OPTIMAL WEATHER ROUTEING PROCEDURES FOR VESSELS ON TRANS-OCEANIC VOYAGES

SIMON CALVERT

A thesis submitted in partial fulfilment of the requirements of the Council for National Academic Awards for the degree of Doctor of Philosophy

March 1990

Polytechnic South West in collaboration with Oceanroutes (UK) Ltd and Oceanroutes Inc, USA

This copy of the thesis has been supplied on condition that anyone who consults it is understood to recognise that its copyright rests with its author and that no quotation from the thesis and no information derived from it may be published without the author's consent.
This thesis is dedicated to my parents.
CONTENTS.

Contents. (iii)
CNAADeclarations. Published papers. (viii)
Acknowledgements. (lx)
Nomenclature (x)
Abstract. (xvi)

1. Introduction.
   1.1 Introduction and Outline of the Chapters. 1
   1.2 The Advent of Ship Weather Routeing - Climatological Weather Routeing. 3
   1.3 Weather Bulletins and Forecast Weather Routeing. 4
   1.4 Automatic Weather Routeing. 7
   1.5 The Benefits of Weather Routeing. 8
   1.6 Automated Routeing Algorithms. 11
       1.6.1 Calculus of Variations or Extremals. 12
       1.6.2 Mathematical Heuristics. 12
       1.6.3 Isochronal and Modified Isochronal. 13
       1.6.4 Dynamic Programming (DP). 15

   2.1 On-Board Weather Routeing Systems. 18
   2.2 Automatic Weather Routeing Control. 21
       2.2.1 The Global (Entire Route) System. 22
       2.2.2 The Local (Route Segment) System. 23
   2.3 Data Storing and Transmission for an On-Board System. 24
   2.4 On-Board Motion Sensing and Prediction. 25
   2.5 Description of the Work in this Study and a Comparison to Previous Systems. 26

   3.1 Introduction. 30
   3.2 James Speed Loss Curves. 32
   3.3 Aertssen's Speed Loss Algorithm. 33
   3.4 Townsin et al Speed Loss Algorithm. 35
   3.5 Babbedge Speed Loss Algorithm. 38
       3.5.1 Effects of Variables on the Babbedge Algorithm. 39
       3.5.2 Powering From The Babbedge Equations. 41
   3.6 Semi-Empirical/Theoretical Ship Powering Algorithm. 42
   3.7 Calm Water Resistances. 46
       3.7.1 Frictional Resistance. 47
       3.7.2 Form Factor of Hull. 49
       3.7.3 Appendage Resistance. 50
       3.7.4 Wave Making Resistance. 51
       3.7.5 Additional Resistance Due to Bulbous Bow. 54
Chapter 3. The Theoretical Seakeeping Program - BRITSEA.

3.13.1 Ship Data Requirements.

3.13.2 Weight Distribution Data.

3.14 Combination of RAO Files to Sea Spectrum.

3.15 Britsea Output Files.

3.16 Critical Motions.

3.16.1 Deck Wetness.

3.16.2 Slamming.

3.16.3 Subjective Motion.

3.16.4 Propeller Emergence.

3.16.5 Roll Motion.

3.16.6 Master's Action.

3.17 Generated Databases of Critical Motions and Added Resistance from BRITSEA and CRISK.

3.18 Expansion of Database Results to Account For Wave Period.

3.19 Ship Powering.

3.19.1 Taylor Wake Fraction.

3.19.2 Thrust Deduction.

3.19.3 Hull Efficiency.

3.19.4 Relative-Rotative Efficiency of the Propeller.

3.19.5 Open Water Efficiency.

3.19.6 Powering Limitations.

3.20 Engine Characteristics.

3.20.1 Specific Fuel Consumption -(SFC).

3.20.2 Brake Power.

3.21 Semi Empirical/Theoretical Algorithms.

3.21.1 Interpolation of The Database.

3.21.2 Computation of Database Values.

3.21.3 Constant Speed Model.

3.21.4 Constant Power Model.

3.22 Chapter Summary.

Chapter 4. The Environmental Algorithm and Environmental Data Arrays.

4.1 Introduction.

4.2 Numerical Atmospheric Prediction Models.

4.3 Oceanographic Wave Prediction Models.

4.3.1 Significant Wave Height Empiricisms.
4.3.2 Spectral Wave Algorithms.

4.4 The Global Spectral Ocean Wave Model - (GSOWM).

4.4.1 GSOWM Environmental Data Points and Condensed data.

4.4.2 Validity of the GSOWM.

4.4.3 Study on the Quality/Reliability of the GSOWM Data.

4.5 GSOWM Data Sets for On-Board Optimum Ship Routeing.

4.6 Extensions of the GSOWM Data Sets.

4.6.1 Climatology.

4.6.2 Generation of +96 and +120 Hour Wave Fields from the +96 and +120 Hour GSOWM Wind Fields Using Simple Empiricisms.

4.6.3 Generation of +96 and +120 Hour Wave Fields From the +96 and +120 Hour GSOWM Wind Fields Using Simple Empiricisms with Fetch and Duration.

4.6.4 Extension of Data Beyond +120 Hours.

4.6.5 Generation of Extended Wave Parameters With Running Climatology.

4.6.6 Generation of Extended Wind and Wave Parameters from the ECMWF Digitised Pressure Arrays.

4.6.7 0000Z Forecast Series : Interpolation of a Cubic-Spline Fit.

4.6.8 Extension of Polynomial Curves Through the Data Time Series.

4.7 Interpolation of the Data Arrays Through Time and Space.

4.8 Chapter Summary.

5. The Optimisation Algorithm.

5.1 Introduction.

5.2 Statement of The Routeing Problem.

5.3 The Stage Variable.

5.3.1 Stage Variable - Time.

5.3.2 Stage Variable - Voyage Progress or Vessel Distance.

5.3.3 Further Computations from the Pre-Defined Grid Points.

5.4 The State Space and Computational Burden.

5.5 Objective Function and Recursive Algorithm Definition.

5.5.1 Stage Variable - Voyage Progress.

5.5.2 Stage Variable - Time.

5.6 Problems With Backwards Recursion.

5.7 Forwards Iteration.

5.7.1 Stage Variable - Voyage Progress.

5.7.2 Stage Variable - Time.

5.8 Stochastic Optimum Weather Routeing.

5.9 Dynamic Grid Systems.

5.9.1 Rectangular Grid Systems.

5.9.2 Spherical Grid Systems.

5.10 Grid Constraints.
5.10.1 Land or Geography of the Ocean Basin. 180
5.10.2 Allowable Ship Headings/Course Constraints (Controls). 181
5.11 Further Grid Considerations. 184
5.12 Route Evaluation Using varying Grid Types. 186
5.13 The Effect of The Size of The Positional State Discretisation (Stage and Lateral Positional State) Spacings. 188
5.14 Ship Control Constraints in the Operational Algorithm. 191
5.15 Positional State Indexing. 192
5.16 Time States. 193
5.16.1 Fixed Terminal Times. 194
5.16.2 Fixed Departure Time. 197
5.17 The Effect of Discretisation in The Time Domain (T-States). 197
5.18 The General Cost Function. 197

6.1 Introduction. 199
6.2 Time Optimisation Model. 199
6.3 Minimum Time Studies. 204
6.3.1 Routes with the Aertssen Algorithm. 206
6.3.2 Routes with the Townsin Algorithm. 210
6.3.3 Routes with the Babbedge Algorithm. 212
6.3.4 Routes with the Semi-Empirical/Theoretical Algorithm. 223
6.4 Chapter Conclusions. 227

7.1 Introduction. 230
7.2 Fuel Optimisation Model. 230
7.3 Minimum Fuel Studies. 234
7.3.1 Routes with the Babbedge Algorithm. 234
7.3.2 Routes with the Semi-Empirical/Theoretical Algorithm. 243
7.4 Chapter Conclusions. 250

8. The Minimum Costing Optimisation Model and Routeing Studies.
8.1 Introduction. 252
8.2 Minimum Cost Model 252
8.2.1 Cost Functions. 252
8.2.2 Port Costs. 256
8.2.3 Operating Costs. 257
8.3 Minimum Cost Studies. 259
8.3.1 Routes with the Babbedge Algorithm. 259
8.3.2 Routes with the Semi-Empirical/Theoretical Algorithm. 264
8.4 Chapter Conclusions. 269
9.1 Conclusions. 271
9.2 An On-Board Weather Routeing Model. 282
9.3 Recommendations for Further Work. 282


11. Appendices
A.1 OCL Containership Body Plans. DART ATLANTIC Dimensions. 294
A.2 Example Response Amplitude Operators. 296
A.3 Sulzer 1ORND90 Diesel Engine. 299
A.4 Comparison of plots of +96 and +120 hour Generated Wave Fields from Simple Empiricisms to Analysis. 300
A.5 Comparison of a Time Sequence of Generated Wave Fields from Simple Empiricisms to Analysis. 303
A.6 Comparison of Running Climatology Wave Fields to Analysis with contour diagrams. 305
A.7 Time Series Plots of Degredation of ECMWF Surface Pressure with contoured diagrams of the generated Wave Fields. 306
A.8 Comparison of Contoured GSOWM Analysis Wave Fields to Contoured Wave Fields Generated from the ECMWF Surface Pressure Fields. 309
A.9 Comparison Plots of Interpolated +84 and +108 Wave Fields to Analysis. 313
No part of this thesis has been submitted for any award or degree at any other institute.

Whilst registered as a candidate for the degree of Doctor of Philosophy, the author has not been a registered candidate for another award of the CNAA or of any other academic institute.

**PUBLISHED PAPERS.**


ACKNOWLEDGEMENTS.

I would like to express my gratitude to the following persons and organisations for their help and consideration in the production of this thesis.

Prof. R. H. Motte, Head of Marine Science and Technology, PSW, for his continual support and guidance and for his persistance during troubled times, acting as Director of Studies.

Dr. E. Deakins, PSW, for his encouragement and help, acting as supervisor.

Mr. J. R. Marshall, PSW and Dr. M. J. Dove, PSW for their advice, acting as supervisors.

Prof. D. H. Moreby, for his help and guidance on costing functions.

Dr. C. T. Stockel, for proof reading and useful preparation advice.

Dr. R. S. Burns, for his help.

Those staff of the Department of Marine Science and Technology, PSW, who gave advice in their relative fields and for proof reading the thesis.

Those staff at OCEANROUTES (UK) LTD, Scotland :-

Mr. J. Thomson, Mr. A. B. Webb, Mr. R. Keeler, for their help and for providing the means to collect environmental data.

Those staff at OCEANROUTES INC, CA :-

Mr. J. Slosar, Mr. N. Stevenson, Mr. B. Selfridge, Mr R. Fazal, and all those who contributed in our discussions during my visit. I would like to extend my thanks to OCEANROUTES for their support and for providing the environmental data on which this work is based.

To friends and family who have provided encouragement, and to Sarah, for her support and patience during the period of this research.

PSW - Polytechnic South West, Devon, England.
**NOMENCLATURE.**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMT</td>
<td>British Maritime Technology Ltd.</td>
</tr>
<tr>
<td>BN</td>
<td>Beaufort Number.</td>
</tr>
<tr>
<td>BSRA</td>
<td>British Ship Research Association, now known as BMT Ltd.</td>
</tr>
<tr>
<td>BRITSEA</td>
<td>Seakeeping computer algorithm developed by BMT Ltd.</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processor Unit.</td>
</tr>
<tr>
<td>CRISK</td>
<td>Critical motion computation program Dr. E. Deakins.</td>
</tr>
<tr>
<td>ECMWF</td>
<td>European Centre for Medium range Weather Forecasts.</td>
</tr>
<tr>
<td>FDS</td>
<td>Fully Developed Seastate.</td>
</tr>
<tr>
<td>FNOC</td>
<td>Fleet Numerical Oceanography Center.</td>
</tr>
<tr>
<td>FP</td>
<td>Forward Perpendicular of ship.</td>
</tr>
<tr>
<td>G</td>
<td>Data derived from the GSOWM.</td>
</tr>
<tr>
<td>GC</td>
<td>Great Circle.</td>
</tr>
<tr>
<td>GCR</td>
<td>Great Circle Route.</td>
</tr>
<tr>
<td>GSCLI</td>
<td>Global Surface Layer Interface model.</td>
</tr>
<tr>
<td>HWSR</td>
<td>Heavy Weather Ship Route.</td>
</tr>
<tr>
<td>INT</td>
<td>Data derived from an interpolation.</td>
</tr>
<tr>
<td>ITTC</td>
<td>International Towing Tank Conference.</td>
</tr>
<tr>
<td>LCG</td>
<td>Longitudinal Centre of Gravity.</td>
</tr>
<tr>
<td>LDR</td>
<td>Least Distance Route.</td>
</tr>
<tr>
<td>MCRt</td>
<td>Maximum Continuous Rating.</td>
</tr>
<tr>
<td>MCR</td>
<td>Minimum Cost optimisation Route.</td>
</tr>
<tr>
<td>MFR</td>
<td>Minimum Fuel optimisation Route.</td>
</tr>
<tr>
<td>MTR</td>
<td>Minimum Time optimisation Route.</td>
</tr>
<tr>
<td>NOGAPS</td>
<td>Navy Operational Global Atmospheric Prediction System.</td>
</tr>
<tr>
<td>NSMB</td>
<td>Netherlands Ship Model Basin.</td>
</tr>
<tr>
<td>NWP</td>
<td>Numerical Weather Prediction.</td>
</tr>
<tr>
<td>PC</td>
<td>Personal Computer.</td>
</tr>
<tr>
<td>PM</td>
<td>Pierson-Moskowitz. FDS Spectral seaway representation.</td>
</tr>
<tr>
<td>POL</td>
<td>Data derived from a polynomial extension.</td>
</tr>
<tr>
<td>QPC</td>
<td>Quasi-Propulsive Coefficient.</td>
</tr>
<tr>
<td>RAO</td>
<td>Response Amplitude Operator.</td>
</tr>
<tr>
<td>RC</td>
<td>Running Climatology, data extension method.</td>
</tr>
<tr>
<td>SFC</td>
<td>Specific Fuel Consumption.</td>
</tr>
<tr>
<td>SMB</td>
<td>Sverdrup-Munk-Bretschneider. Spectral seaway representation.</td>
</tr>
<tr>
<td>SOWM</td>
<td>Spectral Ocean Wave Model. Prior to the GSOWM.</td>
</tr>
<tr>
<td>SP</td>
<td>Environmental data derived from surface pressure.</td>
</tr>
<tr>
<td>SSR</td>
<td>Ship Survival Route.</td>
</tr>
<tr>
<td>TOSR</td>
<td>Tactical Ocean Ship Routeing.</td>
</tr>
<tr>
<td>TSEA</td>
<td>Trigger file for the BRITSEA seakeeping program.</td>
</tr>
<tr>
<td>VCG</td>
<td>Vertical Centre of Gravity.</td>
</tr>
<tr>
<td>WMO</td>
<td>World Meteorological Office.</td>
</tr>
<tr>
<td>ZERO</td>
<td>Zero data field extensions.</td>
</tr>
<tr>
<td>3-D</td>
<td>Three Dimensional.</td>
</tr>
</tbody>
</table>

(x)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Earth's radius.</td>
</tr>
<tr>
<td>ABT</td>
<td>Transverse, slice area of ship's bulb, m².</td>
</tr>
<tr>
<td>A_p/A_0</td>
<td>Propeller blade area ratio.</td>
</tr>
<tr>
<td>A_L</td>
<td>Lateral projected area of ship, m².</td>
</tr>
<tr>
<td>A_T</td>
<td>Area of the ship's transom, m².</td>
</tr>
<tr>
<td>A_TS</td>
<td>Transverse projected area of ship, m².</td>
</tr>
<tr>
<td>B</td>
<td>Ship's beam, m.</td>
</tr>
<tr>
<td>C()</td>
<td>Optimal 'cost' value.</td>
</tr>
<tr>
<td>C</td>
<td>Distance from bow of centroid of lateral projected area.</td>
</tr>
<tr>
<td>C_a</td>
<td>Model ship resistance coefficient.</td>
</tr>
<tr>
<td>C_arrport</td>
<td>Cost function of destination port, $.</td>
</tr>
<tr>
<td>C_B</td>
<td>Ship's block coefficient.</td>
</tr>
<tr>
<td>C_berth</td>
<td>Berthing cost function, $.</td>
</tr>
<tr>
<td>C_BTO</td>
<td>Frictional coefficient of the ship's bow thruster openings.</td>
</tr>
<tr>
<td>C_cap</td>
<td>Capital repayment cost function, $.</td>
</tr>
<tr>
<td>C_cargo</td>
<td>Cargo handling cost function, $.</td>
</tr>
<tr>
<td>C_crew</td>
<td>Crew wages cost function, $.</td>
</tr>
<tr>
<td>C_depport</td>
<td>Departure port cost, $.</td>
</tr>
<tr>
<td>C_destport</td>
<td>Cost function of arrival port, $.</td>
</tr>
<tr>
<td>C_f</td>
<td>Specific frictional coefficient or drag coefficient.</td>
</tr>
<tr>
<td>CF</td>
<td>Cumulative frequency/spectrum bandwidth correction factor.</td>
</tr>
<tr>
<td>Cf2</td>
<td>Two-dimensional frictional coefficient, Hughes (1954).</td>
</tr>
<tr>
<td>C_ins</td>
<td>Insurance cost function, $.</td>
</tr>
<tr>
<td>C_m</td>
<td>Ship's mid-ship coefficient.</td>
</tr>
<tr>
<td>C_n</td>
<td>Coefficient of turning moment due to wind.</td>
</tr>
<tr>
<td>C_operating</td>
<td>Operating costs, $.</td>
</tr>
<tr>
<td>C_p</td>
<td>Ship's prismatic coefficient, based on L (Holtrop).</td>
</tr>
<tr>
<td>C_pilot</td>
<td>Pilotage cost function, $.</td>
</tr>
<tr>
<td>C_pen</td>
<td>Port penalty cost function, $.</td>
</tr>
<tr>
<td>C_stern</td>
<td>Ship's stern shape coefficient, Holtrop (1982).</td>
</tr>
<tr>
<td>C_tow</td>
<td>Cost function for towage by tugs.</td>
</tr>
<tr>
<td>C_v</td>
<td>Viscous resistance coefficient.</td>
</tr>
<tr>
<td>C_wp</td>
<td>Ship's waterplane coefficient.</td>
</tr>
<tr>
<td>C_wve</td>
<td>High wave encounter penalty cost function, $.</td>
</tr>
<tr>
<td>C_x</td>
<td>Coefficient of fore and aft wind resistance.</td>
</tr>
<tr>
<td>C_y</td>
<td>Coefficient of lateral wind resistance.</td>
</tr>
<tr>
<td>C0.75R</td>
<td>Chord length of propeller disc at 0.75 of radius.</td>
</tr>
<tr>
<td>C_10</td>
<td>Wind drag coefficient at 10m height.</td>
</tr>
<tr>
<td>d</td>
<td>Ship's bow thruster diameter, m.</td>
</tr>
<tr>
<td>D</td>
<td>Propeller diameter, m.</td>
</tr>
<tr>
<td>e</td>
<td>Earth's eccentricity.</td>
</tr>
<tr>
<td>E</td>
<td>Spectral energy, J.</td>
</tr>
<tr>
<td>E_engine</td>
<td>Ship engine constraint vector.</td>
</tr>
<tr>
<td>Et</td>
<td>Duration time for heave-to, hrs.</td>
</tr>
<tr>
<td>f</td>
<td>Earth flattening.</td>
</tr>
<tr>
<td>f</td>
<td>Coriolis parameter.</td>
</tr>
<tr>
<td>F</td>
<td>Fetch length, m or Nm, defined in text.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$f_e$</td>
<td>Effective freeboard of ship, m.</td>
</tr>
<tr>
<td>$F_e$</td>
<td>Effective fetch length.</td>
</tr>
<tr>
<td>$F_{gen}$</td>
<td>Generator fuel cost, $.</td>
</tr>
<tr>
<td>$F_{lim}$</td>
<td>Limiting fetch length for FDS, Km.</td>
</tr>
<tr>
<td>$F_{main}$</td>
<td>Main engine fuel cost, $.</td>
</tr>
<tr>
<td>$F_n$, $F_{nl}$, $F_{nT}$</td>
<td>Froude number, Froude number based on immersion of bulb, and based on immersion of transom.</td>
</tr>
<tr>
<td>GRT</td>
<td>Ship's gross registered tonnage.</td>
</tr>
<tr>
<td>$h_B$</td>
<td>Height of bulb, m.</td>
</tr>
<tr>
<td>$H(\cdot)$</td>
<td>Heave cost function.</td>
</tr>
<tr>
<td>$I_e$</td>
<td>Half angle of entrance, °.</td>
</tr>
<tr>
<td>$J$</td>
<td>Advance coefficient</td>
</tr>
<tr>
<td>$K_Q$</td>
<td>Torque coefficient.</td>
</tr>
<tr>
<td>$K_{Th}$</td>
<td>Thrust coefficient.</td>
</tr>
<tr>
<td>$k_w$</td>
<td>Wave number.</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Hull roughness length, μm.</td>
</tr>
<tr>
<td>$I_{cb}$</td>
<td>% value of longitudinal centre of buoyancy from mid-ships.</td>
</tr>
<tr>
<td>LDF</td>
<td>Loaded factor of the ship, %.</td>
</tr>
<tr>
<td>$L$</td>
<td>Ships waterline length, m.</td>
</tr>
<tr>
<td>$L_R$</td>
<td>Ship's length of run, m.</td>
</tr>
<tr>
<td>Loa</td>
<td>Ship's length overall, m.</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Meridional parts.</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of distinct groups of masts or kingposts.</td>
</tr>
<tr>
<td>$\tilde{M}$</td>
<td>Motion constraint vector, av.roll, slamming, deckwetness, SM, propeller emergences.</td>
</tr>
<tr>
<td>$M_{state}$</td>
<td>Maximum number of positional states per stage.</td>
</tr>
<tr>
<td>$m_0$, $m_2$, $m_4$</td>
<td>Moments of spectra.</td>
</tr>
<tr>
<td>$n$</td>
<td>Engine revolutions per second.</td>
</tr>
<tr>
<td>$N$</td>
<td>Engine revolutions per minute.</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Turning moment, at centre of gravity due to wind.</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of propeller emergences per hour.</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of slams per hour.</td>
</tr>
<tr>
<td>$N_{stage}$</td>
<td>Number of stages, k=1,2,...,$N_{stage}$ - 1, $N_{stage}$</td>
</tr>
<tr>
<td>$N_w$</td>
<td>Number of deck wetnesses per hour.</td>
</tr>
<tr>
<td>$O_{cost}$</td>
<td>Operational cost.</td>
</tr>
<tr>
<td>$p$</td>
<td>Air pressure, Kgm⁻³.</td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>Angular value at pole.</td>
</tr>
<tr>
<td>$P$</td>
<td>Propeller pitch.</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Ship's brake power, KW.</td>
</tr>
<tr>
<td>$P_{cost}$</td>
<td>Penalty cost.</td>
</tr>
<tr>
<td>$P_{d}$, $P_{d1}$, $P_{d2}$</td>
<td>Ship's delivered power to propeller, KW, from engine polynomial, from resistance/thrust.</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Ship's effective power, KW.</td>
</tr>
<tr>
<td>$P_{pow}$</td>
<td>Engine power, bhp.</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Ship's shaft power, KW.</td>
</tr>
<tr>
<td>$Q$</td>
<td>Ship's propeller torque, KNm.</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of curvature of isobars, units defined in text.</td>
</tr>
</tbody>
</table>
\( R(\varnothing) \quad \) Radius of the local meridian.
\( R_a \quad \) Model-ship correlation resistance, KN.
\( R_{app} \quad \) Appendage resistance, KN.
\( R_{AR} \quad \) Added ship resistance due to the seaway, KN.
\( R_b \quad \) Additional ship resistance due to the bulbous bow, KN.
\( R_f(Hk) \quad \) Ship viscous resistance, as a function of form factor, KN.
\( R_f \quad \) Ship flat plate frictional resistance, KN.
\( R_n, R_{no.75R} \quad \) Reynolds number, Reynolds number at 0.75 radius of prop.
\( R_T \quad \) Total ship resistance, KN.
\( R_{tr} \quad \) Additional ship resistance due to immersed transom, KN.
\( R_{wind} \quad \) Ship wavemaking resistance, KN.
\( R_x \quad \) Fore and aft wind resistance component, KN.
\( R_y \quad \) Lateral wind resistance component, KN.
\( R_{yaw} \quad \) Ship resistance due to yawing motion of the vessel, KN.
\( S \quad \) Wetted surface area of the hull, m².
\( S(\varnothing) \quad \) Sea state.
\( S_{app} \quad \) Wetted surface area of ship's appendages, m².
\( SFC \quad \) Specific fuel consumption of main engine, g/KWh.
\( SFC_{gen} \quad \) Specific fuel consumption of generator, g/KWh.
\( SM \quad \) Subjective motion value.
\( S_p \quad \) Length of perimeter of lateral projection excluding masts.
\( th \quad \) Thrust deduction fraction.
\( t \quad \) Time or wind duration, hrs.
\( t_{dep} \quad \) Arrival time at departure port, hrs.
\( t_{dest} \quad \) Destination time, hrs.
\( T_{exp} \quad \) Exposure to high sea states, multiple of Et.
\( T_{lim} \quad \) Limiting duration
\( T \quad \) Total time, hrs.
\( T_m \quad \) Ship's mean draft, m.
\( T_a \quad \) Ship's draft at the after perpendicular, m.
\( T_{air} \quad \) Mean air temperature, °C.
\( T_{airc} \quad \) Eddy component of air temperature, °C.
\( T_f \quad \) Ship's draft at the forward perpendicular, m.
\( T_p \quad \) Ship's propeller thrust, KW.
\( T_{dep} \quad \) Depth of propeller shaft, m.
\( T_{dest} \quad \) Scheduled departure time, hrs.
\( T_{sea} \quad \) Scheduled destination time, hrs.
\( T_{wavep} \quad \) Primary wave period, see \( T_0 \), secs.
\( T_{waves} \quad \) Secondary wave period, secs.
\( T_{wavep} \quad \) Mean wave period, secs.
\( T_0 \quad \) Modal wave period, secs. Also primary wave period.

(xiii)
$U_{\text{wind}}$ True wind speed, Knots or $\text{m} \cdot \text{s}^{-1}$, defined in text.

$U_{\text{mean}}$ Mean wind speed, Knots or $\text{m} \cdot \text{s}^{-1}$, units specified in text.

$U$ Ship control vector, heading, power, engine revolutions.

$U_{\text{GEO}}$ Geostrophic wind speed, $\text{m} \cdot \text{s}^{-1}$.

$U_{\text{GR}}$ Gradient wind, $\text{m} \cdot \text{s}^{-1}$.

$U_{\text{rel}}$ Relative wind speed, Knots.

$U_z$ Wind at height above surface, $z$, m.

$U_{10}$ Wind speed at 10m height.

$V_{\text{a}}$ Speed of advance into the propeller disc, $\text{m} \cdot \text{s}^{-1}$.

$V_{\text{b}}$ Wind velocity in baroclinic zone, $\text{m} \cdot \text{s}^{-1}$.

$V_{\text{c}}$ Threshold velocity of bow.

$V_{\text{calm}}$ Ship speed in calm water, Knots or $\text{m} \cdot \text{s}^{-1}$.

$V_{\text{est}}$ Estimated ship speed ove ground, Knots or $\text{m} \cdot \text{s}^{-1}$, defined in text.

$V_{\text{reqd}}$ Required ship speed, Knots or $\text{m} \cdot \text{s}^{-1}$, defined in text.

$V_{\text{ship}}$ Ship speed through water, Knots or $\text{m} \cdot \text{s}^{-1}$, specified in text.

$V_{\text{stream}}$ Current speed, Knots or $\text{m} \cdot \text{s}^{-1}$, specified in text.

$\alpha$ Coefficient value defined within text.

$\chi$ Penalty cost function for early/late arrival at destination, $\$$.

$\Delta$ Ship displacement, m$^3$.

$\nabla$ Ship displacement, tonnes.

$\gamma$ Cost value of loading/discharging containers, $\$$.

$\eta_s$ Shafting efficiency.

$\eta_p$ Ship's propulsive efficiency.

$\eta_o$ Ship's propeller open water efficiency.

$\eta_b$ Ship's propeller behind the ship efficiency.

$\eta_r$ Ship's propeller relative rotative efficiency.

$\theta_a$ Pitch amplitude, o.

$\phi$ Latitude, o.

$\phi_a$ Roll amplitude, o.

$\phi_c$ Latitude of central stage points on GCR.

$\lambda$ Longitude, o.

$\rho_{\text{corr}}$ Townsin et al (1982) correction coefficient for encounter angles other than head seas, see further definitions in text.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{wave}}$</td>
<td>Encounter angle to waves.</td>
</tr>
<tr>
<td>$\mu_{\text{stream}}$</td>
<td>Encounter angle to current.</td>
</tr>
<tr>
<td>$\mu_{\text{wind}}$</td>
<td>Encounter angle to wind.</td>
</tr>
<tr>
<td>$M$</td>
<td>Assembly term for ship motion.</td>
</tr>
<tr>
<td>$M_{\text{interpolated}}$</td>
<td>Assembly of interpolated motion data values from databases.</td>
</tr>
<tr>
<td>$M_{\text{rqd}}$</td>
<td>Assembly of computed motion data values.</td>
</tr>
<tr>
<td>$v$</td>
<td>Kinematic Viscosity coefficient.</td>
</tr>
<tr>
<td>$\rho_{\text{air}}$</td>
<td>Density of air at the surface, Kg m$^{-3}$.</td>
</tr>
<tr>
<td>$\rho_{\text{sea}}$</td>
<td>Density of seawater, Kg m$^{-3}$.</td>
</tr>
<tr>
<td>$\omega_{e}$</td>
<td>Frequency of encounter, Hz.</td>
</tr>
<tr>
<td>$\omega_{p}$</td>
<td>Frequency of the seaway spectral peak, Hz.</td>
</tr>
<tr>
<td>$\omega_{\text{wave}}$</td>
<td>Circular frequency of waves, Hz.</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Mean wave height, m.</td>
</tr>
<tr>
<td>$\zeta_a$</td>
<td>Wave amplitude, m.</td>
</tr>
<tr>
<td>$\zeta_{1/3}$</td>
<td>Significant wave height, m.</td>
</tr>
</tbody>
</table>
ERRATA.

OPTIMAL WEATHER ROUTEING PROCEDURES FOR VESSELS ON TRANS-OCEANIC VOYAGES

(26) Addition. For, Optimum Course Route. Perie, Petrie.

(33) 2nd Para, line 1. For, underestimate the speed, speed loss.

(63) 1st Para, line 7. For, combines, combines.

(63) 1st Para, line 8. For, spreading, spreading.

(134) Equation 4.6.6. For, Vb, delete from =>

(138) 1st Para, line 1. For, separation, separation. (Occurs twice).

(138) TITLE FOR 4.6.6. For, DIGITISED, DIGITISED.

(139) Equation 4.6.7. For, Ω - 2π/3600, Ω - 2π/(3600 x 24).

(146) Equation 4.7.1. Equation is to be read as determinants, not matrices.

(150) 1st Para, line 1. For, correction to ship motions, database values.

(153) Equation 5.2.3. For, dt, read dt.

(157) 2nd Para, line 4. For, 5.8.6.(a) - 5.8.6.(d), 5.9.6.(a) - 5.9.6.(d).

(158) Equation 5.3.3. For, e sin² Φ, e sin Φ.

(165) 2nd Para, line 2. For, pursued, pursued.

(168) 1st Para, line 4/5. For, on the spheroid to the geodetic on the spheroid.

(173) 1st Para, line 4. For, Fifield, Fifield.

(174) Equation 5.9.6.(c). For, cos θ, sin φ cmg.

(199) 2nd Para, line 1. For, Set engine power and engine revolutions, Set engine power or revolutions.

(199) Equation 6.2.1 For, Vship, Vship.

(200) Eqn 6.2.2.(b) and .(d) For, Vship, Vship.

(200) Eqn 6.2.2.(c) and .(e) For, a Vship, a Vship.

(201) Equation 6.2.5. The condition is for ≥ 0.01.

(204) 2nd Para, line 4. For, were used route, were used.

(234) 3rd Para, line 7. For, exceedances, degrees.

(254) Equation 8.2.3.(c) For, a Coperating, a Coperating (a).

(255) Equation 8.2.3.(d). For, a Coperating, a Coperating (a).

(271) 5th Para, line 1. For, have been researched, have been considered.

(273) 11th Para, line 1. For, Off-line processing, On-line processing.

(281) 4th Para, line 3. For, Bhattachyrya, Bhattacharyya. And through text.

(286) Line 1.
ABSTRACT.

S. Calvert. B.Sc (Hons).

'OPTIMAL WEATHER ROUTEING PROCEDURES FOR VESSELS ON TRANS-OCEANIC VOYAGES'.

Three sets of algorithms are formulated for use in a variety of models :-


Optimisation models are constructed for deterministic minima, with time, fuel and cost objective functions. Models are constructed for an actual ship, (M. V. DART ATLANTIC), and realistic working solutions are obtained based on real-time weather information, simulating an actual on-board, computer based system, using dynamic programming. Several combinations of algorithm types are used in the the models, enabling comparisons of effectiveness. Thus, the ship performance algorithms incorporate severally; simple ship speed loss curves, ship resistance, ship motions and ship motion criteria databases devised from a linear seakeeping model. Limitations of the models are discussed from the routeing examples given.

State space restrictions and originally devised methods to aid convergence in the models are discussed. Extension of the forecasted environmental data is achieved by a variety of methods and comparisons sought. In particular ECMWF surface pressure files are interrogated to produce sea wave fields over the extended period, establishing main disturbance centres.

The variety of algorithms formulated in this work has facilitated real-time comparisons, this is particularly effective in route-updating. The development of these models and the methods used to extend the forecast period, and the comparisons and associated results stemming from these models are viewed as an original contribution to real-time weather routeing of ships.

(xvi)
1. INTRODUCTION

1.1. INTRODUCTION AND OUTLINE OF THE CHAPTERS.

Miller (1983), quoting Bowditch, (American Practical Navigator), defines weather routeing as:—

' ... a procedure whereby an optimum route is developed based on the forecasts of weather and seas and the ship's characteristics for a particular route ...'

Weather routeing involves the use of a model that defines an objective function relative to the environment that a ship is forecasted to encounter. Algorithms are constructed for the models to optimise objective functions severally for time, fuel consumption, cost or other. This study considers such a process for on-board application. Since the on-board computer is required to complete many computations in its task, study has concentrated on the sensitivity of the individual components that make up the overall routeing model. These are:—

1. A routeing model, incorporating optimum track selection, whereby the optimum objective is satisfied for a given weather situation;
2. A ship speed and powering algorithm, which enables those objectives to be specified as a function of the environment;
3. An environmental set of data, and an algorithm to generate any missing data or further data beyond the length of the forecast.

Chapter 1 describes the general background to weather routeing and offers solutions to the question, why weather route? The process of manual weather routeing is briefly outlined in order that the automatic methodologies and their general mathematical treatments can be appreciated.
Chapter 2, describes the on-board weather routeing problem in more detail, highlighting previous studies and attempts. This chapter outlines the work that is considered in this study and proposes a system for on-board routeing, emphasising the relative sensitivity studies that were felt to be necessary.

Chapter 3, describes the ship responses, from simple speed loss curves, which define the expected speed of a vessel in a given sea state, ranging up to a ship resistance and powering algorithm constructed in this study. Models are used to determine the amount of fuel the vessel consumes, which are utilised for fuel and cost routeing objectives. This study has also concentrated on developing a 'simple', ship motion database, from which a routeing model can deduce expected levels of critical motions that are used to determine a route or not. This has required the inclusion of seakeeping computer models developed by British Maritime Technology, (BMT), and local computer programs developed in Plymouth.

Chapter 4, describes the environmental data that affects a ship at sea. It is through this data that the on-board system determines the optimum route. Digitised data is provided from sophisticated oceanographic models, however, only to a specified forecast length. Typically, the length of the voyage in time, will exceed the length of the forecast, and this chapter explores ways to extend the data from a synoptic approach, since it is important to maintain the physical parameters, rather than resorting to long-term statistical information.

Chapter 5, describes the mathematical treatment of the routeing strategy. This is dynamic programming, and sensitivity studies of the discrete system have been performed in order that the system be best configured for on-board use. Several routeing examples are shown to this effect. This mathematical treatment results in a gridding area of the ocean and methods are shown that reduce both the feasible positional state space and the time state space, since the vessel moves along trajectories between these states. It is in this chapter that the objective functions are described.
Chapter 6 describes the minimum time routeing algorithm, and proceeds to depict routeing studies which culminate from the previous work. Similarly, chapters 7 and 8 describe the minimum fuel and overall cost algorithms along with several routeing studies. These chapters show the effects of the previous methods of expansion of the data sets and the sensitivity studies of the ship models.

Chapter 9 concludes the work in this study and critiquely reviews the results from chapters 6, 7 and 8. The effects which describe the differences and similarities between the routes obtained under different objectives are fully discussed. The model limitations and error sources are highlighted, in order that areas of further research, that the author envisages expanding the concept of on-board weather routeing, be pursued. A joint venture between Polytechnic South West, OCEANROUTES (UK) LTD and OCEANROUTES INC, USA is now underway to produce a computerised on-board weather routeing/ship performance system.

This study is based on a container vessel, for reasons that will be explained. These ships are more likely to be routed, since they generally carry more sensitive or expensive cargo.

1.2. THE ADVENT OF SHIP WEATHER ROUTEING - CLIMATOLOGICAL WEATHER ROUTEING.

There are three chronological approaches to weather routeing; climatological weather routeing, weather bulletins with forecast weather routeing and automatic weather routeing.

The origins of ship weather routeing can be traced back many hundreds of years, but it was in 1770 that Benjamin Franklin produced the first climatological charts, showing how eastbound crossings of the North Atlantic could take advantage of the Gulf stream. This approach was extended by an American naval lieutenant, Matthew Maury (1855), who, in 1852 published the first Sailing Directions, showing a general view of the kind of weather to expect in particular seaways at
particular times of the year. In other words, he produced a climatological or seasonal average of the weather for specific areas of the world's oceans. This kind of information is still made available to mariners in the form of monthly pilot charts from the United States Hydrographic Office, and detailed route recommendations in the Passages of the World, from the British Admiralty (1989). Such routeing is regarded as a 'passive status' art.

1.3 WEATHER BULLETINS AND FORECAST WEATHER ROUTEING. This involves the use of radio facsimile transmissions, radio reports and transmitted weather satellite pictures for on-board routeing. Weather routeing in this situation is regarded as an 'active status' practice, since it tries to encompass on-going weather effects.

Such information as synoptic facsimile charts, for upper air, surface pressure, and oceanographic phenomena, together with radio bulletins and extended forecasts all aid the mariner to decide upon the course that he should take. The master is able to apply his knowledge of the capabilities of his ship to an assessment of the immediate and forecast weather information. With the extension in the forecast range of meteorological and oceanographic models and the improvements in the accuracy of their outputs, the master of a vessel, or the shore-bound router, is better equipped to make more enlightened decisions regarding the route to be taken.

Forecast weather routeing is provided by land-based routeing companies, who assimilate a much larger variety of forecast meteorological data and ship data. Typically, routes are compiled by the assessment of the storm track, Francis (1971), Korevaar (1976), and based on the collective appraisal of a team of experienced meteorologists and mariners.

With modern wind and wave forecasts, it is possible to choose a route in much the same manner as previously but with improved prospects of success. The
routeing agency today, aims to minimise the cost or transit time of a vessel. For example, OCEANROUTES constantly evaluate the minimum time tracks for 17 common trade routes in the Pacific and Atlantic oceans and up to eight different alternatives on each, for every month of the year. There are considerable time differences in each route, see figure 1.1, and the minimum time track varies daily being therefore non-seasonal in contrast to climatological ship weather routeing.

**FIGURE 1.1 EVALUATION OF MINIMUM TIME TRACKS FOR THE ROUTE BETWEEN ROTTERDAM AND NEW YORK DURING DECEMBER 1975-1979.**
1.4. AUTOMATIC WEATHER ROUTEING.

The digital computer has been used to compute optimum routes based on hindcast and forecast data, generated from atmospheric and oceanographic models.

Typically, routeing agencies today assimilate weather information and apply mathematical algorithms, such as speed reduction curves, James (1957), Howard’s curves (used by OCEANROUTES (1989)), in order to assess the progress that a vessel could make in the predicted sea conditions, and as such these algorithms have been adopted by early computer models. However, these are now being superseded by more sophisticated ship models, Journée et al (1980), Petrie et al (1984), Hagiwara et al (1987), Hagiwara (1989), which predict the power, speed, fuel consumption and motions of the vessel.

The accuracy of these models is determined by the accuracy of the inputs. Since both environment and ship inputs contain errors, both deterministic and stochastic routeing models have been developed. It is a pre-requisite that the environmental data should cover the transit time, which is not the case for ocean crossings, particularly at the outset of the voyage, (N. Atlantic, 7-10 days, N. Pacific 11-15 days). The extension of data, beyond that of the forecast, has been achieved by several methods, notably climatological, Zoppoli (1972), Clune (1975), Klapp (1979), Petrie et al (1984), based on the particular weather pattern that has been evolving, typically evaluated by computer-searching of a past 28-year history library for analogous weather patterns. However, extension by the use of the storm track approach utilising the 500mb forecast charts was favoured by several authors, Moens (1980), Motte (1981), by concentrating on forecast surface pressure charts to describe the ocean environment. Further extension can be seen to be a function of the 500mb, steering level forecasts, or even the mean 5-day zonal index correlation to the 5-day mean sea states, Hagiwara (1989).

The computation of optimum routes has been achieved by several mathematical means as expanded later. The on-set of automatic routeing, and weather routeing
as a whole has resulted from the benefits that it can offer to the ship owner or mariner.

1.5. THE BENEFITS OF WEATHER ROUTEING.

The benefits of weather routing a vessel have long been established, although more recent examples are given by, Francis (1971), Korevaar (1976), Mackie (1975,1981), Dooley (1978), Easter (1981), Miller (1983), Motte (1981,1987), Richards (1978), Wagland (1985). Early benefits were established as long ago as the Clipper tea trading routes in the 19th century, later in the 1970's, the oil crisis forced mariners and ship owners/operators alike to seek economies in their ships. such as :-

(i) Improvements in ship propulsion, particularly those in engine design, slow speed engines, propeller design, automatic pilots, hull design and coatings;
(ii) Weather routing;
(iii) Reduction in manning levels, with improved machinery;
(iv) Integrated or modal transportation strategies;
(v) Optimal ship capacities between terminals.

The main aim during the 1970's was to reduce the cost of the most expensive item, fuel. It is possible to reduce the consumption by efficiencies, by reducing the travel distance between transit ports, or to maintain the best course available under the prevailing weather conditions. The latter strategy aims to reduce the encounter of heavy seas which incur, delays to schedules, heavy consumption of fuel or more critically heavy damage to the ship and cargo.

OCEANROUTES (1989) and Motte (1981,1987), have studied the effects of heavy weather damage and weather induced losses to ships, see figure 1.2, and although these amount to a small percentage of total ship losses, by whatever reason, the quantitative monetary loss is large.
Prior to 1973, when oil bunker price was low, approximately US$4/barrel of heavy fuel oil, (HFO), a saving of one day on the total transit time for an average 165,000dwt steam tanker, steaming at 16.5 knots and consuming two barrels of oil per mile, meant only a saving of US$8/mile or US$3,300/day. However, the same ship operating at a price of US$18/barrel, would realise a saving of US$14,500/day. It is easy to appreciate that economies were sought in all aspects of fuel conservation, ranging from slow speed steaming, reduction in operating speed, to low grade fuel oil engines. Many cases of the benefits of weather routeing are documented, particularly by routeing companies who equate a possible cost saving on a route to the cost of their service. The most involved study was performed by Miller (1983), commissioned by OCEANROUTES, who considered savings as widespread as reduction in casualties, damage, cost and time.
There are therefore several objectives that weather routeing can be applied to:

(i) Minimisation of the transit time;
(ii) Minimisation of the transit cost;
(iii) Minimisation of the damage to the vessel;
(iv) Safety for both crew and ship;
(v) As an input to the total fleet management system;
(vi) As a means of determining the most advantageous routes for sensitive cargoes or passenger ships;
(vii) As a means of determining the most advantageous routes or sites during tactical manouevres of warships.

The benefits to the master of weather routeing his vessel are appreciated when the savings in time or fuel or damage are equated to monetary values. Constantine (1981), shows an OCEANROUTES study into the cost of ships damage,
figure 1.3, however, it would be unwise to adopt this function for all vessels due to the variability in ship design, cargoes, weather situations and so on. Figure 1.3, therefore, is interpreted as a pointer to possible damage costs.

The cost/time savings are typically those values that the vessel would make over an expected great circle crossing during the same period. However, several authors have shown savings to be those over an arbitrarily chosen route through a calm seaway, Francis (1971).

Routeing benefits are also drawn from legal implications:

'As a matter of English law, the master has comparative freedom regarding which route to take in order to get from one port to another whilst performing voyages under a time charter party ...'; Williamson (1981).

The master is under a duty to exercise the utmost despatch within reasons of safety, the test becomes what was reasonable under the circumstances. Time charterers were quick to realise that weather routeing information could be used to confirm a vessel's position, performance and utmost despatch, Asian Shipping (1986); ocean routeing material is now used as evidence in court cases.

1.6. AUTOMATED ROUTEING ALGORITHMS.

In order to find the optimum, many different routes have to be attempted, in simulation. Usually, this process takes the form of a recursive algorithm, where the vessel's progress from departure to destination is split up into a number of stages. The recursive algorithm, termed an N-stage decision process, Zoppoli (1972), is bounded by constraints and error sources, Motte et al (1988). In designing such an algorithm for on-board routeing, several aspects become apparent, dependant upon the capabilities of the computer and its memory size.
There is a need for a simple recursive algorithm using accurately predicted sea state data and ship performance data, and there have been many suggested models for optimisation. The most publicised routeing models fall loosely into the following categories:-

(i) Calculus of variations or extremals, Bleick et al (1964), Bijilsma (1975), Marks et al (1968);
(ii) Mathematical heuristics, De Wit (1971);
(iii) Isochronal, James (1957), and modified isochronal, Hagiwara et al (1987), Spaans et al (1987), Hagiwara (1989);

These are briefly described in sections 1.6.1 - 1.6.4.

1.6.1. CALCULUS OF VARIATIONS OR EXTREMALS

From the aspect of minimum time ship routeing, the method has been described to be the most attractive, but since ship speed is used to find the time objective, problems can be encountered when deriving second order differential equations. Bijilsma (1975), also states that since errors are to be expected in the environmental data, the differential equation errors can become expanded to an unacceptable level, (second-order equations could lead to errors squared). Since speed of computation and simplicity are important considerations, it was felt that this model would not be suitable for on-board use.

1.6.2. MATHEMATICAL HEURISTICS.

This method is very similar to the isochronal method, section 1.6.3. Time fronts are expanded from the departure to the destination point and route tracing involves a clever but complex indexing system. Problems have been found in the constraints, since the time fronts can become looped, see figure 1.4.
Many solutions to this problem have been proposed, for example, Marks et al (1968) and Bijlsma (1975), based on Pontryagin's maximum principle. The methodology can only handle one objective at a time, and involves a complex and cumbersome algorithm.

1.6.3. ISOCHRONAL AND MODIFIED ISOCHRONAL.
First proposed by James (1957), the isochrone system was designed as a manual method to solve the optimum route. It involves the selection of discrete headings that the ship can sail from the departure point for a certain length of time. The line or contour that joins the ends of these radials forms a time front, from which the process is repeated until the destination is met. The system was limited and with the introduction of computers the method has been superseded.

Hagiwara et al (1987), Hagiwara (1989) and Spaans et al (1987), have developed a modified isochronal algorithm. Route calculation is based on great circle tracks (least distance routes), the time-front theory and the remaining great circle
distance to the destination point. (This has been amended to comply with Pontryagin's maximum principle as pointed out by De Wit (1989), and is the maximum distance from the departure. It was seen that differing destinations could have an effect on the fronts). It is a recursive algorithm and suffers from an expanding computational effort as the departure is left. However, constraints on the algorithm are designed to reduce the problem. It is assumed that the vessel completes the journey under constant propeller revolutions, then by referring to figure 1.5, the vessel moves from \( X_0 \) to \( X_e \).

\[
\begin{align*}
&\text{FIGURE 1.5 MODIFIED ISOCRONE. AFTER HAGIWARA ET AL (1987).} \\
&
\end{align*}
\]

The initial track followed is the great circle route, defined by \( C_0 \), which is updated by small increments, \( \pm i\Delta C \), where \( i = 0,1,2, \ldots, m \), along which the ship's performance is evaluated. The vessel transit time can be deduced and recorded at each point as the course change is made. The set of points at \( (t_0 \pm \Delta t) \), derived from \( X_0 \) are defined by \( X(i) \) and termed an isochrone.

These points now define new departures, where \( C_0 \) is the great circle route between \( X(i) \) and \( X_e \), and a new isochronal set is defined at \( (t_0 + 2\Delta t) \). The
points on this isochrone are defined by $X(i,j)$, so that route tracing can be performed. If this method were to continue across the entire ocean for a complete crossing, and the time increment say 6 hours, then the number of points at each isochrone would become enormous. In this respect there is a need to reduce the number to a feasible limit and is performed by the use of lanes, see figure 1.5. From each point on the second isochrone the 'lateral (cross-directional)' distance $D_2(i,j)$, is calculated. This distance is that from the point $X(i,j)$ to the original great circle route. The remaining great circle distance from points $X(i,j)$ to $X_e$ is found and labelled $R_2(i,j)$. The lane width is divided into a number of sub-lanes $p$, and the same method is applied, however, at the third and subsequent isochrones, only the point with the minimum $R$ is taken as the departure point, and similarly the lane and sub-lane widths are recalculated.

The accuracy and also the speed of this method lies in the width of the sub-lane, Hagiwara et al (1987), Hagiwara (1989). In a simple manner, like dynamic programming, the modified isochronal method can incorporate system restraints. The problem of minimum fuel routeing is formulated by continually iterating the overall passage time as a function of the propeller revolutions, so as to bring it close to the specified passage time. Hagiwara et al (1987), suggest that Newton's Regula-Falsi method be used to correct the number of propeller revolutions. This concept as discussed by De Wit (1988), is expanded in section 7.3.

1.6.4. DYNAMIC PROGRAMMING (DP).

Dynamic programming, was originally developed by Bellman (1957), and updated in 1962. It has consequently been used by many authors Bleick et al (1964), Nagle (1972), Zoppoli (1972), Chen et al (1976), Chen (1978), Frankel et al (1980), Kahlilov (1980), Foo (1985), Motte et al (1985,1987,1988), Higham (1988). The procedure as developed by Bellman (1957) is based on his PRINCIPLE OF OPTIMALITY, which states,
'An optimum policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimum policy with regard to the stage resulting from the first decision.'

The procedure can be regarded as a series of legs between the departure and destination point. The DP method can therefore be split up into a number of stages, evenly spaced along a central route and is referred to as an N-stage decision process, Zoppoli (1972). Each stage is made up, like an isochrone, of a number of discrete points, and the whole network constitutes a grid over the ocean. Travel between points is found simply by integration of the environment and the ship performance data. The shape of the grid can be set by limitations on the movements of the vessel, and the optimum route found through the system, Zoppoli (1972), Frankel et al (1980), Calvert (1988), Motte et al (1990).

As shown in section 5.4, the total number of possible routes that could be singularly computed through the grid is not feasible for a computer program and certain constraints are imposed to reduce this 'brute-force' calculation, affectionately termed the 'curse of optimality', Bellman (1957).

Zoppoli (1972), Chen (1978) and Motte et al (1987), describe the constraints placed on the DP method under the headings environmentally induced constraints and control constraints, which loosely cover the following:

**Environmentally Induced Constraints.**

These are essentially natural obstacles such as land, fog areas, ice areas, and possibly areas of high seas and winds. Clearly some constraints are permanent, but it is possible to place time limits on them.

**Ship Control Constraints.**

These are principally concerned with the operating envelope of the
ship's engine, ie maximum power output, smoke limit, maximum continuous rating and so on. However, others such as maximum course deviation, steering rates and ship motion criteria are considered.

These limitations are therefore aimed at reducing the 'sledgehammer' approach, Klapp (1979).

