bmatrix} \ddots & & & \\ & \omega_l^2 & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \quad (20)$$

where  $I$  is a  $N \times N$  identity matrix,  $m_l$  is the  $l^{\text{th}}$  modal mass and  $\omega_l$  is the  $l^{\text{th}}$  natural frequency.

Premultiply and postmultiply both sides of the inverse of equation (18) by the modal matrices  $\Phi^T$  and  $\Phi$  respectively, it becomes

$$\Phi^T ([k_{ij}] - \omega^2[m_{ij}]) \Phi = \Phi^T R(\omega)^{-1} \Phi \quad (21)$$

Introducing the orthogonality relations into (21), the receptance matrix becomes:

$$R(\omega) = \Phi \begin{bmatrix} \ddots & & & \\ & (\omega_l^2 - \omega^2)^{-1} & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \Phi^T \quad (22)$$

The element  $R_{ij}$  in the receptance matrix  $R(\omega)$  means the ratio of the displacement response in the direction of the  $i$ th degree of freedom to unit force in the direction of the  $j$ th degree of freedom. From (22), the receptance coefficient can be expressed as,

$$R_{ij}(\omega) = \sum_{l=1}^N \frac{\Phi_{il} \Phi_{jl}}{\omega_l^2 - \omega^2} = \sum_{l=1}^N \frac{A_{ijl}}{\omega_l^2 - \omega^2} \quad (23)$$

where  $A_{ijl} = \frac{\Phi_{il} \Phi_{jl}}{m_l}$ . For damped MDOF system, equation (23) becomes

$$R_{ij}(\omega) = \sum_{l=1}^N \frac{A_{ijl}}{(\omega_l^2 - \omega^2) + i \left( \frac{\omega c_l}{m_l} \right)} \quad (24)$$

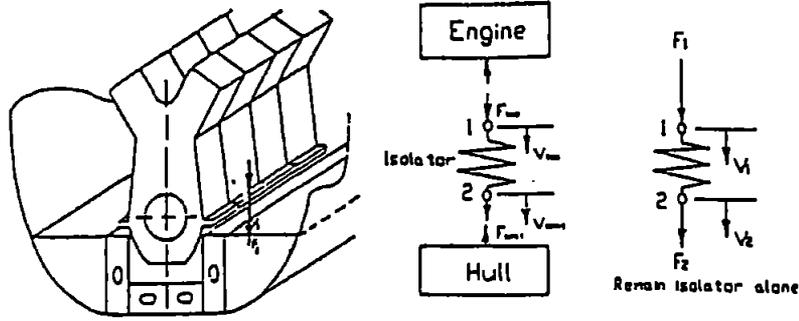


Fig.2 Modelling the engine-hull coupling as a 1-D isolator suspension system

The mobility coefficient can be expressed as

$$M_{ij}(\omega) = \sum_{i=1}^N \frac{i\omega A_{ijl}}{(\omega_l^2 - \omega^2) + i\left(\frac{\omega c_l}{m_l}\right)} \quad (25)$$

From equation (25), it can be seen that the mobility coefficient of an  $N$  degrees of freedom system can be expressed as the sum of the  $N$  modal mobilities.

In reality, mechanical structure system has infinite degrees of freedom. It can be approximated by a  $N$  degrees of freedom system, where some higher modes must be approximation. The mobility of the real system has to add a residual term  $C + iD$  to equation (25), i.e.,

$$M_{ij}(\omega) = \sum_{i=1}^N \frac{i\omega A_{ijl}}{(\omega_l^2 - \omega^2) + i\left(\frac{\omega c_l}{m_l}\right)} + C + iD \quad (26)$$

### 3. Impedance of Multipole Coupling System

Structure-borne sound excitation caused by mechanical coupling between engine and hull structure is shown in Figure 2. By assuming it can be simplified to a 1-direction coupling then it may be modelled as a isolator suspension system. The dynamic behaviour of this isolator element can be expressed in terms of frequency response characteristics. If the responses are defined as velocities  $V_1, V_2$  the frequency response function will take the form of mobility or impedance relationships. Thus,

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (27)$$

or

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} \quad (28)$$

So the mobility matrix  $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$  is the inverse of impedance matrix

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

i.e.

$$M = Z^{-1} \quad (29)$$

When the isolator is attached to a sub-structure, the input impedance ( $Z_{seat}$ ), before attachment of the isolator/machine assembly, is related to the force and velocity by

$$F = Z_{seat}V \quad (30)$$

Then, after attachment the following relationship can be established [6]:

$$\begin{aligned} \frac{F_{top}}{V_{top}} &= Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_{seat}} \\ &= \frac{M_{seat} + M_{22}}{M_{11}(M_{22} + M_{seat}) - M_{12}M_{21}} \end{aligned} \quad (31)$$

$$\begin{aligned} V_{seat} &= \frac{-Z_{21}}{Z_{22} + Z_{seat}} V_{top} \\ &= \frac{M_{21}M_{seat}}{M_{11}(M_{22} + M_{seat}) - M_{12}M_{21}} V_{top} \end{aligned} \quad (32)$$

and

$$\begin{aligned} \frac{F_{seat}}{V_{top}} &= \frac{-Z_{21}}{1 + \frac{Z_{22}}{Z_{seat}}} \\ &= \frac{M_{21}}{M_{11}(M_{22} + M_{seat}) - M_{12}M_{21}} \end{aligned} \quad (33)$$

Where  $F_{top}, V_{top}, F_{seat}$  and  $V_{seat}$  are the force and velocity at the engine side and seating side of the isolator respectively.  $Z_{11}, Z_{22}$  are point

impedances and  $Z_{12}$ ,  $Z_{21}$  are transfer impedances.

Generally an isolator will couple many degrees of freedom (MDOF) so that the order of the matrix impedance terms defined above will increase. It is assumed that the connection sites can be viewed as point connections, at which six degrees of freedom are required, thus equations (27) and (28) will involve  $12 \times 12$  matrices. Hence the impedance characteristics of the isolator can be expressed in the form of partitioned matrices [37].

$$\begin{Bmatrix} F_{\text{top}} \\ F_{\text{bottom}} \end{Bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} \\ Z_{ba} & Z_{bb} \end{bmatrix} \begin{Bmatrix} V_{\text{top}} \\ V_{\text{bottom}} \end{Bmatrix} \quad (34)$$

and that of the seating by the equation:

$$\{F_{\text{seat}}\} = [Z_{\text{seat}}] \{V_{\text{seat}}\} \quad (35)$$

When the isolator is coupled to a seating, the overall impedance relationship can be written in the form,

$$\begin{Bmatrix} F_{\text{source}} \\ 0 \end{Bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} \\ Z_{ba} & Z_{bb} \end{bmatrix} \begin{Bmatrix} V_{\text{source}} \\ V_{\text{seat}} \end{Bmatrix} \quad (36)$$

where

$$\begin{aligned} \{F_{\text{source}}\} &= \{F_1 \ F_2 \ F_3 \ F_4 \ F_5 \ F_6\}^T \\ &= \text{the forces at the source} \\ \{V_{\text{source}}\} &= \{V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6\}_{\text{source}}^T \\ &= \text{the velocities at the source} \\ \{V_{\text{seat}}\} &= \{V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6\}_{\text{seat}}^T \\ &= \text{the velocities at the seating} \end{aligned}$$

On the overall impedance matrix,  $[Z_{aa}]$ ,  $[Z_{ab}]$  and  $[Z_{ba}]$ , will each be  $6 \times 6$  matrices defined purely by the properties of the isolator.  $[Z_{bb}]$  will be the matrix sum of the two  $6 \times 6$  matrices defined at the base of the isolator and at the input seating point (i.e.  $[Z_{BB}] = [Z_{bb}] + [Z_{\text{seat}}]$ ). From equations (34)-(36) it follows that the velocities and forces at the base of the isolator will be given by:

$$\{V_{\text{seat}}\} = -[Z_{BB}]^{-1} [Z_{ba}] \{V_{\text{source}}\} \quad (37)$$

$$\begin{aligned} \{F_{\text{seat}}\} &= -[Z_{\text{seat}}] ([Z_{\text{seat}}] + [Z_{bb}])^{-1} \\ &\quad [Z_{ba}] \{V_{\text{source}}\} \end{aligned} \quad (38)$$

and

$$\begin{aligned} \{F_{\text{source}}\} &= ([Z_{aa}] - [Z_{ab}] + [Z_{BB}])^{-1} \\ &\quad [Z_{ba}] \{V_{\text{source}}\} \end{aligned} \quad (39)$$

It should be noted that the presence of off-diagonal terms within impedance matrices complicate any definition of vibration transmission paths. These terms imply that motion in one direction is capable of transmitting power in all coordinate directions. Thus, power measured in one direction at the base of the isolator will be composed of components associate with all six direction or rotations at the top of the isolator. It must be established how important the cross-coupling terms are with respect to power before they are omitted from any calculations.

Multi-point isolation systems will lead to the introduction of additional terms into the impedance relationships. In particular, the seating impedance equations may include significant coupling terms between sites so that the overall form of equation (39) may be fully populated and of order  $6 \times$  number of isolators. The explicit solution of the resultant equation will involve considerable matrix manipulation and the probability of introducing numerical errors because of ill-conditioning will increase. Thus, every effort must be made to simplify the problem as possible without introducing significant errors. Symmetry, cross-coupling, geometric shapes and sizes and frequency dependent effects should be considered during the simplification phase.

#### 4. Parallel Measurement Technique

When an engine is installed onboard ship and the alignment is made, then only the overall point and transfer mobilities of the system to combine engine, elastic mounting and foundation structure together can be measured. There are some difficulties in directly measuring the point and transfer mobilities simply on both sides (i.e. the driving side or the engine side and the receiving side or the seating side) of the contact elastic mounting. To overcome the above difficulties in making precise spot measurements

Table 2 Range of audible structure-borne sound wave length in ship structure

Structure-borne Sound	Sound Velocity $c$ (cm/s)	Sound Wave Length (cm)	Audible Sound Frequency Range (Hz)	Range of Wave Length (cm)
Longitudinal sound wave in rod	$C_L = \sqrt{\frac{E}{\rho}} = 5.17 \times 10^4$	$\lambda = \frac{2\pi c_L}{\omega} = \frac{32.5 \times 10^4}{\omega}$	20-20000	16-16000
Longitudinal sound wave in plate	$C_L = \sqrt{\frac{E}{\rho(1-\nu^2)}} = 5.43 \times 10^4$	$\lambda = \frac{2\pi c_L}{\omega} = \frac{34.1 \times 10^4}{\omega}$	20-20000	17-17000
Shear or twisting sound wave in rod	$C_T = \sqrt{\frac{G}{\rho}} = 3.19 \times 10^4$	$\lambda = \frac{2\pi c_T}{\omega} = \frac{20.1 \times 10^4}{\omega}$	20-20000	10-10000
Bending sound wave in beam	$C_B = \sqrt{\frac{EI}{m}} \sqrt{\omega} = 136\omega^{0.5}$	$\lambda = \frac{2\pi c_B}{\omega} = \frac{20.1 \times 10^4}{\omega}$	20-20000	6-190

\* Note:

All the figure for material constants are based on structure steel.

$$E = \text{Young's modulus} = 21 \times 10^9 \left( \frac{\text{g}}{\text{cm} - \text{s}^2} \right)$$

$$\nu = \text{Poisson's ratio} = 0.3$$

$$\rho = \text{Steel density} = 7.85 \left( \frac{\text{g}}{\text{cm}^3} \right)$$

$$G = \text{Shear modulus} = 8 \times 10^9 \left( \frac{\text{g}}{\text{cm} - \text{s}^2} \right)$$

$$I = \text{Moment of inertia of beam (cm}^4)$$

$$m' = \text{Mass per unit length of beam (g/cm)}$$

For optimally designed I-beam [3]:

$$I = 0.575 Z_{req}^{1.5} t_w^{0.5} \text{ (cm}^4)$$

$$w = 136 Z_{req}^{1.5} t_w^{0.5} \text{ (g/cm)}$$

where

$$Z_{req} = \text{Required sectional modulus of beam}$$

$$t_w = \text{Web thickness of beam}$$

$$w = \text{Weight per unit length}$$

Take

$$Z_{req} = 500 \text{ (cm}^3)$$

$$t_w = 1 \text{ (cm)}$$

$$I = 6430 \text{ (cm}^4)$$

$$m' = \frac{w}{g} = 3.04 \times 10^4 \left( \frac{\text{g}}{\text{cm}} \right)$$

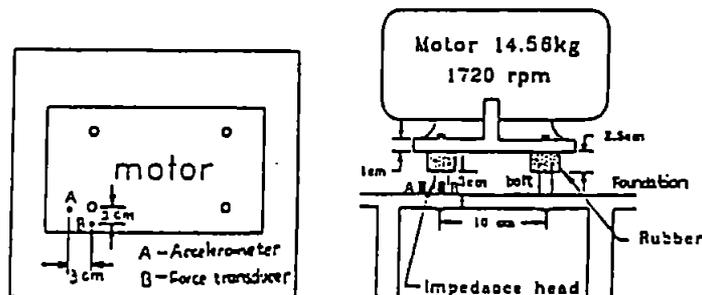


Fig.3 Experiment setup for PMT

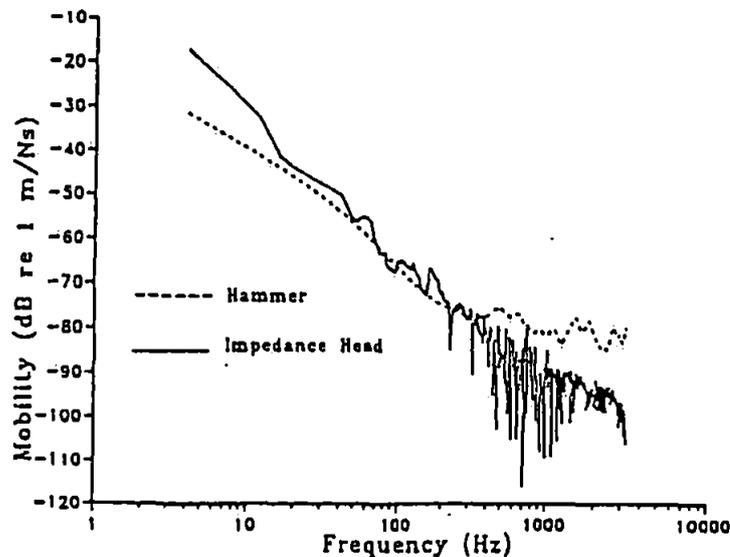


Fig.4 Mobility spectra measured by impedance head and PMT

onboard a vessel, a parallel measurement technique (PMT) can be employed to obtain the mobility and/or impedance data. However, a problem with this approach is in the determination of the positions of the sensors (accelerometer and force transducer) with respect to the spot being considered.

For a ship structure is mainly constructed of plates and stiffeners, therefore the structure-borne sound transmission includes longitudinal wave, shear or twisting propagation wave, and bending wave propagation in plates, bars and beams. Table 2 shows the typical range of audible structure-borne sound wave length in a ship structures [35]. By appreciating that the higher the frequency the shorter the wave length, the feasible region for applying PMT, it can be established that sensors should be placed less than quarter the possible shortest wave length away from the measurement point. From Table 2, it is suggested to be within 3-5 cm for PMT.

To validate the correctness of this technique, the mobility measurements for a motor of 0.5 HP with elastic mounting were undertaken in a laboratory. Figure 3 shows the experiment setup. The motor weighs 14.56 kg and the speed of revolution is 1720 rpm. 2 types of the measurement method were adopted. The first method utilized an impedance head, which was fastened coaxially to the bolt and welded to the

foundation, to measure the mobility at the connected point of the bolt to the foundation when the motor is running. The second method utilized a hammerkit to excite point B (2 cm away from the bolt) and put an accelerometer at A (3 cm away from the bolt) to measure the acceleration. Both the mobility spectra by direct measurement and PMT are shown in Figure 4. These two spectral curves coincide very well between 20 Hz to 400 Hz. In the higher frequency range (i.e. over 400 Hz), the running motor can not stimulate any significant higher mode vibration and the signal noise becomes obvious. Whilst in the lower frequency range, the difference may be due to the measurement errors. This test validates the availability by using a PMT for onboard measurement provided the sensors are close enough to the measured point and within a quarter wave length.

#### 5. Vibration Power Input to a Structure

The instantaneous vibration power input ( $P_{inst}$ ) to a structure is defined as the product of the excitation force ( $f$ ) and the associated velocity ( $v$ ). Thus, for an harmonic excitation  $|F|\sin\omega t$  applied at a point on a structure of mobility  $M = |M|e^{i\theta}$  causes a velocity of  $|V|\sin(\omega t + \theta)$  at that point.  $\theta$  is the phase angle between the driving excitation forces and the associated velocity. The instantaneous vibration power input will be:

$$P_{\text{inst}} = \mathbf{f} \cdot \mathbf{v} = |F||V| \sin(\omega t + \theta) \quad (40)$$

In general, it is of more interest to know the time averaged power input which can be calculated by integrating equation (40) over one period of vibration to yield:

$$\langle P \rangle = \frac{1}{T} \int_0^T |F||V| \sin(\omega t + \theta) dt \quad (41)$$

or

$$\langle P \rangle = \frac{1}{2} |F||V| \cos \theta \quad (42)$$

From equation (42) the averaged power input is related to the phase angle  $\theta$ .

At any point on a structure, the velocity and force can be related via mobility (or impedance) frequency response functions so that equation (42) may be rewritten in the following forms [4]:

$$\langle P \rangle = \frac{1}{2} |F|^2 \text{Re}[M] \quad (43)$$

or

$$\langle P \rangle = \frac{1}{2} |V|^2 \text{Re}[Z] \quad (44)$$

Where  $\text{Re}[M]$  and  $\text{Re}[Z]$  are the real part of the point mobility function and the point impedance at the excitation site respectively. The choice of which equation to apply in a practical situation is controlled by the nature of the vibration source and the ability to measure either the force or velocity. In many cases, equation (44) can be applied if a machine is unaffected by the method of attachment so that it behaves on site as if it was freely suspended. For a machine mounted on an isolation system this condition is usually fulfilled.

The above equations only apply to an harmonically varying point force. In general, machinery is supported at a number of sites and each site will have some contact area over which both direct forces and moments can be transmitted. This will lead to a very complicated set of inter-related expressions for the total input power. If it is assumed that there are  $N$  coordinates involved on the coupling of the machine to the sub-structure, then it will be necessary

to defined an  $N \times N$  matrix of mobility functions. Equations (43) and (44) can be rewritten in matrix form as:

$$\langle P \rangle = \frac{1}{2} |F|^T \text{Re}[M] \{F\} \quad (45)$$

or

$$\langle P \rangle = \frac{1}{2} |V|^T \text{Re}[Z] \{V\} \quad (46)$$

The prediction, or measurement, of these full matrix relationships can be extremely difficult. This is particularly true in such incidents where moment excitations represent a major source of the resultant vibrational power.

From the expressions (43) and (45), it is clear that the power injected to a structure by a machine is dictated by the excitation force or velocity exerted by a machine and the mobility or impedance characteristics of the supporting structure. The characterization of an engine as "velocity source" will also be considered here.

In most practical excitation sources, the driving forces are random. In this circumstance, the input power can be found by using of the force spectrum density function  $G_{ff}$  or the vibration velocity spectrum density function  $G_{vv}$  or the cross power spectrum density function  $G_{fv}$ , which can be expressed as:

$$\begin{aligned} \frac{P}{H_z} &= G_{ff} \text{Re}[M] = \text{Re}[G_{fv}] \\ &= G_{vv} \text{Re}[Z] \end{aligned} \quad (47)$$

The driving point impedance matrix and the receiving point impedance matrix for the well-installed system can be derived from equation (29):

$$Z_{\text{input}} = \frac{M_{22}}{M_{11}M_{22} - M_{12}M_{21}} \quad (48)$$

$$Z_{\text{output}} = \frac{M_{11}}{M_{11}M_{22} - M_{12}M_{21}} \quad (49)$$

Substituting  $Z_{\text{input}}$  and  $Z_{\text{output}}$  into equation (47), the input power and output power can be calculated respectively. Equations (47)-(49) will provide the theoretical basis for the measurements.

## MEASUREMENT OF POINT AND TRANSFER MOBILITIES OF MACHINERY TO FOUNDATION ONBOARD A FAST PATROL VESSEL

The fast patrol vessel being investigated in this study is of 100 gross tonnage, under the maximum continuous output of main diesel engine of 3000 Ps \*2 sets with rpm=2100 and gear ratio=2.8, shaft rpm=750, it can achieve a maximum speed of approximately 32 knots. The measured sound pressure levels are:

- 105 dB in engine room
- 90 dB in captain room
- 85 dB in pilothouse

The main structure-borne sound sources are from:

- Main engines
- Reduction gears
- Generators

In order to achieve an in depth investigation of the structure-borne sound power transmitted, the following measurements have been undertaken onboard:

- 3 dimensional accelerations on the baseplate of the engine and the top of the seating structure at the sites near both ends of each elastic engine mounting bolt, the vibration is excited by the impulsive force from a hammerkit under the conditions of the vessel rested on land and afloat in water respectively.
- 1 dimensional vertical accelerations on the baseplate of the reduction gear box and the top of the seating structure at the sites near both ends of each mounting bolt matted by Chockfast, the vessel on land and in water.
- 1 dimensional vertical accelerations on the baseplate of the generator and the top of the seating structure at the sites near both ends of each elastic mounting bolt, excited by hammerkit when the vessel on land and in water.

- All the 1-dimensional vertical accelerations at the sites mentioned in the above first 3 items when the vessel at full running speed. In this condition, the vibration is mainly excited by the main engines, generators and reduction gears. The vibration acceleration of the exhaust pipe is too hot to measure.

### 1. Principal Particulars of the Vessel

Figure 5 shows the general arrangement of the vessel in which the structure-borne noise sources will be studied. The principal particulars of the vessel are:

Length overall	30.50 m
Breadth moulded	6.80 m
Depth moulded	3.55 m
Draft moulded	1.70 m
Maximum speed	31.55 knots
Cruising speed	28 knots

The specifications of the machineries in engine room are:

#### Main diesel engine

Model	MTU 16 V 396
Revolution speed	2100 rpm
Continuous rating	3000 P <sub>s</sub> * 2 sets
Weight	7.5 tons × 2 sets

#### Generator

Revolution speed	1800 rpm
Output	23 kw × 2 sets
Reduction gear	
Reduction gear ratio	2.8
Shaft speed	750 rpm
Type	V drive

### 2. Experiment Layout and Measurement Points

For the purpose of calculating the vibratory power transmission of the noise sources in the engine room, it is necessary for the transfer mobilities and vibration velocities at all the power transmission paths which link the vibration source and the hull structure to be known. The main sources considered being the propulsion diesel engine, generator, reduction gear and exhaust pipe.

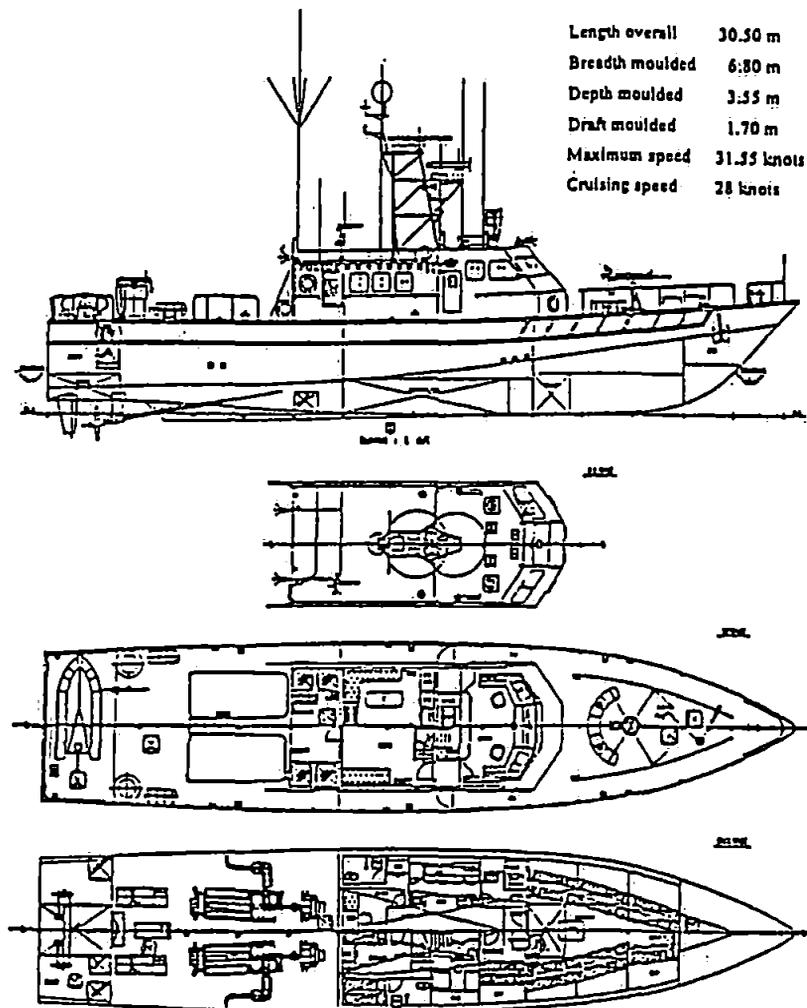


Fig.5 General arrangement

The mobility and vibration velocity measurements were taken at the following points:

**a. Point and transfer mobilities measurement of main diesel engine**

8 points located at both sides of 4 elastic mountings as shown in Figures 6 and 7, and 3 directions along longitudinal ( $X$ -), athwartship ( $Y$ -) and vertical ( $Z$ -) axes are measured.

**b. Point and transfer mobilities measurement of generator**

8 points located at both ends of 4 elastic mountings as shown in Figures 6 and 18, and only the vertical ( $Z$ ) direction are measured.

**c. Points and transfer mobility measurement of reduction gear**

8 point located at both sides of 4 selected positions as shown in Figures 6 and 9, are measured.

All these measurements were taken whilst the vessel rested on land and then afloat in water respectively.

The vibration velocity is measured at the same points in all the cases mentioned above except for the exhaust pipe, which could only be obtained when the vessel was travelling at full speed ahead.

Summarising the above definition of coordinate systems for measurement, primed systems are used to describe the foundation side and unprimed systems are for machinery side.  $X$  and  $X'$  axes are along longitudinal direction,  $Y$  and  $Y'$  athwartship,  $Z$  and  $Z'$  vertical as shown in Figure 10.

The unified notation of the mobility coefficient  $m_{ij}^{kl}$ , where  $i, j = X, Y, Z, X', Y', Z'$  and  $k, l = 1, 2, 3, 4$  means the ratio of the velocity along  $i$ -direction at the mounting position  $k$  to

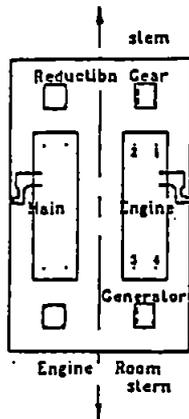


Fig.6 Numbering of the mounting positions for mobility measurement

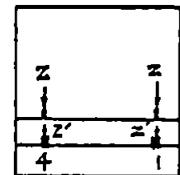
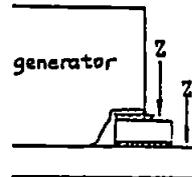
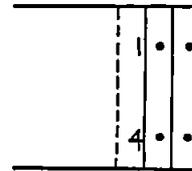


Fig.8 Coordinates define the measurement directions on generator

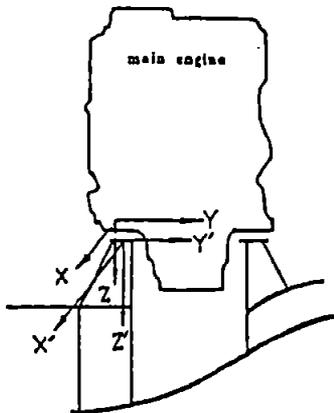


Fig.7 Coordinates define the measurement directions on diesel engine

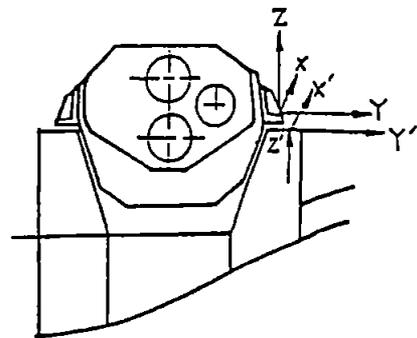


Fig.9 Coordinates define the measurement directions on reduction gear

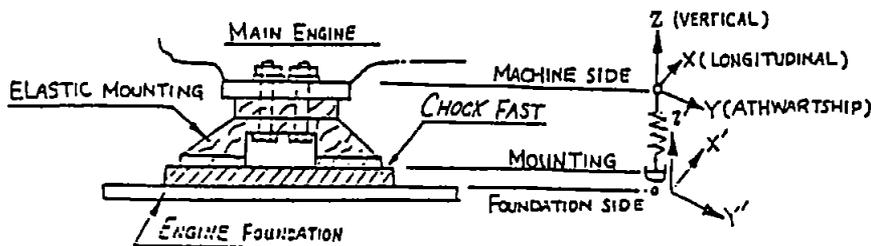


Fig.10 Unified notation for coordinate system used in a measurement position

the force along  $j$ -direction at the mounting position  $l$ .

### 3. Calibration on the Measurement System

The accelerometers, charge amplifiers, force transducer attached to the hammerkit and the digital tape recorder are utilized together to

measure the necessary data which relates to the mobility. To ensure correct measurements, the calibrations of each of the instruments system shown Figure 11 were made in the laboratory.

The calibration method adopted to determine the system calibration factor was by using of the identification of apparent mass. The procedure being as follows:

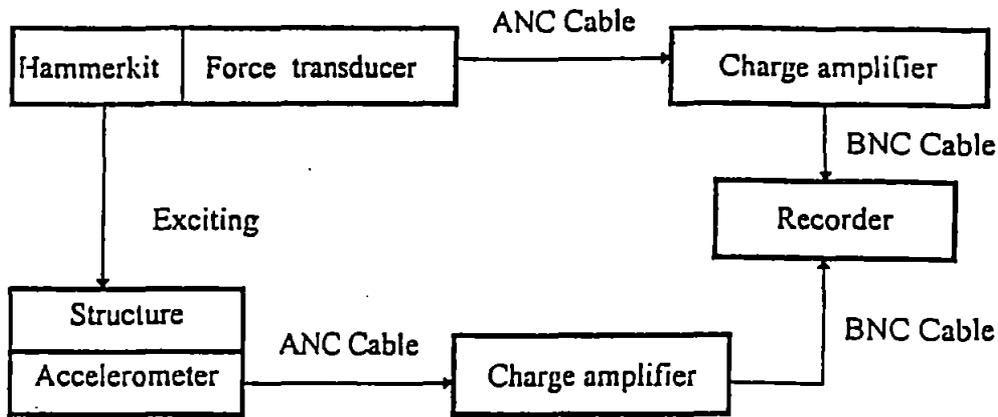


Fig.11 Instrument arrangement for mobility measurement

- (1) Find a stiff body of given mass.
- (2) Put the accelerometer on the stiff body and excite it colinearly with the accelerometer by hammerkit.
- (3) Numbering the instruments of a combination set, and arranging the connections of the force transducer, accelerometer, ANC, BNC cables, charge amplifiers and recorder as that shown in Figure 11.
- (4) Link the digital tape recorder to the frequency analyzer via BNC cable and undertake the apparent mass analysis of the stiff body after hitting of the hammerkit.
- (5) Compare the apparent mass and the actual measured mass to obtain the calibration factor of one set of the measurement instrument system. The actual measured mass should be the sum of the masses of the stiff body, the accelerometer and the hammerhead.

In this research programme, the actual measured mass is equal to the mass of the stiff body (122g) + the mass of accelerometer (54g) + the mass of hammerhead (10g) = 186g. The obtained calibration factors of each set of the measurement system are listed in Table 3.

#### 4. Measurement Result of Mobility and Velocity

Some of the measured data results of the mobility and vibration velocity are shown in Figures 12-63.

Table 3 Calibration factor of measurement instrument system

Instrument Set	1	2	3	4	5	6
Apparent Mass(g)	164	160	163	161	198	162
Calibration Factor	0.88	0.86	0.88	0.87	1.06	0.87

To survey the measurement results of the mobility spectrum, the following concluding remarks can be attained:

1. Water has a significant effect on the mobilities of ship structure. Comparison of the measured mobility data of several foundation structures of the engine room machineries shows that the frequency average mobilities of the vessel in water are 5-30dB lower than those of vessel on hand at most part of the measuring positions. Nevertheless, there are a small part of measurement points show that the frequency average mobility for vessel in water is 2-20 dB higher than those of vessel on-hand conditions. This phenomenon is mainly due to the changes of vibration modes of the structure for vessel on land and in water conditions. The hydrodynamic force effects should be engaged in for vessel in water.
2. Almost all the mobility spectrum curves have a tangent slope approximately equal to -1 in the low frequency region i.e.,  $\omega < 10$  Hz. This very good coincidence to the relation of equation (9) illustrates the correctness of the measurements.

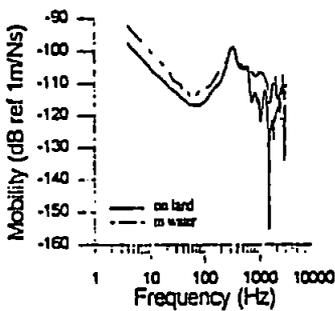


Fig.12  $M_{ZZ}^{11}$  of main diesel engine

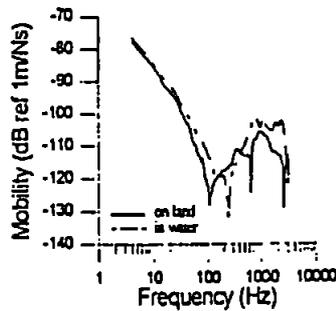


Fig.16  $M_{ZZ}^{22}$  of main diesel engine

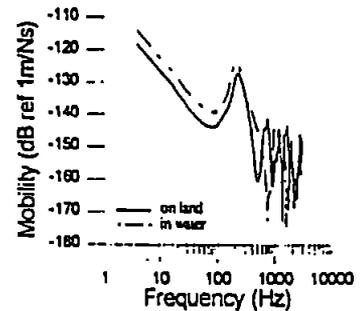


Fig.20  $M_{ZZ}^{33}$  of main diesel engine

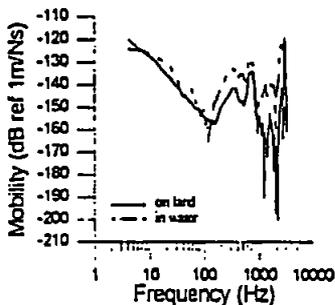


Fig.13  $M_{ZZ}^{11}$  of main diesel engine

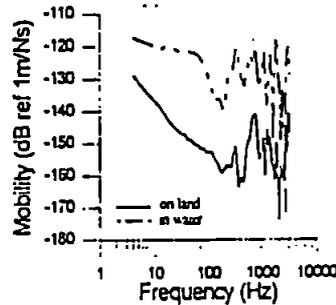


Fig.17  $M_{ZZ}^{22}$  of main diesel engine

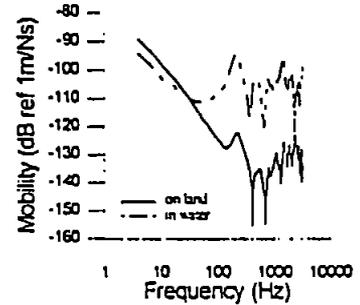


Fig.21  $M_{ZZ}^{33}$  of main diesel engine

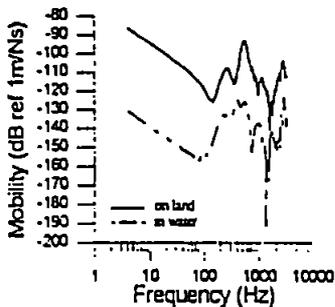


Fig.14  $M_{ZZ}^{11}$  of main diesel engine

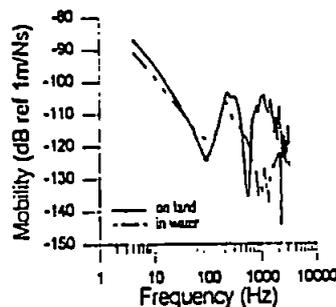


Fig.18  $M_{ZZ}^{33}$  of main diesel engine

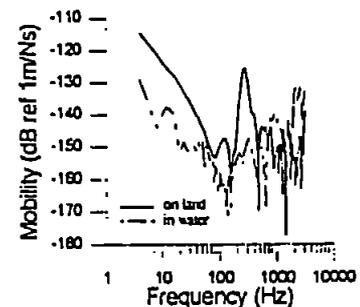


Fig.22  $M_{ZZ}^{44}$  of main diesel engine

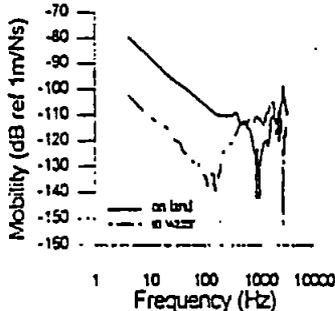


Fig.15  $M_{ZZ}^{11}$  of main diesel engine

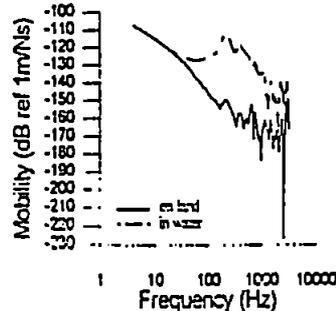


Fig.19  $M_{ZZ}^{33}$  of main diesel engine

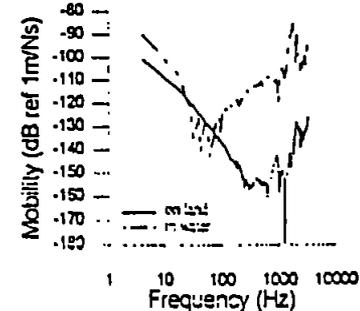


Fig.23  $M_{ZZ}^{44}$  of main diesel engine

### 5. Calculation of Structure-borne Noise Power Transmission via Mountings

All mountings can be considered as the paths of the initial vibrational power transmissions. The structure-borne noise power flow via mountings of the machineries in the engine room can be calculated by equations (47)-(49). By using the relevant measured mobility-spectra com-

bined with the vibrational velocity spectra during the vessel being full speed ahead which are described in the last paragraph, the transmitted power spectra via machine mountings are shown in Figures 64-77.

- Figure 64 - Figure 69 represent the input and output power spectra at both ends of the engine mountings.

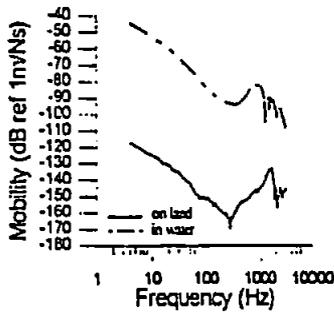


Fig. 24  $M_{Z'Z}^{11}$  of reduction gear

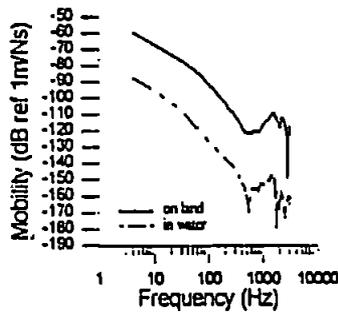


Fig. 29  $M_{Z'Z}^{22}$  of reduction gear

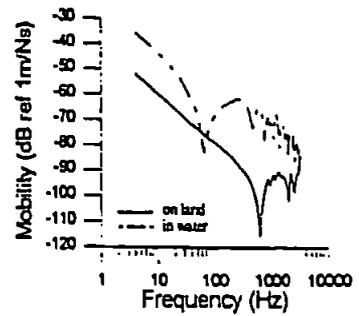


Fig. 34  $M_{Z'Z}^{33}$  of reduction gear

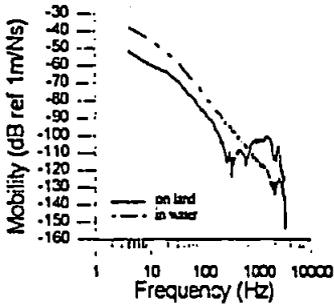


Fig. 25  $M_{Z'Z}^{41}$  of reduction gear

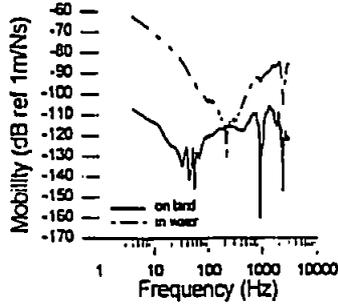


Fig. 30  $M_{Z'Z}^{12}$  of reduction gear

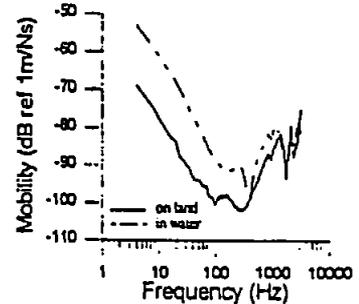


Fig. 35  $M_{Z'Z}^{23}$  of reduction gear

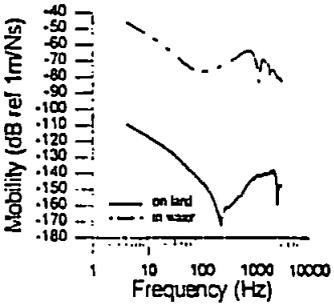


Fig. 26  $M_{Z'Z}^{11}$  of reduction gear

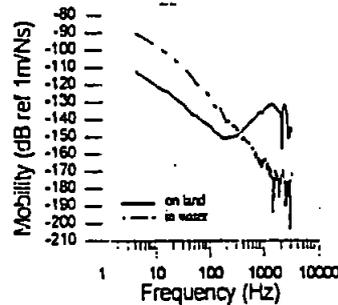


Fig. 31  $M_{Z'Z}^{32}$  of reduction gear

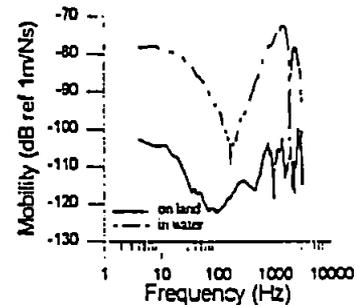


Fig. 36  $M_{Z'Z}^{34}$  of reduction gear

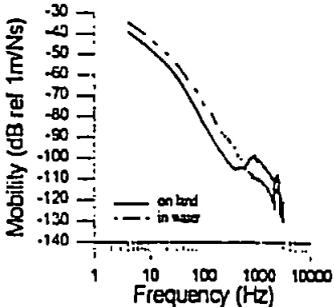


Fig. 27  $M_{Z'Z}^{41}$  of reduction gear

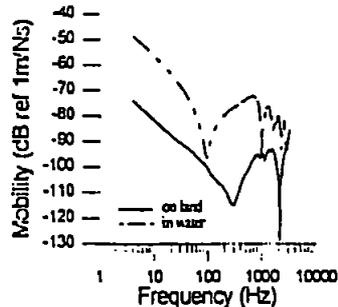


Fig. 32  $M_{Z'Z}^{33}$  of reduction gear

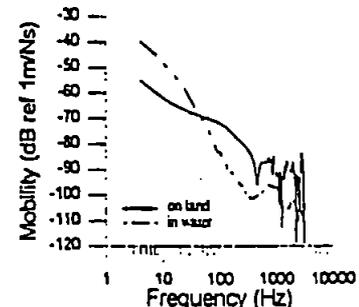


Fig. 37  $M_{Z'Z}^{44}$  of reduction gear

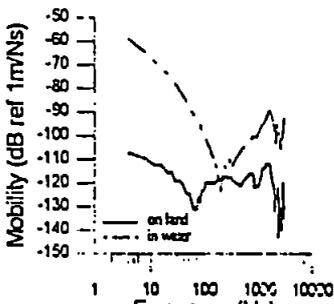


Fig. 28  $M_{Z'Z}^{12}$  of reduction gear

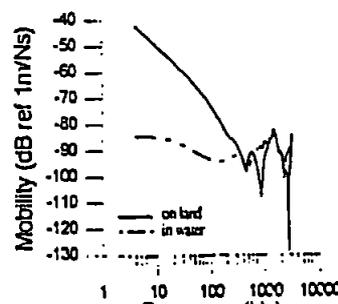


Fig. 33  $M_{Z'Z}^{23}$  of reduction gear

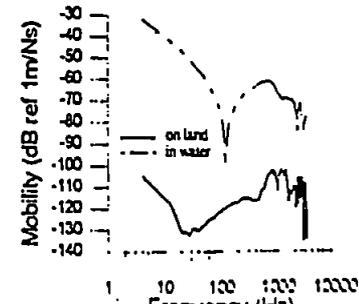


Fig. 38  $M_{Z'Z}^{34}$  of reduction gear

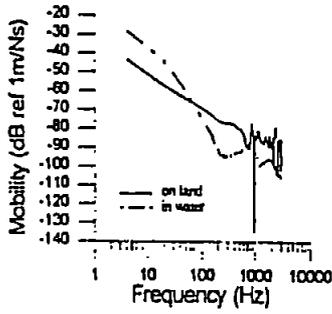


Fig.39  $M_{ZZ}^{44}$  of reduction gear

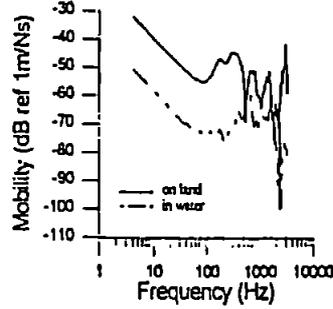


Fig.44  $M_{ZZ}^{22}$  of generator

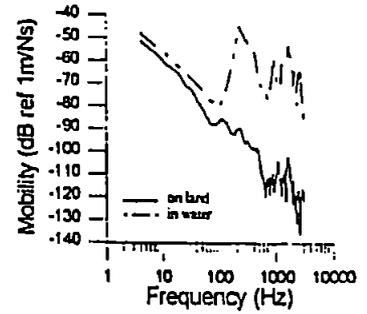


Fig.49  $M_{Z'Z}^{44}$  of generator

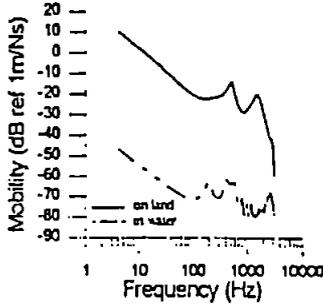


Fig.40  $M_{ZZ}^{11}$  of generator

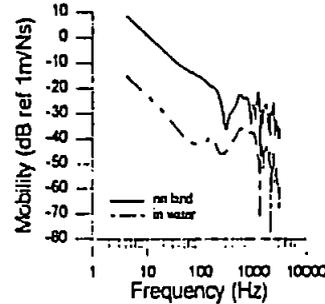


Fig.45  $M_{Z'Z}^{22}$  of generator

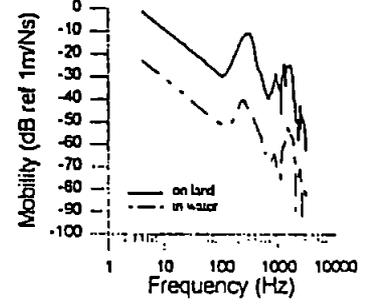


Fig.50  $M_{Z'Z}^{44}$  of generator

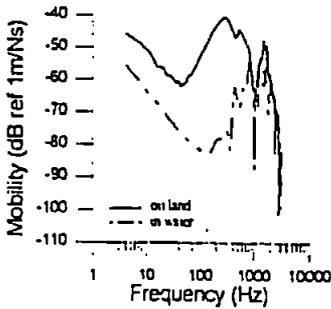


Fig.41  $M_{Z'Z}^{11}$  of generator

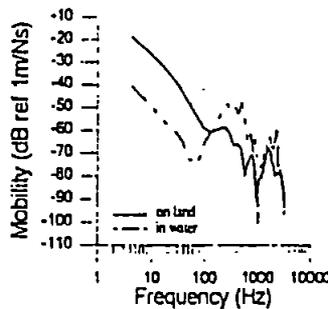


Fig.46  $M_{Z'Z}^{33}$  of generator

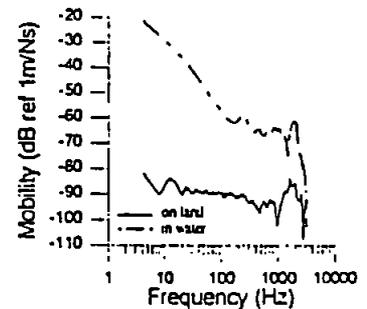


Fig.51  $M_{Z'Z}^{44}$  of generator

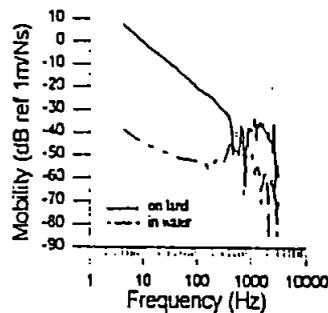


Fig.42  $M_{ZZ}^{22}$  of generator

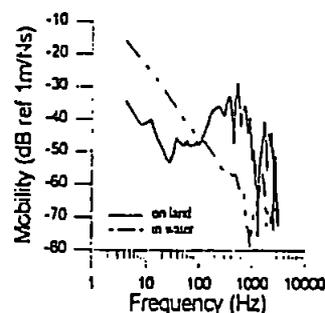


Fig.47  $M_{ZZ}^{33}$  of generator

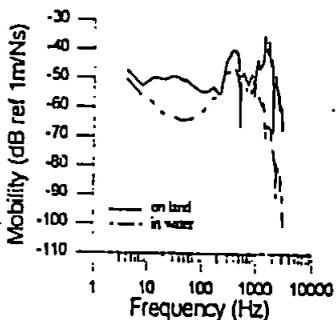


Fig.43  $M_{Z'Z}^{22}$  of generator

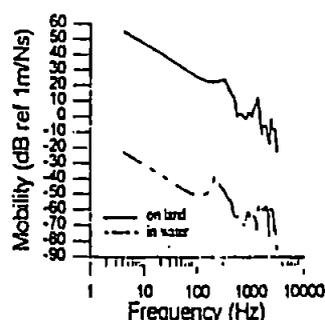


Fig.48  $M_{ZZ}^{44}$  of generator

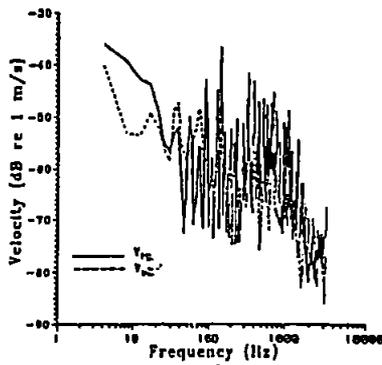


Fig.52  $V_{1z}$  and  $V_{1z}'$  of main diesel engine during vessel full speed ahead

