Channel Coding Techniques for a Multiple Track Digital Magnetic Recording System.

Paul James Davey

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by Paul James Davey

Abstract

In magnetic recording greater areal bit packing densities are achieved through increasing track density by reducing space between and width of the recording tracks, and/or reducing the wavelength of the recorded information. This leads to the requirement of higher precision tape transport mechanisms and dedicated coding circuitry.

A TMS32010 digital signal processor is applied to a standard low-cost, low precision, multiple-track, compact cassette tape recording system. Advanced signal processing and coding techniques are employed to maximise recording density and to compensate for the mechanical deficiencies of this system. Parallel software encoding/decoding algorithms have been developed for several Run-Length Limited modulation codes. The results for a peak detection system show that Bi-Phase L code can be reliably employed up to a data rate of 5kbits/second/track. Development of a second system employing a TMS32025 and sampling detection permitted the utilisation of adaptive equalisation to slim the readback pulse. Application of conventional read equalisation techniques, that oppose inter-symbol interference, resulted in a 30% increase in performance.

Further investigation shows that greater linear recording densities can be achieved by employing Partial Response signalling and Maximum Likelihood Detection. Partial response signalling schemes use controlled inter-symbol interference to increase recording density at the expense of a multi-level readback waveform which results in an increased noise penalty. Maximum Likelihood Sequence detection employs soft decisions on the readback waveform to recover this loss. The associated modulation coding techniques required for optimised operation of such a system are discussed.

Two-dimensional run-length-limited \((d, k_y)\) modulation codes provide a further means of increasing storage capacity in multi-track recording systems. For example the code rate of a single track run length-limited code with constraints \((1, 3)\), such as Miller code, can be increased by over 25% when using a 4-track two-dimensional code with the same \(d\) constraint and with the \(k\) constraint satisfied across a number of parallel channels. The \(k\) constraint along an individual track, \(k_x\), can be increased without loss of clock synchronisation since the clocking information derived by frequent signal transitions can be sub-divided across a number of, \(y\), parallel tracks in terms of a \(k_y\) constraint. This permits more code words to be generated for a given \((d, k)\) constraint in two dimensions than is possible in one dimension. This coding technique is furthered by development of a reverse enumeration scheme based on the trellis description of the \((d, k_y)\) constraints. The application of a two-dimensional code to a high linear density system employing extended class IV partial response signalling and maximum likelihood detection is proposed. Finally, additional coding constraints to improve spectral response and error performance are discussed.
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Declaration

I declare that this thesis is the result of my own investigation, and is not submitted in candidature for the award of any other degree. At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award.

During the research programme I undertook a course of advanced studies. These included the extensive reading of literature relevant to the research project, and attendance of international conferences and seminars on signal processing, coding and magnetic recording.

Papers were presented at the Euromicro’92 Conference, Paris, France, 1992 and at the IEEE International Magnetics Conference (Intermag ’94), Albuquerque, New Mexico 1994.

Paul James Davey

Date 27/10/94
I dedicate this thesis to my children

Christine and Michael Davey

and to the memory of my grandparents

Rose & Jim Phillips
CHAPTER 1

Introduction

The advent of the information age brings an enormous demand for storage of digital data, along with the demands for processing and transmission of such data. For each of the past three decades the capacity of magnetic storage devices has risen by an order of magnitude. Most of this increased storage density has resulted from improvements in the part of the system we call "the channel", which includes the storage medium itself, the read/write heads with associated electronics, and the positioning of these heads.

If we restrict attention to linear density gains, the progress due to advances in signal processing and coding technology has also made a significant improvement to the linear density achievable with a typical set of recording components. However, instead of utilising modern modulation and coding methods that would yield performance closer to the channel capacity, the design engineers for storage systems have taken the alternative approach of increasing the channel capacity itself. This development has resulted in the utilised channel capacity being well below that which is theoretically possible.

Magnetic recording is by far the most popular technique for storing information, in particular magnetic tape has become the predominant means for mass storage of both digital and analogue data. The principal advantage of magnetic tape as a data storage medium is it can store large amounts of information in a relatively small amount of space, and at low cost per bit. In spite of the great progress in optical and electronic
technology, it seems that no viable substitution for magnetic memory, tape or disk, for mass storage will become available in the near future. Therefore, the future trend will be to improve the capacity of current recording techniques through the application of advanced signal processing.

Modern communication theory has played a major role in increasing the efficiency and reliability of communication systems. The aim of this research is to achieve analogous increase in reliability for the storage and retrieval of digital data in magnetic recording systems. More specifically, here the aim is to increase the capacity or packing density of a digital magnetic storage system by increasing the amount of data that can be stored in a unit area of magnetic media. This can be achieved by increasing linear density and/or by increasing the track density. All of this must be accomplished without sacrificing the reliability of the retrieved data.

Recent work at the University of Plymouth has demonstrated the effectiveness of employing a programmable device in the data channel of a recording system. By harnessing the flexibility of a microprocessor in an adaptive manner excellent error/data rate performance has been achieved using a standard cassette mechanism.

It is therefore proposed to extend this work and examine, in part, the application of advanced digital signal processing and, in particular, channel coding schemes to a digital magnetic tape recording system as a means of continuing the increase in density. Techniques are investigated that can more efficiently utilise the available spatial bandwidth of the magnetic recording channel, leading to the desired density increases. A further aim is also to offset the mechanical vagaries of a low-cost tape transport and tape, such as the compact-cassette format, by intelligent software algorithms. The
problem of low signal to noise ratios and high error rates can be alleviated by employing
more sophisticated coding schemes that will exploit the multiple-track capabilities of
such a system.

Chapter 2 describes the basic elements of a digital magnetic recording channel. The
chapter proceeds to describe the sources of error present in the recording channel and
some channel coding and equalisation techniques that have been applied to overcome
them. Important properties of modulation codes are discussed, in particular run-length
limited, and charge constrained modulation codes and their application to the magnetic
recording channel. A systematic review is presented of an assortment of various codes
that have been adopted in practical storage systems, some of which were also
implemented by the author in the peak detect system. The chapter concludes with a
description of some alternative modulation coding techniques that include error
correction capabilities and multi-level recording.

Chapter 3 describes the initial hardware and software employed in the first stage of the
investigation. This system is categorised by the type of detection that was implemented,
namely peak detection. The experimental procedure and results pertaining to this system
are also given. The chapter continues to discuss the application of a second digital
magnetic recording system designed and constructed by the author. This recording
system differs from the previous design in that it incorporates a sampling detection
system that periodically samples the amplitude of the readback signal and converts the
result from the analogue to digital domain, after suitable amplification and filtering.
Finally an equalisation technique employing a digital filter to pulse-slim the readback signal and increase the data rate for a given error rate is also described.

Chapter 4 discusses the application of advanced signal processing techniques, such as Partial Response Signalling and Maximum Likelihood Sequence Detection as a means to further increase linear recording density. The code constraints and several coding techniques, including trellis codes and Matched Spectral Null codes, pertaining to such a system are described in some detail.

Chapter 5 describes a new class of modulation codes that exploits the two-dimensional properties of multi-track digital magnetic recording systems to provide increased areal storage density. Techniques for constructing multi-track codes are discussed and a new technique for implementing a two-dimensional code is presented. The chapter ends by discussing the application of a new two-dimensional code to a multi-track system employing extended class IV partial response and maximum likelihood detection.

Finally the author's conclusions are presented in Chapter 6, together with areas of further work.
CHAPTER 2

Background to the Investigation

2.1 Magnetic Recording of Digital Information

The magnetic recording process is based on the interaction between the magnetic storage medium and the magnetic head; usually the two are moving with respect to each other. In the recording process the head magnetises the medium, whilst during replay the head\(^1\) generates an induced voltage reflecting the rate of change of magnetisation recorded on the surface.

The magnetic recording channel can be viewed as a communication channel that has a band-pass frequency response which suffers from both amplitude and timing instability and is non-linear due to the hysteresis exhibited by the magnetic medium. Transmission of d.c. is prevented since the read head output voltage is proportional to the derivative of the head flux. The low frequency limit is directly related to the overall physical dimensions of the read head, whereas the high frequency response is limited by the read head gap null, the inductance of the read head and the existence of spacing losses. At

\(^1\) This discussion assumes an inductive head. Magneto-Resistive heads produce a signal proportional to the flux rather than the rate of change of flux.
low and medium frequency a recording channel may be rendered linear by the use of a.c. bias.

2.1.1 Digital Recording Theory

A general block diagram of a digital magnetic recording system is shown in Figure 2.1.

Binary source data are first encoded with an error correction code (ECC), then modulation encoded\(^2\). The modulation encoded data are then sent to a write driver which produces a two level waveform that is written onto the magnetic medium. The magnetisation on the medium is then read back and this *readback* waveform is assumed to have been corrupted by noise. The readback waveform is passed through an equaliser which shapes the spectrum to match a desired target response. Next, the equalised

---

\(^2\) A considerable amount of work has been done on combining the ECC and the modulation codes discussed later, however, most current recording systems keep the two processes separate.
waveform is sent to the detector which produces an estimate of the channel data. The estimate is then decoded by the modulation decoder and finally an error correction decoder is employed to recover any errors.

A multiple-track recording system consists of two or more basic systems in parallel, each track usually working independently of the others. However, the multi-track recording system used throughout this thesis employs the operation of several tracks simultaneously. It will be discussed later in this dissertation how parallel channels offer distinct advantages, in terms of recording density and clock recovery, over conventional serial operation.

**Channel Capacity**

Shannon [1] proved that the channel capacity $C$, could be calculated such that if the maximum information rate $R$, at which information can be transmitted is less than $C$, data can be sent error free through a noisy channel. Channel capacity $C$ is defined as

$$C \text{ (bits/second)} = B \log_2 (1 + \text{SNR})$$  \hspace{1cm} \text{eqn{1}}$$

where

$B = \text{channel Bandwidth (Hz)},$

$\text{SNR} = \text{Signal to Noise Ratio} = \text{Signal power (watts)} / \text{Noise power (watts)}.$

This classic law assumes Additive White Gaussian Noise (AWGN), i.e. the noise is additive, covers all frequencies and has a Gaussian distribution.

This shows that the bandwidth of a channel and the signal to noise ratio (SNR) may be traded off against each other in achieving the desired capacity.
Mallinson [2] shows that the SNR for a tape of width $W$ may be approximated as

$$\text{SNR} = W N_p (\lambda_{\text{min}})^2 / 2 \pi$$

where

- $\lambda_{\text{min}} =$ minimum recorded wavelength,
- $N_p =$ number of magnetic particles per unit volume.

Therefore if the width of the tape is divided into $y$ tracks (ignoring guard-bands) the SNR of each track is reduced by $1/y$, giving a total channel capacity of

$$C_y = y B \log_2(1 + \text{SNR}/y).$$

Therefore, from equation (3) the capacity will increase linearly with the number of tracks as will the areal packing density. However, this is at the expense of reduced SNR. In addition, equation (2) shows that doubling the number of tracks reduces the SNR by 3dB, whilst halving the minimum recorded frequency reduces the SNR by 6dB. Hence, it seems beneficial to have a multi-track system operating at a reduced data rate. However, as track widths narrow substantially other sources of noise such as crosstalk and dropouts become more relevant.

Systems that approach the Shannon bound usually incorporate error correction coding, where enough redundancy has been added to the transmitted signal to allow the decoder to detect and correct any errors that might occur.

One of the problems which makes research in signal processing for magnetic channels difficult is that entirely satisfactory models have not been found. The mathematical models which are used today range from finite difference methods requiring super-computers to linear systems approaches running on personal computers. The models
which accurately capture the essence of a magnetic recording system tend to be far too complex for signal processing applications, whilst the models that are simple enough for use in signal-processing algorithms generally have extreme restrictions on the operating conditions in which they are valid. In addition the noise which one must deal with is non-stationary and not very well characterised and in some cases, such as thin -film media, is data dependent.

Therefore, in this research we have tried to limit channel modelling to a minimum, using real or sampled signals whenever possible. However, this is not always convenient for predicting results.

**Pulse Superposition**

Linear pulse superposition [3] applied to magnetic recording states that the voltage waveform produced by a series of flux reversals is the algebraic sum of a series of isolated pulses, centred on the flux reversals.

Expressed mathematically, the combination of a number \( x \), of isolated pulses \( p(t) \), separated by \( T/2 \) is

\[
e(t) = \sum_{x} (-1)^x \cdot p(t + x \cdot \frac{T}{2})
\]

where \( T = 1/\text{data rate} \).

Once the shape of the isolated pulse \( p(t) \), has been determined, the voltage of the replay signal can be generated by combining pulses with the appropriate spacing for any recorded data sequence at any desired packing density.
Non-linear Distortion

The principle of linear superposition is very accurate for systems operating at low densities and at low data rates. However, it has been well documented that the linear superposition model begins to break down as the spatial separation between recorded transitions becomes smaller [4,5]. The most common form of non-linear distortion is non-linear bit shift or non-linear inter-symbol interference. Non-linear bit shift is due to the magnetic field interaction between adjacent recorded transitions and the head field as they are being recorded. The non-linear interaction depends upon the previously written data and as such can usually be precomputed and compensated on the write side with write-precompensation (discussed later).

As the spatial density of the recording system is further increased other non-linear distortions can occur such as partial erasure [6]. These non-linearities are beyond the scope of this investigation and as such are neglected.

Lorentzian Model

The accuracy of modelling the channel using pulse superposition is dependent on the shape and width of the isolated pulse used. A simple and frequently used expression to represent the replay signal is given by the Lorentzian pulse described in Equation (5)

\[ p(t) = \frac{1}{1 + \left(\frac{t}{PW_{50}}\right)^2} \]  

where

\( PW_{50} \) is the pulse width at half (50%) height.
From the work of Karlquist, for distance $y > g/2$, the head field components, illustrated in Figure 2.2, are:

$$H_x = \frac{H_g}{\pi} \left[ \tan^{-1}\left(\frac{g/2 + x}{y}\right) + \tan^{-1}\left(\frac{g/2 - x}{y}\right) \right]$$  \hspace{1cm} \text{eqn} \{6\}

$$H_y = -\frac{H_g}{2\pi} \ln \frac{(g/2 + x)^2 + y^2}{(g/2 - x)^2 + y^2}$$  \hspace{1cm} \text{eqn} \{7\}

![Record Head geometry](image)

**Figure 2.2 Record Head geometry**

A purely longitudinal field results in a symmetrical pulse and a purely perpendicular field produces a dipulse. The resultant response to the current transition is a combination of the two fluxes from the longitudinal and perpendicular component.

Further analysis and development of models for the read/write process is beyond the scope of this research.
2.1.2 Practical Limitations

The recording channel differs from most communication systems in a number of respects. Reliability requirements are usually much higher for magnetic recording systems. For instance, an error rate of $10^{-12}$ or less is not uncommon, also signal power is limited and can not be increased with respect to noise. It is therefore essential to completely define and understand all the error causing mechanisms.

Noise

Many communications’ systems have only a single type of noise, whereas in magnetic recording channels there are several major sources of error:

a) media noise,

b) electronics noise,

c) read/write noise.

Despite limited amounts of available quantitative data, the behaviour of the noise can be roughly described as follows.

Media noise is due to the manufacturing irregularities and to weak recorded tracking information. Media noise may contain a spike component, due to impurities, which results in such a large noise level that no data bits can be written at those geographic locations on the magnetic surface. The noise also contains a continuous component, which may be due to the non-uniformity of the ferromagnetic material.

All components with resistance generate noise according to their temperature, the electronics and more importantly the read/write head are no exception. Over the normal
operational temperature range this noise is more or less constant. The reading and writing noise contain an electrical component of uncertain spectrum, possibly white, and a mechanical component of coloured spectrum, resulting from the variations in distance between the head, the medium and neighbouring head/medium signals. In general, for a given recording on tape, a better signal to noise ratio will be obtained by moving the tape relative to the head at a higher speed, since the head noise is constant and the signal induced is proportional to speed. This is one advantage that rotary head recorders have over stationary head recorders.

*Inter-Symbol Interference*

Due to the finite gap of the replay head, increasing the density of recorded data stored on the magnetic medium causes closely spaced magnetic transitions to interfere with one another. This phenomenon, known as Inter-Symbol Interference (ISI) results in peak shift distortion, also known as pulse crowding, and reduction in signal amplitude causing timing and detection errors.

The recording density effectively specifies a length of track or time interval in which each flux reversal is contained. This is termed the bit-cell or bit interval. In digital recording, a large number of output pulse patterns arise. At low densities each pulse is individually resolved. With higher densities the dispersivity of the channel causes each flux reversal to spill over from its cell into the cells of its neighbours. In this way the flux reversals or symbols interact in such a way as to cause distortion.

The adverse effects of ISI are manifested in two ways:-
a) Firstly the ISI causes the position of the replay pulse peaks to be shifted, known as 'peak shift'. This causes the signal to be less resilient to the addition of noise and gives rise to a discrepancy in pulse location timing with respect to the original clock time period on writing. Figure 2.3 illustrates the output signal when there is an isolated pair of magnetisation changes separated by a distance $t$ appreciably less than $\text{PW}_{10}$ (Pulse Width at 10% of its height). The individual signal from each saturation reversal is shown as well as the resultant voltage, obtained by the superposition of the individual pulses.

b) Secondly since the replay pulses alternate in polarity they interfere destructively resulting in a reduction of signal amplitude. This causes the SNR to be reduced and hence causes the probability of error to increase. When a stream of four successive saturation reversals are recorded, the inner pulses are surrounded by two adjacent pulses of opposite polarity and therefore are reduced in peak amplitude to a greater degree than the pulses on the waveforms' extremities. The net effect is to give the waveform the appearance of a 'droop' as shown in Figure 2.4. It is clear that there is also a baseline shift, down for the first half of the waveform and up for the second. Droop is a major disadvantage in peak and threshold detection techniques because at a critical packing density the peaks will no longer cross the zero threshold and as such can no longer be detected.
Figure 2.3 Inter-Symbol Interference induced Peak Shift

Figure 2.4 Inter-Symbol Interference induced Droop
Azimuth Variation (Skew)

The initial task of the channel decoding process is to detect transitions in the data stream and from this decide at what point the data should be sampled. If all tracks are sampled simultaneously, tape-azimuth variations, also known as skew, need to be taken into account. Ideally, the tape should pass perpendicularly to the head, in a straight line. However, in practice, due to the limited tape guidance system and imperfect slit edges, the tape weaves across the head producing a constantly varying skew angle, illustrated in Figure 2.5.

At low linear packing densities this is not a significant problem as the magnitude of skew is insufficient to cause the sample point to drift into the next bit cell. However, as the linear packing density increases, the magnitude of the data-skew (measured in bits) between tracks increases up to a point where it can become the most significant cause of errors.

Several attempts [7-10] to compensate for the azimuth variation have been reported which involve dynamically rotating the read head to correspond with the maximum amplitude of the readback signal. However, this method is mechanically expensive and generally outweighs the cost of a better tape guidance system. An alternative software solution employed by Donnelly [11] involves initially sampling the readback signals, computing the degree of skew and then re-aligning them digitally. Although more computationally intensive, the electronics to perform this technique are relatively inexpensive but are limited by speed of operation. Since azimuth variations change slowly with respect to the data rate this does not pose a major problem.
**Figure 2.5 Azimuth Variation as Tape passes Read Head**

**Jitter**

In analogue audio equipment timing instability of the readback waveform gives rise to wow and flutter: in digital recording this term is referred to as *jitter*. In the main jitter is largely due to head-tape velocity variation. Magnetic tape transports are designed to provide a linear motion of the tape with as little variation as possible. However, in practice the combination of mechanical and electrical tolerances for the tape drive in conjunction with environmental factors such as vibration, etc., result in velocity variations exceeding the specified tolerance [12].

Static friction (stiction) between the in-contact head and slow moving tape can cause micro velocity changes which are excited by the flexibility and surface irregularities of the tape. Together these are also responsible for the most significant amount of jitter.
Figure 2.6 illustrates how jitter causes uncertainty about the signal voltage relative to a stable time reference for a signal with a finite rate of change of voltage; this has the same effect as noise. Since jitter is directly proportional to the rate of change of voltage in the readback signal the degree to which it affects the error rate increases with frequency. This introduces an important channel code parameter, the *jitter margin* $T_w$, also termed *window margin* or *phase margin*, that determines the tolerance in locating a transition in a bit cell. It is defined as the permitted range of time over which a transition can still be received correctly, divided by the data bit cell period $T$. In general the larger the value of $T_w$ the better since it has the effect of reducing peak shift distortion.

![Diagram showing voltage error and timing uncertainty.](image)

**Figure 2.6** Jitter results in amplitude error due to timing uncertainty.

In a sampling system, a finite amount of variation in the sampling clock can lead to *timing jitter*. This has to be carefully controlled since this has the potential to cause more catastrophic errors than in the previous type of jitter. Since the theory underlying the correct operation of most digital signal processing techniques, such as filtering etc., relies on samples at regular intervals.
**Dropouts**

Many shallow drops in the envelope of the readback signal can be observed in high density digital magnetic recording. Asperities and discontinuities in the recording medium cause *hard* dropouts which have a drastic effect on the recorded signal, however these are minimised by modern medium manufacturing techniques. More common are losses in level caused mainly by instantaneous increases in the separation between the tape and the head due to surface roughness of the tape and debris such as dust and oxidised particles. These are termed *soft* dropouts. Increase in separation causes not only a drop in the envelope but also a loss of amplitude in the higher frequencies of the readback signal. Perry et al. [13] have demonstrated how a microprocessor based system can be used to characterise the effect of dropouts for a high density digital magnetic tape system. Meeks [14] further describes how an appropriate error correction strategy can be selected once the dropout characteristics are determined.

**Crosstalk**

To increase track density care must be taken that the tracks are not so closely spaced that serious pickup leakage interference arises from adjacent recorded tracks. Since the amount of leakage increases with the wavelength of the recorded signal, (similar to the separation loss), the leakage can become pronounced potentially resulting in problems at low frequencies. This type of noise is termed crosstalk and can be considered as similar to the influence of Gaussian noise, since the leakage power is restricted to lie within the low frequency range and the random data signals recorded on the tracks are presumed to have no correlation with each other. Hence channel codes that have little or no low frequency response are less prone to crosstalk induced errors.
2.2 Channel Equalisation

To enhance the performance of the magnetic recording channel at high packing densities appropriate equalisation techniques must be applied to match the frequency and phase response of the recording channel to that of the recorded signal. Equalisation is the process that modifies the transfer function of the analogue channel to provide more reliable data detection by compensating for the channel distortions such as ISI and peak shift. However, the improvement in the channel transfer function is usually accompanied by a degradation in signal to noise ratio, resulting from boosting the high frequencies where there is little signal and much noise. Therefore, although any degree of equalisation can be theoretically applied, the equaliser design will usually be a compromise between the required transfer function and the resultant loss in SNR due to equalisation.

The choice of equalisation depends upon the following:

a) the amount of inter-symbol interference to be compensated,

b) the modulation code,

c) the detection technique used,

d) the signal to noise ratio,

e) the noise spectrum shape.

Channel equalisation may be implemented either prior to the recording process, in which case it is referred to as write equalisation or write precompensation, or during the replay process where it is termed read or post equalisation or simply just equalisation.
2.2.1 Write Equalisation

Write equalisation attempts to modify the spectral components of the signal to be recorded or written to match the frequency response of the channel. The principle benefit of write equalisation is that all the signal conditioning is performed before noise is introduced into the system, hence reducing noise in the readback signal relative to post-equalisation.

However, in practice it is difficult to provide correct equalisation using a write equaliser alone since the channel response is constantly varying due to tape surface asperities, substrate irregularities and the intimacy of the head contact and wear. These variations attenuate the high frequencies much more than the low frequencies which undermines any predicted or fixed equalisation scheme.

Write equalisation can be sub-divided into three different techniques:

Amplitude write equalisation

This method varies the amplitude and/or shape of the write current so that it is no longer a binary signal. Jacoby [15] suggested a cosine equaliser, which is sometimes used for read equalisation, to shape the write current. This technique is not popular due to the need for complex write-side electronics.
**Precompensation**

Precompensation is the favoured technique for write equalisation in the magnetic recording channel since it is data dependent and maintains a binary write signal and is therefore relatively easy to implement. Equalisation is achieved by repositioning the write transitions to offset peak shift in the readback signal [16]. For example, consider the binary signal 0110, as shown in Figure 2.7. The first transition could be written slightly later than its normal clock time and the transition of the second bit slightly earlier in time. This precompensation effectively uses a controlled amount of intersymbol interference to improve the overall readback performance by reducing peak shift of the two pulse peak relative to the clock window centres. Since the data to be recorded is known, a set of precompensation rules can be implemented to deal with the data patterns that create the worst-case performance.

Other methods include the use of redundant magnetic transition pairs at strategic locations on the write waveform [17,18].

![Figure 2.7 Write Precompensation used to equalise a readback signal](image)
**Precoding**

Both Tomlinson [19] and Harashima and Miyakawa [20] independently proposed precoding as a means of equalisation over twenty years ago. This technique employs a feedback transversal filter whose impulse response is the inverse of the channel and which can also maintain a binary write signal through the use of modulo-two arithmetic. This technique although not yet fully utilised in magnetic recording has been used, in conjunction with Trellis Coded Modulation, with great success for achieving increased data rates in telephone-line modems [21].

### 2.2.2 Read Equalisation

This method refers to the use of a linear filter at the output which changes the overall impulse response of the system before the peak detector.

Since read equalisers operate on the signal after noise has been added by the channel the noise spectrum is modified as well as the signal. Therefore an improvement in distortion is usually accompanied by a degradation in signal to noise ratio.

The equaliser increases the bandwidth of the signal by compensating for the loss in high frequencies produced by the recording channel. The increase in bandwidth results in a narrower pulse in the time domain, which has the effect of reducing pulse interaction and hence reduces ISI induced peak shift. However, the amplification of the higher frequency components of the signal spectrum extends the noise power spectral density causing an increase in average noise power. Hence, noise induced peak shift is increased. This trade off between ISI and noise-induced peak shift is the basis of equaliser design.
Pulse Slimming

Pulse slimming is a time domain approach to read equalisation, its purpose being to create a channel whose isolated transition response is a thinner pulse than that produced by the unequalised channel. Many different types of pulse-slimming filters have been used to reduce adjacent pulse interference [22-28].

Jacoby [23] describes one of the more classic equaliser circuits, achieved by a resistor, inductor, capacitance network which controls signal amplitude and phase separately.

Kameyama [25] described a cosine equaliser, that slimmed an isolated pulse by approximately 30% when the SNR at the input is 35dB. The basic circuit is composed of a delay line, an amplitude divider and a differential amplifier. The delay line is terminated with a matching impedance at the input and is open ended at the output so that the signal is completely reflected. This causes the incoming and reflected pulse to add constructively and therefore double in amplitude. For an input signal of $f_i(t+\tau)$ the output of the equaliser $f_o(t)$ is expressed as

$$f_o(t) = 2f_i(t)-K(f_i(t+\tau)+f_i(t-\tau)) \quad \text{eqn}(8)$$

where $\tau$ is the delay and $K$ is the ratio of the amplitude divider.

The resultant transfer function of this equaliser described in the frequency domain is

$$F(\omega) = 1 - K \cos (\omega \tau) \quad \text{eqn}(9)$$

hence the name cosine equaliser.
2.3 Signal Detection

Whereas there have been significant technology developments in read/write heads, magnetic media and servo systems, the analogue Peak Detection [29] method of data recovery has remained largely unchanged for over 25 years.

Data detection in the conventional peak detection magnetic recording channel is achieved by first differentiating the analogue signal and then processing the differentiated signal with a zero crossing detector to determine the presence or absence of a zero crossing event within the detection window. In the absence of noise or other imperfections the zero crossing of the derivative signal in peak detection occur only at times corresponding to the clock times at which a transition was written. Enhancements such as precompensation, Run-Length-Limited codes and more sophisticated detectors have extended the performance of peak detection systems.

The peak detector has the advantage of being both robust and extremely simple to implement. However, by its very nature it performs best at low linear densities. The underlying technique used in all peak detection systems is termed a hard limited process. The output is determined to be either above or below a decision threshold and performs hard decisions on each bit (symbol-by-symbol) coming out of the channel. In doing so this method of detection looses information about how close the signal is to the threshold and as such how good the decision was. If the data bits passing through a channel are independent of one another the performance of the detector is optimum. However, modulation coding introduces correlation among the bits in a sequence and therefore a hard-decision detection is not optimum.
An optimum detection process for correlated data uses a *soft decision* algorithm that decides the result based on past, present and future decisions being above or below a decision threshold and then gives a measure of "goodness" or confidence that determines how close the result was to the threshold level.

Therefore, whilst the peak detector has been acceptable in the past in terms of simplicity, low cost and speed, it has become increasingly apparent that to make more efficient use of available bandwidth a more sophisticated detection method is required. Recent attention has focused on sampling detection and an entirely new type of modulation coding and signal processing, a revolutionary rather than evolutionary approach. These techniques are described further in chapter 4.

A prime requirement in a digital communication channel employing soft decision detection is that the amplitude of the readback signal is periodically sampled. Historically this has prevented the graduation to digital signal processing due to the high speed requirements of many storage systems and the high cost of fast silicon. Hence, only recently, with the introduction of high speed compact mixed digital and analogue signal processors, such techniques have been made feasible. Conversion from the analogue to digital domain also permits other advanced DSP techniques to be performed on the signal such as adaptive equalisation and clock recovery.
2.4 Modulation Coding

In coding for a magnetic recording channel using saturation recording, engineers are presented with overcoming an array of potential problems, some of which are not clearly understood. As opposed to most communication systems:

(a) timing recovery must be obtained from the modulated data,

(b) the noise is non-white and may have a (pattern dependent) multiplicative nature,

(c) most detection systems are sub-optimal,

(d) uncertainty in channel characteristics must be tolerated,

(e) inter-symbol interference dominates at high recording densities.

Due to the differentiating nature of the read head, only polarity changes in the magnetic-medium saturation produce significant energy at the output of the read electronics. As such, the magnetic channel is often modelled as the linear superposition of these step responses. Since timing recovery must be obtained from the channel response to the modulation code all modulation codes require relatively frequent changes in magnetic saturation.

Two common notations used to designate magnetic modulation codes are Non-Return to Zero (NRZ) and Non-Return to Zero Increment (NRZI). Actual recording bits consist of saturation in one direction (+1) or in the other (-1). In NRZ notation a charge in one direction of saturation is referred to as a '1', whilst a charge in the opposing direction is referred to as a '0'. In NRZI notation a change in direction of saturation is referred to as a '1' and no change i.e. constant state of saturation is referred to as '0'. In
NRZI coding, the modulation code first undergoes a precoding operation to convert to a
NRZ bit stream. The NRZ bit stream is level shifted and amplified to produce the actual
recorded bits (write current), illustrated in Figure 2.8.

\[\text{Figure 2.8 NRZ & NRZI coding schemes}\]

Note that the magnetic recording channel converts a positive-going transition into a
positive pulse and a negative-going transition into a negative pulse. We could observe
the presence and polarity of peaks, assigning 1, -1, or 0 to a bit period according to the
presence of a positive or negative peak, or the absence of a peak (peak detection). Or,
we could equalise the channel to force the output signal to be normalised sample values
1, -1 and 0 at the centre of the appropriate clock intervals (sampled amplitude detection).

The use of a recording code is beneficial for three reasons:

a) to maintain frequent transitions suitable for timing clock recovery,

b) to increase the density of recorded data above that which could be achieved
without the use of a suitable code,

c) to match the frequency spectra of the raw digital data to the response of the
magnetic recording channel.
These criteria have lead to the development of several categories of coding techniques:

a) Run Length Limited (RLL) codes,

b) Charge constrained (d.c.-free or d.c.-balanced) block codes,

c) Trellis codes.

The following sections will describe run length limited codes and charge constrained code since these are most commonly used in conventional peak detection systems. Chapter 4 will deal with trellis codes since this type of coding is used in channels employing partial response signalling and maximum likelihood detection.

2.4.1 Run-Length Limited (RLL) Codes

Run-Length-Limited codes are usually classified according to their construction, implementation or certain desirable properties that they possess. They have found almost universal application in magnetic and optical disk recording systems [30].

Whatever the notation, the requirement of relatively frequent changes in magnetic saturation for efficient timing clock recovery is satisfied by limiting the maximum run of zeros in the code to some integer $k$. Also, it is often desirable to limit the effects of ISI and improve performance of the peak detector by imposing a separation between transitions. This is achieved by imposing a $d$ constraint that relates to the minimum wavelength $\lambda_{\text{min}}$ which determines the highest transition frequency and thus is a measure of the code’s susceptibility to ISI over the band-limited magnetic recording channel. Conversely, the maximum runlength parameter $k$, controls the lowest transition frequency, which ensures frequent transitions for synchronisation of the read clock.
Obviously, \( k \) and \( d \) are positive integers where \( k > d \). Such codes are termed Run-Length Limited (RLL) or \((d, k)\) codes, where the \( d \) and \( k \) represent the minimum and maximum number of zeros between adjacent changes in level (transitions) respectively. The vast majority of codes used in magnetic recording fit some \((d, k)\) constraint, selected according to the channel response, information density, jitter and noise characteristics.

Coding is achieved by mapping \( m \) information symbols into \( n \) binary code symbols. A measure of efficiency for a particular code is given by

\[
\text{Code Rate } R = \frac{m}{n}, \quad \text{where } R < 1. \quad \text{eqn}(10)
\]

The rate of a code also completely determines the available time for detecting the presence or absence of a transition, called the Detection Window, \( T_w \) (normalised), usually measured in terms of bit cell duration \( T \). For any given density, codes with high rates are less sensitive to timing jitter, caused by noise and peak shift, due to a reduced detection window.

It can be readily verified that the minimum \( (T_{\text{min}}) \) and maximum \( (T_{\text{max}}) \) distance between consecutive transitions for any RLL sequence is given by \( R.(d+1) \) and \( R.(k+1) \) respectively. This gives rise to an important measure of recording efficiency, defined by the ratio of data density versus the highest density of recorded transitions, termed the Density Ratio \( (DR) \), or packing density. In fact the \( DR \) is numerically equal to \( T_{\text{min}} \) and is therefore defined as

\[
T_{\text{min}} = DR = (d+1).\frac{m}{n}. \quad \text{eqn}(11)
\]
However, a more realistic measure of code performance can be obtained by considering both the density ratio and the jitter margin. Therefore a figure of merit (FoM), defined as

$$\text{FoM} = DR \times T_w = (d+1) \times m^2/n^2$$

is often used as a yard stick for code comparison.

RLL codes are generally characterised by five basic parameters $m/n(d, k, c)$, where the final parameter $c$ is a measure of the charge constraint. The charge constraint is assigned the value derived from the modulus of the maximum Digital Sum Value (DSV) also known as the Running Digital Sum (RDS). The DSV is defined as the running integral of the area beneath the code or more simply the accumulated sum of the recorded data bits, counted from the start of a recording sequence, assuming the binary levels to be $\pm 1$. If the DSV is bounded the code is d.c.-free. Codes that do not posses any charge constraint i.e. $c = \infty$, usually omit this parameter. This parameter will be discussed in further detail in the following sections.

The information capacity of an unconstrained binary sequence is 1 bit/symbol, therefore that of a constrained sequence is necessarily less than 1bit/symbol. As coding constraints are increased so the information capacity of each symbol decreases. A measure of the information carrying capacity of a RLL sequence is the entropy per bit. The maximum entropy per bit is defined as the capacity.

In his classic paper Shannon [1] showed that the rate $R$, of any constrained code is bounded by the Code Capacity (asymptotic information rate). It defines the theoretical
upper limit of the maximum number of information bits that can be represented by a
consstrained sequence and is defined as

\[ C = \lim_{n \to \infty} \frac{1}{n} \log_2 N(n) \]  

where \( N(n) \) is number of binary sequences of length \( n \).

Tang et al [31] calculated the maximum code rate that can be achieved by any runlength limited code \( C(d,k) \), as

\[ C(d,k) = \log_2 \lambda \]  

where \( \lambda \) is given by the largest real root of

\[ x^{k+2} - x^{k+1} - x^{k+1}d + 1 = 0 \]  

When operating at capacity the sequences that are produced are called Maxentropic
[32]. In practice, a rational number \( m/n \leq C(d,k) \) is chosen for the rate of the code. The
code efficiency is a measure of how close the code rate is to the code capacity, defined
as

\[ \text{Code efficiency} = \frac{\text{Code Rate}}{\text{Code Capacity}}. \]

Franaszek [33] found that practical codes could easily achieve efficiencies in the range
90-95%. From equation {15}, increasing the \( d \) constraint leads to a decreased code rate.
However, the higher values of \( d \) provide greater separation between consecutive
transitions, therefore allowing an increased packing density. In reality the increased
packing density has to be traded off against a decrease in detection window.
A problem encountered when coding using RLL coding is the number of available code words for any given value of \( d \). It can be intuitively seen that as the value of \( d \) increases the number of available code words for mapping user data onto decreases. For instance when \( d = 1 \) no code word can contain two consecutive 1's. The number of available \( d \) constrained sequences in a code word of length \( n \) is given by the set of Fibonacci numbers [34] as shown in Table 2.1.

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</table>

There is an alternative technique [35,36] to derive the channel capacity based on the representation of the \((d, k)\) constraints by a Finite State Transition Diagram (FSTD), described in Appendix D.
2.4.2 Charge Constrained (D.C.-Free) Codes

The importance of d.c.-free codes in magnetic recording has been recognised for a number of reasons;

a) The use of a d.c.-free code accommodates detection by integration, which for some noise spectra may be preferable to differentiation.

b) In a.c.-coupled magnetic channels, the transformer or coupling capacitor prevents d.c. response. Codes which are not d.c.-free may exhibit baseline wander on readback, making detection more difficult, or in the case of a.c.-coupled write drivers, may fail to saturate the medium. In particular, d.c.-free codes are used in nearly all helical scan rotary head recorders employing rotary transformers.

c) One of the limiting factors in increasing the areal density of magnetic disk drives is the inability to precisely servo the head positioning mechanism on a given track. In rotary head tape transports multiple heads with different azimuth angles often write over neighbouring tracks which slightly overlap, this deliberate overlap method is called perpetual overwrite. In both these recorders the problem of crosstalk, that is the side-reading of an adjacent track, which is primarily a low frequency phenomenon, is diminished by limiting the low frequency content of the modulation code.

d) Inherently, the read head of many magnetic recording channels is essentially a flux differentiator and therefore the channel exhibits a spectral null at d.c.
D.c.-free codes are not usually employed in current computer peripheral or disk drives. However, almost all rotary head recorders employ d.c.-free codes to alleviate the problem of track mis-registration. It is in many ways, a much more difficult engineering task to “stay on track” with a thin flexible tape wrapped around a helical drum than it is in other types of magnetic recorders. Therefore a great deal of attention has been paid to the basic problem of deriving continuous reproduce-head position error signal in rotary head machines. Rotary head machines also use d.c.-free modulation and channel codes specifically in order to provide empty or clear parts at the bottom of the spectrum in which other useful information may be recorded.

