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OPTIMISATION MODELS FOR CORPORATE TAXATION IN CAPITAL BUDGETING

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OPTIMISATION MODELS FOR CORPORATE TAXATION IN CAPITAL BUDGETING

by

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Abstract

In this study, mathematical programming models are developed which aim to determine simultaneously the optimum combination of investment decisions, financing methods and tax strategy for capital budgeting, taking into account tax-induced interactions between cash flows. The tax treatment of finance leases and the corporate group tax relief provisions are included. Shareholder risk considerations are taken into account in deriving an appropriate discount rate, using an iterative procedure based on the Capital Asset Pricing Model. The commercial mathematical programming software XPRESS-MP is utilised to achieve operational use of the models in complex tax situations.

A mathematical analysis of certain patterns of accelerated tax depreciation recently available in the UK for capital expenditure is presented. This analysis shows that, where there is a time lag between asset purchase and the incidence of tax relief, an optimal cost of capital may be derived at which the incremental value of the accelerated allowances is at a maximum. In the case of declining balance depreciation for plant and machinery, it is shown that this optimal cost of capital is independent of the proportion of the asset cost that may be allowed against tax in the first year.

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Chapter 1

Introduction

The system of corporate taxation in the UK may have a significant impact on optimal investment and financing decisions in terms of maximising shareholder wealth. The tax rules can create interdependencies between capital investment projects and the associated financing decision, and between the firm's existing activities and incremental projects under consideration, where none existed before tax. The result may be that a project which appears worthwhile when evaluated in isolation, with a simple treatment of the tax cash flows, has a negative impact on the value of the firm as a whole after tax. Conversely, a project may appear unattractive by itself but the pattern of its cash flows may change the firm's overall tax situation so that its acceptance has a positive value. Similarly, the tax rules may affect the value of combinations of incremental projects so that the after-tax value of a combination is not the same as the sum of the after-tax values of the individual projects. Within the tax rules themselves there is a certain amount of flexibility of treatment, particularly for the setting off of trading losses and surplus Advance Corporation Tax (ACT). The methods of treatment will also interact, creating further complexities for the investment and financing decision.

Although the firm which is aiming to maximise the wealth of its shareholders will wish to make optimal decisions about its investment, financing and tax strategy, its tax situation may be so complex that suboptimal decisions may be made because of the difficulty in considering the enormous number of potential alternatives. This difficulty may be overcome by the use of a computerised optimisation model which can make investment and financing choices in the presence of complex tax situations. In recent years, enough progress has been made in information technology to enable this type of model to be developed and solved within reasonable times on a desktop computer.

The primary objective of this thesis is therefore to develop mathematical programming models which determine the optimum combination of investment decisions and financing methods for capital budgeting on a post-tax basis, incorporating specific important areas not previously examined in the literature.

A secondary objective is to achieve operational experience of these models, in order to gain insights into:

(a) the impact of taxation on project appraisal in complex situations where several potentially distorting tax effects operate simultaneously, and

(b) the general practical feasibility of operational use.

Structure of the thesis

The structure of the thesis is represented diagrammatically in Figure 1.

It should be noted that the thesis is based on the tax legislation in force throughout the tax year 1994/95, unless it is stated otherwise in the text.

The system of corporate taxation in the UK is discussed in Chapter 2, particularly those features which may distort project appraisal. It is shown with a number of examples that the value of an incremental project may change in the presence of marginal tax rates and restricted setoff of advance corporation tax, and that the choice of tax treatment may have a significant impact on the value of the firm particularly when group relief is available.

In Chapter 3 the relative merits of the different methods of project appraisal often used in practice are considered in the light of the complex interactions that taxation creates between otherwise independent cash flows, and the assumed objective of shareholder wealth maximisation. Since this objective must include some measure of the risk of the cash flows accruing to the shareholder, methods of risk analysis are also examined. For comparison purposes, the empirical evidence on the investment appraisal and risk analysis methods used in practice by firms is reviewed.

Chapter 4 considers the investment decision in more detail. In particular, the value of tax allowances which may be claimed on capital expenditure on plant and machinery and buildings is considered. It is shown that for certain patterns of accelerated depreciation which have been available in the UK, an optimal cost of capital exists which maximises the present value of the series of capital allowances. In the case of plant and machinery, it is shown that the optimal cost of capital is independent of the proportion of the asset cost which may be allowed against tax in the first year. The analysis is extended to the situation where the firm has losses for a number of years following the investment decision.

Chapter 5 reviews the impact of the firm's financing decision and distribution policy on the objective of maximising shareholder wealth under the UK system of corporate and personal taxes. Methods of incorporating the tax effects of debt finance into project appraisal are evaluated, and the potential benefits of finance leasing are reviewed in a UK context. The relative valuation of dividends and capital gains under the imputation system is discussed. Because of the tax-induced interactions that may occur between payments of dividends and debt interest, it is argued that there is a need for the firm's investment, financing and tax decisions to be considered simultaneously.

In Chapter 6, the programming models of financing and investment decisions in the literature are reviewed. Although the early models addressed the issue of project selection under capital rationing and did not consider the effects of tax interactivities between the cash flows, the technique of mathematical programming has been used by a number of authors to model aspects of the tax system in differing degrees of complexity. It is argued that there is a need for an operational optimisation model which is capable of appraising investment and financing choices in the context of the firm's overall tax situation and simultaneously determining optimal investment, financing and tax decisions. Important areas are identified which have not yet been treated fully in the existing mathematical programming literature: the group relief provisions for losses and ACT, the differing tax treatment of finance leasing as opposed to asset purchase, and the need to include procedures to estimate the risk of the shareholders' returns.

In Chapter 7 a model is developed which aims to meet the criteria identified above. The initial formulation of constraints to express the logic of the relevant tax rules, and their representation in a conventional mathematical programming format, are discussed, and the necessary adaptations to meet the requirements of the software. Subsequent modifications to incorporate new areas include an iterative procedure to estimate the cost of capital. The implications of a dividend policy of constant stable growth are considered, and an expression is derived for the minimum horizon valuation at which the rate of dividend growth may be maintained into perpetuity. Finally, a number of features are presented which aim to improve the efficiency of the solution process.

Operational experience of a number of versions of the final model is presented and discussed in Chapter 8. Examples are given of the model's output and decisions under different assumptions about the firm's structure, existing cash flows, debt management policy and dividend policy. Lease or purchase decisions are included.

The values of incremental projects, as determined by the model, are compared with their values under conventional net present value analysis. A very significant effect on solution times is demonstrated by the inclusion of features to improve the model's efficiency.

The conclusions of the thesis are contained in Chapter 9, in which the model's capabilities, limitations and potential future directions are examined.

Fig. 1. Structure of the thesis

Chapter 1
Introduction

Chapter 2
The UK corporate tax system

Chapter 3
Capital investment appraisal

Chapter 4
Tax and capital investment

Chapter 5
Tax and optimal financing

Chapter 6
Optimisation models in capital budgeting

Chapter 7
The mathematical programming model

Chapter 8
Operational use of the model

Chapter 9
Conclusion

Chapter 2

The UK corporate tax system

2.1 Introduction

A corporate tax system may be based on profits or on cash flow, or have elements of both types of system. A firm's profit may be defined as its net operating cash flow, less depreciation charges, plus the periodic investment in net working capital. A tax on profits may mean that the firm has insufficient cash flow to pay its tax liability, depending on its changes in working capital and cash outflows for the purchase of new fixed assets. By contrast, under a cash flow tax system cash inflows are taxed and full immediate relief is given for cash outflows.

The current system of UK corporate taxation is closer to a profits tax than a cash flow tax. Corporation tax is levied on accounting profits, after certain adjustments. Accounting depreciation for fixed assets is replaced by a system of capital allowances which spread the relief for capital expenditure over future periods. A system of marginal tax rates is applied, determined by the level of taxable profit in each period. Losses may be set against certain other sources of income or against previous or future periods' profits. Because taxable capacity is determined from profits, any investment in net working capital is effectively subject to corporation tax. A part of the corporate tax liability is paid in advance shortly after a dividend is paid, which may be set against the shareholder's personal tax liability; the remainder is paid after the period to which it relates. Although dividend payments are not a tax-deductible expense, debt interest payments are, and the implications for a firm's optimal financing policy are discussed in Chapter 5.

In the following sections, the most important aspects of the UK tax system as it applies to project appraisal will be discussed and demonstrated.

2.2 Project appraisal and taxable profit

The acceptance of incremental projects changes a company's tax situation, because the pattern of tax-deductible items will change. This in turn may affect the company's marginal tax rate. The most significant of the tax-deductible items associated with project acceptance are discussed below.

Capital allowances

The UK tax system is broadly based on profits, but does not conform exactly with a profits base. The depreciation charge for fixed assets in the profit and loss account is added back to accounting profits and capital allowances on the firm's capital expenditure are deducted to arrive at the tax base, which is then subject to further adjustments. Capital allowances are calculated on the basis of a specified percentage, currently 25 per cent, of the cost or written down value of the asset. Capital allowances are treated as a trading expense of a company in arriving at taxable profit (or loss).

The pattern of capital allowances for plant and machinery is based on the concept of 'pools' of expenditure on which a writing down allowance (WDA) of 25 per cent on the reducing balance method is granted. A WDA is calculated on the basis of the opening pool balance plus expenditure on new assets during the year, less the disposal value (limited to the original cost) of assets sold during the year. An adjustment or 'balancing charge' will be made if the disposal value is more than the qualifying expenditure. Balancing charges are treated as trading receipts.

Most items of plant and machinery may, if the firm chooses, be 'depooled' and treated in a separate pool as a short life asset irrespective of its actual life. If the asset is sold within five years an adjustment is made so that the WDA on the asset for that year is the asset's residual pool balance. The effect of this provision is to align the tax relief on capital expenditure more closely with an asset's life. Because of the time value of money there will generally be an advantage in depooling, unless the disposal value is expected to be significant.

Losses

Where there are insufficient profits to absorb the available capital allowances, a Schedule D case I trading loss will result. Unrelieved debenture interest carried forward may also result in a loss.

A hypothetical tax system which allowed perfect loss relief would need to provide either an immediate tax repayment, irrespective of the firm's previous taxable profits, or the carry forward of losses at a rate of interest which would compensate the shareholder for the dividend payments or reinvestment opportunities foregone. The UK tax system does not treat corporate profits and losses in this symmetrical way, and the tax deductibility of a loss depends on the firm having other taxable profits against which the loss may be set. A trading loss may be set against a firm's other income in a number of ways, the most important of which are:

(i) The loss may be carried forward and set against future trading income from the same source. There is no time limit beyond which the loss cannot be set off, but it must be done as soon as possible.

(ii) It may be set against other income and gains of the same accounting period.

(iii) Where as much as possible of the loss has been offset by method (ii), any remainder may be carried back and set off against trading profit and other income and gains of the preceding three years, with profits of more recent accounting periods being used first. The profits available in the earlier years are the amounts left after deducting trading charges on income and group relief claimed for those years, and losses brought forward to those years.

(iv) It may be surrendered as group relief to another member of a 75 per cent group, that is, where the parent is entitled to 75 per cent or more of the profits distributable to equity holders. The claiming company may set the group relief against its total profits, after taking into account losses brought forward, the available relief for its own losses whether claimed or not, and all charges on income.

The treatment in (ii) and (iii) above relates to the whole loss and a partial carry back is not permitted, but a partial claim is possible in the case of group relief. The whole of the remaining amount of the surrendering company's loss is treated as in (i), (ii) and (iii) above.

Generally it is of more value to carry back the loss and claim a rebate, obtaining a cash inflow at the present time which may be distributed to shareholders, or used to improve liquidity or for new investments in a capital rationing situation. But where the tax rate is expected to be significantly higher in future years, the option of carrying the loss forward may be preferable. If the firm is 'tax exhausted' with no taxable capacity, it may not be able to claim any relief for its losses until it regains a taxpaying position in future years. Tax exhaustion has been prevalent in the UK over the last ten to fifteen years (Devereux, 1987; Young, 1992).

Excess annual interest payments (trade charges on income)

These are set off against the firm's total profits after deduction of any losses brought forward. If the charges exceed the profits, the excess cannot be carried back but may be carried forward as if it were a trading loss. In a 75 per cent group structure the excess charges can be surrendered wholly or partly as group relief in the same way as a trading loss.

This treatment of interest relating to long-term loans is in contrast to that of bank interest incurred for trade purposes, which is regarded as a trading expense and can therefore form part of a trading loss which may be carried back. Depending on the interest rates associated with each form of borrowing, there may be an incentive for a firm in a non-taxpaying position to maximise its use of bank overdraft facilities.

Marginal tax rates

The structure of the UK corporation tax rates is illustrated in Figure 2 below. A reduced rate of corporation tax, T_s, applies to companies whose taxable profits fall below the lower marginal limit M_l. Above the upper marginal limit M_u the full rate of corporation tax, T_f, applies. Between the marginal limits profits are taxed at a higher marginal rate T_m, where

$$T_m = \frac{T_f M_u - T_s M_l}{M_u - M_l}.$$

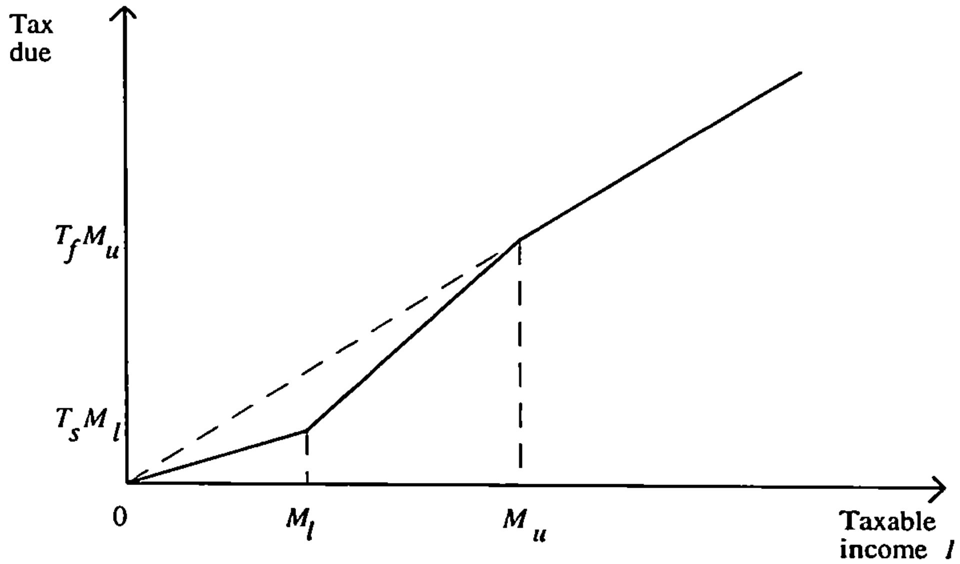


Fig. 2 The relationship between corporation tax and corporate taxable income

Under the current (1994/95) income limits,

M_l =£300,000

M_u =£1,500,000

T_s =25 per cent

T_m =35 per cent

T_f =33 per cent

In a corporate group structure an anti-avoidance provision applies to prevent the splitting up of a company's business among several companies so that each would benefit from the lower small companies tax rate. The upper and lower marginal thresholds are divided between the group members, so that for each member of a group of n firms the lower and upper marginal income limits are M_l / n and M_u / n respectively. The relevant tax rates are then applied to these smaller bands of taxable income.

Where a firm has taxable profits above M_u, heavy capital allowances from additional projects under consideration may reduce the taxable profit to a level where the higher marginal rate applies, particularly if the income from the additional projects is low in the early stages when capital allowances are at their highest. The offset of losses from other years may have the same effect. Conversely, the impact of the incremental projects may be in the other direction so that the firm moves out of the higher marginal rate. Similar effects will occur around the M_l threshold. As a result, if projects are assessed sequentially, suboptimal decisions may be made (Grundy and Burns, 1979). An individual project may have a positive NPV at a 25 per cent tax rate, but not at 33 per cent or 35 per cent, while a project which appears unattractive in isolation may become attractive if the tax effects of other projects reduce the relevant tax rate.

Grundy and Burns pointed out that the tax system can therefore create economic interdependencies between projects which were otherwise independent, since the costs and benefits of one are affected by the acceptance or non acceptance of the other. Another potential difficulty is that the firm's overall tax rate may change over a project's life, so that the timing of project acceptance affects its value.

Marginal tax rates and project appraisal

The impact of the system of tax rates on project appraisal may be illustrated by means of an example (Table 2.1). A firm is considering a project which involves an initial outlay of £1m on plant and machinery and is expected to generate future cash inflows of £65,000, £700,000 and £500,000 arising midway through years one, two and three respectively. The scrap value of the machinery at the end of the three years is nil. Capital allowances are available at 25 per cent on the reducing balance basis, with a balancing adjustment in year 3. The amounts of the allowances in the three years will therefore be £250,000, £187,500 and £562,500. Mainstream corporation tax is payable nine months after the year end. The firm's cost of capital is 10 per cent.

Table 2.1

Incremental project value after tax (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Initial outflow			(1m)	(1m)	(1m)
Year 1					
Additional cash inflow			65,000	61,975 [1]	61,975
Net taxable income for year	1,700,000		1,515,000		
Gross CT liability	561,000	474,816 [2]	499,950	423,145	51,671
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income for year	1,700,000		2,212,500		
Gross CT liability	561,000	431,651	730,125	561,781	(130,130)
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income for year	1,700,000		1,637,500		
Gross CT liability	561,000	392,410	540,375	377,983	14,427
Incremental NPV					(1,315)

Notes

1 65,000/(1.1)^0.5 = 61,975

2 561,000/(1.1)^1.75 = 474,816

Table 2.2

Effect of tax relief at marginal rate (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Initial outflow			(1m)	(1m)	(1m)
Year 1					
Additional cash inflow			65,000	61,975	61,975
Net taxable income for year	700,000		515,000		
Gross CT liability	215,000[1]	181,971	150,250	127,168	54,803
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income for year	1,700,000		2,212,500		
Gross CT liability	561,000	431,651	730,125	561,781	(130,130)
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income for year	1,700,000		1,637,500		
Gross CT liability	561,000	392,410	540,375	377,983	14,427
Incremental NPV					1,816

Note

1 (25% x 300,000)+[35% x (700,000-300,000)] = 215,000.

Columns (i) and (iii) represent the firm's tax situation with and without the additional project. The project will increase taxable profit in each year by the value of the additional cash inflow less the additional capital allowance. Columns (ii) and (iv) show the present value of the relevant tax effects. The final column gives the changes in the present values of the cash flows which result from undertaking the project. The sum of these gives the post tax incremental net present value of the project, which is negative.

Table 2.2 shows the same example, but with the assumption that the firm has £1m of losses and/or excess charges on income brought forward and set off against year 1's profits, so that the taxable profit is now £700,000. The project's post tax NPV is now positive. Taxable profit for year 1 now falls into the marginal tax band so that relief for the additional capital allowance is at the marginal rate of 35 per cent, rather than 33 per cent as before. The extra overall saving of £3,132 switches a negative NPV of £1,315 to a positive value of £1,816, allowing for a rounding error of £1.

2.3 Dividend taxation and project appraisal

Under the UK imputation tax system, a part of the firm's tax liability (Advance Corporation Tax or ACT) is treated as a prepayment of personal tax on dividends paid to shareholders. Restrictions apply, however, to the setoff of ACT against mainstream tax liability. Where there is restricted setoff of ACT, there can be further complexities for project appraisal.

The imputation system

Shareholders are given credit for ACT paid by the firm on distributed profits and receive a dividend payment $(1-b)D$ net of b, gross equivalent of the imputation rate (*see Note 2.1*). A corresponding tax credit is given, equal to the difference between the gross and net value of the dividend payment. The shareholder's actual tax liability is assessed on the gross dividend at the marginal personal tax rate m, so that the dividend receipt net of all taxes is $(1-m)/(1-b)$ per £1 distributed net of ACT by the firm. Although under the UK system a part of the firm's tax liability is imputed to the shareholder, a net mainstream corporation tax liability may still remain. The effective tax rate on distributions may therefore be greater than the shareholders' marginal tax rate.

ACT is regarded both as a payment of personal tax on dividend income and an advance payment of the firm's corporation tax. The amount of ACT that can be offset against mainstream corporation tax (MCT) is limited to the amount that would have been due if the firm had elected to distribute the whole of its taxable profits as a gross dividend. In other words, the ACT setoff must be no greater than the taxable profits for the relevant year multiplied by b. This restriction ensures that shareholders cannot receive credit against their income tax liability on dividend receipts unless the firm has actually paid at least an equivalent amount of tax. If there were no restriction on ACT setoff, King (1983) has shown that shareholders would be able to profit from tax arbitrage and investors who were exempt from tax could gain indefinitely from dividend returns resulting from new share issues.

Under the imputation system the effective cost to the firm of dividend payments reduces to the net dividend, except for a timing difference, which may be significant. Firms may be unable to offset the whole of their ACT because of an insufficiency of taxable profits, possibly resulting from high capital allowances and/or debt interest deductibility. The offset of ACT will be delayed to future profitable periods, thus decreasing the effective value of b and increasing the effective cost of a dividend payment (Kent and Theobald, 1980; Goudie, 1982). In the extreme case of permanent

Table 2.3

The effects of a group income election for dividend payments, with surplus FII

(i) No election

	Parent	*Subsidiary*	*Total tax*
	£	£	£
Income on which corporation tax is payable	1,700,000	1,500,000	
Franked payments (gross dividends)	250,000	450,000	
Franked investment income	450,000	0	
ACT payable, 20% of (FP - FII)	0	90,000	90,000
Corporation tax payable at 33%	561,000	495,000	
less: ACT setoff	0	(90,000)	
Mainstream corporation tax payable	561,000	405,000	966,000
Surplus franked investment income	200,000		

(ii) Group income election

	Parent	*Subsidiary*	*Total tax*
	£	£	£
Income on which corporation tax is payable	1,700,000	1,500,000	
Franked payments	250,000	0	
Group income	450,000		
Group payment		450,000	
ACT payable, 20% of FP	50,000	0	50,000
Corporation tax payable at 33%	561,000	495,000	
less: ACT setoff	(50,000)	0	
Mainstream corporation tax payable	511,000	495,000	1,006,000
Surplus franked investment income	0		

Table 2.4

The effects of a group income election for dividend payments, with no surplus FII

(i) No election

	Parent	*Subsidiary*	*Total tax*
	£	£	£
Income on which corporation tax is payable	1,700,000	1,500,000	
Franked payments (gross dividends)	500,000	450,000	
Franked investment income	450,000	0	
ACT payable, 20% of (FP - FII)	10,000	90,000	100,000
Corporation tax payable at 33%	561,000	495,000	
less: ACT setoff	(10,000)	(90,000)	
Mainstream corporation tax payable	551,000	405,000	956,000
Surplus franked investment income	0		

(ii) Group income election

	Parent	*Subsidiary*	*Total tax*
	£	£	£
Income on which corporation tax is payable	1,700,000	1,500,000	
Franked payments	500,000	0	
Group income	450,000		
Group payment		450,000	
ACT payable, 20% of FP	100,000	0	100,000
Corporation tax payable at 33%	561,000	495,000	
less: ACT setoff	(100,000)	0	
Mainstream corporation tax payable	461,000	495,000	956,000
Surplus franked investment income	0		

ACT-exhaustion, the imputation system operates effectively as a 'classical' system, where companies are taxed separately from shareholders.