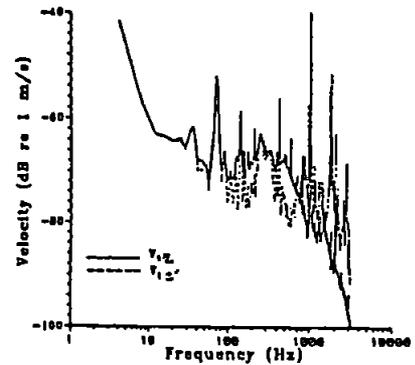


Fig.56  $V_{1z}$  and  $V_{1z}'$  of reduction gear during vessel full speed ahead

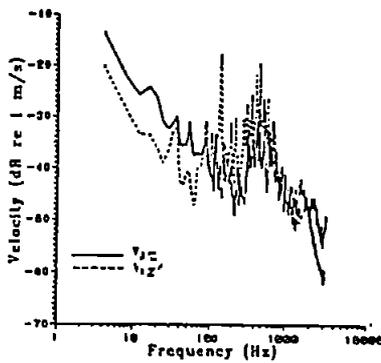


Fig.53  $V_{2z}$  and  $V_{2z}'$  of main diesel engine during vessel full speed ahead

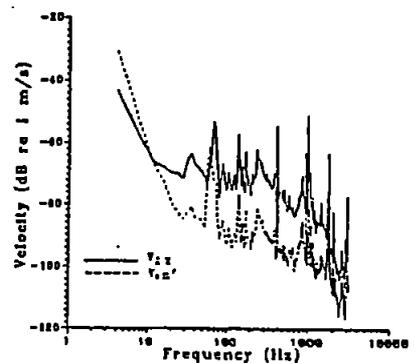


Fig.57  $V_{2z}$  and  $V_{2z}'$  of reduction gear during vessel full speed ahead

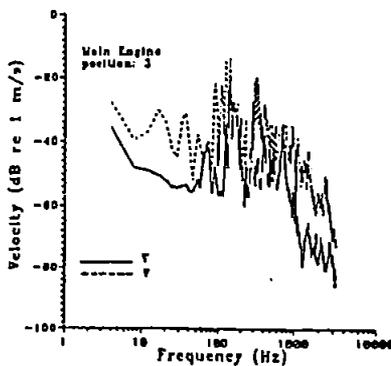


Fig.54  $V_{3z}$  and  $V_{3z}'$  of main diesel engine during vessel full speed ahead

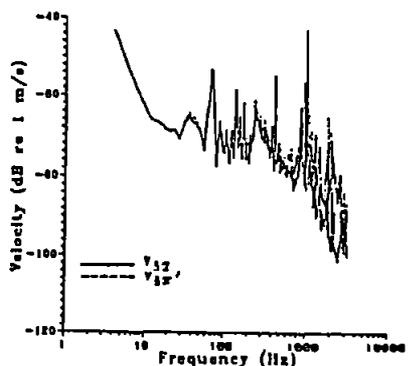


Fig.58  $V_{3z}$  and  $V_{3z}'$  of reduction gear during vessel full speed ahead

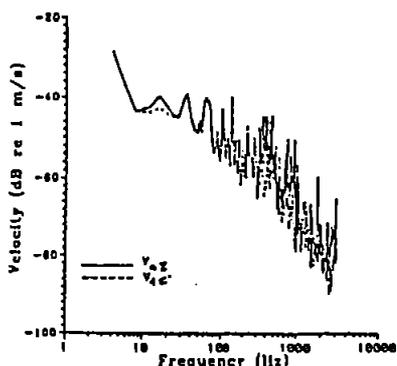


Fig.55  $V_{4z}$  and  $V_{4z}'$  of main diesel engine during vessel full speed ahead

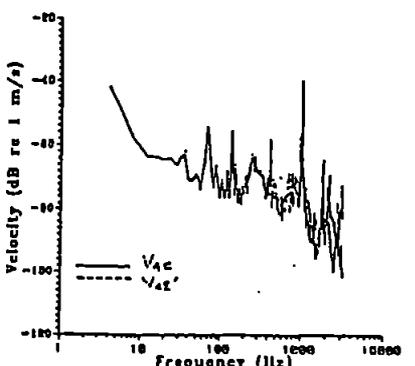


Fig.59  $V_{4z}$  and  $V_{4z}'$  of reduction gear during vessel full speed ahead

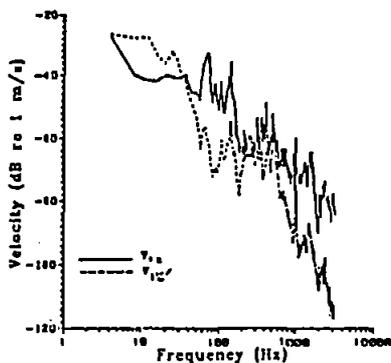


Fig.60  $V_{1z}$  and  $V_{1z}'$  of generator during vessel full speed ahead

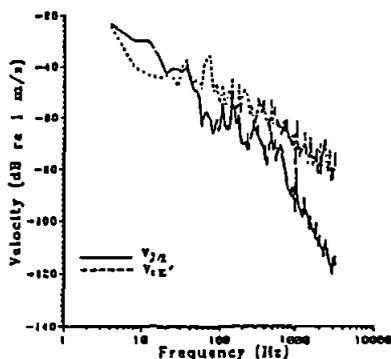


Fig.61  $V_{2z}$  and  $V_{2z}'$  of generator during vessel full speed ahead

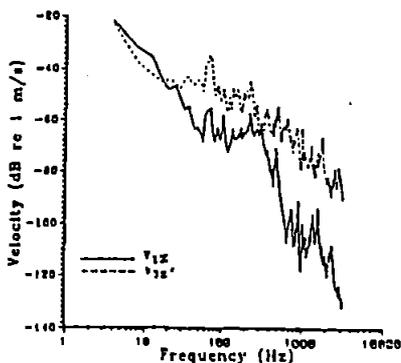


Fig.62  $V_{3z}$  and  $V_{3z}'$  of generator during vessel full speed ahead

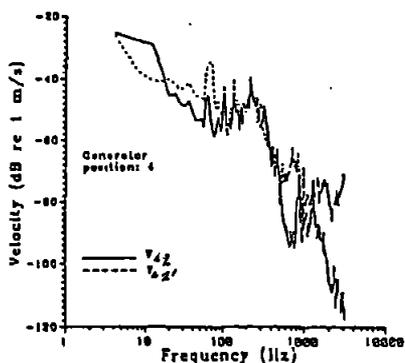


Fig.63  $V_{4z}$  and  $V_{4z}'$  of generator during vessel full speed ahead

- Figure 70 - Figure 73 express the input and output power spectra at both ends of the reduction gear seatings.
- Figure 74 - Figure 77 express the input and output power spectra at both ends of the generator mountings.

From these calculated results of the power-spectra, it can be concluded obviously as follows :

1. Vertical power transmission via the mounting blocks of the main diesel engine constitute the most dominant structure-borne noise source in the engine room, second is the power via the seatings of reduction gear, and finally, the most unimportant source power is that via the generator mountings.
2. Again, the vertically transmitted power is much higher than that transversely transmitted power via each mounting of the main engine.
3. It can be clearly seen that the vertical power transmission via mounting can be either downward or upward. The downward power transmission means that the vibrational power is transmitted from the machinery to the foundation block, while the upward power transmission is power transmitted in the reverse direction. In the latter case, the driving vibration power is mainly due to that induced to the bottom structure by the shaft vibration and the surrounding water pressure fluctuation from the propeller.

### CONCLUDING REMARKS

From the deliberations in this research project, the following conclusions are forwarded:

1. The model and method for identifying the vibrational power flow transmission and characterising the structure-borne noise source have been established. To enhance the correctness, both measurement and analysis are combined to deal with the power flow transmission via mountings of machinery.
2. The parallel measurement technique (PMT) has been validated and it has been found fea-

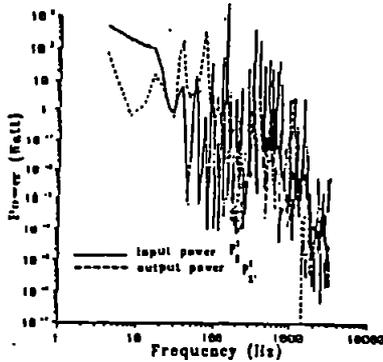


Fig.64 Vibration power  $P_Z^1$  and  $P_Z^2$ , at position 1 engine mounting, vessel full ahead

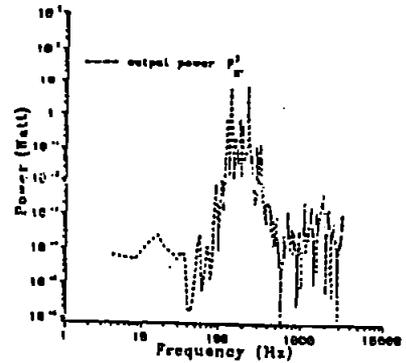


Fig.68 Vibration power  $P_X^3$ , at position 3 engine mounting, vessel full ahead

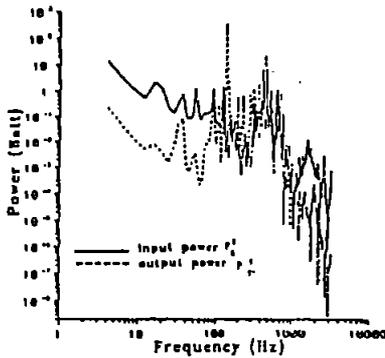


Fig.65 Vibration power  $P_Z^2$  and  $P_Z^2$ , at position 2 engine mounting, vessel full ahead

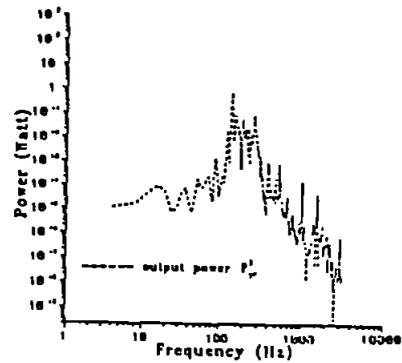


Fig.69 Vibration power  $P_Y^3$ , at position 3 engine mounting, vessel full ahead

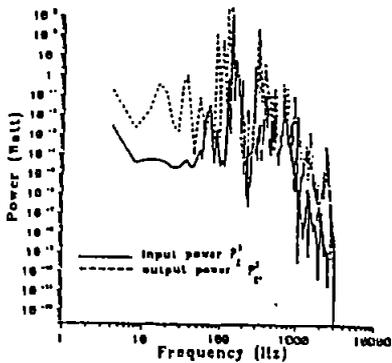


Fig.66 Vibration power  $P_Z^3$  and  $P_Z^2$ , at position 3 engine mounting, vessel full ahead

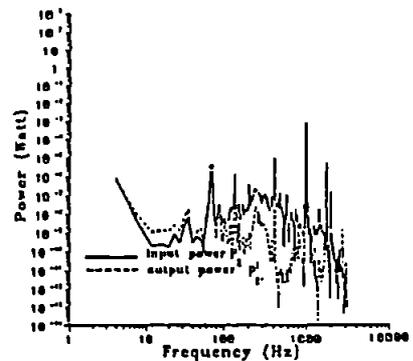


Fig.70 Vibration power  $P_Z^1$  and  $P_Z^2$ , at position 1 reduction gear seating, vessel full ahead

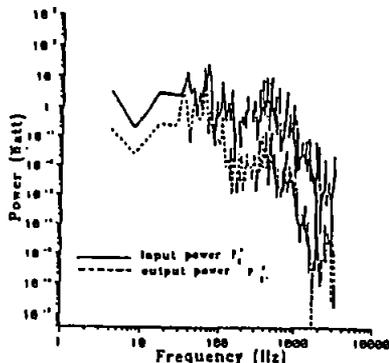


Fig.67 Vibration power  $P_Z^1$  and  $P_Z^2$ , at position 4 engine mounting, vessel full ahead

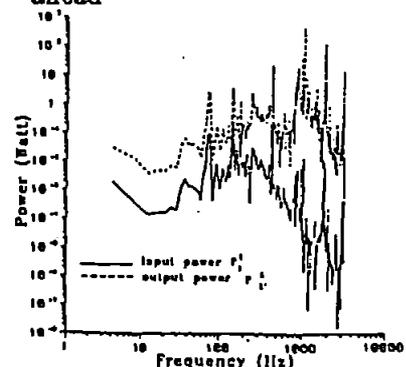


Fig.71 Vibration power  $P_Z^2$  and  $P_Z^2$ , at position 2 reduction gear seating, vessel full ahead

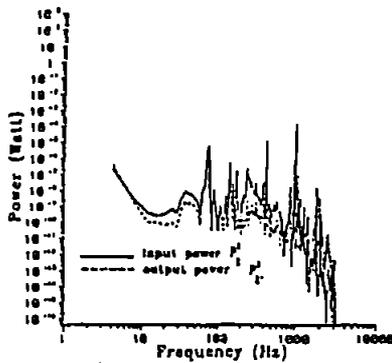


Fig.72 Vibration power  $P_2^3$  and  $P_1^3$ , at position 3 reduction gear seating, vessel full ahead

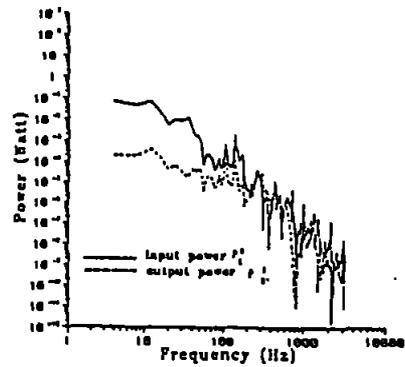


Fig.75 Vibration power  $P_2^2$  and  $P_1^2$ , at position 2 generator mounting, vessel full ahead

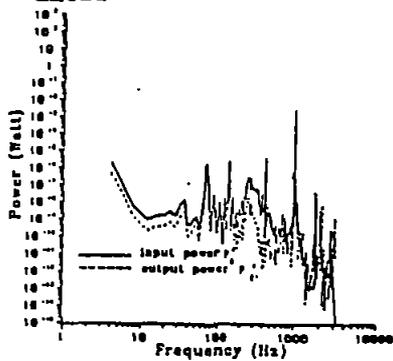


Fig.73 Vibration power  $P_2^4$  and  $P_1^4$ , at position 4 reduction gear seating, vessel full ahead

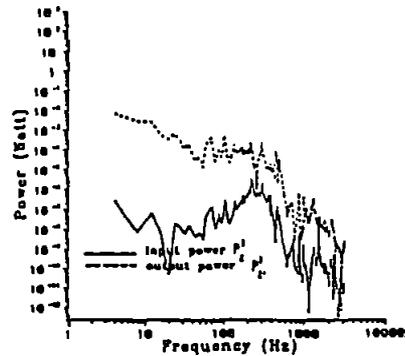


Fig.76 Vibration power  $P_2^3$  and  $P_1^3$ , at position 3 generator mounting, vessel full ahead

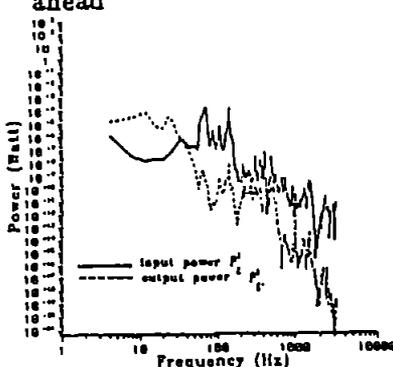


Fig.74 Vibration power  $P_2^1$  and  $P_1^1$ , at position 1 generator mounting, vessel full ahead

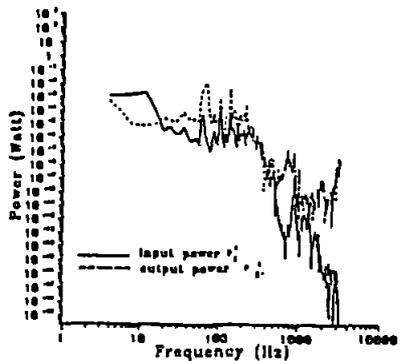


Fig.77 Vibration power  $P_2^4$  and  $P_1^4$ , at position 4 generator mounting, vessel full ahead

sible to obtain mobility measurement during a sea trial.

3. A simple method of checking the mobility spectral data has been derived, shown as equations (13), (15) and (17), to confirm the measured mobility.

4. Form the actual mobility measurement and the calculation of vibration power transmission via the mountings of machinery for a 100 gross tonnage fast patrol boat, it is shown

that the transmission direction via mounting can be either upward or downward. The severity of the structure-borne noise sources in the engine room of this vessel has been verified as:

Main diesel engine > Reduction gear  
> Generator

5. The point mobility at each mounting of the main diesel engine has the characteristic of a weak coupling.

## ACKNOWLEDGMENTS

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## Structure-Borne Noise Control for a Marine Engine

Wei-Hui Wang\* Robert Sutton\*\* Ben Dobson\*\*\*

- \* Department of Naval Architecture, National Taiwan Ocean University, Keelung 202, Taiwan, ROC.
- \*\* Institute of Marine Studies, University of Plymouth, UK
- \*\*\* Devonport Management Limited, Devonport Royal Dockyard, Plymouth, UK

### Abstract

The control of structure-borne noise or high frequency vibration is of paramount importance in the design of high performance ships. In the past, most fast vessels were designed with a small blocks coefficient, compact engine room arrangements and were of light weight construction. Thus powerful high speed marine engines were installed in confined spaces which, as a consequence, caused an extremely high level noise source. Often structure-borne noise and vibration power were transmitted to a sound carrying structure from a source via a number of contact points. In turn, the noise and vibrations are propagated in the structure and could possibly cause an undesired noise radiation. In principle, this may be avoided by taking appropriate measures at the source, in the transmission, during propagation or at radiation. It is, of course, preferable to cope with the problems at the generation sites and thereby avoid more comprehensive and expensive actions later in the chain.

Besides considering the resilient engine mounts, the fundamentals of the control of the engine induced structure-borne noise by the method of branch structure close to the foundation have been discussed so as to enhance the isolation of noise.

### 1. Introduction

For a single degree of freedom spring-mass-dashpot system there exists the relationship that the transmissibility may decrease with increasing frequency ratio. Whilst the measured transmissibility spectra always exhibit multi-peaks in the higher frequency range for an actual engine-resilient mount-foundation system. The main reasons of this phenomenon are due to (1) the standing wave effect in the resilient mount; (2) the influence effect of the finite mass and finite stiffness of the seat structure; and (3) the airborne sound coupling with the foundation structure. These effects thus result in attenuating the isolation effectiveness of the high frequency vibrations of the resilient mount.

In most cases however, the structural components of the foundation are compound, which means that the interactions between the contact points may be reduced due to intermediate stiffeners and/or energy transmission to adjacent structural parts. Basically a structure which is excited, e.g. by means of a point force will oppose motion. For the description of the dynamic behavior of the structure, the mobility is used here, The characteristics of the structure such as mass (inertia), stiffness and internal loss are used to determine the mobility.

The examples of measured point and transfer mobilities for the structure studied in [1] [2] show that the point mobilities are generally greater than the transfer mobilities because of the inhomogeneity of the structures. Also, the point mobilities are found to be mainly determined by the local dynamic properties of the structure.

Both in predictions of the effect of design modifications for vibration isolation and in predictions of structure-borne noise propagation through complex systems, the structural characteristics of the source and the receiver are required. The quantity primarily sought is the active sound power transmitted to the supporting structure. The problem of finding an adequate acoustical description of the structures involved is studied in this paper.

## 2. Elastic Bending Wave in Finite Beam and Plate

For the radiation of airborne noise or hydroacoustics, the elastic bending wave in the structure is the most important wave pattern owing to its larger lateral displacement. The one dimensional bending wave equation in beam and plate can be expressed as [3]:

$$\frac{B}{m'} \frac{\partial^4 v_y}{\partial x^4} + \frac{\partial^2 v_y}{\partial t^2} - \left[ \frac{\theta'_z}{m'} + \frac{B}{K} \right] \frac{\partial^4 v_y}{\partial x^2 \partial t} + \frac{\theta'_z}{K} \frac{\partial^4 v_y}{\partial t^4} = 0 \quad (1)$$

where the coordinate system is shown in Figure 1, and

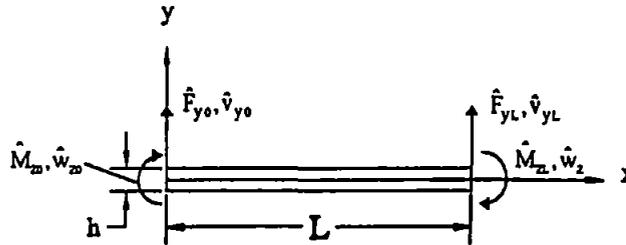


Figure 1 Coordinate system of a beam or plate

$$B = \text{the bending rigidity} = \begin{cases} EI = \frac{bh^3 E}{12} (N - m^2) & \text{for a rectangular beam} \\ \frac{EI'}{1 - \mu^2} = \frac{Eh^3}{12(1 - \mu^2)} (N - m^2) & \text{for plate} \end{cases}$$

$$m' = \text{unit mass} = \begin{cases} \rho S & \text{mass per unit length of beam (kg/m)} \\ \rho h & \text{mass per unit area of plate (kg/m)} \end{cases}$$

$v_y$  = velocity along y axis (m/sec)

$\theta'_z = I \rho dx$  = rotary mass moment of inertia of dx about z (kg-m)

$K = \frac{GS}{\kappa}$  = the shear rigidity (N - m)

$E$  = Young's modulus ( $N/m^2$ )

$S$  = Cross-sectional area ( $m^2$ )

$h$  = thickness of plate or web height of beam (m)

$\kappa$  = shear distribution factor ( $m^{-1}$ )

$\mu$  = Poisson's ratio

$t$  = time (sec)

In equation (1) the rotary inertia and the shear effects have to be taken into account for the bending wave propagation in the beam and plate, especially in the high frequency range. The first two terms represent the pure bending wave, and the last three terms represent the modifications of the rotary inertia and the shear deformation effects. In the case of a steel beam or plate, if the bending wavelength to the thickness (or web height) ratio is less than 6 then the last three terms in equation (1) should be maintained. Otherwise, there exists an error of around 10% to the solution of phase velocity [3].

The phase velocity  $C_B$  and the elastic bending wavelength  $\lambda_B$  can be expressed as:

$$C_B = \sqrt{\frac{B}{m'}} \sqrt{\omega} \quad , \quad \lambda_B = \frac{2\pi C_B}{\omega} \quad (2)$$

where  $\omega$  is the circular frequency of the bending wave.

For a 3% error of phase velocity, the ratio of  $\lambda_B/h$  should be greater than 10, thus according to equation (2) the frequency range has to be less than 3.2 kHz for a steel beam and less than 1 kHz for a steel plate of the scantling for ship structure type [4]. In these cases equation (1) can be reduced to the pure bending wave equation:

$$B \frac{\partial^4 v}{\partial x^4} + m' \frac{\partial^2 v}{\partial t^2} = 0 \quad (3)$$

In an actual vibration supporting structure, there always exists the boundaries. The boundaries may change the wave direction, phase and amplitude, furthermore, they may even change the wave pattern. To consider the progressive wave, the reflection wave and the exponentially decayed near-field wave due to the boundaries, the solution of equation (3) with a circular frequency  $\omega$  is:

$$v = \hat{v}_+ e^{-jkx} + \hat{v}_- e^{jkx} + \hat{v}_{-j} e^{-kx} + \hat{v}_j e^{kx} \quad (4)$$

where  $v = \hat{v}(t)$  has the form of harmonic function of time, and

$$k = \text{bending wave number} = \frac{2\pi}{\lambda_B} = \frac{\omega}{C_B} = (\omega^2 m' / B)^{1/4} \quad (5)$$

$\hat{v}_+$  = amplitude of the progressive wave along the positive x direction

$\hat{v}_-$  = amplitude of the progressive wave along the negative x direction

$\hat{v}_{-j}$  = amplitude of the decayed near-field wave along the positive x direction

$\hat{v}_j$  = amplitude of the decayed near-field wave along the negative x direction

From equation (4), the angular velocity  $\hat{w}_z$ , the bending moment  $\hat{M}_z$ , the shear force  $\hat{F}_y$  can be obtained from:

$$\hat{w} = \frac{\partial \hat{v}}{\partial x}, \quad \hat{M}_z = -\frac{B}{j\omega} \frac{\partial \hat{w}}{\partial x} = -\frac{B}{j\omega} \frac{\partial^2 \hat{v}}{\partial x^2}, \quad \hat{F}_y = -\frac{\partial \hat{M}_z}{\partial x}, \quad \frac{\partial \hat{F}_y}{\partial x} = -j\omega m' \hat{v} \quad (6)$$

For a beam element of length L and state parameters of both ends are  $\langle \hat{v}_{y_0}, \hat{w}_{z_0}, \hat{F}_{y_0}, \hat{M}_{z_0} \rangle$  and  $\langle \hat{v}_{y_L}, \hat{w}_{z_L}, \hat{F}_{y_L}, \hat{M}_{z_L} \rangle$ . The transfer relationships between the end parameters can be determined by:

$$\langle \hat{v}_{y_0}, \hat{w}_{z_0}, \hat{v}_{y_L}, \hat{w}_{z_L} \rangle^T = [\hat{H}_{ij}]_{4 \times 4} \langle \hat{F}_{y_0}, \hat{M}_{z_0}, \hat{F}_{y_L}, \hat{M}_{z_L} \rangle^T \quad (7)$$

where  $\hat{H}_{ij}$  is the complex transfer functions.

Setting  $v = \hat{v}_{y_0}$  and  $\hat{w} = \hat{w}_{z_0}$  at  $x = 0$ ;  $\hat{v} = \hat{v}_{y_L}$  and  $\hat{w} = \hat{w}_{z_L}$  at  $x = L$ , from equations (4), (6) the coefficients  $\hat{v}_+, \hat{v}_-, \hat{v}_{-j}$  and  $\hat{v}_j$  can be obtained as:

$$\begin{Bmatrix} \hat{v}_+ \\ \hat{v}_- \\ \hat{v}_{-j} \\ \hat{v}_j \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -jk & jk & -k & k \\ e^{-jkl} & -e^{jkl} & e^{-kl} & e^{kl} \\ -jke^{-jkl} & jke^{jkl} & -ke^{-kl} & ke^{kl} \end{bmatrix}^{-1} \begin{Bmatrix} \hat{v}_{y_0} \\ \hat{w}_{z_0} \\ \hat{v}_{y_L} \\ \hat{w}_{z_L} \end{Bmatrix} \quad (8)$$

Using the relationships in equation (6) and setting  $\hat{F}_{y_0} = 1, \hat{M}_{z_0} = 0, \hat{F}_{y_L} = 0$  and  $\hat{M}_{z_L} = 0$  into equation (7),  $\hat{H}_{1j}$  can be derived.

Similarly, by setting  $\hat{F}_{y_0} = 0, \hat{M}_{z_0} = 1, \hat{F}_{y_L} = 0, \hat{M}_{z_L} = 0$ ;  $\hat{F}_{y_0} = 0, \hat{M}_{z_0} = 0, \hat{F}_{y_L} = 1, \hat{M}_{z_L} = 0$ ; and  $\hat{F}_{y_0} = 0, \hat{M}_{z_0} = 0, \hat{F}_{y_L} = 0, \hat{M}_{z_L} = 1$  respectively, the transfer functions  $\hat{H}_{2j}, \hat{H}_{3j}$  and  $\hat{H}_{4j}$  can be obtained. In which,  $\hat{H}_{11}, \hat{H}_{33}$  and  $\hat{H}_{22}, \hat{H}_{44}$  are the force and moment impedance functions at both ends respectively. The final form of  $[\hat{H}_{ij}]$  can be derived as:

$$[\hat{H}_{ij}] = \frac{j\omega}{Bk^2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -jk & jk & -k & k \\ e^{-jkl} & -e^{jkl} & e^{-kl} & e^{kl} \\ -jke^{-jkl} & jke^{jkl} & -ke^{-kl} & ke^{kl} \end{bmatrix} \begin{bmatrix} jk & -jk & -k & k \\ 1 & 1 & -1 & 1 \\ jke^{-jkl} & -jke^{jkl} & -ke^{-kl} & ke^{kl} \\ e^{-jkl} & e^{jkl} & -e^{-kl} & -e^{kl} \end{bmatrix}^{-1} \quad (9)$$

To derive the transfer functions of a finite rectangular thin plate, the two dimensional bending

wave equation is [3]:

$$\Delta\Delta\eta - k^4\eta = 0 \quad \text{or} \quad \Delta\Delta v - k^4v = 0 \quad (10)$$

where  $\eta$  and  $v$  are the lateral displacement and velocity of the plate particle in  $y$  direction respectively,  $\Delta = (\partial^2/\partial x^2 + \partial^2/\partial z^2)$  is the Laplace operator, and  $k^4 = \omega^2 m''/B'$  is the wave number.

Assuming the boundary conditions as :

1. there is no rotation of the plate particle at the site of excitation, in other words, this implies that the vibration is of finite amount and the first derivatives of the particle displacement with respect to  $x$  and  $z$  are equal to zero;
2. the excitation force is equal to the sum of the shear force  $Q_r$  in the plate around the excitation site;
3. Faraway from the excitation point, a decayed wave field occurs.

Then the solution if equation (10) can be expressed as:

$$\hat{\eta} = C_1 H_0^{(2)}(kr) + C_2 H_0^{(2)}(-jkr) \quad (11)$$

where  $\hat{\eta}$  is the complex amplitude of the sinusoidal time displacement  $\eta$ , and  $r$  is the distance from the excitation point.  $H_0^{(2)}(kr)$  and  $H_0^{(2)}(-jkr)$  are the Hankel functions of the second kind. The expanded form of  $H_0^{(2)}(kr)$  is:

$$H_0^{(2)}(kr) = J_0(kr) - jY_0(kr) \quad (12)$$

where  $J_0(kr)$  is the Bessel function of the first kind of order zero and  $Y_0(kr)$  is the Bessel function of the second kind of order zero. They are:

$$J_0(kr) = 1 - \frac{(kr)^2}{2^2(1!)^2} + \frac{(kr)^4}{2^4(2!)^2} - \frac{(kr)^6}{2^6(3!)^2} + \dots \quad (13)$$

$$Y_0(kr) = \frac{2}{\pi} \left[ J_0(kr) \left( \ln \frac{kr}{2} + \gamma \right) + \sum_{m=1}^{\infty} \frac{(-1)^{m-1} h_m}{2^{2m}(m!)^2} (kr)^{2m} \right] \quad (14)$$

where  $\gamma \approx 0.5772$  is the Euler constant;

$$h_m = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{m} \quad (15)$$

Using an asymptotic expansion,  $H_0^{(2)}(kr)$  and  $H_0^{(2)}(-jkr)$  have the following characteristics:

$$H_0^{(2)}(kr) \approx \begin{cases} -\frac{2j}{\pi} \ln \frac{kr}{2}, & \text{for } |kr| \ll 1 \\ \sqrt{\frac{2}{\pi kr}} e^{-j(kr - \frac{\pi}{4})}, & \text{for } |kr| \gg 1 \end{cases} \quad (16)$$

$$H_0^{(2)}(-jkr) \approx \sqrt{\frac{2}{-j\pi kr}} e^{j\frac{\pi}{4}} e^{-kr} = j\sqrt{\frac{2}{\pi kr}} e^{-kr}, \quad \text{for } |kr| \gg 1 \quad (17)$$

so,  $H_0^{(2)}(-jkr)$  represents a near-field wave which obeys the exponential decay law.

As  $r \rightarrow 0$  and according to the first boundary condition, it leads to:

$$\frac{\partial \hat{\eta}}{\partial r} = C_1 \left[ -\frac{2j}{\pi kr} + \dots \right] - jC_2 \left[ \frac{2}{\pi kr} + \dots \right] = -\frac{2j}{\pi kr} (C_1 + C_2) + \dots = 0$$

then  $C_1 = -C_2$  and equation (11) becomes:

$$\hat{\eta} = C_1 [H_0^{(2)}(kr) - H_0^{(2)}(-jkr)] \quad (18)$$

owing to

$$\hat{\eta}(0) = \hat{\eta}_0 = C_1 \left[ \frac{2j}{\pi} \ln \left( \frac{kr}{2} \right) - \frac{2j}{\pi} \ln \left( -\frac{jkr}{2} \right) \right] \approx C_1$$

hence

$$\hat{\eta} = \hat{\eta}_0 [H_0^{(2)}(kr) - H_0^{(2)}(-jkr)] = \hat{\eta}_0 \Pi(kr) \quad (19)$$

where

$$\Pi(kr) = H_0^{(2)}(kr) - H_0^{(2)}(-jkr) \quad (20)$$

while  $\hat{\eta}_0$  can be expressed on term of the excitation force  $\hat{F}_0$  according to the second boundary condition. If a perpendicular force  $\hat{F}_0$  is applied to one corner of a rectangular plate, then  $F_0$  can be considered as applied in a quarter of small circular area of radius  $r_0$  and with center at the corner. The internal shear force per unit arc length  $Q_{r_0}$  around the small quarter circumference is

$$Q_{r_0} = B' \frac{\partial}{\partial r} (\Delta \eta) \Big|_{r=r_0} \quad (21)$$

Since

$$\begin{cases} \Delta H_0^{(2)}(kr) = -k^2 H_0^{(2)}(kr) \\ \Delta H_0^{(2)}(-jkr) = k^2 H_0^{(2)}(-jkr) \end{cases} \quad (22)$$

Using equation (22) and substituting equation (19) into equation (21), then

$$Q_{r_0} = \frac{4jB'k^2}{\pi r_0} \hat{\eta}_0 \quad (23)$$

According to the second boundary condition:

$$\hat{F}_0 = \frac{1}{2} \pi r_0 Q_{r_0} = 2jB'k^2 \hat{\eta}_0$$

thus

$$\hat{\eta}_0 = \frac{\hat{F}_0}{2jB'k^2} \quad (24)$$

Now, equation (18) becomes:

$$\hat{\eta} = \frac{\hat{F}_0}{2jB'k^2} [H_0^{(2)}(kr) - H_0^{(2)}(-jkr)] \quad (25)$$

Considering that  $\hat{v}_0 = j\omega \hat{\eta}_0$ , the driving point force impedance  $\hat{Z}_{F_0}$  at one corner of a rectangular plate regarding to the bending wave is:

$$\hat{Z}_{F_0} = \frac{\hat{F}_0}{\hat{v}_0} = \frac{2B'k^2}{\omega} = \frac{2\omega m''}{k^2} \quad (26)$$

The lateral velocity distribution of the plate particle is :

$$\hat{v}_z(x,y) = \frac{\hat{F}_0 \omega}{2B'k^2} [H_0^{(2)}(kr) - H_0^{(2)}(-jkr)] = \frac{\hat{F}_0}{\hat{Z}_{F_0}} \Pi(kr) \quad (27)$$

Therefore, equation (27) can be adopted as a shape function in the finite element applications. For a finite rectangular plate element with state parameters  $(\hat{v}_z, \hat{w}_x, \hat{w}_y, \hat{F}_z, \hat{M}_x, \hat{M}_y)$  at each nodal point as shown in Figure 2, the transfer relationships between the nodal parameters can be determined by:

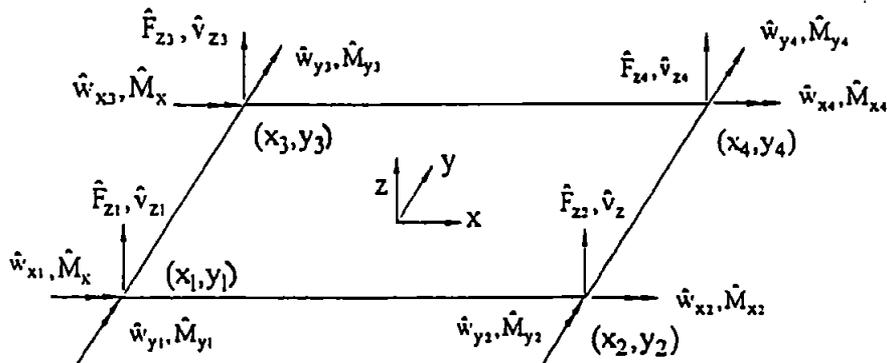


Figure 2 State nodal parameters of the bending wave pattern in a rectangular plate element

$$\langle \hat{v}_z \dots \hat{v}_{z4} \hat{w}_{x1} \dots \hat{w}_{x4} \hat{w}_{y1} \dots \hat{w}_{y4} \rangle^* = [\hat{H}_{ij}] \langle \hat{F}_{z1} \dots \hat{F}_{z4} \hat{M}_{x1} \dots \hat{M}_{x4} \hat{M}_{y1} \dots \hat{M}_{y4} \rangle^T \quad (28)$$

in which the rotation speed  $w_x$  and  $w_y$  can be expressed as:

$$\hat{w}_x(x,y) = \frac{\partial \hat{v}_z}{\partial x} = \frac{\partial \hat{v}_z}{\partial r} \frac{\partial r}{\partial x} = \frac{\hat{F}_0 \omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(kr)}{\partial r} - \frac{\partial H_0^{(2)}(-jkr)}{\partial r} \right] \cdot \frac{x}{r} \quad (29)$$

$$\hat{w}_y(x,y) = \frac{\partial \hat{v}_z}{\partial y} = \frac{\partial \hat{v}_z}{\partial r} \frac{\partial r}{\partial y} = \frac{F_0 \omega}{2B'k^2} \left[ \frac{\partial H_0^{(2)}(kr)}{\partial r} - \frac{\partial H_0^{(2)}(-jkr)}{\partial r} \right] \cdot \frac{y}{r} \quad (30)$$

By using equation (27), (28), (30), setting  $F_0 = 1$  and substituting the coordinates of the nodal point into them,  $H_{ij}$ ,  $i = 1,2,3,4$  and  $j = 1,2,\dots,12$ , in equation (28) can be obtained.

For an excitation moment  $\hat{M}_0$  applied to one given corner, the moment can be replaced by a set of two coupling forces with a small distance  $2a$  apart each other as shown in Figure 3.

$$\hat{M}_0 = 2a\hat{F} \quad (31)$$

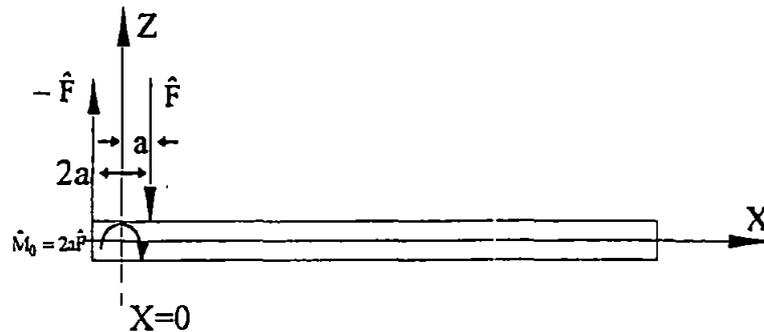


Figure 3 Simulation of excitation moment at a corner of a rectangular plate

From equation (27), it can be seen by superposition principle that the lateral velocity of the plate particle is :

$$\hat{v}_z(x,y) = \frac{\hat{F}\omega}{2B'k^2} [\Pi(kr_1) - \Pi(kr_2)] \quad (32)$$

where  $r_1$ ,  $r_2$  are the distances from the excitation point  $x=a$  and  $x=-a$  to any point  $(x,y)$  in the plate respectively.

For the points along  $x$  axis, there are:

$$\begin{cases} \Pi(kr_1) = H_0^{(2)}[k(x-a)] - H_0^{(2)}[-jk(x-a)] \\ \Pi(kr_2) = H_0^{(2)}[k(x+a)] - H_0^{(2)}[-jk(x+a)] \end{cases} \quad (33)$$

Expanding the Hankel functions with respect to the small quantity of  $k(x-a)$  or  $k(x+a)$ , then

$$\begin{cases} \Pi(kr_1) \approx 1 + 2k^2(x-a)^2 \left[ \alpha_1 + \alpha_2 \ln \frac{\gamma k(x-a)}{2} - \frac{1}{2} \alpha_2 \ln(-j) \right] \\ \Pi(kr_2) \approx 1 + 2k^2(x+a)^2 \left[ \alpha_1 + \alpha_2 \ln \frac{\gamma k(x+a)}{2} - \frac{1}{2} \alpha_2 \ln(-j) \right] \end{cases}$$

where  $\alpha_1 = -\frac{1}{4} - \frac{j}{2\pi}$ ,  $\alpha_2 = \frac{j}{2\pi}$ ,  $\gamma = 1.781$ .

Thus, for  $a \rightarrow 0$ , the angular velocity at the excitation corner can be obtained from equation (32) as:

$$\hat{w}_y(0) = \frac{\partial \hat{v}_z}{\partial x} \Big|_{a \rightarrow 0} = \frac{\hat{F}a\omega k^2}{2B'k^2} \left( 1 - \frac{4j}{\pi} \ln \frac{\gamma ka}{2} \right) = \frac{\hat{M}_0 \omega}{4B'} \left( 1 - \frac{4j}{\pi} \ln \frac{\gamma ka}{2} \right) \quad (34)$$

Using equation (32) and the relations  $\hat{w}_x = \partial \hat{v}_z / \partial y$ ,  $\hat{w}_y = \partial \hat{v}_z / \partial x$ ; setting  $\hat{M}_0 = 1$  and substituting them by the coordinates of nodal points, then  $H_{ij}$ ,  $i = 5,6,7,8$ ;  $j = 1,2,\dots,12$ , in equation (28) can be obtained.

A similar procedure can be followed to derived the transfer functions  $H_{ij}$ ,  $i = 9,\dots,12$ ;  $j = 1,2,\dots,12$  in equation (28) along  $y$  axis.

### 3. Longitudinal Wave and Torsional Wave in a Finite Beam

The longitudinal stress wave pattern in a finite bar element as shown in Figure 4, can be expressed as:

$$\hat{v}_x(x) = \hat{v}_+ e^{-jkx} + \hat{v}_- e^{jkx} \quad (35)$$

In equation (35) the progressive waves both in the positive and negative x directions have been taken into consideration.

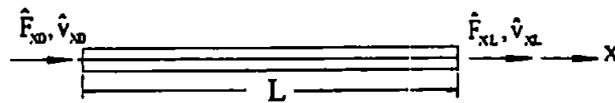


Figure 4 Longitudinal wave propagation in a rod

The transfer relationships between the nodal parameters can be expressed as:

$$\langle \hat{v}_{x0} \hat{v}_{xL} \rangle^T = [\hat{H}_{ij}] \langle \hat{F}_{x0} \hat{F}_{xL} \rangle^T \quad (36)$$

where

$$[\hat{H}_{ij}] = Z^{-1} \begin{bmatrix} 1 & 1 \\ e^{-jkL} & e^{jkL} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ e^{-jkL} & -e^{jkL} \end{bmatrix}^{-1} \quad (37)$$

$$Z = \sqrt{E\rho} s \quad (38)$$

The transfer matrix for the torsional sound wave propagation in a finite bar element is of the same form as equation (37), but with

$$Z = \left( \frac{\rho K G (h^2 + b^2) S^3 b}{12h} \right)^{1/2} \quad (39)$$

for a rectangular bar of section of width b and height h. K is a form factor depending on the height-width ratio (b/h) as shown in Table 1.

