The Drift Angle Theory Applied To Ship Manoeuvring Models

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University of Plymouth

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The Drift Angle Theory Applied To Ship Manoeuvring Models.

by

Matthew Paul Russell BSc.

A thesis submitted to the University of Plymouth in partial fulfilment for the degree of

DOCTOR OF PHILOSOPHY

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Faculty of Science

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Matthew Paul Russell BSc.

ABSTRACT.

A marine vehicle manoeuvring model is concerned with the ability to simulate the status of a vehicle to various demanded controls on a digital computer. Such models have both shore based and sea going applications that are beneficial to the mariner, enhance safety of life at sea and aid in protecting the marine environment. The mathematical representation of marine vehicles has generally been conducted by the measurement of the forces and moments that are experienced by a vehicle, in terms of a series of numbers collectively known as hydrodynamic coefficients. This has resulted in the non-linear force modular model which is considered to be the most accurate and versatile mathematical modelling technique.

This thesis presents the results from research conducted into the construction of an accurate mathematical model of a patrol craft Picket Boat Nine. The non-linear force modular modelling technique was initially adopted. The required hydrodynamic coefficients were evaluated by the use of full scale sea trials, scale model testing techniques and by semi-empirical methodologies; by the installation of a towing tank, a data monitoring and acquisition system onboard Picket Boat Nine.

An alternative new method for mathematically describing marine vehicles has also been developed based upon the drift angle theory. The existence and magnitude of the drift angle has been transformed into a set of hydrodynamic curves that mathematically represent a marine vehicle’s manoeuvrability and into a method of determining the track history of a marine vehicle when underway. These two components have been developed into a new form of mathematical model. This new approach to mathematical modelling has been tested by full scale sea trials in Picket Boat Nine and with comparison to a force modular model that demonstrates the stature and potential of this method. The results indicate that further research is required to include external disturbances and to prove its validity to other marine vehicles.
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To Sandra.
AUTHOR'S DECLARATION

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award.

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Date: 18th April 1994
Signed: Matthew_Dell
Chapter One.

MARINE VEHICLE SIMULATION BY MATHEMATICAL MODELLING.

1.1 INTRODUCTION

The digital computer and the advances in the understanding of the hydrodynamics of marine vehicles and structures that have occurred over the last few decades have enabled mathematical modelling techniques to be applied in simulation. This thesis considers computer simulation of marine vessels for shore based and sea going applications designed to give benefits for the whole maritime industry. This following chapter describes the requirement for simulation of marine vehicles and the application to which they have been deployed.

1.2 THE NEED FOR SIMULATION.

The automation which has arisen from an ever increasing need for efficient and economical operations has led to a general reduction in the number of crew required to man a vessel, Tebay (1990). Since it is cost effective to install a 'machine'; and such equipment can do the tasks expected of two or three people in any one twenty four hour period, modern ships are now becoming more sophisticated in terms of automation, with a result that fewer man hours are spent at sea. Further to this, during the sixties and seventies, there was a large increase in the number of commercial vessels navigating the waters with the gross tonnage steadily increasing, Frankel (1987). These trends have recently reversed due to the economic climate and there no longer exists the need for large Naval forces due to improving political relations. However, there are now more pleasure and luxury craft in operation than before and so there are probably more vessels navigating the waters of the Earth than in the past. This leads to congestion of the waterways and an increase in the likelihood of a catastrophic accident by collision or grounding. A result of the above mentioned
factors and the catastrophic disasters that have occurred, as described by Cahill (1990); Torrey Canyon 1976 (stranded due to human error in trying to make the tide); Aegean Captain and Atlantic Empress 1979 (two VLCC's colliding resulting in the largest oil spill in history); European Gateway 1980 (two vehicle ferries collided with each other in congested waters); Exxon Valdez 1989 (stranding attributed to human error as execution of orders to alter course was unaccountably delayed). There has been a general awareness for the need to quantify the manoeuvrability of large marine vessels and reduce human error.

Over the past two decades national legislation has been introduced on a regional basis to try and reduce the likelihood of disasters occurring. This has resulted in the International Maritime Organisation (1985) introducing an internationally agreed standard for all new gas and chemical carriers and all vessels whose length is greater than one hundred metres. These standards require that every vessel within these categories should be able to:

- Stop - in a tactical diameter of less than five ship lengths and the distance of advance should not exceed four vessel lengths.
- Possess the Yaw checking ability to conduct a 10° - 10° zig-zag where the first overshoot does not exceed 15 degrees and the second and third overshoots do not exceed 20 degrees. Whilst for the 20° - 20° zig-zag the first overshoot should not exceed 25 degrees.
- Possess an initial turning ability such that when 10 degrees of rudder is applied to both port and starboard, the vessel should have changed direction by 10 degrees before two and a half ship lengths have been travelled.
- Possess a course keeping ability so that no more than 30 degrees of phase advance should be required of the helm to maintain an adequate course.

Vessels' stopping and backing ability, performance during an accelerated turn and the rate of application of rudder, which are also relevant for the assessment of a ships manoeuvrability, were discussed but no recommendations were passed.

These guide-lines for new vessels therefore require manoeuvrability assessment techniques during the design stage. At present this can only be assessed by
the use of scale model testing techniques or full scale trials once the vessel is constructed and sea trails conducted. A vessel’s manoeuvring characteristics cannot be determined to a sufficient accuracy from basic design parameters alone.

An added benefit, when these guide-lines are enforced and vessels have been assessed during the acceptance sea trials is that there will be more manoeuvrability data than presently available. There exists to date only a few readily available data sets concerning ships manoeuvring characteristics. Two in particular are used by most researchers in the field: sea trials of the Mariner class tanker USS Compass Island, Morse and Price (1961); and, the ESSO Osaka, Crane (1979), of which the ESSO Osaka trials, conducted in the Gulf of Mexico in 1979 provides the most comprehensive information with data related to both deep and shallow water manoeuvres.

The Torrey Canyon and Exxon Valdez maritime disasters, attributed to human error, have led to the need for extra training of crew at a time when manning levels and the hours available at sea are in decline. The use of nautical simulators for training has over the past ten years has progressed to a point where it is becoming a prerequisite for seagoing certiﬁates of competency. The most sophisticated bridge and engine room simulators are constructed in the shape and form of an actual vessel, they permit day and night sailing via three dimensional projection to the visual horizon. The bridge and engine room equipment displays accordingly the state of the vessel as derived from a mathematical model. This enables the trainee to take command with the simulator responding to and displaying the vessel’s response to the given orders. Further to basic requirements, specialisation upon specific equipment is also possible according to a MARIN Report (1991).

1.3 SIMULATION APPLICATIONS.

Mathematical modelling of marine vessels can, in this context, be separated into two categories even though similar methodologies are involved:

1. Shore based nautical simulators for design and training,
2. Sea going applications to improve efficiency of shipping operations.

The use of shore based nautical simulators and trainers has become a commercially viable industry over the past two decades with applications stretching to all aspects of the maritime industry and are particularly important in the following activities:

- training of pilots and dual purpose bridge crews when there is a general reduction in the available number of hours at sea. The scope, to which these facilities have risen, is represented by the fact that most industrial countries not only accept nautical simulator training as a beneficial instructional aid but also require a minimum attendance period at the helm of a simulator during training. The degree of simulator training, accepted as valid, varies with the sophistication of the available simulator, but in most advanced institutes simulator training constitutes approximately half of the hands on experience required, with the other half obtained from actual sea going voyages, Spaans (1992);

- port, channel, fairway and waterway design. This allows a greater throughput of vessels as their requirements are known to a greater degree permitting larger vessels to penetrate further inland in restricted waters, Daggett (1992);

- registration of ships by classification societies. This enables evidence of a vessel's superior handling capabilities for which a higher grade of classification may be awarded with economical benefits for the owner. A policy incorporating this procedure has been adopted by Lloyd's Register and is known as the "Provisional Rules for the Classification of Ships' Manoeuvring Capability". This policy involves the use of full scale trials to validate the use of a simulator to determine the manoeuvrability for the construction of a wheel house poster containing manoeuvrability data for the vessel under various conditions. If the vessel complies with the requirements then it is eligible to be assigned the "LMA" (Lloyd's Manoeuvring Assessment) notation, Mikelis (1987);

- assessment of ship manoeuvrability at the design stage so that the required performances are ascertained before the lines plans are committed to fabrication, Marinussen and Linnerud (1987).

The sea going applications to which the ability to mathematically model and simulate a marine vessel’s response to various disturbances (wind, current and tide) and control inputs (demanded rudder and engine speed) accurately and so determine the vessels response to such inputs, is currently being examined in the following areas:
• autopilot and control to reduce the number of crew required for a twenty four hour vigilance, Burns (1984);

• collision avoidance; by predicting the most appropriate response based upon the rules of the road when in the vicinity of other vessels and land, Blackwell, Rangachari and Stockel (1991);

• electronic Chart Display and Information System, (ECDIS), where the vessel's track and course are plotted electronically on a digital chart for display and recorded, Atkinson (1991);

• the integrated bridge system that permits all the information available for the watch keeper to be processed, removing all the irrelevant data, and thus producing only the data which is specifically required for the task to be conducted, Chudley (1991);

• navigation, most navigational aids at present are not sufficiently accurate and suffer from daily fluctuations or do not give total daily or global cover. By using all the navigational aids available, to obtain various differing fixes for the same position a best estimate of position can be deduced from selected filtering of all the data, Miller (1990).

These applications are at present being researched by various institutes throughout the world who are endeavouring to produce economically viable and reliable systems that will ensure safety of life at sea and enhance protection of the marine environment.

1.4 MODELLING TECHNIQUES EMPLOYED TO DATE.

To represent the vessel mathematically in a manner suitable for the above applications the vessel's manoeuvring characteristics are described mathematically as a set of related equations for each motion using dimensionless hydrodynamic coefficients to describe the vessels unique manoeuvring characteristics. This is conducted within a computerised mathematical model that replicates and simulates the parent vessels' reactions to the inputs of demanded speed and rudder angle along with an option to accommodate the environmental disturbances such as wind, current, tide and sea state.

To date four different methods of mathematically modelling marine vessels have been explored. These four models have been well documented by Chudley,
Dove and Tapp (1989) and are: The Holistic, Force, Input-Output (Transfer function) and Modular approach, with the modular model being preferred for this programme of research over the other three for its versatility, ease of modification and implementation to the applications already mentioned above.

The modular approach considers a vessel to be composed of individual and integrated components from the hull, rudder, propeller and the environmental disturbances, that can be researched and modelled independently to the desired sophistication for a specific application.

1.5 AIMS OF THE PROGRAMME OF RESEARCH.

The primary objective of this programme of research has been the development of an accurate mathematical model of a twelve metre vessel, Picket Boat Nine (main particulars are detailed in Appendix A). This can then be used to enhance and authenticate the applications that were described previously using Picket Boat Nine for corroborative full scale sea trials. The modular modelling approach has been followed with the research concentrating on the hull component and the manner in which a hull form may be mathematically modelled. This is conducted by the use of a set of numbers, known as hydrodynamic coefficients. These are based upon the measurement of a marine vehicle's basic parameters; the magnitude of the forces in action and, the motions in terms of velocities, that a marine vehicle experiences. Methodologies for determining the properties and values attributable to these hydrodynamic coefficients are determined from; full scale sea trials; physical scale model testing techniques; mathematical and semi-empirical methods.

This thesis is separated into two parts. Part One is structured around the establishment of a modular model and the current methodologies employed to determine the hydrodynamic coefficients, that this modelling technique requires to describe the hull. Part Two then proposes an alternative method to this, based upon the drift angle and the drift angle theory.
In endeavouring to develop an accurate mathematical model of Picket Boat Nine, the following objectives have been undertaken in Part One:

- the identification of the required hydrodynamic coefficients for an accurate force non-linear mathematical modular model of the full scale vessel - Picket Boat Nine;
- the identity of the methodologies and techniques required for the evaluation of the hydrodynamic coefficients from full scale sea trials, scale model testing techniques, mathematical and semi-empirical methods;
- the establishment of a physical scale model environment suitable for the evaluation of these hydrodynamic coefficients;
- the evaluation of the surge hydrodynamic coefficients from the methodologies available and an assessment of the measurement of force and the force modelling technique to describe a marine vehicle.

The results obtained from the above list of objectives have indicated several discrepancies with this form of mathematical modelling and with the use of forces to describe marine vehicles that has resulted in the development of the drift angle theory, that forms the basis of Part Two, where the following have been undertaken:

- the development of the drift angle theory, a new method of quantifying and mathematically describing a marine vehicle, based upon the existence of the drift angle, and its application to marine vehicles;
- the development of a method of determining the track history of a marine vehicle utilising the drift angle theory and based upon basic vessel parameters;
- the development of a mathematical model based upon the drift angle theory, utilising the method of determining a marine vehicle's track history;
- the construction and implementation of a mathematical model of Picket Boat Nine based upon this new form of modelling technique and validation by conducting full scale sea trials with this vessel.

Chapter two introduces the force modular modelling technique and describes each of the component modules, in terms of hydrodynamic coefficients that are required to describe a marine vehicle. The theory that these hydrodynamic coefficients are based upon and the methods currently available for their identification, are detailed in chapter three. These identification methods include
conducting full scale sea trials; physical scale model testing techniques; and employing semi-empirical and mathematical methodologies so that sufficient information about a marine vehicle can be determined.

The surge velocity hydrodynamic coefficients that describe mathematically the relationship that exists between a vehicle's forward velocity and the force required to produce this motion have been determined for Picket Boat Nine as described in chapter four. The values attributable to these hydrodynamic coefficients are determined by measuring the resistance to motion for a range of speeds and describing the relationship between these two parameters mathematically. This has been conducted by the use of the scale model test facility at Britannia Royal Naval College (the installation of which is described in Appendix B) and by conducting full scale sea trials in Picket Boat Nine. Trials include: measured mile and bollard pull manoeuvres; and, towing Picket Boat Nine behind another vessel. The results from these experiments have indicated several discrepancies that exist with this form of mathematical modelling, which are discussed in chapter five.

Part Two, reviews the manoeuvrability of marine vehicle's in chapter six, with the view to indicating the significance of the centre of pressure and the existence of the drift angle, that are produced by the forces that are in action upon a marine vehicle. Chapter seven introduces the drift angle theory where the existence of the drift angle is related to the magnitude to the yaw rate of a marine vehicle. The drift angle theory is then employed to describe the manoeuvrability of a marine vehicle in terms of its turning circle characteristics, that demonstrates its validity. A method of determining a marine vehicle's track history is then developed as an application of the drift angle theory in chapter eight, where from a known point of origin a vehicle's path may be determined. This is supported by full scale sea trials conducted in Picket Boat Nine, where during a series of manoeuvres the position of this vessel was determined by the use of DECCA and GPS navigational systems and a Trisponder positioning system, that demonstrates the validity of this method.
The structure of a new form of mathematical model is then described in chapter nine. This utilises the method of determining the track history of a marine vehicle and a series of hydrodynamic curves, that are derived from applying the drift angle theory to describe a marine vehicle's manoeuvrability. Chapter ten proceeds by describing how a set of hydrodynamic curves are produced for Picket Boat Nine, this is achieved by analysing data obtained from full scale sea trials on this vessel. These hydrodynamic curves in association with the method for determining the track history of a marine vehicle are employed so that a mathematical model of Picket Boat Nine could be constructed. The validity of this new form of mathematical model is then described with reference to a set of full scale sea trials and compared to a modular model of this vessel, the results from these mathematical manoeuvres demonstrate the stature of this mathematical modelling technique.

The conclusion to this thesis discusses the results from this research with a view to indicating areas that require to be investigated further, so that this new form of mathematical model may be improved. The application of the drift angle theory is also discussed with reference to the determination of a marine vehicle's manoeuvrability from scale model testing techniques. This might permit the hydrodynamic curves to be produced, without the need for full scale sea trials to be conducted, that would allow more sophisticated and accurate mathematical models to be developed, by combining this new method with the modular modelling technique.
Chapter Two.

MATHEMATICAL MODELLING OF MARINE VESSELS USING THE MODULAR MODEL.

2.1 INTRODUCTION.

The principle mathematical elements and theories for the non-linear modular model for the representation of the six degrees of freedom that a marine vessel experiences are given in this chapter. These six degrees of freedom are then shown to be reduced to the three degrees of motion Surge, Sway and Yaw as the Heave, Pitch and Roll motions can be considered negligible. Then using a Taylor's multi-variate expansion upon each of the considered motions how the unique handling characteristics of any vessel may be represented mathematically.

2.2 THE CO-ORDINATE SYSTEM.

The six degrees of freedom that a marine vessel experiences are described with reference to three mutually perpendicular axes, (X, Y and N), that represent the vessel's longitudinal axis, lateral axis and vertical axis respectively, through which all the rotational (Pitch, Roll and Sway) and translational (Heave, Surge and Yaw) motions can be expressed. As shown in Figure 2.1, these axes form a consistent right-handed co-ordinate system where the origin is taken at the centre of gravity, G, and the intersection of these axes, C, is taken as the point of intersection between the mid point of the vessel's longitudinal, lateral axes and the water-line plane in the vertical axis.
The six degrees of freedom of a vessel.

Figure 2.1

A second fixed system of orthogonal axes $X_0, Y_0$ referred to as the Earth's co-ordinate system, forms a second right-handed system of axes as shown in Figure 2.2.

The vessel's axis relative to Earth's axis.

Figure 2.2

The $X_0$ axis is so aligned in the direction of True North to conform with standard navigational practice adopting the sign convention where the positive direction of all the variables shown on the diagram are as represented.
2.3 FUNDAMENTAL MATHEMATICS OF MODELLING.

As marine vessel has six degrees of freedom the total force experienced is a result of the surge, sway, and heave forces and the roll, yaw and pitch moments respectively and can be represented as:

\[ \text{Total force} = X + Y + Z \]
\[ \text{Total moment} = K + M + N \]

Equations 2.1.

A mathematical description of the hull may be obtained by considering forces and applying Newton's second law of motion:

\[ \frac{d}{dt} \text{momentum} \]
\[ \frac{d}{dt} \text{angular momentum} \]

Equations 2.2.

The application of this law of motion, for a vessel operating as a rigid body in a fluid medium to include the control and disturbance forces, was shown by Abkowitz [1969], and transformed into a set of Eulerian equations, (Equation set 2.3).

FORCE EQUATIONS

SURGE \[ X = m[u + qw - rv - x_o(q^2 + r^2) + y_o(pq - r)] + z_o(pr + \dot{q})] \]

SWAY \[ Y = m[\dot{v} + ru - pw - y_o(r^2 + p^2) + z_o(qr - \dot{p}) + x_o(qp + \dot{r})] \]

HEAVE \[ Z = m[\dot{w} + pv - qu - z_o(p^2 + q^2) + x_o(rp - \dot{q}) + y_o(rq + \dot{p})] \]

MOMENT EQUATIONS

ROLL \[ K = I_z\dot{\phi} + (I_z - I_y)qr + m[y_o(\dot{w} + pv - qu) - z_o(\dot{v} + ru + pw)] \]

PITCH \[ M = l_y\dot{\psi} + (l_x - l_y)rp + m[z_o(\dot{v} + qw - rv) - x_o(\dot{w} + pv + qu)] \]

YAW \[ N = l_y\dot{\psi} + (l_y - l_z)qr + m[x_o(\dot{v} + ru - pw) - y_o(\dot{u} + qw + rv)] \]

Equation set 2.3.
The involvement of all the six degrees of motion into a mathematical model has been conducted by Matthews (1984) and is required when considering shallow and restricted waters, but for deep and open waters it is common in practice to only consider the equations relating to the horizontal plane of the vessel, namely SURGE, SWAY and YAW as it can be assumed that a vessel maintains a degree of constant trim (heave and pitch) and possesses both longitudinal (pitch) and transverse stability (roll) that permit these motions to be considered small in comparison, thus reducing the Eulerian equations, Equation set 2.3 to:

\[
\begin{align*}
\text{SURGE } X &= m[\dot{u} - rv - r^2x_c] \\
\text{SWAY } Y &= m[\dot{v} + ru + rx_c] \\
\text{YAW } N &= I_r \dot{r} + mx_c(v + ru)
\end{align*}
\]

Equation set 2.4

Since the forces and moments acting upon the vessel are independent of its position and the vessel is assumed to be operating in the horizontal plane in deep waters and that the mass of fuel consumed is small in comparison to the mass of the vessel, these parameters may be considered constant, Tapp (1989). Further to this, if the origin of the ships orthogonal co-ordinate system, is selected to coincide with the vessel's longitudinal centre of mass of the vessel, then the Equation set 2.4 reduce to:

\[
\begin{align*}
\text{SURGE } X &= m\dot{u} - mrv \\
\text{SWAY } Y &= m\dot{v} + mru \\
\text{YAW } N &= I_r \dot{r}
\end{align*}
\]

Equation set 2.5

Where the forces and moments on the left hand side represent the complex hydrodynamic and aerodynamic reactions of the vessel to the control and disturbance forces that the vessel may experience which are assumed to be dependent on the dynamic fluid control variables \( n, \delta \) and the response variables \( u, v, r, \dot{u}, \dot{v}, \dot{r} \).
2.4 THE MODULAR MODEL.

Information concerning the modular model was first publicised by Ogawa and Kasai (1978) from the Mathematical Modelling Group (MMG) of the Society of Naval Architects of Japan. Since then many papers on the subject have either improved or refined this modelling technique to a state whereby the model is the most malleable and versatile for the purpose of being suitable for almost all ship designs, propulsive and control systems.

The modular modelling technique is structured so that the forces acting upon the hull, rudder and propeller and any external disturbances (wind, current, tide) are treated as self contained interactive modules. Therefore the complete system is modelled by combining all the individual components which can be separated, investigated and mathematically modelled separately.

The advantages of this technique are self evident. The model can be developed over a period of time into a sophisticated form by incorporating more modules containing specialised attributes as the application requires. Individual aspects or components of the model can be investigated separately without having to re-define the whole hydrodynamic model as major re-fits or modifications to the vessel occur. With the other main modelling techniques, namely the Hollisic, Force, Quasi-linear or Transfer function approach, that require the vessel to be treated as a complete entity, Chudley et al (1989). Further to this, once the modular model is developed for one vessel its application is also practical via module modification to either sister or similar ships. Another important advantage of this form of modelling is the ability to allow research into more elaborate models via the addition of component modules that will include shallow water effects, Dand (1987); unconventional hull forms, Hirano (1992); propulsive and steering systems Chudley (1991) which are at present being developed.

The governing factor at present is the ability to successfully determine the relationships between each component and then successfully integrate all the modules together so that they adhere as a single unit that represents the vessel's entirety.
2.5 APPLICATION OF MODELLING MATHEMATICS TO A MODULAR MODEL.

The reduced Eulerian equations, Burns (1984), for the modular model are generally expressed in the form:

\[ X = \mu - m_r v = X_H + X_p + X_R + X_E \]
\[ Y = m_r v + m_r u = Y_H + Y_p + Y_R + Y_E \]
\[ N = I_r = N_H + N_p + N_R + N_E \]

Equation set 2.6

Where the suffixes H, P, R, E denote the hull, propeller, rudder and the external forces respectively. These modules can subsequently be subdivided into smaller modules as required. For example, the hull of a tanker that has a bulbous bow fitted may result in the hull and bulbous bow being modelled separately yet being combined as a single module concerning the hull. Further to this if bow and stern thrusters are fitted extra modules may be included.

2.6 APPLICATION OF THE TAYLOR'S MULTI-VARIATE EXPANSION TO THE EULERIAN SET OF EQUATIONS.

As digital computers cannot integrate or differentiate to date an approximation is employed in the form of a series expansion. The most favoured for this purpose is the Taylor's multi-variate expansion, Abkowitz (1969) of the form:

\[ f(x,y) = e^{\Delta x x + \Delta y y} f(x_0, y_0) \]

which can be expanded to:

\[ f(x,y) = f(x_1, y_1) + \Delta x \frac{\partial f(x,y)}{\partial x} + \Delta y \frac{\partial f(x,y)}{\partial y} + \Delta x^2 \frac{\partial^2 f(x,y)}{2! \partial x^2} + \Delta y^2 \frac{\partial^2 f(x,y)}{2! \partial y^2} + \ldots \]

Equation set 2.7

This form of expansion consists of the addition of the terms from zero to infinity which for practical purposes is not possible due to processing time required, therefore the expansion is truncated so as to only include the important terms. Research conducted by Burns (1984) has shown that for satisfactory results particular terms up to and including third order are required. While inclusion of the higher order
terms improves the accuracy, the increase in processing time does not warrant their use.

2.7 THE COMPONENTS OF EACH MODULE FROM THE FORCE MODULAR MODEL.

The implementation of the Taylor's multi-variate expansion for each direction of motion was conducted by Abkowitz (1969) and has been improved by the inclusion of some of the non-linear terms up to and including third order, that has shown to extend the accuracy of the mathematical model, Burns (1984).

The following shorthand notation is introduced for the velocity and acceleration terms, where \( v, u, r \) are the velocity terms for their respective axis and \( \dot{v}, \dot{u}, \dot{r} \) are the symbols used for the acceleration derivatives of these terms representing:

\[
\dot{v} = \frac{dv}{dt} \\
\dot{u} = \frac{du}{dt} \\
\dot{r} = \frac{dr}{dt}
\]

And where the following standard notation is used to represent the partial derivatives from the Taylor's multi-variate expansion:

\[
X_u = \frac{\partial X}{\partial u} \\
X_{\nu} = \frac{\partial^2 X}{2 \cdot \partial \nu \partial t} \\
Y_{\nu v} = \frac{\partial^2 Y}{2 \cdot \partial \nu \partial v^2} \\
Y_\nu = \frac{\partial Y}{\partial \nu}
\]

These partial derivatives represent the relationship between the hydrodynamic and aerodynamic forces or moments that the vessel experiences and the perturbation variables. They are referred to as the dimensionalised hydrodynamic coefficients that enable the different characteristics of every vessel to be represented mathematically as no two vessels have exactly the same handling characteristics.
2.7.1 The Hull Module.

The Taylor's multi-variate expansion for the hull forces and moments, Equation set 2.6, are generally expressed by the following equations as described by Oltmann and Sharma (1984):

\[
\begin{align*}
X_H &= X_\ddot{u} + X_\nu v + X_\nu v^2 + \frac{u}{|u|} X_\nu r^2 + R_H \\
Y_H &= Y_\ddot{v} + Y_\nu r + Y_\nu r + Y_\nu v^3 + Y_\nu r v^2 \\
N_H &= N_\ddot{r} + N_\nu \dot{v} + N_\nu r + \frac{u}{|u|} N_\nu v + N_\nu v^3 + N_\nu r v^2
\end{align*}
\]

Equation set 2.8

where the term \( R_H \) from the surge hull module represents the ship's longitudinal resistance to motion and is modelled by the following:

\[
R_H = X_u u + \frac{u}{|u|} X_u u^2 + X_u u^3
\]

Equation 2.9

The multiplier \( \frac{u}{|u|} \) has been included to correct the sign of the derivatives during the astern motion of the vessel.

2.7.2 The Propeller Module.

The properties of the propeller module were shown by Oltmann and Sharma (1984) to depend upon the physical parameters (pitch, blade area, the number of blades per propeller and the distance from the centre of gravity), the number of propellers in situ and the manner in which they rotate:

A single right or left handed propeller

Or two inward or outward rotating propellers

To resolve the forces and moments resulting from the propeller into their component surge, sway and yaw motions that the vessel experiences. The thrust coefficient \( C_T \) is implemented for the whole range of hydrodynamic advance angles as published by Oltmann et al (1984) and Mikelis (1985). The more conventional
methods of modelling propellers using the advance coefficient $J$ and the thrust and torque coefficients $K_T$ and $K_Q$ are unsatisfactory as they do not represent the breaking force that exists when the propeller revolution rate tends to zero or when the propeller is set for reverse motion, for example during an emergency stop. The derivation of the individual propeller modules as described by Tapp (1989) for each of the three degrees of freedom and are summarised in the following sections:

For a single screw vessel.

Surge.

$$X_p = (1-t_p)T_p$$

where $(1-t_p)$ is the thrust deduction factor and in order to simulate ahead and astern motion two parameters respectively are required.

Sway.

$$Y_p = Y_{mn}n^2$$

Yaw.

$$NP = N_{mn}n^2$$

where $N_{mn} = Y_{mn} \frac{L}{2}$ and that it is assumed that the screw is located at a distance $\frac{L}{2}$ from the LCB.

For a twin screw vessel.

Surge.

The surge module is treated as consisting of two single screws and the thrust developed by each propeller is determined separately, as mentioned for the single screw ship, then combined.

$$T_{total} = T_{portcrew} + T_{starboardscrew}$$

$$X_p = [(1-t_p)T_{pportcrew} + (1-t_p)T_{pstarboardscrew}]$$

Page 18.
Sway.

As with most twin screw vessels the propellers are set to either rotate inwards or outwards thus removing the paddle wheel effect that occurs with single screws and twin screws set so that one rotates outwards whilst the other rotates inwards inducing a large sway force to either port or starboard. In order to model the propeller each of its five conditions, whether it be full ahead, full astern, port ahead and starboard astern or vice versa, have to be accommodated as follows:

Both ahead at equal rotations rates

\[ Y_p = Y_m n_1^2 - Y_m n_2^2 \]

If \( n_1 = n_2 \): \( Y_p = 0 \)

Starboard propeller stopped:

\[ Y_p = Y_m n_1^2 \]

Vessel will turn to port.

Port propeller stopped:

\[ Y_p = Y_m n_2^2 \]

Vessel will turn to starboard.

Port propeller ahead; starboard propeller astern.

\[ Y_p = Y_m n_1^2 - (-Y_m) n_2^2 \]

If \( n_2 = 0.5n_1 \),

\[ Y_p = 1.25Y_m n_1^2 \]

Vessel will turn to port at a faster rate.

Port propeller astern; starboard propeller ahead.

\[ Y_p = -Y_m n_1^2 - Y_m n_2^2 \]

If \( n_1 = 0.5n_2 \)
Y_t = -1.25Y_{aw}n_2^2

Vessel will turn to starboard at a faster rate.

Yaw.

The arrangement of most twin screw vessels is such that there is no paddle wheel effect as the yawing moment of each individual propeller counteracts the other, therefore the yawing coefficient N_p is equal to zero.

2.7.3 The Rudder Module.

The rudder as a control mechanism operates in two complex hydrodynamic environments: in the slip-stream of the propeller and due to hull geometry the inflow to the rudder is different to that around the hull. To this extent the propeller can not be treated as a complete separate module but as a module directly related to the hull and propeller. As explained by Tapp (1989) the forces and moments induced upon the vessel due to rudder action, derived from Hirano, Takashina and Moriya (1987), using the sign convention, are as follows:

\[
X_R = (1-t_R) F_N \sin(\delta)
\]

\[
Y_R = -(1-a_h) F_N \cos(\delta)
\]

\[
N_R = (1-a_h) F_N X_R \cos(\delta)
\]

a_h and t_R are correction factors from the open water tests to the behind hull condition, where a_h the sway hull/rudder interaction constant can be deduced from the form factor C_n and t_R the surge hull/rudder interaction constant can be estimated from the reduction in forward speed of the vessel when turning.

Where F_N is the normal force produced by the rudder, defined as:

\[
F_N = \frac{D}{6.13} \cdot \frac{6.13\lambda}{\lambda + 2.25} A_R \bar{u}r^2 \sin(\alpha_R)
\]
where $\lambda$ is the rudder aspect ratio, and (\(\alpha_r\)) is dependent upon the angle of rudder applied in relationship to the direction of water inflow, thus for negative rudder angles the attack is positive resulting in a starboard turn:

$$\alpha_r = \delta - \beta_r$$

$$\alpha_r = \delta - \arctan \left[ \frac{v}{u} + \frac{x_r f}{u} \right]$$

and for positive rudder angles the resulting turn is negative with a turn to port:

$$\alpha_r = \delta + \beta_r$$

$$\alpha_r = \delta + \arctan \left[ \frac{v}{u} + \frac{x_r f}{u} \right]$$

where $x_r$ is the distance of the LCG to the rudder centre of pressure.

2.7.4 Motion Due To Wind.

The effect of the external disturbance wind upon the superstructure and hull of a vessel was investigated by Isherwood (1983) and found to be dependent upon the projected area that the disturbance encounters, the relative air speed and its angle of incidence to the bow of the vessel, which are generally expressed for the three degrees of freedom in the form:

$$C_x = \frac{F_x}{\frac{1}{2} \rho_a U_{wr}^2 A_T}$$

$$C_y = \frac{F_y}{\frac{1}{2} \rho_a U_{wr}^2 A_L}$$

$$C_n = \frac{N}{\frac{1}{2} \rho_a U_{wr}^2 A_L A_{oa}}$$

where $F_x$ = the longitudinal component of the wind force

$\rho_a$ = the air density

$U_{wr}^2$ = the transverse projected area of the ship
The lateral component of the wind force is denoted as $F_y$.

The lateral projected area of the ship is $A_L$.

The overall length of the vessel is $L_{AO}$.

$N$ represents the wind induced moment about the midships.

Further to this, the above disturbance equations have been translated into the following, by Isherwood (1983):

\[
C_X = A_0 + A_1 \left( \frac{2A_L}{L_{OA}^2} \right) + A_2 \left( \frac{2A_T}{B^2} \right) + A_3 \left( \frac{2L_{OA}}{L_{OA}^2} \right) + A_4 \left( \frac{S}{L_{OA}} \right) + A_5 \left( \frac{C}{L_{OA}} \right) + A_6 M
\]

\[
C_Y = B_0 + B_1 \left( \frac{2A_L}{L_{OA}^2} \right) + B_2 \left( \frac{2A_T}{B^2} \right) + B_3 \left( \frac{2L_{OA}}{B} \right) + B_4 \left( \frac{S}{L_{OA}} \right) + B_5 \left( \frac{C}{L_{OA}} \right) + B_6 \left( \frac{A_{SS}}{L_{OA}} \right)
\]

\[
C_N = C_0 + C_1 \left( \frac{2A_L}{L_{OA}^2} \right) + C_2 \left( \frac{2A_T}{B^2} \right) + C_3 \left( \frac{2L_{OA}}{B} \right) + C_4 \left( \frac{S}{L_{OA}} \right) + C_5 \left( \frac{C}{L_{OA}} \right)
\]

Where: $B$ is the beam of the vessel

$A_{SS}$ is the lateral projected area of the superstructure.

$S$ is the length of the perimeter of lateral projection of the ship, excluding the water-line and slender bodies (masts and ventilators).

$C$ is the distance from the bow to the centroid of the projected area.

$M$ is the number of distinct groups of masts or kingposts in the lateral projection, that relies upon a data base for the constants $A_0$-$A_6$, $B_0$-$B_6$, $C_0$-$C_5$ for the relative wind angle of the bow, $q$. The independent variables of the for $\frac{2A_L}{L_{OA}^2}$ are also determined from a reference source as shown by Tapp (1989).
2.8 SUMMARY OF THE MODULAR MODEL.

The representation of the modules discussed in this chapter can be summarised as being either empirical formulations and expressions based upon vessel design parameters, the rudder and wind modules, or modules that are based upon actual vessel characteristics represented by non-dimensionalised hydrodynamic coefficients namely the hull and propeller modules. The latter two at present require physical observations and measurements to be conducted as there exist no conclusive empirical methods exist for their complete evaluation for all vessels.

The following chapters investigate the methodologies that are presently available for the evaluation of the hydrodynamic coefficients for the modular model and in particular for the hull module.
Chapter Three.

THE FORCE HYDRODYNAMIC COEFFICIENTS.

3.1 INTRODUCTION.

The partial velocity and acceleration derivatives from the Taylor’s multi-variate expansion are the constants of proportionality between the forces and moments that act upon the vessel (and the equilibrium and state). These are known collectively as the hydrodynamic coefficients and are used to represent a particular hull form. They are unique for any one vessel as they allow a vessel’s handling characteristics to be described mathematically by resolving the forces and moments that a vessel experiences into component contributions from each velocity and acceleration for each degree of freedom considered. This chapter begins by describing principle of the hydrodynamic coefficients and then proceeds by outlining the methodologies by which the values are attributed to the required hydrodynamic coefficients, by means of full scale sea trials and scale model testing techniques. This is then followed by a review of the semi-empirical and mathematical methodologies that have been proposed for their evaluation.