If the recipient of the dividend is a UK resident company, the amount of the net dividend together with the associated tax credit is known as a franked payment (FP) of the company making the distribution and as franked investment income (FII) of the recipient. A company pays ACT on a quarterly return system on the excess of its franked payments over its franked investment income, and an excess of FII is carried forward to the next quarter. Although FII is not subject to corporation tax, it is included in the determination of the level of profits for the purpose of determining the company's marginal tax rate. Dividends paid between member firms in a 51 per cent group structure may be made without the payment of ACT and the associated tax credit under a group income election. Companies are members of a 51 per cent group if one controls the other, or both are under the control of another company, where 'control' may also mean ownership of at least 51 per cent of the share capital or votes. Dividends received are not then treated as FP/FII but as 'group income' which is ignored in determining the marginal corporation tax rate. The differing tax treatment of dividends with and without a group election is shown in Tables 2.3 and 2.4. If the subsidiary pays its dividend at an earlier date than the parent, there will be a cash flow timing advantage in a group election since no ACT need be paid by the group until the parent pays its own dividend. If the parent has surplus FII, the advantage is greater since there is a reduction in the ACT payable under a group election, with a corresponding increase in MCT at a later date.

Surplus ACT

If the firm has insufficient taxable profit to offset fully its ACT then a part of it will be unrelieved, or 'surplus'. Surplus ACT relating to the current year's dividends may be treated in a number of ways:

(i) It can be carried back and set against the corporation tax liabilities of previous periods beginning within six years prior to the year in which the surplus arose, on a last-in-first-out basis. The offset can only occur in a particular previous year if its profits chargeable to corporation tax, taking into account any losses or ACT already carried back, meet the maximum setoff criteria relating to that period.

(ii) It may be carried forward without time limit and set as soon as possible against the corporation tax liabilities of future years, after taking into account the offset of ACT in respect of those future years' dividend payments.

(iii) Any amount of the ACT relating to the current year's dividends may be surrendered to a 51 per cent subsidiary company by its parent, but this cannot occur in the other direction. The ACT surrendered by the parent is set off against the subsidiary's corporation tax liability first, and any excess may be carried forward but not carried back. If the surrendered ACT does not exceed the subsidiary's maximum setoff criteria, some or all of the subsidiary's own ACT arising from its distributions for that period may be set off and any surplus may be treated as in (i) and (ii) above.

Unlike the carry back and carry forward provisions for losses, the amount of surplus ACT may be split between a carry forward and a carry back claim. Where there is surplus ACT which must be carried forward, its present value will be reduced.

The surplus ACT problem

Freeman and Griffith (1993) estimate that of a panel of large industrial and commercial firms sampled by the Institute for Fiscal Studies, between 25 and 40 per cent had substantial surplus ACT in each of the years between 1982 and 1992. Firms may have surplus ACT if they pay dividends out of reserves when taxable profits are low, possibly as a result of high levels of capital allowances on investment. Surplus ACT may also be a problem for firms which make a large proportion of their profits abroad. If a UK resident company has overseas income that is subject to both overseas tax and UK tax, relief may be claimed for the overseas tax against the UK tax payable on this income. This relief is prescribed by double taxation agreements or other legislation. The relief obtained may be lower than the overseas tax paid, since the credit is restricted to the UK tax payable on the overseas income. Unutilised double tax credits cannot be carried back or forward, or set against any tax liabilities on other income. However, double tax relief is set off against corporation tax in priority to ACT and may lead to a surplus of ACT, which can be carried back and forward to other periods.

But for companies which earn much of their profits abroad, surplus ACT may be a problem. These companies still pay ACT on the dividends paid out of overseas profits, but may have a very low mainstream tax liability after double tax relief. It is likely that surplus ACT will arise in most periods, and so there will be little chance of carrying it back or forward. This appears inconsistent with the principle of double tax relief since firms are effectively penalised because of the structure of their business, but it is consistent with the idea that ACT is essentially a credit against the income tax of shareholders living in the UK, even if that income originates abroad. In practice, however, surplus ACT can create a number of problems such as reduced earnings per share, a bias against overseas investment, deterrence of international holding companies, encouraging takeovers, and the shifting of cost-centres abroad in order to increase UK taxable profits (Freeman and Griffith, 1993). Further, the increasing integration of markets particularly within Europe is making this issue an international tax problem rather than simply a domestic one.

These concerns led to a number of steps being taken by the Government to reduce the size of the problem. Firstly, there was a reduction of the rate of ACT to 20/80. Secondly, a scheme was introduced to allow the payment of dividends to foreign parent companies, out of foreign profits, without ACT. Thirdly, under the Foreign Income Dividend (FID) scheme, UK-based international companies may pay a dividend out of foreign profits with the surplus ACT on the FID being repaid to the company. FIDs do not carry a tax credit for shareholders.

At present it is too soon to review the effect of these changes on companies' surplus ACT position, although cutting the rate of ACT must reduce the tax penalty on dividends paid with surplus ACT. Freeman and Griffith (1993) commented that only taxpaying shareholders would gain from the FID scheme, while pension funds would be no better off and for exempt investors, 'the scheme simply robs its shareholders to pay the company, leaving the overall tax position unchanged'.

ACT setoff restrictions and project appraisal

Buckley (1975) showed that the UK imputation tax system may distort the incidence of cash flows in project appraisal where firms have substantial unrelieved ACT. In a number of scenarios, based on the assumption of 100 per cent first year allowances which were available at that time, the net present value of a project under evaluation was found to be less than its value under the 'usual' assumption of taxable profits and no unrelieved ACT.

The impact of the current treatment of ACT on project appraisal is shown by extending the example developed in section 2.2. Annual cash inflows are assumed to arise mid-year and discounted half yearly at 10 per cent per annum. It is assumed that the firm pays gross dividends of £1.6m, equivalent to £1.28m net of ACT, in each year. The associated ACT payments fall due at the end of June. Table 2.5 shows the

Table 2.5

Surplus ACT arising from project acceptance (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Initial outflow			(1m)	(1m)	(1m)
Year 1					
Additional cash inflow			65,000	61,975	61,975
Net taxable income for year	1,700,000		1,515,000		
Gross CT liability	561,000		499,950		
Dividends	1,600,000		1,600,000		
ACT	320,000	(305,108)	320,000	(305,108)	
ACT setoff	320,000		303,000		
Net MCT	241,000	(203,976)	196,950	(166,694)	37,283
Unrelieved ACT	0		17,000		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income for year	1,700,000		2,212,500		
Gross CT liability	561,000		730,125		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	0		17,000		
Net MCT	241,000	(185,433)	393,125	(302,483)	(117,050)
Balance of unrelieved ACT	0		0		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income for year	1,700,000		1,637,500		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	0		0		
Net MCT	241,000	(168,576)	220,375	(154,149)	14,427
Balance of unrelieved ACT	0		0		
Value of relieved ACT (yr 4)		0		0	0
Incremental NPV					(2,623)

Table 2.6

Project value with no surplus ACT (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Initial outflow			(1m)	(1m)	(1m)
Year 1					
Additional cash inflow			65,000	61,975	61,975
Net taxable income for year	1,700,000		1,515,000		
Gross CT liability	561,000		499,950		
Dividends	1,500,000		1,500,000		
ACT	300,000	(286,039)	300,000	(286,039)	
ACT setoff	300,000		300,000		
Net MCT	261,000	(220,904)	199,950	(169,233)	51,671
Unrelieved ACT	0		0		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income for year	1,700,000		2,212,500		
Gross CT liability	561,000		730,125		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	0		0		
Net MCT	241,000	(185,433)	410,125	(315,563)	(130,130)
Balance of unrelieved ACT	0		0		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income for year	1,700,000		1,637,500		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	0		0		
Net MCT	241,000	(168,576)	220,375	(154,149)	14,427
Balance of unrelieved ACT	0		0		
Value of relieved ACT (yr 4)		0		0	0
Incremental NPV					(1,315)

Table 2.7

Effect of loss offset (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Initial outflow			(1m)	(1m)	(1m)
Year 1					
Additional cash inflow			65,000	61,975	61,975
Net taxable income for year	700,000		515,000		
Gross CT liability	215,000		150,250		
Dividends	1,600,000		1,600,000		
ACT	320,000	(305,108)	320,000	(305,108)	
ACT setoff	140,000		103,000		
Net MCT	75,000	(63,478)	47,250	(39,991)	23,487
Unrelieved ACT	180,000		217,000		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income for year	1,700,000		2,212,500		
Gross CT liability	561,000		730,125		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	20,000		122,500		
Net MCT	221,000	(170,044)	287,625	(221,308)	(51,263)
Balance of unrelieved ACT	160,000		94,500		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income for year	1,700,000		1,637,500		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	20,000		7,500		
Net MCT	221,000	(154,586)	212,875	(148,903)	5,683
Balance of unrelieved ACT	140,000		87,000		
Value of relieved ACT (yr 4)		89,025		55,323	(33,702)
Incremental NPV					6,921

Table 2.8

Effect of loss offset and lower dividend payment (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Initial outflow			(1m)	(1m)	(1m)
Year 1					
Additional cash inflow			65,000	61,975	61,975
Net taxable income for year	700,000		515,000		
Gross CT liability	215,000		150,250		
Dividends	1,500,000		1,500,000		
ACT	300,000	(286,039)	300,000	(286,039)	
ACT setoff	140,000		103,000		
Net MCT	75,000	(63,478)	47,250	(39,991)	23,487
Unrelieved ACT	160,000		197,000		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income for year	1,700,000		2,212,500		
Gross CT liability	561,000		730,125		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	20,000		122,500		
Net MCT	221,000	(170,044)	287,625	(221,308)	(51,263)
Balance of unrelieved ACT	140,000		74,500		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income for year	1,700,000		1,637,500		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	
ACT setoff	320,000		320,000		
Relief for surplus ACT	20,000		7,500		
Net MCT	221,000	(154,586)	212,875	(148,903)	5,683
Balance of unrelieved ACT	120,000		67,000		
Value of relieved ACT (yr 4)		76,307		42,605	(33,702)
Incremental NPV					6,921

relevant cash flows. The incremental net present value of minus £2,623 indicates that the project is more unattractive than before.

By comparison with Table 2.6, it may be shown that this deterioration arises as a result of the net decrease in taxable profit in year 1 if the project is undertaken, so that £17,000 of unrelieved ACT arises which is not relieved until year 2. Table 2.6 shows the tax situation as it would be if a slightly lower dividend of £1.5m gross (£1.2m net of ACT) were paid in year 1, so that surplus ACT does not arise. In this case the project's incremental NPV, minus £1,315, is the same result that was obtained in table 2.1 where dividends were ignored. The two outcomes in Tables 2.5 and 2.6 may be reconciled by the timing difference in the discounted relief on ACT of £17,000:

$$\left[17{,}000/(1.10)^{1.75}\right]-\left[17{,}000/(1.10)^{2.75}\right]=1{,}308$$

Even if no surplus ACT arises, the project is still unattractive. If, however, the effects of loss offset are taken into account, the situation may change significantly. Tables 2.7 and 2.8 show the impact of a loss offset of £1m in year 1 which reduces taxable profit to £700,000. Surplus ACT will arise whether or not the project is undertaken, although the amounts will differ. It is assumed that any surplus ACT which remains unrelieved after year 3 will be relieved in year 4. In this situation, acceptance of the project will increase the present value of the firm after tax by £6,921 regardless of whether year 1's gross dividend is £1.6m (Table 2.7) or £1.5m (Table 2.8).

The outcome in Table 2.7 may be reconciled with Table 2.2 by considering the pattern of relief of surplus ACT with and without the incremental project. If the project is not undertaken, £180,000 of surplus ACT will arise in year 1 of which £20,000 will be offset in year 2, £20,000 in year 3 and £140,000 in year 4. If no dividend had been paid, that £180,000 would have been paid as mainstream corporation tax on year 1's taxable income. Allowing for the nine month tax lag, the effect of the delayed offset in present value terms is

$$£\left[20{,}000/(1.10)^{2.75}+20{,}000/(1.10)^{3.75}+140{,}000/(1.10)^{4.75}-180{,}000/(1.10)^{1.75}\right]$$

$$=-£33{,}944$$

Similarly, if the project is undertaken, £217,000 of surplus ACT will arise in year 1. The amounts offset in years two, three and four will be £122,500, £7,500 and £87,000 respectively. The present value of the delayed offset will be

$$£\left[122{,}500/(1.10)^{2.75}+7{,}500/(1.10)^{3.75}+87{,}000/(1.10)^{4.75}-217{,}000/(1.10)^{1.75}\right]$$

$$=-£28{,}839$$

The difference of £5,105 accounts for the difference between the project's incremental value in Table 2.2, where no dividend is paid, and in Table 2.7.

By comparison, Table 2.8 shows the situation assuming that the gross dividend payment in year 1 is £1.5m. The incremental net present value is exactly the same as in Table 2.7. The reason is that, with or without the additional project, the unrelieved ACT arising in year 1 is now reduced by £0.20(1,600,000-1,500,000), or £20,000. As a result, the amount of surplus ACT relieved in year 4 is £20,000 less than in the previous example whether or not the project is undertaken. The net incremental effects are therefore the same as in Table 2.7.

The above analysis demonstrates that ACT setoff restrictions can create an interaction between dividend policy and capital budgeting decisions where the surplus ACT would arise only as a result of undertaking a particular project. Where the firm's existing activities would lead to a surplus of ACT in any case, this interaction may not occur.

The ACT setoff restrictions may interact with the rules concerning the carry back of trading losses. Where a loss is carried back and offset against profits of a previous year, part or all of the ACT offset in that year may be displaced. This occurs because the maximum amount which may be offset is a proportion of taxable profits, which are reduced by the loss carry back claim.

2.4 Group relief implications

The provisions for transfer of trading losses, excess debt interest and ACT between group members have important implications for investment appraisal in group companies.

Group relief interdependencies

The amount of trading losses surrendered from one group member to another will affect the total taxable profits, and therefore the maximum ACT offset, in both member firms. But the surrender of ACT from a parent to a subsidiary company may occur at the same time as the transfer of a trading loss. The extent to which the subsidiary can utilise the ACT from the parent depends on the subsidiary's taxable profits, after taking account of any trading losses transferred to the subsidiary from another group member. These interdependencies were examined by Hodgkinson (1987), using a spreadsheet simulation model incorporating the rules for group relief for losses and surrender of ACT, and allowing for the tax effects of the timing of capital investments. The model was designed as a decision aid for project appraisal with maximisation of net present value after tax being achieved by sensitivity analysis. In this type of model the decision space is explored by the decision maker: by altering the variables, the model can be re-run and the effects on net present value determined. Because the range of potential strategies is enormous, since any chosen quantity of ACT and losses may be transferred, it is desirable for the model to have some implicit assumptions so as to limit the complexity of the decision-maker's search. For instance, the Hodgkinson model assumes that all losses and ACT are relieved as soon as possible, since this is likely to be the best strategy in present value terms.

The impact of group relief provisions

The group is assumed to consist of a parent company and a single 75 per cent subsidiary. The marginal income limits for the small company tax rate and the full tax rate are therefore divided between the two firms so that each has $M_L = £150,000$ and $M_U = £750,000$. Each firm uses a discount rate of 10 per cent. The parent is considering the marginal project example given in section 2.2. Tables 2.9a and 2.9b show the relevant cash flows for the parent and subsidiary with and without the incremental project undertaken by P Ltd, ignoring group relief. It is assumed that the parent's trading loss in year 1 cannot be carried back, and must therefore be carried forward. Comparison of the discounted cash flows indicates that the project should be undertaken since it has an incremental NPV of £1,939. The cash flows in the subsidiary are unaffected.

With group relief, there is a large number of potential setoff strategies. Three possible ones are considered below.

Strategy A

The subsidiary has sufficient profits in year 1 to set against the parent's loss. Tables 2.10a and 2.10b show the impact on the two member firms, with and without P's additional project, under the assumption that the parent surrenders any losses in year 1. Under this strategy, the project would have an incremental NPV of only £1,205.

Strategy B

The parent's losses are not surrendered, but instead the whole amount of its surplus ACT in year 1 is surrendered to the subsidiary (tables 2.11a and 2.11b). On this basis, the project now has a negative incremental NPV of minus £1,943.

Strategy C

Tables 2.12a and 2.12b show the effect on the group if only half of the parent's loss and half of its ACT were surrendered, the remainder being carried forward by the parent. The incremental NPV of the project is now £5,607.

The group's choice of tax strategy can therefore affect its decision to accept or reject marginal projects. But it can also have a significant effect on the overall net present value of the group. The cash flows under each of the three strategies, with and without the additional project, can be compared to the cash flows in Tables 2.9a and 2.9b where no group relief is claimed. The increases in the NPV of the parent and subsidiary under each group relief strategy are shown in Table 2.13. With or without the additional project, it may be seen that Strategy B increases the NPV of the group more than Strategy A, and Strategy C increases it more than Strategy B. Finally, Table 2.14 shows the reconciliations of the incremental project values in Tables 2.13 and Table 2.9a, where no group relief is claimed. Many other possible tax offset strategies exist in a situation of this kind, because of the flexibility of the offset

Table 2.9a

Value of an incremental project with no group relief

Cash flows in parent company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Initial outflow			(1m)	(1m)	(1m)
Additional cash inflow			65,000	61,975	61,975
Net taxable income/(loss) for year	(300,000)		(485,000)[1]		
Loss surrendered	0		0		
Gross CT liability	0		0		
Dividends	1,600,000		1,600,000		
ACT	320,000	(305,108)	320,000	(305,108)	0
ACT surrendered	0		0		
ACT setoff	0		0		
Net MCT	0	0	0	0	0
Unrelieved ACT	320,000		320,000		
Loss carried forward	300,000		485,000		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income/(loss) for year	1,700,000		2,212,500[2]		
Profit after losses brought forward	1,400,000		1,727,500		
Loss surrendered	0		0		
Gross CT liability	462,000		570,075		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	0
ACT surrendered	0		0		
ACT setoff	280,000		320,000		
Relief for surplus ACT brought forward	0		25,500		
Net MCT	182,000	(140,037)	224,575	(172,795)	(32,759)
Unrelieved ACT carried forward	360,000		294,500		
Loss carried forward	0		0		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income/(loss) for year	1,700,000		1,637,500[3]		
Profit after losses brought forward	1,700,000		1,637,500		
Loss surrendered	0		0		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	0
ACT surrendered	0		0		
ACT setoff	320,000		320,000		
Relief for surplus ACT brought forward	20,000		7,500		
Net MCT	221,000	(154,586)	212,875	(148,903)	5,683
Unrelieved ACT carried forward	340,000		287,000		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		216,204		182,502	(33,702)
Value of relieved loss (year 4)		0		0	0
Incremental NPV					1,939

Table 2.9b

Value of incremental project with no group relief

Cash flows in subsidiary company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(28,604)	30,000	(28,604)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Net MCT	130,000	(110,029)	130,000	(110,029)	0
Unrelieved ACT	0		0		
Loss carried forward	0		0		
Year 2					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(26,004)	30,000	(26,004)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	0		0		
Net MCT	130,000	(100,026)	130,000	(100,026)	0
Unrelieved ACT carried forward	0		0		
Loss carried forward	0		0		
Year 3					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(23,640)	30,000	(23,640)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	0		0		
Net MCT	130,000	(90,933)	130,000	(90,933)	0
Unrelieved ACT carried forward	0		0		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		0		0	0
Value of relieved loss (year 4)		0		0	0
Incremental NPV					0

Notes

Table 2.9a:

1 -300,000+65,000-250,000 (capital allowance) = -485,000
2 +1,700,000+700,000-187,500(capital allowance) = +2,212,500
3 +1,700,000+500,000-562,500(capital allowance) = +1,637,500

Table 2.9b:

1 25% x 150,000+35% x (500,000-150,000) = 160,000.