Table 1 Relation between form factor K and the ratio h/b of a rectangular bar

h/b	1.0	1.5	2.0	2.5	3.0	5.0	6.0	10	$\infty$
K	0.141	0.196	0.229	0.249	0.263	0.291	0.298	0.312	0.333

### 4. Longitudinal Wave Pattern in a Rectangular Plate

Analogous to the longitudinal wave pattern in a finite rod, the longitudinal wave patterns in both x and y directions of a rectangular plate can be expressed as:

$$\hat{v}_x(x,y) = \hat{v}_{x+} e^{-jkx} + \hat{v}_{x-} e^{jkx} \quad (40)$$

$$\hat{v}_y(x,y) = \hat{v}_{y+} e^{-jky} + \hat{v}_{y-} e^{jky} \quad (41)$$

The transfer matrix between the nodal parameters, which includes the nodal forces and nodal velocities as shown in Figure 5, can be followed by:

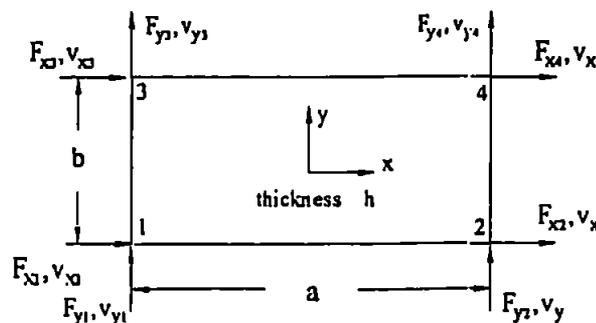


Figure 5 State nodal parameters of the longitudinal wave pattern in a rectangular plate element

$$\begin{Bmatrix} v_{x1} \\ v_{x2} \\ v_{x3} \\ v_{x4} \\ v_{y1} \\ v_{y2} \\ v_{y3} \\ v_{y4} \end{Bmatrix} = \begin{pmatrix} H_{AB} & 0 & 0 & 0 \\ 0 & H_{CD} & 0 & 0 \\ 0 & 0 & H_{EF} & 0 \\ 0 & 0 & 0 & H_U \end{pmatrix} \begin{Bmatrix} F_{x1} \\ F_{x2} \\ F_{x3} \\ F_{x4} \\ F_{y1} \\ F_{y2} \\ F_{y3} \\ F_{y4} \end{Bmatrix} \quad (42)$$

where

$$[H_{AB}] = [H_{CD}] = Z_1^{-1} \begin{bmatrix} 1 & 1 \\ e^{-jka} & e^{jka} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ e^{-jka} & -e^{jka} \end{bmatrix}^{-1} \quad (43)$$

$$[H_{EF}] = [H_U] = Z_2^{-1} \begin{bmatrix} 1 & 1 \\ e^{-jka} & e^{jka} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ e^{-jkb} & -e^{jkb} \end{bmatrix}^{-1} \quad (44)$$

in which

$$k = \frac{\omega}{\sqrt{D/\rho}}, \quad D = \frac{E}{1-\mu^2}, \quad Z_1 = \frac{1}{2} \sqrt{D\rho} ah, \quad Z_2 = \frac{1}{2} \sqrt{D\rho} bh$$

### 5. Structure-Borne Noise Control by Branch Structure

A typical branch structure appearing in a ship is the stiffened plate or plates joined in right angle. Consider the joint structure of two plates 1 and 2 rigidly joined in right angle as shown in Figure 6.

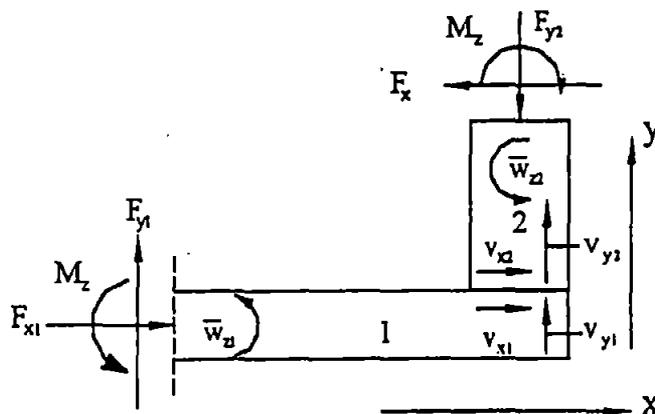


Figure 6 Boundary conditions at the joint of two plates rigidly joined in right angle

As a incident bending wave of plate 1 is oncoming to the joint, the bending wave field in plate 1 should include the incident field, the reflection field and the near-field decayed wave. Whilst the bending wave field in plate 2 is composed of the progressive wave along y-axis and the near-field wave. Since at the joint, the bending wave in plate 1 can induce longitudinal wave in plate 2, these two kind of wave pattern are then coupled together. In an energy point of view, the transmission coefficient  $\tau$  is defined as the ratio of the transmitted sound power  $P_2$  to the incident sound power  $P_1$ , i.e.,

$$\tau = \frac{P_2}{P_1} \quad (45)$$

where

$$P_2 = 2m_2^2 C_{B2} v_{x2}^2 \quad (46)$$

$$P_1 = 2m_1^2 C_{B1} v_{y1}^2$$

In which the state variables  $v_{y1}$  and  $v_{x2}$  at node points can be solved by a finite element algorithm using the method of transfer matrix. Then the structure-borne noise reduction level can be

expressed as:

$$R = 10 \log \frac{1}{\tau} \quad (47)$$

Figure 7 shows the noise reduction level of the structure-borne bending wave transmitted through the junction of two plates joined in right angle. Different combination of plate thickness has different effectiveness of noise reduction. It can be seen that all of them get a reduction level over 5dB in the frequency range above 400Hz and at least 3dB below 400Hz.

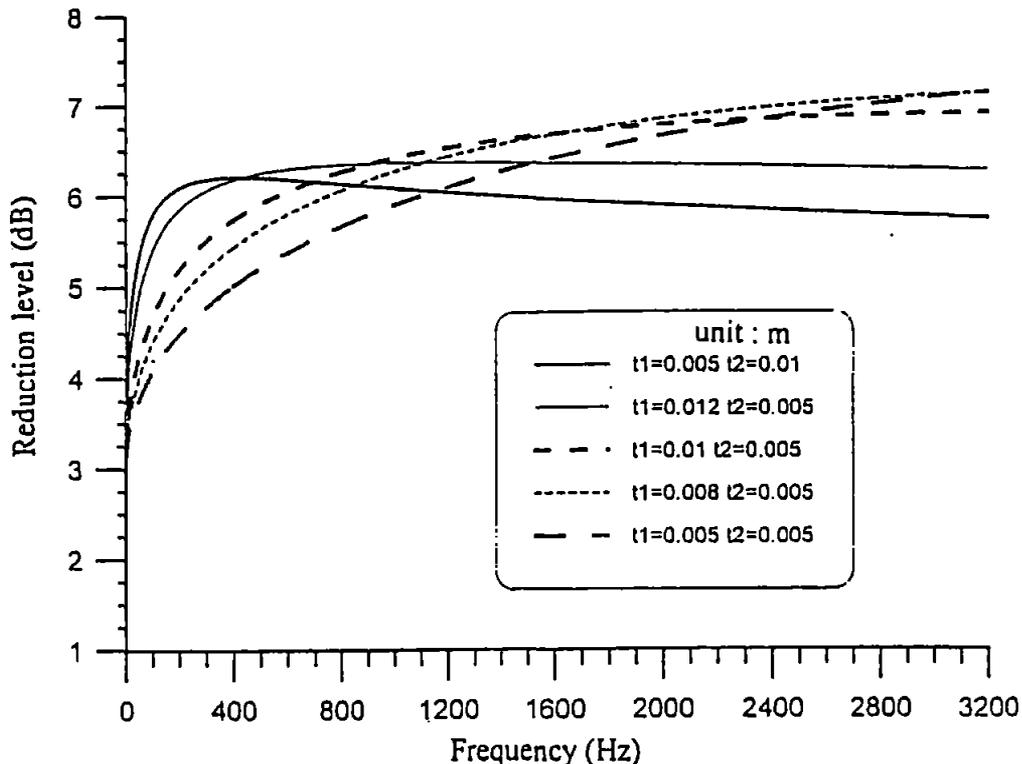


Figure 7 Structure-borne bending wave reduction level through the junction of two plates joined in right angle with different combination of plate thickness

## 6. Concluding Remarks

From the present study, the main contributions in this paper are outlined in the following paragraphs:

A finite element model for analyzing the structure-borne noise and vibration transmission in the stiffened plate type structure has been established. This model is especially suitable for use in the higher excitation frequency range which is below 3.2 kHz for a steel beam and below 1 kHz for a steel plate. Derivations of the transfer matrices between the state parameters of the nodal points for beam and plate elements are concentrated on the elastic pure bending wave pattern. In the derivations the combination of exponential functions are used as the vibration shape function which composes of the progressive wave, the reflection wave and the near-field decayed wave for a finite beam element. Whilst the derivation for the rectangular plate element the used vibration shape function is composed of the Hankel function of the second kind in which the progressive wave and the near-field wave have been taken into consideration. Also the coupled effects of the bending wave and the longitudinal wave have been considered for the branch structure joined in right angles.

Besides considering the use of resilient engine mounts, the fundamentals of the structure-borne noise control by branch structure have been established. In which, the derivations are concentrated on the pure bending waves (including the incident wave, reflection wave and transmitted wave) and the coupling longitudinal waves. The title of the paper implies that the noise frequency range induced by marine engine always below 3 kHz. So the consideration of pure bending waves in beam and plate should be sufficient. Above that frequency, the rotary inertia and the shear

deformation effects should be taken into consideration for the bending wave equations of beam and plate, otherwise there exists an error bigger than 3%.

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## BEHAVIOUR OF STRUCTURE-BORNE NOISE ATTENUATION IN A RESILIENT MOUNT

Wei-Hui Wang<sup>1</sup> Robert Sutton<sup>2</sup> Ben Dobson<sup>3</sup>

<sup>1</sup>Department of Naval Architecture, National Taiwan Ocean University, Keelung 202, Taiwan, ROC.

<sup>2</sup>Institute of Marine Studies, University of Plymouth, UK

<sup>3</sup>Devonport Management Limited, Devonport Royal Dockyard, Plymouth, UK

### ABSTRACT

To increase their performance envelope, most fast vessels are designed with a small block coefficient, compact engine room arrangement and are of light weight construction. Powerful main and/or auxiliary engines are thus installed in confined spaces which, as a consequence, cause an extremely high level noise source. This source can propagate structure-borne noise via multipaths, such as the mounts, pipes, ..., to remote cabins or compartments. Inevitably the structure-borne noise will eventually be radiated as acoustic waves within the vessel.

To characterize the vibration reduction behavior of the resilient mounts of an engine, a model can be established by utilizing the mechanical four-pole parameters method, in which the parameters of the two ends of the mount are expressed in terms of mobility function. The influence of the parameters on the vibration reduction effect is investigated using a sensitivity analysis. To validate the analytical model, comparisons are made with mobility results obtained from experimental trials.

### 1. INTRODUCTION

For the low noise design problem for a simple engine room onboard a vessel, Figure 1 represents the analysis models which can be used to model the structure-borne sound transfer between the primary sources and the radiating surface. For a uni-directional simple harmonic excitation force  $F_1$ , the engine structure vibrates with a simple harmonic velocity  $V_1$  at the driving point. Structural waves propagate from the excitation point to the adjacent bottom plate, where bending waves generate a velocity field  $V_2(x,y)$ . This vibration field radiates a sound field  $p(x,y,z)$ .

The linear equations which relate the radiated sound pressure with the force and the velocity at the driving point are:

$$p(x, y, z, f) = H_{1F} F_1(f) \quad ; \quad p(x, y, z, f) = H_{1V} V_1(f) \quad (1)(2)$$

$H_{1F}$  and  $H_{1V}$  are so-called frequency response functions, describing the sound transfer for force and velocity excitation respectively.

The force and velocity at the driving point are related according to the driving point mobility, which is a measure of vibration velocity for a given  $F_1$  :

$$M_{11} = \frac{V_1}{F_1} \quad (3)$$

In many practical situations instead of linear equations, equations in mean square quantities or so-called "energetic" quantities are used. In principle these formulations are

exact, if derived directly from equations (1) and (2). However, for practical purposes approximations of transfer functions are used, which relate 1/3-octave band levels of  $F_1^2$  and  $V_1^2$  with the radiated sound power  $P_{rad}$ . Using the scheme of Figure 1(b) and 1(c) the "energetic" models can be presented as:

$$H_{iF}^2 = \frac{\overline{V_2^2}}{\overline{F_1^2}} \quad ; \quad H_{iV}^2 = \frac{\overline{V_2^2}}{\overline{V_1^2}} \quad (4)$$

The function  $H_{iF}^2$  is called the mean square transfer mobility and the function  $H_{iV}^2$  is called the velocity transfer function. In the second transmission chain, it has the relation:

$$\rho c S \sigma = \frac{P_{rad}}{V_2^2} \quad (5)$$

The left term of equation (5) contains the characteristic impedance  $\rho c$  of the surrounding fluid (air, water, etc), the area  $S$  of the radiating surface and the radiation efficiency  $\sigma$ .  $\sigma$  is a measure of the efficiency with which a structure converts vibrations into sound.

The factors which determine the sound transfer and which will be influenced by design are seen from the following equations:

$$\frac{P_{rad}}{F_1^2} = (H_{iF}^2 S \sigma) \rho c = (M_{11}^2 H_{iV}^2 S \sigma) \rho c \quad ; \quad \frac{P_{rad}}{V_1^2} = (H_{iV}^2 S \sigma) \rho c \quad (6) \quad (7)$$

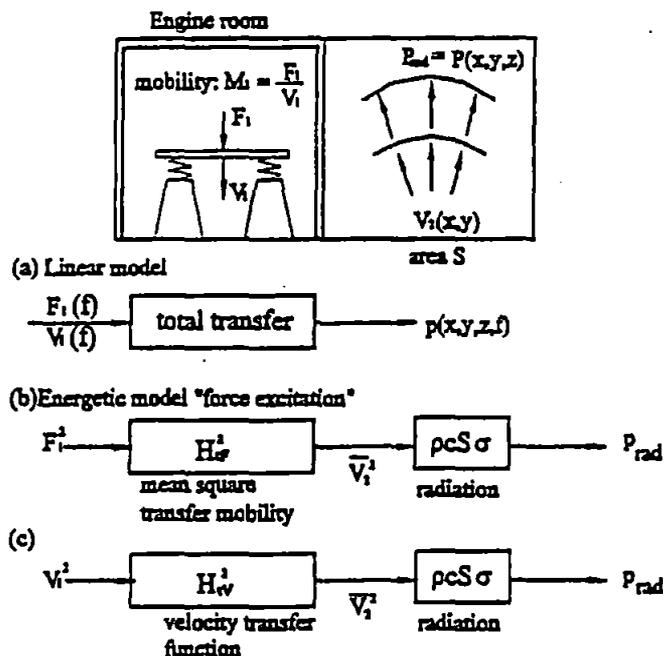


Figure 1 Schematic for the analysis of structure-borne noise transfer

Design measures to decrease  $S$  significantly, have very limited applications. However, measures to decrease  $M_{11}$  and  $H_{iV}^2$  are very important and will be discussed extensively in this paper.

The influence of the dynamic parameters of the mount on the mobility functions as

well as the vibration isolation effect can be recognized from a sensitivity analysis.

## 2. MOBILITY REDUCTION OF STRUCTURE-BORNE SOUND TRANSFER

Structural design measures which affect the structure-borne sound transfer between the point of excitation and the radiating surface can be classified into two categories. There are localized measures close to the point of excitation and measures distributed over the whole system or applied on the radiation surface. To judge whether measures which influence  $M_{11}$  or  $H_{iv}^2$  are more effective, one should start by looking at the frequency range in which noise reduction is needed.

### 2.1 Relation Between $|\hat{M}|$ and $M_{\infty}$ of Beam and Plate

Not only for beams, but also for structure like plates, cylinders, etc., the asymptotic approximation that is obtained by averaging over a frequency is closely equal to the mobility of the corresponding infinitely extended system [1] [2]. Such infinite system mobilities are often applied in machinery acoustics calculations. The reason is that for many problems the main interest is not in a need for a detailed frequency analysis.

The driving point mobilities for infinite beams and plates are given by Cremer et al., [1]:

$$\hat{M}_{\infty} = \frac{0.19(1-j)}{\rho S \sqrt{C_{Lb}} hf} \quad ; \quad M_{\infty} = \frac{0.125}{\sqrt{B' m''}} \approx \frac{0.453}{\rho_p c_{Lp} h^2} \approx \frac{5.41}{\omega \rho_p \lambda_B^2 h} \quad (8) (9)$$

where  $m''$ : mass per unit area;  $B'$ : bending stiffness per unit width;  $\rho/\rho_p$ : density of beam/plate material;  $c_{Lp}, c_{Lb}$ : longitudinal wave speed in plate/beam;  $h$ : thickness of plate / beam;  $\lambda_B$ : bending wavelength.

### 2.2 Velocity Transfer Function and Transfer Mobility

The velocity transfer function  $H_{iv}^2$ , depends strongly on the size and nature of a structure. For noise reduction purposes the main interest is in the eigenfrequency range. Here again the discussion will be limited to the elementary case of a finite plate. In principle  $H_{iv}^2$  can be calculated with the aid of an eigenfunction model and finite element method.

The derivation of  $H_{iv}^2$  can be used by the equality of injected and dissipated power

$$\hat{P}_{in} = P_{diss} \quad (10)$$

The power injected by a point source into the plate at a single frequency is given by :

$$\hat{P}_{in} = \frac{1}{2} \text{Re}\{\hat{F}_1 \hat{V}_1^*\} = F_{1,rms}^2 \text{Re}\{\hat{M}_1\} = V_{1,rms}^2 \text{Re}\left\{\frac{1}{\hat{M}_1}\right\} \quad (11)$$

For averaging over frequencies  $\hat{M}_1$  may be replaced by  $M_{\infty}$ . Using equations (9) and (11) this leads to :

$$\hat{P}_{in} = \frac{0.453 F_{1,\Delta f}^2}{\rho_p c_{Lp} h^2} = 2.2 \rho_p c_{Lp} h^2 V_{1,\Delta f}^2 \quad (12)$$

The power dissipation from the plate is caused by material damping, by energy transportation across the plate boundaries into the support and by sound radiation from the plate. A well known parameter which includes all these damping mechanisms is the

( apparent ) loss factor  $\eta$ . For each frequency or frequency band it may be defined as:

$$\eta(f) = \frac{P_{\text{diss}}}{\omega m'' S V_2^2} = \frac{\text{energy dissipation per vibration period}}{2\pi(\text{reversible}) \text{ mechanical energy}} \quad (13)$$

In equation (13) it is assumed that the total mechanical energy ( i.e. the sum of kinetic and potential energy ) is twice the kinetic energy, i.e.

$$E = m'' S \overline{V_2^2} \quad (14)$$

Therefore, the dissipated power may be written as :

$$\hat{P}_{\text{diss}} = \eta \omega m'' S \overline{V_2^2} \quad (15)$$

Using equations (10), (12) and (15) one obtains

$$H_{iv}^2(f) = \frac{0.35 c_{LP} h}{f S \eta(f)} \quad (16)$$

The squared transfer mobility  $H_{iv}^2$  in the eigenfrequency range follows from equations (9), (10) and (15) :

$$H_{iv}^2 = \frac{V_{2\Delta f}^2}{F_{1\Delta f}^2} = \frac{M_{\text{pos}}}{\eta \omega m'' S} \approx \frac{2.85}{\eta f \rho_p^2 c_{LP} h^3 S} \quad (17)$$

Comparing equations (16) and (17) reveals that the influence of damping upon  $H_{iv}^2$  and  $H_{if}^2$  is the same, but the influence of plate thickness variation is quite different. Therefore, it is of vital importance for a designer to have appropriate knowledge on the source ( or component ) which drives the structure.

### 2.3 Source Characterization : Force or Velocity Source

Using the scheme of Figure 2, a separation has been made between source and transfer system. The transfer functions are solely determined by the part of the engine structure "downstream" of the source connection point. However, for the evaluation of the effectiveness of noise reduction measures in the transfer path, which has happened to the excitation  $F_1$  and  $V_1$  need to be known. This can be easily illustrated from the result of equations (16) and (17) for  $H_{iv}^2$  and  $H_{if}^2$  for a plate. According to equation (16),  $H_{iv}^2$  is doubled, but according to equation (17),  $H_{if}^2$  decreases with a factor 8.

Therefore, the noise reduction due to alteration of a design parameter is not only dependent on the transfer properties of the structure, but also on the nature of the source.

## 3. MECHANICAL FOUR-POLE PARAMETERS METHOD IN SYSTEM MOBILITY VALUATION

During the last decade, a powerful method used to calculate the transmission and isolation of structure-borne vibration and noise has been successfully developed. This is the so-called mechanical four-pole parameters method. Recently, it has been also successfully applied in the vibration and noise control field [3].

### 3.1 Basic Theory of the Four-Pole Parameters Method

Assume a mechanical system, as shown in Figure 2, in which the harmonic force  $F_1$  and the harmonic velocity  $V_1$  were applied to the left end ( or the input end ) of the system and the force  $F_2$  and the velocity  $V_2$  were induced on the right end ( or the output end ). The relations between  $F_1$ ,  $V_1$  and  $F_2$ ,  $V_2$  are :

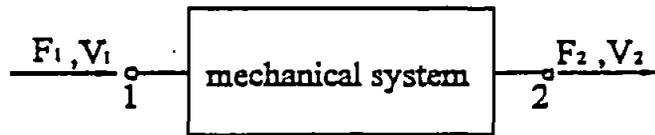


Figure 2 Input and output of a mechanical system

$$\hat{F}_1 = \alpha_{11} \hat{F}_2 + \alpha_{12} \hat{V}_2 \quad , \quad \hat{V}_1 = \alpha_{21} \hat{F}_2 + \alpha_{22} \hat{V}_2 \quad (18)$$

The coefficients  $\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}$  in equation (18) are called the four-pole parameters. Their definitions are:

$$\alpha_{11} = \left. \frac{\hat{F}_1}{\hat{F}_2} \right|_{\hat{V}_2=0} \quad , \quad \alpha_{12} = \left. \frac{\hat{F}_1}{\hat{V}_2} \right|_{\hat{F}_2=0} \quad , \quad \alpha_{21} = \left. \frac{\hat{V}_1}{\hat{F}_2} \right|_{\hat{V}_2=0} \quad , \quad \alpha_{22} = \left. \frac{\hat{V}_1}{\hat{V}_2} \right|_{\hat{F}_2=0} \quad (19)$$

$\hat{V}_2 = 0$  means that the output end is fixed, while  $\hat{F}_2 = 0$  means that the output end is free. From equation (22) it can be seen that  $\alpha_{11}, \alpha_{22}$  are dimensionless,  $\alpha_{12}$  has the dimension of impedance and  $\alpha_{21}$  has the dimension of mobility. In general, all four-pole parameters are frequency dependent complex quantities. They are relevant to the characteristics of the mechanical system itself.

From equation (18), the driving point and transfer mobilities can be obtained as:

$$M_{11} = \frac{\hat{V}_1}{\hat{F}_1} = \frac{\alpha_{21} \hat{F}_2 + \alpha_{22} \hat{V}_2}{\alpha_{11} \hat{F}_2 + \alpha_{12} \hat{V}_2} = \frac{\alpha_{21} + \alpha_{22} M_{22}}{\alpha_{11} + \alpha_{12} M_{22}} \quad (20)$$

$$M_{12} = \alpha_{21} + \alpha_{22} M_{22} \quad ; \quad M_{21} = \frac{\hat{V}_2}{\hat{F}_1} = \frac{1}{\alpha_{12}} - \frac{\alpha_{11} \hat{F}_2}{\alpha_{12} \hat{F}_1} \quad (21) (22)$$

where  $M_{22} = \frac{\hat{V}_2}{\hat{F}_2}$ . Also, the force and velocity transmissibility can be expressed as:

$$T_{F_{21}} = \left| \frac{\hat{F}_2}{\hat{F}_1} \right| = \left| \frac{1}{\alpha_{11} + \alpha_{12} M_{22}} \right| \quad ; \quad T_{V_{21}} = \left| \frac{\hat{V}_2}{\hat{V}_1} \right| = \left| \frac{M_{22}}{\alpha_{21} + \alpha_{22} M_{22}} \right| \quad (23) (24)$$

By equations (23) and (24), the system transmissibilities can be found providing the four-pole parameters and the receiving point mobility are known.

### 3.2 Four-Pole Parameters of a Two Degrees of Freedom Mount System

If the four resilient mounts which link the engine and the seating structure in series are composed of elastic and viscoelastic materials, they can be idealized as a two degrees of freedom (TDOF) system. When the exciting forces  $F_i(t) = \hat{F}_i e^{j\omega t}$ ,  $i = 1, 2$ , are applied at the upper point 1 and the lower point 2 of the mounts, the equations of motion for the TDOF system are:

$$[m_{ij}]\{\ddot{x}\} + [c_{ij}]\{\dot{x}\} + [k_{ij}]\{x\} = \{F_i(t)\}.$$

The vibration displacements can be represented as

$$X_i(t) = \hat{X}_i e^{j\omega t}, \quad i = 1, 2. \quad (25)$$

in which  $\hat{X}_1, \hat{X}_2$  are called complex response amplitudes. The force amplitudes and the complex response amplitudes can be related by:

$$S_{ij}(\omega) \hat{X} = \hat{F}_i \quad , \quad i, j = 1, 2. \quad (26)$$

Defining the dynamic stiffness functions  $S_{ij}(\omega)$  of the system as

$$S_{ij}(\omega) = (-\omega^2 m_{ij} + j\omega c_{ij} + k_{ij}) \quad , \quad i, j = 1, 2 \quad (27)$$

then, equation (26) becomes

$$\hat{X}_j = S_{ij}^{-1}(\omega) \hat{F}_i \quad , \quad i, j = 1, 2. \quad (28)$$

The vibration velocity vector is

$$V_i = \dot{X}_i(t) = j\omega \hat{X}_i e^{j\omega t} = \hat{V}_i e^{j\omega t} \quad , \quad i = 1, 2. \quad (29)$$

The mobility matrix for a coupled TDOF engine/mount/seat system is related by :

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{Bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (30)$$

where the mobility functions  $M_{ij}$  ( $i=1,2$  and  $j=1,2$ ) can be expressed in terms of the dynamic stiffness functions :

$$M_{11} = \frac{j\omega S_{22}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad ; \quad M_{12} = \frac{-j\omega S_{12}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (31) \quad (32)$$

$$M_{12} = \frac{-j\omega S_{21}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad ; \quad M_{22} = \frac{j\omega S_{11}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (33) \quad (34)$$

Expanding equation (30) and rearranging gives:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (35)$$

where

$$\alpha_{11} = \frac{M_{11}}{M_{12}} \quad , \quad \alpha_{12} = \frac{M_{12}M_{21} - M_{11}M_{22}}{M_{21}} \quad , \quad \alpha_{21} = \frac{1}{M_{12}} \quad ; \quad \alpha_{22} = -\frac{M_{22}}{M_{12}} \quad (36)$$

### 3.3 Connection of Mechanical four-pole Systems

Analogous to an electric circuit system, there are two basic types of connection for the mechanical four-pole systems which are the series and parallel connections. If the output end of one system is rigidly joined to the input end of another system, it is said to be in series connection. While the input (output) ends of two more mechanical systems are rigidly joined together and move with the same velocity, then it is said to be in parallel connection.

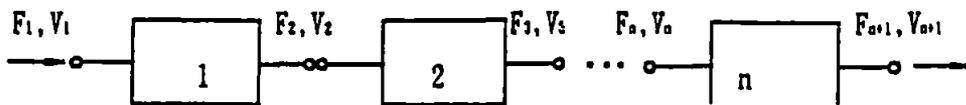


Figure 3 Four-pole subsystems in series connection

As the four-pole mechanical subsystems are connected in series , as shown in Figure 3, then the output of one subsystem is just the input of the subsequently adjacent

subsystem . For each subsystem , the matrix form expression in terms of the four-pole parameters is :

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{V}_1 \end{Bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_2 \\ \hat{V}_2 \end{Bmatrix}, \quad \begin{Bmatrix} \hat{F}_2 \\ \hat{V}_2 \end{Bmatrix} = \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_3 \\ \hat{V}_3 \end{Bmatrix}, \quad \dots \quad \begin{Bmatrix} \hat{F}_n \\ \hat{V}_n \end{Bmatrix} = \begin{bmatrix} \alpha^n_{11} & \alpha^n_{12} \\ \alpha^n_{21} & \alpha^n_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_{n+1} \\ \hat{V}_{n+1} \end{Bmatrix}$$

Therefore,

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{V}_1 \end{Bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \dots \begin{bmatrix} \alpha^n_{11} & \alpha^n_{12} \\ \alpha^n_{21} & \alpha^n_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_{n+1} \\ \hat{V}_{n+1} \end{Bmatrix} \quad (37)$$

Figure 4 shows the n subsystems in parallel connection, in which the combined input or output forces is equal to the sum of the input and output forces of each individual subsystems. Then

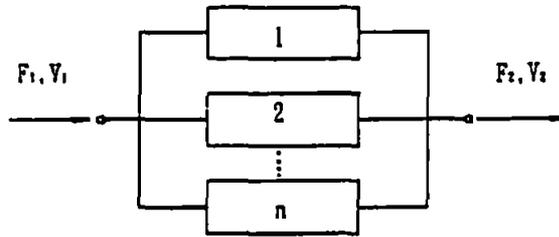


Figure 4 Four-pole subsystems in parallel connection

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{V}_1 \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_2 \\ \hat{V}_2 \end{Bmatrix} \quad (38)$$

where

$$\beta_{11} = \frac{A}{B}, \quad \beta_{12} = \frac{AC - B^2}{B}, \quad \beta_{21} = \frac{1}{B}, \quad \beta_{22} = \frac{C}{B} \quad (39)$$

and

$$A = \sum_{i=1}^n \frac{\alpha^i_{11}}{\alpha^i_{21}}, \quad B = \sum_{i=1}^n \frac{1}{\alpha^i_{21}}, \quad C = \sum_{i=1}^n \frac{\alpha^i_{22}}{\alpha^i_{21}} \quad (40)$$

For example, if the system is composed of two subsystems in parallel connection, then

$$A = \frac{\alpha'_{11}\alpha''_{21} + \alpha'_{21}\alpha''_{11}}{\alpha'_{21}\alpha''_{21}} = \frac{\varphi}{\varepsilon}, \quad B = \frac{\alpha'_{21} + \alpha''_{21}}{\alpha'_{21}\alpha''_{21}} = \frac{\theta}{\varepsilon}, \quad C = \frac{\alpha'_{22}\alpha''_{21} + \alpha'_{21}\alpha''_{22}}{\alpha'_{21}\alpha''_{21}} = \frac{\lambda}{\varepsilon} \quad (41)$$

thus,

$$\beta_{11} = \frac{A}{B} = \frac{\varphi}{\theta}, \quad \beta_{21} = \frac{1}{B} = \frac{\varepsilon}{\theta}, \quad \beta_{12} = \frac{AC - B^2}{B} = \frac{\varphi\lambda - \theta^2}{\varepsilon\theta}, \quad \beta_{22} = \frac{C}{B} = \frac{\lambda}{\theta} \quad (42)$$

where

$$\varphi = \alpha'_{11}\alpha''_{21} + \alpha'_{21}\alpha''_{11}, \quad \theta = \alpha'_{21} + \alpha''_{21}, \quad \varepsilon = \alpha'_{21}\alpha''_{21}, \quad \lambda = \alpha'_{22}\alpha''_{21} + \alpha'_{21}\alpha''_{22} \quad (43)$$

Figure 5 shows a dynamic absorber system, which is composed of the set of a spring and a damper in parallel connection then connected to a mass in series. Using the equations for four-pole parameters of subsystems in parallel and series connection, one can obtain:

$$\alpha_{11} = 1, \alpha_{12} = j\omega m, \alpha_{21} = \frac{j\omega}{k + j\omega C}, \alpha_{22} = 1 - \frac{\omega^2 m}{k + j\omega C} \quad (44)$$

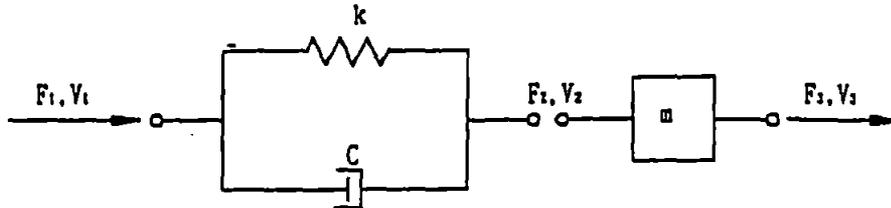


Figure 5 Dynamic absorber system

### 3.4 Explanation of Reduction in Transfer Mobility by Resilient Mount

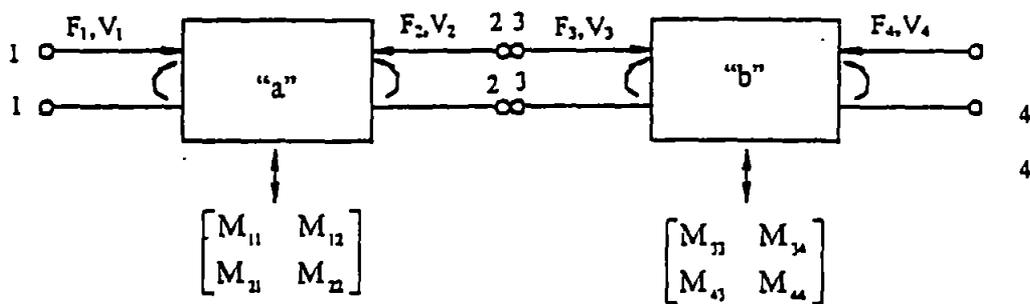


Figure 6 Mobility of integrated system combined by mobility matrices of the subsystems

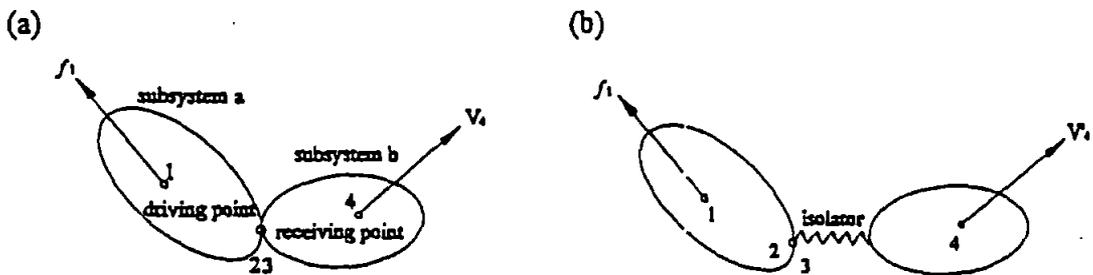


Figure 7 System before and after installation of isolator

If the engine/mount/seat system can be considered as a TDOF system, the mobility of the integrated total system can be derived in terms of the mobility matrices of the individual subsystems. If the two ends of the subsystems a,b are 1,2 and 3,4 respectively, as shown in Figure 6, in which F represents force and V represents velocity. When the subsystems link together, then  $V_3 = V_2$  and  $F_3 = -F_2$ , equation (37) becomes

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{33} & \alpha_{34} \\ \alpha_{21} & \alpha_{22} & -\alpha_{43} & -\alpha_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{14} \\ \alpha'_{41} & \alpha'_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} \quad (45)$$

It requires that  $F_4 = 0$  since 4 is the receiving point and 1 is the driving point. Thus,

$$\frac{1}{\alpha'_{41}} = M_{41} = \frac{V_4}{F_4} \quad (46)$$

From equation (45),

$$\alpha'_{41} = \alpha_{21}\alpha_{33} - \alpha_{22}\alpha_{43} = \frac{M_{22} + M_{33}}{M_{12}M_{34}}, \text{ or } M_{41} = \frac{M_{12}M_{34}}{M_{22} + M_{33}} \quad (47)$$

From equation (47), it can be seen that the transfer mobility of the combined system ( $M_{41}$ ) is not simply the combination of the transfer mobilities ( $M_{12}$  and  $M_{34}$ ) of the two subsystems. The linking point mobilities ( $M_{22}$  and  $M_{33}$ ) of the subsystems play a predominant role to determine the total transfer mobility. As an isolator is connected to the input end of the subsystem b, such as Figure 7(b) which represents the improved system of Figure 7(a), then the mobility  $M_{33}$  significantly. It follows from equation (47) that the total transfer mobility  $M_{41}$  is reduced. In other words, to reduce the total transfer mobility, it requires the introduction of a larger mobility than the original one. This is the reason of the basic principle for a good structure-borne noise attenuation result that has to properly design the structure on both sides of a resilient mount with small mobility than that of the mount itself.

#### 4. MEASUREMENT OF FOUR-POLE PARAMETERS AND APPLICATION

In practical application of the four-pole parameters method to the solution of mechanical vibration and structure-borne noise problem, first it is most important to validate that the four-pole parameters of the system are correct. Undoubtedly, in order to match the derived model, a mobility measurement technique should be developed.

Figure 8 shows the experimental layout and instrumentation for the mobility measurement. The test mount is placed in the middle part of the H-frame of a universal testing machine. In between the pedestal and the mount and in between the press and the mount there are rubber and steel block insertions. Accelerometers are installed on the top and bottom side of the mount. The input excitation force is applied to the top and bottom side of the mount by a hammer kit.

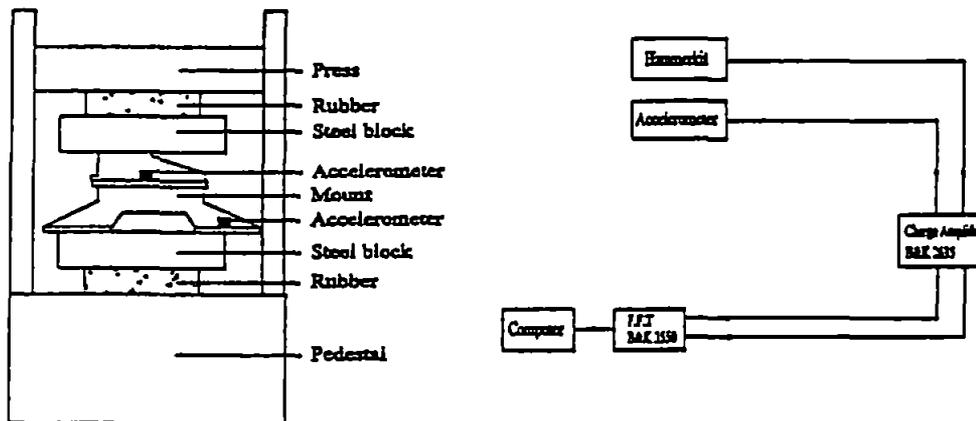


Figure 8 Experiment layout and Instrumentation for mobility measurement

In this case, the mobility measurements were carried out on the motor-bedplate-resilient mount-seat structure system of a scale model as shown in Figure 9. The transfer



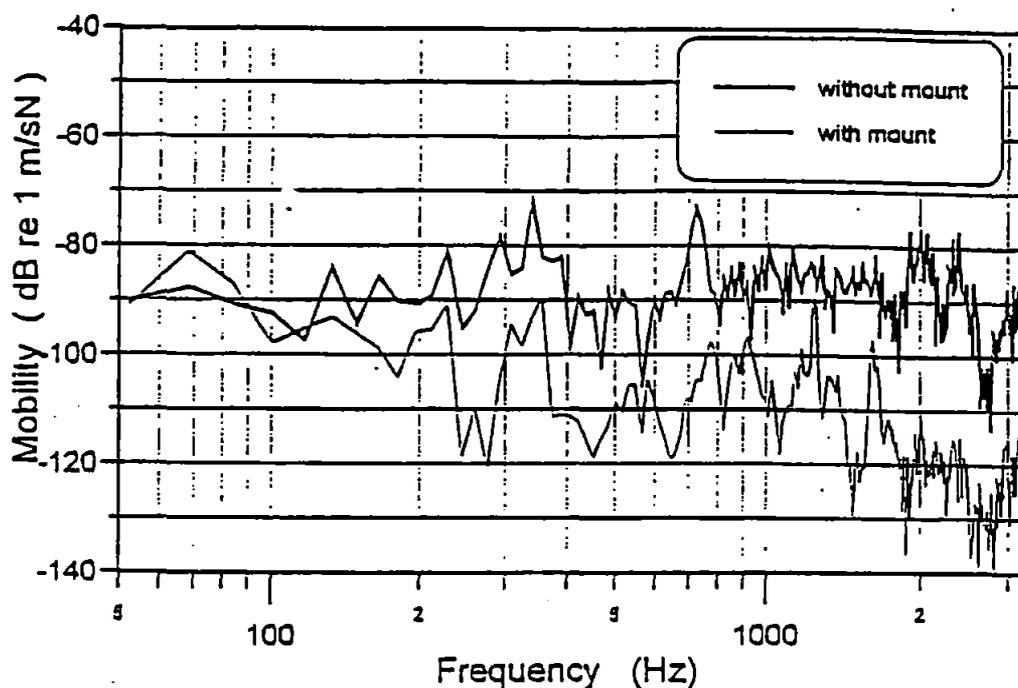


Figure 10 Predicted attenuation of structure-borne noise radiation from a machine by introduction of resilient mount

### 5. CONCLUDING REMARKS

From the present study, the following conclusions are forwarded:

In order to obtain acceptable structure-borne noise attenuation, it is of fundamental importance that the structure in both sides of a resilient mount should have small mobility characteristics in comparison with the mount itself. In other words, the desired mobility changes for the coupled system of engine-resilient mount-foundation in series are: "heavy-compliant-heavy" and / or "stiff-compliant-stiff".

The mechanical four-pole parameters method has been shown to be effective for the experimental analysis in the mobility prediction. As a consequence, it can be applied as a generic design tool to the structure-borne vibration and noise control field as a whole.

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## BEHAVIOUR OF STRUCTURE-BORNE NOISE ATTENUATION IN A RESILIENT MOUNT

W.H. Wang\* R. Sutton\*\* B. Dobson\*\*\*

\* Department of Naval Architecture  
National Taiwan Ocean University  
Keelung 202, Taiwan, ROC

\*\* Institute of Marine Studies University of Plymouth, UK

\*\*\* Devonport Management Limited  
Devonport Royal Dockyard, Plumouth, UK

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### ABSTRACT

To increase their performance envelope, most fast vessels are designed with a small block coefficient, compact engine room arrangement and are of light weight construction. Powerful main and/or auxiliary engines are thus installed in confined spaces which, as a consequence, cause an extremely high level noise source. This source can propagate structure-borne noise via multipaths, such as the mounts, pipes, ..., to remote cabins or compartments. Inevitably the structure-borne noise will eventually be radiated as acoustic waves within the vessel.

To characterize the vibration reduction behavior of the resilient mounts of an engine, a model can be established by utilizing the mechanical four-pole parameters method, in which the parameters of the two ends of the mount are expressed in terms of mobility functions. The influence of the parameters on the vibration reduction effect is investigated using a sensitivity analysis. To validate the analytical model, comparisons are made with mobility results obtained from experimental trials.

### 結構噪音在彈性墊中之削減行爲

王偉輝\* 羅伯沙頓\*\* 班道生\*\*\*

\* 國立台灣海洋大學造船工程系

\*\* 英國普利茅斯大學海洋研究所

\*\*\* 英國丹福浦皇家造船廠

關鍵詞：結構噪音，彈性墊，四端參數，動性，削減

### 摘 要

高速艦艇為提高其綜合性能，大都設計成小方塊係數的船形，緊湊的機艙佈置及採用輕構造。為達高速而選用的大馬力主/副機，勢必按裝於有限的機艙空間中，因此之故造成了一個極高分貝的噪音源。此噪音源可經由多重通道(如彈性墊、管路...等)而對外傳播出結構噪音，以達於相距很遠的房艙，隨之，無可避免的，結構噪音又會輻射成空氣噪音。為了對機器彈性墊之減振行爲加以定性，本文利用機械四極參數法建立了分析模式，其中四極參數是利用彈性墊兩端之動性函數來表示。透過敏感度分析，亦探討了各相關參數對減振效應之影響。為了驗證分析模式之正確性，對所得之動性結果亦與實驗加以比對。

## INTRODUCTION

Often structure-borne noise and vibration power is predominantly transmitted to a sound carrying structure from a source via a number of contact points. In turn, the noise and vibrations are propagated in the structure possibly causing sensitivity equipment to vibrate or to cause an undesired noise radiation. In principle, this may be avoided by measures at the source, in transmission, during propagation or at radiation. It is, of course, preferable to handle the problem at the generation sites and thereby avoid more comprehensive and expensive measures later in the chain.

Kihlman [1] developed a simple model to calculate the vibrational power fed into a structure for a multipoint mounting system based on the concept of effective overall mobility of the source and the foundation and the effective transfer mobility of the vibration isolators. But the vibration source was considered to be under rigid body motion only.

Verheij [2] has discussed an experimental method in which the internal excitation of a machine was replaced by an equivalent external force. Application of this method to three diesel engines for the octave bands with central frequencies from 125 to 2kHz showed that the real excitation may be replaced by an external equivalent force, which generates within 3dB the same response as that of the running engine, for various receiver positions, various sound transmission paths, various ways of mounting and various environments. There are some limitations of such a method, i.e., (1) the vibrational energy should be distributed over the respective transmission paths in the same ratio; (2) the equivalent forced measured with the aid of a loudspeaker can be easily recognized by their narrow band signature, for broad band sources, this will be less easy; (3) the transmission system would be reciprocal.

The works of Middleton [3], Kihlman [4] and Plunt [5] have discussed the practical methods to define the structure-borne noise source strength

of machinery and which could be measured by reference to the response of a typical foundation to the force source.

The transmission in turn, is completely determined by the source strength together with the dynamic characteristics of the source, of the receiver and of the coupling elements in between the two subsystems. A series of research works, such as Pinnington [6][7][8], Pinnington and While [9], Pinnington and Pearce [10] and Dobson et al. [11]; and Janssen and Buiten [12], Buiten [13], Ohlrich [14][15], Petersson [16][17], Petersson and Plunt [18][19][20][21], Plunt [22], Pulunt and Odegaard [23], have been undertaken in United Kingdom and Nordic Cooperative Organization respectively since 1980. Such works mainly concentrated on the parameters controlling power transmission from a vibrating machine to the seating structure, via spring-like vibration isolators. The low frequency range was considered where the machine moves as a rigid body. It was shown that the finite seating structure can be modelled by an equivalent structure of infinite extent for frequency averaged power transmission calculations. The power transmission was measured by two proposed methods; the first involves the real component of the seating impedance, and the second the transfer impedance of the isolator. Most experimental measurements in these research works were undertaken in laboratory. While in the situation of ship structure, the characterization of power flow of structure-borne noise source by measurements will be much more complicated.

A comprehensive review clearly highlights the fact that there are very few validated models for predicting the propagation of structure-borne noise with medium frequency range, i.e. from 125 Hz to 1kHz, to be found in the published literature. The mobility model to calculate the foundation structure is decided to use and a systematical and comprehensive approach is offered in this paper for calculating the point and transfer mobilities of various source-isolator-receiver coupled systems.

In order to evaluate the mobility reduction behaviour of structure-borne sound transfer from a marine engine via resilient mounts, the mechanical four-pole parameters method is adopted in this paper to explain the theory.

### LOGIC OF STRUCTURE-BORNE SOUND REDUCTION

The low noise design problem for a simple engine room onboard a vessel is schematically shown in Fig.1. The primary noise generating mechanism is the main engine that will inject vibrations into the structure via the mounts and foundation. These vibrations are transmitted throughout the structure and may eventually radiate to the surroundings.

The vibration reduction behaviour of the resilient mounts of an engine may be simulated by utilizing the input/output transfer functions. Fig.2 represents the analysis models which can be used to model the structure-borne sound transfer between the primary sources and the radiating surface. For a uni-directional simple harmonic excitation force  $F_1$ , the engine structure vibrates with a simple harmonic velocity  $V_1$  at the driving point. Structural waves propagate from the excitation point to the adjacent bottom plate, where bending waves generate a velocity field  $V_2(x, y)$ . This vibration field radiates a sound field  $p(x, y, z)$ .