3.2 THE FORCE HYDRODYNAMIC COEFFICIENTS.

As a marine vessel is situated at the air and sea interface it possesses the ability to move in all the six degrees of freedom possible even though for simplicity it can be considered that heave and pitch are insignificant. Thus the motions that a vessel experiences are in general cross coupled.

- For example - during a standard turn to port the vessel heels to port initially due to the rudder force and generated moment about the centre of gravity
but then heels to starboard due to the hydrodynamic forces and moments that result from the execution of the turn. Thus yaw is coupled with roll.

To complicate the issue further environmental disturbing forces such as wind, current and tide also act upon the vessel and thus it is impossible to measure directly the separate component forces and moments that are experienced by the vessel due to the rudder and propeller action.

At present six basic methods exist for the observation of a vessel's manoeuvring characteristics:

1. System identification of the full scale vessel during standard manoeuvres, Abkowitz (1980).
5. Semi-empirical equations based upon design parameters, Clarke, Gelding and Hine (1982).

Of these the most accurate methods for the determination of the hydrodynamic coefficients is by captive tests. Physical measurements are taken from captive sea trials or scale model testing techniques, results are then verified by undertaking manoeuvres on the full scale vessel during sea trials. Even though some semi-empirical and mathematical evaluation techniques have been established they are only suitable for the vessels that these theories were derived from and not applicable to all vessels.
The captive methods for evaluation of the hydrodynamic coefficients is to identify the direction and magnitude of the forces and moments that are exerted upon the vessel as a result of the velocities and accelerations involved during a manoeuvre and to equate them together. This is conducted by measuring the force produced when the vessel is moving at a known speed and if this is repeated for all the speeds that the vessel operates over then the speed produced by a given force can be determined. This force could be for example the component from the propeller thrust, giving the surge force that is required to maintain a constant speed, the surge velocity. This form of relationship graphically shown results as a two dimensional plot of the measured Surge force (X) against a known surge velocity (u), Figure 3.1:

![Graphical relationship between surge force and surge velocity](image)

By determining the equation of the line produced the Surge velocity for any given propeller thrust can then be made. The accuracy of the equation depends mainly on the characteristics of the relationship between these two properties whether they are linear or non-linear and the ranges being considered. Small increases in propeller thrust may result in a small change in speed where the relationship between the two can be considered linear resulting in linear hydrodynamic coefficients of the form:

\[ u = mX \]

where \( m \) is the hydrodynamic coefficient \( X_u \) that has the dimensions of force and velocity.
When considering the complete range of speeds that the vessel operates over as shown in Figure 3.1, a polynomial equation is required which results in a greater number of hydrodynamic coefficients so that the relationship may be described accurately of the form:

\[ u = a_1 X + a_2 X^2 + a_3 X^3 + \ldots + a_n X^n \]

Where \( a_1, a_2, a_3 \) and \( a_n \) are the hydrodynamic coefficients \( X_u, X_{u\theta}, X_{\theta u} \ldots \ldots X_{n(u)} \) respectively.

By conducting similar measurements for all the forces and moments for the range of velocities and accelerations that the vessel possesses the hydrodynamic coefficients for each degree of freedom can be established. Further to this, by adding a third dimension, for example the sway velocity, then the cross-coupled hydrodynamic coefficients can be established which will represent in this case the contribution from the sway velocity to the reduction in surge velocity. By extending this to include further dimensions all the cross coupled-motions and the associated cross-coupled hydrodynamic coefficients for a particular degree of freedom can be resolved.

The only practical method available to date for separating and measuring these forces and moments along with their associated velocities and accelerations is via captive scale model testing techniques whereby a physically scaled model is tested in experimentally sterile conditions that are governed by the operator and where the model conducts standard manoeuvres at preset velocities and accelerations; details as given in section 3.5. This permits the separate forces and moments experienced by the model at particular velocities and accelerations to be measured directly, these can then be related to the full scale vessel.

Research into the manoeuvrability of marine vessels has led to the development of a range of open water manoeuvres that can be conducted with a full scale vessel and a series of scale model tests. These permit the hydrodynamic coefficients to be determined and are described in sections 3.4 and 3.5 respectively.
3.3 THE LINEAR AND NON-LINEAR HYDRODYNAMIC COEFFICIENTS REQUIRED FOR THE FORCE MODULAR MODEL FOR THE HULL.

The non-dimensionalised hydrodynamic coefficients that were identified in chapter two for the modular mathematical model can be separated into their respective Surge, Sway and Yaw motions. Further to this they can be divided into either being linear or non-linear and subdivided again into being either independent of all other motions or cross coupled with another motion. The significant hydrodynamic coefficients as identified by Burns (1984) are described in sections 3.3.1 to 3.3.3.

3.3.1 The Surge Hydrodynamic Coefficients For The Hull.

- Independent Linear Velocity Derivatives.
  • $X_u$, The linear component of the hydrodynamic drag force due to the Surge velocity ($u$).
- Cross Coupled Linear Velocity Derivatives.
  • $X_r$, Surge force produced by the action of the vessel during a turn that results in a drag force on the vessel produced by the sway (lateral) and yaw (rotational) velocities.
- Independent Non-Linear Velocity Derivatives.
  • $X_m$, Surge force that results from the yaw velocity.
  • $X_{mu}$, $X_{uu}$, Surge force required to propel the vessel with longitudinal velocity ($u$).
- Independent Linear Acceleration Derivatives.
  • $X_0$, Surge force required to produce the longitudinal acceleration ($u$).

3.3.2 The Sway Hydrodynamic Coefficients For The Hull.

- Independent Linear Velocity Derivatives.
  • $Y_v$, The hydrodynamic sway force that results from the sway velocity ($v$).
  • $Y_r$, The hydrodynamic sway force resulting from the yaw velocity ($r$).
- Independent Non-Linear Velocity Derivatives.
  • $Y_{vv}$, The hydrodynamic sway force due to the lateral velocity ($v$).
- Cross Coupled Non-Linear Velocity Derivatives.
  - $Y_{mv}$ The hydrodynamic sway force due to the lateral velocity ($v$) produced by the rotational velocity ($r$).

- Independent Linear Acceleration Derivatives.
  - $Y_{v}$ The hydrodynamic sway force required to produce a lateral acceleration ($v$).
  - $Y_{r}$ The hydrodynamic sway force produced by the rotational acceleration ($r$).

3.3.3 The Yaw Hydrodynamic Coefficients For The Hull.

- Independent Non-Linear Velocity Derivatives.
  - $N_{v}$ The hydrodynamic yawing moment that results from the sway velocity ($v$).
  - $N_{r}$ The hydrodynamic yawing moment resulting from the yaw velocity ($r$).

- Independent Non-Linear Velocity Derivatives.
  - $N_{vv}$ The hydrodynamic yawing moment due to the lateral velocity ($v$).

- Cross Coupled Non-Linear Velocity Derivatives.
  - $N_{mr}$ The hydrodynamic yawing moment due to the lateral velocity ($v$) produced by the rotational velocity ($r$).

- Independent Linear Acceleration Derivatives.
  - $N_{v}$ The hydrodynamic yawing moment required to produced by the lateral acceleration ($v$).
  - $N_{r}$ The hydrodynamic yawing moment produced by the rotational acceleration ($r$).

These non-dimensionalised hydrodynamic coefficients once evaluated and multiplied with their respective velocities and accelerations can then summed for each particular motion which gives a result for the total force exerted upon the hull in each particular degree of freedom.

The following three sections describe the practical full scale sea trials and scale model testing techniques, empirical and mathematical methodologies that exist for
the determination and evaluation of the dimensionalised hydrodynamic coefficients. These are required to be non-dimensionalised either by the "prime", Tapp (1989), or "bis", Norrbin (1971), notation systems which permits a marine vessel to be expressed mathematically, as described in chapter two and simulated for the applications discussed in chapter one.

### 3.4 FULL SCALE TRIALS FOR THE EVALUATION OF THE FORCE HYDRODYNAMIC COEFFICIENTS.

The following outlines a series of standard manoeuvres along with the observations required to gather sufficient information about a vessel's response during set manoeuvres that permit some of the hydrodynamic coefficients to be physically evaluated for use in the force mathematical model.

The evaluation of the hydrodynamic coefficients from manoeuvring trials, as described by Abkowitz (1980), requires the vessel to be equipped with an onboard monitoring and data acquisition system. That collects information concerning the demanded inputs of rudder settings and propeller shaft speed, along with the vessel's response to these inputs, namely position, speed through the water, heading and rate of change of heading and with the environmental conditions, wind current and tide. The equipment required for monitoring these variables independently is under maritime law carried by most ships, but a suitable data acquisition system, Mayo (1993) is not, therefore very little data is actually available. Furthermore, during the manoeuvres the vessel is effectively removed from service and is therefore not economically profitable for the owner.

The manoeuvres that can be conducted which are of importance for the evaluation and validation of the hydrodynamic coefficients are:

- The measured mile.
- The spiral manoeuvre.
• The inclining experiment.
• The hollard pull.
• The Kempf Zig-Zag manoeuvre.
• Turning circles.
• Emergency stops.
• Accelerated turns.

The first four from the above list have been directly related to the evaluation of the hydrodynamic coefficients, by Abkowitz (1969) and will be discussed in the following sections. The remaining manoeuvres permit the validation of the coefficients once incorporated into the mathematical model, the model should simulate the vessel’s response during these manoeuvres.

Ideally all the manoeuvres should be conducted under standard conditions in deep waters and under environmental conditions that do not exceed a Beaufort scale four as recommended by the IMO Interim Guide-lines for Estimating Manoeuvring Performance in Ship Design (1985). This approach of modelling does not attempt to include sea state, therefore the environmental conditions should be less than the equivalent sea state of a Beaufort scale four for a one hundred metre vessel.

3.4.1 The Measured Mile Manoeuvre.

This manoeuvre involves the vessel to be set with the rudder amidships and steaming along a measured distance 'mile' at various engine settings according to the ITTC Guide for Measured-Mile Trials (1969). The speed of the vessel, shaft speed and time to complete the distance are recorded. Runs are repeated immediately in the opposite direction so as to remove the environmental effects of tide and wind. By conducting a Bollard Pull (section 3.3.4) the thrust produced at a given engine speed can be determined where at constant velocity the thrust produced by the propellers is equal to the total resistance to motion for the vessel. From this data a plot of the total resistance against the speed of the vessel can be plotted, Figure 3.2.
The surge dimensional hydrodynamic coefficients are evaluated by fitting a cubic polynomial to this curve of the form as described by Burns (1984):

\[ Y = aX^3 + bX^2 + cX \]

\[ a = X_{\text{UUU}} \]
\[ b = X_{\text{UU}} \]
\[ c = X_{\text{U}} \]

The coefficients \((a,b,c)\) from the cubic polynomial are the surge hydrodynamic coefficients, which are probably the easiest to obtain from the full scale trials.

### 3.4.2 The Spiral Manoeuvre.

The spiral manoeuvre, Abkowitz (1969), is conducted by steaming the vessel in a straight line until constant velocity is attained and then putting the rudder hard to port. When the vessel is in steady state and constant angular velocity is obtained the rudder angle and the angular velocity are measured. Then the rudder angle is reduced by a decremental step of five degrees with similar measurements taken. This is performed for a range of rudder angles from hard to port to hard to starboard in decrements of five degrees or less with the vessel following paths as indicated in Figure 3.3.
From the data obtained a plot of rudder angle against angular velocity will result in one of two plots, either a plot for a directionally stable vessel, Figure 3.4, or for a directionally unstable vessel, Figure 3.5.

Rudder angle and angular velocity diagram for:

(A) Directionally stable vessel.  
(B) Directionally unstable vessel.

If the vessel is directionally stable then the following equation:

\[
\left( \frac{\partial r}{\partial \delta} \right)_{\delta=0} = \frac{N_v Y_\delta - Y_v N_\delta}{Y_v(N_t - mx_G u_0) - N_v(Y_t - mu_0)}
\]
will represent the gradient of the curve that passes through the origin and its components once resolved will give the hydrodynamic coefficients:

\[ Y_v - \text{hydrodynamic swaying force due to sway velocity} \]
\[ Y_y - \text{hydrodynamic swaying force due to yaw angular velocity} \]
\[ Y_\delta - \text{hydrodynamic swaying force due to rudder force} \]
\[ N_v - \text{hydrodynamic yawing moment due to sway velocity} \]
\[ N_y - \text{hydrodynamic yawing moment due to yaw angular velocity} \]
\[ N_\delta - \text{hydrodynamic yawing moment due to rudder force} \]

The spiral manoeuvre also presents a measure of the directional stability of the vessel as deviation from that of Figure 3.4 to Figure 3.5 indicates a degree of dynamic instability in that the vessel has different handling characteristics when turning to port or starboard therefore being unable to steer in a straight line when demanded.

3.4.3 The Inclining Experiment

The inclining experiment, Lewis ed. (1988) is a method of determining a vessel's static stability by determination of the metacentric height. It involves the application of a heeling moment (HM) to the vessel of known magnitude that causes the vessel to heel to an angle \( \theta \). The righting moment and the associated righting lever are then evaluated from the equation:

\[ W.d = HM = RM = \Delta GZ \]

Where:
\[ W - \text{The weight moved transversely across the deck.} \]
\[ d - \text{The distance that the weight is moved.} \]
\[ HM - \text{The heeling moment caused by moving the weight.} \]
\[ RM - \text{The righting moment caused by the vessel.} \]
\[ \Delta - \text{The displacement of the vessel.} \]
\[ GZ - \text{The righting lever.} \]
This is repeated for all angles of heel for port to starboard. A plot of righting lever against heel angle (GZ curve) result will produce one of two plots the first for a statically stable vessel, Figure 3.6 and the second for a statically unstable vessel, Figure 3.7, where the all values measured to starboard are positive and those to port are negative.

*Graphical representation of GZ curves for:*

(A) Statically stable vessel. (B) Statically unstable vessel.

Figure 3.6

Figure 3.7

Figure 3.6 represents the positive transverse static stability of a vessel where the gradient of an initial linear approximation to this curve is equal to the to product of the displacement ($A$) and the metacentric height ($GM$). As the criteria for static stability is a positive metacentric height, which has been proven analogous to the plot of rudder angle against angular velocity (Figure 3.4) gained from the spiral manoeuvre, Nomoto (1969). Then as the dynamic stability is dependent upon the static stability of a vessel the gradient of this linear approximation may be equated to the yaw and sway hydrodynamic coefficients due to the rudder angle, the sway velocity and the yaw angular velocity by the following equation:

$$
\left( \frac{-\partial \delta}{\partial r} \right)_{t=0} = \left[ \frac{Y_s(N_r - m_k u_0) - N_s(Y_r - m_k u_0)}{Y_s N_\delta - N_s Y_\delta} \right]
$$

Which is the inverse of the equation for that describes the linear approximation of the plot obtained from the results of the spiral manoeuvre as the axis are reversed.
3.4.4 The Bollard Pull.

The Bollard pull involves the vessel being physically restrained to a fixed rigid object via a cable with a stain gauge linked in series so as to measure the pull of the vessel at various demanded engine speeds, Isin (1987). The data obtained from the Bollard pull can then be graphically displayed as shown in Figure 3.8.

From this sea trial two observations are possible, the first being that the engines at similar engine settings create the same thrust and so the vessel does not suffer from being directionally unstable ie pull to one side. The second observation being the thrust delivered by the propellers, \( T_p \) at different engine speeds that permits the surge propeller module, \( X_p \) to be evaluated according to Harvald (1983), which is found to be equal to:

\[
X_p = (1-t_p)T_p
\]

where \( (1-t_p) \) is the thrust deduction coefficient deduced from:

\[
t_p = \left( \frac{T_p - R_T}{T_p} \right)
\]

and where \( R_T \) is the total resistance, which can be deduced from the measured mile manoeuvre (section 3.4.1) and from the paravane tests of section 3.10.

3.4.5 The Kempf Zig-Zag Manoeuvre.

The Kempf Zig-Zag manoeuvre involves the ship steaming at constant engine revolutions with the rudder amidships and then the rudder put to one side to a
predetermined angle. Then once the ship has responded accordingly, the rudder is put
to the other side to the same predetermined angle, Norrbin (1966). The vessel’s
heading is recorded and then plotted with the demanded rudder angle against time so
as to give two plots as shown in Figure 3.9 and Figure 3.10.

![Diagram of vessel's heading and rudder angle](image)

**Figure 3.9**

*The Kemf Zig-zag manoeuvre for a dynamically stable vessel.*

![Diagram of vessel's heading and rudder angle](image)

**Figure 3.10**

*The Kemf Zig-zag manoeuvre for a dynamically unstable vessel.*
This manoeuvre has not been to date related to the evaluation of the hydrodynamic coefficients, but permits the ability of a vessel to rectify motion to be examined over a period of time. Figure 3.9 illustrates a vessel which is capable of conducting similar manoeuvres or counter manoeuvres by turning to both port and starboard equally and is therefore dynamically stable according to Norrbin (1966). Whilst Figure 3.10 represents a dynamically unstable vessel that is not capable of producing similar manoeuvres to both port and starboard, resulting in the time to complete a counter manoeuvre increasing and with a steadily increasing percentage overshoot. This manoeuvre is particularly valuable in the validation of the mathematical model and its ability of the model to simulate the vessel's behaviour to these demanded rudder angles by demonstrating the same percentage overshoot as the full scale vessel.

3.4.6 The Turning Circle Manoeuvre.

The turning circle manoeuvre consists of steaming the vessel at constant approach speed then the rudder is set to a particular angle and is maintained whilst the vessel completes two full turns thus enabling environmental compensation to be considered and steady state turning to be established. The vessel's position is recorded as a series of X and Y co-ordinates and plotted accordingly in the x-y plane, from which the radius of turn can be determined. This is repeated incrementally for successive rudder angles from port to starboard and for a range of approach speeds.

This manoeuvre has been related to the steady state hydrodynamic coefficients according to Lewis ed. (1989) for a vessel when turning by the following equation:

$$ R = \frac{L}{\delta R} \left[ \frac{Y_v N_\delta - N_v (Y_\delta - \Delta)}{Y_v N_\delta - N_v Y_\delta} \right] $$

where $R$ is the radius of the turning circle and $L$ is the length of the vessel in question. The values obtained from this manoeuvre are only suitable for describing the vessel when conducting a turning circle manoeuvre.
3.4.7 The Emergency Stop.

The emergency stop manoeuvre requires the vessel to be set steaming on a straight course at constant velocity, with the data acquisition system commencing prior to the order to stop being given. The order to stop is given and the engines are either placed at zero revolution or at full astern and the rudder is either left amidships or placed hard over to port or starboard to turn the vessel and increase its resistance to motion thus reduce the stopping distance. The x-y co-ordinates for the stopping distance and time for the vessel to come to a complete halt are recorded and then plotted in an x-y plane. This procedure is repeated for all speeds of advance and in both directions so as to remove environmental effects. From the data obtained the directional stability of the vessel may be deduced as a directionally unstable vessel will veer either to port or starboard and similar plots should result for port and starboard for a directionally stable vessel. This manoeuvre may also be used to validate the mathematical model and its validity for this manoeuvre.

3.4.8 Full Scale Resistance To Motion Measurement.

Measurement of the full scale resistance to motion of a vessel can be conducted by towing the vessel in question behind another vessel via the use of a tow-line that is linked in series with it a strain gauge, Saunders (1957b). The strain gauge measures the force required to tow the vessel at a particular speed, by conducting this manoeuvre for the range of speeds that the vessel operates over a resistance curve can be plotted as shown in Figure 3.11.

![Graphical illustration of a resistance curve.](Figure 3.11)
This resistance curve can then be used to determine the surge velocity hydrodynamic coefficients as described in section 3.4.1.

3.5 SCALE MODEL TESTING TECHNIQUES FOR THE EVALUATION OF THE FORCE HYDRODYNAMIC COEFFICIENTS.

This form of investigation involves the use of geometrically, dynamically and kinematically similar scale models based upon the full scale parent vessel. The model is then tested under experimental conditions in various test facilities that include, towing tanks, circulating water channels, model basins, manoeuvring tanks, cavitation tunnels and wind tunnels. These scale models are linked to a monitoring and data acquisition system that allows various observations and measurements to be taken concerning the model's behaviour during the test.

The ability to physically restrain scale models and to force motions allows the six degrees of freedom experienced by a vessel to be separated, into their components and the contributing factors of these components to be investigated individually and resolved. For example, tests with and without the rudders attached to the model allows the bare hull resistance to be measured and the contribution from the rudders to the overall resistance to be evaluated.

3.5.1 The Use Of Towing Tanks For The Evaluation Of The Hydrodynamic Coefficients.

A towing tank is a large contained volume of water through which scale models are towed at various speeds from which observations and measurements about the model's behaviour are recorded. Strain gauges orientated with the model's surge (X), sway (Y) and yaw (N) motions permit measurements to be taken of the magnitude of the forces and moments that exist at various speeds. From analysis of these measurements the hydrodynamic coefficients may be evaluated for the mathematical model.
3.5.1.1 Towing the model longitudinally down the tank.

This test involves towing the model at various Froude numbers, speeds that are equivalent to that of the parent vessel, and in various states, with and without appendages. From the data obtained not only can the various components be separated but this test can also be used to validate the use of scale models with respect to that of the parent vessel in two ways. Firstly, the comparison of the measured resistance with that of the parent vessel at particular speeds, Saunders (1957b). Secondly, by comparing the surge hydrodynamic coefficients obtained from the model with those of the parent vessel which are obtained by the fitting a cubic polynomial to the resistance curve, as described in section 3.4.1.

\[
\text{Gradient} = aX^3 + bX^2 + cX
\]

where:
\[a = X_{UL}\]
\[b = X_{UU}\]
\[c = X_U\]

The resistance curve, Figure 3.12, is obtained by towing the model with its centre line orientated longitudinally down the tank, the model is connected to the towing mechanism (gantry) and a strain gauge that measures the longitudinal resistance to motion for a range of speeds.

![Graphical representation of a scale model resistance curve.](Figure 3.12)
3.5.1.2 Towing the model longitudinally down the tank with the rudders at an angle of attack.

This test is similar to that of the resistance test except that the rudder is set at various angles of attack and a further two strain gauges measure the lateral sway force \((Y)\) and the rotational yawing moment \((N)\) that results from involvement of the rudder, Figure 3.13.

Once these tests have been completed for all the required speeds and rudder angles the sway force and the yawing moment due to the rudder, Lewis ed. (1989) can be evaluated by plotting these measured values against their respective rudder angles and determining the gradient of each curve as described in section 3.5.1.1.

The gradient of the yaw moment versus the rudder angle profile, Figure 3.14, gives the yawing moment due to the rudder hydrodynamic coefficient \(N_b\).
A second similar plot of the rudder angle against the measured sway force Figure 3.15, and its associated gradient gives the sway force hydrodynamic coefficient due to the rudder \( Y_e \)

![Diagram showing measured sway force and rudder angle.](image)

*Figure 3.15*

The surge hydrodynamic coefficient due to the application of the rudder manifests itself as an increase in the surge resistance to motion. The rudder surge component required for the modular mathematical model can be resolved by subtracting the values obtained from the resistance test conducted with a bare hull with those obtained when the rudders are applied as shown in Figure 3.16.

![Diagram showing scale model resistance curve.](image)

*Figure 3.16*

The surge hydrodynamic coefficients due to the rudder are then calculated in a similar manner to those conducted for the sway and yaw motions.
3.5.1.3 Towing the model longitudinally down the tank at an angle of attack.

Towing the model longitudinally at an angle of attack permits the separation of the rudder forces from the surge, sway and yaw motions. This is achieved by towing the model with and without the rudder and with the rudder either in line with the centre line of the model, Abkowitz (1969), Figure 3.17, or with the rudder in line with the longitudinal centre line of the tank Figure 3.18.

Diagram showing model orientation at an angle of attack with:

(A) Rudders aligned with model centreline. Figure 3.17

(B) Rudders aligned with towing tank centreline. Figure 3.18

The procedure involves placing the model at known angles of attack to the centre line, Figure 3.17, of the tank and towing the model as described for the longitudinal tow. Where similar measurements from the (X-Y-N) strain gauges are
recorded and can be plotted in the form of sway force and yaw moment against the lateral sway velocity (v), Figure 3.19 and Figure 3.20 respectively.

*Illustration of measured scale model:*

![Illustration of measured scale model](image)

Where the lateral sway velocity (v) is a component of the longitudinal velocity (U) as calculated from the equation:

\[ v = U \sin(\Psi) \]

where \( \Psi \) is the angle of attack that the model is placed at.

Determination of the gradient for each curve as it passes through the origin allows the hydrodynamic coefficients due to the lateral velocity for the sway motion, \( Yv \), and for the yaw motion, \( Nv \), to be determined.

**3.5.2 The Use Of The Planar Motion Mechanism For The Determination Of The Hydrodynamic Coefficients.**

The planar motion mechanism (PMM), Grim, Oltmann, Sharma and Wolff (1977) was devised for the use at establishments that did not possess a tank sufficient in size to accommodate the rotating arm facility yet had a long narrow towing tank. The PMM apparatus consists of two oscillators, one producing a transverse oscillation
at the bow and the other a transverse oscillation at the stern of the model as the model progresses down the tank at a speed $U_0$. Lewis ed. (1989). The bow and stern of the model are forced to oscillate with an amplitude of (a) either out of phase or in phase with each other with a circular frequency of ($\omega$) but with an oscillation phasing of the stern relative to the bow indicated by the phase angle ($\psi$) as shown in Figure 3.21.

\[ Y_s = a \cos(\omega t + \psi) \]

Illustration of Planar Motion Mechanism

Figure 3.21

Then if the phase angle is zero the bow and stern are forced to oscillate in phase with each other with the same lateral displacement, and following a path down the tank as shown in Figure 3.22.

Track of model with zero phase difference between bow and stern.

Figure 3.22

Where dynamometers at the bow and stern measure the oscillatory sway forces at the bow and stern, $Y_n$ and $Y_s$ respectively then the oscillation experienced is of the form:
\[ \psi = 0 \]

\[ y = a \cdot \cos(\omega t) \]

\[ \frac{\partial y}{\partial t} = v = -a_0 \omega \cos(\omega t) \]

\[ \dot{v} = -a_0 \omega^2 \cos(\omega t) \]

and the derivatives \( Y_v \) and \( N_v \), according to Abkowitz (1969) are then obtained from the following relationship:

\[ Y_v - m = \pm \frac{\text{Out of phase amplitude of } (Y_b + Y_s)}{-a_0 \omega} \]

\[ N_v - m x_0 = \pm \frac{\text{Out of phase amplitude of } (Y_b - Y_s)d}{-a_0 \omega} \]

Further to this the derivatives \( Y_v \) and \( N_v \) are obtained from measuring the in phase components of \( Y_b \) and \( Y_s \) at various frequencies \( \omega \) where:

\[ Y_v - m = \pm \frac{\text{In phase amplitude of } (Y_b + Y_s)}{-\omega^2 a_0} \]

\[ N_i - m x_0 = \pm \frac{\text{In phase amplitude of } (Y_b + Y_s)}{-\omega^2 a_0} \]

From which the inertial effects \( (m, m x_0) \) have to be removed.

The derivatives \( Y_r, Y_i \) and \( N_r, N_i \) can also be found using the PMM, by towing the model sinusoidally down the tank, so that the centre line of the model is constantly tangential to its path, as shown in Figure 3.23.
This ensures that the angular velocity and acceleration of the model are equal to zero and that the longitudinal velocity \((U)\) is constant throughout the length of the tank, which is the state when a model is towed in a circle at constant velocity and radius. This motion of the model is obtained when the following relationship for the PMM oscillator is satisfied:

\[
\frac{\Psi}{2} = \tan^{-1}\frac{\omega d}{U}
\]

The out of phase measurements of \(Y_R\) and \(Y_S\) will provide the forces and moments due to \(r\), whilst the in phase measurements will give the values for the components due to \(r\), providing that the orientation angle \(\psi\) satisfies:

\[
\psi = \psi_0 \cos(\omega t)
\]
\[
r = -\psi_0 \omega \sin(\omega t)
\]
\[
r = -\psi_0 \omega^2 \cos(\omega t)
\]

Therefore:

\[
Y_r \mu_0 = \pm \frac{\text{Out of phase amplitude of } (Y_B + Y_S)}{-\psi_0 \omega}
\]
\[
N_r \mu \omega u_0 = \pm \frac{\text{Out of phase amplitude of } (Y_B - Y_S)}{-\psi_0 \omega}
\]

and the acceleration derivatives are found from:

\[
Y_r - m x_G = \pm \frac{\text{In phase amplitude of } (Y_B + Y_S)}{-\psi_0 \omega^2}
\]
\[
N_r - I_z = \pm \frac{\text{In phase amplitude of } (Y_B - Y_S)}{-\psi_0 \omega^2}
\]

From which the inertial effects \((\mu u_m, m x_G u_m, m x_G I_z)\) have to be removed.

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3.5.3 The Use Of Rotating Arm Facilities For The Determination Of The Hydrodynamic Coefficients.

The rotating arm facility, Lewis ed. (1989) allows models to be towed in a circular path rather than the straight path that the normal towing tank permits, this can be used to examine the rotational motions of a vessel.

The sway and yaw hydrodynamic coefficients due to the rotational yawing velocity may be determined from the use of the rotating arm facility, Lewis ed. (1989). This involves towing the model in a set diameter circle, and taking measurements before the model has completed a turn through 360 degrees so as to prevent the interference from the wave system created by the model. The measurements taken are the sway force and the yaw moment required to turn the vessel in a circle of known diameter and at constant linear speed \( U \) as shown in Figure 3.24.

![Rotating arm motion of model.](image)

*Figure 3.24*

The yawing moment and sway force hydrodynamic coefficients due to the angular velocity of the vessel are determined from plotting the measured sway force
and yawing moment against the angular velocity as shown in Figure 3.25 and Figure 3.26 respectively.

Diagram showing measured sway force against angular velocity.  

Figure 3.25

Diagram showing measured yawing Moment against angular velocity.  

Figure 3.26

Where the angular velocity \( (r) \) is determined from the linear speed \( (U_0) \) divided by the radius of the circle \( (R) \).

\[
r = \frac{U_0}{R}
\]

The gradients of the two profiles will give the hydrodynamic coefficients for the sway force and yawing moment due to the rotational yawing velocity of the vessel.
3.5.4 The Use Of Circulating Water Channels For The Evaluation Of
The Hydrodynamic Coefficients.

A circulating water channel is the opposite of a towing tank in that the water is
forced to flow past a rigidly held model by the use of large impellers. Observations
and measurements similar to those conducted in a towing tank are made as the water
passes the model, but as the water is continually circulated and the wave profile
removed, then measurements may be made over a longer period of time. Experiments
have been conducted successfully using a PMM in this facility for the measurement of

3.5.5 The Use Of Wind Tunnels For The Evaluation Of
The Hydrodynamic Coefficients.

The use of wind tunnels for the evaluation of the hydrodynamic coefficients
for a force mathematical model has been conducted successfully by Pourzanjani
(1987). Similar experiments to those undertaken when using a towing tank were
conducted, where the model was placed within the mainstream of air at various angles
of attack ensuring no boundary layer effects are encountered from the sides of the
wind tunnel and where measurements of lift, pitch and drag are recorded. These tests
are normally conducted at equivalent Reynolds numbers to that of the parent vessel
as it is possible to vary the speed of the air that the model is tested in and that the
properties of this fluid medium, viscosity and density are less than that of water. As
there is only one fluid medium present when testing scale models in wind tunnels the
interface between the water and the air which results in the production of waves and
the associated increase in resistance that the full scale vessel experiences are not
included within any measurements.

This removed air/sea interface permits the characteristics of the hull and
superstructure to be examined individually. Placing two identical hull forms or
superstructures together to provide a profile which is symmetrical about the interface
physically removes the interface, thus measurements taken are valid for the hull or
superstructure alone when measuring the skin and residual resistances. This method
however, does not permit the effect of wave making resistance to be investigated as the interface is not present. This facility is ideal for the study of submerged bodies (submarines, torpedos, ROV's) and vehicles that operate in one fluid medium only (aircraft), but is of limited use for surface marine vehicles.

3.5.6 Alternative Scale Model Testing Facilities.

Other facilities that permit the use of scale models to be employed for the evaluation of the hydrodynamic coefficients include Cavitation Tunnels, Phillips-Birt (1970), Depressurised Towing Tanks, Lightelijin (1992), Seakeeping Basins and Manoeuvring Basins, Lloyd (1989), where the latter two permit the use of free sailing models.

3.6 MATHEMATICAL AND SEMI-EMPIRICAL METHODS FOR THE EVALUATION OF THE FORCE HYDRODYNAMIC COEFFICIENTS.

The hydrodynamic coefficients for any vessel are unique and as a result are required to be evaluated for each vessel being mathematically modelled. The only satisfactory method to date for their evaluation is via the use of scale model testing techniques supported by full scale trials as described in the previous two sections. From analysis of such results, various relationships between the hydrodynamic coefficients and basic vessel parameters that have been investigated, with the aim of providing simple quick and economic evaluation for any vessel.

3.6.1 Slender Body And Strip Theory Methods.

Strip theory divides the body into many small strips. Determining the property of each strip and integrating for the whole of the body gives the total response for the body. The slender body theory employs the same approach as with the strip theory but assumes perturbations of the surface of the body are small tangentially to the strip compared to those in the plane of the strip, Lewis ed. (1989).

Attempts have been made to determine the hydrodynamic coefficients for marine vessels from both slender body, Wellicome (1987) and strip theory, Clarke
(1972) by considering the vessel to be a simplified low aspect ratio wing rotated through 90 degrees. Using the Jones low aspect ratio wing theory, Clarke (1972) extended a set of equations based on a length to breadth ratio of the vessel to include, instead of a constant multiplier, a term that was dependent upon the hull form. The formulae are as follows:

\[
\begin{align*}
Y'_{\text{v}} &= -\pi \left[ \frac{T}{L} \right] \int_{\text{Stern}}^{\text{Bow}} C_H dX' \\
Y'_{\text{r}} &= -\pi \left[ \frac{T}{L} \right]^2 \int_{\text{Bow}}^{\text{Stern}} C_H X'dX' \\
N'_{\text{v}} &= -\pi \left[ \frac{T}{L} \right] \int_{\text{Stern}}^{\text{Bow}} C_H X'dX' \\
N'_{\text{r}} &= -\pi \left[ \frac{T}{L} \right]^2 \int_{\text{Bow}}^{\text{Stern}} C_H X'^2 dX' \\
Y'_{\text{v}} &= -\pi \left[ \frac{T}{L} \right] |C_H|_{\text{Stern}} \\
Y'_{\text{r}} &= -\pi \left[ \frac{T}{L} \right]^2 |C_H X'|_{\text{Stern}} \\
N'_{\text{v}} &= -\pi \left[ \frac{T}{L} \right] \left| |C_H X'|_{\text{Stern}} + \int_{\text{Stern}}^{\text{Bow}} C_H X'dX' \right| \\
N'_{\text{r}} &= -\pi \left[ \frac{T}{L} \right]^2 \left| |C_H X'|_{\text{Stern}} + \int_{\text{Bow}}^{\text{Stern}} C_H X'dX' \right|
\end{align*}
\]

Where \( C_H \) is the zero frequency added mass coefficient at station \( X' \), which is the non-dimensional distance \( \left( \frac{X}{L} \right) \) distance to station length of vessel.

These however have proved unsuccessful due in part to the irregular form of the vessel and the viscous properties of the fluid medium in which the vessel operates with particular attention to the complex fluid flow phenomena that occurs at the stern in the vicinity of the propeller, rudder and hull and their interactions. Other similar ideas have been proposed as stated by Clarke (1982), but with limited success at the present. Future experimental results may resolve the need for practical measurements.