Table 2.10a

Value of an incremental project with group relief for losses

Cash flows in parent company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Initial outflow			(1m)	(1m)	(1m)
Additional cash inflow			65,000	61,975	61,975
Net taxable income/(loss) for year	(300,000)		(485,000)		
Loss surrendered	300,000		485,000		
Gross CT liability	0		0		
Dividends	1,600,000		1,600,000		
ACT	320,000	(305,108)	320,000	(305,108)	0
ACT surrendered	0		0		
ACT setoff	0		0		
Net MCT	0	0	0	0	0
Unrelieved ACT	320,000		320,000		
Loss carried forward	0		0		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income/(loss) for year	1,700,000		2,212,500		
Profit after losses brought forward	1,700,000		2,212,500		
Loss surrendered	0		0		
Gross CT liability	561,000		730,125		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	0
ACT surrendered	0		0		
ACT setoff	320,000		320,000		
Relief for surplus ACT brought forward	20,000		122,500		
Net MCT	221,000	(170,044)	287,625	(221,308)	(51,263)
Unrelieved ACT carried forward	300,000		197,500		
Loss carried forward	0		0		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income/(loss) for year	1,700,000		1,637,500		
Profit after losses brought forward	1,700,000		1,637,500		
Loss surrendered	0		0		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	0
ACT surrendered	0		0		
ACT setoff	320,000		320,000		
Relief for surplus ACT brought forward	20,000		7,500		
Net MCT	221,000	(154,586)	212,875	(148,903)	5,683
Unrelieved ACT carried forward	280,000		190,000		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		178,050		120,820	(57,230)
Value of relieved loss (year 4)		0		0	0
Incremental NPV					(40,094)

Table 2.10b

Value of incremental project with group relief for losses

Cash flows in subsidiary company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	200,000		15,000		
Gross CT liability	55,000		3,750		
Dividends	150,000		150,000		
ACT	30,000	(28,604)	30,000	(28,604)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		3,000		
Net MCT	25,000	(21,159)	750	(635)	20,525
Unrelieved ACT	0		27,000		
Loss carried forward	0		0		
Year 2					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(26,004)	30,000	(26,004)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	0		27,000		
Net MCT	130,000	(100,026)	103,000	(79,251)	20,775
Unrelieved ACT carried forward	0		0		
Loss carried forward	0		0		
Year 3					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(23,640)	30,000	(23,640)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	0		0		
Net MCT	130,000	(90,933)	130,000	(90,933)	0
Unrelieved ACT carried forward	0		0		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		0		0	0
Value of relieved loss (year 4)		0		0	0
Incremental NPV					41,299

Table 2.11a

Value of an incremental project with group relief for ACT

Cash flows in parent company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Initial outflow			(1m)	(1m)	(1m)
Additional cash inflow			65,000	61,975	61,975
Net taxable income/(loss) for year	(300,000)		(485,000)		
Loss surrendered	0		0		
Gross CT liability	0		0		
Dividends	1,600,000		1,600,000		
ACT	320,000	(305,108)	320,000	(305,108)	0
ACT surrendered	320,000		320,000		
ACT setoff	0		0		
Net MCT	0	0	0	0	0
Unrelieved ACT	0		0		
Loss carried forward	300,000		485,000		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income/(loss) for year	1,700,000		2,212,500		
Profit after losses brought forward	1,400,000		1,727,500		
Loss surrendered	0		0		
Gross CT liability	462,000		570,075		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	0
ACT surrendered	0		0		
ACT setoff	280,000		320,000		
Relief for surplus ACT brought forward	0		0		
Net MCT	182,000	(140,037)	250,075	(192,416)	(52,379)
Unrelieved ACT carried forward	40,000		0		
Loss carried forward	0		0		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income/(loss) for year	1,700,000		1,637,500		
Profit after losses brought forward	1,700,000		1,637,500		
Loss surrendered	0		0		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	0
ACT surrendered	0		0		
ACT setoff	320,000		320,000		
Relief for surplus ACT brought forward	20,000		0		
Net MCT	221,000	(154,586)	220,375	(154,149)	437
Unrelieved ACT carried forward	20,000		0		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		12,718		0	(12,718)
Value of relieved loss (year 4)		0		0	0
Incremental NPV					(1,943)

Table 2.11b

Value of incremental project with group relief for ACT

Cash flows in subsidiary company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(28,604)	30,000	(28,604)	0
ACT claimed from parent	320,000		320,000		
ACT setoff	100,000		100,000		
Net MCT	60,000	(50,782)	60,000	(50,782)	0
Unrelieved ACT	250,000		250,000		
Loss carried forward	0		0		
Year 2					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(26,004)	30,000	(26,004)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	70,000		70,000		
Net MCT	60,000	(46,166)	60,000	(46,166)	0
Unrelieved ACT carried forward	180,000		180,000		
Loss carried forward	0		0		
Year 3					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(23,640)	30,000	(23,640)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	70,000		70,000		
Net MCT	60,000	(41,969)	60,000	(41,969)	0
Unrelieved ACT carried forward	110,000		110,000		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		69,948		69,948	0
Value of relieved loss (year 4)		0		0	0
Incremental NPV					0

Table 2.12a

Value of an incremental project with some group relief for losses and ACT

Cash flows in parent company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Initial outflow			(1m)	(1m)	(1m)
Additional cash inflow			65,000	61,975	61,975
Net taxable income/(loss) for year	(300,000)		(485,000)		
Loss surrendered	150,000		242,500		
Gross CT liability	0		0		
Dividends	1,600,000		1,600,000		
ACT	320,000	(305,108)	320,000	(305,108)	0
ACT surrendered	160,000		160,000		
ACT setoff	0		0		
Net MCT	0	0	0	0	0
Unrelieved ACT	160,000		160,000		
Loss carried forward	150,000		242,500		
Year 2					
Additional cash inflow			700,000	606,749	606,749
Net taxable income/(loss) for year	1,700,000		2,212,500		
Profit after losses brought forward	1,550,000		1,970,000		
Loss surrendered	0		0		
Gross CT liability	511,500		650,100		
Dividends	1,600,000		1,600,000		
ACT	320,000	(277,371)	320,000	(277,371)	0
ACT surrendered	0		0		
ACT setoff	310,000		320,000		
Relief for surplus ACT brought forward	0		74,000		
Net MCT	201,500	(155,041)	256,100	(197,051)	(42,011)
Unrelieved ACT carried forward	170,000		86,000		
Loss carried forward	0		0		
Year 3					
Additional cash inflow			500,000	393,993	393,993
Net taxable income/(loss) for year	1,700,000		1,637,500		
Profit after losses brought forward	1,700,000		1,637,500		
Loss surrendered	0		0		
Gross CT liability	561,000		540,375		
Dividends	1,600,000		1,600,000		
ACT	320,000	(252,155)	320,000	(252,155)	0
ACT surrendered	0		0		
ACT setoff	320,000		320,000		
Relief for surplus ACT brought forward	20,000		7,500		
Net MCT	221,000	(154,586)	212,875	(148,903)	5,683
Unrelieved ACT carried forward	150,000		78,500		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		95,384		49,918	(45,466)
Value of relieved loss (year 4)		0		0	0
Incremental NPV					(19,077)

Table 2.12b

Value of incremental project with some group relief for losses and ACT

Cash flows in subsidiary company (£)

	Before incremental project	Present value	After incremental project	Present value	Incremental value
	(i)	(ii)	(iii)	(iv)	(iv) - (ii)
Year 1					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	350,000		257,500		
Gross CT liability	107,500		75,125		
Dividends	150,000		150,000		
ACT	30,000	(28,604)	30,000	(28,604)	0
ACT claimed from parent	160,000		160,000		
ACT setoff	70,000		51,500		
Net MCT	37,500	(31,739)	23,625	(19,996)	11,743
Unrelieved ACT	120,000		138,500		
Loss carried forward	0		0		
Year 2					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(26,004)	30,000	(26,004)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	70,000		70,000		
Net MCT	60,000	(46,166)	60,000	(46,166)	0
Unrelieved ACT carried forward	50,000		68,500		
Loss carried forward	0		0		
Year 3					
Net taxable income/(loss) for year	500,000		500,000		
Adjusted profit/(loss)after group relief	500,000		500,000		
Gross CT liability	160,000		160,000		
Dividends	150,000		150,000		
ACT	30,000	(23,640)	30,000	(23,640)	0
ACT claimed from parent	0		0		
ACT setoff	30,000		30,000		
Relief for surplus ACT brought forward	50,000		68,500		
Net MCT	80,000	(55,959)	61,500	(43,018)	12,940
Unrelieved ACT carried forward	0		0		
Loss carried forward	0		0		
Value of relieved ACT (year 4)		0		0	0
Value of relieved loss (year 4)		0		0	0
Incremental NPV					24,684

Table 2.13

Present value effects of group relief for losses and ACT on parent and subsidiary

	Cash flow changes, £	*Present value, £*
Strategy A without incremental project		
decrease in ACT relieved in parent, year 4	60,000	(38,154)
increase in MCT in parent, year 2	39,000	(30,008)
decrease in MCT in subsidiary, year 1	105,000	88,869
Net increase/(decrease) in NPV		20,708
Strategy A with incremental project		
decrease in ACT relieved in parent, year 4	97,000	(61,682)
increase in MCT in parent, year 2	63,050	(48,513)
decrease in MCT in subsidiary, year 1	129,250	109,394
decrease in MCT in subsidiary, year 2	27,000	20,775
Net increase/(decrease) in NPV		19,974
Strategy B without incremental project		
increase in ACT relieved in subsidiary, year 4	110,000	69,948
decrease in MCT in subsidiary, year 1	70,000	59,246
decrease in MCT in subsidiary, year 2	70,000	53,860
decrease in MCT in subsidiary, year 3	70,000	48,964
Net increase/(decrease) in NPV		28,533
Strategy B with incremental project		
decrease in ACT relieved in parent, year 4	287,000	(182,502)
increase in MCT in parent, year 2	25,500	(19,621)
increase in MCT in parent, year 3	7,500	(5,246)
increase in ACT relieved in subsidiary, year 4	110,000	69,948
decrease in MCT in subsidiary, year 1	70,000	59,246
decrease in MCT in subsidiary, year 2	70,000	53,860
decrease in MCT in subsidiary, year 3	70,000	48,964
Net increase/(decrease) in NPV		24,650
Strategy C without incremental project		
decrease in ACT relieved in parent, year 4	190,000	(120,820)
increase in MCT in parent, year 2	19,500	(15,004)
decrease in MCT in subsidiary, year 1	92,500	78,290
decrease in MCT in subsidiary, year 2	70,000	53,860
decrease in MCT in subsidiary, year 3	50,000	34,974
Net increase/(decrease) in NPV		31,300
Strategy C with incremental project		
decrease in ACT relieved in parent, year 4	208,500	(132,584)
increase in MCT in parent, year 2	31,525	(24,256)
decrease in MCT in subsidiary, year 1	106,375	90,033
decrease in MCT in subsidiary, year 2	70,000	53,860
decrease in MCT in subsidiary, year 3	68,500	47,915
Net increase/(decrease) in NPV		34,968

Table 2.14

Reconciliation of incremental project values under group relief for losses and ACT

	£
Strategy A	
Increase in overall group NPV, with incremental project	19,974
less: increase in overall group NPV, without incremental project	20,708
	734
Base value of project, without allowing any group relief*	1,939
	1,205
Strategy B	
Increase in overall group NPV, with incremental project	24,650
less: increase in overall group NPV, without incremental project	28,533
	3,883
Base value of project, without allowing any group relief*	1,939
	(1,944) **
Strategy C	
Increase in overall group NPV, with incremental project	34,968
less: increase in overall group NPV, without incremental project	31,300
	(3,668)
Base value of project, without allowing any group relief*	1,939
	5,607

* This is the value determined in Table 2.9a.

** Subject to a rounding error of £1.

provisions: any amount of ACT can be surrendered by the parent to the subsidiary, and any partial claims are possible for group relief for losses. The carry back provisions extend the possibilities further.

2.5 Conclusion

It has been shown in this chapter that the corporate tax system can create distortions in investment appraisal as a result of marginal tax rates, imperfect relief for losses and the restricted setoff of ACT. The setoff of losses, or increased capital allowances from additional investment projects, may reduce the firm's taxable profits to the extent that its marginal tax rate falls into the lower small companies rate band or the higher marginal rate between the marginal limits, rather than the 'full' rate. The effect of this may be to change the sign of the project's net present value after tax. It was also demonstrated that ACT setoff restrictions can lead to interactions between dividend policy and capital investment: reduced taxable profits as a result of increased capital allowances may create surplus ACT which, unless it can be carried back and set against previous years' tax liabilities, must be carried forward, reducing its present value. The setting off of losses may have the same effect. The group relief provisions were shown to provide a particularly wide range of possible offset strategies, with a correspondingly varied impact on the firm's net present value.

Other things being equal, a project should be accepted if its cash flows, in combination with those of the firm's other activities, increase the value of the firm (or the group as a whole) after tax. The view expressed at the beginning of this chapter was that project appraisal should incorporate all the relevant cash flows, including taxes, in order to be consistent with an objective of maximising shareholder wealth. To be consistent with this criterion, the tax cash flows used in project appraisal should include the effects of the firm's global optimal tax strategy if the project is accepted. In turn, this should be compared with the firm's optimal situation after tax without the additional investment.

Where a number of projects are being appraised at the same time, each possible combination of accepted projects will have a corresponding optimal tax strategy which will be determined by the interactions between the cash flows, rather than by an additive process. A spreadsheet model may be used to explore these interactions, but although experience and common sense will rule out a large number of inferior strategies, there remains an element of 'trial and error'. This is particularly true where there is flexibility in determining the amounts which can be treated under certain provisions, such as group relief for trading losses and ACT. In a centralised capital budgeting function there will be considerable scope for an optimal tax strategy which maximises the potential value to be gained from these provisions. But in complex tax situations, the determination of a global optimal solution within a reasonable time requires a more complex treatment than a simulation approach. This requirement may be met by the use of an optimisation model based on mathematical programming techniques. A mathematical programming model may be capable of determining an optimal combination of capital investments and the associated tax strategy, in terms of maximising a given objective subject to constraints representing the tax rules and other limits on the firm's behaviour. But a model of this type which aims to maximise shareholder wealth also needs to take into account further complexities which will be addressed in the following two chapters: the effects of taxation on the risk of the return to shareholders, and the impact of taxes on the firm's optimal financing choices. In Chapter 7, an optimisation model will be presented which aims to meet these criteria.

Note 2.1

The rate of ACT was aligned to the basic rate of income tax until April 1993 and dividends were paid net of tax at the basic rate. In the tax year 1994/95 the imputation rate is 20 per cent of the gross dividend, which is the lowest rate applicable to the first £3,000 of personal tax liability, rather than the 25 per cent 'basic' rate. Basic rate taxpayers now pay tax at only 20 per cent on the gross dividend and there is no further liability unless the shareholder's income, including dividend income, is greater than the limit above which the higher personal tax rate of 40 per cent applies. At the time the change was made it was envisaged that the 20 per cent rate would later be applied to the whole of the basic rate tax band, so that the imputation rate would again be aligned to the basic rate of personal tax.

Chapter 3

Capital investment appraisal

3.1 Introduction

Having demonstrated that corporate taxation creates interactions between the firm's activities and incremental capital investment, this chapter will evaluate a number of commonly used methods of project appraisal and risk analysis in terms of shareholder wealth maximisation where complex tax situations exist. The chapter will also consider the role of financial project appraisal in the context of the firm's overall strategy, and review the empirical evidence on firms' capital budgeting techniques.

3.2 The strategic context of project appraisal

Although financial appraisal is often presented as an adequate basis for deciding whether or not to undertake a capital investment, it is not a complete framework for analysis especially for major strategic decisions. Factors such as improvements in the firm's competitive position may be difficult to quantify in financial terms. The key steps in strategic planning may be defined as:

a) selecting the proper strategic goals after scanning the firm's internal and external environment;
b) considering the financial implications of such a strategy and how it will lead to increased profits, cash flows and shareholder wealth;
c) setting realistic and achievable organisational goals and objectives;
d) successful introduction and implementation of the strategic plan;

e) evaluating and monitoring the strategic plan in the light of changes in the firm's environment.

(Shaw, 1993)

The capital budgeting decision is linked to step (*b*) of the strategic planning process and must be consistent with the overall corporate strategy of the firm. The strategic planning literature (for example, Johnson and Scholes, 1993) is concerned with techniques for identifying strategies that will provide the firm with a sustainable competitive advantage in a complex and highly uncertain business environment. In long run competitive equilibrium, all projects have a zero net present value assuming all markets, financial and non-financial, are perfect. A positive net present value implies either a short term deviation from equilibrium or a long term competitive advantage. However, projects with a marginal or negative net present value may sometimes be accepted for strategic reasons, such as establishing the firm in a new commercial market.

If the firm is assumed to be simultaneously pursuing a number of goals, some of which may be in conflict, multi-criteria decision making methods may be used to evaluate and select projects. Goal programming models aim to minimise the deviations from several goals according to a priority ranking scheme. Their disadvantage is that goals and priorities must be specified by the decision maker before the model is used. The analytic hierarchy process (AHP) methodology of Saaty (1980) has been applied to complex decision problems. The AHP structures the goals, attributes, stakeholders and other aspects of the problem in a hierarchy and determines priorities for decision variables. The elements of each level in the hierarchy are evaluated based on their relationship to a particular element in the level above. Through matrix algebra, this comparison process yields local priorities at each level. The AHP methodology has been proposed by Khorramshahgol, Azani, and

Gousty (1988) and Liberatore (1988) for project selection in the context of organisational objectives. Liberatore, Monahan and Stout (1992) suggested practical approaches for structuring hierarchies to deal with non-financial benefits and to link the capital budgeting decision formally with organisational strategy.

Real options in capital budgeting

Although the value-maximising paradigm is a sound basis for rational business decisions, the standard methods in capital budgeting do not capture sources of economic value such as synergies and interdependencies between projects, growth opportunities and the real options which comprise managerial operating flexibility. Investment opportunities can be seen as collections of options on real assets. Strategic planning can be regarded as the explicit recognition, creation, and management (optimal exercise) of the firm's portfolio of real options. Underestimation of the value of options arising from investment decisions may create a bias against strategic or long-term projects (Myers, 1984).

Although traditional DCF methods treat the pattern of investment as fixed, investment decisions and outlays are often made sequentially and can be adjusted as new information arrives. Since uncertainty reduces over time, the firm considering making a new product may prefer to wait and see whether a similar product made by a competitor is successful or not. Because of economic uncertainty and the possibility of delaying undertaking the project, waiting has a positive value if an investment involves a sunk cost, provided the opportunity to invest remains available. The value of this flexibility is like an American call option: a discretionary investment opportunity gives management the right, but not the obligation, to acquire the project's benefits if they exceed the required investment outlay before the opportunity disappears. The underlying asset is valued at the present value of expected cash flows generated by the project, not including its outlay. Flexibility to switch investment from the current to its best future alternative use, redeploy assets or abandon project is analogous to an American put option. Majd and Pindyck (1987) use contingent claims analysis to derive optimal decision rules for investments involving flexibility. Comparisons between this approach and a dynamic programming methodology are expanded in Dixit and Pindyck (1994). With a risk free discount rate, the same differential equations follow from the two approaches. Where assets can be 'spanned', that is, replicated by assets available in the market, then the CAPM derived rate can be an input within this option pricing methodology. Trigeorgis (1991) describes a numerical method based on option pricing for valuing complex investments with multiple interacting options, such as capital budgeting.

An important difference between a real investment opportunity and a call option is the influence of market competition on the value of the real investment (Smit and Ankum, 1993). The anticipated actions of competitors imply a change in the value of the postponement option. For example, in duopoly, both firms may find it valuable to make a coordinated decision to defer investment when there is low project value and uncertain market demand.

Projects may spawn new growth options over time, which also adds strategic value. Trigeorgis and Kasenen (1991) argue that these option values create a beneficial asymmetry in the probability distribution of project value, which can be captured through an expanded NPV criterion which takes into account the value of operating options from active management and any interaction effects between projects. Kasenen (1993) developed a deterministic dynamic model of optimal growth which analyses the generation of investment opportunities.

Project appraisal

Project appraisal is one part of the whole capital investment decision making process, whose stages may be briefly defined as follows:

(i) the generation of potential project ideas;

(ii) the production of cash flow forecasts;

(iii) project appraisal and selection;

(iv) implementation of selected capital investment projects;

(v) post completion audits.

This thesis aims to present an optimisation model for the financial aspects of project appraisal and selection, taking into account tax-induced cash flow interactions (see Chapter 7). The data inputs to the model would arise from the first two steps of the capital budgeting process, and its decisions would be an input to subsequent steps. The capital budgeting process is itself linked to the firm's overall strategy, and in any practical application the model's decisions would be seen in this wider context rather than as being prescriptive and final.

The conventional goal of the firm is prescribed by financial theory as the maximisation of shareholder wealth, which is assumed to be achieved in a capital budgeting context by maximising the firm's discounted future cash flows. This goal has been regarded as unrealistic by some writers. Cyert and March (1963) originally put forward the view that the firm is a coalition of suppliers of capital, suppliers of labour, suppliers of goods and services, suppliers of managerial skill, and customers. This view implies that the firm should aim to 'satisfice' or provide a satisfactory return for the providers of all these inputs, rather than maximising the return to just one group. This goal is not necessarily in conflict with the maximisation of shareholder wealth. For instance, a dissatisfied workforce may lead to poor motivation, high staff turnover and strikes, leading to decreased profits and greater uncertainty about future profits. The satisficing theory of the firm is also based on the idea that firms operate in a highly uncertain environment, and continually adjust their activities to meet targets such as market share, profits, and sales. Day (1967) argued that if these targets are consistently revised upwards to create an incentive for improved performance, a satisficing strategy eventually converges on the long term profit-maximising strategy.

3.3 Capital budgeting techniques

Most capital investment projects involve an initial cash outflow, followed by a series of anticipated, uncertain future net cash inflows. From this cash flow stream, a criterion must be derived upon which project acceptance decisions can be made. The criterion should include the opportunity cost of the project, since the shareholder can lend cash at a positive rate of interest, and a measure of the risk associated with the uncertain future cash flows. Cash flow forecasts should include an evaluation of different possible outcomes for the project itself, the industry and the state of the economy as a whole, in order to assess the riskiness of the cash flows. The project appraisal and selection process should take this into account, along with the effects of any debt financing in the projects' funding.

The techniques developed for capital budgeting and project appraisal fall into two groups: the financial appraisal methods which generally treat projects in isolation, and the programming techniques developed in operational research.

3.3.1 Financial appraisal methods

Payback period

The payback period is the time taken for a project's initial investment to be repaid by the subsequent net cash inflows, with the decision rule involving a comparison with some target period. Its use is often criticised since it ignores cash flows outside the payback period and the timing of returns within it, and because of the lack of objectivity in the choice of target period. Also, if the firm's market value accurately reflects its future cash flows, shareholders will be indifferent between projects with the same net present value but different payback periods. The widespread use of this method may be the result of conflict between managerial goals and shareholder wealth maximisation: a study by Pike (1985) found a significant negative association between the importance of the payback method in investment decision making and the importance of the shareholders' wealth objective.