The linear equations which relate the radiated sound pressure with the force and the velocity at the driving point are:

$$p(x, y, z, f) = H_{tF} F_1(f) \quad (1)$$

$$p(x, y, z, f) = H_{tV} V_1(f) \quad (2)$$

$H_{tF}$  and  $H_{tV}$  are so-called frequency response functions, describing the sound transfer for force and velocity excitation respectively. Basically, these frequency response functions have a complex value at each frequency, i.e. magnitude and phase.

The force and velocity at the driving point are related according to the driving point mobility  $M_{11}$ , which is a measure of vibration velocity for a given  $F_1$ :

$$M_{11} = \frac{V_1}{F_1} \quad (3)$$

In many practical situations instead of linear equations, equations in mean square quantities or so-called "energetic" quantities are used. In principle these formulations are exact, if derived directly from equations (1) and (2). However, for practical purposes approximations of transfer functions are used, which relate 1/3-

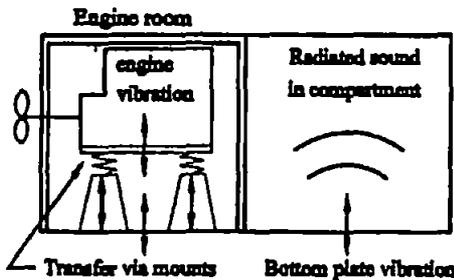
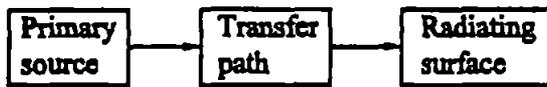
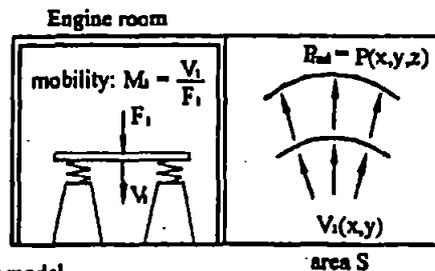


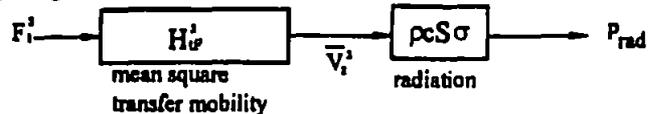
Fig.1 Schematic of machinery noise analysis for low noise design



(a) Linear model



(b) Energetic model "force excitation"



(c) Energetic model "velocity excitation"

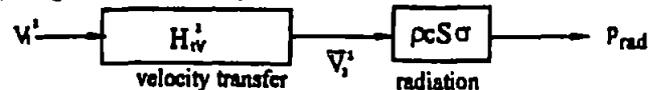


Fig.2 Schematic for the analysis of structure-borne noise transfer

octave band levels of  $F_1^2$  and  $V_1^2$  with the radiated sound power  $P_{\text{rad}}$ . Using the scheme of Fig.2(b) and 2(c) the "energetic" models can be presented. The transmission chain is divided into two blocks. The first block represents the structure and gives the ratio between the spatially averaged mean square velocity  $\bar{V}_2^2$  of the bottom surface and  $F_1^2$  or  $V_1^2$ :

$$H_{iF}^2 = \frac{\bar{V}_2^2}{F_1^2}; \quad H_{iV}^2 = \frac{\bar{V}_2^2}{V_1^2} \quad (4)$$

The function  $H_{iF}^2$  is called the mean square transfer mobility and the function  $H_{iV}^2$  is called the velocity transfer function.

The second block represents the sound radiation and gives the ratio between the radiated sound power and  $\bar{V}_2^2$ :

$$\rho c S \sigma = \frac{P_{\text{rad}}}{\bar{V}_2^2} \quad (5)$$

The left term of equation (5) contains the characteristic impedance  $\rho c$  of the surrounding fluid (air, water, etc), the area  $S$  of the radiating surface and the radiation efficiency  $\sigma$  which is a measure of the efficiency with which a structure converts vibrations into sound.

The factors which determine the sound transfer and will be influenced by design are seen from the following equations:

$$\frac{P_{\text{rad}}}{F_1^2} = (H_{iF}^2 S \sigma) \rho c = (M_{11}^2 H_{iV}^2 S \sigma) \rho c \quad (6)$$

$$\frac{P_{\text{rad}}}{V_1^2} = (H_{iV}^2 S \sigma) \rho c \quad (7)$$

If the nature of the excitation is such that for different engine design  $F_1$  is unaffected, equation (6) implies that sound reduction is obtained by: decrease of driving point mobility; reduction of radiating surface area; decrease of velocity transfer function; decrease of radiation efficiency.

If the nature of the excitation is such that for different engine designs  $V_1$  is unaffected, equation (7) implies that the sound reduction is obtained by: reduction of radiating surface area;

decrease of velocity transfer function; decrease of radiation efficiency.

Design measures to decrease  $S$  significantly, have very limited applications. However, measures to decrease  $M_{11}$  and  $H_{iV}^2$  are very important and will be discussed extensively in this paper.

The influence of the dynamic parameters of the mount on the mobility functions as well as the vibration isolation effect can be recognized from a sensitivity analysis.

## MOBILITY REDUCTION OF STRUCTURE-BORNE SOUND TRANSFER

Structural design measures which affect the structure-borne sound transfer between the point of excitation and the radiating surface can be classified into two categories. There are localized measures close to the point of excitation and measures distributed over the whole system or applied on the radiation surface. To judge whether the measures which influence  $M_{11}$  or  $H_{iV}^2$  are more effective, one should start by looking at the frequency range in which noise reduction is needed.

### 1. Relation Between $|\hat{M}|$ and $M_{\infty}$ of Beam and Plate

Not only for beams, but also for structure like plates, cylinders, etc., the asymptotic approximation that is obtained by averaging over frequency is closely equal to the mobility of the corresponding infinitely extended system [23] [24]. Such infinite system mobilities are often applied in machinery acoustics calculations. The reason is that for many problems the main interest is not in a need for a detailed frequency analysis. Often the noise spectrum is broadband and/or varying with rotation speed. If such an excitation generates structural responses in the frequency range far above the lowest natural frequency, a change of structural design parameters will affect many relevant natural frequencies simultaneously. Averaging over frequency bands is then profitable in analysis procedures.

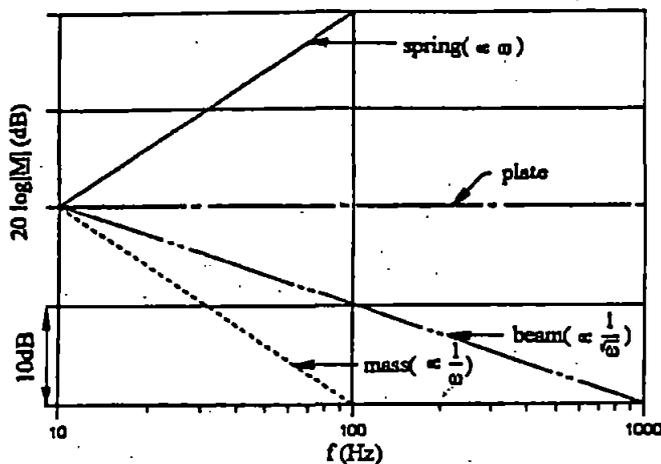


Fig.3 Mobility from idealized structural elements

The driving point mobilities for infinite beams and plates are given by Cremer et.al., [23]:

$$\hat{M}_{b\infty} = \frac{0.19(1-j)}{\rho S \sqrt{C_{Lb} h f}} \quad (8)$$

$$|M|_{p\infty} = \frac{0.125}{\sqrt{B' m''}} \approx \frac{0.453}{\rho_p c_{Lp} h^2} \approx \frac{5.41}{\omega \rho_p \lambda_B^2} \quad (9)$$

where  $m''$ : mass per unit area;  $B'$ : bending stiffness per unit width;  $\rho/\rho_p$ : density of beam/plate material;  $c_{Lp}, c_{Lb}$ : longitudinal wave speed in plate/beam;  $h$ : thickness of plate/beam;  $\lambda_B$ : bending wavelength;  $j$ : unit imaginary number.

Fig.3 shows the frequency behaviour of  $20 \log |\hat{M}|$  for a mass [25], a spring [25], an infinite beam (equation (8)) and an infinite plate (equation (9)). Note that for  $|M|_{p\infty}$  to be approximately valid not only averaging over frequencies is required, but also at least a distance of  $\lambda_B/2$  from plate boundaries.

## 2. Velocity Transfer Function and Transfer Mobility

The velocity transfer function  $H_{iV}^2$  depends strongly on the size and nature of a structure. For noise reduction purposes the main interest is in the natural frequency range. Here again the discussion will be limited to the elementary case of a finite plate. In principle  $H_{iV}^2$  can be calculated with the aid of an eigenfunction model and

finite element method. Again it can be said that if in the frequency range of interest a large number of eigenfrequencies is involved the calculation become time consuming. Moreover, from the viewpoint of the designer, it is important to look at smoothed data and to understand the relation between design parameters and these data.

The derivation of  $H_{iV}^2$  uses the equality of injected and dissipated power

$$P_{in} = P_{diss} \quad (10)$$

The power injected by a point source into the plate at a single frequency is given by:

$$\begin{aligned} P_{in} &= \frac{1}{2} \text{Re} \{ \hat{F}_1 \hat{V}_1^* \} = F_{1,rms}^2 \text{Re} \{ \hat{M}_1 \} \\ &= V_{1,rms}^2 \text{Re} \left\{ \frac{1}{\hat{M}_1} \right\} \end{aligned} \quad (11)$$

For averaging over frequencies  $\hat{M}_1$  may be replaced by  $M_{p\infty}$ . Using equations (9) and (11) this leads to:

$$\hat{P}_{in} = \frac{0.453 F_{1,\Delta f}^2}{\rho_p c_{Lp} h^2} = 2.2 \rho_p c_{Lp} h^2 V_{1,\Delta f}^2 \quad (12)$$

The power dissipation from the plate is caused by material damping, by energy transportation across the plate boundaries into the support and by sound radiation from the plate. A well known parameter which includes all these damping mechanisms is the (apparent) loss factor  $\eta$ . For each frequency or frequency band it may be defined as:

$$\begin{aligned} \eta(f) &= \frac{P_{diss}}{\omega m'' S \bar{V}_2^2} \\ &= \frac{\text{energy dissipation per vibration period}}{2\pi \times \text{mechanical energy (reversible)}} \end{aligned} \quad (13)$$

In equation (13) it is assumed that the total mechanical energy ( i.e. the sum of kinetic and potential energy ) is twice the kinetic energy, i.e.

$$E = m'' S \bar{V}_2^2 \quad (14)$$

Therefore, the dissipated power may be written as:

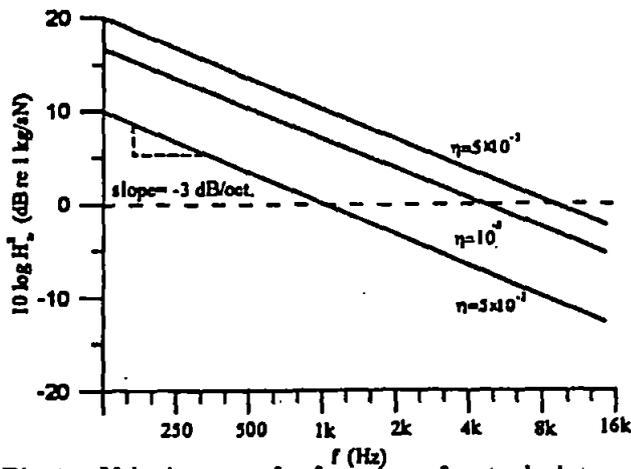


Fig.4 Velocity transfer function of a steel plate (480 mm×340 mm×5 mm ) calculated according to equation (16).

$$\hat{P}_{diss} = \eta \omega m'' S \bar{V}_2^2 \quad (15)$$

Using equations (10), (12) and (15) one obtains

$$H_{iv}^2(f) = \frac{0.35 c_{Lp} h}{f S \eta(f)} \quad (16)$$

Fig.4 shows the effect on  $H_{iv}^2$  of different loss factor. It is seen that at low frequencies  $H_{iv}^2 \gg 1$ . The frequency at which  $H_{iv}^2 = 1$ , decreases with increasing  $\eta$  and decreasing plate thickness. The squared transfer mobility  $H_{if}^2$  in the eigenfrequency range follows from equations (10), (11), (12) and (15):

$$H_{if}^2 = \frac{\bar{V}_{2,\Delta f}^2}{F_{1,\Delta f}^2} = \frac{M_{pco}}{\eta \omega m'' S} \approx \frac{2.85}{\eta f \rho_p^2 c_{Lp} h^3 S} \quad (17)$$

Comparing equations (16) and (17) reveals that the influence of damping upon  $H_{iv}^2$  and  $H_{if}^2$  is the same, but the influence of plate thickness variation is quite different. Therefore, it is of vital importance for a designer to have appropriate knowledge on the source (or component) which drives the structure.

### 3. Source Characterization: Force or Velocity Source

Using the scheme of Fig.2, a separation has been made between source and transfer system. The transfer functions are solely determined by

the part of the engine structure “downstream” of the source connection point. However, for the evaluation of the effectiveness of noise reduction measures in the transfer path, it has also to be known what happens with the excitation  $F_1$  or  $V_1$ . This can be easily illustrated from the result of equations (16) and (17) for  $H_{iv}^2$  and  $H_{if}^2$  for a plate. What is the effect of doubling the plate thickness  $t$ ? According to equation (16),  $H_{iv}^2$  is doubled, but according to equation (17),  $H_{if}^2$  decreases with a factor 8.

Therefore, the noise reduction due to alteration of a design parameter is not only dependent on the transfer properties of the structure, but also on the nature of the source.

## MECHANICAL FOUR-POLE PARAMETERS METHOD IN SYSTEM MOBILITY VALUATION

During the last decade, a powerful method used to calculate the transmission and isolation of structure-borne vibration and noise has been successfully developed. This is the so-called mechanical four-pole parameters method. It has been also successfully applied in the vibration and noise control field [26].

### 1. Basic Theory of the Four-Pole Parameters Method

Assume a mechanical system, as shown in Fig.5, in which the harmonic force  $\hat{F}_1$  and the harmonic velocity  $\hat{V}_1$  were applied to the left end (or the input end) of the system and the force  $\hat{F}_2$  and the velocity  $\hat{V}_2$  were induced on the right end (or the output end). The relations between  $\hat{F}_1, \hat{V}_1$  and  $\hat{F}_2, \hat{V}_2$  are:

$$\begin{aligned} \hat{F}_1 &= \alpha_{11} \hat{F}_2 + \alpha_{12} \hat{V}_2 \\ \hat{V}_1 &= \alpha_{21} \hat{F}_2 + \alpha_{22} \hat{V}_2 \end{aligned} \quad (18)$$

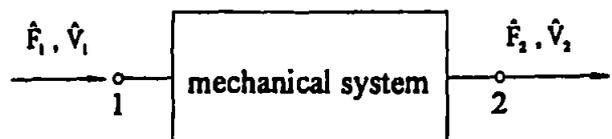


Fig.5 Input and output of a mechanical system

The coefficients  $\alpha_{11}$ ,  $\alpha_{12}$ ,  $\alpha_{21}$ ,  $\alpha_{22}$  in equation (18) are called the four-pole parameters. Their definitions are:

$$\begin{aligned} \alpha_{11} &= \left. \frac{\hat{F}_1}{\hat{F}_2} \right|_{\hat{V}_2=0}, \\ \alpha_{12} &= \left. \frac{\hat{F}_1}{\hat{V}_2} \right|_{\hat{F}_2=0}, \\ \alpha_{21} &= \left. \frac{\hat{V}_1}{\hat{F}_2} \right|_{\hat{V}_2=0}, \\ \alpha_{22} &= \left. \frac{\hat{V}_1}{\hat{V}_2} \right|_{\hat{F}_2=0} \end{aligned} \quad (19)$$

$\hat{V}_2 = 0$  means that the output end is fixed, while  $\hat{F}_2 = 0$  means that the output end is free. From equation (19) it can be seen that  $\alpha_{11}$ ,  $\alpha_{22}$  are dimensionless,  $\alpha_{12}$  has the dimension of impedance and  $\alpha_{21}$  has the dimension of mobility. In general, all four-pole parameters are frequency dependent complex quantities. They are relevant to the characteristics of the mechanical system itself.

From equation (18), the driving point mobility can be obtained as:

$$\begin{aligned} M_{11} &= \frac{\hat{V}_1}{\hat{F}_1} = \frac{\alpha_{21}\hat{F}_2 + \alpha_{22}\hat{V}_2}{\alpha_{11}\hat{F}_2 + \alpha_{12}\hat{V}_2} \\ &= \frac{\alpha_{21} + \alpha_{22}M_{22}}{\alpha_{11} + \alpha_{12}M_{22}} \end{aligned} \quad (20)$$

where  $M_{22} = \frac{\hat{V}_2}{\hat{F}_2}$ . The derived transfer mobilities are:

$$M_{12} = \alpha_{21} + \alpha_{22}M_{22}; \quad (21)$$

$$M_{21} = \frac{\hat{V}_2}{\hat{F}_1} = \frac{1}{\alpha_{12}} - \frac{\alpha_{11}\hat{F}_2}{\alpha_{12}\hat{F}_1} \quad (22)$$

Also, the force and velocity transmissibility can be expressed as:

$$T_{F_{12}} = \left| \frac{\hat{F}_2}{\hat{F}_1} \right| = \left| \frac{1}{\alpha_{11} + \alpha_{12}M_{22}} \right|; \quad (23)$$

$$T_{V_{12}} = \left| \frac{\hat{V}_2}{\hat{V}_1} \right| = \left| \frac{M_{22}}{\alpha_{21} + \alpha_{22}M_{22}} \right| \quad (24)$$

By equation (23) and (24), the system transmissibilities can be found providing the four-pole parameters and the receiving point mobility or impedance are known.

## 2. Four-Pole Parameters of a Two Degrees of Freedom Mount System

If the four resilient mounts which link the engine and the seating structure in series, are composed of elastic and viscoelastic materials, they can be idealized as a two degrees of freedom (TDOF) system as shown in Fig.6. The dynamic properties of this system are the mass per unit height of the mount ( $m_o$ ), the stiffness ( $k$ ) and the damping coefficient ( $c$ ) for each mount; the lumped mass ( $m_e$ ) of the engine, the effective lumped mass ( $m_s$ ), the effective damping coefficient ( $C$ ) and the effective stiffness ( $K$ ) of the seating structure. When the exciting forces  $F_i(t) = \hat{F}_i e^{j\omega t}$ ,  $i = 1, 2$ , are applied at the upper point 1 and the lower point 2 of the mounts, the equations of motion for the TDOF system are

$$[m]\{\ddot{X}\} + [c]\{\dot{X}\} + [k]\{X\} = \{F(t)\} \quad (25)$$

The vibration displacements can be represented as

$$X_i(t) = \hat{X}_i e^{j\omega t}, \quad i = 1, 2. \quad (26)$$

in which  $\hat{X}_1$ ,  $\hat{X}_2$  are called complex response amplitudes. The force amplitudes and the complex response amplitudes can be related by:

$$S_{ij}(\omega)\hat{X} = \hat{F}_i, \quad i, j = 1, 2. \quad (27)$$

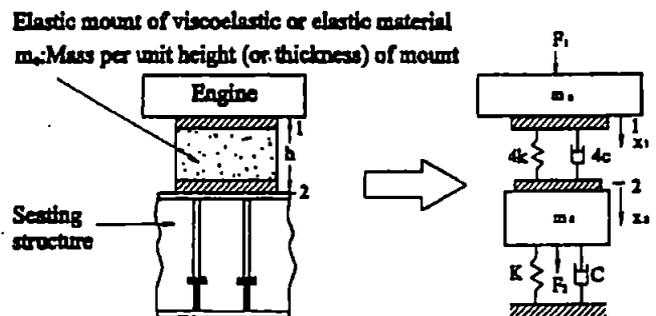


Fig.6 Idealization of the elastic mount as a two degrees of freedom system

Defining the dynamic stiffness functions  $S_{ij}(\omega)$  of the system as

$$S_{ij}(\omega) = (-\omega^2 m_{ij} + j\omega c_{ij} + k_{ij}), \quad i, j = 1, 2. \quad (28)$$

then, equation (27) becomes

$$\hat{X}_j = S_{ij}^{-1}(\omega) \hat{F}_i, \quad i, j = 1, 2. \quad (29)$$

The vibration velocity vector is

$$V_i = \dot{X}_i(t) = j\omega \hat{X}_i e^{j\omega t} = \hat{V}_i e^{j\omega t}, \quad i = 1, 2. \quad (30)$$

The mobility matrix for a coupled TDOF engine/mount/seat system is related by:

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{Bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (31)$$

where the mobility functions  $M_{ij}$  ( $i = 1, 2$  and  $j = 1, 2$ ) can be derived from equations (28) to (31) and expressed in terms of the dynamic stiffness function  $S_{ij}$ :

$$M_{11} = \frac{j\omega S_{22}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (32)$$

$$M_{21} = \frac{-j\omega S_{12}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (33)$$

$$M_{12} = \frac{-j\omega S_{21}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (34)$$

$$M_{22} = \frac{j\omega S_{11}(\omega)}{S_{11}(\omega)S_{22}(\omega) - S_{12}(\omega)S_{21}(\omega)} \quad (35)$$

Expanding equation (31) and rearranging gives:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{Bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{Bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (36)$$

where

$$\begin{aligned} \alpha_{11} &= \frac{M_{11}}{M_{12}}, \\ \alpha_{12} &= \frac{M_{12}M_{21} - M_{11}M_{22}}{M_{21}}, \\ \alpha_{21} &= \frac{1}{M_{12}}, \\ \alpha_{22} &= -\frac{M_{22}}{M_{12}}. \end{aligned} \quad (37)$$

### 3. Reciprocal Principle of the Four-Pole Parameters

By the inverse operation of equation (18), the output unknowns  $\hat{F}_2$  and  $\hat{V}_2$  are:

$$\hat{F}_2 = \frac{\alpha_{22}}{H} \hat{F}_1 - \frac{\alpha_{12}}{H} \hat{V}_1, \quad (38)$$

$$\hat{V}_2 = -\frac{\alpha_{21}}{H} \hat{F}_1 + \frac{\alpha_{11}}{H} \hat{V}_1$$

where

$$H = \alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21} \quad (39)$$

To reverse the input and output ends of the original system, i.e. reversing the force direction, equation (38) can also be applied. It follows that:

$$-\hat{F}_2 = \frac{\alpha_{22}}{H} (-\hat{F}_1) - \frac{\alpha_{12}}{H} \hat{V}_1,$$

$$\hat{V}_2 = -\frac{\alpha_{21}}{H} (-\hat{F}_1) + \frac{\alpha_{11}}{H} \hat{V}_1$$

or

$$\hat{F}_2 = \frac{\alpha_{22}}{H} \hat{F}_1 + \frac{\alpha_{12}}{H} \hat{V}_1, \quad (40)$$

$$\hat{V}_2 = \frac{\alpha_{21}}{H} \hat{F}_1 + \frac{\alpha_{11}}{H} \hat{V}_1$$

By the reciprocal principle, the transfer mobility and transfer impedance between the end couple are irrelevant to which one is to be considered as the input or output end. Therefore, the following relationships exist: for the transfer mobility,

$$\left. \frac{\hat{V}_1}{\hat{F}_2} \right|_{\hat{F}_2=0} = \left. \frac{\hat{V}_2}{\hat{F}_1} \right|_{\hat{F}_2=0} \quad (41)$$

for the transfer impedance,

$$\left. \frac{\hat{F}_1}{\hat{V}_2} \right|_{\hat{F}_2=0} = \left. \frac{\hat{F}_2}{\hat{V}_1} \right|_{\hat{F}_2=0} \quad (42)$$

By using equations (41), (42) and (18), (40) it can be proven that:

$$H = (\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}) = 1 \quad (43)$$

Substituting equation (43) into (40), it becomes:

$$\begin{aligned} \hat{F}_2 &= \alpha_{22}\hat{F}_1 + \alpha_{12}\hat{V}_1, \\ \hat{V}_2 &= \alpha_{21}\hat{F}_1 + \alpha_{11}\hat{V}_1 \end{aligned} \tag{44}$$

The relation exhibiting in equation (43) is called the reciprocal principle of the four-pole parameters. By this relation the system characteristic can be completely determined providing any three of the four-pole parameters are known.

#### 4. Connection of Mechanical Four-Pole Systems

Analogous to an electric circuit system, there are two basic types of connection for the mechanical four-pole systems which are the series and parallel connections. If the output end of one system is rigidly joined to the input end of another system, it is said to be in series connection. While the input (output) ends of two more mechanical systems are rigidly joined together and move with the same velocity, then it is said to be in parallel connection.

As the four-pole mechanical subsystems are connected in series, as shown in Fig.7, then the output of one subsystem is just the input of the subsequently adjacent subsystem. For each subsystem, the matrix form expression in terms of the four-pole parameters is:

$$\begin{aligned} \begin{Bmatrix} \hat{F}_1 \\ \hat{V}_1 \end{Bmatrix} &= \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_2 \\ \hat{V}_2 \end{Bmatrix}, \\ \begin{Bmatrix} \hat{F}_2 \\ \hat{V}_2 \end{Bmatrix} &= \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_3 \\ \hat{V}_3 \end{Bmatrix}, \\ &\vdots \\ \begin{Bmatrix} \hat{F}_n \\ \hat{V}_n \end{Bmatrix} &= \begin{bmatrix} \alpha^n_{11} & \alpha^n_{12} \\ \alpha^n_{21} & \alpha^n_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_{n+1} \\ \hat{V}_{n+1} \end{Bmatrix} \end{aligned}$$

Therefore,

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{V}_1 \end{Bmatrix} = \begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{bmatrix} \alpha''_{11} & \alpha''_{12} \\ \alpha''_{21} & \alpha''_{22} \end{bmatrix} \dots \begin{bmatrix} \alpha^n_{11} & \alpha^n_{12} \\ \alpha^n_{21} & \alpha^n_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_{n+1} \\ \hat{V}_{n+1} \end{Bmatrix} \tag{45}$$

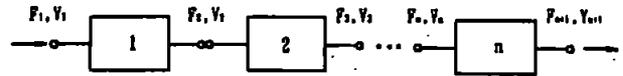


Fig.7 Four-pole subsystems in series connection

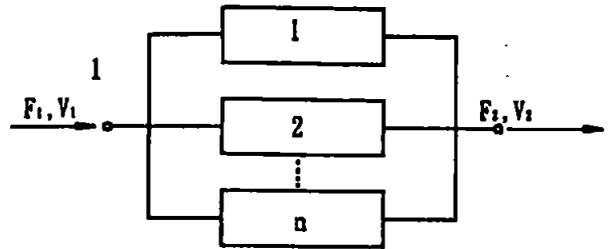


Fig.8 Four-pole subsystems in parallel connection

Fig.8 shows the  $n$  subsystems in parallel connection, in which the combined input or output forces is equal to the sum of the input and output forces of each individual subsystems. Then

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{V}_1 \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{Bmatrix} \hat{F}_2 \\ \hat{V}_2 \end{Bmatrix} \tag{46}$$

where

$$\begin{aligned} \beta_{11} &= \frac{A}{B} \\ \beta_{12} &= \frac{AC - B^2}{B} \\ \beta_{21} &= \frac{1}{B} \\ \beta_{22} &= \frac{C}{B} \end{aligned} \tag{47}$$

and

$$\begin{aligned} A &= \sum_{i=1}^n \frac{\alpha^i_{11}}{\alpha^i_{21}}, \\ B &= \sum_{i=1}^n \frac{1}{\alpha^i_{21}}, \\ C &= \sum_{i=1}^n \frac{\alpha^i_{22}}{\alpha^i_{21}}. \end{aligned} \tag{48}$$

For example, if the system is composed of two subsystems in parallel connection, then

$$\begin{aligned}
 A &= \frac{\alpha'_{11}\alpha''_{21} + \alpha'_{21}\alpha''_{11}}{\alpha'_{21}\alpha''_{21}} = \frac{\varphi}{\varepsilon}, \\
 B &= \frac{\alpha'_{21} + \alpha''_{21}}{\alpha'_{21}\alpha''_{21}} = \frac{\theta}{\varepsilon}, \\
 C &= \frac{\alpha'_{22}\alpha''_{21} + \alpha'_{21}\alpha''_{22}}{\alpha'_{21}\alpha''_{21}} = \frac{\lambda}{\varepsilon}
 \end{aligned}
 \tag{49}$$

thus,

$$\begin{aligned}
 \beta_{11} &= \frac{A}{B} = \frac{\varphi}{\theta}, \\
 \beta_{21} &= \frac{1}{B} = \frac{\varepsilon}{\theta}, \\
 \beta_{12} &= \frac{AC - B^2}{B} = \frac{\varphi\lambda - \theta^2}{\varepsilon\theta}, \\
 \beta_{22} &= \frac{C}{B} = \frac{\lambda}{\theta}
 \end{aligned}
 \tag{50}$$

where

$$\begin{aligned}
 \varphi &= \alpha'_{11}\alpha''_{21} + \alpha'_{21}\alpha''_{11}, \\
 \theta &= \alpha'_{21} + \alpha''_{21} \\
 \varepsilon &= \alpha'_{21}\alpha''_{21} \\
 \lambda &= \alpha'_{22}\alpha''_{21} + \alpha'_{21}\alpha''_{22}
 \end{aligned}
 \tag{51}$$

Fig.9 shows a dynamic absorber system, which is composed of the set of a spring and a damper in parallel connection then connected to a mass in series. Using the equations for four-pole parameters of subsystems in parallel and series connection, one can obtain:

$$\begin{aligned}
 \alpha_{11} &= 1, \\
 \alpha_{12} &= j\omega m, \\
 \alpha_{21} &= \frac{j\omega}{k + j\omega C}, \\
 \alpha_{22} &= 1 - \frac{\omega^2 m}{k + j\omega C}
 \end{aligned}
 \tag{52}$$

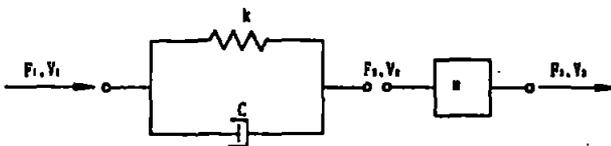


Fig.9 Dynamic absorber system

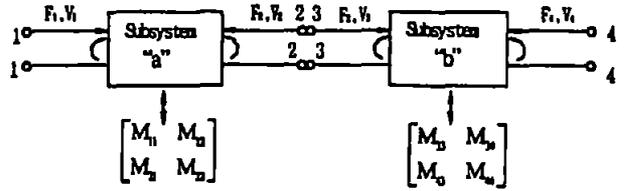


Fig.10 Mobility of integrated system combined by mobility matrices of the subsystems

### 5. Explanation of Reduction in Transfer Mobility by Resilient Mount

As show in Fig.6 the engine/mount/seat system can be considered as a TDOF system. The mobility of the integrated total system can be derived in terms of the mobility matrices of the individual subsystems. If the two ends of the subsystems a,b are 1,2 and 3,4 respectively, as shown in Fig.10, in which  $F$  represents force and  $V$  represents velocity. When the subsystems link together, then  $V_3 = V_2$  and  $F_3 = -F_2$ , equation (45) becomes

$$\begin{aligned}
 \begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} &= \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \\
 &= \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \alpha_{33} & \alpha_{34} \\ -\alpha_{43} & -\alpha_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} \\
 &= \begin{bmatrix} \alpha'_{11} & \alpha'_{14} \\ \alpha'_{41} & \alpha'_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix}
 \end{aligned}
 \tag{53}$$

It requires that  $F_4 = 0$  since 4 is the receiving point and 1 is the driving point. Thus,

$$\frac{1}{\alpha'_{41}} = M_{41} = \frac{V_4}{F_4}
 \tag{54}$$

From equation (53)

$$\begin{aligned}
 \alpha'_{41} &= \alpha_{21}\alpha_{33} - \alpha_{22}\alpha_{43} = \frac{M_{22} + M_{33}}{M_{12}M_{34}}, \\
 \text{or } M_{41} &= \frac{M_{12}M_{34}}{M_{22} + M_{33}}
 \end{aligned}
 \tag{55}$$

From equation (55), it can be seen that the transfer mobility of the combined system ( $M_{41}$ ) is not simply the combination of the transfer mobilities ( $M_{12}$  and  $M_{34}$ ) of the two subsystems. The linking point mobilities ( $M_{22}$  and  $M_{33}$ ) of the subsystems play a predominant role to determine the total transfer mobility. As an isolator is connected to the input end of the subsystem b, such as Fig.11(b) which represents the im-

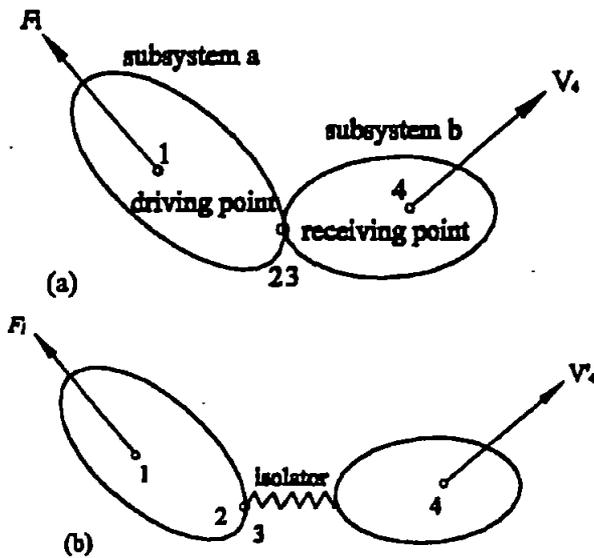


Fig.11 System before and after installation of isolator

proved system of Fig.11(a), then the mobility  $M_{33}$  increases significantly. It follows from equation (55) that the total transfer mobility  $M_{41}$  is reduced. In other words, to reduce the total transfer mobility, it requires the introduction of a larger mobility than the original one. This is the basic principle for a good structure-borne noise attenuation result that has to properly design the structure on both sides of a resilient mount with small mobility than that of the mount itself.

### MEASUREMENT OF FOUR-POLE PARAMETERS AND APPLICATION

In practical application of the four-pole parameters method to the solution of mechanical vibration and structure-borne noise problem, at first it is most important to validate that the four-pole parameters of the system are correct. Undoubtedly, in order to match the derived model, a mobility measurement technique should be developed.

For the resilient mount system considered it has different four-pole parameters under the circumstance of different static load conditions, therefore the measuring setup should take in to account the static preload so as to enhance the measurement accuracy.

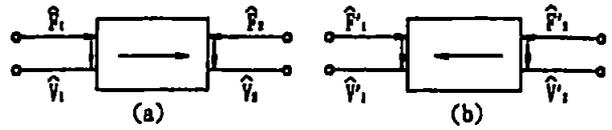


Fig.12 Measurement network for four-pole parameters (a) for normal position (b) for reverse position

The measurement should be undertaken twice. The first measurement is undertaken whilst the test piece is in its normal position, while the second measurement is taken in the reverse position, i.e. to turn over the test piece 180°, as shown in Fig.12.

According to the first measurement data, there exists the relationship as in equation (18). For the second measurement, it has

$$\begin{aligned} \hat{F}'_1 &= \alpha_{22}\hat{F}'_2 + \alpha_{12}\hat{V}'_2; \\ \hat{V}'_1 &= \alpha_{21}\hat{F}'_2 + \alpha_{11}\hat{V}'_2 \end{aligned} \tag{56}$$

By the reciprocal principle,

$$\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21} = 1 \tag{57}$$

From equations (18), (55), (56), the four-pole parameters can be obtained by:

$$\begin{aligned} \alpha_{12} &= \frac{\left(\frac{\hat{F}'_1}{\hat{F}'_2} - \frac{\hat{F}_2}{\hat{F}_1}\right)}{\left(\frac{\hat{V}_1}{\hat{F}_1} + \frac{\hat{V}'_2}{\hat{F}'_2}\right)}, \\ \alpha_{11} &= \frac{\hat{F}_1}{\hat{F}_2} - \frac{\hat{V}_2}{\hat{F}_2}\alpha_{12}, \\ \alpha_{22} &= \frac{\hat{F}'_1}{\hat{F}'_2} - \frac{\hat{V}'_2}{\hat{F}'_2}\alpha_{12}, \\ \alpha_{21} &= \frac{\hat{V}_1}{\hat{F}_2} - \frac{\hat{V}_2}{\hat{F}_2}\alpha_{12} \end{aligned} \tag{58}$$

For instance, if the test piece is an ideal spring, the input and output forces are almost identical with a certain frequency range, which means  $\hat{F}_1 \approx \hat{F}_2$  and  $\hat{F}'_1 = \hat{F}'_2$ ,  $\hat{F}_2 = \hat{F}'_2$  as well. These relations lead the value of the numerator of the first equation of (58) to approach zero, thus it makes  $\alpha_{12}$  to become numerically unstable. Whilst  $\alpha_{11}$ ,  $\alpha_{22}$  and  $\alpha_{21}$ , calculated by the last three equations of (58) in which they are

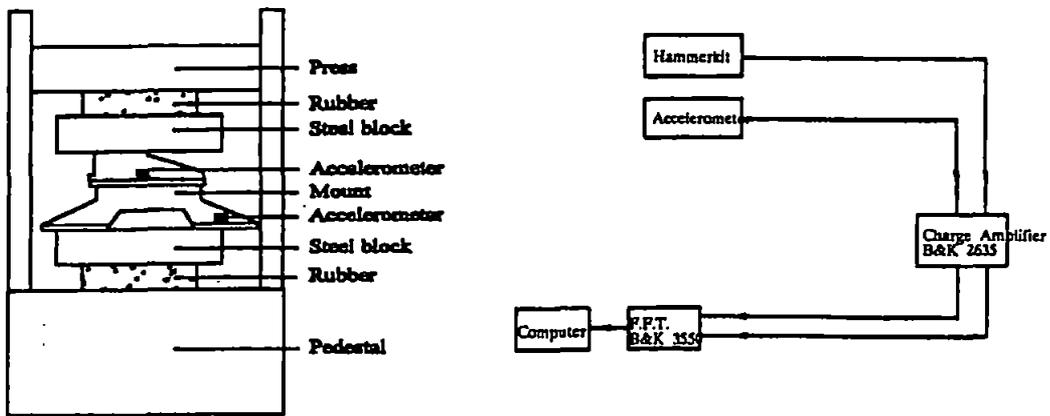


Fig.13 Experiment layout and instrumentation for mobility measurement

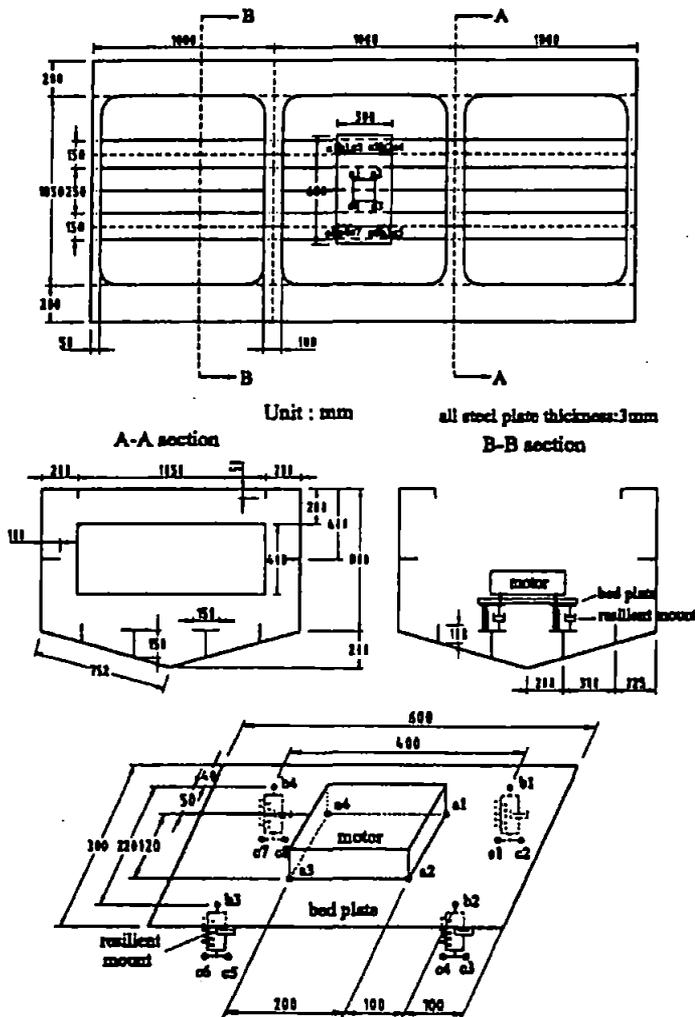


Fig.14 Test model of motor-bedplate-resilient mount-seat structure system

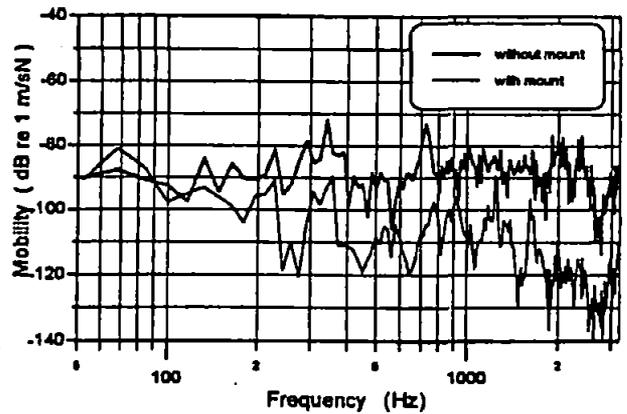


Fig.15 Predicted attenuation of structure-borne noise radiation from a machine by introduction of resilient mount

expressed in terms of  $\alpha_{12}$ , still attain the accuracy required. This is because the following approximate expressions hold for a spring at least in the frequency range out of resonance.

$$\alpha_{11} \approx \frac{\hat{F}_1}{\hat{F}_2} = \alpha_{22}, \tag{59}$$

$$\alpha_{21} \approx \frac{\hat{V}_1}{\hat{F}_2} - \frac{\hat{V}_2 \hat{F}_1}{\hat{F}_2 \hat{F}_2}$$

Using the measured  $\hat{F}_1$ ,  $\hat{V}_1$ ,  $\hat{F}_2$ ,  $\hat{V}_2$  and substituting them into equation (59),  $\alpha_{11}$ ,  $\alpha_{22}$  and  $\alpha_{21}$  can be found. Thus  $\alpha_{12}$  is determined by the reciprocal relation (43).

Fig.13 shows the experimental layout and instruments for the mobility measurement. The test mount is placed in the middle part of the H-frame of a universal testing machine. In between

the pedestal and the mount and in between the press and the mount there are rubber and steel block insertions. Accelerometers are installed on the top and bottom side of the mount. The input excitation force is applied to the top and bottom side of the mount by a hammer kit.

In this case, the mobility measurements were carried out on the motor-bedplate-resilient mount-seat structure system of a scale model as shown in Fig.14. The transfer mobility attenuation of the resilient mount can be calculated by using equation (55), in which  $M_{33}$  and  $M_{34}$  take the data before and after the introduction of resilient mount respectively. As a consequence, from equation (6) the structure-borne vibration will reduce the same amount as the transfer mobility attenuation. The predicted attenuation of the structure-borne noise is shown in Fig.15.

### CONCLUDING REMARKS

From the present study, the following conclusions are forwarded:

In order to obtain acceptable structure-borne noise attenuation, it is of fundamental importance that the structure in both sides of a resilient mount should have small mobility characteristics in comparison with the mount itself. In other words, the desired mobility changes for the connection system of engine-resilient mount-foundation in series are: "heavy-compliant-heavy" and/or "stiff-compliant-stiff".

The mechanical four-pole parameters method has been shown to be effective for the experimental analysis in the mobility prediction. As a consequence, it can be applied as a generic design tool to the structure-borne vibration and noise control field as a whole.

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## VISCOELASTIC PASSIVE DAMPING TECHNOLOGY ON SHIP'S VIBRATION AND NOISE CONTROL

Wei-Hui Wang, Rong-Juin Shyu and Jiang-Ren Chang

Department of Naval Architecture  
National Taiwan Ocean University  
Keelung 202, Taiwan, R.O.C.

Vibration and noise control of structures are vital to many high performance ships. This article first outline the most important physical aspect of structure-borne sound, including the generation, transfer and control of such phenomena. Also, the mechanical four pole system is used to introduce the effect of damping mechanism into the structural model. This article also outlines and summarizes most important aspects of the viscoelastic passive damping technologies associated with vibration and noise control onboard ship. Besides the fundamentals of damping material properties, vibrations of discrete damped systems and fundamentals of vibration control technique have been discussed, approach concentrated on the vibrational behaviour of ship's structures with constrained viscoelastic layers is undertaken. Finite element model for beams, plates like a ship's structure were constructed using PATRAN. Modelling code was then generated for MSC/NASTRAN to compute the driving point impedance. Experiments were also carried out in an engine room structure. A shaker driven by a power amplifier with band-limited white noise signal was used as the input excitation, the driving force signal was measured by an impedance head. Accelerometers were used to pick up the acceleration signals. Input and response signal were analyzed by a spectrum analyzer to estimate the driving point and transfer impedances. The comparisons of the impedances from FEM modelling show good agreement with the experimental data. Also it is clear that the constrained damping layer is good for controlling the resonant vibration response for ship's stiffened plate.

### 1. INTRODUCTION

Structure-borne noise and vibration power transmitted to a sound carrying structure form a source predominantly via a number of contact points. Noise and vibrations propagating in the structure possibly will cause sensitive equipment to vibrate or undesired noise radiation. In principle, this may be avoided by countermeasures at the source, in transmission paths, or at radiation surfaces. It is, of course, preferable to handle the problem at the source and thereby avoid more extensive and expensive measures in the propagation paths.

Viscous damper in a dynamic system provides a mechanism for energy dissipation and results in amplitude reduction at resonance. Problems caused by resonance include noise, fatigue, performance, and human discomfort. The utilization of damping treatment techniques can be a relatively simple solution to resonant vibration problem. It can be

incorporated in design level or can be used after the facts with minimal system alternation and relatively inexpensive alternative which means the low material cost, low application cost and no maintenance cost.

### 2. LOGIC OF STRUCTURE-BORNE SOUND REDUCTION AND VIBRATION CONTROL

The low noise design problem for a simple engine room onboard a vessel is schematically shown in Figure 1. The primary noise generating mechanism is the main engine that will inject vibrations into the structure via the mounts and foundation. These vibrations are transmitted throughout the structure and may eventually radiate to the surroundings.

To reduce the transmitted power, viscoelastic material such as rubber is used inevitably in a mount. Owing to the high damping ratio and loss factor, the rubber has a very good performance for reducing

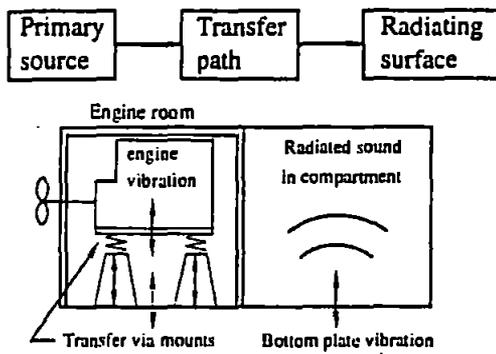


Figure 1. Schematic of machinery noise analysis for low noise design

vibration and structure-borne noise from a mechanical vibrating source. The vibration reduction behaviour of the resilient mounts of an engine may be expressed in terms of the input/output transfer functions.

