

2015-09

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Vlasenko, Vasyl

<http://hdl.handle.net/10026.1/4404>

10.1016/j.dsr.2015.05.003

Deep Sea Research Part I: Oceanographic Research Papers

Elsevier BV

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Internal tides near the Celtic Sea shelf break: a new look at a well known problem

Vasiliy Vlasenko^a, Nataliya Stashchuk^a,

^a*School of Marine Science and Engineering, University of Plymouth, Drake Circus, Plymouth, PL4 8AA, UK*

Abstract

Internal waves generated by tides in the Celtic Sea were investigated on the basis of in-situ data collected at the continental slope in July 2012, and theoretically using a weakly nonlinear theory and the Massachusetts Institute of Technology general circulation model. It was found that internal solitary waves generated over the shelf break and propagated seaward did not survive in the course of their evolution. Due to the large bottom steepness they disintegrated locally over the continental slope radiating several wave systems seaward and transforming their energy to higher baroclinic modes. In the open part of the sea, i.e. 120 km away from the shelf break, internal waves were generated by a baroclinic tidal beam which was radiated from the shelf break downward to the abyss. After reflection from the bottom it returned back to the surface where it hit the seasonal pycnocline and generated packets of high-mode internal solitary waves. Another effect that had strong implications for the wave dynamics was internal wave reflection from sharp changes of vertical fluid stratification in the main pycnocline. A large proportion of the tidal beam energy that propagated downward did not reach the bottom but reflected upward from the layered pycnocline and

returned back to the surface seasonal pycnocline where it generated some extra higher mode internal wave systems, including internal wave breathers.

Key words: Baroclinic tides, internal solitary waves, the Celtic Sea

1. Introduction and motivation

The Celtic Sea (CS) shelf break is one of the “hot spots” of the global ocean where barotropic tidal energy is converted into a baroclinic component (Baines, 1982) making a great contribution to the sustainability of the meridional overturning circulation. This is the reason why much attention is focussed on this site with the aim of quantifying baroclinic processes that develop there. The earliest works by Pingree and Mardell (1981, 1985) followed by more recent studies (Pingree and New, 1995; Holt and Thorpe, 1997; Huthnance et al., 2001; Hopkins et al., 2012) reported the characteristics of internal waves generated by tides over the Celtic Sea shelf break.

The most recent observations were conducted on the 376-th cruise of the RRS “Discovery” (hereafter D376) in June 2012 in the slope-shelf area. The task of the cruise was to quantify the cross shelf transport on the NE Atlantic Ocean margin. In doing so, several long-term moorings with thermistor chains and ADCPs were deployed in the area (some of them are shown in Fig.1), accompanied by CTD surveys and glider missions.

The data collected in-situ revealed evidence of a strong semi-diurnal baroclinic tidal signal that was accompanied by packets of short-period internal solitary waves with amplitudes up to 100 m. A detailed analysis of the characteristics of these waves, their spatial structure and dynamics was reported by Vlasenko et al. (2014) who replicated the generated wave fields numerically

22 using the Massachusetts Institute of Technology general circulation model
23 (MITgcm). The model results were validated against the observational data
24 collected during the D376 cruise.

25 Two classes of tidally generated internal waves were identified in the area
26 with highly corrugated topography shown in Fig.1. Spiral-type internal waves
27 similar to those typical for isolated underwater banks were generated over the
28 headland. The other type was a system of quasi-planar internal wave packets
29 that were generated in the area of several canyons. The spatial structure of
30 these two wave systems is shown in Fig.1 a (see Vlasenko et al. (2014) for
31 more details). Note that the water stratification during the experiment was
32 characterised by a relatively sharp interface at the depth of 50 m and less
33 pronounced main pycnocline located between 500 and 1200 metres (Fig.2 a).

34 Vlasenko et al. (2014) concluded that the strongest internal wave system
35 is a superposition of a 20 m amplitude semi-diurnal baroclinic tidal wave and
36 a series of internal solitary waves (ISW). These waves were generated over the
37 top of the headland (just in the place of the mooring ST2 deployed at isobath
38 185 m, see Fig.1 a) and radiated to the shelf and to the deep water towards
39 mooring ST1. The isotherm time series, Fig.3 a, shows the vertical structure
40 of the internal waves recorded at mooring ST1, and Fig.3 b represents a 5-
41 hour fragment with the strongest leading ISW of 105 m amplitude.

42 The normalized vertical profile of the largest ISW recorded at mooring
43 ST1 is shown in Fig.3 c. It was built by calculation of the displacement of
44 the chosen isotherm from its equilibrium depth before the ISW arrival and
45 normalized by the wave amplitude. Fig.3 c shows that the wave profile reveals
46 the properties of the second baroclinic mode that produces counter-phase

47 displacements of isotherms in the surface and bottom layers. To get a more
 48 statistically justified result on the possible appearance of second-mode ISWs
 49 at mooring ST1, another 45 of the largest ISWs, with amplitudes larger than
 50 30 m, were analysed in a similar way. Considering the whole cluster of the
 51 dots together it was expected to find a general tendency of their distribution
 52 that is noise free and statistically significant. It is clear from Fig.3 d that all
 53 ISWs are waves of depression in the surface 120 m layer. Below this depth
 54 the dots are randomly distributed across the whole range between -1 and 1,
 55 so that both the waves of depression and elevation were equally observed.

56 To make the point clearer, the eigenfunctions of the boundary value prob-
 57 lem (BVP)

$$\frac{d^2\Phi}{dz^2} + \frac{N^2(z)}{c_i^2}\Phi = 0, \quad \Phi(0) = \Phi(-H) = 0. \quad (1)$$

58 were calculated. Here $\Phi(z)$ is the vertical modal structure function, c_i is the
 59 phase speed of the i -th mode, $N(z)$ is the buoyancy frequency shown in
 60 Fig.1 d, H is the water depth.

61 Two first eigenfunctions of the BVP (1) are presented in Fig.3 d. It is clear
 62 that the vertical structure of the ISWs recorded at ST1 does not fit either the
 63 first or the second baroclinic mode. However, initially at the place of their
 64 generation (in the area of the mooring ST2, see Fig.1), the internal waves
 65 had the structure of the first baroclinic mode (for the details see Vlasenko
 66 et al. (2014)). Thus, it is unclear what happened to these waves on their
 67 way from shallow mooring ST2 to the deeper ST1, i.e. in the course of their
 68 seaward propagation (hereafter, “antishoaling”). What is the ultimate fate
 69 of the waves generated on the CS slope: Do they dissipate locally or radiate
 70 far away from the place of generation? These fundamental questions on the

71 mechanisms of the tidal energy conversion and its dissipation were a strong
 72 motivation for the present study.

73 The paper is organized as follows. Section 2 describes the antishoal-
 74 ing process of ISWs in terms of a weakly nonlinear theory and using fine-
 75 resolution modelling based on a 2D version of the MITgcm. Section 3 sum-
 76 marises the finding from the antishoaling study and formulates some further
 77 questions to be answered. Section 4 reports results of a high-resolution mod-
 78 elling of baroclinic tides in the area. Section 5 outlines the main findings.

79 **2. Does the antishoaling kills all ISWs?**

80 *2.1. Weakly nonlinear analysis*

81 Evolution of seaward propagating first mode ISW can be investigated in
 82 terms of the Gardner equation:

$$\frac{\partial \eta}{\partial t} + (\alpha \eta + \alpha_1 \eta^2) \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0. \quad (2)$$

83 Here η is the displacement of the isopycnals; x is the spatial variable in the
 84 direction of wave propagation, and t is the time; α and α_1 are the coefficients
 85 of quadratic and cubic non-linearities, respectively; β is the coefficient of dis-
 86 persion. Note that α , α_1 , and β depend on the water depth and stratification
 87 as follows (Grimshaw et al., 1997):

$$\alpha = \frac{3c \int_{-H}^0 \left(\frac{d\Phi}{dz}\right)^3 dz}{2 \int_{-H}^0 \left(\frac{d\Phi}{dz}\right)^2 dz}, \quad \beta = \frac{c \int_{-H}^0 \Phi^2 dz}{2 \int_{-H}^0 \left(\frac{d\Phi}{dz}\right)^2 dz},$$

$$\alpha_1 = \frac{3 \int_{-H}^0 \left\{ c^2 \left[3 \frac{dT}{dz} - 2 \left(\frac{d\Phi}{dz}\right)^2 \right] \left(\frac{d\Phi}{dz}\right)^2 - \alpha^2 \left(\frac{d\Phi}{dz}\right)^2 + \alpha c \left[5 \left(\frac{d\Phi}{dz}\right)^2 - 4 \frac{dT}{dz} \right] \frac{d\Phi}{dz} \right\} dz}{2c \int_{-H}^0 \left(\frac{d\Phi}{dz}\right)^2 dz}.$$

88 Here $\Phi = \Phi_1$ and $c = c_1$ are defined from the BVP (1), T is a normalized
 89 solution of the following boundary value problem

$$c \frac{d^2 T}{dz^2} + N^2(z)T = -\alpha c \frac{d^2 \Phi}{dz^2} + \frac{3c}{2} \frac{d}{dz} \left[\left(\frac{d\Phi}{dz} \right)^2 \right], \quad (3)$$

$$T(0) = T(-H) = 0, \quad (4)$$

90 The steady state solutions of the Gardner equation (2) depend on the sign
 91 of the coefficients α and α_1 . Fig.4 shows the dependence of the coefficients α
 92 (red line) and α_1 (blue line) from depth H . It is clear that in the near-shore
 93 zone (shallower than 2 km) the coefficient of the quadratic nonlinearity α
 94 changes its sign twice. The coefficient of the cubic nonlinearity α_1 is positive
 95 on the shelf but negative over the slope deeper than 1260 m.