### 3.6.2 Semi - Empirical Methods.

The semi-empirical method for the evaluation of the hydrodynamic coefficients is based upon evaluation of the hydrodynamic coefficients by whatever method available, relating the results found to simple equations that correlate well with basic hull form parameters. The following basic vessel parameters; length,
breadth, draught and block coefficient have been used to express data obtained from the use of planar motion mechanisms (PMM) and rotating arm tests for a variety of hull forms and load conditions, as presented below in a dimensionless form Clarke (1972).

Data obtained from PMM techniques by Wagner-Smitt (1970) that has related the hydrodynamic coefficients to draught and length of the vessel.

\[
\begin{align*}
Y'_v &= -5.0(T/t)^2 \\
Y'_r &= 1.02(T/t)^2 \\
N'_v &= -1.94(T/t)^2 \\
N'_r &= -0.65(T/t)^2
\end{align*}
\]

In a similar study using PMM Norrbin (1971) related the hydrodynamic coefficients to the draught, length, breadth and block coefficients of the vessel and the density of the water.

\[
\begin{align*}
Y'_v &= -\pi(T/t)^2 \left[ -1.69 + 0.08 \left( \frac{C_b}{\rho} \right) \frac{B}{T} \right] \\
Y'_r &= -\pi(T/t)^2 \left[ -0.645 + 0.38 \left( \frac{C_b}{\rho} \right) \frac{B}{T} \right] \\
N'_v &= -\pi(T/t)^2 \left[ +0.64 - 0.04 \left( \frac{C_b}{\rho} \right) \frac{B}{T} \right] \\
N'_r &= -\pi(T/t)^2 \left[ +0.47 - 0.18 \left( \frac{C_b}{\rho} \right) \frac{B}{T} \right]
\end{align*}
\]

Data obtained by Inoue, Hirano and Kijima (1981) from rotating arm techniques for a wide range of hull forms and load conditions have been related to the hydrodynamic coefficients by the draught, length, breadth and block coefficient of the vessel and the density of the water.

\[
\begin{align*}
Y'_v &= -\pi(T/t)^2 \left[ 1.0 + 1.4 \left( \frac{C_b}{\rho} \right) \frac{B}{T} \right] \\
Y'_r &= -\pi(T/t)^2 \left[ -0.5 \right] \\
N'_v &= -\pi(T/t)^2 \left[ \frac{2.0}{\rho} \right] \\
N'_r &= -\pi(T/t)^2 \left[ \frac{1.04}{\rho} - \left( \frac{4.0}{\rho} \right) \frac{1}{T} \right]
\end{align*}
\]

While these equations are suitable for certain forms or design of vessel they would not be universal for all vessels and should only used as an estimate unless the
vessel in question had similar form and dimensions to those used in the data gathering process.

3.6.3 Multiple Regression Analysis.

The use of full scale trials and model testing techniques requires the resources to conduct such tests. Applications are varied and individuals requiring mathematical models do not always have direct access to such facilities and so cannot evaluate the hydrodynamic coefficients. Clarke (1982) therefore, has pooled the data obtained from full scale trials and model testing techniques together for a variety of vessels, which he expressed in the form of graphs of non-dimensionalised hydrodynamic coefficients against basic vessel particulars namely length, breadth, draught and block coefficient. By regression analysis upon this data an equation for a line of best fit was evaluated as shown below.

\[
\begin{align*}
Y'_{\nu} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 1 + 0.16C_{BT} - 5.1 \frac{B}{L} \right) \\
Y'_{r} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 0.67 \frac{B}{L} - 0.0033 \frac{C_B}{L} \right) \\
N'_{\nu} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 0.67 \frac{B}{L} - 0.04 \frac{C_B}{T} \right) \\
N'_{r} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 0.083 + 0.017 \frac{B}{L} - 0.33 \frac{B}{L} \right) \\
Y'_{\nu} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 1 + 0.4C_{BT} \right) \\
Y'_{r} &= -\pi \left[ \frac{L}{T} \right]^2 \left( -0.5 + 2.2 \frac{B}{L} - 0.08 \frac{B}{T} \right) \\
N'_{\nu} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 0.5 + 2.4 \frac{L}{C} \right) \\
N'_{r} &= -\pi \left[ \frac{L}{T} \right]^2 \left( 0.25 + 0.039 \frac{B}{T} - 0.56 \frac{B}{L} \right)
\end{align*}
\]

These equations being based upon a range of tests conducted for a variety of vessels can be considered more accurate than the semi-empirical methods that were previously described in section 3.6.2 which were based upon individual tests.

3.6.4 Future Computer Aided Evaluation Techniques.

At present research is being conducted into the use of numerical codes in the form of Computational Fluid Dynamics (CFD), Cardena (1992), for the analysis and evaluation of ship hydrodynamics. These codes are based upon a modified Navier-Stokes theorem, Bird, Stewart and Lightfoot (1960) which has led to the Potential and viscous flow theories, Douglas (1985). The application of these theories
has culminated in establishment of the Numerical towing Tank, Maeda (1991). At present the flow around any section of the vessel can be studied in detail, and the pressure distribution and flow fields established, with theoretical measurements calculated.

This software tool is still in its infancy and further research and validation of results is required to be conducted before its true potential can be established and employed for the purpose of this study of the required hydrodynamic coefficients.

3.7 SUMMARY OF THE METHODS OF EVALUATION FOR THE FORCE HYDRODYNAMIC COEFFICIENTS.

The most accurate method of determining the hydrodynamic coefficients for a marine vehicle is from conducting full scale trials (as the measurements are directly related to the vehicle). Such trials are not always possible as they are both time consuming and financially unviable, and in some cases impractical to conduct due to vessel availability and the sheer size of a vessel. Therefore scale model testing techniques are a vital tool for the determination of the hydrodynamic coefficients for such vessels. The use of scale model testing techniques has an added advantage in that the six degrees of freedom that a vessel experiences can be separated so that each can be investigated individually, which is not possible when conducting full scale trials. Furthermore, environmental disturbances that are in action during full scale trials have to be taken into consideration especially where small vessels are concerned. This however is not a problem when scale model testing is conducted as sterile conditions exist and the action of external disturbances have to be created if these are to be examined.

Implementing scale model testing techniques requires extensive facilities, not only a contained volume of water but the monitoring and data acquisition system, which are not always available to establishments conducting such research. Furthermore, it is not possible at present to create perfectly scaled models and an environment according to the laws of similarity, Rawson and Tupper (1984). This
inability to satisfy these laws of similarity has to be considered when implementing scale model testing techniques.

The mathematical and semi-empirical methods, as described in section 3.6, are based upon scale model testing techniques and have been proposed so that the hydrodynamic coefficients for a vessel can be determined. This is invaluable when either full scale trials or scale model testing techniques cannot be employed. These methods however, are based upon scale model testing techniques which therefore suffer from the laws of scaling, as described above, and with the exception of the multiple regression analysis method are based upon singular studies, which invokes a question of their suitability to other marine vehicles.

In conclusion then, full scale trials conducted on the vessel that is being modelled are taken as essential for determining the hydrodynamic coefficients. Results obtained from conducting full scale trials can then be utilised to validate scale model data. The scale model testing techniques can then be implemented to separate the motions and allow evaluation of the hydrodynamic coefficients which cannot be evaluated from conducting full scale trials. Thus both the full scale trials and scale model testing techniques can then be employed to validate the mathematical and semi-empirical methods, and data from all sources should be pooled to construct a model.
Chapter Four.

EVALUATION OF THE HYDRODYNAMIC COEFFICIENTS FOR PICKET BOAT NINE.

4.1 INTRODUCTION.

The surge velocity hydrodynamic coefficients, as detailed in section 3.3.1 represent the relationship between a vessel's longitudinal velocity and its resistance to motion in this direction. In order to obtain the surge velocity linear and non-linear hydrodynamic coefficients for the hull of Picket Boat Nine, using the methods described in chapters three, both scale model and full scale sea trials were conducted. The results of which have lead to the surge velocity hydrodynamic coefficients being determined. This chapter then proceeds to determine the sway and yaw hydrodynamic coefficients according to the semi-empirical methods described in chapter three as the facilities required for their derivation are not available for this research programme.

4.2 SCALE MODEL MEASUREMENT OF TOTAL RESISTANCE TO MOTION.

The towing tank at Britannia Royal Naval College the installation of which is described in Appendix B, was implemented to evaluate the surge velocity coefficients for Picket Boat Nine by the measurement of the total resistance to motion by the use of scale models. This involved the construction of two geometrically similar models; one a twenty-fifth scale and the other a sixteenth scale model as shown in Plate C.1 (Plates associated with chapter four are held in Appendix C). The scaling factors of these two models were dictated by the blockage factor in the larger of the two models, where the blockage factor is the ratio of the cross-sectional areas of the model compared to that of the towing tank. A blockage factor of 0.02% according to Phillips-Birt (1970) is sufficient to prevent the effect of the side wall interference due to reflection of the wave profile and the shallow water effects due to the depth of
water below the keel can be considered negligible. The smaller model scaling factor was dictated by the maximum speed that the gantry and full scale vessel operated at (1m/s and 10knots respectively) as the smaller model was required to replicate the range of operating speeds that the full scale vessel possessed. As with most test facilities the scale models were towed at equivalent Froude numbers (Fn) which led to the lengths of the models being 0.72m and 0.45m. The equivalent full scale speed for each model at the test speed is given in Table D.1 (Tables associated with model measurements are held in Appendix D).

To satisfy the Reynolds number and scaling ratios at these speeds studs were positioned on the surface of the model at a distance 0.2% of the length of the model aft of the bow, as shown in Plate C.2, causing the flow to trip from laminar to turbulent as dictated by the use of the Critical Reynolds number (Rnc) when transition occurs at Rn equals 1x10⁶, Vardy (1990).

The determination of the surge hydrodynamic coefficients involved measuring the total longitudinal resistance of each of the two different scale models both in the bare hull condition over the range of speeds that the towing tank permitted at an increment of 0.05cm/s per run, as given in Table D.1. These tests were repeated three times for each model with the total resistance at each speed taken as the average of the five runs as shown in Table D.2 and D.3, which are illustrated graphically as a plot of measured resistance against their respective Froude number in Figure 4.1, shown over the page.

The above procedure for the determination of the total resistance to motion was repeated for the larger of the two models with the turbulence stimulators in place (Plate C.2) and in a separate series of tests rudders were installed onto this model in the bare hull condition as shown in Plate C.3. The tests were repeated three times with the average values for the three runs taken as the total resistance for each test as shown in Figure 4.2 and tabulated in Tables D.4 and D.5.
Graph of average measured resistance against Froude number for the 1:16 and 1:25 scale models in the bare hull condition.  

Figure 4.1

Graph of average measured resistance against Froude number for the 1:16 scale models in the bare hull condition with turbulence stimulators and rudders attached.  

Figure 4.2
4.2.1 Analysis Of The Scale Model Results.

The results from the scale model tests conducted when using both the one to twenty-fifth and one to sixteenth scale models show the basic relationship between the total resistance to motion and the speed of the vessel, where the total resistance to motion increases approximately to the square of the speed. The results from the two different sized models when towed in the bare hull condition, Figure 4.1, shows that the larger model has a greater measured total resistance than that of the smaller model at a given speed as expected; this is due to the size of the models and their respective wetted surface areas. The increase in the total resistance of the 1:16 scale model when the rudders were attached also compares favourably with the same size model when in the bare hull condition as the rudders possess both frictional and residual resistance that contribute to the total resistance of the model. The method chosen for tripping the flow from laminar to turbulent however can be seen from Figure 4.2 to be inconsistent with the theory. The increase in resistance at the lower speeds is due to the residual and frictional resistance of the studs as laminar flow predominates around the hull. This therefore questions the measurements taken for the higher speeds of operation when turbulent flow was expected and an associated increase in total resistance. This increase in total measured resistance may be a result of studs and not the effect of turbulent flow.

The model tests conducted in the conditions mentioned above show that the equipment possesses the ability and accuracy to measure the total resistance of these models in various conditions.

4.2.2 Transfer Of Scale Model Results To Full Scale.

For the surge hydrodynamic coefficients to be resolved from the data collected the scale model total resistances had to be scaled to that of the full scale vessel. This was conducted using the standard 1957 ITTC method, Harvald (1983) and implemented on a spreadsheet. The results of transferring the measured model values to that of the full scale vessel, Tables D.6 - D.8 are illustrated graphically as shown in Figure 4.3 and Figure 4.4 for both models in the bare hull condition along with the larger model with turbulence stimulation and rudders attached respectively.
Graph of transferred measured resistances to full scale against Froude number for the 1:16 and 1:25 scale models in the bare hull condition.  

Figure 4.3

Graph of transferred measured resistances to full scale against Froude number for the 1:16 scale models in the bare hull condition with turbulence stimulators and rudders attached.  

Figure 4.4
4.2.3 Analysis Of The Transferred Scale Model Results To Full Scale.

The transferred measured resistances for twenty-fifth and sixteenth scale models when towed in the bare hull condition, Figure 4.3 shows a large degree of discrepancy between the two, perfect data sets would be expected to be superimposed. This discrepancy is further compounded when the transferred values for the sixteenth model scale model with the rudders attached and the turbulent stimulators are present, shown in Figure 4.4, as the twenty-fifth scale model shows a much greater resistance for all the speeds considered, Figure 4.3. In comparison to the actual measured scale model values, Figure 4.1, where the twenty fifth scale model values were less than those of the sixteenth scale model, there appeared to be a large degree of inaccuracy with the 1957 ITTC method of transferring the values to the full scale vessel.

4.3 FULL SCALE SEA TRIALS FOR THE MEASUREMENT OF THE TOTAL RESISTANCE TO MOTION.

The required relationship for the evaluation of the surge hydrodynamic coefficients between the total resistance to motion and speed for Picket Boat Nine has been determined from three full scale sea trials. The first being a series of measured miles, the second involving a series of bollard pulls and the third sea trial consisting of towing Picket Boat Nine behind another at various speeds.

4.3.1 The Measured Mile Manoeuvre Using Picket Boat Nine.

This sea trial, as described in chapter three, involves the vessel to be set steaming with the rudders amidships at a constant speed with the speed of the engine and the speed of the vessel through the water being recorded simultaneously. This was conducted by setting the engine speed of both engines to the minimum obtainable 400rpm according to the bridge sensor. A trailing log towed behind the vessel permitted the speed of the vessel through the water to be known. Once steady state was achieved the engine speed was recorded from the bridge indicator and the vessel's speed from the trailing log. This procedure was then repeated immediately in the opposite direction with similar measurements taken so that the influence of the
tide, current and wind could be removed from the data. This complete procedure was then repeated at increments of 100rpm for the range of operable engine speeds from 400rpm to 1900rpm with similar measurements recorded as shown in Table D.9. This data can be shown graphically as a plot of engine speed against vessel speed as shown in Figure 4.5.

These results agree with the theory that a greater engine speed is required to produce a higher velocity and that an increase in engine speed when travelling at slow velocities results in a greater increase in vessel speed compared to a similar increase at higher velocities.

4.3.2 The Bollard Pull Manoeuvre Using Picket Boat Nine.

As described in chapter three this manoeuvre involved the thrust produced by the propellers at various engine speeds to be recorded. This was conducted by restraining the vessel via a stern rope to a ridge pontoon and the configuration of ropes involved permitted a two kilo-Newton strain gauge to be mounted in series.
As a tidal current was present the vessel’s engines were set to idle and the gearbox put into neutral thus allowing the vessel to align herself with the flow of the current and at this stage the strain gauge was offset to zero thus removing the effects of external disturbances. The vessel’s engines were then set to 400rpm (the slowest possible according to the bridge indicator), and the thrust produced measured from the strain gauge. This procedure was then repeated for the range of engine speeds from 400rpm to 1200rpm at increments of 100rpm with the thrust produced at each engine speed recorded manually as tabulated in Table D.10. This data can be shown graphically as a plot of measured thrust against engine speed in Figure 4.6.

The total resistance to motion of Picket Boat Nine is equal to the thrust produced by her propellers for her to travel at constant speed. Combining the results from the measured mile and bollard pull manoeuvres allows the total resistance to be determined. As shown in Figure 4.7 and tabulated in Table D.11 the thrust required at any given speed increases approximately with the square of the speed.
Graph of bollard thrust against respective Froude number for Picket Boat Nine from measured mile and bollard pull manoeuvres.

Figure 4.7

4.3.3 Measurement Of The Full Scale Resistance To Motion Of Picket Boat Nine.

The determination of the total resistance to motion of Picket Boat Nine at various speeds through the water was conducted by towing Picket Boat Nine behind another vessel via a single rope that permitted a two kilo-Newton strain gauge to be held in series. The towing vessel was set steaming at constant engine speed 600rpm that related to a speed of two knots in Picket Boat Nine according to the trailing log when both strain gauge and trailing log were constant. This was immediately repeated in the opposite direction so as to remove environmental disturbances and the average for both runs was taken. This whole procedure was then repeated for the range of engine speeds of 600rpm to 2000rpm at intervals of 100rpm that related to the range of speeds of two knots to eight knots in Picket Boat Nine with the average resistance motion and speed noted for each run shown in Table D.12 and shown graphically in Figure 4.8. This complies with the expected relationship between the speed of the vessel and the total resistance to motion as previously mentioned for the other tests conducted.
4.3.4 Analysis Of The Full Scale Sea Trial Results.

The measured mile and bollard pull method for determining the resistance to motion of Picket Boat Nine in comparison to the towed vessel measured resistance illustrates the difference between these two methods, as shown in Figure 4.9.

**Graph of resistance to motion from towed vessel manoeuvre against vessel speed.** Figure 4.8

**Graph of towed measured resistance and combined measured mile and bollard pull manoeuvres against Froude number for Picket Boat Nine.** Figure 4.9
This indicates that at the lower speeds of operation the combined data from the measured mile and bollard pull manoeuvre is erroneous whilst at the higher range of operating speeds the error appears to reduce. This is attributed to the lack of data available from the measured mile manoeuvre as this vessel was not controllable at an engine speed less than 600rpm, whilst during the Bollard pull trials an engine speed greater than 1200rpm was not possible. Furthermore, the vessel employed to tow Picket Boat Nine did not possess sufficient power to sustain an average speed greater than seven knots and resistance measurements were not possible for speeds greater than this. However, by extrapolation of the two data sets for the complete range of speeds that Picket Boat Nine operates over, as shown in Figure 4.10 the error at the higher range of speeds tends to increase more significantly than at the lower speeds of operation.

Extrapolated towed vessel and combined measured mile and bollard pull data against velocity for Picket Boat Nine.  

Figure 4.10

This increase in error can be attributed to the vessel being stationary during the bollard pull trials as the velocity of the water entering the propeller is less than that when the vessel is moving, therefore by continuity the extra thrust produced by this extra velocity of water is not present in the results. Further to this, the velocity
measurement for both sea trials was taken from an analogue meter connected to the trailing log that was towed at the stern in the wake of the vessel, where the recorded speed was higher than expected due to the wake and propeller race. The extrapolated line for the towed vessel increases with speed as expected rather than diminishing. Therefore, as the combined Bollard pull and measured mile manoeuvre is a method of determining the total resistance to motion that has been devised for vessels that are too large to be towed, the actual resistance values obtained from towing Picket Boat Nine are taken as being the most accurate.

4.4 COMPARISON OF THE SCALE MODEL AND FULL SCALE RESULTS FOR THE TOTAL RESISTANCE OF PICKET BOAT NINE.

The transferred scale model values for the total resistance to motion of Picket Boat Nine compared with those obtained from towing the full scale vessel, as shown in Figure 4.11, indicates that the sixteenth scale model in the bare hull condition possesses the best correlation, even though error is still apparent.

![Graph showing full scale resistance to motion of Picket Boat Nine and 1:16 scale model resistance in the bare hull condition.](Figure 4.11)
These erroneous observations concerning the comparison between the transferred scale model values and the full scale values are possibly attributable to the method of scaling and its suitability to the size of the models employed. There appears to be no reason for questioning the magnitude of the measurements, except with respect to their accuracy. Because scale model testing techniques are a method of determining the total resistance to motion of a vessel when trials cannot be conducted, then the values obtained from towing Picket Boat Nine are taken as being more accurate.

4.5 EVALUATION OF THE LINEAR & NON-LINEAR SURGE VELOCITY HYDRODYNAMIC COEFFICIENTS.

The procedure for determining the linear and non-linear surge velocity hydrodynamic coefficients consisted of applying a least squares regression third order polynomial, Charpra and Canale (1988), to all the data collected of the form:

\[
na_0 + a_1\Sigma x + a_2\Sigma x^2 + a_3\Sigma x^3 = \Sigma y
\]

\[
a_0\Sigma x + a_1\Sigma x^2 + a_2\Sigma x^3 + a_3\Sigma x^4 = \Sigma xy
\]

\[
a_0\Sigma x^2 + a_1\Sigma x^3 + a_2\Sigma x^4 + a_3\Sigma x^5 = \Sigma x^2y
\]

\[
a_0\Sigma x^3 + a_1\Sigma x^4 + a_2\Sigma x^5 + a_3\Sigma x^6 = \Sigma x^3y
\]

By the use of Gaussian elimination with partial pivoting this equation was solved for the constants \(a_0\), \(a_1\), \(a_2\) and \(a_3\) from the above least squares regression expansion which relate to \(a\), \(b\), \(c\) and \(d\) from the cubic equation:

\[
y = ax^0 + bx^1 + cx^2 + dx^3
\]

The constants '\(a_0\)' and '\(a\)' have been made equal to zero in the above equations as the curve of total resistance against speed is forced to pass through the origin since at zero speed there is zero resistance to motion. The cubic multipliers (\(b\), \(c\) and \(d\)) then relate to the dimensional linear \(X_u\), and non-linear \(X_{uu}\), \(X_{uuu}\) surge velocity hydrodynamic coefficients.
The data collected from both the full scale sea trials and the scale model tests, once these values had been transferred to their full scale equivalents, were resolved into the cubic, square and normal multipliers taking the values given in Table 4.1. These values are the dimensionalised surge hydrodynamic coefficients that represent numerically the best fit cubic polynomial for each data set and where the correlation coefficient represents the accuracy of the equation to the data.

<table>
<thead>
<tr>
<th></th>
<th>Normal. $X_u$</th>
<th>Square. $X_{uu}$</th>
<th>Cubic. $X_{uuu}$</th>
<th>Correlation Coefficient (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:25 Scale model, (bare hull condition)</td>
<td>56.875561</td>
<td>6.631807</td>
<td>59.98522</td>
<td>0.999975</td>
</tr>
<tr>
<td>1:16 Scale model, (bare hull condition)</td>
<td>12.509607</td>
<td>14.715781</td>
<td>28.7779</td>
<td>0.999949</td>
</tr>
<tr>
<td>1:16 Scale model, (with stimulation)</td>
<td>14.00946</td>
<td>120.13485</td>
<td>6.337293</td>
<td>0.999976</td>
</tr>
<tr>
<td>1:16 Scale model, (with rudders attached)</td>
<td>13.442473</td>
<td>-2.176327</td>
<td>43.347092</td>
<td>0.999951</td>
</tr>
<tr>
<td>Full scale sea trials, (towed vessel approach)</td>
<td>0.832379</td>
<td>20.529585</td>
<td>8.247568</td>
<td>0.999997</td>
</tr>
<tr>
<td>Full scale sea trials, (measured mile-bollard)</td>
<td>7.361263</td>
<td>85.983459</td>
<td>-7.251429</td>
<td>0.999989</td>
</tr>
</tbody>
</table>

_Evaluated surge velocity hydrodynamic coefficients._ Table 4.1

These hydrodynamic coefficients represent the normal, square and cubic multipliers for a cubic polynomial that describes the relationship between the surge velocity of the vessel and its corresponding total resistance to motion in this direction. Figure 4.12, graphically illustrates this relationship and demonstrates the error associated with the scaling laws that were used for the transfer of the scale model values to that of the full scale vessel, as discussed in section 4.2.2. Figure 4.12 also highlights that the bollard pull and measured mile combination of manoeuvres is a reasonable approximation to the total resistance to motion at the lower speeds of operation but not at the higher range, as mentioned in section 4.3.4.
Graph showing accuracy of hydrodynamic coefficients to describe the measured resistance against velocity for Picket Boat Nine.

The error contained within each of the data sets has already been reasoned in the preceding sections for both the full scale sea trials and the scale model tests that were conducted. This error has then been transmitted through to the values allotted to these hydrodynamic coefficients. The magnitude of the error can be seen from Figure 4.12 in which the hydrodynamic coefficients determined from towing the full scale vessel have to be taken as the most accurate with the other methods requiring further investigation.

4.6 THE SWAY AND YAW HYDRODYNAMIC COEFFICIENTS.

The remaining sway and yaw hydrodynamic coefficients required for mathematically modelling Picket Boat Nine have been calculated from the three semi-empirical methods and the equations based upon multiple regression analysis as detailed in chapter three section 3.6.2 and 3.6.3 respectively and written in to a computer program. These equations are derived from observations and measurements similar to those conducted in the previous chapter for the surge
hydrodynamic coefficients for various vessels and represent the only method for their derivation without extensive model tests and sea trials.

The values ascertained from the studies conducted by Wagner-Smitt, Norrbin, Inoue et al and Clarke as detailed in chapter three (section 3.6) are shown in Table 4.2.

<table>
<thead>
<tr>
<th>Multiple regression.</th>
<th>Semi - empirical.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y',</td>
<td>-0.052534</td>
</tr>
<tr>
<td>Y,</td>
<td>0.003677</td>
</tr>
<tr>
<td>N',</td>
<td>-0.024713</td>
</tr>
<tr>
<td>N,</td>
<td>-0.006645</td>
</tr>
<tr>
<td>Y',</td>
<td>-0.028224</td>
</tr>
<tr>
<td>Y,</td>
<td>-0.006065</td>
</tr>
<tr>
<td>N',</td>
<td>-0.002453</td>
</tr>
<tr>
<td>N,</td>
<td>-0.000582</td>
</tr>
</tbody>
</table>

Sway and yaw evaluated hydrodynamic coefficients. Table 4.2.

Which were based upon the following basic parameters of Picket Boat Nine:

Length ($L_{w}$) -------------- 11.683m
Draught (T) -------------- 1.2m
Beam (B) -------------- 3.2m
Block Coefficient ($C_{b}$) --- 0.55
Water density ($\rho$) -------------- 1025.0kgm$^{-3}$

The values attributed to these hydrodynamic coefficients are presented in their non-dimensionalised form by the use of the prime system notation, Tapp (1989) and illustrates the large degree of discrepancy between findings. This highlights the associated problems and questions the suitability of these methods for the evaluation of the hydrodynamic coefficients for Picket Boat Nine. The main cause of concern arises from the vessels used in these studies being of a size much greater and their
form dissimilar to that of the vessel at the centre of this research programme; especially with regards to the semi-empirical solutions (section 3.6.2) that were derived from a single series of tests. Those derived from the multiple regression analysis (section 3.6.3) involved a multitude of data from various tests for different vessels (which is probably the most reliable).

The hydrodynamic coefficients held in Table 4.2 are the linear sway and yaw hydrodynamic coefficients. The mathematical modelling approach adopted for the research programme, however requires both the linear and non-linear hydrodynamic coefficients to accurately model the vessel over a wide range of operating conditions. Furthermore, as the modular model approach is being followed, the contributions from the propellers and rudders are required to be separated from the hull whilst these linear terms were probably evaluated with these appendages in place and thus the hydrodynamic coefficients obtained are unsuitable for this study.

4.7 SUMMARY OF THE HYDRODYNAMIC COEFFICIENTS FOR PICKET BOAT NINE.

The most accurate sway and yaw hydrodynamic coefficients for Picket Boat Nine are taken as those evaluated from Clarke’s (1982) multiple regression analysis as they were formulated from a multitude of pooled data concerning a variety of tests and different vessels. As there are no theoretical or empirical methods for the determination of the surge hydrodynamic coefficients this programme has assumed that those derived from the towing the actual vessel, Picket Boat Nine, during the sea trials are the most accurate than the other proposed methodologies used when the actual vessel is either to large to be towed or when it is not possible to conduct sea trials. Therefore from the research conducted to most comprehensive surge, sway and yaw hydrodynamic coefficients for Picket Boat Nine are as given in Table 4.3.
Hydrodynamic Attributed value.

<table>
<thead>
<tr>
<th>Hydrodynamic coefficient</th>
<th>Attributed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_u$</td>
<td>0.832379</td>
</tr>
<tr>
<td>$X_{uu}$</td>
<td>20.529585</td>
</tr>
<tr>
<td>$X_{uuu}$</td>
<td>8.247568</td>
</tr>
<tr>
<td>$Y_v'$</td>
<td>-0.052534</td>
</tr>
<tr>
<td>$Y_r'$</td>
<td>0.003677</td>
</tr>
<tr>
<td>$N_v'$</td>
<td>-0.024713</td>
</tr>
<tr>
<td>$N_r'$</td>
<td>-0.006645</td>
</tr>
<tr>
<td>$Y_v''$</td>
<td>-0.028224</td>
</tr>
<tr>
<td>$Y_r''$</td>
<td>-0.006065</td>
</tr>
<tr>
<td>$N_v''$</td>
<td>-0.002453</td>
</tr>
<tr>
<td>$N_r''$</td>
<td>-0.000582</td>
</tr>
</tbody>
</table>

*The hydrodynamic coefficients for Picket Boat Nine.*

Table 4.3

Page 75.
Chapter Five.

APPRAISAL OF AND CONCLUSION TO THE NON-LINEAR MODULAR FORCE MODEL.

5.1 INTRODUCTION.

The preceding four chapters have concentrated upon the implementation of the non-linear modular model with reference to the forces and moments to describe the vessel and her motions. This following chapter discusses the problems encountered with this form of mathematical modelling and indicates primary areas where the use of the hydrodynamic coefficients to describe a vessel's hull does not agree with trials results. This chapter also describes the attempts made to improve and resolve these discrepancies which has culminated in the drift angle theory, and this forms the second part of this thesis.

5.2 SUMMARY OF THE FORCE METHOD OF MATHEMATICALLY DESCRIBING A VESSEL.

The non-linear modular modelling technique employed with this programme of research is based upon Newtonian mechanics where the force and moment contributions from the hull, propeller, rudder and any external disturbances that exist are separated and mathematically modelled individually, as described in chapter two. The motion and response of a vessel to such forces and moments is then determined by combining these forces and moments from each module for each of the motions considered. These resultant forces and moments for each motion are then resolved mathematically into displacements, velocities and accelerations and integrated into a computer program. A diagram of this form of mathematical modelling is shown in Figure 5.1.
The rudder and propeller modules along with the external disturbance due to wind action upon the vessel have to date been successfully related to empirical
formulæ. These empirical formulæ have been obtained from research previously conducted from scale model testing and full scale sea trials and relate to a vessel's particular characteristics, as described in chapter two. The hull module treats the hull as a rigid body that operates in a fluid medium and possesses six degrees of freedom. The motion of this body is a direct result of the forces that act upon it and the resultant force or moment for each degree of motion considered is used to determine the response in terms of displacements, velocities or accelerations.

This programme of research has concentrated in particular upon the methods of characterising the hull of a vessel, mathematically, in terms of the forces and moments that act upon the hull and the determination of the vessel's response to such forces and moments. To date there exists no mathematical methodology for determining a vessel's response to a variety of forces from basic design data or vessel particulars.

The current method of describing the hull mathematically in a manner suitable for mathematical modelling is conducted via the use of hydrodynamic coefficients which, as described in chapter three, are the constants of proportionality between the hull and the environment in which it operates. These hydrodynamic coefficients permit the relationships between the forces and moments experienced by the vessel and the associated velocities and accelerations that the vessel produces in response to these forces and moments to be described mathematically.

The various methods presently available for determining these hydrodynamic coefficients, as described in chapter three, have been highly successful in producing a relatively reliable and accurate linear mathematical model but when considering the vessel as a non-linear system error becomes apparent. This error has been deduced and is considered significant as a result of this research programme where the surge velocity hydrodynamic coefficients for Picket Boat Nine from both full scale sea trials and scale model testing techniques that have been conducted, as described in chapter four. The experiments conducted have indicated that large errors are involved with the measurement process in determining the magnitude of the forces and moments.
along with the their associated velocities and accelerations that the vessel possesses. This error is clearly shown by the results as tabulated in Table 4.1 where a large difference exists in the values attributed to the cubic, square and normal hydrodynamic coefficients. This error which is attributed primarily to the measurement process, as described in chapter four, may also be considered to be present in all the other hydrodynamic coefficients as similar methods of determination are employed.

The following three sections summarise the problems and errors that have been encountered in this research programme for the determination of the hydrodynamic coefficients for Picket Boat Nine from full scale sea trials, scale model testing techniques and the already existing semi empirical and mathematical methods.

5.2.1 The Errors Associated With Conducting Scale Model Testing.

The creation of the scale model test facilities, as detailed in Appendix B and the tests conducted, as described in chapter four, have indicated that the primary cause of error in the values attributed to the hydrodynamic coefficients was a result of the physical dimensions of the scale model test facilities available for this research programme. As the scale models were limited to a length of less than one meter, then the forces that acted upon these models were insufficient in magnitude to be easily and accurately measured. Furthermore, as dynamic similarity was not obeyed the flow regime around the hull of the model was very dissimilar to that of the parent vessel. The flow regime around the hull was in fact determined as being laminar rather than turbulent, by comparison of Reynolds numbers, this resulted in an increase in the measured total resistance. This increase in the total resistance was a result of the frictional drag being excessive as described in chapter four. Therefore, it was concluded that the models, even with turbulence stimulators, could not replicate the flow regime of the parent vessel thus the forces measured would not represent those experienced , using the existing scaling laws.
5.2.2 The Errors Associated With Conducting Full Scale Sea Trials.

As the full scale sea trials were conducted in open waters the vessel was free to move in all directions, thus the forces and moments and their associated velocities and accelerations could not be separated and measured independently. Furthermore when conducting these full scale sea trials in open waters the external disturbances due to the wind, waves and tidal currents affected the accuracy of the measurements taken. Thus, the sea trials should be conducted under ideal conditions when these external disturbances are not present. These two factors illustrate the main problems that were encountered when conducting both the measured mile manoeuvre and the towed vessel approach for determining the total resistance to motion for Picket Boat Nine.

More specifically the towed vessel approach possessed erroneous measurements as a result of insufficient power being produced by the towing vessel that prevented the complete range of operating speeds that Picket Boat Nine possessed being tested. Furthermore, as Picket Boat Nine was towed in the wake of the towing vessel the correct flow regime around Picket Boat Nine was not established, had the length of rope been greater then the drag produced by the tow rope would have required determination and subtracting from the resistance measurements.

5.2.3 The Errors Associated With Employing Mathematical And Semi-Empirical Solutions.

The empirical and semi-empirical methods for the determination and evaluation of the hydrodynamic coefficients, as described in chapter three, are founded upon both the full scale sea trials and scale model testing techniques and, as described above, both these methods possess measurement error to a certain degree. These empirical methods are however derived from data collected from sophisticated and well equipped research facilities. The physical dimensions of these test facilities allow the problems associated with this research programme and a small tank and vessel to be considered negligible and overcome. These research institutes are primarily concerned with large commercial vessels, of much greater physical
dimensions than the vessel available for this research programme, then these empirical methods can be considered unsuitable and inappropriate for this research programme.

5.3 THE PROBLEMS WITH EMPLOYING A TRUNCATED TAYLOR'S MULTI-VARIATE EXPANSION FOR THE HULL MODULE.

The use of the Taylor's multi-variate expansion as described in chapter two is so that the force and moment equations that are employed to describe a vessel may be separated down and resolved into the component accelerations, velocities and distances travelled in a particular direction of motion. The accuracy of this mathematical model of the hull depends largely upon the number of hydrodynamic coefficients included from such an expansion. Burns (1984) improved upon the linear model by the inclusion of selected terms up to and including third order with an increase in accuracy of the model. Increasing the number of terms however increases the processing time and this increase in processing time compared with the increase in accuracy did not warrant the inclusion of the terms to the fourth order or higher. The exclusion of the higher terms and those he considered negligible automatically causes error within the mathematical modelling process of the hull module. The physical relationships between the vessel and the medium in which it operates is not completely described by the hydrodynamic coefficients and relevant information may be disregarded by this exclusion of terms.