Dynamic programming is a very powerful tool for determining the optimum route across an ocean, since the algorithm constraints and environmental inputs are handled directly and linearly. It is true that the inputs and grid sizes determine the accuracy, and so complete model description is necessary. In the application of such a method to the on-board approach, consideration has to be given to the complexity of the models and the sensitivity of the system to point spacings, Motte et al (1985,1987,1990). However, the use of DP does not necessarily mean that a constant grid size, such as the 63x63 grid of the Polar Stereographic Projection of the North Atlantic, is required, Chen et al (1976).

The uncertainty in the computation of environmental criteria has been incorporated into the DP method recently, Chen et al (1976), Chen (1978). As Hagiwara (1989) points out, the development of the uncertainties in the ship response and seakeeping algorithms has not been attempted, due to the lack of measured circumstances and their difficulties, and as such a stochastic routeing model is based entirely on the uncertainties in the environmental data. Chen et al (1976), formulated the full stochastic problem, but proceeded to reduce the problem so that it could be easily handled. Zoppoli (1972) states that although the procedure is simple, it requires definite values of the environmental conditions, not at individual graduations, but constantly, and also their corresponding probabilities. The stochastic method and its difficulties are explained in section 5.8.
2. ON-BOARD MICRO-COMPUTER BASED
WEATHER ROUTEING.

2.1. ON-BOARD WEATHER ROUTEING SYSTEMS.

Conflict may arise between the traditional methods of weather routeing, using
shore-based advice or even active on-board weather routeing by the mariner.
Indeed, it may be argued that a shore-based computer algorithm could be
integrated into the subjective team approach.

The computer has the following advantages over the human router:-

1. The processing of routes can take place in a fraction of the time
taken by an experienced mariner;

2. The update of routes can take place at certain time intervals, for
example every twelve hours, (that at which a new set of forecast
environmental data is received);

3. The computer can predict the performance of the vessel for
objectives other than safety or time, and can predict phenomena
such as slamming for inclusion in route decisions;

4. The stochastic processes of the environment can be
accommodated.

On-board weather routeing may be seen to have distinct advantages over
shore-based routeing since the system becomes individual to the ship, and can be
implemented as an adaptive algorithm. The ship model can be fine tuned,
(adaptive), as an on-going process, so that ageing effects on the vessel's engine
and hull can be accommodated. Similarly, should cost or fuel be the master's
priority, then, fuel consumption variations, (effects of engine deterioration, or fuel
quality) can be included. Clearly, shore-based routeing can not offer these, without
extensive data storage and transmission.

\[\text{FIGURE 2.1. SUPER INTEGRATED SHIP OPERATION CONTROL SYSTEM.}\]

\[\text{AFTER KANAMARU ET AL (1988).}\]

Kanamaru et al (1988) developed the routeing system as a continually variable navigational area, along the lines of mathematical heuristics. In an earlier paper, Kanamaru (1984), a routeing system was designed, based on avoidance of sea areas (rectangular blocks of sea) and an algorithm to avoid these areas by selecting minimum transit distances was configured.
The objective function of the former approach was satisfied by a Lagrangian algorithm, which for fuel consumption was determined by the Admiralty coefficient. Fukuda et al (1985), consider the route between ports as a series of sea areas. Both systems consider the objective function to be fuel consumption; however, it is interesting to note that both approaches to model the ship and its engine are tackled in very different ways.

**FIGURE 2.2. SHIP OPERATION SYSTEM. AFTER TAGUCHI ET AL (1980).**
Fukuda et al (1985) explain a (simplified) semi-empirical model incorporating adaptive algorithms and motion prediction. However, Kanamaru et al (1988) adopted a ship algorithm similar to speed loss curves, but with additional modelling of the fuel consumption. Chen (1989), adopted the dynamic programming technique and aligned a ship model which is semi-empirical. It incorporates aspects of the simple regression curves, but also an adaptation of the more sophisticated theoretical models. This system has been installed aboard the American President Lines (APL) container ships and is currently under tests.

Taguchi et al (1980), discussed on-board weather routeing, and postulated a total optimum ship operation system, see figure 2.2. The moving segment, figure 2.2, consists of long and short-range routeing and local optimum routeing. Weather routeing is seen as an integral part of a total optimum strategy.

2.2. AUTOMATIC WEATHER ROUTEING CONTROL.

Kasahara et al (1980) and Kanamaru (1984) have outlined elements of a total on-board system comprising certain elements. These are termed the global and local routeing in this study and are summarised in table 2.1.

<table>
<thead>
<tr>
<th>OBJECT REGION</th>
<th>SAFETY</th>
<th>ECONOMY</th>
<th>PRECISION / FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global. Whole Voyage 1000's nm</td>
<td>Weather Routeing, Navigational Planning</td>
<td>fuel time safety costs</td>
<td>Dependant upon weather accuracy and discretisation. Avoidance of potentially dangerous seaways. Route analysis. Great circle sailing</td>
</tr>
<tr>
<td>Local. Voyage Section 100's nm</td>
<td>Hazard Avoidance, Collision Avoidance</td>
<td>Route tracking, Fuel saving, steering control</td>
<td>Dependant upon course keeping and course changing. Recovery to global route. Local route correction.</td>
</tr>
<tr>
<td>Narrow 10's nm</td>
<td>Collision Avoidance</td>
<td></td>
<td>Dependant upon route tracking. Berthing control. Position keeping.</td>
</tr>
</tbody>
</table>

**TABLE 2.1 NAVIGATION AND MANOEUVRING SYSTEM**.
2.2.1 THE GLOBAL (ENTIRE ROUTE) SYSTEM.

Calculation of an optimal route, based on a series of inputs only forms an open-looped control system, see figure 2.3. Since the environmental data are updated, typically every twelve or twenty four hours, the routeing advice is also updated at these times. In effect the process is continually monitored or controlled, and has been termed an open-loop, closed control system, Chen (1978). The operation is open-loop since the once-off computation has no feedback based on the system output (arrival time), from which the errors, (expected arrival time - required arrival time), update the inputs, (vessel power, heading).

Figures 2.3 and 2.4 depict the system as perceived by this author. At the on-set of the voyage, the optimum routeing algorithms are provided with the optimum strategy and a complete set of environmental data. This data, downloaded from INMARSAT, consists of the US Navy data arrays, which extend to approximately 3
days for waves, and 5 days for winds. The extended forecast data, greater than 3 - 5 days, are also provided. The global optimum route is computed with the integration of the ship model and the environment. Routes are defined by a series of waypoints and engine/navigational requirements. Operation of the vessel between the waypoints is controlled (local optimisation), by a second algorithm, since the vessel may be unable to maintain course, and stray close to other waypoints. Should certain operating criteria be exceeded, control is passed to the main algorithm for further global route calculation. Similarly, should new forecast data become available, control is again passed to the main optimisation algorithm.

2.2.2. THE LOCAL (ROUTE SEGMENT) SYSTEM.

The local algorithm is invoked when the vessel is en-route. It operates as defined previously, and can be used to communicate with the main engine and steering gear in a total control algorithm as shown in figure 2.4.

![Diagram of the local route segment system](image)

**FIGURE 2.4. SYSTEM FOR ON-BOARD MICRO-BASED OPTIMUM WEATHER ROUTEING.**
Kanamaru (1984) and Kanamaru et al (1988), describe how the routeing strategy can be applied to control the engine and steering gear. The required settings are seen to be preset values computed by the global routeing algorithm. Since the transit time between fixed points (say) is given, an algorithm to automatically control the vessel optimally between points (local optimisation) will provide controls for the engine and steering gear. Such control algorithms have been investigated in detail by Burns (1984), for calm water port approach, and its extension to the open ocean is merely an expansion of the algorithm.

2.3. DATA STORING AND TRANSMISSION FOR AN ON-BOARD SYSTEM.

Chen (1985,1987,1989) has described the transmission and storing of environmental data as part of his system on the APL ships. Those data files, explained in chapter 4, contain weather parameters for an ocean (or part) at specific grid points (spatial variation) and each represents one forecast in a twelve hour sequence up to 3-5 days ahead, (time variation). Data resulting from the transmission of crunched (data compression) files via INMARSAT and modems, to the on-board micro computer is described. The computer is able to error check, and request re-transmission of the whole or part, should errors be detected. However, Chen (1987) points out that the service is not free unlike facsimile weather charts and so is difficult to impress upon mariners. Facsimile charts are transmitted at fixed times, and receivers have to be alerted to this time, otherwise the transmission is lost. The system on ship is therefore receive only. The new electronic mail, EMAIL systems and modems have transmit and receive capabilities. Chen (1987) states that the most effective rates of transmission and reception were found to be 8,000 BAUD and 3,000 BAUD respectfully. Data compression is performed by the Huffman code and error checking by the checksum or cylindrical redundancy checking algorithms.

Only the most recent data files need be held in store, since it is these that the routes are generated from. Similarly, there is a need to be stringent in the
amount of data as transmission via satellite is costly and slow, in comparison to land lines. For this reason, the environmental data is reduced from a directional matrix for each grid point, plus wind and current, to a series of parameters, see section 4.4.1 and described by Petrie et al (1979) and Hoffman et al (1980). However, these parameters are used to re-generate a representative spectrum of the ocean for motion prediction, see section 3.9 and 3.10. Similarly, Kanamaru et al (1988), indicates that only a section of the complete data file need be taken, corresponding to an area commensurate with the furthest extent of travel of the vessel within the 12 hour period, as in figure 2.5.

![Environmental Data File Area](image)

**FIGURE 2.5. ENVIRONMENTAL DATA FILE AREA, AFTER KANAMARU (1988).**

2.4. ON-BOARD MOTION SENSING AND PREDICTION.

It is possible to measure the real-time motion of the ship and operate the ship to minimise damage. Local routeing strategy will offer alternative courses based on motion exceedance criteria, Deakins (1988), to reduce the danger but also offer a course that is an integral part of the total optimum strategy. Similarly, the prediction of ship motions can be applied to the global routeing algorithm.
Such motion prediction is based on semi-theoretical ship models, as a combination of response amplitude operators of the vessel, (RAO), and the spectral representation of the seaway. The RAO is a function of the frequency of encounter and the non-dimensional amplitude of the motion of the vessel, see section 3.9. The integration of the two, through linear superposition, results in a prediction of vessel motion.


Comstock et al (1980) described the total system under several headings depending upon the objective; TOSR, tactical ocean ship routeing, OTSR, optimum track ship routeing, HWSR, heavy weather ship routeing and SSR, survival ship routeing. The sensing of motions is linked to the latter two whereas prediction of motions are highlighted in the former two. Comstock et al (1980) also described their perception of an RAO data base, graphically displayed as polar diagrams with operating envelopes.

2.5. DESCRIPTION OF THE WORK IN THIS STUDY AND A COMPARISON TO PREVIOUS SYSTEMS.

The purpose of this study, is to assess the concept of weather routeing as an individual optimum system, by concentrating on the global algorithms. Unlike Taguchi et al (1980), Kanamaru (1984) and Kanamaru et al (1988), the system uses well founded routeing techniques, and removes the shore-based operator from all but total fleet routeing management and optimisation. It does not consider routeing in conjunction with an optimum bunkering strategy. Similarly, the routeing advice is based on data generated at specific grid points for the N. Atlantic ocean, unlike Fukuda et al (1985), who optimise objectives based on the functional form for a sea area consisting of many hundreds of square miles. Optimum fuel
and cost routeing is developed from a semi-empirical approach, since the vessel is modelled by resistance terms, whereas the engine characteristics are taken from regression analyses of trials and dry-docking data.

Any computed route can only be regarded as an estimate of the true optimum, since the function of the routeing objective is, in reality, a continuous integral. The mathematical treatment to the solution of the objective involves the discretisation of the continuous function, therefore treating it as a step-wise series. Similarly, since the environment and prediction of the ship responses involves many sources of unquantifiable error, the computed optimum is only an estimation. Hagiwara (1989) has termed such a computed value as the sub-optimum, which is technically correct. However, this could be interpreted from the algorithms as a search for a sub-optimal value; the term quasi-optimum is preferred since this states that an optimal solution is attempted although the result is not truly optimal.

An on-board, micro-computer based system is proposed by the author in this study for both cost/fuel and time optimisation. The investigations made within this thesis have concentrated in providing a working model, from which a practical approach can be achieved. With regard to table 2.1, the global routeing model is produced, for those economies stated.

Investigations have been made into the most appropriate ship model algorithm, by studying the simple regression analyses of James (1957), Townsin (1982), Aertssen (1969), and Babbedge (1975). These models are referred to as empirical. Further to these, a more sophisticated, semi-empirical/theoretical model has been developed, and designed to emulate the container ship considered by Babbedge (1975). The limitations of the models are shown with regard to their adaptability to the objective functions and the ease and speed of computation.
The Babbedge (1975) and the semi-empirical models are considered in detail as they can be used to optimise for cost/fuel as well as time, since engine, (shaft), power appears in the functions.

The semi-empirical model has been constructed in conjunction with the interrupted outputs from a linear strip theory seakeeping program, (BRITSEA from BMT). The RAOs have been used to produce databases for added resistance and critical values of ship motions, as used in the decision status of the routeing algorithms. Without extensive testing, one cannot say that one model is better than the other; however, route simulations have been attempted with both models, in order that some judgement be made.

The sensitivity of dynamical programming has been investigated, by concentrating on the choice of discrete grid and spacings. The effect on the objective function is also highlighted, and recommendations for a routeing model are proposed. The grid system is considered with regard to the operating and environmental constraints, and several algorithms are shown that can restrict the extent of the grid, and the computational effort. Further to this study, is the interconnection between the (sub) optimal grid spacings and those constraints, which can hinder a converging quasi-optimal route, and also lead to discontinuous routes (large course changes).

The routeing strategies under the objectives are described in chapters 6, 7 and 8, and concentrate on both constant power, and speed situations. However, the strategies to maintain fixed departure and destination times have been viewed both with variable engine power, through the Petrie et al (1984) proposals and also an iterative procedure to determine a constant engine setting. The former approach has been investigated more thoroughly in this study, and several new extensions are provided.
Since the environment forms the major input to the system it has been studied extensively, by investigating the complex integration of analysis and forecast data arrays from the US navy models at the Fleet Numerical Oceanographic Centre, Monterey, (FNOC). The extension of the data beyond the forecast length has been studied. Several mathematical, and physical approaches have been investigated, with regard to storm-track avoidance. There is evidence from experience as presented in this study that due to the poor accuracy of extended data, the prediction and movement of storm centres is more important.
3. SHIP PERFORMANCE ALGORITHMS.

3.1. INTRODUCTION.

The simplest ship algorithm, and one that is still used today for routeing purposes, is that constructed from analyses of ship's logs, James (1957), Babbedge (1975). Graphical representations are based on encounters with waves and the significant wave height, $\zeta_{1/3}$. Typical examples are shown in figure 3.1.

Curves are drawn as least-squares polynomials of many points, and are regarded as the mean response for that ship. Further to this approach, certain empirical formulae have been devised to predict the speeds, for particular ships, based on a series of coefficients and Beaufort numbers, (BN). Such formulae have been proposed by Aertssen (1969,1975) and Townsin et al (1983).

The curves and formulae proposed by Babbedge (1975), were studied extensively, in the early stages of this project. Their construction is similar to that of James.
(1957), whence a linear multi-regression analysis was performed on data recorded on several voyages aboard four ships. The equations incorporate aspects of the environment as well as ship parameters.

Since a powering algorithm is required to compute ship operating costs, one of the simplest functional forms can be regarded in the generalised power diagram originally developed by Telfer in 1922, Townsin et al (1975), Monk (1984). This algorithm was not investigated.

Finally, a ship powering algorithm has been constructed based on the summation of the components of ship resistance and the thrust of the propeller. This formulation requires a knowledge of the added resistance of the ship in irregular seaways. This was conceived to be a pre-calculated database of values, derived from BRITSEA. Calm water resistances are also pre-calculated and based on the regression analyses of Holtrop et al (1980,1982) and Holtrop (1984). The motions of the vessel are used to determine the probabilities of exceedance of motion criteria which are used in the decisions and weightings of the routeing model.

The effect of current set and drift has been accounted for in the routeing models. The ship powering and speed models, in this study, have been used in the routeing algorithms to ascertain their relative advantages or disadvantages. More importantly they are used to determine whether the quasi-optimum route differs qualitatively rather than quantitatively. Table 3.1 summarises the distinct advantages and disadvantages of the ship algorithms.

All routeing examples and ship algorithms have been based on containerships, in particular the DART ATLANTIC/ DART EUROPE, as described in appendix A.1. These sister ships have been described by Babbedge (1975) and Aertssen (1972), thereby making algorithm comparison more accurate. The Aertssen (1972) paper describes performance tests aboard the DART EUROPE in the north Atlantic, which provided some degree of quality check.
<table>
<thead>
<tr>
<th>METHOD</th>
<th>REMARKS</th>
<th>PARAMETERS</th>
<th>OPTIMISATION OBJECTIVES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babbedge (1975)</td>
<td>Significant wave height, ship speed, power, wave encounter angle, wind encounter angle, wind speed, Short calculation time. Model is ship specific and includes added resistance and masters reduction. No motions. Reasonable accuracy. Not accurate for high wave states</td>
<td></td>
<td>Minimisation of time, safety, fuel and cost</td>
</tr>
</tbody>
</table>

**TABLE 3.1 SHIP SEAKEEPING AND SPEEDKEEPING MODELS.**

### 3.2 JAMES SPEED LOSS CURVES.

The speed of the vessel is described as a function of the encounter angle to waves and the significant wave height, and determined from the analyses of ship's logs. Any deduced loss of speed from the curves are only 'accurate' so long as the mean sea state period is not close to the pitch or heave resonance period of the ship, since maximum added resistance due to waves occurs in these
circumstances. It is therefore likely that the curves may seriously underestimate the speed in these cases. Also, Titlow states that although these curves have proved durable for performance assessment, they do not permit the proper treatment of a confused seaway, OCEANROUTES (1986).

Typically three curves, one each for head, beam and following sea encounters are depicted, see figure 3.1, thereby introducing speed estimation errors when encounter angles are other than these idealised ones. Further, since the curves are only functions of two parameters of the sea, differing seaways will still result in the same speed reduction, although they are of equal significant wave height. However, the curves, based on a constant power, are easily determined, quick, and incorporate the master's voluntary reduction, if sufficient observations are available in the regression of the log data.

The speed of the vessel can be represented by:

\[ V_{est} = f\left( \zeta_{1/3}, \mu_{wave}, \psi \right) \]

\[ f\left( \zeta_{1/3}, \mu_{wave}, \psi \right) = \alpha_{1,0} + \alpha_{1,1} \zeta_{1/3} + \alpha_{1,2} \zeta_{2/3} + \cdots + \alpha_{1,n} \zeta_{1/3}^n \]

Where:

- \( \alpha_1 \) are coefficients of the polynomial for
- \( j = 0,1,2,\ldots, n-1,n \)
- \( i = 1; \) Head sea case.
- \( i = 2; \) Beam sea case.
- \( i = 3; \) Following sea case.

3.3. AERTSSEN'S SPEED LOSS ALGORITHM.

Aertssen (1969), proposed an algorithm to predict the speed loss of a vessel, as a function of the environment. The algorithm was based on regression analyses of many hundreds of ship's logs and is similar to the James curves. However, instead of modelling a particular vessel, it uses the length between perpendiculars, \( L_{PP} \), as the ship descriptor. Therefore, all ships of the same length will have the same speed reduction, which seriously limits the accuracy of the algorithm.
The environment is modelled as a function of the Beaufort number, BN, in polynomial form. There is considerable error in basing the algorithm on BN since this covers a band of possible sea heights, therefore leading to a non-smooth speed loss curve, see figure 3.2. Wave height is the primary cause of ship speed loss, and therefore a closer representation of the seaway is required.

The percentage speed loss of the vessel, (knots) is defined as:

\[
\frac{\Delta V_{\text{ship}}}{V_{\text{ship}}} \times 100\%
\]

Aertssen (1969,1975), proposed the percentage loss as:

\[
\frac{\Delta V_{\text{ship}}}{V_{\text{ship}}} \times 100\% = \frac{m}{L_{pp}} + n
\]

<table>
<thead>
<tr>
<th>HEAD SEA 30° off Bow</th>
<th>BOW SEA 30° - 60° off Bow</th>
<th>BEAM SEA 60°-150° off Bow</th>
<th>FOLLOWING SEA 150°-180° off Bow</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
<td>m</td>
<td>n</td>
<td>m</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>5</td>
<td>900</td>
<td>2</td>
<td>700</td>
</tr>
<tr>
<td>6</td>
<td>1300</td>
<td>6</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>2100</td>
<td>11</td>
<td>1400</td>
</tr>
<tr>
<td>8</td>
<td>3600</td>
<td>18</td>
<td>2300</td>
</tr>
</tbody>
</table>

 TABLE 3.2 PERCENTAGE SPEED LOSS COEFFICIENTS AFTER AERTSSEN (1969).

The algorithm does not account for the fullness of the ship's hull or underwater shape and could lead to serious error, Townsin et al (1983), since the frictional resistance to motion is a direct function of the wetted surface area. Routeing examples using the Aertssen formulae have been attempted and compared with those algorithms defined by Babbedge (1975) and Townsin et al (1983), for the DART ATLANTIC/EUROPE container ship. Figure 3.2 gives the speed loss of the DART ATLANTIC/EUROPE computed by the Aertssen algorithm.
3.4 TOWNSEND ET AL SPEED LOSS ALGORITHM.

Townsend et al (1975,1983), devised a speed loss algorithm based upon the results of the addition of resistance terms. The methods employed were; wind resistance using Van Berlekom’s (1981), algorithm, wave reflection resistance and ship motion resistance, but only for a head sea encounter situation. Speed loss at other encounter angles is computed as a function of the head sea case by adopting a correction factor based upon the work of Aertssen (1969). This algorithm is based on much theoretical work and does not require still water powering or speed computation, since it assumes a ship speed.

Formulae were derived for container vessels and both laden and ballasted tanker ships, with results compared to full-scale trials. The formulae account for ship fullness by adopting volume of displacement as the principle ship dimension, which determines more accurately, the wetted surface area of the hull, and therefore the frictional resistance.
The percentage speed loss is given by,

$$\mu_{\text{corr}} = \frac{\Delta V_{\text{ship}}}{V_{\text{ship}}} \cdot 100\%$$  \hspace{1cm} (3.4.1)

Where, \( \mu_{\text{corr}} \) - Correction factor for encounter angles other than head sea.

The loss of speed, in a head sea is computed by:

- **Tankers laden.**
  \[
  0.5 \text{ BN} + \frac{\text{BN}^{6.5}}{2.7 \ \sqrt[V]{\text{g} / \text{m} / \text{s}^2}}
  \hspace{1cm} (3.4.2)
  \]

- **Tankers ballasted.**
  \[
  0.7 \text{ BN} + \frac{\text{BN}^{6.5}}{2.7 \ \sqrt[V]{\text{g} / \text{m} / \text{s}^2}}
  \hspace{1cm} (3.4.3)
  \]

- **Container Vessels.**
  \[
  0.7 \text{ BN} + \frac{\text{BN}^{6.5}}{22 \ \sqrt[V]{\text{g} / \text{m} / \text{s}^2}}
  \hspace{1cm} (3.4.4)
  \]

The weather correction factors, \( \mu_{\text{corr}} \) are:

- \( 2 \mu_{\text{corrbow}} = 1.7 - 0.03 (\text{BN} - 4)^2 \) for 30° - 60°  \hspace{1cm} (3.4.5)

- \( 2 \mu_{\text{corrbeam}} = 0.9 - 0.06 (\text{BN} - 6)^2 \) for 60° - 150°  \hspace{1cm} (3.4.6)

- \( 2 \mu_{\text{corrfollowing}} = 0.4 - 0.03 (\text{BN} - 8)^2 \) for 150° - 180°  \hspace{1cm} (3.4.7)

Townsin et al (1983) compared results to theoretical values using a full resistance algorithm and also to full-scale results from Aertssen (1972) for the DART EUROPE, indicating reasonable accuracy. Figure 3.3 shows a comparison to the speed loss computed for the DART ATLANTIC from equation 3.5.2, although this was difficult due to differing wave height, wind speed and BN relationships.

The use of BN to determine the environmental factors is prone to inaccuracy, see section 3.3 for a discussion. The incorporation of any effects due to the master cannot be made and likewise any speed reductions resulting from long-crested seaways. It was felt that the equations were not applicable to the routeing strategy since :-
1. They apply only to environments described by Beaufort number, which describe a very wide variety of possible seaways;

2. They yield an approximation of loss of speed, based only on the volume of displacement of the ship;

3. They only hold for $BN < 7/8$, whereas there may be a situation where a calculation is required outside these values;

4. It may be incorrect to assume that the approximations hold good for all those vessels which come under the categories;

5. Routeing objectives involving cost or fuel or power cannot be accommodated, since speed is the only dependant variable.

![SPEED LOSS FOR THE DART ATLANTIC AFTER BABBEDGE (1975) AND TOWNSIN (1983)](image)

**FIGURE 3.3 SPEED LOSS OF THE DART EUROPE/ATLANTIC AFTER BABBEDGE (1975) AND AFTER TOWNSIN ET AL (1983).**

A constant speed algorithm could be generated by using the Townsin (1983) speed loss model, to predict the fraction of speed loss. By computing the calm water powering of the vessel from the Holtrop (1984) algorithms, it would be possible to increase the calm water power by that fraction to maintain speed. The required power becomes the sum of the calm water and the fraction values.
3.5. BABBEDGE SPEED LOSS ALGORITHM.

The Babbedge (1975) algorithms for the loss of speed combine the advantages of the previous algorithms, since they are particular to the ship. The formulae, incorporate many environmental parameters, ship's power (assumed as shaft power) and displacement. The algorithms can therefore be adapted for optimum fuel or cost routeing. One would expect the algorithms to be more accurate than previous models, since the seaway is better described. However, significant wave height, and encounter angle, together with relative wind speed and direction, still cover a multitude of possible seaway scenarios.

The regression analyses of log data were carried out in order to establish the main causes for the loss of speed and thus incorporate them in any equation. The form of these are, (speed units knots):

\[ V_{\text{ship}} = V_{\text{calm}} - (\text{wave factor}) - (\text{wind factor}) \]

Equation 3.5.2 was derived for the DART ATLANTIC. (Speed-knots, Power-Bhp).

\[
V_{\text{ship}} = 19.93 + 6.44 \left( \ln P_{\text{pow}} - \ln 20000 \right) - 10.2 \frac{V_{\text{calm}}}{P_{\text{pow}}} \left[ \cos \left( \frac{\mu_{\text{wave}}}{2} \right) + 0.3 \right] \\
- 0.28 \frac{V_{\text{calm}}}{P_{\text{pow}}} \left[ \left( \frac{U_{\text{relw}}}{V_{\text{calm}}} \right)^2 \cos \mu_{\text{relw}} - 1 \right] + 0.04 (T_{\text{sea}} - 12) \\
- 0.00012 (N - 37000) \pm 0.27
\]

Since power features in the algorithm it is possible to compute the calm water speed for a given engine setting and displacement. Figure 3.4 illustrates the speed loss at constant power, however, an increase or decrease in power will shift the origin of the y-axis.
FIGURE 3.4 SPEED LOSS OF THE DART ATLANTIC AFTER BABBEDGE (1975).

3.5.1 EFFECTS OF VARIABLES ON THE BABBEDGE ALGORITHM.

Equations of the form 3.5.1 and 3.5.2 are ideal for micro-based weather routeing, since they are simple and can be used to route for cost, fuel or time. Similarly, they can be accepted as reasonably accurate since they are based on regressional interpretations of measured data.

Since the routeing model requires many iterations, the performance algorithm has to be efficient. Due to the burden placed on the ship algorithm within the routeing algorithm, a series of tests were carried out to determine the contribution to the speed loss from the individual units of equation 3.5.2. A series of curves were obtained by varying $\zeta_{1/3}$, $\mu_{\text{wave}}$, $U_{\text{wind}}$, $\mu_{\text{wind}}$ and removing variables in turn. Wind speed was computed from wave height by the Scott (1962) formula, equation 4.3.5, as used by Babbedge (1975), with $\mu_{\text{wave}} = \mu_{\text{wind}}$ (note, the Scott formula used by Babbedge did not include the constant value for swell). It was decided not to investigate the effect of different wind/wave formulae on equation 3.5.2, since wind values are provided in the routeing algorithm, from the data source.
Figure 3.5 shows these results. Speeds estimated by equation 3.5.2 have been extended beyond the measured period and are expected to overestimate, since it is likely that the master would reduce engine power in higher sea states. Without any knowledge of this, any extension was assumed to be true and no further reduction applied. Since the curves pertain to the ship, they incorporate master's effects within the measured region, and so algorithm the system in a more accurate manner than previous algorithms.

Sea temperature has a small effect upon speed estimation, and could therefore be removed from the equation. Similarly, since it is not readily accessible and would add to data storage and transmission problems, it was deemed unnecessary. Any weather routeing for avoidance of heavy weather will override the effects of sea temperature. Ship displacement, however, is more readily accessible, and has a greater effect upon the route between laden and light ship, most likely because of ship motions.

**Figure 3.5 Effects of removing variables from the Babbedge (1975) algorithm.**
3.5.2. POWERING FROM THE BABBEDGE EQUATIONS.

Powering of the DART ATLANTIC results from a rearrangement of equation 3.5.2, involving an iterative procedure, since calm water speed has to be computed from a given power to obtain speed loss. Figure 3.6 illustrates the iteration scheme.

**FIGURE 3.6 POWERING ALGORITHM FROM EQUATION 3.5.2.**

**Figure 3.7** INCREASE IN POWER TO MAINTAIN SPEED AFTER BABBEDGE (1975)

**FIGURE 3.7 INCREASE IN POWER DERIVED FROM EQUATION 3.5.2, IN ORDER TO MAINTAIN SPEED.**
Such a simple algorithm has been used in conjunction with the engine operating envelope and polynomials describing the propeller law and specific fuel consumption, equations 3.20.1, 3.20.2, to compute the minimum fuel and cost routes. Examples of such routes are shown in chapters 6, 7 and 8.

3.6. SEMI-EMPIRICAL/THEORETICAL SHIP POWERING ALGORITHM.

There are three analytical methods to compute the powering and speed of a vessel in a seaway, Bhattacharyya, (1978), under either, constant speed or constant thrust.

1. Direct power method.
2. Torque-revolutions per minute method.
3. Thrust method.

This study assumes constant thrust is constant power, however, in reality engine power or revolutions will vary under the influence of the random seaway, due to fluctuating propeller load, Vassilopoulos (1971).

Each method is computed from;

1. Predictions of power from measurements on the shaft at the propeller, therefore incorporating any effects for the seaway;
2. Predictions of power from measurements of the engine torque and revolutions, again incorporating any effects for the seaway;
3. Predictions of power from measurements of the resistance/thrust of the vessel. This means no fluctuations are accounted for.

The theoretical computation of powering and speed is found from a balance of the thrust through the propeller to the resistance to motion. The total resistance is augmented from its individual parts.
Total resistance:

\[ R_T = R_f(1+k) + R_{rudder} + R_{yaw} + R_w + R_{tr} + R_{app} + R_a + R_b + R_{AR} + R_{wind} \]

\[ R_T = R_{CALM} + \Delta R_{SEAWAY/WIND} \]

Where:

- \( R_f \) - Frictional resistance according to the ITTC - 1957 formula.
- \( 1+k \) - Form factor of the hull. \( R_f(1+k) \) - Viscous resistance.
- \( R_{rudder} \) - Rudder resistance.
- \( R_{yaw} \) - Yaw resistance.
- \( R_w \) - Wave making resistance.
- \( R_{tr} \) - Additional pressure resistance due to transom immersion.
- \( R_{app} \) - Appendage resistance.
- \( R_a \) - Model-ship correlation resistance.
- \( R_b \) - Additional pressure resistance of bulbous bow near the water surface.
- \( R_{AR} \) - Added resistance due to seaway.
- \( R_{wind} \) - Wind induced resistance.

The thrust produced through the propeller is developed from the engine power, propeller and hull efficiencies. In regular or irregular seaways the thrust can be regarded as \( T + \Delta T \), in a similar fashion to the total resistance.

Added resistance, \( (AR) \), is independant of calm water resistance, Lewis (1988), and is attributable to the motions of ships and to voluntary reductions in engine power, in order that violent ship motions are eased. The various factors responsible for \( AR \) are:

1. Wind on the hull and superstructure;
2. Increased resistance due to motions (heaving and pitching have a greater effect since they are longitudinal forces);
3. Wave reflection on the hull;
4. Drift angle or side slip, or yawing and swaying caused by rudder movement;
5. Loss of propulsive efficiency, since the propeller acts in a random seaway. The calm water characteristics are altered. Bhattachyyra (1978) states those effects which cause changes in the calm water operating conditions;
6. Increased hull and propeller roughness.

Voluntary reductions in engine power are made from observations of the exceedance of critical motions resulting from :

1. Shipping of green water, resulting in deck wetness, per hour;
2. Slamming of the bow, usually taken as the third station from the forward perpendicular, (FP). The slams per hour, was amended to whipping acceleration experienced over the full ship’s length. Apparant improvements could have been made by moving the bridge to a point where no response was experienced, Andrew et al (1981);
3. Excessive roll, and acceleration causing cargo shifting. Number of excessive rolls per hour;
4. Propeller racing, or propeller emergence per hour;
5. Subjective motion at the wheelhouse;
6. Track keeping.

This list is not comprehensive and it is important to stress that not all are particular to any one ship. Speed loss is power-limited when motions are acceptable; however a ship is motion-limited beyond those criteria. Speed loss or added power in a seaway can therefore be regarded graphically in figure 3.8.

The motions of the vessel, including AR, and subsequent motion exceedance criteria, are computed from an integration of the response amplitude operators,
(RAOs), of the vessel and an idealised spectrum of the seaway. Such integration is performed by linear superposition, St. Denis et al (1953).

**FIGURE 3.8. SPEED LOSS OR ADDED POWER IN A SEAWAY AFTER BHATTACHYRRA (1978).**

Linear superposition assumes the mean response to be directly proportional to the wave height for a given wavelength, (except for added resistance). This only applies to the powering assumptions at constant speed, that is, at constant power the vessel will lose speed and therefore reduce encounter frequency to waves in increasing waveheights. (It is stressed that the theory is linear).

This becomes more involved in an irregular seaway, since each wave component will cause fluctuations in power and speed. Any increase in power to overcome one component will not be that required to maintain speed against another. Similarly, thrust and resistances will vary randomly, causing changes in propeller loading and engine power. However, Bhattachyrra (1978) and Lewis (1988) state that the mean characteristics of the propeller in the seaway do not vary significantly from the calm water situation.
The 1978-ITTC recommended a power-prediction method for single screw ships, Lindgren et al (1980), based on three steps. Firstly, computation of the total resistance of the ship, including allowances for hull roughness, based upon the summation of the individual resistance parts. Secondly, computation of the propeller characteristics, thrust and torque coefficients, $K_T$, $K_Q$ and the open water efficiency, $\eta_0$, corrected for propeller size and roughness. Finally, computation of shaft power $P_S$ and engine revolutions from full-scale resistance, propeller and propulsive characteristics. Lindgren et al (1980) pointed out the short-comings in this method which are highlighted in the forthcoming text. Models developed along these lines include those by Journée (1976), Journée et al (1980).

A similar method has been adopted in this study to compute either ship speed when operating with constant revolutions, or increase in power and engine revolutions to maintain speed in varying seaways. The algorithm is based on the Holtrop et al (1980,1982) and Holtrop (1984) calm water resistance and propulsive coefficients, wind resistance after Isherwood (1972), added resistance from BRITSEA, propeller characteristics from Oosterveld et al (1975) and engine characteristics taken from the full-scale test bed data, Sulzer (1989). The algorithm is similar to the thrust method since resistance is used to compute the thrust of the propeller, based on set engine revolutions/power and vessel speed.

Several assumptions are made in the algorithm that can account for inaccurate prediction. However, without recourse to self propulsion tests, the 1978-ITTC method suffices. Primarily, the assumption that open water characteristics of the propeller remain unaffected by the seaway is incorrect and can lead to error, approximately 40% in extreme cases, Bhattachyrra (1978).

3.7. CALM WATER RESISTANCES.

The resistance to motion of the vessel in a calm seaway is regarded as a summation of the individual parts, Holtrop et al (1982,) and Holtrop (1984):
\[ R_{\text{total}} = R_f (1.0 + k_l) + R_{\text{app}} + R_w + R_b + R_{tr} + R_a \]

Units defined in section 3.6.

Such an algorithm was investigated based on regression analyses of random model and full-scale test data, Holtrop et al (1980, 1982) and Holtrop (1984). There follows a brief discussion of each resistance and powering element with results deduced from the study. The integration of the elements into an algorithm are shown in sections 3.21.3 and 3.21.4 for constant speed or power solutions.

3.7.1. FRICTIONAL RESISTANCE.

Frictional resistance is determined by integrating the tangential stresses over the wetted surface of the ship's hull, in the direction of motion. The flow of fluid around the hull is not ideal but is turbulent, (boundary layer), and shears along the normal from the hull. Ideal flow along the hull has negligible friction, whereas turbulent flow caused by the drag of water by the hull has frictional resistance.

The frictional resistance is defined as:

\[ R_f = C_f (0.5 \rho V_{\text{ship}}^2) S \]

From which, \( C_f \) - Specific frictional resistance or drag coefficient.

\( S \) - Wetted surface area of the hull.

There were many proposals for formulae to compute \( C_f \), as defined in figure 3.9. The adopted proposal was:

\[ C_f = \frac{0.075}{(\log_{10} R_n - 2.0)^2} \]

'The ITTC-1957 model-ship correlation line', yields a two-dimensional flow value and is an interim solution to the viscous frictional resistance, \((1+k)R_f\), of the hull. Hughes (1954) proposed a system to compute the \( R_f \), based on the ITTC-1957 line and a hull form factor.
Where $C_f^2$ - Two-dimensional frictional coefficient defined by Hughes (1954).


$R_f$ is dependant upon the Reynold number, $R_n$, which is computed by:-

$$R_n = \frac{V_{\text{ship}} L}{v}$$

Where $v$ - Viscous coefficient = $1.188 \times 10^{-6} \text{ m s}^{-1}$.
The wetted surface area of the hull, \( S \), can be approximated, Holtrop et al (1982), by:

\[
S = L(2.0T+B)\sqrt{C_m(0.453+0.4425C_B-0.2862C_m-0.003467\frac{B+0.3696C_{wp}}{T_m})+2.38\frac{A_B}{C_B}}
\]

3.7.6

Figure 3.10 indicates the viscous resistance for the DART ATLANTIC/EUROPE.

3.7.2. FORM FACTOR OF THE HULL.

This factor, assumed independent of Reynolds number, Lewis (1988), and recommended by the ITTC-1978, relates the two-dimensional frictional resistance \( R_f \) to the three-dimensional ship's hull. \((1+k_f)\) after Holtrop et al (1980).

\[
(1 + k_f) = C_{13} \left[ 0.93 + C_{12} \left( \frac{B}{L} \right)^{0.92497} \left( 0.95-C_p \right)^{-0.521448} (1-C_p+0.0225/cb)^{0.6906} \right]
\]

3.7.7
The length of run parameter, \( L_R \), and coefficients, \( C_{12} \) and \( C_{13} \) are computed according to:

\[
L_R = L \left\{ 1.0 - C_p + \frac{0.06C_{plcb}}{4C_p - 1.0} \right\} \quad \text{and} \quad C_{12} = \left( \frac{T_m}{L} \right)^{0.2228446} \quad \frac{T_m}{L} > 0.05
\]

\[
C_{12} = 0.479948 \quad \frac{T_m}{L} < 0.02
\]

\[
C_{12} = 48.20 \left( \frac{T_m}{L} - 0.02 \right)^{2.078} + 0.479948 \quad 0.02 < \frac{T_m}{L} < 0.05
\]

\[ C_{13} = 1.0 + 0.003C_{\text{stern}} \text{ where } C_{\text{stern}} \text{ was taken as 0.0, therefore, } C_{13} = 1.0 \]

Lindgren et al (1980) states that due to a scatter of the resistance values at lower ship speeds, the determination of form factor is difficult, and proposed several alternatives to its determination, particularly, Prohaska (1966). The assumption that form factor is independent of the Reynolds number could lead to error as it is a simplification.

3.7.3. APPENDAGE RESISTANCE.

The appendage resistance, in single screw ships, is due to the bilge keel and rudder, Lewis (1988), who also states that it can be kept to little more than the additional wetted surface area, (roughness), approximately 1-3% of the total. The rudder resistance varies depending upon the velocity of water over the surface, (wake effects). For model propulsion tests, it is usual to ignore rudder resistance in the race as the propulsive efficiency of the propeller absorbs the error.

\[
R_{\text{app}} = 0.5 \rho_{\text{sea}} V_{\text{ship}} S_{\text{app}} (1 + k_2)_{\text{eq}} C_f
\]

Where, the appendage resistance form factor is seen to be a summation.

\[
(1 + k_2)_{\text{eq}} = \frac{\Sigma (1.0 + k_2) S_{\text{app}}}{\Sigma S_{\text{app}}}
\]

The appendage resistance is increased by bow thruster openings, according to:

- 50 -
Where \( d \) = tunnel diameter and \( C_{BTO} = 0.003 - 0.012 \). Since no knowledge of bow thrusters was available, it was ignored. Values for bilge keel height and length were given, therefore the appendage resistance as is shown in figure 3.11.

![Viscous Appendage Resistance Graph](image)


### 3.7.4. WAVE MAKING RESISTANCE.

Wave making resistance comprises of two parts, wave making and wave breaking terms. Harvald (1978), states that only the former term, the largest, is considered. Wave making resistance has been calculated theoretically by:

1. The fore and aft components of the normal pressure distribution to the ship's hull;
2. The flow of energy necessary to maintain the wave pattern far astern of the ship.
Figure 3.12 indicates the resistance due to wave making for the DART ATLANTIC/EUROPE from the Holtrop (1984) algorithm.

Increases in ship speed causes generated waves to have a larger wavelength. Since there are two systems, bow and stern waves, there are critical speeds where constructive or destructive interference occurs. At such points there is an increase or decrease in the relative resistance leading to humps and hollows in the total resistance curve. Holtrop (1984) reanalysed the wave making regression formula, to better match this phenomenon. Better correlation was found by analysing two speed regimes, above and below $F_n = 0.5$.

\[ F_n > 0.55 \quad R_{w-b} = C_1C_2C_5\varphi_{\text{sea}} g e^{-\left( m_3F_n^d + m_4\cos(\lambda F_n^2) \right)} \]

\[ F_n < 0.4 \quad R_{w-a} = C_1C_2C_5\varphi_{\text{sea}} g e^{-\left( m_1F_n^d + m_4\cos(\lambda F_n^2) \right)} \]
For the range $0.4 < F_n < 0.55$, an interpolation is required.

$$R_w = R_{w-a_{0.4}} + \frac{(10.0F_n - 4.0)(R_{w-b_{0.55}} - R_{w-a_{0.4}})}{1.5}$$  \hspace{1cm} 3.7.15

A reduction in resistance due to the bulbous bow is accomplished with $C_3$, equation 3.7.17, since the bulbous bow reduces wave breaking resistance at low speeds, and improves wave interference at high speeds, Harvald (1978). The immersion of the transom causes additional resistance due to an increased wetted surface area. This resistance term is accounted for in the $C_5$ coefficient. The Holtrop coefficients for $R_{w-a}$ and $R_{w-b}$ are:

$$C_1 = 2223105C_7^{3.78613}(\frac{T}{B})^{1.07961}(90.0-i_0)^{-1.37565}$$  \hspace{1cm} 3.7.16

$$C_2 = e^{(-1.89\sqrt{C_3})} ; \quad C_3 = \frac{0.56A_{BT}^{1.5}}{\{BT(0.31\sqrt{A_{BT}} + T_F - h_B)\}} ; \quad C_5 = 1.0 - \frac{0.8A_T}{(BTC_M)}$$  \hspace{1cm} 3.7.17 : 3.7.18 : 3.7.19

$$C_7 = 0.229577(\frac{B}{L})^{0.33333} \text{, for } \frac{B}{L} < 0.11, \quad C_7 = \frac{B}{L} \text{, for } 0.11 < \frac{B}{L} < 0.25, \text{ or}$$

$$C_7 = 0.5 - 0.0625 \frac{L}{B} \text{ when } \frac{B}{L} > 0.25$$  \hspace{1cm} 3.7.20

$$C_{15} = -1.69385, \text{ when } \frac{L^3}{\nabla} < 512.0 \text{ or } \quad C_{15} = -1.69385 + \left(\frac{\left(\frac{L}{\nabla^{1/3}} - 8.0\right)}{2.36}\right), \text{ when}$$

$$512 < \frac{L^3}{\nabla} < 1726.91, \text{ or } C_{15} = 0 \text{ when } \frac{L^3}{\nabla} > 1726.9$$  \hspace{1cm} 3.7.21

$$C_{16} = 8.07981C_p - 13.8673C_p^2 + 6.984388C_p^3, \text{ for, } C_p < 0.8, \text{ or,}$$

$$C_{16} = 1.73014 - 0.7067C_p \text{ when } C_p > 0.8$$  \hspace{1cm} 3.7.22

$$C_{17} = 6919.3C_m^{-1.3346}\left(\frac{\nabla}{L^3}\right)^{2.0097}\left(\frac{L}{B} - 2.0\right)^{1.40692}$$  \hspace{1cm} 3.7.23

- 53 -
\[ m_1 = 0.0140407 \frac{L}{T_m} - 1.75254 \frac{\nabla^{1/3}}{L} - 4.79323 \frac{B}{L} - C_{16} \quad 3.7.24 \]

\[ m_3 = -7.2035 \left( \frac{B}{L} \right)^{0.326869} \left( \frac{T_m}{B} \right)^{0.605375} \quad 3.7.25 \]

\[ m_4 = C_{15} 0.4e^{-0.034F^{-3.29}} \quad 3.7.26 \]

All other parameters are accounted for by:

\[ \lambda = 1.446C_p - 0.03 \frac{L}{B}, \text{ for, } \frac{L}{B} < 12.0, \text{ or, } \lambda = 1.446C_p - 0.36, \text{ for } \frac{L}{B} > 12.0 \quad 3.7.27 \]

The half angle of entrance, \( i_e \), is that made at the waterline, by the bow, by neglecting the stem, and unless known, it is approximated by:

\[
\left( \frac{L}{B} \right)^{0.80856} \left( 1.0 - C_{wp} \right)^{0.30484} \left( 1.0 - C_p - 0.0225/cb \right)^{0.63767} \left( \frac{L}{B} \right)^{0.34574} \left( \frac{100 \nabla}{L} \right)^{0.16302} \\

i_e = 1 + 0.89e \quad 3.7.28
\]

Holtrop (1984) states that care has to be taken as \( i_e \) yielded negative values for exceptional hull form parameters.

### 3.7.5. ADDITIONAL RESISTANCE DUE TO BULBOUS BOW

Lewis (1988), describes in detail the effect that the bulbous bow has on reducing the wavemaking resistance, however, additional resistance is attributed to the wetted surface area of the bulb. The value is dependant on speed as determined by the Froude number. According to Holtrop et al (1982), the additional resistance due to the bulb near the surface is determined from:

\[
R_b = \frac{0.11 e^{-3.0P_B^{-2}} F_{ni} A_B r_{sea} g}{(1.0 + F_{ni}^2)} \quad 3.7.29
\]

\( P_B \) is a measure of the emergence of the bow, and \( F_{ni} \), is the Froude number based on the immersion. Thus:
The additional resistance due to the bulbous bow is shown in figure 3.13.

\[ P_B = \frac{0.56v_A}{1.5h_B} \quad \text{for} \quad F_{ni} = \frac{V_{ship}}{\sqrt{g(T_F - h_B - 0.25v_A + 0.15v^2)}} \]

3.7.30 : 3.7.31

The additional pressure resistance of Dart Atlantic/Europe due to bulb near free surface from Holtrop algorithms.


3.7.6. ADDITIONAL PRESSURE RESISTANCE FROM THE IMMERSED TRANSOM.

The immersion of the transom causes additional resistance and is a function of vessel speed and transom area. Holtrop et al (1982) define the resistance as:

\[ R_{TR} = 0.5\rho_{sea} \frac{V_{ship}^2}{A_T} C_6 \]

3.7.32

The coefficient \( C_6 \) is determined from:

\[ C_6 = 0.2(1.0 - 0.2F_{ni}) \quad \text{when} \quad F_{ni} < 5.0, \quad \text{or} \quad C_6 = 0.0 \quad \text{when} \quad F_{ni} \geq 5.0 \]

3.7.33
Similarly,

\[ F_n = \frac{V_{\text{ship}}}{\sqrt{\frac{2.0gA_T}{(B + B_{\text{wp}})}}} \]  

Figure 3.14 shows the additional resistance due to the immersed transom.


3.7.7. MODEL-SHIP CORRELATION RESISTANCE.

This resistance, \( R_a \), is designed to account for the still air and hull roughness resistances, Holtrop et al. (1982). Lewis, (1988) states that the resistance is a correction made to the cumulative totals of all other resistances, since these describe a smooth hull. It also accounts for errors in the extrapolation method of other resistances, therefore, it can only be used in the formula for \( R_a \) specifically.
Model-ship resistance is regarded to be a correction for the lack of knowledge in other resistance fields. Hull roughness accounts for the greatest effect, and is a function of hull corrosion, time in port, marine growth, and paint roughness. Lewis (1988) states that Atlantic liners do not foul greatly, since their time in port is limited, however, the length of time since drydock has a considerable effect.

\[
R_a = 0.5 \rho_{\text{sea}} V_{\text{ship}}^2 SC_a
\]

\[
C_a = 0.0006(L + 100.0)^{-0.16} - 0.00205 + 0.003 \left( \frac{L}{0.75} \right)^{0.4} C_2 (0.04 - C_4)
\]

Where, \(C_2\) is defined earlier, and \(C_4 = \frac{T_F}{L}\) when \(\frac{T_F}{L} \leq 0.04\) and \(C_4 = 0.04\) when \(\frac{T_F}{L} > 0.04\).

Lewis (1988) states that \(C_a\) can be used in conjunction with the 1978-ITTC line. Holtrop et al (1982) show that this can be modified by an expression to increase the hull roughness for values beyond \(k_s = 150 \mu m\).

\[
\text{increase in } C_a = \frac{(0.105 k_s^{1/3} - 0.005579)}{L^{1/3}}
\]

The roughness length, \(k_s\), can only be determined by measurements of the hull profile. Since the roughness of the hull cannot be deduced in this study, it was assumed that the hull has a roughness parameter of 150\(\mu m\), recommended by the 1978-ITTC, Lindgren (1980). Svenson et al (1987) expand upon the effect of increased hull and propeller roughness, typically, the effect on the propulsive efficiency.

Since hull roughness was unknown, the simple additional resistance for fouling was investigated. Evans et al (1984, 1989), consider fouling and hull deteriorative effects for computation of service margins on certain trade routes.
Figure 3.15 shows the model-ship correlation resistance for the DART ATLANTIC/EUROPE.

![Graph of Model Ship Correlation Resistance](image)

**Figure 3.15. Model Ship Correlation Resistance.**

3.7.8. FOULING RESISTANCE.

If no knowledge of the hull roughness is known it is possible to include a fouling resistance, based on time out of dry dock. Journée et al (1980), defined the resistance due to fouling as:

\[
\frac{\Delta R_f}{R_f} \times 100\% = 3.6y_a + \frac{40y_d}{1 + 2y_d}
\]

Where

- \( y_a \) - ship age in years.
- \( y_d \) - years since the last dry docking.

This function was deduced by Aertssen (1969), for an Atlantic route. Effects of fouling depend upon the contribution to the total resistance that the friction resistance makes. Similarly, the contribution will vary depending upon the loaded
state of the vessel. The effect on the flat plate frictional resistance is shown in figure 3.16 for the DART ATLANTIC/EUROPE.

![Flat Plate Resistance with Fouling Compensation](image)

**FIGURE 3.16. EFFECT OF ADDING FOULING RESISTANCE TO THE FLAT PLATE FRICIONAL RESISTANCE.**

**3.7.9. ADDED RESISTANCE DUE TO STEERING.**

Waves cause the heading to be offset, and rudder is required to counteract both this and the wind effect. Van Berlekom (1981) has defined a rudder resistance and yawing resistance as functions of yaw amplitude and rudder angle. Journee et al (1980) defines a steering resistance as a function of yaw amplitude and displacement. These resistance terms were not considered as it was felt that they are not as important as calm water and added resistances, (Journee et al (1980) quotes ~ 1.3% of calm water resistance for 2° yaw amplitude per minute.
3.8 WIND RESISTANCE.