Figure 2 represents the analysis models which can be used to model the structure-borne sound transfer between the primary sources and the radiating surface. For a uni-directional simple harmonic excitation force  $F_1$ , the engine structure vibrates with a simple harmonic velocity  $V_1$  at the driving point. Structural waves propagate from the excitation point to the adjacent bottom plate, where

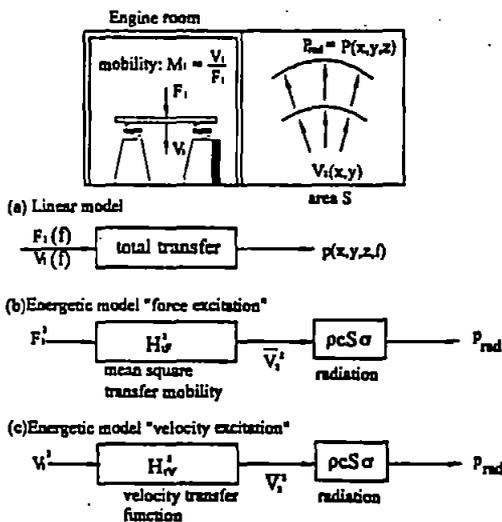


Figure 2. Schematic for the analysis of structure-borne noise transfer

bending waves generate a velocity field  $V_2(x,y)$ . This vibration field radiates a sound field  $p(x,y,z)$ .

The linear equations which relate the radiated sound pressure with the force and the velocity at the driving point are:

$$p(x, y, z, f) = H_{iF} F_1(f); \tag{1}$$

$$p(x, y, z, f) = H_{iV} V_1(f) \tag{2}$$

$H_{iF}$  and  $H_{iV}$  are so-called frequency response functions, describing the sound transfer for force and velocity excitation respectively.

The factors which determine the sound power transfer and will be influenced by design are seen from the following equations:

$$\frac{P_{rad}}{F_1^2} = (H_{iF}^2 S \sigma) pc = (M_{11}^2 H_{iV}^2 S \sigma) pc; \tag{3}$$

$$\frac{P_{rad}}{V_1^2} = (H_{iV}^2 S \sigma) pc \tag{4}$$

where the driving point mobility  $M_{11}$  is defined by :

$$M_{11} = \frac{V_1}{F_1} \tag{5}$$

If the nature of the excitation is such that for different engine design  $F_1$  is unaffected, equation (3) implies that sound reduction is obtained by: decrease of driving point mobility; reduction of radiating surface area; decrease of velocity transfer function; decrease of radiation efficiency.

If the nature of the excitation is such that for different engine designs  $V_1$  is unaffected, equation (4) implies that the sound reduction is obtained by: reduction of radiating surface area; decrease of velocity transfer function; decrease of radiation efficiency.

Design measures to decrease  $S$  significantly, have very limited applications. However, measures to decrease  $M_{11}$  and  $H_{iV}^2$  are very important and will be discussed in this paper.

### 3. MOBILITY REDUCTION OF STRUCTURE-BORNE SOUND TRANSFER

Not only for beams, but also for structure like plates, cylinders, etc., the asymptotic approximation that is obtained by averaging over a frequency is

closely equal to the mobility of the corresponding infinitely extended system [1] [2]. Such infinite system mobilities are often applied in machinery acoustics calculations. The driving point mobilities for infinite beams and plates are given by Cremer et al., [1]:

$$\hat{M}_{b_{\infty}} = \frac{0.19(1-j)}{\rho S \sqrt{C_{Lb}} hf}; \tag{6}$$

$$|M|_{P_{\infty}} = \frac{0.125}{\sqrt{B'm''}} = \frac{0.453}{\rho_p c_{Lp} h^2} = \frac{5.41}{\omega \rho_p \lambda_B^2} \tag{7}$$

where  $m''$ : mass per unit area;  $B'$ : bending stiffness per unit width;  $\rho, \rho_p$ : density of beam/plate material;  $C_{Lp}, C_{Lb}$ : longitudinal wave speed in plate/beam;  $h$ : thickness of plate / beam;  $\lambda_B$ : bending wavelength;  $j$ : unit imaginary number.

The velocity transfer function  $H_{IV}^1$  depends strongly on the size and nature of a structure. For noise reduction purposes the main interest is in the natural frequency range. Here again the discussion will be limited to the elementary case of a finite plate.

In principle  $H_{IV}^2$  can be calculated with the aid of an eigenfunction model and finite element method. Again it can be said that if in the frequency range of interest a large number of eigenfrequencies is involved the calculation become time consuming. Moreover, from the viewpoint of the designer, it is important to look at smoothed average data and to understand the relation between design parameters and these data.

The derivation of  $H_{IV}^2$  uses the equality of injected and dissipated power

$$P_{in} = P_{diss} \tag{8}$$

The power injected by a point source into the plate at a single frequency is given by :

$$P_{in} = \frac{1}{2} \text{Re}(\hat{F}_1 \hat{V}_1^*) = F_{1,rms}^2 \text{Re}(\hat{M}_1) = V_{1,rms}^2 \text{Re}\left(\frac{1}{\hat{M}_1}\right) \tag{9}$$

For averaging over frequencies  $\hat{M}_1$  may be replaced by  $M_{p_{\infty}}$ . Using equations (7) and (9) this leads to :

$$\hat{P}_{in} = \frac{0.453 F_{1,\Delta f}^2}{\rho_p c_{Lp} h^2} = 2.2 \rho_p c_{Lp} h^2 V_{1,\Delta f}^2 \tag{10}$$

The power dissipation from the plate is caused by material damping, by energy transportation across the plate boundaries into the support and by sound radiation from the plate. A well known parameter which includes all these damping mechanisms is the ( apparent ) loss factor  $\eta$ . For each frequency or frequency band it may be defined as:

$$\eta(f) = \frac{P_{diss}}{\omega m'' S V_2^2} = \frac{\text{energy dissipation per vibration period}}{2\pi \times \text{mechanical energy (reversible)}} \tag{11}$$

In equation (11) it is assumed that the total mechanical energy ( i.e. the sum of kinetic and potential energy ) is twice the kinetic energy, i.e.

$$E = m'' S V_2^2 \tag{12}$$

Therefore, the dissipated power may be written as :

$$\hat{P}_{diss} = \eta \omega m'' S V_2^2 \tag{13}$$

Using equations (8), (10) and (13) one obtains

$$H_{IV}^2(f) = \frac{0.35 c_{Lp} h}{f S \eta(f)} \tag{14}$$

Figure 3 shows the effect on  $H_{IV}^2$  of different loss factor. It is seen that at low frequencies  $H_{IV}^2 \gg 1$ . The frequency at which  $H_{IV}^2 = 1$ , decreases with increasing  $\eta$  and decreasing plate thickness.

The squared transfer mobility  $H_{IV}^2$  in the eigenfrequency range follows from equations (8), (9) (10) and (13) :

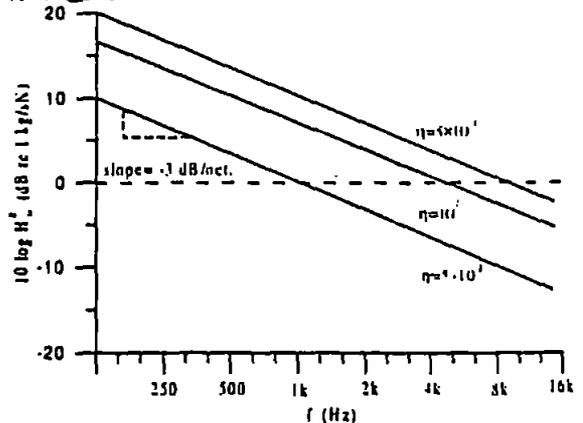


Figure 3. Velocity transfer function of a steel late (480mm×340mm×5 mm ) calculated according to equation (14).

$$H_{1F}^2 = \frac{V_{2,\Delta f}^2}{F_{1,\Delta f}^2} = \frac{M_{p_m}}{\eta\omega m^2 S} \approx \frac{2.85}{\eta f p_p^2 c_{LP} h^3 S} \quad (15)$$

Comparing equations (14) and (15) reveals that the influence of damping upon  $H_{1V}^2$  and  $H_{1F}^2$  is the same, but the influence of plate thickness variation is quite different. Therefore, it is of vital importance for a designer to have appropriate knowledge on the source ( or component ) which drives the structure.

**4. MECHANICAL FOUR-POLE PARAMETERS METHOD IN SYSTEM MOBILITY VALUATION**

Assume a mechanical system, in which the harmonic force  $F_1$  and the harmonic velocity  $V_1$  were applied to the left end ( or input end ) of the system and the force  $F_2$  and the velocity  $V_2$  were induced on the right end ( or output end ). The relations between  $V_1, F_1$  and  $V_2, F_2$  are[3]:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (16)$$

where  $\alpha_{11}, \alpha_{12}, \alpha_{21}$  and  $\alpha_{22}$  are called the four-pole parameters[4], and

$$\alpha_{11} = \frac{M_{11}}{M_{12}}, \quad \alpha_{12} = \frac{M_{12}M_{21} - M_{11}M_{22}}{M_{21}} \quad (17)$$

$$\alpha_{21} = \frac{1}{M_{12}}, \quad \alpha_{22} = -\frac{M_{22}}{M_{12}}$$

and  $M_{ij}$  ( $i=1,2$  and  $j=1,2$ ) are the mobility functions.

If the two ends of the subsystems a,b are 1,2 and 3,4 respectively, as shown in Figure 4, in which F represents force and V represents velocity. When the subsystems link together, then  $V_3 = V_2$  and  $F_3 = -F_2$ , equation(16) becomes

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \alpha_{31} & \alpha_{32} \\ -\alpha_{41} & -\alpha_{42} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix}$$

$$\begin{bmatrix} \alpha'_{11} & \alpha'_{12} \\ \alpha'_{21} & \alpha'_{22} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} \quad (18)$$

It requires that  $F_4 = 0$  since 4 is the receiving point and 1 is the driving point. Thus,

$$\frac{1}{\alpha'_{41}} = M_{41} = \frac{V_4}{F_1} \quad (19)$$

From equation (18)

$$\alpha'_{41} = \alpha_{21}\alpha_{33} - \alpha_{22}\alpha_{43} = \frac{M_{22} + M_{33}}{M_{12}M_{34}}$$

$$\text{or } M_{41} = \frac{M_{12} + M_{34}}{M_{22} - M_{33}} \quad (20)$$

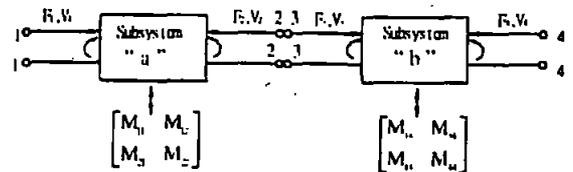


Figure 4. Mobility of integrated system combined by mobility matrices of the subsystems

From equation (19), it can be seen that the transfer mobility of the combined system ( $M_{41}$ ) is not simply the combination of the transfer mobilities ( $M_{12}$  and  $M_{34}$ ) of the two subsystems. The linking point mobilities ( $M_{22}$  and  $M_{33}$ ) of the subsystems play a predominant role for determining the total transfer mobility. As a damping material is introduced into the input end of the subsystem b, such as Figure 5(b) which represents the improved system of Figure 5(a), then the mobility  $M_{33}$  increases significantly. It follows from equation (19) that the total transfer mobility  $M_{41}$  is reduced. In other words, to reduce the total transfer mobility, it requires the introduction of a larger mobility than the original one. This is the basic principle for a good structure-borne noise attenuation result that has to properly design the structure on both sides of a resilient mount with small mobility than that of the mount itself.

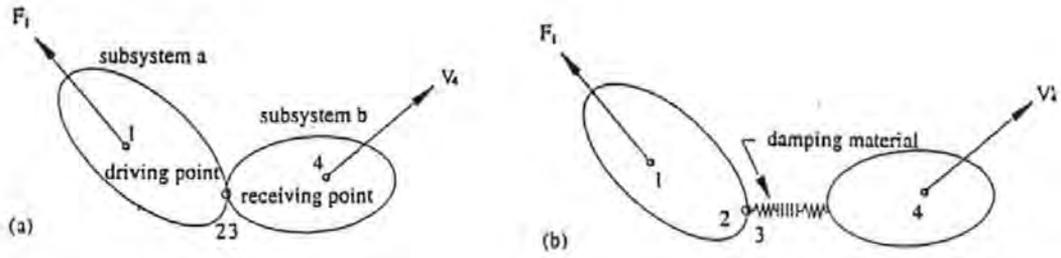


Figure 5. System before and after installation of isolator damping material

The other way of reducing structure-borne sound is the insertion of damping material between point 3 and 4. This is equivalent to the application of a damping material on the surface of the foundation girder of an engine. The type of damping material used is the constrained damping layer. That is, a damping layer sandwiched between the foundation girder and a steel-like constrained plate. This

material is good for providing vibration and noise attenuation in higher frequency ranges. Coupled with resilient mount which provided good vibration attenuation characteristics in low frequency, these two viscoelastic passive damping technology provide a broader frequency range solution to the vibration problems.

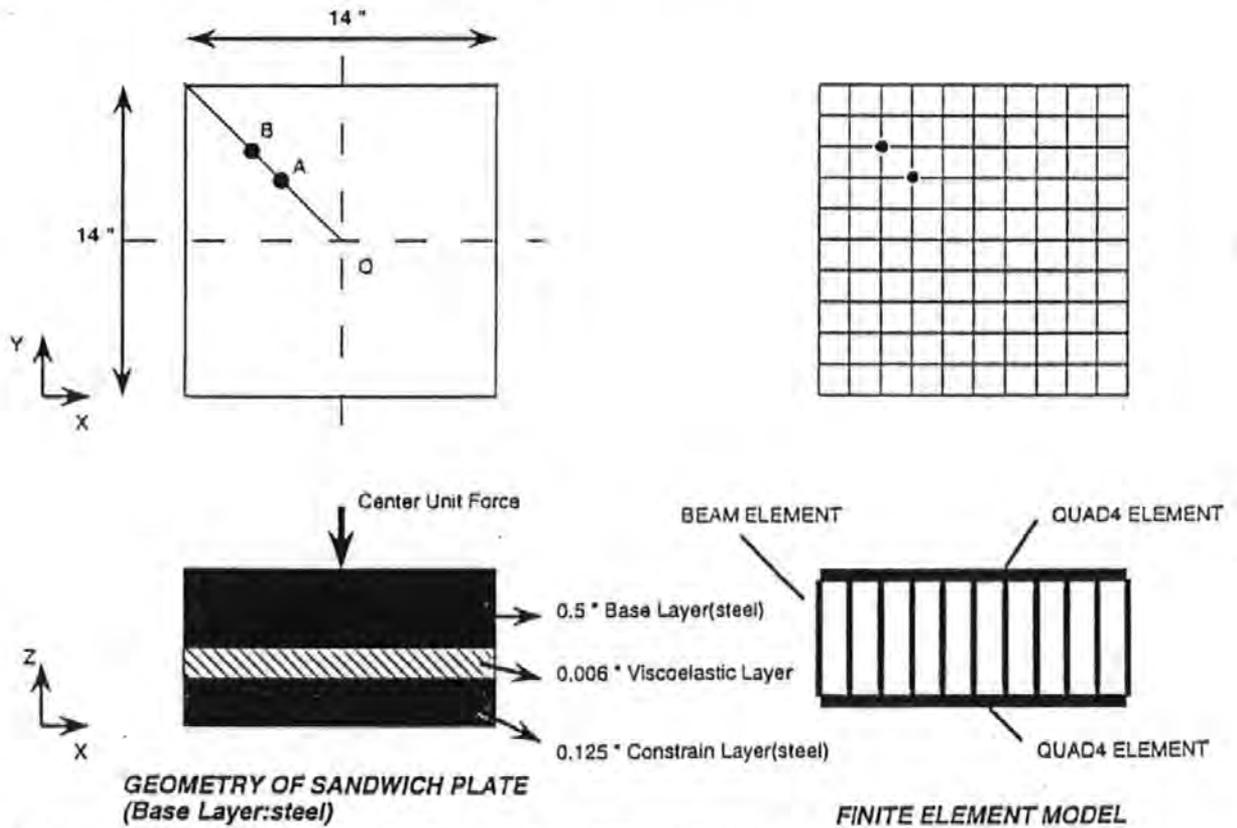
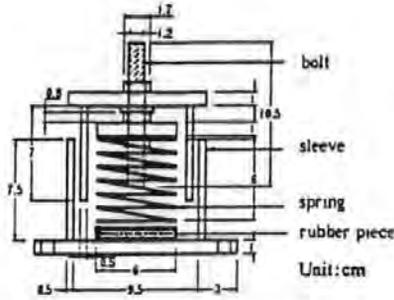
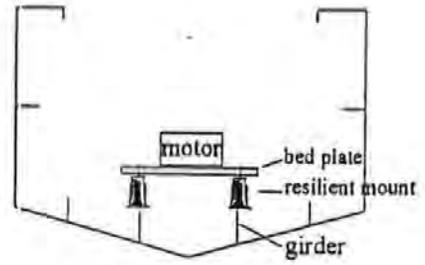


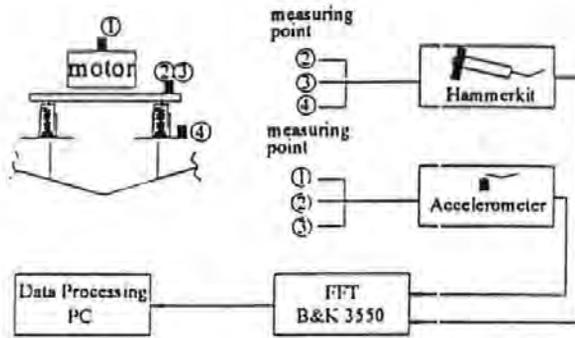
Figure 6. Finite element model of the driving point mobility analysis of a 14" x 14" sandwich steel plate



(a) Rubber piece specimen in a sleeve type resilient mount



(b) Motor-resilient mount-girder system



(c) Mobility measuring points and instrumentation  
Figure 8. Experiment set-up

**5. FEM MODELING OF CONSTRAINED DAMPING LAYER**

To be more flexibly modeling damping layer, a FEM procedure proposed by Lu [5] was used to simulate the dynamic behavior of the damped structure. As

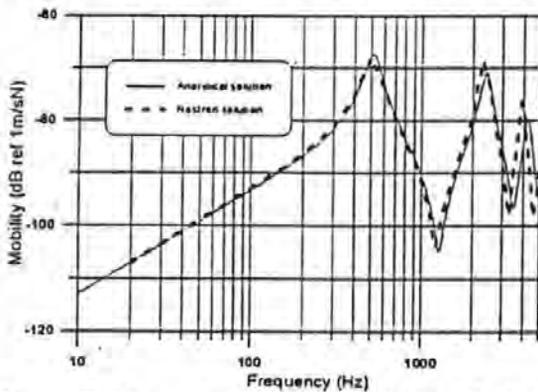


Figure 7. Comparison of driving point mobility by FEA and analytical solution

usual, in FEM model, the base structure and the constraint plate were modeled as quadrilateral plate elements. But the damping material was modeled as beam element that can withstand extension and shear. A numerical experiment was performed on a 14" x 14" steel plate with damping layer. The FEM model is shown in Figure 6, and the result compare with an analytical solution[6] is shown in Figure 7. It is clear that the FEM model shows good approximation with the exact solution.

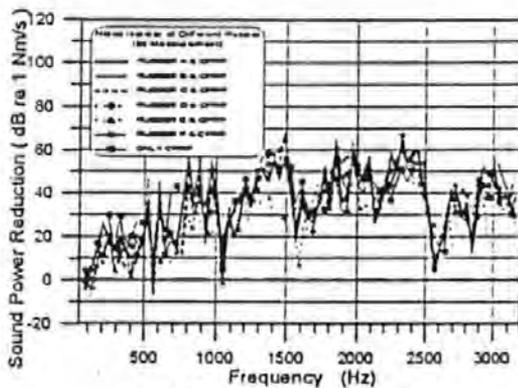
**6. EXPERIMENTS**

The experiments on the damping technology were carried out on a ship model. The dimension of this model is approximately a 3m long and 1.5m wide box girder. There are two longitudinal girders on the ship bottom which served as the engine foundation. An electric motor with a bed plate was used to simulate the actual mounting. The

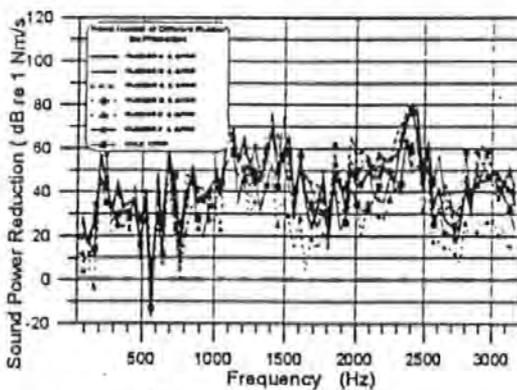
experimental set up and the resilient mount are shown in Figure 8. The types of rubber used in the test are summarized in Table 1. Experiment was also performed on one of the longitudinal girder to obtain its mobility. The mobility is used to correct the FEM prediction model for later use in the prediction after damping treatment.

Table 1. Properties of rubber specimens

Type No.	Material	Specific Weight	Diameter (mm)	Thickness (mm)
A	Butyl	1.251	60	10
B	Butyl	1.251	60	5
C	Silicone	1.315	60	10
D	Silicone	1.315	60	5
E	Styrene Butadiene Rubber (SBR)	1.151	60	5.2
F	Neoprene	1.223	60	12.1

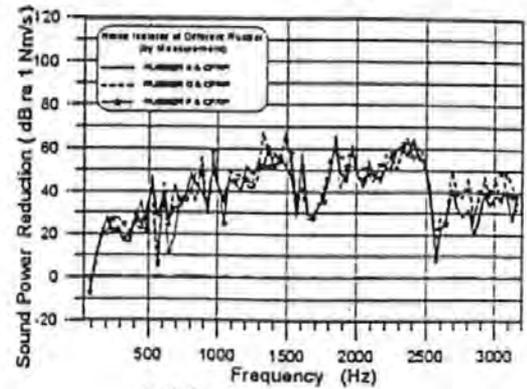


(a) Measurement results

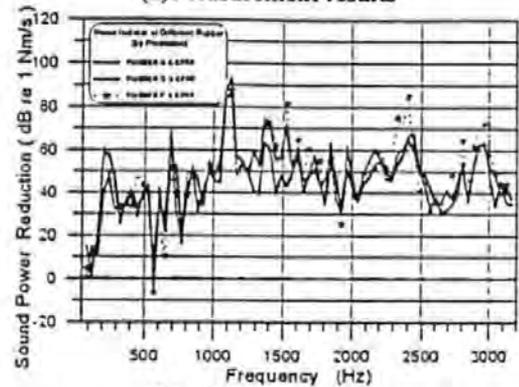


(b) Prediction results

Figure 9. Sound power reduction by insertion of resilient mount with different rubbers



(a) Measurement results



(b) Prediction results

Figure 10. Sound power reduction by insertion of resilient mount and sandwich beam with damping layer

## 7. RESULTS AND DISCUSSION

Consider the motor-resilient mount-girder system shown in Figure 8(b). Comparing to the case of the motor bed plate linked to girder directly the sound power reduction of structure-borne noise transmitted from motor to the girder by insertion of resilient mounts and insertion of resilient mounts and sandwich beam with damping layer simultaneously are shown in Figures 9 and 10. The results by measurement and prediction show good agreement. The results reveal that all the four kinds of rubber have an effectiveness to attenuate the structure-borne noise level 20~30 dB in the frequency range of 800Hz to 3,200Hz. While the sandwich beam with damping layer has the capability to reduce 5-10dB structure-borne noise transmission in the same frequency range.

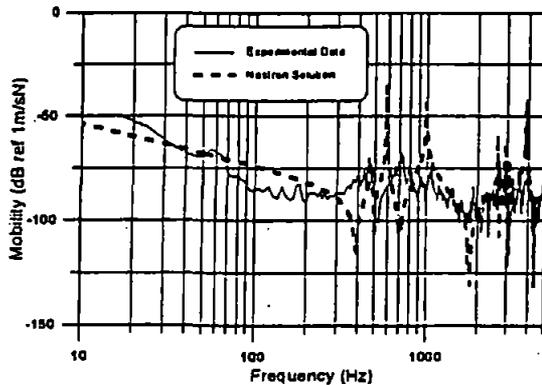


Figure 11 Driving point mobility at the center of face plate

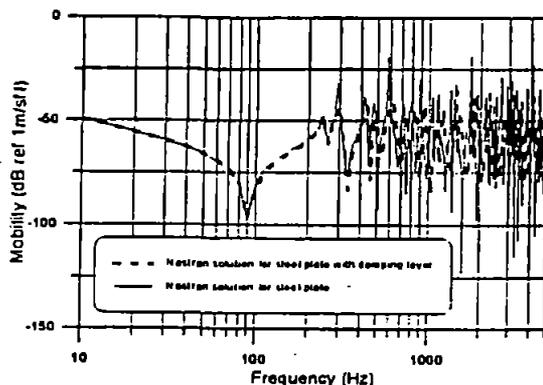


Figure 12 Driving point mobility at  $1/6 W$  from one side of the face plate ( $W$ :width of face plate)

Figure 11 shows both the analytical and experimental driving point mobilities at the midpoint of the longitudinal girder, the measuring position is at the center of the face plate. The purpose of this is to check the validity of the FEM model. Figure 12 shows the analytical results from FEM predictions with and without damping layer, the point being computed is at  $1/6$  of the width from one side of the face plate. The result shows a 5-10 dB reduction of vibration at higher frequency ranges.

## 8. CONCLUSIONS

From the works and results in this paper, it is

concluded that:

1. The prediction model for the transfer mobility function of structure-borne noise from the exciting source to a receiving structure shows good agreement with measurements.

2. The sound power reduction using resilient mounts with four types of rubber material was shown to attenuate the structure-borne noise level of about 20-30 dB in the frequency range of 800 Hz to 3,200 Hz.

3. Damping layer treatment can reduce substantial amount of vibrational energy, usually, 5-10 dB in the magnitude of the mobility. This can be an effective way for reducing the energy transfer from structural vibration into airborne sound.

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# MOBILITY ANALYSIS OF THE IMPROVED BOTTOM STRUCTURE BY SQUEEZE FILM FOR STRUCTURE-BORNE NOISE REDUCTION

Wei-Hui Wang, Tend-Chich Yang  
National Taiwan Ocean University, Keelung, ROC

## ABSTRACT

Machinery or foundation panels can be damped by attaching an auxiliary plate parallel to the surface, thereby trapping a thin layer of air. Relative vibration of the plates pump this air at high velocities, resulting in energy loss due to the air viscosity. Thus, the loss factor of this type of damping mechanism can achieve to the order of 0.1 without the sensitivity to heat and oil associated with viscoelastic materials.

Estimation of the loss factor of the squeeze film is using a more sophisticated incompressible flow model. In which, the Navier-Stokes and continuity equations in two-dimensional system are utilized by neglecting the thermal effect. In other words, the acoustic process is assumed to be nearly adiabatic. This means that no significant thermal flux occurs during the time of the disturbance.

For the purpose of noise attenuation in ship's cabins, the squeeze film technology is discussed to reduce the structure-borne sound transmission from the diesel engines. By using FEA model, the mobility at each site of the engine girder below the resilient mounts can be analyzed for the conditions before and after the installation of squeeze plates. Comparisons of the mobilities show that peak levels reduced by 5-10 dB after the installation of the squeeze films. Thereby the airborne noise radiation has been improved by:

Compartment	Airborne noise level. dB (A)	
	Original	Improved
Engine room	115	112
Captain's room	90	86
Pilothouse	85	74

# 1. INTRODUCTION

The noise onboard a ship generated by a machine can diffuse into surroundings via multi-paths. To evaluate and influence machine noise a method to detect and influence noise generation can be used in the design phase. This kind of method is based on a model of the causal chain of noise generation which describes the path by which sound travels from the source to the radiating surface. The causal chain, as shown in Figure 1, of noise generation is of importance to the designer because it allows assignment of mechanisms to machine structures and leads to the setup of an acoustical model of the machine.

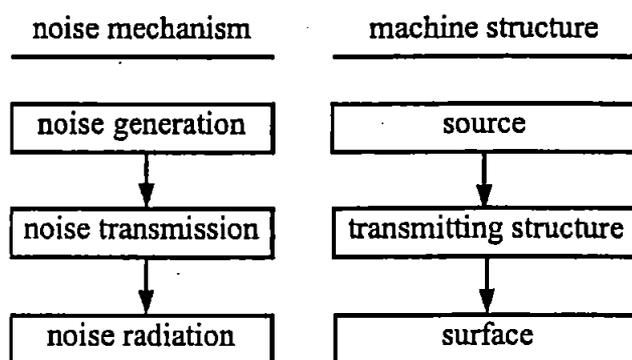


Figure 1 The causal chain of noise generation

There is frequently a need to reduce sound radiation due to resonant flexural motion of stiff machinery panels. This can be achieved by applying squeeze-film damping to the vibrating panel by attaching an auxiliary plate parallel to the surface, thereby trapping a thin layer of air [1,2]. Relative vibration of the plate pumps this air in high velocities to dissipate vibrational energy. Besides, along with other design considerations such as the viscoelastic passive damping technology [3], the structure-borne noise level can be effectively reduced.

In this study, emphasis is put on the prediction of loss factor induced by a squeeze film on a structure. Mass impedance of the attached plate and blocked impedance of the squeeze air film between two plates have been combined to form the treatment impedance. The blocked impedance of squeeze film is approached by incompressible and compressible flow theory respectively. Also the treatment impedance obtained in this way can be compared with that by statistical energy analysis to validate the correctness of the prediction model.

Squeeze film technique has the merit to increase the loss factor of a finished local structure. Design parameters such as the temperature, plate thickness and clearance related to the loss factor have been investigated. As an example, the effectiveness for applying of this technique to improve the bottom structure of a 100 GT fast vessel for noise reduction has carried out.

## 2. Loss Factor of Air Squeeze Film Damping Plates

### 2.1 Mass Impedance of Attached Plate and Blocked Impedance of Squeeze Air Film

Consider two plates which are separated by an air gap (see Figure 1). It is assumed that the thick plate 1 is the excited plate and the thin plate 2 is the attached plate or the damping treatment plate.

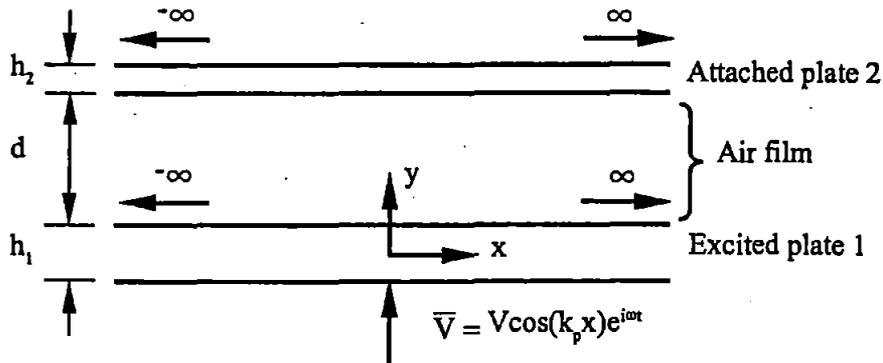


Fig. 1 Coordinates of the flow system between plates

The bending stiffness of the attached plate is assumed to be less than that of the excited plate. Both plates therefore move with the same wavenumber  $k_p$ , which is assumed to be very close to the free wavenumber of the stiffer excited plate. At this particular wavenumber the impedance of the attached plate is mass controlled. Thus the attached plate can be considered as a number of small independent masses (see Figure 2).

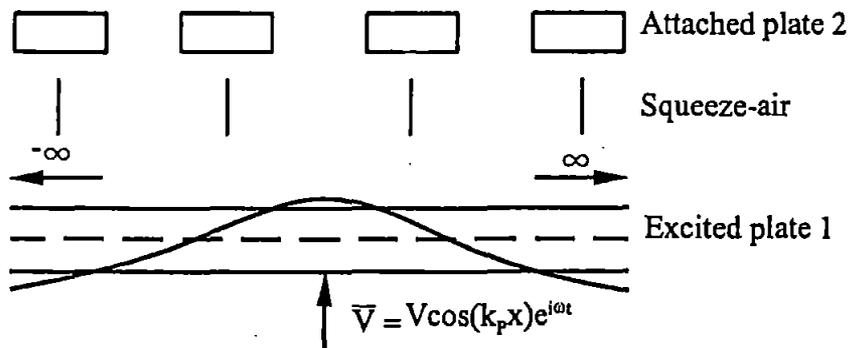


Fig. 2 The representation of the attached plate in the system

Thus, the impedance per unit area of the attached plate is

$$\bar{Z}_2 = i\omega m_2 \tag{1}$$

where  $m_2 = \rho h_2$  is the mass per unit area of the attached plate and  $h_2$  is the thickness of the attached plate.

When the velocity of the excited plate is  $V(k_p, \omega) = V \cos(k_p x) e^{i\omega t}$ , it may cause a pressure distribution  $p(k_p, \omega) = p \cos(k_p x) e^{i\omega t}$  in the air layer. The treatment impedance is therefore defined as:

$$Z_t(k_p, \omega) = \frac{\bar{P}}{\bar{V}}$$

For simplicity, only a velocity variation in the x-direction is considered. It is assumed that plates are separated by a distance  $d$ . The fluid in the gap is assumed to be viscous, incompressible and flowing with a parabolic velocity profile. By considering the flow continuity, it can be shown that the blocked impedance per unit area of the air layer is [4]:

$$\bar{Z}_b'' = \frac{1}{k_p^2 d} \left( \frac{12\mu_a}{d^2} + i\omega\rho_a \right) \quad (2)$$

where  $\mu_a$  and  $\rho_a$  are the dynamic viscosity and the density of air, and  $k_p = (\omega^2 m_1 / B')^{1/4}$ ;  $m_1$  and  $B'$  are the mass per unit area and the flexural rigidity of the thick plate respectively. As the pressure is constant across the gap, the treatment impedance per unit area  $\bar{Z}_t$  (Figure 3) can be written as:

$$\bar{Z}_t'' = \frac{\bar{P}}{\bar{V}} = \frac{\bar{Z}_2 \bar{Z}_b''}{\bar{Z}_2 + \bar{Z}_b''} \quad (3)$$

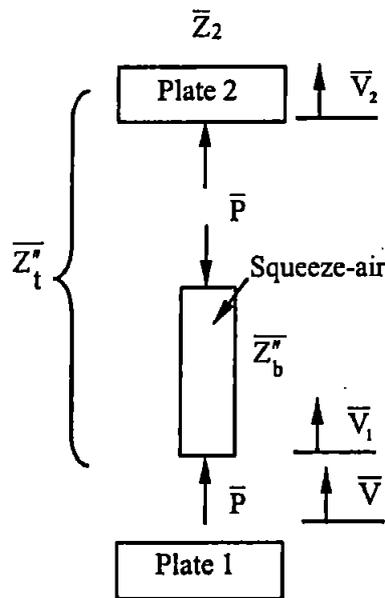


Fig. 3 The model of the treatment impedance of the system

## 2.2 Relation between Treatment Impedance and Loss Factor

Loss factor  $\eta$  is defined by:

$$\eta = \frac{\text{Energy dissipation / circular frequency}}{\text{Storage energy / area}} \quad (4)$$

The total energy dissipated by the coupled plates in area  $A$  may therefore be calculated by:

$$\frac{1}{2} \int_A \bar{P} \bar{V}^* dA = \frac{1}{2} \int_A |\bar{V}|^2 \operatorname{Re}\{\bar{Z}_t\} dA \quad (5)$$

where  $Z_t$  is the treatment impedance per unit area. However a calculation of loss factor require the peak energy stored in the whole system, including the attached plate, excited plate and air layer. Neglecting the minor parts of energy stored in the thin attached plate and air layer the energy stored can be approximated by:

$$\frac{1}{2} \int_A m_1 |\bar{V}|^2 dA \quad (6)$$

From equations (4), (5) and (6), the loss factor can be expressed in terms of the treatment impedance:

$$\eta = \frac{\operatorname{Re}\{\bar{Z}_t\}}{\omega m_1} \quad (7)$$

### 2.3 Estimation of Treatment Impedance Accounting for Air Mass and Incompressible Fluid

In the previous section the velocity profile in the gap was assumed to be parabolic. This is true provided the fluid inertia is not dominant; i.e., from equation (2) when  $\omega \rho_a < 12\mu_a/d^2$ . However increasing the gap  $d$  or the frequency  $\omega$  will violate this assumption. In this circumstance, the velocity profile shape is not specified. The modified analysis consists of determining the time dependent pressure and velocity fields generated in the incompressible moving air between the surface of the two plates of which one is in oscillatory motion. Thus the Navier-Stokes and continuity equations in two-dimensional Cartesian form can be utilized:

$$\frac{\partial \bar{U}_x}{\partial t} = -\left(\frac{1}{\rho}\right) \frac{\partial \bar{p}}{\partial x} + \nu \left[ \frac{\partial^2 \bar{U}_x}{\partial x^2} + \frac{\partial^2 \bar{U}_x}{\partial y^2} \right] \quad (8)$$

$$\frac{\partial \bar{U}_y}{\partial t} = -\left(\frac{1}{\rho}\right) \frac{\partial \bar{p}}{\partial y} + \nu \left[ \frac{\partial^2 \bar{U}_y}{\partial x^2} + \frac{\partial^2 \bar{U}_y}{\partial y^2} \right] \quad (9)$$

$$\frac{\partial \bar{U}_x}{\partial x} + \frac{\partial \bar{U}_y}{\partial y} = 0 \quad (10)$$

where  $\nu = \mu_a/\rho_a$  is the kinematic viscosity of air,  $\bar{p}$  is the pressure,  $\bar{U}_x$ ,  $\bar{U}_y$  are the air velocities in the gap along  $x$  and  $y$  directions respectively. The thermal effect is assumed to be small and is neglected. Since the equations (8), (9), (10) are linear, the equivalent analysis for a travelling wave follows immediately by considering only one component of the individual waves composing the standing wave. Assuming a standing wave is generated on the thick plate with

$$\bar{V}_p = V \cos(k_p x) e^{i\omega t} \quad (11)$$

The air motion and pressure in the gap is assumed to be of the form:

$$\bar{U}_x = Ae^{-i\bar{k}y}e^{i\omega t}(e^{ik_p x} - e^{-ik_p x}) \quad (12)$$

$$\bar{U}_y = Be^{-i\bar{k}y}e^{i\omega t}(e^{ik_p x} + e^{-ik_p x}) \quad (13)$$

$$\bar{p} = Ce^{-i\bar{k}y}e^{i\omega t}(e^{ik_p x} + e^{-ik_p x}) \quad (14)$$

Similar expressions are used for waves reflected from the boundary at  $y=d$ . This gives the wavenumbers

$$\bar{k} = \pm ik_p \quad (15)$$

and

$$\bar{k} \cong \pm \frac{i-1}{\delta} \quad (16)$$

where  $\delta = \sqrt{2\nu/\omega}$  is the boundary layer thickness.

The wavenumber inequation (15) is related to the pumping motion of the air layer, i.e., the transfer of air between the displacement nodes and antinodes of the vibrating surface. While the wavenumber in equation (16) is associated with the viscosity of air, and is independent of the density or the pressure fluctuation of air. This kind of viscosity wave is confined to the boundary layer thickness  $\delta$ . This wave ensures that there is zero tangential velocity at the surface, a condition not satisfied if viscosity is neglected.

When the gap is small the boundary layers of the two surface overlap and this gives rise to the parabolic flow profile in the gap, which occurs when viscous forces dominate the flow, rather than inertia or elastic forces.

By substituting these two pairs of wavenumbers back into equations (12)-(14) in conjunction with the boundary conditions: (a) no tangential motion along the surface of the plates, i.e.,

$$\bar{U}_x(x,0) = \bar{U}_x(x,d) = 0 \quad (17)$$

and (b) the normal velocity of the air adjacent to the thick plate has the same velocity as this excited plate whereas the normal velocity on the surface of the other plate is zero,

$$\begin{cases} \bar{U}_y(x,0) = \bar{V}_p = V \cos(k_p x)e^{i\omega t} \\ \bar{U}_y(x,d) = 0 \end{cases} \quad (18)$$

the pressure and the air velocity  $\bar{U}_x, \bar{U}_y$  can be obtained. Therefore, the blocked impedance per unit area can be found as [2]:

$$\bar{Z}'_b = \frac{\bar{p}}{\bar{V}_p} \quad (19)$$

The blocked impedances predicted by equations (2) and (19) have shown to be very close in the frequency range 0~5k Hz [1].

## 2.4. Estimation of Treatment Impedance Using a Compressible Flow Model

At low frequencies the air tends to behave in an incompressible manner. However, the above incompressible model cannot be used in the region of the critical frequency where the elastic forces in the air are comparable with the inertia forces. If the air is compressible, it is necessary to rewrite the Navier-Stokes and continuity equations as:

$$\frac{\partial \bar{U}_x}{\partial t} = -\left(\frac{1}{\rho}\right) \frac{\partial \bar{p}}{\partial x} + \nu \left[ \frac{\partial^2 \bar{U}_x}{\partial x^2} + \frac{\partial^2 \bar{U}_x}{\partial y^2} \right] + \left(\frac{\nu}{3}\right) \left[ \frac{\partial^2 \bar{U}_x}{\partial x^2} + \frac{\partial^2 \bar{U}_y}{\partial x \partial y} \right] \quad (20)$$

$$\frac{\partial \bar{U}_y}{\partial t} = -\left(\frac{1}{\rho}\right) \frac{\partial \bar{p}}{\partial x} + \nu \left[ \frac{\partial^2 \bar{U}_y}{\partial x^2} + \frac{\partial^2 \bar{U}_y}{\partial y^2} \right] + \left(\frac{\nu}{3}\right) \left[ \frac{\partial^2 \bar{U}_x}{\partial x \partial y} + \frac{\partial^2 \bar{U}_y}{\partial^2 y} \right] \quad (21)$$

$$\frac{\partial \bar{U}_x}{\partial x} + \frac{\partial \bar{U}_y}{\partial y} = -\frac{i\omega \bar{p}}{\rho c_a^2} \quad (22)$$

where  $c_a$  is the sound speed in air. Again, the thermal effect is ignored in the analysis.

By assuming the pressure and the air velocity to be of the forms as in equations (12)-(14), it can be shown that two types of waves are generated in the air layer. The wavenumber can be found as

$$\bar{k} \cong \pm \frac{i-1}{\delta} \quad (23)$$

and

$$\bar{k} \cong \pm i \sqrt{k_p^2 + \frac{i\omega^2}{\frac{4}{3}\omega\nu - ic_a^2}} \quad (24)$$

The wavenumber in equation (23) gives a viscous wave whereas the second wavenumber in equation (24) is related to the pumping motion. At low frequency  $k \cong ik_p$  which is the same as for the incompressible solution, i.e., the air is pumped backward and forward. At the critical frequency of the thick plate, the air is pumped very effectively and the plate radiates very

efficiently into the air layer. Above the critical frequency the air is compressed rather than pumped and so the squeeze film losses will be greatly reduced.

The equivalent impedance per unit area of an entrapped air layer, with only compressive motion, is

$$\bar{Z}_b = \frac{\rho_a c_a^2}{i\omega d} = \frac{K}{i\omega} \quad (25)$$

where  $K$  is the bulk stiffness of the air.

## 2.5 Loss Factor of the squeeze Air Film between Infinite Parallel Coupled Plates

The previous sections describe the impedance of the air layer and the attached plate. Therefore the energy dissipated by the coupled plates may be calculated from the velocity of the excited plate, i.e.,

$$\frac{\text{Energy dissipated}}{\text{circular frequency}} = \frac{1}{2} \frac{|\bar{V}_p|^2}{\omega} \text{Re}\{\bar{Z}_t\} \quad (26)$$

An appropriate model for the calculation of loss factor of the complete system is given in Figure 4. The air layer is modelled by a spring-mass-dashpot system, the impedance of which,  $\bar{Z}_b$ , is described approximately by equation (2) for the lower frequency incompressible flow behaviour; which the high frequency compressible behaviour by given approximately by equation (25).

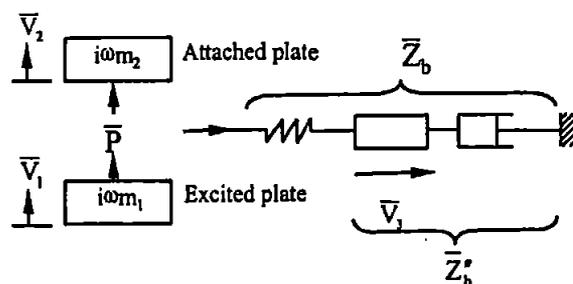


Fig. 4 The model used for the estimation of loss factor

The loss factor of the system is

$$\eta = \frac{1}{2\omega} \frac{|\bar{V}_1|^2 \text{Re}\{\bar{Z}_t\}}{\frac{1}{2} |\bar{V}_1|^2 m_1 + \frac{1}{2} |\bar{V}_2|^2 m_2 + \frac{1}{2\omega} |\bar{V}_3|^2 \text{Im}\{\bar{Z}_b\}} \quad (27)$$

where the numerator is the energy dissipated / circular frequency and the denominator is the kinetic energy of the excited plate, attached plate and air respectively. This reduces to

$$\eta = \frac{\text{Re}\{\bar{Z}_t\}}{\omega m_1 + \frac{|\bar{Z}_t|^2}{\bar{Z}_t'} \text{Im}\{\bar{Z}_t'\}} \quad (28)$$

where

$$\bar{Z}_t = \frac{\bar{Z}_b''}{[1 + \bar{Z}_b''\{(i\omega/K) + (1/i\omega m_2)\}]} \quad (29)$$

and

$$\bar{Z}_t' = \frac{i\omega m_2 \bar{Z}_b''}{i\omega m_2 + \bar{Z}_b''} \quad (30)$$

## 2.6 Loss Factor of the Parallel Coupled Plates Using the Statistical Energy Analysis Method

In the preceding analysis it has been assumed that all motions occur at the wavenumber of the excited thick plate. Energy is stored and dissipated at only one wavenumber and so that the same loss factor could be measured for excitation on either plate. This is a good assumption below the critical frequency. However, above the critical frequency the plates become less well coupled because of the isolating effect of the air compliance, thus the attached plate is free to move with its own wavenumber. These waves represent an additional means of energy storage and dissipation. These waves associated with the attached plate are not well excited in an infinite-two-plates system, but are generated in finite plates when waves in the thick plate meet either the boundaries or coupling bolts.

The loss factor of the coupled system may be estimated by using a two components Statistical Energy Analysis model as shown in Figure 5.