The truncation of the Taylor's multi-variate expansion and the exclusion of selected terms must be considered when this hull module is incorporated into the modular model within an iterative loop of a software program upon the digital computer. This truncation may be regarded as a mathematical rounding off, which constitutes a compounding and accumulating error. This error, due to the processing time requirements and the present day ability of computers, cannot be resolved and therefore challenges the overall performance and reliability of this form of mathematical modelling of the hull.

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5.4 CONCLUSION TO THE FORCE MODULAR MODEL.

This research programme thus far has indicated that the mathematical representation of the hull in terms of the forces and moments that are in action is not an appropriate method of characterising the hull. Errors that become apparent as a more accurate mathematical model is desired and the relationships that govern the response of the hull are erroneous as a result of the factors mentioned above.

During the course of this research programme and the identification and determination of the sway and yaw hydrodynamic coefficients using the facilities available, a new method for describing the hull has been developed. The second part of this thesis explains this theory and employs this new concept for the characterisation, prediction and determination of a vessel's manoeuvrability that may be used to replace the hull module of this non-linear modular force model that is traditionally employed.
PART II.

PREFACE.

In general a marine vehicle, when underway, may be considered in one of three states; stationary, progressing along a straight path or continually turning. The two former situations have successfully been related to Newtons laws of motion and mathematically modelled, but the latter has not.

Chapter five summarised and concluded that the force method of quantifying the hull of a marine vehicle in mathematical terms; as erroneous due to the measurement processes involved with determining the hydrodynamic coefficients and the employment of a mathematical series expansion. In trying to improve and expand this form of mathematical modelling this research programme has succeeded in developing a new method of quantifying the hull of a marine vessel when in a controlled manoeuvre.

The following chapters in this thesis propose and establish a new method of characterising and describing marine vehicles so that they may be mathematically represented on the digital computer for the applications that were introduced in chapter one.
Chapter Six.

THE MANOEUVRABILITY OF A SURFACE MARINE VESSEL.

6.1 INTRODUCTION.

This chapter begins by describing the principle assumptions involved with mathematically describing a surface marine vessel. These assumptions are employed to describe the basic mechanics, especially the forces, that are involved with a vessel's manoeuvrability both from external disturbances and from the application of control surfaces and in doing so establishes the importance of the centre of pressure. The centre of pressure is then utilised in the theory in the following chapters.

6.2 BASIC ASSUMPTIONS INVOLVED WITH THE MATHEMATICAL REPRESENTATION OF SURFACE MARINE VEHICLES.

The design and construction of surface marine vehicles capitalise upon the properties of fluid medium in which they operate, so that these, the largest of man made structures, may be supported and remain in an upright stable condition and float. As a result of this, surface marine vehicles possess six degrees of freedom; three degrees of linear motion: surge, sway and heave and three degrees of circular motion: roll, yaw and pitch. The motions of pitch, roll and heave, even though the most uncomfortable motions for humans, can in many applications be considered negligible in comparison to surge sway and yaw. A surface marine vehicle is designed not to plunge, capsize or bodily sink but operate in the horizontal plane where pitch, roll and heave are generally a result of the external forces due to wind, waves and tidal effects acting upon the vessel. This permits a surface marine vehicle to be considered as only possessing three degrees of motions: surge, sway and yaw as shown in Figure 6.1.
The three degrees of motion.

Furthermore, a surface marine vehicle can also be considered as two inter-related bodies operating at the interface between two fluid mediums, that of water and air, where the hull of a surface marine vehicle operates in the former and the superstructure operates in the latter. The primary objective of a marine vehicle is the transportation of commodities which is conducted at the interface between these two fluid mediums, where the properties of viscosity and density impede this task. Surface marine vehicles are designed to be streamlined so that they may carry out this task efficiently and effectively and therefore may be considered as both a hydrofoil, Gilmer (1982) and aerofoil Pourzanjani (1987), which will be referred to as a foil in the proceeding text as similar principles are involved.

6.3 THE ACTION AND MOTION OF A REGULAR SHAPED BODY ACTED UPON BY AN EXTERNAL FORCE.

A regular shaped body is considered in this context as an object that possesses a symmetry along both the longitudinal and lateral centrelines, and an even weight distribution throughout its entirety where the centroid of such a body is coincidental with the centre of mass.

A symmetrical body that possesses an even weight distribution when acted upon by a longitudinal and lateral force will cause the body to move in the direction
of the forces causing motion in both the directions of the forces, as shown in Figure 6.2.

![Diagram showing resultant force on a regular shaped body.](image)

Motion of a regular shaped body due to external forces. Figure 6.2.

These two forces which are considered to be generated externally from the body may be represented as a single resultant force that acts through the centroid of the body, which for a body that has an even weight distribution is the centre of gravity (G) as shown in Figure 6.3.

![Diagram showing resultant force on a regular shaped body.](image)

Diagram showing resultant force on a regular shaped body. Figure 6.3.

Then the motion of such a body is in the direction of the resultant force.

If the centre of gravity is not the centroid of the body, due to an uneven distribution of weight or if the distribution of the forces acting on the body are not equal, then the resultant force does not act through the centre of gravity but through some other point creating a turning moment about the centre of gravity. This turning moment in association with the longitudinal and lateral forces cause the body to...
rotate whilst moving both longitudinally and laterally, as shown in Figure 6.4.

Motion of an irregular shaped body due to external forces. Figure 6.4.

These two forces and the turning moment can be represented as a single resultant force that acts through some point other than the centre of gravity as shown in Figure 6.5.

Diagram showing resultant force on an irregular shaped body. Figure 6.5.

If this object was held at the centre of gravity and free to rotate, this resultant force would cause the object to spin. As the object possesses six degrees of freedom then this resultant force has components of both the lateral and longitudinal forces and a turning moment that produces the motions of surge, sway and yaw.
6.4 THE ACTION AND MOTION OF AN IRREGULAR SHAPED BODY
ACTED UPON BY AN EXTERNAL FORCE.

An irregular shaped body is considered either as an object that does not possess longitudinal or transverse symmetry or as a body that does not possess an even weight distribution. In either situation the centroid of an irregular shaped body is not coincidental with the centre of gravity. The position that the resultant force acts through, for a body that possesses six degrees of freedom, is primarily concerned with the shape and form of the body. Therefore an irregular shaped body with an even weight distribution could have the resultant force acting through a point other than the centre of gravity. An aerofoil or hydrofoil section that is not symmetrical about the mid-section centreline but symmetrical about the longitudinal centreline is in this context termed an irregular shaped body and is referred to as a foil, as shown in Figure 6.6.

![Diagram showing the shape and form of a foil.](Image)

Figure 6.6.

The only forces which act upon an irregular shaped body when moving longitudinally through a fluid medium are the propulsive force and the drag force, both of which act along the longitudinal centreline and pass through the centre of gravity, as shown in Figure 6.7.

![The primary forces in action on a foil.](Image)

Figure 6.7.
Disturbing this condition requires an external force to be applied, and as described for a regular body in Section 6.2, the resultant force will then act through a point other than the centre of gravity, producing longitudinal and lateral forces and a turning moment that results in the body possessing the motions of surge, sway and yaw.

### 6.5 THE MOTION OF AN IRREGULAR SHAPED BODY IN A FLUID MEDIUM UNDER THE ACTION OF CONTROL SURFACES.

An external lateral force may be produced by the application of control surfaces. For example, tail fins cause a rotation of the body about the centre of gravity, as shown in Figure 6.8.

![The application of control surfaces to a foil](image)

*Figure 6.8.*

The tail fin may be considered as a smaller foil situated at the rear of the main foil. When this control surface is applied to an angle of attack, as shown above, a difference in the velocity of the fluid flowing over the lower and upper surfaces is created. This difference in the two fluid flows creates a pressure difference between the two surfaces that produces a resultant force in the direction as illustrated above.

The external lateral force caused by the application of the tail fin only disturbs the body causing an angle of attack to be produced between the instantaneous velocity (V) and the longitudinal centreline of the body. Once the body has been disturbed and the angle of attack created, the body no longer remains symmetrical about its direction of motion and an uneven pressure distribution is formed around
the body which is produced by the passage of the fluid medium past the body. This uneven pressure distribution is generally expressed as the two forces of lift and drag Houghton and Carruthers (1982), as illustrated in Figure 6.9.

![Graphical representation of lift and drag forces](image)

*The graphical representation of lift and drag forces.* Figure 6.9.

The magnitude of these two forces, lift and drag, are taken as being dependent upon the angle of attack and the properties of the fluid medium, but independent of the velocity. Experiments conducted in wind tunnels and circulating water channels, upon foil sections Houghton *et al* (1982) and Saunders (1965) respectively, has lead to this information being presented in a standard format as illustrated in Figure 6.10.

![Variation of lift and drag with angle of attack](image)

*The variation of lift and drag with angle of attack.* Figure 6.10.

The resultant of these two forces that acts upon this body does not necessarily pass through the centre of gravity but some other point, causing the body to possess lateral and longitudinal forces and a turning moment about the centre of gravity, that result in variations in the motions of surge, sway and yaw.
6.6 THE CENTRE OF PRESSURE.

The point through which the resultant force from both the lift and drag forces acts is known as the centre of pressure and has been well documented both in the aerospace industry by Anderson ed. (1991) and nautical industry by Lammeren ed., Troost and Koning (1948) for aerofoil and hydrofoil shaped objects respectively. It has been found to be dependent upon the shape and form of the body and the angle of attack.

The construction of aerofoils and hydrofoils does not necessarily lead to the centre of gravity being the centroid of the body or the centre of pressure and centre of gravity being coincident, even though an even weight distribution may be possessed. Therefore the lift and drag forces may be replaced by a single resultant force acting through the centre of pressure, as shown in Figure 6.11.

This resultant force acting through some other point which is not coincident with the centre of gravity will cause the body to move laterally, longitudinally and to rotate simultaneously, as previously described for a regular shaped body.

The position of the centre of pressure according to Houghton et al (1982) from the aerospace industry is given as:

"The point about which the pitching moment coefficient is zero, and that the aerodynamic effects on the aerofoil section may be represented by the lift and drag forces alone."
This may be taken as the same for the nautical industry when considering hydrofoils, where the pitching moment described in the above statement refers to the vertical motion of the foil. For all the other planes of motion there then exists similar centres of pressure through which the resultant force acting upon this body may be taken.

The location of the centre of pressure for a foil is generally stated as forwards of both the centre of gravity and of the mid-section station situated on the fore and aft centreline (chord), Lammeren ed. et al (1948). The centre of pressure can be considered as only existing when the foil is placed at an angle of attack and has been found to progressively move along the chord as the angle of attack increases until stall occurs.

This resultant force, acting not through the centre of gravity but through the centre of pressure, will cause the body to move laterally, longitudinally and to rotate simultaneously, possessing the motions of surge, sway and yaw as previously described for a regular shaped body.
Chapter Seven.

THE DRIFT ANGLE THEORY.

7.1 INTRODUCTION.

The previous chapter introduced and described the generation of lateral and longitudinal forces upon three different bodies that combine to produce a resultant force that acts through the centre of pressure. This following chapter utilizes this resultant force and the fact that it acts through the centre of pressure to derive a relationship between the rate of rotation of a body about a point of origin and the magnitude of the drift angle, a definition of which detailed in Appendix E. This relationship, which is referred to as the drift angle theory, is then discussed with a view to indicating how the manoeuvrability and handling characteristics of marine vehicles may be described.

The drift angle theory is then implemented so that a marine vehicle's turning circle ability may be quantified by calculation of the turning circle diameters for a Mariner hull class tanker, the USS Compass Island, about which where there exists sufficient full scale data that supports and validates these predictions.

7.2 THE IMPORTANCE OF THE CENTRE OF PRESSURE AND THE DRIFT ANGLE.

By definition the resultant force that acts upon a body whose centre of gravity and centre of pressure do not coincide causes the heading of the body to change. If a constant resultant force is applied then the heading of the body will change at a constant rate and, as the body is free to move, then the body will scribe a circular path as shown in Figure 7.1.
Diagram showing the circular path of a body where the resultant force acts through the centre of pressure. Figure 7.1.

This resultant force which acts through the centre of pressure, is perpendicular to the longitudinal centreline of the body along which the centre of gravity is also situated is shown in Figure 7.2.

Diagram showing the position of the centre of pressure. Figure 7.2.

The angle that is created between the centreline of the body and the instantaneous tangential velocity (V) is referred to as the angle of drift or drift angle (β), Lammeren ed. et al (1948) and represents a change in the heading of the body. When this drift angle is constant for a given period of time then the heading of the
body is constantly changing and the body will move in a circular path, Saunders (1965), as illustrated graphically in Figure 7.3.

\[ \beta = \text{Drift angle} \]
\[ \Theta = \text{Angle GOP} \]
\[ P = \text{Centre of pressure} \]
\[ G = \text{Centre of pressure} \]
\[ O = \text{Centre of curvature} \]
\[ V = \text{Instantaneous velocity} \]
\[ R = \text{Radius of curvature} \]

*Diagram showing the geometry of the centre of pressure.*  

Figure 7.3

This body may then be taken as rotating about the origin (O) of a circle of radius (R). A perpendicular line to the instantaneous tangential velocity (V), originating at the centre of gravity (G), and a perpendicular line to the centreline of the body at the centre of pressure (P) intercept at a point that is the centre of a circle scribed by the centre of gravity.

With reference to Figure 7.3, the two perpendicular lines that intercept at the centre of curvature (O), creates the angle GOP (\( \Theta \)), then if a constant resultant force acts upon this body over a period of time, then this body will continue to move in the circular path. As this body is moving in a circular path about the centre of curvature with respect to time, then this angle GOP (\( \Theta \)) can be expressed as proportional to the circular velocity (\( \omega \)) of this body about the centre of curvature:

\[ \Theta \propto \omega \]

Equation 7.1.
This equation may be transformed from a proportionality equation to an equivalence equation via incorporating a constant of proportionality (K):

\[ K \cdot \Theta = \omega \]

Equation 7.2.

From dimensional analysis, circular velocity is the relationship between the rate of change of an angle when moving in a circular path with regards to the rate of change of time required for this angle to change, which is expressed as:

\[ \frac{\delta \Theta}{\delta t} = \omega \]

Equation 7.3.

The units of the numerator on the left hand side of Equation 7.3 are radians and the denominator are seconds, therefore for Equation 7.2 to be valid the constant of proportionality (K) must possess units of time, and so can be written as follows:

\[ \frac{\Theta}{K} = \omega \]

Equation 7.4.

The following diagram, Figure 7.4, which is a simplified version of Figure 7.3 permits by the use of similar triangles the angle GOB (Θ), to be compared with the drift angle (β).

\[ G = \text{Centre of gravity} \]
\[ P = \text{Centre of pressure} \]
\[ O = \text{Centre of curvature} \]
\[ V = \text{instantaneous velocity} \]
\[ R = \text{Radius of curvature} \]
\[ \beta = \text{Drift angle} \]
\[ \Theta = \text{Angle GOP} \]

*The geometry of the centre of pressure.*

Figure 7.4.

Page 96.
As the resultant force acts through the centre of pressure (P) and is perpendicular to
the centreline along which centre of gravity (G) is also situated and that the bodies
instantaneous velocity (v) is perpendicular to the radius of curvature, then the angles
GOP (Θ) and the drift angle (β) can be seen, from the use similar triangles, to be
equal to each other, and written mathematically as:

Θ = β

Equation 7.5.

Substituting the drift angle (β) from Equation 7.5 for the angle GOP (Θ) from
Equation 7.4, then the rate of rotation of the body about the centre of curvature can
be related to the drift angle (β) by the following equation.

\[ \frac{\beta}{K} = \omega \]

Equation 7.6.

Furthermore, if the constant of proportionality (K) is taken as one (ie one
second) then the drift angle can be directly related to the rate of rotation about the
centre of curvature, as shown in Equation 7.7:

\[ \beta = \omega t \]

Equation 7.7.

Expanding the meaning of the drift angle (β) to comply with the fact that the
constant of proportionality (K) is equal to one second, then the drift angle becomes
the instantaneous drift angle. This is related to the change in heading or direction of
the body for the duration of one second, which is in fact the rate of change of heading
of the body with respect to time.

This is supported by the fact that when an irregular shaped body traverses a
circular path through one complete revolution about a point of curvature then the
body returns to the same position and possesses the same orientation. This can be
summarised by the following statement:
"For a body that possesses six degrees of freedom, the rate of change in the horizontal drift angle is equal to the instantaneous horizontal circular velocity when turning in a circular path."

7.3 APPLICATION OF THE DRIFT ANGLE THEORY TO A MARINE VEHICLE.

A surface marine vehicle that does not possess an angle of list and that possesses straight-line stability (Maritime Technology Monograph, 1978) may be considered as a non-symmetrical body fore and aft of amidships but symmetrical about the longitudinal centre-line. It therefore possesses a form similar to that of a hydrofoil. A body that possesses an uneven weight distribution, primarily due to the location of the main propulsion system does not have the centre of gravity and centre of pressure in identical positions, but they will be located along the same longitudinal centreline of the vessel. Therefore, the resultant force acting upon a marine vessel will act through the centre of pressure causing it to possess both lateral and longitudinal forces and a turning moment, which as shown in the previous section, to generate a drift angle.

7.3.1 The Three Phase Turning Theory Of A Surface Marine Vehicle.

The rudder of a marine vehicle is normally taken as the method of changing a ships heading and may be considered as a foil passing through a fluid medium even though other control devices produce the same effect. When the rudder is applied to a given angle the fluid passing by it creates the forces of lift and drag due to the difference in the velocities over the lower and upper surfaces, Gilmer and Johnson (1982). The resultant of these two forces is commonly termed the rudder force, or disturbing force, and acts at a longitudinal distance from the centre of gravity. It causes the stern of the vessel to swing out and the bow in, thus a rotation about the centre of gravity is experienced and the vessel changes heading, Lammeren ed. et al (1948), as illustrated in Figure 7.5.
The initial rotation of a vessel due to the application of the rudder. Figure 7.5.

This initial application of the rudder and the associated rotation about the centre of gravity is referred to as the first phase of a ship during a turn Lammeren ed. et al (1948). Also, in this first phase the application of the rudder causes the vessel to heel into the turn, as described by Lewis ed. (1989), due to the rudder force acting at a vertical distance from the centre of gravity, as shown in Figure 7.6. This is more apparent the larger the vessel and is a result of the time required for the rudder forces and hull forces to reach equilibrium, this concept is discussed further in chapter thirteen.

Phase one, Initial roll motion of a vessel during a turn. Figure 7.6.

Phase two, or the transient stage of a ship during a turn, involves the hull of the vessel being considered as a foil and the flow of water past the hull then produces the lift and drag forces due to the different fluid velocities around the hull. This new flow regime around the hull is however not instantaneously developed but is dependent upon the length and speed of the vessel through the fluid medium; a long vessel will require a greater period of time for the new flow regime to be established than a
shorter vessel. This new flow regime once established then creates the forces of lift and drag the resultant of which then acts through the vessel's longitudinal centre of pressure as shown in Figure 7.7.

Diagram showing the generation of the hull forces. Figure 7.7.

The resultant force acting through the centre of pressure also acts through the vessel's vertical centre of pressure and causes the vessel to heel out of the turn once this transient stage has transformed into steady state as shown in Figure 7.8.

Phase three steady state roll motion during a turn. Figure 7.8.

The third phase of a marine vessel during a turn may be regarded as the steady state when the disturbing rudder force and the lift and drag forces generated by the hull are in equilibrium. When equilibrium is established between the disturbing rudder forces and the resultant hull forces they cause the vehicle to navigate a circular path at a constant rate of rotation about a point of curvature and to possess a constant rate of change of heading, as shown in Figure 7.9.
Disturbing rudder force

Diagram showing resultant force and centre of pressure for a vessel.

Figure 7.9.

This classical description of the three phases that a marine vehicle is considered to possess during a turning circle manoeuvre can be extended with reference to the generation of the drift angle. The two transient states that the vessel undergoes before steady state is established relates to the generation of a drift angle. During steady state turning, phase three, the magnitude of the drift angle becomes constant. The vessel then possesses a constant rate of change of heading, which is referred to as the yaw rate, and the vehicle traverses a circular path about a point of curvature, which is illustrated in Figure 7.10.

Approach course

Rudder execute, rotation about the centre of gravity.

Drift angle

Centre of gravity

Tangent to path

Diagram showing turning circle terminology.

Figure 7.10.

This three phase theory has only been referenced to a particular rudder angle that disturbs the vessel but can also be applied to all the rudder angles possible. Phase three is established only if the rudder angle is sustained for a sufficient period of time which is dependent upon the length of the hull and rudder and the speed of the water.
7.3.2 The Action Of External Environmental Disturbances Upon A Marine Vehicle.

Marine vehicles are not symmetrical fore and aft of the mid-section both above and below the water line, therefore the hull is acted upon by the forces due to the waves, tides and currents, whilst the superstructure is affected by wind. These forces do not necessarily act symmetrically in magnitude about the vessel or in direction therefore an unequal distribution of forces about the vessel generally causes the resultant force to act through a point other than the centre of gravity. This causes a turning moment and induces a change in heading, and so the vessel turns in a circular path as shown in Figure 7.11.

![Diagram showing action of external disturbances upon a vessel.](image)

Figure 7.11.

This external force will not only change the heading, but may also change the forward velocity of the vessel.

7.3.3 An Explanation Of A Dynamically Unstable Marine Vehicle By The Drift Angle Theory.

A marine vessel that does not possess an angle of list and that is geometrically similar both to port and starboard along the centre line and possesses a propeller or propellers that does not induce the paddle wheel effect may be considered as dynamically stable. This is not always possible due to slight variations in hull construction (the size of the weld or number of rivets), the location of various equipment along the hull (log and sonar), and the distribution of hull roughness along the hull (marine fouling, rust and plate buckling) all of which induce resistance to the
motion of the hull. If this is unequal both to port and starboard then unequal forces are established that cause the vessel to possess a turning moment and change its heading. This is also caused by the action of the propeller being inward or outward turning, twin or single screw, inducing the paddle wheel effect. If for a vessel, operating under perfectly still environmental conditions (mill pond conditions), these variations require the application of control devices to maintain a constant heading, but not necessarily along the original path, the vessel is said to be directionally unstable. Whilst a vessel is termed directionally stable if under the same conditions these control devices are not required to be applied to maintain a constant heading along an original path.

A marine vessel that is dynamically unstable will naturally possess a constant drift angle and therefore continually move in a circular path either to port or to starboard obeying the drift angle theory, depending upon the magnitude of the constant drift angle.

7.4 THE DRIFT ANGLE THEORY APPLIED TO DETERMINING THE TURNING CIRCLE DIAMETER OF A MARINE VEHICLE.

The final phase of the turning circle theory can be summarised, with regards to the drift angle theory as:

'The rate of rotation about the point of curvature is equal to the rate of change of heading for the vehicle during the same period of time.'

So for a time interval of one second, the vehicle would have rotated about the point of curvature by the same angle that the heading of the vehicle had changed. By definition this is the instantaneous drift angle, and as shown previously can be expressed mathematically as:

$$\omega = \dot{\beta}$$

Equation 7.8
This instantaneous drift or rate of change of heading for a marine vehicle is known as the yaw rate ($\dot{\beta}$) and can determined from a rate gyro or by differentiating the change in compass heading with regards to time.

Applying circular motion theory, Duncan (1982) to a marine vehicle traversing a circular path about a point of curvature permits the drift angle or yaw rate from Equation 7.8 to be employed in the following equation:

$$v = \omega r$$

Equation 7.9

Which is graphically illustrated for a regular shaped body rotating about a point of curvature as shown in Figure 7.12.

This results in the following equation:

$$v = \dot{\beta} r$$

Equation 7.10

As most marine vehicles are required by maritime law to possess both a compass and a log, then Equation 7.9 can be implemented so that the steady state turning circle diameter at any given speed and for any given rudder angle may be calculated. The magnitude of the turning circle diameter is only dependent upon the
speed of the vehicle and the magnitude of the drift angle, which can be illustrated as shown in Figure 7.13.

![Diagram showing turning circle magnitude.](image)

Figure 7.13.

This shows that from a common point of origin a vessel that possesses a large drift angle (yaw rate) will produce a turning circle whose diameter will be less than the same vessel at the same speed when the drift angle (yaw rate) is reduced. The opposite will also apply when the drift angle is constant and the speed changes.

### 7.4.1 The Maximum Drift Angle And The Minimum Turning Circle Diameter For A Marine Vehicle.

The rudder as a control device for marine vehicles is limited to a maximum angle of deployment. This is to prevent stall occurring and the rudder becoming inoperative, Clayton and Bishop (1982) and in extreme cases to prevent the drag force from shearing the rudder off from the stern post leaving the vehicle uncontrollable. This maximum rudder angle causes the vessel to turn in the tightest possible circle with a minimum diameter.

As described in Section 7.3, the rudder only disturbs the vehicle inducing lift and drag forces, where the resultant acts through the centre of pressure causing the generation of the drift angle and if the disturbing force is a maximum then the
resulting drift angle will also be a maximum. Therefore, the minimum turning circle
diameter ($D_{\text{min}}$) for a marine vehicle can be calculated from sea trials by evaluating
the maximum yaw rate ($\dot{\psi}_{\text{max}}$) which is also the maximum drift angle ($\beta_{\text{max}}$) and can
then be applied to the following equation:

$$D_{\text{min}} = \frac{2.7}{\frac{\dot{\psi}_{\text{max}}}{\beta_{\text{max}}}}$$

Equation 7.11.

This ability to determine a vessel's turning circle diameter can then be
employed as a method for quantifying and characterising the manoeuvrability of
marine vehicles.

7.4.2 Initial Results From Employing The Drift Angle Theory For Calculating
The Turning Circle Diameter Of A Marine Vehicle.

The previous sections derived, introduced and explained the drift angle theory
with respect to marine vehicles. This theory is implemented in the following section
so that a prediction of the, USS Compass Island's, manoeuvrability may be quantified
from basic data. Turning circle diameters are calculated for a range of engine speeds
and rudder angles. In order to validate this technique, results are compared with those
obtained from full scale sea trials for this vessel.

The Mariner hull class ship, the USS Compass Island, a five hundred and fifty
eight foot tanker, underwent extensive manoeuvring sea trials in 1961, as detailed by
Morse and Price (1961). The vessel was equipped with a data monitoring and
acquisition system. These trials consisted of the following manoeuvres: the
Dieudonne spiral; turning circles; the Kemf zig-zag manoeuvre; deceleration and
acceleration manoeuvres, all of which were conducted at various engine speeds and
rudder angles.

The turning circle manoeuvres consisted of the vessel steaming at constant
speed and the order to put the rudder to a predetermined angle was given. The surge,
sway and yaw velocities, rudder angle and engine speed during the manoeuvre were
recorded. This turning circle manoeuvre was repeated at each of the following approach speeds: five; ten; fifteen and twenty knots and for the following rudder angles: five; ten; twenty and thirty-five degrees both to port and starboard.

The following information was held within the report as single averaged values for each run: approach speed; desired rudder angle; actual rudder angle; surge; sway and yaw velocities; wind speed; actual engine speed, and the following concerning the size and magnitude of the turning circle: advance; transfer; tactical diameter; and final diameter. All this information was given in yards and feet which has been transferred into standard units SI, as shown in Table F.1 in Appendix F.

The drift angle theory, that has been introduced and then applied to marine vehicles in the preceding sections, has been shown to permit the turning circle diameter of a marine vehicle to be calculated from basic sea trial data. The information, held in Figure 7.14 concerning the USS Compass Island's turning circle trials, has permitted the drift angle theory to be implemented. Applying the modified circular motion equation, Equation 7.11, for the steady state turning circle diameter of the form:

\[
D = \frac{2V}{\beta}
\]

Equation 7.11

Substituting the average measured velocity, and drift angle (average yaw rate) for each of the turning circle trials conducted, the diameter for each respective turning circle was calculated.

These calculations were conducted upon a standard spreadsheet using SuperCalc5, and are shown in a tabulated form in Table 7.1 on the following page. Where the letter following the rudder angle relates to a port (P) or starboard (S) turn and are denoted as negative or positive respectively for both the averaged yaw rates and the turning circle diameters.
<table>
<thead>
<tr>
<th>Average Speed (kts.)</th>
<th>Rudder angle (deg)</th>
<th>Yaw rate (deg s⁻¹)</th>
<th>Calculated diameter (m)</th>
<th>Actual final diameter (m)</th>
<th>Difference (m)</th>
<th>Percentage error (%)</th>
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</thead>
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<tr>
<td>10.1</td>
<td>20P</td>
<td>-0.8</td>
<td>-745.05</td>
<td>-731.5</td>
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<td>-1202.4</td>
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<tr>
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<td>1422.7</td>
<td>1403.6</td>
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<td>1124.1</td>
<td>1133.9</td>
<td>9.8</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Measured and calculated turning circle diameters for the USS Compass Island.

Table 7.1

The percentage errors attributed to these calculated turning circle diameters were determined from dividing the difference between the calculated and actual measured diameters by the latter as shown by the following equation:

Page 108.
The percentage error column from Table 7.1 shows that all the calculated diameters were accurate to within ten percent of the official measured diameters and the majority were accurate to within three percent. The most accurate of the predictions for this vessel's turning circle diameter, a turn conducted at an approach speed of ten knots with a ten degree starboard rudder angle, relates to an error of less than three meters. This translates to less than the beam of the vessel, for a turning circle diameter greater than one kilometre.

7.4.3 Summary To The Determination Of A Marine Vehicles Turning Circle Diameter.

The error associated with the calculation of the turning circle diameters for this vessel, the USS Compass Island, for a variety of rudder angles and speeds can be attributed to the original data. The original report does not indicate whether or not the environmental conditions due to tides, currents, waves and wind have been taken into consideration. These average values can also be considered as causing error since the yaw rate was only given to an accuracy of two decimal places and the average speed values to only one decimal place.

Furthermore, the velocity measurement was conducted using four logs, none of which were placed at the centre of gravity. Therefore the average speed values employed to determine the turning circle diameters was not necessarily the vessel's actual instantaneous tangential velocity at the centre of gravity, but an averaged value at a point forwards of the centre of gravity.

7.5 CONCLUSION.

The existence of the drift angle has been demonstrated and related to the yaw rate, which is the basis for the drift angle theory. The principle of which is that for marine vehicle that possesses six degrees of freedom, the rate of rotation about a
point of curvature is equal to the rate of change of heading. The three phase turning theory that is used to describe, the turning characteristics of, marine vehicles has thus been extended to include the generation of the drift angle and that during steady state turning the drift angle and yaw rate are constant and equal. The drift angle theory also explains why some marine vehicles possess a degree of dynamically instability which could be used to quantify this characteristic.

The drift angle theory has also been related to a method by which a marine vehicles turning ability can be quantified in terms of the magnitude of a turning circles diameter. Furthermore, the drift angle for a marine vehicle has been described as being limited in magnitude which corresponds to a minimum turning circle diameter, due to the maximum rudder angle possible. The former application has been validated by calculation of the turning circle diameters for a Mariner class tanker, the USS Compass Island, with an accuracy of less than half a percent, which relates to an error of less than three meters for a turning circle diameter greater than fourteen hundred meters.

These results contained within this chapter have indicated that the drift angle theory possesses validity and stature as a method of describing, characterising and quantifying marine vehicle and is the basis for the remaining chapters in this thesis on mathematical modelling.
Chapter Eight.

THE TRACK HISTORY OF A MARINE VEHICLE BY THE DRIFT ANGLE THEORY.

8.1 INTRODUCTION.

The track of a marine vehicle is the path along which the centre of gravity of a ship moves and over a given period of time can be used to illustrate the route history for a marine vehicle. With reference to a known point this can show the location of the vehicle at or after a given time interval.

The drift angle theory is developed in this following chapter to allow the path of the centre of gravity for a marine vehicle to be described mathematically with reference to a set of cartesian co-ordinates. This method of describing the route history is then applied to a marine vehicle underway and is shown to be a valid navigational procedure for determining a vehicle's position and location when underway.

8.2 THE APPLICATION OF THE DRIFT ANGLE THEORY TO A ROUTE HISTORY OF A MARINE VEHICLE.

The drift angle theory states that the rate of rotation of a body about the point of curvature is equal to the rate of change of heading, shown in chapter seven to be the yaw rate of a marine vehicle, given this and the forward velocity of the vehicle a relationship to the diameter of a turning circle was provided.

To employ the drift angle theory for the purpose of mathematically modelling the track of a marine vehicle then the turning circle is considered as consisting of a series of arcs, as shown in Figure 8.1.
The turning circle illustrated as a series of arcs. Figure 8.1

Where the length of the radius is taken as solely dependent upon the velocity and yaw rate of the vehicle. Since the radius of turn is proportional to the velocity and inversely proportional to the yaw rate this is mathematically represented by Equation 7.4 from chapter seven:

\[ r = \frac{\nu}{\psi} \]

Equation 7.4

Then for a vehicle that possesses a large yaw rate the vehicle will turn more quickly than a vehicle that possesses a small yaw rate for a similar speed, and the faster the velocity the larger the radius of turn as shown in chapter seven. Furthermore, as an arc is a fraction of the total circumference of a circle then for a vehicle that changes its heading, which as the drift angle theory implies is the rate of rotation about a point of curvature, then the angle of the arc scribed in any one second is equal to the yaw rate. Where the circumference of a circle is calculated by the following equation:

\[ \text{Circumference} = 2\pi r \]

Equation 8.1
Then the length of the arc is dependent upon the yaw rate and the radius of turn as shown by:

$$\text{Arc length} = \Psi \cdot r$$

*Equation 8.2*

where the yaw rate ($\Psi$) has units of radians per second. This arc length can then be transformed into a distance moved in the X direction and the Y direction using the cartesian co-ordinates of the form X and Y as shown in Figure 8.2.

The vehicle has traversed from an origin at position A to position B at a constant velocity in a time interval of one second which leads to the yaw rate being equal to the drift angle, ($\Psi = \psi$) and the radii ($r_1$ and $r_2$) are also equal resulting in the distances in both the X and Y directions being described as follows:

$$X_1 = \sin(\psi) \cdot r_2$$

*Equation 8.3*

$$Y_1 = r_1 - \cos(\psi) \cdot r_2$$

*Equation 8.4*
Where the radii \( r_1 \) & \( r_2 \) are determined from Equation 8.4

The subsequent position of the vehicle after a further change in heading relative to position A, which is taken as the origin, in terms of these cartesian co-ordinates is no longer determined from Equations 8.3 and 8.4 as position B becomes the new origin, relative to the X and Y distances to position C as shown in Figure 8.3.

![Diagram](image-url)

The geometry of an arc from point B to point C relative to a set of X&Y axis. Figure 8.3

These new distances \( X_2 \) & \( Y_2 \) are then summed with the original \( X_1 \) & \( Y_1 \) distances to give the position of C relative to the origin at position A.

The magnitude of these new distances \( X_2 \) & \( Y_2 \) are determined from the geometry contained within Figure 8.4.
Diagram showing the geometry required for calculation of the X&Y distances traversed. Figure 8.4

Where the distances $X_2$ & $Y_2$ are found from the following two equations that are based upon the structure and geometry of the arcs shown in the Figure above.

\[ X_2 = BC \cos(\alpha) \]  
\[ Y_2 = BC \sin(\alpha) \]

Equation 8.5
Equation 8.6

Where the line $BC$ is the hypotenuse of the triangle $BCZ$ and is determined from the following relationships taken from Figure 8.4:

\[ ZC = \sin(\Psi_2).r_3 \]
\[ ZB = r_2 - \cos(\Psi_2).r_3 \]

\[ BC = \sqrt{ZC^2 + ZB^2} \]
and where the angle ($\alpha$) created by the intercept of the line $b'$ and the line $BC$ which is determined from the following:

$$\alpha = b' B x' + Z CB$$

Where

$$b' B x' = \Psi_1$$

and

$$Z CB = \tan^{-1}\left(\frac{ZB}{ZC}\right)$$

The subsequent X and Y distances for successive changes in heading are then calculated in the same manner with the values attributed to the X&Y co-ordinates being the cumulative total of these individual distances. If this procedure is repeated over a large time interval then the path of the centre of gravity of the vehicle can then be mapped with reference to a known origin, that of position A, as shown in Figure 8.5.