There have been two significant proposals for the estimation of resistance due to the above water structure of the ship; Isherwood (1972) and Van Berlekom (1975, 1978, 1981). Lewis (1988) states that the resistance depends upon the ship's speed, projected area of the above water structure and wind speed and direction. Beukelmann et al (1974) found reasonable results from Isherwood’s algorithm (1972) for containerships. The apparent wind is given as the vectoral summation of the ship’s speed and true wind speed, see figure 3.17.

\begin{itemize}
  \item $U_{\text{wind}}$ - True wind speed.
  \item $U_{\text{ship}}$ - Ship's speed.
  \item $U_{\text{relw}}$ - Relative wind speed.
\end{itemize}

**FIGURE 3.17. RELATIVE WIND FORCE.**

For a head wind, the resistance will be totally in the X-plane or longitudinal plane and dependant upon the transverse area, $A_T$. For beam winds the longitudinal area, $A_L$, is dependant and does not act in the direction of motion, moreover it results in a sideways slip or leeway. Van Berlekom (1981) concluded that the leeway is less important than the resistance to motion. The fore and aft, lateral and yawing moment components of wind resistance are defined by:

\begin{align*}
  R_x &= 0.5C_x p_{\text{air}} A_T S U_{\text{relw}}^2 \\
  R_y &= 0.5C_y p_{\text{air}} A_L S U_{\text{relw}}^2 \\
  N_m &= 0.5C_n p_{\text{air}} A_L S U_{\text{relw}}^2 L_oa
\end{align*} 

\[3.8.1 \quad 3.8.2 \quad 3.8.3\]
Where

\[ C_x = \text{Fore and aft resistance coefficient.} \]
\[ C_y = \text{Lateral resistance coefficient.} \]
\[ C_n = \text{Coefficient of wind-induced yawing moment.} \]

The yawing moment term, \( N_{ym} \), along acts from a point amid-ships with wind speeds defined at a height, \( z \). Isherwood (1972) took the results of 107 ship models and through a multiple regression analysis defined equations to compute the coefficients in equations 3.8.1, 3.8.2 and 3.8.3.

The additional resistance caused by a counter-rudder to offset the yawing moment has not been considered, therefore, only the fore and aft component is computed.

\textbf{Fore and Aft Component of Wind Force.}

\[ C_x = A_0 + A_1 \frac{2.0A_L S}{L^2} + A_2 \frac{2.0A_T S}{B^2} + A_3 \frac{L}{B} + A_4 \frac{S_p}{L} + A_5 \frac{C}{L} + A_6 M \pm 1.96S.E \]

Where,

\( S_p \) - Length of perimeter of lateral projection excluding waterline and slender bodies such as masts or kingposts.

\( C \) - Distance from bow of centroid of lateral projected area.

\( M \) - Number of distinct groups of masts or kingposts seen in the lateral projection.

\( A \) - Isherwood (1972) coefficients. \( S.E \) - Standard error.

The constants, \( A_0 - A_6 \), are tabulated by relative encounter angle, see Isherwood (1972). Since some of the independent variables, above, are difficult to determine, Isherwood (1972) defines mean values for several ship types. Unfortunately, container ships were not included, although the values for passenger ships and ferries were assumed due to their hull and superstructure similarities when loaded. Therefore :-
### TABLE 3.3. VALUES OF INDEPENDENT VARIABLES.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>$\frac{2.0A_{LS}}{B^2}$</th>
<th>$\frac{2.0A_{TS}}{B^2}$</th>
<th>$\frac{L}{B}$</th>
<th>$\frac{S}{L}$</th>
<th>$C_L$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isherwood values</td>
<td>0.192</td>
<td>1.95</td>
<td>7.66</td>
<td>1.44</td>
<td>0.492</td>
<td>2.0</td>
</tr>
<tr>
<td>Computed values if available</td>
<td>0.131*</td>
<td>1.61*</td>
<td>7.81</td>
<td>--</td>
<td>0.500£</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>0.119ⅱ</td>
<td>1.60ⅱ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key:
- * - Value computed assuming 3 layers of containers on the deck.
- ⅱ - Value computed assuming 1 layer of containers on deck.
- £ - Value computed assuming $C=0.5L$.

Aertssen (1972), in Isherwood’s paper (1972), compared the results of model tests of the DART EUROPE to those computed from equation 3.8.1, for 0,1 and 3 layers of containers on the deck. The maximum fore and aft error occurs as the height of the containers increases, reaching a maximum of 41% for a wind 30° off the bow. These errors are attributed to the velocity gradient of the wind around the superstructure. This is the turbulent wind speed in the boundary layer of the ship as opposed to the uniform free flow. The fore and aft lateral coefficient computed from Isherwood’s equation, is shown in figure 3.18 using those values for 3 containers on the deck.

In a strong beam wind the ship will make leeway. Wieghardt (1973) measured such results on a passenger ferry concluding that there is an important increase in hydrodynamic resistance, however, more importantly, and observed by Jorgensen et al (1966), the existence of leeway can influence the wake, and hence propulsive efficiency. However, Van Berlekom (1981) concluded that the effect of leeway is less important than the wind force, which is of the same order of magnitude as the added resistance due to the waves.

Leeway effect was not considered in this study, due to the randomness of the wind force.
FIGURE 3.18. LATERAL FORE AND AFT COEFFICIENT FOR DART ATLANTIC/EUROPE AS A FUNCTION OF ENCOUNTER ANGLE.

3.9. STRIP THEORY.

The object of strip theory is to predict the motions of the vessel in a seaway. The seaway is assumed to be sinusoidal and the ship made up of a series of strip sections (21), where each strip is considered to be cylindrical with the axis on the still water surface. The motions of each strip are considered, and the total ship response found by integration along the length. The motion is predicted for a unidirectional wave train, and the application of linear superposition, proposed by St. Denis et al (1953) combines the two-dimensional wave response to the three-dimensional seaway, with the introduction of a spreading function.

The cross-section of each cylinder can be represented by a 'close-fitting' technique with multi-coefficients, Tasai (1960), however, Katory (1974) extended this theory for use in the BRITSEA programs. Each section of the ship is represented by a Lewis form, which has the same beam, draft and sectional area but not necessarily the same shape. Odabasi et al (1977) recognises this
assumption to be accurate, except for sections with small sectional area or large bulbs. Program BRITSEA, uses strip theory to produce the RAOs of the vessel to unidirectional longcrested, regular sea waves, through the bandwidth of the frequency of encounter of the ship to waves.

In order to implement strip theory, information regarding the ship's hull has to be supplied usually as a table of sectional offset distances, or sectional geometric coefficients, Strom-Tejsen et al (1973). BRITSEA requires such information provided as a trigger file (TSEA), see section 3.13.1.

3.10. MOTIONS OF SHIPS IN A SEAWAY.

Motions of the vessel in a random seaway have been computed using the RAOs coupled to a sea spectrum, defined by certain statistical sea parameters. Motions of vessels are termed the six degrees of freedom, three rotational and three translational, see figure 3.19.

Each of the motions are coupled, and cannot technically be treated individually. It is possible to obtain the spectral density of any ship motion either from theoretical calculations or by model tests in regular waves, as a function of the encountering wave frequency. These transfer functions or RAOs, are usually depicted in non-dimensional forms, see appendix A.2, for examples.

\[
\text{Translational motions,} \quad \text{RAO} = \left( \frac{\text{vessel motion}}{\text{wave amplitude}} \right)^2 \quad 3.10.1
\]

\[
\text{Rotational motions,} \quad \text{RAO} = \left( \frac{\text{vessel motion}}{\text{max wave slope}} \right)^2 \quad 3.10.2
\]

Therefore examples are,

\[
\text{RAO} = \frac{z_a}{\zeta_a} \quad (\text{heave response}) \quad = \frac{\Phi_a}{k_w\zeta_a} \quad (\text{roll response}) \quad = \frac{\vartheta_a}{k_w\zeta_a} \quad (\text{pitch response})
\]

\[
= \frac{R_{aw}}{\rho_{\text{sea}}g \zeta_a (B^2/L)} \quad (\text{AR}) \quad \text{Note AR is a function of wave height squared.} \quad 3.10.3
\]
The spectral density of ship's motions are deduced from the RAOS and the encountering wave spectra. This short-crested wave system is a function of the two-dimensional spectra given in section 4.3.2 and a spreading function. Therefore, the spectral density of the 3-D wave spectrum is:

\[ S(\omega_e, \mu_{\text{wave}}) = S(\omega_e) \times f(\mu_{\text{wave}}) \] 3.10.4

Where
- \( \omega_e \) - frequency of encounter.
- \( \mu_{\text{wave}} \) - Primary wave encounter angle.
- \( S(\cdots) \) - spectral energy density function.

The frequency of encounter of the ship to wave is given by:

\[ \omega_e = \omega_{\text{wave}} \left( 1.0 - \frac{\omega_{\text{wave}}}{g} \sqrt{V_{\text{ship}} \cos \Psi} \right) \] 3.10.5

Where
- \( \omega_{\text{wave}} \) - circular frequency of waves.
- \( \Psi \) - heading.
The spreading function, $f(\mu_{\text{wave}})$, is an assumption made to convert the 2-D spectrum to a 3-D seaway, and the integral of the spreading function is unity. Therefore the energy under the spectrum is:

$$m_0 = \int_0^\infty \int_{-\pi/2}^{\pi/2} S(\omega_{\text{wave}}) f(\mu_{\text{wave}}) d\omega d\mu_{\text{wave}} = \int_0^\infty \int_{-\pi/2}^{\pi/2} S(\omega_{\text{wave}}) f(\mu_{\text{wave}}) d\omega_{\text{wave}}d\mu_{\text{wave}}$$

3.10.6

$$f(\mu_{\text{wave}}) = \frac{2}{\pi} \cos^2 \mu_{\text{wave}}$$

3.10.7

The spectral density of the ship's response is given by:

$$S_r(\omega_{\text{e}}, \mu_{\text{wave}}) = (S(\omega_{\text{e}})f(\mu_{\text{wave}}))(\text{RAO}(\omega_{\text{e}}, \mu_{\text{wave}}))$$

3.10.8

The area under the response spectrum is given by:

$$m_o = \int_0^\infty \int_{-\pi/2}^{\pi/2} S_r(\omega_{\text{e}}, \mu_{\text{wave}}) d\omega_{\text{e}} d\mu_{\text{wave}} = \int_0^\infty \int_{-\pi/2}^{\pi/2} (S(\omega_{\text{e}})f(\mu_{\text{wave}}))(\text{RAO}(\omega_{\text{e}}, \mu_{\text{wave}}))$$

3.10.9

and the significant relative motion is given by:

$$2.0\sqrt{m_o \times CF}$$

3.10.10


$m_o$ - The zeroth moment of the response spectrum, area under the curve.

Ochi et al (1977) have studied the effects of the different spectral formulations on the prediction of the mean responses through linear superposition.
The rms values of the responses are given by \((m_0)^{0.5}\), whereas the apparent mean square value is the area under the response spectrum curve, \(m_0\).

### 3.11. ADDED RESISTANCE (AR)

There have been many theories for the prediction of AR, beginning with Havelock (1942) who showed the importance of pitch and heave motions, but did not include their cross-coupling. When both are in phase, maximum resistance to motion occurs. AR is explained as:

1. The energy loss in damping and the work needed to maintain a phase relationship between the seaway (forcing function) and the motions;
2. The equalisation of the work done by the forcing functions and that necessary to tow the ship through the seaway.

Bhattachyyra (1978) states:

'... all the quantities needed for the prediction of added resistance can be obtained from the computation of ships motions using strip theory,... by means of ... motion amplitudes and damping coefficients determined from strip theory.'

However, linear strip theory is an assumption to the non-linear case, but is recognised as a practical approach, Strom-Tejsen (1973). The added resistance of a ship consists of three parts, Strom-Tejsen (1973):

1. Resistance resulting from the interference between incident waves and waves generated by the ship, due to heave and pitch.
2. A component which results from the damping force to pitching and heaving motions.
A component arising from the reflection of waves on the hull-diffraction effect.

The AR of a vessel in a seaway has the properties that it is proportional to the square of wave height, is independent of the calm water resistance, and is a function of the motion amplitudes, being particularly sensitive to the accuracies of the latter. The value of AR computed is the mean AR in the seaway.

AR has been extensively discussed by Strom-Tejsen et al. (1973) who discuss the methods for analytical and theoretical AR computation, beginning with the derivation of the RAO and subsequent descriptions of the theories to compute AR. They also show the difference in AR resulting from the differing theories.

It has been stated that the prediction of AR cannot be treated with the same confidence as other motions computed from BRITSEA, Deakins (1988). Similarly, Liyin et al. (1974) point out that the mean AR is a rough estimation of the instantaneous AR.

3.12. DATABASES OF SHIP MOTIONS

Due to the complexities of computing vessel motions in a 3-D spectrum, many authors have only considered the head sea encounter. Obviously, due to the enormous burden that computation of real-time motions would place on a PC system, it was realised that a series of pre-computed databases of motions or critical motion criteria could be made. Loukakis et al. (1975) developed a database, termed seakeeping tables, for a series of hull forms (series 60). The results in the tables, are given as functions of hull geometry, sea state and ship speed. One could apply a spreading function to the results in order to obtain a quantitative value for other encounter angles.

Loukakis et al. (1975) state that the results can be applied to hull forms other than Series 60, since local hull geometry changes have very little effect upon the
response. Difficulties in variable weight distribution between ships was overcome to some extent by analysis of hull parameters of many vessels, and the use of average values for the distribution. The sea spectrum was represented by the Pierson-Moskowitz formula, equation 4.3.20.

The series 60 hull forms are those with cruiser sterns, where draft is constant along the length of the ship, all beams and Icb data are consistent so that the series 60 is representative of many hull types.

There are many databases of ship motions and motion criteria for example Blume (1978) as used extensively at BMT, Evans et al (1984,1989) and Svenson (1987). All are either non-dimensionalised ship motions as a function of the ship’s encounter angle, speed and wave parameters.

The size of any data base is subject to the number of dependant variables, particularly those defining the sea spectrum. The main criticism is the computation of response or motion exceedance criteria to a seaway built up of sea and swell. This is highlighted later.

3.13. THE THEORETICAL SEAKEEPING PROGRAM - BRITSEA

3.13.1. SHIP DATA REQUIREMENTS.

Best accuracy is obtained from BRITSEA by defining the following, Deakins (1988).

(a) General:

Length overall and between perpendiculars.

Beam.

Block coefficient.

Drafts and trims at mid-section.

(b) Hull particulars.

Table of hull offsets, stern and stem profile data.

Bilge keel/fin extent.
(c) Lightship weight distribution:

- Weight distribution per section, as fixed items.
- LCG and VCG. (Longitudinal and Vertical Centre of Gravity).
- Distance of after end from the after perpendicular.

(d) Loaded condition details:

- Weight distribution per section.
- VCG and LCG.
- Free surface moments.

Such information provides the trigger file, (TSEA), to BRITSEA.

By varying ship's speed a series of RAO data bases can be obtained from the seakeeping program, BRITSEA. The bandwidth of the spectrum is 0.3-1.3Hz, which is unfortunately narrow, however, this does include the region of resonance at which maximum motion amplitude occurs, Deakins (1988). It was decided that for the PC, it would be necessary to use these RAO files to compute vessel motions and exceedance criteria, and use the latter to form a database.

BRITSEA is divided into two sub-sections which compute the longitudinal and transverse motions. Each requires the trigger file, and since no sectional information was known for the DART ATLANTIC/EUROPE, data was generated from the OCL body plans, (particularly the LIVERPOOL BAY), Meek (1970), Meek et al (1971), see appendix A.1. As shown in table 3.4, the OCL ship is representative of the DART ATLANTIC/EUROPE hull form.

The TSEA file which describes the hull geometry at each 'strip' is first used to compute the hydrodynamic characteristics, such as added mass and damping coefficients, for the transverse and longitudinal responses. Calculations are performed for each frequency of encounter (0.3-1.3Hz), and computation of the RAO databases follows.
3.13.2. WEIGHT DISTRIBUTION DATA.

Weight distribution for the DART ATLANTIC/EUROPE was computed using the distribution curve for the OCL container ship, Meek (1970), with further information taken from Taggart (1980), giving general arrangements and weights for three containership types.

<table>
<thead>
<tr>
<th></th>
<th>OCL CONTAINERSHIP</th>
<th>DART ATLANTIC/EUROPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>30.5m</td>
<td>30.48m</td>
</tr>
<tr>
<td>Length btwn prps</td>
<td>213.414m</td>
<td>218.0m</td>
</tr>
<tr>
<td>Design Draft</td>
<td>9.144m</td>
<td>9.144m</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>0.60</td>
<td>0.60</td>
</tr>
</tbody>
</table>

TABLE 3.4. COMPARISON OF DART ATLANTIC/EUROPE AND OCL HULL FORMS.

Since both hull forms are similar it was felt that the weight distribution curve for the OCL ship could be used. However, the OCL ship has a larger engine, 32,000
bhp (MCR) compared to 29,000 bhp (MCR), leading to errors in the after sections. Figure 3.20 illustrates the weight distributions.

The breakdown of individual weights is defined in appendix A.1.

3.14 COMBINATION OF RAO FILES TO SEA SPECTRUM.

The number of parameters that define the spectrum will increase the size of any database, however, any deduced motion will improve in accuracy as the seaway is better defined. The most appropriate spectrum is Ochi (1976), since this defines a family of spectra, from which the most likely can be deduced for sea and swell. This database would be large and unfeasible. Bretschneider's two-parameter spectra (1959), would involve a 3-D database, defined by speed and encounter angle for each wave period at a central wave height value.

In order to overcome storage problems of a large database it was decided to use the ITTC, (Pierson-Moskowitz) spectrum, defined only by wave height. It is possible to include period as shown later for certain motions, as a correction factor.

It is important to consider sea and swell excitation to the vessel for computation of motions and as such any database should be able to cope with both. All databases based on spectral formulations are prone to the same problem, the combination of sea and swell responses. Hagiwara (1989) uses an RAO database to compute responses to sea and additionally to swell by assuming a Bretschneider two-parameter spectrum for both. The combination of the two responses is then an integration of the two response spectra. The main criticism of this approach is the use of Bretschneider's spectrum to emulate a sea and a swell. It can be argued that the spectrum is a combination of both, in that sea+swell builds up the seaway spectrum. It was also felt that the additional computational burden at this stage is not feasible for a micro-based system. The use of a database describing
critical motion exceedance values assumes a spectrum whereby the predominant sea and swell directions are superimposed. It is seen that the swell height is minimal compared to the sea height, and that the swell direction can be applied as a correction factor to the database values at a later stage.

3.15. BRITSEA OUTPUT FILES.

The longitudinal and transverse RAO outputs from BRITSEA are used by a seakeeping program CRISK, Deakins (1988). The sequence of events are:

Note: Prefix L - Longitudinal response T - Transverse responses.

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>INPUT</th>
<th>OUTPUT</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. LRA</td>
<td>TSEA</td>
<td>LRAUT</td>
<td>Added mass and damping values.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>XLRA</td>
<td></td>
</tr>
<tr>
<td>2. TRA</td>
<td>TSEA</td>
<td>TRAUT</td>
<td>Added mass and damping values.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>XTRA</td>
<td></td>
</tr>
<tr>
<td>4. LRB</td>
<td>XLRA</td>
<td>LRBUT</td>
<td>Amplitudes and phases for longitudinal motions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>XLRB</td>
<td></td>
</tr>
<tr>
<td>5. TRB</td>
<td>XTRA</td>
<td>TRBUT</td>
<td>Amplitudes and phases for transverse motions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>XTRB</td>
<td></td>
</tr>
<tr>
<td>6. ARC</td>
<td>XLRB</td>
<td>ARCUT</td>
<td>Added resistance coefficients.</td>
</tr>
<tr>
<td>7. Rename LRBUT and TRBUT as LRBUTnn and TRBUTnn for nn = speed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. For each LRBUTnn and TRBUTnn file, rename to LRAOnn and TRAOnn.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. CRISK</td>
<td>CRISKR</td>
<td>Datafile containing all motion criteria and added resistance results, computed from the RAOs coupled to the Pierson-Moskowitz spectrum and spreading function.</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 3.5. SEQUENCE FOR BRITSEA AND CRISK CALCULATIONS.**
3.16. CRITICAL MOTIONS.

Critical motions considered in this study are mentioned previously in section 3.6 and table 3.6 summarises the critical motion limits by author, with reference to the type of ship studied.

<table>
<thead>
<tr>
<th>AUTHOR</th>
<th>MOTION CRITERIA (refer to list)</th>
<th>SHIP TYPE REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. 2. 3. 4. 5.</td>
<td></td>
</tr>
<tr>
<td>Aertssen (1968,1972)</td>
<td>0.05</td>
<td>Various ships</td>
</tr>
<tr>
<td></td>
<td>0.07-0.08 max</td>
<td>Values are probabilities</td>
</tr>
<tr>
<td></td>
<td>0.03 t/bc</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04 gc</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05 f</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.06 tr</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3-4 per 100 oscilltns</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25 per 100 oscilltns</td>
<td></td>
</tr>
<tr>
<td>Andrew et al (1981)</td>
<td>average interval 100secs</td>
<td>assumed crew distribution over ship length two warships</td>
</tr>
<tr>
<td></td>
<td>0.18g</td>
<td>Values to overcome problems</td>
</tr>
<tr>
<td></td>
<td>12 as an average crew</td>
<td></td>
</tr>
<tr>
<td></td>
<td>over ship length interval</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30secs</td>
<td></td>
</tr>
<tr>
<td>Lloyd et al (1977)</td>
<td>0.15g per 15mins</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15 value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>for 1 crew</td>
<td></td>
</tr>
<tr>
<td>Lloyd (1981)</td>
<td>1 per min at FP</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 per min at 0.15L from FP</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15 value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>for 1 crew</td>
<td></td>
</tr>
<tr>
<td></td>
<td>interval</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40 secs</td>
<td></td>
</tr>
<tr>
<td>Kehoe (1973)</td>
<td>5 per 100 oscilltns pitch</td>
<td></td>
</tr>
<tr>
<td>Ferdinande (1974)</td>
<td>0.4g at bow.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*1 - slam given as impact > 0.093(g/l)^0.5

*2 - slam given as whipping stress > 5.9MN/m²

| t - tankers; gc - general cargo; f - ferries; tr - trawlers; bc - bulk carriers. |

Nb. See also Cox (1987) and Lloyd (1988) for a tabular presentation.

Where 1 - Deck wetness.

2 - Slamming.

3 - Subjective motion at the wheelhouse.

- 74 -
There are many suggestions for the limits to critical motions which are those which may cause damage to the vessel's structure or cargo, or cause crew discomfort, Pingree (1988). The limits vary depending upon vessel type and action of the master, Van Sluijs (1974), Svenson et al (1987). Pingree (1988), Lloyd et al (1977,1988), Ferdinande (1978) and Lindemann (1976) have discussed the motion criteria problem, to which the reader is referred.

It is difficult to relate criteria from differing sources, due to ship type, but also due to variations in the point of measurement. Lloyd et al (1977) proposed several values to overcome these difficulties. Buxton et al (1986) studied criteria which are applicable at different speeds and states that for a sea conditions below BN=4, and light ship displacement, no criteria should be exceeded. However, as Lindemann (1976) points out, all criteria are very subjective and subject to the 'mood' of the master; values are required for an 'average seaman'.

3.16.1. DECK WETNESS.

The number of deck wetnesses per hour is obtained, Bhattachyrra (1978), from:

\[
N_w = \frac{\text{Probability of Deck Wetness}}{\text{Average Rel. Motion Period at } P_{pp}} \cdot 3600
\]

\[
= \frac{P_w}{T_r} \cdot 3600 = \frac{1.0}{2\pi} \frac{\sqrt{m_2}}{m_0} \cdot 3600 = \frac{3600}{2\pi} \frac{\sqrt{m_2}}{m_0} \exp\left(\frac{\varepsilon^2}{2m_0 CF^2}\right)
\]

Where, \( f_e \) = Effective freeboard at the bow accounting for static and dynamic swell up, \( f_e = f - h \), where \( h = \xi_b + \xi_s \), the bow wave height and sinkage phenomena. Tasaki (1969) has given an approximate formula for statical swell up, \( h = 0.75 B \frac{LF_P}{L_e} \), where \( L_e \) is the length of entrance. Dynamic swell up is ignored (Formula from Ochi et al (1974)).

\( CF \) = Spectrum bandwidth correction factor \( = \left(1 - \varepsilon^2\right)^{0.5} \), \( \varepsilon = 1 - \frac{m_2}{m_0 m_4} \)

\( m_0, m_2, m_4 \) = Moments of the relative motion spectrum.
The critical limits are however, very subjective since it is difficult to include bow flare. Svenson (1987) states that deck wetness has the greatest impact on operating speed, in complete contrast to the results from Lloyd (1988).

Containerships typically have a large bow flare, therefore any relative motion at the bow although large, may result in water being thrown outwards rather than over the bow, although Lloyd (1985) states that large bow flares are susceptible to deck wetness due to increased spray. Similarly, the large bow flare alters the predicted (and actual) motions, since strip theory assumes the submerged bow shape to remain constant. However, a large flare, typically high above the waterline, will drastically alter the submerged area during extreme pitching motions. Further research is necessary in this area, but it is envisaged at this stage that a correction factor will be used for each vessel, dependant upon the pitch, and bow vertical acceleration.

Van Sluijis (1974) found large discrepancies between deck wetness computations from various methods, therefore care must be taken with decisions based on the wetness of the deck per unit time. Aertssen (1972) gives limited results for the DART EUROPE for which certain comparisons could be made from the databases and propulsion models.

3.16.2. SLAMMING.

Slams are encountered when the forefoot hits the water surface under violent pitching motions. Damage may result in plate buckling and whipping stress on the main structure. Usually a slam occurs when the forefoot emerges. The number of slams per hour is given by:

\[ N_s = \frac{\text{Probability of a slam}}{\text{Average Rel. Motion period at } F_{pp}} \times 3600 \]

The probability of a slam is built up from,
\[ \text{prob} \left( \text{forefoot emergence} \right) = e^{-\left( \frac{T_{\text{bow}}}{2m_0CF_1^2} \right)} \]

Where, \( T_{\text{bow}} \) - Draft at the bow.
\( m_0 \) - Zeroth moment of the bow motion response spectrum.
\( CF_1 \) - Motion spectrum bandwidth correction factor.

The probability that the relative bow velocity exceeds the threshold is given by:

\[ \text{prob} \left( \text{Rel, Vel} > V_c \right) = e^{-\left( \frac{V_c^2}{2m_2CF_2^2} \right)} \]

where
\( V_c \) - Threshold velocity.
\( m_2 \) - Second moment of the relative bow motion spectrum.
\( CF_2 \) - Vertical velocity spectrum bandwidth correction factor.

Then the probability of a slam is given by:

\[ \text{prob} \left( \text{slam} \right) = e^{-\left( \frac{T^2}{2m_0CF_1^2 + V_c^2/2m_2CF_1^2} \right)} \]

The number of slams per hour is now given by:

\[ N_s = \frac{3600}{2\pi} \sqrt{\frac{m_2}{m_0}} e^{-\left( \frac{T^2}{2m_0CF_1^2 + V_c^2/2m_2CF_1^2} \right)} \]

Using Ochi (1964), the value of \( V_c = 0.093\sqrt{gL_{pp}} = 0.093\sqrt{9.81 \times 218} = 4.30 \text{ms}^{-1} \)
although this value was ship specific.

Ferdinand (1978) describes the localised damage to bow flare plating that may result from a slam, but assigns no limits for this.
3.16.3. SUBJECTIVE MOTION.

Subjective motion, measured at the wheelhouse, is a measure to relate to the other criteria. In essence it is a value which depicts the master's perception of motion critical to the survival of his ship, Deakins (1988). The subjective motion, (SM), magnitude is given by :-

\[
SM = \left( 3.087 \ln\left(\frac{1}{2\pi}\right) \right)^{\frac{m_0}{m_4}} m_4^{0.715} \quad \text{after Lloyd (1977)}.
\]

The critical value of SM is 12, Deakins (1988), although this value was taken for a fisheries protection vessel. Subjective motion has been computed using the assumption that it is measured at the wheelhouse, and represents the master's opinion.

3.16.4. PROPELLER EMERGENCE.

An emergence of the propeller has deemed to have taken place when a quarter of the blade diameter has been exposed. The number of propeller emergences per hour is given by :-

\[
N_e = \frac{\text{Probability of Prop emergence}}{\text{Average Rel. Motion period at prop}}
\]

\[
N_e = \frac{3600}{2\pi} \sqrt{\frac{m_2}{m_0}} e^{\frac{-\left(\frac{T_p-D}{4}\right)^2}{2m_0CF^2}}
\]

Where, 

\[T_p\] - The depth of the propeller shaft. 

\[m_0\] - Variance of relative motion at the propeller. 

\[m_2\] - Variance of relative velocity at the propeller.

A value of 120 emergences per hour, Deakins (1988), is taken as the critical limit.
3.16.5. ROLL MOTION.

A maximum roll angle of 15 degrees is taken as the critical limit beyond which danger to cargo and ship is more likely to occur, and that the master will try to reduce, Deakins (1988). It is difficult to evaluate a critical roll angle for containerships, as this will depend upon :-

1. Height of containers on the deck, and cargo carried on deck for example yachts, OCEAN ROUTES (1989);
2. Lashings. Containers below deck are held rigidly by the guides, whereas any cargo on deck needs to be tightly secured.

3.16.6. MASTER'S ACTION.

Within the routeing algorithm, at each interrogation of the environmental data, and subsequent speed and encounter angle deduction, a call is made to a routine to deduce the values of the critical motions, from the databases. Should any limit be exceeded, then the route is subject to further action; Ochi et al (1974) state that any motion exceedance should constitute an action. No reduction for the master's action is implemented due to the subjectivity of the problem. It is envisaged that there are sufficient other routeing legs that do not impinge on the exceedance of the critical motions. Vercoe (1975) states that routeing is an alternative to voluntary speed reduction. Similarly, heavy weather damage although costly is difficult to quantify, Vercoe (1975), and as such, critical exceedance criteria are used to omit any likely occurrences. Further to this, it is unknown to what extent the master will reduce speed, or make a course alteration instead.

3.17. GENERATED DATABASES OF CRITICAL MOTIONS AND ADDED RESISTANCE FROM BRITSEA AND CRISK.

Program CRISK, Deakins (1988), was adapted to produce motion exceedance criteria and added resistance information for each speed (3-27 Knts, in steps of 3), and primary wave encounter angle (0-180°, in 15° steps), based on the Pierson-Moskowitz spectrum and cosine spreading information. Databases were
Several problems were encountered with BRITSEA, based on the trigger file, TSEA. It was discovered that computation of longitudinal gyr-radius and yaw gyr-radius values could not be made for small sectional areas at the ends of the ship, stations 0 and 21, therefore, small adjustments had to be made. Similarly, transverse motions could not be computed above 18 knots. This problem is assumed to be due to stern wave effects, since the ship is literally overtaking the waves. Since the transverse motions are not so important for a routeing model, the solutions at 18 knots were assumed for all speeds above this value.

3.18. EXPANSIÓN OF DATABASE RESULTS TO ACCOUNT FOR WAVE PERIOD

Balee (1985) has shown how the motion information derived from the Pierson-Moskowitz spectra can be adjusted for wave period.

The adjustment of the single-parameter spectrum takes account of decaying or developing seaways, with a relatively higher or lower spectral peak.

Given the wave period, $T_{\text{wavep}}$, a (fully developed seaway) significant wave height can be computed, $\zeta_{1/3}$, using the relationships:

$$\omega_p = \frac{0.4013 \sqrt{g}}{\zeta_{1/3}}$$

$$\omega_p = \frac{2\pi}{T_{\text{wavep}}}$$

Where
- $\omega_p$ - Frequency at the spectral peak.
- $T_{\text{wavep}}$ - Primary wave period.

Since the significant wave height is already defined, $\zeta_{1/3}$, with subsequent motions, $M$, the corrected motions are:

$$M^* = M \times \left(\frac{\zeta_{1/3}}{\zeta_{1/3}}\right)^2$$

for added resistance.
where:

\[ M^* = M \times \left( \frac{\zeta_{1/3}}{\zeta_{1/3}^*} \right) \], for all other responses.

3.18.4

Where:

- \( M \) - Assembly of ship's motions.
- \( \zeta_{1/3} \) - Defined significant wave height.
- \( \zeta_{1/3}^* \) - Significant wave height, computed using defined wave period.

Corrections can be made to critical motion criteria as they are linear functions of wave height, and are based on probabilities of exceedance and moments of the response spectrum for that wave height. Difficulties were encountered, however, especially with AR, since periods yielding wave heights smaller than the defined value gave large AR increases. Care has to be applied, and since no thorough testing was made, it was decided not to include the facility at this stage.

3.19. SHIP POWERING.

The power developed by the prime mover, or engine, is usually termed the indicated power, \( P_i \) or brake power, \( P_b \), depending upon the type of engine; steam reciprocating engines or internal-combustion engines respectfully. The Sulzer 10RND90 supplied in the DART ATLANTIC, is an internal-combustion engine and it is assumed that the power given by the tests, section 3.20.2, are the brake powers. Aertssen (1975) and Beukelmann et al (1974) have conducted tests on containerships, and in particular Beukelmann et al (1974) have compared mathematical models to the full-scale tests.

The brake power at the crank-shaft coupling is transferred along the shaft to the propeller. It is usual to refer to the shaft power, \( P_s \), instead of the brake power, which is transmitted to the developed power, \( P_d \), at the propeller. In transmission along the shaft, there is a loss of power attributable to the gearing and shaft resistance, (value assumed as 0.98 = 2% loss). As the propeller advances through the water it delivers a thrust power, \( P_t \), which through the efficiency of the
propeller and hull, is balanced by the effective power, $P_e$, required to tow the vessel, at the resistance. Figure 3.21, indicates these quantitative values.

The power delivered to the propeller can be determined by the torque, $Q$ and engine revolutions, $n$, (assuming gearing ratio 1:1), but also through the shaft efficiency.

$$P_d = \eta_s P_s$$

$$P_d = 2\pi n Q$$ \hspace{1cm} 3.19.1$$

\[ P_s = 2\pi n M \hspace{1cm} M = \eta_s Q \] \hspace{1cm} 3.19.2

The shaft power can be represented by :-

\[ P_s = \frac{1}{1 - \frac{1}{U_s}} \]

\[ R_l = \frac{P}{U_s} \]

**FIGURE 3.21. SHIP - PROPELLER - PRIME MOVER RELATIONSHIPS.**

Thrust power is related to the delivered power by the relative rotative efficiency, $\eta_r$, which is an overall term relating the open water efficiency, $\eta_o$, and the behind efficiency, $\eta_b$, of the propeller. Typically, the propeller is tested in open water situations, where the thrust is derived from accelerating the undisturbed fluid. However, when behind the ship, the propeller is advancing into turbulent water.
which has a forward movement, (known as the wake). The relative advance speed is therefore reduced, (known as the speed of advance, \( V_a \), units ms\(^{-1} \)).

The open water efficiency, \( \eta_o \), is given by,

\[
\eta_o = \frac{Th V_a}{2\pi n Q_o} = \frac{P_t}{P_s} \quad 3.19.3
\]

It is stressed that the open water efficiency is determined by the open water torque, therefore, when working behind a hull, and advancing at the same speed, the thrust and revolutions are associated to some different torque value. Therefore,

\[
\eta_o = \frac{Th V_a}{2\pi n Q} \Rightarrow \frac{\eta_o}{\eta_o} = \frac{Q_o}{Q} \quad 3.19.4
\]

When the hull is propelled, the action of the propeller reduces some of the pressure over the stern, Lewis (1988). Since the pressure has a forward component, it reduces the total resistance, however, this is in turn increased by the action of the propeller, which results in an increased resistance. This augmentation of resistance is viewed as a deduction of thrust from the propeller.

\[
R_t = (1 - th)Th \quad 3.19.5
\]

Lewis (1988), states that the work done in moving a ship at speed \( V_{ship} \) against a resistance, \( R_t \), is proportional to the product, \( R_t V_{ship} \), (effective power, \( P_e \)). The relationship between the effective power and the thrust power is given by the hull efficiency, \( \eta_h \). (Note \( V_{ship} \) - units ms\(^{-1} \)).

\[
\eta_h = \frac{P_e}{P_t} = \frac{R_t V_{ship}}{Th V_a} = \frac{1 - th}{1 - w} \quad 3.19.6
\]

The propulsive efficiency, \( \eta_d \), relates the effective power to the delivered power.
This is termed the Quasi-Propulsive Efficiency, (QPC), when taken in conjunction with the shafting efficiency.

The sources used to determine the efficiencies and the power of the prime mover for the DART ATLANTIC/EUROPE in this study are:

1. Holtrop et al (1982) and Holtrop (1984) who determined regression analyses for the thrust deduction fraction, wake fraction and relative rotative efficiency. However, further sources were investigated, Wright (1967);
2. Oosterveld et al (1975), who determined regression analyses for the propulsive characteristics, open water thrust and torque coefficients, $K_T$ and $K_Q$, from which the open water efficiency can be determined:

$$\eta_o = \frac{J}{2\pi} \frac{K_T}{K_Q}$$

Where $J$ - Advance coefficient given by:

$$J = \frac{V_a}{nD}$$

Further examples of regression analyses to determine the open water efficiency were investigated, Wright et al (1971);
3. Sulzer (1989) who provided information on the SULZER 10RND90 engine from which polynomial expressions were derived for specific fuel consumption, SFC and power, $P_s$, as functions of engine revolutions. Without this knowledge it is possible to determine the developed power through the following expression.
3.19.1. TAYLOR WAKE FRACTION.

The wake is the speed of advance of the water into the propeller disc, and is a summation of the potential wake, the frictional wake, the wave wake plus an additional correction allowance wake. Potential wake is that of a ship moving in a fluid without wave making and friction, whereas frictional wake is due to the boundary layer effect. Waves generated by the ship at the stern produce a wake when the motion of particles in the wave is in the direction of motion of the ship.

The wake is slightly less than the ship's speed due to the drag of water by the hull.

\[ w = \frac{V_{\text{ship}} - V_a}{V_{\text{ship}}} \quad \text{or} \quad V_a = (1 - w)V_{\text{ship}} \quad (\text{SI units}) \]

Equation 3.19.11, defines the wake under Taylor notation, and as measured by pitot tubes, varies over the propeller disc and is usually depicted as contoured charts. Propellers are designed to suit the average wake at a particular radius, Lewis (1988), normally taken at 0.75 of the disc radius. Wake fraction has been shown to be susceptible to the after body shape of the ship, the position of the propeller and the trim and displacement of the vessel, Harvald (1978).

The regression analyses performed by Holtrop et al (1982) and Holtrop (1984) produced the following expression for the wake fraction.

\[ w = C_9C_{20}C_v \frac{L}{T_a} \left( 0.050776 + 0.93405C_{11} \frac{C_v}{(1 - C_{p1})} \right) + 0.27915C_{20} \sqrt{\frac{B}{L(1 - C_{p1})}} C_{19}C_{20} \]

This expression defines that for single screw ships with conventional sterns,

\[ C_8 = \frac{BS}{LDT_a} \quad \text{when} \quad \frac{B}{T_a} < 5, \quad \text{or} \quad C_8 = \frac{S(\frac{7B}{T_a} - 25.0)}{LD(\frac{B}{T_a} - 3.0)} \quad \text{when} \quad \frac{B}{T_a} > 5 \]
\[ C_g = C_8 \text{ when } C_8 < 28, \text{ or } C_g = 32.0 - \frac{16.0}{(C_8 - 24.0)} \text{ for } C_8 > 28 \]

\[ C_{11} = \frac{T_a}{D} \text{ when } \frac{T_a}{D} < 2.0 \text{ or } C_{11} = 0.0833333 \left( \frac{T_a}{D} \right)^3 1.33333 \text{ when } \frac{T_a}{D} > 2.0 \]

\[ C_{19} = \frac{0.12997}{(0.95 - C_B)} - \frac{0.11056}{(0.95 - C_p)} \text{ when } C_p < 0.7 \text{ or } C_{19} = \frac{0.18567}{(1.3571 - C_m)} - 0.71276 + 0.38648C_p \text{ when } C_p > 0.7 \]

\[ C_{20} = 1.0 + 0.015C_{\text{stern}} \text{ and } C_{p1} = 1.45C_p - 0.315 - 0.0225/cb \]

The coefficient \( C_v \) is the viscous resistance coefficient where,

\[ C_v = (1.0 + k)C_f + C_a \]

The value of \( C_f \) is computed from equation 3.7.3 and \( C_a \) from equation 3.7.36.

Typical average values of wake fractions for single screw ships are given by Taylor (1933), and reproduced in table 3.7.

<table>
<thead>
<tr>
<th>BLOCK COEFFICIENT</th>
<th>TAYLOR WAKE FRACTION SINGLE SCREW SHIPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.230</td>
</tr>
<tr>
<td>0.55</td>
<td>0.234</td>
</tr>
<tr>
<td>0.60</td>
<td>0.243*</td>
</tr>
<tr>
<td>0.65</td>
<td>0.243</td>
</tr>
<tr>
<td>0.70</td>
<td>0.283</td>
</tr>
<tr>
<td>0.75</td>
<td>0.314</td>
</tr>
<tr>
<td>0.80</td>
<td>0.354</td>
</tr>
<tr>
<td>0.85</td>
<td>0.400</td>
</tr>
</tbody>
</table>

* - Value compares to 0.2716 @ 2.57ms\(^{-1}\) to 0.2627 @ 10.3ms\(^{-1}\) computed for the containership in this study.

TABLE 3.7. AVERAGE TAYLOR WAKE FRACTIONS. AFTER TAYLOR (1933).
Further to Holtrop (1984), the algorithm defined by Wright (1971), has been investigated, yeiding 0.3813 @ 2.57ms\(^{-1}\) to 0.3058 @ 10.5ms\(^{-1}\).

3.19.2. THRUST DEDUCTION.

The thrust deduction, defined in equation 3.19.5, is in part due to the propeller working astern of the vessel, Harvald (1978). Thrust deduction can be regarded as a summation of three parts, the potential, the frictional and the wave thrust deduction factors. The propeller thrust is not equal to the resistance of the ship to motion because the propeller acts in the potential velocity field of the stern, and the acceleration of water into the stern increases the frictional resistance. Similarly the interference by the propeller on the stern wave system may cause a change in the wave-making resistance. Thrust deduction is in fact an augmentation of resistance, resulting from an incomplete knowledge of the total resistance to motion of the ship.

Holtrop et al (1982,1984) defined the following expression for the thrust deduction fraction for a single screw ship:

\[
\theta_n = \frac{0.25014 \left( \frac{B}{L} \right)^{0.2956} \left( \frac{BT}{D} \right)^{0.2624}}{(1.0 - C_p + 0.0225/cb)^{0.01762} + 0.0015C_{\text{stern}}} \tag{3.19.20}
\]

This equation emphasises those parameters that affect the thrust deduction fraction described by Harvald (1978), the fullness of the hull form, \(C_p\), and the breadth, length ratio and the shape of the afterbody, in accordance with the frictional and wave making components.

It is assumed that the thrust deduction does not vary with propeller loading, Beukelmann et al (1974), therefore even in the random seaway as the loading varies slightly, the thrust deduction fraction remains constant. Similarly, there is
only a slight change in quantity with speed variation within the operation of the propeller. By considering equations 3.19.5, and 3.19.8, the advance coefficient has an effect, however, only appreciably outside the propeller’s normal operation, Harvald (1978) and Bhattachyrrra (1978).

Thrust deduction computed by the Holtrop (1984) algorithm was found to be 0.1854 compared to the value of 0.2246 using the BSRA algorithm, Wright (1971).

3.19.3. HULL EFFICIENCY.

The values computed using the BSRA solutions are 1.188 @ 5.1ms⁻¹ and 1.107 @ 12.4ms⁻¹, compared to 1.111 and 1.103 from the holtrop (1984) equations and the relationship in equation 3.19.6. It would be possible to model hull efficiency as a polynomial, however, this was not pursued as the function for wake fraction is also subject to leeway effects, which may be applied at a later date. Similarly little computational time would be gained as many coefficients for wake fraction can be processed off-line. Similarly it was possible to pre-compute the thrust deduction, as this is independent of vessel speed or propeller revolutions.

3.19.4. RELATIVE-ROTATIVE EFFICIENCY OF THE PROPELLER.

Holtrop (1984), defines the relative-rotative efficiency by the following expression.

\[ \eta_r = 0.9922 - 0.05908 \frac{A_e}{A_o} + 0.07424(C_p - 0.0225cb) \]

3.19.21

The blade area ratio \( \frac{A_e}{A_o} \) may be expressed by Keller’s formula:

\[ \frac{A_e}{A_o} = K + \frac{(1.3 + 0.3Z)Th}{D^2(p_0 + \rho_{sea}gh - \rho_v)} \]

Where \( K = 0.2 \) for single screw ships.

\[ p_0 - \rho_v = 9904.7 \text{Nm}^{-2} @ 15^\circ\text{C}. \]

3.19.22

This equates to 0.82 measuring \( h \) as the height from the shaft line to water surface from the body and lines planes of the OCL ship, see appendix A.4.
Assuming the thrust, \( T_0 \), to be the average value at design speed, the value of \( A_e/A_0 \) is easily computed, off-line from the propulsion algorithm.

Bhattachyrra (1978) states that the propeller efficiency varies slightly with loading due to the augmentation of resistance, but concludes that the calm water prediction is adequate for seaways. Lewis (1988) states that \( \eta_p \), the ratio of behind the ship to open water efficiencies, does not depart to any great extent from unity, and can be deduced to between 1.0 and 1.1 for single screw ships.

Wright (1971) also gives a regression analysis formula for the relative rotative efficiency which gave a value of 1.0 compared to 1.005 from the Holtrop (1984) algorithm for an assumed blade area ratio of 0.82.

3.19.5. OPEN WATER EFFICIENCY.
The open water efficiency, defined earlier in equations 3.19.3 and 3.19.8 has been determined from the polynomial expressions for the thrust and torque coefficients, \( K_T \) and \( K_Q \), as defined by Oosterveld et al (1975) for the Wageningen B-series propeller. Further to this, expressions given by Munsen et al (1968) for the Troost B-series, or Wageningen B-Series standard propellers have been studied.

The Wageningen B-series are frequently used in practice, Lewis (1988) and Oosterveld et al (1975). It is assumed that the propeller on the DART ATLANTIC is of a similar design to the B-series. This assumption is given further credence since the containership was built in 1975. Particulars of the B-series can be found in Oosterveld et al (1975) and Lewis (1988).

The NSMB, produced a polynomial expression for the open water efficiency of the B-series, Munson et al (1968), where :-
\[ \eta_0 = a_0 + a_1 \Delta' + a_2 \Delta'^2 + a_3 \Delta'^3 + a_4 \Delta'^4 + a_5 \Delta'^5 + a_6 \Delta'^6 + \alpha (b_0 + b_1 \Delta' + b_2 \Delta'^2 + b_3 \Delta'^3 + b_4 \Delta'^4 + b_5 \Delta'^5) + \alpha^2 (c_0 + c_1 \Delta' + c_2 \Delta'^2 + c_3 \Delta'^3 + c_4 \Delta'^4) + \alpha^3 (d_0 + d_1 \Delta' + d_2 \Delta'^2 + d_3 \Delta'^3) + \alpha^4 (e_0 + e_1 \Delta' + e_2 \Delta'^2) + \alpha^5 (f_0 + f_1 \Delta') + \alpha^6 g_0 \] 

Where,

\[ \Delta' = \delta - 2.27993827 \times 10^2 \] 
\[ \delta = \frac{nD}{V_a} \] 
\[ \alpha = \frac{A_e}{A_0} - 6.42592593 \times 10^{-1} \]

and,

<table>
<thead>
<tr>
<th>Coeff</th>
<th>N^0</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>5.185502301 x 10^{-1}</td>
<td>-1.29661847 x 10^{-4}</td>
<td>2.19222123 x 10^{-7}</td>
<td>3.33718647 x 10^{-10}</td>
<td>9.49118643 x 10^{-13}</td>
<td>1.52722314 x 10^{-16}</td>
<td>1.02722314 x 10^{-19}</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-5.80554808 x 10^{-2}</td>
<td>-2.76674116 x 10^{-4}</td>
<td>4.9874625 x 10^{-6}</td>
<td>2.6545844 x 10^{-8}</td>
<td>-1.75836409 x 10^{-11}</td>
<td>-8.98262419 x 10^{-14}</td>
<td>-8.98262419 x 10^{-17}</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>-6.88683585 x 10^{-2}</td>
<td>2.48658590 x 10^{-4}</td>
<td>2.29551019 x 10^{-6}</td>
<td>-7.1039129 x 10^{-8}</td>
<td>-7.9411238 x 10^{-10}</td>
<td>-7.9411238 x 10^{-13}</td>
<td>-7.9411238 x 10^{-16}</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 3.8. BSRA OPEN WATER EFFICIENCY POLYNOMIAL COEFFICIENTS.**

The above formulation is restricted to limits of the original data, therefore,

\[ \delta_{\text{max}} = 133.33 \frac{A_e}{A_0} + 273.33 \quad \text{and} \quad \delta_{\text{min}} = 95. \]

The open water efficiency cannot be computed at low speeds, typically 2.5ms\(^{-1}\), it therefore poses a difficulty in the propulsion program. Since the algorithm has been superseded by that of Oosterveld et al (1975), and because of the problems outlined, it was not used.
The Oosterveld et al (1975) algorithm is based on regression analysis of 120 propellers. The effect of the Reynold's number has been included with correction factors obtained by means of an equivalent profile method developed by Lerbs (1951). The Reynold's effect is attributed to the errors on open-water tests carried out at differing rotational speeds. The Lerbs profile assumes the blade section at 0.75R to be representative of the whole blade, Oosterveld et al (1975) and Lewis (1988).

The polynomials were developed for $R_n = 2 \times 10^6$, chosen as a characteristic of the model scale. $R_n$ effects are then deduced from separate polynomials.

\[
K_{Th}^* = \sum_{stuv} C_{stuv} J^5 P D^5 A_e A_o^u Z^v
\]
\[
K_Q^* = \sum_{stuv} C_{stuv} J^5 P D^5 A_e A_o^u Z^v
\]

3.16.29
3.19.28

Where

- $J$ - Advance coefficient, equation 3.16.9.
- $P, D$ - Pitch and diameter of the propeller.
- $A_e / A_o$ - Expanded blade area ratio, see equation 3.16.22.

A correction for the effect of Reynold's number is made by:

\[
\Delta K_{Th} = a_1 + a_2 \left( \frac{A_e}{A_o} \right)^2 + a_3 \left( \frac{A_e}{A_o} \right) \left( \frac{P}{D} \right) + a_4 \left( \log R_n - 0.301 \right)^2 \left( \frac{A_e}{A_o} \right) \left( \frac{P}{D} \right)^2
\]
\[
+ a_5 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_6 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_7 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_8 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)^2
\]
\[
+ a_9 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_{10} \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_{11} \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_{12} \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right) + a_{13} \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)^2
\]

3.19.29

\[
\Delta K_Q = b_1 + b_2 \left( \frac{P}{D} \right) + b_3 \left( \frac{P}{D} \right)^2 + b_4 \left( \frac{A_e}{A_o} \right)^2 + b_5 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)
\]
\[
+ b_6 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)^2 + b_7 \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)^2 + b_8 \left( \log R_n - 0.301 \right) \left( \frac{A_e}{A_o} \right) \left( \frac{P}{D} \right)
\]
\[
+ b_9 \left( \log R_n - 0.301 \right) \left( \frac{A_e}{A_o} \right) \left( \frac{P}{D} \right) + b_{10} \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)^2 + b_{11} \left( \log R_n - 0.301 \right) \left( \frac{P}{D} \right)^2 + b_{12} \left( \log R_n - 0.301 \right) \left( \frac{A_e}{A_o} \right)^2 + b_{13} \left( \log R_n - 0.301 \right) \left( \frac{A_e}{A_o} \right)^2
\]

3.19.30
The coefficients are:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th></th>
<th>a</th>
<th>b</th>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00353485</td>
<td>-0.000591412</td>
<td>5</td>
<td>0.00000643192</td>
<td>-0.000938091</td>
<td>9</td>
<td>0.0000032049</td>
<td>-8.8528x10^-7</td>
</tr>
<tr>
<td>2</td>
<td>-0.00333758</td>
<td>0.00696898</td>
<td>6</td>
<td>-0.000010636</td>
<td>-0.00059539</td>
<td>10</td>
<td>2.30171x10^-5</td>
<td>-8.8528x10^-6</td>
</tr>
<tr>
<td>3</td>
<td>-0.00478125</td>
<td>-0.0000666654</td>
<td>7</td>
<td>-0.0000276305</td>
<td>0.0000782099</td>
<td>11</td>
<td>-1.84341x10^-6</td>
<td>-1.84341x10^-5</td>
</tr>
<tr>
<td>4</td>
<td>0.000257792</td>
<td>0.0160818</td>
<td>8</td>
<td>0.00000954</td>
<td>0.0000052199</td>
<td>12</td>
<td>-0.00400252</td>
<td>0.000220915</td>
</tr>
</tbody>
</table>

**TABLE 3.9. COEFFICIENTS FOR EQUATIONS 3.19.29 AND 3.19.30 CORRECTIONS FOR REYNOLDS NUMBER EFFECTS AFTER OOSTERVEELD ET AL (1975).**

Reynolds number for the propeller is computed from:

\[
R_{n0.75R} = \frac{C_{0.75R} \sqrt{V_a^2 + (0.75\pi n D)^2}}{\nu}
\]  

3.19.31

Where 

\[
C_{0.75R} = \frac{2.073A_e/A_o D}{Z}
\]  

3.19.32

\[V_a\] - Advance speed given by equations 3.16.9, 3.16.11.

\[\nu\] - Kinematic viscosity = 1.1881 x 10^-6.