D.c.-free code sequences have a balance of charge, that is they exhibit an RDS with an average of zero. Justesen [37] has shown that to produce a code with a spectral null at d.c. is equivalent to producing a code with a bounded RDS. To produce a d.c. null with more rapid roll-off, tighter bounds on the maximum allowable RDS may be introduced [38], or higher order charge statistics may be constrained.

Norris and Bloomberg [39] have determined the information carrying capacity of \((d, k)\) codes with bounded RDS, which they refer to as charge constrained run length limited (CCRLL) codes. Fredrickson [40] describes an alternative method of determining the capacity of (CCRLL) codes and derives some new charge constrained \((d, k)\) codes.

The **disparity** of a code word is defined as the difference between the number of 0's and the number of 1's in a code word; thus code words 000011, 00111, 001111 have disparity -2, 0 and 2 respectively.

There are basically three approaches that have been used to create d.c.-balanced codes:
a) The most obvious method for the construction of d.c.-free codes is to employ zero disparity code words. However, this severely limits the code rate, due to the limited number of zero disparity code words for any given length of code.

b) The next logical step is to extend the above technique to a low disparity code. Where zero disparity code words are uniquely allocated to the source data words and pairs of opposite low disparity code words are also used to represent a single source data word. During recording, the choice of a specific translation is made in such a way that the accumulated disparity, or RDS is minimised.

c) Another special case of low-disparity codes, known as polarity bit code, was devised by Bowers [41] and Carter [42]. In their method a group of \((n-1)\) source symbols are supplemented by a symbol 1. The encoder has the option to record the resulting \(n\)-bit word without modification or to invert all symbols. Similarly to the previous method, the choice is made to minimise the accumulated disparity. The last symbol of the code word, called the polarity bit, is used by the decoder to identify whether the transmitted code word has been inverted or not.

Recently a new algorithm for generating zero disparity code words has been devised by Knuth [43]. This method translated the set of \(m\)-bit source words to \((m+p)\)-bit balanced code words. The translation is achieved by selecting a bit position within the \(m\) bit code word that defines two segments each having half of the total disparity. A d.c.-free code word is generated by the inversion of all bits within one segment. The remaining \(p\) bits of the code word contain a balanced encoding of the bit position that defines the two segments.
2.5 Coding Schemes

A number of attempts have been made to compare different digital recording codes [44-47]. Mackintosh [46] concludes that there is little to chose between all popular codes as regards maximum packing density, whilst according to Kiwimagi [47] there is no single best choice for all situations.

It is difficult to place codes in league order when considering them in isolation, therefore a summary of some of the main characteristics of several of the codes to be discussed is presented in Table 2.2. Examining this table clearly shows that d.c. free codes are less efficient than others that do not possess this property.

Consideration must be given to the cost and complexity of implementation and since the objective is high data rate, error-free recording, compatible equalisation, clock recovery, error propagation and error detection and correction (EDAC) techniques must be considered.

Following is a description of some popular coding schemes used for digital recording.
Table 2.2 A Comparison of Various Channel Code Parameters

<table>
<thead>
<tr>
<th>Code</th>
<th>d,k</th>
<th>Rate (m/n)</th>
<th>Density Ratio</th>
<th>FoM</th>
<th>Detection Window</th>
<th>Capacity</th>
<th>Efficiency</th>
<th>d.c. Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRZ</td>
<td>0,∞</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>T</td>
<td>1</td>
<td>100%</td>
<td>no</td>
</tr>
<tr>
<td>NRZI</td>
<td>0,∞</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>T</td>
<td>1</td>
<td>100%</td>
<td>no</td>
</tr>
<tr>
<td>E-NRZ</td>
<td>0,7</td>
<td>7/8</td>
<td>0.875</td>
<td>0.766</td>
<td>7T/8</td>
<td>0.9971</td>
<td>88.75%</td>
<td>no</td>
</tr>
<tr>
<td>FM</td>
<td>0,1</td>
<td>1/2</td>
<td>0.5</td>
<td>0.25</td>
<td>T/2</td>
<td>0.6942</td>
<td>72%</td>
<td>yes</td>
</tr>
<tr>
<td>GCR 4/5</td>
<td>0,2</td>
<td>4/5</td>
<td>0.8</td>
<td>0.64</td>
<td>4T/5</td>
<td>0.8792</td>
<td>91%</td>
<td>no</td>
</tr>
<tr>
<td>8/10</td>
<td>0,3</td>
<td>8/10</td>
<td>0.8</td>
<td>0.64</td>
<td>4T/5</td>
<td>0.947</td>
<td>84.5%</td>
<td>yes</td>
</tr>
<tr>
<td>8/9</td>
<td>0,3</td>
<td>8/9</td>
<td>0.89</td>
<td>0.79</td>
<td>8T/9</td>
<td>0.947</td>
<td>93.9%</td>
<td>no</td>
</tr>
<tr>
<td>MFM</td>
<td>1,3</td>
<td>1/2</td>
<td>1</td>
<td>0.5</td>
<td>T/2</td>
<td>0.5515</td>
<td>90.6%</td>
<td>no</td>
</tr>
<tr>
<td>ZM</td>
<td>1,3</td>
<td>1/2</td>
<td>1</td>
<td>0.5</td>
<td>T/2</td>
<td>0.5515</td>
<td>90.6%</td>
<td>yes</td>
</tr>
<tr>
<td>Miller^2</td>
<td>1,5</td>
<td>1/2</td>
<td>1</td>
<td>0.5</td>
<td>T/2</td>
<td>0.6509</td>
<td>76.8%</td>
<td>yes</td>
</tr>
<tr>
<td>1,7</td>
<td>1,7</td>
<td>2/3</td>
<td>1.33</td>
<td>0.88</td>
<td>2T/3</td>
<td>0.6793</td>
<td>98.1%</td>
<td>no</td>
</tr>
<tr>
<td>2,7</td>
<td>2,7</td>
<td>1/2</td>
<td>1.5</td>
<td>0.75</td>
<td>T/2</td>
<td>0.5174</td>
<td>96.6%</td>
<td>no</td>
</tr>
<tr>
<td>HDM-1</td>
<td>2,8</td>
<td>1/2</td>
<td>1.5</td>
<td>0.75</td>
<td>T/2</td>
<td>0.6266</td>
<td>79.8%</td>
<td>no</td>
</tr>
<tr>
<td>3PM</td>
<td>2,11</td>
<td>1/2</td>
<td>1.5</td>
<td>0.75</td>
<td>T/2</td>
<td>0.545</td>
<td>91.7%</td>
<td>no</td>
</tr>
</tbody>
</table>
2.5.1 Scrambled NRZ (S-NRZ)

It is possible to convert raw data into a channel code simply by randomising or scrambling the data. This early technique is used to break up the possible long runs of zeros in NRZ, which causes loss of synchronisation. Figure 2.9 illustrates how incoming NRZ is scrambled by performing modulo-2 addition (XOR) with a Pseudo-Random Binary Sequence (PRBS), which is generated by an optimum feedback shift register. If the register length is $L$, $2^L - 1$ pseudo-random $L$-bit words are generated. On replay, an identical PRBS must be produced, synchronised to the readback data so that repeated modulo-2 addition will unscramble the data.

![Diagram of Scrambled NRZ](image)

**Figure 2.9 Scrambled NRZ**

The major advantage of S-NRZ is that it retains unity rate and therefore requires no more bandwidth than NRZ. Randomised data also maintains a unity density ratio and window margin which results in a Figure of Merit of 1, which is better than all block codes. However, its big weakness and the reason why it has never been used in computer storage products, is that it fails when the user data contains the complementary sequence of pseudo-random words. However, it has been adopted in many applications such as in the D-1 video tape recorder and is very popular in satellite communication channels.
2.5.2 Block Codes

Block codes have proved very successful in digital recording systems. Coding is achieved by partitioning the source data into blocks of length $m$, which are mapped onto code words of length $n$ using a conversion table known as a codebook.

*Manchester Code (Bi-Phase, PE, FM)*

The Manchester group of codes, known also as Bi-phase, Phase Encoding (PE) and Frequency Doubling, together with Frequency Modulation (FM), were the first self clocking codes to be developed for direct digital recording. The coding rules for this family of modulation codes are very simple to implement. For example PE uses a positive transition at the bit cell centre to represent a logic 1 and a negative transition corresponds to a logic 0. Where two or more 1’s or 0’s occur in succession, an extra transition is inserted between them at the bit cell boundary. Similarly for Bi-Phase the code always changes state at the centre of a bit-cell; the first half of the bit-cell is encoded as the complement of the data, whilst the second half is the actual data value. Whereas FM maintains a transition at the bit-cell boundary and employs a transition at the bit cell centre to represent a logic 1 and no transition to represent a logic 0.

All these codes have run-length constraints of $(0, 1)$ and a code rate of $1/2$. However, self clocking is achieved by sacrificing the detection window to half of one bit cell and a reduction of the density ratio to a $1/2$. Therefore the FoM for these code is reduced to $0.25$. Whilst the low $k$ value facilitates data recovery, the low value of $d$ implies a potential for excess inter-symbol interference when recording at high data rates.
Never-the-less due to the simplicity of these codes and the added advantage that they possess no d.c. content means they remain widely used today where recording density is not of prime importance.

**Miller Code (MFM, DM)**

In Miller code [48], also known as Modified Frequency Modulation (MFM) or Delay Modulation (DM), the highly redundant clock content of FM was reduced by the use of a phase-locked loop (PLL) in the receiver that could flywheel over missing clock transitions. A technique which is implicit in more advanced codes. In Miller code the presence or absence of a transition at the bit-cell centre, corresponding to a logic 1 or 0 respectively, was retained but the bit-cell boundary transition is now only required between successive 0's. There are still two channel bits for every data bit, hence the code rate = 1/2, but adjacent channel bits will never be one, doubling the minimum time between transitions and giving a DR of 1. Miller code, unlike FM, is not d.c.-free and with run-length constraints of (1,3) the increased value of k makes data recovery slightly more difficult.

MFM has been considered a standard code for several years and was adopted for many hard disks at the time of development and remains in use on double-density floppy disks. However, it was found that this was not an optimum code and additional improvements have been realised which, make it d.c.-free by bounding the charge constraint (Zero Modulation [49]), widen the detection window (GRC 4/5 [50]) and improve the density ratio. These improvements are achieved by an increase in complexity of encoding/decoding and clock recovery circuits.
2.5.3 Variable Length RLL Coding

Attempts to improve the efficiency of fixed length state-dependent codes result in increased code word length and thus, increased encoder and decoder complexity. Variable length codes offer the possibility of using short words more frequently than those of longer length. This is usually profitable in terms of a marked reduction in the encoder/decoder hardware complexity relative to a fixed length code with the same rate and run-length constraints.

Manufacturers of magnetic recording systems resist variable rate codes mostly because they desire a block of data that fits within a fixed size record on the medium and are concerned that they must accommodate for the worst case data block where the code has a minimum rate. To overcome this objection the codes are constructed to achieve a minimum coding rate equal to that of the industry-accepted code rate for a given \((d, k)\) constraint. If the code designer has additional knowledge about the statistics of non-random binary digits to be recorded favourable assignment of data sequences to code sequences can be listed to achieve average rates higher than the \((d, k)\) constraint capacity.

The structure of variable length codes required to satisfy the run-length constraints is similar to that for fixed length codes. However, various special features arise from the presence of words with different lengths. The requirement of synchronous transmission, coupled with the assumption that each word carries an integer number of information bits implies that the code word lengths are integer multiples of a basic word length \(n\), where \(n\) is the smallest code word length. Each code word may be of length \(jn, j = 1, 2, \ldots, M\), where \(Mn\) is the maximum code word length.
Franaszek [51] laid the basis of variable length synchronous RLL codes and although only a few practical examples exist they are widely used in magnetic recording products [52]. Table 2.3 presents the parameters of variable length codes that have been published by Gabor [53], Franaszek [54, 55], Kobayashi [56], Horiguchi and Morita [44], Eggenberger and Hodges [57].

### Table 2.3 Parameters of variable length RLL codes

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>d</th>
<th>k</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>4</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>17</td>
<td>6</td>
</tr>
</tbody>
</table>

**Franaszek's Rate 1/2 (2,7) Code**

The construction of this Franaszek code is based on the selection of a set of terminal states. Just as for the Freiman-Wyner [58] block codes, the code words begin and end in a terminal state, however now the code words don’t all possess the same block-length.

The shortest fixed length block code that generates a rate = 1/2, (2,7) code has a code word length of 34 bits. Therefore a variable length RLL code is much more attractive with respect to encoder/decoder hardware requirements.
The encoding of incoming data is achieved by dividing the source sequence into two, three and four bit partitions to match the entries given in the code table, Table 2.4, and then mapping them into the corresponding channel code word.

For example the data sequence 11010100010 would be split into

11 000 10 0010

which is translated, according to Franaszek's code, into the code word

0100 100100 1000 00001000

Table 2.4 gives two different coding permutations for a given data sequence as described by Franaszek [55] and Eggenberger [57]. There are a total of 24 permutations of the code word assignments and although at first sight they seem quite arbitrary, they require careful choice for optimum performance. This is highlighted by the fact that the Eggenberger code can be decoded using a sliding block decoder with an 8-bit decoder window length which limits error propagation to a maximum of four bits. Franaszek's code, however, has the drawback that it requires a 12-bit shift register which increases error propagation to at most six decoded symbols. This example demonstrates that the allocation of code words in a variable-length code has a crucial effect on the degree of error propagation experienced. The optimum code word assignment that minimises error propagation, as yet, can only be found by trial and error.
Table 2.4 Variable length, rate 1/2 (2, 7) code

<table>
<thead>
<tr>
<th>Data</th>
<th>Franaszek Code Word</th>
<th>Eggenberger Code Word</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1000</td>
<td>0100</td>
</tr>
<tr>
<td>11</td>
<td>0100</td>
<td>1000</td>
</tr>
<tr>
<td>011</td>
<td>000100</td>
<td>001000</td>
</tr>
<tr>
<td>010</td>
<td>001000</td>
<td>100100</td>
</tr>
<tr>
<td>000</td>
<td>100100</td>
<td>000100</td>
</tr>
<tr>
<td>0011</td>
<td>00100100</td>
<td>00001000</td>
</tr>
<tr>
<td>0010</td>
<td>00001000</td>
<td>00100100</td>
</tr>
</tbody>
</table>

2.5.4 Look-Ahead RLL Codes

A class of run-length limited codes is called look-ahead or future dependent if the encoding and decoding of current source data requires a knowledge of upcoming data [59].

Rate 2/3 (1,7) Code

It can be shown that a code with (1, 6) RLL constraint has the information capacity of 0.699 bits per symbol, therefore a 2/3 rate code with such constraints is theoretically possible. However, known algorithms [44,60] for such codes are unattractive both in terms of complexity and error propagation. Therefore a code with the next best set of RLL constraints is the rate 2/3 (1,7) code, which is arguably the most popular run-length limited code in use today, in fact several different variations of this RLL code exist [33,61-64].
The ISS 2/3 code is the (1, 7) code invented by Cohn, Jacoby and Bates [65] whilst working at ISS Sperry Univac. The most simple and elegant method for implementing the ISS 2/3 code was discovered by Jacoby and Kost [66], where the coding is performed using a lookup table shown in Table 2.5a. The basic encoding table is used to encode 2 bits of user data into 3 code bits, where code bit 1 represents a transition. A violation check must also be made to ensure the next block of code bits will not break the \( d = 1 \) constraint. If a violation appears imminent then the present two user bits combined with the next two user bits are coded according to a violation substitution table, Table 2.5b.

Table 2.5Rate 2/3 (1,7) Encoding and Substitution Table

<table>
<thead>
<tr>
<th>a) Encoding Table</th>
<th>b) Substitution Table</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td><strong>Code word</strong></td>
</tr>
<tr>
<td>00</td>
<td>101</td>
</tr>
<tr>
<td>01</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>001</td>
</tr>
<tr>
<td>11</td>
<td>010</td>
</tr>
</tbody>
</table>

RLL (1,7) has the appealing feature for high density recording that the maximum transition density is 3/4, which is lower than that of the user data. The 2/3 (1, 7) code has a 98% efficiency, the detection window is 0.66 of one source bit cell and the clock rate is 1.5 times the user data rate.
Three Position Modulation (3PM)

The basic coding scheme for Three Position Modulation [67,68], abbreviated to 3PM, relies on a coding table illustrated in Table 2.6. The source data are divided into three-bit words which are mapped onto corresponding six bit code words, giving a code rate \(=1/2\) and \((d, k) = (2, 11)\). The positions, \(P_1\) to \(P_6\), in the code word at which transitions occur are assigned a 1. Each code word possesses, at least, one or two transitions separated by a minimum of two zeros and position \(P_6\) has no transition unless a special circumstance occurs. The special circumstance being when the concatenation of two code words results in two transitions being separated by only a single zero. In this case a "Merging" technique, illustrated in Figure 2.10, is adopted whereby the transitions at position \(P_5\) of the current word and \(P_1\) of the next word combine to form a single transition at position \(P_6\).

Table 2.6 3PM Look-Up Table

<table>
<thead>
<tr>
<th>Data</th>
<th>Transition Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_1 D_2 D_3)</td>
<td>(P_1)</td>
</tr>
<tr>
<td>000</td>
<td>0</td>
</tr>
<tr>
<td>001</td>
<td>0</td>
</tr>
<tr>
<td>010</td>
<td>0</td>
</tr>
<tr>
<td>011</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>110</td>
<td>1</td>
</tr>
<tr>
<td>111</td>
<td>1</td>
</tr>
</tbody>
</table>

Cohn and Jacoby [69] have also discovered a way of reducing the maximum run-length constraint to \(k=7\), but the complexity of the encoding and decoding grew by the addition
of another look-ahead code word. This code possesses the same characteristics as Franaszek’s variable length 1/2 rate (2,7) code described earlier.

![Figure 2.10 Example 3PM code showing Merging](image)

**2.5.5 Charge Constrained Codes**

*Miller Squared Code* $R=1/2, (1,5;3)$

The Miller Squared code is a rate 1/2 RLL modulation code devised by J.W. Miller in the mid 1970’s. Miller Squared code is the result of a direct extension on A. Miller’s code, described above. It removes the d.c. component by modifying the sequences that have a non-zero DSV. To understand how such identification is achieved, the input data stream is categorised into the concatenation of the following three variable length sequences.

a) $1, 11, 111, \ldots$ {any number of 1’s, but no 0’s, in a row}

b) $00, 010, 01110, \ldots$ {a pair of 0’s separated by an odd number or no 1’s}
c) 011, 01111, 0111111,... {a even number of consecutive 1's preceded by a 0}

Note class (c) must be followed by a sequence whose first bit is zero.

When Miller code is applied to the three sequences, the integral of the resulting waveforms for class (a) and (b) always equals zero at the end of the sequence, however, for class (c) the integral reaches a value of \( \pm T \). Therefore the Miller code rules apply for all bits except the last 1 of the class (c) sequence, whose transition is suppressed. This is illustrated in Figure 2.11.

![Figure 2.11 Miller Squared Encoding Sequences](image)

Figure 2.11 Miller Squared Encoding Sequences

Encoding using the above algorithm can be achieved with a two bit look ahead scheme to determine whether the additional Miller Squared rule will apply or whether conventional Miller coding is applied. However, an easier way is to use the finite state transition diagram shown in Figure 2.12.
8/10 Rate Code

In certain applications, specifically digital magnetic tape recording, a rate $R = 8/10$ charge constrained code has attractive features [70,71]. There are several variations of this code; most of the implementations translate one byte (8 bits) of source data into ten channel symbols. Clearly a zero disparity block code is impossible since there are only 252 zero-disparity code words. A two-state encoder offers the freedom of at maximum $252 + 210 = 462$ code words, since only 256 of these are required, this method offers a large variety of choices to suit particular channel requirements. Table 2.7 shows the main parameters of selected d.c.-constrained codes with rate 8/10 documented in the literature (for a more comprehensive survey of such codes see Tazaki [70]).
Table 2.7 Summary of parameters of selected rate 8/10 codes

<table>
<thead>
<tr>
<th></th>
<th>Max. Charge $c$</th>
<th>$D_{SV_{\text{max}}}$</th>
<th>$k$</th>
<th>$T_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baldwin [72]</td>
<td>9/2</td>
<td>10</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Morizono [73]</td>
<td>9/2</td>
<td>10</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Shirota [74]</td>
<td>3</td>
<td>7</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Widmer [75]</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Fukuda [76]</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Fukuda [76]</td>
<td>3</td>
<td>7</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Immink [77]</td>
<td>5/2</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Parker [78]</td>
<td>5/2</td>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Fredrickson [79]</td>
<td>5/2</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

*block code with code word length, $n = 20$.*

Both the Rotary head Digital Audio Tape (R-DAT) and Digital Compact Cassette (DCC) digital audio tape recorders employ variations of this modulation code. The R-DAT system uses azimuth recording which is susceptible to low-frequency crosstalk. The rate 8/10 code used in the R-DAT [76] is therefore designed to suppress low frequencies as well as being d.c.-free. A further issue is that erasure is achieved by overwriting and since the heads are optimised for short-wavelength operation, the best erasure will occur when the ratio between the longest and shortest recorded wavelength is small. Finally it is desirable to limit the maximum run-length constraint to improve overwrite and clock synchronisation. In the 8/10 code employed by R-DAT no more than three channel zeros are permitted between transitions, $k = 3$, which makes $T_{\text{max}}$ only four times $T_{\text{min}}$. There are only 153 ten-bit code words which satisfy all these constraints, therefore the remaining 103 source words are allocated two non-zero disparity code words that have a $D_{SV}$ of +2 and -2. For simplicity, the only difference between them is that the first bit is inverted.
2.5.6 Convolutional Codes

Convolutional codes have mainly been used as error control codes and not as modulation codes due to their uncontrollable run-length constraints. As such many convolutional codes are discovered by computer searches.

Convolutional codes have an additional parameter known as the constraint length, $L_e$. The constraint length, usually measured in source data bit periods $T$, describes the maximum number of prior bits which affect the encoding/decoding of the current data bit.

This parameter directly relates to the error propagation experienced if a single bit is detected in error. Therefore if $L_e$ is long all errors will be burst errors.

Two codes that can best be described as convolutional [80] due to their complex encoding algorithms that require numerous exceptions, look-ahead and look back will be summarised below.

The High Density Modulation (HDM) family of codes [81] has been developed specifically for Digital Audio Stationary Head (DASH) recorders. HDM-1 code has a rate 1/2 and (2, 8) run-length constraint which results in a density ratio of 1.5. The coding algorithm basically treats the source data 01 as a single symbol which has a transition recorded at the centre of the one. Further transitions are recorded between lengths of ones and zeros depending on a complex set of conditions requiring a constraint length of $5.5T$. A derivative of this code is HDM-2 which reduces the $k$ constraint to 7 at the expense of an increased constraint length $L_e = 7.5T$. 

52
The $2/4M$ [82] is another code that is best described as convolutional. This code has a code rate $= \frac{1}{2}$, run-length constraints of (2,15) and identical density ratio and window margin to HDM-1. When encoding a pair of data bits this code requires a look-ahead and look-back of four source bits. This code has been adopted by the Mitsubishi ProDigi quarter-inch digital audio stationary head recorder.

2.6 Combined Error Correcting and RLL Coding

In magnetic recording there are many mechanisms available to corrupt data, from the mechanical problems such as media dropouts and poor head contact, to Gaussian thermal noise in the replay circuits and heads. Whatever the cause of error the result will be that the received data will be corrupted with respect to the recorded data. In some two-way communication channels it is sometimes enough to know that an error has been detected in which case the receiver can request a retransmission (ARQ). However, this is quite inappropriate for the digital magnetic recording channel.

A typical encoding configuration for a magnetic recording channel consists of encoding the bits of information with an error correcting code (ECC) [83] followed by a run length limited modulation code. The error-correction code is selected according to the statistics of errors produced by the channel.

One of the problems with this approach is that the demodulator propagates errors whilst attempting to estimate the correct recorded sequence, therefore a single bit error may become a burst. Hence the error correction strategy must be able to correct burst errors even when the noise in the channel is dominated by random errors.
Run-length limited codes do have a limited capability for error detection during transmission by identifying violations of the run-length constraints. However, not all errors result in a run-length violation, therefore several authors have explored the possibility of producing RLL codes with error correction capabilities.

Ferreira [85] introduced the concept of Hamming-distance preserving RLL codes that have a Hamming distance between any two encoded sequences greater than or equal to the Hamming distance of the corresponding input sequences.

An alternative to the traditional concatenation of error correction coding and modulation coding was proposed by Lee and Wolf [86,87]. The authors use the symbols of a block modulation code in order to construct a general algorithm for a single error-correcting RLL code. Based on this technique a number of other authors [88-90] have presented papers describing combined ECC/RLL trellis codes based on the concatenation of convolutional codes [91] with run-length limited codes. Immink [92] investigated the Euclidean distance between ECC/RLL sequences for the noisy digital magnetic recorder.

Another alternative approach taken by Fredrickson and Wolf [93] concentrated on RLL codes that can detect transition shifts in a \( (d, k) \) constrained sequence. Recently Ytrehus [94] utilised an exhaustive computer search program to establish the upper and lower bounds on the size of run-length constraints for error-controlling block codes that have a minimum specified Hamming distance.
2.7 Multi-Level Signalling

Other communications channels, such as the telephone line, have achieved significant increases in performance from the use of multi-level, also known as $M$-ary signalling techniques [95,96]. $M$-ary coding schemes for magnetic recording utilise one or more of the three degrees of freedom that are possible: position, polarity and amplitude. In the case of binary codes ($M=2$) the source data are represented using the position of flux transitions.

Due to the hysteresis of the magnetic recording medium it is not possible to record directly a multi-level signal. If the recording medium was allowed to take on $M$-ary states, the effect of a non-saturating record current would depend not only on the magnitude of the recording field, but also on the magnitude of the existing magnetisation state. Since, when recording, it is not possible to know the position on the hysteresis curve of the existing state of magnetisation for the magnetic medium, the remanent state of magnetisation after applying a known field would also be unknown.

Mackintosh and Jorgensen [97] have conducted a theoretical investigation into the use of multi-level coding, the result of which suggests that $M$-ary signalling techniques are less efficient than binary codes under most conditions. However, these results were for a peak detect system and as such did not take into consideration additional benefits that can be gained from a Viterbi detector.

Over the years several different multi-level techniques have been proposed [98-104], however, these approaches to increase storage density have been resisted in commercial
products. The following is a brief review of some multi-level signalling techniques considered for implementation by the author.

Chi [105] proposed a $M$-ary recording scheme called Controlled Return to AC (CRA) which utilises a change in the polarity sequence of two recorded dibits or doublets$^3$. A high frequency a.c. signal, which in fact erases any previous data, is continuously recorded, interrupted by short durations that represent the $M$-ary source symbol. The advantages of this code are, it is d.c.-free, it has excellent overwrite properties, it offers 50% improved packing density over Miller code. Chi later revised this code [106] by varying the duration of the doublets to exploit amplitude variation in the readback signal. This further improved the density ratio to 130% over Miller code. However, the main disadvantage of this code due to the multi-level signal is that the amplitude of the readback signal is attenuated by 6dB relative to a conventional recording code.

Jacoby [107] describes a multi-level code based on the 3PM coding scheme that doubles the packing density relative to Miller code. This technique employs the peak shift between two closely spaced flux transitions to provide the third code state. Therefore three source words 0, 1 and 2, are represented by the absence and presence of a transition and a doublet respectively.

A particular ternary modulation technique called controlled polarity modulation [104-109] can be interpreted as a form of write equalisation.[110] and is similar to that proposed by Veilard [111].

---

$^3$ Dibit or Doublet is the name given to two closely-spaced transitions.
Phase modulation techniques are often chosen for use in systems with non-linear behaviour because of the constant amplitude envelope. The additional constraint of continuous phase leads to signals which make full use of the available channel bandwidth. Recently Weathers [112] has combined conventional RLL codes with a bias signal to achieve greater packing densities. This work was furthered [113] using a conventional modem to record data on a rotary head tape with surprisingly good results. The modem linearises the channel by modulating data on to a carrier signal and uses quadrature amplitude modulation (QAM) in conjunction with trellis coded modulation (TCM) to obtain increased performance. A similar technique has also been proposed by Kobayashi et al. [114] to increase the recording efficiency of a video cassette recorder.
2.8 Summary

Modulation codes have been used with great success in magnetic and optical storage to increase linear density and improve performance. These codes represent the result of a steady evolution of channel code design for a particular detection technique, namely peak detection, employed in most storage systems. It is difficult to quantify the increase in storage capacity that RLL coding affords since it varies with the detection circuitry used. By way of an example Immink calculates that for a compact disc system the RLL scheme increases capacity by approximately 25% over that of the uncoded medium.

Several different run-length limited and charge constrained modulation coding techniques have been described that are or have been used in various magnetic recording channels. The choice of the optimum modulation code for any particular device involves trade-offs between the various properties of the modulation code and the characteristics of the channel in which it is to be applied. A prime example is the choice of modulation coding for magnetic hard disk drives.

Research on RLL coding schemes continues, however most current work aims at combining error correction codes with RLL codes. These integrated codes offer increased efficiency and reliability over individual error correction and modulation codes applied consecutively to the source data. However, no such code has yet been included in a commercial device. The author believes that if peak detection is to maintain a position in commercial storage devices combined ECC/RLL codes will have to be employed.
2.9 References


CHAPTER 3

Experimental Development

3.1 Introduction

This chapter describes the initial hardware and software employed in the first stage of the investigation. This system is categorised by the type of detection that was implemented, namely peak detection. Finally the experimental procedure and results pertaining to this system are given.

The analogue readback signal from the head must be amplified and converted back to its digital form so that the original binary data can be restored. There are several ways of performing this conversion [1-4], however most detectors use one or a combination of two basic techniques.

a) Amplitude Detection: relies on the readback signal crossing a threshold voltage to discriminate the presence of a logic 1. The main cause of errors in amplitude detection results from dropouts causing missing bit errors.

b) Peak Detection: because of the shape of the readback pulse it is possible to detect the presence of a logic 1 by determining if a pulse peak occurred during the bit interval of interest. Peak detection is implemented by differentiating or integrating [5] the readback signal and detecting any
resultant zero crossings. The primary cause of errors arising from this method of detection is due to peak shift, where the pulse peak is shifted outside the detection window of interest due to excessive inter-symbol interference.

3.2 Gated Peak Detection System

The design of the modulation, coding and signal processing in previous magnetic recording products has been largely influenced by the detection method chosen to determine the presence or absence of a transition in the readback waveform. The detector, called the Peak Detector [6] or Gated Cross Over Detector has the advantage of being both robust and extremely simple to implement. However, by its very nature, it works best at low linear densities. A block diagram of a typical peak detector is shown in Figure 3.1.

![Figure 3.1 Block Diagram of Gated Peak Detector](image)

There are two paths through the detector. One path is used to qualify a peak, i.e., to ensure that the peak has sufficient amplitude, the other is used to locate the peak by differentiating the signal and then passing it through a zero crossing detector. The detector only accepts a peak if the peak amplitude was large enough to pass the qualification test.
The main objectives for using peak detection in a magnetic recording systems are to:

a) compress the signal bandwidth,

b) optimise the trade-off's between inter-symbol interference, detection window size and data density,

c) support data-driven timing and gain control,

d) minimise implementation cost by reduced complexity.

3.2.1 Apparatus

The experimental apparatus illustrated in Figure 3.2 provides a multi-track magnetic storage system that permits the recording of digital data onto compact cassette tape, to be subsequently replayed and analysed.

Figure 3.2 Photograph of Experimental Apparatus
The basic items of equipment used in this investigation consist of a compact-cassette tape transport interfaced to a microprocessor control board to form a simple direct digital recording system. The interface elements comprise read/write amplifiers plus tape transport circuitry.

A circuit was designed and constructed to contain the digital electronics which include: the microprocessor; the program memory stored in Erasable Programmable Read Only Memory (EPROM); a 16 bit programmable up/down counter; an interface for a IBM compatible host computer; an interface for the read/write electronics; and the control logic. The basic elements of the microprocessor control board are shown in Figure 3.3. The read/write electronics are housed on a small wire-wrap board which could be mounted near the head to minimise noise.

![Figure 3.3 Basic Elements of Recording System 1](image-url)
Compact Cassette

The tape transport comprised a commercial 4-track, audio-frequency, inductive head in conjunction with a standard solenoid controlled compact cassette mechanism. The head, tapes and mechanism were employed to provide a cost effective means of achieving a multi-track system.

The tape speed for a compact cassette tape transport is 4.75 cm/s with a tolerance ranging up to ±2%, depending on the quality of the device. The low-cost mechanism employed had poorly controlled motor velocity, eccentricity of the capstan and capstan shaft, as well as inconsistent friction between bearings, drive belts, and between the tape and its pressure pad and tape guides. However for the purpose of this investigation these abnormalities proved ideal to test the ability of intelligent software techniques.

The compact cassette tape transport mechanism employed was solenoid controlled by the microprocessor via a custom built interface containing solenoid driver amplifiers.

All experimental work was carried out using the 4-track head, however it was envisaged that heads with greater track densities would be available for experiment during the investigation. Therefore all the electronics were designed to allow additional channels to be included as they became available.

TMS 32010 Digital Signal Processor

The Texas Instruments TMS32010 is a first generation digital signal processor [7]. The structure of the TMS32010 takes advantage of the Harvard Architecture, in which program and data memory area are separate to permit a full overlap of fetch and execution of instructions. Texas Instruments further modified this architecture to allow
transfers between 4K words of external program memory and 144 words of internal RAM so that large tables could be stored in program memory. The pipeline architecture enables the TMS32010 to execute up to 5 million instructions per second (5 MIPS), based on a 20 MHz clock giving a 200 ns instruction cycle. These operations have been incorporated into a, relatively limited, instruction set optimised for DSP applications. However most instructions can be executed in a single cycle, making it extremely fast and allowing digital filtering at a rate of up to 2.5 million samples per second.

**Write Amplifier**

The design of the read/write amplifiers have been conditioned by the requirement to provide a multi-track capability at a low cost. Consideration has also been given to the need to interface these items to the TMS32010.

Figure 3.4 shows the basic design as proposed by Donnelly [8]. Since a record current of only a few milliamps is required it was possible to use logic gates to drive the record head. A series resistor was used to limit the record current.

![Figure 3.4 Simple Write Amplifier Circuit](image)
Read Amplifier including Gated Peak Detector

The basic read amplifier design, as described in [9], is illustrated in Figure 3.5. The signals at each stage of the amplifier are shown graphically in Figure 3.6.

The reproduced signal is sliced to convert the peaks into pulses. These are gated with a coincident pulse derived from the differential of the signal. First a pre-amplifier increases the signal level and converts the high output impedance of the read head to a low impedance. The signal is then passed through a low pass filter to remove any high frequency noise. A further stage of amplification follows after which the signal path divides. Along the first path the signal is sliced at a positive and negative level and the two outputs are applied to two NAND gates. Along the second path the signal is differentiated to shift it by 90 degrees; this converts the peaks of the signal to zero crossing points. These points are detected by a comparator that converts them to signal transitions. The signal to noise ratio at the comparator input is degraded by 6 dB due to the differentiator. To prevent the noise signal generating a false output, threshold hysteresis is employed. The edges are converted to pulses which are applied to the second input of the NAND gates. The output of the NAND gates are used to control a set/reset flip-flop that delivers a pulse train corresponding to the direction of the second recorded flux.

The double detection action of the circuit reduces the possibility of false triggering of the flip-flop due to points of inflection of the differentiated signal. The slice levels are adjusted to allow for variations in signal amplitude with record frequency.
Figure 3.5 Read Amplifier including Gated Peak Detector

Figure 3.6 Read Amplifier / Peak Detector Waveforms
Host Computer

An IBM compatible personal computer (PC) was interfaced to this system to provide a means to store and analyse the results. The host computer was also used to develop and debug all software written in assembly language for the TMS 32010 microprocessor and Pascal for the IBM.

The host computer contained a General Purpose Input/Output (GPIO) card that could be interfaced directly to a parallel port on the microprocessor board to allow bi-directional communications. A “shell” program (see Appendix B) allowed the PC to perform as a user interface, where instructions could be entered via the PC keyboard to select the desired function of the microprocessor controlled recording system. The PC’s monitor was also used to display any data or results obtained from the recording system, so the user was informed of the system’s status at all times.

Finally, the host computer was used to load the TMS 32010 with the programmed information via an EPROM Emulator. The EPROM emulator provides a quick and simple solution to rapidly changing the program of a target microprocessor without the need to repeatedly program and erase EPROM’s. The EPROM emulator was connected to the parallel port of the PC, by which it was programmed, and also to the EPROM socket on the target microprocessor control board. Once the software for the recording system had been completely debugged the recording system could be used in Stand-alone operation, where the EPROM emulator is replaced with EPROM’s programmed with the correct code.
3.2.2 Software

Two completely separate and different types of software programs were developed for this digital recording system.

a) Software written in TMS 32010 assembly language [7] was used to control the systems operation and functions. It can be divided into several distinct sections:

i) Hardware control,

ii) Coding,

iii) Decoding,

iv) IBM PC interface protocol.

b) Software written in Pascal [10], for the IBM compatible PC, was used to provide a user interface to; and analyse data from; the microprocessor control board.

The assembly language programs (given in Appendix C) for the TMS 32010 often had to perform several task simultaneously. In RECORD mode for instance it had to communicate at regular intervals with the PC whilst recording so that the user could interrupt the process. In PLAY mode it had to maintain synchronisation with the readback signal, determine the optimum sampling point, decode the data sequence and then relay the data back to the PC for processing. Due to the large number of tasks involved in the PLAY mode this was accomplished in quasi-real-time. Where a block of data would be detected and decoded in real time and then these processes would stop whilst the data were relayed back to the host computer for processing. Once all the data had been transferred to the PC the process would repeat. The error rate was analysed
using the host computer. Simple analysis was achieved in real-time whilst the recording system was busy gathering data, more complex analysis required the data being stored on the host computer's hard disk and processed off-line.

**Record Software**

The record software converts the binary data stream into a corresponding pattern of states of magnetic surface saturation separated by transition regions. This process maps the data stream into the write current waveform required to produce the desired magnetisation pattern on the storage medium and is known as channel, or modulation, coding. Many channel codes have been developed to optimise the performance of particular digital magnetic recording systems, a review of channel coding techniques is described in chapter 2.