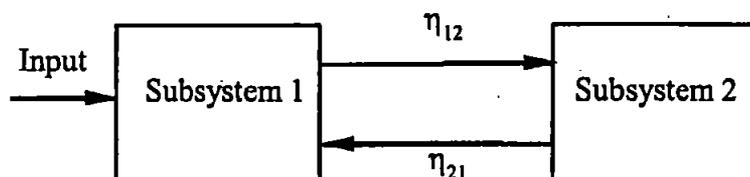


Fig. 5 Two subsystems model for the total loss factor estimation

If the two components are moderately well coupled, it is permissible to assign a single loss factor to the coupled system, defined as

$$\eta_{\text{TOT}} = \frac{\eta_1 e_1 + \eta_2 e_2}{e_1 + e_2} \quad (31)$$

where  $e_1$  and  $e_2$  are the time averaged energies of the thick and attached plates, and  $\eta_1$  is the loss factor for squeeze film damping (equation (28)) for wavenumbers close to the thick plate wavenumber. In these circumstances a single loss factor would be measured on both plates because there is rapid exchange of energy between the two systems via the discontinuities, as at low frequencies when the two plates moved vibrationally as a single system. The loss factor  $\eta_2$  of the attached plate due to squeeze film damping is calculated in the same manner as for the excited plate (equation (27)). At the free wavenumbers of the attached plate, the excited plate is comparatively very stiff and so it can be appropriately considered as a rigid or blocked boundary. The loss factor  $\eta_2$  is

$$\eta_2 = \frac{\text{Re}\{\bar{Z}_b\}}{\omega m_2 + \left|\frac{\bar{Z}_b}{Z'_b}\right|^2 \text{Im}\{\bar{Z}'_b\}} \quad (32)$$

where  $Z_b$  is calculated at the free wavenumber of the excited plate.

Equation (31) can be rewritten in terms of the coupling loss factor [2]:

$$\eta_{\text{TOT}} = \frac{(\eta_1 + \eta_{12})(\eta_2 + \eta_{21}) - \eta_{12}\eta_{21}}{\eta_2 + \eta_{12} + \eta_{21}} \quad (33)$$

For point force connection between two plates and upon assuming there is no rotation of the connection, the coupling loss factor [5] is:

$$\eta_{12} = \frac{4}{\sqrt{3}} \frac{h_1 C_{L1}}{\omega A_1} \frac{(m_1 h_1 C_{L1})(m_2 h_2 C_{L2})}{(m_1 h_1 C_{L1} + m_2 h_2 C_{L2})^2} \quad (34)$$

where  $A$ ,  $h$ ,  $m$  and  $C_L$  are the area, thickness, mass per unit and the longitudinal wave speed of the component. For  $N$  connections and upon assuming there is no coupling between connections, the coupling loss factor is  $\eta'_{12} \approx N\eta_{12}$ . This is valid provided that there is more than half a wavelength between the coupling points.

The coupling loss factor for the solid line connection [5] is :

$$\eta_{12} = \left(\frac{2}{3}\right)^{\frac{1}{4}} \ell \frac{1}{A_1} \left(\frac{h_1 C_{L1}}{\omega}\right)^{\frac{1}{2}} \frac{(m_1 \sqrt{h_1 C_{L1}})(m_2 \sqrt{h_2 C_{L2}})}{(m_1 \sqrt{h_1 C_{L1}} + m_2 \sqrt{h_2 C_{L2}})} \quad (35)$$

where  $\ell$  is the total length of the connection. This is derived on the assumption of a diffuse field. The factor  $\eta_{21}$  is given by:

$$\eta_{21} = \frac{n_1}{n_2} \eta_{12} \quad (36)$$

where  $n_1$ ,  $n_2$  are the modal densities of the components 1 and 2. For a plate, the modal density is given by

$$n(f) = \frac{\sqrt{3}A_i}{h_i C_{L1}} \quad (37)$$

A simple estimate of total loss factor could be made by assuming strong coupling between the plates. Therefore put  $e_1 = e_2$  in equation (31) and so

$$\eta_{TOT} = \frac{1}{2}(\eta_1 + \eta_2) \quad (38)$$

### 3. Parametric Study of the Loss Factor for Squeeze Film Damping

To predict the increased loss factor of squeeze film damping plates in practice, the established model is applied to approach the web panels of the engine girders of a 100 GT gross tonnage fast vessel with the following data:

thick plate panel	:	800 mm × 500 mm × 6 mm ;
attached plate	:	300 mm × 200 mm × 1 mm ;
gap between plates	:	0.25 mm ;
plate density	:	7850 Kg/m <sup>3</sup> ;
Young's modulus of plate	:	2.1×10 <sup>11</sup> N/m <sup>2</sup> ;
Poisson's ratio	:	0.3 ;
air density	:	1.164 kg/m <sup>3</sup> (at 30°C) ;
dynamic viscosity of air	:	1.86×10 <sup>-5</sup> Ns/m <sup>2</sup> (at 30°C) ;
kinematic viscosity	:	1.60×10 <sup>-5</sup> m <sup>2</sup> /s (at 30°C) ;
sound speed in air	:	349.3 m/s (at 30°C) ;

The results of loss factors, obtained by using equation (27) for the compressible flow model of the air layer between two infinite parallel coupled plates and equation (33) for the statistical energy analysis method of two finite parallel coupled plates respectively, are compared as shown in Figure 6. It can be seen that these two approaches appear coincident very well in the frequency range higher than 300 Hz but deviate in the lower frequency range. In lower frequency range the precise model, equation (33), would be recommended. From Figure 6, it can also be seen that the loss factor raises to the level of 0.01~0.1 for such a squeeze film damping plate mechanism. This magnitude of loss factor is 5~50 times of that of steel.

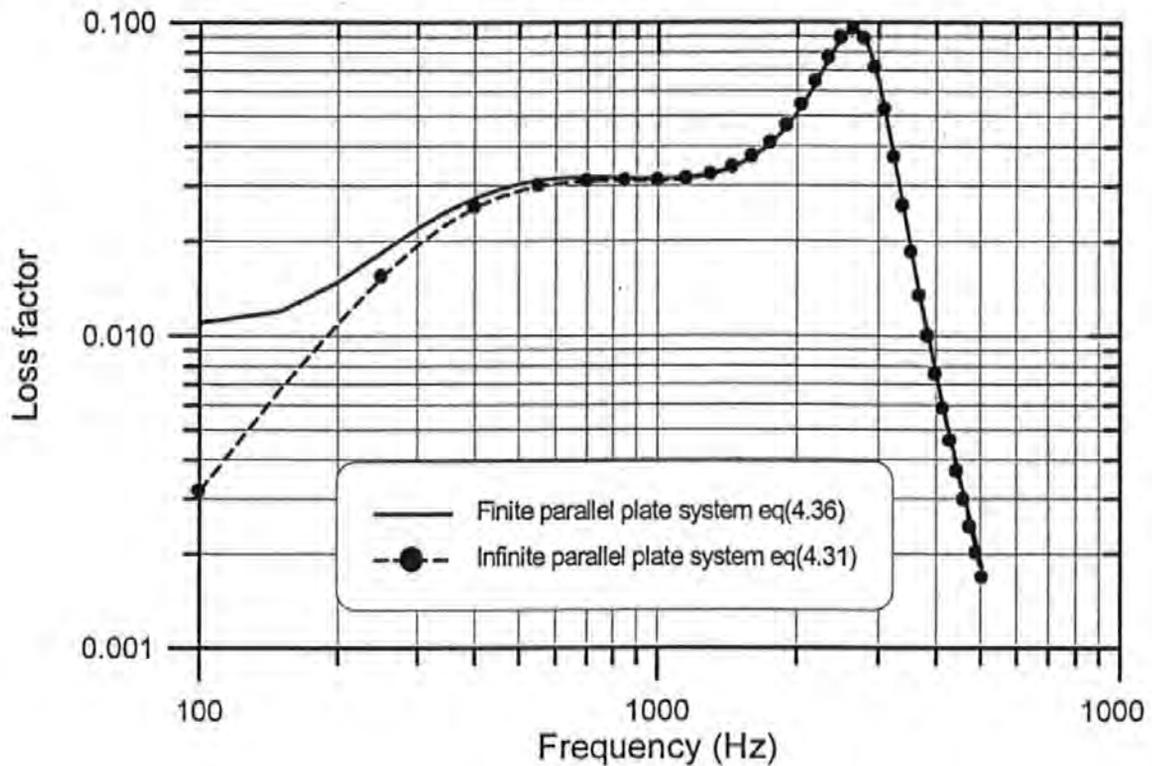


Fig. 6 Comparison of loss factor by the SEA model between two finite parallel coupled plates and the compressive flow model between two infinite parallel coupled plates

For a systematical parameter study, the sensitivity analyses of air temperature, thickness of plates, and gap clearance between coupled parallel plates have been carried out. For the variation of temperature from  $20^{\circ}\text{C}$  to  $60^{\circ}\text{C}$ , the properties of density, viscosity and sound speed of air are shown in Table 1. Figure 7 shows the calculated loss factor with respect to different temperature. Put the gap clearance  $d=0.25\text{ mm}$ ,  $0.5\text{ mm}$  and  $1.0\text{ mm}$ ; the thickness of attached plate  $h_2=0.5\text{ mm}$ ,  $1.0\text{ mm}$  and  $2.0\text{ mm}$ ; the thickness of excited thick plate  $h_1=3\text{ mm}$ ,  $6\text{ mm}$  and  $12\text{ mm}$  to undertake systematic loss factor analysis. Figure 8 shows the loss factor variation vs. the gap clearance between two parallel coupled plates. Figure 9 shows the loss factor variation vs. the thickness of attached plate where the thickness of the excited plate keeping to be  $h_1=6\text{ mm}$ . Figure 10 shows the loss factor variation vs. the thickness of excited plate where the thickness of the attached plate keeping to be  $h_2=1\text{ mm}$ . From Figure 7-10, it reveals that temperature is less sensitive to loss factor of the squeeze film.

Table 1 The properties of air at different temperature

Temperature	air density ( $\text{kg/m}^3$ )	dynamic viscosity of air ( $\text{Ns/m}^2$ )	Kinematic viscosity of air ( $\text{m}^2/\text{s}$ )	Sound speed in air (m/s)
$20^{\circ}\text{C}$	1.204	$1.81 \times 10^{-5}$	$1.51 \times 10^{-5}$	343.5
$30^{\circ}\text{C}$	1.164	$1.86 \times 10^{-5}$	$1.60 \times 10^{-5}$	349.3
$40^{\circ}\text{C}$	1.127	$1.91 \times 10^{-5}$	$1.69 \times 10^{-5}$	355.0
$50^{\circ}\text{C}$	1.092	$1.95 \times 10^{-5}$	$1.79 \times 10^{-5}$	360.6
$60^{\circ}\text{C}$	1.060	$2.00 \times 10^{-5}$	$1.89 \times 10^{-5}$	366.2

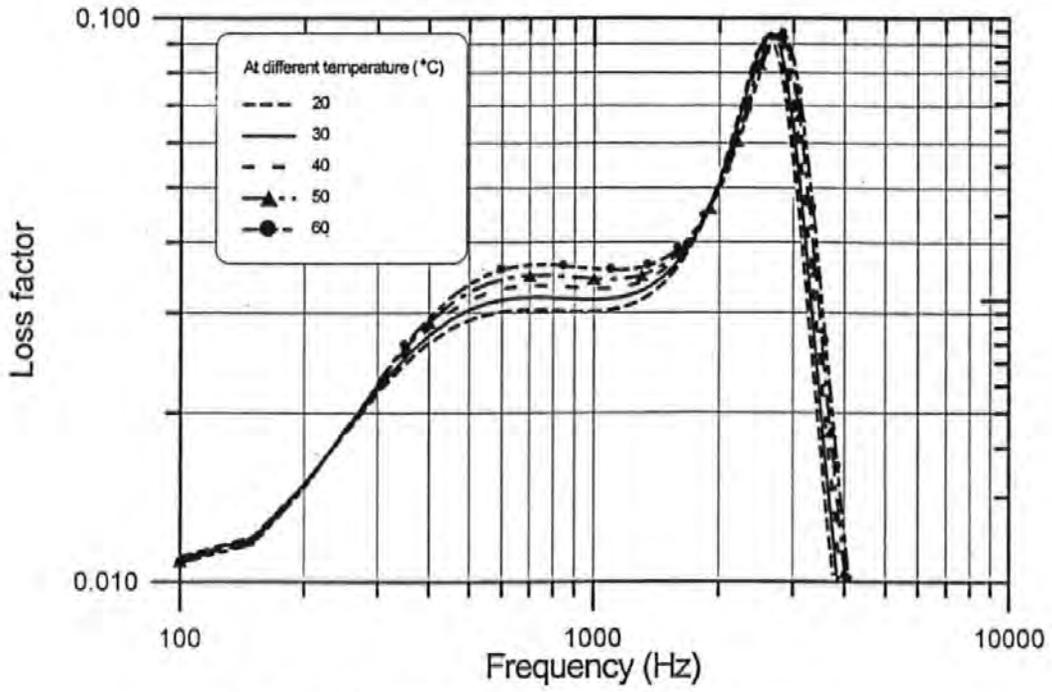


Fig.7 Loss factor variation vs. temperature

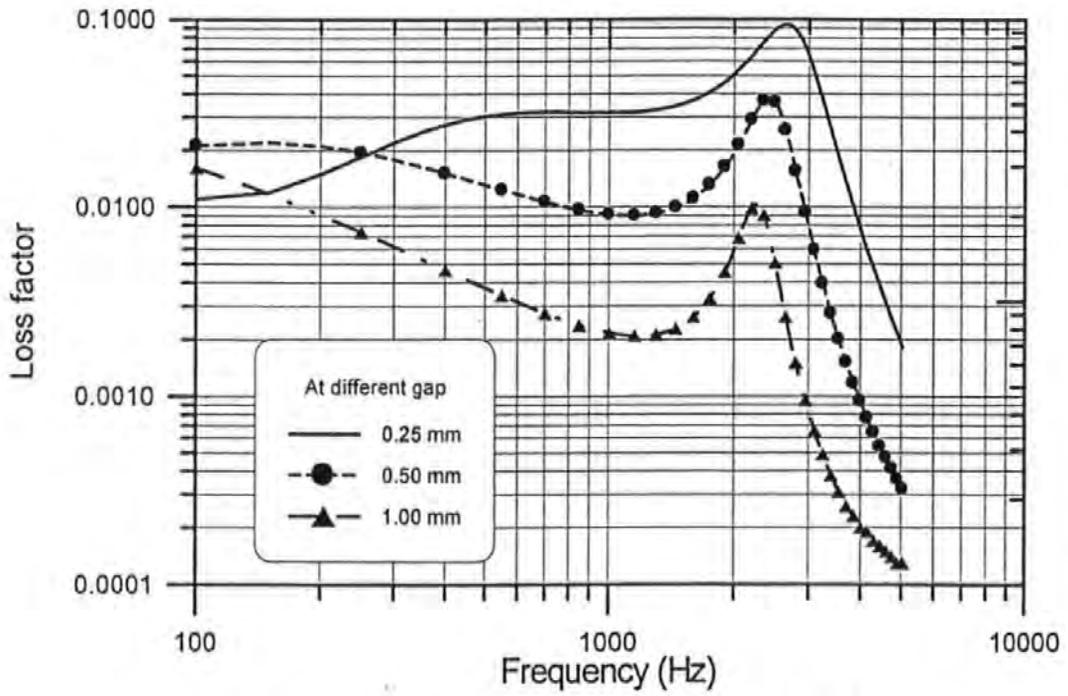


Fig. 8 Loss factor variation vs. gap clearance

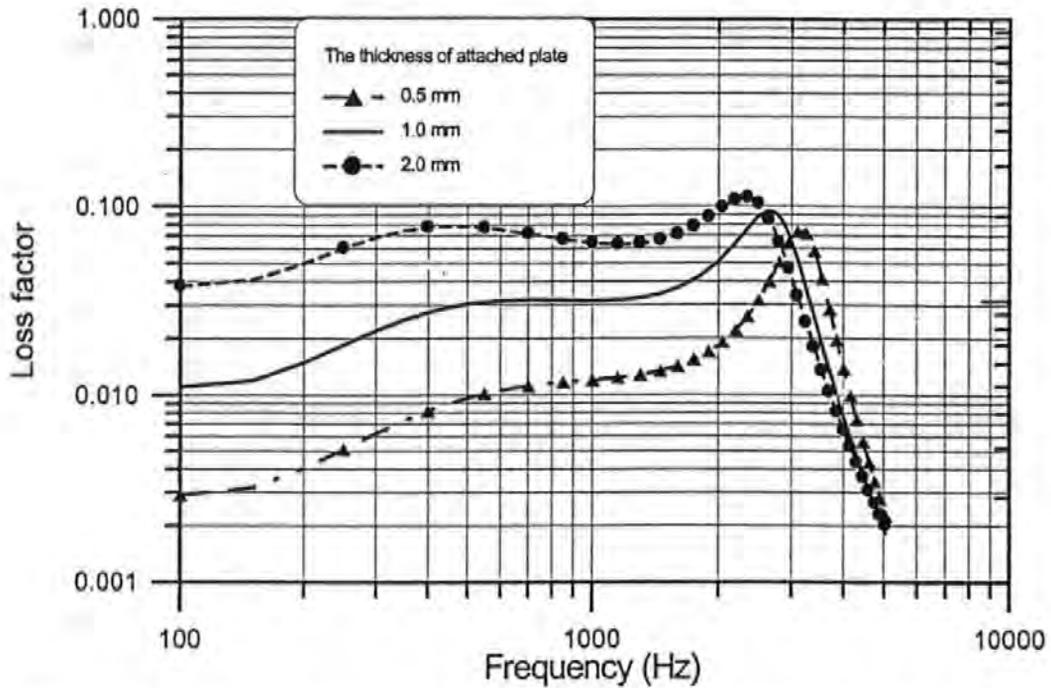


Fig. 9 Loss factor variation vs. thickness of attached plate keeping  $h_1=6\text{mm}$ .

#### 4. Effect of squeeze film on Mobility of Engine Room Structure

A major part of the structure-borne noise onboard a vessel is transmitted from the engine vibration via multiple mounts [6]. Take the bottom structure of engine room of a 100 gross tonnage fast vessel as an example to illustrate the mobility variation after the installation of squeeze films on the webs of the bottom girders. Figure 11 shows the FEA model of the bottom structure for mobility analysis. In this model, there are

5218	nodes
7260	shell elements
1249	beam elements
500	master degrees of freedom

In order to increase the damping of the bottom structure for reducing structure-borne noise transmission, several rectangular thin plates of size  $300\text{ mm} \times 200\text{ mm} \times 1\text{ mm}$  (thick) as shown in Figure 12 and 13 are attached on the engine girders A and B by tack welding. It constitutes the squeeze film damping plates. Thus, the damping ratio at these portion of attached plates raises to 0.01, corresponding to loss factor 0.02 as in Figure 6, from 0.001 of the original structure. Figure 14-17 show the analysis results of the driving point mobility at the location of each mount. For installation of squeeze film damping plates, there are 5-10 dB reduction in peak mobilities.



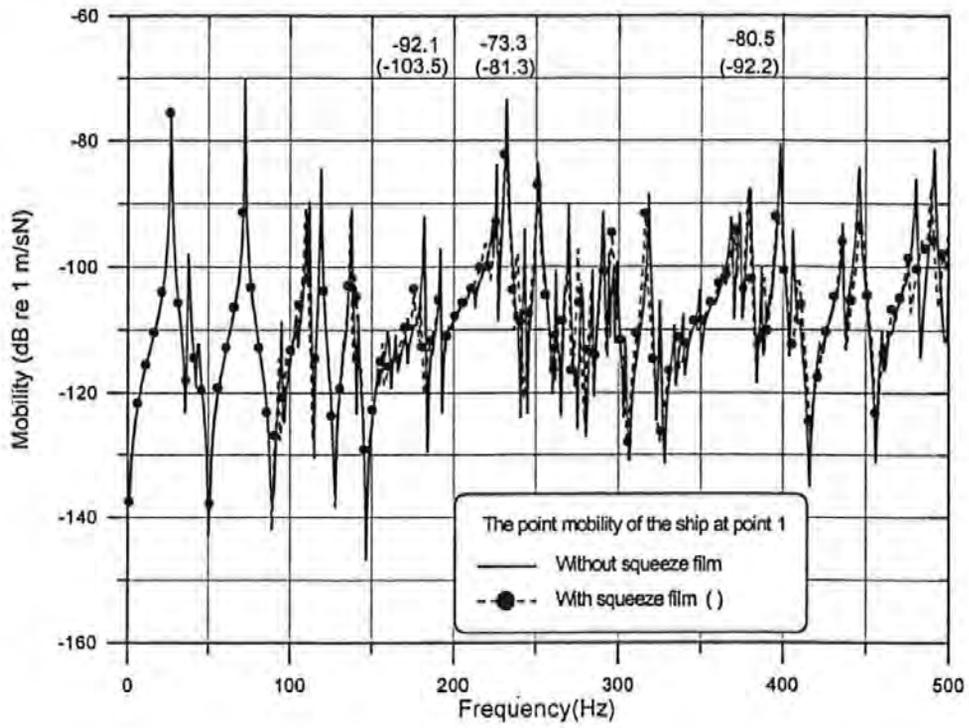


Fig. 14 Comparison of mobility at mount 1 with and without squeeze film

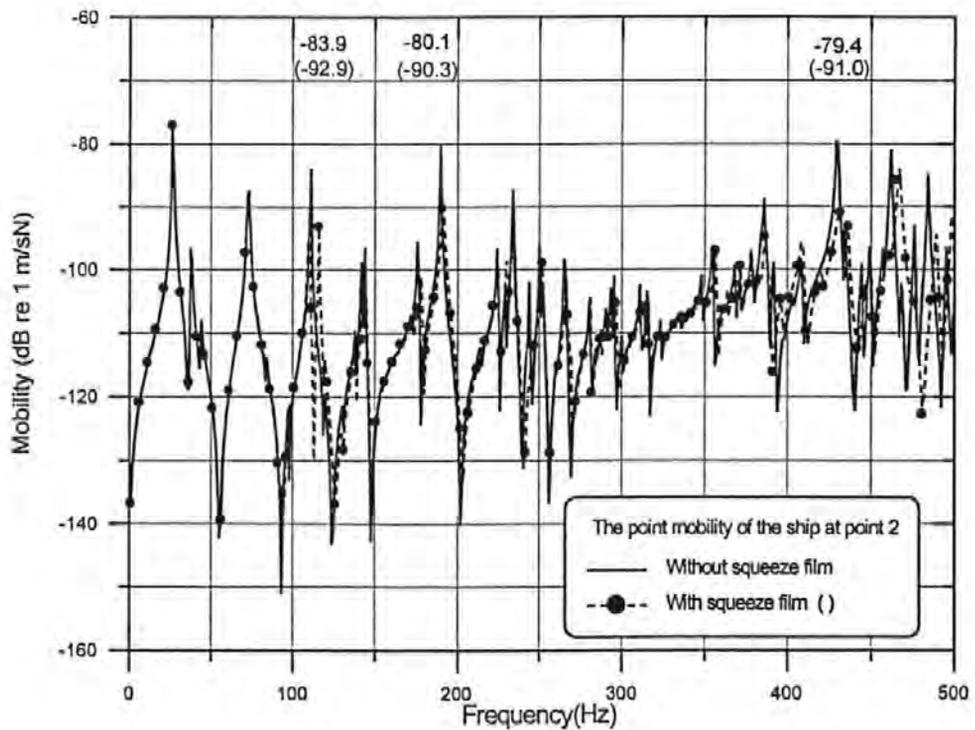


Fig. 15 Comparison of mobility at mount 2 with and without squeeze film

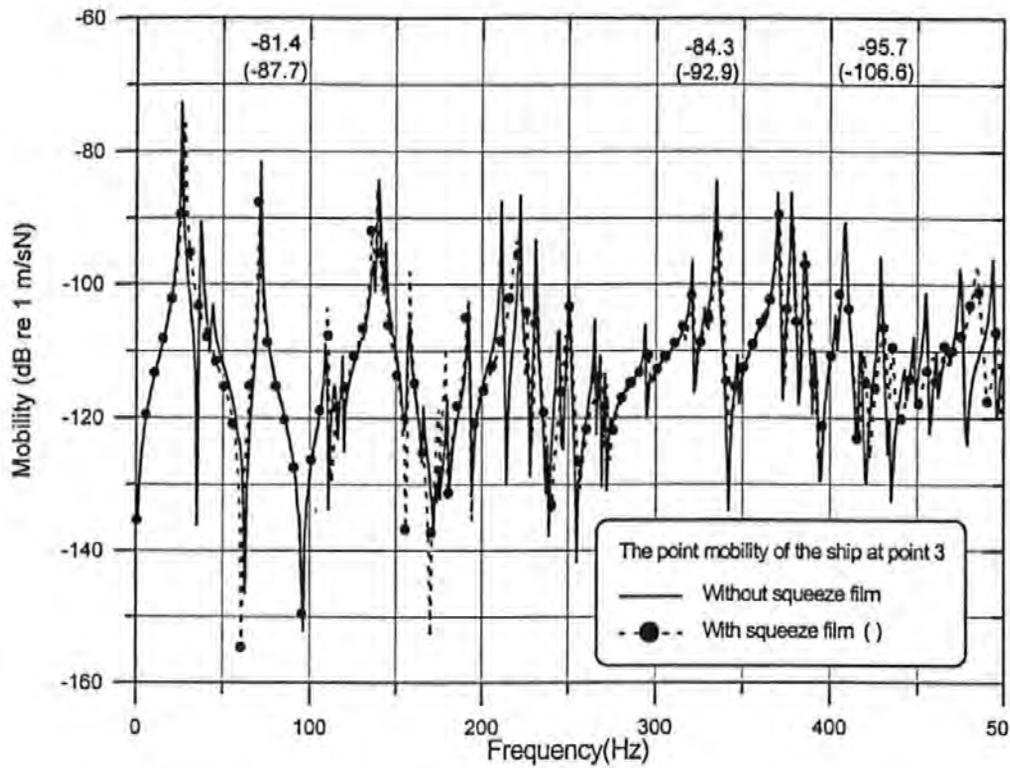


Fig.16 Comparison of mobility at mount 3 with and without squeeze film

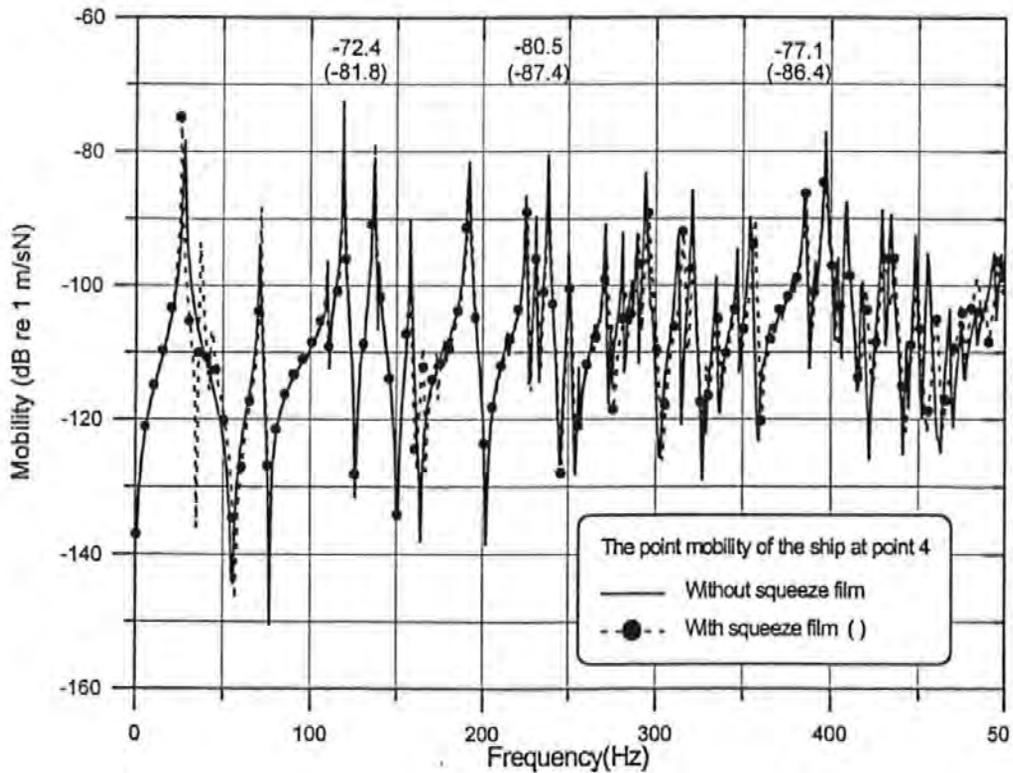


Fig. 17 Comparison of mobility at mount 4 with and without squeeze film

## 5. Concluding Remarks

From the work of this study, it forwards to the following concluding remarks:

-A more precise model of the loss factor prediction for two finite parallel coupled plates has been established by use of the statistical energy analysis method. Nevertheless, the compressible flow model of the air layer between two infinite coupled plates for the loss factor estimation has shown to be very satisfactory in the medium and higher frequency range.

-It is easy to raise the loss factor to the extent of 0.001-0.1 in the portion of structure installed with attached damping plate.

-Squeeze film damping technology is a economical and efficient measure to reduce structure-borne noise 5-10 dB.

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## MOBILITY ANALYSIS OF THE IMPROVED BOTTOM STRUCTURE BY SQUEEZE FILM FOR STRUCTURE-BORNE NOISE REDUCTION



**WEI-HUI WANG,**  
*Tend-Chich Yang National Taiwan Ocean University, Keelung, ROC*

Machinery or foundation panels can be damped by attaching an auxiliary plate parallel to the surface, thereby trapping a thin layer of air. Relative vibration of the plates pump this air at high velocities, resulting in energy loss due to the air viscosity. Loss factors can be achieved to

the order of 0.1 without the sensitivity to heat and oil associated with viscoelastic materials.

For the purpose of noise attenuation in ship's cabins, the squeeze film technology was adopted to reduce the structure-borne sound transmission from the diesel engines. Estimation of the loss factor of the squeeze film is using a more sophisticated incompressible flow model. In which, the Navier-Stokes and continuity equations in two-dimensional system are utilized by neglecting the thermal effect. In other words, it is assumed that the acoustic process is nearly adiabatic. For the acoustic disturbance of the trapping air to be adiabatic, neighbouring elements of the fluid must not exchange thermal energy. This means that no significant thermal flux occurs during the time of the disturbance.

By the FEA model, the mobilities at each site on the girder below the resilient engine mounts can be analysed for the conditions before and after the installation of the squeeze plates. The comparisons of the mobilities show that peak levels reduced by 5-10 dB after the installation of the squeeze films. Meanwhile the airborne noise radiation has been improved by:

Compartment	Airborne noise level, dB (A)	
	Original	Improved
Engine room	115	112
Captain's room	90	86
Pilothouse	85	74

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Автор статьи - один из старейших профессоров факультета системной инженерии и кораблестроения Национального Тайваньского Университета Океана. С 1981 г. его преподавательская карьера в университете непосредственно связана с административной руководящей работой. В настоящее время профессор Ванг является председателем Академического Совета Обществ кораблестроителей и морских энергетиков Тайваня, принимает участие в работе Международных конгрессов по судовым и шельфовым конструкциям (сооружениям).

# REWARD

Wei-Hui Wang

National Taiwan Ocean  
University,  
ROC, Taiwan

Who took the 1 place in Third International Students'  
Congress of The Asia-Pacific Region Countries

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Conference Chairman



Gennady Turmov

# MACHINE VIBRATION INDUCED UNDERWATER ACOUSTIC RADIATION

Wei-Hui Wang\* Jia-Horng Liou\*\* Robert Sutton\*\*\* Ben Dobson\*\*\*\*

Keywords: fluid-structure interaction, coupled FEM/BEM, radiation pattern.

## ABSTRACT

A coupled FEM/BEM for solving fluid-structure interaction problem in structure-borne noise propagation and underwater acoustic radiation is present in this paper. In order to improve the habitability and stealth low noise condition is required in various types of ships. One of the dominant components is the radiated airborne and underwater noise caused by ship hull vibration. Accordingly, the estimation of structure-borne sound characteristics of a ship is important in the design of a quieter ship-structure. The hull vibration response and pressure distribution on the wetted surface of a ship subjected to excitation sources can be analyzed by utilizing the developed method. Further discussions on the evaluation of surface complex power, radiation and reactance power have been conducted.

Two cases were studied, a submerged spherical shell subjected to a uniformly harmonic pressure excitation and a scale model of engine room structure subjected to machine excitations, to illustrate the applications of the analytical model. The surface pressure, the normal velocity on the wetted surface, the surface complex power, the radiation power, the reactance power, the vibrational displacement and the radiated underwater sound pattern have been computed. Comparison of the predicted results and that by measurement indicates that the numerical model is creditable. In addition, the effect on underwater acoustic radiation at the elastic mounts for the engine is discussed.

## INTRODUCTION

Structure-borne sound power is predominantly transmitted to a sound carrying structure from a source via a number of contact points. In turn, the noise and vibrations are propagated in the structure possibly causing sensitive equipment to vibrate or to cause undesired radiated noise. In principle, this may be avoided by

measures at source, in transmission, during propagation or at radiation. It is, of course, preferable to cope with the problems at the generation sites and thereby avoid more comprehensive and expensive measures later in the chain. To this point, the resilient mounts are utilized as an effective countermeasure to reduce noise and vibrational power transmitted from the source to sound receiver.

In reviewing noise transmission paths, we note that from the machinery room to other areas of the ship, there are three parallel paths: air, water, and structure, the latter being the most important. While structure-borne noise is familiar, it takes an uncommon form in ship hulls; compared to industrial environments, the hull is relatively homogeneous and long wave-guide whose transmission characteristics are affected by spatially periodic stiffening frames. Furthermore, the waterborne path is intimately couple to the structure-borne path, partially short-circuiting structural impedance discontinuities such as bulkheads, which, in the atmosphere, would constitute effective barriers to structure-borne noise propagation. Another peculiarity of hull is that, over much of the relevant frequency range, radiation loading is associated with inertia forces, exerted by the entrained mass of water, which modify hull plating response. Because of the high sound velocity of water, the coincidence frequency of surface ship hull plating lies in the ultrasonic range. Flexural waves therefore radiate from areas where wavenumber conversion occurs, viz. driving points and impedance discontinuities. Once launched, the waterborne acoustic signal suffers propagation losses which are small compared to those encountered in the atmosphere.

The main features of shipboard noise have been studied in the last decades. The power flow concept was proposed and used to explain the relation between the mobility and structure-borne sound transmission [1]. The effectiveness of resilient mounts to abate the engine vibration was modeled in [2, 3]. Furthermore a coupled multi-path mobility model using the four-pole parameter method was established to predict the structure-borne sound transmission between a source structure

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\*Professor, Department of System Engineering and Naval Architecture  
National Taiwan Ocean University, R.O.C.  
\*\*Ship Inspector, Keelung Harbor Bureau, R.O.C.  
\*\*\*Reader, Department of Mechanical and Marine Engineering, University of Plymouth, UK  
\*\*\*\*Chief Design Engineer, Devonport Royal Dockyard, Plymouth, UK.

and a receiving structure in [4, 5]. The sound radiation from a vibrating body was firstly dealt with by helmholtz equation in [6]. Meanwhile the boundary integral equations for vibrating sources located in interior or exterior domain and on boundary were established separately. Some fictitious eigenfrequencies and singular values could be appeared in dealing with the exterior domain problems. This phenomenon is essentially due to the numerical method. To improve the solution method a combined Helmholtz integral equation formulation (CHIEF) has submitted in [7, 8]. Underwater sound radiation from various vibrating submerged structures was studied in [9-16] by using a coupled finite element and variational boundary element technique. As a summary of these studies, the fluid-structure interaction effect showed predominant important in predicting the underwater sound radiation especially in low frequency vibrational radiation. It has been validated by experimental measurement in [17].

In this paper three main aspects of machine vibration radiating underwater sound are predicted and measured. They are the transfer mobility and power flow from a motor via resilient mounts, fluid-structure interaction effect in underwater acoustic radiation and the underwater radiation pattern.

### POWER FLOW ANALYSIS VIA RESILIENT MOUNTING

Operational motions of engine and other machineries start the vibration, and the resilient mounts under the machinery seatings initiate the reduction efforts of noise control. The effectiveness of isolation is defined as the measure of the power transmitted to the structure. This single quantity includes both the vibration parameters of force and velocity. Minimizing the power transmitted through an isolator is also consistent with the first step in any vibration control exercise, i.e. to deal with the problem at the source. Normally, the resilient mounts are composed of elastic and viscoelastic materials and designed in various configurations.

The vibrational power fed into a structure for a multipoint mounting system can be calculated by utilizing the concept of effective mobility [1-5] of the source/mount/receiver system and the source characterization as a force or a velocity. Furthermore, if the relations between the mobility function pertaining to the coupled and decoupled source/mount/receiver systems can be derived, then the effectiveness and performance of the mount can be identified.

#### 1. Mobility Theory

Assume a mechanical system in which the har-

monic force  $F_1$  and the harmonic velocity  $V_1$  were applied to the input end of the system and the force  $F_2$  and the velocity  $V_2$  were induced on the output end. The relations between state variables  $V_1, F_1$  and  $V_2, F_2$  are [3]:

$$\begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \quad (1)$$

where  $\alpha_{11}, \alpha_{12}, \alpha_{21}$  and  $\alpha_{22}$  are called the four-pole parameters [4] and

$$\alpha_{11} = \frac{M_{11}}{M_{12}}, \quad \alpha_{12} = \frac{M_{12}M_{21} - M_{11}M_{22}}{M_{12}},$$

$$\alpha_{21} = \frac{1}{M_{12}}, \quad \alpha_{22} = -\frac{M_{22}}{M_{12}} \quad (2)$$

and  $M$  ( $i = 1, 2; j = 1, 2$ ) are the mobility functions.

If the subsystems  $a$  and  $b$  are linked in series, the two ends of each subsystems are 1, 2 and 3, 4 respectively, as shown in Fig. 1(a), in which  $F$  represents force and  $V$  represents velocity. When the subsystems link together, then  $V_3 = V_2$  and  $F_3 = -F_2$ . The transfer relation of the state vectors between points 1 and 4 is:

$$\begin{aligned} \begin{Bmatrix} V_1 \\ F_1 \end{Bmatrix} &= \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ F_2 \end{Bmatrix} \\ &= \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \alpha_{33} & \alpha_{34} \\ -\alpha_{43} & -\alpha_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} \\ &= \begin{bmatrix} \alpha'_{11} & \alpha'_{14} \\ \alpha'_{41} & \alpha'_{44} \end{bmatrix} \begin{Bmatrix} V_4 \\ F_4 \end{Bmatrix} \end{aligned} \quad (3)$$

It requires that  $F_4 = 0$  since 4 is the receiving point and 1 is the driving point. Thus,

$$M_{41}(\omega) = \frac{V_4}{F_1} = \frac{1}{\alpha'_{41}} = \frac{1}{\alpha_{21}\alpha_{33} - \alpha_{22}\alpha_{43}} = \frac{M_{12} + M_{34}}{M_{22}M_{33}} \quad (4)$$

From equation (4), it can be seen that the transfer mobility of the combined system ( $M_{41}$ ) is not simply the combination of the transfer mobilities ( $M_{12}$  and  $M_{34}$ ) of the two subsystems. The linking point mobilities ( $M_{22}$  and  $M_{33}$ ) of the subsystems play a predominant role for determining the total transfer mobility. As a damping material is introduced into the input end of the subsystem  $b$ , such as Fig. 1(b) which represents the improved system of Fig. 1(a), then the mobility  $M_{33}$  increases significantly. It follows from equation (4) that the total transfer mobility  $M_{41}$  is reduced. In other

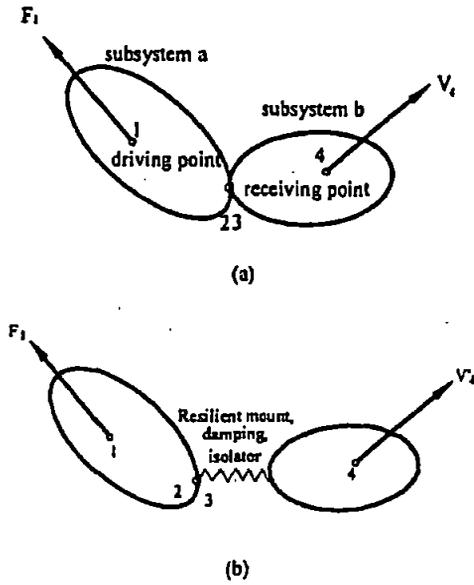


Fig. 1. System before and after installation of isolator damping material.

words, to reduce the total transfer mobility, it requires the introduction of a larger mobility than the original one. This is the basic principle for a good structure-borne noise attenuation result that has to properly design the structure on both sides of a resilient mount with small mobility than that of the mount itself.

2. Power Flow

The instantaneous vibration power input ( $P_{inst}$ ) to a structure is defined as the product of the excitation force ( $F$ ) and the associated velocity ( $V$ ). At any point on a structure, the velocity and force can be related via frequency response mobility (or impedance) functions, so that the power can be expressed in the following forms:

$$P = \frac{1}{2} \{F\}^T \text{Re}[M] \{F\} \tag{5}$$

or

$$P = \frac{1}{2} \{V\}^T \text{Re}[Z] \{V\} \tag{6}$$

From expressions (5) and (6), it is clear that the power injected into a structure by the machine is dictated by the excitation force or velocity exerted by a machine and the mobility or impedance characteristics of the supporting structure. The characterization of an engine as a "velocity source" will now be considered .

In practical measurements by utilizing a FFT

analyser, the power can be found by using the force spectrum density function  $G_{ff}$ , the vibration velocity spectrum density function  $G_{vv}$ , or the cross power spectrum density function  $G_{fv}$ , which can be expressed as:

$$P(\omega) G_{ff} \text{Re}[M] = \text{Re}[G_{fv}] = G_{vv} \text{Re}[Ze] \tag{7}$$

The driving point impedance matrix and the receiving point impedance matrix and the transfer impedance matrix for well-installed system can be obtained as:

$$[Z_{input}] = [Z_{11}] = \frac{[M_{44}]}{[M_{11}][M_{44}] - [M_{14}][M_{41}]} \tag{8}$$

$$[Z_{output}] = [Z_{44}] = \frac{[M_{11}]}{[M_{11}][M_{44}] - [M_{14}][M_{41}]} \tag{9}$$

$$[Z_{transmit}] = [Z_{41}] = [Z_{14}] = \frac{-[M_{14}]}{[M_{11}][M_{44}] - [M_{14}][M_{41}]} \tag{10}$$

Substituing  $[Z_{input}]$ ,  $[Z_{output}]$  and  $[Z_{transmit}]$  into equation (7), the input power, output power and transmitted power can be calculated respectively by:

$$P_{input} = G_{v_A} v_A \text{Re}[Z_{input}] \tag{11}$$

$$P_{output} = G_{v_B} v_B \text{Re}[Z_{output}] \tag{12}$$

$$P_{transmit} = G_{v_A} v_A \text{Re}[Z_{transmit}] \tag{13}$$

Thus the general methodology for obtaining the power fed into a receiving structure from the excitation sources of a engine via the contact points is derived.

MODELLING SOUND RADIATION FROM A SHIP STRUCTURE

1. Fluid-Structure Interaction

When a ship is floating in water and subjected to machine induced vibrating forces  $\{f\}$ , then the equations of motion of the ship structure can be expressed in matrix form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} - \{p\} \tag{14}$$

where  $[M]$ ,  $[C]$  and  $[K]$  represent the mass, damping and stiffness matrices respectively.  $\{f\}$  is the acoustic loading transmitted from the machine mountings which is gained by the mobility theory and characterising the vibrational sources as velocity sources. While  $\{p\}$  represents the hydrodynamic pressure which can be

only applied to the surfaces of the structure contacted with water and shall be analysed by the coupling algorithm which is a combination of finite element (FE) and boundary element (BE) methods.

If the internal structural damping effect can be neglected and the displacement vector  $\{x\}$  of the structure is divided into  $\{x_n\}$ , ie, the displacement of the interior structure as well as the tangential displacement on the water contacting surface as shown in Fig. 2, then equation (14) becomes:

$$\begin{bmatrix} M_{nn} & M_{ni} \\ M_{in} & M_{ii} \end{bmatrix} \begin{Bmatrix} \ddot{x}_n \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} K_{nn} & K_{ni} \\ K_{in} & K_{ii} \end{bmatrix} \begin{Bmatrix} x_n \\ x_i \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_i \end{Bmatrix} - \begin{Bmatrix} p \\ 0 \end{Bmatrix} \quad (15)$$

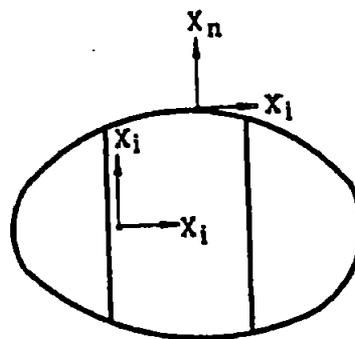


Fig. 2. Definition of  $\{x_n\}$  and  $\{x_i\}$ .

2. Coupling of the FE and BE Methods

Consider a linear structure system put  $\{x\} = \{X\}e^{i\omega t}$  into equation (15) then,

$$\begin{bmatrix} K_{nn} & K_{ni} \\ K_{in} & K_{ii} \end{bmatrix} \begin{Bmatrix} X_n \\ X_i \end{Bmatrix} - \omega^2 \begin{bmatrix} M_{nn} & M_{ni} \\ M_{in} & M_{ii} \end{bmatrix} \begin{Bmatrix} X_n \\ X_i \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_i \end{Bmatrix} - \begin{Bmatrix} p \\ 0 \end{Bmatrix} \quad (16)$$

The second equation in (16) is:

$$([K_{in}] - \omega^2[M_{in}]) \{X_n\} + ([K_{ii}] - \omega^2[M_{ii}]) \{X_i\} = \{f_i\} \quad (17)$$

and  $\{X_i\}$  can be expressed in terms of  $\{X_n\}$  from equation (5.62) as :

$$\{X_i\} = ([K_{ii}] - \omega^2[M_{ii}])^{-1} (\{f_i\} - ([K_{in}] - \omega^2[M_{in}]) \{X_n\}) \quad (18)$$

Substituting (18) into equation (16),

$$([K_{nn}] - \omega^2[M_{nn}]) \{X_n\} + ([K_{ni}] - \omega^2[M_{ni}]) \{X_i\} = \{f_n\} - \{p\}$$

or

$$\begin{aligned} &([K_{nn}] - \omega^2[M_{nn}]) - ([K_{ni}] - \omega^2[M_{ni}]) ([K_{ii}] - \omega^2[M_{ii}])^{-1} ([K_{in}] - \omega^2[M_{in}]) \{X_n\} \\ &= - ([K_{in}] - \omega^2[M_{in}]) ([K_{ii}] - \omega^2[M_{ii}])^{-1} \{f_i\} + \{f_n\} - \{p\} \end{aligned} \quad (19)$$

Since the underwater acoustic pressure distribution radiated from a vibrating hull surface is governed by the Helmholtz equation:

$$\nabla^2 p(\vec{r}, \omega) + k^2 p(\vec{r}, \omega) = 0 \quad (20)$$

and  $p(\vec{r})$  can be expressed as :

$$P(\vec{r}) = \frac{1}{2\pi} \int_{S_s} [P(\vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n(\vec{r}_s)} - G(\vec{r}, \vec{r}_s) \frac{\partial P(\vec{r}_s)}{\partial n(\vec{r}_s)}] dS_s \quad (21)$$

where

- $\vec{r}$  = position vector in water;
- $\vec{r}_s$  = position vector on immersed hull surface;
- $n(\vec{r}_s)$  = normal direction vector at  $\vec{r}_s$ ;
- $S_s$  = immersed hull surface area;
- $P(\vec{r})$  = underwater acoustic pressure amplitude at  $\vec{r}$ ;
- $G(\vec{r}, \vec{r}_s)$  = Green's function or the fundamental solution to Eq.(20), and

$$G = \frac{e^{-jk r(x,y)}}{4\pi r(x,y)} \quad (22)$$

where

$$r(x, y) = |x - y| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$$

in which  $x$  is the field point and  $y$  is the source point.