![Figure 8.5](image)

*An illustrated track history of a vehicle by a series of arcs.*

Where after a time interval of 'n' seconds the vehicle is at position 'n'.
8.3 A COMPUTER MODEL FOR THE TRACK HISTORY OF A MARINE VEHICLE.

A marine vehicle when underway may be considered in one of two states either as traversing a straight course along a known heading or as turning. The former situation can be mathematically modelled in terms of the vehicle's position after a given period of time has elapsed provided that the velocity of the vehicle is known. Since velocity is the distance travelled in a unit of time then for a marine vehicle whose velocity is known the distance travelled at this velocity can be calculated from the following equation:

\[ s = v \cdot t \]

where \( s \) = the distance travelled
\( v \) = the velocity of the vehicle
\( t \) = the time interval

The previous section illustrated a procedure for transforming the drift angle theory into cartesian co-ordinates where only the yaw rate and the instantaneous tangential velocity of the vehicle at the centre of gravity was required to be known. The ability to map the history of the vehicle is therefore only dependent upon the collection of the vehicle's velocity and heading with respect to time. This data must be in a suitable format and as most marine vehicles possess both a log and compass for conventional navigational purposes the information is readily available. To employ this theory however, the output from the log and compass, namely speed and yaw rate, are required to be in a digital format that can be processed and manipulated by a digital computer. This requires both the log and compass to be electronic with preferably a coded digital output.

This information from the log and compass can either be utilised as a real time navigational system whilst underway or as a record of the voyage providing the required information is stored in a suitable format. The digital computer is ideal for
converting the compass heading into the yaw rate and then for the transformation of
the yaw rate and speed of the vehicle into cartesian co-ordinates as it requires an
reiterative calculat i on. Where the individual distances both in the X&Y directions are
determined and summated the following diagram, Figure 8.6, illustrates the structure
of such a iterative program for use on a digital computer.

\[
\begin{align*}
\text{Radius of turn} & \quad r_i = \frac{\psi_i}{\Psi_i} \\
\text{IF } n = 0 \text{ then} & \quad X_i = 10 \cdot \sin(\psi_i) \\
\text{ELSE} & \quad \beta_i = \text{Heading relative} \\
& \quad \text{to original heading} \\
\text{IF } n > 1 \text{ then} & \quad X_i = r_i \cdot \sin(\psi_i) \\
& \quad Y_i = r_i \cdot \cos(\psi_i) \\
\text{Flow diagram showing the X}& \quad \text{Y} \\
& \quad \text{calculation of successive points.} \\
\end{align*}
\]

Figure 8.6

Page 118.
The only information required to determine the position of a vehicle, relative to a known point of origin is from a compass and a log.

### 8.4 APPLICATION OF THE TRACK HISTORY OF A MARINE VEHICLE TO PICKET BOAT NINE DURING A MANOEUVRE.

To validate this procedure for determining the track history of a marine vehicle when underway sea trials have been conducted in Picket Boat Nine, where only the forward velocity and heading of this vessel were measured with respect to time. The first sea trial consisted of a series of turning circle manoeuvres which were conducted in the River Dart, where Picket Boat Nine was set steaming at an engine speed 1200 revolutions per minute with the helm applied to twenty degrees to starboard. This rudder angle was maintained until three starboard turning circles had been completed where upon the rudder was then applied to twenty degrees to port for three complete turns.

To collect the required yaw rate and forward velocity during these sea trials, Picket Boat Nine was fitted with a commercial data monitoring and acquisition system that comprised of the Workbench 3.1 software package and associated hardware that was installed upon a stand alone IBM compatible personal computer. This software package permitted both digital and analogue electronic signals to be analysed and measured by simply connecting the output channels from various electronic equipment to the associated hardware. The software package then collected and correlated the information from these channels which permitted via a Windows operating system to manipulate the data. The heading of the vehicle was collected from a Seatrek flux gate compass which produced a sixteen character code that contained the required information and was in turn connected to the acquisition package via a standard RS232 nine pin serial interface port. The vehicle’s speed was measured by the use of a trailing edge log that was towed behind the vehicle and recorded by intercepting the voltage signal that was used to move the arm of a pointer across a scale. This analogue voltage had previously been calibrated for range of operating speeds via the use of a wind tunnel. Both the heading and forward velocity
of Picket Boat Nine were recorded with respect to the time interval between the measurements and stored upon a computer disk. These measurements of heading and forward velocity are illustrated graphically with respect to time, as shown in Figure 8.7.

![Graph of measured heading and forward velocity of Picket Boat Nine.](image)

This graph demonstrates that from an original heading of 245 degrees the heading of Picket Boat Nine increased as a linear function with respect to time to 360 degrees. At which point the heading changes to 000 degrees and again increases to 245 degrees which represents a complete turning circle as the heading of Picket Boat Nine has changed by three hundred and sixty degrees. This linear relationship is repeated for a further two turns and demonstrates that the rate of change of heading is constant during these starboard turns. After a time interval of one hundred and eighty seconds the gradient of the heading versus time profile changes and becomes negative which represents the application of the rudder from starboard to port. This is repeated over the following one hundred and seventy seconds during which time two and a half turns to port are completed.

The forward velocity of Picket Boat Nine during this manoeuvre averaged six and a half knots with a fluctuation about this mean value attributed to the
measurement process and the external disturbances that were in action during the sea trial.

To determine the X&Y co-ordinates relative to a known position from the compass heading and speed of the vessel the former was converted into the yaw rate of the vessel by determining the difference between successive measurements and dividing by the time interval between these two measurements, these values are illustrated graphically with respect to time as shown in Figure 8.8.

![Graph of calculated yaw rate and measured forward velocity of Picket Boat Nine.](Figure 8.8)

This diagram demonstrates that the yaw rate during the first one hundred and eighty seconds possesses a positive mean value of approximately 0.12 rad/s where a low frequency oscillation about this value is again attributed to the environmental disturbances. The high frequency oscillation observable in the yaw rate is a result of the compass only measuring the heading of the vessel to the nearest integer. After one hundred and eighty seconds has elapsed the yaw rate changes to a similar negative value which signifies the application of the helm from starboard to port. This negative yaw rate the oscillates in a similar manner, except that the low frequency oscillation possesses is greater amplitude which represents an increase in the magnitude of the external disturbances.
To conduct the transformation of the yaw rate and speed of the vessel into the series of X&Y co-ordinates, by the method described earlier in this chapter, a computer program, referred to as TRACK, was written in 'C' to perform the necessary series of calculations. By reading the forward velocity and yaw rate values for this manoeuvre into this computer program, distances traversed in the X and Y directions during each successive time interval for this manoeuvre were calculated. The results from this software produce a series of X&Y co-ordinates relative to a point of origin that are displayed as a two dimensional cartesian plot showing the track of the vessel during the manoeuvre as shown in Figure 8.9.

![Diagram](image)

_X&Y Track history of Picket Boat Nine for a series of port and starboard turning circles._

This X&Y track history of Picket Boat Nine illustrates that from a point of origin (0.0,0.0) the vessel conducted three turns to starboard in a clock wise direction from...
an initial heading of 245 degrees followed by two and a half turns to port in an
anti-clockwise direction.

The constant forward velocity and rate of change in heading that were
apparent in Figure 8.7 are demonstrated in the X&Y track history for this manoeuvre
by the fact that all the turning circles are all similar in diameter, and that the low
frequency oscillation that was apparent in the yaw rate and forward velocity profile,
Figure 8.8 which were attributed to the action of environmental disturbances upon
Picket Boat Nine, generates an offset to each of the consecutive turning circles in the
direction of 045 degrees.

8.5 VALIDATION OF THE TRACK HISTORY OF A MARINE VEHICLE
BY COMPARISON WITH CONVENTIONAL NAVIGATION AIDS.

To validate this Track History procedure for determining the position of a
marine vehicle during a manoeuvre from a known point of origin; a radio navigation
system (DECCA) and a satellite navigation system (Global Positioning System) were
installed onboard Picket Boat Nine; along with the necessary data acquisition system
to record the positional information from both pieces of equipment.

A second set of sea trials were conducted, which consisted of steaming Picket
Boat Nine at an engine speed of 1000 rpm with ten degrees of starboard helm applied
from an initial heading of 090 degrees. After the heading had changed to 270 degrees
the helm was altered so that Picket Boat Nine could steam along a constant heading
of 320 degrees. This constant heading was maintained for twenty five seconds; after
which the helm was applied to thirty degree to starboard and maintained whilst two
complete turns were executed. Throughout this manoeuvre positional data in the
form of Northerly's and Easterly's were recorded digitally from both the DECCA and
GPS navigational systems, whilst the forward velocity and heading were recorded via
the data monitoring and acquisition system previously described and transformed into
a series of X&Y co-ordinates by the afore mentioned software.
The recorded Northerly's and Easterly's from the DECCA Navigator are illustrated graphically as a two dimensional plot, as shown in Figure 8.10. A large turning circle is apparent whilst the two smaller turning circles are obscured by the lack of positional data which is a result of insufficient data points being recorded due to the slow update of the receiver.

Furthermore, with reference to the Ordnance Survey grid of the United Kingdom, these Northerly and Easterly measurements place this manoeuvre forty kilometres in error in a northerly direction and four kilometres in an easterly direction. This error has been attributed to the equipment receiving the wrong information from the DECCA chain as the trials were conducted in the shelter of the River Dart, where the banks of the river prevented a direct line of sight to the transmitters.
The northerly and easterly co-ordinates recorded from the GPS receiver are illustrated in a similar manner, as shown in Figure 8.11.

![Diagram showing GPS positional data for a manoeuvre of Picket Boat Nine.](https://example.com/figure8.11)

*GPS positional data for a manoeuvre of Picket Boat Nine.*

Figure 8.11

This demonstrates more clearly a large turn followed by two consecutive smaller turning circles than the DECCA data, Figure 8.10. The apparent spikes that occur with this data is attributable to the linear filter within GPS receiver. However, this GPS positional data, with reference to an Ordnance Survey Chart, sites the trials within the River Dart and therefore can be considered more accurate than the DECCA positional data.
The measured forward velocity and heading of Picket Boat Nine during this manoeuvre was transformed into a series of X&Y co-ordinates, by the method previously described, and is illustrated graphically as shown in Figure 8.12.

From a point of origin, Northerly 53075m Easterly 287700m (taken from the United Kingdom Ordnance Survey grid) the initial turn at ten degrees to starboard from a heading of 090 degree, followed by the approach heading of 320 degrees and the two consecutive turning circles at thirty degrees to starboard are displayed. This track history of Picket Boat Nine also indicates the environmental disturbances that were present during the manoeuvre as the two consecutive smaller turning circles possess an off set in an northerly direction.
This track history of Picket Boat Nine in comparison with the DECCA and the GPS positional data, is more representative of the manoeuvre that was conducted. A vessel when underway can only move in an uniform, smooth, regular un-fluctuating path. This is not represented by the DECCA data, but is illustrated by the GPS positional data when the spikes at the beginning of each successive series of data points are removed. Furthermore, the GPS data does show an offset in the second of the two smaller turning circles due to the environmental disturbances that the track history exhibits, with the former of the two circles being smaller than the latter.

The DECCA and GPS positional data then places the two consecutive turning circles at a distance from the point of origin, whilst the track history indicates that the two turning circles occurred at a point very close to the point of origin. This discrepancy is either a result of the residual error that accompanies both navigational aids or is due to the environmental disturbances physically moving the vessel without altering the forward velocity or rate of change of heading. This is considered possible as the forward velocity measured is the through the water and not over the ground; where the velocity measurement may not include a tidal current, which physically moves the vessel and the trailing log simultaneously. In conclusion the position of a marine vehicle when underway during a manoeuvre can be determined by the track history method.

To improve upon the accuracy of measuring the position of Picket Boat Nine during a manoeuvre so that this method of determining the track history could be validated further, two portable (microwave) Trisponder transmitters were positioned so that the receiver, on board Picket Boat Nine, would continuously be in a direct line of sight during the third set of sea trials. The Trisponder receiver was set up so that the positional data would be recorded digitally to computer every second during the manoeuvre. The manoeuvre conducted consisted of steaming Picket Boat Nine upon a heading of 355 degrees and then the helms man proceeded to conduct half a turn to starboard followed by a turn to port without given instructions for demanded engine speed or rudder angle. The forward velocity and heading of Picket Boat Nine during the manoeuvre were recorded digitally, as described previously. At the start of the
manoeuvre both data acquisition systems were initiated simultaneously and three hundred and fifty eight lines of data were recorded from each, which relates to approximately six minutes of steaming.

The heading of Picket Boat Nine was then converted into the yaw rate and is illustrated graphically with the measured forward velocity as shown in Figure 8.13. Throughout the manoeuvre an average forward velocity of four knots was maintained and the yaw rate changes from being positive during the starboard turn (as shown by time interval A-B in Figure 8.13) and becomes negative during the turn to port (time interval B-C). After two hundred and sixty seconds the yaw rate changes and becomes positive over a period of thirty seconds (time interval C-D) and then returns to being negative for the following one hundred seconds (time interval D-E) and then oscillates from being positive to negative for the remaining part of the manoeuvre.

The forward velocity and yaw rate values for this third manoeuvre were then transformed by the Track software into a series of X&Y co-ordinates which are plotted with the Trisponder positional data as shown in Figure 8.14, which illustrates the manoeuvre conducted with regards to a common origin (0.0,0.0). The track history of Picket Boat Nine during this manoeuvre follows a uniform, smooth, regular un-fluctuating path whilst the Trisponder data points demonstrate a degree of scattering about such a path where both agree strongly with each other. However the maximum difference between the two data sets during any part of the manoeuvre is twenty meters and as the Trisponder system has a standard deviation of three meters (assuming the system was calibrated accurately) this therefore indicates that the track history is also in error. This error can be attributed to the heading of the vessel only being measured to the nearest integer, the use of a trailing log to determine the forward velocity and the presence of external disturbances that did not register on either the compass or the trailing log.
Plot measured forward velocity and yaw rate for a manoeuvre of Picket Boat Nine.

Figure 8.13

X&Y Track history for a manoeuvre of Picket Boat Nine.

Figure 8.14

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8.6 CONCLUSION TO THE TRACK HISTORY OF A MARINE VEHICLE BY THE DRIFT ANGLE THEORY.

The transformation of the yaw rate and forward velocity into a route history of a marine vehicle when underway has been demonstrated by the use of DECCA and GPS navigational systems and a Trisponder positioning system. The sea trials conducted with the DECCA and GPS navigational systems have already been discussed and concluded in that; the former was erroneous due to sheltering from the bank of the River Dart and that the results from the latter indicated that this application of the drift angle theory could indeed be implemented so that the path of a marine vehicle could be determined. The Trisponder positioning system, with an accuracy of less than 10m, proved the validity of this method of determining the track history of Picket Boat Nine as during a manoeuvre very similar positional data was recorded, which was illustrated graphically in Figure 8.14, even though error between them was still apparent. The difference between the results obtained indicates that external disturbances may have been present but were not detected by the compass and log, thus causing Picket Boat Nine to possess the motion of sway. The importance and significance of the motion of sway to this method is not known.

The sea trials conducted prove that the track of a marine vehicle, in terms of distances traversed in an X&Y direction, can be determined from information concerning the forward velocity and compass alone to an accuracy acceptable for this programme of research.
Chapter Nine.

A MATHEMATICAL MODEL OF MARINE VEHICLES FOUNDED UPON THE DRIFT ANGLE THEORY.

9.1 INTRODUCTION.

Mathematical modelling of marine vehicles concerns the ability to simulate and replicate the track, course, speed and yaw rate of such vehicles on the digital computer when only the commands governing the engine speed and demanded rudder angle are known. The drift angle theory which was introduced and implemented in chapter seven, and applied to the track history of a marine vehicle when underway in the previous chapter, can be utilised as the basis for a mathematical model for marine vehicle.

This following chapter describes a method and possible structure for such a mathematical model employing this new theory which is then implemented in the following chapters to simulate, replicate and predict the track and course of a particular marine vehicle given only fundamental data, namely the engine speed and rudder angle.

9.2 THE REQUIREMENTS OF A MATHEMATICAL MODEL

A mathematical model of a marine vehicle for the purposes and applications described in chapter one, should by definition be able to replicate, simulate and predict the motions of the marine vehicle given only the inputs of rudder angle, engine speed and all the relevant information about the external disturbances that affect the vehicle. For a true and accurate mathematical model to be established and suitable to perform these tasks the engine speed, rudder angle and environmental disturbances have to be related to the response of the vehicle, so that the state of the vehicle with regards to the actual engine speed, rudder angle, heading, velocity, and
position can be determined with respect to time. A schematic representation of this form of mathematical model is illustrated in Figure 9.1.

This is a repetitive process with the control unit being either controlled by a human operator or a computer which may demand an alteration in the engine speed or rudder angle and where the environmental conditions may change over a period of time. Any variation in these parameters will cause the vehicle to alter course and track thus requiring re-computation of the vehicle’s speed, heading, and position.
9.3 THE DRIFT ANGLE THEORY IN TERMS OF STEADY STATE MATHEMATICAL MODELLING.

The previous chapter illustrated how the drift angle theory could be implemented for the determination of a marine vehicle's position from actual information concerning a vehicle's log and compass. Therefore for an accurate mathematical model to be developed so that the tasks described in chapter one may be conducted and the requirements set out above fulfilled. The speed of the vehicle and yaw rate are required to be related to the two control inputs engine speed and rudder angle.

The speed of the vehicle through the fluid medium in which it operates can be related to the speed of the engine as illustrated in Figure 9.2.

![Diagram showing variation of engine speed with forward velocity](Figure 9.2)

Where the steady state speed of a vehicle at a given engine speed can be determined for a given vehicle by conducting the measured mile manoeuvre over the range of operating speeds for the vehicle, which as described in chapter three, can be mathematically described by a polynomial of a suitable order. This does not however, describe the transient state, where the vehicle accelerates from a steady state speed to another steady state speed, which will be covered in section 9.4.
Similarly the yaw rate can be related to the rudder angle by conducting a series of turning circle manoeuvres at a constant approach speed for the range of rudder angles possible. The yaw rate at a given rudder angle is measured during steady state turning, which can be graphically illustrated as shown in Figure 9.3.

![Diagram showing relationship between rudder angle and yaw rate.](image)

For a given rudder angle and approach speed the steady state yaw rate can be determined mathematically by fitting a suitable polynomial to that data. As with the mathematical description of the speed of the vehicle the transient phase where the drift angle develops and the yaw rate increases up to a maximum is not described. Furthermore, as the development of the drift angle is dependent upon the generation of the lift and drag hull forces, as described in chapter six, then this drag force causes a reduction in the vehicle’s forward velocity. This reduction in the forward velocity becomes a minimum during the steady state phase of turning and the vehicle possesses a constant velocity with a magnitude less than that predicted from the method described above for the determination the vehicle’s speed at a given engine speed. This reduction in the vehicle’s forward velocity due to the application of the rudder and the generation of the drift angle can be determined by measuring the speed of the vehicle during the turning circle manoeuvres that are conducted for the determination of the yaw rate described above.
As the drift angle is dependent upon the rudder angle then so too is this reduction in the vehicle's forward velocity also a consequence of this drag force, even though the engine speed is a constant. This relationship can be illustrated graphically as shown in Figure 9.4.

![Figure 9.4](image)

*Figure 9.4. Yaw rate and forward velocity variation with rudder angle for a single engine speed.*

For a given rudder angle and approach speed the steady state yaw rate and forward velocity of a vehicle can be determined and mathematically described via the use of suitable polynomials.

If a series of turning circle manoeuvres were conducted for the range of engine speeds possible then the steady state yaw rate, approach speed and the final forward velocity can be determined for the range of rudder angles possible. Then the magnitude of the yaw rate and the forward velocity of the vehicle can be related to the control inputs of rudder angle and engine speed as shown in Figure 9.5.
An illustration of five hydrodynamic curves. Figure 9.5

Where the numbers 1, 2, 3, 4 and 5 in the above diagram represent a series of five turning circle manoeuvres conducted at five different engine speeds that relate to five approach speeds when the rudders are at amidships, in this case:

1. = five knots;
2. = ten knots;
3. = fifteen knots;
4. = twenty knots;
5. = twenty five knots.

Each of the five manoeuvres would have consisted of altering the rudder angle incrementally from hard to port to hard to starboard, and measuring the steady state yaw rate and final velocity. The curves are then produced by plotting the steady state measured yaw rate against the measured final forward velocity for the vehicle with the rudder angle for each manoeuvre plotted against same the forward velocity.
With reference to Figure 9.5, by entering the curves at a given engine speed at position A, which intercepts curve 4 at position B and relates to the forward velocity of the vehicle when the rudders are at amidships, the approach speed of the vehicle is found at position C. Then without the rudders being applied or the engine speed being altered the vehicle will travel at this speed.

When the rudders are applied the vehicle begins to turn and a reduction in speed is experienced. The final velocity and yaw rate for a given rudder angle can also be determined by entering these curves. Position D represents a port rudder angle, which intercepts curve 4 at position E and a perpendicular line to DE then intercepts the solid line at position F. By continuing along this perpendicular line the forward velocity axis is intercepted at position H, which relates to the final forward velocity of the vehicle during steady state turning. Furthermore, a horizontal line through position F cuts the right hand axis at position G which relates to the vehicle's steady state yaw rate. Therefore the yaw rate and final forward velocity for a vehicle whose engines are set to a given speed and whose rudders are applied to a known angle can be determined once a set of curves have been produced for a particular vehicle.

The USS Compass Island sea trial results that were employed in chapter thirteen for the determination of the steady state turning circle diameter for a given velocity and yaw rate have also been utilised for the generation of a set of hydrodynamic curves based upon these actual results as shown in Figure G.1 (held in Appendix G). The USS Compass Island being a single screw vessel is therefore dynamically unstable and possesses the paddle wheel effect which is illustrated by the fact that these hydrodynamic curves are not symmetrical for both port and starboard turns.

The hydrodynamic curves, as illustrated in Figure 9.5 and Figure G.1, can be mathematically described by fitting a polynomial of a suitable order of the form:

\[ y = a_0 + a_1 X + a_2 X^2 + a_3 X^3 + \ldots + a_n X^n \]
to each individual set of data for each approach speed considered. In doing so the above described procedure for determining the vehicle's final velocity and steady state yaw rate from only the information concerning the engine speed and rudder angle can be conducted mathematically.

Examination of the USS Compass Island results have indicated that for this particular vessel the reduction in final velocity with respect to the rudder angle is a close approximation to a linear relationship as shown in Figure G.2 (Appendix G). This can be mathematically expressed by the following equation:

\[ V_f = c - m(\alpha_f) \]
Equation 9.1

where \( C \) is the intercept upon the \( Y \) (\( V_r \)) axis which relates to the forward velocity (approach speed) of this vessel when the rudders are at amidships for this particular engine speed. Hence for a given rudder angle the final velocity can be determined. This final forward velocity can then be employed to determine the vehicle's steady state yaw rate by the following equation:

\[ (\psi_f)^2 = a - b(V_f) \]
Equation 9.2

which appears as shown in Figure G.3 (Appendix G) to be a reasonable approximation to the plot of final forward velocity against yaw rate. By combining the engine speed versus forward velocity relationship, Equation 9.1, and both of the above two equations, Equations 9.1 and Equation 9.2, the final forward velocity and yaw rate can be determined from information about the engine speed and the rudder angle.

These curves however only describe the steady state turning of a marine vehicle and so can only be practically utilised for the determination of a marine vehicle's minimum turning circle diameter as the transient phase when the yaw rate is increasing and the speed of the vehicle is decreasing is not described. These curves
can be employed as the basis for a mathematical model of a marine vehicle, as one of the requirements for a mathematical model is for the position of the vehicle. The forward velocity and heading of the vehicle to be determined when only the demanded rudder angle and engine speed are known.

9.4 MATHEMATICAL MODELLING OF THE TRANSIENT PHASE.

The transient phase of a marine vehicle's manoeuvrability is concerned with the first two phases that were described in chapter seven, and explains the processes that the vehicle experiences during the time period from when the rudders are applied to when the steady state yaw rate and final forward velocity are achieved.

As a result of maritime convention, the application of the rudders to a given angle can be taken as occurring over a short period of time. The order to alter the helm is put into action in one swift clean motion of the wheel by the helmsman. The lift and drag forces are then generated upon the rudder and cause a rotation about the centre of gravity which in turn begins the development of the lift and drag forces upon the hull. Equilibrium is only achieved when the disturbing rudder forces equal the hull forces. The time required for the generation of these forces both on the rudder and the hull can be assumed to dependent upon the length of these surfaces and the speed of the fluid medium past them. The greater the length of the vehicle the more time required for the flow regime to establish the forces of lift and drag, as illustrated in Figure 9.6.

![An illustration of length and speed dependency of flow regimes.](Figure 9.6)
From Figure 9.6, the larger body possesses a length seven times greater than that of the smaller one, both are travelling at the same speed, then by the time the fluid medium has passed from bow to the stern of the smaller body, then the fluid medium would only have passed a seventh of the larger body's length. Therefore, it can be assumed that the smaller body will have established the new flow regime and generated the forces of lift and drag in approximately a seventh of the time required for the larger body.

Applying this approximation for the time interval between when the rudders are applied and the time required for a marine vehicle to fully develop the forces of lift and drag can be illustrated graphically as shown in Figure 9.7.

![Diagram showing yaw rate and rudder angle variation with time.](Figure 9.7)

Where the rudder angle changes in the time interval \((t_1-t_0)\) and the steady state yaw rate is established after the time interval \((t_2-t_0)\). This assumption can also be employed for the reverse when the rudder angle is reduced or returned to amidships. This transient phase is also dependent upon the magnitude of the rudder angle deployed. The greater the rudder angle, the larger the disturbing force that will cause the vehicle to possess an initial rotation about the centre of gravity before the lift and drag hull forces can be produced to counteract and produce equilibrium during the steady state. This dependency upon the magnitude of the rudder angle employed can be illustrated graphically for the range of rudder angles possible as shown in Figure 9.8.
A large rudder angle can cause an excessive rotation about the centre of gravity that causes the yaw rate of the vehicle to exceed that of the steady state yaw rate as a result of the hull forces not being developed in a sufficient time interval. This can counteract the disturbing rudder force that causes the initial rotation about the vehicle's centre of gravity. The yaw rate in such a situation then follows an underdamped oscillation with a single overshoot. The two other rudder angles depicted in the diagram illustrate that in the majority of rudder angles deployed the yaw rate follows an overdamped situation and that for all the rudder angles possible a different yaw rate response occurs. Therefore each rudder angle and rate at which the rudder is deployed would have to be mathematically described individually. This relationship between the rudder angle and yaw rate with respect to time is illustrated for the USS Compass Island as shown in Figure G.4 (Appendix G). This shows the yaw rate and rudder angle with respect to time for three turning circles conducted at an approach speed of twenty knots with the rudder put over to ten, twenty and thirty-five degrees respectively.

The reduction in a vehicle's velocity due to the generation of the drag forces upon the hull requires a time interval to reach a minimum during steady state turning. The application of the rudder compared to the reduction in the vehicle's velocity can be graphically illustrated as shown in Figure 9.9.
Diagram showing rudder angle and forward velocity variation with respect to time.

Figure 9.9

where the rudder application can be considered as occurring over a small period of time compared to the reduction in velocity for same reasons as mention above. When the rudder angle is reduced or positioned at amidships again the vehicle's velocity will increase in a reverse manner, assuming that the engine speed remains constant. This relationship between the final forward velocity and the rudder angle with respect to time for the USS Compass Island sea trials is shown in Figure G.5 (Appendix G) for the same rudder angles as mentioned above.

The engine speed of a marine vehicle, as with the application of the rudders described above, can be considered as a rapid change from one setting to another. The reduction or increase in the vehicle's velocity due to a change in the engine speed and can be considered as occurring over a period of time. The engine speed if increased will cause the vehicle to accelerate until equilibrium is achieved between the thrust produced by the increase in propeller speed and the drag generated by the vehicle as it traverses through the fluid media in which it operates. Therefore a similar methodology for mathematically describing the response of the vehicle to a change in the engine speed is required to be employed.
This research programme does not attempt to conduct this mathematical modelling of the yaw rate or reduction in speed concerned with the application of the rudders or the change in velocity of the vehicle due to an alteration in the engine speed with regards to the transient phase. This transient phase of a marine vehicle when the rudders are applied or the engine speed changes can be considered as the change from one steady state to another. This change from one steady state to another will occur over a period of time therefore response times are employed to model such changes. This programme of research employs linear approximations to mathematically describe these changes from one steady state to another by the use of the following equation:

\[ X_t = X_{t-1} \pm \Delta X \]

Equation 9.3

where \( X_t \) is the transient value attributed to either the engine speed, forward velocity, or the yaw rate; \( X_{t-1} \) is the value of either of these before the change in control parameters was executed, and \( \Delta X \) is the increment or decrement associated with the change in control parameters with respect to time.

9.5 THE STRUCTURE OF A MATHEMATICAL MODEL BASED UPON THE DRIFT ANGLE THEORY.

The specifications of a mathematical model of a marine vehicle, can be summarised as the requirement to determine the vehicles: Speed, Heading, Position, Yaw rate Engine speed and Rudder angle at discrete time intervals when only the latter two are actually known. This can be achieved by combining the drift angle theory, the mathematical sequence for the transformation of the vehicles yaw rate and speed in to \( X \& Y \) co-ordinates that was described in chapter eight and the hydrodynamic curves described in this chapter. This following section outlines the possible structure for a mathematical model of a marine vehicle based upon these
strategies but does not attempt to include external environmental disturbances that act upon the vehicle.

The structure of this mathematical model requires that a forward velocity versus engine speed curve when the rudders are at amidships and a complete set of hydrodynamic curves are possessed for the vehicle in question and that all this data is mathematically expressed in a format suitable for manipulation and processing upon the digital computer. This requires that the forward velocity of the vehicle to be expressed in terms of the engine speed by the use of a cubic polynomial of the form:

\[ V_A = a_1(n) + b_1(n)^2 + c_1(n)^3 \]

Equation 9.4

where the constants \( a_1, b_1 \) and \( c_1 \) are the multipliers from the above equation and relate to the hydrodynamic coefficients, as described in chapter three, for the determination of the surge velocity coefficients. The final forward velocity is expressed in terms of the rudder angle by the use of Equation 9.1:

\[ V_F = c - m(\alpha) \]

Equation 9.1

and the steady state yaw rate is then expressed in terms of this final forward velocity as demonstrated by Equation 9.2.

\[ (\Psi_F)^2 = a - b(V_F) \]

Equation 9.2

Therefore the approach speed, final forward velocity and steady state yaw rate can be mathematically determined from information concerning the two control parameters; engine speed and rudder angle. Where an alteration in either these two control parameters is reflected in these steady state variables by the transient equation, Equation 9.3:

\[ X_1 = X_1 \pm \Delta X \]

Equation 9.3
Then combining these four equations within a loop permits the vehicles' forward velocity and yaw rate to be determined as illustrated in Figure 9.10.

**Flow diagram showing the determination of the forward velocity and yaw rate.** Figure 9.10
The only inputs to the loop are the demanded engine speed \((n_d)\) and the rudder angle \((\delta_t)\), and the response times for the engine speed \((\tau_f)\), the reduction in speed \((\tau_v)\) and the yaw rate \((\tau_w)\).

If this loop was repeated at a given time interval then these two variables would be known with respect to time. The frequency that this loop is executed or the magnitude of the time interval chosen depends to a large extent upon the processing power available. A time interval of one second would generate a large amount of data and would be very accurate as the velocity and yaw rate would only change by a small amount in any one second. In comparison to a larger time interval of say one minute where the yaw rate and velocity may have changed significantly. Furthermore, the choice of response times employed to model the transient states is also dependent upon the magnitude of this time period.

The yaw rate and the forward velocity calculated with respect to time by the loop can then be utilised for the determination of a vehicles position relative to a known point of origin in terms of the X\&Y co-ordinates by the method described in chapter thirteen for the Track History of a marine vehicle. By summating the yaw rate with respect to time then the heading of the vehicle relative to a known original heading can also be determined.

The above procedures for transforming the demanded engine speed and rudder angle in to the yaw rate and forward velocity which are then used to determine the position and heading of the vehicle, can then be taken as satisfying the primary objectives required of a mathematical model of a marine vehicle as laid out at the beginning of this chapter. A possible structure for a mathematical model based upon the drift angle can be developed as illustrated in Figure 9.11.
Only the vehicles original position, heading, speed, rudder angle and engine speed are required to which the procedure outlined in Figure 9.10, then increase or decrease, with respect to time, depending upon the demanded engine speed and rudder angle.
Chapter Ten.

A MATHEMATICAL MODEL OF PICKET BOAT NINE BASED ON THE DRIFT ANGLE THEORY.

10.1 INTRODUCTION.

A mathematical model of Picket Boat Nine based upon the drift angle theory requires a set of hydrodynamic curves, as detailed in chapter nine. As there exist at present no scale model tests or empirical methods that can be employed for the generation of a set of hydrodynamic curves, full scale sea trials are required to be conducted. This following chapter describes these full scale sea trials and the method by which a set of hydrodynamic curves for Picket Boat Nine have been produced which are then transformed into coefficients, for implementation within a mathematical model.

This new form of mathematically modelling marine vehicles is then validated by a sequence of mathematical manoeuvres. The first being a simulation of a manoeuvre that was conducted on Picket Boat Nine during a set of full scale sea trials and secondly by a comparison with the output from a force modular model where similar engine speeds and rudder angles are demanded.

10.2 FULL SCALE SEA TRIALS FOR THE GENERATION OF A SET OF HYDRODYNAMIC CURVES.

The hydrodynamic curves, as described in chapter nine, represent the relationships between a vessel's forward velocity, rudder angle, yaw rate and engine speed of a vessel. In order to gather sufficient information so that a set of hydrodynamic curves for Picket Boat Nine could be produced a series of turning circle manoeuvres were conducted at various engine speeds and rudder angles.
The results obtained from conducting sea trials to validate the transformation of the yaw rate and speed a vessel into X&Y co-ordinates, as described in chapter eight, demonstrated that Picket Boat Nine's handling characteristics were affected by the presence of wave motion, wind and tidal currents. As this programme of research does not attempt to include such external disturbances it was required that the sea trials be conducted in the shelter of the River Dart to prevent deep sea waves causing unwanted motion. Furthermore, it was also desirable to conduct trials at high or low water, to remove the tidal effects, and when there was very little wind blowing. An added restriction during the sea trials involved a speed limit within the River Dart of six knots that prevented an engine speed greater than 1200 RPM being applied. Also the safe navigation of Picket Boat Nine in the width of water available prevented rudder angles less than twenty degrees being applied. Therefore, the turning circle manoeuvres were conducted at engine speeds of 800, 1000 and 1200 RPM for the range of rudder angles 20, 30 and 40 degrees both to port and starboard. Trials were undertaken on three different dates so that the environmental conditions described above were satisfied. During each manoeuvre between three and four turns were completed, this ensured steady state turning was established and that sufficient data was collected.

The turning circle manoeuvres consisted of steaming Picket Boat Nine along a constant heading at a given engine speed for a period of time and then the helm was put over to starboard to a given rudder angle, after three consecutive turns the rudder was applied to the same angle to port. During each manoeuvre the engine speed, forward velocity, rudder angle and heading of Picket Boat Nine were monitored and recorded digitally with respect to time via the use of the onboard data acquisition and monitoring system as described in chapter eight.

The required information from each manoeuvre consisted of the initial forward velocity before the rudders were applied, the final forward velocity and the steady state yaw rate during each turn. The yaw rate was determined from dividing the difference between two successive compass measurements by the time interval between them.
As these manoeuvres were conducted in open waters, environmental disturbances were present which affected the measurements by either increasing or deceasing the yaw rate and forward velocity, in a manner as described in chapter eight. This is illustrated graphically as a plot of yaw rate and forward velocity against time, as shown in Figure 10.1 for the port and starboard thirty degree rudder angle turning circle manoeuvre conducted at 1000rpm.