96 In the shallow water zone where $H \leq 850$ m, $\alpha < 0$ and $\alpha_1 > 0$, and
 97 according to (Grimshaw et al., 1999, 2004) either negative algebraic solitons
 98 or breathers are allowed. For the depth $850 \text{ m} < H < 1270$ m both coefficients
 99 are positive, and the weakly nonlinear theory predicts the existence of either
 100 positive algebraic solitons or breathers. Moving further offshore to depths
 101 $1270 \text{ m} < H < 1750$ m the coefficient of the cubic nonlinearity changes its
 102 sign again, Fig.4, allowing only positive ISWs, whereas at deeper the isobath
 103 1750 m only negative ISWs are expected.

104 The spatial variability of the coefficients α and α_1 has strong implications
 105 for the dynamics of internal waves. It was found that ISWs generated at
 106 the shelf break in the area of the mooring ST1 are the waves of depression
 107 (Vlasenko et al., 2014). However, the weakly nonlinear theory predicts that
 108 after passing the turning point at $H=850$ m these waves have to change their
 109 polarity or transform into a breather. Probably the proximity of mooring

110 ST1 to the turning point is the reason why the vertical structure of ISWs
111 shown in Fig.3a does not fit the structure of the first baroclinic mode. To
112 learn more about the wave dynamics in the area and to check whether a
113 dramatic transformation of ISWs propagated seaward as predicted by the
114 weakly nonlinear theory is correct, a series of numerical experiments on the
115 antishoaling of ISWs was conducted. We restrict our interests to a 2D-
116 version of the problem in order to have a fine resolution grid that allows a
117 more accurate reproduction of the cross-slope wave transformation.

118 *2.2. Numerical modelling*

119 In numerical experiments with seaward propagating ISWs the vertical
120 and horizontal grid steps were $\Delta z=10$ m and $\Delta x=7.5$ m, respectively, and
121 the buoyancy frequency was set as that shown in Fig.2a by the thin line.
122 The bottom profile was defined along the cross-section depicted in Fig.1.
123 Initial fields for this series of model runs were prepared as it was done in
124 Vlasenko et al. (2009). An 80 m amplitude first-mode K-dV ISW of depres-
125 sion propagating in a basin of 200 m depth was used for the model initial-
126 ization. Being inserted into the numerical scheme it started to evolve due to
127 strong nonlinearity into a new solitary wave. This new-born stationary 53 m
128 amplitude “numerical” ISW was used afterwards as an initial condition in
129 the experiments on antishoaling.

130 Seaward propagation of ISW is presented in Fig.5. Here the horizontal
131 and vertical velocity fields overlaid with the temperature field at different
132 stages of the wave evolution are shown in the middle and bottom panels,
133 respectively. In order to have a guess whether the propagated ISW can be
134 visible on synthetic aperture radars (SAR) images, the top panel represents

135 the wave-induced horizontal velocity gradient du/dx at free surface $z = 0$.
136 According to Alpers (1985) the radar recognises ISWs as systems of bright
137 and dark bands if the wave signal $du/dx(z = 0)$ is of the order of 10^{-3} s^{-1} .

138 Analysis of Fig.5 shows that in the shelf break area where the water depth
139 is less than 850 m, the ISW behaves as the weakly nonlinear theory predicts,
140 i.e. in the course of evolution a negative ISW adjusts its structure adia-
141 batically to the varying topography. After passing the turning point where
142 the quadratic nonlinearity changes its sign (see Fig.4), the ISW starts to
143 transform, but not exactly as the weakly nonlinear theory predicts. Instead
144 of changing its polarity in the whole water column the wave turns into a
145 second-mode ISW. As a confirmation of that, in the deep part of the basin
146 where the depth exceeds 1100 m the horizontal velocity changes its sign twice
147 from the surface to the bottom revealing properties of the second mode (the
148 most right panel in Fig.5 b). This conclusion is also confirmed by the coun-
149 terphase displacements of the isotherms in the surface and bottom layers,
150 and by the spatial structure of the vertical velocity shown in Fig.5 c.

151 The process of energy conversion from lower to higher modes continues
152 to progress in the course of the wave evolution. Being a second-mode wave
153 at the 1700 m isobath (its structure is shown in two left panels in Fig.6),
154 the seaward propagated ISW transforms into a packet of third-mode internal
155 waves at a depth of 4000 m (two right panels in Fig.6). Thus, it is clear from
156 the comparison of the weakly-nonlinear theory predictions and the fully-
157 nonlinear model output that the theory correctly predicts the positions of
158 the turning points, but fails to describe disintegration of ISWs into packets
159 of higher modes.

160 One of the possible reasons for such unexpected transformation of the
161 propagated ISWs could be inapplicability of the weakly non-linear theory
162 over the continental slope. In fact, the Gardner equation (2) is valid for
163 shallow water systems when $\lambda/H \gg 1$ (λ is the wave length and H is water
164 depth). The horizontal scale of the initial 53 m ISW on the 200 m depth shelf
165 is about 1.5 km (i.e. $\lambda \approx 750$ m), so formally this ISW can be classified as a
166 long wave. However, over the continental slope and in the open part of the
167 sea this condition is not valid. To check whether the ratio of the wavelength
168 to the water depth is a key parameter controlling the wave evolution (and
169 applicability of the shallow water theory), another experiment with a 2 m am-
170 plitude (and three times longer wavelength) ISW was conducted. However,
171 it was found that, similar to the large amplitude ISWs, the 2 m amplitude
172 ISW also reveals strong energy transfer from the first to highest modes in
173 the course of its evolution (not shown here). In fact, the sensitivity runs
174 have confirmed that the energy transfer to the higher modes over an inclined
175 bottom does not depend on the amplitude/wavelength ratio.

176 Another reason for the cross-mode energy transport could be the steep-
177 ness of the bottom topography. It is equal to 0.13 (or 7.5°) between the 600
178 and 1000 m isobaths. In situations when the wave crosses strong horizontal
179 gradients, the transfer of wave energy to higher modes is quite possible. In
180 order to check whether this is the case, some extra numerical experiments
181 were conducted. In a new series of model runs all settings were the same as
182 above except for the bottom steepness which was reduced to 0.01 (i.e. 0.6°).

183 Three fragments of the wave antishoaling over the flat topography are
184 shown in Fig.7. The spatial structure of the horizontal velocity reveals the

185 properties of the first mode at all stages of wave evolution. There is still
186 evidence of the energy leak to the second and third modes in the wave tail
187 (find the wave fragment between 55 and 70 km), but this fragment accounts
188 for only 3% of the total energy of the initial ISW.

189 Thus, this experiment has confirmed that steep topography is the main
190 reason for the energy transfer from the first mode ISW to the higher modes.
191 Presumably, evidence of such a transformation was recorded at the deep
192 mooring ST1, Fig.3d. Note also that collapse of the seaward propagated
193 ISW over steep topography was accompanied by a radiation of several wave
194 systems overtaking the main wave packet. The process of wave radiation is
195 clearly seen in the Hovmöller diagram presented in Fig.8 a. Here the instant
196 profiles of the free surface velocity are shown for 45 hours of wave evolution.

197 Fig.8 a shows that an internal wave train is radiated forward after the
198 ISW passes the 850 m isobath. In the Hovmöller diagram this wave system is
199 marked by the arrow with number 1. In the deep part of the ocean this packet
200 has amplitude less than 10 m, and propagates here with velocity 2.38 m s^{-1} ,
201 which is close to the value 2.45 m s^{-1} predicted by BVP (1) for the first
202 baroclinic mode (the blue line in Fig.8 b). Vertically the structure of this
203 leading wave system (not shown here) resembles first baroclinic mode.

204 A few hours later when the wave system passes the 1800 m isobath a
205 second wave packet detaches from the main wave train and overtakes it (find
206 arrow 2 in Fig.8 a.). An average propagation speed of the new born packet is
207 equal to 1.08 m s^{-1} , which almost ideally coincides with the second eigenvalue
208 of the BVP (1), 1.10 m s^{-1} .

209 It is interesting that the phase speed of the strongest wave fragment over

210 the slope (find arrow 4 in Fig.8 a) remains constant, viz. 0.42 m s^{-1} , during
211 four days of evolution. At the beginning of the ISW antishoaling this velocity
212 was equal to the phase speed of the first baroclinic mode on the shelf (dotted
213 line in Fig.8 b). At the latest stages of ISW evolution, i.e. in the deep water
214 part of the basin, this value is close to the phase speed of the fourth baroclinic
215 mode. Evidence of higher baroclinic modes can be seen in vertical structure
216 of the horizontal and vertical velocity fields shown Figs.6 e and 6 f.

217 **3. T-beam generation of ISWs in the far field**

218 It was shown above that the ISWs generated over the shelf break and
219 propagated seaward disintegrate into packets of the first, second, and higher
220 baroclinic modes losing a large proportion of their initial energy. As a result,
221 the surface signal $du/dx(z = 0)$ of propagated 53 m amplitude ISW drops
222 from initial $1.5 \cdot 10^{-3} \text{ s}^{-1}$ on the shelf to $0.03 \cdot 10^{-3} \text{ s}^{-1}$ in the deep part (see
223 Figs.5 a and 6 d). Therefore, it is unlikely that propagated seaward ISWs can
224 be observed far from the shelf break. Note, however, that the remote sensing
225 data presented by New and Da Silva (2002) for the Bay of Biscay (BB)
226 ($45\text{-}48^\circ\text{N}$, $5\text{-}9^\circ\text{W}$, Fig.1), clearly show evidence of ISWs at a distance of 120-
227 150 km (hereafter the “far field”), see Fig.1 b. Scrutiny of the ISW signature
228 pattern also shows that there is no strong evidence of ISWs between the shelf
229 break area (hereafter “near field”) and the far field. Only a few wave systems
230 are presented there.