The open water characteristic of the propeller can be determined from:

\[K_{Th} = K_{Th}^* + \Delta K_{Th}\text{ and } K_Q = K_Q^* + \Delta K_Q\]

3.19.33

\[\eta_0 = \frac{J K_{Th}}{2\pi K_Q}\]

3.19.34

The assumption that the open water characteristics do not vary in the random seaway is not correct. Vassilopoulos (1971), studied the propulsive performance of ships in irregular seaways by expanding the thrust and torque variables as
functions of the mean value and derivatives of the input functions, $V_a$ and $n$. The main conclusion was that the mean open water value computed through 3.19.34 is adequate. Figure 3.22 indicates the open water characteristics of the propeller.

![Open Water Characteristics @ 100RPM](image)

**FIGURE 3.22. OPEN WATER CHARACTERISTICS COMPUTED FROM THE OOSTERVELD ET AL (1975) REGRESSION EQUATIONS.**

3.19.6. POWERING LIMITATIONS.

Beukelmann et al (1974) conclude several important considerations in their full-scale tests and mathematical modelling of the ATLANTIC CROWN containership.

1. Accurate determination of ship speed is necessary; at high speeds slight changes have a dramatic effect upon the still water powering;
2. Sailing in $\zeta_1=7m$, gave variations of torque to a maximum of 18% of the average value;
3. Still water powering prediction is the most critical;
4. The fraction of added power in waves may be up to 50% in $\zeta_{1/3}=7m$.

There are major assumptions in the propulsive coefficient, chiefly concerned with the open water characteristics, the wake fraction, determined from the speed of advance and the thrust deduction value. The instantaneous values of thrust and torque are complex functions of the open water characteristics and the random seaway, however, the mean open water value can be assumed. Note, no propeller cavitation effects were included. Lily et al (1974) state that the fluctuations in the thrust and torque about the mean results in variations in the revolutions.

For the purposes of an on-board system, one has to consider the application of the system to many vessels, therefore the models have to be generated from limited data and not a series of full-scale trials. Adjustments to the models can be made once installed.

3.20. ENGINE CHARACTERISTICS.

Lloyds (1988), show that the DART ATLANTIC/EUROPE is powered by a SULZER engine. Sulzer (1989), supplied the engine test bed, and trials data, for the DART ATLANTIC SULZER 10RND90 engine, described in appendix A.3. Using the curves of brake power against engine revolutions for the commissioning test, the 1981 test and the specific fuel consumption against engine revolutions from commissioning tests, it was possible to determine the specific fuel consumption curve from assumed contours, for 1981. There is therefore a possibility to determine the specific fuel consumption for different propeller curves as the engine deteriorates, however, the contours are rather subjective. Using least-squares, with Gauss-Sidel elimination method, polynomial fits were deduced for the 1981 curves.

(Note the engine rpm is taken as that at the shaft, therefore, the engine and gearbox are treated as one. No gearing reduction is considered).
Sulzer indicates the maximum continuous rating, MCRt, of the engine at 120 revolutions corresponding to 29,000bhp, however, the deterioration of the engine indicates in 1981, that only 119 revolutions can be achieved, with a reduction in maximum power. Both MCRt and maximum power constitute upper limits in the routeing model, however figure 3.23 shows a typical engine operating envelope, Sulzer (1974).

![Engine Operating Envelope](image)

**FIGURE 3.23. ENGINE OPERATING ENVELOPE, AFTER SULZER (1989).**

Sulzer (1989) state that the tests on specific fuel consumption have to be increased by approximately 10-15gm/Bhp-hr for service conditions. Conversion of bhp to KWh, resulted in the following polynomial expressions.

(i) **Specific Fuel Consumption.** (SFC).

\[
SFC \text{ (gm/KWh)} = 479.81 - 7.4667N + 0.8171N^2 - 95 - 75 - 85 - 90 - 95 - 100 \% EACAC SPEED
\]
(ii) Brake Power. Following an approximate cubic relationship.

\[ P_b = P_s \text{ (KW)} = -0.09 + 30.5699N + 0.00097977N^{3.5} \]

Figures 3.24 and 3.25 represent these relationships.

**FIGURE 3.24. SFC FOR THE DART ATLANTIC SULZER ENGINE.**

**FIGURE 3.25. ENGINE BRAKE POWER FOR THE DART ATLANTIC SULZER ENGINE.**
3.20.1. SPECIFIC FUEL CONSUMPTION (SFC).

Equation 3.20.1 cannot take account of the fuel density or quality, and assumes a constant fuel flow rate at each engine setting. The quality of the fuel, which is not guaranteed even when bunkers are loaded, will vary, and cause variations in the fuel consumption, Hughes (1987).

It is possible to assume SFC to be a function of the form SFC = aPb, Hagiwara (1989), when the engine is restricted to working around the design value. In order to cover the eventuality that the algorithm requires a low engine setting, SFC has been expressed by equation 3.20.1 an extension of the SULZER information.

3.20.2. BRAKE POWER.

Equation 3.20.2 assumes the engine to be as efficient as that in 1981 and no hull or propeller deterioration since that date. Perturbations of the engine power for random seaways are neglected around the mean setting. Similarly, the measured power is assumed to be that at the crank shaft and not the maximum before any power take-off for other machinery.

3.21. SEMI EMPIRICAL/THEORETICAL ALGORITHMS.

In order to increase the computational speed in the algorithms, the total calm water resistances can be modelled by a least-squares polynomial. Therefore,

\[
R_{\text{calm}} = -36.74346188 + 34.64391532V_{\text{ship}} + 2.96730364V_{\text{ship}}^2 + 0.1835885V_{\text{ship}}^3 \\
+ 0.01074761V_{\text{ship}}^4 + 0.00062089V_{\text{ship}}^5 \\
V_{\text{ship}} \text{ – } \text{ms}^{-1} \quad 3.21.1
\]

Algorithms have been constructed for constant speed and constant power, however, Svenson (1987) states that containerships normally operate at constant speed. The processes are iterative around the delivered power, using a Newton-Raphson scheme described in sections 3.21.3 and 3.21.4.
Prior to any computations, Holtrop (1984) coefficients are evaluated, along with any non-speed dependant coefficients such as the wake fraction and thrust deduction factors. Computation of added resistance and critical motions are performed through interpolation of the databases.

3.21.1 INTERPOLATION OF THE DATABASES.

With reference to figures 3.26 and 3.27, two methods were studied to interpolate the databases for the required value, given the speed and the encounter angle to waves, bi-cubic splines and Maud curves.

(i) Bi-Cubic Spline Interpolation Scheme.

Spline curves are described as more efficient to model a series of points, than polynomial representations, Pennington (1974). Depending upon the number of points and the order of the polynomial, the curve can be made to be more exaggerated.

Let the database matrix be made up of MxN elements, then for each interpolation the following routine was made:

(a) Along each row (1..M), constant speed, fit a cubic spline, order N-1. (encounter angle = independant variable).
(b) Interpolate each spline, (1..M), at the heading value and store the result as a vector of M values.
(c) Fit a spline curve along the vector, (constant angle) order M-1. (speed = independant variable).
(d) Interpolate the curve at the required speed.

This algorithm although accurate, was found to be slow, since M+1 splines had to be deduced at each iteration, requiring the computation of many curve coefficients.

Solutions to the problems were found by using a Maud curve fitting routine.
(ii) Maud interpolation Scheme.

Prior to any interpolation, the coefficients of each Maud curve along the rows, 1..M, are stored off-line. Let the database be made up of M×N elements, then for each interpolation the following routine was made:

(a) Interpolate each curve 1..M, for the required heading and store the coefficients in a separate vector, of M elements.

(b) Deduce those elements of the vector, m−2, m−1, m, m+1 around the required speed.

(c) If a change of heading is 1° has occurred, skip to (e).

(d) Compute the coefficients of the curve along the vector and store.

(e) Interpolate the curve for the required speed using either the stored coefficients from the previous iteration or those just computed.
FIGURE 3.27. MAUD CURVE FIT TO INTERPOLATE THE DATABASES.

The difference between values computed from (i) or (ii) was found to be less than 1%. The Maud iteration has the advantage that the coefficients of the curves are precomputed and that only four elements are required to deduce the final value. Should a small change of heading be made from the previous iteration then there is no need to compute any coefficients making the scheme very efficient.

3.21.2. COMPUTATION OF DATABASE VALUES.

Each element in the databases was computed for a 5m significant wave height, since no critical motion responses would be deduced at 0 or very low wave heights. Therefore, the required data value is given by:

\[ M_{\text{reqd}} = \frac{M_{\text{interpolated}} \cdot \zeta_{1/3}}{5.0} \]

Where:

- \( M_{\text{interpolated}} \) - Assembly of interpolated data from the Maud routines.
- \( M_{\text{reqd}} \) - Assembly of computed data values.
3.21.3 CONSTANT SPEED MODEL.

Figure 3.28 shows the iteration scheme to compute the powering, engine revolutions and fuel consumption given a required ship speed.

---

(i) Newton-Raphson Iteration Scheme

The process works by iterating engine revolutions until computation of the delivered power, $P_{d1}$, to the propeller from the engine characteristics and shafting efficiency is matched, by computation of $P_{d2}$ through the relationships given by equation 3.19.7. The effect of the current is introduced before the powering is computed, by a simple vectorial summation. Vessel motions are computed prior to
any powering, so that the algorithm can omit any unnecessary computation should the limits be exceeded.

Iteration procedure.

1. Let

\[ F = P_{d2} - P_{d1} \]  \hspace{1cm} 3.21.3

2. Set low engine revolutions so that \( F \) is positive, increase \( N \), engine setting, until a negative value of \( F \) is found. That is:

\[ N(k) = N(k-1) + \alpha \]  \hspace{1cm} 3.21.4

\( \alpha = 15 \text{rpm} \).

3. Compute the Newton-Raphson iteration scheme.

\[ f = \frac{F(k) - F(k-1)}{N(k) - N(k-1)} \]

\hspace{1cm} 3.21.5

The new engine setting, \( N \) is given by:

\[ N(k+1) = N(k) + \frac{F(k)}{f} \]  \hspace{1cm} 3.21.6

Compute the value of \( F(k+1) \), goto 1, until \( |F(k+1)| \leq 125 \). (Consitutes 0.6\% @ MCR and 1.3\% @ \( N = 90 \%). The error on speed calculation is minimal.

(ii) Current set and Drift.

With reference to figure 3.29, the current speed and heading is a pre-determined value for both time and space from the forecast and analysis data arrays. These data, chapter 4, consider both the wind-driven and Stokes current incorporating the topography, Titlow (1989). The ship's speed and heading is that required to overcome the effects of current. Therefore, after deducing the encounter angle for current, the required speed and heading becomes a vectoral summation, Titlow, (1989).
\[ V_{\text{ship}} = \sqrt{V_{\text{stream}}^2 + V_{\text{rqd}}^2 + 2V_{\text{stream}}V_{\text{rqd}} \cos(\mu_{\text{stream}})} \]

Where:
- \( V_{\text{stream}} \): Current drift speed.
- \( \mu_{\text{stream}} \): Encounter angle to stream on required trajectory.
- \( V_{\text{rqd}} \): Required speed over the ground.
- \( V_{\text{ship}} \): Ship's speed through the water.

The required heading becomes a function of the encounter angle. All values used in the computation are functions of distance and time, and are therefore not affected under the assumption that current remains constant over that interval. So long as the intervals are small enough this assumption is reasonable, only breaking down in regions of strong variation in current, for example across the core of the North Atlantic drift.

3.21.4. CONSTANT POWER MODEL

Figure 3.30 indicates the scheme to compute the speed of the vessel assuming constant power, engine revolutions and specific fuel consumption.
Since the engine setting is constant, the delivered power $P_{d1}$ can be computed initially, before the iteration routines.
By setting a vessel speed, the delivered power, $P_{d2}$, is computed with reference to equation 3.19.7. The effect of current is computed after each speed iteration and corrections are made to vessel heading in order to maintain the required course. Speed has to be re-computed due to changes in the encounter angles.

**Iteration Procedure.**

1. Let

$$F = P_{d2} - P_{d1}$$

2. Set low ship speed so that $F$ is positive, increase speed until a negative value of $F$ is found. That is:

$$V_{\text{ship}}(k) = V_{\text{ship}}(k-1) + \alpha$$

$$\alpha = 4-5 \text{ knts.}$$

3. Compute the Newton-Raphson iteration scheme.

$$f = \frac{F(k) - F(k-1)}{V_{\text{ship}}(k) - V_{\text{ship}}(k-1)}$$

The new ship speed, $V_{\text{ship}}$, is given by:

$$V_{\text{ship}}(k+1) = V_{\text{ship}}(k) - \frac{F(k)}{f}$$

(i) **Current Set and Drift.**

Current set and drift is a pre-determined quantity, however, the problem now is that the speed of the vessel is unknown although its trajectory is. The speed of the vessel is first deduced along the prescribed course and the effect of current set and drift is computed afterwards. An adjustment to the heading is made to counteract the stream and the vessel's speed recomputed. Should the addition of current bring the vessel's heading to within the limits of the required value, the resultant speed is computed, otherwise the iteration is repeated. Figure 3.31 shows this scheme.
Figure 3.31. Computed speed and heading as a function of current.

Equation 3.21.7 is rewritten in the form

$$V_{\text{est}} = \sqrt{V_{\text{stream}}^2 + V_{\text{ship}}^2 - 2V_{\text{stream}}V_{\text{ship}} \cos(\Psi_{\text{stream}})}$$

Where
- $V_{\text{stream}}$ = Current drift speed.
- $V_{\text{est}}$ = Estimated speed over the ground.
- $V_{\text{ship}}$ = Ship's speed through the water.

The new heading of the vessel becomes:

$$\Psi_{\text{reqd}} = \Psi^* \pm \delta\Psi$$

$$\pm \delta\Psi = \Psi \pm \Psi_{\text{cmg}}$$
Where

\[ \psi_{rqd} \] - Required heading. \[ \psi_{r*} \] on first iteration and
\[ \psi_{r*} = \psi_{rqd} \] for next iteration.

\[ \delta \psi \] - Alteration to heading to counteract stream to bring course
made good the required trajectory.

\[ \psi \] - Required trajectory.

\[ \psi_{cmg} \] - Course made good after current effects.

Iteration ceases when:

\[ |\psi - \psi_{cmg}| \leq 0.1^\circ \]

3.21.15

Should the effect of current not cause any further change in heading, iteration
ceases.

3.22. CHAPTER SUMMARY

Several mathematical models of the ship to produce speed, fuel and powering
estimates have been analysed and constructed in this study. Particular attention
has been placed on the Babbedge (1975) and the semi-empirical/theoretical
algorithms. The Babbedge (1975) speed estimations and powering estimations have
been combined with the motion databases in order that a better comparison can
be made between the algorithms. Therefore, the Babbedge (1975) algorithm is used
along with the corrections for current set and drift and the motion simulations in
the same way as the semi empirical/theoretical algorithm.

The limitations of the algorithms and the assumptions made have been highlighted
in the text, however, it is felt that an appraisal of the models can still be made
for a micro-based weather routeing exercise.

On implementation of an on-board system it is envisaged that the algorithm be
tuned to fit the particular ship, Chen (1989).
4. THE ENVIRONMENTAL ALGORITHM AND ENVIRONMENTAL DATA ARRAYS.

4.1 INTRODUCTION.

Definition of the environment is fundamental to the routeing of ocean-going ships since the wind and waves forms the disturbance terms in the models. In order to route a vessel successfully it is a pre-requisite that there be sufficient data defining the environment for the entire time that the ship is at sea, Motte (1981), since prevailing conditions for the latter part of a voyage may influence decisions made at the outset. For a container ship, travelling at 20 knots over a 3000 nautical mile voyage, the ideal requirement is for data defining the ocean and atmosphere at every possible point and time of the 150 hour crossing.

Data necessary for a comprehensive routeing model are derived from numerical atmospheric and ocean models. The US Navy Global Spectral Ocean Wave Model, (GSOWM), is the main source of data for the routeing algorithms in this study.

Since no meteorological or oceanographical organisation produces forecasts of environmental data beyond +120 hours, there is a need to generate data beyond that period. These data, (the extended forecast), are generated within this study and compared to data sets provided at analysis time.

In order to extend the sets of data, beyond the forecast length, there are certain methods to extrapolate or produce these data arrays. Methods include curve fitting and extrapolation along with wave data generated from wind fields which are either provided from the numerical models or, as investigated within this study, from the extended surface pressure information from the European Centre for Medium range Weather Forecasts, (ECMWF). Further to these methods, there is a necessity to refine and smooth the data from within the routeing algorithms and environmental data generation algorithms. Since this study is concerned with
on-board approaches, the problems occuring in data down-loading, generation and handling are discussed. Primarily, the problem of data extension has been approached with a view to the physical nature of the environment, rather than the statistical inference from extended past historical or hindcast data.

In an ideal world data for the full crossing, would contain no errors, and the forecasts would be exact predictions of the actual environment.

In the real world, predictions are not entirely accurate even at analysis time. Predictions of today's numerical models are regarded as adequate to about +72 hours for the wind and wave fields. The correlation of predicted data against analysis has been determined for a selection of time progressive series from the environmental data supplied for this study.

4.2. NUMERICAL ATMOSPHERIC PREDICTION MODELS.
Numerical atmospheric models which predict the movement of the air masses define the primary input for the ocean wave models. The Meteorological Office in Bracknell produces forecast data at grid points in a layer structure, Met' Office (1986,1987). Models are run twice daily with the output being quality controlled and checked by spot measurement data.

The medium range, ECMWF forecast, 10 layer, Numerical Weather Prediction, (NWP), model was introduced in 1972. (Medium range forecasts are defined by the World Meteorological Office (WMO), as greater than 72 hours and up to 10 days). Correlation of the 72 hour numerical forecast was at that time approximately 0.65. The Meteorological office now operate a 20-layer coarse mesh, NWP model which incorporates a fine mesh in local areas, running on an ETA 10 supercomputer. In 1987, the NWP model operated on a series of grid points spaced at 1.5° latitude and 1.875° longitude, and operated a finite difference scheme rather than the spectral approach employed in other meteorological models.
There are three solutions to the prediction of waves, Wiegel (1964), Bishop (1984), involving the wind field as the primary input. These are:

1. Significant wave height, \( \zeta_{1/3} \), empiricisms;
2. Discrete spectral methods;

These models are further characterised by their method of manipulation, either manually or by computer, Bishop (1984). Comparisons of these methods have been made by Bretschneider (1959), and Neumann et al (1966), but this proved to be very difficult since the algorithms were produced for specific sites. Wiegel (1964) states that there is not much difference between the empirical methods and the spectral approach, since the latter can be defined as a function of the total energy, \( E \), of the spectrum - originally found as a function of wind speed for a fully developed seaway, \( \text{FDS} \).

4.3.1. SIGNIFICANT WAVE HEIGHT EMPIRICISMS

Many attempts at forecasting wave parameters from the wind field have been made. Wiegel (1964), Neumann (1966), Motte (1972), have all given extensive historical descriptions and catalogues of these empiricisms. Some examples are:

1. Cornish. 
   \[ \zeta_{1/3} \text{ (feet)} = 0.48 \times U_{\text{wind}} \text{ (knots)} \]

2. Zimmerman.
   \[ \zeta_{1/3} \text{ (feet)} = 0.44 \times U_{\text{wind}} \text{ (knots)} \]

3. Scripps Inst.
   \[ \zeta_{1/3} \text{ (feet)} = 0.026 \times U_{\text{wind}}^2 \text{ (knots)} \]

4. Rossby.
   \[ \zeta_{1/3} \text{ (feet)} = 0.0305 \times U_{\text{wind}}^2 \text{ (knots)} \]

5. Scott.
   \[ \zeta_{1/3} \text{ (feet)} = U_{\text{wind}}^{3/2} \text{ (knots)} + 5.0 \]

These have distinct limitations since they are site-specific and should not be used for areas other than those for which they were designed. The empiricisms assume a fully developed condition, since no account is taken for fetch or duration lengths.
of the wind. These models produce mono-chromatic or two-dimensional wave systems.

Further empiricism were developed to include the fetch and duration of the seaways. The Sverdrup-Munk-Bretschneider method (SMB) method is given by:

\[
\zeta_{1/3} = 0.283 \frac{U_{\text{wind}}^2}{g} \tanh \left( 0.0125 \left( \frac{gF}{U_{\text{wind}}^2} \right)^{0.42} \right)
\]

\[
T_{\text{wavep}} = 2.0 \frac{U_{\text{wind}}}{g}^{1.2} \left[ 0.77 \left( \frac{gF}{U_{\text{wind}}^2} \right)^{0.25} \right]
\]

Where
- \( g \) = Acceleration due to gravity, 9.81 m/s\(^2\).
- \( U \) = Wind speed, m/s.
- \( F \) = Fetch length, m.
- \( T_{\text{wavep}} \) = Primary wave period or modal wave period.

Should fetch be limited by duration then the effective fetch is computed by:

\[
g \frac{t}{U_{\text{wind}}} = 6.5882 \left\{ 0.0161 \left( \ln \left( \frac{gF_e}{U_{\text{wind}}^2} \right)^2 \right) - 0.3692 \ln \left( \frac{gF_e}{U_{\text{wind}}^2} \right) + 2.2024 \right\}^{0.5} + 0.8798 \ln \left( \frac{gF_e}{U_{\text{wind}}^2} \right)
\]

Where
- \( F_e \) = Effective fetch length, m.
- \( t \) = Duration of wind, secs.

Graphical representations of this method have been produced, Perry et al (1977), known as cumulative sea state diagrams which are still used today. The Darbyshire (1963) model, from Hogben (1981), states those parameters above based on observations taken in the North Atlantic and Irish seas.
\[ \zeta_{1/3} = 1.416\left(0.0081y^{3/2}U_{wind}^2\right) = 0.0115y^{3/2}U_{wind}^2 \]

and:
\[ y = \frac{\left(F^3 + 3F^2 + 65F\right)}{\left(F^3 + 12F^2 + 260F + 80\right)} \]

Where:
- \( F \) - Fetch length in nautical miles.
- \( F_e = 1.52t \) if limited by duration.
- \( t \) - Duration of wind in hours.
- \( U \) - Wind speed in Knots.

\[ T_0 = y^{0.75}\left(1.94\sqrt{U_{wind}} + 0.0000025U_{wind}^4\right) \]

Where:
- \( T_0 \) - Modal wave period or primary.

Titov (1971), has shown the production of wave forecasts involving the fetch and duration of wind fields, along with the delimiters defining these parameters.

\[ \overline{\zeta} = 0.029t^{-1.5}U_{wind}^{0.5} \]

Under the limiting conditions for FDS,

\[ F_{lim} = 3.06\ U_{wind}^2 \quad t_{lim} = 1.89\ U_{wind} \]

Where:
- \( F \) - Fetch Length, Km.
- \( t \) - Duration, hours.

These expressions are treated with caution, due to the difficulties in measuring fetch lengths and durations accurately.

**4.3.2. SPECTRAL WAVE ALGORITHMS.**

The formulation of a mono-chromatic wave does not take account of the wave components of the random seaway. Total energy in the seaway can be
represented by a summation of the individual sinusoidal wave train energies that make up that seaway. For ship seakeeping and motion studies it is important that the seaway be represented in this form, Ochi et al (1977) and Deakins (1988).

Pierson and Moskowitz (1964), assumed the sea spectra to be a function:

\[ S(f) = h(f, g, U_{\text{wind}}, F) \]  \hspace{1cm} (4.3.16)

Where \( f = \omega / 2\pi \)

In this formulation, the wind velocity, \( U_{\text{wind}} \), derived from geostrophic flow, is corrected to the friction velocity at the sea surface; also fetch length is removed for fully developed seaways. Then :

\[ S(\omega) = h(g, \omega, U_{\text{wind}}) \]  \hspace{1cm} (4.3.17)

Spectral density is controlled by the single parameter, \( U_{\text{wind}} \), with discrete values for circular frequency, \( \omega \). Many authors have produced spectral representations.

1. Pierson and Moskowitz (1964), studied approximately 400 sea records from the North Atlantic and produced a spectral form of :

\[ S(\omega) = \frac{\alpha \sigma \omega^2}{\omega_0^4} e^{-\beta \left( \frac{\omega_0}{\omega} \right)^4} \]  \hspace{1cm} (4.3.18)

Where \( \beta = 0.74 \); \( \alpha = 8.1 \times 10^{-3} \); \( \frac{\omega_0}{\omega} = \frac{g}{U_{\text{wind}} \omega} \)

This spectrum defines the FDS, and can be expressed as a function of the significant wave height, by assuming the waves to be gaussian and the spectrum to be narrow banded. The area under the spectrum is given by :

\[ m_0 = \left( \frac{\sigma_{1/3}}{4} \right)^2 \]  \hspace{1cm} (4.3.19)

and :
Equation 4.3.20, defines the PM spectrum by wave height. The modal frequency becomes fixed, and given by:

\[ \omega_m = 0.4 \sqrt{\frac{g}{\zeta_{1/3}}} \]  

For the FDS seaway, equation 4.3.18 can be expressed in the empirical form:

\[ \zeta_{1/3} = 2.12 \times 10^{-2} \frac{U_{wind}^2}{\text{SI units}} \]  
\[ \zeta_{1/3} = 1.82 \times 10^{-2} \frac{U_{wind}^2}{\text{Imperial units}} \]  

Wind speed is defined at 19.5m, although it is possible to refer this wind to a value at any height, Pierson (1964). Neumann et al (1966) state that after height corrections for wind, the Pierson-Moskowitz spectrum gives good agreement to the Neumann and Bretschneider spectra. The average period of the wave can be determined from:

\[ T_{wavep} = 0.81 \frac{2\pi U_{wind}}{g} \]

These representations have been used in the development of the extended wave forecast since as Neumann et al (1966) state, they generate the gross features of the wind waves and swell and can be recomposed as the spectral form for the study of ship motions.

2. Bretschneider (1959) developed a spectral representation for developed as well as partially developed seaways, and is analogous to the previous empirical statements.
\[ S(\omega) = 3.437 \frac{F_1^2}{F_2^4} \frac{g^2}{\omega^5} e^{-\left(\frac{g}{F_2 U_{\text{wind}} \omega}\right)} \]  \hspace{1cm} 4.3.24

where:

\[ F_1 = \frac{g \xi}{U_{\text{wind}}} \]
\[ F_2 = \frac{T_{\text{wavep}}}{2\pi U_{\text{wind}}} \]

\[ \int_0^\infty T_{\text{wavep}} S(T_{\text{wavep}}) dT_{\text{wavep}} = \int_0^\infty S(T_{\text{wavep}}) dT_{\text{wavep}} \]

\[ T_{\text{avep}} \quad \text{average wave period.} \]

\[ \zeta \quad \text{average wave height.} \]

\[ \text{and can be expressed in terms of } \zeta_{1/3}. \]

\[ S(\omega) = \frac{1.25}{4.0} \frac{\omega_m^4}{\omega^5} \zeta_{1/3}^2 e^{-1.25 \left(\frac{\omega_m}{\omega}\right)} \]  \hspace{1cm} 4.3.25

This spectrum has two spectral shape parameters, \( \zeta_{1/3} \) and \( \omega_m \), and is defined as a family of spectra, which can be seen to take account of developing and decaying seaways. The FDS, Bretschneider spectrum, or most probable, is identical to the Pierson-Moskowitz. Balee (1985), states that the Bretschneider two-parameter results in a better description of the seaway and therefore an improvement of ship responses. However, as discussed in section 3.15, the FDS spectrum can be corrected for conditions other than FDS. Values of modal frequency, \( \omega_m \), can be obtained from Walden’s all-season data, (1964). Ochi et al (1977) defined a set of nine modal frequencies, along with the most probable, thereby deducing a set of nine possible ship motion values.

Ochi et al (1977) have shown the effect that various spectral representations have on the response spectrum of the vessel. Deakins (1988), concluded that the better spectra is that by Ochi (1976) which defines a family of spectra covering the growth and decay of seaways including swell. This spectrum involves six-parameters, and as explained in section 3.14 was not feasible for a motion database for an on-board computer.
The total energy of the seaway spectrum can be expressed as:

\[
E = \int_{\mu_{\text{wave}} = 0}^{\mu_{\text{wave}} = 360} \int_{\omega = 0}^{\omega = \infty} S(\omega, \mu_{\text{wave}}) \, d\omega \, d\mu_{\text{wave}}
\]

This indicates the spectral limits that are taken by the GSOWM, fifteen frequencies, \(\omega\), by twelve directions, \(\mu_{\text{wave}}\). Many statistical inferences can be deduced from \(E\), for example:

\[
\zeta_{1/3} = 4.0 \sqrt{E \cdot \text{CF}}
\]

Where \(\text{CF}\) - Cumulative frequency.

The Pierson-Moskowitz spectral form has been used to compute the spectral energy, \(E\),

\[
E = \int_{\omega = 0}^{\omega = \infty} S(\omega) \, d\omega = \frac{\alpha U_{\text{wind}}^4}{4\beta g^2}
\]

From equations 4.3.28 and 4.3.29, it can be deduced that:

\[
\zeta_{1/3} = 2.12 \times 10^{-2} \, U_{\text{wind}}^2
\]

This compares with 4.3.22.(a) and 4.3.22.(b).

4.4. THE GLOBAL SPECTRAL OCEAN WAVE MODEL - (GSOWM).

The low-level wind, (forcing function), for the US navy model is produced from the Global Surface Contact Layer Interface, (GSCLI), model, which results from the Navy Operational Global Atmospheric Prediction System, (NOGAPS).

The ECMWF and the US Navy model at the Fleet Numerical Oceanography Center (FNOC), both base their models on the work by Pierson et al (1964).

The FNOC based their models on the spectral method since 1973, These models were referred to as SOWM, and their output has been freely available since 1981.
via the Satellite Data Distribution System, (SDDS), of the US Navy, Lanzanoff (1983). Many studies as to its validity have been made and have been summarised in Pierson et al (1982).

The Global Spectral Ocean Wave Model, (GSOWM), superseded the SOWM in 1984, Clancey et al (1986). The model incorporates the growth, dissipation, angular propagation, decay, and interaction of waves providing a prediction of the directional wave spectrum for locations spread throughout the world.

The GSOWM takes account of the local wind fields, propagation of the wave energy from another storm area, influence of land masses, currents and shallow water effects on the continental shelves.

Each spectral parameter in the seaway is provided by forward time-differencing in three-hourly steps.

The directional spectrum output is provided as a matrix defining 15 frequencies and 12 directions at 12-hourly increments, (cf 13 frequencies and 16 directions in the UK Meteorological Global Ocean Wave Prediction System, Met' Office (1987)).

4.4.1. GSOWM ENVIRONMENTAL DATA POINTS AND CONDENSED DATA.

Figure 4.1 shows the locations of the North Atlantic grid points used in this study. It was decided to concentrate on the North Atlantic, from below the northeast trade winds (~22.5° N) to the northerly limit of the westerlies (60° N). This area of the 'open ocean' is strongly influenced by latitude, Deakins (1988), and incorporates the baroclinic zone, where the highest transfer of energy to the ocean takes place. This region accounts for depressional growth.

The environmental data is supplied from the GSOWM at 2.5° grid points for the whole globe and therefore provides arrays formed by the discrete grid points, (denoted by i and j, see figure 4.1), where each array is defined as an element of
a time series. Procedures were implemented which permit the dissemination of the spectral energy matrix data for commercial users. Petrie et al (1984). OCEANROUTES (1987), operate a distribution system for ocean data through INMARSAT (1987). The parameters available from the GSOWM are numerous and would easily swamp a micro-based routeing system, as well as greatly increasing the cost of the transmission of the data, over land lines.

Data supplied by OCEANROUTES INC was for $i = 13 - 28$ and $j = 89 - 121$, or $i = 60^\circ N - 22.5^\circ N$ and $j = 80^\circ W - 0^\circ$.

**FIGURE 4.1. LOCATION OF THE GSOWM GRID POINTS USED IN THIS STUDY.**
The outputs from GSOWM are:

1. Significant wave height, $\eta_{1/3}$;
2. Maximum wave height, $\eta_{\text{max}}$;
3. Whitecap probability;
4. Primary and secondary wave direction, $\Psi_{\text{wave}}$, $\Psi_{\text{waves}}$;
5. Primary and secondary wave period, $T_{\text{wavep}}$, $T_{\text{waves}}$

The outputs made available to this study are, 1, 4, and 5, but also:

1. Wind speed and direction, $U_{\text{wind}}$, $\Psi_{\text{wind}}$;
2. Current set and drift, (direction and speed), $\Psi_{\text{stream}}$, $V_{\text{stream}}$

Wave height parameters are integral functions of the spectrum, as an assumed Rayleigh distribution, $(CF=1)$, Clancey et al (1986).

4.4.2. VALIDITY OF THE GSOWM MODEL.

Partial validation of the SOWM model, climatological data sets have been carried out, Cummins et al (1980) and Chen (1979), indicating reasonable accuracy. Cummins et al (1980) and Clancey et al (1986), give brief descriptions of the work carried out by Lanzanoff and Stevenson (1975) and Salfi et al (1977), for SOWM validation.

The SOWM can accurately predict the propagation of low frequency energy (swell), can cope with simultaneous wave trains, compares reasonably well with satellite observations and buoy measurements and can permit the realistic growth of high waves in a non-linear situation, when in fact only linear superposition is applied, Cummins et al (1980), Denis (1980). The Royal Netherlands Navy collected data during 1978 and 1979, Clancey et al (1986), and concluded from comparisons of the SOWM forecast and the point spectra that:–
1. The shapes of the spectra are more accurate when significant wave height is large (~10m);

2. Derived spectral parameters are usually a good statistical fit, made on the GSOWM.

Hamilton (1980) states that the standard deviation of error between measured point spectra from NOAA Data Buoys and the interpolated model-predicted data is of the order 0.5m. More recently comparison studies have been made between the GSOWM and SOWM models, Clancey et al (1986), who based the results on statistical derivatives of the outputs and concluded that the poorest prediction is in the eastern Atlantic attributable to the error in the wind forcing function.

4.4.3. STUDY ON THE QUALITY/RELIABILITY OF THE GSOWM DATA

The initial conditions of the GSOWM are not provided by observations, but instead the model provides its own initial conditions based on a hindcast file of predicted wave energy. Forecasts are produced by integrating the model forward in time Clancey (1986) and updated twice daily.

The contoured GSOWM data was compared visually to the sea and swell facsimile charts transmitted from Offenbach. Surprising variations were found between these sources, typically on the eastern seaboard of the North Atlantic, note, Clancey et al (1986). The exact values and positions of high wave activity are not the same, however, it is difficult to draw any firm conclusions from the comparisons without a recourse to their data sources. Care should be applied, especially if statistical inferences are deduced from data taken from these sources.

For a relative assessment in the following sections, the GSOWM output at \( \tau = 0 \) is assumed to have a correlation of unity with the true ocean environment. All correlations result from the whole array and not individual elements or areas.
Figure 4.2 depicts the degradation of GSOWM data as time progresses from the base dates, March 3rd and 26th April 1989. The forecasted data is shown plotted against the analysis (tau=0), that time on from the base date.

Results from the correlations are unfortunately limited, and care has to be applied to any conclusions, since differing meteorological conditions will affect the results, (whether or not the atmosphere and ocean was unstable, semi-stable or stable). Figure 4.2 indicates degredation during relatively stable periods and therefore produces good correlation at extended periods.

4.5. GSOWM DATA SETS FOR ON-BOARD OPTIMUM SHIP ROUTEING.

Earlier studies on the selection of the ship route, see Chapter 1, were based on storm avoidance, Francis (1971). Forecasts of atmospheric pressure were made and the estimated wave activity derived from the estimated wind field. Moens (1980), Motte (1981) favoured the 500mb steering level as a pointer to extended data fields. It was realised that this approach cannot be fully utilised for the
predictions of ship motions and the incorporation of seakeeping and speedkeeping into the routeing algorithms when only considering the single wave parameters. The condensed data available from the GSOWM with wind and current information, see section 4.4.1, are produced at defined forecast periods given in table 4.1.

<table>
<thead>
<tr>
<th>FORECAST HOUR</th>
<th>TAU NUMBER</th>
<th>WAVE PARAMETERS</th>
<th>WIND PARAMETERS</th>
<th>CURRENT PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+12</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+24</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+36</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+48</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+60</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+72</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+84</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+96</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+108</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+132</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+144</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.1. 0000Z (GMT) Data Set.**

<table>
<thead>
<tr>
<th>FORECAST HOUR</th>
<th>TAU NUMBER</th>
<th>WAVE PARAMETERS</th>
<th>WIND PARAMETERS</th>
<th>CURRENT PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+12</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+24</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+36</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+48</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+60</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+72</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+84</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+96</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+108</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+132</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+144</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.2. 1200Z (GMT) Data Set.**

Key:  ■ — data file supplied by OCEANROUTES, from the GSOWM.

Wave parameters are forecast to 72-hours in 12-hourly steps, whilst the wind fields are supplied with two additional forecasts at +96 and +120 hours. Similarly, current data is only provided once, at analysis time. Table 4.2 indicates the data provided at 1200Z or GMT.
These data sets present several problems:-

**For the 0000Z data set.**

1. The extension of the data beyond the +120 hour period at the early stages of the route. Extension for +132 and +144 especially;
2. The fill in of the data sets for wind and waves at +84 and +108 hours;
3. The fill in of the data set for waves at +96 and +120 hours and finally;

**For the 1200Z data set.**

4. The problem of the 1200Z data set only extending to +48 hours, and not including current information.

These problems were tackled in a variety of ways.

### 4.6. Extensions of the GSOWM Data Sets.

Several methods to fill the period beyond the forecast length to the termination of the ship's route have been attempted, notably concentrating on monthly or seasonal mean values, or climatology, however, this study has attempted the:

1. Use of GSOWM forecasted wind data to extend the wave fields at +96 and +120 hour forecasts in the 0000Z data set;
2. Use of spline interpolation for waves at +84, +108 hours for the 0000Z data set;
3. Extension of wind and wave fields from digitised surface pressure fields +120 and +144 hours for 1200Z data sets;
4. Spline Interpolation of the +132 hour wind and wave fields for the 1200Z data set;
5. Generation of a moving or running climatology, (OCEANROUTES) as described later, beyond +144 hours for both sets or as an extension of both data sets in the event that no pressure data is received;
6. Extrapolation of a polynomial beyond the forecast period of waves and winds to compute the required extended data sets.


8. Use of the +144 1200Z forecast transposed to the +132 0000Z data set.

9. Extrapolation of the +144 0000Z pressure array from the preceding +144 1200Z arrays and subsequent generation of other data arrays.

Routing examples contained in chapters 6, 7 and 8, show the effect of using extended data, including an observation on the effect on the route that a blanket zero sea state would have beyond the +120th hour.

The accuracy of the extended data will be relatively poor by whatever means are taken, however, in the light of a routing exercise it is only required to persuade the vessel's route at the termination of its voyage, and not at the critical departure and central sections. Furthermore, a container ship which transits the North Atlantic from east to west, at 25 knots will cover 3000nm in 120 hours, (assuming she maintained that speed under fair weather). The vessel will only use the extended data after 3000nm and in reality will never encounter these predictions, since they will become updated as the route progresses. The extended data will only be required, in the North Atlantic, for those segments of the journey near to shore, since the width of the ocean is of the order 3000nm. For routes which tend equatorwards, the need for extended data is much reduced, since this area is outside of the baroclinic zone, see section 4.6.4.

It was decided that the +96 and +120 0000Z hour wind field forecasts could be utilised with certain empirical wind-wave relationships, sections 4.3.1 and 4.3.2 but also those formulae including fetch and duration of the wind field, section 4.3.1. These parameters were found from the wind forecasts leading to the desired forecast.
The extension beyond the $+120^{th}$ hour $0000Z$ and $1200Z$ forecast requires generation of the wind fields and the wave fields. For this reason OCEANROUTES provided the digitised ECMWF surface pressure data. From such information it is possible to derive the surface wind field and through the wind-wave relationships, deduce the wave fields.

Since the pressure data only extends to $+144$ hours, there is a necessity to have further file extensions. OCEANROUTES provided their 'running climatology' data fields which are simply a running summation of the preceding seven forecast data arrays, assumed constant beyond the forecast length of the two series. Extension of a polynomial or spline curve fit through the data sets was investigated for extended forecast files beyond $+144$ hours.

The $+84$, $+108$ and $+132$ forecasts $0000Z$ forecasts were generated by inference from a spline curve fit through the $+0$ to $+144$ data fields. It was realised that any time point could be deduced, and the polynomial based on the preceding time-series.

The missing data files in the $1200Z$ data sets could be easily computed by assuming the forecasts from the previous $0000Z$ data set, but removed by 12 hours.

Throughout the following sections the direct results from the data extensions for the whole north Atlantic ocean have been correlated with GSOWM forecast and analysis data arrays. It will be seen that poor results appear to have been obtained by reference to the correlation coefficient, however, by referring to the relevant appendices it will be seen that these results are strongly influenced by a few poor predictions. The reasons for these are described.

Unfortunately it was not possible to further the statistical analysis of the results and produce confidence limits or provide correlation results for areas of the ocean
rather than the whole. However, a few selected examples are shown, effectively by removing those poor predictions from the results and correlating the remainder.

4.6.1. CLIMATOLOGY.

Petrie et al (1984), state that the parameters required from climatological studies need to be the same as those from the forecast GSOWM. Wind fields have been archived for up to 18 years hence, Bales et al (1982), and the (G)SOWM has been used to generate the wave parameters for each grid point at 6-hourly intervals. This forms a data set from which the joint probabilities of parameters can be found, for seasonal or monthly periods, Petrie et al (1984).

Climatology is unfortunately a static phenomenon and cannot be used singularly for weather routeing. It is recognised that it tends to mean out the actual wind/wave systems that the vessel is attempting to avoid.

The fill-in from day 4, (+72 hours) is obtained by a computer search of a past 28-year history library of surface pressure and 500mb, upper air charts, Chen (1979), Klapp (1979), Frankel et al (1980) and Clune (1975). The search aims to match the current weather system with an analogous pattern from the history set in order to obtain a logical sequence of events forecast from day 4. The period beyond the 10th day reverts to the SOWM/GSOWM climatological data.

A true pattern match will never be achieved due to the randomness of the ocean and atmosphere. Similarly it is well known that the energy in the ocean dissipates slowly and is greatly influenced by the previous patterns, as shown by the correlations of the wind and wave parameters mentioned in section 4.4.3. Using the atmospheric parameters to decide the relative pattern is therefore prone to difficulties. It should be mentioned that the search only takes place within a 60 day band about the period of the required time. A statistical value is also provided along with the data which describes the skill of the match between the patterns, Klapp (1979) and Clune (1975).
4.6.2 GENERATION OF +96 AND +120 HOUR WAVE FIELDS FROM THE +96 AND +120 HOUR GSOWM WIND FIELDS USING SIMPLE EMPIRICISMS.

The wind fields for the +96 and +120 hour forecasts from the GSOWM data sets were used in conjunction with the Pierson-Moskowitz, (PM), and Scott, empirisms, sections 4.3.1 and 4.3.2.

The wind field was assumed at a height of 19.5m, Clancey et al (1986), therefore, in order to make comparisons, the wind has to be adjusted to the 10m height for the Scott algorithm. Pierson (1964), studied the effect of the wind profile, from the surface to a height z, on the computation of the wave parameters.

Pierson (1964) concluded that the Sheppard formulae (1958), gave rise to more representative values of the open sea conditions since it generated the greatest reduction in discrepancies between the different wind speed, wave height theories. (It was noted that measurements of wind at a constant height from vessels, are prone to error due to the natural ship motions). The Sheppard formulae (1958) is given by:-

\[
U_{10} = \frac{U_z}{\left(1.0 + \frac{C_{10}^{0.5}}{k \ln \frac{z}{10}}\right)}
\]

\[
C_{10} = (0.8 + 0.114U_{10}) \cdot 10^{-3}
\]

Where
- \( k = 0.4 \)
- \( z \) - Height above surface, m.
- \( C_{10} \) - Drag coefficient, at 10m.

Empiricisms were used at each data point, that is the wind/wave function was evaluated for the array matrix and can be expressed as :-

\[
\begin{bmatrix}
\text{wave matrix}
\end{bmatrix}_{i,j} = f(\text{wind})\begin{bmatrix}
\text{wind matrix}
\end{bmatrix}_{i,j}
\]

For \( i = 9, 24 \) and \( j = 0, 32 \), see figure 4.1.
The wave fields were compared directly to the analysis files, +96 and +120 hours from the base date. In an ideal situation the plot of forecasted data against analysis data should yield a straight line (y=x), with a correlation of unity. Such plots were made and several are contained in appendix A.4. Figure 4.3 summarises several correlations and it will be seen that this is fairly low, however, this simple method improves the climatological approach since it uses the predicted atmospheric situation, and has a greater chance of picking up the systems that affect the ocean. It is therefore regarded as an indicative measure.

![Graph showing correlations of +96 and +120 hour wave heights correlated against analysis that time on from the base dates.](image)

**Figure 4.3. Correlations of +96 Hour and +120 Hour Wave Height Predictions to Analysis.**

The correlation is determined by the accuracy of the wind prediction and the accuracy of the wind-wave relationship. Wind accuracy varies considerably as shown in the degradation of the GSOWM in section 4.4.3, and is due to the varying stability of the atmosphere. Stable systems or blocking high pressure situations give rise to low wind speeds or very low wave heights. However, it is usual to experience swell, originating from far off depressions, which simple
empiricisms cannot predict. Unstable or depressional activity cause high wind speeds and high wave heights. The local sea is therefore dominant and predictions are more accurate so long as the fully developed state is achieved. Near shore areas, or those at meteorological fronts, limit the fetch and duration, and therefore the developed sea state.

Figure 4.4 indicates correlations of a time series of wave fields which were generated from the time wind series, using the wave empiricisms, see appendix A.5. Comparisons were made to both analysis and the forecast wave arrays in order that some judgement can be made reflecting the wind accuracy. Correlations are reasonable, and deteriorate as the forecast time increases. Since the atmosphere is so random, it is difficult to determine average correlation values.

With reference to appendix A.4, it can be seen that the correlation coefficients are not a true representation and only determine some straight-line fit, not necessarily $y=x$. Similarly, one or two poor predictions that seriously affect the correlation can be related to high wind fields in near-shore areas, where the waves are fetch limited, although the empiricism assumes an FDS. This occurs on the western seaboard, whereas, poor wind predictions are attributed to poorer predictions of waves on the eastern seaboard.

General high wave centres are maintained, since high waves and depressions are associated with high wind speeds, however, the actual quantitative values of the waves are not entirely accurate. This emphasises that it is the relative change of data that is important, since the vessel will be similarly routed even though the wave parameters may differ.

Predictions of average wave period were obtained from equation 4.3.23, corrected to primary wave period, were not regarded since better predictions were found from climatological extensions.
4.6.3. Generation of +96 and +120 hour wave fields from the +96 and +120 hour GSWM wind fields using simple empiricisms with fetch and duration.

The computation of wave parameters using the wind data at +96 and +120 hours was thought to be improved by incorporating the duration and fetch of the wind, to reduce the poorer near-shore wave predictions. The Darbyshire, Titov and Bretschneider formulae, section 4.3.1, were used.

Titov (1971) states the following bounds to be sufficient to assume or compute fetch length or wind duration:

1. Fetch length is that where the wind direction does not vary more than 2½ compass points (~28 degrees). In the open ocean, the start of the fetch is usually located at the point where a marked change in wind direction or wind speed occurs, for example at meteorological fronts.
2. Duration of the wind is that where the wind speed does not change more than 2 ms\(^{-1}\). Bishop (1986) noted that under GSOWM predictions, the minimum duration is 6 hours, (half the period between forecasts).

It will be appreciated that both parameters are interdependent, since duration will limit fetch, and vice-versa.

In computing the wave parameters, the time series of wind fields from \(\tau=0\) to the forecast time are required. Using wind data at each element of the array, at the forecast time and by back-tracking through the preceding forecast data arrays, it is possible to compute the duration of the wind at that element.

Following the anti-direction of the wind, it is possible to find the closest element to an imaginary point, at which the duration and fetch delimiters are applied, assuming that the data does not vary over the distance between the points. The process is repeated until the fetch conditions are exceeded. By referencing those elements in the array that define the fetch length it is possible to check the data at those elements through the time series, in reverse order from the forecast time to obtain the duration of constant blow.

Titov (1971) states that :-

'... in order to estimate wave dimensions from complicated weather conditions during which the wave-generating factors change ... (fetch/duration/velocity) ... it would be sensible to estimate the development of each individual wave or of each wave system and sum the results ...'

Such a system would be extremely complicated. The summation of waves computed from different fetch lengths at extended lengths would be prone to many errors.
Since the Bretschneider, Darbyshire and Titov algorithms require wind speed to be defined at 10m, it was necessary to use the Sheppard formulae, equation 4.6.1.

It is necessary to assume a minimum fetch length, for the open ocean. Following that of Bishop (1984), this was assumed to be half of the grid point spacing at that latitude. The near shore array elements, on the western coast, are fetch limited by the geography of the land.

Unfortunately poor results were obtained, for the following reasons:

1. The generation of fetch lengths are prone to inaccuracies, and it was found that the minimum fetch length was used frequently;
2. Since the wind array is wide, elements of that array close to the point on the direction of the fetch are not representative. This could only be improved using some non-linear interpolation routine to estimate the wind parameters from the closest elements of the wind array which could require implementation throughout the time-series;
3. Site-specific wave height formulae were used;
4. Poor wave empiricisms.

The accuracy of the calculations can be considered in the general form under the assumption that the main wave parameters are dependant upon the wind speed, duration and fetch, Titov (1971). Thus if:

\[ \zeta = k_1 \frac{U_{\text{wind}}^{0.5}}{t^{0.5}} \quad \text{P(\% \zeta) = 10 + 5} = 15\% \]
\[ \zeta = k_2 \frac{U_{\text{wind}}^{1.5}}{t^{0.5}} \quad \text{P(\% \zeta) = 15 + 5} = 20\% \]

relative error at 10% per parameter.

Titov (1971) states that the relative error in determining \( U_{\text{wind}} \), \( t \) and \( F \) are to be no greater than 10% and that the error in wind speed effects the computation the greatest.
Due to the difficulties, this methodology was disregarded, although there is a possibility to utilise FDS empiricisms for the majority of the ocean and only use fetch and duration empiricisms at the Atlantic, western seaboard. This would induce problems, arising from the use of two separate empiricisms, and could lead to inconsistency in the coastal areas.

4.6.4. EXTENSION OF DATA BEYOND +120 HOURS.

It is necessary to extend the data beyond the forecast length from the GSOWM. It was felt that since the accuracy of predictions would be low, the relative shape of the higher wave areas was more relevant, that is, the positions of high and low pressure areas of extremes of wind and waves. The routeing algorithms should still avoid such areas, due to the relative change of wave heights.