The relation between  $P(\vec{r})$  and the normal displacement  $x_n$  on the hull surface must satisfy:

$$\frac{\partial P(\vec{r}_s)}{\partial n(\vec{r}_s)} = -i\rho_0\omega v_n(\vec{r}_s) = \rho_0\omega^2 x_n \quad (23)$$

Utilizing the discretization technique with interpolating shape functions to the water contacting hull surface, equation (22) becomes :

$$\{A\} \{P\} = \{G\} \{x_n\} \quad (24)$$

Substituting equation (24) into (19), the governing equation for the combined FE and BE methods for the structure-borne noise transmission problem inboard ships can show to be:

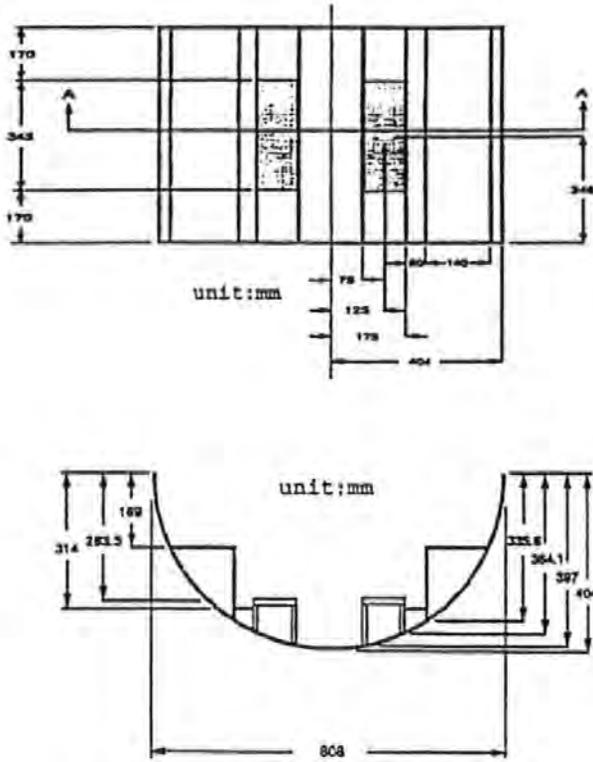


Fig. 3. Experimental model of an engine room.

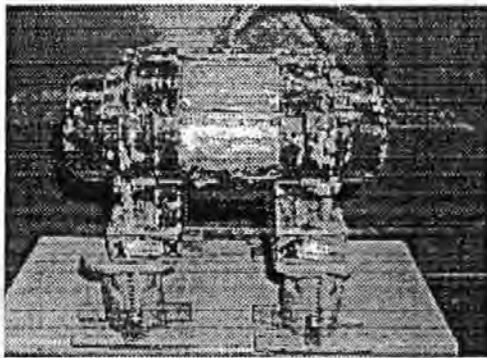


Fig. 4. Instrument arrangement for mobility measurement.

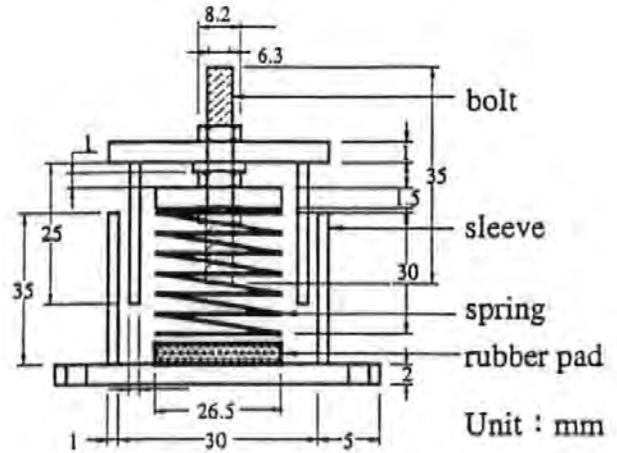
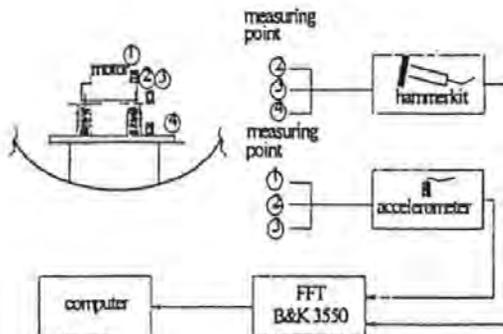


Fig. 5. Configuration of the sleeve type of resilient mount.

$$\begin{aligned}
 & \{([K_{nn}] - \omega^2[M_{nn}] - ([K_{ni}] - \omega^2[M_{ni}])([K_{ii}] \\
 & - \omega^2[M_{ii}])^{-1}([K_{in}] - \omega^2[M_{in}]))\} \{x_n\} \\
 & + [A]^{-1} [G] \{x_n\} = \{f_n\} - ([K_{ni}] - \omega^2[M_{ni}])([K_{ii}] \\
 & - \omega^2[M_{ii}])^{-1} \{f_i\} \quad (25)
 \end{aligned}$$

## COMPARISON OF NUMERICAL ANALYSIS AND MEASUREMENT

### 1. Description of Experiment Design

To validate the accuracy of the mobility analysis and the FEM/BEM noise propagation models experiments were undertaken on a scale model of an engine room structure. This model with a length 808mm, as shown in Fig. 3, consists of two end bulkheads, four girders on the bottom, one bilge tank on each side. On the top of the girders there installed a bedplate and a stepless motor supported by four resilient mounts bolted on the bedplate. The motor was designed with a deliberate eccentric mass on the shaft. The arrangement of mobility measurement and the configuration of the resilient mount are shown in Figs. 4 and 5 respectively.

### 2. Comparisons of Mobility and Vibrational Power

The measured and predicted transfer mobility  $M_{41}$  when the model being put in water and in air with and without rubber pad are compared in Figs. 6-9. The resilient mount effect on transfer mobility in water and in air are compared in Figs.10 and 11. When the motor ran at a speed 1800 rpm, the rubber pad effect on the exciting vibration velocity and the transfer power at the input and output ends are shown in Figs. 12-19. From the results, it is evident that resilient mounts have the effects to reduce transfer mobility, vibratory velocity and vibrational power by an amount of 5-20dB, 2-5dB

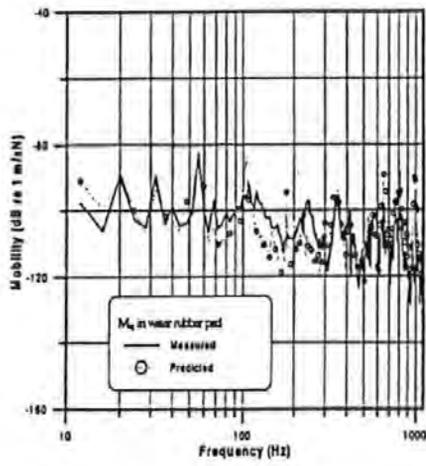


Fig. 6. Transfer mobility  $M_{41}$  in water without rubber pad.

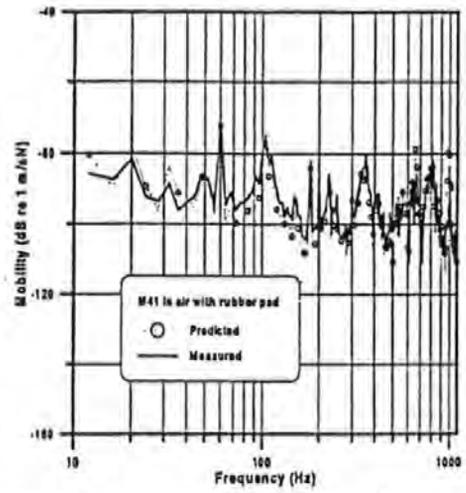


Fig. 9. Transfer mobility  $M_{41}$  in air with rubber pad.

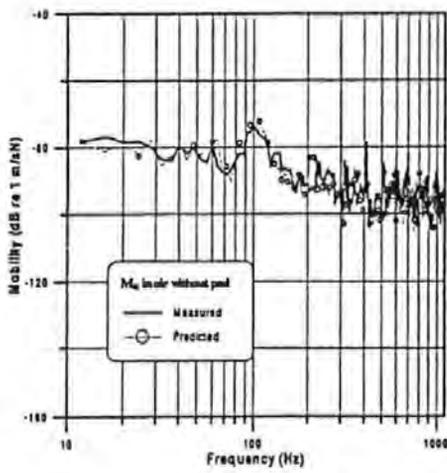


Fig. 7. Transfer mobility  $M_{41}$  in air without rubber pad.

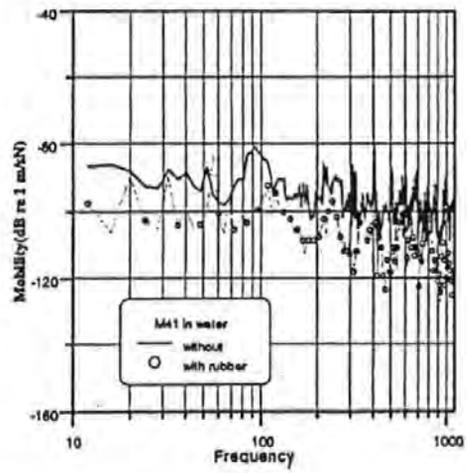


Fig. 10. Resilient mount effect on transfer mobility in water.

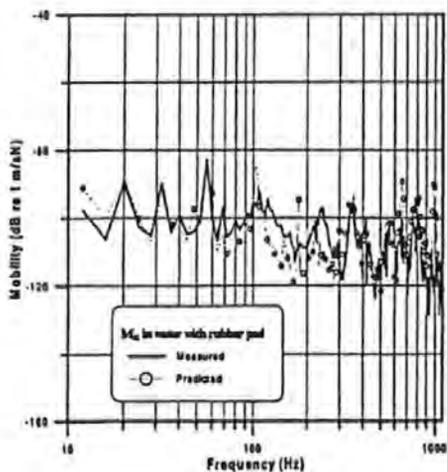


Fig. 8. Transfer mobility  $M_{41}$  in water with rubber pad.

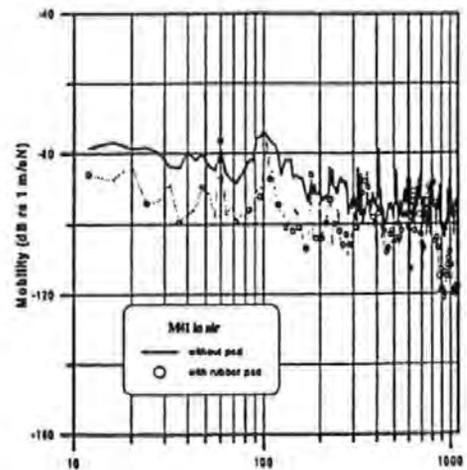


Fig. 11. Resilient mount effect on transfer mobility in air.

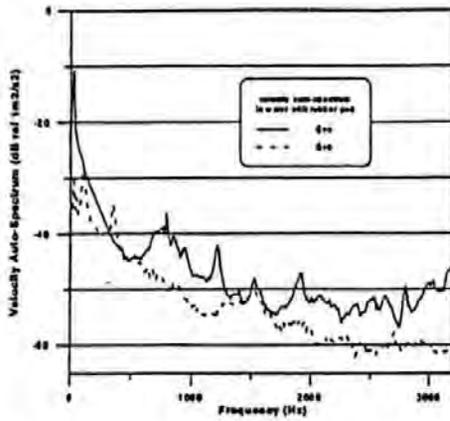


Fig. 12. Velocity auto-spectrum at input and output ends with rubber pad in water.

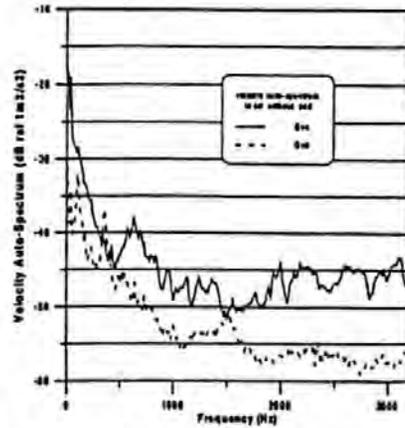


Fig. 15. Velocity auto-spectrum at input and output ends without rubber pad in air.

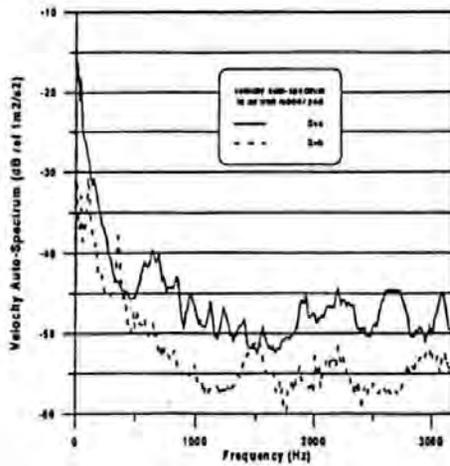


Fig. 13. Velocity auto-spectrum at input and output ends with rubber pad in air.

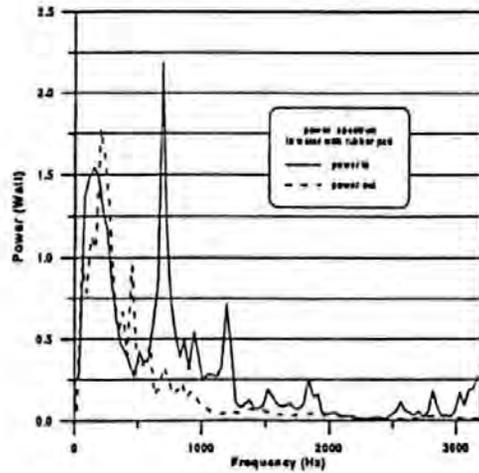


Fig. 16. Power spectrum at input and output ends with rubber pad.

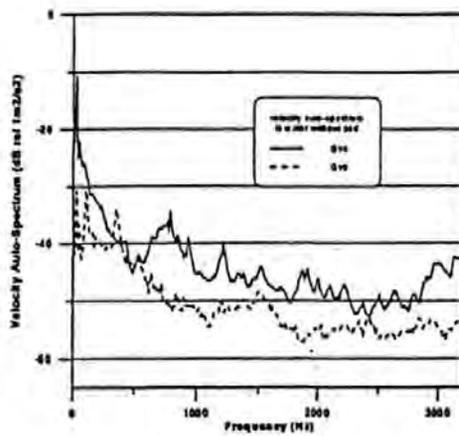


Fig. 14. Velocity auto-spectrum at input and output ends without rubber pad in water.

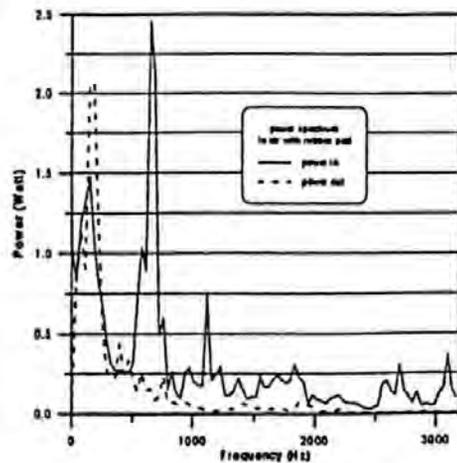


Fig. 17. Power spectrum at input and output ends with rubber pad in air.

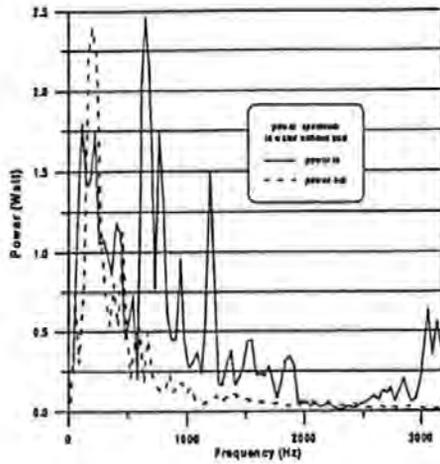


Fig. 18. Power spectrum at input and output ends without rubber pad.

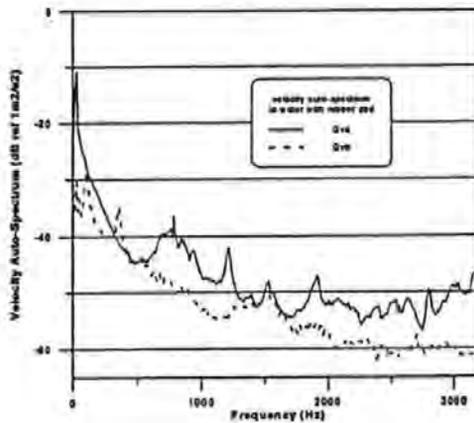


Fig. 19. Power spectrum at input and output ends without rubber pad in air.

and 10-40% respectively. The entrained mass of water apparently reduced the transfer mobility by an amount 5-10dB in the lower frequency range.

### 3. Validation of the Numerical Model

Consider a submerged spherical shell of radius 1m and thickness 3cm. When the shell subjected to a harmonic internal pressure of magnitude 1 Pa, the analytical solution of the sound pressure and normal velocity on the surface of spherical shell can be found in [11] and [12]. Fig. 20 shows the FEM/BEM model. Comparisons of the numerical solutions of the spectra of surface pressure magnitude and the normal velocity magnitude are shown in Figs. 21 and 22. The coincidence of the curves validate the numerical model being credible.

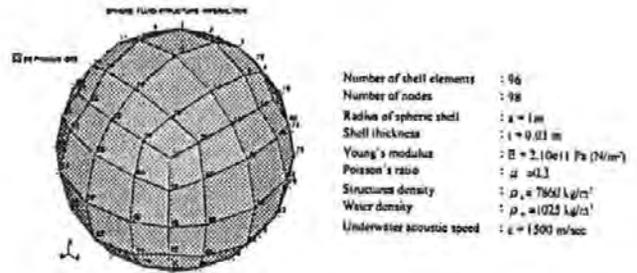


Fig. 20. Numerical model of a spherical shell.

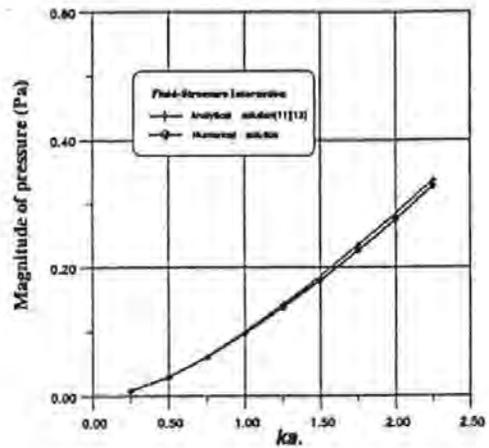


Fig. 21. Surface pressure magnitude spectrum on a spherical shell under unit harmonic internal pressure excitation.

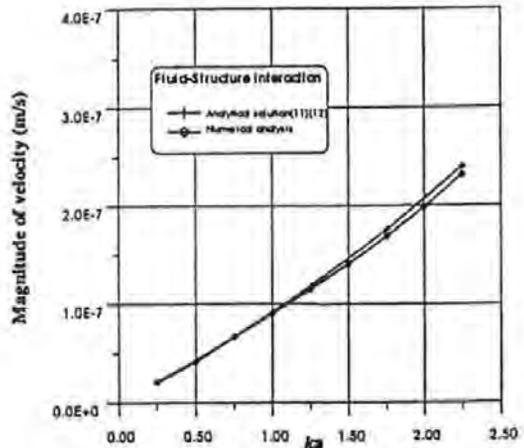
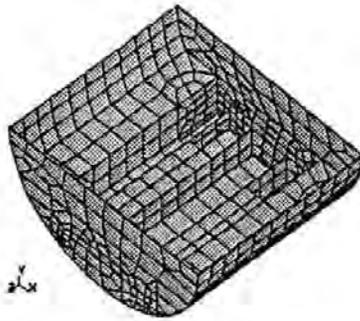


Fig. 22. Surface normal velocity magnitude spectrum on a spherical shell under unit harmonic unit internal pressure excitation.

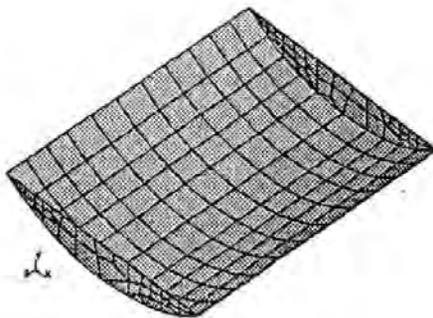
### 4. Predicted and Measured Underwater Sound Pattern

The FEM/BEM mesh of the model structure were established as Figs. 23 and 24. When the scale model



Number of shell elements : 711  
 Number of nodes : 654  
 Young's modulus :  $E = 2.10 \times 10^{11} \text{ Pa (N/m}^2\text{)}$   
 Structural density :  $\rho_s = 7860 \text{ kg/m}^3$   
 Poisson's ratio :  $\mu = 0.3$

Fig. 23. Finite element modelling.



Number of elements : 180  
 Number of nodes : 201  
 Underwater Sound :  $C = 1500 \text{ N/s}$   
 Water density :  $\rho_w = 998 \text{ kg/m}^3$

Fig. 24. Boundary element modeling on wetted surface.

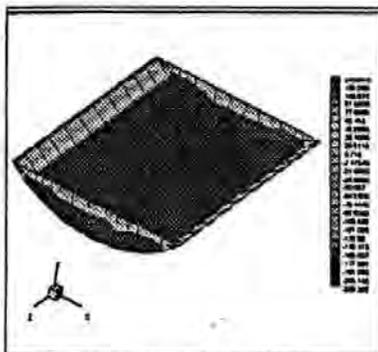


Fig. 25. Predicted sound pressure distribution on the wetted surface.

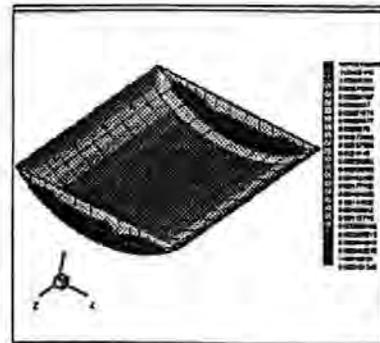


Fig. 26. Predicted normal velocity distribution on the wetted surface.

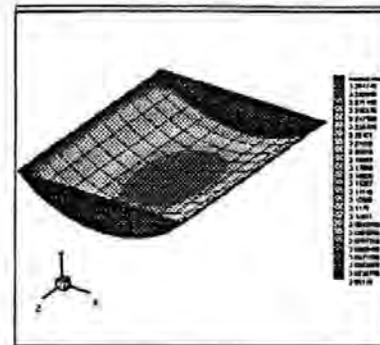


Fig. 27. Predicted sound intensity distribution on the wetted surface.

was floating on the surface center of a rectangular water tank of length 4.2m, breadth 3.6m and water depth 2.4m, the motor ran at 1800 rpm. The predicted distributions of pressure normal velocity, and sound intensity on the wetted surface are shown in Figs. 25-27. The prediction of underwater sound pressure on two symmetrical planes in the fluid domain and a horizontal plane at a depth 1.5m are compared with the measured sound pressure level in Figs. 28-30.

### CONCLUDING REMARKS

From the analysis and measurement works and results, it is concluded that:

1. Four-pole parameter theory for the transfer mobility analysis has validated to be credible.
2. Resilient mounts have the effects to reduce transfer mobility, vibratory velocity and vibration power by an amount of 5-20 dB, 2-5 dB and 10-40% respectively.
3. The entrained mass of water apparently reduced the transfer mobility by an amount of 5-10 dB in the lower frequency range, i.e., below 1000Hz.

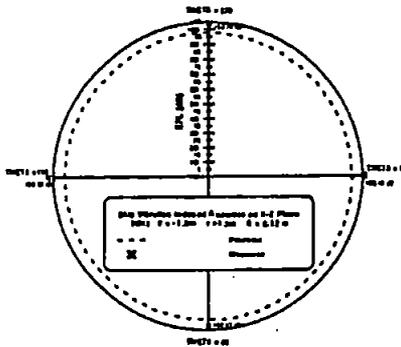


Fig. 28. Machine vibration induced underwater sound radiation at 30Hz on X-Z plane.

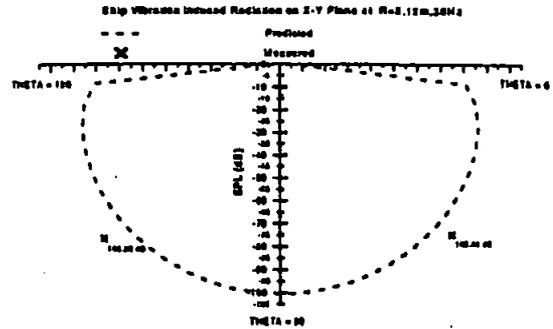


Fig. 30. Machine vibration induced underwater sound radiation at 30Hz at X-Y plane.

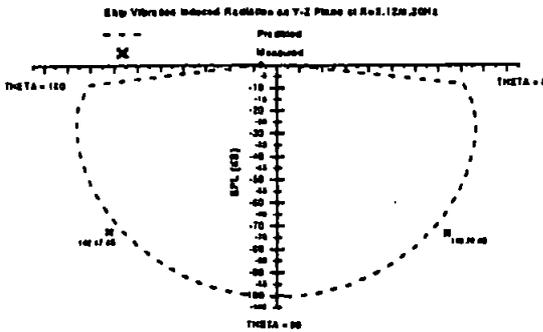


Fig. 29. Machine vibration induced underwater sound radiation at 30Hz on Y-Z plane.

4. The established FEM/BEM model for structure-borne and fluid-borne sound propagation analysis has validated to be precise in comparing with the analytical solution submerged spherical shell subjected to a uniformly harmonic pressure excitation. In the case of the scaled model of engine room, there are some differences and augmentation of the far field sound pressure level between measurement and prediction. This augmentation is due to the reflection effect of the tank wall.

**ACKNOWLEDGEMENTS**

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## 機械振動引發之水下聲音輻射

王偉輝

國立台灣海洋大學系統工程暨造船學系教授

劉嘉洪

基隆港務局舵政組技術課技士

羅伯沙頓

英國普利茅斯大學機械與輪機工程學系資深講師

班道卜生

英國丹佛皇家造船廠設計部總工程師

摘要

本文以偶合的有限元素法/邊界元素法，來處理結構噪音傳播與水中噪音輻射中彈性結構物與流體之互制問題。為改進船舶之適居性與隱匿性，各式船舶均需要靜音之條件，其中由船體振動所輻射出之空氣與水中音是主要的音源成分，因此結構音特性之估算在設計一艘安靜船是很重要的一件事情。利用本文之方法對船體承受激振力後濕表面之壓力分佈及各部振動反應可以分析，進一步亦可評估探討船體表面之複數功率、輻射功率及阻抗功率。

文中舉兩個數值實例，一是浸在水中的球殼受均勻簡諧內壓，及另一浮於水面之機艙結構模型受馬達激振，分別計算其表面聲壓、法向速度、表面複數功率、輻射功率、阻抗功率、表面振動位移響應及在遠場的水中輻射聲型，將此數值解與實驗量測值比較，顯示建立之預估模式可信。另者，對主推進引擎加裝彈性墊對水下噪音輻射之降低效果，亦加以探討。

## **Appendix E**

### **FORTRAN Program for Structure-Borne Noise Propagation Analysis**

PROGRAM MAIN

```

C
C PURPOSE:THIS PROGRAM IS DEVELOPED TO PREDICT
C PROPAGATION OF STRUCTURE-BORNE NOISE
C NW IS FREQUENCY
C A_M IS MASS
C DB IS THE RIGID OF THE BENDING
C A_K IS K
C N_FORC(NT,3) IS THE NUMBER OF THE FORCE
C N_DISP(NT,2) IS THE NUMBER OF THE DISPLACE
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (NT=100,CI=(0.0,1.0),CJ=(1.0,0.0))
DIMENSION A_NODE(NT,4),N_ELEM(NT,5),CAHM(NT,NT),CHR1(24,24)
+K(4),N_FORC(NT,3),N_DISP(NT,3),NIN(NT)
C OPEN(17,FILE='TEST_1.TXT')
OPEN(18,FILE='T33.TXT')
C OPEN ELEM,A_NODE FILES
OPEN(12,FILE='INPUT.TXT')
READ(12,*) NE,NN,E,U,RO,TH,N_FOR,N_DIS
CLOSE(12)
OPEN(13,FILE='ELEM.TXT')
OPEN(14,FILE='NODE.TXT')
DO 10 I=1,NE
READ (13,*) N_ELEM(I,1),N_ELEM(I,2),
+N_ELEM(I,3),N_ELEM(I,4),N_ELEM(I,5)
10 CONTINUE
DO 20 I=1,NN
20 READ(14,*) A_NODE(I,1),A_NODE(I,2),A_NODE(I,3),A_NODE(I,4)
CLOSE(13)
CLOSE(14)
C READ THE BOUNDARY CONDITION
C N_ALL IS THE NUMBER OF ALL NODES
C N_UKN IS THE NUMBER OF THE UNKNOW DEGREE OF FREEDOM
C NIN IS THE INDEX AFTER CHANGING THE LOCATION
OPEN(15,FILE='FORCE.TXT')
OPEN(16,FILE='DISP.TXT')
DO 50 I=1,N_FOR
50 READ(15,*) N_FORC(I,1),N_FORC(I,2),N_FORC(I,3)
DO 51 I=1,N_DIS
51 READ(16,*) N_DISP(I,1),N_DISP(I,2),N_DISP(I,3)
CLOSE(15)
CLOSE(16)
N_ALL = NN*6
N_UKN = N_ALL-N_DIS
CALL NINDEX(NT,N_DISP,N_DIS,N_ALL,N_UKN,NIN)
W = 1
DO 500 III=1,1000
A_M = RO*TH
DB = E*TH**3/(12*(1-U**2))
DD = E/(1-U**2)
A_K = (W**2*A_M/DB)**0.25
A_K1= W/SQRT(DD/RO)
DO 21 J=1,N_ALL
DO 21 I=1,N_ALL
21 CAHM(I,J)=(0.0,0.0)
DO 30 L=1,NE
DO 22 J=1,4
K(J)=N_ELEM(L,J+1)
22 CONTINUE
CALL HMATR(W,DB,DD,A_K,A_K1,TH,RO,A_NODE,N_ELEM,CHR1,NT,L)
C ASSEMBLE THE CHR1 MATRIX

```

```

DO 40 JJ=1,4
DO 40 KK=1,4
DO 40 I=1,6
DO 40 J=1,6
CAHM(((K(JJ)-1)*6+I),((K(KK)-1)*6+J))
+= CAHM(((K(JJ)-1)*6+I),((K(KK)-1)*6+J))
+ +CHR1((I+(JJ-1)*6),(J+(KK-1)*6))
40 CONTINUE
30 CONTINUE
C PARTITION CAHM
CALL PARTTT(W,N_FOR,N_DIS,N_UKN,N_ALL,NIN,CAHM,NT,N_FORC,N_DISP)
W=W+1
500 CONTINUE
C CLOSE(17)
CLOSE(18)
STOP
END

```

```

SUBROUTINE NINDEX(NT,N_DISP,N_DIS,N_ALL,N_UKN,NIN)
C THE SUBROUTINE THE INDEX OF THE BOUNDARY CONDITION AFTER
C REARRANGEMENT
C N_DISP IS THE LOCATION OF THE DISPLACE
C N_KN IS THE LOCATION OF THE DIMENSION
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
DIMENSION N_DISP(NT,3),N_KN(N_DIS),N_UN(N_ALL),NIN(NT)
DO 10 I=1,N_DIS
N_KN(I)=(N_DISP(I,1)-1)*6+N_DISP(I,2)
10 CONTINUE
C I SET THE N_UN(I)=0
DO 20 I=1,N_ALL
N_UN(I)=I
DO 30 K=1,N_DIS
IF (I .EQ. N_KN(K)) THEN
N_UN(I)=0
END IF
30 CONTINUE
20 CONTINUE
DO 40 K=(N_ALL-1),1,-1
DO 50 J=1,K
IF(N_UN(J) .GT. N_UN(J+1)) THEN
M = N_UN(J)
N_UN(J) = N_UN(J+1)
N_UN(J+1) = M
END IF
50 CONTINUE
40 CONTINUE
DO 60 I=1,N_UKN
N_UN(I)=N_UN(I+N_DIS)
60 CONTINUE
DO 61 I=1,N_DIS
NIN(I)=N_KN(I)
DO 62 I=1,N_UKN
62 NIN(I+N_DIS)=N_UN(I)
RETURN
END

```

```

SUBROUTINE HMATR(W,DB,DD,A_K,A_K1,TH,RO,TN,NTE,CHR1,NT,L)
C THIS SUBROUTINE IS MOBILITY MATRIX OF FINITE RECTANGULAR

```

```

C      K(4) COULD FIND NODE NUM.
C      T_X,T_Y,T_Z ARE THE GLOBAL COOR.
C      R12 ,R13 AND R14 ARE THE VECTOR FROM 1 TO 2,3 AND 4.
C      RCROS IS R12 X R13.
C      UNIT_X,UNIT_Y,UNIT_Z ARE THE LOCAL UNIT VECTOR.
C      X(I,J) AND Y(I,Y) IS THE MAGNITUDE OF THE UNIT_X AND UNIT_Y
C      FROM NODE I TO NODE J.
C      R(I,J) IS THE DISTANCE FROM NODE I TO NODE J.
C      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
C      IMPLICIT COMPLEX*16 (C,Z)
C      PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0),A=1.0E-5,AMP=1.0,
+PI=3.141592653589793D0)
C      DIMENSION NTE(NT,5),TN(NT,4),CHR(24,24),CHR1(24,24)
+ ,CHL(24,24),CHLB(24,24),CHLL(24,24),CHL1(12,12),CHL2(8,8),K(4)
+ ,T_X(7),T_Y(7),T_Z(7),R12(3),R13(3),R14(3),R(7,4),X(7,4),Y(7,4)
+ ,UNIT_X(3),UNIT_Y(3),UNIT_Z(3),RCROS(3),TL(24,24),CTL(24,24)
+ ,CTTL(24,24)
+ ,CHZ1(2,2),CHZ2(2,2),CHZ1I(2,2),CHZ2I(2,2),CHAB(2,2),CHEF(2,2)
+ ,CHAB1(2,2),CHEF1(2,2)
+ ,CHL2A(8,8),CHL2B(8,8),CHL2C(8,8),IN(8)
C      DO 3 J=1,12
C      DO 3 I=1,12
3     CHL1(I,J)=(0.0,0.0)
C      DO 4 J=1,8
C      DO 4 I=1,8
4     CHL2(I,J)=(0.0,0.0)
C      DO 1 I=1,3
C      RCROS(I)=0
C      UNIT_X(I)=0
C      UNIT_Y(I)=0
1     UNIT_Z(I)=0
C      DO 2 I=1,7
C      DO 2 J=1,4
C      R(I,J)=0
C      X(I,J)=0
2     Y(I,J)=0
C      CALL THE ROTATION TRANSFORMATION MATRIX [T]
C      CTL=TL*i
C      CTTL=CTL TRANSPOSE CTL
C      CALL TRANS(TN,NTE,TL,NT,L)
C      DO 9 I=1,24
C      DO 9 J=1,24
C      CHR1(I,J)=(0.0,0.0)
C      CTL(I,J)=(0.0,0.0)
9     CTTL(I,J)=(0.0,0.0)
C      DO 10 I=1,24
C      DO 10 J=1,24
C      CTL(I,J)=CJ*TL(I,J)
10    CONTINUE
C      CALL TMPY(CTL,CTTL,24,24,24,24)
C      DO 20 J=1,4
C      K(J)=NTE(L,J+1)
20    CONTINUE
C      DO 30 I=1,4
C      T_X(I)=TN(K(I),2)
C      T_Y(I)=TN(K(I),3)
C      T_Z(I)=TN(K(I),4)
30    CONTINUE
C      CALCULATE THE DISTANCE R(I,J)
C      DO 31 I=1,4
C      DO 31 J=1,4
C      D_X=T_X(I)-T_X(J)

```

```

D_Y=T_Y(I)-T_Y(J)
D_Z=T_Z(I)-T_Z(J)
R(I,J)=SQRT(D_X**2+D_Y**2+D_Z**2)
31 CONTINUE
DO 60 I=1,3
T_X(I+4)=T_X(I)
T_Y(I+4)=T_Y(I)
T_Z(I+4)=T_Z(I)
60 CONTINUE
C FIND THE VECTOR X AND Y
DO 70 ID=0,3
R12(1)=T_X(2+ID)-T_X(1+ID)
R12(2)=T_Y(2+ID)-T_Y(1+ID)
R12(3)=T_Z(2+ID)-T_Z(1+ID)
R13(1)=T_X(3+ID)-T_X(1+ID)
R13(2)=T_Y(3+ID)-T_Y(1+ID)
R13(3)=T_Z(3+ID)-T_Z(1+ID)
R14(1)=T_X(4+ID)-T_X(1+ID)
R14(2)=T_Y(4+ID)-T_Y(1+ID)
R14(3)=T_Z(4+ID)-T_Z(1+ID)
RCROS(1)=R12(2)*R13(3)-R12(3)*R13(2)
RCROS(2)=R12(3)*R13(1)-R12(1)*R13(3)
RCROS(3)=R12(1)*R13(2)-R12(2)*R13(1)
AM_R12=SQRT(R12(1)**2+R12(2)**2+R12(3)**2)
AM_R13=SQRT(R13(1)**2+R13(2)**2+R13(3)**2)
AM_RCR=SQRT(RCROS(1)**2+RCROS(2)**2+RCROS(3)**2)
C CALCULATE THE UNIT_X,UNIT_Y AND UNIT_Z
DO 40 II=1,3
UNIT_X(II)=R12(II)/AM_R12
UNIT_Z(II)=RCROS(II)/AM_RCR
40 CONTINUE
UNIT_Y(1)=UNIT_Z(2)*UNIT_X(3)-UNIT_Z(3)*UNIT_X(2)
UNIT_Y(2)=UNIT_Z(3)*UNIT_X(1)-UNIT_Z(1)*UNIT_X(3)
UNIT_Y(3)=UNIT_Z(1)*UNIT_X(2)-UNIT_Z(2)*UNIT_X(1)
X(2+ID,1+ID)=UNIT_X(1)*R12(1)+UNIT_X(2)*R12(2)+UNIT_X(3)*R12(3)
X(3+ID,1+ID)=UNIT_X(1)*R13(1)+UNIT_X(2)*R13(2)+UNIT_X(3)*R13(3)
X(4+ID,1+ID)=UNIT_X(1)*R14(1)+UNIT_X(2)*R14(2)+UNIT_X(3)*R14(3)
Y(2+ID,1+ID)=UNIT_Y(1)*R12(1)+UNIT_Y(2)*R12(2)+UNIT_Y(3)*R12(3)
Y(3+ID,1+ID)=UNIT_Y(1)*R13(1)+UNIT_Y(2)*R13(2)+UNIT_Y(3)*R13(3)
Y(4+ID,1+ID)=UNIT_Y(1)*R14(1)+UNIT_Y(2)*R14(2)+UNIT_Y(3)*R14(3)
70 CONTINUE
X(1,2)=X(5,2)
X(1,3)=X(5,3)
X(2,3)=X(6,3)
X(1,4)=X(5,4)
X(2,4)=X(6,4)
X(3,4)=X(7,4)
Y(1,2)=Y(5,2)
Y(1,3)=Y(5,3)
Y(2,3)=Y(6,3)
Y(1,4)=Y(5,4)
Y(2,4)=Y(6,4)
Y(3,4)=Y(7,4)
C CHL1 IS THE MOBILITY MATRIX OF FINITE RECTANGULAR PLATE
C IN BENDING WAVE
DO 80 I=1,4
DO 80 J=1,4
IF(I.EQ.J) THEN
CHL1(I,J)=W/(2*DB*A_K**2)*CJ*AMP
ELSE
C_A=CJ*A_K*R(I,J)
C_B=-1*CI*A_K*R(I,J)

```

```

CALL HAN0(C_A,CHAN_A)
CALL HAN0(C_B,CHAN_B)
CHAN=CHAN_A-CHAN_B
CHL1(I,J)=W/(2*DB*A_K**2)*CHAN*AMP
END IF
80 CONTINUE
DO 90 I=5,12
DO 90 J=1,4
IF((I-J).EQ.4) THEN
CHL1(I,J)=0*CJ
ELSE IF((I-J).EQ.8) THEN
CHL1(I,J)=0*CJ
ELSE
IF((I.GT.4).AND.(I.LE.8)) THEN
II=I-4
CALL DHAN0(A_K,R(II,J),CDHANA)
CALL DHAN1(A_K,R(II,J),CDHANB)
CDHAN=CDHANA-CDHANB
CHL1(I,J)=W/(2*DB*A_K**2)*CDHAN*Y(II,J)/R(II,J)*AMP
ELSE
II=I-8
CALL DHAN0(A_K,R(II,J),CDHANA)
CALL DHAN1(A_K,R(II,J),CDHANB)
CDHAN=CDHANA-CDHANB
CHL1(I,J)=W/(2*DB*A_K**2)*CDHAN*X(II,J)/R(II,J)*AMP
END IF
END IF
90 CONTINUE
DO 100 I=1,4
DO 100 J=5,8
IF((J-I).EQ.4) THEN
CHL1(I,J)=0*CJ
ELSE
JJ=J-4
R1=DSQRT((Y(I,JJ)-A)**2+(X(I,JJ))**2)
R2=DSQRT((Y(I,JJ)+A)**2+(X(I,JJ))**2)
C_A1 =CJ*A_K*R1
C_A2 = -1*CI*A_K*R1
C_B1 =CJ*A_K*R2
C_B2 = -1*CI*A_K*R2
CALL HAN0(C_A1,CHANA1)
CALL HAN0(C_A2,CHANA2)
CALL HAN0(C_B1,CHANB1)
CALL HAN0(C_B2,CHANB2)
CHANA=CHANA1-CHANA2
CHANB=CHANB1-CHANB2
CHL1(I,J)=(0.5E+5)*W/(2*DB*A_K**2)*(CHANA-CHANB)*AMP
END IF
100 CONTINUE
RGAMA=1.781
DO 110 I=5,8
DO 110 J=5,8
IF((I-J).EQ.0) THEN
CHL1(I,J)=W/(4*DB)*(1-4*CI/PI*DLOG(RGAMA*A_K*A/2))*AMP
ELSE
II=I-4
JJ=J-4
R1=SQRT((Y(II,JJ)-A)**2+(X(II,JJ))**2)
R2=SQRT((Y(II,JJ)+A)**2+(X(II,JJ))**2)
CALL DHAN0(A_K,R1,CDHA0)
CALL DHAN1(A_K,R1,CDHA1)
CALL DHAN0(A_K,R2,CDHB0)