This diagram shows that a reduction in the forward velocity from an approach speed of approximately five and a half knots to four knots after the rudder has been applied to port, this reduction occurs over a time interval of approximately twenty seconds, where after the forward velocity oscillates about this reduced value for a further one hundred and twenty seconds due to external disturbances acting. The increase in forward velocity after a time interval of approximately one hundred and forty seconds represents the change in the demanded helm where the rudders are placed back to amidships and through to starboard thirty degrees. At this point the forward velocity decreases to another reduced value of approximately four and a quarter knots with
the same oscillation. At the end of the manoeuvre the forward velocity increases as the rudders are placed back to amidships. The yaw rate of the vessel during manoeuvre was initially negative for the turns to port and changed to positive for the starboard turns. In a similar manner to that of the forward velocity the yaw rate oscillates due to the external disturbances, where the scattering of the yaw rate about this oscillation is attributed to the compass only measuring the heading to the nearest integer. The severity of the external disturbances can be observed from the X&Y plot, Figure 10.2, in that the starboard turning circles are more consistent and regular than the turns to port.

![X&Y Track history of Picket Boat Nine for port and starboard 30°rudder angle turning circles at 1000rpm.](Figure 10.2)
To remove the environmental disturbances from the measurements an average value was calculated for both the final forward velocity and steady state yaw for each of the turning circle manoeuvres by the use of a spreadsheet. These average values have been employed so that the average turning circle diameter for each manoeuvre could be calculated by the use of the following equation:

\[ \bar{D} = 2 \frac{\bar{v}}{\bar{\psi}} \]

Equation 10.4

The averaged yaw rate, final forward velocity and turning circle diameter values are tabulated as shown in Table H.1, Table H.2 and Table H.3 (that are held in Appendix H) which detail these parameters for the three different engine speeds at which trials were conducted. These average values for a given approach speed demonstrate that as the rudder angle increases the yaw rate increases, creating a larger drift angle, whilst the final forward velocity decreases, resulting in a tighter turning circle with a smaller diameter. This relationship is also apparent when the engine speed is increased with the yaw rate, final forward velocity and turning circle diameters being greater in magnitude.

### 10.3 THE COEFFICIENTS THAT DESCRIBE THE HYDRODYNAMIC CURVES FOR PICKET BOAT NINE.

The required hydrodynamic curves were produced by plotting, the rudder angle against the final forward velocity with the associated yaw rate plotted against the same final forward velocity as shown in Figure 10.3. These hydrodynamic curves illustrate that the relationship between the rudder angle and the final forward velocity is approximately linear and that a parabolic relationship is a reasonable approximation between the final forward velocity and the steady state yaw rate as described in chapter nine.
Hydrodynamic curves for Picket Boat Nine. Figure 10.3

The transformation of these curves into the required coefficients was conducted by solving the following equation for the rudder angle versus final forward velocity data:

\[ V_F = c_{v_F} - m_{v_F}(\alpha_F) \]

Equation 9.1

and in a similar manner Equation 9.2 was solved for the steady state yaw rate versus final forward velocity data:

\[ (\Psi_F)^2 = a_{\Psi} - b_{\Psi}(V_F) \]

Equation 9.2

These equations were solved using the two software programs that utilised the principle of Gaussian elimination, as described in chapter four, for the determination of the surge velocity hydrodynamic coefficients. The resulting coefficients that describe these hydrodynamic curves are shown below in Table 10.1.
The coefficients for the hydrodynamic curves of Picket Boat Nine.

The values attributed to these coefficients demonstrate that at a particular engine speed the order of each coefficient is similar to that at a different engine speed and that as the engine speed increases so does the magnitude of each coefficient. This form of relationship would be expected for the coefficients concerning the operating regime of engine speeds and rudder angles for Picket Boat Nine.

During the collection of the above data the measured mile manoeuvre, as described in chapter three and chapter four, was repeated for the range of engine speeds considered during this set of sea trials using the digital data acquisition system available. In a similar manner to the above procedure for mathematically describing the hydrodynamic curves the engine speed against forward velocity were transformed into a set of coefficients by the use of a cubic polynomial of the form:

\[ V_a = a_n \cdot (\text{RPM}) + b_{nn} \cdot (\text{RPM})^2 + c_{nnn} \cdot (\text{RPM})^3 \]

Equation 9.4

This was solved in a similar manner to that employed for the previous two equations, where the resulting coefficients are shown in Table 10.2.

<table>
<thead>
<tr>
<th>Curve Number</th>
<th>Engine speed (rpm)</th>
<th>COEFFICIENTS</th>
<th>( a_n )</th>
<th>( b_{nn} )</th>
<th>( c_{nnn} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>800</td>
<td>0.02787788</td>
<td>0.01141228</td>
<td>-0.02171461</td>
<td>2.3785218</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>0.0487282</td>
<td>0.01517459</td>
<td>-0.0276259</td>
<td>3.1058041</td>
</tr>
<tr>
<td>3</td>
<td>1200</td>
<td>0.05813255</td>
<td>0.01536032</td>
<td>-0.03670299</td>
<td>3.7688361</td>
</tr>
</tbody>
</table>

The forward velocity and engine speed coefficients for Picket Boat Nine.
The accuracy of all the above coefficients can be observed from Figures H.1, Figure H.2 and Figure H.3 in Appendix H, in that from the limited data obtained these coefficients satisfactorily describe the data.

10.4 INITIAL RESULTS WITH A MATHEMATICAL MODEL BASED UPON THE DRIFT ANGLE THEORY.

The coefficients previously evaluated allow the forward velocity for a given engine speed, the steady state yaw rate and final forward velocity at a given rudder angle to be determined mathematically. Therefore by incorporating these coefficients into a computer program similar to that written for the determination of a vessel's track history, as detailed in chapter eight, allows the vessel's position and heading with respect to time to be determined mathematically. The initial inputs to such a computer program would be:

- The initial heading of the vessel
- The initial position of the vessel
- The initial engine speed
- The initial rudder angle
- The demanded engine speed
- The demanded rudder angle

Information which a navigator or pilot of a vessel would know as matter of routine and where the output from the computer program would be:

- The vessel's heading
- The vessel's position
- The vessel's engine speed
- The vessel's speed
- The vessel's rudder angle
- The vessel's yaw rate

all with respect to time. This is the only information a navigator or pilot would have about a vessel during a controlled manoeuvre so that control may be maintained. Furthermore this list matches the specification of a mathematical model described in chapter one.
A mathematical model based upon the drift angle theory incorporating the coefficients for Picket Boat Nine has been constructed and written as a computer program. This mathematical model has been implemented to determine the position of Picket Boat Nine during a set of manoeuvres conducted for the range of engine speeds and rudder angles considered. Each of these manoeuvres consisted of setting the desired heading to '090' degrees with the rudders at amidships for a time interval of twenty seconds, at which point the rudders were applied to a predetermined rudder angle. At this point the yaw rate increased and the forward velocity reduced under the linear transient phase modelling technique employed until steady state was achieved, where the forward velocity and yaw rate (drift angle) became constant.

The transient phase modelling for the manoeuvre when the engine speed was set to 1000rpm and a rudder angle of thirty degrees to starboard was applied, is illustrated in Figure 10.4,

![Plot showing transient state modelling from mathematical model of Picket Boat Nine.](image)

where, during the first nineteen seconds, the yaw rate increases to a maximum constant value, and the forward velocity decreases to a minimum after a time interval of thirty seconds. Both remain constant during the steady state turning for the
of thirty seconds. Both remain constant during the steady state turning for the duration of two complete turns and then return to their original values under the same modelling conditions. The constant yaw rate and forward velocity values during steady state turning illustrate that the environmental disturbances that are found upon the actual yaw rates and forward velocities measurements (as shown in Figure 10.1) are not present as this programme does not attempt to include them.

This mathematical was so designed that during each successive time interval the heading and the position of the vessel was calculated, as described previously. An example of these manoeuvres are graphically illustrated as a series of positional plots showing the distance traversed in the X&Y directions, as shown in Figure 10.5 for the range of rudder angles considered at a constant engine speed of 1000rpm and in Figure 10.6 for a rudder angle of thirty degrees both to port and starboard for the range of engine speeds considered.

![X&Y plot showing turning circle variation with rudder angle at constant engine speed.](Figure 10.5)
The former of these two X&Y plots, Figure 10.5, demonstrates that from a common point of origin (0.0,0.0) the greater the rudder angle the less the turning circle diameter. Furthermore, Figure 10.6 demonstrates that as the engine speed increases the distance traversed during the first twenty seconds before the rudders are applied also increases, and that the greater the engine speed the greater the turning circle diameter.

The turning circle diameters for all the engine speeds' and rudder angles considered are tabulated in Table 10.3 along with the average turning circle diameters from Table H.1, Table H.2 and Table H.3 calculated from source data and the approximate turning circle diameters measured from the X&Y track histories produced from the original forward velocity and heading measurements.
### TURNING CIRCLE DIAMETERS.

<table>
<thead>
<tr>
<th>Engine speed (rpm)</th>
<th>Rudder angle (deg)</th>
<th>X&amp;Y track History plots (m)</th>
<th>from average yaw rates &amp; velocities (m)</th>
<th>from drift angle model (m)</th>
<th>error with X&amp;Y track histories (m)</th>
<th>error with average values (m)</th>
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</table>

*Turning circle diameters from mathematical model X&Y track histories and average source data.*  

Table 10.3

These turning circle diameters emphasise the relationship between the rudder angle and the engine speed with the associated turning circle diameters for the range of rudder angles and engine speeds considered.

The error values quoted for each turning circle diameter is the calculated difference between the drift angle mathematical model prediction and the measured diameter from the X&Y track history for each manoeuvre and average turning circle diameter calculated from the average yaw rate and forward velocity values. The drift angle mathematical model turning circle diameters illustrate a port and starboard symmetry which is a direct result of the two equations (Equation 9.1, Equation 9.2)
employed to describe the hydrodynamic curves. This port and starboard symmetry is not apparent in either the X&Y track history or the average calculated diameter values and indicates that Picket Boat Nine does not possess straight line stability but has a tendency to turn to starboard which manifests itself as the mathematical model values for the starboard turning circle diameters being greater than those from the other two methods.

10.5 SUMMARY TO THE DRIFT ANGLE BASED MATHEMATICAL MODEL OF PICKET BOAT NINE.

The mathematical representation of a marine vehicle by the use of the yaw rate and forward velocity by the use of the relationships that exist between the engine speed and rudder angle has been demonstrated for Picket Boat Nine. The greater the rudder angle the tighter the turning circle and that the greater the engine speed the larger the diameter, numerical results from this mathematical model that compare favourably with the full scale vessel, as indicated by the error values quoted in Table 10.3.

The accuracy of this form of mathematical modelling is considered dependent upon three factors: Firstly, the data that was collect from the full scale sea trials which were affected by the presence of the external disturbances, (tidal currents, waves and wind) even though the sea trials were conducted in the optimum of conditions so as to reduce them to a minimum. Secondly, the equations chosen to describe the hydrodynamic curves as the forward velocity and rudder angle data may not be exactly linear and the yaw rate with respect to the final forward velocity not a parabolic relationship, but may be some other non-linear relationship. Finally, the resolvement of the coefficients and the values attributed to them as the Gaussian elimination technique employed introduced a port starboard symmetry that is expressed in the turning circle diameters produced by this mathematical model that is not apparent in the track history plots and diameters or the calculated average turning circle diameters from source data.
To improve this mathematical model the sea trials would have to be conducted in ideal conditions where there were no external disturbances and for the trials to be conducted for the complete range of rudder angles and engine speeds.

10.6 MATHEMATICAL MANOEUVRES OF PICKET BOAT NINE.

The previous section only referenced the turning circle diameters produced by this new form of mathematical modelling marine vehicles. It compared them to the turning circle diameters calculated from the source data and those measured from the original X&Y track history of the Picket Boat Nine during manoeuvres. This demonstrates that from this form of mathematical model the yaw rate and forward velocity can be employed to determine the distance traversed in the X&Y directions for a given engine speed and rudder angle.

The following describes the implementation of this new mathematical model of Picket Boat Nine and compares the results from a series of manoeuvres with similar manoeuvres conducted on the full scale vessel and with a non-linear modular model of the same vessel.

10.6.1 The Simulation Of A Manoeuvre.

By incorporating an original vessel status (original position, heading, engine speed, forward velocity and rudder angle) into the drift angle mathematical model the path of a vessel during a manoeuvre can be simulated and replicated. To demonstrate this marine vessel simulation, full scale sea trials were conducted and the results were compared with the output from this mathematical model given the same control inputs of original heading, demanded engine speed and rudder angle with respect to time.

The sea trials consisted of steaming Picket Boat Nine at 1200rpm along a constant heading of 328 degrees for a time interval of seventeen seconds (due to the action of the environmental disturbances - a tidal current - a four degree starboard rudder angle had to be applied to maintain this constant heading), at which point the rudders were applied to forty degrees to port and three complete turning circles were
completed. On reaching a heading of 350 degrees of the final port turning circle the rudders were applied to forty degrees to starboard and maintained until three complete turning circles were completed. During this manoeuvre the yaw rate and forward velocity were recorded using the onboard data monitoring and acquisition system. These recorded yaw rate and forward velocity values are illustrated graphically with respect to time, as shown in Figure 10.7. The forward velocity and yaw rate remain constant during the first twenty seconds, which is the lead in to the manoeuvre when the rudders were at a midships.

![Plot showing measured yaw rate and forward velocity from Picket Boat Nine validation manoeuvre. Figure 10.7](image)

The rudders were then placed to forty degrees to port where the yaw rate and forward velocity decrease, over a time interval of eighteen and thirty seconds respectively, to a minimum value and remain constant for the following one hundred and forty seconds. At this point the rudders were then applied to starboard forty degrees. The yaw rate then increases over a time interval of approximately twenty seconds to a maximum. The forward velocity during this change in helm increases during the first fifteen seconds as the rudders are applied back through amidships to starboard, where upon the forward velocity decreases in approximately the same time interval to a similar
reduced value and remains constant for the remainder of the manoeuvre. The apparent oscillation of the speed and yaw rate are attributed to the environmental disturbances acting upon Picket Boat Nine during this manoeuvre.

By employing the software program 'TRACK' which transforms yaw rate and forward velocities into a series of X&Y distances traversed the path of Picket Boat Nine can be illustrated graphically as shown in Figure 10.8.

This diagram illustrates the lead in to the manoeuvre along the heading 328 degree for the seventeen seconds and then the three turns to port followed by the three turns to starboard. The environmental disturbances are acting in the direction 060 degree which cause the vessel to sway in this direction. This sway motion manifests itself in this diagram as a sideways movement "off-set" to each of the turning circles.
To implement this mathematical model based upon the drift angle theory to simulate this full scale manoeuvre conducted in Picket Boat Nine the following original vessel status was entered into the program:

- position 0.0, 0.0
- heading 348 degrees
- engine speed 1200rpm
- rudder angle amidships.

After a time interval of seventeen seconds the rudders were applied to forty degrees to port, this desired rudder angle was maintained until three complete port turns were completed. Once a heading of 350 degrees was achieved during the third turn the rudders were applied to a similar angle to starboard and three turns were completed. To end this manoeuvre the rudders were returned back to amidships during the third starboard turn once a heading of 180 degrees was reached.

Application of the rudder was taken as an occurring rapidly, with the yaw rate and forward velocity increasing or decreasing over a period of time according the transient state modelling employed (Equation 10.3). The response times for these were taken from the full scale sea trials as eighteen and thirty seconds respectively. This transient state modelling is illustrated graphically for the rudder angle and yaw rate with respect to time, as shown in Figure 10.9.
This transient state modelling demonstrates that during the first seventeen seconds the rudders were amidships and that the yaw rate was zero. At which point the rudder angle decreases to port forty degrees and the yaw rate reduces during the following eighteen seconds to a minimum. Both remain constant for the following one hundred and fifty seconds during which time three port turns are completed. The rudder then increases to starboard forty degrees and the yaw rate increases to a maximum value over a period of eighteen seconds when three starboard turns are completed. After which the rudders are returned back to amidships and the yaw rate reduces in a similar manner to zero.

The transient state modelling of the forward velocity and the yaw rate with respect to time are illustrated in Figure 10.10.

![Graph showing transient state of forward velocity and yaw rate from mathematical model.](Figure 10.10)

This shows that the forward velocity reduces as the yaw rate reduces during the turns to port and that forward velocity increases as the rudders are applied back through amidships to starboard forty degrees and then reduces to a minimum once steady state is established. At the end of the manoeuvre when the rudders are applied back to amidships and the yaw rate becomes zero the forward velocity increases to a maximum.
This forward velocity and yaw rate versus time profile from this mathematical model can be compared to that obtained from the full scale sea trials, Figure 10.7. This shows the linear transient phase modelling technique employed for the forward velocity and yaw rate is a close approximation to the actual transient phases except that the actual transient phases appear non-linear. Furthermore, this mathematical model does not include the environmental disturbances that were present during the full scale sea trials which resulted in the actual values for both the forward velocity and yaw rate oscillating about their constant maximum and minimum values and each of the X&Y turning circles possessing an off-set. The absence of these external disturbances in this mathematical model simulation is demonstrated by the plot of the distances traversed in the X&Y directions as shown in Figure 10.11 by the fact that the three turning circles are coincidental with each other and that there is no off-set or lateral displacement observable.

X&Y plot from mathematical model for validation manoeuvre. Figure 10.11
10.6.2 Summary To The Simulation Of A Manoeuvre.

The results obtained from this new form of mathematical model for marine vehicles have demonstrated that the response of a marine vehicle can be represented upon the digital computer and that these results also show a close agreement with the measured response of the vehicle when similar rudder angles and engine speed are demanded. This was illustrated by the forward velocity and yaw rate values from this model when compared to those measured from Picket Boat Nine, with the exception that the latter exhibited the presence of external disturbances. These external disturbances that are not included in this mathematical model manifest themselves in the X&Y track history and are clearly visible due to the off-set in the consecutive turning circles, Figure 10.2 and Figure 10.11. Furthermore, the modelling of the transient phase showed a good approximation in the yaw rates and forward velocities when a change in helm was demanded and that the forward velocity was modified to compensate for the rudders returning to amidships and then when the rudders were placed hard over.

10.6.3 Comparison Of The Drift Angle Based Mathematical Model With A Force Modular Model Of Picket Boat Nine.

The non-linear mathematical modular modelling technique that was described in Part One has been successfully employed for mathematically modelling large marine vessels. Vehicles about which sufficient information has been collected from the various methods that were outlined in chapter three. This has enabled the required hydrodynamic coefficients to be evaluated and implemented within a mathematical model based upon the research conducted by Tapp (1987). The following paragraphs describe the implementation such a mathematical model of Picket Boat Nine and compares this form of mathematical modelling of small marine vessels with the drift angle theory based mathematical modelling technique for the same vessel.
To compare the non-linear modular model with this new form of mathematically modelling a series of simulated turning circle manoeuvres were conducted. Each mathematical model was set with a desired engine speed of 800rpm. After the first second of each manoeuvre the rudders were applied to a an angle of twenty degrees to port. From this desired control input, engine speed and rudder angle, the mathematical models then determined the response of the vessel by computing the yaw rate, forward velocity, heading and position (in terms of a set of X&Y co-ordinates) of the vessel with regards to time.

This manoeuvre was then repeated from the three rudder angles (twenty, thirty and forty degrees) to port and starboard for the different three engine speeds (800rpm, 1000rpm and 1200rpm). The resulting output from these manoeuvres permits the two different modelling techniques and methodologies to be compared. By comparing the yaw rate and forward velocity values with respect to time from the modular model with this new form of mathematical modelling, as shown graphically in Figure 10.12 and 10.4 respectively for the thirty degree port and starboard rudder angle manoeuvres conducted at 1000rpm.

\[ \text{Plot of forward velocity and yaw rate from force modular model.} \]  
\[ \text{Figure 10.12} \]
The modular modelling technique of the forward velocity demonstrates that from an approach speed of approximately six knots the application of the rudders causes the forward velocity to increase, then decrease to a minimum of less than three knots as an overshoot and settle down following an over damped response to a constant steady state value of approximately four and a half knots. The yaw rate in a similar manner begins with an overshoot followed by an over damped response with respect to time after the rudders have been applied.

The drift angle mathematical model forward velocity and yaw rate values with respect to time were shown in section 10.6.1 to be a reasonable linear approximation during the transient phase when the rudders were applied to the full scale vessel. The forward velocity and yaw rate values produced by the modular model demonstrates an overshoot and over damped response with respect to time, as shown in Figure 10.12. This form of transient phase does not appear with this vessel as illustrated by Figure 10.7, even though larger vessels do display such a response when the rudders are applied (Manoeuvring Trials of the 278,00 DWT ESSO OSAKA in Shallow and Deep Waters 1979). The yaw rate and forward velocity values, once steady state is established, illustrates that the vessel then turns at a constant speed in a turning circle of constant diameter. This is illustrated graphically by the calculated X&Y co-ordinates as shown in Figure 10.13. This diagram illustrates that during the transient phase when the rudders are applied the overshoot and overdamped response causes the vessel to turn producing an initial turning circle diameter which is smaller than the steady state diameter. This results in the vessel not traversing an adequate distance in the X&Y directions after one turning circle is completed as shown in Figure 10.14 for the drift angle X&Y positional plot. The transient phase causes the vessel to traverse a distance both in the X&Y directions from the point of origin.

The modular modelling technique for all the rudder angles and engine speeds considered demonstrated similar characteristics during the transient phase.
$X$-direction (m).

$Y$-direction (m).

$X$&$Y$ plot from force modular model. Figure 10.13

$X$&$Y$ plot from drift angle based model. Figure 10.14

Page 170.
Furthermore, from the X&Y positional plots from both mathematical models the steady state turning circle diameters were measured and are tabulated in Table 10.4 along with the measured track history and calculated average turning circle diameters from the sea trials conducted on Picket Boat Nine.

<table>
<thead>
<tr>
<th>Engine speed (rpm)</th>
<th>Rudder angle (deg)</th>
<th>X &amp; Y track History plots (m)</th>
<th>Calculated from average yaw rates &amp; velocities (m)</th>
<th>Drift angle model turning circles (m)</th>
<th>Modular model turning circles (m)</th>
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Tabulated turning circle diameters showing comparison of mathematical models with full scale results. Table 10.4

Where the turning circle diameters from the modular model are generally less than the diameters predicted from the drift angle mathematical model, as can be observed from the error values quoted that refer to the differences between these two different mathematical models; and those measured from the track history and calculated from the average yaw rate and forward velocity values.
10.7 CONCLUSION.

Initial results with this new form of mathematical model have indicated a good agreement with full scale measurements for the magnitude of the turning circle diameters associated with different demanded rudder angles and engine speeds. Furthermore, the requirements of a mathematical model and its ability to determine the position, heading, velocity, yaw rate, engine speed and rudder angle, when only the latter two are known, is satisfied. This new method has been compared to both full scale measurements conducted with the vessel being modelled and with a modular model. The results of which prove the validity and accuracy of this method to describe and simulate a marine vehicle.
Chapter Eleven.

CONCLUSION.

11.1 INTRODUCTION.

The primary objective of this research programme has been the investigation of mathematical modelling techniques for marine vehicles. When this research was commenced, the non-linear force based modelling technique was considered the most versatile and accurate method of describing marine vehicles during a manoeuvre. During the construction of such a mathematical model, errors have become apparent and in resolving these errors a new method of describing marine vehicles has been developed. This new method of describing marine vehicles is the basis for a new form of mathematical model that has been implemented in this thesis. This final chapter discusses the findings of this research programme, outlines the applications to which these may be utilised, indicates areas which require further research and development to extend the drift angle theory and its applications to enhance safety of life at sea and to the protection of the marine environment.

11.2 DISCUSSION OF RESULTS.

The non-linear force mathematical model described in part one was concluded as being erroneous. This is based upon the results from the research undertaken and may only apply to this programme of research. This form of mathematical modelling has been implemented successfully by other establishments, Tapp (1989), Chudley (1991), who required extensive trials to conduct the model but have limited validation results.

This conclusion to the force based mathematical model discussed in chapter five and the salient factors being: the use of scale models that were less than a metre in length; the effect of external disturbances upon the full scale vessel during sea
trials; and the semi-empirical and mathematical method for their evaluation being based upon completely different vessels to that of Picket Boat Nine. In attempting to resolve these problems the definitive nature of the values attributed to the hydrodynamic coefficients and the accuracy attainable with these methodologies is considered significant when an accurate force mathematical model is being sought. Furthermore, the truncation of the Taylor's multi-variate expansion and the inclusion of selected terms, demonstrates that the accuracy of these hydrodynamic coefficients is not that significant to the overall performance of the mathematical model.

The use of forces and moments to describe marine vehicles is considered to arise from the fact that for the last two hundred years or more forces have been the only easily measured parameter with an accuracy that has improved throughout the years. Therefore, the most versatile and accurate method of describing marine vehicles was based upon the measurement of forces and moments and has resulted in the non-linear force mathematical model utilising the modular approach.

The results from the second part of this thesis have indicated that the measurement of force is not the most suitable method for the mathematical representation of a marine vehicle's manoeuvrability and handling characteristics. Advances in technology, especially in the electronics industry and the advent of the digital computer, has permitted the more fundamental parameters of heading and velocity to be measured more readily and precisely with respect to time. This measurement of heading and velocity is the basis for this new method of quantifying and mathematically describing marine vehicles as the research conducted has shown that a marine vehicle can be considered either as stationary, moving along a constant track or as turning, causing a change in heading occur. This change in heading has been related to the generation of the drift angle. The generation of which, as described in chapter six, is a direct result of the forces that are in action upon a marine vehicle.
The drift angle theory, described in chapter seven, is based upon the relationship that has been shown to exist between the magnitude of the drift angle and the yaw rate of a marine vehicle during a turn which is equal to the rate of rotation about a point of curvature. Furthermore, the drift angle of a marine vehicle during a steady state turn has been shown to be constant, as is the yaw rate (assuming there are no external disturbances present), and that a different drift angle is generated for each demanded rudder angle and engine speed. The generation of the drift angle has been applied to a possible explanation of why some marine vehicles possess a degree of straight-line, directional or dynamic instability; the action of a marine vehicle when acted upon by external disturbances, namely current and wind; and as method of predicting and quantifying the turning ability of a marine vehicle, as identified in, section 7.3.

The measurement of the velocity and heading with respect to time has been made possible by the use of electronics and computers; these two parameters have been shown to permit the calculation of the diameter of a turning circle for a marine vehicle during steady state turning in chapter seven. This has been demonstrated in section 7.4.1 and is supported in section 7.4.2 by data collected from sea trials conducted with the USS Compass Island, Morse et al (1961), where from this data the turning circle diameter was calculated to an accuracy of less than half a percent, which relates to an error of less than three metres in a turning circle diameter greater than thirteen hundred meters. This error is attributable to the method in which the forward velocity and yaw rate of the vessel was recorded, as described in chapter eight, and to the external disturbances that were present during these sea trials that were not taken into consideration in the presentation of the data. These initial results indicate that a marine vehicle can be quantified and described mathematically by the use of these two parameters.

The natural extension to calculating the turning circle diameter has produced a method by which the position of a marine vehicle from a known point of origin can be determined, as described in chapter eight. This method, referred to as the track
history of a marine vehicle, calculates the distance traversed from a point of origin from data collected from the log and compass of a marine vehicle and by geometric calculations within a piece of specifically written software, as demonstrated in section 8.3. The accuracy of this method for determining the position of a vehicle has been compared to the data collected from two navigational aids, DECCA and GPS, and to a further high accuracy position fixing system, Trisponder, during a series of sea trials conducted with Picket Boat Nine. The results from these sea trials, which are presented in section 8.4, demonstrates that an accuracy of less than twenty metres is attainable with this method.

The Track History method determining a vehicle's position, as with the drift angle theory, is solely based upon the measurement of the forward velocity and compass heading with respect to time. Therefore part of the track history error is considered to be attributable to the log and compass that were employed, both of which were commercial products that were not designed with the accuracy required for this programme of research. The former consisted of a spinning vane where the number recorded pulses per seconds was calibrated to measure the vessel's speed and that the latter only recorded the compass heading to the nearest integer. The accuracy of the track history method for determining a vehicle's position could be improved by the use of a more accurate log and with a compass that recorded the heading to an accuracy of three or more decimal places.

The presence of the external disturbances; currents, tides, waves and wind, upon Picket Boat Nine were shown to be taken into consideration by the track history method of determining the position of a marine vehicle during manoeuvres in section 8.4. As the external disturbances were shown to manifest themselves in the measured forward velocity and yaw rate values, by the fact that a low frequency oscillation was exhibited in these parameters with respect to time. This low frequency oscillation correlates to an off set or displacement of consecutive turning circles, as was shown in Figure 8.9. The accuracy of this method has to be considered with reference to the significance of the motion of sway, as this motion is not measured by the compass or
the log. The results obtained, from the track history manoeuvres conducted with Picket Boat Nine, which is a responsive vessel compared to much larger vessels, indicates that the motion of sway is negligible. This can be explained by the fact that sway motion will only occur, as described in chapter seven, when the forces acting upon a vessel are symmetrical about the centre of gravity and thus can be taken as solely acting through this point. Otherwise, the motions of yaw and surge coupled with sway will occur, the former two being recorded by the compass and log respectively. Therefore, the motion of sway for Picket Boat Nine during the manoeuvres conducted is considered negligible.

The drift angle theory and the ability to determine the position of a vehicle is the basis for this new method of mathematically modelling marine vehicles, as described in chapter nine. This new method, which has been implemented so that a mathematical model of Picket Boat Nine could be established, is based upon a set of hydrodynamic curves, that graphically relate a discrete engine speed and rudder angle to the yaw rate and forward velocity. The generation of these hydrodynamic curves is based upon data collected during a standard series of turning circle manoeuvres, those which are generally conducted as part of the builders and acceptance trials, where the forward velocity and compass heading are recorded with respect to time, for the range of rudder angles and engine speeds possible. The accuracy of these hydrodynamic curves are dependent upon the measurement of the velocity and compass heading with the problems that are associated with them and the presence of external disturbances as described previously.

These hydrodynamic curves are then represented mathematically by a set of coefficients determined from two inter-related equations. The accuracy of these coefficients is not only dependent upon the measurements taken from the log and compass but by the choice of equation used to describe the data. The data collected from Picket Boat Nine indicated that based upon the line of best fit the relationship between the forward velocity and the rudder angle can be described as linear, whilst that between the rudder angle and yaw rate exhibits a parabolic relationship. This
choice of equation may not be the most suitable for all vehicles but is supported by the data collected from the USS Compass Island sea trials as similar characteristics were observed. Furthermore, the values attributable to the coefficients is also dependent upon the method selected for their determination. Gaussian elimination was chosen for this programme of research due to the principle of least squares regression and that this produces a line of best fit through all the data. This choice, in hindsight, may not be the most appropriate as the results from the full scale sea trials have indicated that Picket Boat Nine possesses a slight degree of straight-line instability (a tendency to veer to port), which is not displayed by the coefficients. This would be rectified by a more appropriate method for their determination.

The coefficients once evaluated then allow the steady state yaw rate and forward velocity to be determined from a demanded rudder angle and engine speed. These calculated yaw rate and forward velocity values are then utilised so that the distance traversed from a point of origin can be determined, in a similar manner to that for the track history of a marine vehicle. Therefore this new form of mathematical modelling can be considered as fulfilling the fundamental requirements of a mathematical model. From an input of demanded engine speed and rudder angle the position of a marine vehicle with respect to time can be determined.

The modelling of the transient states, by the application of the rudder, and the effect that this has on the yaw rate and forward velocity of Picket Boat Nine, either to increase or decrease each, according to the helm demanded, has in the case of the rudder been conducted as an rapid application in comparison with the change in yaw rate and forward velocity. The yaw rate and forward velocity parameters have been found, from the measurements taken from Picket Boat Nine, to vary in a linear manner with respect to time, where response times based upon measurements are used. This method of transient state modelling may not be appropriate to other marine vehicles, as mentioned in chapter eight, as the time for the drift angle to become constant is considered to be dependent upon the length of the vehicle and the size of the rudder. Initial results with this mathematical model have indicated that the
heading, rudder angle, engine speed, velocity, position and yaw rate can all be determined by this proposed method, to an accuracy that will be discussed in the following section, but these parameters provide sufficient information to pilot or navigate a vessel when underway and therefore can be considered as achieving the primary objective of mathematically modelling a marine vehicle.

This mathematical model of Picket Boat Nine has been employed to simulate a manoeuvre conducted in Picket Boat Nine, where identical control commands, of initial heading, engine speed and rudder angle are instructed to both the mathematical model and to the helmsman of Picket Boat Nine. Measurements taken during the manoeuvre are compared to predictions from this mathematical model. The results have demonstrated that, with the exception of the inclusion of the external disturbances that were in action upon Picket Boat Nine during the sea trials, this method is capable of simulating a manoeuvre given the demanded controls. These simulation results however, contain a degree of error, in comparison to the measured values from Picket Boat Nine, which are considered attributable to the problems previously described with the measurements of the velocity and yaw rate. The determination of the coefficients, and as the results demonstrate a port and starboard symmetry, which has not been found to occur with the full scale vessel. This port and starboard symmetry would not be apparent if more accurate data could be collected and a more appropriate method for determining the coefficients was employed, but this programme of research has not been concerned with the definitive nature of the coefficients or the values that they possess, but in providing a format and method that has stature and validity for mathematical modelling marine vehicles in the future.

A comparison of this new form of mathematical modelling has also been conducted with a force based mathematical model using the modular approach for the same vessel. The results have demonstrated that given the same demanded control inputs of engine speed and rudder angle, that the force modular model possesses a large degree of error in comparison to the magnitude of the turning circle diameters.
produced when turning circle manoeuvres are conducted. Furthermore, the transient state modelling technique employed with the modular model is not representative of that which was exhibited by Picket Boat Nine. Therefore, this new form of mathematical modelling is considered as more representative of Picket Boat Nine than this force model, but comparisons with other vessels and forms of modelling are required to prove its comprehensiveness.

The modular modelling approach however, is considered important to the development of this new form of mathematical modelling by using this new method to replace the force method of describing the hull, rudder and propeller. The action of the external disturbances; wind, current, tides and waves can then be modelled in a similar manner to that which the force modular model utilises and included as separate modules that increase or decrease the forward velocity or yaw rate accordingly, as described in section 8.4. Furthermore, once the basic mathematical model has been established this method could then be extended to include bow and stern thrusters, the action of tow boats and any other additional modules that can be considered to cause a vessel to handle differently; ie the action of otter boards and drag nets for fishing boats.

In this investigation the drift angle model has only been implemented for three discrete engine speeds and large rudder angles. An attempt was made to keep the external disturbances to a minimum, and in doing so the full scale sea trials were conducted in the shelter of the River Dart. This prevented engine speeds greater than 1200rpm being applied as a speed limit of six knots existed in the river and due to the width of the river it was not possible to obtain rudder angles of less than twenty degrees.

An added advantage of this form of mathematical modelling is the size of the computer program that conducts the required calculations; the smaller the program the less time that is required for processing. The size of this program is considerably smaller than that for the force method, and so this software, if written correctly could
be employed to determine the position of a vehicle for a range of rudder angles and engine speeds in a very small period of time.

11.3 FURTHER RESEARCH.

The primary objective of this research programme has been the establishment of an accurate mathematical model of Picket Boat Nine, the results obtained and that were discussed previously have indicated two separate areas that are considered to require further research. These are:

- the mathematical model and its structure;
- the drift angle theory and its applications.

Further research in these two main areas is considered to enhance this form of mathematical modelling and so that a greater understanding of the principles involved with the manoeuvrability of marine vehicles can be gained.

The mathematical model and its structure.