Since all coding/decoding is implemented in software the complexity of the channel encode process depends on the channel code chosen. In general the record process is much less problematical than the replay process. Parallel track software algorithms have been developed for a number of codes including:

- Bi-Phase-L (Manchester Code)
- Miller (MFM) Code
- Miller Squared Code
- ISS 2/3 (1, 7) Code
- 1/2 (2, 7) Code
- 3 PM Code
In each case the encoding/decoding process utilises the arithmetic and logical functions of the processor to generate code directly from data without resort to look-up tables. For example the Boolean equations, developed by the author, for the 3PM code are:

\[
\begin{align*}
P_1 &= \overline{P_5} \cdot (D_1 \cdot (D_2 + D_3)) \\
P_2 &= \overline{D}_1 \cdot D_2 \\
P_3 &= D_1 \cdot \overline{D}_2 \cdot \overline{D}_3 \\
P_4 &= D_3 \cdot (\overline{D}_1 \oplus D_2) \\
P_5 &= (D_1 \cdot D_2 \cdot D_3 + D_2 \cdot (D_1 \oplus D_3)) \cdot \overline{P_1} \\
P_6 &= P_5 \cdot P_1^{+1}
\end{align*}
\]

where \( P_1^{+1} \) is the \( P_1 \) condition of the next word and \( P_5^{-1} \) is the \( P_5 \) condition of the previous word.

By performing the Boolean expressions on hexadecimal numbers, coding can be achieved for all four tracks of the compact cassette system simultaneously.

Following channel encoding, data streams are written to the recording medium, broadside, across the tape.

**Decoding (Replay) Software**

The problems of skew and velocity variation were addressed in software by optimising the data decoding techniques as the data rate was increased. Bit-cell identification is achieved by timing the intervals between successive transitions and computing the ratio. Each code employed required the identification of one or more unique ratios that determined the relative position within the bit cell.
For example when employing Bi-Phase-L code, the third transition of a 1:2 ratio always occurs at the bit cell centre whilst that of a 2:1 ratio always marks the bit-cell boundary. The different types of data detection techniques used for Bi-Phase-L code are described in the following sections. Similar techniques, varying the identification ratio were applied to all codes Therefore we had to ensure the data to be recorded contained the appropriate ratios.

**Timing Recovery**

The specified tolerance on the 4.75 cm/sec velocity of compact cassette tape is between ±0.5% to ±2% depending on the quality of the tape-transport mechanism. This variation will directly affect the time interval of each bit cell. It is required to accurately synchronise to the replay data rate so that data can be interpreted correctly.

The clock recovery mechanism was achieved purely by software in an attempted to reduce complexity and cost of the system. Several methods were implemented to maintain clock synchronisation depending on the code employed.

If the code employed was not self clocking then a separate clocking track was recorded so that synchronisation could be maintained, however this technique proved to be prone to skew-induced errors. Another method of maintaining clock synchronisation was recording timing information in header blocks so that the software could synchronise to this and *flywheel* whilst reading data. Again this technique was prone to skew and tape velocity variations which resulted in poor error rate performance. Attempts, described below, were made to improve this technique by identifying the optimum sampling point and hence reduce the errors due to skew. However, even with sophisticated software
clock recovery algorithms, the non self-clocking codes did not perform as well as those
that were. This has resulted in a further research into software clock recovery techniques

When a self clocking code is used such as Bi-Phase-L and Miller Squared the clock
recovery was achieved using the following technique.

The software allows for the tape velocity variation by computing the ratio of successive
transition intervals on replay, because ratios are used the technique is independent of
tape speed. The robustness of the software is improved by incorporating a flywheel
effect into the sampling process. The possibility of measuring an incorrect bit-cell
duration due to system perturbations is eliminated by comparing a reference bit-cell
duration with the currently measured value before it is updated. This enables the
detection software to function as an intelligent Phase-Locked Loop (PLL) by rejecting
short-term variations in signal frequency whilst tracking longer-term changes.

*Fixed Track*

The decoding software polls the output of a single read amplifier until a transition is
detected. A counter timer circuit is then reset to zero. The software continues to poll the
same channel until another transition is detected. The counter/timer is then interrogated
and reset to zero. This is repeated and the ratio of the two time intervals is compared
until a 2:1 or 1:2 ratio is detected. Once bit-cell identification has been achieved the
playback software continuously monitors the ratios of successive intervals to maintain
synchronism throughout replay.
To decode the data, the outputs of all four tracks are simultaneously sampled at the 3/4 point of each bit-cell. The sample point can be derived from a knowledge of the duration of the current bit cell. This is stored and updated when future 2:1 or 1:2 ratios are encountered. However, before the new sample point is updated the value of the new time interval is compared to that previously measured by the software. This is to ensure it has not changed dramatically, since a large increase or decrease in the bit-cell period highlights a dropout or glitch respectively.

**Leading Edge**

In the playback software described previously, the sample point is determined using the information from a single track. Since the outputs of all four tracks will be out of phase, due to tape skew, the sample window is limited to 1/4 of a bit cell. Typically, the bit cell displacement across the tape at 2.5 kbits/sec is of the order of a 1/4 of a bit cell, therefore accurate data detection could not be achieved above this rate using fixed track method.

Figure 3.7 illustrates how the sample window was doubled by programming the software to respond to the skew dynamics of the tape. This was accomplished using a method proposed by Donnelly [9] that synchronised to the leading track, whose bit-cell centre transition occurred first. Once the leading track was determined, the software sampled all four tracks as late as possible after the detection of the leading track bit-cell centre. Provided the other tracks have changed within 1/2 a bit-cell the sample will be valid. Since the tape skew oscillates between positive and negative values this sampling arrangement effectively doubles the tolerance to tape skew compared to the previous method.
Although the Leading Edge method, described above, improved on the Fixed Track method it was difficult to implement efficiently due to inter-symbol interference. The ISI altered the bit-cell period, which made it impossible to accurately predict when the end of a bit-cell would occur. Therefore the leading edge technique which relied on sampling the data just before the end of the bit-cell was modified to allow for this fluctuating bit-cell period. An improved technique was devised that identified the trailing edge of the parallel data, the bit-cell centre transition that occurred last, and sampling immediately after. Provided the other tracks have not changed within 1/2 a bit-cell the sample will be valid. This method, shown in Figure 3.8, relies on fast polling of the data tracks, since the error will be the time between successive reads of the data tracks.
**Oversample Read**

Finally to try and completely eliminate any skew-induced errors from the system’s raw error rate an oversample technique was developed. Instead of accurately timing the length of a pulse contained on a single track and then estimating the read position to counter the effects of skew, a timed "interrupt read" oversamples each track individually. Each track was assigned its own software counter to time the duration of a particular binary state. Again the ratio of time or "pulse counts" was used to determine the position and thus the data value.

This technique, illustrated in Figure 3.8, solved several problems encountered by the previous detection techniques, the main one being skew. By treating each track individually skew could be completely eliminated. However, the TMS32010 could not cope with the signal processing required to decode four tracks in real time at the data rates required. By looking at the pulse count for each track it could be seen that as the data rate was increased the distribution of the pulse counts spread. This method was able to highlight the fact that data could be detected up to about 7 kbits/sec/track, beyond this point inter-symbol interference becomes predominant and makes it almost impossible to determine 2:1 & 1:2 ratios from 1:1 ratios.
Error Rate Analysis

Previous work on open-reel multiple track digital tape recorders has indicated that the distribution of errors are largely confined to single track events [13]. Similar tests using a compact cassette system [9] have verified these results and have shown that as track density is increased so too does the number of multi-track errors.

The errors were detected by recording a Maximum Length Sequence, also termed Pseudo Random Binary Sequence\(^1\) (PRBS), of various lengths and comparing it with

\(^1\) Binary maximum length sequences (PRBS) have the property that for an \(n\)-stage linear feedback shift register the sequence repetition period \(p = 2^n - 1\). The PRBS's pass several statistical tests for randomness since there are always \(2^n - 1\) 1's & \(2^n - 1\) 0's which implies equal probabilities for large \(n\). Also the autocorrelation function of a PRBS closely approximates that of white noise.
the received data on a bit by bit basis. The PRBS were generated by software simulation of a shift register with feedback from various stages to the input as illustrated in Figure 3.9. The length of the PRBS is determined by varying stages in the feedback path as shown in Table 3.1.

![4 Track PRBS Generator](image)

**Figure 3.9 PRBS generator**

<table>
<thead>
<tr>
<th>Length of Shift Register</th>
<th>PRBS Length</th>
<th>Exclusive OR Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>15</td>
<td>4,3</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>5,3</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>6,5</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>7,6</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>8,6,5,4</td>
</tr>
<tr>
<td>9</td>
<td>511</td>
<td>9,5</td>
</tr>
<tr>
<td>10</td>
<td>1023</td>
<td>10,7</td>
</tr>
<tr>
<td>11</td>
<td>2047</td>
<td>11,9</td>
</tr>
<tr>
<td>12</td>
<td>4095</td>
<td>12,11,8,6</td>
</tr>
<tr>
<td>13</td>
<td>8191</td>
<td>13,12,11,9</td>
</tr>
<tr>
<td>14</td>
<td>16383</td>
<td>14,13,8,4</td>
</tr>
<tr>
<td>15</td>
<td>32767</td>
<td>15,14</td>
</tr>
<tr>
<td>16</td>
<td>65535</td>
<td>16,15,13,4</td>
</tr>
</tbody>
</table>

The same reference PRBS was synchronised with the replayed data. As each four bit word is read from tape it is compared to the reference PRBS, if the two words are not equal then the software logs the error.
3.2.3 Experimental Procedure

Prior to operation the azimuth angle and lateral displacement of the read/write head were set-up with the aid of a commercial Test Tone Compact Cassette tape. All tapes were bulk erased before being used to store data. Data were not stored until a period of 20 seconds had elapsed after starting the tape to allow sufficient time for the tape leader to pass and the tape speed to stabilise. From error rate analysis performed on the tapes it could be seen that the section of tape following the lead-in splice was prone to a significantly increased number of errors. This is probably due to mechanical damage and debris deposited around the splice. The surface of the recording head, pinch roller and capstan were cleaned using a cotton bud with Iso-Propyl Alcohol (IPA) before recording and at regular intervals between replays.

A number of standard, commercially available compact cassette tapes were used throughout the experimentation. In an attempt to maintain consistent results all tapes used were C90, where the number relates to the total playing time of 90 minutes (45 minutes per side). Since compact cassettes tapes of different lengths have varied thickness of polyester backing, this inevitably leads to significant differences in the transport properties of the tape due to stretching. Apart from their playing times the tapes used can be categorised in terms of the coercivity of their magnetic coating. The magnetic coatings range from the low coercivity (250-350 oersted) gamma ferric oxide, Fe$_2$O$_3$, to chromium dioxide, CrO$_2$, and latterly the highest coercivity (800-1500 oersted) metal particle tapes. The higher coercivity tapes exhibit higher output and improved frequency response compared to Fe$_2$O$_3$. 

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The tapes used were;

- Sony HFS 90 \((\text{Fe}_2\text{O}_3)\)
- Thats TX 90 \((\text{Fe}_2\text{O}_3)\)
- Maxell UDI 90 \((\text{Fe}_2\text{O}_3)\)
- Maxell UDI 90 \((\text{CrO}_2)\)
- Maxell SXII 90 \((\text{CrO}_2)\)
- Sony UX-Pro90 \((\text{CrO}_2)\)
- Thats MG-X90 \((\text{Metal})\)

### 3.2.4 Results for System Characterisation

Initially characterisation experiments were performed to determine the optimum write current and system bandwidth.

The write head must produce a field greater than the coercivity of the magnetic medium in order to record information. The magnitude of the write field is dependent on the write current. Three different types of tape coating were investigated. Each cassette tape was recorded with a range of write currents from 0.1 mA to 2 mA and the signal amplitude of the replayed waveform was measured. Figure 3.10 illustrates how the signal amplitude varies with write current for the coercivities of each cassette. It can be seen that the higher the tape's coercivity the greater the maximum recorded field, therefore the replay head will sense an increased rate of change of flux which results in an increased magnitude replay signal.

The frequency response of the cassette system using the three different types of tape coatings was also measured. A square wave was recorded on each cassette varying in frequency from 100 Hz to 15 kHz. A write current of 1 mA was determined to be
optimum for this experiment to record sufficient field for all coercivities of tape used. The results are displayed in Figure 3.11.

![Figure 3.10 Signal Amplitude as a function of Write Current](image)

![Figure 3.11 Frequency Response of Compact Cassette System with various tapes](image)
From these results a write current just large enough to saturate the medium is required; this is not necessarily the case for high density recording since increasing the write current also increases the width of the replayed pulse [12]. Whilst the recording current must be high enough to exceed the coercivity of each tape coating a low value is desirable to minimise peak shift. Since the replay channel was not equalised a write current of 0.35mA was chosen to minimise inter-symbol interference at the expense of signal amplitude.

3.2.5 Peak Detection Performance

The performance of the system was determined by the raw (uncorrected) error rate measured for a range of data rates from 500 bits/second to 6 kbits/second using several different coding strategies.

A Bi-Phase-L encoded pseudo-random binary sequence was recorded onto ferric tape which was then replayed. On playback all four tracks were sampled simultaneously by the replay software and decoded. Clock recovery and data synchronisation was achieved through computation of the transition ratio. The synchronisation strategies described earlier were employed, the results are illustrated in Figure 3.12.

The detected data were compared to regenerated PRBS and the errors were logged. To obtain accurate error rates the measurements were made over four complete passes of the tape. After each pass the tape was rewound rather than turned over to prevent additional errors due to head skew and mis-alignment occurring.

The graph in Figure 3.13 shows the comparison of results obtained previously by Donnelly [9] and the results obtained with the above system using the Trailing edge
detection technique on Bi-Phase-L code. The curves highlight two distinct regions; where the error rate increases linearly and where the error rate increases logarithmically. This indicates that the errors caused at the lowest data rates were probably due to defects on the medium. Therefore the error rate would be expected to increase linearly with the data rate as the defective area on the magnetic tape affects a higher number of bits.

From data rates above 4500 bits/second errors start to increase dramatically. This was mainly due to a combination of peak shift and droop caused by inter-symbol interference.

![Graph showing error rate performance for various synchronisation techniques.](image)

**Figure 3.12 Comparison of Error Rate Performance for various Synchronisation techniques**
Figure 3.13 Error Rate Performance for a 4 track Digital Compact Cassette Tape Storage System Employing Bi-Phase-L Code

The above process using the trailing edge synchronisation technique was repeated for the range of data rates using the following codes.

- Miller Code
- Miller Squared Code
- 3PM Code
- ISS rate2/3 (1,7) RLL Code
- 1/2 rate (2,7) Code

Figure 3.14 displays the results of applying these more efficient channel codes. Since the maximum frequency of these codes is half that of Bi-Phase-L it was expected to double the data rate with respect to error rate. In practice this was not the case, in fact
the results from using these codes were worse than those using Bi-Phase-L. The reason for this being the software algorithm used to synchronise favours codes with few ratios. Bi-Phase-L code is very robust having only three different ratios, (1:1, 1:2, 2:1), two of which were used for synchronisation. However the other codes have many more ratios that are spaced closer in terms of bit period $T$, therefore at recording rates above 2 kbits/sec/track ISI effectively merged these ratios making it very difficult to synchronise the replayed data. Also synchronisation for these codes relied on identifying unique bit ratios which occurred less often than the 2:1 and 1:2 ratios of Bi-Phase-L code. The PRBS was modified slightly to incorporate more of these ratios but yielded negligible improvement in high density performance.

![Figure 3.14 Comparison of Error Rate performance for various Coding schemes](image)

Figure 3.14 Comparison of Error Rate performance for various Coding schemes

The results from replaying the tape using the oversample detection technique are displayed in Figure 3.15. They show that at low data rates the distribution of pulse
counts (pulse widths) is closely spaced due to little inter-symbol interference. However at double the data rate ISI causes the distribution of pulse counts to spread. This creates problems when trying to determine the 2:1 and 1:2 ratios needed for synchronisation, since once the pulse count for a single pulse exceeds 2/3 the pulse count for a double pulse all ratios become indistinguishable.

These results confirm the fact that for any dramatic improvement in recording density some form of equalisation would be required in addition to azimuth correction. This led to the design of a new system that would incorporate a more powerful DSP device and also replace the peak detection.
Figure 3.15 Distribution of Pulse Count (Width) for Bi-Phase-L at a) Low Data Rate, b) High Data Rate
3.3 Sampling Detection System

To maximise the use of the magnetic tape channel a new system, shown in Figure 3.16, was developed which included a more powerful processor. A second generation TMS320C25 digital signal processor [14] was chosen since it was upwardly compatible with the TMS32010, that provided a more sophisticated instruction set and executed code greater than twice the speed of the previous system. Since the exact amount of processing power required by the computationally intensive tasks to be undertaken is unknown the system was designed so that several processors could be linked together in parallel to share the work load, if required. Another option could also be to upgrade the TMS320C25 to the TMS320C50 when it becomes more commercially available.

Figure 3.16 Photograph of Recording System employing Sampling Detection
3.3.1 Apparatus

**TMS320C25 Processor**

The TMS320C25 can execute up to 12.5 million instructions per second (MIPS) based on a 50 MHz clock giving an instruction cycle of 80 ns. Most instructions take only one instruction cycle. The TMS320C25 uses a 16-bit program and data word size. The Central Arithmetic Logic Unit (CALU) has one 32-bit accumulator register and two registers used in conjunction with the hardware multiplier. The instruction set for the TMS320C25 contains 133 instructions, over half of which involve the CALU to incorporate significant parallelism that speeds up DSP algorithms.

The TMS320C25 has two separate 64K-word memory spaces: program memory and data memory. Some on-board zero wait state memory is provided in both the memory spaces. Internal data memory consists of 544 words of RAM. Internal memory accesses are performed using two internal 16-bit address buses and two internal 16-bit data buses. One of each is used for program memory accesses and the other of each is used for data memory accesses. External memory (both program and data) are accessed using a single external 16 bit address bus and single external 16 bit data bus.

Memory addressing can be either direct or indirect. In the case of direct addressing, memory is paged such that 128 words of data memory are addressable at any one instant. Indirect addressing is provided using eight 16-bit auxiliary registers which are also used for looping control. Seven addressing modes are provided which include two types of address update arithmetic. One of the auxiliary registers is available as an index register and must be shared between the seven other auxiliary registers. The Auxiliary
Register Arithmetic Unit (ARAU) performs all address updates, operating in parallel with the CALU. Six of the addressing modes (which encompass both types of address arithmetic) allow memory updates to be read or written and the auxiliary register to be updated all in the same instruction cycle.

Circuitry

The electronics for this system was constructed on two separate printed circuit boards (PCB's); a digital circuit containing the TMS320C25 processor, RAM, ROM, address decoding logic and peripherals illustrated in Figure 3.17; an analogue interface circuit containing the Analogue to Digital Converters, solenoid drivers, read and write amplifiers shown in Figure 3.18. Each board has an edge connector that connects it to the other via a small single sided 'link' PCB. The system was constructed on two separate boards for three reasons:

a) to facilitate a multi-processor system if additional processing power was required. The processor circuit was designed so that it could pass information between similar or duplicate processors boards thereby creating an easily upgradable system,

b) to separate the analogue and digital electronics to minimise noise between the two,

c) to accommodate heads of greater track density as they become available via the addition of duplicate read amplifier circuits.
Figure 3.17 Block Diagram of Digital PCB containing TMS 320C25

Figure 3.18 Block Diagram of Analogue PCB
**Write Amplifier**

Since the bandwidth of the write amplifier is limited by the time constant of the write-head inductance and its series resistance, the write amplifier used in the first system experienced a poor rise-time. This could be improved in two ways: either increase the voltage across the head and series resistor to increase the rise time or increase the series resistor to improve the time constant. Several circuits adopting one or both of the improvements were designed and tested. The circuit finally chosen sacrificed optimum performance for simplicity. This circuit makes use of the MC1488 quad line driver, which converts standard TTL input logic levels of 0v and 5v through one stage of inversion to output levels of +12v and -12v respectively. Therefore a single, low cost, chip can provide two write amplifiers with the addition of some series resistors.

**Digital "Sampling" Detector**

The term sampling detector is used here to denote the detection method that samples the read-head output with an analogue-to-digital converter. The hope is that the accuracy introduced by using sampling detection and the ensuing digital signal processing can be used to compensate for inter-symbol interference more successfully than is possible with peak detection, thereby increasing density. The density increases have been verified in a number of theoretical and experimental works [3,4].

As before, each channel is pre-amplified, low pass filtered and then amplified again. The signal is then passed through a second order anti-aliasing filter. Band limiting the signal reduces aliasing errors caused by sampling the input signal at a rate less than twice the highest frequency of the signal.
Several basic circuit configurations using operational-amplifiers are available to perform the filtering operation. These circuits allow the choice of external components to determine the passband characteristics and cut-off frequency of the filter. The Infinite-Gain Multiple-Feedback (IGMF) filter was chosen since it required the fewest external components whilst maintaining good stability. It also gives an inverted gain and has a low output impedance.

Finally, the signal is digitised using a 10 bit analogue to digital converter (ADC) with conversion times better than 2 μs, thus permitting a sampling rate of 100 ksamples/s.

3.3.2 Read Equalisation

Read equalisation was employed to improve recording density by reducing the effects of inter-symbol interference. Since the sampling detection system digitised all the readback signals, a digital signal processing technique was employed to slim the readback signals.

*Finite Impulse Response (FIR) Transversal Filter*

Many electrical designs can produce the desired equalisation, however an efficient equalisation method for the applied cassette tape recorder seems to be a tapped-delay-line equaliser or transversal filter, illustrated in Figure 3.19.

The slimmed pulse as a result of the summed filter waveforms, shown in Figure 3.20, also introduces *under-shoots*. These under-shoots increase in magnitude as the resultant pulse width is reduced. If two slimmed pulses are placed in close proximity then these under-shoots will over-lap causing interference that effectively represents high
frequency noise. It is this trade between the increase in noise and the reduced pulse width that determines the degree of equalisation.

The circuit delays the input signal by multiples of the clock period, multiplies the delayed versions by coefficients $W_i$, also termed the tap weights, and sums them to produce the equalised output signal. Design of the circuit involves choosing the coefficients to minimise inter symbol interference at a finite number of points in the time domain or to shape the overall frequency response in the frequency domain. Both amplitude and phase response can be equalised by the transversal filter. Digital Signal Processors are designed specifically to implement such filters, therefore these filters were implemented in software using the TMS320C25.

In the time domain the output signal of the circuit $y(t)$ is

$$y(t) = -W_0.x(nT) + W_1.x((n-1)T) - W_2.x((n-2)T)$$

where $n=0,1,..N$  \hspace{1cm} eqn(1)

By means of variable time delays $z^{-1}$ and adjustable gains $W_0, W_1$ and $W_2$ at each tap an isolated read pulse $x(nT)$ can be slimmed and made symmetrical.

Appropriate adjustments of $W_0$ and $W_2$ force zero-crossings of signal $y(nT)$ at $n=0$ and $n=2$. The transfer function yields

$$H(j\omega) = -W_0 + W_1.e^{j\omega T} - W_2.e^{j2\omega T}$$

$$= (W_1 - W_0.e^{j\omega T} - W_2.e^{j2\omega T}).e^{j\omega T}$$

Suppose $W_0 = W_2 = W$ and $W_1 = 1$ then

$$H(j\omega) = (1 - 2.W\cos(\omega T)).e^{j\omega T}$$

Amplitude: $|H(j\omega)| = 1 - 2.W\cos(\omega T)$ \hspace{1cm} eqn(5)

Phase: $\arg (H(j\omega)) = -\omega.T$ \hspace{1cm} eqn(6)
Due to the relatively low number of coefficients initially employed an iterative technique that measured the pulse width for a given set of coefficients was used to determine optimum equalisation. This method proved adequate for a 3-tap filter, however, using an equaliser of longer length required an alternative approach. Since the variety of tapes used possessed different frequency responses an adaptive equaliser was employed to obtain the optimum coefficients.
3.3.3 Adaptive Equalisation

There are a number of advantages to using an adaptive equaliser in a magnetic recording system, in addition to the potential for density increase:

a) The channel characteristics can vary significantly with time (due to wear), temperature and good detection schemes should adapt to such changes.

b) An adaptive equaliser can compensate for the variations in heads and recording mediums that inevitably occur during the manufacturing process.

c) The effect of mechanical tolerances causing mis-alignment of the read head with respect of the write head can also be minimised.

d) An adaptive equaliser can significantly reduce maintenance and installation costs by eliminating the need for individual product adjustment, which are instead performed automatically by the equaliser.

e) Finally, an appropriately modified adaptive equaliser can compensate for non-linear and data-dependent noise effects that can not be accurately anticipated or eradicated in a fixed design.

The adaptive equaliser, shown in Figure 3.21 consists of two distinct parts; a digital transversal filter with adjustable coefficients and an adaptive algorithm which is used to modify the coefficients of the filter. In 1960 Widrow and Hoff [16] presented a Least-Mean-Squared (LMS) error algorithm for general purpose adaptive filtering, when a training sequence is available. Lucky [17] further observed that once reliable decisions are available at the receiver output, these decisions could replace the training sequence
so that adaptation may be continuous in order to track slow changes in the channel. There are a variety of different structures and related algorithms for adaptive equalisation in saturation recording channels [18,19]. However, the LMS algorithm is generally accepted as one of the most computationally efficient and easiest to implement and hence was adopted for this purpose.

Two input signals, $x_k$ and $s_k$ are applied simultaneously to the adaptive equaliser: the readback signal $x_k$ (to be slimmed) and $s_k$ is the training sequence of desired outputs. For stationary\(^2\) inputs the mean-square value of the difference between the desired response $s_k$ and the equaliser output $y_k$, the mean-squared error (MSE), is precisely a second order function of the tap weights in the transversal filter. The dependence of the MSE on the unknown tap weights may be viewed in the form of a multidimensional

---

\(^2\) When the statistical characteristics of the sample function do not change with time, a random process is said to be stationary.
paraboloid (like a punch bowl), with a uniquely defined bottom or minimum point. We refer to this paraboloid, illustrated in Figure 3.22, as the error performance surface where the optimum coefficients correspond to the minimum point of the surface. The LMS adaptive process derives optimum coefficients iteratively, starting at an initial set of values and continuously updating the coefficients. The algorithm employs a steepest descent gradient procedure to solve the minimum value of hyperparabolic error surface and thus converge onto the optimum operating point.

The procedure may be described mathematically as

\[ W_{k+1}(i) = W_k(i) + 2\mu e_k x_{k,i} \]

\[ i = 0, 1, \ldots, L \]  

\text{eqn}(7)

where \( \mu \) is the step size or convergence factor and \( e_k \) is the error signal given by

\[ e_k = y_k - s_k \]

\text{eqn}(8)
3.3.4 Results for Pulse Slimming

A simple 3-tap FIR filter was implemented to equalise the channel by slimming the pulse. The resultant waveform shown in Figure 3.23 is approximately 20% slimmer than the original playback signal.

![Isolated Lorentzian Pulse Slimmed with a 3-tap Transversal Filter](image)

**Figure 3.23 Isolated Lorentzian Pulse Slimmed with a 3-tap Transversal Filter**

Since the replay signal is not a perfect Lorentzian pulse it is necessary to increase the length of the filter to improve the response. By oversampling the readback signal at 100 k.samples/sec. and employing a 5-tap filter it was possible to slim the pulse by approximately 30%.

At high recording densities\(^3\) the readback signal is the linear super-position of the isolated pulses. Figure 3.24 illustrates how, at these high densities, ISI causes errors

\[^3\] A data rate of 5.5 kbits/s/track using Bi-phase-L code translates to a recording density, \(PW_{50}/T=3.9\), where \(T\) is the minimum time between transitions.
due to peak shift and droop. However, when the readback signal is equalised using the FIR filter the peaks are shifted back and the droop is removed, thus permitting improved detection. Figure 3.25 shows that an unequalised Lorentzian signal of density $PW_{50}/T=3$ has comparable peak position and amplitude to an equalised Lorentzian signal of density $PW_{50}/T=3.9$. Since the most dominant cause of errors in the peak detector are ISI-induced, the SNR degradation due to undershoots, caused by the equaliser, can be largely disregarded. Therefore the equalised signals yield an approximate 30% increase in performance relative to the unequalised signals.

Figure 3.26 illustrates how a real readback signal from the cassette system (top) has been slimmed after passing through a 5-tap transversal filter (bottom). Further increases in the tap length will result in improved slimming but the law of diminishing returns applies. A 10 tap equaliser has been reported as being optimum for slimming the pulse whilst minimising the effects of undershoot. However, applying equalisers of greater length to the above experimental system gave negligible measurable improvement.

Applying pulse slimming to the readback signals of the previous system would also result in at least a 30% increase in data rate for the same error rate as illustrated in Figure 3.27.

A paper based on these results [20], titled "Pulse Slimming in Magnetic Recording using Digital Signal Processing Techniques" (Appendix A), was presented at Euromicro '92 Conference.
Figure 3.24 Modelled Equalisation of a Lorentzian Readback Signal at High Recording Density (PW50/T=3.9)

Figure 3.25 Comparison of Unequalised and Equalised Lorentzian Signal of Density PW50/T=3.9 and Unequalised Lorentzian Signal of Density PW50/T=3
Figure 3.26 Equalisation of Readback Signal Slimmed via 5-tap Transversal Filter

(a) Read signal from head  (b) Read signal after pulse slimming

Figure 3.27 The Effects of Pulse Slimming on Error/Data Rate for a Multi-Track Tape System
3.4 Discussion

It can be seen from Figure 3.14 that Bi-Phase-L code by far exceeded the error rate performance of all other codes. This was attributed to the minimal number of integer bit-cell ratios, namely 2:1, 1:2 and 1:1, which permitted the code to operate at a normalised density $\text{PW}_{50}/T = 3$. This achievement was possible because the ISI was controlled much more than in other codes which possess higher or non-integer bit-cell ratios. This resulted in fewer ISI induced amplitude variations which the peak detector was able to detect. Thus even if the other codes were used with pulse-slimming equalisation they could never achieve normalised densities much above 1.5 without experiencing severe ISI that would cause a peak detector to malfunction.

It was therefore concluded that pulse slimming and other types of equalisation that try to resist the effects of ISI, by increasing the bandwidth of the signal to compensate for the loss in high frequencies produced by the recording channel, were not optimal. Hence further attention was concentrated on a different type of equalisation, such as Partial Response Signalling, that controlled ISI and associated detection methods, namely Maximum Likelihood Detection.
3.5 References


15. Troullinos, G., & Bradley, J., "Hardware interfacing to the TMS320C25 - Product Application", Texas Instruments,


CHAPTER 4

Increased Recording Density using One Dimensional Coding Techniques

4.1 Introduction

In conventional magnetic recording channels employing peak detection, RLL constrained codes have played a crucial role in achieving maximum linear bit density with low error rate. The \((d, k)\) constraints reduce inter-symbol interference whilst ensuring adequate self-clocking characteristics of the data signal. Recently, a different approach to combating ISI, referred to as PRML (Partial Response (PR) signalling with Maximum Likelihood (ML) sequence detection) has demonstrated increased storage potential. Therefore, a new breed of constrained codes have been developed for this channel that limited the complexity of the computationally intensive Viterbi decoder, as well as providing timing and gain control. Also, since PRML allows for controlled ISI, the commonly used \(d\) constraint need not be greater than zero; indeed \(d = 0\) allows for a higher code rate and thus a lower clock rate for a given source data rate.
4.2 Maximum Likelihood Sequence Detection

Maximum Likelihood Sequence Detection (MLSD) was cited by Forney [1] for optimum data detection in bandlimited data communication channels. For a given received sequence, the maximum likelihood detector selects (decodes) the output sequence which is most likely, on the basis of observation, to have been the transmitted sequence. An example of the complexity associated with a brute force approach to MLSD can be achieved by realising that if \( L \) is the number of different output levels, and \( N \) is the length of the transmitted sequence then there are \( L^N \) different possible transmitted signals. This would suggest an unreasonably complex detector since \( L^N \) likelihood values would have to be computed and compared with one another. For \( L=2 \) and \( N=1000 \) this would mean \( 2^{1000} \) computations, an impossibly large number. The virtue of the Viterbi Algorithm is that the number of computations necessary for MLSD grows linearly with \( N \) rather than exponentially.

4.2.1 Viterbi Algorithm

In 1967, Viterbi [2] introduced a decoding algorithm for convolutional codes which has since become known as the **Viterbi Algorithm** (VA). Omura [3] showed that the Viterbi algorithm was equivalent to a dynamic programming solution to the problem of finding the shortest path through a weighted graph. Forney [4] later recognised that it was in fact a maximum likelihood decoding algorithm for convolutional codes and pointed out that the VA could also be used to produce the maximum likelihood estimate of the transmitted sequence over a channel with inter-symbol interference. A comprehensive description of the Viterbi algorithm is also given in [5].
This technique can be contrasted with conventional threshold detection, such as peak detection, where valuable information is otherwise lost. Viterbi Detection is illustrated in Figure 4.1 where an equalised playback waveform fails to cross the threshold of a peak detector at a given point due to noise, thus resulting in erroneous data. However by using Viterbi detection it can be seen in Figure 4.2 that the hard decisions of the peak detector are replaced by a number of intelligent soft decisions, which result in the correct data sequence being identified. In Viterbi detection, the signal is sampled and its value interpreted relative to those of surrounding samples, rather than by measuring whether the signal has crossed a threshold with a fixed value. This enables otherwise ambiguous samples to be correctly interpreted, yielding a lower error rate or a higher bit-packing density.

The Viterbi algorithm is simply a fast algorithm for searching a labelled trellis for a path that most closely agrees with a given path. The VA operates iteratively frame by frame, tracing through the trellis in the hope of finding the correct path. For each branch (path) of the frame it computes the squared error between the received sequence and each possible output sequence (weight). For each node (state), the branches entering the node with the greatest cumulative weight are rejected. At any frame of the trellis the Viterbi detector does not know which node the true data sequence has reached, nor does it try to decode it immediately. A symbol is successfully decoded when only one survivor path emanating from a node is remaining.
Figure 4.1 Equalised Playback waveform

The paths to each node with the greatest weights are least likely, therefore these are rejected in favour of those with the MAXIMUM LIKELIHOOD of being correct.

A path can only be eliminated when all paths from a previous node result in the greatest cumulative weight.

The path with the smallest cumulative weight is not always the chosen path. Here the path with the smallest cumulative weight is eliminated in the next frame.

It may be necessary to proceed a long way along the trellis before a definite decision can be made.

Figure 4.2 Detection of playback waveform using Viterbi Detection
Kobayashi [6] first suggested the use of the Viterbi Algorithm in the magnetic recording channel to detect NRZ and NRZI coded sequences. A number of published works [7-9] have analysed the performance of maximum likelihood detection for magnetic recording. Cioffi and Melas [10] have concluded that it offers up to a four-fold improvement over that achievable with their peak detection system. French [11] has devised a software implementation of the VA for the (1-D) partial response magnetic recording channel. Whilst Schneider [12] has proposed a novel Viterbi equivalent decoder that takes advantage of the channel response and RLL code constraints to reduce hardware complexity. Shafiee & Moon [13] have investigated methods to reduce the complexity of the Viterbi detector for a magnetic channel employing EPR4, whereas Wood [14] has suggested a reduced-complexity algorithm that may be implemented more efficiently than the VA.

4.2.2 Sequence Detection

A Maximum Likelihood Detector effectively searches the entire code for the best approximation, whereas sub-optimal detectors such as a sequential decoder restricts its attention to those sections of the code that appear most likely to contain the actual message. A sequential detector performs well when there is a low level of noise present, but under more severe noise conditions generally fails. In a sense, a ML detector is designed for the worst case noise event and does not take advantage of lulls in noise levels, whereas a sequential detector does not burden itself with a large amount of work when the sequence is fairly obvious, but expends the greatest effort when a sequence is not so obvious.
Individual bit decisions made by a sequential detector must be considered tentatively at each step of the decoding process. The decoder must maintain the ability to reverse any or all of the bit decisions at a later time if it seems necessary. Sequential decoding algorithms are thus sub-optimal tree searching procedures that share two features. Specifically, a sequential decoder attempts to quickly recognise that it has departed from the correct path in the code tree and uses a sequence of test criteria on path metrics to determine the relative "correctness" of the path being pursued. A sequential decoder attempts to find the best path in the code tree by searching for the path with the largest metric.

_Fano Algorithm_

A very popular sequential decoding algorithm is the one developed by Fano [15]. This technique can be used for hard or soft decoding. The Fano decoder searches for the most likely transmitted message moving along the code tree one node at a time. The direction of movement, forward or backwards, is determined by the behaviour of the metric, therefore the selection of the metric used for sequential decoding is most crucial for correct operation. Fano proposed the best metric function for sequential decoding is the logarithm of the probability that a particular output is observed given that a certain symbol was transmitted, normalised by the total probability of the observed output. The metric suggested by Fano has the property that correct path metrics tend to increase at a moderate rate whereas incorrect path metrics decrease relatively quickly.
4.3 Partial Response Signalling (Correlative Level Coding)

Nyquist [16] investigated the transmission of data through a bandlimited channel. He concluded that for any bandwidth $B$ Hz it is theoretically possible to transmit $2B$ symbols/second, commonly referred to as the *Nyquist Rate*, without inter-symbol interference. In other words, he showed that a maximum of $B$ symbols/second can be transmitted across a communication channel with a rectangular frequency response cutting-off at $B/2$ Hz, the so-called *Nyquist Bandwidth*. The pulse shape required to satisfy the Nyquist criteria is a sinc $\left(\frac{\sin(x)}{x}\right)$ function that decays at a rate of $1/t$. A sequence consisting of sinc pulses experiences considerable ISI, however the resultant signal always pass through zero at the detection instances.

The $\frac{\sin(x)}{x}$ pulse shape is not practical for two main reasons:

a) The slow rate of decay makes the pulse nearly impossible to generate and implies that extreme peak amplitudes can occur from the addition of delayed *sidelobes* (tails) in phase.

b) The inter-symbol interference is only low within a small range around the detection instances and is therefore critically dependent on accurate timing synchronisation. Practical timing offset and jitter would generate considerable errors.