![Diagram of Baroclinic Zone Model](image)

**FIGURE 4.5. BAROCLINIC ZONE MODEL.**

By considering the baroclinic zone of the ocean and the baroclinic model, it will be appreciated that reasonable correlations of the mean or climatological 'type' data extensions will have a reasonable correlation with the ocean environment. However, the depressions will be omitted. This can be represented in figure, 4.5.
The baroclinic zone is formed by the temperature gradient existing between tropical and polar regions. This may be regarded as consisting of two components, the mean temperature difference, $\Delta T_{air}$, and the eddy component, $\Delta T_{air}'$, as a result of transient depression activity, thus:

$$\Delta T_b = \Delta T_{air} + \Delta T_{air}'$$  \hspace{1cm} (4.6.5)

The mean temperature difference varies seasonally, obviously being much greater in winter months. This thermal difference component yields a zonal wind value, $\bar{U}$, varying directly as the inducing thermal gradient. As depressions are generated along the polar front eddy fluctuations occur in the temperature gradient moving in and out of phase with the mean zonal component. These baroclinic eddies introduce both zonal and meridianal components, $(u' \text{ and } v')$, which may be considered to be superimposed on the mean seasonal value. Thus:

$$v_b = \bar{U}_{wind} + u'v' \Rightarrow v_b = \bar{U}_{wind} + (u' \times v')$$  \hspace{1cm} (4.6.6)

A meaningful or averaging of data over periods relating to the time scale of eddies, (one week), will eventually remove these eddy component with only the mean seasonal value emerging in such a climatological process. The very depressional activity which gives rise to the extreme seaway systems that routeing plans to avoid, will be removed.

During blocking and when there is little wave or depression activity, it is quite likely that climatological correlation to the environment will be high, and even during periods when there are depressions of sufficient strength, climatology will correlate with the underlying steady components but not the actual storm centres.

The dissipation period of the eddy wind components is very short in comparison to the eddy wave components generated from them, therefore, in a climatological
sense it is quite likely that better correlation will be obtained for wave fields than for wind fields, as graphically shown in section 4.6.5, figures 4.6 and 4.7.

By considering the ECMWF surface pressure, a representation of the wind is obtained and therefore high wave centres are located. The correlation value of the generated waves may well be low, and the value of climatological data higher during low depressional activity, since the baroclinic system is more stable. The opposite is the case when deep depressions are traversing the ocean. This can be seen by considering figures 4.7, 4.8 and 4.9. During April 21st, 22nd correlation values from the ECMWF are better than climatological approaches, due to oceanic depressions, but becomes poorer later when the depressions have decayed.

A further possibility would be to combine the two systems and use the pressure arrays or storm centres to artificially increase the climatological wave arrays in local areas.

4.6.5. GENERATION OF EXTENDED WAVE PARAMETERS WITH RUNNING CLIMATOLOGY.

(i) Running climatology over seven days.

OCEANROUTES, supplied a series of data arrays which are used for the extension of the data, beyond the GSOWM outputs, and are termed running climatology, (RC). Quite simply this involves taking the mean of the previous seven forecast data arrays.

By using a short climatological period, there is a greater chance of modelling the eddy wind fluctuations in the baroclinic model. Since the typical period of a depression to develop and decay across the north Atlantic is a week, it is felt that averaging over seven previous days is likely to pick up this effect.

The computed RC array is assumed to be constant for all extended periods. This can be potentially erroneous for a routeing algorithm, since the environmental
system is certainly not static, and developing depression activity will be omitted. However, the RC data is based on the assumption that the wave field has a long energy dissipation period. Correlations of the RC arrays to analyses have been produced. Figure 4.6 summarises the coefficients, and appendix A.6 contains diagrams corresponding to these correlations which compare RC data to actual conditions. Correlation of the RC arrays are relatively good, but centres of high waves are only maintained when the developed depression is slow moving and of long life.

![Graph showing correlation of extended RC arrays to GSWM analysis.](image)

**FIGURE 4.6. CORRELATION OF EXTENDED RC ARRAYS TO GSWM ANALYSIS.**

Running climatological wind arrays bear little resemblance to actual conditions, due to the averaging out of the eddy fluctuations. This poor correlation is true of all data extensions and for this reason, more consideration is paid in the routeing, to the wave fields, since most routeing effects upon the vessel are wave induced.

The running climatological data is only considered from +144 hours and not +72 (0000Z data sets), since the use of empiricisms provides better correlations but also for those reasons outlined above.
(ii) Running climatology over one forecast period.

The latest GSOWM forecast file was assumed to be constant for those times extended beyond the forecasts. In other words, the +72 hour wave arrays were assumed constant for extended periods, and similarly, the +120 hour wind arrays were assumed constant beyond this time.

By assuming the last forecast file to be the RC data file, the averaging process is considered over a very short time, and the eddy components are modelled more closely. Both wind and wave fields correlate better with the environment, although the RC wind field loses accuracy quickly, when the depressional activity is high and the eddy components are changing rapidly. Figure 4.7 summarises the correlation of the singular RC extended arrays against the analyses.

![Diagram](RUNNING CLIMATOLOGICAL DATA COMPARED TO ANALYSIS RUNNING CLIMATE USING LAST DATA FILE ONLY)

**Figure 4.7. Correlation of the latest forecast file (assumed constant for extended period) against the analysis files.**

The correlation of these RC wave data arrays to the analyses shows remarkable results due to slow ocean energy dissipation and low depressional activity at this time, it is therefore not surprising that the latest forecast file should retain high
accuracy. Since the element separation is 2.5°, and the array time separation is 12 hours it can be expected that the wave parameters will not vary significantly (assuming no dramatic change in wind speed or interaction of wave trains, resulting from fast moving storm centres or swell propagation).

4.6.6 GENERATION OF EXTENDED WIND AND WAVE PARAMETERS FROM THE ECMWF DIGITISED PRESSURE ARRAYS.

The ECMWF digitised data is only transmitted for the 1200Z period and extends to +144 hours. Only the +120 and +144 files need be considered for the 1200Z data set. The +144 ECMWF for the 0000Z data sets can be deduced from a polynomial extension through the preceding 1200Z time series. Table 4.3 indicates the periods of the pressure charts and those that are utilised.

<table>
<thead>
<tr>
<th>Forecast</th>
<th>0000Z Forecast</th>
<th>1200Z Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GSOWM Data</td>
<td>ECMWF Surf P</td>
</tr>
<tr>
<td>+0</td>
<td>Wind - Wave</td>
<td>Wind - Wave</td>
</tr>
<tr>
<td>+12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+108</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+144</td>
<td>£ - ££</td>
<td>£ - ££</td>
</tr>
</tbody>
</table>

Key: * Production of wind data from ££. ** Production of wave data from £. & Extrapolated +144 ECMWF pressure. £ Production of wind data from &. ££ Production of wave data from £. ECMWF surface pressure array.

TABLE 4.3. PRODUCTION OF EXTENDED DATA FILES FROM THE ECMWF SURFACE PRESSURE ARRAYS.

Surface pressure charts provide a better representation of the forcing system in the oceans than running climatological methods. The generated charts are time
specific and should pinpoint centres of high wave activity more accurately since the eddy wind components are modelled in some way. Since it is expected that a fast containership will have completed the majority of its Atlantic crossing in this period, arrays deduced at +120 and +144 are only regarded as an indicator to potential storm centres, in order that specific routeing decisions can be made earlier in the route.

There are two main drawbacks to the digitised data, firstly that it is only generated every 24 hours at midday, and secondly, that it is supplied at 5° degree spacing rather than at 2.5°.

The latter problem was overcome by subjecting the 5° degree array to a bi-cubic spline. Cubic splines were fitted along the row of the array and the intermediate points evaluated. For every column, including those evaluated, further cubic splines were fitted and the intermediate points evaluated, see section 3.21.1. Since the elements of the arrays represent grid points, distance from column 1 was used as the ordinate and distance from row 1 used as the abcissa, both evaluated by formulae, given in section 5.9.2.

Surface pressure can be converted to geostrophic wind through the following representation, which is a balance of the pressure gradient force, \( p_\text{air} \), and the Coriolis force.

\[
\frac{-1}{\rho_{\text{air}}} \frac{\delta p_{\text{air}}}{\delta n} + f U_G = 0 \quad \text{ie} \quad U_G = \frac{1}{f \rho_{\text{air}}} \frac{\delta p_{\text{air}}}{\delta n}
\]

Where \( f \) - The Coriolis parameter, \( = 2 \Omega \sin \Phi \)

\( \Omega \) - \( 2\pi/3600 \), \( \Phi \) - latitude. \( \rho_{\text{air}} \) - Air density, Kg m\(^{-3}\).

\( p_{\text{air}} \) - Air pressure, N m\(^{-2}\). \( n \) - Distance, m.

McIntosh et al (1983) state that the empiricism gives a good estimate of...
geostrophic wind, although the value cannot be applied directly to the surface because of the frictional retarding force exerted on air motion. This force causes isobaric winds to be backed by 10 degrees over the sea and wind speed reduced by approximately one third. Such simple assumptions make no account of the mixture of air through the velocity gradient from the surface to higher levels, which can unfortunately not be accounted for. Since high wave areas are to be emphasised, no retardation of the wind speed is considered, and the geostrophic wind is assumed to be generated at 19.5m.

The anticyclonic and cyclonic flow of air acquires a cyclostrophic force that causes the flow to become sub- and super-geostrophic. Such winds are termed gradient winds and can be found from the following:

\[
\frac{-1}{\rho_{\text{air}}} \frac{\delta P_{\text{air}}}{\delta n} + f \frac{U_{\text{GR}}}{r} = \frac{U_{\text{GR}}^2}{r} \rightarrow \text{ANTICYCLONIC} \quad U_{\text{GR}} = \frac{fr}{2} - \sqrt{\frac{f^2 r^2}{4} - \frac{r \delta P_{\text{air}}}{\rho_{\text{air}} \delta n}}
\]

\[
\frac{-1}{\rho_{\text{air}}} \frac{\delta P_{\text{air}}}{\delta n} + f \frac{U_{\text{GR}}}{r} = -\frac{U_{\text{GR}}^2}{r} \rightarrow \text{CYCLONIC} \quad U_{\text{GR}} = -\frac{fr}{2} + \sqrt{\frac{f^2 r^2}{4} + \frac{r \delta P_{\text{air}}}{\rho_{\text{air}} \delta n}}
\]

Where \( r \) = Radius of curvature, m. \( U_{\text{GR}} \) = Gradient wind ms\(^{-1}\).

Overland (1979) states further corrections to the geostrophic/gradient winds to account for fast-moving storms, since the streamline pattern as evidenced by a representation at an instant in time will not indicate a true trajectory or air movement for these storms.

By neglecting friction and cyclostrophic forces it is realised that errors are introduced, (although cyclostrophic amounts to approximately 10% of the total force and friction removes up to 30%). The storm centre will, however, still be identifiable and the relative differences of sea state be maintained. It is paramount that the ship route analyst recognises the importance of this. It would be pointless to introduce fine tuning of the formulae operating on crude forecast.
data. What is important is to continue to identify storm centres and associate gradients of pressure so that relative sea state gradients will be maintained in the environmental algorithms ensuring that the various optimisation models are truly sensitive to the likely conditions to be encountered. The ECMWF, 2.5° array was interpolated by a central difference routine, such that, the pressure gradient, at each element becomes a resultant of :

\[
\frac{\delta P_{\text{air}}}{\partial x} = \frac{P(i+1) - P(i-1)}{dx} \quad \text{and} \quad \frac{\delta P_{\text{air}}}{\partial y} = \frac{P(j+1) - P(j-1)}{dy}
\]

4.6.11

The pressure gradient is resolved such that the direction is that along the maximum that is :

\[
\frac{dp_{\text{air}}}{\partial n} = \text{max} \left( \frac{dp_{\text{air}}}{\partial x} \sin(1) + \frac{dp_{\text{air}}}{\partial y} \cos(1) \right) \quad \text{for } l=0,\ldots, 359^\circ.
\]

4.6.12

Geostrophic air flow is assumed with a crude frictional correction, therefore any deduced wave parameters are prone to many error sources resulting from the accuracy of the forecast pressure, geostrophic wind calculation and FDS empiricisms. Appendix A.7 contains examples of the degradation of pressure arrays and the generated wave files. Figure 4.8 summarises the correlation of the forecasted ECMWF pressure arrays compared to analysis arrays and figure 4.9 summarises the correlation of both the forecasted wind to analyses, as well as the wave parameters deduced by applying the FDS empiricisms. As stated in the section introduction 4.6.4, there was no time available to remove the poor correlation areas, or use confidence limits. Therefore it can be appreciated that the correlation values are influenced by certain poor predictions and do not give a clear representation.

Appendix A.8 contains examples of contoured plots of the wave heights, deduced from the pressure arrays, along with the corresponding contoured pressure array and analysis wave array. It can be seen that the maximum heights correspond closely to the centres of the depressions. A vessel could be routed by assuming a
certain wave height for the extension and weighting that value depending upon the local pressure gradient at the nearest or interpolated point. In effect a characteristic central wave height could be assumed and corrected according to the inferred pressure gradient. Therefore:

\[
\text{Route decision or ship states } = f_1 \left( \zeta_{1/3} f_2 \left( \delta p \over \delta n \right) \right)
\]

\[4.6.14\]

**Figure 4.8. Correlation of ECMWF Forecasted Surface Pressure to Analyses.**

In order to obtain the +144 hour 0000Z pressure array an \( n^{th} \) order least squares polynomial utilising the Gauss-Sidel method was fitted, for each element, through \( n+1 \) preceeding, +144 hour 1200Z pressure arrays. It is realised that extension of polynomials is subject to gross errors particularly when the curve is highly undulating, subjective care has to be applied.

In a similar manner, a \( 5^{th} \) order polynomial was fitted to obtain the +144 hour 1200Z pressure array, should it be missing, by utilising the +0, +24, +48, +72,
+96 and +120, 1200Z arrays. In the event of intermediate arrays missing, through bad reception or transmission, a bi-cubic spline can be fitted, section 3.21.1, or the former polynomial utilised to interpolate.

![CORRELATION OF WIND AND WAVE PARAMETERS FROM ECMWF SURFACE PRESSURE ARRAYS TO ANALYSIS](image)

**FIGURE 4.9. CORRELATION OF GENERATED WIND AND WAVE PARAMETERS (BY INFEERENCE FROM THE DIGITISED ECMWF DATA) TO ANALYSIS FILES.**

Since polynomial extension is subject to unquantifiable error it was felt that the Offenbach facsimile charts could be used in some manner, as a +144 forecast is provided at 0000Z. This was not investigated thoroughly, but was found to involve many difficulties:

1. Digitisation of the facsimile chart, in order that the array be representative. Since contours are only provided, there is little variation in pressure in between those contours.
2. Computation of the pressure gradient. Gradients are only found when the central difference scheme straddles isobars.
3. Computation of wind and wave parameters from very inaccurate pressure digitisation.
The method was not considered, although representative high wave centres could still be generated, when no other course of action is suitable.

4.6.7. 0000Z FORECAST SERIES: INTERPOLATION OF A CUBIC-SPLINE FIT.

A cubic spline curve, see section 3.21.1, was applied to the 0000Z forecast data set to interpolate the +84, +108 hour data files. The +84 and +108 files were deduced from the time series to +120 hours. Since the +144 hour forecasts were deduced from inferred pressure arrays and the +120 hour wave forecasts deduced from inaccurate FDS empiricisms it was felt that interpolation of the +132 hour forecast would contain very large inaccuracies indeed. This array set is transposed from the +144 hour forecast from the previous 1200Z data set. The correlation of deduced arrays to the analyses files are summarised in figure 4.10. Plots are given in appendix A.9.

There are two interpolation schemes:

---

+84 AND +108 INTERPOLATED WINDS AND WAVES COMPARED TO ANALYSIS

Legend
- +84 WINDS
- +84 WAVES
- +108 WINDS
- +108 WAVES

FIGURE 4.10. CORRELATION OF INTERPOLATED +84/+108 ARRAYS FOR THE 0000Z DATA SET USING CUBIC SPLINES.

---
1. Interpolation of the +84, +108 wind fields and computation of the wave parameters, section 4.6.3, from these;
2. Interpolation of the +84, +108 wave fields and wind fields.

The latter was felt more appropriate due to the variability of wind which may cause large fluctuations in a spline at the end values. This method gives satisfactory results, however, splines are strongly influenced by the accuracy of end values since these could invoke a 'ripple' type effect. Therefore the accuracy of the latest forecast files should be considered, and could induce large fluctuations in the spline if highly inaccurate.

4.6.8 EXTENSION OF POLYNOMIAL CURVES THROUGH THE DATA TIME SERIES.

**FIGURE 4.11. CORRELATION OF EXTRAPOLATED DATA ARRAYS FOR THE 0000Z AND 1200Z DATA SETS USING LEAST SQUARES.**

- 145 -
The effect of extending the polynomial or spline curves fitted at array elements through the 0000Z or 1200Z forecasts, has been investigated.

It was realised that such a curve extension is erroneous since the bias for the curves are only set at earlier points. However, using the time series of forecasts, several evaluations were made. Figure, 4.11 summarises the correlation of +132 and +144 hour forecasts from the 0000Z data set and the +84, +96, +108 and +120 hour forecasts from the 1200Z data set.

4.7. INTERPOLATION OF THE DATA ARRAYS THROUGH TIME AND SPACE

Chen et al (1976), Chen (1978) and Hagiwara (1989), show methods to interpolate environmental data, defined at specified grid points, for any point by assuming all datapoints to be on a linear plane surface. Since this study only concentrates on discrete routeing there is no need to incorporate the stochastical nature of the environment into the interpolation scheme. The reader is referred to Frankel et al (1980) and Hagiwara (1989) for a complete description.

Since the data arrays are defined every 12-hours, there is a requirement to interpolate the data for intermediate times. This can be performed by curve fit or by a simpler interpolation of the data arrays.

(i) Spatial Interpolation.

By computing the position at which the data is required, it is a simple process to determine those elements in the array that surround the data point. Figure 4.1 indicates the scheme. The required data is computed by:

\[
\begin{align*}
\Phi_2 - \Phi_1 & \quad X_2 - X_1 \\
\Phi_3 - \Phi_1 & \quad X_3 - X_1
\end{align*}
\]

\[
(\lambda - \lambda_1) \left( \begin{array}{c} X_2 - X_1 \\ X_3 - X_1 \end{array} \right) (\Phi - \Phi_1) + \left( \begin{array}{c} \lambda_2 - \lambda_1 \\ \lambda_3 - \lambda_1 \end{array} \right) \left( \begin{array}{c} \Phi_2 - \Phi_1 \\ \Phi_3 - \Phi_1 \end{array} \right) (X - X_1) + D = 0
\]

Where 1, 2, 3 - Denote nearby points in figure 4.12.

X - Assembly of data.
This formulation can be applied to wind speeds and direction, wave heights and periods, but can not be assumed for wave directions, since primary wave direction is not a resultant. In this respect, the closest element of the data array is assumed to be representative of the required data point.

(ii) Time interpolation.

Since elements of the GSOWM arrays have been determined for the spatial interpolation, see figure 4.12, there is a requirement to produce the data at these points, as a function of time, prior to the computation of equation 4.7.1. This can be achieved by fitting least-squares polynomials through the data arrays, assuming that the required time is 'surrounded'. Figure 4.13 indicates this process, which requires much computation, since many interpolations are required in a vessel trajectory, see chapter 5. The environment contains many unquantifiable errors and not all arrays have the same error, so it is difficult to accept this computational expense. Similarly, wave parameters are only statistical representations, and inference from a polynomial may be just as erroneous as a simpler scheme.
The data between forecasts are assumed to be a linear fraction of the difference between forecast values. Figure 4.14 indicates this process, and figure 4.15 shows the entire interpolation scheme.

**FIGURE 4.13. TIME INTERPOLATION OF GSOWM DATA FILES.**

**FIGURE 4.14. SIMPLE TIME INTERPOLATION OF THE GSOWM FILES.**
4.8. CHAPTER SUMMARY

Several methods for the extension of the data beyond the forecast period have been investigated. Although the methods are somewhat rudimentary, good results are obtained for the routing of vessels, see Chapters 6, 7 and 8. This study has been concerned with the direction of the quasi-optimal route, and not the quantitative evaluation of the routing policies. The forms of data extension used in this study are shown in Table 4.5.

The route is influenced by the values of the policies, see Chapter 5, but one must remember that, (for the North Atlantic liner trade), the vessel has completed most of its theoretical journey in the baroclinic zone before the extended data is required. Improvements in the data fields and extensions can only be made by improvements in the numerical atmospheric and oceanographic models.
<table>
<thead>
<tr>
<th>DATA SET</th>
<th>+0</th>
<th>+12</th>
<th>+24</th>
<th>+36</th>
<th>+48</th>
<th>+60</th>
<th>+72</th>
<th>+84</th>
<th>+96</th>
<th>+108</th>
<th>+120</th>
<th>+132</th>
<th>+144</th>
<th>&gt;+144</th>
</tr>
</thead>
<tbody>
<tr>
<td>wind 1200Z wave</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>SP</td>
<td>INT</td>
<td>G</td>
<td>SP</td>
<td>RC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wind 0000Z wave</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>G</td>
<td>INT</td>
<td>G</td>
<td>INT</td>
<td>FDS</td>
<td>POL</td>
<td>RC</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Key:**
- **FDS** - Data derived from FDS empiricisms.
- **G** - Data given from GSOWM and ECMWF.
- **INT** - Data derived from a polynomial or cubic spline interpolation.
- **POL** - Data derived from a polynomial extension.
- **RC** - Data derived from running climatology.
- **SP** - Data derived from an ECMWF surface pressure array.
- - Data transposed from preceding data set.

**TABLE 4.5. DATA GENERATION SCHEME.**

Since it was decided not to include the correction to ship motions for wave period, section 3.18, and only to use the PM spectra, no wave period data was investigated, although it is felt that RC data would suffice rather than the PM empiricism, equation 4.3.23.
5. THE OPTIMISATION ALGORITHM.

5.1 INTRODUCTION.

Several optimisation algorithms are described in chapter 1, however, the requirements of an on-board optimisation algorithm are :

1. It has to be relatively simple and efficient in handling the many system parameters. A complex system may result in a cumbersome algorithm that overrides route smoothness, and computer speed.
2. It should not interfere with the objective function and handle any objective, in this case, fuel or cost or time.
3. It should handle system constraints in a simple and straight-forward manner, since complexity will add inefficiency.
4. The sensitivity of the algorithm should be known so that correct tuning can be applied. Sensitivity studies are particularly required to distinguish the effects of discrete step-sizes.
5. The algorithm should be adaptive so that any real-world changes of the predicted data values can be accounted for. Should the environmental data differ from that expected, the system should provide alternative routeing strategies based upon this information.

Dynamic programming has been adopted as the optimisation algorithm since it satisfies the above requirements, however, the algorithm has been modified with additions and enhancements to the strategy to provide an effective means for the solution of a route. Importantly, the reduction in the brute-force computation of many routes is paramount. The vessel's routeing arena, (the ocean area that is used to compute the quasi-optimum route), subject to the routeing constraints defines the feasible state space, and it is a balance in its size that determine both an efficient algorithm and one that does not impose unrealistic routeing solutions.
In ship routeing, dynamic programming represents the problem as a sequence of stages from the departure point to the destination point. Each stage consists of a series of states which are the position, time or other operating value of the vessel. Connection between the states is determined by the transfer function. At each stage and state, the objective function is described by some form of the transfer function and/or states. The quasi-optimal route, which is defined by a series of states from departure to destination, is formulated as a multi-stage decision process, Zoppoli (1972), Chen (1978), Frankel et al (1980).

The states of the vessel are primarily represented by position and therefore depicted as a grid point in the ocean. Such states are vectoral matrices of latitude, $\Phi$, and longitude, $\lambda$.

$$\begin{pmatrix} \Phi \\ \lambda \end{pmatrix}$$

Each stage represents a series of grid points, and the whole system can be regarded as a two-dimensional grid system, the positional state space, over the ocean. It is possible to define stage separations by time or distance. Similarly, the positional state space can be expanded, to include time, therefore adding a third element. In this case the two-dimensional grid becomes three-dimensional and it is necessary to find the connection of the states in the box-matrix that represents the quasi-optimal trajectory. Understandably, this vastly increases the total state space and computational speed.

The connection of states, performed through the transfer function, takes place as a series of discrete steps, involving computation of the objective function. The routeing algorithm constantly checks the values of states and objectives, ensuring that the system constraints are not exceeded. Similarly, the connection of the set of states on stage $k$ to stage $k+1$, is subject to further ship control constraints.
5.2. STATEMENT OF THE ROUTEING PROBLEM.

The following notation has been adopted from Frankel et al (1980). The general functional form of the routeing algorithm, Frankel et al (1980), is:

\[
\begin{pmatrix}
\Phi \\
\lambda \\
T
\end{pmatrix}
= f_t\left( \Phi', \lambda', T', \tilde{U}, \tilde{M} \right)
\]

Where
- \(\Phi\) - Latitude.
- \(\lambda\) - Longitude.
- \(\tilde{M}\) - Constraints.
- \(T\) - Time.
- \(\tilde{U}\) - Controls.
- ' - Denotes previous state.

The ship arrives at an assembly of states, \((\Phi, \lambda, T)\), from a previous assembly, \((\Phi', \lambda', T')\), under controls \(\tilde{U}\) and provided that the constraints of the system, \(\tilde{M}\) were not infringed. The stepwise procedure to connect the states can be determined by:

\[
T' = T - \alpha \Delta T
\]

Where
- \(\alpha\) - Constant to depict number of small inter-state steps.

However, in a similar manner, steps of distance, \(\Delta X\), can be used, where \(X\) is the distance between the positional states. The two are similar, since they involve vessel speed.

The objective function, which can be labelled at each state, and defined here as \(C(\Phi, \lambda, T)\), becomes a summation of the costs for each state connection, with additional costs for the terminals. That is:

\[
C(\Phi, \lambda, T)_{\text{des}} = \int_{T_{\text{dep}}}^{T_{\text{des}}} \alpha(\Phi, \lambda, \tilde{U}, \tilde{M}, \tau) \, dt + \beta(\Phi, \lambda, T)_{\text{des}}
\]

Where
- \(\text{des}\) - Destination point.
- \(\text{dep}\) - Departure point.
- \(\alpha\) - Scalar function of cost per unit time.
- \(\beta\) - Scalar function of cost of arrival.
- \(\tau\) - Dummy time integration variable.
The routeing problem now becomes: What is the ship's trajectory to minimise the total transit objective or policy, given the state system and control constraints?

Clearly, equation 5.2.3 is rewritten in a form for stepwise summation, as an approximation of the continuous function.

5.3. THE STAGE VARIABLE.
Chen (1978), Frankel et al (1980) and Foo (1985) all go to some length in describing the stage separation variable. There are two choices; time or a measure of the progress of the vessel from departure. Both are increasing variables from the departure point and both could be used to formulate the recursive decision algorithm. However, fundamental to the choice is the objective function and its representation to the stage variable, see figure 5.1.

- Time, means stage points and vessel headings are not predefined. Direct comparison of the objective function is made between the ends of trajectories which terminate in roughly the same area.
- Progress measure, means stage points and headings can be pre-defined. Direct comparison of the objective function is made at the stage point reached from previous stage points.

5.3.1. Stage Variable - Time.
Frankel et al (1980) state that the interpretation of the routeing strategies imply controls provided only at each stage point. In other words, time as the stage variable will produce decisions made at regular time intervals, whereas distance implies decisions made as the vessel progresses on its journey. This latter strategy relates more to the master's approach to routeing, as course changes are usually made when certain positions or distances are achieved.
Similarly, the objective function is explicitly related to time and not to position, therefore an area has to be defined on the terminus of each trajectory from the previous stage, which becomes the state on the next stage. The objective function will vary over this area to an unknown degree and it is difficult to determine from which 'position' on the previous stage, the optimal trajectory was made from. This problem is represented in the figure 5.1.

![Diagram of stage variable](image)

**FIGURE 5.1. CHOICE OF STAGE VARIABLE.**

The stage spacing \( \delta t \), would be expected to be some value consistent with the period between forecasts.

5.3.2 Stage variable - Voyage Progress or Vessel Distance.

Figure 5.1 shows that distance measure produces direct or explicit comparisons of the objective function at pre-defined grid points.
There are two options for distance measure, longitude or distance. A meridional value suggests the states of the vessel to be latitude and time, however, it is more feasible and more general to use voyage distance.

As will be shown later, in section 5.8.(c), it is usual to assume the great circle from departure to destination as the reference for the construction of the routeing grid system, and the stage variable becomes a unit of distance along this route. The states of the vessel can become latitude points along either perpendiculars to the great circle, or along further great circles which intersect the main route perpendicularly. Each latitude point is a unit of distance from the main great circle.

Since 'waypoints' of the vessel's journey are chosen **optimally** from the grid, the vessel's movements become more restricted than previous. Each heading from stage to stage is expressed explicitly, and it is expected that the vessel is tightly controlled along this trajectory, Frankel et al (1980). (In the routeing model it is expected that the local optimisation routine manages this concept). This is a reasonable assumption, considering automatic pilotage.

Consideration of the stage and state spacings, \( \delta x \) and \( \delta y \), has to be made since these have a marked effect upon the quasi-optimal solution and the smoothness of the route. It is true to say that there is a limit to the size of these spacings to maintain accuracy, and one should consider an **optimal** grid for the purposes of the application.

5.3.3. Further Computations from the Pre-defined Grid Points.

On movement between grid points, the solution of the transfer function requires an interpolation of the environmental data. Since the GSOWM data is given at 2.5° points, an interpolation scheme is used, see section 4.14, to compute the data for the time and position of the ship. The state of the sea can be represented by :-
\[
S( X_{\text{ship}}, t ) = f \left( S( X_{\text{GSOWM}}, t_i ), S( X_{\text{GSOWM}}, t_{i+1} ), X_{\text{GSOWM}}, X_{\text{ship}}, t \right)
\]

Where

- \( S \) - State of the sea
- \( X \) - Positional state, (ship) of the ship, (GSOWM) of nearby GSOWM data points.
- \( t \) - Time, (i) previous forecast, (i+1) previous forecast +12 or +24 hrs. Therefore \( t_i < t \) (of ship) < \( t+1 \).

In this study the grid system uses a distance stage variable, where each stage is constructed from the main Great Circle Route, (GCR), consisting of a number of positional states, constructed on perpendicular Great Circles, (GC), to the GCR. The control variables, \( U \), consist of the course to be made, \( \psi_{\text{cmg}} \), and required speed, \( V_{\text{reqd}} \), or power, \( P \). Transitions from stage to stage are subject to the constraints, \( M \), comprising the vessels motion extremes, and any other environmental or geographical constraint.

It is shown in sections 6, 7 and 8 that the objective can be computed by considering steps or intervals along the state trajectory, in time or distance. Both require the computation of intermediate positional states, between the stages, for which the Mercator formula is used with equations 5.8.6.(a) - 5.8.6.(d). The Mercator formula provides a means of accurate course angle from which accurate positions can be determined, Fifield (1980).

Therefore, the vessel heading, \( \psi_{\text{cmg}} \), is given by, see figure 5.6 :-

\[
\psi_{\text{cmg}} = \tan^{-1} \left( \frac{\lambda_{k+1} - \lambda_k}{m_p(\Phi_{k+1},c\pm L) - m_p(\Phi_k,c\pm L)} \right)
\]

Where,

- \( c \) - Central stage value.
- \( L \) - Positional state, from central value.
- \( m_p \) - Meridional parts of latitude.
\begin{equation}
mp(\Phi) = \ln\left(\tan\left(\frac{\Phi}{2} + 45\right)\right) - \frac{e}{2} \ln \left\{ \frac{1 + e \sin^2 \Phi}{1 - e \sin^2 \Phi} \right\}
\end{equation}

after Hagiwara (1989).

\textbf{e} - Eccentricity of Earth, \quad e^2 = 2f - f^2

\textbf{f} - Flattening = 1/298.26 for WGS -72 Geoid.

This formula provides a means of deducing the correction to the spheroid for flattening, and is sufficiently accurate for routeing purposes, Hagiwara (1989). In fact simpler equations could be used without a severe loss in accuracy.

5.4. THE STATE SPACE AND COMPUTATIONAL BURDEN.

It is necessary to consider the size of the state space as it will define the computational burden of the optimal algorithm.

Consider a voyage of length 2500nm and a vessel travelling constantly at 25 knots, the journey takes 100 hours. If the stage variable is 250nm, the state space contains 11 stages. Similarly, each stage consists of say 50 pre-defined positional states. The number of possible routes through this grid system will be \(50^{11}\), and is referred to as the brute-force computation. Obviously there is a need to reduce this computational burden.

If the stage separation was 5 hours, then the state space would contain 21 stages. Each stage would consist of a theoretical area, see figure 5.1, of say Znm x 50nm, where Z is of increasing size. Since there are many trajectories each ending somewhere on the next stage, the number of states increases dramatically. In using micro-computers it becomes necessary to reduce the brute-force computation, which may be achieved by dynamic programming techniques, but also the limitations to the state space that are described in this study.
5.5. OBJECTIVE FUNCTION AND RECURSIVE ALGORITHM DEFINITION.

5.5.1 Stage Variable - Voyage Progress.

The objective of the routeing algorithm has been defined by the continuous function, equation 5.2.3. To facilitate a solution, the function is re-written as a stepwise summation.

\[ C(\vec{x}, N_{\text{stage}}) = \sum_{k=1}^{k=N_{\text{stage}}-1} \alpha_k(\vec{x}(k), U(k), M(k)) + \beta(\vec{x}, N_{\text{stage}}) \]

The cost to arrive at stage \( N_{\text{stage}} \), the terminal, is a summation of the costs to each intermediate stage point on the optimal trajectory, with additional terminal costs on reaching stage \( N_{\text{stage}} \), termed the boundary function.

Equation 5.5.1 can be defined as a summation of the operating costs and the port costs. Based on Bellman's principle of optimality, section 1.5.5, the recursive algorithm takes on a reverse form and the operating cost in equation 5.5.1 becomes, after Frankel et al (1980) :-

\[ C(\vec{x}(k), k) = \min_{U(k)} \left\{ \sum_{j=k}^{j=N_{\text{stage}}} \alpha_j(\vec{x}(j), U(j), M(j)) \right\} \quad \text{for } k=N_{\text{stage}}, \ldots, 2, 1. \]

This statement reads; the transition cost to reach stage \( k \), is a summation of the costs from stage \( N_{\text{stage}} \), via stages, \( N_{\text{stage}}-1, \ldots \) to \( k \). This is re-written to be a summation of the journey cost to stage \( k+1 \) plus the additional cost of the trajectory \( k+1 \) to \( k \), under the controls \( U \), set at stage \( k \). If :-

\[ C(\vec{x}(k), k) = C(\vec{x}_{k+1}) \text{ and } \alpha_k(\vec{x}(j), U(j), M(j)) = \alpha_k(\vec{x}, U, M) \]

Then,

\[ C(\vec{x}, k) = \min_{U(k)} \left\{ \alpha_k(\vec{x}, U, M) \right\} + C(\vec{x}, k+1) \]

Where

\[ C(\vec{x}, k+1) = \sum_{j=k+1}^{j=N_{\text{stage}}} \alpha_j(\vec{x}, U, M) \]
According to the principle of optimality, the cost to stage \( k \) is independent of any controls made previously, hence reverse computation, since controls made in the future have no relevance to those made at present. Therefore the cost function can be regraded in two parts, where the cost to stage \( k \) is a minimisation of the costs to stages \( k+1, k+2, \ldots, N_{\text{stage}} \) under the controls set at those stages, and the cost to stage \( k \) under the controls set at stage \( k \). This assumes that the controls are constant for the trajectory from stage to stage, and assumes that any control of the vessel takes place at these discrete events. Equation 5.5.3 can be written to encapsulate this description.

\[
C(\vec{x}, k) = \min_{U(k)} \min_{U(j)} \left\{ \alpha_k(\vec{x}, \vec{u}, \vec{m}) + \sum_{j=k+1}^{j=N_{\text{stage}}} \alpha_j(\vec{x}, \vec{u}, \vec{m}) \right\}
\]

The costs over the trajectory, \( k \) to \( k+1 \) under controls set at \( k \) are not subject to the controls set at \( k+1 \) or any other controls at subsequent stages. The first term in equation 5.5.5 relates the state transition equation or transfer function to the cost value. In other words 5.5.5 can be written as:

\[
C(\vec{x}, k) = \min_{U(k)} \left\{ \alpha_k(\vec{x}, \vec{u}, \vec{m}) \right\} + \min_{U(j)} \left\{ \sum_{j=k+1}^{j=N_{\text{stage}}} \alpha_j(\vec{x}, \vec{u}, \vec{m}) \right\}
\]

By referring to equation 5.5.4, it can be seen that the second term in equation 5.5.6 is the minimum cost function to stage \( k+1 \). Therefore, considering individual stages, the recursive algorithm becomes:

For the final stage.

\[
C^*(\vec{x}, N_{\text{stage}}) = \beta(\vec{x}, N_{\text{stage}})
\]

For intermediate stages.

\[
C^*(\vec{x}, k) = \min \left\{ \alpha_k(\vec{x}, \vec{u}, \vec{m}) + C^*(\vec{x}, k+1) \right\}
\]

for \( k = N_{\text{stage}}-1, N_{\text{stage}}-2, \ldots, 3, 2. \)
For the first stage.

Petrie et al (1984) state a terminal or boundary function for departure as well as destination points.

\[ C^*(x, 1) = \min_\alpha \left\{ \alpha \left( x, u, m \right) + C^*(x, 2) \right\} + \chi(x, 1) \]

Where - \( C^* \) - The policy decision to the state on the stage. This refers to the minimum over the trajectories to stage \( k \).

\( \alpha \) - The scalar function for cost.
\( \beta \) - The scalar function for destination cost.
\( \chi \) - The scalar function for departure cost.

Equations 5.5.7.(a)-5.5.7.(c), involving three cost functions, can be interpreted as the costs of early, or late departures and arrivals. Any cost function on the terminals are additions performed in conjunction with Bellman's principle.

5.5.2, Stage Variable - Time.

The objective function remains the same, however, the stages are defined as :-

\[ N_{\text{stage}} = t_{\text{dep}} + \alpha \delta t \quad \text{or} \quad k = N_{\text{stage}}, \ldots, t_{\text{dep}} + 2 \delta t, t_{\text{dep}} + 3 \delta t, t_{\text{dep}} \]

Where \( \alpha \) - Number of time increments.

Re-introducing the definition of states and controls by stage \( k \), the state transfer function is given by :-

\[ X(k+1) = f(X(k), U(k), M(k), k) \]

The positional state \( X \), is a function of the previous state, controls and time, (stage), note also, the constraints, \( M \), would not form a new state \( X(k+1) \) if
exceeded. The recursive cost function becomes, after Chen et al (1976) :-

\[ C(\vec{X}(N_{\text{stage}}), N_{\text{stage}}) = \sum_{j=0}^{j=N_{\text{stage}}} \alpha_j(\vec{X}(j), \vec{U}(j), \vec{M}(j), j) + \beta(\vec{X}(N_{\text{stage}}), N_{\text{stage}}) \]  

5.5.10

The cost to positional state and stage \( N_{\text{stage}} \) is a summation of the costs to previous states under controls, and time, with additional terminal costs, (positional states are not explicit and are therefore defined by stage or time).

The recursive algorithm becomes :-

**Final stage.**

\[ C^*(\vec{X}(N), N_{\text{stage}}) = \beta(\vec{X}(N_{\text{stage}}), N_{\text{stage}}) \]  

5.5.11.a

**Intermediate stages.**

\[ C^*(\vec{X}(k), k) = \min_{U(k)} \left\{ \alpha_k(\vec{X}(k), \vec{U}(k), \vec{M}(k), k) + C^*(f(\vec{X}(k), \vec{U}(k), \vec{M}(k), k), k+1) \right\} \]  

5.5.11.b

**First stage.**

\[ C^*(\vec{X}(1), 1) = \min_{U(1)} \left\{ \alpha_k(\vec{X}(1), \vec{U}(1), 1) + C^*(f(\vec{X}(1), \vec{U}(1), 1), 2) \right\} + \chi(\vec{X}(1), 1) \]  

5.5.11.c

Cost is a function of cost over \( k \) - \( k+1 \) and cost at \( k+1 \), where positional states at \( k+1 \) are a function of the previous states, controls and time.

**5.6. PROBLEMS WITH BACKWARDS RECURSION.**

Backwards recursion, poses certain difficulties for minimum time optimisation, using both stage variables since the arrival time at the destination is unknown, although the departure time is. Similarly, time influences the very disturbance mechanisms routing aims to avoid.
By using the great circle route as a reference value with forwards iteration it is possible to have an estimation of the destination time. From this a recursive procedure is invoked; the destination time is manipulated in order that after deduction of the transit time, the departure time is matched.

Chen et al (1976) and De Wit (1989), have defined forwards iteration routines to overcome these difficulties, however, Chen et al (1976) define a system with the stage variable as time, which may be considered to confuse the issue.

5.7. FORWARDS ITERATION.
5.7.1 Stage Variable - Voyage Progress.
For minimum time routeing, forwards iteration is very simple, requiring only a re-arrangement of the formula, since for deterministic routeing, both forwards and backwards iteration are equivalent, Frankel et al (1980), (controls are limited), when the same pre-defined states are used.

\[
C^*(\vec{X}, k) = \min \left\{ C_k(\vec{X}, \vec{U}, \vec{M}) + C^*(\vec{X}, k-1) \right\}
\]

for \( k = 2, 3, \ldots, N_{\text{stage}} - 2, N_{\text{stage}} - 1 \).

5.7.1

The iterative procedure can now be deduced as in equations 5.5.8.(a) - (c).

5.7.2 Stage Variable - Time.
This procedure is more complicated, but is basically the same as backwards iteration. The problem is the definition of the state, at an unknown time. The positional state at time, \( \text{stage}, k \), becomes, after Chen et al (1976) :-

\[
\vec{X}(k) = f(\vec{X}(k-1), \vec{U}(k-1), \vec{M}(k-1), k-1)
\]

5.7.2

This is interpreted with Pontryagin's maximum principle, where the state is that at a maximum along the trajectory, or is the latest that can be reached, Chen et al
Since the positional state is not explicit, the cost at present stage \( k \), and state, is given as:

\[
C(\bar{X}, k) = \min_{U(k-1)} \left\{ \alpha_{k-1}(\bar{X}, U(k-1), M(k-1), k-1) + \sum_{j=0}^{j=k-1} \alpha_j(\bar{X}(j), U(j), M(j), j) \right\}
\]

The cost to a positional state at time or stage \( k \), is a function of the operating costs to a previous state, plus the cost to reach the present state \( \bar{X} \), under controls set previously. Note, this is not referenced by \( k-1 \), since it is not the previous state, but is an expected state at \( k \).

It is necessary to define the positional state, \( \bar{X} \) at stage or time \( k \), as an inverse function of the previous positional state, controls and time. This can be interpreted as a definition of a positional state \( \bar{X} \) at present time \( k \), used to determine from which 'previous' state it could have been reached. If a function \( f' \), describes state \( \bar{X}(k) \), from previous states \( \bar{X}(k-1) \), using equation 5.7.2, an inverse function \( f'' \) will describe state \( \bar{X}(k-1) \) from state \( \bar{X}(k) \). Therefore \( \bar{X} \) can be construed as:

\[
\bar{X} = f\left( f'\left( \bar{X}(k-1), U(k-1), k-1 \right) \right)
\]

The iteration procedure, is similar to equation 5.7.1 and becomes:

\[
C^*(\bar{X}, k) = \min_{U(k-1)} \left\{ \alpha_k(f''(\bar{X}, U(k-1), M(k-1), k), k-1) + C^*(f'\left( \bar{X}, U(k-1), M(k-1), k \right)), k-1) \right\}
\]

This equation describes the cost to present stage \( k \) and state \( \bar{X} \), as a function of:

1. The previous state, \( \bar{X}(k-1) \), which is a function of the present position time and previous controls, at the previous stage \( k-1 \).
2. The previous cost at state \( X(k-1) \), which is a function of present state, time and previous controls, at the previous stage \( k-1 \).

The derivations of the functional forms of the cost functions are shown for time, fuel or cost objectives in sections 6.2, 7.2 and 8.2. However, the terminal functions are only considered for optimum cost routeing strategies.

5.8. STOCHASTIC OPTIMUM WEATHER ROUTEING.

The formulation towards a deterministic recursive algorithm has been described. Due to lack of time, stochastic routeing has not been pursued in this study, however, the reader is referred to Devanny Ill (1971), Chen et al (1976), Chen (1978), Frankel et al (1980) and Hagiwara (1989). As yet the author is unaware of any studies which compare directly, the routeing solutions, in real-time, of stochastic and deterministic methods.

Any stochastic algorithm only considers errors in forecast environmental data and not ship models. This restricts the application of stochastic algorithms. Similarly those errors in forecast information constitute an enormous increase in the volume of data that an algorithm has to contend with. There is a requirement to deduce the standard deviation of the environmental data at each specific area of the ocean and each time interval. This becomes more complicated when one considers the expected duration of a sea state and the expected ship speed. Similarly, the deviation may vary depending upon the sea state.

All stochastic information has to be deduced from observations of the environment over a sufficiently long period. Hagiwara (1989) considered five years, whereas the information derived by Hogben et al (1967) has been deduced over many more years. Similarly, Comstock et al (1980) have discussed the application of forecast and hindcast statistical data to routeing.
All approaches to stochastic routeing have been required to reduce the size of the problem. For example Hagiwara (1989), deduced the standard deviation for the environment as a value for an area of the ocean. Over the North Pacific, he used twenty-five such areas, which represent the mean of that area. This approach is questionable as it removes the clarity of stochastic routeing.

5.9 Dynamic Grid Systems

Having defined the stage and state variables, consideration has to be made of the make-up of the grid and its spacings. For the purposes of optimal control, the grid should be adaptive, updating as time and route progress, providing a feedback in the control-loop. Several grid types have been investigated:

1. Rectangular grid system, on a Mercator projection. Therefore, stages are defined as meridians and states or points defined at intervals of latitude, Higham (1988). Stage spacing, δλ, and state spacings, δΦ.

2. Grid system constructed from the great circle route between departure point and destination point.

(a) Stages defined as meridians and states or points defined at intervals of latitude. Stage spacing δλ, (d.longitude), along the GCR and state spacing, δΦ.

(b) Stages defined at δλ spacing along the GCR and states perpendicular to it, defined at intervals along this line. Stage spacing, δλ, and state spacing, δφ, or δy (distance measure).

(c) Stages defined at δx spacing along the GCR and perpendicular to it, with states defined as in (b). Stage spacing δx and state spacing δφ, or δy.
(d) Stages defined at $8x$ spacing along the GCR and along further great circles, (GCs), intersecting the GCR at $90^\circ$.
Stage spacing $8x$, and state spacing, $8^\circ$, or $8y$.

5.9.1. RECTANGULAR GRID SYSTEMS.

A 'rectangular' grid system was developed by Motte et al (1985), (termed rectangular, by its appearance on a Mercator projection). Such a system is fixed and requires no computation of the positional states. The voyage progress is represented by longitude.

A one by one degree system would bias routes towards the parallels of latitude since this becomes the shortest distance between the two points. The great circle fit on this grid becomes very jagged or discontinuous and the overall distance greater than that along the parallels.

If the lateral spacing of grid points on a stage is large and the allowable course change, under the constraints, $M_c$, is large, then clearly strange results will emerge. In reality the ship's master will not alter course dramatically unless such a circumstance warrants this drastic strategy. This problem was easily resolved by reducing the lateral separation of the positional states.

Motte et al (1985) showed that an elongation of the east-west spread reduces the discontinuities of any route, since the fit of the great circle becomes closer. This system incorporated a stage separation of four degrees and positional states separated by a half a degree, $(8:1)$. A route on this grid system is shown in figure 5.2.
5.9.2. SPHERICAL GRID SYSTEMS.

The great circle is used as the basis for the computation of the grid system between departure and destination points on a spheroid, and forms the central line of a system that can be regarded as a rectangular area. Hagiwara (1989) has shown that little error results from considering the GCR on the spheroid to the geodetic on the geoid. By considering the spherical triangles shown in figure 5.3, a great circle is computed by:

\[
\cos(A) = \left( \cos(PB) - \cos(PA)\cos(AB) \right) \csc(PA)\csc(AB)
\]

Where -

PB - 90 - latitude of B.
PA - 90 - latitude of A.
AB - Angular distance from A to B.
FIGURE 5.3. GREAT CIRCLE COMPUTATION.

The GCR is represented as a series of stage points, from which it is possible to compute the lateral positional states, or grid points. With reference to figure 5.3 the central positional states per stage, subscript C, are found by:

\[
\Phi_c = 90 - \cos^{-1}\left(\cos(AC)\cos(PA) + \sin(AC)\sin(PA)\cos(A)\right)
\]

5.9.2

Where, \( AC \) - Incremental stage variable, distance or degree values.

\[
\lambda_c = \cos^{-1}\left(\frac{\cos(AC) - \cos(A)\cos(PC)}{\sin(A)\sin(PC)}\right)
\]

5.9.3

The extent of the grids are limited by the geography, terminal states and set of controls, but within the limits there is a degree of subjectivity to the stage and state spacings. It was one aim of this study to construct the most suitable grid, as a result of performing sensitivity studies.
There are several spherical grid systems to be considered:–

(i) States Formed on Pre-Defined Meridians.

Constructing the states or grid points along meridians requires little computation since longitude remains constant and state refers only to latitude. Once the central points have been computed, the remaining stage points are found by increments of $\delta \Phi$, (or $\delta y$), either side of the central stage point latitude, $\Phi_c$. It is possible to define the stages at increments of distance along the GCR or increments of $\delta \lambda$, of the angle at P in figure 5.3.

Lateral positional state spacing was taken between $0.3^\circ-0.75^\circ$, (18-45nm) since a larger value causes discontinuities. Up to 25 stage points were computed either side of the central stage point, giving a 900 to 2250nm wide grid system.

There are advantages for both methods of stage spacing, distance, $\delta x$, or $\delta \lambda$. Firstly by taking a constant $\delta \lambda$, the number of stages, $N_{\text{stage}}$, is given by:

$$N_{\text{stage}} = \text{int} \left( \frac{\hat{P}}{\delta \lambda} \right) + 2$$  \hspace{1cm} (5.9.4)

Where $\hat{P}$ – Difference in longitude between departure and destination.

The formulation simplifies calculation time and complexity, therefore making recomputation quick since it is envisaged that the grids are updated as the route progresses. As shown in figure 5.4.(b), stage distance becomes variable, dependant upon the latitude of the grid points. The figure illustrates the range of distance between the stage points throughout the grid, under the course constraints, as explained in section 5.10.2. Therefore, since many course headings are considered near terminals, the extent is larger when at these points. The spread of distance is quite large, up to a value of 80% in some cases, of the central stage distance.
FIGURE 5.4.(a) MERIDIANAL GRID SYSTEM WITH 2.5° LATITUDE SPACING

FIGURE 5.4.(b) STAGE DISTANCES ON THE MERIDIANAL GRID SYSTEM
Using constant distance, $\delta x$, requires more computation, but improves the grid, since it is important to maintain consistency in stage distance throughout the grid.

By defining $\delta x$, $N_{\text{stage}}$ is given by:

$$N_{\text{stage}} = \text{int} \left( \frac{\text{GCR dist}}{\delta x} \right) + 2$$

The system and the stage distance graph for this grid are shown below in figures 5.5.(a) and 5.5.(b).

**Figure 5.5.(a). Grid system using constant stage distance on GCR and states along meridians.**
(ii) States formed along perpendiculars to the GCR.