The new form of mathematical modelling is still in its infantile stages of development and requires further research to complement and to improve this method so that its validity and comprehensiveness to other vessels can be determined. Further research upon this model is considered to be required in the following five areas:

1. A comprehensive assessment of the significance of the motion of sway to the track history method for determining the position of a vehicle.
2. A more accurate method for measuring the forward velocity and yaw rate of a marine vehicle is employed and that the sea trials be conducted for all the rudder angles and engine speeds possible. This would enable a complete set of hydrodynamic curves to be generated and in doing so the validity of this form of mathematical modelling to other vehicles could be established.
3. The selection of more appropriate set equations to describe the hydrodynamic curves and that a more suitable method for determining the values attributed to the coefficients should be examined.
4. An improvement to the transient state modelling technique presently employed.
5. The inclusion of external disturbances due to the action of tidal current, waves and wind to the mathematical model.
1. The track history method for determining the position of a marine vehicle has been shown to be accurate to within twenty meters by the use of a Trisponder positioning system. The accuracy is expected to be greater than this, but the equipment required to prove this has not been available. Therefore a series of sea trials with sufficient instrumentation to permit accurate measurement of the forward velocity and yaw rate along with a more accurate positioning system, Differential GPS or a laser tracking system, would enable the significance of the motion of sway to be determined, and the degree of definiteness that this method can be awarded.

2. A more accurate measurement of the forward velocity could also include the measurement of the velocity across the ground instead of through the water, as has been the case during the programme of research. This more accurate measurement of velocity could be conducted either with an electromagnetic log or with a Doppler log, the former measures the velocity through the water whilst the latter across the ground. A measurement of velocity across the ground would enable the magnitude of sway and its significance to be examined. To improve upon the yaw rate measurement a more accurate compass would be required that could measure the heading to at least three decimal places. Furthermore, both the log and compass should be located at a central point, known in respect to the dimensions of the vessel. This would enable the length of the vessel and its breadth to be taken into consideration for very large vessels.

The collection of more accurate data for a greater range of engine speeds and rudder angles for the generation of a set of hydrodynamic curves is considered to not only validate this method but would enable the relationship between the discrete engine speeds to be investigated. Thus a single hydrodynamic curve for a particular vessel may be presented and this would enhance this form of mathematical modelling.

3. The choice of equations to describe the hydrodynamic curves and the method for determining the values attributed to the coefficients is considered to be dependent upon the accuracy of the forward velocity and yaw rate measurements. Insufficient data exists for the precise nature of these mathematical relationships to be
determined. It is however thought that the forward velocity and rudder angle relationship is, non-linear and requires investigation to determine a more appropriate relationship. Furthermore, a more superior method of evaluating the coefficients from the data is also considered important for improving the accuracy of the coefficients.

4. The transient state modelling technique employed with this research has been linear and is not considered applicable to all vehicles. This therefore requires to be researched further and it is expected to be a non-linear relationship dependent upon the magnitude of the rudder angle demanded and the speed of the vehicle. Furthermore, only discrete engine speeds have been considered and the relationship between the engine speed and forward velocity requires to be examined, to represent accelerated and decelerated turns. During most manoeuvres however, the rudder angle is actuated more frequently than a change in engine speed. This is common navigational practice where the mariner is concerned fuel costs and engine wear, as most vessels steam at an operating or service speed.

5. The inclusion of external disturbances is considered to be the most important factor for improving this form of mathematical modelling. The results obtained have indicated that for Picket Boat Nine, which is a small responsive vessel, these disturbances are constantly in action. The most appropriate method for including these disturbances is considered to be by utilising the modular approach employed for the force model. Where this mathematical model would be the hull, propeller and rudder modules combined and that a different module would be created for each disturbance. Each module would then either increase or decrease the forward velocity of yaw rate accordingly depending upon the direction and magnitude of each.

This form of mathematical modelling can be extended further to include shallow and restricted water manoeuvring and slow speed steaming. It is considered that this would require the six degrees of freedom to be modelled rather than the two degrees considered in this programme of research, as heave and sway then are more
prominent. Furthermore, a six degree of freedom mathematical model will permit external disturbances to be included as wind acts upon the superstructure and tidal currents act upon the hull, both of which cause the motion of roll to occur. Wave motion will not only cause roll but the motion of pitch which will have an effect upon the forward velocity. Therefore, by implementing the drift angle theory to the other degrees of freedom a comprehensive mathematical model can be established.

The drift angle theory and its applications.

The basis of this mathematical model has been the drift angle theory which is considered to be a direct result of the existence of a centre of pressure. The elusive centre of pressure, as it has been referred to by Anderson (1991), is generally quoted as being situated along the centre of a vehicle, as described in chapter seven, but has been found to move along the centre-line. If the location of the centre of pressure can be found from either semi-empirical, mathematical or experimental methodologies then it is believed that the magnitude of the drift angle can be determined. Then the manoeuvrability of a marine vehicle can be determined before the vehicle is built with a high degree of accuracy.

Initial results have been obtained from conducting scale model tests for the determination of the location of the centre of pressure and the magnitude of the drift angle, as described in Appendix I. The results indicate that the magnitude of the drift can be determined if the location of the centre of pressure can be found.

Furthermore, as a marine vehicle's manoeuvrability can at present only be estimated, where the builders and the owners acceptance sea trials either disprove or confirm these predictions; either way the vehicle has to be built. Extending this application to its natural conclusion if the centre of pressure and the magnitude of the drift angle can be determined, by applying the drift angle theory, then marine vehicles could be designed to possess the manoeuvrability and handling characteristics desired, and thus, be included as one of the design criteria.
11.4 SYSTEM APPLICATION DEVELOPMENT.

The initial findings and results from this research programme have indicated several applications that might be beneficial to the maritime industry but require developing into comprehensive commercial systems. These include:

1. the presentation of the hydrodynamic curves as a wheel house method for aiding navigation and piloting, as the required rudder angle and or engine speed could be calculated to give a desired rate of turn or turning circle diameter for a manoeuvre or passage through restricted or confined waters.
2. the magnitude of the drift angle, when quoted with basic vessel parameters, engine speed and rudder angle can be used to compare and quantify the manoeuvrability of different vessels.
3. the track history as a second form of navigational aid, once its validity has been proven, to support conventional radio and satellite navigational systems or as part of the information recorded by an onboard Black Box data logging system.

1. The use of the wheel house data concerning the manoeuvrability of a vessel is already common practice among some merchant fleets. This data is illustrated graphically by diagrams showing the region that the vessel is likely to turn in and is left to the user to interpret. The hydrodynamic curves could be determined during sea trials, checked and if necessary updated during routine voyages, as most ships carry a log and compass. This would allow the navigator or pilot a high degree of accuracy in determining the required rudder angle and engine speed of a given turn or manoeuvre.

2. The requirement for international standards governing a vessel's manoeuvrability have been considered by the IMO, as discussed in section 1.2. At present the only accepted, but not agreed, method for quantifying a vessel's turning ability has been that a vessel should be able to turn in a circle whose diameter does not exceed four or five ship lengths. This does not take into consideration the paddle wheel effect of single screw vessels or vessel's that possess a natural degree of straight line instability. The magnitude of the drift angle along with basic vessel parameters, engine speed and rudder angle could be employed to present a dimensionless ratio, that can quantify and signify such manoeuvring characteristics. This would enable
national and international regulations to be laid down that would prevent vessels without a certain degree of manoeuvrability from causing disastrous accidents.

3. The determination of a vessel's position by the track history method has not been completely validated but is considered to be an important method that could be used to determine the position of a vehicle to a high degree of accuracy. Therefore, for vessels that use satellite navigation, this method could be employed to indicate the position of a vehicle when satellite cover is not available or when a vessel is out of the range of radio navigation. Furthermore, as vessels that carry the Black Box data recorder normally store information from the log and compass, this method of determining the track history of a vehicle could be employed to verify a vessel's position, in the event of inquiry.

11.5 CONCLUDING SUMMARY.

The drift angle theory proposed in this thesis is based upon the existence and magnitude of the drift angle, which is shown to be a product of the forward velocity and yaw rate of a marine vehicle and has been related to a method of mathematically describing the manoeuvrability and handling characteristics of marine vehicles. This has been developed into a new form of mathematical model during the course of this research and is a consequence of the implementing the methodologies for quantifying the hull of a marine vehicle in terms of the forces and moments that are experienced by a vessel. From the installation of a towing tank and the implementation of scale model testing techniques supported by full scale sea trials, the use of forces and moments has been found to be an inadequate method of describing a marine vehicle so that an accurate mathematical model can be constructed.

This new form of mathematical model is structured around a set of hydrodynamic curves that permit the rudder angle and engine speed to be mathematically related to the forward velocity and yaw rate of a vehicle. These two parameters are then utilised to determine the track history of the vehicle in terms of the distance traversed from a known point of origin. A mathematical model of Picket
Boat Nine has been constructed using this new technique and has been tested and validated by full scale sea trials in this vessel and in comparison to a force modular model.

The drift angle theory and this new mathematical model are still in their infancy and require further development to enhance their accuracy and validity to other marine vehicles as results have only been obtained from two vessels; Picket Boat Nine, a patrol craft, and the USS Compass Island, a seventeen thousand tonne tanker. These results however demonstrate the stature and rigidity of these methods for describing a marine vehicle mathematically. The research has indicated that development of this model requires the inclusion of external disturbances before its full potential can be evaluated for the applications that were discussed in chapter one.
APPENDIX A.

Picket Boat Nine particulars.

Plate A.1 Photograph of Picket Boat Nine.
Plate A.2 Photograph of Picket Boat Nine.
Introduction to Picket Boat Nine.
Vessel main particulars.
Leading particulars.
Figure A.1 Side View of Picket Boat Nine.
Figure A.2 Plan View of Picket Boat Nine.
PICKET BOAT NINE

Plate A.1

Plate A.2

Page 189.

Appendix A.
Picket Boat Nine, as illustrated on the previous page (Plate A.1 and A.2), is a twelve meter twin screw patrol craft that this programme of research, into the mathematical modelling of marine vehicles, has been based upon. This vessel is currently employed in two roles, the first as a training vessel by which Royal Naval Officers under training, at Britannia Royal Naval College, Dartmouth, learn basic seamanship skills and the other being available for full scale sea trails. The following lists the main particulars of this vessel and the equipment that has been employed.

VEssel Main PARTICULARS.

The vessel is constructed from glass reinforced plastic and consists of six compartments, separated by transverse marine grade plywood bulkheads as illustrated in Figures A.2-A3.

- Fore peak,
- Stowage locker,
- Fore compartment, including the WC and gallery space,
- Machinery compartment, with wheel house above,
- After compartment,
- After peak.

Leading PARTICULARS.

- CONSTRUCTION MATERIAL ....................... Glass fibre reinforced plastic.

- MAIN DIMENSIONS
  - Length overall ....................... 12.547m
  - Length at water line ................. 11.683m
  - Breadth overall ..................... 3.658m
  - Draft forward ...................... 1.066m

- LIFTING WEIGHT ............................. 11.73 tonnes

- PERFORMANCE
  - Unladen, sustained speed........... 12 knots
  - Fully laden, sustained speed ....... 8 knots
  - Endurance, full power .............. 24 hours

- MAIN ENGINES
  - Type ...................................... Perkins straight six, four Stroke diesel
• Number of engines .......................... 2
• Bore ........................................... 98.4mm
• Stroke ........................................ 127mm
• Cubic capacity ............................. 5.8litre

• SPEED AND DIRECTION CONTROLS........ Morse, model MT twin, Individual engine control.

• PROPELLERS
  • Type........................................ Propulsion Ltd, 3 bladed 520.7mm diameter x 520.7mm pitch.
  • Number of propellers ................. Two.
  • port propeller - left handed.
  • starboard propeller - right handed.
  • Weight per unit ........................... 11.34kg

• RUDDERS
  • Type ....................................... Spade type
  • Number ..................................... 2

• STEERING GEAR
  • Type ........................................... Wills-Ridley hand hydraulic
  • Transmitter .................................. Triton 2.1, operated by Steering wheel
  • Receiver .................................... Hydraulic cylinder and ram T-100
  • Steering system oil capacity ........ 2.95litres

• ELECTRICAL SYSTEM
  • System voltage .................... 24 volts
  • Batteries ............................. Two banks of 12 volt, lead acid
  • Voltage, each bank ............... 24volts
  • Alternators ......................... CAV AC/524/Y30M
  • Regulators .......................... Lucas 440
  • AC generator

• SENSORY EQUIPMENT.
  • GPS, Global Position System
  • DECCA Navigator system.
  • Gyro Compass
  • Trailing edge log.
  • Strain Gauge, 2kN.
  • Standalone Personal Computer.
  • Workbench 3.1 software
  • Strawberrytree data acquisition hardware

REFERENCES:

Side view of Picket Boat Nine.  

Figure A.1
Plan view of Picket Boat Nine. Figure A.2
APPENDIX B.

The installation of a towing tank at Britannia Royal Naval College.

Part I.

The towing tank.

Part II.

The creation of an accurate scale model of the parent vessel.
THE INSTALLATION OF A NEW TOWING TANK AT BRITANNIA ROYAL NAVAL COLLEGE.

ABSTRACT

A new ship tank has recently been installed at Britannia Royal Naval College (BRNC) Dartmouth for the purpose of instructing Young Naval Officers in 'Ship Technology' and as a research tool for the evaluation of ship manoeuvring characteristics. Part one of this report introduces this technical facility in terms of the tank itself and the dynamic equipment employed to simulate the motion of a marine vehicle when underway and then proceeds to describe the problems that have been encountered and that are associated with the creation of a suitable dynamic test facility for accurately measuring and observing ship model behaviour.

Part two of this report explains the factors of scaling that have to be obeyed and satisfied so that scale models can be employed within this ship tank to illustrate the dynamic behaviour of full scale vessels.

PART I.

THE TOWING TANK.

The ship tank at BRNC Dartmouth is a free standing concrete structure (length 15m, breadth 2.72m and depth 1.2m) containing chemically treated fresh water that supports two parallel rails upon which a computer controlled, mechanically driven gantry runs the length of the tank as shown in Plate 1.
THE FLUID MEDIUM WITHIN THE TOWING TANK.

The water level in the tank is automatically topped up by a ball-valve system to the required depth with fresh water that is maintained at a constant pH value of 7.5 by the addition of:

- Sodium bisulphate - acidic additive
- Sodium dichloroisocyanate - bacterial agent
- Calcium chloride - water hardener
- Sodium bicarbonate - alkaline additive

The water within the tank is circulated through filters for ten hours in every twenty four hours to prevent stagnation and the growth of algae which has been the cause of some erratic data and commonly known as the 'green monster' [1]. Being located in a centrally heated laboratory prevents the temperature from varying not more than 0.2 degrees Celsius in any twenty four hour period and not more than four degrees Celsius annually as shown in Figure 1. The temperature variation with respect to position in the tank (longitudinal, lateral and depth) was found to be negligible when measured with a salinity, temperature and density (STD) probe. This is critical as half a degree temperature difference within a volume of water has been found to cause convection currents known as tank storms [2].

The density of the fluid was investigated by weighing known volumes of the fluid but no satisfactory value could be found as the accuracy required in both the measurement of volume and mass were not possible with the instrumentation available; and no significant difference could be determined when a hygrometer was used to compare the density of fresh water with the fluid in the towing tank. The viscosity of the fluid in the towing tank was also investigated by a modified Stokes experiment [3] where the fluid as a result of a head of water was made to exit, under laminar flow, from a capillary tube of known diameter and length, with the time to collect a known volume permitting the flow rate to be calculated. From this information the coefficient of viscosity was determined from the following equation:

\[ \eta = \frac{8\cdot OB}{\pi r^4 \cdot AB} \]

Where:
- \( \eta \) = Viscosity of water.
- \( r \) = Radius of the needle.
- \( l \) = length of needle.
- \( OB \) = Head of water.
- \( AB \) = Flow rate at this head of water.

Inaccuracy with the instrumentation and measurements taken only gave an indication to the viscosity of the fluid as did a viscosometer. As a result of the above mentioned inaccuracies (in instrumentation and experimentation) the density and viscosity of the fluid medium contained within the tank were taken as the standard published values for fresh water [4].
Temperature profile for the year April 1991 to April 1992 for the BRNC ship tank.
THE DYNAMIC FACILITY FOR MODEL TESTING.

The mechanism for towing the scale models consists of an 'A' framed structure with two driven wheels at its base and an idle wheel at the apex known as a gantry of carriage. The structure is driven along two RSJ girders optically lined with the longitudinal of the tank and held in place by two pairs of rigidly mounted guidance wheels located at the base of the structure as shown in Figure 2 and Figure 3.

The velocity of the gantry was dictated by the operator between the limits of five and one hundred centimetres per second. The chosen velocity is maintained by a positional feedback controller that consists of a resolver and an encoder positioned on the back of the motor, where the resolver is linked to a Vickers amplifier and the encoder is linked to a TRIO processor. The amplifier relays the positional data to the processor which then calculates the required output signal for the motor, as shown in Figure 4.

The amplifier also possesses a personality card that was pre-programmed for the expected system loading with physically adjustable pots for proportional gain and off-set following system installation. The TRIO processor possessed a digital controller accessible via an external computer through which the operators desired speed is entered. Further to this the gain settings of the controller are also accessible for tighter control.

THE MEASUREMENT OF LOAD.

A single component strain gauge (load cell) was installed and consists of a variable potentiometer rigidly mounted upon the gantry with a moving arm linked to a sprung plate that was attached to the model, as shown in Figure 5. The measurement information from this load cell is then sent to a dedicated computer as an analogue signal. This dedicated computer calculates the load exerted by the towed model upon the load cell from the analogue signal received, then stores this information in the computers memory during the run and writes it to disk once the test is complete.

MODEL MOUNTING.

The scale models were rigidly mounted to the load cell via a telescopic arm (sting). The model mount positioned at the models longitudinal centre of rotation also possesses a sealed bearing joint that permits the model to heave and pitch to its natural vertical position when creating the bow wave system but prevented the remaining motions.
TRANSVERSE SECTION OF TOWING TANK

- Motor
- Load cell
- Driven wheels
- Driving belts
- Guidance wheels
- Sting
- Idle wheel
- Proximity switch
- Emergency stop button
- Power supply and data exchange cable
Figure 3.

Block diagram of the control system for the towing tank.

Figure 4.

M. Russell, Department of Engineering Science.
Britannia Royal Naval College. 1992
Sprung mechanism
supporting load cell

Load cell

'\( h \)-plate'

Gantry structure

Sting

Sealed bearing
pivot joint

MODEL
INTRODUCTION - AREA FOR CONCERN.

The equipment once installed, was tested for accuracy, reliability and repeatability. These initial results, as shown in Figure 6, proved that when no model was towed and a zero resistance expected the system possessed substantial noise that obscured the true signal. Further to this when a model was towed, as shown in Figure 7, a similar obscured signal was also produced (when a value greater than zero was expected).

SOURCES OF NOISE.

The apparent noise of the measured signal from initial observations was found to be a resident 10 Hertz frequency and was attributed to the natural oscillation of the load cell. This was confirmed via the use of a stroboscope set to 10 Hertz directed at the load cell. The oscillation of the load cell was then found to be directly related to the following four main factors:

1. Inertial forces concerned with acceleration and deceleration of the mass of the gantry.
2. Physical imperfections in the rails.
3. Torsional forces acting as the gantry moved down the tank.
4. The driving motor control system.

The first two factors from the list can be seen from Figure 8 as definite excitations to the system; the initial acceleration and final deceleration phase of the gantry is evident at the beginning and end of each plot and the half way excitation in the trace attributed to a physical joint in the rails.

The torsional forces acting upon the gantry as it moved down the tank were deduced by the use of a laser plot, where a low powered infra red laser was mounted rigidly to the gantry, aligned in the longitudinal plane of the tank and the path of the laser observed as the gantry moved. The torsional effects were found to be attributed to the alignment of the wheels and the constant correction that was being applied by the guidance wheels in order that the gantry maintained the demanded straight path along the rails. Further to this as the centre of gravity of the gantry was not central over the driving wheels the apex of the gantry tended to lag behind during the acceleration phase setting up an initial torsional motion and once the constant velocity phase had begun the side with the idle wheel continued to accelerate thereby causing the opposite to occur. This initial torsional motion invoked the use of the guidance wheels which constantly corrected the gantry along its path causing the gantry to zig-zag down the tank.

The control system of the gantry during the constant velocity phase was also found to be another cause of the load cell oscillation as it 'hunted' to maintain the demanded velocity through a positional feedback loop and inducing small inertial forces that were transmitted to the load cell.

Graph of measured resistance against distance travelled when no model was present and travelling at 20cm/s.

Measured values (N).

Distance travelled (m).
Graph of measured resistance against distance travelled when a model was present and travelling at 20cm/s.

Measured values (N).

Distance travelled (m).
Graph indicating mechanical excitations to the gantry when no model is present at 20cm/s.

Measured values (N).

Guidance wheels correct the path of gantry.

Initial Acceleration phase.

Joints in the rails.

Deceleration phase.

Distance travelled (m).
CORRECTIONS OF THE DYNAMIC FACILITIES.

The contributing factors as outlined above were resolved in three basic ways: firstly by the optimisation of the mechanical system, secondly fine tuning of the computer control system and thirdly a reduction of the residual oscillations associated with a mechanical system.

The optimisation of the mechanical system was initiated by the correct alignment of all the wheels. This was established by aligning the two driving wheels and the idle wheel to be parallel with each other and with the longitudinal line of the tank. This ensured that once the gantry was in the constant velocity phase the guidance wheels were deemed redundant except as a safety measure for preventing the gantry from leaving the rails or entering the tank. The alignment of the wheels was complemented by the adjustment in the tensioning of the driving belts to ensure that the driving wheels pulled true and did not oppose each other.

This correct alignment of the wheels and the associated redundant guidance wheels reduced the torsional effects during the constant velocity phase but the initial acceleration of the gantry from rest induced an initial torsional moment and required the guidance wheels to be employed. This was reduced by employing the guidance wheels only during the acceleration phase and was implemented by increasing the width of the rails for the distance when these forces were in operation, thus holding the gantry rigid. This not only reduced the initial torsional effects but ensured that the gantry began the correct straight course during the constant velocity phase whereby the lateral variation of the rails became insignificant further reducing the need for the guidance wheels except as a safety factor.

The control system consisted of a controller, amplifier and a motor with two feedback loops from the motor, one to the amplifier and the other to the controller, as illustrated previously in Figure 4. The resident hunting for the demanded velocity was accredited to the incorrect setting of the off-set and proportional potentiometers on the amplifier and the following variable parameters contained within the TRIO controller:

- proportional gain
- integral gain
- derivative gain
- hysteresis band width
- following error limit
- output velocity gain
- following error
- force output dive

The TRIO controller was accessible via an RS232 interface cable connected to an external computer that housed the TRIO set-up software.

To allow the set-up software to communicate with the controller the communications channel was opened by changing the hex dial situated on the
controller from channel 0 to channel 2, then force output drive (parameter 16) was changed from 10,000 to 0. This forced zero volts to the motor and the off-set pot was adjusted manually so that the motor was stationary and no creep was evident.

The proportional pot on the amplifier was set by initiating a computer program from the set-up software that forced a square wave of +5volts and -5volts to the motor which drove the gantry backwards and forwards. The x-y channels of an oscilloscope were then connected to the tacho and zero volts on the amplifier which displayed visually the response of the motor to a square wave. The amplifiers proportional pot was then adjusted manually so that the oscilloscope displayed the best attainable square wave at which point the motor followed the demanded input without oscillating and the amplifier was installed correctly.

The digital TRIO controller was then optimised by introducing a 10 second delay into the square wave program that caused the gantry to move a greater distance before stopping and then returning. This permitted the parameters stated above to be adjusted with the response noted whilst the gantry was in motion. The optimal setting for the parameters was deduced by the use of the oscilloscope when the system was not over or under damped but followed the demanded input as closely as possible. Once the best estimate for these parameters was deduced the square wave program was altered so that they could be validated for the complete range of speeds that the gantry operated over.

The final parameters derived were found suitable for the complete range of speeds of operation even though at certain speeds the system was apparently more stable than at others.

RESIDUAL NOISE REMOVAL.

The residual oscillation that which accompanies any dynamic mechanical system was investigated, in order to reduce the noise generated and so give a more accurate signal.

Initially various forms of physical viscous damping were introduced in the form of air and oil dashpots mounted rigidly to the gantry and damping the load cell motion. An improvement was observed in the form of the scattering of the measurements about the mean (standard deviation), but deterioration of the fluid medium and the associated friction with an actual physical link proved unreliable and unrepeatable.

The use of damping became evident from these initial results and a non-viscous damper was installed in the form of an eddy current damper that possessed no physical links with the associated friction.

The principle of the eddy current damper is that the magnetic flux produced between opposite poles (N-S) creates an induced electromagnetic current in a conductive plate that is moved longitudinally between the poles. This electromagnetic current in turn produces an electromagnetic force that opposes the motion of the plate hence damping the oscillation of the load cell in a non-viscous manner and
without friction, as no physical links existed. The use of an eddy current damper was investigated with a permanent horseshoe shaped magnet, having a 20mm fixed pole separation, which was rigidly mounted to the gantry and the aluminium plate attached at one end to the load cell passed through the poles.

The results obtained showed an improvement in the repeatability of the tests, but the effectiveness of the damping was required to be increased. The theory of the eddy current damper relies upon the density of the magnetic flux being cut, the area over which the flux flows and the conductivity of the plate passing through the poles. As the density of the flux is dependant upon the separation of the two permanent magnets, the use of electromagnets where the flux density and poles separation was variable was investigated, but this proved to be unsuccessful with the resources available. The solution to maximising the flux density was by the use of two permanent U-shaped magnets each with a 677N holding force physically separated within a cradle, as shown in Figure 9, where the separation of the poles could manually be adjusted. The initial results showed a marked improvement in the accuracy of the results. Further tests concluded that a minimum separation between the plate and the poles was required, aluminium was more effective than copper and that the plate should also cut the flux that flows around the magnets.

The second method of physical damping employed removed the rigidity of the system by placing the load cell upon a dampened bed which consisted of two solid layers - the lower of which was attached to the gantry and the other supported the load cell with a layer of expanded polyurathane foam between the two. This isolated the load cell from the small perturbations of the gantry, and is shown in Plate 2 along with the installed eddy current damper.

![Image](https://example.com/image.jpg)
Figure 9.

- Motion of h-plate.
- Induced electromagnetic force (EMF).
- Magnetic field.

EDDY CURRENT DAMPER

LOAD CELL

GANTRY STRUCTURE

MODEL
Furthermore electronic and software filtering, once the measurements had been made, was also investigated. Initially a software filter was introduced into the sampling program that smoothed the results before they were recorded. This was deemed unsatisfactory as it prevented the raw data from being recorded and further analysis being conducted. This was also the case with electronic signal processing and filtering.

The final method of software filtering implemented to increase the accuracy of the results involved processing the raw data once it had been recorded digitally. Since only one result for each run was necessary and the sampling rate was ten samples every second, between 100 and 2000 samples were collected each run, depending upon the speed of the run. The initial acceleration and final deceleration phases were removed and the mean and the standard deviation were calculated with in an iterative loop, where the standard deviation was taken as a measure of the accuracy about the mean. The effectiveness of each method of damping employed can be seen in Table 1.

<table>
<thead>
<tr>
<th>Attribute / Speed</th>
<th>5cm s⁻¹</th>
<th>30cm s⁻¹</th>
<th>65cm s⁻¹</th>
<th>100cm s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>No damping, no model, initial measurements.</td>
<td>std = 13%</td>
<td>std = 43%</td>
<td>std = 141%</td>
<td>std = 262%</td>
</tr>
<tr>
<td>No damping, with model, initial measurements.</td>
<td>std = 14%</td>
<td>std = 42%</td>
<td>std = 130%</td>
<td>std = 240%</td>
</tr>
<tr>
<td>Oil dashpot, no model.</td>
<td>std = 12%</td>
<td>std = 66%</td>
<td>std = 123.3%</td>
<td>std = 162%</td>
</tr>
<tr>
<td>Air dashpot, no model.</td>
<td>std = 5%</td>
<td>std = 29.7%</td>
<td>std = 57.5%</td>
<td>std = 87.8%</td>
</tr>
<tr>
<td>One fixed pole magnet, no model.</td>
<td>std = 11%</td>
<td>std = 52%</td>
<td>std = 122%</td>
<td>std = 169.9%</td>
</tr>
<tr>
<td>Two U-shaped magnets with Al. plate, no model.</td>
<td>std = 10%</td>
<td>std = 25%</td>
<td>std = 50%</td>
<td>std = 64%</td>
</tr>
<tr>
<td>Final solution, no model.</td>
<td>std = 8%</td>
<td>std = 22%</td>
<td>std = 43%</td>
<td>std = 61%</td>
</tr>
</tbody>
</table>

Table 1.

Where the final chosen method consisted of the two U-shaped magnets held apart with in a cage and an aluminium plate, which was attached to the H-plate, was free to move between the magnets. This structure was then supported upon a bed of foam.

**CALIBRATION TO ZERO.**

The load cell and the sampling PASCAL program when installed was not set to zero which led to all the measurements being positive by 1.89 Newtons. This was initially remedied by a physical adjustment to zero but a constant zero reading was not possible due to the sensitivity of the load cell which was affected by slight wind currents within the laboratory and spikes in the power supply. Therefore the zero
calibration was determined by entering an iterative loop into the sampling program that calculated the mean and standard deviation of 100 samples. The mean calculated was then subtracted from each of the measurements once they had been recorded and before the mean resistance was determined for each run.

**DETERMINATION OF THE ACCURACY AND RELIABILITY OF THIS SCALE MODEL TESTS FACILITY.**

The reliability and accuracy of the equipment was established by a series of standard test runs. The initial program consisted of repeated dry runs (without a model) over the range of speeds from 5 cm s\(^{-1}\) to 100 cm s\(^{-1}\) where the speed was increased by 5 cm s\(^{-1}\) every run. This was repeated 10 times with the mean and standard deviation calculated for the 10 runs as given in Table 2.

<table>
<thead>
<tr>
<th>Speed (cm s(^{-1}))</th>
<th>Average Resistance (N)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>10</td>
<td>0.004</td>
<td>0.0016</td>
</tr>
<tr>
<td>15</td>
<td>-0.006</td>
<td>0.0019</td>
</tr>
<tr>
<td>20</td>
<td>-0.018</td>
<td>0.0024</td>
</tr>
<tr>
<td>25</td>
<td>0.012</td>
<td>0.0023</td>
</tr>
<tr>
<td>30</td>
<td>0.024</td>
<td>0.0022</td>
</tr>
<tr>
<td>35</td>
<td>-0.007</td>
<td>0.0023</td>
</tr>
<tr>
<td>40</td>
<td>0.041</td>
<td>0.0029</td>
</tr>
<tr>
<td>45</td>
<td>-0.004</td>
<td>0.0029</td>
</tr>
<tr>
<td>50</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>55</td>
<td>0</td>
<td>0.0035</td>
</tr>
<tr>
<td>60</td>
<td>0.059</td>
<td>0.0038</td>
</tr>
<tr>
<td>65</td>
<td>0.062</td>
<td>0.0043</td>
</tr>
<tr>
<td>70</td>
<td>0.03</td>
<td>0.004</td>
</tr>
<tr>
<td>75</td>
<td>0.038</td>
<td>0.0043</td>
</tr>
<tr>
<td>80</td>
<td>-0.063</td>
<td>0.0055</td>
</tr>
<tr>
<td>85</td>
<td>0.088</td>
<td>0.0058</td>
</tr>
<tr>
<td>90</td>
<td>0.011</td>
<td>0.0054</td>
</tr>
<tr>
<td>95</td>
<td>-0.016</td>
<td>0.0059</td>
</tr>
<tr>
<td>100</td>
<td>-0.028</td>
<td>0.0061</td>
</tr>
<tr>
<td>Mean</td>
<td>0.01165</td>
<td>0.0037</td>
</tr>
</tbody>
</table>

Table 2

The accuracy of the mean zero was then determined to be within the tolerance range 0.0 ± 0.01 Newtons and as this was repeated ten times the system can be considered as accurate and reliable within resource limitations.
A second set of standard runs was then conducted using a 1:16 scale model of Picket Boat Nine always ensuring that the water was 'calm' before each run was commenced and that each series of runs was completed within a single power up of the equipment. In total ten tests were conducted for the same speeds as with the dry runs and Table 3 gives the results for the standard model tests.

<table>
<thead>
<tr>
<th>Speed (cm s⁻¹)</th>
<th>Average resistance (N)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.011</td>
<td>0.001</td>
</tr>
<tr>
<td>10</td>
<td>0.012</td>
<td>0.003</td>
</tr>
<tr>
<td>15</td>
<td>0.021</td>
<td>0.002</td>
</tr>
<tr>
<td>20</td>
<td>0.028</td>
<td>0.004</td>
</tr>
<tr>
<td>25</td>
<td>0.039</td>
<td>0.002</td>
</tr>
<tr>
<td>30</td>
<td>0.049</td>
<td>0.006</td>
</tr>
<tr>
<td>35</td>
<td>0.056</td>
<td>0.006</td>
</tr>
<tr>
<td>40</td>
<td>0.078</td>
<td>0.012</td>
</tr>
<tr>
<td>45</td>
<td>0.084</td>
<td>0.009</td>
</tr>
<tr>
<td>50</td>
<td>0.135</td>
<td>0.005</td>
</tr>
<tr>
<td>55</td>
<td>0.119</td>
<td>0.01</td>
</tr>
<tr>
<td>60</td>
<td>0.144</td>
<td>0.003</td>
</tr>
<tr>
<td>65</td>
<td>0.246</td>
<td>0.019</td>
</tr>
<tr>
<td>70</td>
<td>0.237</td>
<td>0.001</td>
</tr>
<tr>
<td>75</td>
<td>0.368</td>
<td>0.015</td>
</tr>
<tr>
<td>80</td>
<td>0.365</td>
<td>0.029</td>
</tr>
<tr>
<td>85</td>
<td>0.5</td>
<td>0.032</td>
</tr>
<tr>
<td>90</td>
<td>0.563</td>
<td>0.031</td>
</tr>
<tr>
<td>95</td>
<td>0.541</td>
<td>0.025</td>
</tr>
<tr>
<td>100</td>
<td>0.696</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 3.

The results of the series of dry runs, when the control (no model present) (Table 2) and when a sixteenth scale model was towed (Table 3), for the range of speeds that the tank permitted are shown graphically in a plot of resistance measured against speed, in Figure 10. These results prove that the equipment is capable of measuring the small forces that are experienced when a scale model in the bare hull condition is towed at a constant velocity in the longitudinal plane.
Graph showing increase in resistance due to a model being present compared to when no model was towed.

Resistance (N)

- No model present  →  1:16 scale model

Velocity (m/s)
VALIDATION OF EQUIPMENT.

The equipment described above has been validated by the Defence Research Agency (DRA), based at Haslar, where a sister model to the one used in the above series of tests was towed under their operating conditions and with their facilities. The results of this series of trials can be seen in Table 4.

<table>
<thead>
<tr>
<th>Velocity (ms⁻¹)</th>
<th>Drag (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.054</td>
<td>0.0316</td>
</tr>
<tr>
<td>0.143</td>
<td>0.0323</td>
</tr>
<tr>
<td>0.241</td>
<td>0.0573</td>
</tr>
<tr>
<td>0.34</td>
<td>0.0616</td>
</tr>
<tr>
<td>0.438</td>
<td>0.0948</td>
</tr>
<tr>
<td>0.538</td>
<td>0.1468</td>
</tr>
<tr>
<td>0.636</td>
<td>0.209</td>
</tr>
<tr>
<td>0.737</td>
<td>0.3171</td>
</tr>
<tr>
<td>0.836</td>
<td>0.4618</td>
</tr>
<tr>
<td>0.934</td>
<td>0.6406</td>
</tr>
<tr>
<td>1.005</td>
<td>0.7675</td>
</tr>
</tbody>
</table>

Table 4.

When the measurements from DRA (Table 4) are compared to those from the new towing tank at BRNC (Table 3) there is a high degree of correlation, as shown in Figure 11, which further indicates the reliability and repeatability of this new towing tank.
Scale model resistance tests conducted upon sister models in different towing tanks on different dates.

- BRNC Dartmouth.
- DRG Haslar.
PART II.

THE CREATION OF AN ACCURATE SCALE MODEL OF THE PARENT VESSEL.