Thus waveform design with sinc pulses to make full use of the available channel bandwidth, though mathematically simple, is impractical to implement.
In 1963, Lender [17,18] demonstrated that it is possible to transmit at the Nyquist rate of 2B symbols /second with no inter-symbol interference using the theoretical minimum bandwidth of B hertz. Lender pioneered a technique termed *Duobinary signalling* where the "duo" implies the doubling of the transmission capacity (double the speed or half the bandwidth) when compared to the raised cosine filter. The technique, more commonly called *Partial Response Signalling* or *Correlative Level Coding*, relied on introducing a controlled amount of ISI into the data stream rather than trying to eliminate it completely. By introducing correlated interference between the pulses the resultant effect "cancelled out" the interference at the detector. The term Partial Response arises from the idea of conveying information in a channel subject to controlled amounts of inter-symbol interference, conversely a full response assumes zero ISI. Thus the peak detection channel previously described is a full response channel.

There are a number of partial response schemes, categorised by Kretzmer [19], that are suitable for digital communication systems. Kabal and Pasupathy [20] have produced an excellent tutorial paper in which they define nine different partial response channels in terms of the characterising polynomial $F(D)$, transfer function $H(\omega)$, impulse response $h(t)$, and number of output signal levels. A selection of these PR schemes are given in Table 4.1, where the system polynomials are of the form

$$F(D) = \sum_{n}^{N-1} f_n D^n \tag{1}$$

where $D$ is the delay operator equal to one symbol delay, and $f_n$ are the sample values of the desired impulse response that effectively define the correlation between symbols in the information sequence.

From equation (1) the frequency response, $H(\omega)$ is obtained by substituting $D = e^{j\omega T}$. 123
One of the merits of PR signalling is that the introduction of correlation imparts an overall structure to the data sequence which can be used to shape the frequency spectrum of the system. For instance certain partial response schemes have desirable attributes such as nulls in the frequency response, at the Nyquist frequency and at d.c., which can make the system less sensitive to timing errors. This allows practical channels using PR signalling to transmit at the Nyquist rate, a feat not possible with ordinary pulse amplitude modulation.
The main disadvantage of partial response signalling is that it increases the number of output levels to be larger than the number of input levels. Thus, a detector using symbol by symbol detection will experience a reduced signal to noise ratio. Eye diagrams for PR4 and EPR4 waveforms are shown in Figure 4.3 and Figure 4.4 respectively. These diagrams represent the overlaying of the channel output signal seen in each time interval \( T \), assuming a random binary input sequence. One can clearly see the nominal three (respectively, five level) set of values at the sampling instances for PR4 (respectively, EPR4) response. The eye diagrams provide some useful, qualitative indication of the robustness of the sample values at the bit cell boundaries in the presence of additive noise and jitter.
Figure 4.3 Eye Diagram for PR4

Figure 4.4 Eye Diagram for EPR4
4.3.1 Application to Magnetic Recording

Conventionally, partial response signalling has been considered as a transmission technique, where the shape of the response is applied at the transmitter. However, in magnetic recording, the channel with appropriate equalisation, is used to give the desired pulse response. Kobayashi & Tang [21] first discovered that the step response to a transition and the pulse response to a dibit bear close resemblance to class IV partial response (PR4) and extended class IV partial response correlative encoding schemes.

To understand why partial response signalling applies well to magnetic recording, consider the digital data, represented as an impulse train, is first applied to a zero order hold circuit to form the write current in NRZ format. The write current records transitions on the medium which are sensed by the readback process, resulting in an analogue signal. The readback signal has a bandpass spectrum which can be approximated by

\[(af)\exp(-\mid bf\mid)\] eqn\{2\}

where \(f\) is the frequency, and \(a\) and \(b\) are constants.

The combined transfer function of the zero order hold and the write process is similar to the (1-D) partial response system. Thus, partial response polynomials for saturation recording must include a (1-D) factor. Since the amplitude response of the (1-D) system is high pass, it must, for purposes of bandwidth efficiency and spectral matching with the overall channel transfer function, be moulded into a bandpass response. The PR4 class introduces a spectral null at the Nyquist frequency to achieve the bandpass spectrum. The spectral requirements of the PR4 detector can be satisfied over a range of
recording density by suitably equalising the readback signal. Beyond that range, however noise enhancement penalty in equalisation and the lower readback SNR, due to the peak power limitation of the channel, cause the input SNR to the detector to drop below the theoretical level for acceptable performance.

Thapar & Patel [22] noted that the class of partial response systems characterised by the following polynomial,

\[ P_n(D) = (1-D)(1+D)^n \quad n=0,1,2,3,\ldots \quad \text{eqn}\{3\} \]

where \( P_n(D) \) defines the input-output relationship, are particularly suited to the magnetic recording channel.

The signal spectrum, \( H_n(\omega) \), for the polynomial in equation \{3\} is obtained by calculating its Fourier Transform, and this is simply achieved by substituting \( D=e^{j\omega T} \), which yields

\[ H_n(\omega) = jT2^n\cos^{n-1}(\omega T/2)\sin(\omega T). \quad \text{eqn}\{4\} \]

Figure 4.5 displays \( H_n(\omega) \) for a normalised frequency and various \( n \). As shown, the signal energy shifts towards the lower frequencies with increasing \( n \). If, for increasing \( n \), the symbol interval \( T \) is reduced so that roughly the same spectral region is occupied, the data rate (and thus the data storage density) increases correspondingly.
The coefficients of $D$ in the polynomial $(1+D)^n$ represent the relative sample values of the step (single transition) response at the sampling instants. This also describes the low pass characteristic that limits the bandwidth more with increasing $n$. The effect of the $(1-D)$ factor is to convert the step response into a pulse (dibit) response. Therefore, the magnetic recording channel is equivalent to a dispersive transmission filter which approximately shapes the recorded signals into partial response waveforms.

As with all partial response signalling schemes the output signals are related by the linear superposition to the particular sequence of the appropriate Nyquist channel outputs. Thus, the PR4 step-function response, illustrated in Figure 4.6, may be obtained by adding two Nyquist step-function responses (sinc pulses) one bit interval apart.
Increasing the value of $n$ and altering the signalling rate effectively packs more bits into the natural width of the step pulse. Alternatively, if $n$ is increased for a constant signalling rate a broader step pulse is required, with spectral energy being distributed to the lower frequencies.

The performance of the system employing partial response signalling depends mainly on the choice of $n$. How $n$ is chosen, in turn, depends upon the channel response, the desired data rate or density, and the noise characteristics.
For higher values of \( n \) the frequency response of the polynomial in equation (3) is a better match to that of the magnetic recording channel, hence reducing the noise enhancement penalty in channel equalisation. However, as \( n \) is increased more controlled ISI is introduced resulting in a larger number of output levels at the sampling instants.

Forney [1] describes how a maximum likelihood detector can be employed, where the number of trellis states also increases with \( n \), to limit the SNR loss due to a greater number of levels.

Using the Viterbi Algorithm the loss due to ISI is approximated by the ratio \( d_{\text{min}}^2/E_p \), where \( d_{\text{min}}^2 \) is the squared minimum distance between soft decision sequences and \( E_p \) is the energy in the partial response pulse. Table 4.2 shows the loss in SNR for various values of \( n \).

<table>
<thead>
<tr>
<th>PR</th>
<th>( P_a(D) )</th>
<th>( n )</th>
<th>No of Levels</th>
<th>( E_p )</th>
<th>( d_{\text{min}}^2 )</th>
<th>( d_{\text{min}}^2/E_p )</th>
<th>SNR loss (dB) due to ISI</th>
</tr>
</thead>
<tbody>
<tr>
<td>dicode</td>
<td>1-D</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>PR4</td>
<td>1-D^2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>EPR4</td>
<td>1+D-D^3-D^4</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>16</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>E^2PR4</td>
<td>1+2D-2D^3-D^4</td>
<td>3</td>
<td>7</td>
<td>10</td>
<td>24</td>
<td>0.6</td>
<td>2.2</td>
</tr>
<tr>
<td>E^3PR4</td>
<td>1+3D+2D^2-2D^3-3D^4-D^5</td>
<td>4</td>
<td>13</td>
<td>28</td>
<td>48</td>
<td>0.4</td>
<td>3.7</td>
</tr>
<tr>
<td>E^4PR4</td>
<td>1+4D+5D^2-5D^4-4D^3-D^6</td>
<td>5</td>
<td>19</td>
<td>84</td>
<td>120</td>
<td>0.3</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 4.2 Parameters of Higher-Order Partial Response systems
4.4 PRML Coding

4.4.1 Precoding

Precoding is a non-redundant transformation on a bit stream prior to entering a channel that cancels another transformation that takes place on the bit stream within the channel (or modulator/demodulator). The cascade of the precoder and the channel will return the binary sequence to its original form.

This technique is used to alleviate error propagation at the decoder by eliminating the effect of previous symbols at the source where they are known precisely. In essence this is equivalent to making the decoder memoryless.

For instance, the recorded binary sequence $a_k$ replayed through a magnetic recording channel equalised to PR4 ($1-D^3$), shown in Figure 4.8, can be represented by

$$c_k = a_k - a_{k-2}$$  \hspace{1cm} eqn{5}

Therefore each transmitted symbol is correlated with a prior symbol which suggests that the original binary recorded sequence can be decoded as

$$\hat{a}_k = \hat{c}_k + \hat{a}_{k-2}$$  \hspace{1cm} eqn{6}

From this equation it can be seen that if noise corrupts $\hat{c}_k$ then not only will $\hat{a}_k$ be in error but so also will $\hat{a}_{k+2}, \hat{a}_{k+4}, \hat{a}_{k+6},...$

Figure 4.9 illustrates how Lender [23] cleverly solved this problem by introducing a precoder, using modulo-2 arithmetic, that has a transfer function equal to the inverse of the channel.
Figure 4.8 Propagation of Errors in PR4 channel

Figure 4.9 Use of Precoder to prevent Error Propagation
4.4.2 Randomisation

Early attempts at applying partial response and maximum likelihood detection to the magnetic recording channel involved the use of data randomisation [24, 25]. As described in chapter 2, randomisation or scrambling is a non-redundant technique employed to alleviate undesirable properties of the source data, e.g., d.c. content, and long run lengths. This coding technique was also employed in the first known commercial product to employ PRML [26-28], a "black box" flight recorder produced by Ampex.

The main reason this method of coding was chosen instead of a conventional 1/2 rate RLL codes [29,30], was to enable the channel to be considered as two interleaved (1-D) channels. Thus allowing the Viterbi detector to be split into two independent detectors, that could proceed completely separately on the two, odd and even, data streams. From this separation three distinct advantages accrue:

a) Both data streams can be processed at one-half the data rate. At high data rates this is a vital consideration and is in sharp contrast to the Viterbi detection of Miller Squared code [31] where similar processing must proceed at twice the source data rate.

b) The channel memory which determines the complexity of the Viterbi detector is halved.

c) Two-state detectors are more easily implemented than detectors involving an increased number of states such as would be required for a single Viterbi detector [32].
4.4.3 (0, G/I) Codes

The previous section highlights the advantages in Viterbi detection by considering the PR4 channel to be two interleaved (1-D) channels. This group of codes was specifically designed for this type of channel implementation.

In a PRML channel, a channel code can also be used to provide clocking and automatic gain control (AGC) information. Since the maximum run length of nominally zero samples must be limited, the \( k \) constraint is still appropriate when specifying the channel code requirements for PRML channels. However, RLL codes with \( d \) greater than zero are not necessary in PRML channels because compensation for ISI is inherent in the ML detector. Thus, there is no need to reduce interference by coding with a \( d \) constraint. On the other hand the \( k \) constraint in not the only constraint required for the PRML channel. Since ML detection requires that more than one option be kept open with respect to recent past data estimates, an additional constraint is desired to limit both detectors' delay and hardware complexity. If a data sequence of the input signal is demultiplexed into an odd and even indexed sample sub-sequence, and ML detection is applied to each sub-sequence independently, a constraint on the number of successive nominally zero samples in each sub-sequence adequately limits the detector delay and hardware.

Codes for PRML can therefore be designed as (0, G/I) codes where 0 is the value for the conventional \( d \) constraint; \( G \) is analogous to the \( k \) constraint, and represents the maximum runlength of zero samples in the Global output data stream; and \( I \) represents the maximum number of consecutive zero samples in each of the Interleaved branches of the output data stream. A small value \( G \) is desirable for accurate timing and gain.
control, and a small value of $I$ reduces the size of the path memory required in the ML detector.

These modulation codes improve performance of the timing and gain control circuits of the channel by providing frequent non-zero samples. In addition, they limit the complexity of the ML detector by forcing the path merging in the path memory during processing of data estimators.

Finally, another constraint, $M$, may be added to limit the number of consecutive non-zero symbols in the ternary readback sequence. This constraint is required to provide good discrimination of encoded user data from the preamble field of non-zero symbols used in most systems [33].

The channel memory makes it cumbersome to assign the $(0, G/I)$ constraints to the encoder map, therefore the channel memory is nullified by using a precoder.

_Eggenberger 8/9 (0,4/4) PRML code_

Both the one gigabit per square inch experiment [34] and the first disk drive products employing PR4 signalling [35,36] utilise different versions of a particular rate 8/9 (0, 4/4) code, devised by Eggenberger and Patel [37].

According to the present invention, the smallest code value of the parameters $G$ and $I$ for which a rate 8/9, $(0, G/I)$ block code exists are $(0,3/6)$ and $(0,4/4)$. A rate 8/9 RLL

*Preamble is the pattern used for training the synchronisation of the Phase Locked Loop employed in clock recovery, usually a long sequence of nonzero symbols which implies a long series of transitions.
block code having \((0,4/4)\) constraints provides 279 9-bit code words from 8-bit data bytes, see Table 4.3. Thus, at least 256 code words of 9 bits each can be uniquely defined where all concatenations of such code words comply with the \(G/I\) constraint. The code provides for specific assignment of 8-bit data bytes to 9-bit code words which preserves read-backwards symmetry and creates partitions of bytes and code words with similar structure. The partitions of bytes are uniquely identifiable and overall mapping of the code words is produced by gating partition bits according to simple Boolean functions.

If \(Y\) denotes a 9-bit code word in the \((0,G/I)\) code then

\[
Y = \{Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9\} \quad \text{eqn}\{7\}
\]

The constraint \(G = 4\) in the overall coded sequence can be produced by eliminating 9-bit sequences with run lengths of 3 zeroes at either end and run lengths of 5 zeroes within each 9-bit sequence. Such a constraint is given by the following Boolean relation,

\[
(Y_1+Y_2+Y_3)(Y_2+Y_3+Y_4+Y_5+Y_6)(Y_3+Y_4+Y_5+Y_6+Y_7)(Y_4+Y_5+Y_6+Y_7+Y_8)(Y_7+Y_8+Y_9) = 1 \quad \text{eqn}\{8\}
\]

Similarly, the constraint \(I = 4\) is described by the following two equations for the sequence of all odd bit positions and the sequence of all even bit positions respectively,

\[
(Y_1+Y_3+Y_5)(Y_5+Y_7+Y_9) = 1 \quad \text{eqn}\{9\}
\]

\[
(Y_2+Y_4+Y_6)(Y_4+Y_6+Y_8) = 1 \quad \text{eqn}\{10\}
\]

Two hundred and seventy nine valid 9-bit binary sequences satisfy equations \{8\}, \{9\}, and \{10\}, the hexadecimal equivalents of which are given in Table 4.3.
The patent [37] also describes a similar set of Boolean equations that describes 272 9-bit code words derived for a rate 8/9 (0, 3/6) code. The (0, 3/6) code has a smaller value of \( G = 3 \) and will therefore provide more accurate timing and gain control than the (0, 4/4) code. However it has a disadvantage relative to the (0, 4/4) code in that \( I = 6 \) which indicates it requires more hardware in the Viterbi detector path memory. Since these codes cannot be used simultaneously, a design choice must be made depending on the relative importance of the advantages inherent in each code. It was this compromise that led to other (0, \( G/I \)) codes being sought.

Table 4.3 Hexadecimal Code word of (0, 4/4) Code

<table>
<thead>
<tr>
<th>049</th>
<th>04B</th>
<th>04C</th>
<th>04D</th>
<th>04E</th>
<th>04F</th>
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<th>05B</th>
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<th>05F</th>
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<td>0B5</td>
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<td>0BB</td>
<td>0BC</td>
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<td>1E6</td>
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<td>1F2</td>
<td>1F3</td>
<td>1F4</td>
<td>1F5</td>
<td>1FF</td>
<td>1FF</td>
</tr>
</tbody>
</table>

Marcus rate 8/9 (0, 3/5) PRML code

Marcus et al., [38] realised that with added encoder complexity it was possible to produce a rate 8/9 (0, 3/5) code. Such a code has the timing and gain control advantages
of the (0, 3/6) code and the reduced Viterbi detector path memory requirement advantage of the (0, 4/4) code.

As in the previous section, if we let Y denote a 9-bit code word as described by equation \(7\). Then the \(G = 3\) constraint can be produced by eliminating sequences with run lengths of three zeros at the left end, run lengths of two zeros at the right, or run lengths of four zeros within each 9-bit code word. Such a constraint is given by the following Boolean equation

\[
(Y_1+Y_2+Y_3)(Y_2+Y_3+Y_4+Y_5)(Y_3+Y_4+Y_5+Y_6) \\
(Y_4+Y_5+Y_6+Y_7)(Y_5+Y_6+Y_7)(Y_8+Y_9) = 1
\]  

Equation \(11\)

Similarly the constraint \(l = 5\) is described by equations \(9\) and \(10\) of the previous section.

Although such a block code provides at most two hundred and fifty one 9-bit code words from 8-bit data bytes, two hundred and fifty six code words can be derived by excluding the all-ones code word and adding 6 state-dependent code word pairs for two state encoding. The two states are identified by the value of the last bit of the previous code word concatenation. Table 4.4 contains the possible values of code words which may result.
Table 4.4 List of State Dependent Code Words

<table>
<thead>
<tr>
<th>State dependent code words</th>
<th>State 0 (PAST = 0)</th>
<th>State 1 (PAST = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0A01B0101</td>
<td>010100101</td>
<td>000110101</td>
</tr>
<tr>
<td>0A01B0110</td>
<td>010101101</td>
<td>000111101</td>
</tr>
<tr>
<td>0A01B0111</td>
<td>010100110</td>
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<tr>
<td>0A01B1101</td>
<td>010101110</td>
<td>000111110</td>
</tr>
<tr>
<td>0A01B1110</td>
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<td>000110111</td>
</tr>
<tr>
<td>0A01B1111</td>
<td>010101111</td>
<td>000111111</td>
</tr>
</tbody>
</table>

where $A = (PAST)$ and $B = (PAST)$

PAST being a binary variable that defines the state and is given by the last digit of the preceding code word.

Capacity of $(0, G/I)$ Codes

Marcus et al, [39, 40] describe how a $(0, G/I)$ constraints code can be represent by diagrams based on states which reflect the three relevant quantities: the number $g$ of zero symbols since the last non-zero symbol in the global string; and the number of 0’s since the last 1 in each of the two interleaved sub-strings denoted by $i$ and $j$. Note that $g$ is a function of $i$ and $j$, denoted $g(i, j)$

$$g(i, j) = \begin{cases} 2i + 1 & \text{if } i < j \\ 2j & \text{if } i \geq j \end{cases}$$ eqn{12}

Each state is labelled with 2-tuples $(i, j)$, where $i$ is the number of zero symbols in the interleaved sub-string containing the next to last bit, and $j$ is the number in the sub-string containing the last bit. In the $(i, j)$ notation, the set of states $S$ for a $(0, G/I)$ constraint is given by

$$S = \{(i, j): 0 \leq i, j \leq 1 \text{ and } g(i, j) \leq G\}$$ eqn{13}
and the transitions between states are given by the rules

"0": \((i, j) \rightarrow (j, i+1)\), provided \((j, i+1) \in S\)

"1": \((i, j) \rightarrow (j, 0)\)

As an example consider the \((0, 3/3)\) code. The state set \(S\) consists of the 12 states

\[
\{ (0, 0), (0, 1), (0, 2), (0, 3), \\
(1, 0), (1, 1), (1, 2), (1, 3), \\
(2, 0), (2, 1), (3, 0), (3, 1) \}
\]

The finite state transition matrix for this code is

\[
B = \begin{bmatrix}
(0,0) & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(0,1) & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
(0,2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
(0,3) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
(1,0) & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(1,1) & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
(1,2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
(1,3) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
(2,0) & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(2,1) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
(3,0) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(3,1) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The code capacity of the \((0, 4/4)\) code can thus be calculated as the largest real Eigenvalue of the matrix. A brief summary of capacities and other parameters is given in Table 4.5. For a more detailed table of capacities of \((0, G/I)\) codes see [41].
Table 4.5 Parameters of some rate 8/9 (0, G/I) codes

<table>
<thead>
<tr>
<th>Code Rate</th>
<th>Capacity</th>
<th>Efficiency</th>
<th>Encoder States</th>
<th>Decoder Look-Ahead (bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 4/4)&quot;</td>
<td>8/9</td>
<td>0.961</td>
<td>92.5%</td>
<td>1</td>
</tr>
<tr>
<td>(0, 4/3)</td>
<td>8/9</td>
<td>0.939</td>
<td>94.7%</td>
<td>3</td>
</tr>
<tr>
<td>(0, 3/6)&quot;</td>
<td>8/9</td>
<td>0.944</td>
<td>94.1%</td>
<td>1</td>
</tr>
<tr>
<td>(0, 3/5)</td>
<td>8/9</td>
<td>0.942</td>
<td>94.4%</td>
<td>2</td>
</tr>
<tr>
<td>(0, 3/4)</td>
<td>8/9</td>
<td>0.934</td>
<td>95.1%</td>
<td>3</td>
</tr>
<tr>
<td>(0, 3/3)</td>
<td>8/9</td>
<td>0.915</td>
<td>97%</td>
<td>4</td>
</tr>
</tbody>
</table>

" Eggenberger codes see [37]

A partial ordering of the afore-mentioned states of the PRML (G, I) code enables a significant simplification in the construction process and in the final code implementation. A partial ordering can be achieved by considering two states, \( S_1 = (i_1, j_1) \) and \( S_2 = (i_2, j_2) \). Then

\[
S_1 \leq S_2 \quad \text{if either} \quad i_1 \geq i_2 \text{ and } j_1 \geq j_2 \quad \text{or} \quad j_1 \geq j_2 \text{ and } g(i_1, j_1) = G \quad \text{eqn (14)}
\]

The ordering is interpreted geometrically by placing the states on the integer lattice in the plane. Each state \( (i, j) \) with \( g(i, j) < G \) is placed at the grid point with co-ordinates \( (i, j) \) while states with \( g(i, j) = G \) are placed at grid point \( (i, j) \). The ordering can then be described by the simple rule:

\[
S_1 < S_2 \text{ if } S_2 \text{ is below and to the left of } S_1.
\]

The lattice of states for the (0, 3/3) code is illustrated in Figure 4.10. Note that states (1, 2) and (1, 3) are shifted from their normal grid position to the far right edge because the global run achieves the maximum value \( g = 3 \). It is shown in [39] that this ordering is
the key to reducing the encoder complexity when designing codes using the state
splitting sliding block algorithm [42].

\[(0, 3) \bullet \quad (1, 3) \bullet \]
\[(0, 2) \bullet \quad (1, 2) \bullet \]
\[(0, 1) \bullet \quad (1, 1) \bullet \quad (2, 1) \bullet \quad (3, 1) \bullet \]
\[(0, 0) \bullet \quad (1, 0) \bullet \quad (2, 0) \bullet \quad (3, 0) \bullet \]

Figure 4.10 Lattice of States for (0, 3/3) code

Finally, by applying an additional constraint, $M$, on the maximum runlength of non-zero
samples, it has been shown in [43] that rate 8/9 codes remain viable at the expense of an
increased decoding window size. This result of adding the additional constraint can be
seen by comparing Table 4.5 with Table 4.6, for a modest increase in encoder/decoder
complexity an increase in code capacity can be realised.

Table 4.6 Properties of rate 8/9 (0, G/l) codes with $M$ constraint

<table>
<thead>
<tr>
<th>$(0, \text{G/l, M})$</th>
<th>Capacity</th>
<th>Efficiency</th>
<th>Encoder States</th>
<th>Encoder Look Ahead (bits)</th>
<th>Decoder Window (bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 4/4, 9)$</td>
<td>0.9603</td>
<td>92.56%</td>
<td>1</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>$(0, 4/5, 6)$</td>
<td>0.9624</td>
<td>92.36%</td>
<td>1</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>$(0, 4/6, 5)$</td>
<td>0.9587</td>
<td>92.71%</td>
<td>1</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>$(0, 4/6, 4)$</td>
<td>0.9437</td>
<td>94.19%</td>
<td>2</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>$(0, 4/4, 5)$</td>
<td>0.9457</td>
<td>93.98%</td>
<td>4</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>$(0, 4/5, 4)$</td>
<td>0.9395</td>
<td>94.6%</td>
<td>3</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>
4.4.4 Trellis Codes

It has been recognised [44] for a long time that the functions of coding and modulation could be combined to effectively utilise available channel bandwidth and signal-to-noise ratio. Over the past decade this has been achieved in the form of Trellis Coded Modulation (TCM) [45], which has revolutionised the communications industry by yielding data rates within 15-20% of the Shannon Capacity. TCM codes are designed for transmitting multi-level symbols over memoryless channels, and as such are not applicable to saturation recording where only binary input signals are permitted.

Spurred by the impressive performance of TCM on spectrally flat channels efforts have been made to apply trellis coding to partial response channels. Trellis coding techniques for binary partial response channels have recently been proposed by Wolf and Ungerboeck [46], Calderbank, Heegard, and Lee [47], Immink [48], and Karabed and Siegel [49]. In all of these, the constructions are based upon binary codes, typically convolutional, that have attractive Hamming distance properties for a given rate and decoder complexity.

The Asymptotic Coding Gain (ACG) for channels with binary input restriction is given by

$$ACG = 10 \log_{10} R \frac{d_{\text{free}}(\text{coded})}{d_{\text{free}}(\text{uncoded})}$$
Trellis Coded Modulation

Ungerboeck [50] has proposed a channel coding technique that achieves coding gains without sacrificing the data rate or expanding the bandwidth of the transmitted signal. The basic idea is that by trellis coding onto an expanded modulation set (relative to that needed for uncoded transmission) and by designing the trellis code to maximise the minimum free Euclidean distance between allowable code sequences asymptotic (high signal-to-noise ratio) coding gains of 3-6 dB compared to an uncoded system can be achieved without bandwidth expansion. This is accomplished by encoding \( m \) information bits, by means of a rate \( R = m/(m+1) \) convolutional encoder, into \( m+1 \) bits, which select points from one of the conventional \( 2^{m+1} \) signal constellations. The mapping of information bits to coded bits is achieved by set partitioning.

The mapping ensures that the free Euclidean distance \( d_{\text{free}} \) between any two possible sequences is greater than the minimum distance \( d_0 \) between any two points in the \( 2^m \) signal constellation. Such memory can then be exploited by a maximum likelihood decoder, yielding a coding gain of \( d_{\text{free}}^2/d_0^2 \).

The coding gain is a function of the amount of memory introduced by the encoder, i.e., the constraint length, and of the positioning of the signal points in the signal space, i.e., the signal constellation.

In summary, Ungerboeck’s approach uses two key ideas:

a) In an uncoded system transmitting \( m \) bits of information in an interval \( T \), one of \( 2^m \) signals are sent each \( T \) seconds. Ungerboeck has available \( 2^{m+1} \) signals from which he chooses one every \( T \) seconds.
b) The rate \( R = \frac{m}{m+1} \) code limits the choice of signals to be transmitted. In any key interval \( T \) only \( 2^m \) of the \( 2^{m+1} \) are candidates for transmission. The set partitioning rules specify which \( 2^m \) signals are available at any instance in time.

**Wolf-Ungerboeck Rate 4/5 Trellis Code**

Wolf and Ungerboeck [46] have described a coded system for improving the reliability of digital transmission over a noisy partial-response channel with transfer function \((1-D)\). The \((1-D)\) system uses a convolutional encoder that generates a binary convolutional code with good free Hamming distance, followed by a precoder to increase the free Euclidean distance between permitted channel output sequences. The addition of the precoder causes the channel to resemble a spectrally flat channel. The maximum likelihood decoder matched to the encoder, precoder and channel requires, in general, \( 2^{v+1} \) states where \( v \) is the constraint length of the convolutional encoder. Recently, Zehavi and Wolf [51] showed that for a class of convolutional encoders the number of states in the decoder is only \( 2^v \).

The rate 4/5 Wolf-Ungerboeck code can be described as having parameters \((0, G/1) = (0, 44/22)\) and gives a coding gain of 3dB. Its trellis is shown in Figure 4.11 and the associated path outputs are given in Table 4.7.
Figure 4.11 Trellis for Rate 4/5 Wolf-Ungerboeck Code
Wolf and Ungerboeck concluded that the use of well known convolutional codes in combination with a precoder provide a desirable solution to achieving a coding gain. However, they noted that coding gains are less than when convolutional codes are used with a comparable decoder for channels without ISI. This was attributed to the fact that partial response signalling already represents a simple form of coding.

### Table 4.7 Sets and Subsets of Five Consecutive (1-D) channel outputs

<table>
<thead>
<tr>
<th>Starting at state - Ending at state -</th>
<th>Starting at state - Ending at state +</th>
<th>Starting at state + Ending at state -</th>
<th>Starting at state + Ending at state +</th>
</tr>
</thead>
<tbody>
<tr>
<td>0) 10-100 1-101-1</td>
<td>8) 00100 010-11</td>
<td>16) 00-100 0-101-1</td>
<td>24) -10100 -110-11</td>
</tr>
<tr>
<td>1) 1-11-1-1 10001</td>
<td>9) 01-110 0001</td>
<td>17) 0-11-1-1 0000-1</td>
<td>25) -11-11 1001</td>
</tr>
<tr>
<td>2) 01-100 0001-1</td>
<td>10) 1-1100 100-11</td>
<td>18) -11-1100 -1001-1</td>
<td>26) 0-1100 000-1</td>
</tr>
<tr>
<td>3) 001-10 0100-1</td>
<td>11) 10-110 1-1001</td>
<td>19) 101-110 -1100-1</td>
<td>27) 0-11-10 0-1001</td>
</tr>
<tr>
<td>4) 00101-1 010-10</td>
<td>12) 10-101 1-1010</td>
<td>20) -1010-1 -110-10</td>
<td>28) 0-1101 0-1001</td>
</tr>
<tr>
<td>5) 01-11 10000</td>
<td>13) 1-11-11 10000</td>
<td>21) -11-11-11 -10000</td>
<td>29) 0-11-11 00000</td>
</tr>
<tr>
<td>6) 1-110-1 100-10</td>
<td>14) 01-101 00010</td>
<td>22) 0-110-1 000-10</td>
<td>30) -11-101 00010</td>
</tr>
<tr>
<td>7) 10-111-1 1-1000</td>
<td>15) 001-11 01000</td>
<td>23) 0-11-1-1 0-1000</td>
<td>31) -101-11 1-1000</td>
</tr>
</tbody>
</table>

4.4.5 Matched Spectral Null Code

A new family of codes, Matched Spectral Null (MSN) codes [52,53], have recently been proposed. These MSN codes capture the basic essence of trellis coded modulation by providing a means to combine the functions of coding and modulation to improve the reliability of digital communication over noisy partial response channels. MSN codes
are constructed to have nulls* in the power spectrum precisely corresponding with those of the channel transfer function. Class IV Partial Response channels have first order nulls at zero frequency (d.c.) and at the Nyquist frequency. Unlike other trellis codes for PR channels [46-49,], by matching these null frequencies, the MSN codes exploit, rather than nullify, the inherent channel memory producing enhanced coding gains. Additional coding gain is achieved by increasing the order of the spectral null of the code strings at these frequencies.

The code described is designed for a (1-D) channel, but may be applied to a PR4 channel by interleaving, bit-wise, the code words. When interleaved the code satisfies the constraints \((0, G/l) = (0, 10/5)\). The code is also designed to be 180 degrees phase-shift invariant without requiring a precoder circuit. In other words the channel output sequences \(a\) and \(-a\) both decode to the same data sequence \(b\). The code is implemented in a sliding block decoder that requires 4 states and one look-ahead code word. Therefore the maximum length a single bit error from the detector can propagate is limited to 2 bytes.

The full trellis for the matched spectral null code is quite complicated since a rate 8/10 code has \(2^8=256\) branches leaving every state. Thus an alternative to true maximum likelihood decoding was achieved by using a trellis structure derived from the spectral null diagram, illustrated in Figure 4.12, that has a maximum DSV=6. The code words of the rate 8/10 MSN code are a subset of the code words of a slightly larger code which has a much simpler trellis. However, for any MSN code word, no word in the larger

---

* Those frequencies at which the power spectrum value is zero, due to no energy transmission.
code is closer to that word (in Euclidean distance) than the free Euclidean distance of
the MSN code. Thus the detector uses the Viterbi algorithm to find the closest MSN
code word on the simpler trellis of the larger code.

![State Diagram for sequences with $DSV \leq 6$]

In the unlikely event that the word produced by the decoder is not in the MSN code
alphabet, a sliding block decoder is used to decode, thereby limiting error propagation.
The maximum possible length of a minimum distance error event is limited to 42 bits;
this is important for limiting the required path memory in the Viterbi detector.

Figure 4.13 shows the reduced complexity Viterbi detector trellis structure for the 8/10
MSN code. The edges are labelled by $u_1u_2/v_1v_2$ where $u_1u_2$ are code symbols and $v_1v_2$
are channel output symbols. From this the free Euclidean distance of the trellis can be
calculated as $d^2_{free}=4$, indicating a potential 3dB coding gain over the uncoded (1-D)
channel. For each of the six trellis states, the Viterbi detector recursively generates a
code sequence called the survivor. Among all sequences generated by a trellis path
ending in a specific state, the output sample sequence corresponding to the survivor
provides the best fit (in the sense of squared error) to the noisy received output
sequence.
The Viterbi detector contains a branch metric calculation for each branch labelled \( v_1 v_2 \), which is the squared Euclidean distance between the channel output symbols \( v_1, v_2 \) and the received signals \( s_1, s_2 \), expressed as,

\[
bm_{v1v2} = (v_1 - s_1)^2 - (v_2 - s_2)^2
\]

Since \( s_1^2 \) and \( s_2^2 \) are common for all branch metrics, they can be ignored in the subsequent minimisation. The branch metrics then become

\[
bm_{v1v2} = v_1^2 + v_2^2 - 2v_1s_1 - 2v_2s_2
\]

The path metric for each state is the accumulation of branch metrics, shown in Table 4.8.
Path metric update is the key operation to determine the extension of the survivor sequence. It involves adding the old path metric to the corresponding branch metric of all incident branches, comparing their sums and selecting the minimum. Hence the name Add-Compare-Select (ACS) is adopted for performing this operation.

The 8/10 MSN code and the Viterbi detector was implemented in software (given in Appendix B), with the view of extending it to the sampling detection compact cassette system (described in chapter 2). In doing so, an error was identified in the finite-state-machine encoder and the Boolean equations for the look-ahead encoder presented in [54].

Table 4.10 shows the structure of the corrected finite state machine encoder and the corrected Boolean equations are given in Table 4.9. For a further description of the encoding/decoding rules and tables refer to [54].

---

Table 4.8 Viterbi Algorithm Metric Update Equations

<table>
<thead>
<tr>
<th>Metric Update</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{n+1}(1)$</td>
<td>$\min M_n(1) + 2 + 2z_1 - 2z_2, M_n(2) + 1 - 2z_2$</td>
</tr>
<tr>
<td>$M_{n+1}(2)$</td>
<td>$\min{M_n(1) + 1 + 2z_2, M_n(2) + 2 - 2z_1 + 2z_2, M_n(3) + 1 + 2z_1, M_n(4)}$</td>
</tr>
<tr>
<td>$M_{n+1}(3)$</td>
<td>$\min{M_n(1), M_n(2) + 1 - 2z_1, M_n(3) + 2 + 2z_1 - 2z_2, M_n(4) + 1 - 2z_2}$</td>
</tr>
<tr>
<td>$M_{n+1}(4)$</td>
<td>$\min{M_n(3) + 1 + 2z_2, M_n(4) + 2 - 2z_1 + 2z_2, M_n(5) + 1 + 2z_1, M_n(6)}$</td>
</tr>
<tr>
<td>$M_{n+1}(5)$</td>
<td>$\min{M_n(3), M_n(4) + 1 - 2z_1, M_n(5) + 2 + 2z_1 - 2z_2, M_n(6) + 1 - 2z_2}$</td>
</tr>
<tr>
<td>$M_{n+1}(6)$</td>
<td>$\min{M_n(5), + 1 + 2z_2, M_n(6) + 2 - 2z_1 + 2z_2}$</td>
</tr>
</tbody>
</table>
Table 4.9 Boolean equation for look-ahead decoder

| U(y,z) | \[\text{if } V(y) = 5 \& \ y_1 y_2 = 11 \& \ ((V(z) = 4) \text{ OR } (V(z) = 5 \& (z_1 z_2 z_3 z_4 = 0111 \text{ OR } 0100))) \text{ OR} \]
| | \[\text{if } V(y) = 5 \& \ y_1 y_2 = 00 \& \ ((V(z) = 6) \text{ OR } (V(z) = 5 \& (z_1 z_2 z_3 z_4 = 1000 \text{ OR } 0011))) \]
| | otherwise

where \(V(y)\) = Hamming weight for current codeword,

\(V(z)\) = Hamming weight for look ahead codeword,

\(z = z_1, z_2, \ldots, z_{10}\) look ahead 10-bit code word,

\(y = y_1, y_2, \ldots, y_{10}\) current 10-bit code word,

\(000\) = set of binary 3-tuples excluding 000, similarly for \(111\).

Table 4.10 Finite State Machine Encoder for rate 8/10 MSN code.