231 New and Pingree (1990, 1992) and Pingree and New (1989, 1991) ex-
232 plained the appearance of internal waves in the far field in terms of local
233 generation. In short, this mechanism suggests that the tidal internal waves

234 generated over a supercritical topography are radiated from the shelf break
 235 to the abyss in the form of a tidal beam (hereafter “T-beam”) along one of
 236 the characteristics lines

$$\int \frac{dz}{\gamma(z)} = \pm x + \text{const}, \quad \gamma(z) = \sqrt{(\sigma^2 - f^2)/(N^2(z) - \sigma^2)}, \quad (5)$$

237 of the hyperbolic wave equation:

$$w_{xx} - \gamma^2(z)w_{zz} = 0. \quad (6)$$

238 Here w is the vertical velocity, σ is the M_2 tidal frequency, f is the Coriolis pa-
 239 rameter. After reflection from the bottom the T-beam returns to the surface
 240 120 km away from the shelf break where it hits the pycnocline and generates
 241 internal waves. Gerkema (2001) confirmed the possibility of this mechanism
 242 for the BB theoretically using simplified stratification with mixed surface
 243 layer and two underlying layers with constant stratification. Schematically,
 244 the idea of the T-beam generation is presented in Fig.1 c.

245 Note that considered here area is next to the BB, Fig.1. As such two
 246 basins should have similar conditions for internal wave generation, viz. fluid
 247 stratification, bottom profiles, and tidal forcing. If so, a similar appearance
 248 of ISWs in the far field of two basins is expected. To investigate the local
 249 generation of ISWs by a T-beam in the far field a two-dimensional version
 250 of the fully nonlinear nonhydrostatic MITgcm was applied.

251 The model domain was chosen to be long enough to reproduce the in-
 252 ternal waves in the far field. The bottom profile was similar to those used
 253 in the experiments with ISW antishoaling, Fig.1. The spatial resolution
 254 was $\Delta x=15$ m and $\Delta z=10$ m. The tidal forcing was defined using the ADCP
 255 measurements for the period of 25-27 June 2012 (days 177-179 of 2012) when

256 the strongest ISWs were recorded at mooring ST1, Fig.3. The amplitude of
257 tidal discharge in northern and eastern directions was equal to $60 \text{ m}^2 \text{ s}^{-1}$. The
258 background vertical viscosity and diffusivity coefficients were taken at a mini-
259 mum level, i.e. $10^{-5} \text{ m}^2 \text{ s}^{-1}$, but with the Richardson number parametrization
260 for extra mixing in the areas with hydrodynamic instabilities produced by
261 internal waves. The coefficients of horizontal viscosity and diffusivity were
262 set constant, $0.1 \text{ m}^2 \text{ s}^{-1}$.

263 Two buoyancy frequency profiles, (i) a smoothed climatic one based on
264 the Boyer et al. (2009) data set, and (ii) an instant profile taken from the
265 D376 yo-yo CTD cast to 1200 m depth extended by the climatic data below
266 1200 m, were used in modelling. These two profiles are shown in Fig.2 a
267 by the thick and thin lines, respectively. The difference between the model
268 outputs obtained for both cases is discussed below.

269 *3.1. Smoothed stratification*

270 The dynamics of internal waves can be studied using the Hovmöller di-
271 agram, Fig.9 a. It shows the evolution of vertical displacement (left axes)
272 of the 13°C isotherm during the time span 240-252 h (right axes) after the
273 beginning of the experiment. Fig.9 b represents the amplitude of the hori-
274 zontal velocity during the 21-st tidal cycle. A long term spin-up of the model
275 was required to allow higher baroclinic tidal modes to propagate through the
276 whole model domain. Their superposition results in the formation of a tidal
277 beam that is clearly seen in both the near and far fields, Fig.9 b.

278 The T-beam is quite a narrow band with a high intensity of baroclinic
279 tidal energy. It starts at point (a) on the shelf edge and propagates downward
280 to the abyss along the characteristic line (5) that is depicted in Fig.9 b by

281 the dashed white contours. After reflection from the bottom at point (b) the
282 baroclinic tidal energy returns back to the surface at point (c).

283 In the shelf break area (between 10 and 20 km) Fig.9 a shows evolution
284 of two systems of short waves generated over the shelf break. The first
285 group comprises two wave packets (1)-(2)-(3) highlighted by yellow colour.
286 It seems that both packets, one directed to the shelf and another to the open
287 sea, were generated according to the lee wave mechanism. The other wave
288 system shown in orange (fragments (4)-(7)) was developed due to steepening
289 and disintegration of propagating internal tidal wave (Vlasenko et al., 2005).
290 Both systems remain visible during one additional tidal cycle and attenuated
291 in the deep part of the sea according to the mechanism of disintegration
292 discussed in Section 2.

293 On the other side of the model domain, i.e. in the far field, the wave
294 motions are also well developed. The strongest wave system (8)-(9) (in Fig.9 a
295 it is marked by blue colour) starts to develop at point (c) where the tidal
296 beam hits the pycnocline located just below the free surface (at 50 m depth,
297 see Fig.2 a). At the first stage the generated wave looks like a bore propagated
298 seaward. It becomes steeper in the course of nonlinear evolution gradually
299 transforming into a packet of rank-ordered ISWs.

300 The propagation speed of the wave fragment (8)-(9) calculated on the
301 basis of the Hovmöller diagram, Fig.9 a, is equal to 0.61 m s^{-1} , which is well
302 below the phase speed of the first mode ISWs, but close to the velocity of
303 the third mode (first four eigenvalues of the BVP (1) for the depth 4.3 km
304 are equal to 2.52, 1.23, 0.68, and 0.45 m s^{-1} , respectively). Thus, the slow
305 velocity of propagation suggests that specific features of the high baroclinic

306 modes should be evident in the vertical structure of the wave packet (8)-(9).

307 Fig.10 represents the displacements of isotherms and the vertical velocity
308 of the wave packet depicted in right bottom corner of Fig.9 a by a magenta
309 rectangle. It is clear from Fig.10 a that the leading wave in this packet is the
310 wave of depression in the surface 200 m layer (maximum displacements up to
311 35 m), but it is the wave of elevation between 800 m and 1100 m depths. The
312 high mode wave structure is more evident in Fig.10 b which shows that the
313 vertical velocity changes its sign twice along the dashed line in panel Fig.9 b,
314 which is a specific feature of the third mode.

315 One can also identify in Fig.9 b a secondary tidal beam originated at point
316 (d) located in the layer of the main pycnocline, i.e. around 900 m depth, see
317 Fig.2 a. This beam propagates to the surface along the characteristic line,
318 where it is reflected from the surface downward at point (e), and ultimately
319 ends up at the bottom at point (f). The explanation of this phenomenon
320 can be found in terms of the mechanism of scattering of the main tidal beam
321 from layers with a sharp change of vertical stratification discussed below.

322 *3.2. D376 buoyancy frequency profile*

323 The buoyancy frequency profile recorded during cruise D376 was used in
324 the next series of numerical experiments. As seen from Fig.2 a, the instant
325 profile (shown by the thin line) is highly corrugated in the layer of the main
326 pycnocline, specifically between 750 m and 1200 m depths. High intermit-
327 tency of vertical fluid stratification creates favourable conditions for internal
328 wave reflection from layered fluid structures. The effect of tidal beam re-
329 flection from a pycnocline was reproduced numerically by Gerkema (2002)
330 and proven in laboratory experiments by Wunsch and Brandt (2012). There

331 are only a few specific profiles of the buoyancy frequency (see (Magaard,
332 1962; Vlasenko, 1987)) that allow internal wave propagation without reflec-
333 tion. Sharp increase or decrease of the vertical density gradients can lead to
334 a massive reflection of the tidal beam energy (Grimshaw et al., 2010)

335 The T-beam energy reflection from the layers with sharp changes of buoy-
336 ancy frequency is illustrated in Fig.11. The intermittent layered structure of
337 the main pycnocline between 750 m and 1200 m depths resulted in a stronger
338 secondary tidal beam (d)-(e), Fig.11 b, than it was for the smoothed pro-
339 file, Fig.9 b. Moreover, an additional tidal beam, viz. (g)-(h), has appeared
340 in Fig.11 b. It was generated by reflection of the tidal energy of the main
341 beam (a)-(b) from a number of layers of intermittent stratification in the
342 main pycnocline. As a consequence of this strong internal energy reflection,
343 the main tidal beam (a)-(b) below 1500 m is much weaker in Fig.11 b than
344 it was in Fig.9 b. In other words, a large part of the beam energy returns
345 to the surface in the area between the far- and near fields. In this area the
346 secondary tidal beams (d)-(e) and (g)-(h) interact with the subsurface pyc-
347 nocline and generate locally two new wave systems depicted in Fig.11 a by
348 numbers (1)-(2) and (3)-(4).

349 The intensity of the signals produced by wave systems (1)-(2) and (3)-
350 (4) at the free surface is strong enough to be visible from space by SARs.
351 The derivative $du/dx(z = 0)$ of the model output at $t = 252$ h is shown in
352 Fig.12 a. Note, however, that the spatial and temporal characteristics of these
353 wave two systems are quite different. The speed of propagation of the wave
354 packet (1)-(2) is equal to 0.61 m s^{-1} , which coincides with the phase speed
355 of the ISW packets (6)-(7) and (8)-(9) shown in Fig.9 a. Moreover, spatially

356 all these wave packets are mostly rank-ordered (see, for instance, Fig.12 b)
 357 resembling a third baroclinic mode in the vertical direction, Fig.12 d.

358 Wave system (3)-(4) is quite different from packet (1)-(2). It propagates
 359 much slower, i.e. with average speed 0.17 m s^{-1} (see Fig.11 a). Further
 360 analysis of its spatial structure has shown that the wave system (3)-(4) can
 361 be classified as an internal wave breather (Lamb et al., 2007), i.e. a localised
 362 wave packet with a sinusoidal periodic carrier that is restricted in space by a
 363 soliton-shape envelope, as shown Fig.12 c. Weakly nonlinear theory predicts
 364 such quasi-stationary solutions, internal breathers, that remain stable in the
 365 course of their propagation. Fig.11 a shows that wave packet (3)-(4) does
 366 keep its form at least one and a half tidal period until it was overtaken by
 367 another wave group.