By referring to figure 5.6, positional states are computed along a line that is perpendicular to the GCR from the central state. By using the Mercator algorithms, the triangle of representation can be considered over lengths to approximately 600 nm, Fifield (1980) and Hagiwara (1989), with sufficient accuracy. This is however, extended for extremity points. The latitude and longitude of the stages can be computed from the following, having determined the heading angle accurately, equation 5.3.2 and 5.3.3, see figure 5.6.

\[ \Phi_c + L = \Phi_c + \frac{\delta y \cos \psi_{cmg}}{R(\Phi_c)} \]  

5.9.6 (a)

\( \delta y \) - Incremental state distance spacing, \((0.3^\circ - 0.75^\circ)\) or \(\frac{\delta x}{60}\)

\( R(\Phi_c) \) - Local radius of the meridian.
\[ R(\theta_c) = \frac{a(1.0 - e^2)}{(1.0 - e^2 \sin^2(\theta_c))^{3/2}} \]

\[ a = \text{Radius of the Earth} = 6378135\,\text{m WGS -72 Geoid.} \]

\[ e = \text{Earth's eccentricity. See section 5.3.3.} \]

\[ \lambda_{c+L} = \lambda_c + \frac{\delta y \cos \theta}{v(\phi_m)} \]

Where \( v(\phi_m) = \text{Local radius of the latitude.} \)

\[ v(\phi_m) = \frac{a}{(1.0 - e^2 \sin^2(\theta_c))^{1/2}} \]

**Figure 5.6. Positional State Computation Along Perpendiculars to the GCR.**

- Course to waypoint \( k+1, c+1 \)
- Additional course angle to \( k+1, c+1 \)
- Intermediate point on state trajectory
- Positional states
Figures 5.7.(a) and 5.7.(b) indicate a grid computed using stages defined at $\delta\lambda$ and its corresponding stage distance spread. As can be appreciated, the extent of distance spread between the states on two stages has improved; the distance between two stages is almost constant for the whole routing arena, under the allowable course constraints. However, the stage distance understandably increases as the route progresses south and in order to overcome this problem, the grid is recomputed using a $\delta x$, stage spacing as indicated previously. The grid and the stage distance plot are shown in figures 5.8.(a) and 5.8.(b).
DISTANCE BETWEEN STAGES \((K)-(K+1)\)  
GCR FROM 49.8 14.0  
TO 41.2 63.0

FIGURE 5.7.(b). STAGE DISTANCE SPREAD FOR THE GRID IN FIGURE 5.7.(a).

DP DISCRETE GRID

FIGURE 5.8.(a). GRID SYSTEM COMPUTED USING \(\delta x\) STAGE SPACING AND  
POSITIONAL STATES ALONG A PERPENDICULAR.
(iii) States formed on further GCs to the GCR.

De Wit (1989), states that the most useful grid system is that computed using a GCR reference route, but incorporating positional states defined along further GCs perpendicular to the GCR. One can regard such a system as a skewed equator, over the earth, De Wit (1989). By reference to the figure, 5.9, the positional states are easily and quickly computed through spherical trigonometry.

The positional state vector is computed by:

\[ \Phi_D = 90.\cos^{-1}\left(\cos(\Phi_A)\cos(\Delta D) + \sin(\Phi_A)\sin(\Delta D)\cos(\hat{\Delta} - \hat{\Delta}_A)\right) \]

\[ \Phi_D = 90.\cos^{-1}\left(\cos(\Phi_A)\cos(\Delta D) + \sin(\Phi_A)\sin(\Delta D)\cos(\hat{\Delta} - \hat{\Delta}_A)\right) \]

\[ \Phi_D = 90.\cos^{-1}\left(\cos(\Phi_A)\cos(\Delta D) + \sin(\Phi_A)\sin(\Delta D)\cos(\hat{\Delta} - \hat{\Delta}_A)\right) \]

Where \( \Phi_A = 90. - \Phi_A \)
**FIGURE 5.9. COMPUTATION OF POSITIONAL STATES ALONG FURTHER GCs PERPENDICULAR TO THE GCR.**

\[ AD = \cos^{-1}\left(\cos(AC)\cos(CD) + \sin(AC)\sin(CD)\cos(C)\right) \]
\[ = \cos^{-1}\left(\cos(AC)\cos(CD)\right) \quad \text{(5.9.8)} \]

Where

- AC = Pre-defined variable = \( \frac{\delta x}{\delta \theta} \), stage variable.
- CD = Pre-defined variable = \( \frac{\delta y}{\delta \theta} \), lateral positional state variable.

The variable \( \delta A \) is computed as:

\[ \delta A = \cos^{-1}\left(\frac{\cos(CD) - \cos(AD)\cos(AC)}{\sin(AD)\sin(AC)}\right) \quad \text{(5.9.9)} \]

Then,

\[ \lambda_D = \lambda_A + \cos^{-1}\left(\frac{\cos(AD) - \cos(PD)\cos(PA)}{\sin(PD)\sin(PA)}\right) \quad \text{(5.9.10)} \]
FIGURE 5.10(a). GRID SYSTEM COMPUTED USING δx STAGE SPACING AND POSITIONAL STATES ALONG A FURTHER GC.

FIGURE 5.10(b). STAGE DISTANCE THROUGHOUT THE GRID IN FIGURE 3.10(a).
This system has been adopted in the routeing studies and is easily computed when departure and destination points are defined. As the vessel progresses, the departure point is modified, until there exists only one clear route to follow, the rhumb line to destination.

Several grids have been shown, all are slightly different and lead to differing routes because:-

- The positional states of the grids differ, leading to slightly different policy values, note the variation of policy over the area of positional state considered under a time stage variable.
- The environmental data used in the state trajectories differs due to differing states and differing interpolated points.
- There may be numerous routes within the grid approximating to the policy on the quasi-optimum.

The true optimum will only be found by considering infinitely small state and stage spacings.

5.10. GRID CONSTRAINTS.

5.10.1. Land or Geography of the Ocean Basin.

The routeing arena was considered as a 0.5° grid matrix where each element is flagged as land or sea, (or any other obstacle). Any positional states that interfere with these obstacles are simply ignored.

It was felt that such a structure is sufficiently fine to cope with the land perimeter on oceanwide scales after discussion with OCEANROUTES. The routeing algorithm also inspects the state trajectories for land interference and should such a situation occur, the routeing algorithm simply nulls that particular segment of the route. This will have a 'knock-on' effect since trajectories, (k-1 to k), are only
considered if the state, $\tilde{X}(k-1)$, was achieved in previous recursions. Areas of ice flow, particularly near Newfoundland and Canada in winter, are treated as an extension of the land.

5.10.2. Allowable Ship Headings/Course Constraints. (Controls).

Within the routeing strategy, there is a set of allowable headings, compatible with the capability of the vessel. Frankel et al (1980) state that a value of $35^\circ$, is the maximum alteration of course, from the previous heading. This value commands the on-going controls of the vessel, and reduces the number of states on stage $k$, that are considered from stage $k-1$. With reference to figure 5.11, this constraint is perceived as a course deviation and limits, in the same manner, the number of allowable state trajectories, from a positional state to the subsequent stage. These can have large effects upon the smoothness of the quasi-optimum route and in extreme cases can lead to serious discontinuities, in conjunction with stage and state separations.

![Figure 5.11: Definition of course deviation, or set of allowable state transitions per stage.](image)
By defining these course deviations, effectively a routeing envelope is described, which further limits the size of the state space. Figures 5.12.(a), 5.12.(b) and 5.12.(c), show pictorially, the size of the two-dimensional, positional state space, using large course deviations small course deviations and a combination.
FIGURE 5.12.(c). ROUTE ENVELOPE DEFINING POSITIONAL STATE SPACE UNDER VARIABLE COURSE DEVIATIONS.

Figure 5.12.(c) indicates the system adopted in this study. It is interpreted as:

1. Large initial and final course deviations allow a vessel to avoid large depressions centred close to the terminals, or gives sufficient northerly or southerly deviation to avoid any large centrally placed depression activity later on in the route.

2. Small course deviation, on central route states reduces the computational burden, but more importantly, once under way, there should be no large course alterations since this is sub-optimal. However, in real-time routeing, this is still permissible, since the vessel will always be at the departure point of the constantly updated grid.
The course deviation places further limits upon the vessel control than just the maximum course change criteria, since the state trajectories are pre-defined from $X(k-1)$. If the maximum course change was solely used, it could lead to discontinuous routes and an inefficient algorithm. However, states $X(k)$ would be considered, which are described by the heading to state $X(k-1)$ and the maximum course deviation. This could yield a large state space, unless a variable value is defined by stage and similarly, could lead to non-convergence. This returns us to the original statement and problem. When large course deviations are considered near the grid terminals, the maximum course change is invoked. The use of limited course deviations in central stages forces the algorithm to search for a route that makes as much distance in the direction of the intended journey as possible, but allows any maximum route changes in early or late stages.

There exist, therefore a set of vessel controls, $U_a(k)$, defined by stage, and the optimal control at each stage form a sub-set of this, ie. $U_{k,i} \subseteq U_a(k)$.

5.11. FURTHER GRID CONSIDERATIONS.

The reference great circle, between the terminal positional states will undoubtedly interfere with land obstacles. Therefore there are two further considerations in the computation of the grid system.

1. Should the reference route be defined by a Least Distance Route, (LDR), which does not interfere with the land, and construct the positional states on GCs, or other, perpendicular to this?

2. Should the reference route be maintained as the GCR, regardless of land and omit any positional states on land? Thereby a least distance route could be computed through the remaining grid.

Petrie et al (1984) adopted the approach in point 1, but it was decided that this system may be difficult to compute quickly and simply, since the waypoints of the
LDR have to be defined by the master. Similarly, a situation may occur where positional states on separate stages interfere with each other.

Point 2 maintains simplicity, however, the system is constructed under the assumption that sufficient positional states either side of the GCR are computed, so that the state space is not severely restricted. With the set of allowable transitions, and the redefined number of states per stage because of geography, it is possible to pre-compute the route envelope, thereby restricting the feasible positional state space. The route envelope becomes modified by time invariant obstacles, or constraints, and an example is shown in figure 5.13.

**Figure 5.13. Modified Route Envelope for a North Atlantic Crossing.**

For comparison to the quasi-optimum route the LDR is found through a restricted central band of the grid, usually taken between ±8 states from the central GCR value. A quick adaptation of dynamic programming is used to compute the LDR.
It is possible to include time variant restrictions on the state space, for example fog, or high wind/wave activity. The maximum allowable wave height, that the master will allow without any heaving of his vessel is pre-set and automatically shields the route from considering positional and time states that define this value. One has to consider the set of allowable state transitions, the physical length of the stage (number of positional states) and the size of the encountering wave system, when deciding upon the size of the positional state space.

5.12. ROUTE EVALUATION USING VARYING GRID TYPES.

The previously defined grids have been used to evaluate a particular north Atlantic route, without any data extension.

--- OPTIMAL PATH --- Mercator Projection

- KEY -

* LEAST COST ----------
GREAT CIRC -------
ADvised RT -------

--- REVIEW ---

DART ATLANTIC
DEF TIME : 12/04/89 0000 HRS
DEPARTURE : 35.5 N 15.0 W
ARRIVAL : 40.0 N 72.0 W
DISPLCMT : 36458.0m*3
SPD RNGE : 17.0 - 23.0 Knts
MX EN WVE : 7.0m
ROUTE DIST TIME AvSPEED COSTS
GCR 2680.1 139.6 19.2 49874.6
OCR 2698.5 138.3 19.5 49431.5
SAVINGS : 443.1

POSITION : 35.5N 15.0W
HEADING : 280.2
ETA : 17/04/89 1820 HRS
DIST END : 2698.5 Nm

FUEL COST : $ 95.0/tonne FUEL CONS : 520.3tonnes BUNKER : 2077.7tonnes RPM:106.0
SFC : 247.7g/Kwh

* LEast cost refers to least time, great circle is the least distance travelled.
* Advised rts is that given by OCEAN ROUTES if provided.

FIGURE 5.14.(a) NORTH ATLANTIC ROUTE FOUND USING THE GCR GRID WITH STATES COMPUTED ALONG MERIDIONALS.
**FIGURE 5.14. (b). NORTH ATLANTIC ROUTE FOUND USING THE GCR GRID WITH STATES COMPUTED ALONG PERPENDICULARS.**

**FIGURE 5.14. (c). NORTH ATLANTIC ROUTE FOUND USING THE GCR GRID WITH STATES COMPUTED ALONG FURTHER GCs.**
There are slight variations between the routes resulting from the different grids. Those points outlined in section 5.9.2, part (iii), are responsible for the route changes. The question becomes, which of the grid systems yields a more representable value of the true-optimum? This can be answered by the previous routeing examples, and by concluding that a system that represents the physical shape of the earth is more likely to achieve better results. There is therefore some degree of subjectivity in the results, although all grids will produce an LDR route under calm conditions. A summary of the routes in figure 5.14. is given below. Only grids on δx stage spacing were considered, for the route. Since the GCR interferes with land, the LDR fit through the grid systems is slightly different.

<table>
<thead>
<tr>
<th>GRID TYPE</th>
<th>DISTANCE MTR (NM)</th>
<th>DISTANCE LDR (NM)</th>
<th>ETA MTR (HRS)</th>
<th>ETA LDR (HRS)</th>
<th>LAT AT ~ 30°W</th>
<th>LAT AT ~ 50°W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meridional δx=200nm</td>
<td>2698.45</td>
<td>2688.17</td>
<td>138.331</td>
<td>139.5708 (+1.2398)</td>
<td>38.31</td>
<td>41.73</td>
</tr>
<tr>
<td>GC on GCR δx=200nm</td>
<td>2711.32</td>
<td>2687.52</td>
<td>138.6091</td>
<td>139.4157 (+0.8066)</td>
<td>37.54</td>
<td>41.73</td>
</tr>
<tr>
<td>Perpendicular δx=200nm</td>
<td>2708.61</td>
<td>2689.00</td>
<td>138.5298</td>
<td>139.6547 (+1.1249)</td>
<td>37.60</td>
<td>41.73</td>
</tr>
</tbody>
</table>

**Table 5.1. Summary of the Routes Computed Using Certain Grid Types**

Table 5.1 indicates the perpendicular grids to the GCR to be very similar. It is expected that these results would vary depending upon the environmental state, although, it is felt that the GC perpendicular to the GCR should be adopted for the routeing models.

### 5.13. The Effect of the Size of the Positional Discretisation (Stage and Lateral Positional State) Spacings.

The stage and lateral positional state spacings have a marked effect upon the quantitative value of the objective function. Frankel et al (1980) have shown the effect of reduction of these spacings, concluding that finer spacings result in larger values of the objective function. Similarly, should several routes be of similar cost, since it is likely that a 'smooth' crossing will produce this, then it is plausible to have a variable quasi-optimal route, depending upon grid spacings.
Finer spacings inevitably increase the size of the state space, and consequently hugely increase the computational burden. There is therefore a compromise between the state spacings and the estimated central processor unit, (CPU), time. Since forecast data is updated every 12 hours it is taken as the upper limit for stage spacings, assuming the data to be constant between forecast times; for a 20 knot ship, this would equate to a stage distance of 240nm. A study of the effect of grid point spacings was made, Motte et al (1990), with the previous grid systems. A rough guide of the order 5:1, stage to lateral positional state spacings was deduced. Although stage and lateral state spacings increase, the trajectory step-length does not. Therefore, the positional states only act as decision nodes, which, because of the relatively inactive sea states for the period of data in this study, does not influence the policy values significantly. However, should there be storms, then accuracy would be improved by using shorter stage spacings.

**Figure 5.15.** MTRs. - VARYING STAGE DISTANCES.

Figure 5.15 indicates ETAs for the LDR and MTR of a journey, at 19 and 17 knots, departing on 15-4-89 and a further route at, 19 knots departing on 17-4-89.
(constant 45nm lateral state spacing). As stage separation increases, the MTR policy departs from the LDR policy values at 150 - 200 nm. Slight curve variations are due to the fit of the step length. At 19 knots, the vessel will cover 114nm, in the calm, resulting in data iterations at 114 + the remaining stage distance. It can be concluded that stretching the stage distance reduces the change in heading, (restricting vessel control). Finer course changes give a greater possibility of an MTR different to the LDR, which appears from 200nm, (a stage to lateral state spacing ratio of approximately 4.5 : 1). Figure 5.16 indicates the effect of increased state spacings, for the same routes in figure 5.15. Stage distance is constant (150nm), therefore the LIDR ETAs remain constant. Below approximately 30 to 45nm there is a slight difference between the MTR and LDR policy values, (occurring at approximately 4.5 : 1). As the stage spacing is increased, larger course changes are made, and since there is only a small optimum saving, the increased distance to the waypoints does not offer an optimal solution. Similarly, it has been found that larger separations result in jagged or discontinuous routes.

![Variation of LDR and MTR ETAs for varying lateral state separation](image)

**FIGURE 5.16. MTRs: VARYING LATERAL POSITIONAL STATE SPACING.**
5.14. SHIP CONTROL CONSTRAINTS IN THE OPERATIONAL ALGORITHM.

Frankel et al (1980) state that the maximum change in heading from one stage to another is to be $35^\circ$, however, it has been found that localised depressional activity in the vicinity of the terminals warrants increasing this value to between $40^\circ$ and $50^\circ$, and at each stage there are a pre-defined set of allowable transitions.

Further to these constraints, the set of motion constraints, $M$, are provided for each state trajectory. These ship motion exceedance criteria, defined in section 3.6, simply invoke further action on this trajectory, rather than cause omission.

In the event of a vessel encountering seaways that produce exceedance values outside the permitted range, or where the vessel encounters wave systems greater than the set maximum, then the vessel is simply "hove-to". This can be regarded as a penalty function, which is a measure of time or fuel or cost, however, the vessel is not permitted to continue the trajectory until the constraints are not exceeded. These penalty values are determined in time increments of one hour, and initially no such increments are allowed by the algorithm. In the hove-to mode the vessel is given a nominal steerageway speed of 4 knots, as being practicable under prevailing severe conditions. (This will be modified by impressed current).

Should the depressional activity be large enough to constrain every state transition, then the algorithm re-invokes this stage, and allows the transitions to take place without the master's pre-set maximum wave height. However, the motion criteria may still prevent any stage transitions and therefore the algorithm increases the maximum allowable number of heave increments. When all transitions have been attempted, the algorithm will repeat the latter process until a certain number of state transitions are made.
This system is realistic, since it ensures that the route is geared towards safety and ensures the vessel is hove-to until, and only when, the sea state permits it to recommence it's journey. In the event of large slow moving depressions, the algorithm will eventually converge on a route that involves the least number of stops and therefore the minimum exposure to heavy weather under the circumstances, assuming the penalty function to be large enough.

Penalties are assigned to the vessel for each increment of the heave to, in order that the route is weighted to the minimum.

The penalty function is simply expressed as a representation of the cost function, therefore, it can be regarded as :-

\[ H(\bar{X}, k, t) = f(\text{operating}, \text{wve}, \alpha \delta Et) \]

Where :-

- **H** - Heave-to cost function, defined at states \( \bar{X} \), \( t \) on stage \( k \).
- **C_{\text{operating}}** - Operational cost of holding the vessel into wind and waves, which is a function of the fuel consumption/crew wages/maintenance/insurance/capital costs.
- **C_{\text{wve}}** - Penalty cost, either pre-set or some function of the damage likely to be suffered by the ship, Constantine (1980). This is a function of ship's strength, type of cargo, hull form, lashings strength, container strength, maximum expected wave height, duration of sea state and so on.
- **Et** - Duration time considered for the heave to.

**5.15. POSITIONAL STATE INDEXING.**

In order that the quasi-optimal route be defined after computation, it is necessary to index the states which make up that route as the computation proceeds. Using De Wit's (1989) notation, but incorporating equation 5.5.7.(b), the positional states
or waypoints of the quasi-optimal route comprise a sequence from an index matrix, of the same size as the state space. In other words for each state, positional and time, at stage \( k \) there is a previous state, on stage \( k+1 \), (reverse computation), from which the optimal to the state on stage \( k \) was found.

Let the feasible two-dimensional state space be made up of a matrix comprising, \( k=1, 2, \ldots, N_{\text{stage}}-1, N_{\text{stage}} \) stages by \( l=1, 2, \ldots, M_{\text{state}}-1, M_{\text{state}} \) states per stage. The recursive equation is given by:

\[
C^*(k, l) = \min \left\{ f(k+1, l \pm p, k, l) + C^*(k+1, l \pm p) \right\}
\]

Where \( p \) - Value associated with maximum course deviation or allowable transitions.

The index to the optimal cost at point \((k, l)\) is defined by \( \text{index}(k, l) \). Therefore for each point in the grid system, there is an index value given by \( l \pm p \), since the previous stage is inherent. This can be viewed as, 'the point, \( l \pm p \), on the previous stage (either forwards or backwards recursion), from which the optimal functional value to point \( l \), on stage \( k \), was achieved from'. Therefore, upon reaching the terminal point, there is a unique set of positional indeces which define the quasi-optimal track.

Similarly, the extension of the two-dimensional state space in the time dimension requires a subsequent extension to the indeces. Two index systems operate, one which defines the previous positional state from which the quasi-optimal was deduced and the second defining the time state at this position.

5.16. **TIME STATES.**

So far only positional states have been described, whereas it is possible to expand the feasible state space in time. Under constant engine setting for the whole
voyage, one cannot define time states, since time of arrival to stage is one-dimensional and unknown. In other words, there is only one specific time of arrival in each positional state which is a function of the sea state, $S(k,t)$. Therefore, for minimum fuel, cost, and time routeing, under set engine revolutions or power, there is only a need to compute the positional states.

There is a further approach to routeing that allows a variable engine power or revolution setting at each positional state, under the engine constraints, $E$. Petrie et al (1984) defines $t$-states, which are compatible with the maximum and minimum vessel speeds. Chen et al (1978) and Frankel et al (1980), show an alternative solution, by defining a set of discrete power or engine settings between the maximum and minimum, so that a set of arrival times are defined at each positional state. The following scenarios were regarded for optimal cost routeing, although it may be applied to optimal fuel routeing.

5.16.1 Fixed Terminal Times.

The vessel operates between fixed terminal times, as on a liner service, and operates to match both, or, operates from one fixed time at an envisaged average speed, (therefore fixed terminal times). An operating time envelope or time state space is defined, by stating the maximum and minimum vessel speeds. The master is left with two extreme options, Laurence (1987).

1. Operate from departure at maximum speed, then minimum speed towards the destination.

2. Operate from departure at minimum speed, then maximum speed towards the destination.

The following figure shows an example of the limits of the time state space per stage for the LDR route, however, for each route through the positional grid there will be a similar envelope.
The maximum and minimum times of arrival in a positional state are a function of the maximum and minimum speeds of the ship, and is given by:

\[
t_{\text{max}}(k, l) = \min \left( t(N_{\text{stage}}, C) - \frac{\text{GCR}_{\text{dest}}}{V_{\text{max}}}, \frac{\text{GCR}_{\text{dep}}}{V_{\text{min}}} + t(1, C) \right) \tag{5.16.1}
\]

\[
t_{\text{min}}(k, l) = \max \left( t(N_{\text{stage}}, C) - \frac{\text{GCR}_{\text{dest}}}{V_{\text{min}}}, \frac{\text{GCR}_{\text{dep}}}{V_{\text{max}}} + t(1, C) \right) \tag{5.16.2}
\]

Where

- \( k \) - Stage variable = 1, 2, ..., \( N_{\text{stage}}-1, N_{\text{stage}} \)
- \( l \) - State variable \( C \) = Central value on LDR.
- \( \text{GCR} \) - Distance on the great circle between the point \((k, l)\) and \( \text{dest} \) = destination; \( \text{dep} \) = departure points.

Computation of the t-state extremes is quick and simple. There are two methods of inter-discretisation between the extremes.
1. Define a certain number of time states, for example Petrie et al (1984) stated that as many as $T = 20$ were required. The set of time states at each positional state becomes:

$$t(k, l, m) = m \left( \frac{t_{\text{max}}(k, l) - t_{\text{min}}(k, l)}{T} \right)$$

For $m= 1, 2, \ldots, T-1, T$.

2. Define the time state, at a certain interval of time from the minimum, plus the maximum value. The set of time states at each positional state become:

$$t(k, l, m) = \text{Min} \left( t_{\text{max}}(k, l), t_{\text{min}}(k, l) + m\delta t \right)$$

Both approaches were used, however, it was found that the former method requires many time states if strange results are to be overcome around the central stages where the inflection in the curves occur, see figure 5.17. Similarly, a small value of $\delta t$ is required in the second method. Typically it was found that 10-15 time states or 1-3 hours was sufficient to overcome encumberment problems and refinement.

Secondly, but more importantly, the former method defines $T$, $t$-states at each position, therefore the method is inefficient in the vicinity of the terminals where the difference between maximum and minimum times is very small. In central stages, each $t$-state becomes wide, unless $T$ is large. Clearly, the situation is subjective.

The latter method, overcomes the problem at terminal stages, but a sufficiently small value of $\delta t$ is required in central stages in order that they do not become too wide. Similarly, the matrix of $t$-states is variable, reaching a maximum at the central stage. It is possible to have a variable $\delta t$, value for each stage, proportional with the difference between the extreme values.
5.16.2 Fixed Departure Time.

If only the departure time is pre-set, along with the extremes of vessel speed, then it is possible to employ an open ended t-state envelope at the destination.

Such a t-state envelope can be used to route vessels which do not operate a liner service, but still wish minimum time (variable engine power), cost or fuel routes. This may result in minimum fuel and time routes coinciding.

Care has to be taken with the definition of t-states, since the difference between the extreme times of the envelope are increasing with each stage away from the departure, reaching a maximum at the destination. Such a representation of the t-state envelope or space is best manipulated by using method 2, in 5.16.1.

5.17 THE EFFECT OF DISCRETISATION IN THE TIME DOMAIN. (T-STATES).

The effect of discretisation in the t-states has not been investigated fully, but suffice it to say that their inclusion vastly increases the overall state space and computational burden.

Problems occur in the 'connection' of t-states, when operating near the maximum speed of the vessel. If states become unachievable, it may cause a 'ripple effect' through the system which reduces the number of good states as the recursion continues. This is complicated when the t-states are widely separated.

Furthermore, when using a constant number of t-states per stage, this value has to be sufficiently large so as to overcome the rippling from the central stages, therefore introducing vast inefficiency at terminal stages.

5.18 THE GENERAL COST FUNCTION.

Spaans et al (1987) and Hagiwara (1989) have both shown that the general cost function can be made up of a summation of the cost centres which describe the
daily operating costs of the vessel, the penalty costs at the terminal and port costs. The overall cost function, \( C \), concluded in this study, can be regarded as:

\[
C = \omega_1 t + \omega_2 (t_{\text{dest}} - t_{\text{sdest}}) + \omega_3 (t_{\text{dep}} - t_{\text{sdep}}) + \omega_4 \beta_{\text{main}} t + \omega_5 C_{\text{cap}} t + \omega_6 C_{\text{ins}} t + \omega_7 C_{\text{wwe}} t + \omega_8 C_{\text{crew}} t + \omega_9 (t_{\text{dep}} - t_{\text{adep}}) C_{\text{depport}} + \omega_{10} (t_{\text{dest}} - t_{\text{sdest}}) C_{\text{arrport}}
\]

Where \( t \) - Time. Subscript: \( \text{dest} \)-Destination, \( \text{sdest} \)-Scheduled destination,
\( \text{dep} \)-Departure, \( \text{sdep} \)-Scheduled departure, \( \text{adep} \)-Arrival departure.
\( \text{exp} \)-Exposure to high sea states.
\( \omega \) - Weighting. \( C_{\text{main}} \)-Main engine fuel cost function.
\( C_{\text{cap}} \)-Capital repayment cost function. \( C_{\text{crew}} \)-Crew wages cost function. \( C_{\text{gen}} \)-Generator fuel cost function. \( C_{\text{ins}} \)-Insurance cost function.
\( C_{\text{wwe}} \)-Penalty cost function for wave damage/heave-to

\[
= C_{\text{crew}} + C_{\text{main}} + C_{\text{cap}} + C_{\text{ins}} + C_{\text{pen}}
\]

\( C_{\text{pen}} \)-Penalty value.
\( l \) - Penalty cost function on early/late departure.
\( x \) - Penalty cost function on early/late arrival at destination.
\( C_{\text{arrport}} \)-Arrival port cost function. \( C_{\text{depport}} \)-Departure port cost function.

\[
= C_{\text{port}} (C_{\text{crew}} + C_{\text{ins}} + C_{\text{cap}} + C_{\text{gen}}) + C_{\text{berth}} + C_{\text{pilot}} + C_{\text{cargo}} + C_{\text{tow}} + l(x)
\]

\( C_{\text{berth}} \)-Berthing cost. \( C_{\text{pilot}} \)-Pilotage cost. \( C_{\text{cargo}} \)-Cargo handling cost.
\( C_{\text{tow}} \)-Tugs cost.

It can be seen that the overall cost function contains expressions for port costs, daily operating costs and penalty costs for exposure to heavy weather, or arrival or departure at times other than the scheduled times.

<table>
<thead>
<tr>
<th></th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>( \omega_3 )</th>
<th>( \omega_4 )</th>
<th>( \omega_5 )</th>
<th>( \omega_6 )</th>
<th>( \omega_7 )</th>
<th>( \omega_8 )</th>
<th>( \omega_9 )</th>
<th>( \omega_{10} )</th>
<th>( \omega_{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>COST</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FUEL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TIME</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 5.2. COST FUNCTION WEIGHTING VALUES.**
6. THE MINIMUM TIME OPTIMISATION
MODEL AND ROUTEING STUDIES.

6.1. INTRODUCTION.
Minimisation of vessel transit time is performed by combinations of the ship
performance algorithm, environmental algorithm and optimisation algorithm. The
dynamic programming routine has been adapted to formulate the minimisation of
time over the journey. Two routeing strategies were considered:

1. Set engine power and engine revolutions, with a pre-defined
departure time but no arrival time. There is no requirement for
time states, since the vessel control is restricted to heading only,
therefore the quasi-optimal policy is time;

2. Variable engine power and engine revolutions, with a pre-defined
departure time, but no arrival time. The time states are
pre-computed since the vessel control is widened to include
vessel speed. The quasi-optimal policy is time. Since this strategy
is shown in minimum cost, chapter 8, results are not given.

Several routeing studies are performed to investigate the effects of motion
criteria, ship algorithm complexity and the influence of the environmental extension
methods.

6.2. TIME OPTIMISATION MODEL.
With reference to equation 5.18.1, the optimal policy becomes :-

\[ C = \frac{v_{\text{ship}}}{\delta x} \]

The recursive iteration algorithm, equations 5.5.8.(a) - 5.5.8.(b) can be written
for forwards computation as :-
For the first stage,

\[ C^*\left( \vec{X}, 1 \right) = t_0 \quad 6.2.2.(a) \]

For intermediate stages,

\[ C^*\left( \vec{X}, k \right) = \min \left\{ \left( \frac{V_{\text{ship}}}{\delta x} + \alpha \delta t \right) + C^*\left( \vec{X}, k-1 \right) \right\} \quad 6.2.2.(b) \]

for \( k = 2, 3, 4, ..., N_{\text{stage}}-2, N_{\text{stage}}-1 \)

The state trajectory becomes a summation of a certain number of time intervals plus the remaining amount where \( \frac{V_{\text{ship}}}{\delta x} < \delta t \). If distance intervals are taken, 6.2.2.(b) can be regarded as the summation of time over intervals \( \delta x \), plus the remaining value \( \delta t \), where \( V_{\text{ship}} \delta t < \delta x \).

\[ C^*\left( \vec{X}, k \right) = \min \left\{ \left( \frac{V_{\text{ship}}}{\delta x} + \delta t \right) + C^*\left( \vec{X}, k-1 \right) \right\} \quad 6.2.2.(c) \]

for \( k = 2, 3, 4, ..., N_{\text{stage}}-2, N_{\text{stage}}-1 \)

There are advantages in both methods.

For the final stage,

\[ C^*\left( \vec{X}, N_{\text{stage}} \right) = \min \left\{ \left( \frac{V_{\text{ship}}}{\delta x} + \alpha \delta t \right) + C^*\left( \vec{X}, N_{\text{stage}}-1 \right) \right\} \quad 6.2.2.(d) \]

\[ C^*\left( \vec{X}, N_{\text{stage}} \right) = \min \left\{ \left( \frac{V_{\text{ship}}}{\delta x} + \delta t \right) + C^*\left( \vec{X}, N_{\text{stage}}-1 \right) \right\} \quad 6.2.2.(e) \]

Obviously, the two methodologies will result in slightly different policy quantities, in a similar way to the variations achieved by altering the size of the distance or time intervals along the state trajectory.

The state transfer function, 6.2.1, is deduced from an interpolation of the local environmental data, and the ship speed algorithm. Therefore, the sea state, is defined by :—
The assumption that the data is constant for the length of the state trajectory becomes necessary. However, if the interval is distance, the data can be found for the mid-point of the distance interval along the trajectory, or, if the interval is time, then the data can be found for the mid-point of the time interval along the trajectory, and at the ship's position.

Clearly, both have in-built inaccuracies, which can only be overcome by shortening the trajectory interval. De Wit (1989) suggested, that the estimated ship's speed along the trajectory, found from the function of ship's position and time at the start of the trajectory interval, be corrected by the estimated speed deduced from the function of position and time at the end of the trajectory interval.

Let the arrival time in \( k+1 \) be:

\[
t_{k+1} = t_k + \frac{\delta x}{v_{\text{ship}, k}}
\]

then the speed is estimated at \( k+1 \), \( v_{\text{ship}, k+1} \).

Since \( t_{k+1} = t_k + \Delta t \), the transit time \( \Delta t = \frac{\delta x}{v_{\text{ship}, k+1} - v_{\text{ship}, k}} \ln \left( \frac{v_{\text{ship}, k+1}}{v_{\text{ship}, k}} \right) \)

for

\[
\left| \frac{v_{\text{ship}, k+1}}{v_{\text{ship}, k}} - 1.0 \right|
\]

or

\[
\Delta t = \frac{2\delta x}{v_{\text{ship}, k+1} - v_{\text{ship}, k}}
\]

Clearly this introduces further computational burden, since the environment and ship algorithms have to be interpolated twice for each iteration along the trajectory interval, when in fact one could simply reduce the interval size.
Figure 6.1 gives an indication of the computational flow of the algorithm to compute minimum transit time. It is envisaged that the LDR be computed prior to the OCR, since the destination LDR policy forms further restrictions to the feasible state space and the total data requirement of the OCR.

![Diagram of Minimum Time Operational Algorithm]

**FIGURE 6.1. MINIMUM TIME OPERATIONAL ALGORITHM.**
It was stated in section 5.14 that the algorithm requires stage recalculation if insufficient state trajectories were made. There were found to be three reasons for such an occurrence.

- Wave, wind and current causing a speed below the minimum;
- Wave activity above the desired maximum;
- Wave activity causing exceedance of ship motion criteria.

The minimum number of state trajectories has been set at 1, although this may be incrementally increased. Therefore in the event of incompletion of state trajectories, the algorithm is left with four options for the stage.

1. Re-compute by relaxing the maximum wave height criteria;
2. Re-compute with increased course deviation, see section 5.10.2.
3. Re-compute, after a one-hour hove-to penalty;
4. Re-compute by increasing the motion criteria or omitting altogether.

This order of priority was considered suitable for a working algorithm, however, to maintain smooth routes, incremental course deviation is allowed only after several iterations of other criteria. The stage is re-computed initially without the maximum wave height criteria, which allows the vessel to enter seaways, otherwise avoided. The ship performance algorithm will still choose trajectories with the highest speed, or lowest wave heights. Should the stage still require re-computation, this will be due to low achievable speed, or excessive motions. The stage is re-computed by allowing a one hour waiting period, in the event of decaying seaways. The maximum wave height criteria is then re-initiated. If the stage still requires re-calculation, then the motion criteria are increased. It was found that to emphasise route safety, motion criteria should not be relaxed. Motion limits are set at 1.0, and increased to 1.5 for recalculation with incremental 0.5 increases as required. The above procedure is a multiple iteration, repeated until sufficient
state trajectories are made. Any negative savings computed are simply due to the much larger increase in motion limits on the LDR. In order to maintain the LDR, motion limits are increased above those used for the minimum time route, (MTR). Therefore, the MTR although arriving later, does so with improved safety.

6.3 MINIMUM TIME STUDIES.

The following study considers a voyage between Le Havre, France and Cristobal in the Caribbean. Only initial MTRs, were deduced with the Aertssen (1969) and Townsin et al. (1982) algorithms, whereas the Babbedge (1975) and semi-empirical/theoretical algorithms were used route with and without the motion criteria. In order that a subjective comparison of the forthcoming routes be made, the following sequence of figures, 6.2.(a) - (f) indicates the depressional activity in the central North Atlantic, at analysis time, as given by the ECMWF bulletins. Throughout, in the figures; LEAST COST refers to the MTR, GREAT CIRC refers to the LDR and ADVISED RT is that given by OCEANROUTES if provided.

![Figure 6.2(a)](image-url)

FIGURE 6.2.(a). SURFACE PRESSURE 15-4-89, 1200Z.
FIGURE 6.2 (b). SURFACE PRESSURE 16-4-89 1200Z.

FIGURE 6.2 (c). SURFACE PRESSURE 17-4-89 1200Z.
FIGURE 6.2.(d). SURFACE PRESSURE 18-4-89, 1200Z.

FIGURE 6.2.(e). SURFACE PRESSURE, 19-4-89, 1200Z.
FIGURE 6.2 (f). SURFACE PRESSURE 20-4-89, 1200Z.

FIGURE 6.2 (g). SURFACE PRESSURE 21-4-89, 1200Z.
6.3.1 ROUTES WITH THE AERTSSEN ALGORITHM.

Figure 6.3 indicates a route deduced with the Aertssen (1969) algorithm. A blanket zero sea state was used for the extended periods. No motion criteria were used, since introduction of these may cloud the results of the performance of the different algorithms. (Speed directly influences motion criteria).

--- OPTIMAL PATH --- Mercator Projection

<table>
<thead>
<tr>
<th>- KEY -</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEAST COST</td>
</tr>
<tr>
<td>LEAST COST: $ 95.0/tonne FUEL CONS: 747.8tonnes BUNKER: 1850.2tonnes RPM:106.0 FUEL: 247.7g/Kwh</td>
</tr>
</tbody>
</table>

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS: 747.8tonnes BUNKER: 1850.2tonnes RPM:106.0
SFC : 247.7g/Kwh

--- REVIEW ---

DART ATLANTIC
DEP TIME : 15/04/89 0000 HRS
DEPARTRE : 49.7 N 2.4 W
ARRIVAL : 25.8 N 80.0 W
DISPLCMNT: 36458.0m*3
SPD RNGE : 17.0 - 23.0 Knts
MX EN WVE: 7.0m
ROUTE DIST TIME AvSPEED COSTS
GCR 3778.5 199.4 18.9 71262.3
OCR 3798.9 198.8 19.1 71037.7
SAVINGS: 0.63 224.6
POSITION: 49.7N 2.4W
heading 264.0
ETA 23/04/89 0640 HRS
DIST END: 3798.9 Nm

FIGURE 6.3. ROUTE DEDUCED WITH THE AERTSSEN ALGORITHM.

It can be seen that the vessel route is south of the LDR, between the terminals, which seems to be contrary to the depressional activity. The time saving is very minimal, and several routes probably exist close to the LDR.

The sensitivity of the algorithm to the environment is poor, therefore, medium sea states will not be picked up, as in the case of the seaway in these examples. The vessel is able to maintain high speeds, in low beaufort numbers. It is interesting to note that the arrival times are consistent with those determined by the Babbedge...
(1975) and semi-empirical/theoretical models. Data for the extended period was with the ECMWF and RC data, which was low during this period, enabling the vessel to maintain its required speed easily.

Loss of speed during extended periods is due to an assumed current, taken as that at analysis time. The following can be concluded from the route.

1. Loss of speed due to Beaufort number leads to inaccurate results, since BN describes several seaways. Ship performance model is therefore not sensitive to the environment unless clear distinctions are seen, unless data covers the whole extended period.

2. A stage re-computation scheme ensures a converging solution even for a single LDR through extreme seaways. If heave-to occurs on the initial trajectories, it can be treated as a delayed departure time, until the seaway permits the journey to continue.
6.3.2. ROUTES WITH THE TOWNSIN ALGORITHM.

Figure 6.5 indicates a route deduced using the Townsin et al (1982) algorithm, using zero data extension and no motion criteria.

--- OPTIMAL PATH --- Mercator Projection

| - KEY - |
| LEAST COST --- | GREAT CIRC --- | ADVISED RT --- | REVIEW --- |

--- DART ATLANTIC ---

DEP TIME: 15/04/89 0000 HRS
DEPARTURE: 49.7 N 2.4 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMT: 36458.0 m
SPD RNGE: 17.0 - 23.0 Knts
MAX EN WVE: 7.0 m
ROUTE DIST TIME AVG SPEED COSTS
- GCR 3778.5 199.9 18.9 71433.1
- OCR 3808.4 199.1 19.1 71158.0
SAVINGS: 0.77 275.2

POSITION: 49.7N 2.4W
HEADING: 264.0
ETA: 23/04/89 0700 HRS
DIST END: 3808.4 Nm

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS: 749.0 tonnes BUNKER: 1849.0 tonnes RPM: 106.0
SFC: 247.7 g/Kwh

* Least cost refers to least time, great circle is the least distance fit
Advised rise is that given by OCEANROUTES if provided.

FIGURE 6.5. ROUTE DEDUCED WITH THE TOWNSIN ET AL ALGORITHM.

The MTR is predicted south of the LDR, in much the same manner as the Aertssen (1969) algorithm. For the same reason, due to Beaufort number, the algorithm is not sensitive to the environment. By referring to the Babbedge (1975) algorithm routes, in section 6.3.3, the time saving is small, over a long distance.

The Aertssen (1969) and Townsin (1982) algorithms appear to overpredict speed. Hence, the vessel is able to maintain higher speeds through the higher sea states. The accuracy of the determined speed is dependant upon the polynomial describing the BN. Since BNs are described as that for a range of wind speeds, the possible speed error can be large, when using either the minimum or maximum values in the range.
The following can be concluded from the Aertssen (1969) and Townsin et al (1982) algorithms, further to those in section 6.3.1, which lead to the differences between the routes.

1. The algorithms predict inaccurate speeds based on BN, which can lead to problems in restricted waters, where stage re-calculation is forced.

2. Ship performance accuracy becomes paramount, especially, if predictions are close to the speed limits, since a small error may mean the difference between a good or a bad state connection.

3. It is difficult to conclude which algorithm is more accurate, since it is impossible to evaluate the extent by which the master would influence the respective 'speed loss curves'.

4. Realistic routes can not be expected, since the algorithms only indicate the gross relative change of ship speed as a function of the environment.

5. Introduction of motion criteria may possibly improve route decisions based on the very simple algorithms.
Further to these examples, routes have been attempted during more stable periods. It has been found that these routes are very similar since no stage re-computation is used. Furthermore, the routes become similar when the stage is not severely restricted, since state trajectories are made in roughly the same regions. If a high wave system is placed centrally on the LDR, the OCRs will tend closer to the LDR as one algorithm predicts greater speeds than the other. Figure 6.7 depicts this graphically.

![Diagram of routes around a central depression with various ship performance algorithms.]

**WAVE REGIME**

- **-1-** Route with algorithm that predicts >14 Knts in high wave heights, >8m.
- **-2-** Route with algorithm that predicts 14 Knts below 8m but above 6m.
- **-3-** Route with algorithm that predicts 14 Knts below 6m but above 2m.

**FIGURE 6.7, ROUTES AROUND A CENTRAL DEPRESSION WITH VARIOUS SHIP PERFORMANCE ALGORITHMS.**

### 6.3.3. ROUTES WITH THE BABBEDGE ALGORITHM.

The following sequence of routes indicates those deduced by the Babedge (1975) algorithm, using:

1. Zero data extension.
2. RC data extension.
3. ECMWF and RC data extension.

The routes have been deduced with and without motion criteria.
<table>
<thead>
<tr>
<th></th>
<th>LEAST COST</th>
<th>GREAT CIRC</th>
<th>ADVISED RT</th>
<th>REVIEW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DART ATLANTIC</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEP TIME</td>
<td>15/04/89</td>
<td>0000 HRS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEPARTURE</td>
<td>49.7 N 2.4 W</td>
<td>25.8 N 80.0 W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARRIVAL</td>
<td>25.8 N 80.0 W</td>
<td>25.8 N 80.0 W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DISPLCMNT</td>
<td>36458.0m³</td>
<td>36458.0m³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPD RNGE</td>
<td>17.0 - 23.0 Knts</td>
<td>17.0 - 23.0 Knts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MX EN WVE</td>
<td>7.0m</td>
<td>7.0m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROUTE DIST TIME</td>
<td>195.4</td>
<td>194.1</td>
<td>19.3</td>
<td>19.7</td>
</tr>
<tr>
<td>AvSPEED</td>
<td>69839.6</td>
<td>69358.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COSTS</td>
<td>481.4</td>
<td>481.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAVINGS</td>
<td></td>
<td></td>
<td>1.35</td>
<td>1.62</td>
</tr>
<tr>
<td>POSITION</td>
<td>49.7N 2.4W</td>
<td>49.7N 2.4W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEADING</td>
<td>289.3</td>
<td>289.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETA</td>
<td>23/04/89</td>
<td>23/04/89</td>
<td>0200 HRS</td>
<td></td>
</tr>
<tr>
<td>DIST END</td>
<td>3814.7 Nm</td>
<td>3814.7 Nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FUEL COST</td>
<td>$ 95.0/tonne</td>
<td>$ 95.0/tonne</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FUEL CONS</td>
<td>730.1tonnes</td>
<td>730.1tonnes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BUNKER</td>
<td>1867.9tonnes</td>
<td>1868.8tonnes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RPM</td>
<td>106.0</td>
<td>106.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SFC</td>
<td>247.7g/Kwh</td>
<td>247.7g/Kwh</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 6.8. BABBEDGE ROUTE WITH ZERO DATA EXTENSION.**

--- OPTIMAL PATH --- Mercator Projection

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS: 730.1tonnes BUNKER: 1867.9tonnes RPM: 106.0
SFC: 247.7g/Kwh

**FIGURE 6.9. BABBEDGE ROUTE WITH RC DATA EXTENSION.**

--- OPTIMAL PATH --- Mercator Projection

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS: 730.1tonnes BUNKER: 1868.8tonnes RPM: 106.0
SFC: 247.7g/Kwh
During the extended period, >+144 hours, there was little wave activity on the western seaboard of the Atlantic. For this reason, there is only a minimal difference between the arrival times, from using the three data extension methods. As time progresses, every 12 hours, a new set of environmental data is used to compute a new route from a position 12 hours along the previous MTR. Figures 6.11.(a) and 6.11.(b) indicate the variation of arrival times as the vessel progresses along both the LDRs and the MTRs, whilst using the three data extension methods.

As indicated, during initial routes, the arrival time is earliest when zero sea state data extensions are used, although this value reaches a steady value between 192 and 193 hours, when the extended data is no longer utilised. It is interesting to note that the variation of arrival time from other methods varies only slightly. However, it must be remembered that little wave activity was experienced at this time. Results would be expected to change depending upon the state of the ocean at extended periods, although, the arrival time should steady when only forecast...
data is used. In defence of this example it should be stated that these conditions realistically examine the sensitivities of the three methods used; which, of course, is the main object of the exercise.

![Variation of ETA as the route progresses along the LDRs](image)

**Figure 6.11(a). Variation of arrival times on the LDR as the route progresses.**

By referring to figures 6.11(a) and figure 6.11(b), it will be seen that at certain times the ETAs concluded with RC and ECMWF data extensions are earliest, for example at +36 hours, when the RC data extension arrives earliest. This is attributed to an increase in speed achieved in the extended data by following winds and seas. At +72 hours all routes are deduced with the same forecast data, as the extended data is no longer utilised. Therefore, all ETAs are identical.

The slight variations in arrival times in figures 6.11(a) and 6.11(b) also indicate the changes in forecasted data as the routes progress. The vessel encounters two depressions, although these are not severe, upon encounter a speed reduction results. The model indicates slight increases in ETA for the initial route at 0.
(savings of +1.4759, +1.6169, +1.3470 for ECMWF, RC and ZERO data extensions respectfully), and also when at central stages, at +48 hours. It is at these points that a departure from the LDR is noticed and a time saving is realised. This can be found by referring to figures 6.11(a) and figure 6.11(b).

![Variation of ETA as the route progresses along the MTRs](image)

**Figure 6.11(b). Variation of Arrival Times on the MTR as the route progresses.**

Figures 6.11(a) and 6.11(b) both indicate a reduction in the ETA as the route progresses, with an exception at +96 hours. This can be attributed to model accuracy and forecasted data accuracy.

The step length, taken as 6 hours will undoubtably affect the results. Differing environmental data will be interpolated, since the step length will yield differing spatial and time interpolated points which can only be overcome by shortening the step length. Unfortunately the environmental data was not severe, however, it is believed that these results would be typical of other situations. It is difficult to conclude the RC extension from that using the ECMWF, since these examples indicate either could be used. However, in more extreme situations it is believed that pinpointing the storm centres would be more advantageous.
### FIGURE 6.12(a). BABBEDGE ROUTE WITH MOTION CRITERIA.

<table>
<thead>
<tr>
<th>Route</th>
<th>Dist</th>
<th>Time</th>
<th>Ave Speed</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCR</td>
<td>3778.5</td>
<td>205</td>
<td>18.4</td>
<td>3937.8</td>
</tr>
<tr>
<td>OCR</td>
<td>3842.8</td>
<td>195.2</td>
<td>19.7</td>
<td>6750.2</td>
</tr>
<tr>
<td>SAVINGS</td>
<td>10.59</td>
<td></td>
<td></td>
<td>1787.6</td>
</tr>
</tbody>
</table>

**Position:** 49.7N 2.4W

**Heading:** 288.6

**ETA:** 23/04/89 0310 HRS

**Distance End:** 3842.8 Nm

**Fuel:**
- Cost: $95.0/tonne
- Cons: 734.2 tonnes
- Bunker: 1163.8 tonnes
- RPM: 106.0
- SFC: 247.7g/Kwh

---

### FIGURE 6.12(b). BABBEDGE ROUTE 24HRS FROM (a).

<table>
<thead>
<tr>
<th>Route</th>
<th>Dist</th>
<th>Time</th>
<th>Ave Speed</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCR</td>
<td>3305.0</td>
<td>175.2</td>
<td>18.9</td>
<td>67621.6</td>
</tr>
<tr>
<td>OCR</td>
<td>3320.6</td>
<td>169.0</td>
<td>19.6</td>
<td>60398.2</td>
</tr>
<tr>
<td>SAVINGS</td>
<td>6.22</td>
<td></td>
<td></td>
<td>215.4</td>
</tr>
</tbody>
</table>

**Position:** 50.9N 14.4W

**Heading:** 266.4

**ETA:** 23/04/89 0050 HRS

**Distance End:** 3320.6 Nm

**Fuel:**
- Cost: $95.0/tonne
- Cons: 635.7 tonnes
- Bunker: 1963.3 tonnes
- RPM: 106.0
- SFC: 247.7g/Kwh
--- OPTIMAL PATH --- Mercator Projection

--- KEY ---

LEAST COST --- --- ---
GREAT CIRC --- --- ---
ADvised RT --- --- ---

- REVIEW -

DART ATLANTIC
DEP TIME: 17/04/89 0000 HRS
DEPARTURE: 51.1 N 26.4 W
ARRIVAL: 51.8 N 80.0 W
DISPLCMNT: 36458.0m³
SPD Rnge: 17.0 - 23.0 Knts
MX En Wve: 7.0m

ROUTE DIST TIME AvgSPEED COSTS

GCR: 2857.6 144.7 19.7 517.05.2
OCR: 2863.7 144.7 19.8 517.05.2
SAVINGS: 0.00

------ POSITION ------

DEP-ARTRE 51.1 N 26.4 W
ARRIVAL . 15.8 N 80.0 W

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS: 544.3 tonnes BUNKER: 2053.7 tonnes RPM: 106.0
SFC : 247.7g/Kwh

FIGURE 6.12. (c). BABBEDGE ROUTE 48HRS FROM (a).

--- OPTIMAL PATH --- Mercator Projection

--- KEY ---

LEAST COST --- --- ---
GREAT CIRC --- --- ---
ADvised RT --- --- ---

- REVIEW -

DART ATLANTIC
DEP TIME: 18/04/89 0000 HRS
DEPARTURE: 48.3 N 37.5 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMNT: 36458.0m³
SPD Rnge: 17.0 - 23.0 Knts
MX En Wve: 7.0m

ROUTE DIST TIME AvgSPEED COSTS

GCR: 2366.1 120.7 19.9 112.43.3
OCR: 2412.1 121.0 19.9 122.45.5
SAVINGS: 0.32 -107.1

------ POSITION ------

DEP-ARTRE 48.3 N 37.5 W
ARRIVAL 25.8 N 80.0 W

------ FUEL ------

FUEL COST: $ 95.0/tonne FUEL CONS: 455.12 tonnes BUNKER: 2142.7 tonnes RPM: 106.0
SFC : 247.7g/Kwh

FIGURE 6.12. (d). BABBEDGE ROUTE 72HRS FROM (a).