INTRODUCTION.

The equipment as described in Part one permits the use of scale models to accurately measure the forces and moments experienced during motion and so replicate the parent vessel's behaviour. Under experimental conditions instrumentation can be used to allow the six degrees of motion that the vessel experiences to be separated and investigated individually. To ensure the observations and measurements taken are a valid representation of the full scale vessel the scale model must satisfy the laws of scaling and comparison.

THE FACTORS OF SCALING.

The properties of a marine vessel may be expressed by its size, length, breadth and draft in metres and with the mass in Kilograms. The mass also has a distribution about the vessel characterised by the position of the centre of gravity \((X_G, Y_G, Z_G)\) with moments of inertia \((I_{XX}, I_{YY}, I_{ZZ})\). The shape of the vessel is normally depicted by the use of form coefficients namely the area of midships, block and prismatic coefficients. This vessel then operates at the interface between two fluid mediums, water and air. In order to reproduce an identical scale model that performs and behaves in a manner that replicates that of the vessel, the vessel and its environment have to be scaled accordingly.

THE LAWS OF COMPARISON.

The ability to examine the behaviour of marine vessels by the use of scale model testing techniques is governed by the three laws of comparison [5], Geometric similarity, Kinematic similarity and Dynamic similarity. These have to be satisfied in order that the scale model is a true representation of the full scale vessel in all its operations and direct measurements can then be taken from the model and scaled accordingly to that of the vessel.

Geometric similarity refers to a constant ratio between the model and the parent vessel with regards to all physical dimensions. This is achieved by the use of a scaling factor \(\lambda\) where any dimensional length of the parent vessel is scaled according to:

\[
\lambda = \frac{L_S}{L_M}
\]

where \(L_S\) is the length from the parent vessel and \(L_M\) is the model's equivalent length. If all dimensions are scaled accordingly then a perfect 'small' vessel is created, hence a scale model.
Kinematic similarity between the model and the parent vessel is concerned with the motion of both without reference to force or mass [6]. Using the law of geometric similarity the model should possess a velocity scaled to:

$$\lambda = \frac{V_s}{V_M}$$

where $v_s$ and $v_M$ are the respective speeds of the vessel and scale model. This is not possible as the parent vessel operates in either fresh or salt water and the scale model normally operates in fresh water and not a fluid medium that is scaled according to the law of geometric similarity.

Dynamic similarity is concerned with the forces that are in action with regards to the model, parent vessel and the two fluid mediums in which they operate, and which should also be geometrically scaled according to the scaling ratio ($\lambda$).

In practice it is assumed that geometric similarity is satisfied by the use of the scaling ratio and then either the kinematic law or the dynamic law of similarity can be satisfied as it is not possible to satisfy both. The inability to satisfy both the kinematic and dynamic laws of similarity for the model is because no suitable fluid medium has been found that will satisfy all three laws when fresh or salt water is taken as the parent vessels fluid medium. That is to say the main properties of the fresh or salt water that are of importance when considering a marine vessels characteristics are the viscous, inertial and gravitational forces. However to satisfy the correct relationship the scale model fluid medium would require a decreased viscosity and/or an increased density to that of the water and would be totally dependant upon the scaling factor and therefore for every different geometrically scaled model a similar scaled fluid medium would be required.

**RESOLVEMENT OF THE LAWS OF COMPARISON.**

Scale model testing techniques normally assume that geometric law of similarity between the parent vessel and the model is satisfied by taking the full scale vessels parameters and applying a scaling factor ($\lambda$).

The second and third laws of comparison are mainly concerned with fluid flow phenomena due to the inertial and gravitational forces and the viscous properties of the fluid mediums. Since no fluid medium has been found to date that possesses the required scaled properties for each different model, the resolvement of these two laws of similarity has been in general conducted by the use of two dimensionless numbers, Froudes number ($Fn$) [4]:

$$Fn = \frac{V}{\sqrt{gL}}$$

which is the ratio of \(\frac{\text{The inertial forces}}{\text{The gravitational forces}}\)

and Reynolds number ($Rn$)[4]

$$Rn = \frac{\rho V L}{\eta}$$

which is the ratio \(\frac{\text{The inertial forces}}{\text{The viscous forces}}\)
These are derived from simple dimensional analysis of the properties and forces that act upon a vessel. Other dimensionless ratios have also been proposed for describing the properties and can be used in the scaling of the fluid mediums; Cauchy number (Cn), Euler number (En), Planing number (Pn), Weber number (Wn) and the Boussinesq number (Bn) but the properties they refer to can be considered negligible for large vessels operating in deep unrestricted waters. [7]

To satisfy both Froudes number and Reynolds number, the model has to be towed at two different speeds simultaneously. Therefore in the majority of model testing techniques the model is towed at an equivalent Froudes number to that of the parent vessel, satisfying the inertial and gravitational forces, and resulting in a similar wave profile being established by the model to that of the parent vessel. This leaves the viscous forces unsatisfied. As a result of the viscous properties the molecules of water in contact with the parent vessels hull are assumed to be motionless compared to that of the main stream, and that at some distance from the hull the mainstream velocity is achieved [8]. This is generally represented by the velocity profile, as shown in Figure 12.

![Figure 12](image)

These viscous properties of the fluid medium are responsible for the generation of a boundary layer around the vessel that consists of either laminar or turbulent flow or both depending upon the speed of the fluid and the length of the vessel in question.

From the work conducted by Osbourne Reynolds in 1883 [9] it is assumed that for a given fluid in contact with a smooth solid boundary, when the ratio of the inertial to viscous forces of a fluid approaches a value in the order of $2 \times 10^6$, then transition from laminar to turbulent flow occurs and that for ratios below this, laminar flow predominates, whilst turbulent flow prevails above this figure and that for any roughened surface laminar flow always exists below a figure of $0.5 \times 10^6$. In the case of the parent vessel, transition generally falls shortly aft of the bow depending upon the speed of the vessel or fluid and the roughness of the hull, leaving the remaining length of the hull in turbulent flow with a viscous sub layer, as shown in Figure 13.

---

The flow around the hull of the model in general exists in a state of laminar flow which is a result of the length of the model and the speeds that are used to satisfy Froude's number.

**FLOW REGIMES AND FLOW TRIP FROM LAMINAR TO TURBULENT FLOW.**

As mentioned above, most of the model testing techniques conducted at various research establishments are conducted at equivalent Froude numbers to that of the parent vessel, leaving the viscous forces unsatisfied and the model in a state of laminar flow. To this extent physical mechanisms have been implemented to force the transition from laminar to turbulent flow around the model by either the addition of a roughened surface or a series of pins that protrude from the surface of the model hull. At the point of transition causing the flow to trip from laminar to turbulent flow by the creation of eddies, as shown in Figure 14.

Various other methods are also used to trip the flow; these include combes, rakes and fine wire positioned forward of the bow and towed along with the model. Other methods have also been investigated and include the use of chemical additives to the water to simulate the required viscous effects.

**THE STATE OF THE FLOW.**

---

_M. Russell, Department of Engineering Science._
_Britannia Royal Naval College. 1992_
As mentioned above, the state of the flow around the scale model has to transform from laminar to turbulent flow at the correct position along the model to that of the parent vessel. The location of the transition point upon the parent vessel can be found from the use of the critical Reynolds number [7]:

\[ \text{Rn}_C = \frac{v \cdot L_c}{\nu} \]

Where:
- \( \text{Rn}_C \) = Reynolds number for the given hull roughness.
- \( v \) = Speed of fluid or speed of vessel.
- \( L_c \) = Position of transition point from the bow.
- \( \nu \) = Kinematic viscosity \( \left( \frac{\text{Density}}{\text{Coefficient of viscosity}} \right) \)

and then can be geometrically scaled to that of the model, but is normally taken at a position two percent of the vessel's length aft of the bow.

To ensure that the flow has tripped and that just aft of the bow the flow is turbulent, the following visual techniques are normally implemented. Tufts of wool [12] attached to the hull that are free to move, will in laminar flow align themselves with the direction of flow, whilst in turbulent flow they will move in all directions. Coloured ink mixed with oil adheres to the hull [12], yet does not protrude, and the length of the streak produced by turbulent flow is greater than that of laminar flow and also the rate of removal in turbulent flow is predominantly faster than in laminar. The use of long chain polymers [13] ejected from the hull that solidify upon contact with the water and align themselves with the state of flow are used in a similar manner to that of the tufts of wool. Further to this the size of the boundary layer and the volume of water, whose velocity is affected by the presence of a solid boundary, can be investigated by the use of Pitot tubes placed at varying distances from the hull measuring the dynamic pressure which is dependant upon the velocity of the stream.

**THE STATE OF THE HULL.**

The transition from laminar to turbulent flow on the parent vessel is also dependant upon the condition of the hull [14], (hull roughness) and as with most marine structures corrosion and biological growth occur that alters the surface with respect to time. Research conducted upon the effect of hull roughness has found that roughness protruding above a smooth surface will cause eddies to form in laminar flow, below the critical Reynolds number causing the flow to become turbulent [1].

It is assumed that the hull of the parent vessel is smooth compared to being rough and that the hull of the model should also be as smooth as possible, since geometrically scaling hull roughness of a parent vessel requires a finish not easily attained. Even though the roughness of the hull affects the flow pattern to the extent that the transition point upon a rough hull will be further forward than that of a smooth hull, this is normally taken into consideration when determining the transition point upon the scale model.

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Britannia Royal Naval College. 1992*
THE USE OF A FAMILY OF MODELS TO RESOLVE FROUDE AND REYNOLDS NUMBER.

In large establishments that possess the facilities, the assumption is made that a family of models [15] will comply with the geometric law of similarity and when each model is towed at equivalent Froude then Reynolds numbers, then the kinematic and dynamic similarity is also achieved, where direct measurements from the models relate to the parent vessel as described previously.

EXTENSION OF THIS TEST FACILITY.

This model test facility has since been improved upon and includes the installation of a new data acquisition system for collecting the scale model measurements. This new system consists of a commercial software and hardware package, called Work Bench, which utilises a windows environment upon a dedicated computer for the acquisition and processing of the measurements. Further to this, extra strain gauges have been employed, which are model mounted, and removes most of the unwanted noise produced by the sprung 'H-plate' and sting. These new strain gauges are of a suitable range so that the sway and yaw components when either the rudders are applied or when the model is towed at an angle of attack can be measured, by changing the orientation and location of these strain gauges. The latest addition to the tank has been the installation of a gantry mounted video camera that permits the model to be recorded when in motion.

THE TANK AS A RESEARCH TOOL.

This new ship tank and a family of scale models is currently being employed in a collaborative research programme between Britannia Royal Naval College and Plymouth University to determine the hydrodynamic coefficients for a twelve meter twin screw vessel that are required for a three degrees of freedom, non-linear mathematical model, using a modular approach.
REFERENCES.

APPENDIX C.

Plates referenced in chapter five.

Plate C.1 1:25 & 1:16 scale models.
Plate C.2 1:16 scale model with turbulent stimulators.
Plate C.3 1:16 scale model with rudders attached.
Photograph showing the hulls for both the 1:16 and 1:25 scale models in the bare hull condition along side a thirty centimetre ruler.
Plate C.2.

Photograph showing the 1:16 scale model with turbulence stimulators, studs, attached.
Plate C.3.

Photograph showing the 1:16 scale model with scale rudders attached.
APPENDIX D.

Tables associated with scale model and full scale results referenced in chapter four concerning the surge velocity hydrodynamic coefficients.

Table D.1 Equivalent scale model and full scale speeds.
Table D.2 Measured total resistance for 1:25 scale model.
Table D.3 Measured total resistance for 1:16 scale model.
Table D.4 Measured total resistance for 1:16 scale model with turbulence stimulators in place.
Table D.5 Measured total resistance for 1:16 scale model with rudders attached.
Table D.6 Listing of Super Calc5 spreadsheet for 1:16 scale model values transferred to full scale equivalents.
Table D.7 Listing of Super Calc5 spreadsheet for 1:25 scale model values transferred to full scale equivalents.
Table D.8 Scale model values transferred to full scale equivalent values for 1:25 and 1:16 scale models in bare hull condition, and the 1:16 scale model with turbulence stimulation and rudders attached.
Table D.9 Full scale speed measurement at known engine speed from measured mile manoeuvre.
Table D.10 Full scale measurement of bollard thrust at known engine speed from bollard pull manoeuvre.
Table D.11 Combined measured results for measured mile and bollard pull manoeuvres.
Table D.12 Full scale total resistance measurements from towed vessel approach.
<table>
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<th>1:16 scale model. (m s⁻¹)</th>
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<th>1:25 scale model. (m s⁻¹)</th>
<th>Equivalent full scale. (kts)</th>
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Table D.1

Equivalent scale model and full scale speeds.
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<th>Speed of run. (m s⁻¹)</th>
<th>Run 1. (N)</th>
<th>Run 2. (N)</th>
<th>Run 3. (N)</th>
<th>Average value</th>
<th>Standard deviation</th>
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<tr>
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<td>0.124</td>
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<td>0.198</td>
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Table D.2

*Measured total resistance for 1:25 scale model.*
<table>
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<th>Speed of run. (m s⁻¹)</th>
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<th>Run 2. (N)</th>
<th>Run 3. (N)</th>
<th>Average value.</th>
<th>Standard deviation.</th>
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<td>0.01</td>
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<td>0.001</td>
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<td>0.019</td>
<td>0.021</td>
<td>0.002</td>
</tr>
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<td>0.028</td>
<td>0.004</td>
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</tr>
<tr>
<td>0.25</td>
<td>0.041</td>
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<td>0.039</td>
<td>0.002</td>
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<td>0.049</td>
<td>0.006</td>
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</tr>
<tr>
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<td>0.056</td>
<td>0.006</td>
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</tr>
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<td>0.012</td>
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Table D.3

Measured total resistance for 1:16 scale model.
### Table D.4

Measured total resistance for 1:16 scale model with turbulence stimulators in place.

<table>
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<th>Speed of run. (m s(^{-1}))</th>
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<td>Run 2. (N)</td>
<td>Run 3. (N)</td>
<td>Average value.</td>
<td>Standard deviation.</td>
</tr>
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<td>-----------</td>
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<td>---------------------</td>
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</table>

Table D.5

Measured total resistance for 1:16 scale model with rudders attached.
### Table D.6

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<th>Scale</th>
<th>Lm</th>
<th>LS</th>
<th>Vs</th>
<th>Rs</th>
<th>RTm</th>
<th>CTm</th>
<th>CFm</th>
<th>CRs</th>
<th>CRTs</th>
<th>Sm</th>
<th>SS</th>
<th>Rs1</th>
<th>Rs2</th>
<th>Frm</th>
<th>Fns</th>
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**Subscripts:**
- **m**: Denotes scale model
- **s**: Denotes full scale

**Spread sheet listing showing 1:16 scale model resistances transferred to full scale values.**
Spread sheet listing showing 1:25 scale model resistances transferred to full scale values.

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<th>Scale</th>
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<th>Ls</th>
<th>Rs</th>
<th>Vm</th>
<th>Vs</th>
<th>Rm</th>
<th>Ts</th>
<th>Ct</th>
<th>Ct*</th>
<th>Rs</th>
<th>Ts</th>
<th>S</th>
<th>S</th>
<th>m</th>
<th>s</th>
<th>Fm</th>
<th>Fns</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>L</th>
<th>Length (m)</th>
<th>CT</th>
<th>Coefficient of total resistance</th>
<th>S</th>
<th>Wetted surface area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Velocity (m s⁻¹)</td>
<td>CF</td>
<td>Coefficient of frictional resistance</td>
<td>Rn</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>RT</td>
<td>Total resistance (N)</td>
<td>CR</td>
<td>Coefficient of residual resistance</td>
<td>Fn</td>
<td>Froude number</td>
</tr>
</tbody>
</table>

Subscripts: m Denotes scale model  
s Denotes full scale

Table D.7
1:16 Scale model.

1:25 Scale model.

Bare hull +
Bare hull + Full scale
stimulation. (N) rudders. (N) speed. (kts)

Full scale
Speed. (kts)

Bare hull.

7.79

2234.871

2539.402

2934.005

9.73

8404.212

7.4

1632.994

2027.598

2443.647

9.25

7298.951

7.02

1787.944

2116.92

2126.788

8.76

6577.028

6.63

1575.926

1751.782

1781.807

8.27

4843.655

6.24

1052.669

1391.513

1485.875

7.78

4069.287

5.85

1118.86

1234.668

1208.933

7.3

3202.916

5.46

607.808

976.676

865.158

6.81

2521.804

5.07

694.701

1256.582

986.365

6.33

2157.239

4.68

302.913

718.962

19.828

5.84

1087.388

4.29

238.754

521.839

393.164

5.35

1063.725

3.9

347.758

523.614

459.276

4.87

976.817

3.51

166.628

479.737

363.93

4.38

782.755

3.12

175.677

308.641

201.412

3.89

714.944

2.73

113.18

246.144

160.36

3.41

466.7

2.34

111.997

163.467

142.021

2.92

548.7 1

1.95

94.8

163.427

73.354

2.43

303.906

1.56

70.011

35.697

57.143

1.95

242.875

1.17

58.87

110.34

20.267

1.46

219.272

0.78

35.349

95.397

35.349

0.97

174.172

0.39

41 .86

89.041

24.703

0.49

165.07

0

0

0

0

0

0

(N)

Bare hull.
(N)

Tahle D.8
Scale model values transferred to full scale for the 1:25 and 1:16 scale model
in the bare hull condition, and the 1:16 scale model with turbulence
stimularion and rudder auached.

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Appendix D.


<table>
<thead>
<tr>
<th>Engine speed. (rpm)</th>
<th>Run 1. (kts)</th>
<th>Run 2. (kts)</th>
<th>Average value.</th>
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</thead>
<tbody>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
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<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>600</td>
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</tr>
<tr>
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<tr>
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<tr>
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<td>6.1</td>
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<tr>
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<tr>
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<td>8</td>
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Table D.9

*Full scale speed measurement at known engine speed from measured mile manoeuvre.*
<table>
<thead>
<tr>
<th>Engine speed (rpm)</th>
<th>Run 1 (kN)</th>
<th>Run 2 (kN)</th>
<th>Average value (kN)</th>
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<tbody>
<tr>
<td>100</td>
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<tr>
<td>600</td>
<td>0.38</td>
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<td>700</td>
<td>0.41</td>
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<td>0.51</td>
<td>0.505</td>
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<tr>
<td>900</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>1000</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
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<tr>
<td>1100</td>
<td>0.73</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>1200</td>
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<td>1.13</td>
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Table D.10

Full scale measurement of bollard thrust at known engine speed from bollard pull manoeuvre.
<table>
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<th>Speed (m s(^{-1}))</th>
<th>Thrust (N)</th>
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</thead>
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<td>1.545</td>
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</table>

Table D.11

*Combined measured results from measured mile and hollard pull manoeuvres.*
<table>
<thead>
<tr>
<th>Vessel speed measured with trailing log. (kts)</th>
<th>Measured resistance of PB9 when towed. (kN)</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<td>2.5</td>
<td>0.05</td>
</tr>
<tr>
<td>3.2</td>
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<td>0.62</td>
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<tr>
<td>7</td>
<td>0.72</td>
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</tbody>
</table>

Table D.12

*Full scale total resistance measurements from towed vessel approach.*
APPENDIX E.

The Drift Angle and its True Definition.
THE DRIFT ANGLE and its true definition.

The definition of the drift angle from the ITTC Dictionary of Ship Hydrodynamics (1978) is as follows:

"The horizontal angle between the instantaneous direction of motion of the centre of gravity of a ship and its longitudinal axis. It is positive in the positive sense of rotation about the vertical axis."

Where in the following diagram the drift angle according to the above definition is referenced by the symbol (\(\beta\)).

Where \(U\) is the longitudinal direction of motion,

\(V_T\) is the instantaneous tangential velocity of the vessel,

\(G\) is the centre of gravity,

\(V\) is the lateral velocity of the vessel,

\(U\) is the longitudinal velocity of the vessel,

\(\beta\) is the drift angle

The drift angle from the above diagram can then be determined from measuring the two velocities, surge and sway respectively, during sea trails, and then calculated from the trigometric function:

\[ \beta = \tan^{-1}\left(\frac{V}{U}\right) \]

Equation 1

The area for concern arises from the fact that when full scale sea trials are conducted, for example the sea trails of the USS Compass Island, Morse and Price (1961), the two measured velocities also include external components due to the
action of the wind, current, tide and wave motion upon the vessel, as shown in Figure E.2.

![Diagram](image)

Figure E.2

Where $V_1$ is the measured lateral velocity of the vessel ideal under mill pond condition.

$V_2$ is the measured lateral velocity that includes an external component,

$\beta$ is the drift angle according to the ITTC definition,

$B$ is the drift angle according to the trigometric calculation (Equation 1)

As shown above the trigometric calculation for the drift angle ($B$) does not necessarily equal the ITTC definition for the drift angle ($\beta$) and so the drift angle ($B$) determined using this trigometric calculation can either be greater or less than the definitive drift angle ($\beta$) depending upon the direction and magnitude of the external disturbances.
APPENDIX F.

USS Compass Island sea trial data Table referenced in chapter seven.

Table 7.1 USS Compass Island turning circle data.
<table>
<thead>
<tr>
<th>EM Log approach speed</th>
<th>Electromagnetic log measured approach speed (knots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal rudder angle</td>
<td>Desired rudder angle (degrees)</td>
</tr>
<tr>
<td>Measured rudder angle</td>
<td>Actual rudder angle during manoeuvre (degrees)</td>
</tr>
<tr>
<td>Advance distance</td>
<td>The distance that the centre of gravity moves during the first quadrant of a turn, measured parallel to approach path (m).</td>
</tr>
<tr>
<td>Transfer distance</td>
<td>The lateral offset of the centre of gravity during the first quadrant of a turn (m).</td>
</tr>
<tr>
<td>Tactical diameter</td>
<td>Diameter of turning after heading has changed by 180 degrees (m)</td>
</tr>
<tr>
<td>Final diameter</td>
<td>Maximum diameter of turning circle once steady state is achieved (m).</td>
</tr>
<tr>
<td>Uf Inertial Kts</td>
<td>Forward velocity measured by an inertial system (kts).</td>
</tr>
<tr>
<td>Uf EM log Kts</td>
<td>Forward velocity measured by an electromagnetic log (kts).</td>
</tr>
<tr>
<td>Vf kts</td>
<td>Lateral velocity (kts)</td>
</tr>
<tr>
<td>β Deg.</td>
<td>Drift angle calculated as a function of the lateral and forward velocity (degrees).</td>
</tr>
<tr>
<td>ψ Deg/s</td>
<td>Average yaw rate for turning circle (deg s⁻¹).</td>
</tr>
<tr>
<td>Run number</td>
<td>Nominal approach speed</td>
</tr>
<tr>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
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Table 7.1
USS Compass Island turning circle data, Morse and Price (1961)

Page 217, Appendix F.
APPENDIX G.

Figures referenced in chapter nine concerning the USS Compass Island tanker.

Figure G.1 Hydrodynamic curves for the USS Compass Island

Figure G.2 Graph showing relationship between rudder angle and forward velocity for the USS Compass Island.

Figure G.3 Graph showing relationship between yaw rate and forward velocity for the USS Compass Island.

Figure G.4 Graph showing variation of yaw rate and rudder angle with respect to time for the USS Compass Island.

Figure G.5 Graph showing variation of forward velocity and rudder with respect to time for the USS Compass Island.
Hydrodynamic curves for the USS Compass Island  

Figure G.1

Graph showing the relationship between rudder angle and forward velocity for the USS Compass Island.  

Figure G.2
Graph showing relationship between the yaw rate and forward velocity for the USS Compass Island.  Figure G.3

Graph showing variation of yaw rate and rudder angle with respect to time for the USS Compass Island.  Figure G.4
Graph showing variation of forward velocity and rudder angle with respect to time for the USS Compass Island. Figure G.5
APPENDIX H.

Figures and Tables referenced in chapter ten.

Table H.1 Average measured values for Picket Boat Nine conducting turning circle manoeuvres at 800rpm.

Table H.2 Average measured values for Picket Boat Nine conducting turning circle manoeuvres at 1000rpm.

Table H.3 Average measured values for Picket Boat Nine conducting turning circle manoeuvres at 1200rpm.

Figure H.1 Plot showing calculated and measured velocity against engine speed for Picket Boat Nine.

Figure H.2 Plot showing calculated and measured forward velocity against rudder angle for Picket Boat Nine.

Figure H.3 Graph showing calculated and measured yaw rates against forward velocity for Picket Boat Nine.
<table>
<thead>
<tr>
<th>Engine speed (rpm)</th>
<th>Velocity (m s(^{-1}))</th>
<th>Rudder angle (deg)</th>
<th>Yaw rate (rad s(^{-1}))</th>
<th>Turning circle radius (m)</th>
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Table H.1

*Average measured values for Picket Boat Nine conducting turning circle manoeuvres at 800rpm.*
<table>
<thead>
<tr>
<th>Engine speed (rpm)</th>
<th>Velocity (m s(^{-1}))</th>
<th>Rudder angle (deg)</th>
<th>Yaw rate (rad s(^{-1}))</th>
<th>Turning circle radius (m)</th>
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Table H.2

*Average measured values for Picket Boat Nine conducting turning circle manoeuvres at 1000rpm.*
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<tr>
<th>Engine speed (rpm)</th>
<th>Velocity (m s(^{-1}))</th>
<th>Rudder angle (deg)</th>
<th>Yaw rate (rad s(^{-1}))</th>
<th>Turning circle radius (m)</th>
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Table H.3

*Average measured values for Picket Boat Nine conducting turning circle manoeuvres at 1200rpm.*
Plot showing calculated and measured forward velocities against engine speed for Picket Boat Nine

Figure H.1

Plot showing calculated and measured forward velocities against rudder angle for Picket Boat Nine

Figure H.2
Plot showing calculated and measured forward velocities against yaw rate for Picket Boat Nine. Figure H.3
APPENDIX I.

Initial results on the location of The Centre of Pressure and the magnitude of The Drift Angle.
Initial results on the location of the centre of pressure and the magnitude of the drift angle.

INTRODUCTION.

The generation of the drift angle has been shown in chapter nine and ten to be dependent upon the magnitude of the disturbing rudder force, as external forces are not included in this study, and possibly the speed of the vehicle through the water. This dependence of the drift angle upon the disturbing force and the location of the centre of pressure are demonstrated by captive scale model tests. The following describes a series of experiments that have been conducted utilising the facilities that are at the disposal of this research programme so that the magnitude of the drift angle could be determined, without the need for full scale sea trials.

THEORY.

A vessel that is symmetrical to port and starboard will possess directional stability when the rudder are at amidships. Applying the drift angle theory for when the rudders are applied during steady state turning, then the vessel will rotate about a point of curvature with a constant radius, yaw rate and forward velocity that generates a constant drift angle.

This is caused by the rudders producing lift and drag forces that combine to produce a disturbing force that acts at a distance from the centre of gravity. this disturbing force causes an initial rotation about the centre of gravity, which generates lift and drag forces upon the hull. At this point steady state turning is achieved and the vessel will be acted upon by two forces and a turning moment creating the motions of surge, sway and yaw. If steady state is maintained then these motions become constant and a constant drift angle is generated.

The resultant acting upon the vessel from the hull and rudder forces then acts through the centre of pressure which is taken as being situated along the centreline of the vessel. Furthermore, as there is no turning moment about the centre of pressure
then this resultant force can be separated into a longitudinal force and lateral force that acts through the centre of pressure, as shown in Figure 1.1.

Diagram showing the forces in action upon a scale model. Figure 1.1

The longitudinal and lateral forces can then be represented by the forces of surge and sway as shown in Figure 1.2.

The forces of surge and sway at the centre of pressure. Figure 1.2

Applying this to the use of scale models, indicates that by towing a model at the centre of pressure, once equilibrium has been achieved, that there will be no rotation and that only surge and sway forces will be evident at this point. Furthermore, by towing a model in a longitudinal towing tank, where the motion of sway is restricted but the other five degrees of freedom are permitted, then the model will assume equilibrium about this point of tow that will result in a constant drift angle being established that will be displayed continually as it traverses the length of
the tank. Therefore by towing a model at the centre of pressure and preventing the motion of sway will enable the magnitude of the drift angle to be determined and measured for the range of rudder angles and forward velocities possible.

IMPLEMENTATION.

In order to demonstrate this principle of towing a scale model with the drift angle being displayed the following attachment was constructed so that all but the motion of sway were present, as illustrated in Figure I.3.

![Diagram of model attachment for towing at the centre of pressure.](image)

*Model attachment for towing at the centre of pressure.* Figure I.3.

The ball joint permits the yaw and roll motions and the roller braces the motions of pitch and heave. The screw thread, positioned along the centreline of the model, was employed so that the point of tow to be varied as the location of the centre of pressure was not known, except that it is taken as being situated on the longitudinal centreline and forwards of the centre of gravity, and that at the centre of pressure there is no rotational motion as the forces that are in operation are in equilibrium.

This method of towing a scale model has been implemented for a sixteenth scale model of Picket Boat Nine. This consisted of aligning the model with the centreline of the tank, with the rudders applied to twenty degrees to starboard. The point of tow was taken at a point 25cm forwards of the centre of gravity as shown in Plate I.1.
Photograph showing model point of tow for drift angle and centre of pressure tests. Plate I.1

This model was then towed at a speed of 40 cm s\(^{-1}\) down the tank and during the test aerial photographs were taken to record the orientation of the model as it progressed, these are shown by the series of photographs in Plates I.2.1 to Plate I.2.8.

This series of photographs illustrate that from the initial alignment with the centreline of the tank, Plate I.2.1, that as the model begins to move down the tank that a rotation occurs about the point of tow as can be seen from plates I.2.2 and Plate I.2.3. This is considered to be a result of the disturbing rudder forces that are in action upon the model. The remaining photographs then demonstrate that equilibrium is then established between the disturbing rudder forces and the hull forces that are created that results in the generation of a constant drift angle. Measurement of this angle between the centreline of the model and its direction of motion, gives a drift angle of approximately ten degrees. This drift angle is then constant for the remaining duration of the test. These initial results have demonstrated that the model attachment permits all but the motion of sway and that a constant drift angle is displayed for the duration of the tow, once equilibrium has been established.
A second experiment was then conducted in exactly the same manner except that the point of tow was at a position 18cm forwards of the centre of gravity, half way along the screw thread. This second tests demonstrated in a similar manner to the fist test that the model rotated about the point of tow until equilibrium was established and a constant drift angle was displayed, as shown by the series of photographs in Plate I.3.1 to plate I.3.3. Measurement of this drift angle gave a value of thirteen degrees, which is larger than that measured from the previous test.
This experiment was then repeated a third time, but with the point of tow positioned 11cm forwards of the centre of gravity, with the series of aerial photographs as shown in Plate 1.4.1 to 1.4.4. The model during this last test is observed in these photographs to rotate from an initial heading (Plate 1.4.1) and to continue to rotate through to an angle of approximately eighty degrees as illustrated by the series of photographs in Plate 1.4.4. The magnitude of this drift angle is considered excessive and as impossible to exist for a vehicle during a controlled manoeuvre.

By closer examination of this set of photographs held in Plates 1.4, Plate 1.4.2 illustrates a flow regime similar to that which can be seen in the previous Plates for the former two tests, which is considered to represent steady state, as the water is able to pass by the hull of the model in a regular flow, that generates the lift and drag forces upon the hull that counteract disturbing rudder forces. Plate 1.4.3 and Plate 1.4.4 however, do not display this flow regime but the demonstrate a completely different one. The flow regime around the hull in these latter two photographs is considered to be represent stall, when the model can no longer maintain a flow regime that produces lift and drag forces upon the hull to counteract the disturbing rudder forces. When the model is in this condition, there is not a constant drift angle displayed and the model is not representative of steady state turning. Therefore, the point of tow is considered to be between the centre of pressure and the centre of gravity.

CONCLUSION.

This method of towing scale models has demonstrated that a constant drift angle is observable and that its magnitude is dependent upon the point of tow. The last test conducted demonstrated that if the model is towed between the centre of pressure and the centre of gravity, that an excessive drift angle will result due the flow regime that is established around the hull, as the lift and drag hull forces are not created. Whilst the former two tests have demonstrated that the point of tow was probably forwards of the centre of pressure, as the first test displayed a constant drift angle that was smaller than that observed from the second test. Furthermore the
centre of pressure for this model, at this particular rudder angle and forward velocity is thought to be between the 18cm position and the 11cm position. As at some point between these two positions the transition from displaying a constant drift angle and the model possessing a rotation would occur, and that this transitional point would identify the centre of pressure.

These results have demonstrated that equilibrium is established between the disturbing rudder forces and the hull forces and that a drift angle is displayed, when a model is towed in this manner. However these are only initial results that require further development, as the position of the elusive centre of pressure is not known, so that the magnitude of the drift angle could be determined.

Furthermore, the problems associated with the use of scale model test facilities and the scaling laws that would be required to relate such findings to a full scale vessel are not known and would be required to be investigated. But it is thought that by towing models in this manner has more to yield than has been concluded here.
## ABBREVIATIONS.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>ATTC</td>
<td>American Towing Tank Conference.</td>
</tr>
<tr>
<td>BRNC</td>
<td>Britannia Royal Naval College.</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>ECDIS</td>
<td>Electronic Chart Display &amp; Information System.</td>
</tr>
<tr>
<td>GPS</td>
<td>Globe Positioning System.</td>
</tr>
<tr>
<td>IMO</td>
<td>International Maritime Organisation.</td>
</tr>
<tr>
<td>IMSF</td>
<td>International Marine Simulator Forum.</td>
</tr>
<tr>
<td>ITTC</td>
<td>International Towing Tank Conference.</td>
</tr>
<tr>
<td>LMA</td>
<td>Lloyds Manoeuvring Assessment.</td>
</tr>
<tr>
<td>MARIN</td>
<td>Marine Research Institute Netherlands.</td>
</tr>
<tr>
<td>MARSIM</td>
<td>Marine Simulation</td>
</tr>
<tr>
<td>PB9</td>
<td>Picket Boat Nine.</td>
</tr>
<tr>
<td>PMM</td>
<td>Planar Motion Mechanism</td>
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<tr>
<td>RINA</td>
<td>Royal Institute of Naval Architects.</td>
</tr>
<tr>
<td>ROV</td>
<td>Remotely Operated Vehicle</td>
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<tr>
<td>SNAME</td>
<td>Society of Naval Architects and Marine Engineers.</td>
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<tr>
<td>VLCC</td>
<td>Very Large Crude Carrier.</td>
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</table>
NOMENCLATURE.

Characters:

B  Beam.
C_B Block coefficient.
D  Diameter.
F  General Force.
G  Centre of gravity.
I  Moment of inertia.
K  Roll motion, Roll force.
L  Length.
M  Pitch Motion, Pitch force.
N  Yaw motion, Yaw moment.
n  Engine speed.
O  Centre of curvature.
p  Roll Velocity.
P  Centre of pressure.
q  Pitch rate.
r  Rotational velocity (yaw).
R  Resultant force.
R  Radius.
T  Thrust.
T  Draught.
t  Time.
u  Longitudinal velocity (surge).
v  Lateral velocity (sway).
V  Instantaneous tangential velocity.
w  Heave velocity.
X  Surge motion, direction, Surge force.
Y  Sway motion, direction, Sway force.
Z  Heave motion, direction, Heave force.

Symbols:

\( \alpha \)  Angle of attack.
\( \beta \)  Drift angle.
\( \Delta \)  Displacement.
\( \delta \)  Rudder angle.
\( \eta \)  Dynamic viscosity.
\( \lambda \)  Rudder aspect ratio.
\( \nu \)  Kinematic viscosity.
\( \rho \)  Density.
\( \Psi \)  Ships heading.
\( \psi \)  Yaw rate.
**Subscripts:**

- d Demanded values.
- E External.
- H Hull.
- P Propeller.
- R Rudder.
- FW Fresh water.
- SW Sea water.

Other subscripts when present represent their respective meanings.

**Other notation:**

- Prime representing non-dimensionalised coefficients.
- When above a character indicates derivative values (acceleration terms).
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