368 In fact, the phase speed of the carrier wave should not necessarily coincide
 369 with the propagation velocity of the breather, i.e. the group speed. For the
 370 surface wave in deep water, for instance, the phase speed of the breather's
 371 carrier is double as high as the group speed (Zakharov, 1968; Slunyaev and
 372 Shrira, 2013). In the present case the phase speed of the carrier is equal to
 373 0.30 m s^{-1} , which is almost twice as high as the breather's speed. This value
 374 was calculated using the frequency of the carrier wave 0.0025 s^{-1} (found from
 375 the model by sampling at several fixed points that the breather crosses) and
 376 an average wavelength 660 m of the wave carrier detected from Fig.11 (see
 377 also Fig.12).

378 It is interesting to validate the model-predicted parameters of the breather
 379 against some theoretical values obtained from the boundary value problem:

$$\frac{d^2\Phi_j}{dz^2} + \frac{k_j^2}{\gamma^2(z)}\Phi_j = 0, \quad \Phi_j(0) = \Phi_j(-H) = 0. \quad (7)$$

380 Here $\Phi_j(z)$ is the vertical structure function and k_j is the wave number of
 381 the j -th mode. As distinct from (1), BVP (7) calculates wave numbers and
 382 vertical structure of periodic internal waves. The dispersion relations of the
 383 first three baroclinic modes are shown in Fig.13. For the wave carrier with
 384 frequency 0.0025 s^{-1} and the fluid stratification shown in Fig.2 a the wave
 385 length, phase speed, and the group speed of the first three modes ($j=1,2,3$)
 386 are shown in the Table. The model-predicted parameters of the numerical
 387 breather carrier from the Hovmöller diagram, Fig.11 a, are as follows: the
 388 wavelength - 600 m, phase speed - 0.30 m s^{-1} , and group velocity 0.17 m s^{-1} .
 389 It is clear from the Table that the third baroclinic mode provides the best fit
 390 to the model predicted parameters of the numerical breather carrier wave.
 391 This is also consistent with Fig.12 e, that shows the vertical structure of the
 392 breather.

393 New and Da Silva (2002) analysed parameters of more that 100 ISWs
 394 observed in SAR images of the BB (see Fig.1 b), which is next to the con-
 395 sidered here area. This data set can be taken as indirect validation of the
 396 model output. It was found that the wavelengths of the whole ensemble of
 397 observed ISWs ranged from 0.6 to 3.0 km, with a mean value of 1.35 km.
 398 The wavelengths $\lambda_1, \lambda_2, \lambda_3$ shown in Fig.12 b are 1.35, 1.15, and 0.85 km, re-
 399 spectively, that is in agreement with the observations by New and Da Silva
 400 (2002). It is also interesting that 10 % of all reported ISWs were quite short.
 401 They had their wavelengths in the range between 600 and 800 m which is
 402 consistent with the wavelength $\lambda_4=\lambda_5=660 \text{ m}$ of the internal breather shown
 403 in Fig.12 c.

404 4. Discussion and conclusions

405 The present study was motivated by the fact that majority of ISWs
406 recorded at the mooring deployed on the continental slope of the Celtic Sea
407 in July 2012 exhibited properties of the second baroclinic mode. In light of
408 the most recent modelling efforts by Vlasenko et al. (2014) who reproduced
409 numerically the three-dimensional dynamics of the baroclinic tides in the
410 area and showed that first-mode ISWs are generated over the shelf break,
411 this observational evidence of higher baroclinic modes over the continental
412 slope required further scrutiny.

413 A preliminary analysis of the wave dynamics in terms of a weakly nonlin-
414 ear theory has shown that all ISWs generated over the shelf break and prop-
415 agated seaward over an inclined bottom must change their form after passing
416 the turning points, i.e. the isobath where the coefficients of the quadratic
417 and cubic nonlinearities of the Gardner equation (2) change their sign. This
418 non-adiabatic behaviour of ISWs was reproduced numerically using the fully
419 nonlinear nonhydrostatic MITgcm. The numerical runs convincingly showed
420 that steepness of the bottom topography is the major factor controlling the
421 process of wave disintegration. It was found, however, that the seaward prop-
422 agated ISWs do not evolve exactly as the weakly non-linear theory predicts.
423 Quite opposite, after passing the turning point where the coefficient of the
424 quadratic nonlinearity $\alpha=0$, the ISW did not change its polarity but started
425 to transfer its energy to higher modes. This process was accompanied by
426 forward radiation of low-mode internal wave systems. The first-mode wave
427 packet is radiated first, then the second mode, etc, as shown in Fig.8.

428 It was found that the reason for the energy transfer to higher modes is the

429 steep gradient of the continental slope. Sensitivity runs with much smaller
430 bottom inclinations have shown that ISW really reverses its polarity after
431 passing the turning point according to a classical mechanism predicted by
432 the weakly nonlinear theory.

433 Overall, the seaward propagated ISWs do not escape far from the shelf
434 break and cannot be observed on SAR images far away from the shelf break.
435 Figs.6 a and 6 d clearly show that in the open part of the sea the surface
436 signal produced by seaward propagated ISWs is much weaker than 10 s^{-3}
437 which is considered as a threshold for detection of ISWs by SARs (Alpers,
438 1985). Note, however that these high-mode waves, being invisible from space,
439 are still able to form strong circulation cells at the depths of 1000-1500 m
440 (see Figs.6 b-c and 6 e-f). Ultimate disintegration and wave breaking can
441 lead to strong local water mixing and formation of the intermittent layered
442 stratification of the main pycnocline shown in Figs.2 b-e (Boyer et al., 2009).

443 The appearance of internal waves in the far field is explained here in
444 terms of the spatial structure of baroclinic tides. A tidal beam is generated
445 over supercritical topography and propagates downward from the shelf break
446 to the abyss along the characteristic line $\int 1/\gamma(z)dz = x + \text{const}$ of the
447 hyperbolic wave equation (5). After reflection from the bottom the tidal
448 beam returns to the free surface in the far field where it hits the pycnocline
449 and generates internal waves.

450 The MITgcm reproduced the tidal beam over the shelf break of the Celtic
451 Sea quite accurately. The model predicted wave velocities inside the tidal
452 beam (up to 16 cm s^{-1} during the spring tide) and its spatial position were
453 consistent with that recorded at the deep water mooring deployed in the BB

454 58 km from the shelf break (Pingree and New, 1991). The new element found
455 in the present study concerns the vertical structure of the ISWs in the far
456 field. The model reproduced third-mode ISW packets generated locally. An
457 example of such a wave packet is shown in Fig.12.

458 It should be noted here that Grisouard and Staquet (2010) in their nu-
459 merical analysis of the T-beam generation of internal waves in the BB also
460 reproduced the second mode in the far field (their Figs.2 and 3). It is in-
461 teresting that in the earliest publications by New and Pingree (1990, 1992),
462 and in a more recent interpretational paper by New and Pingree (2000) the
463 observed in the BB ISWs were treated as the first-mode waves. Unfortu-
464 nately, it is not possible to confirm or disprove this fact from the provided
465 observational data that cover only a surface 150 m layer. The temperature
466 time series presented in Fig.3 a also shows all ISWs as waves of depression
467 near the surface, but they clearly reveal properties of higher modes in the
468 deep, Fig.3 d.

469 An indirect confirmation of the modal structure of ISWs can be deduced
470 from their phase speed calculated based on SAR images. New and Da Silva
471 (2002) analysed satellite images assuming the first-mode phase speed of the
472 observed waves which is close to 1 m s^{-1} . However, Envisat-ASAR image
473 from the middle of the BB dated 12 August 2005 and published in (Muacho
474 et al., 2014) (their Figure 2) allows one to calculate this value very accurately.
475 The image presented in Fig.14 clearly shows at least nine circular wave fronts
476 with an average distance between them of about 25 km. Assuming a semidi-
477 urnal tidal periodicity of the observed waves it can be found that their phase
478 speed must be equal to 0.55 m s^{-1} , which is close to 0.6 m s^{-1} , i.e. the model

479 predicted phase speed found from Fig.11. It is interesting that Grisouard
480 and Staquet (2010) in their numerical experiment E2 also found that the two
481 wave trains were separated by 25-30 km, meaning that these ISWs propagate
482 at almost half of the speed estimated by New and Pingree (1990); New and
483 Da Silva (2002).

484 A series of sensitivity runs with smoothed and realistically intermittent
485 fluid stratification has shown that sharp changes of the vertical fluid stratifi-
486 cation are the sites of internal wave reflection. As a result of strong internal
487 wave reflection from a layered main pycnocline quite a substantial part of
488 the T-beam energy propagated from the shelf break downward to the abyss
489 returns to the surface in the area between the near- and the far fields where
490 it generates locally some extra internal wave systems. Numerical evidence of
491 this effect is shown in Fig.11. This could be a reasonable explanation of why
492 ISW packets are still visible in SAR images between the near- and far fields.

493 Note also that the backward reflection of the T-beam from the layers
494 with intermittent stratification was conducted here for a relatively smoothed
495 stratification (see Fig.2 a) that was found by averaging of 14 instant buoyancy
496 frequency profiles recorded in D376 cruise at the yo-yo CTD station. The
497 effect of T-beam reflection can be even stronger in case of real instant profiles
498 that are much more “fuzzy”, Figs.2 b-e. Four of them acquired from Boyer
499 et al. (2009) are shown in Fig.2 b-e. They reveal very strong local layering
500 between 500 m and 2000 m depths that can provide much stronger T-beam
501 reflection.

502 Another finding from the present study is shown in Figs.12 c and e. Anal-
503 ysis of all characteristics of this wave system generated by the secondary tidal

504 beam, Fig.11, has shown that this wave packet can be classified as an internal
505 wave breather. First of all, it is not rank-ordered; maximal amplitudes are
506 located in the middle of the packet. It was found that this wave system prop-
507 agates as a periodic wave train with a soliton envelope. Most importantly,
508 this form is stable in space and time (see Fig.11, wave (3)-(4)). Secondly, this
509 packet propagates much slower than the rank-ordered wave packets. More-
510 over, the group speed of this packet is twice as small as the phase speed of
511 the carrier wave. Theory-wise, all these characteristics are associated with
512 internal wave breathers that are probably generated in the far field, but some
513 further research is required.