<table>
<thead>
<tr>
<th>Current State</th>
<th>Data</th>
<th>Next State</th>
<th>Code word</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 - 99</td>
<td>3</td>
<td>A(100)</td>
</tr>
<tr>
<td>0</td>
<td>100 - 142</td>
<td>1</td>
<td>B(43)</td>
</tr>
<tr>
<td>0</td>
<td>143 - 255</td>
<td>2</td>
<td>C(113)</td>
</tr>
<tr>
<td>1</td>
<td>0 - 76</td>
<td>2</td>
<td>D(77)</td>
</tr>
<tr>
<td>1</td>
<td>77 - 119</td>
<td>0</td>
<td>E(43)</td>
</tr>
<tr>
<td>1</td>
<td>120 - 127</td>
<td>1</td>
<td>F(8)</td>
</tr>
<tr>
<td>1</td>
<td>128 - 135</td>
<td>2</td>
<td>\overset{\phi}{F}(8)</td>
</tr>
<tr>
<td>1</td>
<td>136 - 255</td>
<td>0</td>
<td>\overset{\phi}{C}(120)</td>
</tr>
<tr>
<td>2</td>
<td>0 - 76</td>
<td>1</td>
<td>\overset{\phi}{D}(77)</td>
</tr>
<tr>
<td>2</td>
<td>77 - 119</td>
<td>3</td>
<td>\overset{\phi}{E}(43)</td>
</tr>
<tr>
<td>2</td>
<td>120 - 127</td>
<td>2</td>
<td>F(8)</td>
</tr>
<tr>
<td>2</td>
<td>128 - 135</td>
<td>1</td>
<td>\phiF(8)</td>
</tr>
<tr>
<td>2</td>
<td>136 - 255</td>
<td>3</td>
<td>G(120)</td>
</tr>
<tr>
<td>3</td>
<td>0 - 99</td>
<td>0</td>
<td>A(100)</td>
</tr>
<tr>
<td>3</td>
<td>100 - 142</td>
<td>2</td>
<td>B(43)</td>
</tr>
<tr>
<td>3</td>
<td>143 - 255</td>
<td>1</td>
<td>C(113)</td>
</tr>
</tbody>
</table>
To implement this code on the sampling compact cassette system would require:
additional equalisation to optimise the channel response to that of PR4; a separate clock
recovery circuit and a more powerful processor to implement the real-time computation
required by the Viterbi Algorithm. Therefore this code was not successfully applied to
the experimental recording system. However, it was implemented, in Pascal, using
simulated Lorentzian pulses. It would be unfair to compare the simulated results against
those obtained from practical experimentation since realistic noise models have not been
developed.
4.4.6 Coding for EPR4 Channels

The choice of class IV partial response is not a good match to very high density magnetic recording channels or to channels using minimum runlength constraints. In such cases, higher order PR polynomials must be used, leading to exponential growth in the complexity of the Viterbi detector.

(1,7)ML Code

Patel [55,56] has recently proposed a detection scheme for channels with a minimum run-length constraint of \(d = 1\) two bit intervals between transitions. Instead of equalising the channel to a PR4 response, the channel is equalised to an enhanced class IV partial response transfer function \((1 + D - D^2 - D^3)\). This scheme achieves near maximum likelihood performance without Viterbi computations by addressing only the most likely error events. This leads to a detector which requires 15 adders and a relatively simple finite state machine. However, the optimality of the equaliser extends only over a limited range of densities.

By employing a \(d = 1\) constrained code the density ratio on the magnetic medium is improved by a factor of 1.5, from 8/9 in conventional PRML channels to 4/3 in the new channel. It also controls the ISI and increases decoding reliability by removing many error prone paths from the trellis.

The trellis diagram for an EPR4 channel sequence, as shown in Figure 4.14a, has eight states. By applying (1,7) code constraints two states, \((010), (101)\), are eliminated, and a further two, \((001), (110)\), become transitional only resulting in the trellis illustrated in
Figure 4.14b. This would effectively yield a 37.5% reduction in the number of computations required for optimal Viterbi detection. The remaining four states are processed with a look ahead decoder that obtains maximum likelihood performance without use of the Viterbi algorithm. Figure 4.15 presents the simplified state diagram for the input/output sequence of the (1,7) EPRML code.

Figure 4.14 Trellis diagram for a) EPR4 channel b) d=1 constrained EPR4 channel
The ML algorithm employed is a six state iterative algorithm in which the states are denoted by a three bit binary number representing the binary logic level of the write current as illustrated in Figure 4.16.

<table>
<thead>
<tr>
<th>Clock</th>
<th>1 1 0 0 1 0 0 0 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1 1 0 0 1 0 0 0 0</td>
</tr>
<tr>
<td>(1,7) Sequence</td>
<td>0 1 0 1 0 1 0 0 0</td>
</tr>
<tr>
<td>Write Waveform</td>
<td>0 1 1 0 0 1 1 0 0</td>
</tr>
<tr>
<td>State Values (3 bits)</td>
<td>0 1 1 0 0 1 1 0 0</td>
</tr>
</tbody>
</table>

Figure 4.16 (1,7) EPR4 Sequence Detection
The states (100), (000), and (001) correspond to the detection of a positive read pulse and possess mirror image symmetry (+/- sign) with states (011), (111), and (110) respectively that correspond to the negative pulse.

The detector is state dependent and requires a look ahead of five samples, i.e. given the current state \(a_0b_0c_0\) and the sample values \(y_0, y_1, y_2, y_3,\) and \(y_4\), the detector determines the noise free sample value \(y_0\), and the resultant next state \(a_1b_1c_1\). More recently the detector has been increased to a six sample look ahead to reduce errors caused by certain dibit combinations at high density [59].

The detector makes its decision on the outcome of three basic checks, namely:

a) the baseline check,

b) the peak-position check,

c) the phase check.

Each check is a comparison of a specific function of the sampled values against a fixed threshold, and is represented by a Boolean variable. Each threshold is represented by a corresponding constant that is strongly related to the shape of the readback signal. The linear functions and their thresholds are chosen so as to minimise the overall mean squared error in the sequence detection process. Table 4.11 lists these functions with the decision boundaries and the functions are classified in accordance with their functional role in the detection process. Table 4.12 shows the ML state decoder driven by the decision function \(AB+X+YZ\) and phase check \(p\).
Fable 4.11 Linear Functions of Look Ahead Sample Values and Decision Boundaries for States in Positive Phase

<table>
<thead>
<tr>
<th>Functions of look ahead sample values</th>
<th>Nominal decision boundary at 100</th>
<th>000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Check</td>
<td>A = ( F_a \leq 4 )</td>
<td>A = ( F_a \leq 6 )</td>
</tr>
<tr>
<td>( F_a = y_0 + 2y_1 + y_2 )</td>
<td>B = ( F_b \leq 5 )</td>
<td>B = ( F_b \leq 7 )</td>
</tr>
<tr>
<td>( F_b = y_0 + 2y_1 + y_2 - y_3 )</td>
<td>X = ( F_x \leq -2 )</td>
<td>X = ( F_x \leq -0 )</td>
</tr>
<tr>
<td>Peak Position Check</td>
<td>Y = ( F_y \leq 0 )</td>
<td>Y = ( F_y \leq 2 )</td>
</tr>
<tr>
<td>( F_x = y_0 + y_1 - y_2 - y_3 )</td>
<td>Z = ( F_z \leq -4 )</td>
<td>Z = ( F_z \leq -2 )</td>
</tr>
<tr>
<td>( F_y = y_0 + y_1 - y_2 - y_3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F_z = y_0 + y_1 - y_2 + y_4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase Check</td>
<td>p = ( F_p \leq -7 )</td>
<td></td>
</tr>
<tr>
<td>( F_p = y_0 + y_1 - 2y_2 + y_4 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.12 ML Decision from Current State to Next State.

<table>
<thead>
<tr>
<th>Current State</th>
<th>Sample Value/ Next State/ Decoded Data Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0b_0c_0 )</td>
<td>( (AB+X+YZ)=1 ) and p=0 ( (AB+X+YZ)=0 ) and p=0 ( (Phase \ Check) p=1 )</td>
</tr>
<tr>
<td>000</td>
<td>0/000/0</td>
</tr>
<tr>
<td>001</td>
<td></td>
</tr>
<tr>
<td>011</td>
<td>2/111/0</td>
</tr>
<tr>
<td>100</td>
<td>-2/000/0</td>
</tr>
<tr>
<td>110</td>
<td></td>
</tr>
<tr>
<td>111</td>
<td>0/111/0</td>
</tr>
</tbody>
</table>

A similar coding scheme employing peak detection has also been proposed by Armstrong and Wolf [58]. They employ a \( 1/(1+D^2) \) modulo two precoder to reduce the number of levels at the detector from 5 to 3, thereby permitting reliable peak detection.
4.5 Discussion

Analytical studies have shown that PRML can potentially increase the recording density by 80-100% when compared to MFM recording [59], and by 30-50% when compared to RLL (1,7) or (2,7) coding [59,33].

Several codes applicable to a digital magnetic recording channel employing partial response signalling and maximum likelihood detection have been presented.

The Eggenberger 8/9 (0, 4/4) codes have been adopted by hard disk drives due to their high code rate and their ability to simplify the Viterbi detector for class IV partial signals.

More advanced trellis codes have been proposed by Wolf and Ungerboeck that yield approximately 3dB advantage over baseline class IV partial response at the expense of a reduction in code rate and a dramatically increased complexity Viterbi detector. The matched spectral null code goes some way to alleviating the problem of the complexity of the Viterbi detector as well as matching the spectral properties of the channel.

The theory of MSN codes is appealing from the intuitive standpoint. It is consistent with the well known adage that “the code spectrum should match the channel transfer characteristics”. A recent paper [60] comparing the rate 8/9 (0, 4/4) code against the rate 8/10 MSN code for an experimental hard disk system has shown that the 8/10 MSN code reduces the “On-track Error Rate” by at least 2 orders of magnitude and gives a 10-20% increase in “Off-track Capability” at bit error rate of $10^{-6}$. The overall improvement
in linear density is in excess of 17% which, when combined with superior track pitch requirements yields approximately 20% increase in the areal density.

Higher orders of partial response schemes have also been proposed that would require a greater complexity Viterbi detector. The RLL 2/3 (1,7) code has been proposed to simplify the Viterbi detection for the EPR4 channel. Employing this code would result in a code rate loss with respect to other codes described. However, this is balanced by reduced loss through the equaliser, due to the fact that EPR4 is a better match than PR4 to the magnetic recording channel at higher densities. Another advantage of this code is that at high linear densities non-linear errors occur due to the medium not being able to support such closely spaced transition. Hence, by employing a \( d=1 \) constrained code the transitions are further apart therefore permitting greater capacity.

Attempts made to employ 8/10 MSN on the compact cassette magnetic recording system high-lighted the fact that a great deal of signal processing power is required for successful PRML systems, for maximum-likelihood detection, and for equalisation, as well as the coding. Therefore more efficient codes are sometime rejected in favour of those that simplify the detection process. Hence, commercial disk drive applications have chosen to go with 8/9 (0, 4/4) code. However, as processing power becomes more widely available and less costly other more efficient coding schemes may be adopted.
4.6 References


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CHAPTER 5

Two Dimensional (Multi-Track) Coding

5.1 Introduction

This chapter describes a new class of modulation codes that exploits the two-dimensional properties of a multi-track digital magnetic recording systems. Marcellin & Weber [1] first recognised that Two Dimensional or Multi-track Modulation codes, as they are known, provide the capacity to substantially increase data storage density by operating on several tracks in parallel. Their paper demonstrates how the run-length, of a recording code, could be shared over several channels in a multi-channel environment by relaxing the \( k \) constraint along each individual track whilst maintaining a common \( k_y \) constraint across the tracks. Since multiple tracks are read out in parallel, it is not necessary for each individual track to provide all the clocking information required to maintain clock synchronisation. Instead, the clocking information derived from frequent transitions and defined by the \( k \) constraint, can now be sub-divided over several tracks giving a \( k_y \) constraint. By allowing the \( k_y \) constraint to be met across the tracks many more code words are possible for a given \( d \), and \( k_y \) in two dimensions than was possible in one dimension.
Swanson & Wolf [2] noted that the prime disadvantage of the proposed codes was their vulnerability to the loss of a channel due to poor clock recovery. In magnetic tape recording the loss of a channel for long periods of time is not uncommon due to problems such as head clog or medium dropouts. Therefore, in the event of a burst error on a particular track containing the majority of timing information (transitions), the result would be fatal since the clock recovery would not function correctly causing mis-synchronisation. They noted for practical implementation, it is essential to distribute the timing information equally across all channels, so that in a single-track error situation, clock recovery is maintained by transitions from other tracks. To address this problem they proposed the addition of a $k_x$ constraint, to impose a maximum run-length along each track without a transition, assuring sufficient timing information even if one channel is completely lost.

This type of coding scheme is thus characterised by $(d, k_x, k_y, y)$. Where the $d$ constraint must be satisfied for each track independently to avoid inter-symbol interference, $k_x$, $k_y$ are the $k$ constraints satisfied along, and across the $y$ number of tracks respectively. The inclusion of the cross channel constraint $k_y$ allows the $k_x$ constraint to be much larger than the customary $k$ so that capacity can be increased over conventional, single channel, RLL codes.

Finally, this chapter will describe the author's work in designing a two-dimensional coding scheme for the multi-track compact cassette system described in chapter 3. Further, the development of a multi-dimensional coding scheme for a high density, multi-track system that employs partial response and maximum likelihood detection along the tracks is described. A discussion on how error correction could also be incorporated into two-dimensional codes is also presented.
5.2 Code Design

Proving the existence of a code and ignoring the decoder complexity does not result in a usable code. The performance measures obtained from theoretical and mathematical predictions, such as those in information and communication theory, should never be used solely to assess a channel code, since theories can never predict or explain the complexities of the real world and therefore can never be substituted for experience or experimental results. However such theories do provide code designers with tools that can determine the code performance with respect to the theoretical limits thereby saving time investigating impractical codes.

As a simple example of the density increase that can be expected from a two-dimensional code, consider the case when \( (d,k) = (0,1) \), that of the popular, rate=1/2, Bi-Phase-L code that gave such good results on the original peak-detect compact cassette system described in chapter 3.

The coding for a single-track Bi-Phase-L code is achieved by the following mapping of source bits to code bits.

\[
\begin{align*}
0 & \rightarrow 10 \\
1 & \rightarrow 01
\end{align*}
\]

Now consider a two-dimensional code for a four track system with \( (d,k) = (0,1) \), where six source bits \( (a_0, a_1, a_2, a_3, a_4, a_5) \) are mapped onto seven code bits \( (b_0, b_1, b_2, b_3, b_4, b_5, 1) \) as shown,

\[
\begin{align*}
a_0, a_1, a_2, a_3, a_4, a_5 & \rightarrow b_0 b_4 \\
b_1 b_5 & \\
b_2 1 & \\
b_3 & \\
\end{align*}
\]

following code words

where \( b_i = a_i \).
The column of code bits represents the code symbol written across all tracks simultaneously, and hence the rows represent the code bits written along each individual track. This simple code can be described as a rate = 6/7 (0,6,1;4) code. It maintains the same $k=1$ constraint, satisfied across the tracks by $k_r$ as Bi-Phase-L code but allows a 58% increase in data rate. This is equivalent to a saving of five channel bits of information, alternatively five bits of redundancy can be added to this code to achieve the same rate as Bi-Phase-L code. If a similar technique was applied to a system with many more tracks, the code rate would rapidly approach capacity.

The above code achieves this increase in data rate at the expense of the desirable charge constrained sequence along each track. Therefore another multi-track code was devised that retained the d.c.-free property whilst increasing data rate.

The code word mapping for this rate 3/4, (0, 4, 2; 2) code is as follows:

- $000 \rightarrow 10$
- $001 \rightarrow 01$
- $010 \rightarrow 10$
- $011 \rightarrow 01$

- $01 \rightarrow 10$
- $010 \rightarrow 10$
- $011 \rightarrow 01$

- $100 \rightarrow 00$ or $11$
- $101 \rightarrow 00$ or $11$
- $110 \rightarrow 10$ or $01$
- $111 \rightarrow 01$ or $10$

The encoding scheme monitors the digital sum variation (DSV) in each track and selects the choice of code word accordingly to minimise that value. Decoding can be achieved by a simple look-up table. This code permits a 50% increase in storage capacity over Bi-Phase-L code, however, the $k_r$ constraint has increased marginally but should not cause any major problems to the clock recovery system.

Early attempts to devise a more efficient two-dimensional code for a multi-track compact cassette system, described in chapter 3, resulted in the creation of several
simple two-dimensional codes, similar to those described above. Most of the
aforementioned codes had a \( d = 0 \) constraint, however for increased recording density \( d > 0 \) is required. Unfortunately, channel codes that have an increased \( d \) constraint usually require a longer length code word to achieve a code rate close to the capacity. Therefore more sophisticated two-dimensional coding algorithms were sought.

5.2.1 Capacity of Two-Dimensional Codes

We consider the case when \( k_y < k_x \) since when \( k_x \leq k_y \) the capacity is less than or equal to that of a one-dimensional \((d, k)\) code. While [1] allows for \( k_y < d \), we assume \( 0 \leq d \leq k_y \). During each code bit interval, \( T \), one bit must be produced for each track. For simplicity, we group these bits into a \( y \)-tuple which we shall refer to as a \textit{code symbol}, also called code vector in [1]. Swanson and Wolf [2] calculate the capacity of codes that have both \( k_y \) and \( k_x \) constraints. The \( k_x \) constraint was included to maintain adequate clock synchronisation in the event of a burst error to a single track containing all clocking information (transitions). However, if the coding scheme guarantees transitions distributed over more than one track the \( k_x \) constraint could be infinite. Orcutt and Marcellin [3] describe a method, whereby adding redundancy, any subset of the \( y \) number of tracks can satisfy the \( k_y \) constraint. The redundancy is given by the number of defective tracks that can be tolerated, while maintaining synchronisation, without reduction in the clocking information. Since the \( k_x \) constraint can be infinite for given tracks the method to calculate capacity is confined to \( k_y \) limited codes.

The states of a multi-track \((d, k_y; y)\) code take the form \( S_d(u_1, u_2, \ldots, u_y; v) \). The value \( u_i \) describes the number of consecutive 0's in the \( i^{th} \) track and can take on any value in \( \{0, 1, \ldots, d\} \) with \( u_i = d \) meaning that \( d \) or more consecutive zeros have occurred in the \( i^{th} \)
track. Hence, $u_i$ monitors the $d$ constraint of the $i^{th}$ track. The value $v$ relates to the $k_y$ constraint and indicates the number of consecutive times that all tracks can be 0 simultaneously. Therefore the value $v$ falls in the range \{0, 1, .., $k_y$\}. The number of states, $S_j$, is calculated as

$$S_j = (d + 1)^y + k_y - d$$ eqn\{1\}

The first $D = (d + 1)^y - 1$ of these states are labelled $S_j$ for $j = 0, 1, .., D$, and possess the associated code symbol $c_j = (u_{j1}, u_{j2}, .., u_{jk})$ which when interpreted as a $(d+1)$-ary number has decimal value $j$. The remaining $(k-d)$ states are labelled as $S_{D+i}$ for $i = 1, .., k-d$.

The general form of the one-step finite state transition matrix $B_y(d, k_y)$ is given by

$$B_y(d, k_y) = 
\begin{bmatrix}
S_0 & S_1 & \cdots & S_{y-1} & S_y & \cdots & S_{D+y-1} \\
S_0 & \alpha & & & & & \\
S_{y+1} & & \ddots & & & & \\
S_{D+y-1} & & & \beta & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
\end{bmatrix}
$$ eqn\{2\}

The sub-matrix $\alpha_x$ is given by
where

\[
A_y = \begin{bmatrix}
0 & A_{y-1} & 0 & \cdots & 0 \\
0 & 0 & A_{y-1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & A_{y-1} \\
A_{y-1} & 0 & 0 & \cdots & A_{y-1}
\end{bmatrix}
eqn{4}
\]

with initial condition given by the \((d+1) \times (d+1)\) matrix

\[
A_0 = B_j(d, \infty) = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 1
\end{bmatrix}
eqn{5}
\]

The block labelled \(\beta_y\) in equation \(\{2\}\) is obtained by simply copying \((k-d)\) times the last row of \(\alpha_y\). The sub-matrix \(I\) is the identity matrix.

The capacity of this new class of codes can most easily be determined by constructing the one-step finite state transition matrix for \(y\) tracks and finding the largest positive eigenvalue \(\lambda\), for the matrix as described in Appendix D. The capacity \(C_y\) of a \(y\)-track \((d, k_y)\) coding matrix is computed as:

\[
C_y(d, k_y) = (\log_2 \lambda) / y
\]
An example demonstrating the stages involved in calculating the capacity of a two-dimensional (1,3) code for 4-track system follows.

Initially a finite state transition diagram for a single channel run-length limited \((d, k) = (1, \infty)\) sequence is constructed, as shown in Figure 5.1.

From Figure 5.1 the finite state transition matrix (FSTM) is derived as

\[
B_{1}(1,\infty)=\begin{bmatrix}
0 & 1 \\
1 & 1 \\
\end{bmatrix}
\]

Marcellin & Weber [1] describe how this can be developed into the FSTM for a two-dimensional code, this technique is displayed graphically in Figure 5.2. From the FSTM the largest real Eigenvalue, \(\lambda = 6.849\), is computed. By substituting this value into equation \(5\) we calculate the capacity of a 4-track (1,3) two-dimensional code as

\[
C_{4}(1,3) = \frac{\log_{2} 6.849}{4} = 0.694
\]
Figure 5.2 Construction of Two-Dimensional finite state transition matrix
Table 5.1 shows the capacities for two-dimensional codes with various \((d, k)\) runlength constraint when \(k\) is distributed across a number of tracks. Studying this table it can be seen that the capacity for a two-dimensional code using only one track is the same as the capacity of conventional one-dimension RLL codes. However, as the number of tracks increase significant improvements in capacity are obvious. For instance, the capacity of a one-dimensional code with runlength constraint \((1, 3)\), such as Miller code, is 0.552, however when the \(k\) constraint is distributed across 4 tracks the new capacity becomes 0.694. Thus, greater than 25% improvement is realisable.

An alternative comparison can be made on a one-dimensional code with rate \(= 1/2\) and runlength constraint of \((2,7)\). If the code rate is maintained, the \(k\) constraint can be reduced by over 50% for 2 tracks. Therefore, for a 2-track system employing two-dimensional coding, a rate 1/2 code can be devised with runlength constraint \((2,3)\). If this is extended further to a 3 track system a rate 1/2 \((2,2)\) code would be possible, which is totally impossible for a one-dimensional code.

Finally, further study of the table shows that the influence of \(y\) diminishes at about 4 channels. In a practical system this means that one can design codes near the one dimensional capacity limit, whilst being assured of ample timing pulses, for a system of around 4 channels. However, additional channels would be useful in providing timing redundancy in the event of a temporary or permanent loss of one or more channels. Also, extra channels would provide additional code words that can be used to improve the code’s spectral characteristics i.e. by making it d.c. free.
<table>
<thead>
<tr>
<th>$d$</th>
<th>$k_y$</th>
<th>$y=1$</th>
<th>$y=2$</th>
<th>$y=3$</th>
<th>$y=4$</th>
<th>$y=5$</th>
<th>$y=6$</th>
<th>$y=7$</th>
<th>$y=8$</th>
<th>$y=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.793</td>
<td>0.936</td>
<td>0.977</td>
<td>0.991</td>
<td>0.996</td>
<td>0.998</td>
<td>0.999</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.694</td>
<td>0.961</td>
<td>0.993</td>
<td>0.999</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>1</td>
<td>0.599</td>
<td>0.655</td>
<td>0.681</td>
<td>0.689</td>
<td>0.692</td>
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<tr>
<td>1</td>
<td>2</td>
<td>0.406</td>
<td>0.653</td>
<td>0.686</td>
<td>0.692</td>
<td>0.694</td>
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<td>1</td>
<td>3</td>
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<td>0.68</td>
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<td>0.692</td>
<td>0.694</td>
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</tbody>
</table>
5.2.2 Global Clock Recovery

Timing is an essential part of any modulation scheme. The two-dimensional codes described link the timing of individual channels to each other so that a more robust means of clock recovery must be devised.

Howell [4] in his paper on statistical properties of run-length limited codes finds that, for example, in a rate 1/2 (1,3) code the run-length probabilities are:

\[
\begin{align*}
    p(\text{run of length } 1) &= \frac{1}{2}, \\
    p(\text{run of length } 2) &= \frac{1}{3}, \\
    p(\text{run of length } 3) &= \frac{1}{6}.
\end{align*}
\]

This indicates that as more channels are used the exposure of clocking disruption is reduced since more runs of shorter lengths are added.

Figure 5.3 and Figure 5.4 show respectively the block diagram and associated waveforms of a global clock recovery scheme for a two-dimensional code, based on allied research by Smithson [5]. The digital data from each track are delayed and passed to an Exclusive-OR gate together with the original data which generates a pulse for every transition. The generated pulses from each track are combined using an OR gate which are then used to trigger an Infinite Impulse Response (IIR) filter that is set to oscillate at the desired recovered clock frequency. If we assume the tracks are not skewed with respect to one another then this technique will suffice. However, if skew is present a microprocessor could be included to determine the respective skew between tracks, as described by Donnelly [6], and delay the recovered clock accordingly. Also, if the frequency of the recovered clock is not known prior to playback a microprocessor could be used to derive this by timing the intervals between known synchronisation words to set the corresponding coefficients for the IIR filter.
Figure 5.3 Block Diagram of a global clock recovery system

Figure 5.4 Global clock recovery waveforms
5.3 Two-Dimensional Coding Algorithms

The following methods were investigated as a means to achieve more efficient two-dimensional codes. All methods have previously been employed for single track codes but can be extended for multi-track codes. The culmination of this investigation is the development of a reverse enumeration technique that has several advantages over other two-dimensional coding algorithms.

5.3.1 Block Coding Technique

Work by Freiman and Wyner [7] on block code construction for single track codes is the most easily adapted for two-dimensional codes. By raising the FSTM to the $n^{th}$ power, the number of sequences of length $n$ can be calculated for any starting and terminating states. For each starting state there are a unique set of resulting sequences and a set of acceptable termination states that produce no constraint violations when sequences are concatenated.

Consider again the example 4-track (1,3) code used to illustrate the computation of the capacity for a two-dimensional code. Figure 5.2 shows the one-step FSTM for this code, and Figure 5.5 shows the same FSTM raised to the 4th power.

For a starting state of $S_{15}$ or $S_{16}$ and termination state of $S_{15}$, Figure 5.5 shows there are 544 and 529 sequences respectively that satisfy the (1, 3) runlength constraints for a 4-track code. Therefore nine source bits can be encoded uniquely and without fear of concatenation, since nine source bits only generate $2^9 = 512$ source sequences. As a result the channel rate for such a code is $9/16 = 0.5625$. As in the case of our example
there are usually more code words available than required. In that event any desired criteria, such as weight, may be applied to select the $2^m$ code sequences.

| $S_{15}$ | 16 | 24 | 24 | 36 | 24 | 36 | 36 | 36 | 54 | 24 | 36 | 36 | 36 | 54 | 36 | 54 | 54 | 65 | 15 | 0 |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 24      | 40 | 36 | 60 | 36 | 60 | 54 | 90 | 36 | 60 | 54 | 90 | 54 | 90 | 81 | 111 | 23 | 1 |
| 24      | 36 | 40 | 60 | 36 | 54 | 60 | 90 | 36 | 54 | 60 | 90 | 54 | 81 | 90 | 111 | 23 | 1 |
| 36      | 60 | 60 | 100 | 54 | 90 | 90 | 90 | 150 | 54 | 90 | 90 | 150 | 81 | 135 | 135 | 135 | 189 | 32 | 3 |
| 24      | 36 | 36 | 54 | 40 | 60 | 90 | 60 | 90 | 36 | 54 | 34 | 81 | 60 | 90 | 60 | 90 | 111 | 23 | 1 |
| 36      | 60 | 54 | 90 | 60 | 100 | 90 | 150 | 54 | 90 | 81 | 135 | 90 | 150 | 135 | 135 | 189 | 32 | 3 |
| 36      | 54 | 60 | 90 | 60 | 100 | 150 | 54 | 81 | 90 | 135 | 90 | 135 | 150 | 189 | 32 | 3 |
| 54      | 90 | 90 | 150 | 90 | 150 | 150 | 250 | 81 | 135 | 135 | 225 | 135 | 225 | 225 | 321 | 46 | 7 |
| 24      | 36 | 36 | 54 | 36 | 54 | 36 | 54 | 81 | 40 | 60 | 60 | 90 | 60 | 90 | 60 | 90 | 111 | 23 | 1 |
| 36      | 60 | 54 | 90 | 54 | 90 | 81 | 135 | 60 | 100 | 90 | 150 | 90 | 150 | 135 | 189 | 32 | 3 |
| 36      | 54 | 60 | 90 | 54 | 90 | 81 | 135 | 60 | 90 | 100 | 150 | 90 | 135 | 150 | 189 | 32 | 3 |
| 54      | 90 | 90 | 150 | 81 | 135 | 135 | 225 | 90 | 150 | 150 | 250 | 135 | 225 | 225 | 321 | 46 | 7 |
| 36      | 54 | 54 | 81 | 60 | 90 | 90 | 90 | 135 | 60 | 90 | 90 | 135 | 100 | 150 | 150 | 189 | 32 | 3 |
| 54      | 90 | 81 | 135 | 90 | 150 | 150 | 225 | 90 | 150 | 135 | 225 | 150 | 250 | 225 | 321 | 46 | 7 |
| 54      | 81 | 90 | 135 | 90 | 135 | 150 | 225 | 90 | 135 | 150 | 225 | 150 | 250 | 225 | 321 | 46 | 7 |
| 80      | 134 | 134 | 224 | 134 | 224 | 224 | 374 | 134 | 224 | 224 | 224 | 224 | 224 | 224 | 374 | 374 | 544 | 65 | 15 |
| 80      | 133 | 133 | 221 | 133 | 221 | 221 | 367 | 133 | 221 | 221 | 221 | 221 | 221 | 367 | 367 | 367 | 367 | 367 | 529 | 65 | 15 |
| 65      | 111 | 111 | 189 | 111 | 189 | 189 | 321 | 111 | 189 | 189 | 321 | 189 | 321 | 321 | 479 | 50 | 15 |

**Figure 5.5 4th Order FSTM for a 4-track (1,3) Two-Dimensional Code**

Block coding/decoding is simply achieved with the aid of look up tables, which can be implemented in hardware via read-only-memory (ROM). However, to achieve efficient codes, with code rate approaching capacity, long code sequences are required. For instance the example code has been shown to have a capacity of 0.694, therefore the above technique produces a code that is only 81% efficient. To increase the efficiency above 95% would require a code length $n = 18$, with a look-up table containing $2^{18}$ entries. This makes it impractical to use look-up tables as a means of encoding/decoding with current memory devices. Although it is possible to trade code word length, $n$, against an increased number of tracks, $y$, it will still result in a large look-up table of impractical proportions. However, the technique described is very useful for determining the number of available code sequences for a given track density, $y$, and code word length, $n$. 

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5.3.2 Sliding Block Technique

The sliding block code construction technique, pioneered by Adler et al. [8], involves state splitting of the FSTM. Swanson and Wolf [2] provide an excellent detailed description of how this technique can be applied to a two-dimensional code, therefore this technique will be dealt with rather briefly high-lighting only the main points of concern. This technique results in a finite state machine solution to the problem of encoding, and a combinational logic decoder. Although this process provides a practical means to encode/decode multi-track codes the method by which the finite state machine is derived is very complex and does not always yield an optimal solution, therefore several iterations for the same code generation are sometimes required. Finally, the example chosen in [2] generates a very simple, inefficient code. Again, by increasing the efficiency of the code by increasing track density or code word length, the FSTM becomes very large which drastically increases the complexity involved in deriving the finite state machine solution.

5.3.3 Enumeration

Recently, Orcutt & Marcellin [9] have described an enumeration technique which is used to implement a simple two-dimensional code based on a trellis. This scheme reduces, considerably, the amount of memory required in code implementation with respect to that of block coding via look-up tables, where memory requirements increase exponentially with block length. However, this method can be unsuitable for certain applications since code words with minimal transitions are generated, and no limit has been placed on the $k_x$ constraint.
Enumeration is a method for indexing the elements of a given set of code words according to their lexicographical order [10,11]. This involves organising the code words by their numerical magnitude under the interpretation that \(0 \leq 1\). The concept of enumerating code words for implementation of single track RLL codes was introduced by Tang & Bahl [12] and extended by Beenker & Immink [13].

The enumeration scheme described in [9] assigns the source words in turn to the lowest magnitude code word available, i.e. the code word closest to the all zero code word. This results in undesirable code words that contain the least number of transitions per track, occurring early on in the translation. In their paper Orcutt also determines the worst case memory requirements for the enumeration technique employing a trellis to be given by

\[ M_{\text{trellis}} = ((n-1)((d+1)^y + k - d) +1)(2^y -1). \]

Compared to the memory requirements for a look-up table, given by

\[ M_{\text{lookup}} = 2^m = 2^{y \cdot n \cdot R}. \]

Thus, for a given code rate \(R\) it can be seen that \(M_{\text{lookup}}\) is exponential in block length \(n\), whilst \(M_{\text{trellis}}\) is linear in \(n\).

Let us consider the example in [9] for a two track \((1,3)\) constrained code where the \(k_y\) constraint is met across the tracks. This trellis results in the encoding table illustrated in Table 5.2 where a source word, length \(m = 4\) is mapped onto two code words of length \(n = 4\), therefore the code rate

\[ R = \frac{m}{(n,y)} = \frac{4}{(2 \cdot 4)} = 1/2. \]
Table 5.2 Encoding Table for 2-track rate 1/2 (1,3) code with n=4

<table>
<thead>
<tr>
<th>Source Word</th>
<th>Decimal Equivalent</th>
<th>Code Word</th>
</tr>
</thead>
<tbody>
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<td>0000</td>
<td>0</td>
<td>0100</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>0120</td>
</tr>
<tr>
<td>0010</td>
<td>2</td>
<td>0200</td>
</tr>
<tr>
<td>0011</td>
<td>3</td>
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<tr>
<td>0110</td>
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<td>1020</td>
</tr>
<tr>
<td>0111</td>
<td>7</td>
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</table>

<table>
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<th>Source Word</th>
<th>Decimal Equivalent</th>
<th>Code Word</th>
</tr>
</thead>
<tbody>
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<tr>
<td>1001</td>
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<td>1210</td>
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<td>1010</td>
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<td>2010</td>
</tr>
<tr>
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<td>11</td>
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<tr>
<td>1111</td>
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<td>3010</td>
</tr>
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</table>

Unused Code Words: 3020, 3030.

The unused code words have more desirable properties than some of those that have been employed. For instance the code words (0100) & (0200) are very undesirable since they only have a single transition. If a single bit error occurred whilst reading the transitions in these code words then loss of synchronisation could occur.

A more suitable encoding table can be constructed from reversing the order in which the code words are assigned to source words as illustrated Table 5.3. Note although this technique removes undesirable code words that are initially generated by the enumeration scheme proposed in [3] some undesirable code words will still be present in the mapping achieved by the trellis. For example the source word (15) will be mapped onto (0200). Therefore another technique must be introduced to ensure source words are not mapped onto undesirable code words. This can be achieved by mapping
source words onto other out of range source words with more desirable code words. Therefore source word (15) can be mapped onto source word (16) prior to enumeration.

Table 5.3 Revised Encoding Table for 2-track rate 1/2 (1,3) code with n=4

<table>
<thead>
<tr>
<th>Source Word</th>
<th>Decimal Equivalent</th>
<th>Code Word</th>
</tr>
</thead>
<tbody>
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<td>2020</td>
</tr>
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<td>0111</td>
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<th>Code Word</th>
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<td>0200</td>
</tr>
<tr>
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<td>16</td>
<td>0120</td>
</tr>
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</table>

5.3.4 Reverse Enumeration

The following example demonstrates how reverse enumeration can be employed to develop a 1/2 rate multi-track code for a 3-track system with constraints \( d=1 \) and \( k_f=2 \). The number of tracks and code word length have deliberately been made small for the purpose of illustration.

Initially a single-step finite state transition matrix given in Figure 5.6a, is constructed, as described above, to represent legal paths between states. The matrix is raised to increasing powers until sufficient states are available to achieve a desired code rate.
Figure 5.6b. shows the fourth order FSTM that has 91 valid code words, starting from state \( S_6 \) and ending in state \( S_7 \) or \( S_8 \).

\[
B_3^4 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
\end{bmatrix}
\]

Figure 5.6 a) Single-Step FSTM; b) Fourth order FSTM;

for 3-track two-dimensional code with RLL constraints (1,2)

The trellis, illustrated in Figure 5.7, is used for both encoding and decoding. Each node contains an octal number representing the code symbol, \( c_n \), across \( y \) tracks at time \( t \), for all paths entering that state. Each path is assigned an integer value to represent the cumulative number of paths from the present state \( S_t \) to the next state \( S_{t+1} \). For reasons of clarity only selected path values are labelled to demonstrate the encoding-decoding process. The dotted line illustrates valid paths that are not used, and the bold line highlights the following example.

The 64 source words \{(000000),...,\( (111111) \}\) are represented as the decimal numbers \( \{0...63\} \) respectively. To encode the source word, the branch with the lowest value that is greater than the decimal equivalent of the source word is traversed. The code symbol, \( c \), is obtained from the value contained within the node that the path leads to. The value of the branch that is just less than the decimal source word is subtracted from it. This new value is then used as before to select the next path.
The following stages describe how a reverse enumeration technique operates.

**Encoding Scheme**

Consider the source word 011111, which is represented by the decimal value 31. Beginning at state \( S_8 \) we select the path with the lowest cumulative weight which is greater than the decimal equivalent of the source word. The path with weight 46 is the smallest value that is greater than 31, therefore we traverse this path to state \( S_3 \). This state has a code symbol of 4 allocated to it, hence the first code symbol, \( c_1 \), of the code word is assigned the value 4. The path which has the greatest weight less than 31 is then subtracted from the decimal source word to give the new decimal source word. Hence 31-29 = 2. The process is then repeated, therefore the next path traversed has the weight 6, this leads to state \( S_5 \) with value 2 which is assigned to \( c_2 \). The new decimal source word is then calculated as 2-2 = 0. The process continues to state \( S_7 \), giving the third code symbol, \( c_3 = 5 \), and ends in state \( S_7 \) allotting the final code symbol, \( c_4 = 0 \). Hence (011111) is encoded as

\[
1010
\]

\[
c_1, c_2, c_3, c_4 = 4250 = 0100
\]

\[
0010
\]

**Decoding Scheme**

To decode our example code word, (4250), sum the values on each of the branches that have the largest value below that of the branch taken. More
simply, sum the values on each of the branches directly above the branch taken. Where there are no branches above the one traversed just add zero.

**Substitution**

As previously mentioned, code words with all transitions occurring on a single track are undesirable, since a burst error whilst reading that track would result in all timing information being lost. Therefore in our example we wish to prevent code word (4040) occurring. We achieve this by employing a substitution table, which prior to encoding, maps the source word onto another, arbitrary, unused source word that has a more desirable code word. The same table is used inversely after decoding to regenerate the original source word.