However, the payback method may provide a useful perspective for managers on, for example, projects involving products or production methods in areas of rapidly evolving technology.

Net present value

A project's net present value (NPV) is the sum of its cash flows, expressed in present value terms by discounting at a rate which allows for the project's risk:

$$NPV = \sum_{t=0}^{n} F_t (1+k)^{-t} \qquad (3.1)$$

where F_t is the cash inflow or outflow at time t , n is the terminal period of the project's life and k is the discount rate, which in practice will normally vary over the project's life. The NPV technique has the advantages of incorporating all the project cash flows and adjusting them for risk and the time value of money. k is often taken to be the firm's weighted average cost of capital, given by the sum of the after tax cost of debt, and the after tax cost of equity, multiplied by their respective proportions in the firm's capital structure. However, the project may have different financing and risk characteristics to that of the firm as a whole. This issue is discussed more fully in Chapter 5.

Project yield

A project's yield is measured by its internal rate of return (IRR) defined as that value of k, determined iteratively, which reduces the net present value formula to zero. At this discount rate the investor is indifferent between investing in the project and not investing in it. If the yield exceeds the discount rate that would have been used to determine NPV, then the project should be accepted. This technique has two main difficulties. Firstly, a project may have more than one IRR if some of the future cash flows are negative, although this may be accommodated by discounting positive and negative terms separately (Wilkes, 1978, chapter 1). Secondly, the IRR does not give a measure of the actual size of the financial benefit that would result from the project's acceptance. When several projects are competing for funds, either because of capital rationing or because they are mutually exclusive, the use of IRR can therefore lead to suboptimal decisions.

A difficulty with the approaches discussed above is that the project's cash flows are assumed to be separable from those of the firm as a whole. However, it may be difficult or arbitrary to identify particular cash flows with a specific project because of interdependencies between an individual project and other projects being considered at the same time, and the existing activities of the firm. This is especially so after tax effects are taken into account. As shown in Chapter 2, the tax rules concerning marginal tax rates and the treatment of capital allowances, debt interest and ACT setoff apply to the firm's total cash flows including those of the incremental project, so that interdependencies exist between all the firm's activities.

3.3.2 Operational research techniques

Linear and integer programming

Linear programming (LP) aims to allocate scarce resources in such a way as to maximise a specified objective. The characteristics of LP problems may be summarised (for example, Salkin and Kornbluth, 1973) as follows:

(i) the system can be described in terms of a series of possible activities;

(ii) the decision maker has to choose the most appropriate levels for each of these activities;

(iii) his choice is restricted by the availability of scarce inputs;

(iv) there is a well defined quantity (money, profit, cost) which can be used to compare the desirability of different strategies.

The technique is therefore suitable for capital budgeting applications where a number of potential projects are being considered and where the decision maker's choice may be restricted by capital rationing or other constraints, such as the tax rules. Linear programming assumes that all variables are continuously divisible, and so the optimal solution may include fractional projects. Where this is not acceptable, variables must be restricted to integer values, but the solution process becomes computationally much more difficult. Binary (zero-one) variables can be used to model logical conditions, such as the conditions relating to the setoff of losses and other tax effects. This topic will be discussed further in Chapter 6 and Chapter 7.

Dynamic programming

Dynamic programming (DP) improves the solution efficiency of some mathematical programming problems by decomposing the original problem and solving it as a series of interdependent subproblems, or stages. DP can be applied to simple capital budgeting problems (for example, Taha (1987)). For larger problems, however, such as capital budgeting in an environment of complex tax flows, the number of state variables would increase greatly the number of evaluations for the different alternatives at each stage, leading to computational inefficiency.

Network flow models

Some linear, integer and mixed-integer models can be converted into network flow models. These have a number of advantages over the corresponding LP or MIP formulation: solution times are often much faster, and binary variables do not have such a detrimental effect on the solution efficiency in a network as they do in a mixed integer programming model. This might be an important advantage in a capital budgeting model with logical conditions relating to the tax rules. However, the formulation of a network model becomes increasingly difficult as the size and complexity of the problem increases. Where reasonable solution times can be obtained for a linear or mixed integer programming model, there is little advantage in formulating the problem as a network flow model.

3.4 Risk and project appraisal techniques

Risk analysis in the capital budgeting literature broadly falls into two groups: the operational research based approaches such as probabilistic analysis of cash flows, and the applications of concepts of financial theory such as the capital asset pricing model (CAPM). The usual operational research assumption is that the relevant measure of risk may be obtained by modelling the project on its own, leaving the final accept/reject decision to managerial judgement or 'utility'. By contrast, the finance literature normally assumes that maximisation of the diversified shareholder's wealth is the correct criterion, with the relevant measure of risk derived from a comparison with the market portfolio or from option pricing.

3.4.1 Operational research techniques

Probabilistic methods

Hillier (1963, 1969) took an analytical approach to the evaluation of risky investments, using statistical methods to derive probabilistic information about capital investments. Hertz (1964) proposed a probabilistic simulation approach to risk analysis in capital budgeting, where uncertainty is recognised by assigning probability distributions to factors affecting the various components of project cash flow. This 'Monte Carlo' approach can give a summary joint distribution of either rates of return or net present value for a project, with the net present value being derived using the risk free rate (Hertz and Thomas, 1983).

It would be possible to design a model of this type to incorporate the firm's cash flows from its ongoing activities as well as the potential investment project, including features of the tax system. But although this type of model could provide the distribution of returns after tax given a particular set of decisions regarding project acceptance and tax strategy, it could not consider a range of possible decisions simultaneously and make an optimal choice. Also, dependencies between cash flow components and between time periods would considerably complicate the method (Hertz and Thomas, 1983).

The use of certainty equivalents

An expected cash flow $\overline{F}_t$ can be regarded as having equal value to a certain cash flow having the same utility, which is given by the expected flow multiplied by a certainty equivalent factor α_t. The relevant discount rate should be the risk free rate r_f. The present value V of a set of future cash flows for n years is then:

$$V = \sum_{t=0}^{n} (\overline{F}_t \alpha_t) / (1 + r_f)^t . \qquad (3.2)$$

This procedure could be applied to project cash flows or, if interdependencies are assumed, to the residual cash flows accruing to the firm's shareholders. The main difficulty is that a utility function must be specified for each time period and this requires some assumption about the investor's preferences over time.

The mean-variance criterion

The mean-variance criterion trades off risk and return, based on the assumption that the uncertain future net project cash flows may be described in terms of their expected value and standard deviation:

$$\overline{V} = \sum_{t=0}^{n} \overline{F}_t / (1 + r_f)^t \quad \text{and} \quad \operatorname{var}(V) = \sum_{t=0}^{n} \operatorname{var}(F_t) / (1 + r_f)^t \qquad (3.3, 3.4)$$

Assuming project independence, the mean-variance approach allows projects which are not preferred by a risk-averse investor to be identified and rejected. But any further reduction in the remaining efficient set of investments again requires the specification of the investor's utility function, since if one project or set of projects has a higher mean return but a greater risk or variance than another, a choice can only be made with a knowledge of the investor's attitude to risk. If projects' cash flows are not mutually independent, as may be the case after tax, the method cannot be applied.

3.4.2 Portfolio approaches

Portfolio theory

Portfolio theory extended the idea of the mean-variance criterion to combinations of investments in securities or projects. A mean-variance efficient portfolio has the maximum expected return for all levels of risk (as measured by standard deviation) not exceeding its own, and the minimum standard deviation of return for all levels of return not lower than its own. The idea of portfolio selection as originally introduced by Markowitz (1959) is based on the assumption of risk-interdependence. The value of a security to an investor depends on its expected return and variance, and also on the covariances of its return with returns on other securities in the portfolio. By analogy, the value of a project within a firm may be assumed to depend on the covariances of its return with the returns to the firm's other activities (Adelson, 1965).

There are a number of problems with this approach from the perspective of maximising shareholder wealth. In practical terms the number of covariances to be considered becomes far too large for all but a small number of projects to be considered. The technique again requires the separate identification of cash flows from different activities and assumes value-additivity between them, which may not be the case after tax, and a choice between possible mean-variance efficient portfolios requires a knowledge of the shareholders' utility function. Also, although for managers who are making the capital investment decisions there may be an advantage in reducing the firm's risk by considering the covariances between its activities, there is no benefit to the investor with a diversified portfolio of securities.

The capital asset pricing model

The Capital Asset Pricing Model (CAPM) originally developed by Sharpe (1964) recognises that if shareholders are assumed to hold an efficiently diversified portfolio, those aspects of a firm's risk that are unique to the firm are irrelevant. Diversified shareholders are concerned only with non-diversifiable systematic risk, determined by the covariance of cash flows with the returns on the market portfolio. The major advantage is that there is no need to consider the utility functions of investors; all risk-averse wealth-maximising investors will require the same rate of return for the same level of risk.

The CAPM equation measures the expected return on an asset over the next time period. It assumes that there is a risk-free asset, and investors can borrow and lend unlimited amounts at the risk-free rate. The expected rate of return on asset *j*, $\bar{r}_j$, is:

$$\bar{r}_j = r_f + (\bar{r}_m - r_f)\beta_j \tag{3.5}$$

where

r_f = the risk free rate of interest

$\bar{r}_m$ = the expected rate of return on the market portfolio

β_j = asset *j*'s beta, a measure of its systematic risk where $\beta_j = \dfrac{\mathrm{cov}(j,m)}{\mathrm{var}(m)}$ (3.6)

β_j is by definition the measure of risk which determines the equilibrium rate of return for the security.

The CAPM relationship may be drawn as a line with a slope equal to the market risk premium and an intercept at the risk free rate, as shown in Fig. 3.1:

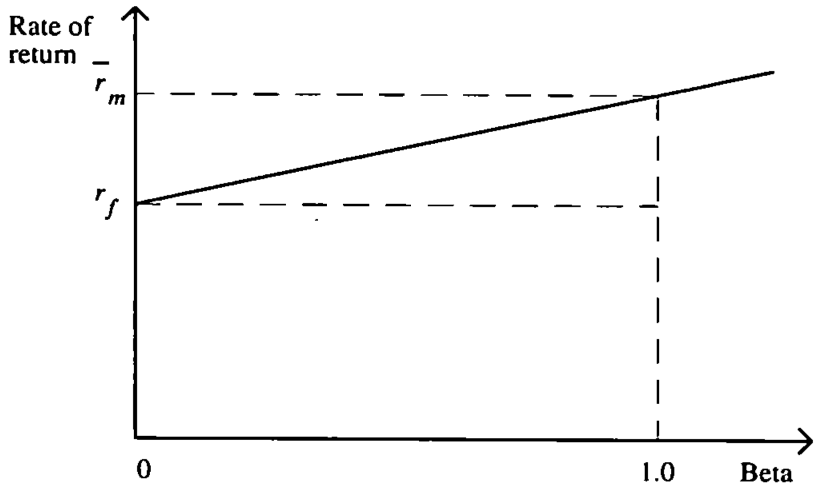


Fig. 3.1 The security market line.

The CAPM is essentially a one-period expectational model and generalising it to a multiperiod application, such as project appraisal, necessitates some assumptions about cash flow expectations (Fama, 1977). Cash flows further into the future are more uncertain, and uncertainty over future expectations may further increase the risk over time. As time passes, uncertainties will be resolved and expectations about future cash flows will be reassessed. It may therefore be inaccurate to assume a constant risk adjusted discount rate over the project's life. However, Kazemi (1991) derived a valuation formula for multiperiod stochastic cash flows, under conditions of equilibrium and rational risk-averse investor behaviour, which showed that the

CAPM need not be sequentially applied and that a single beta can measure the riskiness of an uncertain income stream.

Assumptions of the CAPM

The CAPM is based on assumptions concerning investor behaviour and the market.

Investor behaviour:

- investors are risk averse;
- investors aim to maximise the single-period expected utility of their wealth;
- investors are rational and choose only portfolios which are mean-variance efficient in the sense of Markowitz (1959);
- investors have the same perceptions of the expected return and risk of each traded asset.

Market assumptions:

- unlimited borrowing or lending is available to investors at a risk-free rate of interest;
- no taxes, transaction costs or bankruptcy costs;
- information is freely available to all investors;
- all assets are perfectly divisible.

The return on the market portfolio is after corporation tax and before personal taxes, and hence the appropriate return from an individual security to use as input into the CAPM model is also after corporation tax and before personal taxes. This is the approach which will be followed in the optimisation model developed in Chapter 7.

The CAPM assumes an efficient capital market, so that security prices fully reflect all available information. The CAPM model assumes that there are no market imperfections and information is freely and simultaneously available to investors. In these conditions risk and return are in equilibrium so that the cost of an asset is equal to the present value of the expected receipts. Under market efficiency, the price of an investment with a positive net present value would be driven up as investors recognised its value. As the price rose so the yield would fall, eventually down to the level where the investment's net present value would be zero.

The implied assumption that there is no asymmetry of information between managers and shareholders or other providers of funds is a very strong one. In the UK, empirical evidence (for example, Keane, 1983) indicates that the market is efficient in the semi-strong form with publicly available information being fully reflected in security prices. Although managers who have access to information not yet publicly available may be able to obtain investment returns that are abnormally high, this is difficult to observe empirically in the UK since such insider dealing is illegal. Insider dealing in the USA has been observed to lead to consistently abnormal returns (Jaffe, 1974). However, Heinkel and Kraus (1987) found only weak evidence of insider trading on superior information in the Canadian Stock Exchange. The implications of asymmetric information between managers and shareholders will be discussed further in Chapter 5.

Other assumptions of the CAPM may also be questionable in a practical context. Clearly taxes and transactions costs do exist, and investor perceptions of the same security may differ. The existence of a risk free rate with unlimited borrowing and lending appears unrealistic. However, the invalidity of some of its assumptions may not seriously weaken the model as an explainer and predictor of observed events. The CAPM predicts that the market portfolio is mean-variance efficient, implying that expected returns on a security are linearly related to its beta and that market betas fully explain the cross-section of expected returns. These predictions have been tested by a number of empirical studies.

Sharpe and Cooper (1972) using data for the New York Stock Exchange between 1931-1967, found evidence of a strong, positive linear relationship between a security's return and beta. However, the intercept (alpha) of their estimated SML had a much higher value that the rate on Treasury bills at that time. This supports the idea of a 'zero-beta' form of the CAPM where instead of a risk free rate, the model uses the expected return on the minimum variance portfolio that is uncorrelated with the market portfolio. Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) found further support for the validity of the CAPM. Also Stambaugh (1982) found strong support for the zero-beta form.

A difficulty of the CAPM is that the composition and return on the true, unobservable market portfolio of all risky assets is not known (Roll, 1977). Tests of the theory use portfolios of traded equities as a proxy. If the evidence is consistent with efficiency of the proxy, it does not necessarily mean that the true market portfolio is efficient. Roll showed that the choice between the different forms of CAPM model is highly sensitive to the market proxy measure used, and concluded that equilibrium theory is not testable. Shanken (1987) found evidence to reject the joint hypothesis that the correlation of the proxy with the market portfolio exceeds some limit and that the CAPM is valid.

Other studies have suggested different explanations of observed stock returns. Banz (1981) found that the average returns on securities with a small market value were higher than expected from their betas, and average returns on large market value firms were lower than expected. Lakonishok and Shapiro (1986) also discovered that size was significant. Bhandari (1988) found that more highly geared firms had higher returns than would be expected from their market beta, which should take into account financial risk. As well as size, the ratio of a firm's book value of equity to its market value also appears to be a significant determinant of average returns (Fama and French, 1992). Fama and French (1992), argued that although a correlation exists between beta and observed returns, this is largely due to a correlation of both beta and returns with factors such as firm size.

But Kothari, Shanken, and Sloan (1995) using annual betas, rather than the monthly betas used by Fama and French, showed evidence that average returns do incorporate a substantial compensation for systematic risk over the period 1927 to 1990, suggesting that risk estimation deviations in Fama and French's study could account for their findings. Kothari *et al.* also cast doubt on the positive relationship found between book to market value of equity and average returns. Using an alternative data source (Standard and Poor, rather than Compustat as in Fama and French) they found that the book to market value of equity variable is at best weakly related to average stock return.

Further, a recent study by Pettengill, Sundarem and Mathur (1995) recognised that the CAPM's prediction of a positive relationship between returns and beta is based on expected returns, rather than realised returns. In periods where excess market returns are negative, there should be an inverse relationship between beta and portfolio returns. Pettengill *et al.* adjusted for the expectations concerning negative market excess returns and found a consistent and significant relationship between beta and returns, and evidence that beta risk is rewarded by higher returns.

Lee, Wu and Wei (1990) considered the effect of heterogeneous investment horizons on the CAPM, and argued that some empirical findings which were inconsistent with the traditional CAPM model resulted from the misspecification of the CAPM by ignoring the discrepancy between the observed data periods and the true investment horizons.

If other variables than beta also explain some of the cross-sectional variation in expected returns, deviations may be the result of an inefficient proxy for the true market portfolio instead of or as well as problems with the asset pricing model itself. Even if tests of the CAPM are subject to the criticism of Roll (1977) they may have important implications for behaviour. From the empirical evidence it would generally appear that return and risk are linearly related for securities and portfolios over long periods of time, when risk is defined as systematic risk. On the whole, residual risk does not appear to be important in determining return, and deviations from the model between time periods cannot be used to make an excess return. Although it may not be possible to test if the model is true, it does give insight into capital market observations. Investors are not rewarded for taking non-market risk but are rewarded for systematic risk.

Derivation of the required rate of return

An individual project's beta may be derived by forecasting the distribution of the expected market returns and project returns over the project's life and calculating the betas for each period directly. Alternatively, it can be estimated from the firm's historic beta provided it has the same risk characteristics as the firm's other activities, or from the beta of a suitable proxy firm. The risk free rate may be approximated by the return on government securities, and the market risk premium $(\bar{r}_m - r_f)$ may be approximated using historical averages, under the assumption that these reflect *ex ante* expectations and are an appropriate indication of current and future premiums.

There are a number of potential difficulties in using past or current observed betas to appraise new projects. Firstly, if the beta of a proxy firm is used, differences in the rates of taxation and distribution policy between the proxy firm and the firm undertaking the project would need to be taken into consideration because these affect the rate of return required by shareholders. Secondly, Miller and Modigliani

(1961) showed that a firm's market value has two components, the present value of the cash flows generated by existing assets, and the present value of 'growth opportunities'. Myers and Turnbull (1974) argue that because growth opportunities affect a firm's observed systematic risk, the correct discount rate for an incremental project cannot be determined from the firm's observed beta. Thirdly, observed betas for geared firms reflect not only the systematic risk of the firm's activity, but also the financial risk borne by the equity holders as a result of debt financing and the extent to which this is mitigated by the debt interest tax shield. This will be determined by the firm's ability to set off its debt interest against taxable profits, which will in turn be determined by the firm's overall cash flows including the incremental capital investment, which may be financed differently from the firm's other activities.

Finally, the separate analysis of incremental projects using the CAPM is subject to the same difficulty as other methods discussed in this chapter. Shareholders' returns derive not from the sum of individual projects' cash flows, but from the whole firm's residual cash flows after payments made outside the firm, including taxes. If there are no interdependencies between cash flows, then the relevant risk of a combination of projects or of the entire firm will be the weighted average of the betas of the individual projects. But the effects of taxation-induced interdependencies between projects means that the after tax beta of the combination will not simply be a weighted average of the individual project betas adjusted for tax in isolation. An incremental project's cash flows may change the firm's tax flow pattern and hence change its beta and the rate of return required by shareholders, even if it has the same beta as the rest of the firm's activities before tax and is financed in the same proportions. Tax effects, resulting from marginal tax rates and the carrying forward or back of losses and ACT, will determine the incremental after tax cash flows. These determine the covariance of the project's return with the market portfolio, which in turn determines the firm's after tax beta risk and hence the required rate of return after tax.

Thus a realistic measure of the firm's systematic risk after tax must be assessed in the context of the firm's overall tax situation. The most appropriate asset cash flows to use in the CAPM model are those relating to the firm as a whole rather than individual projects, since investors cannot trade in individual projects and also because tax-induced interactions between projects and between investment and financing determine what is available to shareholders. The required rate of return may be determined directly, using as input to the CAPM model the after-tax cash flows to equity holders for the firm as a whole with and without any additional capital investment.

3.4.3 Call options in capital budgeting

An alternative to portfolio based analysis is to take account of the possibility of constructing riskless or hedged positions in capital assets, via the option pricing model (Black and Scholes, 1973).

An irreversible investment opportunity is analogous to a financial call option. A call option gives the right to pay an 'exercise price' for an asset with value, such as a share, for a specified time period. An investment opportunity gives a firm the option to spend an initial cash outlay, over some period of time into the future, in return for a project's value. Exercising the option, or undertaking the project, involves giving up the option of waiting for new information which enables a more accurate assessment of demand and costs. This implies that the firm should undertake the project when its value is at least as large as the direct cost of the project, plus the opportunity cost of exercising the option to invest (Pindyck, 1991)

Discounted cash flow techniques to capital budgeting have been criticised for neglecting price uncertainty and possible managerial responses to price variations, particularly for natural resource industries where prices vary widely. In this situation the output rate will depend on the stochastic output price and the investment decision may be treated as a problem of stochastic optimal control (for example, Constantinides, 1978; Pindyck, 1980; Brennan and Schwartz, 1985). McDonald and Siegel (1985) develop a methodology to value risky investment projects with the option to shut down production temporarily whenever variable costs are greater than operating revenues, assuming that prices and costs follow a continuous time stochastic process. McDonald and Siegel (1986) studied the optimal timing of investment, assuming that the project's outlay and benefits follow continuous-time stochastic processes.

Paddock, Siegel and Smith (1988) used the analogy between a call option and the value of a resource reserve to develop a model of offshore petroleum leases. In a conventional DCF analysis, the greater the volatility of oil prices, the higher the discount rate and the smaller the value of the undeveloped resource. But the DCF approach does not incorporate the value of the option over when to develop the reserve, so it understates the true value. Because of this option, the value of the reserve increases as the oil price volatility increases, contrary to the DCF result.