514 *Acknowledgments* This work was supported by the Natural Environment
515 Research Council grant FASTNEt (award NE/I030259/1).

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Table 1: Parameters of first three baroclinic modes

Mode	Wavelength (m)	Phase speed (m s^{-1})	Group speed (m s^{-1})
1	2346	0.93	0.27
2	890	0.35	0.21
3	675	0.27	0.13

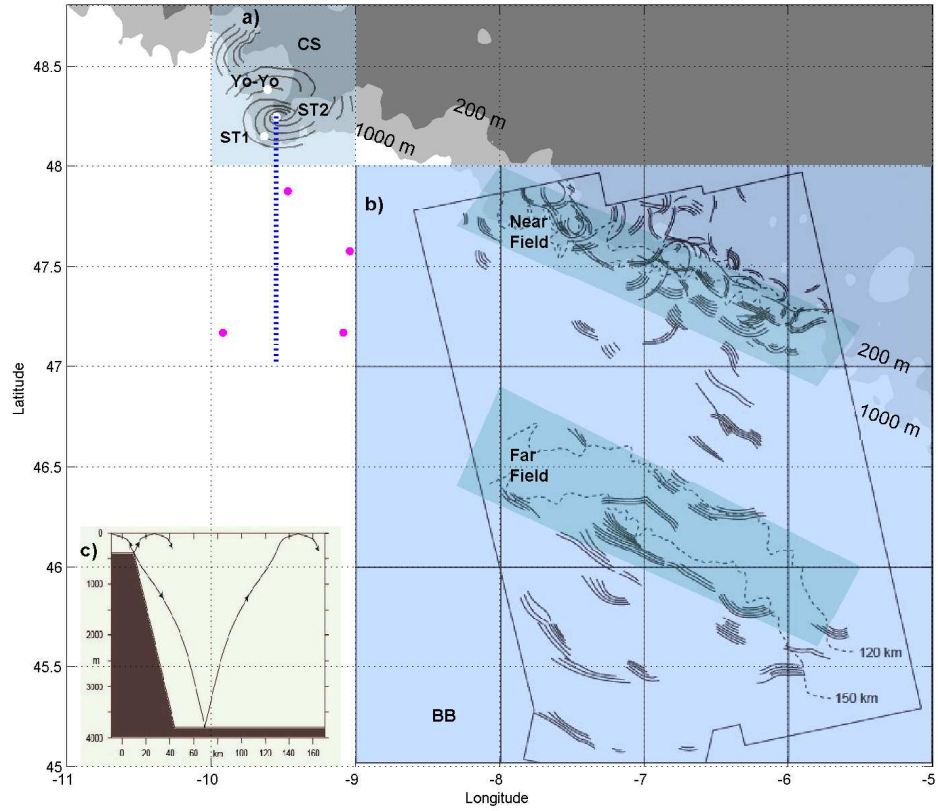


Figure 1: Shelf break of the Celtic Sea presented by the 200 m izobath. Positions of three moorings, Yo-Yo, ST1, and ST2 deployed during cruise D376 are depicted by black dots. Vertical dotted line shows the cross-section used in numerical modelling. a) Plan view of tidally generated internal wave systems predicted by the MITgcm (Vlasenko et al. (2014)). Time interval between two wave fronts equals 2 h. b) Composite image of all ISW packets observed in July 1994 and July-September 1999 in the Bay of Biscay and reported by New and Da Silva (2002). The observational area ranges from 5 to 9°W which is adjacent to the domain considered here. c) The tidal beam that is mentioned in the aforementioned paper as a reason for a local generation of internal waves in the far field.

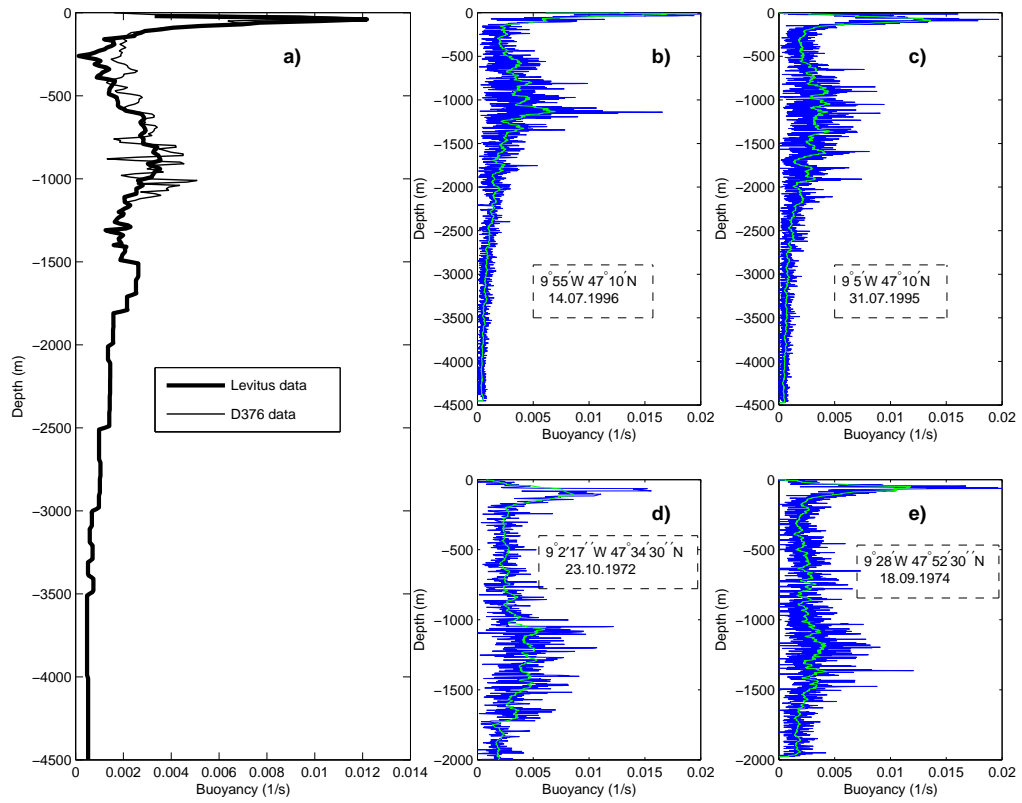


Figure 2: a) Vertical profiles of the buoyancy frequency: recorded at the Yo-Yo station (light line) and produced using the Levitus data set (thick line). Red lines depict the model profiles used in Section 3. b)-e) Buoyancy frequency profiles recorded at four CTD stations shown in Fig.1 by red dots (Boyer et al., 2009).

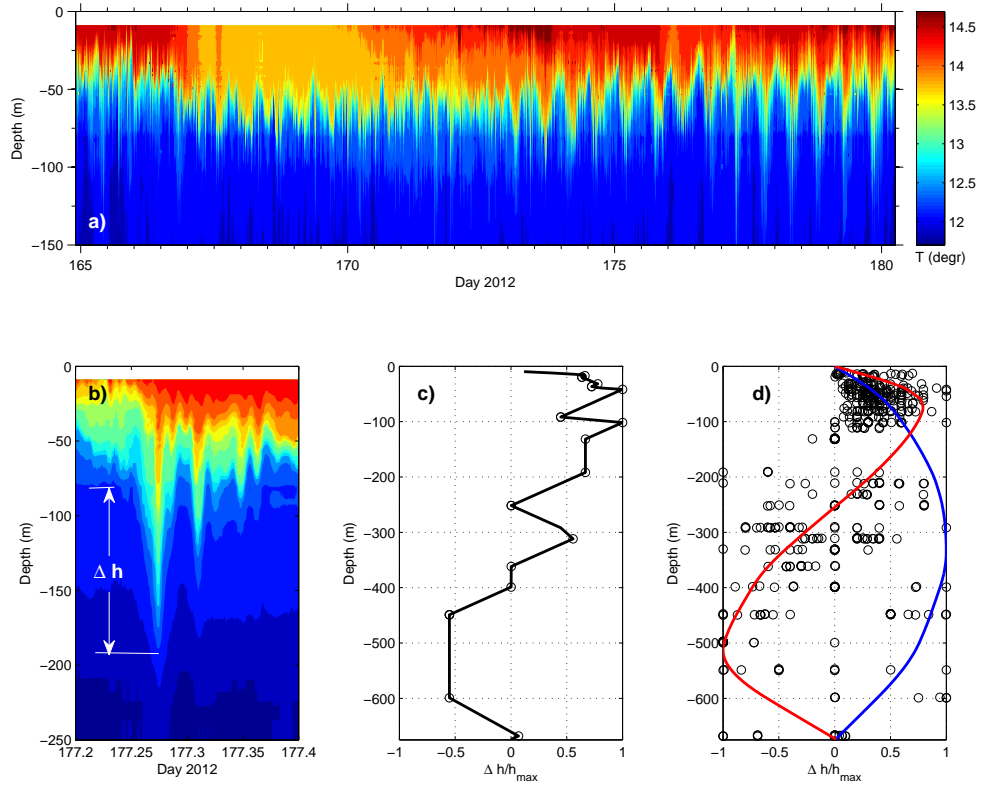


Figure 3: (a,b) Temperature recorded at mooring ST1. (c) Normalized vertical structure function of isotherm displacements of the leading ISW shown in panel (b). (d) The normalised wave displacements of the strongest 45 ISWs recorded at ST1. Blue and red lines show the normalised profiles Φ_1 and Φ_2 of the boundary value problem (1), respectively.