It can be seen from Table 5.4 that in the example code, code word (4040) is generated by decimal source word 42. Since only sixty four of the ninety one available code words are used, there are twenty seven remaining code words from which to chose a substitution. To maintain continuity, a replacement source word is chosen as 64 which gives a code word (2120).
Figure 5.7 Trellis for 3 track rate=1/2 (1,2) constrained code

Table 5.4 Encoding Table for 3-track rate 1/2 (1,2) code

<table>
<thead>
<tr>
<th>Source word</th>
<th>Code word</th>
<th>Source word</th>
<th>Code word</th>
<th>Source word</th>
<th>Code word</th>
<th>Source word</th>
<th>Code word</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7070</td>
<td>16</td>
<td>6020</td>
<td>32</td>
<td>4240</td>
<td>48</td>
<td>3410</td>
</tr>
<tr>
<td>1</td>
<td>7060</td>
<td>17</td>
<td>6010</td>
<td>33</td>
<td>4210</td>
<td>49</td>
<td>3400</td>
</tr>
<tr>
<td>2</td>
<td>7050</td>
<td>18</td>
<td>5250</td>
<td>34</td>
<td>4200</td>
<td>50</td>
<td>3070</td>
</tr>
<tr>
<td>3</td>
<td>7040</td>
<td>19</td>
<td>5240</td>
<td>35</td>
<td>4160</td>
<td>51</td>
<td>3060</td>
</tr>
<tr>
<td>4</td>
<td>7030</td>
<td>20</td>
<td>5210</td>
<td>36</td>
<td>4140</td>
<td>52</td>
<td>3050</td>
</tr>
<tr>
<td>5</td>
<td>7020</td>
<td>21</td>
<td>5200</td>
<td>37</td>
<td>4120</td>
<td>53</td>
<td>3040</td>
</tr>
<tr>
<td>6</td>
<td>7010</td>
<td>22</td>
<td>5070</td>
<td>38</td>
<td>4100</td>
<td>54</td>
<td>3030</td>
</tr>
<tr>
<td>7</td>
<td>6160</td>
<td>23</td>
<td>5060</td>
<td>39</td>
<td>4070</td>
<td>55</td>
<td>3020</td>
</tr>
<tr>
<td>8</td>
<td>6140</td>
<td>24</td>
<td>5050</td>
<td>40</td>
<td>4060</td>
<td>56</td>
<td>3010</td>
</tr>
<tr>
<td>9</td>
<td>6120</td>
<td>25</td>
<td>5040</td>
<td>41</td>
<td>4050</td>
<td>57</td>
<td>2520</td>
</tr>
<tr>
<td>10</td>
<td>6100</td>
<td>26</td>
<td>5030</td>
<td>42</td>
<td>4040</td>
<td>58</td>
<td>2500</td>
</tr>
<tr>
<td>11</td>
<td>6070</td>
<td>27</td>
<td>5020</td>
<td>43</td>
<td>4030</td>
<td>59</td>
<td>2430</td>
</tr>
<tr>
<td>12</td>
<td>6060</td>
<td>28</td>
<td>5010</td>
<td>44</td>
<td>4020</td>
<td>60</td>
<td>2420</td>
</tr>
<tr>
<td>13</td>
<td>6050</td>
<td>29</td>
<td>4340</td>
<td>45</td>
<td>4010</td>
<td>61</td>
<td>2410</td>
</tr>
<tr>
<td>14</td>
<td>6040</td>
<td>30</td>
<td>4300</td>
<td>46</td>
<td>3430</td>
<td>62</td>
<td>2160</td>
</tr>
<tr>
<td>15</td>
<td>6030</td>
<td>31</td>
<td>4250</td>
<td>47</td>
<td>3420</td>
<td>63</td>
<td>2140</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>64</td>
<td>2120</td>
</tr>
</tbody>
</table>
5.4 Two-Dimensional Coding for EPRML Channels

At this point it is necessary to once again consider the $k_x$ constraint imposed along each individual track. A $k_x$ of infinity should not pose a problem to a peak detection channel, assuming clock synchronisation and automatic gain control can be derived from other tracks. This is not the case for a channel employing maximum likelihood detection, where frequent transitions are desirable to force soft decisions.

The $d > 0$ constrained two-dimensional code produced by the reverse enumeration scheme can be employed to simplify the extended class IV partial response channel, as described in chapter 4. The $d = 1$ constraint removes paths from the EPR4 trellis which in turn simplifies the number of calculations required for Viterbi detection. However, the code produced by the reverse enumeration technique does not limit the $k_x$ constraint along each track. A $k_x = \infty$ constraint can result in no transitions at all occurring on some tracks, this would make it impossible for the Viterbi detector working on each individual track to make any decisions. Although, theoretically it should be possible to construct a Viterbi detector that operated on all or many of the tracks in parallel, this would result in an immensely large trellis which would dramatically increase the time and memory required for a practical implementation of such a detector. Therefore, for a practical implementation it was decided to retain individual Viterbi detectors for each track.

To re-introduce the $k_x$ constraint it was decided to opt for a merging bit solution that plays a double role. The inclusion of a single charge control bit, $P$, inserted between code words on each individual track can be used to bound the DSV and thus make the code d.c.-free. Making each track d.c.-free has the advantage of effectively introducing a
constraint along each track. Therefore the Viterbi detector is provided with non-zero samples to enable path merges, which result in decisions. A null at d.c. is desirable to match the spectral response of the EPR4 magnetic recording channel.

By monitoring the digital sum variation of two code words, the charge control bit is chosen to minimise the combined DSV. To prevent the charge control bit violating the $d$ constraint along each track the following substitutions have to be made.

$$
\begin{align*}
000P0 & \rightarrow \text{valid for all } P \\
000P1 & \rightarrow \text{if } P=1 \text{ substitute } 00101 \\
010P0 & \rightarrow \text{valid for all } P \\
010P1 & \rightarrow \text{if } P=1 \text{ substitute } 00100 \\
100P0 & \rightarrow \text{valid for all } P \\
100P1 & \rightarrow \text{if } P=1 \text{ substitute } 10101
\end{align*}
$$

The technique described above provides a $k_x$ constraint and a null at d.c., however this is at the expense of a reduced code rate. The addition of $y$ charge control bits reduces the rate of the two-dimensional code too.

Instead of controlling the DSV an odd parity symbol could be used to disperse long run-lengths of zeros. This would prove more efficient in that it could aid the error detection scheme. The code rate for such a code is given by

$$
\text{Code Rate } R = \frac{m}{y(n+N_p)}
$$

where $N_p$ is the number of parity bits per track.

However, this must be tolerated in order to benefit from increased density offered by the combination of extended class IV partial response and maximum likelihood detection.
5.4.1 Two-Dimensional rate 1/2 (1,8,2;3) code without D.C. null.

The code described above is only one of many possible variations. For instance a code with identical run-length constraints that might suit another system that did not require zero d.c. content along the tracks is described.

From Table 5.4 several code words can be identified that when concatenated would result in an infinite run-length of zeros on one of the three tracks. Therefore if the source data resulted in the concatenation of those two-dimensional code words given in the columns of Table 5.5 one track would contain no transitions.

Table 5.5 Two-dimensional code words without transitions on selected tracks

<table>
<thead>
<tr>
<th>Track 1 (LSB)</th>
<th>Track 2</th>
<th>Track 3 (MSB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6060</td>
<td>5050</td>
<td>3030</td>
</tr>
<tr>
<td>6040</td>
<td>5040</td>
<td>3020</td>
</tr>
<tr>
<td>6020</td>
<td>5010</td>
<td>3010</td>
</tr>
<tr>
<td>4240</td>
<td>4140</td>
<td>2420</td>
</tr>
<tr>
<td>4200</td>
<td>4100</td>
<td>2120</td>
</tr>
<tr>
<td>4060</td>
<td>4050</td>
<td></td>
</tr>
<tr>
<td>4020</td>
<td>4010</td>
<td></td>
</tr>
</tbody>
</table>

There are however a number of replacement code words that allow concatenation ensuring all tracks posses a transition; these are given in Table 5.6.
Table 5.6 Substitution code words that contain transitions on all tracks

<table>
<thead>
<tr>
<th>Code words that contain a transition in every track</th>
</tr>
</thead>
<tbody>
<tr>
<td>2070 1610 1430 1250 1070</td>
</tr>
<tr>
<td>2050 1600 1420 1240 1060</td>
</tr>
</tbody>
</table>

We can use these ten code words that have a transition in every track to design a code that can never have a run-length more than eight ($k_x = 8$). Therefore there is no need to add any additional bits. Table 5.7 gives the substitution code words. Two code words indicate alternate usage which can be applied to Table 5.4 to produce a rate 1/2 two-dimensional code without a d.c. null.

Table 5.7 Revised Substitution Table for 3-track rate 1/2 (1,8,2) Two-dimensional Code without d.c. null

<table>
<thead>
<tr>
<th>Source Word</th>
<th>Code word</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2070</td>
</tr>
<tr>
<td>14</td>
<td>2050</td>
</tr>
<tr>
<td>16</td>
<td>1610</td>
</tr>
<tr>
<td>24</td>
<td>1600</td>
</tr>
<tr>
<td>25</td>
<td>1430</td>
</tr>
<tr>
<td>28</td>
<td>1420</td>
</tr>
<tr>
<td>32</td>
<td>1250</td>
</tr>
<tr>
<td>34</td>
<td>1240</td>
</tr>
<tr>
<td>36</td>
<td>1070</td>
</tr>
<tr>
<td>38</td>
<td>1060</td>
</tr>
<tr>
<td>40</td>
<td>4050 / 6060</td>
</tr>
<tr>
<td>41</td>
<td>4050 / 6060</td>
</tr>
<tr>
<td>42</td>
<td>2120 / 4140</td>
</tr>
<tr>
<td>44</td>
<td>4020 / 5040</td>
</tr>
<tr>
<td>45</td>
<td>4010 / 6040</td>
</tr>
<tr>
<td>54</td>
<td>3030 / 6020</td>
</tr>
<tr>
<td>55</td>
<td>3020 / 4240</td>
</tr>
<tr>
<td>56</td>
<td>3010 / 4200</td>
</tr>
</tbody>
</table>

The number of available concatenable $d$ constrained two-dimensional code words available for this type of code is given by

$$N_d(n) = (N_d(n-d))^2.$$  

Eqn (8)
5.5 Combined Error Correction & Modulation

It has been shown that by alleviating the $k$ constraint of the $(d,k)$ constrained sequences along each individual track, by sharing the $k$ constraint among many parallel tracks, codes with increased code rates approaching that of the code capacity can be realised. Another way of looking at this is, that ones in the code word providing the timing information in each individual track have been dispersed over many tracks thereby leaving space for additional information. By increasing the code rate closer to that of its capacity means that the additional information stored in a code word relates to that of the source data. However this could be replaced with error correction information, that would strengthen the code’s ability to detect and correct any errors that may have occurred in recording, whilst maintaining a constant code rate. If enough tracks were available, a code could be constructed to have a rate approaching that of its capacity as well as containing error correction information. One such application where this type of coding would be of benefit is a stationary head video recorder with several hundred tracks to disperse information over [14,15].

For example take a simple 1/4 rate (1,3) code, where the source data, $x$, bit is mapped on to the first bit of the code word, and the following bits of the code word maintain the $d$ and $k$ constraints. The third bit of the code word is always 1 to maintain the $k$ constraint along the track that provides timing information.

$$x \rightarrow x010$$
If many tracks are now employed, instead of a single track, the timing information can be shared, thus leaving a *hole* in the code word where alternative information could be stored.

\[
\begin{align*}
\text{a}_0 & \rightarrow \text{a}_0010 \\
\text{a}_1 & \rightarrow \text{a}_10\_0 \\
\text{a}_2 & \rightarrow \text{a}_2010 \\
\text{a}_3 & \rightarrow \text{a}_30\_0 \\
\text{a}_4 & \rightarrow \text{a}_40\_0 \\
\text{a}_5 & \rightarrow \text{a}_50\_0 \\
\text{a}_6 & \rightarrow \text{a}_60\_0
\end{align*}
\]

If the information inserted into the *holes* is additional source data then this will have the effect of increasing the code rate form 1/4 to approximately 1/2. Alternatively error correction parity bits could be added to increase the code’s immunity to errors whilst maintaining a constant rate.

If the number of tracks was increased dramatically we could achieve both the above features, increase code rate and also add error correction ability. The addition of the parity bits prevents the code rate ever reaching the capacity, however with large code blocks rational rates close to capacity can be achieved.

Two parity bits can *protect* 3 bits

Three parity bits can *protect* 7 bits

Eight parity bits can *protect* 255 bits
It can be seen that the number of bits, $N_o$, protected by addition of parity bits grows exponentially with the following equation

$$N_o = 2^p - 1 \quad \text{for } p > 1 \quad \text{eqn\{9\}}$$

where $p$ is the number of parity bits.

However, the number of code bits, $N_b$, increase linearly with the addition of each track

$$N_b = y.n \quad \text{eqn\{10\}}$$

where $y$ is the number of tracks, $n$ is the code word length.

For a single bit error correcting code the number of bits protected by parity must be greater than or equal to the total number of bits in all code words.

$$N_o \geq N_b \quad \text{eqn\{11\}}$$

substituting equations \{9\} & \{10\} in \{11\} gives

$$2^p - 1 \geq y.n \quad \text{eqn\{12\}}$$

The rate of a code is given by

$$R = \frac{m}{n.y} \quad \text{eqn\{13\}}$$

The number of combinations of a binary source word of length $m$ is $2^m$.

The total number of combinations of a binary code word of length $n$, constrained by $d$ is given as

$$N_d(n) = n+1 \quad \text{for} \quad 1 \leq n \leq d+1 \quad \text{eqn\{14\}}$$
\[ N_d(n) = N_d(n-1) + N_d(n-d-1) \quad \text{for} \quad n > d+1 \quad \text{eqn}(15) \]

However for concatenable, \( d \) constrained code words the total number of combinations is reduced to

\[ N_d(n - d) \quad \text{eqn}(16) \]

For successful code realisation,

\[ N_d(n-d) \geq 2^m \quad \text{eqn}(17) \]

\[ \log_2 N_d(n-d) \geq m \quad \text{eqn}(18) \]

Substituting \(18\) in \(13\) we get the maximum rate of a code is

\[ R = \frac{m}{n} = \log_2 N_d(n-d)/y.n \quad \text{eqn}(19) \]

If we now add the number of parity bits required for a single bit error correcting code we obtain an expression relating maximum rate to the number of tracks, \( y \), and the code word length \( n \).

\[ R = \log_2 N_d(n-d)/(y.n + \log_2(y.n+1)) \quad \text{eqn}(20) \]

5.6 Discussion

Two-dimensional modulation codes and how such codes can be used to increase the overall capacity of a multiple track digital recording system are presented. The capacity of several codes has been determined and shows improved performance over conventional one-dimensional RLL codes. The issue of global clock recovery for such codes has been addressed and a method based on digital filtering is described. Although
such a scheme has yet to be implemented in practice, simulation on a single track has shown enhanced performance over software techniques previously employed by the author. Various encoding techniques have been detailed including a new reverse enumeration scheme. Example two-dimensional codes for system employing extended partial response and maximum likelihood detection are constructed using a reverse enumeration scheme. These codes have the added ability to either match the d.c. null spectral response of magnetic recording channel or incorporate error detection information in the form of parity bits.

A method of constructing multiple-track RLL block codes, implemented using a reverse enumeration scheme based on the trellis description of the constraints has also been proposed. This scheme results in a complexity which varies only linearly with block length as opposed to the exponential relationship exhibited by a look-up table implementation. This method is rather elegant in its simplicity and requires no knowledge of the code words since all relevant information is contained within the trellis which is relatively easy to construct. However, the state-splitting technique for creating trellis codes generally produce codes which are more efficient and exhibit less error propagation. This result should not be surprising since trellis codes use a greater number of sequences which satisfy the \((d, k)\) constraints than block codes do. This is because block codes are state independent while trellis codes are not. The one thing the enumerable block codes have in their favour is ease of construction and implementation. Since the state partitioning/splitting process performed in construction of trellis codes can be tedious and time consuming, especially when trying to achieve codes near capacity, and the resultant encoder/decoder is generally not trivial.
5.7 References


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CHAPbER 6

Conclusions & Further Work

6.1 Conclusions

The impetus behind the information storage industry is to increase the storage capacity of devices for minimum cost. In magnetic recording this means increasing the areal bit packing density by increasing track density resulting from reducing space between and width of the recording tracks, and/or reducing the wavelength of the recorded information.

The aim of this research was to offset the mechanical vagaries of a low-cost tape transport and tape, such as the compact-cassette format, by sophisticated software signal processing and coding techniques. The problem of low signal to noise ratios and high error rates may be alleviated by the use of more sophisticated coding schemes and signal processing techniques.

The ultimate limits of magnetic recording cannot be reached without sophisticated signal processing and coding techniques. The theme of this dissertation has been to investigate the degree to which channel coding can help reach those limits.
The author initially interfaced a first generation digital signal processor to a standard compact cassette mechanism to form a basic multiple-track digital magnetic tape storage system. Software written in TMS32010 assembly language was developed by the author to control the recording system's functions, such as channel encoding/decoding and data synchronisation, to maintain the low cost. Also, parallel software algorithms were developed which permit each track to be processed concurrently. Several RLL modulation codes were applied to the system and the error rates for a range of data rates were measured. By including a high speed digital signal processor new techniques were developed to more accurately derive the best sampling point to minimise errors caused by skew. A trailing edge detection process, developed by the author, used for Bi-Phase-L code gave an improvement over results obtained on a similar system by Donnelly. From the results of this system it was shown that greater efficiency codes do not always perform as well as predicted. This is because the coding has to be matched to the vagaries of the system and detection process as well as the channel bandwidth. Further, an oversample read technique was developed and implemented to completely eliminate azimuth errors. This technique highlighted the need for post-equalisation to limit the effects of inter-symbol interference.

In the second phase of research a new system was developed by the author that had a faster digital signal processor and replaced the hard limiting peak detectors with analogue to digital converters. Equalisation was employed using a simple 5-tap Finite Impulse Response (FIR) filter that slimmed the readback signal by approximately 30%. The improvement in pulse width was translated into a corresponding increase in data rate for a given error rate. However, it was noted that equalisation techniques also boost
the high frequency noise and therefore there is a limit as to how much equalisation can be applied without dramatically reducing the signal-to-noise ratio.

Partial Response signalling techniques, that introduce a controlled amount of inter-symbol interference to shape the readback pulse, were proposed as an alternative method of increasing recording density to pulse slimming that tries to remove ISI. In particular PR4 and EPR4 techniques, that have spectral nulls at d.c and the Nyquist frequency, were identified as having spectral response similar to the magnetic recording channel.

However, partial response techniques produce multi-level signals and hence also reduce the signal to noise ratio. Therefore Maximum Likelihood sequence detection was proposed that makes deferred decisions on the outcome of a signal instant by considering the relative position in a sequence rather than symbol-by-symbol detection of the peak detector. Maximum Likelihood sequence detection is a computationally intensive task that requires high speed processors to perform the calculations in real time. Recent advances in silicon technology have only now permitted PRML to be considered as a viable alternative to the analogue peak detector.

Investigation into PRML for the magnetic recording channel showed that spectral requirements of the PR4 detector can be satisfied over a range of recording densities by suitable equalisation of the readback signal. Beyond that range, however, noise enhancement penalty in equalisation and the lower signal to noise ratio due to peak power limitation of the channel cause the input SNR to the ML detector to drop below the theoretical level for acceptable performance. Further increase in recording density may be achieved by using a class of partial response that is more closely matched to the
readback signal, thereby reducing the noise enhancement penalty in equalisation. However, due to the increased number of levels in higher orders of partial response the effective SNR at the detector is reduced, although the Viterbi algorithm can recoup some loss through redundant output levels inherent due to correlation in the partial response signal. If we ignore the fact that the complexity of the Viterbi detector increases exponentially with number of output levels then equalising to extended class IV partial response is the best choice for high linear recording densities. EPR4 also has the advantage of permitting \( d>1 \) constrained codes that can be used to simplify the Viterbi detection algorithm.

Conventional coding schemes designed for the peak detect channel are not well suited to channels employing PRML. Therefore a new breed of trellis codes for PRML channels have recently attracted much attention. Trellis codes such as the Matched Spectral Null code give a 3dB coding gain over uncoded sequences, as well as matching the spectrum of digital magnetic channel. Several coding schemes for the magnetic PRML channel were investigated by the author and the associated advantages and disadvantages have been discussed.

Whilst the above techniques significantly increase density along the tracks few attempts have been made to benefit from the redundancy introduced by a multiple-track system. Therefore, the author concentrates the final stage of research on investigating coding techniques that exploit the redundancy of the multi-track recording system.

A method to generate concatenable block codes for arbitrary \( y \)-track \( (d, k_y) \) two dimensional codes is proposed. The resultant code is implemented via a reverse enumeration scheme, developed by the author, based on a trellis which contains all
relevant information about the code words. This allows both encoding and decoding to be executed without explicit knowledge of these code words. The advantage of this scheme over a look-up table implementation is in the amount of memory required. For a look-up table approach, memory increases exponentially for increased code block length $n$ whilst using a trellis approach memory only increases linearly with $n$.

This work has been furthered by the author through the development of a coding technique that produces two-dimensional code words with an increased number of non-zero bits. This technique is desirable for a multi-track magnetic recording system since it results in a higher average number of transitions per code symbol. This is advantageous for two reasons:

a) It provides more information to the clock recovery system, this can be used for either reducing the complexity of the clock recovery algorithm or determining a de-skew technique.

b) It reduces the amount of path memory required in a system employing maximum likelihood sequence detection using the Viterbi algorithm, since transitions cause the paths through the trellis to converge.

A simple two-dimensional code described in chapter 5 has been specifically designed as an example to match several characteristics found in a high density compact cassette system. The $k_y$ constraint has been deliberately made small in an effort to duplicate the performance of Bi-Phase-L code by limiting the number of ratios for synchronisation. The author proposed that PRML would be used in a high density digital magnetic recorder, therefore a $d=1$ constraint has been imposed on the code.
primarily to simplify the Viterbi detection by reducing the number of states in the trellis.

A technique has also been described to make the code d.c.-free in an aim to match the spectral characteristics of both EPR4 and the magnetic recording channel. Only experimentation will determine if the sacrifice of reduced capacity through the addition of the an extra bit in each track to control the d.c.-content, yields an overall improved performance. If the d.c.-free constraint is not required then an alternative two-dimensional code has also been developed.

6.2 Further Work

The research carried out for this thesis has generated a substantial amount of potential further work, described below.

The two-dimensional codes developed in this investigation were developed for a multi-track system employing extended class IV partial response signalling with maximum likelihood detection. Because of time constraints it was not possible to develop a system employing EPRML. This would entail the development of several crucial procedures such as: automatic gain control (AGC), to ensure the playback signal from a variety of different tapes is of equal amplitude; channel equalisation equally distributed between both the write and read process with the post equalisation preferably employing some adaptive equalisation; and also a more effective clock recovery scheme. All these tasks could be accomplished using sophisticated software which would require considerable development. Indeed an allied research project into software clock recovery is in progress.
The sampling detection system developed in this investigation was designed to facilitate the addition of several processor boards capable of working concurrently on individual signal processing tasks and sharing information. It would therefore be possible to use this system as a platform on which to build a multi-track digital tape recorder with a completely adaptive software channel. However, as more advanced high-speed parallel-processing DSP devices, such as the TMS320C40, become more readily available using such a device may prove a more cost effective solution, since a single device has greater than ten fold increase in computational speed.

Finally, all techniques developed in this investigation are for parallel operation and may be applied to any track density provided a microprocessor of comparable word length is employed. If an adequate signal to noise ratio can be maintained, thin-film, magnetoresistive read/write recording heads can be applied with higher track densities. A Philips Digital Compact Cassette recorder has been purchased for the next phase of the investigation. The application of two-dimensional coding with EPRML should yield increased data rates. Also higher track density heads produced for a stationary head digital video recorder will permit further exploration into this area. This will allow further exploration into the possible advantages and disadvantages of higher track densities. Other signal processing algorithms are sure to be developed as a result of such an investigation.
Appendix A

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IEEE Internation Magnetics Conference (Intermag’94),
PULSE SLIMMING IN MAGNETIC RECORDING USING DIGITAL SIGNAL PROCESSING TECHNIQUES

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KEYWORDS. Pulse Slimming, Magnetic Recording, Digital Signal Processing

ABSTRACT. An equalisation method for reducing peak shift caused by inter-symbol interference in high density digital magnetic recording is presented. A TMS320C25 Digital Signal Processor has been applied to a low cost digital magnetic recording system. Software is used to slim the signal from the read head to produce a 30% slimmer pulse width. This is translated into a comparable data rate gain at the same error rate.

1. INTRODUCTION

For each of the past three decades the capacity of magnetic storage devices has risen by an order of magnitude [1]. Most of this increased storage capacity has resulted from improvement in the part of the system called "the channel". This includes the storage medium, the read/write electronics and, not least, in signal processing techniques developed to maximise channel throughput. Progress in this latter area has been supported by the continuous development in microprocessor and Digital Signal Processing (DSP) devices. These improvements are such that some of the functions of a recording system, conventionally implemented in hardware, can be given over to software.

Previous work at the University of Plymouth [2] has demonstrated the effectiveness of employing a programmable device in the data channel of a recording system. This work continues by extending the application of software to a further area of the recording system: pulse slimming.

This paper describes the application of DSP software techniques to the frequency and phase equalisation (pulse slimming) of a digital magnetic recording data channel. Following an explanation of pulse-slimming theory, the system implementation will be covered. The results of applying software pulse-slimming techniques will show how system improvement can be effected, concluding remarks then follow.

2. PULSE SLIMMING

Attempts at recording digital data on a magnetic medium at high linear density causes inter-symbol interference (ISI) as adjacent flux transitions interfere. This is known as pulse crowding or peak shift and is illustrated in Figure 1.

Figure 1. Inter-Symbol Interference induced Peak Shift
Since data are encoded on the medium in terms of the positions at which the pulse peaks occur, peak shift causes mis-reading of the data during playback.

Higher recording densities can be achieved by slimming down the width of the pulses, thus enabling them to be placed closer together before the onset of peak shift. In practice, because the readback process is linear the principle of superposition applies and pulse slimming can take place after the recorded pulses have been read from the recording medium [3].

Pulse slimming can be effected by filtering the recorded signal. A suitable filter type is the tapped-delay-line equaliser or transversal filter, illustrated in Figure 2.

The circuit delays the input signal by multiples of the clock period, multiplies the delayed versions by coefficients \( C_i \), the weights, and sums them to produce the equalised output signal as shown in Figure 3. Design of the circuit involves choosing the coefficients to minimise inter-symbol interference at a finite number of points in the time domain or to shape the overall frequency response in the frequency domain. Both amplitude and phase response are equalised by the transversal filter. Digital Signal Processors are designed specifically to implement such filters, therefore these filters were implemented in software using the Texas Instruments TMS320C25 device.

In the time domain the output signal of the circuit \( y(t) \) is

\[
y(t) = -C_{-1}x(t) + C_0x(t-D) - C_{+1}x(t-2D)
\]

By means of variable time delays \( D \) and adjustable gains \( C_{-1}, C_0 \) and \( C_{+1} \) at each tap an isolated read pulse \( x(t) \) can be slammed and made symmetrical.

Appropriate adjustments of \( C_{-1} \) and \( C_{+1} \) force zero-crossings of signal \( y(t) \) at \( t=0 \) and \( t=2D \). The transfer function yields

\[
H(j\omega) = (C_0 - C_{-1}e^{-jD\omega} - C_{+1}e^{j2D\omega})e^{-jD\omega}
\]

Suppose \( C_{-1} = C_{+1} \) then

\[
H(j\omega) = (C_0 - 2C_{+1}\cos\omega D)e^{-jD\omega}
\]

Amplitude: \( |H(j\omega)| = C_0 - 2C_{+1}\cos\omega D \)

Phase: \( \arg(H(j\omega)) = -\omega D \)

The equaliser increases the bandwidth of the signal by compensating for the loss in high frequencies produced by the recording channel. The increase in bandwidth results in a narrower pulse in the time domain. This has the effect of reducing pulse interaction and hence reduces ISI-induced peak shift. However the amplification of the higher frequency components of the signal spectrum extends the noise power spectral density causing an increase in average noise power. This trade off between ISI and noise is the basis of equaliser design.
3. SYSTEM

The system, illustrated in Figure 4, was constructed around a TMS320C25 digital signal processor which can execute up to 12.5 million instructions per second (MIPS) with most instructions taking only a single clock cycle. The processor is tailored to exploit a high degree of internal parallelism to speed up digital signal processing algorithms.

The system includes two blocks of 25 nS access time static ram. The 'Shadow ROM' block, was loaded with the EPROM's data on initialisation so that the system could run at maximum speed with zero wait states. The 'RAM' block provides a large working area to store variables and buffer incoming and outgoing data. Address decoding and wait state generation is accomplished using a Programmable Array Logic (PAL) device to reduce the number of logic devices thus reducing propagation delay. Each recording channel has its own 10 bit Analogue-to-Digital Converter (ADC), with a conversion time better than 2µs, anti-aliasing filter and amplification circuitry. The system is constructed on two individual boards linked by an edge connector. This keeps the analogue and digital circuits separate to minimise electrical noise and also facilitates system expansion.

4. RESULTS

The filter described in the previous section was implemented in software. The greater the length (number of taps) of the transversal filter the slimmer the output pulse. However the filtering process is effectively a high frequency boost and whilst the pulse width decreases the associated noise increases. Thus a trade off is necessary.

Both 3-tap and 5-tap filters were implemented giving output pulses slimmed by 20% and 30% respectively. Figure 5 shows the plot of an isolated pulse slimmed by the 3-tap filter.

![Figure 5. Isolated Pulse Slimmed via 3-tap Transversal Filter](image)
The effects of applying a 5-tap filter to a readback waveform to slim a stream of data and also remove the hazardous low frequency content is illustrated in Figure 6. An analogue representation of the signal was obtained after slimming for comparison purposes. Normally, the signal would be retained in digital form through the detection and decoding process.

The increase in performance using pulse slimming can be gauged by comparison with results obtained from a microprocessor-based recording system without pulse slimming. This is highlighted in Figure 7. These results are for a multi-track system and the data rates given are on a per track basis.

5. CONCLUSION

It has been shown that by using a relatively simple filter it is possible to achieve an increased Error/Data Rate performance for a given system.

By increasing the length of the filter, to more accurately describe the equalisation, it is possible to further increase performance. However the law of diminishing returns results in an optimum filter length of about 10 taps.

Further work is proceeding to investigate different software equalisation techniques that can successfully be implemented in real time to yield a improved performance.

REFERENCES


Figure 6. Equalisation of Readback Signals Slimmed via 5-tap Transversal Filter. (Top) Read signal from head. (Bottom) After pulse slimming.

Figure 7. The Effect of Pulse Slimming on Error/Data Rate for a Multi-Track Tape Recording System.
Two Dimensional Coding for a Multiple-Track, Maximum-Likelihood Digital Magnetic Storage System

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Abstract—Two dimensional run-length-limited (d, k) modulation codes were recently introduced as a means of increasing storage capacity in multi-track recording systems. This paper furthers this coding technique by describing a reverse enumeration scheme based on the trellis description of the (d, k, ) constraint for a channel employing extended class IV partial response signalling and maximum likelihood detection.

I. INTRODUCTION

Run-Length Limited (RLL) codes have previously been characterised by the (d, k) constraints, which describe the minimum and maximum number of consecutive zeros between transitions, respectively, in a single channel. Marcellin and Weber [1] first recognised that Two Dimensional Modulation Codes provide the capacity to increase data storage density by satisfying the k constraint using a number of parallel channels as in the case of a multi-track tape storage system.

The k constraint along an individual track, k, can be increased without loss of clock synchronisation since the clocking information derived by frequent signal transitions can be sub-divided across a number of, y, parallel tracks in terms of a k constraint. This permits more code words to be generated for a given (d, k) constraint in two dimensions than is possible in one dimension. For example the code rate of a single track RLL code with constraints (1, 3), such as Miller code [2], can be increased by over 25% when using a 4-track two-dimensional code with the same d constraint and with the k constraint distributed across all the tracks.

Swanson and Wolf [3] observed that such codes could have an infinite k constraint on certain tracks. They noted that a disadvantage was the vulnerability of clock recovery to the loss of a channel due to a burst error on a track containing all transitions for a given code word, assuming a low track density where a dropout would only span a single track. However, a k constraint of infinity should not pose a problem to a peak detection channel, assuming clock synchronisation and automatic gain control can be derived from other tracks [4]. This is not the case for a channel employing maximum likelihood detection, where frequent transitions are desirable to force soft decisions.

Orcutt and Marcellin [5] have described a process of enumeration, whereby the code words for a multi-track code have been constructed using a trellis. However, this method can be unsuitable for certain applications since code words with a low density of transitions are generated, and no limit has been placed on the k constraint.

This paper describes a Two Dimensional Code for a multi-track tape system which overcomes the above shortcomings. A process of reverse enumeration is used to give code words which, whilst satisfying the required (d, k) constraint, contain regular transitions conducive to good clock recovery and efficient Viterbi detection. Also described is a method of making the code d-e free in the x-direction by the addition of code-word merging bits.

II. EPRML CHANNEL

Thapar and Patel [6] showed that the extended class IV partial response (EPR4), with transfer function (1+D+D'5'), improves performance over class IV partial response at higher recording densities, assuming perfect equalization.

The finite state diagram for EPR4, illustrated in Fig. 1, shows that the eight states, representing the write current in NRZ format, can be reduced to six by coding with a d = 1 constrained code.

Patel [7] describes how a (1,7) code, for a channel employing EPRML, eliminates two states and reduces two more states to transitional only.

![Fig. 1. State Diagram for d = 1 constrained EPRML channel](image-url)

III. TWO DIMENSIONAL CODING

The states of a multi-track (d, k) code take the form S,(u, u2, ..., u, v). The value u, describes the number of consecutive 0's in the i-th track and can take on any value in {0, 1, ..., d} with u = d meaning that d or more consecutive zeros have occurred in the i-th track. Hence, u, monitors the d constraint of


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the $i$th track. The value $v$ relates to the $k_y$ constraint and indicates the number of consecutive times that all tracks can be 0 simultaneously. Therefore the value $v$ falls in the range \( \{0, 1, \ldots, k_y \} \). The number of states, $S_y$, has been determined [1] as

\[ S_y = (d + 1)^y + k_y - d. \]

IV. REVERSE ENUMERATION

Enumeration is a method, introduced by Tang and Bahl [8] for single track RLL codes, for indexing the elements of a given set of code words according to their lexicographical order. This involves organizing the code words by their numerical magnitude under the interpretation that $0 < 1$.

The enumeration scheme assigns the source words in turn to the lowest magnitude code word available, i.e. the code word closest to the all zero code word. This results in code words that contain the smallest number of transitions per track, occurring early on in the translation. An alternative is reverse enumeration whereby the source words are mapped onto the code words which have a greater number of transitions distributed over several tracks. This is achieved by arranging the code words in reverse lexicographical order, so that code words with the greatest magnitude are used first.

V. EXAMPLE

The following example demonstrates how reverse enumeration can be employed to develop a 1/2 rate multi-track code for a 3-track system with constraints $d=1$ and $k_y=2$. The number of tracks and code word length have deliberately been made small for the purpose of illustration.

Initially a single-step finite state transition matrix (FSTM), given in Fig.2a, is constructed to represent legal paths between states. The FSTM is useful in determining the capacity of the code [9]. The matrix is raised to increasing powers until sufficient states are available to achieve a desired code rate. Figure 2b, shows the fourth order FSTM that has 91 valid code words, starting from state $S_x$ and ending in state $S_y$ or $S_z$.

Fig. 2. a) Single-Step FSTM; b) Fourth order FSTM; for 3-track two dimensional code, with RLL constraints (1,2).

The trellis, illustrated in Fig. 3, is used for both encoding and decoding. Each node contains an octal number representing the code symbol, $c_y$, across $y$ tracks at time $t$, for all paths entering that state. Each path is assigned an integer value to represent the cumulative number of paths from the present state $S_i$ to the next state $S_{i+1}$. For reasons of clarity only selected path values are labelled to demonstrate the encoding-decoding process. The dotted line illustrates valid paths that are not used, and the bold line highlights the following example.

The 64 source words \([000000, \ldots, 111111]\) are represented as the decimal numbers \([0 \ldots 63]\) respectively. To encode the source word, the branch with the lowest value that is greater than the decimal equivalent of the source word is traversed. The code symbol, $c_y$, is obtained from the value contained within the node that the path leads to. The value of the branch that is just less than the decimal source word is subtracted from it. This new value is then used as before to select the next path.

Fig. 3. Trellis for 3-track (1,2) constrained code

<table>
<thead>
<tr>
<th>Source word</th>
<th>Code word</th>
<th>Source word</th>
<th>Code word</th>
<th>Source word</th>
<th>Code word</th>
<th>Source word</th>
<th>Code word</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7070</td>
<td>1</td>
<td>6070</td>
<td>32</td>
<td>4240</td>
<td>48</td>
<td>3410</td>
</tr>
<tr>
<td>1</td>
<td>7060</td>
<td>17</td>
<td>6010</td>
<td>33</td>
<td>4210</td>
<td>49</td>
<td>3400</td>
</tr>
<tr>
<td>2</td>
<td>7050</td>
<td>18</td>
<td>5250</td>
<td>34</td>
<td>4200</td>
<td>50</td>
<td>3070</td>
</tr>
<tr>
<td>3</td>
<td>7040</td>
<td>19</td>
<td>5240</td>
<td>35</td>
<td>4160</td>
<td>51</td>
<td>3060</td>
</tr>
<tr>
<td>4</td>
<td>7030</td>
<td>20</td>
<td>5210</td>
<td>36</td>
<td>4140</td>
<td>52</td>
<td>3050</td>
</tr>
<tr>
<td>5</td>
<td>7020</td>
<td>21</td>
<td>5200</td>
<td>37</td>
<td>4120</td>
<td>53</td>
<td>3040</td>
</tr>
<tr>
<td>6</td>
<td>7010</td>
<td>22</td>
<td>5070</td>
<td>38</td>
<td>4100</td>
<td>54</td>
<td>3030</td>
</tr>
<tr>
<td>7</td>
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<td>55</td>
<td>3020</td>
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<td>5050</td>
<td>40</td>
<td>4060</td>
<td>56</td>
<td>3010</td>
</tr>
<tr>
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<td>57</td>
<td>2520</td>
</tr>
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<td>6100</td>
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<td>5030</td>
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<td>4040</td>
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<td>6050</td>
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<td>4340</td>
<td>45</td>
<td>4010</td>
<td>61</td>
<td>2410</td>
</tr>
<tr>
<td>14</td>
<td>6040</td>
<td>30</td>
<td>4300</td>
<td>46</td>
<td>3430</td>
<td>62</td>
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<td>6030</td>
<td>31</td>
<td>4250</td>
<td>47</td>
<td>3420</td>
<td>63</td>
<td>2140</td>
</tr>
</tbody>
</table>

A. Encode

Consider the source word $m = 011111$, which is represented by the decimal value 31. Beginning at state $S_x$ we select the path with the lowest cumulative weight which is
greater than the decimal equivalent of the source word. The path with weight 46 is the smallest value that is greater than 31, therefore we traverse this path to state $S_1$. This state has a code symbol of 4 allocated to it, hence the first code symbol, $c_1$, of the code word is assigned the value 4. The path which has the greatest weight less than 31 is then subtracted from the decimal source word to give the new decimal source word. Hence 31-29 = 2. The process is then repeated, therefore the next path traversed has the weight 6, this leads to state $S_2$ with value 2 which is assigned to $c_2$. The new decimal source word is then calculated as 2-2 = 0. The process continues to state $S_3$, giving the third code symbol, $c_3 = 5$, and ends in state $S_7$ allotting the final code symbol, $c_4 = 0$. Hence (011111) is encoded as

$$c_1, c_2, c_3, c_4 = 4250 = 0100$$

0010

B. Decode

To decode our example code word, (4250), sum the values on each of the branches that have the largest value below that of the branch taken. More simply, sum the values on each of the branches directly above the branch taken. Where there are no branches above the one traversed just add zero.