3.5 Empirical evidence on investment appraisal

It has been assumed in this chapter that investment appraisal should aim to maximise shareholder wealth, taking into account the relevant measure of risk for a diversified shareholder. The relevant cash flows in this context are the residual flows which accrue to the shareholder, which will be determined partly by the tax situation of the firm as a whole. An optimal approach would aim to take this into account. In practice, however, there is a widespread reliance on techniques which may not maximise shareholder wealth, and tax considerations are often ignored.

The use of project appraisal and risk analysis techniques

The empirical evidence indicates that even the largest firms tend to rely on simple techniques such as payback period, although the use of more sophisticated methods is increasing.

Morgan (1992) reported the results of a survey carried out in 1985-1986 of *Times 1000* firms, excluding those in the oil industry where different tax provisions apply. The survey showed that payback period was the most widely used criterion overall, although rate of return on capital employed was the single most important technique mentioned by the largest number of firms. Discounted cash flow methods were less frequently used.

A survey by Pike (1988) of 100 large UK firms over an 11 year period found very significant increases in the uptake of 'sophisticated' methods, especially in risk analysis. 'Sophisticated' techniques included discounted cash flow methods, risk analysis techniques, and management science techniques such as mathematical programming and computer simulation. 'Naive' techniques as defined in the survey included payback period and accounting rate of return. Pike reported that while only 26% of respondents to a survey in 1975 had formally evaluated project risk, 86% of the same firms were doing so by 1986. Sensitivity analysis and shortening of the required payback period were, however, the usual methods employed. Probability analysis and beta analysis were rarely used.

A later survey by Ho and Pike (1991) of the largest 350 of the firms in the *Times 1000* found that managers most frequently (79 per cent) assessed the risk of each project or project class individually, although 60 per cent also considered the effect of the project's risk on the overall corporate risk and return. 25 per cent of firms stated that they considered the effect of project risk on the shareholder's portfolio.

Although the authors comment that the survey shows a higher usage of the CAPM technique than any prior survey in the UK or US, 73 per cent of managers 'never' used it. An earlier survey by Hodgkinson (1987) also found that the CAPM had not yet received great acceptance by the business world.

This finding may reflect conflict between managerial and shareholder objectives: the application of the CAPM to investment appraisal by firms implies that managers incur a greater risk than shareholders of loss of future consumption opportunities, since their interest in the firm is undiversified and systematic risk may be only a small part of overall risk. Where ownership and control are separated, there may be little incentive for managers to put the interests of the suppliers of equity finance before their own employment security. But if the market's information flows are efficient, shareholders will be aware of the investment opportunities available and may remove directors who are too risk averse to undertake investments which would maximise the wealth of the diversified investor.

Tax and project appraisal

Empirical evidence shows that the impact of taxation is not generally given much importance by firms when appraising capital investments, and there is widespread use of pre-tax appraisal, even by large firms.

A survey by Alam and Stafford (1985) of 249 UK manufacturing firms of different sizes found that taxation and tax incentives were not major factors in the investment decision, although large firms appeared to be more influenced by tax considerations than small firms. At the time of the survey, major changes were taking place in the system of corporate taxation and many respondents considered that the uncertainty associated with forecasting tax changes over the economic life of an investment precluded taking tax into account in project appraisal.

Case studies undertaken by Morgan (1986) suggested that investment decisions by some of the largest firms were taken to meet commercial and strategic needs which were determined on a pretax basis, without using sophisticated methods of investment appraisal. Where tax was incorporated into the appraisals, its treatment varied considerably and the tax rates applied did not necessarily reflect actual tax positions or tax horizons. Similarly, a survey by Hodgkinson (1989) based on a sample of 134 companies from the top 500 of the Times 1000 Index, suggested that a significant number of the companies sampled were either not including taxation in their project appraisals or including taxation at the full statutory rate, despite being in a restricted tax position. It therefore appears that even the largest companies do not always take tax effects into account in their investment decision making, fully or correctly.

A later suvey by Morgan (1992) showed that some of the largest and most sophisticated companies (in terms of their strategic planning procedures and methods of identifying potential investments) considered that tax issues, including possible tax efficient methods of financing, should always follow the investment decision. No link was made between the underlying strategic considerations and possible tax effects. Where assets were leased, the investment and financing decisions were often taken separately, reducing the likelihood of the possible tax benefits of leasing being taken into account. Morgan suggested that tax exhaustion, prevalent in the UK prior to the changes introduced in 1984, was a partial explanation for this attitude. But tax exhaustion may itself have significant tax consequences for investment and financing decisions.

On the whole, the empirical evidence suggests that firms may prefer only to accept projects which are viable when evaluated without their associated tax effects. This implies that the impact of government intervention in business investment, through the manipulation of capital allowances and corporate tax rates, may be limited. As Morgan (1992) commented:

'Neglect of tax considerations and insistence on pre tax 'commercial' viability in project appraisal will reduce the direct impact of government intervention through the tax system on business investment and as a result fiscal investment incentives may be blunter instruments than policy makers have often supposed.'

3.6 Conclusion

In an uncertain business environment, the strategy that maximises value over time is likely to be a flexible one rather than a fixed set of investment decisions. But the overall firm cash flows after tax may create complex interactions between projects. Also, it is the future cash flows from the combination of projects accepted in the current period that will enable the firm to take advantage of investment opportunities arising in the future.

Most project appraisal techniques treat cash flows as independent, which assumes that value additivity exists between new and existing investments. Tax induced interdependencies between the firm's overall cash flows will make this an invalid assumption. An optimal technique to take these interdependencies into account would consider the firm's overall cash flows after tax with and without any additional projects being considered. An appropriate measure of risk, given the firm's objective assumed in this thesis, would be based on the risk of the residual cash flows to the diversified shareholder. Again, this will be determined by the firm's overall situation incorporating any tax-induced interactions between the cash flows. But empirical evidence suggests that, in practice, firms do not take tax considerations fully into account in their capital investment decisions, although there does appear to be a trend towards the use of more sophisticated methods.

If a mathematical programming approach is taken to investment and financing decisions, the firm's real options such as abandonment, delaying implementation or leasing should be included wherever possible. For example, the abandonment option can be included by identifying the points in time where the option would be possible and including all the possible project lives as separate mutually exclusive projects. using binary (zero-one) variables. This is the approach that will be taken in the optimisation model developed in Chapter 7. The model will then choose the optimal project life in terms of the overall after-tax increase in shareholder wealth. Contingency conditions, such as where a project may only be undertaken provided a pilot version is carried out first, can be included using a similar technique. As a project's life proceeds, managers will respond to events, uncertainty will be resolved and the strategic planning horizon will shift in time, so the model would need to be run at regular intervals using updated information inputs.

Chapter 4

Tax and capital investment

4.1 Introduction

The impact of taxation on capital investment decisions will now be reviewed in terms of the wider issues of tax neutrality, capital investment incentives and the impact of taxation on risk. Incentives or disincentives to invest, determined by the present value of tax relief for capital expenditure and by firms' taxable capacity and financing choices, have been studied widely in the literature.

The author examines certain patterns of accelerated depreciation which have been available in the UK for buildings and plant and machinery. It is shown that the present value of the capital allowances obtained reaches a maximum at an optimal cost of capital, which in the case of plant and machinery is independent of the proportion of the asset cost which may be allowed against tax in the first year. The analysis also shows that the optimal cost of capital is reduced if the firm is temporarily non-taxpaying and must carry forward its early capital allowances to a future profitable period.

4.2 Capital investment and tax neutrality

Tax neutrality exists when the tax system does not distort economic choices. With respect to corporate capital investment, this may be defined as a system under which the internal rate of return of a project is unchanged by taxation and the overall cost of financing it is unchanged. Where there is a tax-induced distortion in economic choice, the social disutility which results is known as an excess burden of taxation. For instance, if a project with an initial capital outlay of £10,000 has a positive net present value before tax of £15,000 but a negative one after corporate tax, the firm may decide to distribute £10,000 as dividends to its shareholders rather than retain that amount in order to proceed with the project. Before personal taxes, the shareholders' loss of wealth resulting from the tax-induced distortion is £5,000.

Under a cash flow system, assuming constant marginal tax rates and perfect relief for tax losses, it may be shown (Pointon, 1980) that the rate of return of a capital investment project is unaltered and the government effectively funds a proportion of the project and receives the same proportion of the actual return. Where there is less than 100 per cent instantaneous relief for capital expenditure and capital allowances are spread into future periods instead, there is a potential disincentive to invest under discounted cash flow analysis. There may also be a disincentive if there is imperfect relief for losses arising in periods where new capital expenditure exceeds the taxable receipts. With perfect relief, losses would either receive an immediate rebate or else be carried forward and (in theory) inflated at the firm's required rate of return. The introduction of tax time lags into the cash flow tax system does not alter the analysis, provided that the delay between expenditure on capital assets and the receipt of the associated tax rebate is the same as the delay between a cash inflow and the associated tax payment.

4.3 Capital investment incentives

There will be imperfect relief for capital expenditure if the value of the capital allowances, discounted to the time of purchase, is less than the purchase cost of the asset. Prior to 1984 there was a system of high tax rates and generous allowances on investment, especially plant and machinery, with 100 per cent capital allowances available in the first year. However, this was believed to have distorted investment decisions, encouraging tax driven investment behaviour rather than response to market signals, thus contributing to Britain's poor industrial performance (HMSO, 1984). Also, at that time, many firms with low profits before tax were unable to utilise fully their capital allowances and were in a tax loss position as a result. A survey of *Times 1000* companies by Morgan (1992) carried out in the mid 1980s showed the prevalence of tax exhaustion during the period 1979-84: 46 per cent were paying very little tax or were fully tax-exhausted. Further evidence that the system required reform was given by MacDonald (1981) who examined the pressures of the fiscal system prior to the 1984 Finance Act with particular reference to the source of finance for investment. He concluded that tax-based distortions in the cost of capital gave rise to opportunities to invest which were not necessarily optimal as regards shareholders' interests.

Much of the literature examining the impact of the UK taxation system on investment has been concerned with the major reform of corporate taxation over the period 1984-1986. The 100 per cent capital allowances were withdrawn and replaced by a system of writing down allowances at 25 per cent on a reducing balance basis, with a simultaneous reduction in the corporation tax rate. The general conclusion was that for firms with relatively low levels of capital investment such as the service sector, the net effect would be beneficial, but capital intensive companies would be insufficiently compensated for the reduced capital allowances by the new lower tax rate (Devereux and Mayer, 1984; Morgan, 1986; Moon and Hodges, 1989). Mole (1987) examined the impact of the new system of writing down allowances for the ungeared firm, with projects being financed purely from retentions and under inflationary conditions. He concluded that projects which were profitable before tax would be rejected under the new system even after taking the lower rate of corporation tax into account, and therefore the 1984 Finance Act imposed an 'excess burden' which was increased by any level of inflation. A later paper by Mole (1990) examined the relationship between yield and NPV under taxation and demonstrated that a project's NPV was reduced by the nominal 35 per cent rate of corporation tax only when its pre-tax yield was very high.

Others argued that the post-1986 corporate tax system created too great a disincentive to capital investment and that changes were needed (for example, Devereux and Mayer, 1984; King, 1985). A permanent increase in the level of first year allowances for plant and machinery was advocated by the CBI (1992). But the effectiveness of this policy as an incentive to capital investment was questioned by Young (1992) who examined the impact of a permanent increase in first year allowances on plant and machinery from 25 per cent to 40 per cent, using a model of UK corporation tax based on aggregate estimates. Taking into account the prevalence of tax exhausted firms, Young estimated that a significant proportion of the additional capital allowances would simply be added to the stock of unused allowances, and concluded that 'the revenue cost of policies to increase capital allowances is much smaller than crude calculations suggest when tax exhaustion is prevalent.'

Young also pointed out that although a firm which had previously always had sufficient taxable profits to be able to utilise 100 per cent capital allowances would initially find its capital allowances substantially reduced, in the long run, capital allowances would increase to roughly the level they would have been without the reforms. Each annual 'cohort' of investment continues to attract a writing down allowance in each subsequent year until it is fully depreciated, whereas under the previous system, only the current year's capital investment attracted an allowance.

4.4 Risk, taxation and investment

Domar and Musgrave (1944) showed that where taxes are proportional and losses may be fully offset, there is effectively a partnership between firm and government where risk and return are both shared. Taxation reduces the rate of return and the risk by the same proportion. Where there is imperfect relief for losses, however, the government does not share fully the risk and return. Domar and Musgrave assumed that the choice of investments lay between cash and one risky asset. Tobin (1958) extended the insight of Domar and Musgrave, showing that under certain conditions an investor's preferences could be represented by an indifference map in terms of the investment's expected outcome and its standard deviation. Mossin (1968) proved that where a riskless asset exists, a proportional tax increases risk taking without any restrictions on the subjective probability distribution. Similarly Stiglitz (1969) concluded, without specifying a utility function, that an increase in tax leads to an increase in the demand for the risky asset.

Pointon (1980) analysed the risk-return relationship for capital expenditure under a cash flow system within the framework of the capital asset pricing model (CAPM). Under a cash flow system with constant tax rates, constant tax time lags, perfect loss relief and 100 per cent capital allowances, the rate of return on a capital investment project is unaltered. In a CAPM framework, deviations of rates of return from the mean and covariances also remain the same under this system so there is no tax incentive or disincentive to risky investment.

An excess burden results if tax allowances on capital expenditure are imperfect; for instance, if the firm does not have taxable profits large enough to absorb 100 per cent tax depreciation or if this provision is unavailable. Pointon also shows that if the present value of the relief on capital expenditure is less than 100 per cent, the reduction in risk may be insufficient to compensate for the reduction in the rate of return. In this case the expected rate of return after tax is given by the expected rate of return before tax multiplied by $(1-T)/(1-aT)$ where T is the corporate tax rate and a is the present value of the capital expenditure relief as a proportion of cost. The after tax beta risk is derived from the pretax beta by multiplying by the same factor. An acceptable project with a rate of return and beta represented by point B on Fig. 3 below may drop to point A after tax, which is below the required rate of return shown by the security market line SML.

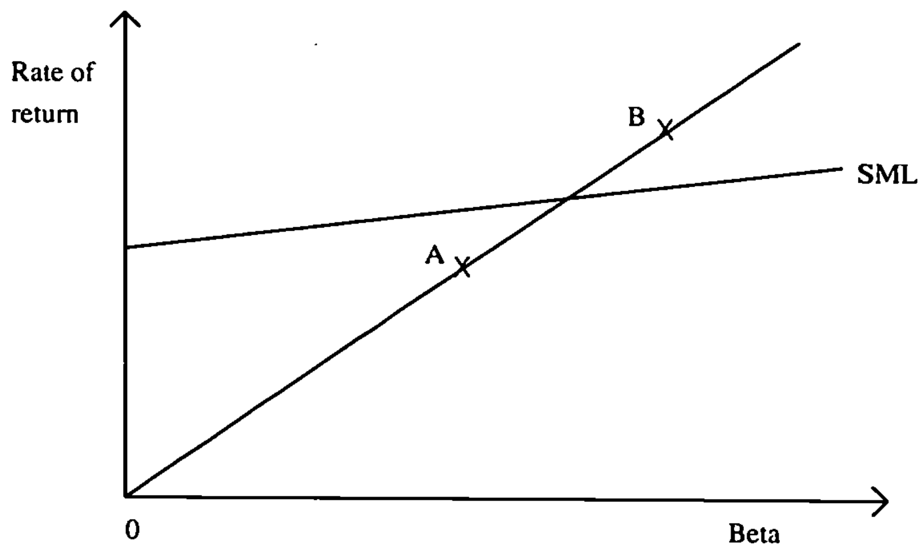


Fig. 3 The effect of tax on a project's risk and return

4.5 Accelerated depreciation

In the 1992 Autumn Statement, a temporary increase in the capital allowances available in the first year was announced. On 1 November 1992 a first year allowance of 40 per cent became available for plant and machinery purchased during the next twelve months. In subsequent years the writing down allowance for assets purchased under this provision is 25 per cent per annum on a reducing balance basis. After the twelve month period, the system reverted to the normal pattern of 25 per cent of cost in the first year, with subsequent writing down allowances at 25 per cent on a reducing balance basis. Bond, Denny and Devereux (1993) suggested that the temporary nature of this increase would provide a more effective short-term stimulus to investment than a permanent increase, creating an incentive to bring investment forward from 1994 into 1993. However, there would be a corresponding depressing effect in the subsequent year. Manufacturing investment statistics quoted by Bond *et al* show that this happened in the mid 1980s, when a similar pattern of allowances was available in the two years following the 1984 tax reform.

This availability of accelerated depreciation in the form of a first year allowance followed by a series of reducing-balance writing down allowances at a lower rate has therefore been an intermittent feature of the UK corporate tax system: between 1984 and 1986, and in 1993-94. The implications of this pattern of tax allowances for the value of the relief on capital expenditure are explored in the following section.

4.6 Accelerated capital allowances and the cost of capital

From the viewpoint of the firm undertaking a discounted cash flow appraisal of an investment, the benefits of an increased first year allowance must be balanced against the lower writing down allowances that will be available in subsequent years. The relevant factor is the overall effect on a project's net present value, which is derived in the following sections.

4.6.1 Plant and machinery

The value of the increased first year allowance, in present value terms, may be calculated as follows. Assuming the asset is purchased at the start of the year, the present value of the allowance available in the first year will be:

$$\frac{\alpha' JT}{(1+k)^{1+L}}$$

where

α' = the proportion of the asset's cost which is the available FYA

J = the cost of the asset

T = the corporate tax rate

k = the appropriate discount rate

L = the time lag between the year-end date and the tax payment date.

The present value of the second year's capital allowance will be

$$\frac{\alpha(1-\alpha')JT}{(1+k)^{2+L}}$$

where

α = the proportion of the written down value at the start of year 2 which is available as the writing down allowance in that year.

Similarly, the third year 's allowance will be

$$\frac{\alpha(1-\alpha')(1-\alpha)JT}{(1+k)^{2+L}}$$

Assuming that k, the discount rate, and T are constant over time, the allowances available after the first year in present value terms form a geometric progression with a common ratio of $(1-\alpha)/(1+k)$. The sum of this progression is therefore

$$\frac{\alpha(1-\alpha')JT/(1+k)^{2+L}}{1-(1-\alpha)/(1+k)}$$

If there is no special FYA available, then the proportion of the asset's cost which is the writing down allowance in year 1 will be α. The present value of the first year's capital allowance will be

$$\frac{\alpha JT}{(1+k)^{1+L}}$$

and the sum in present value terms of the remaining series of allowances will be

$$\frac{\alpha(1-\alpha)JT/(1+k)^{2+L}}{1-(1-\alpha)/(1+k)}$$

The value, V_m, of the increased first year allowance is given by

$$V_m = \left[\frac{\alpha' JT}{(1+k)^{1+L}} + \frac{\alpha(1-\alpha')JT/(1+k)^{2+L}}{1-(1-\alpha)/(1+k)}\right] - \left[\frac{\alpha JT}{(1+k)^{1+L}} + \frac{\alpha(1-\alpha)JT/(1+k)^{2+L}}{1-(1-\alpha)/(1+k)}\right]$$

$$= \frac{\alpha' JT}{(1+k)^{1+L}} - \frac{\alpha JT}{(1+k)^{1+L}} + \frac{\alpha(1-\alpha')JT/(1+k)^{1+L}}{(k+\alpha)} - \frac{\alpha(1-\alpha)JT/(1+k)^{1+L}}{(k+\alpha)}$$

$$= \frac{JT}{(1+k)^{1+L}}\left[(\alpha'-\alpha) + \frac{\alpha(1-\alpha')-\alpha(1-\alpha)}{(k+\alpha)}\right]$$

$$= \frac{JT}{(1+k)^{1+L}}\left[(\alpha'-\alpha)-\alpha(\alpha'-\alpha)/(k+\alpha)\right]$$

$$= \frac{JT}{(1+k)^{1+L}}(\alpha'-\alpha)\left[1-\alpha/(k+\alpha)\right]. \tag{4.1}$$

From (4.1), the value of V_m under the tax rules for plant and machinery acquired during the twelve months from 1 November 1992 may be calculated. Assuming a discount rate of 10 per cent and a tax rate of 33 per cent, V_m is approximately 1.197 per cent of the asset's cost.