- 218 -
The crossing between Le Havre and Cristobal has been used to study the effects of motion criteria on the algorithm and also to compare routes to the semi-empirical/theoretical performance algorithm. Figures, 6.12(a) - (e) indicate a simulated 'real-time' on-board weather routeing journey, with motion criteria. With reference to figures 6.2, it can be seen that the MTR tends to the north of the initial and central developing depressions, tending to a great circle from central stages, as the sea states diminish. The required speed was 20 knots or 106 rpm.

Variations in routes from the initial route are only found when there is a significant change in the forecast sea states. This results from either inaccurate sea height/direction prediction or an inaccurate forecast of the centre of high wave height areas. There is therefore no question that a predicted route needs continual update, based on the results in this study.
Similarly, changes in speed have an effect upon the route, since this predescribes the arrival time at waypoints, and subsequent environmental data. A drawback of the routeling models is the 6 hour step length on the state trajectory, for which the data is assumed constant. This is not the case, especially in the vicinity of fast moving storms. Variation in ship speed, will change the physical length of the step making comparison of routes between speeds extremely difficult.

A comparison between routes with and without motion criteria can be made by referring to table 6.1 which summarises the routes in figures 6.12 and those found without these criteria. The table indicates the route with motion to be further to the north at the outset and initial positions (+1° - 1.5°), tending to the route without motions at later stages. Similarly, as the route progresses and the vessel utilises improved forecast data, the routes resort to the LDR.

### Table 6.1: Variations of MTRs Using the Babbage Algorithm

<table>
<thead>
<tr>
<th>MTR at time</th>
<th>10°</th>
<th>25°</th>
<th>45°</th>
<th>70°</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.89 @ 12.77</td>
<td>52.72 @ 23.88</td>
<td>50.50 @ 45.28</td>
<td>34.39 @ 69.00</td>
<td></td>
</tr>
<tr>
<td>50.88 @ 12.77</td>
<td>3004.61</td>
<td>2191.67</td>
<td>768.75</td>
<td></td>
</tr>
<tr>
<td>3406.53</td>
<td>51.98 @ 23.67</td>
<td>49.15 @ 44.29</td>
<td>35.50 @ 70.15</td>
<td></td>
</tr>
<tr>
<td>2991.99</td>
<td>2164.65</td>
<td>2151.05</td>
<td>776.50</td>
<td></td>
</tr>
<tr>
<td>+12 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.95 @ 13.48</td>
<td>50.4 @ 23.79</td>
<td>46.77 @ 43.44</td>
<td>34.23 @ 69.59</td>
<td></td>
</tr>
<tr>
<td>3348.68</td>
<td>2954.05</td>
<td>2143.47</td>
<td>740.18</td>
<td></td>
</tr>
<tr>
<td>50.96 @ 13.28</td>
<td>50.47 @ 23.79</td>
<td>46.82 @ 43.28</td>
<td>34.24 @ 69.58</td>
<td></td>
</tr>
<tr>
<td>3355.68</td>
<td>2954.05</td>
<td>2151.05</td>
<td>740.54</td>
<td></td>
</tr>
<tr>
<td>+24 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.33 @ 24.89</td>
<td>47.11 @ 44.53</td>
<td>34.85 @ 70.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2916.99</td>
<td>2111.09</td>
<td>705.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.35 @ 24.48</td>
<td>46.11 @ 43.86</td>
<td>33.86 @ 70.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2926.87</td>
<td>2123.88</td>
<td>705.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+36 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.19 @ 25.91</td>
<td>47.01 @ 44.67</td>
<td>33.16 @ 71.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2879.69</td>
<td>2080.21</td>
<td>627.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.21 @ 25.46</td>
<td>46.13 @ 44.69</td>
<td>33.63 @ 70.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2888.43</td>
<td>2085.47</td>
<td>664.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+48 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.81 @ 45.67</td>
<td>45.75 @ 2060.71</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2065.68</td>
<td>657.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+60 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.84 @ 45.76</td>
<td>46.29 @ 2046.73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2065.68</td>
<td>627.81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+72 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.75 @ 45.68</td>
<td>45.15 @ 1996.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>210.75</td>
<td>591.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.15 @ 46.93</td>
<td>32.73 @ 71.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1996.15</td>
<td>602.47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+84 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.47 @ 47.38</td>
<td>45.97 @ 1961.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1963.14</td>
<td>738.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+96 Hrs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.81 @ 45.67</td>
<td>45.15 @ 1996.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>210.75</td>
<td>591.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46.15 @ 46.93</td>
<td>32.73 @ 71.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1996.15</td>
<td>602.47</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key: MTR + motions - Small, Standard text. MTR - motions - Small Italic text.

N lat @ W Long
Nm to go

TABLE 6.1: Variations of MTRs Using the Babbage Algorithm

- 220 -
Typically, it was found that average roll was a major motion criteria that was frequently exceeded, (It was suspected that insufficient accuracy was achieved with the roll damping values), together with deck wetness. Whilst the latter, is not so important, the former could strongly influence ship stability. For this reason, especially, when at critical speeds, strange routeing results can be achieved. This is shown in section 6.3.4.

**Figure 6.13. BABBEDGE ROUTE WITH AVERAGE ROLL CRITERION INCREASED BY 50%.**

Figures 6.13 and 6.14 indicate an increase in average roll criterion by 50% and 100%. Clearly, the introduction of motion criteria has a marked effect upon the routeing decision and care must be taken to ensure that the limiting values are correctly set for the vessel.

For low average roll criteria, the vessel is forced to choose routes to the north of the LDR and central depression, see figures 6.12.(a) and 6.13. Since the vessel
may exceed the motions on all trajectories, it is those to the north that are finally achieved as 0.5 increases are permitted. As the criterion value is increased, more state trajectories are permitted, even after stage re-computation. For a 100% increase, the actual stage re-computation increases four fold over the initial value, allowing more southerly routes. This is evidenced by figure 6.14. When no motions are used, the vessel resorts to a northerly route, or even along the LDR, since the central depression is not severe.

--- OPTIMAL PATH --- Mercator Projection

--- REVIEW ---

dart atlantic
DEP TIME : 15/04/89 0000 HRS
DEPARTRE : 49.7 N 2.4 W
ARRIVAL : 25.8 N 80.0 W
DISPLCNMT: 36458.0m
SPEED RANGS : 17.0 - 23.0 Knts
MX EN WVE: 7.0m

--- FUEL ---
FUEL COST: $ 95.0/tonne FUEL CONS : 729.7 tonnes BUNKER : 1868.3 tonnes RPM: 106.0
SFC : 247.7 g/Kwh

**FIGURE 6.14. BABBEDGE ROUTE WITH AVERAGE ROLL CRITERION**

**INCREASED BY 100%**

Introduction of motion criteria to the prediction of MTRs, has been found to provide reasonable results, sometimes quite dissimilar from an MTR computed without the criteria. Due to the difficulties encountered in providing a reasonable database for the DART ATLANTIC/EUROPE, (difference in shiptype, and unknown BRITSEA accuracies) it has been found that a degree of flexibility has to be introduced to the limits. For this reason, the employment of a steadily increasing non-dimensional limit from 1.0, in 0.5 steps, is again emphasised. This remains
sensitive to the relative difference in seaways that may influence the motion criteria whilst allowing a routeing algorithm to converge on a sensible MTR.

With reference to table 6.2, it can be seen that the MTR computed with motion criteria still maintains a northerly route, whilst the MTR without the criteria tends to an LDR. The motion MTR, is still influenced by the decaying depression in the mid-Atlantic and is regarded as a route with further in-built safety. With reference to figure 6.11, the expected arrival times are not dissimilar, concluding that there are many routes around the LDR and to the north, all of which could be the true MTR. It is difficult to conclude that MTRs should be computed with motion criteria, due to the many error sources from the ship and environmental algorithms, coupled with the unquantifiable master's influence.

6.3.4. ROUTES WITH THE SEMI-EMPIRICAL/THEORETICAL MODEL.

--- OPTIMAL PATH --- Mercator Projection

| - KEY - |
| LEAST COST | GREATEST CIRC | ADVISED RT | REVIEW |

### Figure 6.11. Semi-empirical/theoretical route without motion criteria.

| POSITION | 49.7N 2.4W |
| HEAD | 289.3 |
| ETA | 23/04/89 1510 HRS |
| DIST END | 3812.8 Nm |

**FUEL**

- FUEL COST: $ 95.0/tonne
- FUEL CONS: 767.7 tonnes
- BUNKER: 1830.3 tonnes
- RPM: 106.0
- SFC: 247.7 g/Kwh

--- FUEL ---

These routes are not dissimilar from those computed with the Babbedge (1975) algorithm, since both are influenced by the initial and central depressions, figure 6.2. However, due to the speed errors causing differing encountered seaways, the motion criteria become dissimilar, possibly close to the critical limits. The trend of the routes under increasing average roll criterion is similar to the 'Babbedge' routes, tending to the LDR when no criteria are used.

Since the sensitivity of the algorithms to increasing sea states is different, an initial small difference may lead to much larger divergence as time integration effectively relates the ship to differing environmental conditions.
FIGURE 6.17. SEMI-EMPIRICAL/THEORETICAL ROUTE WITH AVERAGE ROLL CRITERION INCREASED BY 50%.

--- OPTIMAL PATH --- Mercator Projection

--- FUEL ---

FUEL COST: $95.0/tonne FUEL CONS: 779.5 tonnes BUNKER: 1818.5 tonnes RPM: 106.0
SFC: 247.7 g/KWh

FIGURE 6.18. SEMI-EMPIRICAL/THEORETICAL ROUTE WITH AVERAGE ROLL CRITERION INCREASED BY 100%.

--- OPTIMAL PATH --- Mercator Projection

--- FUEL ---

FUEL COST: $95.0/tonne FUEL CONS: 788.4 tonnes BUNKER: 1809.6 tonnes RPM: 106.0
SFC: 247.7 g/KWh

- 225 -
Table 6.2 shows the speed difference/error between the predicted speeds on an LDR. The error is small when in zero sea states, indicating reasonable agreement of the calm water powering to the Babbedge speeds. However, the error becomes more appreciable as the sea state increases. The errors/differences indicate the gross effect, and therefore include real time integration differences due to differing encounter angle and difference in wave height.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Semi-Empirical/ Theoretical</th>
<th>Babbedge</th>
<th>Speed error at stage</th>
<th>% Speed error on Mean Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Knts $\zeta_{1/3}$ $\alpha$</td>
<td>Babbedge Knts $\zeta_{1/3}$ $\alpha$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.14</td>
<td>2.5m $1^0$</td>
<td>19.55</td>
<td>2.5m $11^0$</td>
<td>0.41</td>
</tr>
<tr>
<td>18.61</td>
<td>3.0m $12^0$</td>
<td>19.09</td>
<td>3.0m $13^0$</td>
<td>0.48</td>
</tr>
<tr>
<td>18.20</td>
<td>3.4m $26^0$</td>
<td>18.70</td>
<td>3.4m $26^0$</td>
<td>0.50</td>
</tr>
<tr>
<td>18.15</td>
<td>2.8m $4^0$</td>
<td>18.81</td>
<td>2.8m $0^0$</td>
<td>0.66</td>
</tr>
<tr>
<td>18.20</td>
<td>3.3m $63^0$</td>
<td>19.08</td>
<td>3.8m $59^0$</td>
<td>0.88</td>
</tr>
<tr>
<td>17.83</td>
<td>5.2m $49^0$</td>
<td>18.55</td>
<td>4.9m $49^0$</td>
<td>0.72</td>
</tr>
<tr>
<td>17.86</td>
<td>4.5m $76^0$</td>
<td>19.30</td>
<td>4.5m $78^0$</td>
<td>1.44</td>
</tr>
<tr>
<td>18.98</td>
<td>2.1m $159^0$</td>
<td>19.85</td>
<td>2.4m $155^0$</td>
<td>0.87</td>
</tr>
<tr>
<td>18.70</td>
<td>2.7m $132^0$</td>
<td>19.89</td>
<td>2.7m $148^0$</td>
<td>1.19</td>
</tr>
<tr>
<td>17.92</td>
<td>3.3m $23^0$</td>
<td>19.46</td>
<td>3.1m $71^0$</td>
<td>1.54</td>
</tr>
<tr>
<td>18.51</td>
<td>2.9m $64^0$</td>
<td>19.71</td>
<td>2.4m $60^0$</td>
<td>1.20</td>
</tr>
<tr>
<td>19.95</td>
<td>0.0m</td>
<td>19.77</td>
<td>1.5m $87^0$</td>
<td>0.22</td>
</tr>
<tr>
<td>19.42</td>
<td>0.0m</td>
<td>19.66</td>
<td>0.0m</td>
<td>0.24</td>
</tr>
<tr>
<td>19.56</td>
<td>0.0m</td>
<td>19.78</td>
<td>0.0m</td>
<td>0.22</td>
</tr>
<tr>
<td>19.29</td>
<td>0.0m</td>
<td>19.50</td>
<td>0.0m</td>
<td>0.21</td>
</tr>
<tr>
<td>19.48</td>
<td>0.0m</td>
<td>19.68</td>
<td>0.0m</td>
<td>0.20</td>
</tr>
<tr>
<td>19.50</td>
<td>0.0m</td>
<td>19.70</td>
<td>0.0m</td>
<td>0.20</td>
</tr>
<tr>
<td>19.81</td>
<td>0.0m</td>
<td>20.00</td>
<td>0.0m</td>
<td>0.19</td>
</tr>
<tr>
<td>19.77</td>
<td>0.0m</td>
<td>19.95</td>
<td>0.0m</td>
<td>0.18</td>
</tr>
</tbody>
</table>

**TABLE 6.2. COMPARISON OF THE PREDICTED SPEEDS FROM THE BABBEDGE (1975) AND SEMI-EMPirical THEORETICAL ALGORITHMS ALONG AN LDR.**

The increase in error is directly attributable to the prediction of added resistance of the vessel in the seaway, which relies on the linear superposition principle. When the seaway becomes severe, linear theory becomes unreliable. Similarly, the Babbedge (1975) algorithm has been extended beyond the measured range, and may overpredict speed. It is extremely difficult to ascertain the size of any error without recourse to full-scale trials. In the event of a ship-borne system, a ship algorithm would be 'tuned' and adaptive as more measurements are received. This would hopefully include any power reduction by the master in severe seaways, which cannot be accounted for at present.
A comparison of routes with varying criteria between the Babbedge (1975) and the semi-empirical/theoretical algorithms can be made in table 6.3. The variation between models is confused due a difference of approximately 1 knot average speed.

<table>
<thead>
<tr>
<th>MTR</th>
<th>approximate Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babbedge</td>
<td>0°</td>
</tr>
<tr>
<td>No criteria</td>
<td>49.7</td>
</tr>
<tr>
<td>Semi-Em/Th</td>
<td>49.7</td>
</tr>
<tr>
<td>Av.Roll 15</td>
<td>49.7</td>
</tr>
<tr>
<td>Semi-Em/Th</td>
<td>49.7</td>
</tr>
<tr>
<td>Av.Roll 15</td>
<td>49.7</td>
</tr>
<tr>
<td>Babbedge</td>
<td>49.7</td>
</tr>
<tr>
<td>Av.Roll +50%</td>
<td>49.7</td>
</tr>
<tr>
<td>Babbedge</td>
<td>49.7</td>
</tr>
<tr>
<td>Av.Roll +100%</td>
<td>49.7</td>
</tr>
</tbody>
</table>

**TABLE 6.3. VARIATION OF MTRs COMPUTED WITH AND WITHOUT MOTION CRITERIA AND VARIABLE AVERAGE ROLL MOTION CRITERION.**

Table 6.3 indicates the sensitivity of the algorithms to average roll. In the case of the semi-empirical/theoretical algorithm, due to lower average speeds at the same engine setting, the effect of removing average roll causes a large MTR movement. The vessel enters the fully developed seas of the central depression. Since this depression remains central or moves slightly north, the MTR is found to the south. Faster speeds, cause the vessel to miss the higher sea states, enabling a northerly MTR, in order to counteract the depressional movement from the western seaboard.

For both algorithms and no motion criteria, table 6.3 indicates very similar routes. With average roll set at 15, both models show increased latitude at central stages, in the vicinity of the developing depression. Similarly, as the average roll criteria is increased by 50%, the central latitudes are in general reduced for the
Babbedge (1975) algorithm, and remain relatively similar for the semi-empirical algorithm. Increasing average roll criteria by 100% further reduces the Babbedge (1975) route, to that without criteria. However, the semi-empirical/theoretical algorithm route, moves very far to the south, going underneath the central depression. Accurate determination of motions and motion criteria limits are therefore required.

6.4. CHAPTER CONCLUSIONS.
The following can be concluded regarding the Babbedge (1975) and semi-empirical/theoretical algorithms, noting the increase in speed error as the sea state increases.

1. These algorithms provide better estimates of the objective function (time) than the Aertssen (1969) and Townsin et al (1982) algorithms.
2. The difference in MTR only becomes appreciable when the predicted speeds are close to the motion or other operating limits. A slight variation or error could mean the difference between a good or bad state trajectory. See figure 6.7.
3. The prediction of speeds in calmer seaways are not dissimilar and will yield identical MTRs.
4. Both suffer from reduced accuracy in severe sea states.
5. There may be several routes with very similar policy end values, each of which could be close to the truer optimal route. Speed accuracy is paramount.

Overall conclusions can be made about the ship performance algorithms, with reference to tables 6.2, 6.3 and 6.4.

1. The algorithms generally give similar MTRs, when there are clear distinctions between the depressional centres, since the relative change of the environment will be distinct. In these medium situations,
the sensitivity of the ship performance algorithm will depict clearer MTRs.

2. Low speed predictions incurring stage re-calculation over prolonged periods will drastically reduce MTR accuracy.

3. Motion criteria must be used with flexibility, and treated as a further safety constraint. It is shown, that increasing motion criteria limits has a marked effect upon the MTR. One concludes, therefore, that accurate motion prediction is paramount, since the limits result from several sources, see section 3.16.

4. Accurate prediction of ship speed in all sea severities is essential for the prediction of accurate estimated times of arrival, (ETA).

Use of a semi-empirical/theoretical ship algorithm, and motion calculation can only be recommended when accurate performance predictions, and environmental data are guaranteed. A simpler system will predict a similar route (emphasising the gross OCR), with an unknown reduction, (or increase), in accuracy. Large variations in route are only found in low to medium sea states, when there exist several routes with similar terminal policies. Improved ship and motion algorithms should predict better OCRs, within the limits of the environmental accuracy.

Prediction of motions/criteria, without swell factors could seriously affect results, especially, since this will strongly influence the rolling motion of the vessel.
7. THE MINIMUM FUEL OPTIMISATION MODEL AND ROUTEING STUDIES.

7.1. INTRODUCTION.

Minimisation of vessel transit fuel consumption is performed by combinations of the ship performance algorithm, environmental algorithm and optimisation algorithm.

The dynamic programming routine has been adapted to formulate the minimisation of fuel over the journey. The routeing strategy is:

1. Set both arrival and departure times, which are fixed boundary conditions. Compute a fixed engine revolution, or engine power that will produce a route that satisfies the boundary conditions and the minimisation of fuel.

Routeing studies have been performed to investigate the effects of the ship algorithm complexity.

7.2. FUEL OPTIMISATION MODEL.

With reference to equation 5.18.1, the optimal policy becomes:

\[ C = \text{fuel} = \frac{\delta x}{V_{\text{est}}} \cdot \text{SFC} \cdot P_s \cdot 1 \times 10^{-6} \text{ tonnes}. \]  

7.2.1

Where  

- SFC - Specific fuel consumption, see equation 3.20.1  
- \( P_s \) - Engine power, related to engine speed through equation 3.20.2.

Since engine speed is required to be pre-set and not variable, the minimum fuel route, (MFR), can be computed from the minimum time route, (MTR). Therefore, once the route that satisfies the boundary conditions has been deduced, from an interpretation of a minimum time route, the fuel consumption is deduced.
The recursive algorithm, equation 5.5.8(a) - 5.5.8.(b) can be written for forwards computation as in equations 6.2.2.(a) to 6.2.2.(e). No fuel consumption is considered in ports or terminal states, unlike the full costing function, chapter 8.

The state transfer function is deduced from an interpolation of the local environmental data, and the ship speed algorithm. In a similar method to minimum time, the sea state is derived from 6.2.3 as described in section 4.14.

It would be possible to include the adaptation of De Wit (1989) to adjust the estimated vessel speed, as in section 6.2, however, this was not included, as it was felt that the trajectory interval was sufficient to assume a constant environmental data value.

Figure 7.1 gives an indication of the computational flow of the algorithm to compute minimum fuel consumption. It is envisaged that the LDR be computed prior to the OCR since the destination LDR policy can be used to further restrict the feasible positional state space. Since the terminal boundary times are preset, the environmental data can be 'loaded' prior to any computation.

As stated, the minimum time route is used to deduce the minimum fuel route since they are directly related. Similarly, it is assumed that the minimum time route will utilise lower sea state areas, and can therefore be used as the basis for the minimum fuel route. The algorithm proceeds as :-

1. Given a set engine speed, compute the minimum time route, MTR.
2. Using the MTR, compute the engine revolutions that match both terminal times and deduce the fuel policy. Note MTR = MFR, (Minimum Fuel Route).
3. Since it is possible that there may be a more optimal route using the newly defined engine speed, recompute the MTR. Should a
vast difference occur between the two routes, then return to point 2.

Upon completion of the MTR, the function \( F_n \) is defined.

\[
F_n = t_{\text{dest}} - t_{\text{sdest}} \tag{7.3.1}
\]

Where:
- \( t_{\text{dest}} \) - Arrival time at destination given engine setting, \( N \).
- \( t_{\text{sdest}} \) - Scheduled arrival time.

Therefore it is possible to construct a Newton-like iteration scheme to minimise 7.3.1 by varying the engine speed setting. That is, a search is made until values of \( F_n \) are found which surround the solution that minimises 7.3.1

\[
f = \frac{F_n - F_{n-1}}{N - (N-1)} \tag{7.3.2}
\]

Where \( N \) - Engine revolutions per minute.

The new engine setting is given by:

\[
N + 1 = N - \frac{F_n}{f} \tag{7.3.3}
\]

Iteration continues until \( 7.3.1 \leq 1 \text{ hr} \), De Wit (1989). When the value of \( N \) that satisfies 7.3.1 is found, the MTR is recomputed, and the engine revolutions reiterated if there is a significant change in the route. This is simply performed by checking the positional state indices at each stage. The limits that are taken are if any positional state difference exceeds 3 at 45nm spacing. In some circumstances as seen in the following studies, the iteration procedure cannot converge on the demanded ETA. The routeing model selects the engine revolutions that minimises the terminal time difference. Therefore, some results will be slightly in error. This occurs when the routeing model is restricted by motion criteria, especially, since the motion database is particularly sensitive, as discussed.
FIGURE 7.1: MINIMUM FUEL COMPUTATIONAL ALGORITHM.
There are several problems that befall this algorithm, chiefly concerned with the non-convergence of 7.3.1, which results from large depressional activity, usually close to the terminal states. The inclusion of the heaving routine, section 5.11, and stage recomputation generally overcomes this problem, however, in extreme circumstances, only changes in the boundary times will provide a solution. This is realistic, since a master would delay departure, or arrival in the event of extreme local weather. The program deals with this scenario automatically, should no other solution be available.

Similarly, the iteration of engine revolutions along the MTR may require several stage re-computations, increasing the motion criteria limits. Upon re-iteration of the MTR, a route may be found that gives a slightly higher fuel consumption but utilises a route with lower motion limits. There are therefore many difficulties in an expanding control problem.

7.3. MINIMUM FUEL STUDIES.
The route between Le Havre and Cristobal defined in chapter 6 has been used to compute the MFR by the Babbage (1975) and the semi empirical/theoretical models. One should consider figures 6.2 which indicate the surface pressure, (depression activity), to visualise the systems that cause the following routes. Routes have been computed both with and without motion criteria including an observation on the effect of increasing the average roll criterion, from 15 to 30 exceedances. Throughout, in the figures; LEAST COST referes to the MFR, GREAT CIRC refers to the LDR, and ADVISED RT is that given by OCEANROUTES if provided.

7.3.1 ROUTES WITH THE BABBEDGE ALGORITHM.
(1) Without motion criteria.
The following sequence of routes shows the simulation of a real-time crossing, by re-computing the MFR on each occasion a new set of environmental data is received. These routes have been computed without motion criteria.
--- OPTIMAL PATH --- Mercator Projection

--- KEY ---
LEAST COST ---- ----
GREAT CIRC ---- ----
ADVISED RT ---- ----

- REVIEW -

DART ATLANTIC
DEP TIME : 15/04/89 0000 HRS
DEPARTRE : 49.7 N 2.4 W
ARRIVAL : 25.8 N 80.0 W
DISPLCMNT : 36458.0m
SPD RNGE : 16.1 - 23.0 Knts
MX EN WVE: 7.0m

ROUTE DIST TIME AvSPEED COSTS

GCR 3778.5 193.7 19.5 72969.5
OCR 3823.5 193.2 19.8 72758.1
SAVINGS: 0.56 211.4

POSITION : 49.7N 2.4W
HEADING : 263.9
ETA : 23/04/89 0100 HRS
DIST END : 3823.5 Nm

FIGURE 7.2. (a). MFR COMPUTED WITH THE BABBEDGE (1975) ALGORITHM.

--- OPTIMAL PATH --- Mercator Projection

--- KEY ---
LEAST COST ---- ----
GREAT CIRC ---- ----
ADVISED RT ---- ----

- REVIEW -

DART ATLANTIC
DEP TIME : 15/04/89 0000 HRS
DEPARTRE : 49.6 N 14.4 W
ARRIVAL : 25.8 N 80.0 W
DISPLCMNT : 36458.0m
SPD RNGE : 16.0 - 23.0 Knts
MX EN WVE: 7.0m

ROUTE DIST TIME AvSPEED COSTS

GCR 3304.6 168.7 19.6 63161.8
OCR 3319.9 168.6 19.7 63116.4
SAVINGS: 0.12 45.4

POSITION : 49.6N 14.4W
HEADING : 268.0
ETA : 23/04/89 0030 HRS
DIST END : 3319.9 Nm

FIGURE 7.2. (b). MFR +24 HOURS FROM ROUTE IN (a).

FUEL COST: $ 95.0/tonne FUEL CONS : 765.9tonnes BUNKER : 1832.1tonnes RPM:108.5
SFC : 247.6g/Kwh

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS : 664.4tonnes BUNKER : 1933.6tonnes RPM:108.2
SFC : 247.6g/Kwh

- 235 -
### Figure 7.2 (c). MFR +48 Hours From Route In (a).

<table>
<thead>
<tr>
<th></th>
<th>GCR</th>
<th>OCR</th>
<th>SAVINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ROUTE DIST</strong></td>
<td>2862.8</td>
<td>2878.3</td>
<td>0.71</td>
</tr>
<tr>
<td><strong>TIME</strong></td>
<td>146.0</td>
<td>145.3</td>
<td></td>
</tr>
<tr>
<td><strong>AVSPEED</strong></td>
<td>19.6</td>
<td>19.8</td>
<td></td>
</tr>
<tr>
<td><strong>COSTS</strong></td>
<td>53121.1</td>
<td>52862.4</td>
<td></td>
</tr>
<tr>
<td><strong>POSITION</strong></td>
<td>49.2N 25.7W</td>
<td>48.1N 37.2W</td>
<td></td>
</tr>
<tr>
<td><strong>HEADING</strong></td>
<td>259.2</td>
<td>250.2</td>
<td></td>
</tr>
<tr>
<td><strong>ETA</strong></td>
<td>23/04/89 0110</td>
<td>23/04/89 0120</td>
<td></td>
</tr>
<tr>
<td><strong>DIST END</strong></td>
<td>2878.3 Nm</td>
<td>2410.0 Nm</td>
<td></td>
</tr>
<tr>
<td><strong>FUEL</strong></td>
<td>$95.0/tonne</td>
<td>556.4 tonnes</td>
<td>BUNKER : 2041.6 tonnes RPM: 107.2</td>
</tr>
<tr>
<td><strong>SFC</strong></td>
<td>247.6 g/Kwh</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 7.2 (d). MFR +72 Hours From Route In (a).

<table>
<thead>
<tr>
<th></th>
<th>GCR</th>
<th>OCR</th>
<th>SAVINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ROUTE DIST</strong></td>
<td>2405.7</td>
<td>2410.0</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>TIME</strong></td>
<td>121.3</td>
<td>121.3</td>
<td></td>
</tr>
<tr>
<td><strong>AVSPEED</strong></td>
<td>19.6</td>
<td>19.9</td>
<td></td>
</tr>
<tr>
<td><strong>COSTS</strong></td>
<td>43225.9</td>
<td>43225.9</td>
<td></td>
</tr>
<tr>
<td><strong>POSITION</strong></td>
<td>48.1N 37.2W</td>
<td>48.1N 37.2W</td>
<td></td>
</tr>
<tr>
<td><strong>HEADING</strong></td>
<td>250.2</td>
<td>250.2</td>
<td></td>
</tr>
<tr>
<td><strong>ETA</strong></td>
<td>23/04/89 0120</td>
<td>23/04/89 0120</td>
<td></td>
</tr>
<tr>
<td><strong>DIST END</strong></td>
<td>2410.0 Nm</td>
<td>2410.0 Nm</td>
<td></td>
</tr>
<tr>
<td><strong>FUEL</strong></td>
<td>$95.0/tonne</td>
<td>455.0 tonnes</td>
<td>BUNKER : 2143.0 tonnes RPM: 106.5</td>
</tr>
<tr>
<td><strong>SFC</strong></td>
<td>247.7 g/Kwh</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DART ATLANTIC
DEP TIME: 19/04/89 0000 HRS
DEPARTRE: 45.1 N 47.9 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMT: 36458.0m*3
SPD RNGE: 16.6 - 23.0 Knts
MX EN WVE: 7.0m
ROUTE DIST TIME AVSPEED COSTS
GCR 1930.0 97.2 19.9 35086.0
OCR 1932.9 97.2 19.9 35086.0
SAVINGS: 0.00
POSITION: 45.1N 47.9W
HEADING: 243.0
ETA: 23/04/89 0110 HRS
DIST END: 1932.9 Nm

FUEL COST: $ 95.0/tonne FUEL CONS: 369.3tonnes BUNKER: 2228.7tonnes RPM: 106.9
SFC: 247.7g/Kwh

**Figure 7.3. VARIATION IN ENGINE SETTING FOR THE ROUTES IN FIGURES 7.2.**

--- OPTIMAL PATH --- Mercator Projection

**- KEY -
LEAST COST --- --- ---
GREAT CIRC --- --- ---
ADVISRED RT --- --- ---
**

**- REVIEW -
DART ATLANTIC
DEP TIME: 19/04/89 0000 HRS
DEPARTRE: 45.1 N 47.9 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMT: 36458.0m*3
SPD RNGE: 16.6 - 23.0 Knts
MX EN WVE: 7.0m
ROUTE DIST TIME AVSPEED COSTS
GCR 1930.0 97.2 19.9 35086.0
OCR 1932.9 97.2 19.9 35086.0
SAVINGS: 0.00
POSITION: 45.1N 47.9W
HEADING: 243.0
ETA: 23/04/89 0110 HRS
DIST END: 1932.9 Nm

**Figure 7.2. (e). MFR +96 HOURS FROM ROUTE IN (a).**

Figure 7.3 indicates the variation in engine setting as the route progresses.

**Figure 7.3. VARIATION IN ENGINE SETTING FOR THE ROUTES IN FIGURES 7.2.**
As with the MTRs shown in section 6.3.3, the influence of the initial and central depressions causes the MFR to go to the north of the LDR. Since the arrival time of the MFR was set at a value close to those deduced from the MTRs in section 6.3.3, the engine settings are similar. A slight increase in revolutions is necessary to counteract the increased distance around the developing central depression, but more importantly to move closer to the operating point of the SFC curve, see figure 3.24. Higher revolutions are necessary to counteract the higher sea states on the LDR. The arrival time of the MTR was shown to decrease by approximately 1.5 to 2 hours, figure 6.11, therefore, in order that the vessel arrive at the scheduled time, a reduction in engine revolutions is necessary, figure 7.3. This is due to the calmer waters on the western seaboard, than those predicted.

Table 7.1 indicates the state indices of the MFR and the MTR for the initial route. It is evident that both these routes satisfy the destination times, MTR = 194.3hrs and MFR = 193.15hrs. The MTR was set at 106rpm and the MFR was calculated at 108.47rpm, a difference of 2.47rpm, but an increase in fuel consumption. It would seem that the ± 1 hour limit is too wide, and the algorithm requires further research. This increase in rpm allows the vessel to stray closer to the LDR initially, where greater speed reduction occurs. As the route progresses, and the sea states subside, and the MFR becomes the LDR. With reference to figures 6.12.(a)-(e), 7.2.(a)-(e) and 7.3, slight route variations will be seen, similarly, as the MTR begins to arrive earlier than 193.5 hours, the MFR reduces the revolutions, in order that the destination time is maintained.

| STAGE | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| MTR   | 26| 27| 27| 28| 29| 30| 30| 30| 30| 30| 30| 30| 30| 29| 28| 27| 26| 26|    |
| MFR   | 26| 25| 26| 27| 28| 29| 30| 30| 30| 30| 30| 30| 30| 29| 28| 27| 26| 26|    |

The state indices are numbered 1, 2, .... 50,51 and refer to the positional state on the previous stage which forms the OCR. Number 26 = GCR

**TABLE 7.1 VARIATION OF THE INITIAL MTR AND MFR COMPUTED FROM THE BABBEDGE (1975) ALGORITHM, WITHOUT MOTION CRITERIA.**

- 238 -
(ii) With motion criteria.

--- OPTIMAL PATH -- Mercator Projection ---

<table>
<thead>
<tr>
<th>LEAST COST</th>
<th>GREAT CIRC</th>
<th>ADVISED RT</th>
</tr>
</thead>
</table>

--- REVIEW ---

DART ATLANTIC

DEP TIME: 15/04/89 0000 HRS
DEPARTUE: 49.7 N 2.4 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMT: 36458.0m
MAX EN WVE: 7.0m
SPD RNGE: 16.1 - 23.0 Knts

ROUTE DIST TIME: 15/04/89 0000 HRS
GCR 3778.5 188.3 20.1 76385.9
OCR 3882.0 190.4 20.4 77232.7
Savings: -2.09

POSITION: 49.7N 2.4W
HEADING: 263.9
ETA: 22/04/89 2220 HRS
DIST END: 3882.0 Nm

FUEL COST: $95.0/tonne FUEL CONS: 813.0 tonnes BUNKER: 1785.0 tonnes RPM: 111.2
SFC: 247.7g/Kwh

FIGURE 7.4. MFR COMPUTED WITH THE BABBIDGE (1975) ALGORITHM AND MOTION CRITERIA.

FIGURE 7.5. VARIATION OF ENGINE SETTING FOR THE MFR INITIATED IN FIGURE 7.4.
Figure 7.4 indicates the initial MFR computed with the Babbedge (1975) algorithm with motion criteria. The variation of engine setting as the route progresses is shown in figure 7.5. Table 7.2 indicates the variation of the routes computed with and without motion criteria.

<table>
<thead>
<tr>
<th>MFR at time</th>
<th>approximate Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>20°</td>
</tr>
<tr>
<td>+0 Hrs</td>
<td></td>
</tr>
<tr>
<td>50.25 @ 12.74</td>
<td>51.24 @ 23.47</td>
</tr>
<tr>
<td>3471.09</td>
<td>3050.28</td>
</tr>
<tr>
<td>50.88 @ 12.77</td>
<td>51.98 @ 23.67</td>
</tr>
<tr>
<td>3406.53</td>
<td>2991.99</td>
</tr>
<tr>
<td>+12 Hrs</td>
<td></td>
</tr>
<tr>
<td>56.15 @ 19.92</td>
<td>52.51 @ 41.84</td>
</tr>
<tr>
<td>3221.81</td>
<td>2393.48</td>
</tr>
<tr>
<td>50.82 @ 18.73</td>
<td>48.00 @ 38.83</td>
</tr>
<tr>
<td>3148.41</td>
<td>2345.30</td>
</tr>
<tr>
<td>+24 Hrs</td>
<td></td>
</tr>
<tr>
<td>54.45 @ 24.06</td>
<td>51.64 @ 39.98</td>
</tr>
<tr>
<td>3030.95</td>
<td>2419.45</td>
</tr>
<tr>
<td>50.75 @ 19.63</td>
<td>47.79 @ 39.63</td>
</tr>
<tr>
<td>3114.02</td>
<td>2310.92</td>
</tr>
<tr>
<td>+36 Hrs</td>
<td></td>
</tr>
<tr>
<td>55.20 @ 24.06</td>
<td>52.59 @ 40.61</td>
</tr>
<tr>
<td>3020.64</td>
<td>2410.08</td>
</tr>
<tr>
<td>51.75 @ 25.77</td>
<td>49.64 @ 41.36</td>
</tr>
<tr>
<td>2908.90</td>
<td>2090.63</td>
</tr>
</tbody>
</table>

Key: MFR+motion criteria – Small, Bold text.
MFR−motion criteria – Small, Standard text.

TABLE 7.2. VARIATION OF BABBEDGE MFRs WITH/WITHOUT MOTION CRITERIA.

Within the algorithm, should no convergence be made, the rpm with the closest arrival time to that required is used. This was discovered with the initial route and with motion criteria, see figure 7.4. Large motion increases, yield a route not dissimilar to that without motion criteria. Table 7.2 indicates the strong influence of the motion criteria. Routes incline to the north, and consequently an increased engine setting is necessary to offset the increased distances, see figure 7.5. Routes to the south of the LDR are restricted by motions since the sea states are majority head sea cases. It is expected that longitudinal type motions are exceeded on these legs. Also, the model has chosen a route that minimises all motion criteria or exposure, therefore, it is not uncommon for strange results to emerge. There are clearly, either errors in the computation of motions and their quantities used for the motion database, or inaccurate motion criteria limits.
It has already been stated that roll, computed from BRITSEA, is inaccurate since the damping values were computed only from the input ship data. Discussions with Dr. Deakins, at Polytechnic South West, England, in conjunction with his thesis, Deakins (1988), suggest reasonable accuracy for pitch, heave and yaw, and reduced accuracy for other motions. Similarly, the PM spectra, and non-inclusion of swell all add to inaccurate results. The routeing algorithms clearly operate in a manner which concentrates on a route that minimises these criteria.

Figure 7.6 indicates the effect of increasing the average roll criterion by 50%, and figure 7.7 indicates the same route with average roll criteria increased by 100%. The vessel is permitted into areas which would otherwise cause state trajectory omission. Should stage re-computation still be necessary, only a small increment will yield connections as compared to computation with lower limits.

--- OPTIMAL PATH --- Mercator Projection

<table>
<thead>
<tr>
<th>LEAST COST</th>
<th>GREAT CIRC</th>
<th>ADVISED RT</th>
</tr>
</thead>
</table>

**DART ATLANTIC**

**DEP TIME**: 15/04/89 0000 HRS
DEPARTRE: 49.7 N 2.4 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMNT: 36458.0m³
SPD Rnge: 16.1 - 23.0 Knts
MX EN WVE: 7.0m

**ROUTE DIST TIME AVGSPED COSTS**

<table>
<thead>
<tr>
<th>GCR</th>
<th>OCR</th>
<th>SAVINGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1778.5</td>
<td>193.6</td>
<td>19.5</td>
</tr>
<tr>
<td>3882.8</td>
<td>196.2</td>
<td>19.8</td>
</tr>
<tr>
<td>-2.60</td>
<td>-1029.7</td>
<td></td>
</tr>
</tbody>
</table>

**POSITION**: 49.7 N 2.4 W
HEADING: 263.9
ETA: 23/04/89 0410 HRS
DIST END: 3882.8 Nm

**FUEL**

FUEL COST: $ 95.0/tonne
FUEL CONS: 817.8 tonnes
BUNKER: 1780.2 tonnes
RPM: 110.3
SFC: 247.6g/Kwh

**FIGURE 7.6 MFR WITH AVERAGE ROLL CRITERION INCREASED BY 50%**

- 241 -
There is a vast change in the MFR from figure 7.4, which indicates the route computed with the non-dimensional criteria at 1, with 0.5 increments, per stage. The latter route indicates that computed with the non-dimensional criteria at 1.5 and 2, from the outset. At early stages, where the states are restricted and high sea states are experienced, a northerly component may only have been found, with low limits, obscuring the southerly route altogether. However, increasing the limits re-introduces the southerly components. This is a major drawback, since the vessel may only experience a single high exceedance on this southerly route, but experience medium exceedances on the northerly route. This poses the question, 'which is the more optimal?' The routeing algorithm will at present opt for the northerly route.
The northerly MFR, aims for a route still with the lowest criteria exceedance, however, a blanket increase of the average roll criterion allows the vessel to go under the central depression, whilst not exceeding other criteria.

7.3.2 ROUTES WITH THE SEMI EMPIRICAL/THEORETICAL ALGORITHM

Because of the speed/power differences between the Babbedge (1975) and semi empirical/theoretical algorithms, described in section 6.3.4, there are differences in the MFRs. The variation in engine revolutions, especially in high sea states to maintain the same speeds, means a variation in the routes especially when linked with motions, since the sustainable speed at a certain speed may be just above or below the motion criteria limit.

(i) Without motion criteria.

Figures, 7.8.(a)-(e) indicate the simulated weather routeing operation to determine the MFR with the semi-empirical algorithm, and without motion criteria limits.

![Map of MFR Computed with the Semi-empirical/ Theoretical Model and No Motion Criteria](image-url)

--- OPTIMAL PATH --- Mercator Projection

<table>
<thead>
<tr>
<th>KEY</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEAST COST</td>
</tr>
</tbody>
</table>

| /
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>DEP TIME</td>
</tr>
<tr>
<td>DEPARTRE</td>
</tr>
<tr>
<td>ARRIVAL</td>
</tr>
<tr>
<td>DISPLCMNT</td>
</tr>
<tr>
<td>SPD RANGE</td>
</tr>
<tr>
<td>MX EN WVE</td>
</tr>
<tr>
<td>ROUTE DIST</td>
</tr>
<tr>
<td>TIME</td>
</tr>
<tr>
<td>SAVINGS</td>
</tr>
<tr>
<td>POSITION</td>
</tr>
<tr>
<td>HEADING</td>
</tr>
<tr>
<td>ETA</td>
</tr>
<tr>
<td>DIST END</td>
</tr>
</tbody>
</table>

FIGURE 7.8.(a). MFR COMPUTED WITH THE SEMI EMPIRICAL/ THEORETICAL MODEL AND NO MOTION CRITERIA.

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS: 879.3tonnes BUNKER: 1718.7tonnes RPM: 113.5

SFC: 247.9g/Kwh

- 243 -
--- OPTIMAL PATH --- Mercator Projection

- KEY -

LEAST COST --- --- ---
GREAT CIRC --- --- ---
ADVISED RT --- --- ---

- REVIEW -

DEP TIME: 16/04/89 0000 HRS
DEPARTRE: 49.5 N 14.5 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMNT: 36458.0m²
SPD RNGE: 16.0 - 25.0 Knts
MX EN WVE: 7.0m
ROUTE DIST TIME AvSPEED COSTS
GCR 3301.1 169.2 19.5 73572.5
OCR 3316.5 169.2 19.6 73534.2
SAVINGS: 0.09 38.3
POSITION: 49.5N 14.5W
HEADING: 268.1
ETA: 23/04/89 0100 HRS
DIST END: 3316.5 Nm

- FUEL -

FUEL COST: $ 95.0/tonne FUEL CONS: 774.0 tonnes BUNKER: 1824.0 tonnes RPM: 113.7
SFC: 247.9 g/Kwh

FIGURE 7.8. (b). MFR +24 HOURS FROM ROUTE IN (a).

--- OPTIMAL PATH --- Mercator Projection

- KEY -

LEAST COST --- --- ---
GREAT CIRC --- --- ---
ADVISED RT --- --- ---

- REVIEW -

DEP TIME: 17/04/89 0000 HRS
DEPARTRE: 49.0 N 26.1 W
ARRIVAL: 25.8 N 80.0 W
DISPLCMNT: 36458.0m²
SPD RNGE: 16.1 - 25.0 Knts
MX EN WVE: 7.0m
ROUTE DIST TIME AvSPEED COSTS
GCR 2845.2 145.3 19.6 63182.0
OCR 2851.5 145.3 19.6 63182.0
SAVINGS: 0.00 0.0
POSITION: 49.0N 26.1W
HEADING: 259.0
ETA: 23/04/89 0110 HRS
DIST END: 2851.5 Nm

- FUEL -

FUEL COST: $ 95.0/tonne FUEL CONS: 665.1 tonnes BUNKER: 1932.9 tonnes RPM: 113.7
SFC: 247.9 g/Kwh

FIGURE 7.8. (c). MFR +48 HOURS FROM ROUTE IN (a).
Figure 7.9 indicates the variation of engine setting as the overall MFR progresses.
These routes differ from those computed with the Babbedge (1975) algorithm. By referring to the speed differences at the same engine setting, it can be appreciated that the arrival time of the MTR with the semi-empirical/theoretical algorithm is later. Therefore, in order to maintain the required destination time, an increase in the engine rpm is required, giving the vessel greater speed, in low/medium sea states, (calm water powering is only 1% in error to the Babbedge (1975) algorithm), the vessel is able to bypass the central depression before it has fully developed. The MFR route is therefore similar to the LDR, and a virtually constant engine setting is seen. Also, as there is little variation from the LDR, the MFR has to increase the revolutions as the developing depression is encountered.

(iii) With motion criteria.

Figure, 7.10 indicates the initial MFR determined with the semi-empirical algorithm and motion criteria limits. Table 7.3 indicates the variation of the computed MFRs
with and without motion criteria as the ship’s journey progresses.

FIGURE 7.10. MFR COMPUTED WITH THE SEMI-EMPIRICAL/THEORETICAL ALGORITHM AND MOTION CRITERIA.

<table>
<thead>
<tr>
<th>MFR at time</th>
<th>10°</th>
<th>20°</th>
<th>40°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0 Hrs</td>
<td>50.25 @ 23.47</td>
<td>51.24 @ 23.47</td>
<td>49.65 @ 39.87</td>
<td>41.49 @ 59.91</td>
</tr>
<tr>
<td></td>
<td>3480.99</td>
<td>3060.18</td>
<td>2432.34</td>
<td>1427.44</td>
</tr>
<tr>
<td></td>
<td>49.38 @ 12.71</td>
<td>49.02 @ 22.91</td>
<td>45.72 @ 42.05</td>
<td>36.56 @ 60.71</td>
</tr>
<tr>
<td></td>
<td>3403.64</td>
<td>3002.11</td>
<td>2198.95</td>
<td>1188.32</td>
</tr>
<tr>
<td>+12 Hrs</td>
<td>55.42 @ 19.98</td>
<td>3216.82</td>
<td>50.89 @ 42.02</td>
<td>45.17 @ 58.75</td>
</tr>
<tr>
<td></td>
<td>49.11 @ 23.87</td>
<td>2973.78</td>
<td>45.00 @ 42.56</td>
<td>35.43 @ 61.13</td>
</tr>
<tr>
<td></td>
<td>3216.82</td>
<td>2973.78</td>
<td>2165.56</td>
<td>1155.13</td>
</tr>
<tr>
<td>+24 Hrs</td>
<td>54.47 @ 24.07</td>
<td>3008.72</td>
<td>52.36 @ 39.54</td>
<td>41.45 @ 59.53</td>
</tr>
<tr>
<td></td>
<td>49.37 @ 19.61</td>
<td>3111.04</td>
<td>46.73 @ 39.23</td>
<td>39.40 @ 59.82</td>
</tr>
<tr>
<td></td>
<td>3008.72</td>
<td>3111.04</td>
<td>2305.03</td>
<td>1302.36</td>
</tr>
<tr>
<td>+36 Hrs</td>
<td>52.52 @ 40.70</td>
<td>2405.87</td>
<td>52.52 @ 40.70</td>
<td>41.72 @ 59.68</td>
</tr>
<tr>
<td></td>
<td>47.21 @ 40.07</td>
<td>2285.73</td>
<td>39.76 @ 60.72</td>
<td>1283.24</td>
</tr>
</tbody>
</table>

MFR +motion criteria - Small, Bold text. MFR-motion criteria - Small, Standard text.

N lat @ W long
Nm to go

TABLE 7.3. VARIATION OF THE SEMI-EMPIRICAL/THEORETICAL MFRS WITH AND WITHOUT MOTIONS AS THE VESSEL'S JOURNEY PROGRESSES.
The effect of motion criteria is strongly felt by the MFR. The routeing model requires a route with minimal motion criteria exceedance, but still maintaining the destination time. Therefore, a more northerly route is chosen to offset the criteria, and a higher engine setting required for the extra distance involved. As the vessel's speed increases, more criteria are exceeded, and so the route becomes even more northerly, towards lower sea states. Likewise, the vessel's speed has to be increased. The increase in engine revolutions and shift in the MFR from those computed without the motions, is of similar proportions to the MFRs computed with the Babbedge (1975) algorithm.

**Figure 7.11. Variation of engine setting for the semi-empirical/theoretical algorithm as the route progresses with motion criteria.**

Figures 7.3 and 7.5, for the Babbedge (1975) algorithm and figures 7.9 and 7.11, for the semi-empirical/theoretical algorithm, show the large effect of motion criteria. Increased revolutions offset the increased distance of routeing far to the north, for each algorithm. The relative changes in fuel savings are similar, becoming positive after the initial high departure from the LDR. Differences in
route and engine setting culminate from the speed loss at that engine setting.
Since the MFR is based on MTRs, the main objective is maintaining certain speeds.
Small differences between the algorithms, in speed loss, and deduced motion criteria limits will affect the choice of route and consequently, the engine setting.

Figures 7.6, 7.7, 7.12 and 7.13, indicate the routes with increased average roll.
Since the ship performance algorithms are constrained by the terminal times, speeds are relatively fixed. Therefore, both algorithms are subject to similar motion criteria values, with regard to their speed differences. The difference between the performance algorithms becomes more evident in the difference between the engine settings, and the routes become similar. Unfortunately, as figure 7.12 indicates, the model was unable to converge on a satisfactory terminal time. Since the vessel arrives early, the engine setting is high.
The effect of motion criteria is such as to allow the vessel to enter more and more severe seaways. This is indicated in the movement of the MFR towards the LDR where the depressions are located.

7.4. CHAPTER CONCLUSIONS.

The following conclusions, further to those mentioned in section 6.3.3, and 6.3.4 can be inferred from the varying performance algorithms with and without motion criteria.

1. The effect of motion criteria has an even more marked effect upon the MFR than the MTR, since the vessel's speed becomes a function of the extra distance involved to offset the excessive motions on shorter distance routes.

2. The MFR becomes a balance between the vessel's speed and the motion criteria.
3. The prediction of optimal routes is dependant upon the accuracy of the performance algorithm and the motion database. Unlike the MTR, where the route may stay the same but the ETA may vary, the route or engine setting may vary since the ETA remains constant.

4. In lower sea state areas an increase in accuracy could well vary the MFR or engine setting considerably. In higher sea states the MFR, has to avoid such areas, since the direct route becomes sub-optimal, (increased revolutions).

5. Since the terminal times are fixed, all models are forced to achieve the same speeds, and errors become apparent in the engine setting. However, since relative change of speed to sea state is similar, the routes do not vary significantly. Also the introduction of motion criteria will have the same effect upon the routes with different ship performance algorithms, since each is aiming for similar speeds.

6. It can be concluded that a simple model could be used to determine the MFR initially, and a more complex algorithm introduced to compute the fuel more accurately on a refined but constricted grid on the first MFR.

7. It appears that the ±1 hour terminal boundary condition and the limit taken as that which determines no route change on the MTRs are both too wide. This is evidenced by slight increases in fuel consumption over the same route depicted in chapter 6. Further research is therefore necessary although the routes are similar.
8. THE MINIMUM COST OPTIMISATION ALGORITHM AND ROUTEING STUDIES.

8.1. INTRODUCTION.
Minimisation of vessel transit cost is performed by combinations of the ship performance algorithm, environmental algorithm and optimisation algorithm.

The dynamic programming routine has been adapted to formulate the minimisation of cost over the journey. The routeing strategies is:

1. Set destination time, and departure time which is variable between the set value ± 24 hours. Either by (i) Average speed, and a specified departure time. (ii) Specified terminal times.