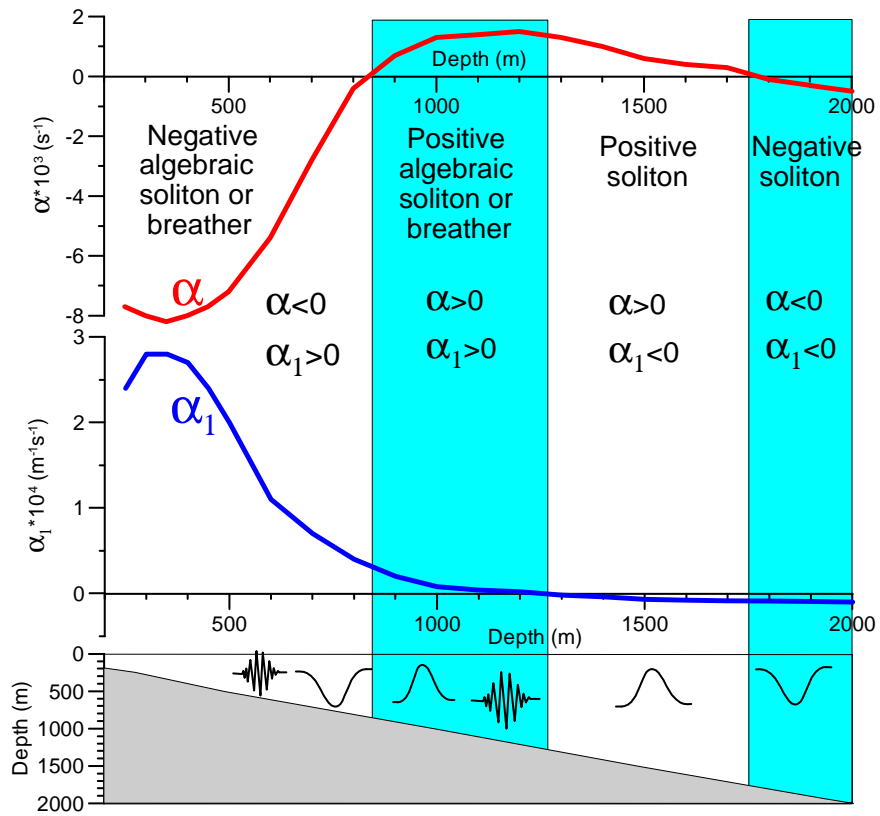


Figure 4: Quadratic α and cubic α_1 coefficients of nonlinearity of the Gardner equation (2) as functions of depth.

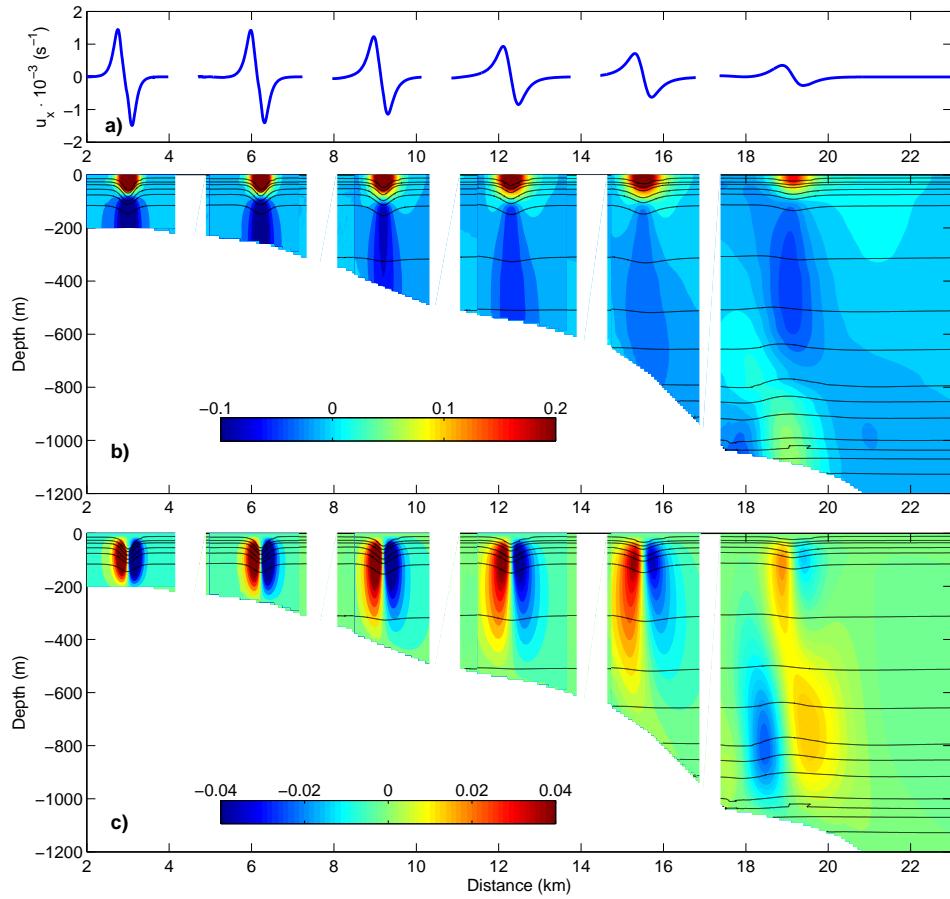


Figure 5: (a) Gradient of the horizontal velocity at the free surface $\partial u/\partial x(z = 0)$, (b) horizontal velocity overlaid with the temperature field and (c) vertical velocity overlaid with the temperature field of the ISW propagated seaward at different stages of its evolution. Colour bars show the velocity in ms^{-1} . Both panels are a composite of six different moments in time. Black solid lines depict isotherms.

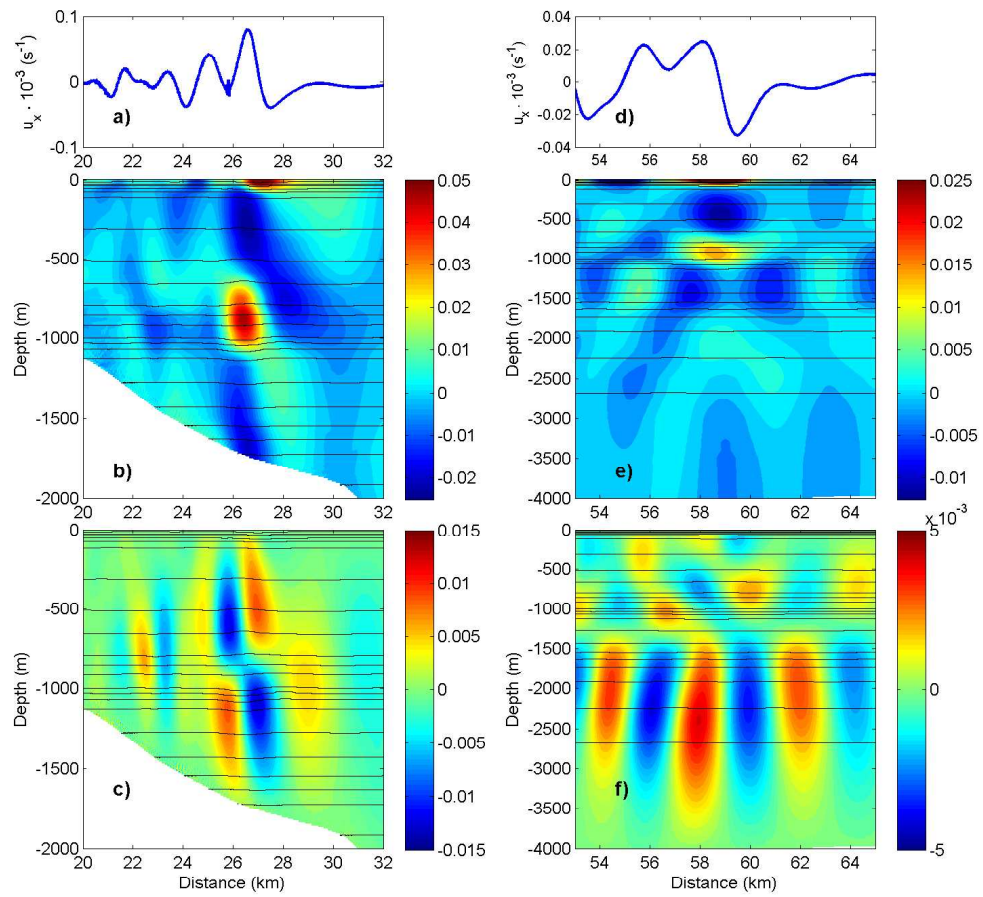


Figure 6: The same as Fig.5 but for the latest stages of ISW evolution.

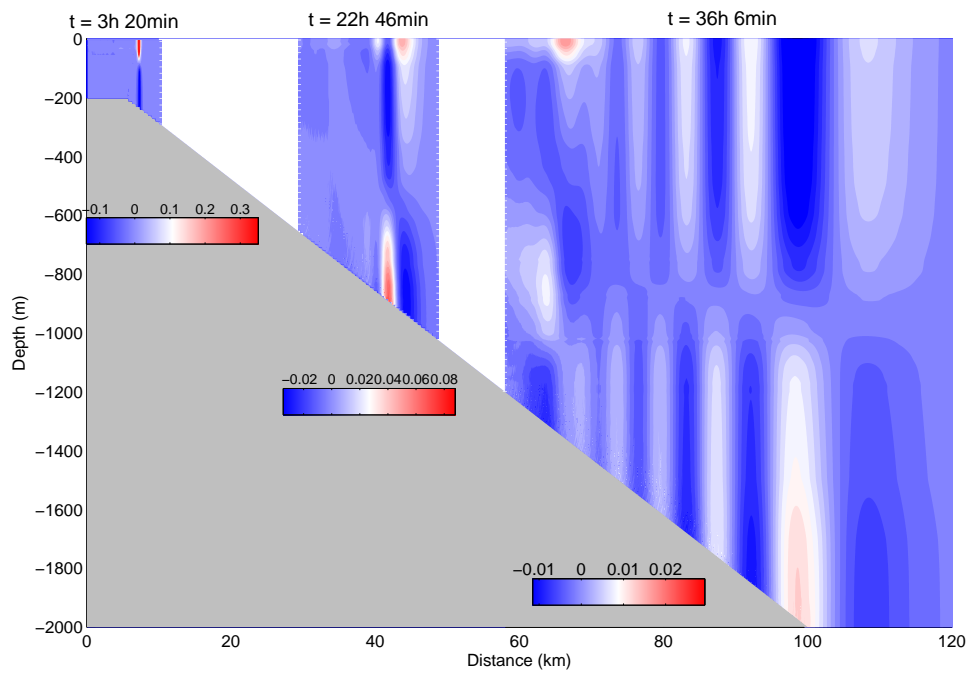


Figure 7: Horizontal velocity of the ISW propagated seaward over a gently sloping topography at three different stages of its evolution.