C. Substitution

As previously mentioned, code words with all transitions occurring on a single track are undesirable, since a burst error whilst reading that track would result in all timing information being lost. Therefore in our example we wish to prevent code word (4040) occurring. We achieve this by employing a substitution table, which prior to encoding, maps the source word onto another, arbitrary, unused source word that has a more desirable code symbol. The same table is used inversely after decoding to regenerate the original source word.

It can be seen from Table I that in the example code, code word (4040) is generated by decimal source word 42. Since only sixty four of the ninety one available code words are used, there are twenty seven remaining code words from which to chose a substitution. To maintain continuity, a replacement source word is chosen as 64 which gives a code word (2120).

VI. CHARGE CONSTRAINT

A null at dc is desirable to match the spectral response of the magnetic recording channel. Therefore, to bound the accumulated charge, a charge control bit, $P$, can be added to each track between two adjacent code words. By monitoring the digital sum variation (DSV) of the two adjacent code words, the charge control bit is chosen to minimise the total DSV. To prevent the charge control bit violating the $d$ constraint along each track, when a non-zero charge control bit is adjacent to a non-zero code bit, the following substitutions have to be made.

$$000P0 \rightarrow \text{valid for all } P$$

$$000P1 \rightarrow \text{if } P=1 \text{ substitute 00101}$$

$$010P0 \rightarrow \text{valid for all } P$$

$$010P1 \rightarrow \text{if } P=1 \text{ substitute 00100}$$

$$100P0 \rightarrow \text{valid for all } P$$

$$100P1 \rightarrow \text{if } P=1 \text{ substitute 10101}$$

Making each track dc-free has the added advantage of effectively introducing a $k$, constraint along each track. This is an important factor in determining the path memory of the Viterbi detector. The Viterbi detector requires non-zero samples to enable path merges, which result in decisions.

VII. CONCLUSIONS

A Two Dimensional code has been described which is suitable for a multi-track tape system. The code words have been generated by reverse enumeration using a trellis. This technique produces code words which, whilst satisfying a given $(d, k)$ constraint, has a sufficient distribution of transitions to maintain detector clock synchronisation in the event of a severe dropout on any one of the tracks.

Where the encoding process results in code words that don’t directly satisfy this latter requirement substitution code words are employed.

Additional charge control bits for each track improve the code’s spectral properties and ensure frequent transitions to enable efficient Viterbi detection.

Alternatively, error correction information could be added to provide a degree of error control without any additional rate loss.

VIII. REFERENCES


Appendix B

Software, written in Pascal, for the IBM PC Compatible Host Computer.
PROGRAM DIGITAL_COMPACT_CASSETTE_RECORDER (input, output);
uses graph, crt, dos, consts, menu, filt, sel_opt, export, display, viterbi;

{$1 convert.inc}

type linestring = string[80];
  num_str = array [1..1000] of integer;

const PLAY = 0; REC = 1; FF = 2;REW = 3; STOP = 4; TEST = 5;
clock = $04; sync = $08; L_edge = $10; oversamp = $18;
maxlen = 501;
maxpass = 3;
MAXTIME = 50000;
clkmsk = $FB;

var
  GrDriver, GrMode: integer;
  option: char;
  CH: data_array;
  data: num_str;
f: int_file;

procedure TestGrMode;
begin
  {GrDriver := Detect;}
  DetectGraph(GrDriver, GrMode);
  InitGraph(GrDriver, GrMode, 'D:\TP\BGI');
  Errcode := GraphResult;
  CloseGraph;
  if Errcode <> grOK then writeln('Graphics Error: ', GraphErrorMsg(ErrCode));
end;

procedure Display_hex(var info: integer);
begin
  convert_hex(info);
  write(' ');
end;

function Getdata(var data: integer): boolean;
var
  lopin: longint;
  inp, val: byte;
  done: boolean;
begin
  lopin := 0;
  REPEAT
    inp := PORT[P2C] AND $01;
    data := PORT[P2B] AND $0F;
    done := inp XOR val = 1;
    inc(lopin);
    UNTIL done or (lopin > maxtime);
  val := inp;
  getdata := done;
end;

function Get_DStr: boolean;

begin
i:= 1;

while (getdata(data[i]) and (i < maxlen)) do inc(i);
if i = maxlen then Get_DStr := true else Get_DStr := false;
end;

{******************************************************************************}
Procedure Viewdata;
var i:integer;
begin
repeat
for i := 1 to maxlen do data[i] := port[p1b] and $0f;
for i := 1 to maxlen do Display_hex(data[i]);
until keypressed = true;
end;

{******************************************************************************}
Procedure Getdval(var v:real; i:integer);
const testdata:ARRAY[1..15] OF REAL = (0.8,-0.3,0.8,0.3,-0.1,-1,-0.5,0.8,-0.3,0.8,0.3,-0.1,-1,-0.5,0);
var r:integer;
begin
r := Random(1024);
v := (r-512)/512;
v := testdata[i];
end;

{******************************************************************************}
Procedure ANALOG_DISPLAY;
const testc1:filt_array = (-0.2,-0.9,1,-0.3,-0.2);
testc2: filt_array = (-0.1,-0.3,1,-0.9,0.2);
testc3: filt_array = (-0.1,-0.9,1,-0.3,0.2);
testc4: filt_array = (-0.5,0,1,0,-0.5);

var coeff: filt_array;
width, slim: real;
x, y, offset: integer;
f: int_file;
rl, r2: text;

begin
if FileExists(f,'c:\pjdd\dcr\SAMPLES2.DAT') then Reset(f)
else Rewrite(f);
InitGraph(GrDriver,GrMode,'d:\tp\bgi');
SetBkcolor (black);
repeat
for x := 1 to 4 do for y := 1 to 2048 do read(f,Ch[x,y]);
{ filt_3tap(4,1); }
{ auto_equalize(4,2,coeff); }
filt_prog(CH,4,5,testc2);
{ slim := get_pw50(2); }
until eof(f) OR Display_Data(Ch,4,5,-1,1,100,false,0);
CloseGraph;
close(f);
end;

{******************************************************************************}
Procedure Save_Sig(var Sigsave:boolean);
var sigfile: string;
done: boolean;

begin
done := false;
WINDOW(14,10,65,11);
TEXTBACKGROUND(RED);
crser;
Sigsave := false; done := false;
write('Do you want to save the readback signals <y/n>?');
if getyn then
repeat
  write('Enter filename to store signals');
  readln(sigfile);
  if (Length(sigfile) > 12) OR (Length(sigfile) < 1) then begin
    clrscr;
    write('Invalid filename!! Do want to try again <y/n>?');
    if not getyn then done:=true;
  end
else if FileExists(f,sigfile) then begin
  clrscr;
  write(sigfile,' already exists do you want to overwrite <y/n>?');
  if getyn then begin
    Sigsave := true;
    Reset(f);
  end;
else begin
  Sigsave := true;
  Rewrite(f);
end;
until done or Sigsave;
End;

{············································································}
Procedure Play_Tape;
var y,x: integer;
Sigsave: boolean;
begin
  Save_Sig(Sigsave);
  Out_Port(PLAY);
  InitGraph(GrDriver,GrMode,'D:\TP\BGJ');
  SetBkcolor(black);
  DELAY(1000); {1 SECOND DELAY}
  for y := 1 to 2048 do for x := 1 to 4 do IN_PORT(Ch[x,y]);
  REPEAT
    for y := 1 to 2048 do for x := 1 to 4 do IN_PORT(Ch[x,y]);
    Plot_Data(ch,1,4,0,1,100,false,0);
    if Sigsave then
      for x := 1 to 4 do for y := 1 to 2048 do write(f,Ch[x,y]);
  UNTIL KEYPRESSED AND (Ord(READKEY) = ESC);
  RESET_SYS;
  CLOSEGRAPH;
  if Sigsave then Close(f);
  {crap := readkey ;}
end;
{············································································}
PROCEDURE WRITE_TAPE;
var i: integer;
begin
  OUT_PORT(REC);
  OUT_PORT(Hi(PERIOD)); {send high byte}
  OUT_PORT(Lo(PERIOD)); {send low byte}
  OUT_PORT(CODE);
  OUT_PORT(D_Type);
  if D_TYPE = SEQ THEN
begin

OUT_PORT (SEQLEN-1);
for i := 1 to seqlen do OUT_PORT(seqval[i]);
end;
crscr;
InitGraph(GrDriver,GrMode,'d:\tp\bg');
SetBkcolor (Red);
SetColor (Yellow);
SetTextStyle(SansSerifFont, HorizDir, 8);
OutTextXY (120,180, 'RECORDING!');
SetTextStyle(DefaultFont,HorizDir,1);
Outtextxy (140,440, 'Hit ESC to Terminate and return to main menu');
while ord(upcase(READKEY)) <> ESC do write(CHR(bell));
closegraph;
reset_sys;
end;

{$I TESTS.INC}

Appendix B

begin
TestGrMode;
port[P2CTRL] := $83;  {SET P2A=0/P P2B=1/P P2C=0/P I/P}
port[P2CTRL] := $0F;  {SET BUSY HIGH}
RESET_SYS;
repeat
  Display_Menu (Main_Menu);
  Get_Menu_Response (Main_Menu, option);
case option of
  'S': reset_sys;
  'R': OUT_PORT (REW);
  'F': OUT_PORT (FF);
  'W': WRITE_TAPE;
  'P': PLAY_TAPE;
  'T': Test_Sel;
  'V': viterbi_det(ch,200,5,6);
  'D': ANALOG_DISPLAY;
  'O': SEL_OPTION;
  end;
until ord(option) = ESC;
clrscr;
halt;
end.
unit consts;
interface
const maxlen = 50;

type data_array = array [1..8,1..2048] of integer;
datarray = array [1..maxlen] of integer;
const
maxdata = 2048;

BELL = 7;
CR = 13;
ESC = 27;
SPACE = 32;
R_ARROW = 77;
L_ARROW = 75;
D_ARROW = 80;
U_ARROW = 72;

PIA = $300; P1B = $301; P1C = $302; P1CTRL = $303;
P2A = $304; P2B = $305; P2C = $306; P2CTRL = $307;

implementation
end.
unit Codes;
interface
uses consts;
type
  seqarray = array [0..100] of byte;
const
datend = 40;
Procedure Miller_Squared_Encoder(var M2:seqarray; data:seqarray);
Procedure Miller_Squared_Decoder(var data:seqarray; M2:seqarray);

implementation

Procedure Miller_Squared_Encoder(var M2:seqarray; data:seqarray);
  type stat_val = (A,B,C,D,E,F,G,H,I,J);
  var M2state : stat_val;
  ii:integer;
  begin
    M2state := C;
    for ii := 0 to datend do
      case M2state of
        A: begin
          if data[ii] = 0 then M2state := B else M2state := I;
          M2[2*ii] := 1;
          M2[2*ii+1] := 0;
        end;
        B: begin
          if data[ii] = 0 then M2state := D else M2state := C;
          M2[2*ii] := 0;
          M2[2*ii+1] := 0;
        end;
        C: begin
          if data[ii] = 0 then M2state := D else M2state := E;
          M2[2*ii] := 0;
          M2[2*ii+1] := 1;
        end;
        D: begin
          if data[ii] = 0 then M2state := A else M2state := B;
          M2[2*ii] := 1;
          M2[2*ii+1] := 1;
        end;
        E: begin
          if data[ii] = 0 then M2state := F else M2state := C;
          M2[2*ii] := 1;
          M2[2*ii+1] := 0;
        end;
        F: begin
          if data[ii] = 0 then M2state := H else M2state := G;
          M2[2*ii] := 0;
          M2[2*ii+1] := 0;
        end;
        G: begin
          if data[ii] = 0 then M2state := H else M2state := J;
          M2[2*ii] := 0;
          M2[2*ii+1] := 1;
        end;
        H: begin
          if data[ii] = 0 then M2state := F else M2state := E;
          M2[2*ii] := 1;
          M2[2*ii+1] := 1;
        end;
  end;
end.
I. begin
  if data[ii] = 0 then M2state := F else M2state := A;
  M2[2*ii] := 0;
  if data[ii] = 0 then M2[2*ii+1] := 0 else M2[2*ii+1] := 1;
end;
J. begin
  if data[ii] = 0 then M2state := D else M2state := G;
  M2[2*ii] := 1;
  if data[ii] = 0 then M2[2*ii+1] := 1 else M2[2*ii+1] := 0;
end;
end;

Procedure Miller_Squared_Decoder(var data:seqarray; M2:seqarray);
var ii:integer;
begin
  for ii := 1 to datend do
    data[ii] := (M2[2*ii] and M2[2*ii+1] and not M2[2*ii+3])
    or (not M2[2*ii] and not M2[2*ii+1] and M2[2*ii+3]);
end.
end.
unit display;

interface
uses consts,graph,crt;

Procedure Plot_Data (var Data:data_array; ChSt,ChEnd:integer; sclY, xscl:real; xoffset:integer; axis:boolean; sample:integer);
Procedure Scope (var data:data_array; ChSt,ChEnd:integer; sclY, xscl:real; xoffset:integer);
Function Display_Data(var Data:data_array; ChSt,ChEnd:integer; sclY,zoom:real; offset:integer; axis:boolean; sample:integer):boolean;

implementation

Procedure Plot_Data (var Data:data_array; ChSt,ChEnd:integer; sclY, xscl:real; xoffset:integer; axis:boolean; sample:integer);

var maxy, miny : real;
    x, y, ysize, yoffset, Xmax, Xlim, yaxis, samp, ysample : integer;
    yscl:real;

begin
    clearviewport;
    Ysize := round((GetMaxY-10)/(Chend-ChSt+1));
    Xmax := GetMaxX; Xlim := Round(Xmax/Xscl);
    for y := ChSt to ChEnd do
        begin
            maxy := DATA[y, l + xoffset]; miny := DATA[y, l + xoffset];
            for x := xoffset+2 to xoffset + Xlim do
                if DATA[y, x] > maxy then maxy := DATA[y, x]
                else if DATA[y, x] < miny then miny := DATA[y, x];
            if sclY = -1 then begin
                yscl := Ysize / (maxy - miny + 1);
                yoffset := Round((y-ChSt)*YSIZE)-(miny*yscl);
            end
            else if sclY = 0 then begin
                yscl := Ysize / 256;
                yoffset := Round((y-ChSt)*YSIZE);
            end
            else begin
                yscl := sclY;
                yoffset := Round(((y-ChSt)*YSIZE)-(miny*yscl));
            end;
            if Xmax > (2048-xoffset)*xscl then xoffset := 2047 - Xlim;
            yaxis := round((ysize*(y-chst+1))-ysize/2);
            if yaxis then 
                begin
                    setcolor(Lightgray);
                    line (0,yaxis,xmax,yaxis);
                end;
            SetColor(black+y);
            moveto (0,round((DATA[y,xoffset]*yscl)+yoffset));
            x := 1;
            repeat
                lineto(round((x-1)*xscl),round((DATA[y,x+xoffset]*yscl)+yoffset));
                inc(x);
            until (round((x-1)*xscl) >= Xmax) or (x > maxdata);
            if sample > 0 then begin
                samp := 0;
            end;
end;
**Appendix B**

Procedure Scope (var data: data_array; ChSt, ChEnd: integer; sclY, xscl: real; xoffset: integer);

```pascal
var maxy, yoffset, ysize, yp1, yp2, yp3: longint;
x, y, xp1, xp2, Xmax: integer;
yscl: array[1..4] of real;
miny: array[1..4] of longint;

begin
  {············································································}
  {············································································}
  clearviewport;
  Ysize := round(GetMaxy - 10 / (ChEnd - ChSt + 1));
  Xmax := GetMaxX;
  for y := ChSt to ChEnd do begin
    maxy := DATA[y, 1]; miny[y] := DATA[y, 1];
    for x := 2 to maxdata do
      if DATA[y, x] > maxy then maxy := DATA[y, x]
      else if DATA[y, x] < miny[y] then miny[y] := DATA[y, x];
    if sclY = -1 then yscl[y] := Ysize / (maxy - miny[y] + 1)
    else if sclY = 0 then yscl[y] := Ysize / 256
    else yscl[y] := sclY;
  end;
  repeat
    if Xmax > (2048 - xoffset) * xscl then xoffset := 2047 - Round(Xmax / xscl);
    x := 1;
    repeat
      for y := Chst to chend do begin
        setcolor (black);
        yoffset := (y - ChSt) * Ysize - 1;
        xp1 := Round((x - 1) * xscl);
        xp2 := Round(x * xscl);
        yp1 := ysize - Round((Data[y, x - 1 + xoffset] - miny[y]) * yscl[y]) + yoffset;
        yp2 := ysize - Round((Data[y, x + xoffset] - miny[y]) * yscl[y]) + yoffset;
        yp3 := ysize - Round((Data[y, x + 1 + xoffset] - miny[y]) * yscl[y]) + yoffset;
        line (xp1, yp1, xp2, yp2);
        setcolor (black + y);
        line (xp1, yp2, xp2, yp3);
      end;
      inc(x);
      until (round((x - 1) * xscl) >= Xmax) or (x > maxdata);
      xoffset := xoffset + 1;
      until xmax > (2048 - xoffset) * xscl;
  end;
  {············································································}
  {············································································}
end;
```

Function Display_Data (var Data: data_array; ChSt, ChEnd: integer; sclY, zoom: real;offset: integer; axis: boolean; sample: integer): boolean;

```pascal
var fin, valid: boolean;
resp: char;
```
begin
valid := true;
REPEAT
if valid then
begin
plot_data(Data,ChSt,ChEnd,scly,zoom,offset,axis,sample);
SetColor(Yellow);
OutTextXY(130,470,'Hit <+/> for Zoom, <SPACE> for Next or <ESC> to Exit');
SetColor(White);
Rectangle(0,0,GetMaxX,GetMaxY-11);
end;
valid := true;
RESP := upcase(READKEY);
case ord(resp) of
  ESC : fin:=true;
  SPACE: fin := false;
  43 : if zoom < 1 then zoom := zoom +0.1
       else zoom := zoom +1;
  45 : if zoom > 1 then zoom := zoom -1
       else if zoom > 0.4 then zoom := zoom - 0.1;
  65 : axis := not axis;
  83 : if sample = 0 then sample := 7 else sample :=0;
R_arrow : inc(offset);
L_arrow : dec(offset);
else begin
  write(CHR(bell));
  valid := false;
end;
end;
UNTIL (ord(resp) = ESC) OR (ORD(REsp)=SPACE);
Display_Data := fin;
end;
{******************************************************}
end.

Appendix B
unit Export;

interface
uses consts, graph, crt;

type  int_file = file of integer;

const timeout = $fffff;

Function FileExists(var f: int_file; FileName: string): Boolean;
Procedure Show_Err(err_no: integer);
Procedure OUT_PORT (data: integer);
Procedure RESET_SYS;
Procedure IN_PORT (var data: integer);

implementation

{-------------------------------------------------------------------}
Function FileExists(var f: int_file; FileName: string): Boolean;
{ Returns True if file exists; otherwise,
it returns False. Closes the file if it exists. }

begin
{SI-}
Assign(f, FileName);
Reset(f);
Close(f);
{SI+}
FileExists := (IOResult = 0) and (FileName <> "");
end; { FileExists }

{-------------------------------------------------------------------}
Procedure Show_Err(err_no: integer);
var resp:char;
begin
  restorecrtmode;
  clrscr;
  writeln; writeln;
  write('ERROR : ');
  case err_no of
    1 : writeln('BUSY - cannot send data');
    2 : writeln('Timeout - no data received');
    3 : writeln('ADC not initialising');
    4 : writeln('External Ram Fault');
  end;
  writeln('Hit any key to continue');
  resp := readkey;
  halt;
end;

{-------------------------------------------------------------------}
Procedure OUT_PORT (data: integer);
var busy: integer;
loop: longint;
begin
  loop := 0;
  port[P2CTRL] := $OF; { set STROBE high }
  repeat
    { wait until BUSY goes low }
    busy := port[P2C] AND $01;
    inc(loop);
    until (busy = 0) or (loop > timeout);
  port[P2A] := data; { WRITE DATA TO PORT A }

  pause;
end;
port[P2CTRL] := $0E;  {set STROBE low}
repeat  
    bus := port[P2C] AND $01;
    inc(loop);
until (bus <> 0) or (loop > timeout);
port[P2CTRL] := $0F;  {set STROBE high}
if loop > timeout then Show_Err(1);
end;

Procedure RESET_SYS;
begin
    port[p2ctrl] := $09;
    port[p2ctrl] := $08;
    DELAY (50);
    port[p2ctrl] := $09;
end;

Procedure IN_PORT (var data: integer);
var strobe: integer;
loop: longint;
begin
    loop := 0;
    repeat
        strobe := port(P2C) AND $01;
        inc(loop);  {wait until strobe set}
    until (strobe <> 0) or (loop > timeout);
    port[P2CTRL] := $0E;  {SET BUSY LOW}
    repeat
        strobe := port(P2C) AND $01;
        inc(loop);
    until (strobe = 0) or (loop > timeout);
    data := port[P2B];  {READ DATA FROM PORT B}
    port[P2CTRL] := $0F;  {SET BUSY HIGH}
if loop > timeout then Show_Err(2);
end;

end.
unit filt;

interface
uses consts;
type  filt_array = array [1..5] of real;

Procedure Filt_3tap(VAR Ch: data_array; s,d: integer);
Procedure Fir_5tap(VAR Ch: data_array; s,d: integer);
Procedure Filt_5tap(VAR Ch: data_array; s,d: integer);
Procedure Filt prog(VAR Ch: data_array; s,d: integer; coeff:filt_array);
Procedure Auto_Equalize(VAR Ch: data_array; s,d: integer; coeff:filt_array);

implementation

Procedure Filt_3tap(var Ch: data_array; s,d: integer);
const coeff:array [1..3] of real = (-0.5,1,-0.5);
var step,x,i:integer;
begin
  step := 6;
  for x:= 1 to 2047 do
  begin
    Ch[d,x] := 0;
    for i := 1 to 3 do
      Ch[d,x] := Ch[d,x] + Round(Ch[s,x+(i-2)*step] * coeff[i]);
  end;
end;

Procedure Fir_5tap(var Ch: data_array; s,d: integer);
const coeff:array [1..5] of real = (-0.1,-0.9,1,-0.3,-0.2);
var step,x,i:integer;
begin
  for x:= 1 to 2047 do
  begin
    Ch[d,x] := 0;
    for i := 1 to 3 do
      Ch[d,x] := Ch[d,x] + Round(Ch[s,x+(i-2)] * coeff[i]);
  end;
end;

Procedure Filt_5tap(var Ch: data_array; s,d: integer);
const coeff:array [1..5] of real = (-0.5,0,1,0,-0.5);
var step,x,i:integer;
begin
  step := 3;
  for x:= 1 to 2047 do
  begin
    Ch[d,x] := 0;
    for i := 1 to 5 do
      Ch[d,x] := Ch[d,x] + Round(Ch[s,x+(i-3)*step]) * coeff[i];
  end;
end;

Procedure Filt prog(VAR Ch: data_array; s,d:integer; coeff:filt_array);
var step,x,i:integer;
begin
  step := 6;
  for x:= 1 to 2047 do
  begin
    Ch[d,x] := 0;
    for i := 1 to 5 do
Ch[d,x] := Ch[d,x] + Round(Ch[s,x+(i-3)*step] * coeff[i]);
end;
end;

procedure Auto_Equalize(var Ch: data_array; s, d: integer; coeff: filt_array);
var step, x, i: integer;
begin
  coeff[1] := -0.5;
  coeff[2] := 0;
  coeff[3] := 1;
  coeff[4] := 0;
  coeff[5] := -0.5;

  step := 6;
  for x := 13 to 17 do
    begin
      Ch[d,x] := 0;
      for i := 1 to 5 do
        Ch[d,x] := Ch[d,x] + Round(Ch[s,x+(i-3)*step] * coeff[i]);
    end;
  for x := 18 to 2000 do
    begin
      Ch[d,x] := 0;
      for i := 1 to 5 do
        coeff[i] := coeff[i] + (0.0003*(Ch[s,x-i]-Ch[d,x-i]));
      for i := 1 to 5 do
        Ch[d,x] := Ch[d,x] + Round(Ch[s,x+(i-3)*step] * coeff[i]);
    end;
end;
((c AND $f=0) OR (c AND $f=2)) OR (c AND $f=4) OR (c AND $f=8)) then
begin
if $c$ AND $040000 <> 0 then inc(bcnt);
inc(count);
write(count,' ');
bin2hex(c);
writeln(g,' ');
end;
end;
writeln(g,'bitcount=',bcnt);
close(g);
end;

begin
'Code_2D';
end.
unit Sel_Opt;

interface
uses crt,consts,menu;
type SEQUENCE = array [1..15] of byte;
const
  Mil2seq:SEQUENCE = (2,9,12,6,11,13,14,15,7,3,1,8,4,10,5);
  Mil2pos:SEQUENCE = (11,1,10,13,15,4,9,12,2,14,5,3,6,7,8);
  Mil2len = 15;
  BiPhase_L = 0; Miller = 1; Miller2 = 2; ISS2_3 = 3; TPM = 4; CRA = 5;
  PRBS = 0; RAMP = 1; SQUARE = 2; SEQ = 3;
  TIMER = $C0; ADCTST = $C1;
var
  seqval,seqpos : SEQUENCE;
  sync_trk,CODE,D_TYPE,seqlen:integer;
  period:longint;

Function GETYN:boolean;
Procedure Get_Freq(var time:longint);
Procedure GET_SEQ;
Procedure Get_Read_Opt ;
Procedure Get_Code_Opt;
Procedure Get_Data_Opt;
Procedure Sel_Option;

implementation

Function GETYN:boolean;
var reply:char;
begin
  repeat
    reply:= readkey;
    reply:= upcase(reply);
  until (reply = 'Y') or (reply = 'N');
  writeln(reply);
  GETYN := reply = 'Y';
end;

Function Get_Trk(line:integer;text:string):integer;
var trk:char;
begin
  WINDOW(l8,line,62,1ine);
  TEXTBACKGROUND(RED);
  clrscr;
  repeat
    write('Enter ',text,' Track <1 . .4> ');
    trk := readkey;
    writeln;
  until (trk >= '1') and (trk <= '4');
  get_trk := ord(trk)-ord('O');
  WINDOW(1,1,80,24);
  TEXTBACKGROUND(BLACK);
end;

Procedure Get_Freq(var time:longint);
var freq:integer;
begin
  WINDOW(IS,I0,62,I0);
  TEXTBACKGROUND(RED);
  clrscr;
  write('Enter ',text,' Track <1..4> ');
  trk := readkey;
  writeln;
  until (trk >= '1') and (trk <= '4');
  get_trk := ord(trk)-ord('0');
  WINDOW(1,1,80,24);
  TEXTBACKGROUND(BLACK);
end;
repeat
  repeat
    write('Enter RECORD frequency <400-20,000> Hz');
    readln(freq);
    until (freq>=400) and (freq<=20000);
    time := round(5000000/freq);
    write('Write period is ',round(time/10),', uS <Y/N> ');
  until:getyn;
  WINDOW(1,180,24);
  TEXTBACKGROUND(BLACK);
end;

(*----------------------------------------------------------)
Procedure GetSeq_val(i:integer);
var seqch:char;
begin
  gotoxy(5,6);
  write('Enter Hex Data at Sequence Value ',i,' '); clreoi;
  repeat
    seqch := upcase(readkey);
    if (ord(seqch)>$21) and (ord(seqch)<$3A)
      then seqval[i] := ord(seqch)-$30
    else if (ord(seqch)>$40) and (ord(seqch)<$47)
      then seqval[i] := ord(seqch)-$37
    else seqval[i] := $ff;
    until seqval[i] <> $ff;
  gotoxy(i*2+6,4);
  write(seqch);
  seqpos[seqval[i]] := i;
end;

(*----------------------------------------------------------)
Procedure GET_SEQ;
const maxslen = 32;
var seqok : boolean;
i:integer;
begin
  WINDOW(4,10,76,16);
  TEXTBACKGROUND(RED);
    repeat
      clrscr;
      writeln; write('Enter Length of Sequence <1..32> '); readln(seqlen);
      until (seqlen <= maxslen) and (seqlen > 0);
    writeln;
  gotoxy(0,4);
  write('Seq = {');
  for i := 1 to seqlen-1 do write(' ,');
  write(' } ');
  for i := 1 to seqlen do getseq_val(i);
  gotoxy(5,6);
  write('Is this sequence correct? '); clreoi;
  seqok := getyn;
  if not seqok then begin
    repeat
      gotoxy(5,6);
      write('Enter position of value to change');
      readln(i);
next line...
if (i>seqlen) OR (i<0) then
begin
  gotoxy(39,6);
  clreol;
  write(CHR(bell));
end;
until (i<=seqlen) and (i>0);
getseq_val(i);
end;
until seqok;
WINDOW(1,1,80,24);
TEXTBACKGROUND(BLACK);
D_Type := SEQ;
end;

Procedure Get_Read_Opt;
var rdsel:char;
begin
  Display_Menu (Read_Menu);
  Get_Menu_Response (Read_Menu, Rdsel);
  case Rdsel of
    'C' : sync_trk := Get_Trk(16,'CLOCK');
    'S': sync_trk := Get_Trk(16,'SYNC');
  end;
end;

Procedure Get_Code_Opt;
var codsel:char;
begin
  Display_Menu (Code_Menu);
  Get_Menu_Response (Code_Menu, Codsel);
  case codsel of
    'B': code:= BiPhase_L;
    'M': code:= Miller;
    'S': code := Miller2;
    'I': code := ISS2_3;
    '3' :code := TPM;
    'C': code := CRA;
  end; {of case}
end;

Procedure Get_Data_Opt;
var datsel:char;
begin
  Display_Menu (Data_Menu);
  Get_Menu_Response (Data_Menu, datsel);
  case datsel of
    'P': D_Type := PRBS;
    'R' : D_Type := RAMP;
    'S': D_Type := Square ;
    'E' : Get_seq;
    'M': begin
      Seqlen := Mil2len;
      seqval := Mil2seq;
      seqpos := Mil2pos;
      D_Type := SEQ;
    end;
  end;
end;
Procedure Sel_Option;
var opt: char;
begin
  repeat
    Display_Menu (Options_Menu);
    Get_Menu_Response (Options_Menu, opt);
    case opt of
      'R': Get_Read_Opt;
      'C': Get_Code_Opt;
      'F': Get_Freq(period);
      'D': Get_Data_Opt;
    end;
    until ord(opt) = Esc;
end;
end.

unit Viterbi;

interface
uses consts, graph, crt;
Procedure Precode(var data: data_array; dlen, din, dout: integer);
Procedure Man_decode(var data: data_array; dlen, din, dout: integer);
Procedure Viterbi_Det(var data: data_array; vlen, vin, vout: integer);
Procedure Detector(ch: data_array; s,d: INTEGER);
Procedure Detector2(ch: data_array; s,d: INTEGER);
Function Peak(var s,x: integer; Ch: data_array): integer;
Function Get_PWSO(ch: data_array; s: integer): real;

implementation

{******************************************************}
Procedure Precode(var data: data_array; dlen, din, dout: integer);
var i: integer;
begin
  for i := 1 to dlen do
    data[dout, i] := (data[din, i] XOR data[dout, i-1]) AND $01;
end;
{******************************************************}
Procedure Man_decode(var data: data_array; dlen, din, dout: integer);
var i: integer;
begin
  for i := 1 to (dlen div 2) do
    data[dout, i] := data[din, i*2];
end;
{******************************************************}
Procedure Viterbi_Det(var data: data_array; vlen, vin, vout: integer);
var f,i,z: integer;
  n,o,p,a,b,c,d,v: real;
State1,State0: array [1..400] of longint;
Wp,Wn: array [1..100] of real;
begin
  clrscr;
  f := 0;
  for i := 1 to vlen do
    begin
      data[vout,i] := -1; {initialise output value}
      v := data[vin,100+(i*7)]/10;
    end;
  end;
\[\begin{align*}
Wp[1] & := 0; \quad Wn[1] := 0; \\
n & := \text{Sqr}(v-1); \quad o := \text{Sqr}(v); \quad p := \text{Sqr}(v+1); \\
a & := Wp[i] + o; \quad b := Wn[i] + n; \quad c := Wn[i] + o; \quad d := Wp[i] + p;
\end{align*}\]

if \(a > b\) then 
begin 
\(Wp[i+1] := b;\) 
\(\text{State}1[i] := 1;\) 
end 
else begin 
\(Wp[i+1] := a;\) 
\(\text{State}1[i] := 0;\) 
end;

if \(c > d\) then 
begin 
\(Wn[i+1] := d;\) 
\(\text{State}0[i] := 1;\) 
end 
else begin 
\(Wn[i+1] := c;\) 
\(\text{State}0[i] := 0;\) 
end;

while \((Wp[i+1] > 1) \text{ AND } (Wn[i+1] > 1)\) do 
begin 
\(Wp[i+1] := Wp[i+1] - 1;\) 
\(Wn[i+1] := Wn[i+1] - 1\)
end;

if \((b > a) \text{ AND } (c > d)\) then 
begin 
\(\text{for } z := f \text{ to } i-1 \text{ do } data[vout,z] := \text{State}1[z];\) \{PrevState1\} 
\(f := i;\) 
end 
else if \((a > b) \text{ AND } (d > c)\) then 
begin 
\(\text{for } z := f \text{ to } i-1 \text{ do } data[vout,z] := \text{State}0[z];\) \{PrevState0\} 
\(f := i;\) 
end; 
write(v:8:3);
end;

writeln;
for \(i := 1 \text{ to } vlen \text{ do } write(data[vout,i]:3);\)
writeln;
precode(data,vlen,vout,vout+1); 
for \(i := 1 \text{ to } vlen \text{ do } write(data[vout+1,i]:3);\)
writeln;
man_decode(data,vlen,vout+1,vout+2); 
for \(i := 1 \text{ to } (vlen \text{ div } 2) \text{ do } write(data[vout+2,i]:3);\)
writeln;
write('Hit <ENTER> to continue'); 
readln;
end;

\{***************************************************\}
Function Peak(var s,x:integer; Ch: data_array):Integer;
var count : integer;
begin 
count := 0;

Appendix B
while \( Ch[s,x] = Ch[s,x+1] \) do inc(x);
if \( Ch[s,x] > Ch[s,x+1] \) then
repeat
\( \text{inc}(x); \)
if \( Ch[s,x] = Ch[s,x+1] \) then inc(count);
until \( (Ch[s,x] < Ch[s,x+1]) \) and \( (Ch[s,x+1] < Ch[s,x+2]) \)
else if \( Ch[s,x] < Ch[s,x+1] \) then
repeat
\( \text{inc}(x); \)
if \( Ch[s,x] = Ch[s,x+1] \) then inc(count);
until \( (Ch[s,x] > Ch[s,x+1]) \) and \( (Ch[s,x+1] > Ch[s,x+2]) \);
\( \text{Peak} := \text{round}(x-(\text{count}/2)); \)
end;

Appendix B

Function Get_PW50(ch: data_array; s:integer):real;
var cnt,pk1,pk2,x1,x2,pulse_no,x:integer;
ph50,pw50,total,a1,a2,m1,m2 :real;
begin
pulse_no := 0; total:=0;
x:=100;
pk1 := \text{peak}(s,x,ch);
repeat
pk2 := \text{peak}(s,x,ch);
ph50 := \( Ch[s,pk1] - (Ch[s,pk1]-Ch[s,pk2])/4; \)
cnt :=1;
if \( Ch[s,pk1] > \) ph50 then
begin
while \( Ch[s,pk1-cnt] > \text{ph50} \) do inc(cnt);
x1 := pk1-cnt;
cnt=1;
while \( Ch[s,pk1+cnt] > \text{ph50} \) do inc(cnt);
x2 := pk1+cnt;
end
else begin
while \( Ch[s,pk1-cnt] < \text{ph50} \) do inc(cnt);
x1 := pk1-cnt;
cnt=1;
while \( Ch[s,pk1+cnt] < \text{ph50} \) do inc(cnt);
x2 := pk1+cnt;
end;
m2 := \( Ch[s,x2]-Ch[s,x2-1]; \)
if \( m2 = 0 \) then \( a2 := 0 \)
else \( a2 := \text{(ph50}-Ch[s,x2])/m2; \)
m1 := \( (Ch[s,x1]-Ch[s,x1+1]); \)
if \( m1 = 0 \) then \( a1 := 0 \)
else \( a2 := \text{(ph50}-Ch[s,x1])/m1; \)
pw50 := \( (a2+x2)-(a1+x1); \)
total := total + pw50;
inc(pulse_no);
\{ \text{writeln}(pk1:5, Ph50, x1:5, x2:5, pw50); \}
pk1 := pk2;
until pk1 > 2000;
\{ \text{writeln}(total/pulse_no); \}
Get_PW50 := total/pulse_no;
end;

Procedure Detector2(ch: data_array; s,d:INTEGER);
var same,x,i:integer;
begin
same:=0;
for x:= 1 to 2047 do
begin
  if (Ch[s,x+1]-Ch[s,x]) > 0 then
    begin
      if (Ch[s,x]-Ch[s,x-4]) >= 0 then Ch[d,x] := $FF
      else Ch[d,x] := 0;
    end
  else if (Ch[s,x+1]-Ch[s,x]) < 0 then
    begin
      if (Ch[s,x]-Ch[s,x-4]) <= 0 then Ch[d,x] := 0
      else Ch[d,x] := $FF;
    end;
if (Ch[s,x+1]-Ch[s,x]) = 0 then
begin
  inc(same);
  Ch[d,x] := Ch[d,x-1];
end
else if same > 0 then
begin
  for i:= 1 to round(same/2) do Ch[d,x-i]:=Ch[d,x];
  same := 0;
end;
end;
end;