If k =0, then $[1-\alpha/(k+\alpha)]$ = 0 and hence V_m= 0 also. For all positive values of k, V_m will be greater than zero, but reaches a maximum at a critical value of k, which may be derived by differentiating V_m with respect to k.

$$V_m = \frac{JT}{(1+k)^{1+L}}(\alpha' - \alpha)[1-\alpha/(k+\alpha)] \tag{4.1}$$

$$\frac{d}{dk}\left[\frac{JT(\alpha'-\alpha)}{(1+k)^{1+L}}\right] = \frac{JT(\alpha'-\alpha)[-(1+L)]}{(1+k)^{2+L}}$$

$$\frac{d}{dk}[1-\alpha/(k+\alpha)] = -\alpha(-1)(k+\alpha)^{-2}$$

$$= \alpha/(k+\alpha)^2$$

$$\frac{dV_m}{dk} = \frac{JT(\alpha'-\alpha)\alpha}{(1+k)^{1+L}(k+\alpha)^2} + \left[1-\frac{\alpha}{(k+\alpha)}\right]\frac{JT(\alpha'-\alpha)(-1-L)}{(1+k)^{2+L}}$$

$$= \frac{JT(\alpha'-\alpha)}{(1+k)^{1+L}}\left\{\frac{\alpha}{(k+\alpha)^2} + \left[1-\frac{\alpha}{(k+\alpha)}\right]\left[\frac{(-1-L)}{(1+k)}\right]\right\}$$

$$= \frac{JT(\alpha'-\alpha)}{(1+k)^{1+L}}\left[\frac{\alpha}{(k+\alpha)^2} - \frac{(1+L)}{(1+k)} + \frac{\alpha(1+L)}{(k+\alpha)(1+k)}\right]$$

$$= \frac{JT(\alpha'-\alpha)}{(1+k)^{1+L}}\left[\frac{\alpha(1+k)-(1+L)(k+\alpha)^2+\alpha(1+L)(k+\alpha)}{(k+\alpha)^2(1+k)}\right]$$

$$= \frac{JT(\alpha'-\alpha)}{(1+k)^{2+L}(k+\alpha)^2}\left[\alpha(1+k)-(1+L)(k+\alpha)^2+\alpha(1+L)(k+\alpha)\right] \tag{4.2}$$

Simplifying the terms in square brackets gives

$$\frac{dV_m}{dk} = \frac{JT(\alpha'-\alpha)}{(1+k)^{2+L}(k+\alpha)^2}\left\{-\left[k^2(1+L)+k\alpha L-\alpha\right]\right\}$$

or,

$$\frac{dV_m}{dk} = \frac{JT(\alpha - \alpha')}{(1+k)^{2+L}(k+\alpha)^2}\left[k^2(1+L) + k\alpha L - \alpha\right]. \tag{4.3}$$

Hence, $\frac{dV_m}{dk}$ must be 0 if $\left[k^2(1+L) + k\alpha L - \alpha\right] = 0$. This solution to this quadratic equation gives the value of k at which $\frac{dV_m}{dk} = 0$:

$$k = \left(-b \pm \sqrt{b^2 - 4ac}\right)\Big/ 2a$$

$$k = \left(-\alpha L \pm \sqrt{(\alpha L)^2 - 4(1+L)(-\alpha)}\right)\Big/ 2(1+L) \tag{4.4}$$

If $\alpha = 0.25$ and $L = 0.75$, assuming tax is paid nine months after the year end, this gives approximate values of $k = 0.3282$ or $k = -0.4353$. Ignoring the negative root, the value of k at which V_m is maximised, k_m, is 32.82 per cent approximately. (Although the second derivative is not shown, a maximum, rather than a minimum, is supported by the graphs that follow.)

This analysis shows that k_m is independent of the proportion α' of the asset cost which is available as a writing down allowance in the first year. Instead, k_m is determined by the value of α and by the time lag before tax relief is obtained.

The implication is that, where there is a time lag between the purchase of an asset and the receipt of the first tax benefits, it would be impossible for government to manipulate the value of α' in order to maximise the benefits of accelerated depreciation at any chosen discount rate.

Timing effects

If the firm has losses for y years, followed by a return to tax paying status in $y + 1$, the tax benefits of the capital allowances for each of the years in which there are losses will be obtained simultaneously in $y + 1$, assuming a sufficiency of profits against which they may be set. In this situation it may be shown that k_m may still be determined from (2).

It is assumed that there are losses for the first y years of the asset's life and that in year $y+1$ there are sufficient profits to set against all the allowances relating to years $1, \ldots, y+1$. Relief for all these allowances will be obtained at time $y+1+L$. The capital allowance for year $y+2$ will be obtained at $y+2+L$.

Temporarily ignoring discounting, the total sum of tax benefits from the asset's capital allowances will be JT. The sum of the geometric progression which starts with the allowance for $y+2$ will be

$$\frac{\alpha(1-\alpha')(1-\alpha)^{y}JT}{1-(1-\alpha)}$$

that is, $(1-\alpha')(1-\alpha)^{y}JT$.

Hence the undiscounted value of the sum of the capital allowances for years prior to $n+2$ is

$$JT-(1-\alpha')(1-\alpha)^{y}JT$$

and so their present value is

$$\frac{JT\left(1-(1-\alpha')(1-\alpha)^y\right)}{(1+k)^{y+1+L}}$$

The present value of the remaining allowances will be

$$\frac{\alpha(1-\alpha')(1-\alpha)^y JT/(1+k)^{y+2+L}}{1-(1-\alpha)/(1+k)}$$

or, $$\frac{\alpha(1-\alpha')(1-\alpha)^y JT/(1+k)^{y+1+L}}{k+\alpha}$$

The total present value of the allowances is therefore

$$\frac{JT\left(1-(1-\alpha')(1-\alpha)^y\right)}{(1+k)^{y+1+L}}+\frac{\alpha(1-\alpha')(1-\alpha)^y JT/(1+k)^{y+1+L}}{k+\alpha} \qquad \text{(i)}$$

The corresponding expression assuming an FYA of α rather than α' would be

$$\frac{JT\left(1-(1-\alpha)^{y+1}\right)}{(1+k)^{y+1+L}}+\frac{\alpha(1-\alpha)^{y+1} JT/(1+k)^{y+1+L}}{k+\alpha} \qquad \text{(ii)}$$

Subtracting (ii) from (i) and simplifying, gives

$$V_m=\frac{JT}{(1+k)^{1+L}}(\alpha'-\alpha)\left[1-\alpha/(k+\alpha)\right]\left[(1-\alpha)/(1+k)\right]^y \qquad (4.5)$$

which is the same as (4.3) where there are no losses, multiplied by $\left[(1-\alpha)/(1+k)\right]^y$.

Differentiating (4.5) in the same way as (4.3) gives

$$\frac{dV_m}{dk} = \frac{JT(\alpha - \alpha')(1-\alpha)^y}{(1+k)^{y+2+L}(k+\alpha)^2}\left[k^2(y+1+L) + k\alpha(y+L) - \alpha\right] \tag{4.6}$$

where the expression between the square brackets is equivalent to that in (2), allowing for the additional time lag.

Defining L' as the time which elapses between the purchase of the asset and obtaining the benefit of the first capital allowance, or set of allowances, V_m will be at a maximum where $\left[k^2 L' + k\alpha(L'-1) - \alpha\right] = 0$.

Table 4.1

Values of k_m obtained under different values for a and L'

α	L' *(years)*	k_m
0.25	1.75	32.82%
0.25	2.75	23.23%
0.25	6.75	11.35%
0.02	1.75	10.27%
0.10	1.75	21.86%
0.40	1.75	35.47%

These results indicate that k_m is more sensitive to the value of L' than to the value of α. Comparison of Figs. 4, 5, and 6 shows that as L' increases, the value of V_m peaks more sharply at a lower value of k.

Fig. 4

Graph of Vm against k, where L'=1.75

asset cost	=	100
tax rate	=	0.33
FYA (%)	=	40
WDA (%)	=	25

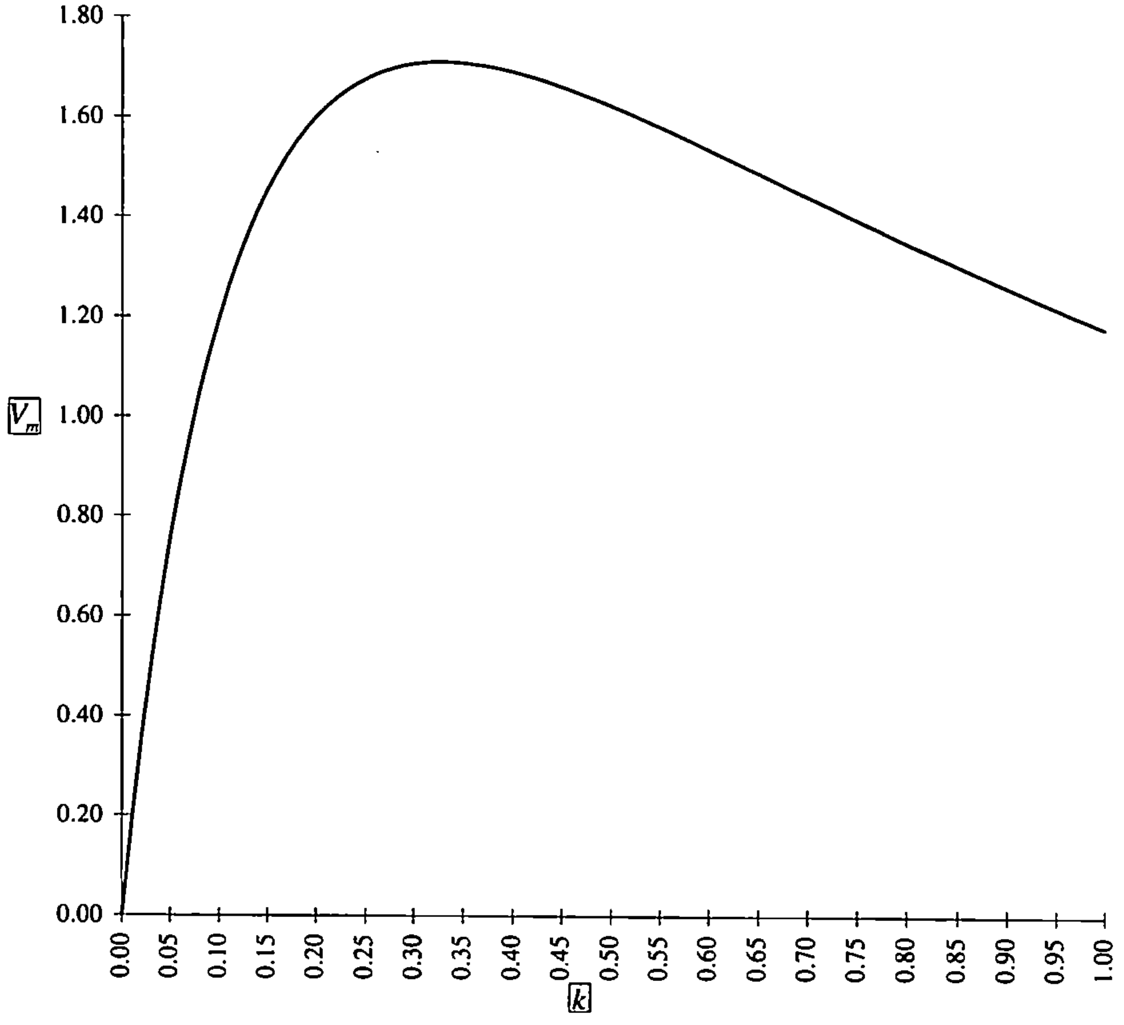


Fig. 5

Graph of Vm against k, where L'=2.75

asset cost	=	100
tax rate	=	0.33
FYA (%)	=	40
WDA (%)	=	25

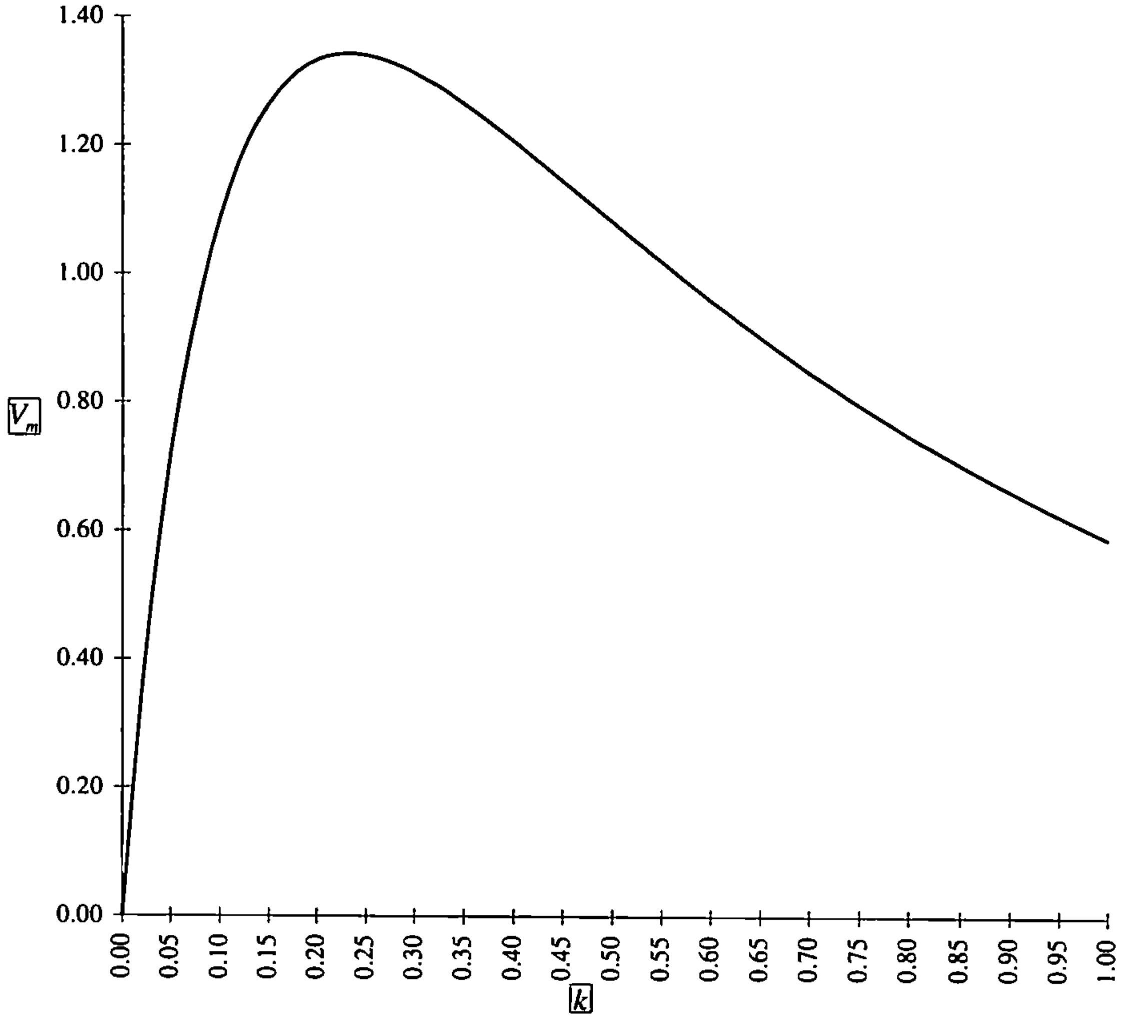
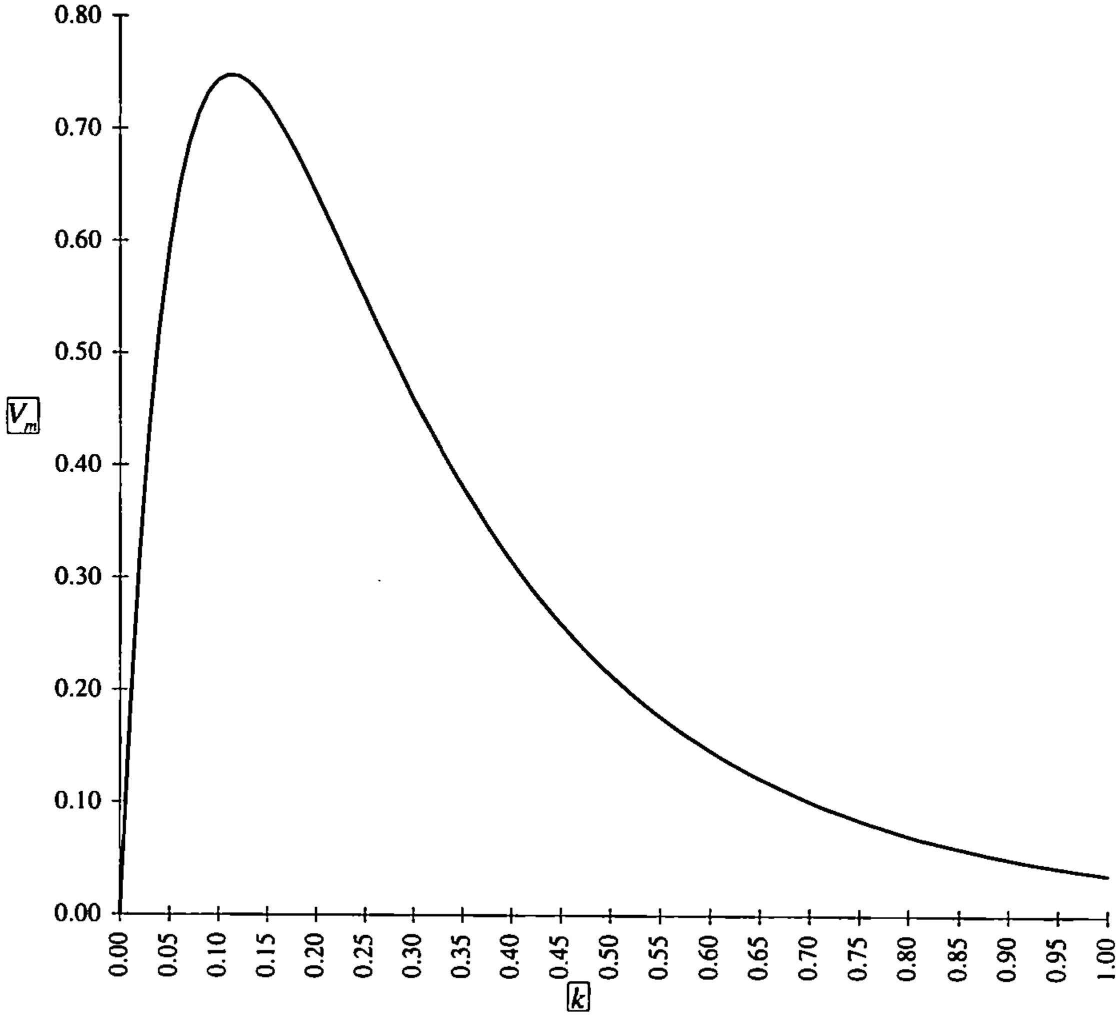


Fig. 6

Graph of Vm against k, where L'=6.75

asset cost	=	100
tax rate	=	0.33
FYA (%)	=	40
WDA (%)	=	25



4.6.2 Buildings

For new industrial or agricultural buildings acquired between 1 November 1992 and 31 October 1993, an initial allowance of 20 per cent of the cost was available, with the balance being written off at 4 per cent per year. In the first year it is possible to claim both the initial allowance and a 4 per cent writing down allowance, depending on when the building first came into use. It is assumed here that both these allowances are claimed.

The present value of the first year's initial allowance is given by

$$\frac{a'JT}{(1+k)^{1+L}}$$

where a' is the initial allowance and L is the time lag between the year end and the obtaining of the tax benefits of the allowance.

The writing down allowances are a geometric progression, with first term $\frac{aJT}{(1+k)^{1+L}}$

and n terms, where $n = (1-a')/a$.

If there were no initial allowance in the first year, the series of writing down allowances would have n' terms, where $n' = 1/a$.

The incremental value, V_b, from introducing the initial allowance for buildings is therefore

$$V_b = \frac{a'JT}{(1+k)^{1+L}} + \frac{\frac{aJT}{(1+k)^{1+L}}\left[1-\left(\frac{1}{(1+k)}\right)^{n}\right]}{1-1/(1+k)} - \frac{\frac{aJT}{(1+k)^{1+L}}\left[1-\left(\frac{1}{(1+k)}\right)^{n'}\right]}{1-1/(1+k)} \tag{4.7}$$

This simplifies (see *Note 4.1*) to:

$$V_b = \frac{JT}{(1+k)^{1+L}} \left\{ a' + \frac{a}{1-1/(1+k)} \left[(1/(1+k))^n \left((1/(1+k))^{a'/a} - 1 \right) \right] \right\} \tag{4.8}$$

Denoting $1/(1+k)$ by x to simplify the differentiation,

$$V_b = JTx^{1+L} \left\{ a' + \frac{a}{1-x} \left[x^n \left(x^{a'/a} - 1 \right) \right] \right\}$$

$$= JTx^{1+L} \left\{ a' + \left[\frac{ax^{n+(a'/a)} - ax^n}{1-x} \right] \right\}$$

$$\frac{dV_b}{dx} = JTx^{1+L} \left[\frac{(1-x)\left[a(n + a'/a)x^{n+(a'/a)-1} - anx^{n-1} \right] + ax^{n+(a'/a)} - ax^n}{(1-x)^2} \right]$$

$$+ JT(1+L)x^L \left[a' + \frac{ax^{n+(a'/a)} - ax^n}{1-x} \right] \tag{4.9}$$

which simplifies (see *Note 4.2*) to:

$$\frac{dV_b}{dx} = \frac{JTx^{L+n}}{1-x} \left\{ a\left(x^{a'/a} - 1\right)\left(n + \frac{x}{1-x} + 1 + L\right) + a'\left[x^{a'/a} + \frac{(1-x)(1+L)}{x^n} \right] \right\} \tag{4.10}$$

V_b will be at a maximum when $\frac{dV_b}{dx} = 0$, that is, when

$$a\left(x^{a'/a} - 1\right)\left(n + \frac{x}{1-x} + 1 + L\right) + a'\left[x^{a'/a} + \frac{(1-x)(1+L)}{x^n} \right] = 0.$$

The value of k at which V_b is maximised, k_b, is therefore determined by the initial allowance a', the writing down allowance a, and the time lag between the purchase of the asset and the obtaining of tax relief.

Losses

If the firm has losses for y years, so that the tax benefits of the allowances for each of the years where there are losses will be obtained simultaneously in year $y+1$, V_b may be determined by a similar procedure.

The first year's initial allowance, and the writing down allowances relating to the years with tax losses, will be obtained when the firm resumes taxpaying. Their present value will be

$$\frac{a'JT + yaJT}{(1+k)^{y+1+L}}$$

The remaining writing down allowances are a geometric progression, with first term

$$\frac{aJT}{(1+k)^{y+1+L}}$$

and $n-y$ terms, where $n = (1-a')/a$.