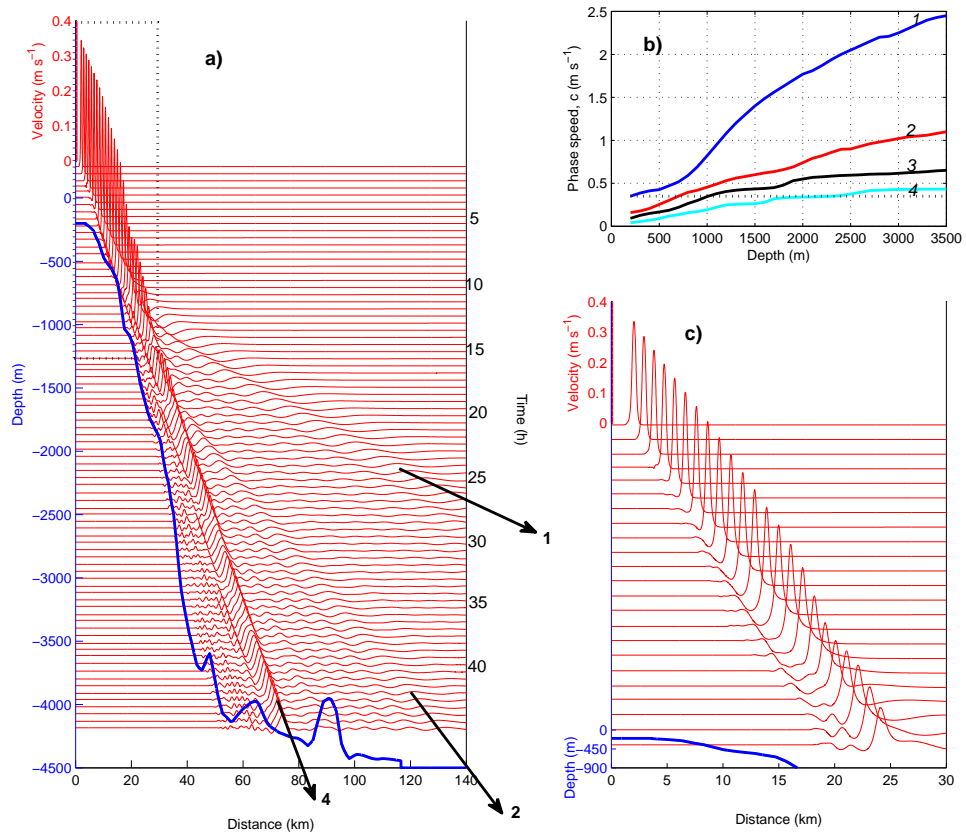


Figure 8: (a) Horizontal velocity produced at the free surface by a 53 m ISW propagated from 200 m deep shelf into the open sea. The time interval between two successive records is 2000 sec. The bottom profile is shown by a blue solid line. Arrows with numbers mark radiated wave packets of the corresponding mode. (b) Phase speed of four first modes calculated using the BVP (1) as a function of depth. (c) Zoom of the fragment depicted in panel (a) by a dashed rectangle.

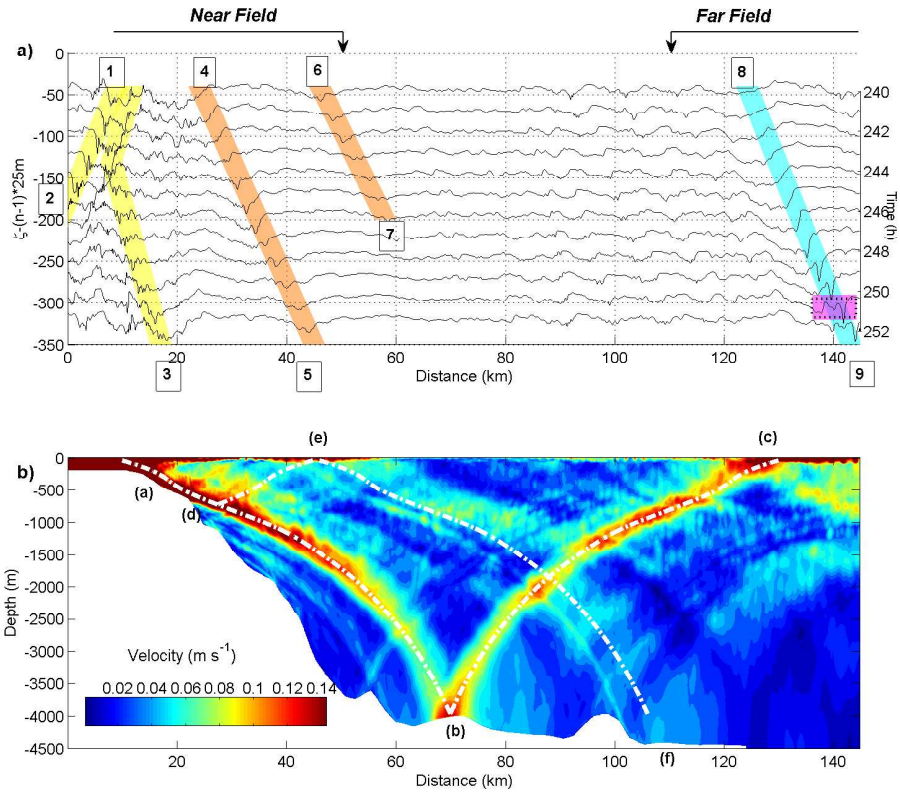


Figure 9: (a) Horizontal profiles of the isotherm 13°C taken with a 1 hour time interval after 20 tidal periods of model time. Vertical displacement ζ of the isotherm from its equilibrium depth of 47 m is calculated using the formula depicted on the vertical axis. Numbers $n = 1, 2, \dots, 12$ are counted from the topmost curve. (b) Amplitude of horizontal velocity calculated for the time span 240-252h. The Levitus based buoyancy frequency profile (see Fig.2 a) was used in this run.

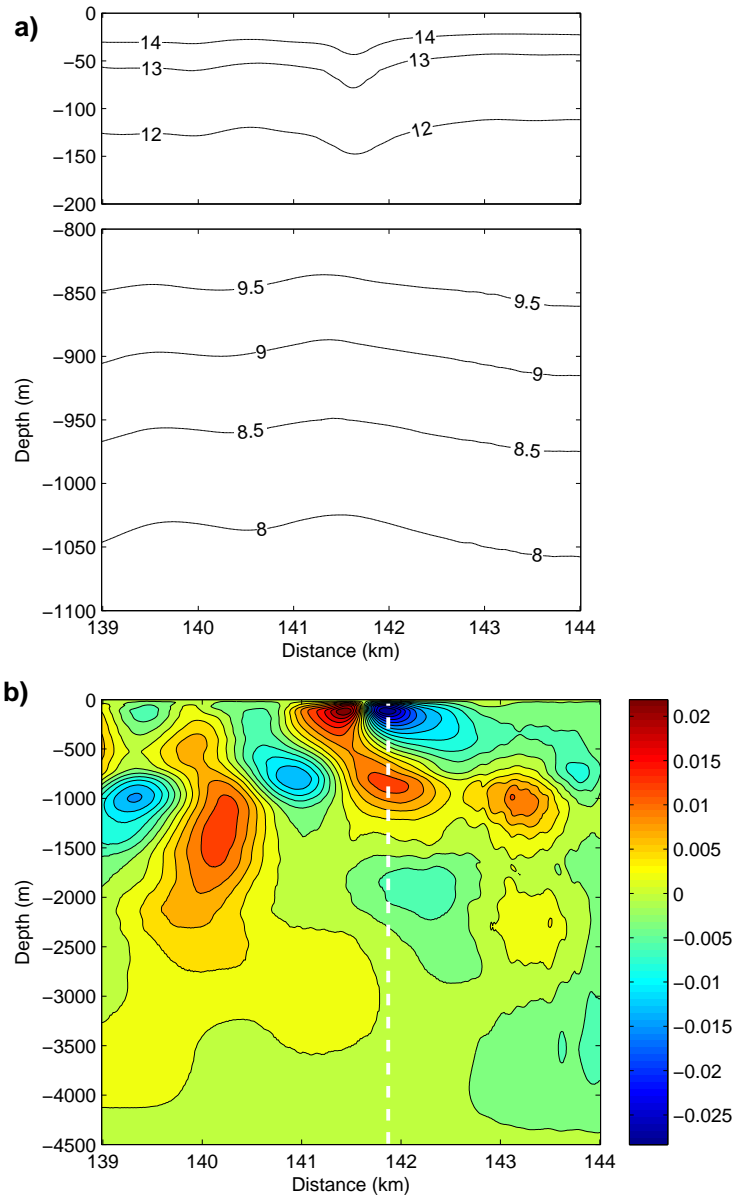


Figure 10: (a) Isotherms and (b) vertical velocity field of the fragment depicted in Fig.9 a by a magenta rectangle.