Furthermore there is a scheduled departure time, denoted $t_{sd}$: The positional state space is expanded to include the time states, as a function of the ship speeds, between the actual required average and a specified maximum. Compute the minimum cost route, so that the arrival time is maintained, under variable engine setting, with the prerequisite that arrival times in positional states are maintained.

Several routeing studies have been performed to investigate the effect of ship algorithm complexity, the influence of environmental data extension and engine powering. Furthermore, since the vessel operates between discrete positional and time states, it is possible to formulate the iterative routines in reverse order.

8.2. MINIMUM COST MODEL.

8.2.1. COST FUNCTIONS.

With reference to equation 5.18.1, the optimal policy becomes:
\[
C_{\text{operating}} = \left( \delta t + t_{\exp} \right) \left( SFC.P_s.1 \times 10^{-6} F_{\text{main}} + C_{\text{crew}} + C_{\text{ins}} + C_{\text{cap}} + C_{\text{wve}} \right)
\]

Where

\(C_{\text{operating}}\) - Operating cost of vessel whilst at sea.

\(\delta t\) - Trajectory time interval, which can also be \(\frac{\delta x}{V_{\text{est}}}\).

\(F_{\text{main}}\) - Main engine fuel cost.

Other parameters defined in 5.18.

In contrast, the departure port cost, \(C_{\text{depport}}\) is given by:

\[
C_{\text{depport}} = \gamma \cdot LDF \cdot \text{TEU}_{\text{ship}} + \gamma \cdot \text{TEU}_{\text{shore}} + \left( t_{\text{dep}} - t_{\text{adep}} \right) \left( C_{\text{crew}} + C_{\text{ins}} + C_{\text{cap}} + F_{\text{gen}} \right) + \left( t_{\text{dep}} - t_{\text{sdep}} \right) t + C_{\text{tow}} + C_{\text{berth}} + C_{\text{pilot}}
\]

Where

\(\text{TEU}_{\text{ship}}\) - Container capacity of ship.

\(\text{TEU}_{\text{shore}}\) - Container capacity to be loaded.

\(LDF\) - Loaded factor of ship to be offloaded.

\(\gamma\) - Cost of loading/discharging per container regardless of number of cranes.

\(C_{\text{tow}}\) - Cost to berth ship by tugs.

\(F_{\text{gen}}\) - Generator fuel cost.

\(C_{\text{pilot}}\) - Pilot cost.

\(C_{\text{berth}}\) - Berthing occupancy cost. (Possibly included in container cost)

Figure, 8.1, indicates the cost of departing from the destination, given the scheduled arrival at the port, and the scheduled departure. There is a minimum departure time which is that at which all containers have been unloaded and loaded. Similarly, the function only applies to a single 24 or 48 hour slot. Therefore, should the function be applied as a destination function, and should the arrival of the vessel be greater than the minimum departure time, the function moves a 24 or 48 hour slot. This is realistic, since the vessel would have to wait for a berth should it miss its scheduled time. The departure cost function, 8.2.2, is adapted for the destination, to depict the expected port cost to scheduled departure, so that maximum cost occurs before scheduled arrival.
The port cost function takes no account of stowage costs of shore containers, crew overtime, maintenance, light dues, or main engine fuel consumption costs. It is assumed that any cost incurred after the scheduled departure cost is subject to a penalty. Equations 5.5.8.(a)-(c) can be written for reverse computation as:

For the final stage,

$$ C^*(X, N_{stage}, t_{dest}) = 0 \quad C^*(X, N_{stage}, dest_{tp}) = 0 $$

8.2.3(a)

For a single, $(t_{dest})$, or variable destination time, $(t_{dest_{tp}})$. It is possible to assign a cost function to the terminal state. That is:

$$ C^*(\vec{X}, N_{stage}, t_{dest}) = C_{destport} \quad C^*(\vec{X}, N_{stage}, t_{dest_{tp}}) = C_{destport} $$

8.2.3(b)

For intermediate stages

$$ C^*(\vec{X}, k, t_r) = \text{Min} \left\{ (\alpha C_{operating} + \delta C_{operating}) + C^*(\vec{X}, k+1, t_{s}) \right\} $$

8.2.3(c)
The cost to a positional and time state at stage $k$, is a function of the cost at a positional state and time at stage $k+1$, and the operating cost to transit under the controls, (heading and required speed), to complete the trajectory. The operating cost is a function of time, and this can be taken as the trajectory interval, or a function of distance (trajectory interval) over the estimated speed.

**First stage.**

\[
C^*(\tilde{x}, 1, t_1) = \min \left\{ (\alpha C_{\text{operating}} + \delta C_{\text{operating}}) + C^*(\tilde{x}, 2, t_2) + C_{\text{depport}} \right\}
\]

A typical connection of two stage points can be viewed as:

![Figure 8.2: State Connections](image)

Clearly, many connections are not feasible, outside the vessel operating range. Within the algorithm, the vessel's required speed is checked, since speed may need to be increased if in the trajectory, the heaving routine was necessary. If a
stage becomes inaccessible under the constraints, further routines are initiated. Finally, should no connection be made, the algorithm resorts to full power, and estimated speeds from these, until the pre-defined t-states can be used again.

Any negative savings are attributable to large increments of motion criteria on the LDR, with no associated penalty values.

It is possible to compute the minimum transit time under varying revolutions by employing an open ended t-state envelope. However, many t-states are evidenced at the destination, requiring many state connections. Only minimum cost is shown, with a 'closed' t-state envelope. Equations 8.2.1 and 8.2.2, indicate the form of the cost functions used in this study. Only departure port cost is considered, since the arrival port becomes the departure port for the next journey. In order to assign realistic figures to the cost function, both capital and insurance charges were included, Moreby (1989).

8.2.2 PORT COSTS.

The port cost function is constructed of elements attributable to the ship for the berthing and unloading of cargo, plus additional costs for reloading. These are:

1. The cost for pilotage, and towing of the vessel, which are represented by, (Galbraith (1984), and increased 5% per annum for inflation, Moreby (1989), and for Le Havre):

   \[ C_{\text{pilot}} = 1600.0 \] \[ C_{\text{tow}} = 2200.0 \]

2. The berthing occupancy, and cargo handling costs. Berthing occupancy is regarded as a once payment included in cargo handling, Hughes (1989).

   \[ C_{\text{cargo}} = 2 \cdot \gamma \cdot \frac{\text{TEU}}{100} \cdot \text{LDF} \]
Where

\begin{align*}
\text{TEU} & - 1300, \text{container capacity, Aertssen (1972)} \\
\text{LDF} & - \text{Loaded factor (\%).}
\end{align*}

\gamma - $47.3 \text{ per container, (Southampton), Hughes (1989). Assume equal quantities of containers are unloaded and loaded. Since these ports are in competition.}


\begin{align*}
\text{Generator fuel cost/hr, } \varphi &= SFC_{\text{gen}} P_{\text{gen}} F_{\text{gen}} \\
&= \$21.875 \\
\text{Crew wages/hr, } C_{\text{crew}} &= \$133.0 \\
\text{Insurance/hr, } C_{\text{ins}} &= \$57.3 \\
\text{Capital/hr, } C_{\text{cap}} &= \$271.0 \\
F_{\text{gen}} &= \$175.0
\end{align*}

8.3.4 (a) - 8.3.4 (e)

4. In addition to operating port costs, a penalty value per hour is levied, which is regarded as lost revenue, ($500/hr).

8.2.3 OPERATING COSTS.

As defined in equation 8.2.1, and those values defined in 8.3.4.(a) - 8.3.4.(e), the operating cost becomes a function of time and the fuel consumption of the main engine. This is determined through equations 3.18.1, and 8.2.1, given the fuelcost, $F_{\text{main}}$, taken from Lloyd's List, ($95/\text{tonne}). The penalty function assigned to exposure to high sea states, $C_{\text{wve}}$, is arbitrary, since it is difficult to conclude any values from studies, due to the vast array of parameters that will result in damage. However, this is set high enough that the routeing algorithm will not include such areas in the minimum route. Figure 8.3 gives an indication of the
computational flow of the algorithm to compute the minimum cost route. It is envisaged that the LDR be computed prior to the MCR, since the terminal LDR policy forms further restrictions on the feasible state space of the MCR.

\[\text{Ship algorithm} \rightarrow \text{Motion limits} \rightarrow \text{Trajectory} = \sum \text{intervals} \rightarrow \text{Optimisation} \rightarrow \text{Sea state algorithm} \rightarrow \text{Wave/other limits} \rightarrow \text{Compute the LDR waypoints} \rightarrow \text{State trajectory policies} \rightarrow \text{Input data requirements} \rightarrow \text{Further OCR constraints} \rightarrow \text{Compute the OCR waypoints} \rightarrow \text{State trajectory policies} \rightarrow \text{Data requirements} \rightarrow \text{Oversee stage computation} \rightarrow \text{Re-initiate under extreme conditions} \rightarrow \text{Trace out OCR waypoints} \rightarrow \text{Graphical output} \rightarrow \text{END} \rightarrow \text{Main iteration} \]

\textbf{FIGURE 8.3. MINIMUM COST COMPUTATIONAL ALGORITHM.}

- 258 -
8.3 MINIMUM COST STUDIES

The route between Le Havre and Cristobal, has been used for the quasi-optimal routes obtained with the Babbedge (1975) and semi-empirical/theoretical ship performance algorithms. Routes are described with and without motion criteria. Note figures 6.2.(a)-(g), which depict the depressional activity during the transit time. Throughout, in the figures; LEAST COST refers to the MCR, GREAT CIRC refers to the LDR and ADVISED RT is that given by OCEANROUTES if provided.

8.3.1 ROUTES WITH THE BABBEDGE ALGORITHM

(i) Without motion criteria.

Figure 8.4 shows the initial minimum cost route, (MCR), computed between Le Havre and Cristobal.

--- OPTIMAL PATH --- Mercator Projection

- KEY -

LEAST COST --- --- ---
GREAT CIRC --- --- ---
ADVISED RT --- --- ---

--- REVIEW ---

DART ATLANTIC
DEP TIME : 15/04/89 0000 HRS
DEPARTRE : 49.7 N 2.4 W
ARRIVAL : 25.8 N 80.0 W
DISPLCMNT: 3458.0m*3
SPD RNGE : 15.6 - 21.5Knts
MX EN WVE: 7.0m

ROUTE DIST TIME AvSPEED COSTS
-------------------------
GCR 3778.4 203.5 18.6 317525.9
OCR 3813.2 203.5 18.7 313914.1
SAVINGS: 0.00 3611.8
-------------------------

POSITION : 49.7N 2.4W
HEADING : 289.4
ETA : 23/04/89 1130 HRS
DIST END : 3813.2nm

--- FUEL ---

FUEL COST: $ 95.0/tonne FUEL CONS : 740.6tonnes BUNKER : 1857.4tonnes RPM: 93.5
SFC : 250.9g/KWh

Figure 8.4. MCR WITH THE BABBEDGE ALGORITHM AND WITHOUT MOTION CRITERIA.
This route is similar to both the MTR and the MFR even though the arrival time was relaxed by ten hours, indicating the influence and sensitivity of the routeing models to the central and initial depressions.

Figure 8.5 shows the vessel to maintain a steady time gradient, on both the LDR and the MCR. In order to achieve these waypoint arrival times, the variations of power per stage are shown in figure 8.6. The power curve for the MCR indicates increased values initially, for the depression at the entrance to the English channel, evening off for the remainder of the route. The power curve for the LDR, on the other hand, indicates increased values for initial and central stages, corresponding to the two depressions. It is also seen that the cost saving becomes noticeable from these power increments. The scheduled departure time was set equal to the departure time, and so the algorithm aims to match this time, since penalty values are associated for later departures.
(ii) With motion criteria.

**FIGURE 8.6. INCREMENTAL COST AND POWER VARIATION ALONG THE LDR AND MCR FOR THE ROUTE IN FIGURE 8.4.**

**FIGURE 8.7. MCR WITH THE BABBEDGE ALGORITHM AND MOTION CRITERIA.**
Figure 8.7 indicates the initial MCR computed with the Babbedge (1975) performance algorithm and the motion criteria. Figure 8.8 indicates the t-state envelope, and figure 8.9 indicates the variation of engine setting and cost per stage. Average roll criteria was set with a 100% increment, after discussions with Dr. E. Deakins with regard to the accuracy of roll angle predictions. Upon re-calculation of the stage, motion limits are increased to a point, from which the vessel is operated at MCRt only, and the arrival time at the stage point is taken as the estimated speed only. The inclusion of this routine is only used as a last resort. (It is felt that the resolution of the model to one hour between t-states may not be fine enough to accommodate subtle speed changes to offset motions.).

The influence of motion criteria, since speeds are pre-set, causes a southerly route under the influence of the central depression, as trajectories to the north are omitted or are less optimal. Figure 8.10 indicates the variation of engine setting for the routes with and without motion criteria.

![Figure 8.8. T-state envelope for the MCR and LDR in Figure 8.7](image-url)
FIGURE 8.9. INCREMENTAL COST AND POWER VARIATION ALONG THE LDR AND MCR IN FIGURE 8.7.

FIGURE 8.10. VARIATION OF ENGINE RPM PER STAGE FOR THE BABBEDGE ALGORITHM WITH AND WITHOUT MOTIONS.
The terminal times for the MCRs were set close to those for the MFRs in chapter 7, it is therefore not surprising that the routes are very similar. However, the variation of engine setting computed from the cost model are different to the constant values for the MFRs. The variation in power is attributed to increments in power to maintain sea speed, whilst aiming for least fuel consumption.

8.3.2 ROUTES WITH THE SEMI-EMPIRICAL/THEORETICAL ALGORITHM.

(i) Without motion criteria.

The initial MCR was computed with and without motion criteria. Figure 8.11 indicates that route without motion criteria. The route is influenced by the depressions, causing it to deviate north of the LDR, in a similar fashion to the route with the Babbedge (1975) algorithm. The costs between the algorithms differ due to the difference in engine settings required to maintain the same speeds. However, the relative cost savings are of similar quantities, considering the size of the total voyage+port costs.
Figure 8.12 indicates the initial t-state envelope and incremental time per stage, whilst figure 8.13 shows the increment of cost and the engine setting.

**Figure 8.12. T-state Envelope for the LDR and MCR in Figure 8.11.**

**Figure 8.13. Incremental Cost and Power Variation along the LDR and MCR in Figure 8.11.**

The variation of engine setting, indicated in figures 8.13 and 8.16, are of a similar form to the MCR computed with the Babbage (1975) algorithm, see figures 8.6
and 8.10. The difference between the algorithms, like those between the MFRs, is indicated in the engine setting. However, the influence of the initial and central depressions can still be seen. Routes could be computed with a simpler algorithm, and the resulting MCR used as the basis for a small restricted grid, with a more complex ship performance algorithm.

(ii) With motion criteria.

When motions were included, although each ship performance algorithm should be influenced to the same degree, the change in cost values, (difference in engine revolutions), at intermediate stage points influences the route, since cost is the objective function. Similarly, since the vessel was operating at the top end of its range, the difference between the algorithms, may be restrictive on the semi-empirical/theoretical algorithm, (maximum required speeds give no power allowance for weather). However, it is shown that the routes with the semi empirical/theoretical algorithm and the Babbedge (1975) algorithm are both to the south of the LDR. Figure 8.14 shows the MCR computed with motion criteria.

--- OPTIMAL PATH --- Mercator Projection

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th>KEY</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEAST COST</td>
<td>GREAT CIRC</td>
<td>ADVISED RT</td>
<td>REVIEW</td>
<td></td>
</tr>
</tbody>
</table>

//\/\/\/\/\/\/\/\/\/\/\/\/\/\/\/\/\/\/\/

DEP TIME : 15/04/89 0000 HRS
DEPARTUE : 49.7 N 2.4 W
ARRIVAL : 25.6 N 80.0 W
DISPLCMT : 36458.0nm
SPD HIGH : 15.6 - 25.0kts
MV EN WOE: 7.0m
ROUTE DIST TIME AVSPEED COSTS
GCR 3778.4 203.5 18.6 318332.3
GCR 3854.8 203.5 18.9 327830.3
SAVINGS : 0.00 9498.0
POSITION : 49.7N 2.4W
HEADING : 264.0
ETA : 23/04/89 1130 HRS
DIST END : 3854.8nm

---------- FUEL ----------
FUEL COST: $ 95.0/tonne FUEL CONS : 833.5tonnes BUNKER : 1764.5tonnes RPM : 104.6
SFC : 247.9g/KWh

FIGURE 8.14, MCR COMPUTED WITH THE SEMI-EMPIRICAL/THEORETICAL ALGORITHM AND MOTION CRITERIA.
Figures 8.15, and 8.16 indicate the t-state envelope for the LDR and the increment of cost per stage with engine powering respectfully, for the route in figure 8.14. Since the MCR involves motions, the algorithm attempts to minimise their exceedance whilst searching for a minimal solution. Figures 8.15 and 8.16, show increases in power and cost to achieve the deviation to the south. This deviation is less than with the Babbedge (1975) algorithm due to the differences in the quantitative values of the objective function. That is, comparison of the objective functions will yield different routes, at intermediate points. Therefore, although the algorithm computes a southerly route for both ship performance algorithms, (lower motion criteria), to the south, the differences in costs between the two routes with the same ship performance algorithm describes the difference between these routes. Should cost values be associated with the increment of motion limits, as a penalty, then routes would differ again. Due to the difficulty in assigning realistic values, this was not undertaken.
Although the routes are different, it can still be argued that the simpler ship performance algorithms can still be utilised to formulate the gross features of a quasi-optimal route. The physical size of the state space of a finer grid system erected on the MCR computed with the Babbedge (1975) algorithm is unknown, although the results in this study suggest a relatively narrow band. This may vary depending upon ship performance sensitivity and complexity as well as the complexity of the routeing model.

![Power and Cost to Stages](attachment:image.png)

*FIGURE 8.16. INCREMENTAL COST AND POWER VARIATION ALONG THE LDR AND MCR IN FIGURE 8.14.*

By referring to figures 8.10 and 8.17, the differences in engine setting accountable to the introduction of motion criteria can be appreciated. It will be seen that engine settings increase on early stages as the vessel attempts to, both minimise the motions, and also, maintain speed to effect the southerly route. The setting remains high, even on the eastern seaboard, to make up the lost time for the initial deviation. Similarly, increments are seen at central stages where the developing depression is evident. Certain low engine settings, can be overcome by setting a lower engine limit.
8.4. CHAPTER CONCLUSIONS.

The initial MCR, for the semi-empirical/theoretical algorithm indicate a fuel consumption of 792.5 tonnes. This compares with the MFR, at constant engine revolutions, arriving ten hours earlier of 879.3 tonnes. Similarly, the initial MCR, for the Babbedge (1975) algorithm, indicates a fuel consumption of 740.6 tonnes, whilst the MFR shows 769.5 tonnes. It is difficult to say that the variation in engine setting enables a more minimal consumption, although it is shown that the algorithm does yield realistic routes comparable with MFRs and MTRs.

The following conclusions can be made further to those in chapters 6 and 7.

1. Simpler ship models incorporating powering can be used to formulate an MCR, since the routes are similar to those with more complex algorithms. The terminal times are fixed, in a closed time envelope and the ship performance algorithms are forced to achieve
the same speeds. Errors become evident in the engine setting, although the relative changes to the sea state remain the same.

2. The complexity of the algorithm means extended calculation times. For each positional state trajectory there is a maximum of 25 x 25 possible connections. Therefore, each MCR requires up to 625 times more computation than a single MTR. In this respect if the MTR takes one minute to complete its computation, then, (at a maximum), as a rough estimate the MCR will take 10.42 hours to complete. For an on-board PC, this is unacceptable, and means of reducing the computational burden are necessary.

3. Based on point 2, the use of a simple algorithm initially and a post computation with a more complex algorithm, is emphasised.

4. The routeing strategy shown in this study computes an MCR with variable engine power, although that variation is restricted, to those intermediate state points and the grid points themselves. In reality, the master is more likely to resort to constant engine settings, and the strategy employed in the MFR algorithm, could be simply amended for cost.

5. Routeing studies with motion criteria, like those in the MTRs and MFRs require accurate determination of motions and criteria limits for the particular ship. Since in these algorithms, they have a large effect upon the routes. With the case of a central depression, there is a possibility of a northerly or southerly route. If the policy values are similar on both, then motions may swing the balance from one to the other.
9. CONCLUSIONS.

9.1 CONCLUSIONS.

This study has concerned itself with the development of realistic working routeing models for containerships, to compute:

1. The minimum transit time route of the vessel between specified terminals under constant engine revolutions or constant vessel power;
2. The minimum fuel consumption route between specified terminals and scheduled departure and arrival times under constant engine revolutions or constant vessel power;
3. The minimum cost route between specified terminals and scheduled departure and arrival times under variable engine revolutions or variable vessel power, constrained within a time envelope, or rhombus.

Further to these solutions, the following strategies have been researched:

1. The minimum time route between specified terminals and a scheduled departure time, under variable engine revolutions or vessel power, constrained within an open-ended time envelope;
2. The minimum cost route between specified terminals and a scheduled departure time, under variable engine revolutions or vessel power, constrained within an open-ended time envelope.

Chapters 1 and 2 demonstrated the need for weather routeing and highlighted those features that are necessary for an on-board system. The overall definition of the global and the local routeing strategies and their integration was thus developed. From these, 'global' models were constructed, and those elements that
contribute to their construction were discussed. These were treated separately, and detailed studies of element components were undertaken in order that an efficient and accurate routeing model be facilitated. The routeing elements are:

1. An accurate ship performance algorithm, incorporating efficient and quick computational speed;
2. Environmental data, including the transmission, handling and generation of missing or extended data, necessary to a routeing exercise involving iteration;
3. A realistic routeing model for the operation of containerships, incorporating an efficient and accurate optimisation algorithm.

The study recognised several approaches in the construction of each element and converged on those aspects that were found to provide more tangible results.

Chapter 3 investigated four approaches to the modelling of a containership, the MV. DART ATLANTIC. The relative advantages and disadvantages of each algorithm were discussed, with particular reference to their computational ease, complexity and adaptability for differing routeing policies.

The construction of a computer program for a relatively sophisticated semi-empirical/theoretical algorithm based on hull and engine particulars of the MV. DART ATLANTIC was undertaken. Possible error sources were highlighted, and are:

1. Inaccurate hull definition in the generation of a trigger file for the seakeeping model. Insufficient definition of section weights and end sectional areas lead to problems at higher ship speeds;
2. The use of the Pierson-Moskowitz spectrum, for sea only. Differing spectra and the inclusion of swell were discussed, with possible solutions;
3. The generation of motion criteria databases based on motion limits for ships of differing hull shape;
4. Unquantifiable error sources in the seakeeping algorithm, in particular the computation of roll, from inaccurate roll damping coefficients. The use of linear theory for higher sea states was discussed;
5. Use of test bed/trials data for the engine operating curves, in particular, assumptions based on the propeller and hull interaction.
6. Assumption of a Wageningen B-Series propeller;
7. Unknown leeway effects on wake and propulsive coefficients;
8. Assumption of constant wind speed and above water areas, for the calculation of wind resistance.

Although the semi-empirical/theoretical performance algorithm lacks finer detail, the method of construction and the degree of complexity required for a routeing model were indicated. Since this model is relatively complex, a simplification process was undertaken in order that its integration to a multiple iteration process be performed within the capabilities of micro-computers. This involved:

1. Generation of calm water resistance components and their summation to form a polynomial;
2. The use of Maud curves to interpolate the databases. Generation of the coefficients of the curves can be made off-line;
3. Construction of two algorithms for constant ship speed or constant engine speed involving Newton-Raphson iteration schemes;
4. Off-line processing of Taylor wake and hull efficiency coefficients;
5. The generation of a series of motion criteria databases, (rather than RAO or response databases), based on ship speed and encounter at a central wave height, using the PM spectrum.
Chapter 4 investigated the environmental data necessary for the routeing algorithms, in conjunction with sophisticated ship performance algorithms. These data, transmitted from the US navy GSOWM, on a 2.5 degree spherical grid for the globe, are provided at 12 hourly intervals. Data reliability and the methods of in-fill for missing, or non-provided forecast times, were investigated. The methods of interpolation and generation of all extended and interpolated data files were based on a real-time scenario. Computer programs were constructed for cubic-spline or polynomial data interpolation, and the possible consequences of these were discussed. It was, however, the development of extended data, (which runs from the end of the forecast period, either +60 or +120 hours, to the projected ETA of the vessel), that was highlighted and several methodologies were investigated:

1. Running climatology, (RC);
2. Generation of gross seaway features from +144 ECMWF surface pressure as a storm avoidance approach, backed up by RC data;
3. Polynomial extensions throughout the time series at each element of the data arrays.

Due to the randomness of the sea and atmosphere, the generation of extended environmental features is fraught with error. However, it has been demonstrated and concluded, that the construction of gross features, (those depressional areas that are the basis of weather routeing), is satisfactory, especially, since the vessel only actually encounters the most reliable forecast or analysis data. The continual update of routes is necessary in order to smooth out inaccuracies inherent within the extended forecast data.

Full discussions of error sources in providing the extended data ensued. It was argued that the prediction of the location of storm centres is paramount, since routeing is based on the relative change of seaway conditions. The exact
prediction of vessel ETAs can only be made with completely accurate forecasts; an accurate ship performance algorithm and an optimisation algorithm that iterates over very small discrete steps. The overall methodology for construction of full data sets at 1200Z and 0000Z was shown.

Chapter 5 described, in detail, the dynamic programming routine, and illustrated two methodologies of a discrete algorithm, based on differing stage and state variables. Reasons were given for the adoption of discrete positional states, defined as a routeing grid, pre-computed from the departure and destination points. The development of these discrete grids involved the choice of stage and lateral state separations for which routeing examples were given. It was concluded that a stage value in the range, \( dx = 150\text{nm} \) to \( 250\text{nm} \) and a lateral state spacing, \( dy = 30\text{nm} \) to \( 50\text{nm} \) should be adopted, \((5:1)\). More importantly, several methods of grid construction were investigated. These were:

1. Rectangular grid systems;
2. Spherical grid systems.

The development of several spherical type grid systems was shown, including a study of their effects on the optimisation objective. From these studies it was concluded that a discrete grid should comprise of:

1. A central reference GCR between the terminals, regardless of land interference;
2. Stages constructed at a constant distance interval along the GCR;
3. Positional states constructed at each stage, along a further GC which intersects the GCR at \( 90^\circ \);
4. An LDR which is computed through the grid system;
5. Restriction of the grid by a routeing envelope, defined by land and the set of allowable state transitions per stage.
The construction of time states on the positional state space was investigated and developed. The construction of time states can be made by considering a restricted time envelope of operation, derived from the maximum and minimum speeds of the vessel. The discretisation of the time states was found to be facilitated using a one hour step, rather than a fixed number of states per positional state or stage. The consequences of these methods were discussed within the context of routeing problems.

It was further concluded that differing routeing strategies could be made by considering a contained routeing envelope, (rhombus), or an open-ended time state envelope. This also has the consequence of reducing computational load.

Chapters 6, 7 and 8 described in detail, the minimum time, fuel and cost routeing algorithms respectively. A description of each routeing model was given, in particular the use of stage re-computation, for a realistic solution of quasi-optimal routes. Construction of each objective policy function was described, within the routeing strategy, of which the limitations are :-

1. The discretisation of a continuous event. Reduction of the positional and time state spacings will increase accuracy, (considering the time and spatial spacings of environmental data, data accuracy and ship performance accuracy);
2. Discretisation of headings. Only predefined headings are permitted;
3. Discretisation of the positional routeing arena. Only heading variations can be made at pre-defined grid points. These variations are also subject to constraints;
3. Ship performance algorithm accuracy;
4. Environmental accuracy. All forecast data are treated as accurate, and furthermore, the interpolation through time and space are regarded as true;
5. Inter-positional trajectory step-lengths. Data is assumed constant for either the time or distance step length;

6. The time and spatial interpolation of data. Correct interpolation of data for the trajectory step sizes is difficult, since data must be assumed for that length. Use of distance or time step lengths both involve difficulties. These can be overcome only by size reduction or the possibility of a correction routine as discussed in section 6.2.

Minimisation of routeing objectives has been conducted to investigate:-

1. The effects of differing ship performance algorithms;
2. The effects of differing data extensions;
3. The effects of motion criteria; with and without all criteria and the effect of increasing average roll criteria.

Routeing studies considered the continual update of routes as time progressed. Therefore, the time-series of routes were designed to simulate real-time crossing.

The studies have concluded that routeing can be performed with both simple and more sophisticated ship performance algorithms, within the limits of their adaptability. Gross routeing features and similar routes are to be found when the sea states are medium or high, since the algorithms operate on relative variations. When in lower sea states, wider differences between quasi-optimal routes will be found. Obviously it was concluded that more accurate ETAs or further policy values are obtained when the ship and engine are modelled more closely.

The routeing strategy for minimum cost (also possibly minimum time or fuel), was developed with variable power or engine revolutions. Conclusions regarding the restrictions of the t-state space have been described. However, this strategy assumes that engine variations are made at points described by the trajectory
terminals and intermediate points. Also, an assumption is made that these values remain constant over the step length.

Reasonable results have been concluded with the cost routeing model, which utilises realistic monetary values for the operating costs and the terminal costs. Only ship performance models which include power or engine fuel consumption can be used for cost objectives. It must be noted that the limitations of this model are expanded by the problems in defining true, (variable), fuel consumption which arise from differing oil densities or qualities, engine load, and engine deterioration.

The use of motion criteria was shown to have considerable effects upon the variation of quasi-optimal routes, for all objectives, (governed by the accuracy of such motion databases). Conclusions made were as follows:

1. Motion criteria have to be accurately developed for particular ship types and crew;
2. The accurate prediction of vessel motions is paramount in the prediction of motion criteria;
3. Variations in criteria limits can have substantial effects on quasi-optimal routes, since many may have the same terminal policy value, and variation of one criteria may swing the emphasis from one route to another;
4. Motion criteria must be viewed as a further safety factor, which can at any time be increased, if complete stages are unachievable when the state space is severely restricted.

The effect of motion criteria on fuel and cost routeing models have a more marked effect, since these operate between fixed boundary times. Introduction of the motion criteria was shown to cause wider deviations from courses with a
subsequent increase in engine revolutions. Furthermore, increasing average roll criteria was shown to affect the MFR in a similar way to the MTR.

Since the terminal times for fuel and cost routeing strategies were fixed, it was shown that both simpler and more complex ship performance algorithms produce the same quasi-optimal routes. The demanded speeds to achieve these boundary times are required from all ship performance algorithms, and errors between the models were shown to manifest themselves in the engine setting. Therefore, differences exist between the fuel or cost policy values. However, since the algorithms respond to the sea state relatively, the same quasi-optimal routes will result. Similarly, since the speed is constant for all ship performance algorithms within the motion databases, their effect will be identical.

The overall conclusions, further to those outlined in sections 6.4, 7.4 and 8.4, emerging from these results were:

1. Realistic gross routeing may be made with simple ship empiricism due to the relative change of sea states. Reasonably accurate ETA estimation cannot be made with these overall algorithms, since they are not ship particular and only involve environmental data described by Beaufort Number;

2. Realistic routeing may be made with polynomial forms describing ship speed as a function of significant wave height. If the power or engine setting is known for the polynomial, then fuel and cost routeing may be performed;

3. Use of polynomial forms indicates an OCR which is individual to the ship and may incorporate the peculiarities of the master;

4. Realistic routeing may be made with a complex ship algorithm incorporating added resistance. However, this requires detailed ship and engine descriptions. Although the computation of calm water
powering and speed is relatively accurate, the computation of added 
resistance is more complex and erroneous at higher sea states; 
6. Routeing studies involving motions and added resistance have to be 
simplified to provide an efficient, quick algorithm for on-board PCs; 
7. More accurate ETAs can be assumed with the resistance type 
algorithms;
8. The use of motion criteria add further safety to the computation 
of OCRs. However, they can only be used when accurate motion 
prediction is guaranteed.
9. Accurate ship motion prediction is required for the computation of 
motion criteria. This involves the introduction of swell to the sea 
spectra, which adds complication, and reduces the speed of 
computation. Furthermore, motion criteria have to be defined for the 
particular vessel, since hull shape may affect their limits.
10. Simpler ship models can be used to compute a gross 
quasi-optimal route. This OCR may then be used to formulate a 
reference route and a fine, but restricted grid system, on which a 
more complex ship performance algorithm can be used.
11. In the routeing examples shown, the environmental data was not 
extreme. This was not planned, since data collection was only 
available at certain times. However, it is interesting to note the 
sensitivity of these routeing models, in these circumstances, where 
slightly varying ETAs are evidenced from different data extensions 
used in the MTR model.
12. The MCRs operate at variable engine revolutions, throughout the 
discrete stages. Although this may differ from the constant value 
used for minimum fuel, this model effectively changes the engine 
setting as the time progresses.
13. The main difference between the MTRs and MFRs/MCRs results 
from the fixed terminal times. MTR obviously operates to maximise
speed, whereas with a fixed terminal time, speed becomes restricted, and the algorithm operates to minimise the engine revolutions (within the usual operating limit fuel \( \alpha \) a.SFC), but still maintain a necessary speed.

14. MCRs and MTRs are very similar since the only true variable is fuel cost. Capital, insurance, wages and so on are fixed costs, dependant only on time. For fixed terminals, this is a fixed cost.

15. The MFR and MCR models operate between pre-set terminal times. However, although the routes may be similar, the MCR model operates on variable and not constant power, as in the MFR model.

Overall, it is concluded that gross feature routeing may be made with very simple performance algorithms, as well as the more complex, although the more complex model will be more superior. The sophistication vastly increases computational time, especially in these recursive algorithms. Therefore, any application of fine 'tuning' to the individual algorithms, must be made with the knowledge of the uncertainties in the environmental data and the ship performance algorithms, together with the accuracy of the discretised optimisation algorithm, (errors may be compounded in their integration).

Introduction of stochastic routeing, would seem to provide a solution to some of these problems. However, the uncertainty of both the environment and the ship performance algorithms, as well as the limitations of predicting their interaction, all restrict stochastic application. For on-board routeing, the increased computational burden and storage of stochastic information for all data points presents, at this time, unreasonable gains for a PC routeing algorithm. However, the process can be simplified, as evidenced by Frankel et al (1980) and Hagiwara (1989). This unfortunately appears to defeat the object. The author is unaware of any real-time studies involving concurrent deterministic and stochastic routeing.
9.2. AN ON-BOARD WEATHER ROUTEING MODEL.

It can be concluded that the following routeing model will be investigated, by the
author, for an on-board approach:

1. Initial global route calculated using simpler ship performance
   algorithms;
   (i) Compute the positional state space and state space
       restrictions as outlined in chapter 5;
   (ii) Compute the minimum policy using simple ship performance
       models, with or without motions;
   (iii) Utilise ECMWF and RC data extensions;
   (iv) Record several minimal policy OCRs, within certain limits,
       OCR(1a), OCR(1b), OCR(1c)....

2. Use OCR(1..) as the central route of a further grid system to
   compute a more accurate OCR based on finer grid point spacings,
   finer step lengths and a more accurate ship model;
   (i) Compute a fine, restricted positional state space around the
       OCR(1..);
   (ii) Use a complex ship model with motion criteria to compute
       the final OCR, OCR(2a);

3. Return to point 2, to compute a route based on OCR(1b) and so
   on;

4. Should OCR(2a) not converge, or be vastly different from
   OCR(1..), or use different engine settings, return to 1, with the new
   settings;

5. Once little route variation is found, the OCR is found.

9.3. RECOMMENDATIONS FOR FURTHER WORK.

Further development of the routeing models may be attained with the adaption of
stochastic processes and associated advances in naval architecture. Therefore,
future work should be concentrated in these areas. As improvements in
environmental data forecasting are made the accuracy of predicted routes will likewise improve. The author intends to investigate the adaption of stochastic routeing and its effect upon the variation of routes from the deterministic case. Although this work has already been pioneered, Chen (1989) and Hagiwara (1989), it’s application to an on-board PC is still in its infancy, certainly as evidenced by published work.

The inclusion of vessel motions and motion-type databases require further work, especially if studies involving swell are to be used. In conjunction with an OCEANROUTES financed project, the author will be developing on-board routeing models with a complex database. This is intended to be adaptable to many ship types and operate on a central value with corrections for variations in hull type. Advancement in the prediction of variable propeller loading within the seaway, and its effect upon the wake and thrust must be achieved in order that more accurate ship speed estimations be made.

Methods to refine the routeing models and improve their efficiency are required for an on-board approach. For this reason, the author intends to investigate the concept of dynamic programming with transputers. Dynamic programming is directly applicable to parallel processing as it involves the use of identical processes, over several differing transit legs. Speed of computation will be vastly increased if several of these transits can be computed concurrently.

The concept of an on-board system has been established, and will be extensively investigated during the OCEANROUTES project. This will involve the introduction of practical routeing models and the performance of real-time routeing studies. Unfortunately, routeing advice can only be verified when two sister ships depart from the same port at the same time and it is unlikely that this will materialise.

Finally, it is hoped that different routeing methodologies be investigated, such as those mentioned in section 1.6. Routeing studies involving all models on the same
set of environmental data, with the same ship performance algorithm, may provide interesting results. It is possible that a combination of two or more such optimisation algorithms may aid micro-computer based weather routeing. For example, if the concept in section 9.2 is expanded:-

1. Utilise dynamic programming to compute a gross OCR based on coarse grid point settings and a simpler ship performance model.

2. Use a routeing arena, defined around the initial OCR as the basis for an isochronal type approach.

In this way, the initial OCR restricts the size of the problem presented to the secondary system.

The concept of a local optimisation algorithm operating between the waypoints determined from the global optimisation algorithm requires investigation and development. Studies at Polytechnic South West have investigated optimal automatic control algorithms. With further advancement, these could be integrated with the weather routeing package. For example, add-on units to the controller would be required constantly in order to check the progress along the desired track and to check the predicted policy values to nearby stage points. Furthermore, should the weather deteriorate beyond that forecasted, then a survival type routeing algorithm, integrated into the local algorithm would be required. This algorithm would operate on actual vessel motions and operate to optimise the best local track to the next stage, in order that motions are limited. Such work has been primarily investigated at Polytechnic South West, in the department of Marine Science and Technology.
10. BIBLIOGRAPHY.

10.1. BIBLIOGRAPHY.


Anon, 1986. 'Meteorological Office Global Wave Prediction System', Private Correspondence.

Anon, 1985. 'An Evaluation of Minimum Time Tracks for Major Routes'. OCEANROUTES (UK) LTD, Private Correspondence.

Anon, 1985. 'Master's Technical Paper', OCEANROUTES (UK) LTD, Private Correspondence.


Anon, 1989. 'Ocean Passages of the World', British Admiralty, HMSO.


Burghes, D; Graham, A, 1980. 'Introduction to Control Theory Including Optimal Control', Ellis Horwood Ltd, Chichester, UK.


Evans, J. P. 1989. 'Ship Performance', Private Correspondence.

Evans, J. P; Svenson, T. E. 1984. 'Rational Selection of Service Margins', British Ship Research Association, Report for General Council of British Shipping, British Shipbuilders, and DTI.


- 287 -


Hogben, N; Lumb, F. E, 1967. 'Ocean Wave Statistics', HMSO.


Hughes, 1954. 'Friction and Form Resistance of Surface vessels and the Skin Friction of Flat Plates', Trans. Soc. Naval Arch and Marine Engineers.


Hughes, R, 1989. 'Port Costs', Private Correspondence.


Mays, J, 1987. 'Performance Analysis', OCEANROUTES (UK) LTD, Private Correspondence.


Miller, J. N, 1983. 'Does Oceanrouteing Reduce Casualties?', Dec, OCEANROUTES (UK) LTD, Private Correspondence.


OCEANROUTES INC, 1987. 'Oceanews, Newsletter of the OCEANROUTES Companies', fall, Private Correspondence.

OCEANROUTES INC, 1989. Private Correspondence.


Sulzer, 1989. 'MV Canmar Ambassador, formally Dart Atlantic, Engine IORND90', Private Correspondence.


Titlow, J, 1989. 'Weather Performance and Ship Propulsion Factors', OCEANROUTES.


Townsin, R. L; Moss, B; Wynne, J. B; Whyte, I. M, 1975. 'Monitoring the Speed Performance of Ships', April, North East Coast Inst. Engineers and Shipbuilders, Vol. 91, 1974-1975, pp159-178.


Van Berlekom, W. B, 1978. 'Involuntary Speed Loss at Sea', 1st WEGEMT School, University of Newcastle upon Tyne, UK.


Walden, H, 1964. 'Die Eigenschaften der Meereswellen im Nordatlantischen Ozean', Deutscher Wetterdienst, Einzelveröffentlichungen, Nr. 46.

Wiegel, R. L, 1964. 'Oceanographical Engineering', Prentice-Hall. NJ, US.


11. APPENDICES.

A.1. OCL CONTAINERSHIP BODY PLANS.

The OCL containership body plans were assumed identical to those of the DART ATLANTIC. The generation of the TSEA file required digitisation of these body plans to obtain the sectional data for the seakeeping program, BRITSEA. Furthermore, hull geometry and sectional weights were required.

The full-loaded condition of the DART ATLANTIC was assumed to be that given by Aertssen (1972).

\[
\text{Full load volume of displacement} = 36458 \text{ m}^3. \\
= 37369.45 \text{ tonnes.}
\]

It was possible to deduce the loading of containers and the overall weight distribution, assuming parameters defining the OCL containership, Meek (1970), Meek et al (1972).

\[
\text{Lightship weight} = 15130 \text{ tons} = 15372.0 \text{ tonnes.} \\
\text{Weight attributable to fuel/stores/cargo} = 21997.5 \text{ tonnes.}
\]

Meek (1970) gives weight values and locations for stores, fuel and water, it was therefore possible to distribute the weights to appropriate section areas, by area digitisation. Fuel oil weight was taken from Lloyds (1988), (Register of Ships).

\[
\begin{align*}
\text{Heavy fuel oil} &= 2598.0 \text{ tonnes,} \\
\text{Water} &= 248.0 \text{ tonnes,} \\
\text{Stores} &= 143.0 \text{ tonnes,} \\
\text{Total for Cargo} &= 19008.5 \text{ tonnes.}
\end{align*}
\]
Further ship details are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loa</td>
<td>231.6m</td>
<td>C_B</td>
<td>0.600</td>
<td>P</td>
<td>5.78</td>
</tr>
<tr>
<td>L_pp</td>
<td>218.0m</td>
<td>C_wp</td>
<td>0.695</td>
<td>D</td>
<td>6.3</td>
</tr>
<tr>
<td>B</td>
<td>30.48m</td>
<td>C_m</td>
<td>0.969</td>
<td>Z</td>
<td>6</td>
</tr>
<tr>
<td>T_mean</td>
<td>9.144m</td>
<td>(\Delta)</td>
<td></td>
<td>Screws</td>
<td>1</td>
</tr>
<tr>
<td>T_f</td>
<td>9.144m</td>
<td>(h_{(b)}) (keel)</td>
<td>0.4m</td>
<td>(k_{yy})</td>
<td>23.8% L_pp</td>
</tr>
<tr>
<td>T_a</td>
<td>9.144m</td>
<td>(l_{(b)}) (keel)</td>
<td>67.6m</td>
<td>KG</td>
<td>12.084m</td>
</tr>
</tbody>
</table>

**TABLE A.1.1 DART ATLANTIC VARIABLES.**

Figure A.1.1 depicts body plans assumed for the DART ATLANTIC, Meek (1970).

**FIGURE A.1.1. OCL CONTAINERSHIP BODY PLANS.**
The following figures indicate sample RAOs devised for the DART ATLANTIC from the outputs from BRITSEA. They indicate :-

1. Pitch RAO at 15Knts and $\mu_{\text{wave}} = 90^\circ$, figure A.2.1.
2. Heave RAO at 15Knts and $\mu_{\text{wave}} = 90^\circ$, figure A.2.2.
3. Yaw RAO at 15Knts and $\mu_{\text{wave}} = 90^\circ$, figure A.2.3.
4. Roll RAO at 15Knts and $\mu_{\text{wave}} = 90^\circ$, figure A.2.4.
5. Sway RAO at 15Knts and $\mu_{\text{wave}} = 90^\circ$, figure A.2.5.
6. Added resistance RAO at 15Knts and $\mu_{\text{wave}} = 90^\circ$, figure A.2.6.

Considering figure A.2.4, the roll amplitude operator indicates large amplitudes at the resonant frequency. It was discovered that the average roll criterion was exceeded frequently, indicating that BRITSEA was overpredicting the roll response. The reasons are thought to be due to the roll damping values, being insufficient.
FIGURE A.2.2. HEAVE RAO.

FIGURE A.2.3. YAW RAO.
ROLL AMPLITUDE OPERATOR (90 DEG. 15KNTS)
DART ATLANTIC/EUROPE

FIGURE A.2.4. ROLL RAO.

SWAY AMPLITUDE OPERATOR (90 DEG. 15KNTS)
DART ATLANTIC/EUROPE

FIGURE A.2.5. SWAY RAO.
A.3. SULZER 10RND90 DIESEL ENGINE.

SULZER UK provided details of the original test bed and test in 1981 on the M.V DART ATLANTIC, now known as the M.V CANMAR AMBASSADOR. However, due to its sensitivity, original details cannot be published, suffice it to say that engine details were used in the construction of the ship performance algorithm.

1. By utilising the test bed details, a plot of power against engine revolutions was made.
2. SFC values were placed on the curve, and idealised iso-SFC contours were drawn.
3. By using the 1981 test, an increase in power to maintain the same revolutions, or a correction to the engine revolutions for the same power was made.
4. By reading values off the SFC contours, it was possible to deduce the engine revolutions against SFC. Similarly, values for engine revolutions against power were obtained.
SULZER (1989) recommended an increase of 10 - 15 gm/BHP-hr, or 13.4 - 20.1 gm/KWh for service conditions. Also, this accounts for...

'an engine installed in a ship burning heavy fuel instead of marine diesel'.

A.4. COMPARISON OF PLOTS OF +96 AND +120 HOUR GENERATED WAVE FIELDS FROM SIMPLE EMPIRICISMS TO ANALYSIS.

Figures A.4.1 - A.4.7 indicate comparisons of the analysis GSOWM data +96 and +120 hours on from the base data plotted against the forecasted wave data generated from the +96 and +120 hour 'Reading' wind field, and the PM empiricism. Since the degradation of wind fields is great at these forecast lengths it is quite surprising to find some agreement. Many forecasted points are very low, this is due to low wind speeds. The empiricisms do not account for swell. Scott (1962) included a swell factor, allowing for better correlations when wind activity is low, since the predominant wave becomes the swell. Similarly, when wind speed is high, the PM spectra will overpredict. Typically this happens in near-shore regions where the wave field is in fact fetch limited. Note, use of fetch and duration was found to be too clumsy at these forecast lengths.

For the purposes of weather routeing, these errors are not too drastic, since little speed loss or power increase is required in low to medium sea states, therefore introducing small or insignificant errors. Overpredictions prove to be more difficult unless these are isolated values, as seemed to be the case. When these large predictions occur at or around the destination points, there is little option left to a routeing algorithm but to heave-to or not converge. For this reason some degree of subjectivity was required, either to amend the data, or aid a routeing algorithm. In open ocean situations, these predictions are more realistic, and a routeing algorithm will be persuaded away from such areas, as required.
WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-12-89 0000HRS (+96 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME. COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.4.1 +96 B-DATE 12/4/89

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-13-89 0000HRS (+96 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME. COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.4.2 +120 B-DATE 12/4/89

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-13-89 0000HRS (+96 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME. COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.4.3 +96 B-DATE 13/4/89

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-16-89 0000HRS (+96 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME. COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.4.4 +120 B-DATE 13/4/89
FIGURE A.4.5. +96 B-DATE 16/4/89

FIGURE A.4.6. +120 B-DATE 16/4/89

FIGURE A.4.7. +96 B-DATE 20/4/89

FIGURE A.4.8. +120 B-DATE 20/4/89
A.5. COMPARISON OF A TIME SEQUENCE OF GENERATED WAVE FIELDS FROM SIMPLE EMPIRICISMS TO ANALYSIS

Since wind field accuracy has a large effect upon the wave field, a sequence from +12 to +96hrs was used to create the wave fields, shown in figures A.5.1 - A.5.7.

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+12 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME, COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.1. +12 B-DATE 11/4/89.

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+24 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME, COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.2. +24 B-DATE 11/4/89.

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+36 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME, COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.3. +36 B-DATE 11/4/89.

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+48 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME, COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.4. +48 B-DATE 11/4/89.
WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+60 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME.
COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.6. +60 B-DATE 11/4/89.

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+72 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME.
COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.7. +72 B-DATE 11/4/89.

WAVE HEIGHT COMPUTED FROM WIND FILE ON 04-11-89 0000HRS (+96 HRS) V ANALYSIS WAVE FILE FOR THE SAME TIME.
COMPUTED BY MOSKOWITZ (Vw AT 19.5m).

FIGURE A.5.7. +96 B-DATE 11/4/89.
As the wind forecast accuracy decreases, see figure 4.2, the accuracy of the prediction through the simple empiricisms reduces. Similarly, the effect of the few poor predictions affects the correlation coefficients. Since the wind speeds at this base date were relatively low, there is an underprediction of the waves heights. This is shown by the cluster of points close to the x=0 line.

A.6. COMPARISON OF PLOTS OF RUNNING CLIMATOLOGY WAVE FIELDS TO ANALYSIS.

Figures A.6.1 - A.6.4 indicate the correlation of the running climatological wave heights to analysis for the base date, 25-4-89. Since the RC wave height file is generated from the previous seven days, the file remains constant, and therefore, as the forecast length increases, the correlations reduce. Remarkable results are found, since the weather is relatively stable, and the data results from an analysis of the GSOWM data.

**FIGURE A.6.1. +96 B-DATE 25/4/89.**

**FIGURE A.6.2. +108 B-DATE 25/4/89.**

**Figure A.6.3.** +120 B-Date 25/4/89

**Figure A.6.4.** +132 B-Date 25/4/89

**Figure A.7.1.** +48 B-Date 24/4/89

**Figure A.7.2.** +72 B-Date 24/4/89
Figures A.7.1 - A.7.5 indicate the correlation of forecasted surface pressure arrays for the base date 24/4/89 to analysis pressure arrays. Correlation is very good through the series, degrading somewhat as the forecast length increases.
Figures A.7.6 - A.7.10 indicate the correlation of the generated wave fields from the surface pressure files in figures A.7.1 - A.7.5.
The environment at this time was relatively stable, and the correlations of the generated wave fields are relatively good. There are certain high overpredictions arising from errors within the algorithm, typically resulting from high wind in coastal areas. The use of geostrophic wind rather than that corrected for friction and curvature, will emphasise high winds.

**A.B. COMPARISON OF CONTOURED GSOWM ANALYSIS WAVE FIELDS TO CONTOURED WAVE FIELDS GENERATED FROM THE ECMWF SURFACE PRESSURE PRESSURE FILES.**

Figures A.8.1 - A.8.14 indicate the contoured +120 and +144 generated wave fields and the contoured GSOWM wave field analysis. The environmental algorithm operates on the pressure gradient, and it will be seen that wave centres are located where the gradients are steepest. The location of high waves centres are good, as shown in the figures, however, the quantitative values are too high. This
is to be expected by using the geostrophic wind and simple wind to wave empiricisms. All contours are depicted in feet x 10.

**CONTOUR KEY**

1 0.0
2 30.0
3 60.0
4 90.0
5 120.0
6 150.0
7 180.0
8 210.0
9 240.0
10 270.0
11 300.0

**FIGURE A.8.1. ANALYSIS 17/4/89.**

**FIGURE A.8.2. EXTENDED +120 12/4/89.**

**FIGURE A.8.3. ANALYSIS 18/4/89.**

**FIGURE A.8.4. EXTENDED +144 12/4/89.**
FIGURE A.8.9, ANALYSIS 25/4/89

FIGURE A.8.10, EXTENDED +120 20/4/89

FIGURE A.8.11, ANALYSIS 26/4/89

FIGURE A.8.12, EXTENDED +144 20/4/89
A.9. COMPARISON OF INTERPOLATED +84 AND +108 WAVE FIELDS TO ANALYSIS.

Figures A.9.1 - A.9.4 indicates the correlation of the +84 and +108 interpolated wave fields to analysis. The correlations are relatively good but deteriorate as the extended data influences the interpolation, at +108.
FIGURE A.9.3. +84 B-DATE 26/4/89

FIGURE A.9.4. +108 B-DATE 26/4/89