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************

Procedure Detector(ch:data_array; s,d:INTEGER);
var x:integer;
begin
for x:= 1 to 2047 do
begin
  if (Ch[s,x+1]-Ch[s,x]) > 0 then Ch[d,x] := $FF
  else if (Ch[s,x+1]-Ch[s,x]) < 0 then Ch[d,x] := 0
  else if (Ch[s,x+1]-Ch[s,x]) = 0 then Ch[d,x] := Ch[d,x-1];
end;
end;
end.
Appendix C

Software, written in TMS320C25 Assembly Language, for the
Digital Compact Cassette Tape Recording System.
Appendix C

TMS32025/

;***+ THIS PROGRAM WAS DEVELOPED BY P. DAVEY WRITTEN IN TMS32025 ASSEMBLY
;***+ CODE IT CONTROLS THE READING & WRITING OF DIGITAL DATA ON TAPE

TITLE "MULTI CHANNEL DIGITAL CASSETTE TAPE DATA RECORDER"

TEXAS

EXTERN _BackChannel,TSTBCHSND,TSTBCHGET

;**. MEMORY MAP

.BLK0 EQU >200 ;INTERNAL RAM >200 - >300
.BLK1 EQU >300 ;INTERNAL RAM >300 - >400
.BLK2 EQU >0060 ;INTERNAL RAM >060 - >07F
.XRAM EQU >8000 ;EXTERNAL RAM >8000 - >9FFF

.PROM EQU >8000 ;FAST PROGRAM ROM >8000 - >9FFF

.ADC1 EQU >2000
.ADC2 EQU >2001
.ADC3 EQU >2002
.ADC4 EQU >2003

.WRITE EQU >2000

;PORTS
.PARPORT EQU 0 ;BIT 0 WRITE/READ RELAY
.CONPORT EQU 1 ;BIT 1-2 SOL1(REV),SOL2(FF)

;**. RESERVED MEMORY LOCATIONS

.DRR EQU >0 ;SERIAL PORT DATA RECEIVE REGISTER
.DXR EQU >1 ;SERIAL PORT DATA TRANSMIT REGISTER
.TIM EQU >2 ;TIMER REGISTER (CUURENT COUNT)
.PRD EQU >3 ;PERIOD REGISTER (STARTING COUNT)
.IMR EQU >4 ;INTERRUPT MASKING REGISTER (6 LSB'S)
; | XINT | RINT | TINT | INT2 | INT1 | INT0 |
.GREG EQU >5 ;GLOBAL MEMORY SPACE

;**. BLK2 MEMORY USAGE

.SEGMENT WORD AT 60-7F 'RAM_B2'

.CONTROL DS.B
.STATUS DS.B
.STATUS1 DS.B
.ACCL DS.B
.ACCH DS.B
.DATA DS.B
.TEMP DS.B
.RDEND DS.B
.WRT_CODE DS.B
.CODE_NUM DS.B
; THIS FILE CONTAINS SOME COMMONLY USED MACROS

; INC: INCREMENT ROUTINE: INCREMENT "PTR" AND TEST ITS WITHIN LIMITS
; PTR = PTR + 1
; IF PTR > END THEN PTR = START

INC MACRO PTR,START,END
LOCAL INCEND

LAC PTR
ADDK 1
SACL PTR
SBLK END
BLEZ INCEND
LALK START
SACL PTR

INCEND MEND

; DEC: DECREMENT ROUTINE: DECREMENT "PTR" AND TEST ITS WITHIN LIMITS
; PTR = PTR - 1
; IF PTR < START THEN PTR = END

DEC MACRO PTR,START,END
LOCAL DECECEND
LAC PTR
SUBK 1
SACL PTR
SBLK START
BGEZ DECECEND
LALK END
SACL PTR

DECECEND MEND

; SET: SET BIT ROUTINE

SET MACRO LOC,BITPOS
LAC LOC
ORK 1,BITPOS
SACL LOC
MEND

; RESET: RESET BIT ROUTINE

RESET MACRO LOC,BITPOS
LAC LOC
ORK 1,BITPOS
XORK 1,BITPOS
Appendix C

SACL LOC
MEND

;**- BIT: TEST BIT ROUTINE
BIT MACRO BITPOS, MEM
; LAC BIT, BITPOS
; AND MEM
; MEND

;**- SRL: SHIFT RIGHT LOGICAL
SRL MACRO MEM, XPLACES
LAC MEM, 16-XPLACES
SACH MEM
MEND

;**- SAVE: SAVE CURRENT PARAMETERS WITHOUT USING STACK
; USES DATA PAGE 0
STORE MACRO
; SST STATUS ; SAVE STATUS REG. ST0
SST1 STATUS1 ; SAVES STATUS REG. ST1
LDPK 0 ; CHANGE TO DATA PAGE 0
SACL ACCL ; SAVE ACCUMULATOR
SACH ACCH
MEND

;**- RESTORE: RESTORES SAVED PARAMETERS
RESTORE MACRO
; LDPK 0 ; GOTO DATA PAGE 1 TO RESTORE PRE-INTERUPT CONDITIONS
ZALH ACCH ; RESTORE ACCUMULATOR
ADD5 ACCL
LST STATUS ; RESTORE ST0
LST1 STATUS1 ; RESTORE ST1
MEND

;**- DELAY1: 1uS DELAY * ACC
DELAY1 MACRO ; 1uS DELAY LOOP UNTIL ACC = 0
LOCAL DELJ1
DELJ1 SUBK 1
RPTK 4
NOP
BNZ DELJ1
MEND

;**- TSTAPE: TEST TAPE IS MOVING
TSTAPE MACRO DEST
LOCAL CHANGED, NOTEND, NOTST
LAC BIT, 4
AND CHANGE
BNZ CHANGED

BANZ NOTST
LAC MOVING
SUB ONE
BGZ NOTEND
CALL STOP
LAC ONE, 10
SACL MOVING
B DEST
CHANGED LAC ONE,10
NOTEND SACL MOVING
NOTST MEND

;************************************************************************************
;** SELFUNC: SELECT A GIVEN FUNCTION
SELFUNC MACRO FNUM,MASK,ROUTINE
LOCAL NOTSEL
LACK >FNUM
SACL TEMP
LACK >MASK
AND FUNC
SUB TEMP
BNZ NOTSEL
CALL ROUTINE
NOTSEL MEND

;*****************************************************************************,**-
NRZI: CONVERT ACC TO NRZI FORMAT
NRZI MACRO
XOR LAST
SACL LAST
MEND

;********************************************************************************
;** ROUTINES TO COMMUNICATE TO EXTERNAL DEVICES VIA PARALLEL PORT
;*****************************************************************************,**-
REPORT: READ PARALLEL PORT

REPORT MACRO PDATA;READ PARALLEL PORT
LOCAL STRBLO,STRBH1,STRBLO2,STRBH2
SXF ;SET BUSY
STRBLO BIOZ STRBLO ;WAIT FOR STROBE HIGH
RXF ;NOT BUSY
STRBH1 BIOZ STRBLO2 ;WAIT FOR STROBE LOW
B STRBH1
STRBLO2 IN PDATA,PARPORT ;GET DATA FROM PARALLEL PORT
SXF ;SET BUSY LINE
LACK >FF
AND PDATA;ONLY 8 BIT DATA SO ZERO OTHER BITS
SACL PDATA
STRBH2 BIOZ STRBH2 ;WAIT FOR STROBE TO GO HIGH
MEND

;*****************************************************************************,**-
WRPORT: WRITE TO PARALLEL PORT (HOST COMPUTER)

WRPORT MACRO PDATA
LOCAL BSYLO,BSYHI,NOTBSY
SXF ;SET STROBE LINE HIGH
BSYHI BIOZ NOTBSY
B BSYHI
NOTBSY OUT PDATA,PARPORT ;SEND DATA TO PARALLEL PORT
RXF ;SET STROBE LINE LOW
BSYLO BIOZ BSYLO ;WAIT FOR HOST TO GET DATA AND BECOME BUSY
SXF ;SET STROBE LINE HIGH
MEND

;*****************************************************************************,**-
GETINP: GET INPUT FORM HOST COMPUTER

GETINP MACRO INPUT ;GET INPUT FROM PARALLEL PORT
LOCAL STRBER, STRBH1, NXTRY, TSTFLG

SET CONTROL, 13
SET CONTROL, 7
OUT CONTROL, LINES ; CONFIG PARALLEL PORT AS BUSY INPUT
RESET FLAG, 1

NXTRY BIT 7, CONTROL
BZ STRBH1
IN DATA, TAPE
LACK > 80 ; TEST STROBE LINE HIGH
AND DATA
BZ STRBER
RESET CONTROL, 7
OUT CONTROL, LINES ; SET BUSY LINE LOW

STRBH1 IN DATA, TAPE
LACK > 80 ; WAIT FOR STRB TO GO LOW
AND DATA
BNZ STRBER
IN INPUT, PARPORT ; GET DATA FROM PARALLEL PORT
SET FLAG, 1
SET CONTROL, 7 ; SET BUSY LINE HIGH
OUT CONTROL, LINES ; ONLY 8 BIT DATA SO ZERO OTHER BITS
SAACL INPUT

STRBER LAC DATA
XOR OLDATA
SAACL CHANGE
LAC DATA
SAACL OLDATA
TSTAPE TSTFLG
TSTFLG BIT 1, FLAG
BZ NXTRY
MEND

; *rieving: SEND AN OUTPUT SIGNAL
OUTSIG MACRO
LAC ONE
XOR CONTROL
SAACL CONTROL
OUT CONTROL, CONPORT
MEND

; ** ERROR: SEND AN ERROR MESSAGE TO HOST
ERROR MACRO ERRNO
LACK > FF
SAACL TEMP
WRPORT TEMP
LACK > 00
SAACL TEMP
WRPORT TEMP
LACK ERRNO
SAACL TEMP
WRPORT TEMP
REPORT TEMP
B START
MEND

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Appendix C

DS.B

;********************************************************************

#include "MACROS.INC"

;********************************************************************

;**+          INTERRUPT VECTORS

SEGMENT WORD AT 0-7 'HARD_INT'

.reset b init ;INIT EXTERNAL RESET SIGNAL
.int0 b isr0 ;EXTERNAL USER INTERRUPT #0
.int1 b isr1 ;EXTERNAL USER INTERRUPT #1
.int2 b isr2 ;EXTERNAL USER INTERRUPT #2

SEGMENT WORD AT 8-17 'RESERVED'

SEGMENT WORD AT 18-1F 'SOFT_INT'

.tint b time ;INTERNAL TIMER INTERRUPT
.rint b rx_int ;SERIAL PORT RECEIVE INTERRUPT
.xint b tx_int ;SERIAL PORT TRANSMIT INTERRUPT
.user b proc ;TRAP INSTRUCTION ADDRESS

;**

;**+          MAIN PROGRAM

SEGMENT WORD AT 20-1FFF 'EPROM'

;**- INIT: INITIALISE INTERNAL RAM & COPY EPROM TO SHADOW ROM

.init rovm ;DISABLE OVERFLOW MODE
.lrpk 0 ;SET DATA PAGE = 0
.larp 0 ;SET AUX REG = 0
.dint
.zac ;LOAD ACC WITH 0
.sacl imr ;DISABLE ALL INTERRUPTS
.rsxm ;DISABLE SIGN EXTENSION MODE
.lack >06 ;STOP TAPE, START SAMPLING, READ TAPE

.sacl control
.out control, conport

.zac
.lark ar0,blk2 ;POINT TO BLOCK B2
.rptk 31
.sacl ++ ;STORE 0 IN ALL 32 LOCATIONS

.lrlk ar0,blk0 ;POINT TO BLOCK B0
.rptk 255
.sacl ++ ;ZERO ALL OF DP 4 & 5

.lrlk ar0,blk1 ;POINT TO BLOCK B1
.rptk 255
.sacl ++ ;ZERO ALL OF DP 6 & 7

.zac
.lark ar1,((high_backchannel)-1) ;31 MAX
.lark ar0,ram ;LOAD AR0 WITH DEST

.nxtread rptk 255 ;READ 256 WORDS IN EPROM
.tblr ++ ;& STORE IN EXTERNAL RAM
.adlk >100 ;NEXT PAGE
LARP 1
BANZ NXTRD,*,0 ;REPEAT 32 TIMES (32*256 = 8K)

LARK AR1,{(high_BackChannel)-1} ;31 MAX
LALK PROM ;LOAD ACC WITH DEST
LRLK AR0,XRAM ;LOAD AR0 WITH SOURCE

NXTWRT RPTK 255 ;READ 256 WORDS IN EXTERNAL RAM
TBLW -*+ ;& STORE IN PROM
ADLK >100 ;NEXT PAGE
LARP 1
BANZ NXTWRT,*+,0 ;REPEAT 32 TIMES (32*256 = 8K)
B START

****
SHADOW ROM
SEGMENT WORD AT 8100-9FFF 'PROM'
**- INTERRUPT ROUTINES

ISR0 RET
ISR1 RET
ISR2 RET
TIME ANDK >0F
SACL *
EINT
RET
RX_INT RET
TX_INT RET
PROC RET
**- START: INITIALISE EXTERNAL RAM
START
ZAC
LARK AR1,31
LRLK AR0,XRAM
CLRXRAM RPTK 255 ;CLEAR 256 WORDS IN EXTERNAL RAM
SACL -*+
LARP 1
BANZ CLRXRAM,+-,0
LDPK 0

LALK >D008
SACL PRBS
**- SEL_FUNC: GET CHOICE OF FUNCTION FROM HOST COMPUTER
SEL_FUNC
REPORT FUNC
LAC FUNC,1
ADLK FUNC_TABLE
CALA
B SEL_FUNC

;************************************************************************************************************
**- FUNC_TABLE: TABLE OF POSSIBLE FUNCTIONS
FUNC_TABLE
B READ
B RECORD
B FF
B REW
B STOP
B TESTSEL

**- DATA_TABLE: TABLE OF DATA TYPES
DATA_TABLE
DATA_0 B WRT_PRBS
DATA_1 B WRT_SAW
DATA_2 B WRT_SQR
DATA_3 B WRT_SEQ

;**- CODE_TABLE: TABLE OF CODING ALGORITHMS
.CODE_TABLE
CODE_0 B MANWRT
CODE_1 B MILLWRT
CODE_2 B MIL2WRT
CODE_3 B ISSWRT
CODE_4 B WRT3PM
CODE_5 B CRAWRT
CODE_6 B NRZIMAN

;**- TEST_TABLE: TABLE OF TEST ROUTINES
.TEST_TABLE
B ECHO
B TSTBTCNGET
B TSTBTCNSND
B TSTXRAM

;****************************************************************************
;**+ READ: READ DATA ON TAPE AND SEND IT TO THE PC
.READ CALL NXTREAD
NXTREAD LRLK AR0,XRAM
LRLK AR7,2047
LARP 1
MORE LRLK AR0,ADC1
LARK AR2,3
CALL SAMPLE
NXTCHAUX *+,0,0
SAACL *+,0,2
BANZ NXTCH,*-,1
LARP7
BANZ MORE,*-,1
LRLK AR0,XRAM
LRLK AR7,8191
LARP0
OUTNXT LARK AR4,7
LAC *+,0,4
FUDGE ROR
SAACL TEMP
LAC DATA
ROL
SAACL DATA
LAC TEMP
BANZ FUDGE,*-
LARP7
WRPORT DATA
BANZ OUTNXT,*-,0
B NXTREAD

.SAMPLE
LACK >0F
AND CONTROL
SAACL CONTROL
OUT CONTROL,CONPORT

C9
CONVERT

RPTK 6 ;DELAY FOR SAMPLE
NOP
LACK >F0 ;SET 4 MSB'S
OR CONTROL
SACL CONTROL
OUT CONTROL,CONPORT
LACK 4
DELAY1 ;DELAY FOR EOC
RET

;***************************************************************************

ROUTINES TO WRITE DATA TO TAPE
;***************************************************************************

**. WRT_INIT: WRITE INITIALIZATION

.WRT_INIT
ZAC
SACL LAST
SACL MASK1
SACL MASK2
SACL PREV1
SACL PREV2
SACL OLDATA
RET

;***************************************************************************

**. RECORD: RECORD A SIGNAL ON TAPE

.RECORD
CALL WRT_INIT
REPORT TEMP ;get high byte
REPORT PERIOD ;get low byte
ADD TEMP,8 ;add them to get Period between writes
SACL PERIOD ;period = 5e6/freq(hz)
SACL PRD ;load the timer

REPORT WRT_CODE ;get code
LACK CODE_TABLE
ADD WRT_CODE,1 ;select correct algorithm
SACL WRT_CODE

REPORT WRT_DATA ;get type of data to record
SUBK >3
BNZ NOSEQ

REPORT SEQLEN
LAR AR6,SEQLEN ;CORRECTED FOR BANZ
LAR AR4,SEQX
LARP 4
NXTSEQ REPORT *
MAR *+,6
BANZ NXTSEQ,*-,4

NOSEQ LRLK AR5,WRITE
LARP 5 ;load aux reg 5 with Write address
LACK >8
SACL IMR ;SELECT timer interrupt
EINT ;enable interrupts
SET CONTROL,0
OUT CONTROL,CONPORT ;switch relay to write driver
CALL PLAY
Appendix C

LALK DATA_TABLE
ADD WRT_DATA,1 ; call routine to write data selected
CALA RET

;*******************************************************
;**- WRT_SQR: WRITE A SQUARE WAVE ON TAPE
.WRT_SQR
  ZAC
  B WRTNXT
  NXTSQR LAC DATA
  CMPL
  WRTNXT SA CL DATA
  LAC WRT_CODE
  CALA B NXTSQR

;*******************************************************
;**- WRT_SEQ: WRITE A SEQUENCE ON TAPE
.WRT_SEQ
  LAR AR4,SEQX
  LAR AR6,SEQLEN
  LARP 4
  OUTSEQ LAC *,0,5 ; LOAD ACC WITH CONTENTS OF AUX, INC AUX,
                     ; LEAVE ARP = 5 FOR INTERRUPT WRITE
                     ; SA CL DATA
  LAC WRT_CODE
  CALA
  LARP 6
  BANZ OUTSEQ,*-,4
  B WRT_SEQ

;*******************************************************
;**- WRT_SAW: WRITE A SAW TOOTH PATTERN ON TAPE
.WRT_SAW
  LACK 1
  ADD DATA
  SA CL DATA
  LAC WRT_CODE
  CALA B WRT_SAW

;*******************************************************
;**- WRT_PRBS: WRITE A PSEUDO RANDOM BINARY SEQUENCE ON TAPE
.WRT_PRBS
  LACK >C
  SA CL PRBS
  LACK 1 ; IF BIT2 <> BIT3 THEN ADD 1
  SAMVAL ADD DATA,1 ; SHIFT 1 PLACE LEFT
  SA CL DATA
  LAC WRT_CODE
  CALA
  LAC DATA
  AND PRBS ; MASK OFF BITS 2 & 3
  BZ SAMVAL ; TEST TO SEE IF THEY ARE BOTH = 0
  XOR PRBS ; TEST TO SEE IF THEY ARE BOTH = 1
  BZ SAMVAL ; IF SO JUST SHIFT 1 PLACE LEFT
  B WRT_PRBS

;*******************************************************
;**- MILLWRT: MILLER CODE WRITE ROUTINE
MILLWRT LAC PREVI ; LOAD ACC WITH 1ST HALF OF PREVIOUS BIT CELL
IDLE ; WAIT FOR INTERRUPT TO OUTPUT ACC
LAC PREV2 ;LOAD ACC WITH 2ND HALF OF PREVIOUS BIT CELL
IDLE ;WAIT FOR INTERRUPT TO OUTPUT ACC
LAC OLDATA ;GET COMPLEMENT OF PREVIOUS DATA
CMPL
SACL TEMP
LAC DATA ;UPDATE OLDATA WITH CURRENT DATA
SACL OLDATA ;GET COMPLEMENT OF CURRENT DATA
AND TEMP ;AND WITH COMPLEMENT OF PREVIOUS DATA
XOR PREV2 ;XOR WITH 2ND HALF OF PREVIOUS BIT CELL
SACL PREV1 ;STORE THE 1ST HALF OF CURRENT BIT CELL
XOR DATA ;XOR CURRENT DATA
SACL PREV2 ;STORE THE 2ND HALF OF CURRENT BIT CELL
RET

;*******************************
;**- MIL2WRT: MILLER SQUARED WRITE ALGORITHM
MIL2WRT LAC PREV1 ;LOAD VAL WITH 1ST HALF OF PREVIOUS BIT CELL
IDLE ;WAIT FOR INTERRUPT TO OUTPUT ACC
CLKIT
LAC OLDATA
AND DATA ;GET TRACKS WITH CONSECUTIVE '1's
AND MASK1 ;ISSOLATE TRACKS WITH EVEN NUMBER OF ONES
XOR PREV2
SACL PREV2 ;XOR WITH 2ND HALF OF PREVIOUS BIT CELL
CELL
IDLE ;WAIT FOR INTERRUPT TO OUTPUT ACC
LAC OLDATA
AND DATA
SACL MASK1 ;UPDATE MASK1
LAC OLDATA
CMPL
SACL TEMP
LAC DATA
SACL OLDATA
CMPL
AND TEMP ;AND WITH COMPLEMENT OF PREVIOUS DATA
XOR PREV2 ;XOR WITH 2ND HALF OF PREVIOUS BIT CELL
SACL PREV1 ;STORE THE 1ST HALF OF CURRENT BIT CELL
XOR DATA ;XOR CURRENT DATA
XOR MASK1
SACL PREV2
RET

;*******************************
;**- MANWRT: WRITE DATA ON TAPE IN MANCHESTER CODE
MANWRT
CMPL LAC DATA
IDLE ;WAIT FOR INTERRUPT TO OUTPUT VAL
LAC DATA
IDLE ;WAIT FOR INTERRUPT TO OUTPUT VAL
RET

;*******************************
;**- NRZIMAN:
NRZIMAN
LAC DATA
CMPL
XOR OLDATA
SACL OLDATA
IDLE
LAC DATA
XOR OLDATA
SACL OLDATA
IDLE
RET

;****************************************************************************************
;**  ISSWRT:  WRITE DATA ON TAPE IN ISS 2/3 RATE CODE

ISSWRT
LAC DATA
SACL OLDATA
OR PREV2
SACL MASK1
LALK ISS_PT2
SACL WRT_CODE
RET

ISS_PT2 LAC PREV1
CMPL
AND MASK2
NRZI
IDLE

LAC MASK1
CMPL
AND PREV1
AND DATA
SACL TEMP
LAC PREV1
AND PREV2
OR TEMP
AND MASK2
NRZI
IDLE

LAC MASK1
CMPL
AND DATA
CMPL
SACL TEMP
LAC PREV2
CMPL
AND TEMP
AND MASK2
NRZI
IDLE

LAC MASK1
SACL MASK2
LAC OLDATA
SACL PREV1
LAC DATA
SACL PREV2
LALK ISSWRT
SACL WRT_CODE
RET

;****************************************************************************************
;**  3PMWRT:  WRITE DATA ON TAPE IN 3PM CODE

.3PMWRT
LAC DATA
SACL PREV1
LALK PM_PT2

C13
SACL WRT_CODE
RET

.PM_PT2
LAC DATA
SACL PREV2
LALK PM_OUT
SACL WRT_CODE
RET

.PM_OUT
LALK WRT3PM
SACL WRT_CODE
LAC PREV2
OR DATA
AND PREV1
AND OLDATA
SACL MASK1

LAC OLDATA
XOR MASK1
NRZI

P5_OUT       IDLE
LAC MASK1
NRZI

P6_OUT       IDLE

P1_CALC       LCAC PREV2
OR DATA
AND PREV1
XOR MASK1
NRZI

P1_OUT       IDLE

P2_CALC       LAC PREV1
CMPL
AND PREV2
NRZI

P2_OUT       IDLE

P3_CALC       LAC PREV2
OR DATA
CMPL
AND PREV1
NRZI

P3_OUT       IDLE

P4_CALC       LAC PREV1
XOR PREV2
CMPL
AND DATA
NRZI

P4_OUT       IDLE

P5_CALC       LAC DATA
OR PREV1
OR PREV2
CMPL
SACL TEMP
LAC PREV1
XOR DATA
AND PREV2
OR TEMP
SACL OLData

RET

*******************************************************************************

;** CRAWRT: WRITE DATA ON TAPE IN CRA CODE
WRT_AC MACRO CYCLES
LOCAL ACWRT
LARK AR7,CYCLES
ACWRT LACK >F
NRZI
IDLE
LARP 7
BANZ ACWRT,*-,5
MEND

.CRAWRT
LAC DATA
SACL PREVI
LALK CRA_PT2
SACL WRT_CODE
RET

.CRA_PT2
LAC DATA
SACL PREV2
LALK CRA_OUT
SACL WRT_CODE
RET

.CRA_OUT
WRT_AC 5

LAC PREVI
CMPL
SACL TEMP
AND PREV2
SACL MASK1

LAC PREV2
CMPL
AND TEMP
SACL MASK2

LAC DATA
CMPL
SACL OLDATA
AND TEMP
SACL TEMP
LAC PREV2
CMPL
AND PREVI
OR TEMP
SACL TEMP
LAC LAST
AND MASK1
XOR TEMP
LARK AR7,15
WRTB1 SACL LAST
IDLE

;temp = not prev1
;mask1 = not prev1 and prev2
;mask2 = not prev1 and not prev2
;olddata = not data
;temp = not prev1 and not data
;temp = (not prev2 and prev1) or (not prev1 and not data)
;mask of last data bits to change
Appendix C

; XOR MASK1
LARP 7
BANZ WRTB1,*-,5

WRT_AC 5

LARK AR7,15
LAC LAST
AND MASK2
XOR OLDATA

WRTB2 SACL LAST
IDLE
XOR MASK2
LARP 7
BANZ WRTB2,*-,5

WRT_AC 5

LALK CRAWRT
SACL WRT_CODE
RET

;*****************************************************************************

CONTROLS FOR THE TAPE DECK

*****************************************************************************

;**-REW: REWIND TAPE
REW CALL STOP
RESET CONTROL,2 ;DISABLE FAST FORWARD SOLENOID
SET CONTROL,1
OUT CONTROL,CONPORT
CALL SOLDLY ;DELAY WHILE SOLENOIDS ENGAGE
RET

*****************************************************************************

;**- FF: FAST FORWARD TAPE
FF CALL STOP
SET CONTROL,2 ;DISABLE REWIND SOLENOID
RESET CONTROL,1 ;ENABLE FAST FORWARD SOLENOID
OUT CONTROL,CONPORT
CALL SOLDLY ;DELAY WHILE SOLENOIDS ENGAGE
RET

*****************************************************************************

;**-STOP: STOPS THE TAPE
STOP SET CONTROL,2 ;DISABLE REWIND SOLENOID
SET CONTROL,1 ;DISABLE FAST FORWARD SOLENOID
OUT CONTROL,CONPORT
CALL SOLDLY
RET

*****************************************************************************

;**-PLAY: PLAY THE TAPE
PLAY RESET CONTROL,2 ;ENABLE REWIND SOLENOID
RESET CONTROL,1 ;ENABLE FAST FORWARD SOLENOID
OUT CONTROL,CONPORT
CALL SOLDLY ;DELAY WHILST SOLENOIDS ENGAGE
RET

*****************************************************************************

;**-SOLDLY: SOLENOID DELAY
SOLDLY LALK >1000,3 ;LOAD ACC WITH >40000 (262144)
DELAY1 ;1us DELAY LOOP REPEATS UNTIL ACC = 0
RET

*****************************************************************************


**MONIT:** MONITOR HALL EFFECT TRANSISTOR TO SEE WHEN TAPE ENDS

**NOTE:** HALL EFFECT TRANSISTOR MUST BE CONNECTED TO BIT 11 OF TAPE READ I/P PORT

.MONITRRK AR1,>FFFF
LARK AR0,2
LRLK AR7,ADC1
LARP 7

BLKD ADC1,TEMP
REPMON LAC TEMP
XOR *,1 ;
ANDK >0800 ;MASK OFF BIT 11
BNZ MONIT ;
BANZ REPMON,*,7
LARP 0
BANZ REPMON,*,7
MONEND CALL STOP ;STOP THE TAPE

---

**TEST ROUTINES**

**TESTSEL:** SELECT TEST ROUTINE

.TESTSEL
REPORT FUNC
LAC FUNC,1
ADLK TEST_TABLE
CALA
RET

**ECHO:** ECHO'S INPUT FROM PC

.ECHO
LARK AR0,255
LARP 0
ECHO REPORT TEMP
WRPORT TEMP
BANZ ECHO1,*-
RET

**TSTOP:** TEST OUTPUT TO PC - SEND 0,1,2...FF.

.TSTOP
LARK AR5,DATA
LARP 5
ZAC
CNTOUT SACL *
WRPORT *
ADDK 1
B CNTOUT
RET

**TSTSTRB:** PULSE STROBE LINE

.TSTSTRB
SXF
RXF
B TSTSTRB

**TSTIME:** TEST INTERNAL TIMER

---

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.TSTIME  LDPK  0
  LACK >10
  SACL  PRD
  LACK  8
  SACL  IMR
REPTIM  IDLE
  B  REPTIM
  
;ENABLE TIMER INTERRUPT ONLY

;**-  TSTCNCL:  PULSE CONTROL LINES
.TSTCNCL  LARP  5
  LARK  AR5,CONTROL
REPTST  LACK  >FF
  SACL *
  OUT * ,CONPORT
  ZAC
  SACL *
  OUT * ,CONPORT
  B  REPTST
  RET

;**-  TSTADC:  TEST ADC SELECT LINES
.TSTADC  LRLK  AR7,ADC1
  LARP  7
  RPTK  3
  LAC *
  B  TSTADC

;**-  TSTXRAM:
.TSTXRAM
  LRLK  AR0,XRAM
  LRLK  AR1,8191
  LARP  0
NXTADDR  LARK  >AAAA
  SACL *
  SUB *
  BNZ  XRAMERR
  LALK  >5555
  SACL *
  SUB *
  BNZ  XRAMERR
  MAR  *+,1
  BANZ  NXTADDR,*,0
  ZAC
XRAMERR  SACL  DATA
  WRPORT  DATA
  RET

;**-  TSTROM:
.TSTROM  LARK  AR1,31
  LALK  PROM
  LRLK  AR0,XRAM
  LARP  0
NXTBLK  RPTK  255
  TBLR *
  LARP  1
  BANZ  NXTBLK,*,0

  LDPK  0
  LRLK  AR0,XRAM
  NXTBYTE  LAC  **
RPTK 7
ROR
SACL DATA
WRPORT DATA
RPTK 7
ROL
SACL DATA
WRPORT DATA
B NXTBYTE:

END
The file contains some commonly used macros.

**INC: INCREMENT ROUTINE**

Increment "PTR" and test its within limits.

```
PTR = PTR + 1
IF PTR > END THEN PTR = START
```

```
INC MACRO PTR,START,END
LOCAL INCEND
LAC PTR
ADDK 1
SACTL PTR
SBLK END
BLEZ INCEND
LALK START
SACL PTR
INCEND MEND
```

**DEC: DECREMENT ROUTINE**

Decrement "PTR" and test its within limits.

```
PTR = PTR - 1
IF PTR < START THEN PTR = END
```

```
DEC MACRO PTR,START,END
LOCAL DECEND
LAC PTR
SUBK 1
SACL PTR
SBLK START
BGEZ DECEND
LALK END
SACL PTR
DECEND MEND
```

**SET: SET BIT ROUTINE**

```
SET MACRO LOC,BITPOS
LAC LOC
ORK I,BITPOS
SACTL LOC
MEND
```

**RESET: RESET BIT ROUTINE**

```
RESET MACRO LOC,BITPOS
LAC LOC
ORK I,BITPOS
XORK I,BITPOS
SACTL LOC
MEND
```

**BIT: TEST BIT ROUTINE**

```
BIT MACRO BITPOS,MEM
LAC BIT,BITPOS
AND MEM
MEND
```

**SRL: SHIFT RIGHT LOGICAL**

```
SRL MACRO MEM,XPLACES
LAC MEM,16-XPLACES
SACH MEM
MEND
```
Appendix C

******************************************************************************
;**-  SAVE: SAVE CURRENT PARAMETERS WITHOUT USING STACK
;  USES DATA PAGE 0

STORE MACRO
  SST  STATUS ;SAVE STATUS REG. ST0
  SST1 STATUS1 ;SAVE STATUS REG. ST1
  LDPK 0 ;CHANGE TO DATA PAGE 0
  SACL ACCL ;SAVE ACCUMULATOR
  SACH ACCH
MEND

******************************************************************************
;**-  RESTORE: RESTORES SAVED PARAMETERS

RESTORE MACRO
  LDPK 0 ;GOTO DATA PAGE 1 TO RESTORE PRE-
  ZALH ACCH ;INTERUPT CONDITIONS
  ADDS ACCL ;RESTORE ACCUMULATOR
  LST  STATUS ;RESTORE ST0
  LST1 STATUS1 ;RESTORE ST1
MEND

******************************************************************************
;**-  DELAY1:  List DELAY * ACC

DELAY1 MACRO ;List DELAY LOOP UNTIL ACC = 0
  LOCAL DELJ1
  DELJ1 SUBK 1
  RPTK 4
  NOP
  BNZ DELJ1
MEND

******************************************************************************
;**-  TSTAPE: TEST TAPE IS MOVING

TSTAPE MACRO DEST
  LOCAL CHANGED,NOTEND,NOTST
  LAC BIT,4
  AND CHANGE
  BNZ CHANGED

  BANZ NOTST
  LAC MOVING
  SUB ONE
  BGZ NOTEND
  CALL STOP
  LAC ONE,10
  SACL MOVING
  B DEST

CHANGED LAC ONE,10
NOTEND SACL MOVING
NOTST MEND

******************************************************************************
;**-  SELFUNC: SELECT A GIVEN FUNCTION

SELFUNC MACRO FNUM,MASK,ROUTINE
  LOCAL NOTSEL
  LACK >FNUM
  SACL TEMP
  LACK >MASK
  AND FUNC
  SUB TEMP
  BNZ NOTSEL
CALL ROUTINE
NOTSEL MEND

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;**- NRZI: CONVERT ACC TO NRZI FORMAT
NRZI MACRO
XOR LAST
SACL LAST
MEND
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

; ROUTINES TO COMMUNICATE TO EXTERNAL DEVICES VIA PARALLEL PORT
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;**- REPORT: READ PARALLEL PORT

REPORT MACRO ;READ PARALLEL PORT
LOCAL STRBLO,STRBHI,STRBLO2,STRBHI2
SXF ;SET BUSY
STRBLO BIOZ STRBLO ;WAIT FOR STROBE HIGH
RXF ;NOT BUSY
STRBHI BIOZ STRBLO2 ;WAIT FOR STROBE LOW
B STRBHI
STRBLO2 IN PDATA,PARPORT ;GET DATA FROM PARALLEL PORT
SXF ;SET BUSY LINE
LACK >FF AND PDATA ;ONLY 8 BIT DATA SO ZERO OTHER BITS
SACL PDATA
STRBHI2 BIOZ STRBHI2 ;WAIT FOR STROBE TO GO HIGH
MEND
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;**- WRPORT: WRITE TO PARALLEL PORT (HOST COMPUTER)

WRPORT MACRO PDATA ;READ PARALLEL PORT
LOCAL BSYLO,BSYHI,NOTBSY
SXF ;SET STROBE LINE HIGH
BSYHI BIOZ NOTBSY
B BSYHI
NOTBSY OUT PDATA,PARPORT ;SEND DATA TO PARALLEL PORT
RXF ;SET STROBE LINE LOW
BSYLO BIOZ BSYLO ;WAIT FOR HOST TO GET DATA AND BECOME BUSY
SXF ;SET STROBE LINE HIGH
MEND
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;**- GETINP: GET INPUT FORM HOST COMPUTER

GETINP MACRO INPUT ;GET INPUT FROM PARALLEL PORT
LOCAL STRBER,STRBHI,NXTRY,TSTFLG
SET CONTROL,13
SET CONTROL,7
OUT CONTROL,LINES ;CONFIG PARALLEL PORT AS BUSY INPUT
RESET FLAG,1
NXTRY BIT 7,CONTROL
BZ STRBHI
IN DATA,TAPE
LACK >80 ;TEST STROBE LINE HIGH
AND DATA
BZ STRBER
RESET CONTROL,7
OUT CONTROL,LINES ;SET BUSY LINE LOW

STRBHI IN DATA,TAPE
LACK >80 ;WAIT FOR STRB TO GO LOW
AND DATA
BNZ STRBER
IN INPUT,PARPORT
SET FLAG,1
SET CONTROL,7
OUT CONTROL,LINES
LACK >FF
AND INPUT
SACL INPUT

;ONLY 8 BIT DATA SO ZERO OTHER BITS

STRBER LAC DATA
XOR OLDATA
SACL CHANGE
LAC DATA
SACL OLDATA
TSTAPE TSTFLG
TSTFLG BIT 1,FLAG
BZ NXTRY
MEND

;********************************************************************
;**- OUTSIG: SEND AN OUTPUT SIGNAL
OUTSIG MACRO
LAC ONE
XOR CONTROL
SACL CONTROL
OUT CONTROL,CONPORT
MEND

;********************************************************************
;**- ERROR: SEND AN ERROR MESSAGE TO HOST
ERROR MACRO ERRNO
LACK >FF
SACL TEMP
WRPORT TEMP
LACK >00
SACL TEMP
WRPORT TEMP
LACK ERRNO
SACL TEMP
WRPORT TEMP
REPORT TEMP
B START
MEND

;********************************************************************
Appendix D

Description of State Transition Matrix and Finite State Transition Diagrams for Run-Length Limited Codes
**State Transition Matrix Description**

The FSTD, illustrated in Figure 1, is a directed graph with \( k + 1 \) states, any path through the FSTD defines an allowed \((d, k)\) sequence. Since the graph is directed, each edge can only be transversed in the direction indicated by the arrow. Edges that start and end in the same state, termed self loops, are permitted. For instance a \( d = 0 \) constraint would be represented as shown in Figure 2. The adjacency or connection matrix is given by the \((k + 1) \times (k + 1)\) array, \( A \), with entries \( a_{ij} \) that represent the number of paths from state \( i \) to state \( j \).

![Figure 1 Finite State Transition Diagram for a \((d,k)\) sequence](image1)

![Figure 2 FSTD for \(d=0\) constraint](image2)
As an illustration, the following example displays the connection matrix for the $(d,k) = (1,3)$ constraints:

The $n$ step state-transition matrix, $A^n$, has $ij$ entries that give the number of distinct sequences from state $i$ to state $j$ that are $n$ bits long.
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