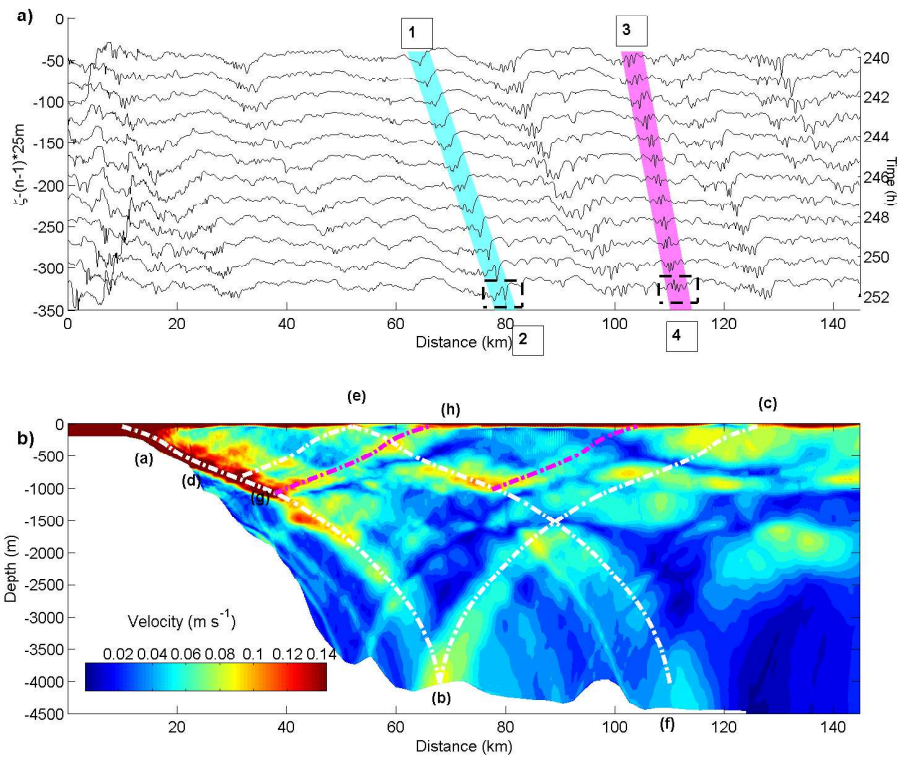


Figure 11: The same as in Fig.9 but for the original buoyancy frequency profile (the thin line in Fig.2 a). Two extra wave systems, (1)-(2) and (3)-(4), are not visible in Fig.9.

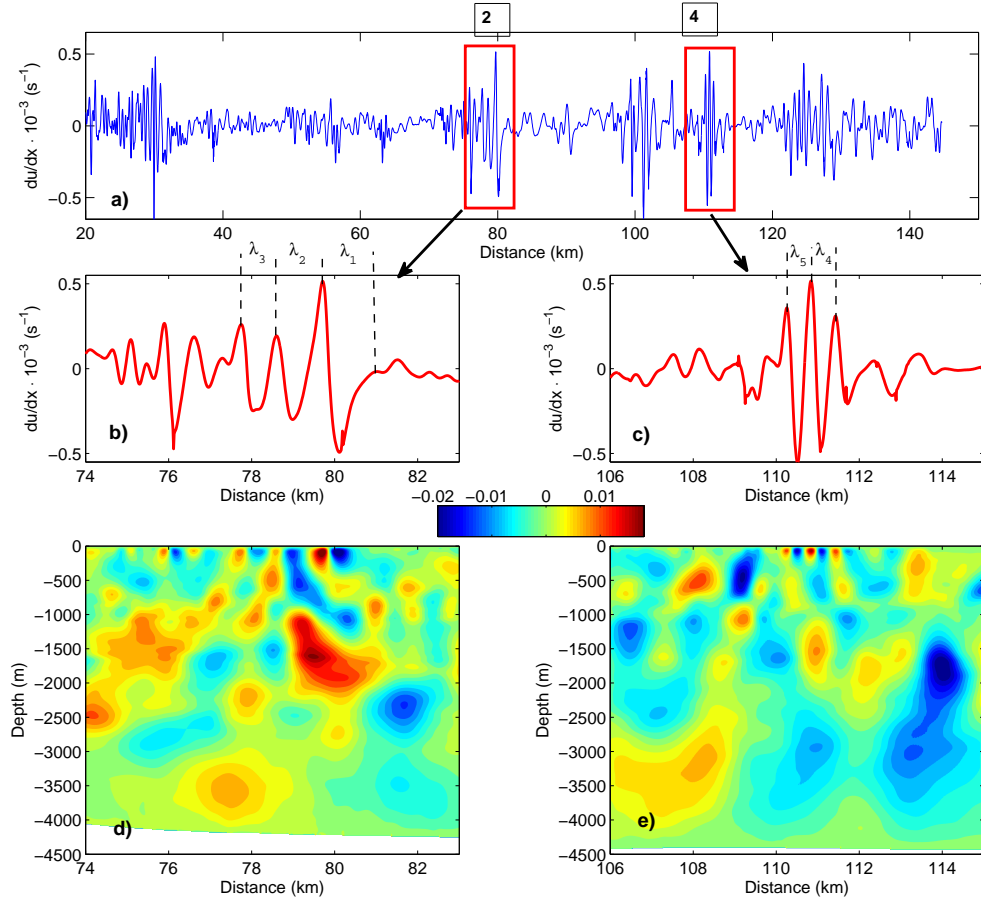


Figure 12: (a) Gradient of the horizontal velocity at the free surface $\partial u / \partial x(z = 0)$ after 252 hours of the model run (see bottom line in Fig.11 a). Wave systems (1)-(2) and (3)-(4) in Fig.11 are marked here by red rectangles with numbers 2 and 4, respectively. (b) and (c) Zoom of two fragments shown in panel (a) by red rectangles. (d) and (f) Vertical velocity of the two wave fragments shown in panels (b) and (c).

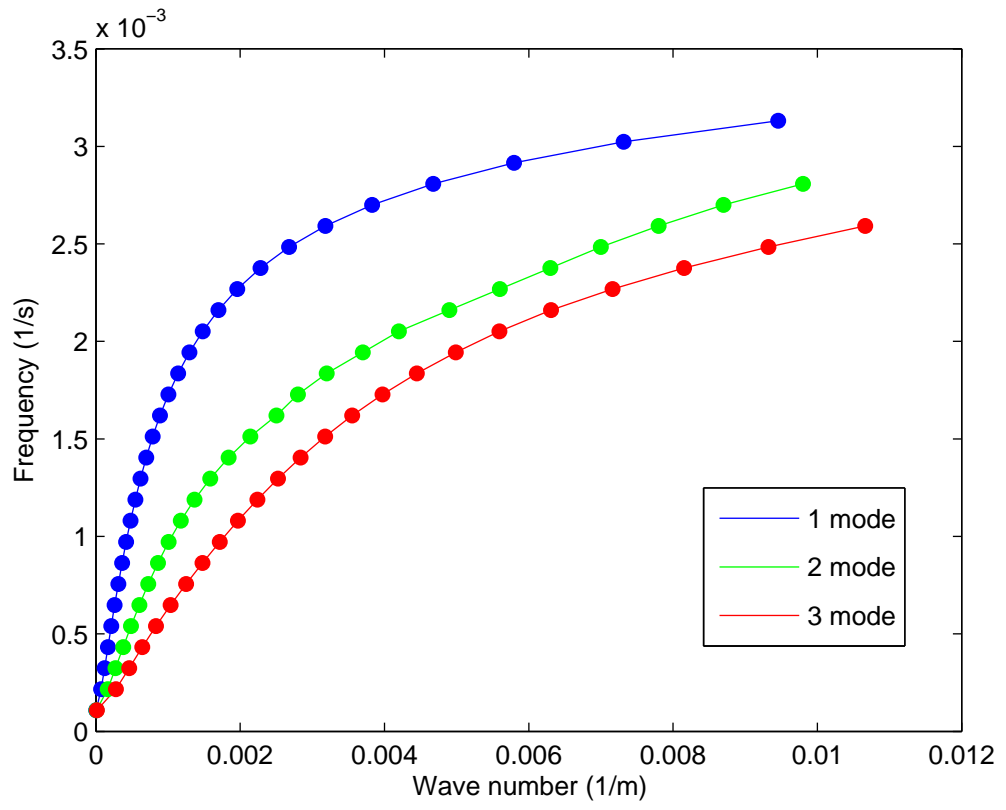


Figure 13: The dispersion relations calculated using the BVP (7) for the first (blue), second (green), and third (red) baroclinic modes.

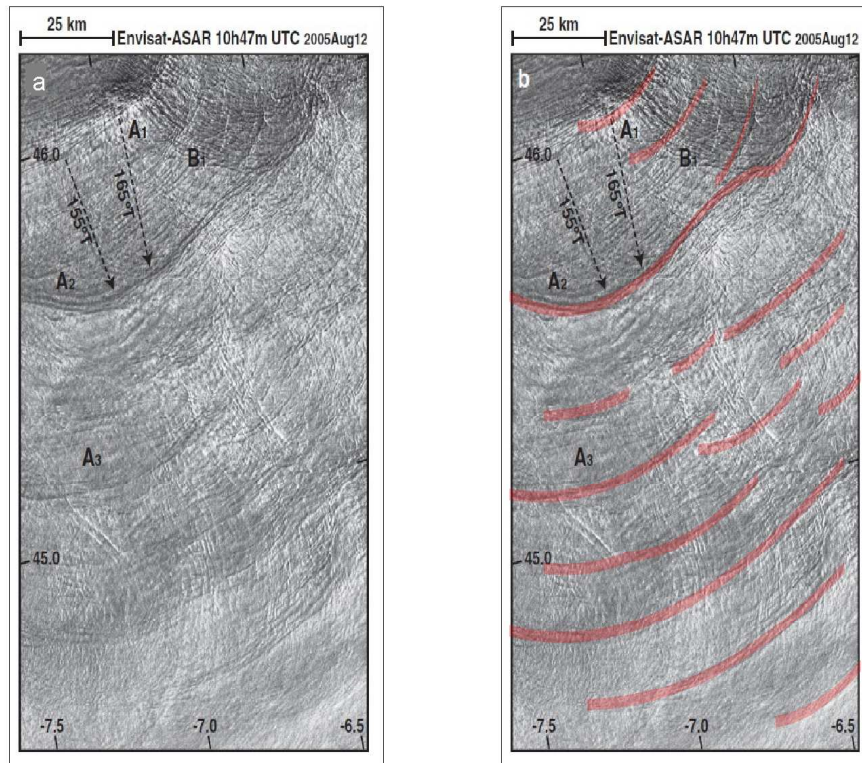


Figure 14: a) Envisat-ASAR image acquired on 12 August 2005 at 10 h 47 m UTC (Muacho et al., 2014). b) The same image with marked by red signatures of ISW packets.