The total present value of the allowances is therefore:

$$\frac{a'JT + yaJT}{(1+k)^{y+1+L}} + \frac{\frac{aJT}{(1+k)^{y+1+L}}\left[1-\left(\frac{1}{(1+k)}\right)^{n-y}\right]}{1-1/(1+k)} \qquad \text{(i)}$$

The corresponding expression where there is no initial allowance would be

$$\frac{yaJT}{(1+k)^{y+1+L}}+\frac{\frac{aJT}{(1+k)^{y+1+L}}\left[1-\left(\frac{1}{(1+k)}\right)^{n'-y}\right]}{1-1/(1+k)} \qquad \text{(ii)}$$

Subtracting (ii) from (i) and simplifying gives V_b', the value of the initial allowance where there are losses for y years:

$$V_b'=\frac{JT}{(1+k)^{y+1+L}}\left\{a'+\frac{a}{1-1/(1+k)}\left[(1/(1+k))^{n-y}\left((1/(1+k))^{a'/a}-1\right)\right]\right\}$$

(4.11)

Differentiating as before and denoting $1/(1+k)$ by x,

$$\frac{dV_b'}{dx}=JTx^{y+1+L}\left[\frac{(1-x)\left[a(n-y+a'/a)x^{n-y+(a'/a)-1}-a(n-y)x^{n-y-1}\right]+ax^{n-y+(a'/a)}-ax^{n-y}}{(1-x)^2}\right]$$

$$+JT(y+1+L)x^{L+y}\left[a'+\frac{ax^{n-y+(a'/a)}-ax^{n-y}}{1-x}\right] \qquad (4.12)$$

which simplifies (see *Note 4.3*) to:

$$\frac{dV_b'}{dx}=\frac{JTx^{L+n}}{1-x}\left\{a\left(x^{a'/a}-1\right)\left(n+\frac{x}{1-x}+1+L\right)+a'\left[x^{a'/a}+\frac{(1-x)(y+1+L)x^y}{x^n}\right]\right\}$$

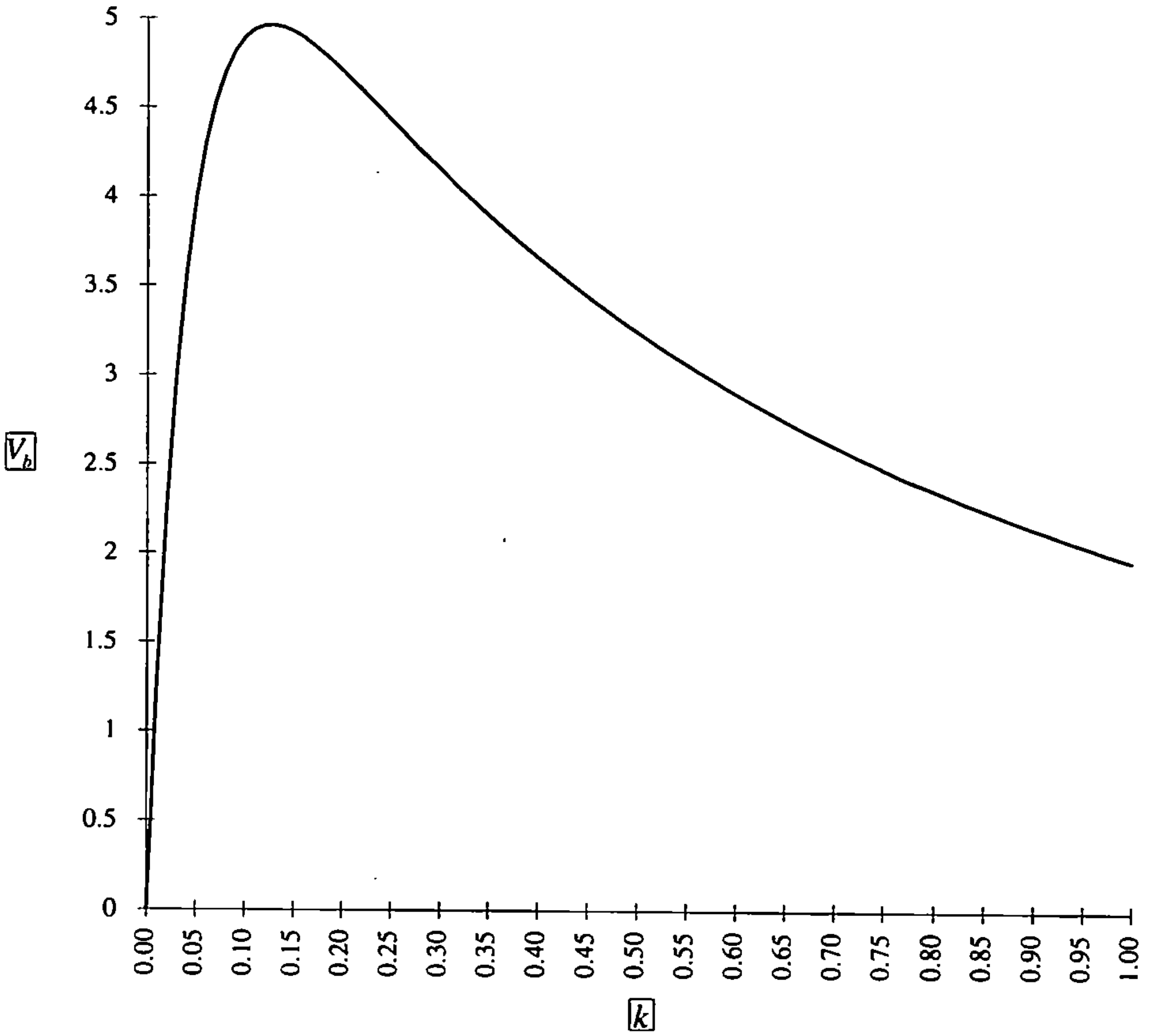
(4.13)

When $y=0$, $V_b'=V_b$ and $\frac{dV_b'}{dx}=\frac{dV_b}{dx}$.

Fig. 7

Graph of Vb against k, with no tax losses

asset cost	=	100
tax rate	=	0.33
IA (%)	=	20
WDA (%)	=	4
tax lag (years)	=	0.75
tax losses (years)	=	0

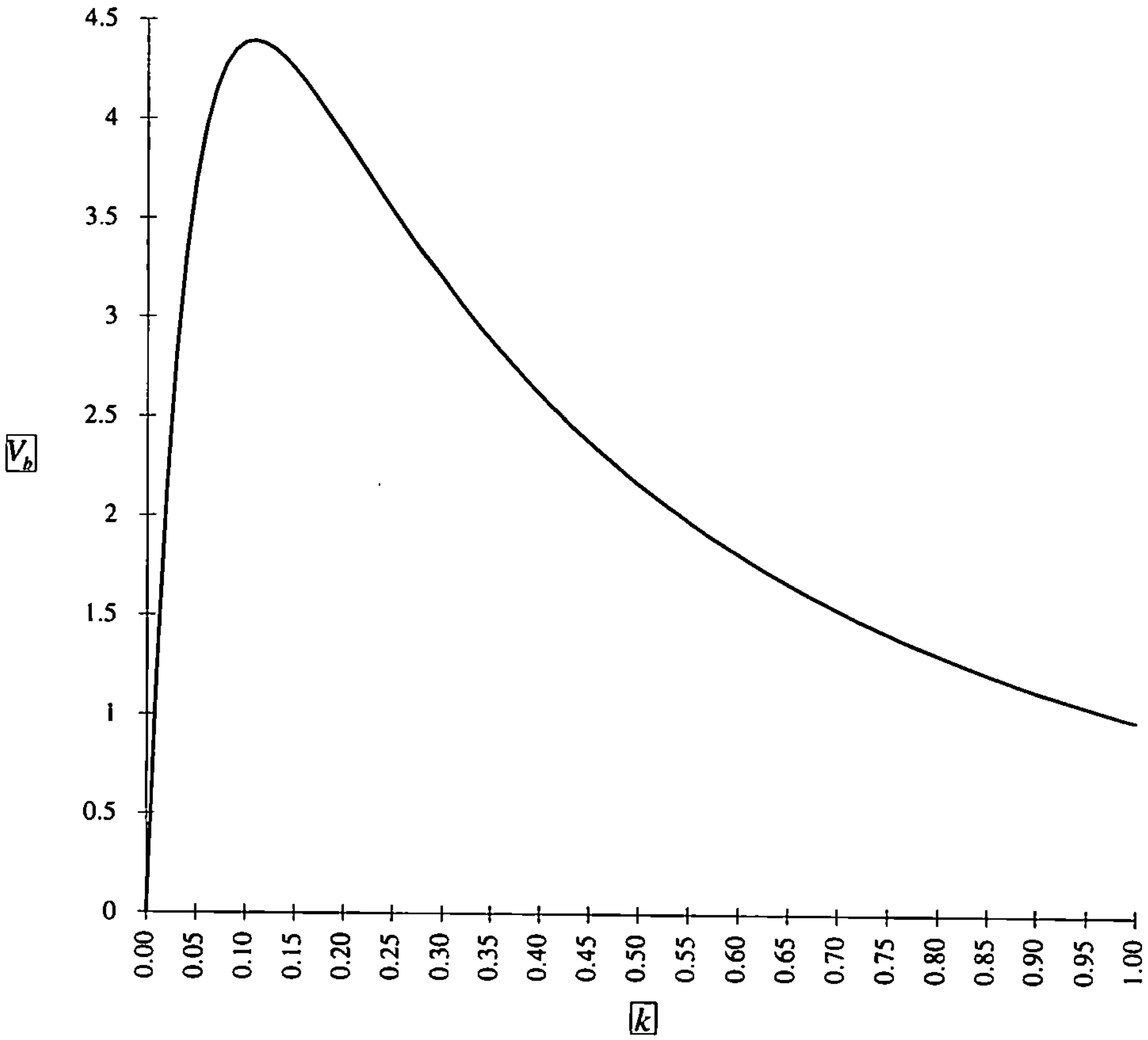


* Length of time between year end and tax due date

Fig. 8

Graph of Vb against k, with one year of tax losses

asset cost	=	100
tax rate	=	0.33
IA (%)	=	20
WDA (%)	=	4
tax lag (years)	=	0.75
tax losses (years)	=	1

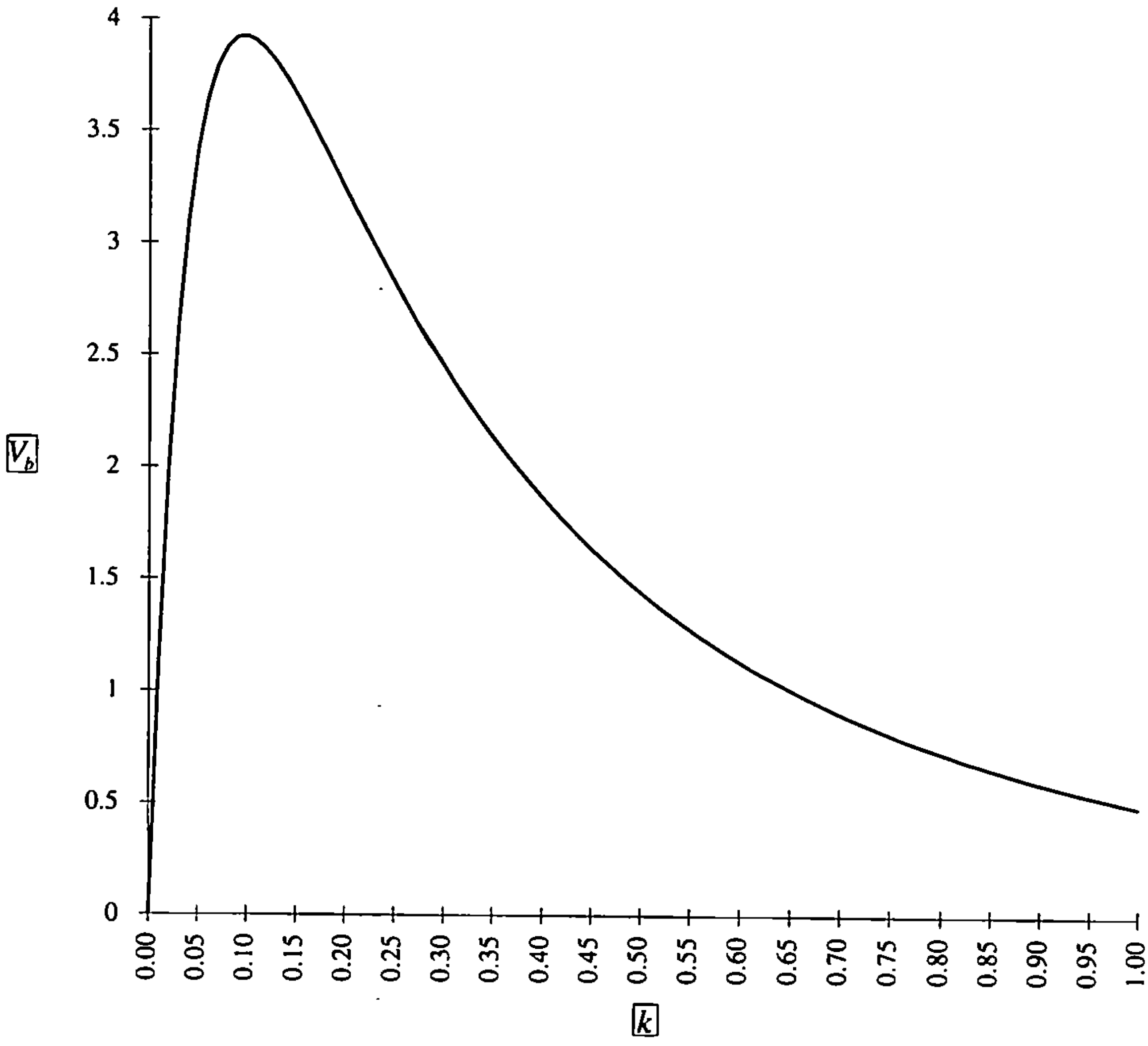


* Length of time between year end and tax due date

Fig. 9

Graph of Vb against k, with two years of tax losses

asset cost	=	100
tax rate	=	0.33
IA (%)	=	20
WDA (%)	=	4
tax lag (years)	=	0.75
tax losses (years)	=	2

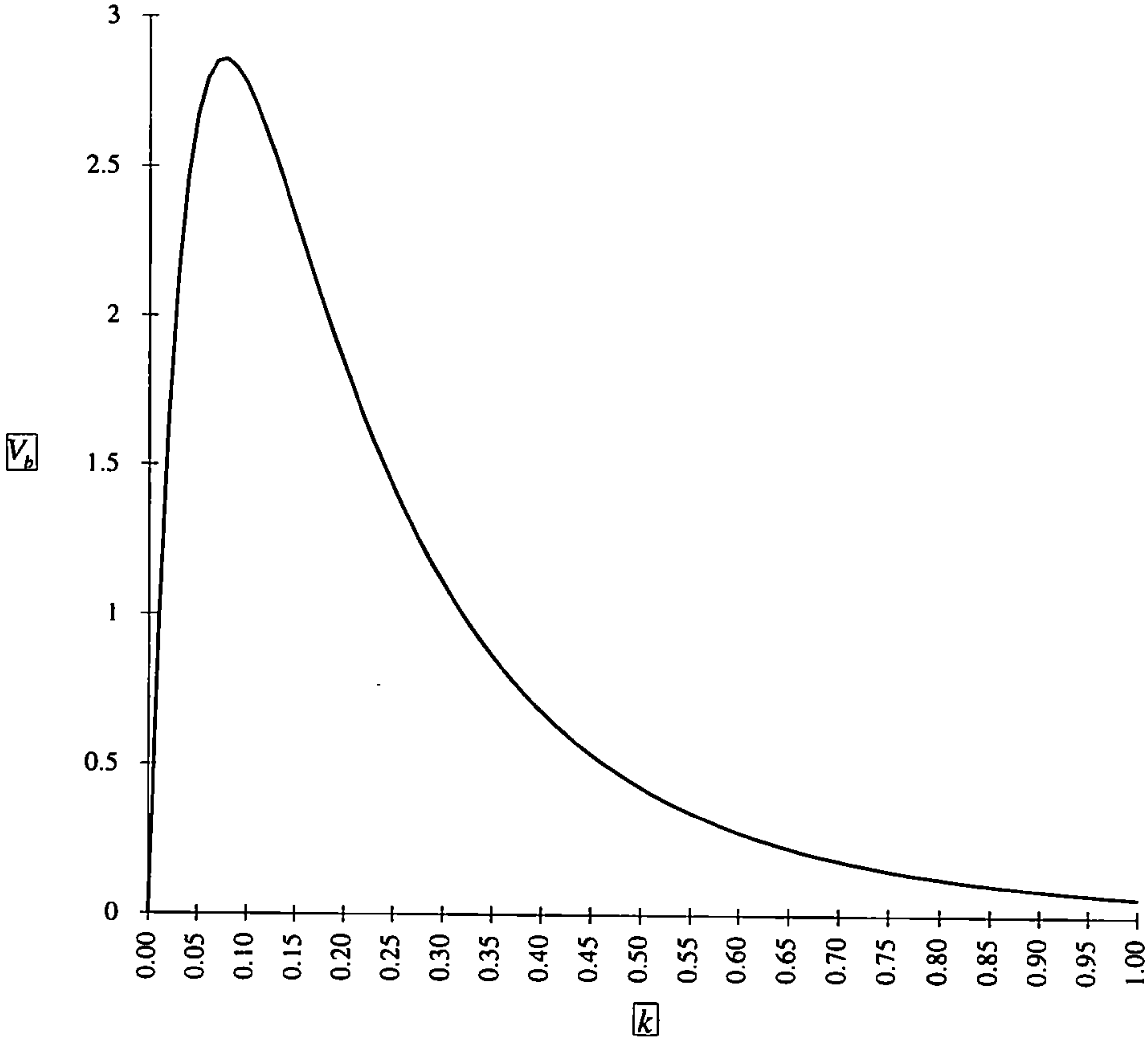


* Length of time between year end and tax due date

Fig. 10

Graph of Vb against k, with five years of tax losses

asset cost	=	100
tax rate	=	0.33
IA (%)	=	20
WDA (%)	=	4
tax lag (years)	=	0.75
tax losses (years)	=	5



* Length of time between year end and tax due date

V_b' will be at a maximum when $\frac{dV_b'}{dx} = 0$, that is, when

$$a\left(x^{a'/a}-1\right)\left(n+\frac{x}{1-x}+1+L\right)+a'\left[x^{a'/a}+\frac{(1-x)(y+1+L)x^{y}}{x^{n}}\right]=0.$$

Table 4.2

Values of k_b obtained under different values for a, a' and L

a	L	a'	y	k_b
0.04	0.75	0.20	0	12.61%
0.04	0.75	0.50	0	14.46%
0.04	0.75	0.10	0	12.12%
0.02	0.75	0.20	0	7.66%
0.10	0.75	0.20	0	23.12%
0.04	0.75	0.20	1	10.82%
0.04	0.75	0.20	2	9.67%
0.04	0.75	0.20	5	7.68%

The position of k_b is more sensitive to the value of a than that of a', and periods of tax losses (where $y > 0$) do not have such a significant effect on the value of k_b as in the corresponding analysis for plant and machinery. Comparison of Figs. 7, 8, 9 and 10 shows that as y increases, V_b' peaks more sharply.

4.7 Summary

The system of tax allowances for depreciation of capital investment has significant implications for the investment decision Tax relief for capital expenditure is imperfect if its present value is less than the cost of the asset, which is normally the case under the UK tax system. Although under proportional taxation it appears that taxes reduce both risk and return in the same proportion, implying a risk sharing by the firm and government, with imperfect capital expenditure relief the rate of return may fall below the CAPM required rate.

In the situation where tax lags exist and a higher proportion of the asset cost may be allowed against tax in the first year than in subsequent years, the author has shown that an optimal cost of capital may be derived at which the present value of the capital expenditure relief is at a maximum. For expenditure on plant and machinery, this optimal cost of capital is independent of the first year allowance but is determined by the subsequent writing down allowance and by the timing of the incidence of tax relief, which may be delayed where the firm has no taxable profits. In the case of buildings, a complex relationship exists between the optimal cost of capital, the initial and subsequent allowances and the timing of relief.