```

```

CALL DHAN1(A_K,R2,CDHB1)
CDHA=CDHA0-CDHA1
CDHB=CDHB0-CDHB1
CHL1(I,J)=(0.5E+5)*W/(2*DB*A_K**2)*(CDHA*(Y(II,JJ)-A)/R1-
+CDHB*(Y(II,JJ)+A)/R2)*AMP
END IF
110 CONTINUE
DO 120 I=9,12
DO 120 J=5,8
IF((I-J).EQ.4) THEN
CHL1(I,J)=0*CJ
ELSE
II=I-8
JJ=J-4
R1=SQRT((Y(II,JJ)-A)**2+(X(II,JJ))**2)
R2=SQRT((Y(II,JJ)+A)**2+(X(II,JJ))**2)
CALL DHAN0(A_K,R1,CDHA0)
CALL DHAN1(A_K,R1,CDHA1)
CALL DHAN0(A_K,R2,CDHB0)
CALL DHAN1(A_K,R2,CDHB1)
CDHA=CDHA0-CDHA1
CDHB=CDHB0-CDHB1
CHL1(I,J)=(0.5E+5)*W/(2*DB*A_K**2)*(CDHA*X(II,JJ)/R1-
+CDHB*X(II,JJ)/R2)*AMP
END IF
120 CONTINUE
DO 130 I=1,4
DO 130 J=9,12
IF((J-I).EQ.8) THEN
CHL1(I,J)=0*CJ
ELSE
JJ=J-8
R1=SQRT(Y(I,JJ)**2+(X(I,JJ)+A)**2)
R2=SQRT(Y(I,JJ)**2+(X(I,JJ)-A)**2)
C_A1=CJ*A_K*R1
C_A2=-1*CI*A_K*R1
C_B1=CJ*A_K*R2
C_B2=-1*CI*A_K*R2
CALL HAN0(C_A1,CHANA1)
CALL HAN0(C_A2,CHANA2)
CALL HAN0(C_B1,CHANB1)
CALL HAN0(C_B2,CHANB2)
CHANA=CHANA1-CHANA2
CHANB=CHANB1-CHANB2
CHL1(I,J)=(0.5E+5)*W/(2*DB*A_K**2)*(CHANA-CHANB)*AMP
END IF
130 CONTINUE
DO 140 I=5,8
DO 140 J=9,12
IF((J-I).EQ.8) THEN
CHL1(I,J)=0*CJ
ELSE
II=I-4
JJ=J-8
R1=SQRT((Y(II,JJ))**2+(X(II,JJ)+A)**2)
R2=SQRT((Y(II,JJ))**2+(X(II,JJ)-A)**2)
CALL DHAN0(A_K,R1,CDHA0)
CALL DHAN1(A_K,R1,CDHA1)
CALL DHAN0(A_K,R2,CDHB0)
CALL DHAN1(A_K,R2,CDHB1)
CDHA=CDHA0-CDHA1
CDHB=CDHB0-CDHB1

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```

      CHL1(I,J)=(0.5E+5)*W/(2*DB*A_K**2)*(CDHA*Y(IL,JJ)/R1-
+CDHB*Y(IL,JJ)/R2)*AMP
      END IF
140  CONTINUE
      DO 150 I=9,12
      DO 150 J=9,12
      IF((I-J).EQ.0) THEN
      CHL1(I,J)=W/(4*DB)*(1-4*CI/PI*DLOG(RGAMA*A_K*A/2))*AMP
      ELSE
      II=I-8
      JJ=J-8
      R1=SQRT((Y(IL,JJ))**2+(X(IL,JJ)+A)**2)
      R2=SQRT((Y(IL,JJ))**2+(X(IL,JJ)-A)**2)
      CALL DHAN0(A_K,R1,CDHA0)
      CALL DHAN1(A_K,R1,CDHA1)
      CALL DHAN0(A_K,R2,CDHB0)
      CALL DHAN1(A_K,R2,CDHB1)
      CDHA=CDHA0-CDHA1
      CDHB=CDHB0-CDHB1
      CHL1(I,J)=(0.5E+5)*W/(2*DB*A_K**2)*(CDHA*(X(IL,JJ)+A)/R1-
+CDHB*(X(IL,JJ)-A)/R2)*AMP
      END IF
150  CONTINUE
C     CHL2 IS THE MOBILITY MATRIX OF FINITE RECTANGULAR PLATE
C     IN LONGITUDINAL WAVE
C     Z1=SQRT(D*RO)*S1
C     Z2=SQRT(D*RO)*S2
      DIS_A=R(2,1)
      DIS_B=R(4,1)
      AZ1=SQRT(DD*RO)*(0.5*DIS_B*TH)
      AZ2=SQRT(DD*RO)*(0.5*DIS_A*TH)
      CHZ1(1,1)=AZ1*CJ
      CHZ1(1,2)=-1*AZ1*CJ
      CHZ1(2,1)=AZ1*CDEXP(-1*CI*A_K1*DIS_A)
      CHZ1(2,2)=-1*AZ1*CDEXP(CI*A_K1*DIS_A)
      CHZ2(1,1)=AZ2*CJ
      CHZ2(1,2)=-1*AZ2*CJ
      CHZ2(2,1)=AZ2*CDEXP(-1*CI*A_K1*DIS_B)
      CHZ2(2,2)=-1*AZ2*CDEXP(CI*A_K1*DIS_B)
      CALL DLINCG(2,CHZ1,2,CHZ1I,2)
      CALL DLINCG(2,CHZ2,2,CHZ2I,2)
      CHAB1(1,1)=1*CJ
      CHAB1(1,2)=1*CJ
      CHAB1(2,1)=CDEXP(-1*CI*A_K1*DIS_A)
      CHAB1(2,2)=CDEXP(CI*A_K1*DIS_A)
      CHEF1(1,1)=1*CJ
      CHEF1(1,2)=1*CJ
      CHEF1(2,1)=CDEXP(-1*CI*A_K1*DIS_B)
      CHEF1(2,2)=CDEXP(CI*A_K1*DIS_B)
      CALL AMPY(CHAB1,CHZ1I,CHAB,2,2,2,2,2,2)
      CALL AMPY(CHEF1,CHZ2I,CHEF,2,2,2,2,2,2)
      DO 160 I=1,8
      DO 160 J=1,8
160  CHL2(I,J)=(0.0,0.0)
      CHL2(1,1)=CHAB(1,1)*AMP
      CHL2(1,2)=CHAB(1,2)*AMP
      CHL2(2,1)=CHAB(2,1)*AMP
      CHL2(2,2)=CHAB(2,2)*AMP
      CHL2(3,3)=CHAB(2,1)*AMP
      CHL2(3,4)=CHAB(2,2)*AMP
      CHL2(4,3)=CHAB(1,1)*AMP
      CHL2(4,4)=CHAB(1,2)*AMP

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```

CHL2(5,5)=CHEF(1,1)*AMP
CHL2(5,8)=CHEF(1,2)*AMP
CHL2(8,5)=CHEF(2,1)*AMP
CHL2(8,8)=CHEF(2,2)*AMP
CHL2(6,6)=CHEF(1,1)*AMP
CHL2(6,7)=CHEF(1,2)*AMP
CHL2(7,6)=CHEF(2,1)*AMP
CHL2(7,7)=CHEF(2,2)*AMP
C   CHANGE THE COOR.
C   1 -> 1 , 2 -> 6 , 3 -> 3
C   4 -> 8 , 5 -> 5 , 6 -> 2
C   7 -> 7 , 8 -> 4
C   IN(1)=1
C   IN(2)=6
C   IN(3)=3
C   IN(4)=8
C   IN(5)=5
C   IN(6)=2
C   IN(7)=7
C   IN(8)=4
C   DO 161 I=1,8
C   CHL2A(3,I)=-1.0*CHL2(3,I)
C   CHL2A(4,I)=-1.0*CHL2(4,I)
C   CHL2A(6,I)=-1.0*CHL2(6,I)
C161 CHL2A(7,I)=-1.0*CHL2(7,I)
C   DO 162 I=1,8
C   CHL2B(I,3)=-1.0*CHL2A(I,3)
C   CHL2B(I,4)=-1.0*CHL2A(I,4)
C   CHL2B(I,6)=-1.0*CHL2A(I,6)
C162 CHL2B(I,7)=-1.0*CHL2A(I,7)
C   DO 163 I=1,8
C   DO 163 J=1,8
C163 CHL2C(I,J)=CHL2B(IN(I),J)
C   DO 164 I=1,8
C   DO 164 J=1,8
C164 CHL2(I,J)=CHL2C(I,IN(J))
C   ASSEMBLY OF MOBILITY MATRIX OF PLATE ELEMENT
      DO 170 I=1,24
      DO 170 J=1,24
      CHL(I,J)=(0.0,0.0)
170  CHL(I,J)=(0.0,0.0)
C   THIS IS LONGITUDINAL WAVE
      DO 180 I=1,8
      DO 180 J=1,8
180  CHL(I,J)=CHL2(I,J)
C   THIS IS BENDING WAVE
      DO 190 I=1,12
      DO 190 J=1,12
190  CHL(I+8,J+8)=CHL1(I,J)
C   GIVE A SMALL NUMBER 1.0E-20
      DO 191 I=21,24
191  CHL(I,I)=1.0E-20
C   CHANGE THE ADDRESS ACCORDING TO NODE 1->4
      DO 200 J=1,24
      CHLB(1,J)=CHL(1,J)
      CHLB(2,J)=CHL(5,J)
      CHLB(3,J)=CHL(9,J)
      CHLB(4,J)=CHL(13,J)
      CHLB(5,J)=CHL(17,J)
      CHLB(6,J)=CHL(21,J)
      CHLB(7,J)=CHL(2,J)
      CHLB(8,J)=CHL(6,J)

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CHLB(9,J)=CHL(10,J)
CHLB(10,J)=CHL(14,J)
CHLB(11,J)=CHL(18,J)
CHLB(12,J)=CHL(22,J)
CHLB(13,J)=CHL(3,J)
CHLB(14,J)=CHL(7,J)
CHLB(15,J)=CHL(11,J)
CHLB(16,J)=CHL(15,J)
CHLB(17,J)=CHL(19,J)
CHLB(18,J)=CHL(23,J)
CHLB(19,J)=CHL(4,J)
CHLB(20,J)=CHL(8,J)
CHLB(21,J)=CHL(12,J)
CHLB(22,J)=CHL(16,J)
CHLB(23,J)=CHL(20,J)
200 CHLB(24,J)=CHL(24,J)
DO 210 I=1,24
CHLL(I,1)=CHLB(I,1)
CHLL(I,2)=CHLB(I,5)
CHLL(I,3)=CHLB(I,9)
CHLL(I,4)=CHLB(I,13)
CHLL(I,5)=CHLB(I,17)
CHLL(I,6)=CHLB(I,21)
CHLL(I,7)=CHLB(I,2)
CHLL(I,8)=CHLB(I,6)
CHLL(I,9)=CHLB(I,10)
CHLL(I,10)=CHLB(I,14)
CHLL(I,11)=CHLB(I,18)
CHLL(I,12)=CHLB(I,22)
CHLL(I,13)=CHLB(I,3)
CHLL(I,14)=CHLB(I,7)
CHLL(I,15)=CHLB(I,11)
CHLL(I,16)=CHLB(I,15)
CHLL(I,17)=CHLB(I,19)
CHLL(I,18)=CHLB(I,23)
CHLL(I,19)=CHLB(I,4)
CHLL(I,20)=CHLB(I,8)
CHLL(I,21)=CHLB(I,12)
CHLL(I,22)=CHLB(I,16)
CHLL(I,23)=CHLB(I,20)
210 CHLL(I,24)=CHLB(I,24)
C   CHR1=[CTL] [CHR] [CTL]
CALL AMPY(CTTL,CHLL,CHR,24,24,24,24,24,24)
CALL AMPY(CHR,CTL,CHR1,24,24,24,24,24,24)
RETURN
END

```

```

SUBROUTINE HAN0(Z1,C_HAN)
C   THIS SUBROUTINE IS FOR HANKEL FUNCTION,
C   X IS INPUT REAL NUMBER
C   C_HAN IS OUTPUT COMPLEX NUMBER
C   H0(X)=J0(X)-J*Y0(X)
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0))
CALL CJY01(Z1,CBJ0,CDJ0,CBJ1,CDJ1,CBY0,CDY0,CBY1,CDY1)
C_HAN=CBJ0-CI*CBY0
RETURN
END

```



```

IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0),PI=3.141592653589793D0,
+ERR = 10E-15)
H1 = 0.0
H4 = 0.0
C ADD IS GAIN NUMBER
C IF ADD IS SMALLER THAN ERR
C THEN THE FUNCTION WILL EXIT DO-LOOP
C ERR IS DEFINED AS 10E-15
DO 10 I=1,100
G_I=I+1
CALL GAMMA(G_I,GA)
A=2.0**(I*2.0)
ADD=(-1)**I*(2*I)*(X)*(X*Y)**(2*I-1)/(A*GA**2)
H1 = H1 + ADD
IF (ABS(ADD) .LT. ERR) GOTO 40
10 CONTINUE
40 C_H1 =CJ*H1
Z = X*Y*CJ
CALL CJY01(Z,CBJ0,CDJ0,CBJ1,CDJ1,CBY0,CDY0,CBY1,CDY1)
C_H2 = -2*CI/PI*CBJ0/Y
C_H3 = -2*CI/PI*(DLOG(X*Y/2)+0.5772)*H1
DO 20 I=1,100
A_H = 0.0
DO 30 J=1,I
A_H=A_H + 1.0/J
30 CONTINUE
G_I=I+1
CALL GAMMA(G_I,GA)
ADD = (-1)**(I-1)*A_H*2*I*X*(X*Y)**(2*I-1)
+/(2.0**(2.0*I)*GA**2.0)
H4=H4+ ADD
IF (ABS(ADD) .LT. ERR) GOTO 50
20 CONTINUE
50 C_H4 = -2.0*CI/PI*H4
CDHAN0 = C_H1+C_H2+C_H3+C_H4
RETURN
END

```

```

SUBROUTINE DHANI(X,Y,CDHAN1)
C THIS IS THE DERIVATIVE OF hANKEL FUNCTION WITH
C RESPECT TO -JKR.
C X IS K
C Y IS R
C C_DHAN IS THE RESULT
C C_K =CI*X
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0),PI=3.141592653589793D0,
+ERR = 10E-15)
C_K = -1*CI*X
C_H1 = (0.0,0.0)
C_H4 = (0.0,0.0)
C ADD IS GAIN NUMBER
C IF ADD IS SMALLER THAN ERR
C THEN THE FUNCTION WILL EXIT DO-LOOP
C ERR IS DEFINED AS 10E-15
DO 10 I=1,100
G_I=I+1

```

```

CALL GAMMA(G_I,GA)
A = 2.0**(I*2.0)
ADD = (-1)**I*(2*I)*(C_K)*(C_K*Y)**(2*I-1)/(A*GA**2)
C_H1=C_H1+ ADD
IF (ABS(ADD) .LT. ERR) GOTO 40
10 CONTINUE
40 Z = C_K*Y*CJ
CALL CJY01(Z,CBJ0,CDJ0,CBJ1,CDJ1,CBY0,CDY0,CBY1,CDY1)
C_H2 = -2.0*CI/PI*CBJ0/Y
C_H3 = -2*CI/PI*(CDLOG(C_K*Y/2)+0.5772156649)*C_H1
DO 20 I=1,100
A_H = 0.0
DO 30 J=1,I
A_H= A_H + 1.0/J
30 CONTINUE
G_I=I+1
CALL GAMMA(G_I,GA)
ADD= (-1)**(I-1)*A_H*2*I*C_K*(C_K*Y)**(2*I-1)
+/(2.0**(2.0*I)*GA**2.0)
C_H4 = C_H4 + ADD
IF (ABS(ADD) .LT. ERR) GOTO 50
20 CONTINUE
50 C_H4 = -2.0*CI/PI*C_H4
CDHAN1 = C_H1 + C_H2 + C_H3 + C_H4
RETURN
END

```

```

SUBROUTINE TRANS(TN,NTE,TL,NT,L)
C THIS PROGRAM TRANSFER LOCAL COOR. TO GLOBAL COOR.
C M(4) COULD FIND NODE NUM.
C T_X,T_Y,T_Z ARE THE GLOBAL COOR.
C R12,R13 ARE THE VECTOR FROM 1 TO 2 OR 3
C RCROS IS R12 X R13
C UNIT_X,UNIT_Y,UNIT_Z ARE THE LOCAL UNIT VECTOR
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
DIMENSION NTE(NT,5),TN(NT,4),TL(24,24),K(4),T_X(6)
+,T_Y(6),T_Z(6),R12(3),R13(3),UNIT_X(3),UNIT_Y(3),
+UNIT_Z(3),RCROS(3)
DO 1 J=1,24
DO 1 I=1,24
1 TL(I,J)=0
DO 20 J=1,4
K(J)=NTE(L,J+1)
20 CONTINUE
DO 30 I=1,4
T_X(I)=TN(K(I),2)
T_Y(I)=TN(K(I),3)
T_Z(I)=TN(K(I),4)
30 CONTINUE
DO 60 I=1,2
T_X(I+4)=T_X(I)
T_Y(I+4)=T_Y(I)
T_Z(I+4)=T_Z(I)
60 CONTINUE
DO 70 ID=0,3
R12(1)=T_X(2+ID)-T_X(1+ID)
R12(2)=T_Y(2+ID)-T_Y(1+ID)
R12(3)=T_Z(2+ID)-T_Z(1+ID)
R13(1)=T_X(3+ID)-T_X(1+ID)

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```

R13(2)=T_Y(3+ID)-T_Y(1+ID)
R13(3)=T_Z(3+ID)-T_Z(1+ID)
RCROS(1)=R12(2)*R13(3)-R12(3)*R13(2)
RCROS(2)=R12(3)*R13(1)-R12(1)*R13(3)
RCROS(3)=R12(1)*R13(2)-R12(2)*R13(1)
AM_R12=SQRT(R12(1)**2+R12(2)**2+R12(3)**2)
AM_R13=SQRT(R13(1)**2+R13(2)**2+R13(3)**2)
AM_RCR=SQRT(RCROS(1)**2+RCROS(2)**2+RCROS(3)**2)
DO 40 II=1,3
UNIT_X(II)=R12(II)/AM_R12
UNIT_Z(II)=RCROS(II)/AM_RCR
40 CONTINUE
UNIT_Y(1)=UNIT_Z(2)*UNIT_X(3)-UNIT_Z(3)*UNIT_X(2)
UNIT_Y(2)=UNIT_Z(3)*UNIT_X(1)-UNIT_Z(1)*UNIT_X(3)
UNIT_Y(3)=UNIT_Z(1)*UNIT_X(2)-UNIT_Z(2)*UNIT_X(1)
DO 50 IA=1,3
TL(1+ID*6,IA+ID*6)=UNIT_X(IA)
TL(2+ID*6,IA+ID*6)=UNIT_Y(IA)
TL(3+ID*6,IA+ID*6)=UNIT_Z(IA)
TL(4+ID*6,IA+3+ID*6)=UNIT_X(IA)
TL(5+ID*6,IA+3+ID*6)=UNIT_Y(IA)
TL(6+ID*6,IA+3+ID*6)=UNIT_Z(IA)
50 CONTINUE
70 CONTINUE
RETURN
END

```

```

SUBROUTINE MOMENT_1(X,W,Y,CMO1)
C THIS IS THE FUNCTION (A-29).
C EXPANDING THE HANKEL FUNCTION H0(KR1) WITH RESPECT TO K(Y+A)
C X IS K
C Y IS R
C W IS X

```

```

IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0),PI=3.141592653589793D0,
+ERR = 10E-15)
C THIS IS (A-29)
A = 10E-5
R1 = SQRT((Y-A)**2+W**2)
R2 = SQRT((Y+A)**2+W**2)
CR11= X*R1*CJ
CR12= X*R1*(-1)*CI
CALL HAN0(CR11,CH11)
CALL HAN0(CR12,CH12)
CANS1=CH11-CH12
C THIS IS (A-29)
CX1 = -0.25-CI/(PI*2)
CX2 = CI/(2*PI)
R = 1.781
CMO1= 1.0+2*X**2*(Y-A)**2*(CX1+
+CX2*DLOG(0.5*R*X*(Y-A))-0.5*CX2*CDLOG(-1*CI))
RETURN
END

```

```

SUBROUTINE CJY01(Z,CBJ0,CDJ0,CBJ1,CDJ1,CBY0,CDY0,CBY1,CDY1)
C

```

C Purpose: Compute Bessel functions  $J_0(z)$ ,  $J_1(z)$ ,  $Y_0(z)$ ,  
 C  $Y_1(z)$ , and their derivatives for a complex  
 C argument  
 C Input :  $z$  — Complex argument  
 C Output: CBJ0 —  $J_0(z)$   
 C CDJ0 —  $J_0'(z)$   
 C CBJ1 —  $J_1(z)$   
 C CDJ1 —  $J_1'(z)$   
 C CBY0 —  $Y_0(z)$   
 C CDY0 —  $Y_0'(z)$   
 C CBY1 —  $Y_1(z)$   
 C CDY1 —  $Y_1'(z)$   
 C

---

```

IMPLICIT DOUBLE PRECISION (A,B,E,P,R,W)
IMPLICIT COMPLEX*16 (C,Z)
DIMENSION A(12),B(12),A1(12),B1(12)
PI=3.141592653589793D0
EL=0.5772156649015329D0
RP2=2.0D0/PI
CI=(0.0D0,1.0D0)
A0=CDABS(Z)
Z2=Z*Z
Z1=Z
IF (A0.EQ.0.0D0) THEN
  CBJ0=(1.0D0,0.0D0)
  CBJ1=(0.0D0,0.0D0)
  CDJ0=(0.0D0,0.0D0)
  CDJ1=(0.5D0,0.0D0)
  CBY0=(1.0D300,0.0D0)
  CBY1=(1.0D300,0.0D0)
  CDY0=(1.0D300,0.0D0)
  CDY1=(1.0D300,0.0D0)
  RETURN
ENDIF
IF (REAL(Z).LT.0.0) Z1=-Z
IF (A0.LE.12.0) THEN
  CBJ0=(1.0D0,0.0D0)
  CR=(1.0D0,0.0D0)
  DO 10 K=1,40
    CR=-0.25D0*CR*Z2/(K*K)
    CBJ0=CBJ0+CR
    IF (CDABS(CR).LT.CDABS(CBJ0)*1.0D-15) GO TO 15
10  CONTINUE
15  CBJ1=(1.0D0,0.0D0)
    CR=(1.0D0,0.0D0)
    DO 20 K=1,40
      CR=-0.25D0*CR*Z2/(K*(K+1.0D0))
      CBJ1=CBJ1+CR
      IF (CDABS(CR).LT.CDABS(CBJ1)*1.0D-15) GO TO 25
20  CONTINUE
25  CBJ1=0.5D0*Z1*CBJ1
    W0=0.0D0
    CR=(1.0D0,0.0D0)
    CS=(0.0D0,0.0D0)
    DO 30 K=1,40
      W0=W0+1.0D0/K
      CR=-0.25D0*CR/(K*K)*Z2
      CP=CR*W0
      CS=CS+CP
      IF (CDABS(CP).LT.CDABS(CS)*1.0D-15) GO TO 35
30  CONTINUE
35  CBY0=RP2*(CDLOG(Z1/2.0D0)+EL)*CBJ0-RP2*CS
  
```

```

W1=0.0D0
CR=(1.0D0,0.0D0)
CS=(1.0D0,0.0D0)
DO 40 K=1,40
W1=W1+1.0D0/K
CR=-0.25D0*CR/(K*(K+1))*Z2
CP=CR*(2.0D0*W1+1.0D0/(K+1.0D0))
CS=CS+CP
IF (CDABS(CP).LT.CDABS(CS)*1.0D-15) GO TO 45
40 CONTINUE
45 CBY1=RP2*((CDLOG(Z1/2.0D0)+EL)*CBJ1-1.0D0/Z1-.25D0*Z1*CS)
ELSE
DATA A/-.703125D-01,.112152099609375D+00,
& -.5725014209747314D+00,.6074042001273483D+01,
& -.1100171402692467D+03,.3038090510922384D+04,
& -.1188384262567832D+06,.6252951493434797D+07,
& -.4259392165047669D+09,.3646840080706556D+11,
& -.3833534661393944D+13,.4854014686852901D+15/
DATA B/.732421875D-01,-.2271080017089844D+00,
& .1727727502584457D+01,-.2438052969955606D+02,
& .5513358961220206D+03,-.1825775547429318D+05,
& .8328593040162893D+06,-.5006958953198893D+08,
& .3836255180230433D+10,-.3649010818849833D+12,
& .4218971570284096D+14,-.5827244631566907D+16/
DATA A1/.1171875D+00,-.144195556640625D+00,
& .6765925884246826D+00,-.6883914268109947D+01,
& .1215978918765359D+03,-.3302272294480852D+04,
& .1276412726461746D+06,-.6656367718817688D+07,
& .4502786003050393D+09,-.3833857520742790D+11,
& .4011838599133198D+13,-.5060568503314727D+15/
DATA B1/-.1025390625D+00,.2775764465332031D+00,
& -.1993531733751297D+01,.2724882731126854D+02,
& -.6038440767050702D+03,.1971837591223663D+05,
& -.8902978767070678D+06,.5310411010968522D+08,
& -.4043620325107754D+10,.3827011346598605D+12,
& -.4406481417852278D+14,.6065091351222699D+16/
K0=12
IF (A0.GE.35.0) K0=10
IF (A0.GE.50.0) K0=8
CT1=Z1-.25D0*PI
CP0=(1.0D0,0.0D0)
DO 50 K=1,K0
50 CP0=CP0+A(K)*Z1**(-2*K)
CQ0=-0.125D0/Z1
DO 55 K=1,K0
55 CQ0=CQ0+B(K)*Z1**(-2*K-1)
CU=CDSQRT(RP2/Z1)
CBJ0=CU*(CP0*CDCOS(CT1)-CQ0*CDSIN(CT1))
CBY0=CU*(CP0*CDSIN(CT1)+CQ0*CDCOS(CT1))
CT2=Z1-.75D0*PI
CP1=(1.0D0,0.0D0)
DO 60 K=1,K0
60 CP1=CP1+A1(K)*Z1**(-2*K)
CQ1=-0.375D0/Z1
DO 65 K=1,K0
65 CQ1=CQ1+B1(K)*Z1**(-2*K-1)
CBJ1=CU*(CP1*CDCOS(CT2)-CQ1*CDSIN(CT2))
CBY1=CU*(CP1*CDSIN(CT2)+CQ1*CDCOS(CT2))
ENDIF
IF (REAL(Z).LT.0.0) THEN
IF (DIMAG(Z).LT.0.0) CBY0=CBY0-2.0D0*CI*CBJ0
IF (DIMAG(Z).GT.0.0) CBY0=CBY0+2.0D0*CI*CBJ0

```

```

IF (DIMAG(Z).LT.0.0) CBY1=(CBY1-2.0D0*CI*CBJ1)
IF (DIMAG(Z).GT.0.0) CBY1=(CBY1+2.0D0*CI*CBJ1)
CBJ1=-CBJ1
ENDIF
CDJ0=-CBJ1
CDJ1=CBJ0-1.0D0/Z*CBJ1
CDY0=-CBY1
CDY1=CBY0-1.0D0/Z*CBY1
RETURN
END

```

```

SUBROUTINE JY01A(X,BJ0,DJ0,BJ1,DJ1,BY0,DY0,BY1,DY1)

```

```

C
C
C Purpose: Compute Bessel functions  $J_0(x)$ ,  $J_1(x)$ ,  $Y_0(x)$ ,
C            $Y_1(x)$ , and their derivatives
C Input:  x  --- Argument of  $J_n(x)$  &  $Y_n(x)$  ( x ?0 )
C Output: BJ0 ---  $J_0(x)$ 
C           DJ0 ---  $J_0'(x)$ 
C           BJ1 ---  $J_1(x)$ 
C           DJ1 ---  $J_1'(x)$ 
C           BY0 ---  $Y_0(x)$ 
C           DY0 ---  $Y_0'(x)$ 
C           BY1 ---  $Y_1(x)$ 
C           DY1 ---  $Y_1'(x)$ 
C

```

```

C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C DIMENSION A(12),B(12),A1(12),B1(12)
C PI=3.141592653589793D0
C RP2=0.63661977236758D0
C X2=X*X
C IF (X.EQ.0.0D0) THEN
C   BJ0=1.0D0
C   BJ1=0.0D0
C   DJ0=0.0D0
C   DJ1=0.5D0
C   BY0=-1.0D+300
C   BY1=-1.0D+300
C   DY0=1.0D+300
C   DY1=1.0D+300
C   RETURN
C ENDIF
C IF (X.LE.12.0D0) THEN
C   BJ0=1.0D0
C   R=1.0D0
C   DO 5 K=1,30
C     R=-0.25D0*R*X2/(K*K)
C     BJ0=BJ0+R
C   IF (DABS(R).LT.DABS(BJ0)*1.0D-15) GO TO 10
C 5  CONTINUE
C 10  BJ1=1.0D0
C     R=1.0D0
C     DO 15 K=1,30
C       R=-0.25D0*R*X2/(K*(K+1.0D0))
C       BJ1=BJ1+R
C     IF (DABS(R).LT.DABS(BJ1)*1.0D-15) GO TO 20
C 15  CONTINUE
C 20  BJ1=0.5D0*X*BJ1
C     EC=DLOG(X/2.0D0)+0.5772156649015329D0
C     CS0=0.0D0
C     W0=0.0D0
C     R0=1.0D0

```

```

DO 25 K=1,30
W0=W0+1.0D0/K
R0=-0.25D0*R0/(K*K)*X2
R=R0*W0
CS0=CS0+R
IF (DABS(R).LT.DABS(CS0)*1.0D-15) GO TO 30
25 CONTINUE
30 BY0=RP2*(EC*BJ0-CS0)
CS1=1.0D0
W1=0.0D0
R1=1.0D0
DO 35 K=1,30
W1=W1+1.0D0/K
R1=-0.25D0*R1/(K*(K+1))*X2
R=R1*(2.0D0*W1+1.0D0/(K+1.0D0))
CS1=CS1+R
IF (DABS(R).LT.DABS(CS1)*1.0D-15) GO TO 40
35 CONTINUE
40 BY1=RP2*(EC*BJ1-1.0D0/X-0.25D0*X*CS1)
ELSE
DATA A/-.703125000000000D-01,.1121520996093750D+00,
& -.5725014209747314D+00,.6074042001273483D+01,
& -.1100171402692467D+03,.3038090510922384D+04,
& -.1188384262567832D+06,.6252951493434797D+07,
& -.4259392165047669D+09,.3646840080706556D+11,
& -.3833534661393944D+13,.4854014686852901D+15/
DATA B/.7324218750000000D-01,-.2271080017089844D+00,
& .1727727502584457D+01,-.2438052969955606D+02,
& .5513358961220206D+03,-.1825775547429318D+05,
& .8328593040162893D+06,-.5006958953198893D+08,
& .3836255180230433D+10,-.3649010818849833D+12,
& .4218971570284096D+14,-.5827244631566907D+16/
DATA A1/.1171875000000000D+00,-.1441955566406250D+00,
& .6765925884246826D+00,-.6883914268109947D+01,
& .1215978918765359D+03,-.3302272294480852D+04,
& .1276412726461746D+06,-.6656367718817688D+07,
& .4502786003050393D+09,-.3833857520742790D+11,
& .4011838599133198D+13,-.5060568503314727D+15/
DATA B1/-.1025390625000000D+00,.2775764465332031D+00,
& -.1993531733751297D+01,.2724882731126854D+02,
& -.6038440767050702D+03,.1971837591223663D+05,
& -.8902978767070678D+06,.5310411010968522D+08,
& -.4043620325107754D+10,.3827011346598605D+12,
& -.4406481417852278D+14,.6065091351222699D+16/
K0=12
IF (X.GE.35.0) K0=10
IF (X.GE.50.0) K0=8
T1=X-0.25D0*PI
P0=1.0D0
Q0=-0.125D0/X
DO 45 K=1,K0
P0=P0+A(K)*X**(-2*K)
45 Q0=Q0+B(K)*X**(-2*K-1)
CU=DSQRT(RP2/X)
BJ0=CU*(P0*DCOS(T1)-Q0*DSIN(T1))
BY0=CU*(P0*DSIN(T1)+Q0*DCOS(T1))
T2=X-0.75D0*PI
P1=1.0D0
Q1=0.375D0/X
DO 50 K=1,K0
P1=P1+A1(K)*X**(-2*K)
50 Q1=Q1+B1(K)*X**(-2*K-1)

```

```

CU=DSQRT(RP2/X)
BJ1=CU*(P1*DCOS(T2)-Q1*DSIN(T2))
BY1=CU*(P1*DSIN(T2)+Q1*DCOS(T2))
ENDIF
DJ0=-BJ1
DJ1=BJ0-BJ1/X
DY0=-BY1
DY1=BY0-BY1/X
RETURN
END

```

```

SUBROUTINE PARTIT(W,N_FOR,N_DIS,N_UKN,N_ALL,NIN,CAHM,
+NT,N_FORC,N_DISP)
C THIS IS THE PARTITION OF THE MATRIX
C GET CH11,CH12,CH21,CH22
C CH22 IS THE INVERSE OF THE CH22
C CHA=CH11-CH12*INV(CH22)*CH21
C CHB=CH12*INV(CH22)
C CHC=-INV(CH22)*CH21
C CHD=INV(CH22)
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0))
DIMENSION CAHM(NT,NT),NIN(NT),C_AHM(N_ALL,N_ALL),N_IN(N_ALL),
+CCAHM1(N_ALL,N_ALL),CCAHM2(N_ALL,N_ALL),
+CH11(N_DIS,N_DIS),CH12(N_DIS,N_UKN),CH21(N_UKN,N_DIS),
+CH22(N_UKN,N_UKN),CH22I(N_UKN,N_UKN),
+CHA(N_DIS,N_DIS),CHB(N_DIS,N_UKN),CHC(N_UKN,N_DIS),
+CHD(N_UKN,N_UKN),
+CHA1(N_DIS,N_UKN),CHA2(N_DIS,N_DIS),CHC1(N_UKN,N_DIS),
+CAM(N_ALL,N_ALL),CAMI(N_ALL,N_ALL),
+N_FORC(NT,3),N_DISP(NT,3),A_NN(N_DIS),CA_NU(N_UKN),
+A_FN(N_UKN),CA_FU(N_DIS),A_NFN(N_ALL),A_F(N_ALL),
+CNODE(N_ALL),CFORC(N_ALL)
C SET C_AHM(N_ALL,N_ALL) IS THE SAME AS CAHM(1->N_ALL,1->N_ALL)
C SET N_IN(N_ALL) IS THE SAME AS NIN(1->N_ALL)
DO 10 I=1,N_ALL
N_IN(I) = NIN(I)
DO 10 J=1,N_ALL
10 C_AHM(I,J)= CAHM(I,J)
DO 20 I=1,N_ALL
DO 20 J=1,N_ALL
20 CCAHM1(I,J)=C_AHM(N_IN(I),J)
DO 30 I=1,N_ALL
DO 30 J=1,N_ALL
30 CCAHM2(I,J)=CCAHM1(I,N_IN(J))
C SET CH11,CH12,CH21,CH22
DO 40 I=1,N_DIS
DO 40 J=1,N_DIS
40 CH11(I,J)=CCAHM2(I,J)
DO 50 I=1,N_DIS
DO 50 J=1,N_UKN
50 CH12(I,J)=CCAHM2(I,N_DIS+J)
DO 60 I=1,N_UKN
DO 60 J=1,N_DIS
60 CH21(I,J)=CCAHM2(N_DIS+I,J)
DO 70 I=1,N_UKN
DO 70 J=1,N_UKN
70 CH22(I,J)=CCAHM2(N_DIS+I,N_DIS+J)
C CALCULATE CHA=CH11-CH12*INV(CH22)*CH21
C CHB=CH12*INV(CH22)
C CHC=-INV(CH22)*CH21

```

```

C   CHD=INV(CH22)
    CALL DLINCG(N_UKN,CH22,N_UKN,CH22I,N_UKN)
    CALL AMPY1(CH12,CH22I,CHA1,N_DIS,N_UKN,N_UKN)
    CALL AMPY1(CH12,CH22I,CHA1,N_DIS,N_UKN,N_UKN)
    CALL AMPY1(CHA1,CH21,CHA2,N_DIS,N_UKN,N_DIS)
    DO 80 I=1,N_DIS
    DO 80 J=1,N_DIS
80   CHA(L,J)=CH11(L,J)-CHA2(L,J)
    CALL AMPY1(CH12,CH22I,CHB,N_DIS,N_UKN,N_UKN)
    CALL AMPY1(CH22I,CH21,CHC1,N_UKN,N_UKN,N_DIS)
    DO 90 I=1,N_UKN
    DO 90 J=1,N_DIS
90   CHC(L,J)=-1*CHC1(L,J)
    DO 100 I=1,N_UKN
    DO 100 J=1,N_UKN
100  CHD(L,J)=CH22I(L,J)
C   ASSEMBLE CHA,CHB,CHC,CHD
    DO 110 I=1,N_ALL
    DO 110 J=1,N_ALL
110  CAM(L,J)=(0.0,0.0)
    DO 120 I=1,N_DIS
    DO 120 J=1,N_DIS
120  CAM(L,J)=CHA(L,J)
    DO 130 I=1,N_DIS
    DO 130 J=1,N_UKN
130  CAM(L,J+N_DIS)=CHB(L,J)
    DO 140 I=1,N_UKN
    DO 140 J=1,N_DIS
140  CAM(I+N_DIS,J)=CHC(L,J)
    DO 150 I=1,N_UKN
    DO 150 J=1,N_UKN
150  CAM(I+N_DIS,J+N_DIS)=CHD(L,J)
C   A_NN IS KNOWN NODE VALUE
C   A_FN IS KNOWN FORCE VALUE
C   A_NFN ASSEMBLE A_NN AND A_FN
    DO 160 I=1,N_DIS
160  A_NN(I)=N_DISP(I,3)
    DO 170 I=1,N_ALL
170  A_F(I)=0.0
    DO 180 I=1,N_FOR
180  A_F(6*(N_FORC(I,1)-1)+N_FORC(I,2))=N_FORC(I,3)
    DO 190 I=1,N_UKN
190  A_FN(I)=A_F(N_IN(N_DIS+I))
    DO 200 I=1,N_DIS
200  A_NFN(I)=A_NN(I)
    DO 210 I=1,N_UKN
210  A_NFN(I+N_DIS)=A_FN(I)
    CALL DLINCG(N_ALL,CAM,N_ALL,CAMI,N_ALL)
C   CALCULATE THE UNKNOWN VALUE A_FU,A_NU
    DO 220 I=1,N_DIS
220  CA_FU(I)=(0.0,0.0)
    DO 230 I=1,N_UKN
230  CA_NU(I)=(0.0,0.0)
    DO 240 I=1,N_DIS
    DO 240 J=1,N_ALL
240  CA_FU(I)=CA_FU(I)+CAMI(I,J)*A_NFN(J)
    DO 250 I=1,N_UKN
    DO 250 J=1,N_ALL
250  CA_NU(I)=CA_NU(I)+CAMI(I+N_DIS,J)*A_NFN(J)
C   THE SOLUTION OF THE NODE AND THE FORCE ARE
C   CNODE AND CFORC
    DO 260 I=1,N_ALL
    CFORC(I)=(0.0,0.0)

```

```

260  CNODE(I)=(0.0,0.0)
      DO 270 I=1,N_DIS
      CFORC(N_IN(I))=CA_FU(I)
270  CNODE(N_IN(I))=A_NN(I)*CJ
      DO 280 I=1,N_UKN
      CFORC(N_IN(I+N_DIS))=A_FN(I)*CJ
280  CNODE(N_IN(I+N_DIS))=CA_NU(I)
      ANS=cdabs(CNODE(33))/1.0
      WRITE(18,*) W,ANS
      RETURN
      END

```

```

      SUBROUTINE AMPY1(CAA,CAB,CAC,N,M,L)
C    THIS IS MATRIX A * B = C
      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
      IMPLICIT COMPLEX*16 (C,Z)
      PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0))
      DIMENSION CAA(N,M),CAB(M,L),CAC(N,L)
      DO 9 J=1,L
      DO 9 I=1,N
      CAC(I,J)=0*CI
      DO 9 K=1,M
9    CAC(I,J)=CAC(I,J)+CAA(I,K)*CAB(K,J)
      RETURN
      END

```

```

      SUBROUTINE AMPY(CAA,CAB,CAC,N,M,L,NA,NB,NC)
C    THIS IS MATRIX A * B = C
      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
      IMPLICIT COMPLEX*16 (C,Z)
      PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0))
      DIMENSION CAA(NA,NA),CAB(NB,NB),CAC(NC,NC)
      DO 10 J=1,L
      DO 10 I=1,N
      CAC(I,J)=0*CI
      DO 10 K=1,M
10  CAC(I,J)=CAC(I,J)+CAA(I,K)*CAB(K,J)
      RETURN
      END

```

```

      SUBROUTINE TMPY(CA,CB,M,N,NA,NB)
C    THIS IS THE TRANSPOSE OF MATRIX
C    B=A'
      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
      IMPLICIT COMPLEX*16 (C,Z)
      DIMENSION CA(NA,NA),CB(NB,NB)
      DO 100 J=1,M
      DO 100 I=1,N
100  CB(I,J)=CA(J,I)
      RETURN
      END

```

```

PROGRAM MAIN
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)

```

```

IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (NT=6,C1=(0.0,1.0),CJ=(1.0,0.0))
DIMENSION A_NODE(NT,2),N_ELEM(NT,3),CAHM(NT,NT),CHR1(4,4)
+N_FORC(NT,3),C_FOR(NT),CAHMI(NT,NT)
PI=DACOS(-1.0d0)
OPEN(12,FILE='INPUT.TXT')
READ(12,*) NE,NN,E,U,RO,THO,THI,N_FOR
CLOSE(12)
OPEN(13,FILE='ELEM.TXT')
OPEN(14,FILE='NODE.TXT')
OPEN(118,FILE='N3.DAT')
OPEN(119,FILE='N4.DAT')
DO 10 I=1,NE
10 READ (13,*) N_ELEM(I,1),N_ELEM(I,2),N_ELEM(I,3)
DO 20 I=1,NN
20 READ(14,*) A_NODE(I,1),A_NODE(I,2)
CLOSE(13)
CLOSE(14)
N_ALL = NN*2
A_L = A_NODE(2,2)-A_NODE(1,2)
A_I = 0.25*PI*((THO/2.0)**4-(THI/2.0)**4)
A_S = PI*((THO/2.0)**2-(THI/2.0)**2)
A_M = RO*A_S
A_B = E*A_I
C A_Z = (E*RO)**0.5*A_S
DO 49 I=1,N_ALL
49 C_FOR(I)=(0.0,0.0)
OPEN(15,FILE='FORCE.TXT')
DO 50 I=1,N_FOR
50 READ(15,*) N_FORC(I,1),N_FORC(I,2),N_FORC(I,3)
CLOSE(15)
C_FOR((N_FORC(1,1)-1)*2+N_FORC(1,2))=CJ*N_FORC(1,3)
F = 1.0
DO 500 III=1,800
W=2.0*PI*F
C A_G = E/(2*(1+U))
C A_KK = 1.18
C A_K = A_G*A_S/A_KK
A_K = (W**2*A_M/A_B)**0.25
DO 21 J=1,N_ALL
DO 21 I=1,N_ALL
21 CAHM(I,J)=(0.0D0,0.0D0)
DO 22 J=1,4
DO 22 I=1,4
22 CHR1(I,J)=(0.0D0,0.0D0)
CALL HMATR(A_B,A_K,A_L,CHR1)
DO 30 L=1,NE
C ASSEMBLE THE CHR1 MATRIX
DO 40 I=1,4
DO 40 J=1,4
CAHM(I+2*(L-1),J+2*(L-1))
+= CAHM(I+2*(L-1),J+2*(L-1))+CHR1(I,J)
40 CONTINUE
30 CONTINUE
CALL DLINCG(N_ALL,CAHM,N_ALL,CAHMI,N_ALL)
CALL SOLVE(F,N_ALL,CAHMI,C_FOR,NT)
F=F+1.0
500 CONTINUE
STOP
END

```

```

SUBROUTINE HMATR(A_B,A_K,A_L,CHR1)
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0D0),CJ=(1.0D0,0.0))
DIMENSION CHR1(4,4),CB1(4,4),CB(4,4),CB2(4,4),CB2I(4,4)
+,CB1I(4,4),CBT(4,4)
PI=DACOS(-1.0d0)
DO 10 I=1,4
DO 10 J=1,4
10  CHR1(I,J)=(0.0D0,0.0D0)
   B_SS=DSIN(A_K*A_L)
   B_LS=DSINH(A_K*A_L)
   B_SC=DCOS(A_K*A_L)
   B_LC=DCOSH(A_K*A_L)
   CB1(1,1)=0*CJ
   CB1(1,2)=1.0D0*CJ
   CB1(1,3)=0*CJ
   CB1(1,4)=1.0D0*CJ
   CB1(2,1)=A_K
   CB1(2,2)=0.0
   CB1(2,3)=A_K
   CB1(2,4)=0.0
   CB1(3,1)=B_SS
   CB1(3,2)=B_SC
   CB1(3,3)=B_LS
   CB1(3,4)=B_LC
   CB1(4,1)=A_K*B_SC
   CB1(4,2)=-1*A_K*B_SS
   CB1(4,3)=A_K*B_LC
   CB1(4,4)=A_K*B_LS
C   DO 20 J=1,4
C20  CB1(3,J)=-1.0*CB1(3,J)
C20  CB1(4,J)=-1.0*CB1(4,J)
   CB2(1,1)=-1*A_K**3
   CB2(1,2)=0.0
   CB2(1,3)=A_K**3
   CB2(1,4)=0.0
   CB2(2,1)=0.0
   CB2(2,2)=A_K**2
   CB2(2,3)=0.0
   CB2(2,4)=-1*A_K**2
   CB2(3,1)=A_K**3*B_SC
   CB2(3,2)=-1*A_K**3*B_SS
   CB2(3,3)=-1*A_K**3*B_LC
   CB2(3,4)=-1*A_K**3*B_LS
   CB2(4,1)=-1*A_K**2*B_SS
   CB2(4,2)=-1*A_K**2*B_SC
   CB2(4,3)=A_K**2*B_LS
   CB2(4,4)=A_K**2*B_LC
DO 25 I=1,4
DO 25 J=1,4
25  CB2(I,J)=A_B*CB2(I,J)
C   DO 29 J=1,4
C   CB2(3,J)=-1.0*CB2(3,J)
C29  CB2(4,J)=-1.0*CB2(4,J)
   CALL DLINCG(4,CB2,4,CB2I,4)
   CALL DLINCG(4,CB1,4,CB1I,4)
C   CALL cinvs(CB2,CB2I,4)
   CALL AMPY1(CB1,CB2I,CB,4,4,4)
   CALL AMPY1(CB2,CB1I,CBT,4,4,4)
C   DO 29 J=1,4

```

```

C   CB(3,J)=-1.0*CB(3,J)
C29  CB(4,J)=-1.0*CB(4,J)
      DO 50 I=1,4
      DO 50 J=1,4
50   CHR1(I,J)=CBT(I,J)
      RETURN
      END

```

```

      SUBROUTINE AMPY1(CAA,CAB,CAC,N,M,L)
C   THIS PROGRAM IS MATRIX A * B = C
      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
      IMPLICIT COMPLEX*16 (C,Z)
      PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0))
      DIMENSION CAA(N,M),CAB(M,L),CAC(N,L)
      DO 9 J=1,L
      DO 9 I=1,N
      CAC(I,J)=0*CI
      DO 9 K=1,M
9     CAC(I,J)=CAC(I,J)+CAA(I,K)*CAB(K,J)
      RETURN
      END

```

```

      SUBROUTINE AMPY(CAA,CAB,CAC,N,M,L,NA,NB,NC)
C   THIS PROGRAM IS MATRIX A * B = C
      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
      IMPLICIT COMPLEX*16 (C,Z)
      PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0))
      DIMENSION CAA(NA,NA),CAB(NB,NB),CAC(NC,NC)
      DO 10 J=1,L
      DO 10 I=1,N
      CAC(I,J)=0*CI
      DO 10 K=1,M
10    CAC(I,J)=CAC(I,J)+CAA(I,K)*CAB(K,J)
      RETURN
      END

```

```

      SUBROUTINE TMPY(CA,CB,M,N,NA,NB)
C   THIS IS FOR MATRIX TRANSPOSE
C   B=A'
      IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
      IMPLICIT COMPLEX*16 (C,Z)
      DIMENSION CA(NA,NA),CB(NB,NB)
      DO 100 J=1,M
      DO 100 I=1,N
100  CB(I,J)=CA(J,I)
      RETURN
      END

```

```

      SUBROUTINE CINVS(H,HINVS,N)
C   FIND INVERSE (HINVS) OF COMPLEX MATRIX H
      COMPLEX*16 H(N,N),HINVS(N,N),A(N,N),B(N,N),SUM
      CALL CSUBN(H,A,N)
      NM1=N-1
      DO 40 I=1,NM1
      SUM=(0.0,0.0)

```

```

DO 41 K=1,N
41 SUM=SUM+A(K,K)
SUM=SUM/I
DO 42 J=1,N
42 A(J,J)=A(J,J)-SUM
IF (I.EQ. NM1) CALL CSUBN(A,HINVS,N)
CALL CMPY(H,A,B,N)
40 CALL CSUBN(B,A,N)
DO 43 I=1,N
DO 43 J=1,N
43 HINVS(I,J)=HINVS(I,J)/A(1,1)
RETURN
END

```

```

SUBROUTINE CSUBN(A,B,N)
COMPLEX*16 A(N,N),B(N,N)
DO 50 I=1,N
DO 50 J=1,N
50 B(I,J)=A(I,J)
RETURN
END
C REAL MATRIX MULTIPLICATION:C=A*B
SUBROUTINE CMPY(A,B,C,N)
COMPLEX*16 A(N,N),B(N,N),C(N,N)
DO 60 I=1,N
DO 60 J=1,N
C(I,J)=0
DO 60 K=1,N
60 C(I,J)=C(I,J)+A(I,K)*B(K,J)
RETURN
END

```

```

SUBROUTINE SOLVE(F,N_ALL,CAHM,C_FOR,NT)
IMPLICIT DOUBLE PRECISION (A,B,D-H,O-Y)
IMPLICIT COMPLEX*16 (C,Z)
PARAMETER (CI=(0.0,1.0),CJ=(1.0,0.0),IPATH=1)
DIMENSION CAHM(NT,NT),C_FOR(NT),CA(N_ALL,N_ALL),
+CB(N_ALL),CX(N_ALL)
PI=DACOS(-1.0d0)
DO 9 I=1,N_ALL
9 CX(I)=0*CJ
DO 10 I=1,N_ALL
DO 10 J=1,N_ALL
10 CA(I,J)=CAHM(I,J)
DO 20 I=1,N_ALL
20 CB(I)=C_FOR(I)
C CALL DLSLCG (N_ALL,CA,N_ALL,CB,IPATH,CX)
DO 30 I=1,N_ALL
DO 30 J=1,N_ALL
30 CX(I)=CX(I)+CA(I,J)*CB(J)
ANS=CDABS(CX(29))
C ANS=CDABS(CX(8))
C WRITE(118,*) F,20*LOG10(CDABS(CX(29)/10E+20))
WRITE(118,*) F,ANS
C WRITE(119,*) F,20*LOG10(CDABS(CX(6)))
C WRITE(119,*) F,CDABS(CX(6))
WRITE(119,*) F,CDABS(CX(30))
RETURN
END

```