Note 4.1

$$V_b = \frac{a'JT}{(1+k)^{1+L}} + \frac{\frac{aJT}{(1+k)^{1+L}}\left[1-(1/(1+k))^n\right]}{1-1/(1+k)} - \frac{\frac{aJT}{(1+k)^{1+L}}\left[1-(1/(1+k))^{n'}\right]}{1-1/(1+k)} \tag{4.7}$$

$$= \frac{JT}{(1+k)^{1+L}}\left\{a' + \frac{a\left[1-(1/(1+k))^n\right]}{1-1/(1+k)} - \frac{a\left[1-(1/(1+k))^{n'}\right]}{1-1/(1+k)}\right\} \tag{4.7a}$$

Since $n' = n + (a'/a)$,

$$V_b = \frac{JT}{(1+k)^{1+L}}\left\{a' + \frac{a\left[1-(1/(1+k))^n\right]}{1-1/(1+k)} - \frac{a\left[1-(1/(1+k))^{n+(a'/a)}\right]}{1-1/(1+k)}\right\} \tag{4.7b}$$

$$= \frac{JT}{(1+k)^{1+L}}\left\{a' + \frac{a}{1-1/(1+k)}\left[(1/(1+k))^n\left((1/(1+k))^{a'/a}-1\right)\right]\right\}. \tag{4.7c}$$

Note 4.2

$$\frac{dV_b}{dx} = JTx^{1+L}\left[\frac{(1-x)\left[a(n+a'/a)x^{n+(a'/a)-1} - anx^{n-1}\right] + ax^{n+(a'/a)} - ax^n}{(1-x)^2}\right]$$
$$+JT(1+L)x^L\left[a' + \frac{ax^{n+(a'/a)} - ax^n}{(1-x)}\right] \tag{4.9}$$

$$= JTx^L\left\{\frac{x\left[a(n+a'/a)x^{n+(a'/a)-1} - anx^{n-1}\right]}{(1-x)} + \frac{x\left(ax^{n+(a'/a)} - ax^n\right)}{(1-x)^2}\right.$$
$$\left. +(1+L)a' + \frac{(1+L)\left(ax^{n+(a'/a)} - ax^n\right)}{(1-x)}\right\} \tag{4.9a}$$

$$= \frac{JTx^{L}}{(1-x)}\left\{x\left[a(n+a'/a)x^{n+(a'/a)-1} - anx^{n-1}\right] + \frac{x\left(ax^{n+(a'/a)} - ax^{n}\right)}{(1-x)}\right.$$

$$\left. + a'(1-x)(1+L) + (1+L)\left(ax^{n+(a'/a)} - ax^{n}\right)\right\} \tag{4.9b}$$

$$= \frac{JTx^{L+n}}{(1-x)}\left\{anx^{a'/a} + a'x^{a'/a} - an + \frac{ax^{(a'/a)+1}}{(1-x)} - \frac{ax}{(1-x)}\right.$$

$$\left. + \frac{a'(1-x)(1+L)}{x^{n}} + \left(ax^{a'/a} - a\right)(1+L)\right\} \tag{4.9c}$$

$$= \frac{JTx^{L+n}}{(1-x)}\left\{a\left[nx^{a'/a} - n + \frac{x^{(a'/a)+1}}{(1-x)} - \frac{x}{(1-x)} + (1+L)\left(x^{a'/a} - 1\right)\right]\right.$$

$$\left. + \; a'\left[x^{a'/a} + \frac{(1-x)(1+L)}{x^{n}}\right]\right\} \tag{4.9d}$$

$$= \frac{JTx^{L+n}}{(1-x)}\left\{a\left[n\left(x^{a'/a} - 1\right) + \frac{x}{(1-x)}\left(x^{a'/a} - 1\right) + (1+L)\left(x^{a'/a} - 1\right)\right]\right.$$

$$\left. + \; a'\left[x^{a'/a} + \frac{(1-x)(1+L)}{x^{n}}\right]\right\} \tag{4.9e}$$

$$= \frac{JTx^{L+n}}{(1-x)}\left\{a\left(x^{a'/a} - 1\right)\left(n + \frac{x}{(1-x)} + 1 + L\right) + a'\left[x^{a'/a} + \frac{(1-x)(1+L)}{x^{n}}\right]\right\}. \tag{4.9f}$$

Note 4.3

$$\frac{dV_b'}{dx} = JTx^{y+1+L}\left[\frac{(1-x)\left[a(n-y+a'/a)x^{n-y+(a'/a)-1} - a(n-y)x^{n-y-1}\right] + ax^{n-y+(a'/a)} - ax^{n-y}}{(1-x)^2}\right]$$

$$+JT(y+1+L)x^{L+y}\left[a' + \frac{ax^{n-y+(a'/a)} - ax^{n-y}}{1-x}\right] \qquad (4.12)$$

$$= JTx^{L}\left\{\left[\left[\frac{x^{y+1}\left[a(n-y+a'/a)x^{n+(a'/a)} - a(n-y)x^{n-y-1}\right]}{(1-x)}\right]\right] + \frac{ax^{n+(a'/a)+1} - ax^{n+1}}{(1-x)^2}\right.$$

$$\left. +(y+1+L)\left[x^{y}a' + \frac{ax^{n+(a'/a)} - ax^{n}}{1-x}\right]\right\} \qquad (4.12a)$$

$$= \frac{JTx^{L}}{1-x}\left\{a(n-y+a'/a)x^{n+(a'/a)} - a(n-y)x^{n} + \frac{ax^{n+(a'/a)+1} - ax^{n+1}}{(1-x)}\right.$$

$$\left. +(y+1+L)x^{y}a'(1-x) + (y+1+L)\left(ax^{n+(a'/a)} - ax^{n}\right)\right\} \qquad (4.12b)$$

$$= \frac{JTx^{L+n}}{1-x}\left\{a(n-y)x^{a'/a} + a'x^{a'/a} - a(n-y) + \frac{ax^{(a'/a)+1} - ax}{(1-x)}\right.$$

$$\left. + \frac{(y+1+L)x^{y}a'(1-x)}{x^{n}} + (y+1+L)\left(ax^{a'/a} - a\right)\right\} \qquad (4.12c)$$

$$= \frac{JTx^{L+n}}{1-x}\left\{a\left(\frac{x}{1-x}\right)\left(x^{a'/a} - 1\right) + a(y+1+L)\left(x^{a'/a} - 1\right) + a(n-y)\left(x^{a'/a} - 1\right)\right.$$

$$\left. +a'x^{a'/a} + \frac{a'(1-x)(y+1+L)x^{y}}{x^{n}}\right\} \qquad (4.12d)$$

$$= \frac{JTx^{L+n}}{1-x}\left\{a\left(x^{a'/a}-1\right)\left(n+\frac{x}{(1-x)}+y+1+L\right)+a'\left(x^{a'/a}+\frac{(1-x)(y+1+L)x^{y}}{x^{n}}\right)\right\}. \tag{4.12e}$$

Chapter 5

Tax and optimal financing

5.1 Introduction

This chapter will examine the issues concerning the optimal financing decision. The literature has analysed the implications of agency theory on the costs of equity and debt, and these will be reviewed briefly before discussing the tax implications in more detail. The tax deductibility of debt interest payments increases the attractiveness of debt financing. However, in the UK the imputation system affects the cost of equity finance. But the asymmetrical treatment of tax losses can increase the cost of both debt and equity finance. A further complexity is the availability of finance leasing as an alternative to debt, which carries similar obligations but leads to a different pattern of cash flows and tax relief over time. In this chapter, the benefits of finance leasing and methods of lease evaluation will be reviewed in the context of the UK tax system.

5.2 Optimal financing and shareholder wealth

There are many theories of capital structure which exclude tax: for instance, agency costs, asymmetric information, product/input market considerations, and corporate control considerations (Harris and Raviv, 1991). The separation of ownership and control of firms, with delegation of decision-making from the shareholders to managers, has implications for the costs of the firm's sources of finance independently of taxation. These will be reviewed below, prior to a discussion of the tax-related issues.

5.2.1 Informational asymmetries

Managers, shareholders and debtholders may not have equal access to information about the firm's opportunities and future cash flows.

Informational asymmetry between managers and shareholders

This situation occurs where managers are better informed than shareholders about their efforts on the shareholders' behalf and about their ability to run the firm. Jensen and Meckling (1976) identified a 'moral hazard' agency cost of equity, that is, the cost of effort aversion and the tendency to consume perks by managers. Managers will attempt to build a reputation for high ability (Hirshleifer, 1993) which has a number of consequences for the wealth-maximising shareholder. There will be an incentive to pursue safe projects, instead of riskier ones which shareholders might prefer. A manager may follow decisions made by other managers rather than acting optimally for the firm based on his or her own information; if the decision is wrong, the manager avoids appearing foolish in isolation. If a manager plans to move from the firm, he or she will choose projects with high cash flows in early years over those with a higher NPV.

Managerial reputation-seeking may be a reason to favour a centralised approach to capital budgeting, as implied in the optimisation model presented in this thesis.

Informational asymmetry and optimal gearing

If shareholders have imperfect information about future cash flows, higher debt ratios may signal management confidence and so conversely an equity issue may be a negative signal that reduces share price (Ross, 1977; Miller and Rock, 1985; Narayanan, 1988). An empirical study by Kim, Chen and Nance (1992) of the information content of gearing showed evidence consistent with the existence of optimal gearing ratios. The market reacts to changes in optimal leverage and the direction of the reaction depends on the position of the gearing relative to its optimal level.

Because their own income depends on their role in the firm, managers may have an incentive to continue the firm's operations beyond the stage where liquidation might be preferred by shareholders. On the other hand, debtholders may be able to force liquidation. This can result in an optimal capital structure as improved liquidation decisions are traded off against higher investigation costs (Harris and Raviv, 1990). Debt payments can reduce free cash flow (cash flow in excess of that required to fund all available projects with a positive NPV), thus reducing the conflict of interest between shareholders and managers over payout policies (Stulz, 1990).

As a borrowing firm achieves a good reputation from its debt repayment history, its cost of borrowing should fall and there should be an incentive for the firm to choose safe projects where debt repayment is assured. Younger firms, conversely, may have an incentive to choose riskier projects (Diamond, 1989).

If asymmetric information means that only insiders know the quality of the firm, debt is preferable to equity. Debt can keep unprofitable firms out of the market, which enhances the average quality of firms in the market. Any project financed with equity will therefore be seen as a 'lemon' (Narayanan, 1988).

Myers and Majluf (1984), assuming insiders have more knowledge than sharheolders about the value of assets, argue that the market may misprice equity. Underpricing of an equity issue means that new investors would gain more from the project's NPV than if the equity issue were accurately priced, while existing shareholders lose out. Internal funds and low risk debt will be preferred to equity, in that order. However, Brennan and Kraus (1987) and Constantinides and Grundy (1989) showed that if the firm is assumed to have a choice of financing strategies, such as debt repurchase or the issue of securities which are neither straight debt nor equity, Myers and Majluf's analysis is not always valid.

Conflicts between debtholders and equity holders

A corporate limited liability borrower will pay more interest as debt becomes more risky, but may default on the loan if it becomes necessary. A lender receives more interest, accepting that there is an increased risk of default as the borrower's gearing level increases. If it can be assumed that there are no additional costs associated with default, risky debt is a zero-sum game which does not change the value of the firm and maximising firm value is equivalent to maximising the value of equity. In practice, however, there are additional costs such as the anticipated costs of liquidation, monitoring costs and 'moral hazard' which lenders may incorporate into interest rates. The holders of debt and equity do not have exactly the same interests in the firm, which implies that maximising the value of the sum of the firm's debt and equity is not achieved by simply trading off tax advantages and the expected costs of financial distress. The optimal capital structure is likely to vary from one firm to the next, depending on its overall characteristics.

If managers act as agents for the shareholders, with decision-making authority delegated from the shareholders, and are aiming to maximise shareholder wealth, then if maximising shareholder wealth and maximising the value of the firm are not the same, the managers' decisions may benefit shareholders at the expense of debt holders. Debt holders may therefore seek to monitor the firm's actions and to impose debt covenants, which represent costs to the firm.

If the firm is financed by equity and risky debt and the holders of debt and equity finance do not have equal access to information aboiut the firm's investment and financing decisions, there are implications for the firm's project decisions. The equity may be viewed as a call option on the firm's total assets (Jensen and Meckling, 1976). Jensen and Meckling argued that if the firm is predominantly financed by debt, managers have an incentive to undertake investments which promise very high pay-offs if successful, even if there is a low probability of success. The shareholders gain high rewards if the risky projects succeed. In the event of failure, limited liability ensures that the shareholders lose only the small amount of equity funds they have provided, while debtholders' claims are reduced by bankruptcy. Suboptimal investments cause a loss in the value of the firm's equity, but this is more than offset for the shareholders by their gain at the debtholders' expense. This 'asset substitution' effect will be anticipated by creditors and reflected in the cost of debt.

Risky debt can also cause 'underinvestment moral hazard'. Myers (1977) shows that in some states of nature, a firm financed with risky debt will not undertake investment opportunities which could increase the value of the firm. The resulting loss in market value is borne by the shareholders.

5.2.2 Debt and equity finance and taxation

Equity financing

The issue of why firms pay dividends to the suppliers of equity finance has been the subject of much analysis and debate. In perfect markets with no personal taxes, it may be shown (Miller and Modigliani, 1961) that dividends and capital gains are valued equally and so dividend policy is irrelevant. But taxes, transaction costs and other market imperfections such as information effects may all modify this conclusion.

The relative valuation of dividends and capital gains

If tax payments are assumed to be immediate, and if £1 of earnings retained by the firm is assumed to have the same value to the investor before personal taxes as £1 of gross dividend paid out by the firm, then the conditions under which dividends are preferred to capital gains may be derived.

Under the classical system, shareholders prefer dividends to capital gains so long as $m > z$, where m is the marginal rate of income tax and z is the rate of capital gains tax. Under the imputation system, ignoring the possibility of tax exhaustion, dividends will be preferred to capital gains if $(1-m)/(1-b) > (1-z)$. A shareholder with marginal tax rate m, where $m < b+(1-b)z$, should prefer dividend income. Ashton (1991) pointed out that in the UK there are many groups of investors who fall into this category: for instance, for pension funds which are tax exempt $m = z = 0$ and for life assurance companies, $m = b$ and $z > 0$. Investment trusts and unit trusts with $m = b$ and $z = 0$ are neutral, with the preference for either income or capital gains depending on the tax position of the fund subscribers. For individual shareholders, $m = z$ which should lead to a preference for dividends. However, under the UK tax system there are two additional factors which would tend to make capital gains more attractive: there is an annual threshold below which no capital gains tax is payable, and indexation of capital gains for inflation means that only real gains are taxed. Individuals for whom $m > b$ and $z = 0$, would therefore have a preference for capital gains.

Rational shareholders with high income tax and low capital gains tax would be expected to invest in high growth companies, and those with low income tax and high capital gains tax should tend to hold shares of low growth firms with high dividend payout. If these tax clienteles exist, firms should be concerned to take into account their shareholders' tax arrangements when setting dividend policy. Tax clienteles of investors would require the firm to pursue a stable ratio of dividend payments to retentions. If transaction costs are greater than zero, this action will be consistent with the maximisation of shareholder wealth criterion.

If shareholders rationally capitalise their dividend receipts on an after personal tax basis, the price behaviour of a share around the time of dividend payment (ratio of the cum-div to ex-div share price change to the net dividend) should be a well defined function of the shareholders' assessment of their personal tax brackets, giving support for the existence of tax clienteles of investors. Empirical evidence is conflicting in the UK. Davidson (1989) and Davidson and Mallin (1989) concluded that there was only weak evidence to reject the view that shareholders value dividends independently of their personal tax liability. However, Crossland, Dempsey and Moizer (1991) found a systematic correlation between the price/earnings ratio of shares and the ratio of the cum-div to ex-div share price change to the net dividend. The issue is complicated by, for example, tax avoidance trading; shareholders may be able to optimise their tax positions by taking a buying or selling position on a cum-div share, which would tend to depress the observed tax clientele effect.

Agency costs may be another determinant of dividend policy. Managers acting as agents for the shareholders of profitable firms may have the opportunity to spend cash on perquisites rather than paying it to the shareholders as dividends. The payment of dividends may act as reassurance to the market that managers are acting in the best interests of the shareholders (Jensen, 1986). In the absence of taxes, dividend payments may reduce agency costs (Easterbrook, 1984). By maintaining a constant payment of dividends, equity funds do not accumulate and the firm is forced to seek external finance, causing a periodic review of the firm's activities by lenders, which results in reduced moral hazard to shareholders. Against this must be traded off the increased transaction costs. This theory may provide an insight into why firms pay dividends and raise external finance at the same time.

Also, the separation of ownership and control leads to an informational asymmetry regarding estimates of future earnings. Managers may not wish to disclose information about planned investments in order to maintain a competitive market position. In this situation, dividends may act as a signal from management to shareholders about future cash flows (Healy and Palepu, 1988). If dividends act as a signalling mechanism, the market should react to unexpected changes in dividends. Empirical evidence indicates that the market does react to unexpected dividend changes, and that it reacts more strongly to unexpected dividend cuts than to increases (Marsh, 1992). This is consistent with recent empirical findings (Green, 1993) that UK managers believe investors require a steadily growing dividend stream, rather than a constant dividend payout ratio.

Debt financing

The use of debt by firms carries the advantage that debt interest payments are tax deductible. However, debt financing increases the level of risk of a firm's equity returns because debt interest charges must be paid whatever the level of profits in any year, increasing the variability of the residual flows to shareholders. Risk-averse shareholders will require a higher rate of return on their equity to compensate them for the extra risk. The total cost of debt consists of the payments of interest to providers of finance, and the additional return required by shareholders to compensate for the increased risk of the earnings stream. These costs must be set against the benefits of tax deductibility of debt interest payments.

The 'traditional' view of capital structure assumes that as the firm's proportion of cheaper debt finance increases, over moderate ranges, so the average cost of capital falls. At higher proportions of debt, the equity holders demand a higher rate of return to compensate for the increased risk of their investment and lenders will also expect a higher rate of return on successive increases of debt. This increasing cost of both debt and equity eventually outweighs the advantage of debt finance and the overall cost of capital begins to rise. An optimal level of gearing therefore exists for each firm at which the overall cost of capital is at a minimum. However, this view implies that investors would accept the same rate of return from otherwise similar firms with different levels of gearing, implying shareholder irrationality.

Debt and taxes in equilibrium

Under a restrictive set of assumptions, Modigliani and Miller (1958) showed that in a world without taxes, the total market value of a firm's securities is unchanged by its level of borrowing. The weighted average cost of capital does not change as the amount of debt increases, but the return on equity is equal to the expected return on the equivalent ungeared firm plus a premium which is a linear function of the amount of debt. In Modigliani and Miller's later (1963) model corporation tax was correctly introduced into the framework, demonstrating that the value of the geared firm is equal to the value of the equivalent ungeared firm plus the tax shield resulting from debt interest deductibility. The higher returns required by equity holders are more than compensated by the tax shield, so that as the proportion of debt funding increases, the weighted average cost of capital decreases. The MM analysis assumes a perfect capital market where personal taxes are either non-existent or equal for both debt and equity income, all cash flows and amounts of debt borrowed are perpetuities, and firms expect to be able to obtain the tax benefits of debt financing in full.

Within an MM framework, a tax advantage to debt can exist only if the tax deductibility of debt interest does not increase total corporate borrowing. Miller (1977) argued that any tax subsidy on debt interest would lead to greater borrowing by corporations and result in an increase in interest rates up to the point where the after-tax cost of debt to firms would equal the cost of equity. For the tax exhausted firm, the analysis implies that debt will be more expensive than equity after tax, since the two costs are equal when tax deductibility of debt finance is taken into account. In Miller's equilibrium there is no tax advantage in corporate borrowing from the firm's point of view provided bankruptcy costs are assumed to be zero, although lenders with marginal tax rates below the corporate tax rate will benefit from a tax subsidy on debt. While capital structure is irrelevant for the individual firm an optimal debt/equity ratio does exist for the economy as a whole, so that the quantity of corporate debt issued increases as the corporate tax rate increases. Miller's analysis implies the specialisation of investors in firms following particular gearing strategies, with firms with lower gearing levels attracting investors with higher marginal tax rates. If shareholders are assumed to trade off their preference for debt or equity for tax reasons against the need to reduce non-systematic risk by diversification, then firms in any particular industry would have an incentive to select extreme capital structures to suit the needs of differing tax clienteles (for instance, Auerbach and King (1983)).

If it is assumed that debt interest is tax-deductible, an expression derived by Miller (1977) incorporates both the MM and Miller models:

$$V_g = V_u + D\left[1 - \frac{(1-T_c)(1-T_e)}{(1-T_d)}\right] \tag{5.1}$$

where V_g = the market value of the geared firm

V_u = the market value of the equivalent ungeared firm

D = amount of debt

T_c = corporate tax rate

T_e = personal tax rate on equity income

T_d = personal tax rate on debt income.

In a Modigliani and Miller world T_e is effectively the same as T_d and the model above becomes $V_g = V_u + T_c D$.

If it is assumed that the operating cash flows of the geared firm remain the same as those of the ungeared firm, then it follows that the benefits from the tax advantage to debt must be paid out immediately to equity holders in the form of increased dividends and T_e is the marginal rate of personal tax on dividend income. If the additional cash flow is assumed to be retained by the firm, then T_e must also take capital gains tax into account.

Tax-induced interactions between optimal financing and dividend payments

The analyses of MM and Miller were based on the assumption of a classical system of dividend taxation, where residual corporate profits after corporate taxes are taxed again at the shareholder's marginal rate on distribution. Retained profits will be subject to the tax on the shareholder's capital gains.

In the UK, the imputation tax system creates interdependencies between personal taxes and optimal financing (for instance, King, 1977; Ashton, 1989a, 1989b). For a firm which is fully taxpaying and can set off its ACT with the minimum delay, dividend policy is irrelevant if the marginal rate of personal tax on dividend income is the same as the capital gains tax rate z:

$$1 - T_e = 1 - z = (1 - m) / (1 - b) \tag{5.2}$$

In a general equilibrium model investors will adjust their portfolios and firms their capital structure until the tax advantage of debt disappears. If returns to shareholders consist only of capital gains, the gain from gearing occurs until $1 - T_d = (1 - T_c)(1 - T_e)$, where $T_e = z$. Hence $T_d = 1 - (1 - T_c)(1 - z)$. If dividends are paid, then under the imputation system, $1 - T_e = (1 - m)/(1 - b)$. If returns to shareholders comprise both dividends and capital gains, there is a gain from gearing until $(1 - T_d) = (1 - T_c)(1 - T_e)$, or, $(1 - T_c)(1 - z) = (1 - T_c)(1 - m)/(1 - b)$, that is, where $m = 1 - (1 - b)(1 - z)$. (for instance, Davis and Pointon, 1984, chapter 12).

The relative advantage of debt financing under the imputation system is less than it would be under the corresponding classical system. For instance, Ashton (1991) examined the value of the geared firm with a constant perpetual earnings stream and a constant level of debt under the imputation system, assuming that capital gains and dividends are taxed at the same rate and that the debt interest tax shields are uncertain and should be discounted at a rate that reflects this risk. He concluded that the tax advantage to debt in the UK is quite small (about 7.5 per cent of its market value under 1994 tax rates). As might be expected, the imputation tax system appears to increase the relative attractiveness of equity financing. Dempsey (1991) extended Ashton's analysis, showing that the market 'spread' between borrowing and lending rates, also reduces the value of the debt interest tax shield.

Where the firm is not perpetually in a full taxpaying position, the relationship between optimal financing and the imputation system becomes complex. Surplus ACT will increase the effective cost of paying a dividend, reducing the attractiveness of equity financing. On the other hand, the after tax cost of debt will be increased, since the present value of the tax savings from interest deductibility will be reduced as the unrelieved interest is carried forward. The firm's ability to set off its ACT may be affected by the level of debt interest payments, which reduces the mainstream corporation tax liability. Restricted setoff of ACT therefore creates an interaction between the investment, financing and dividend decisions, so that a complex relationship exists between the value of the geared and ungeared firms under the imputation system (Pointon, 1981a).

5.3 The asymmetrical treatment of tax losses

Taxable profits and tax losses are not treated symmetrically by the tax system where there is imperfect relief for losses. This imperfection has been studied in terms of optimal debt financing and in an options framework.

5.3.1 Debt interest deductibility and optimal debt financing

The MM model assumes that the firm's debt generates a perpetual, level and certain stream of interest 'tax shields', implying that the firm will always be in a position to obtain the benefit of the interest deductions and that the tax rate will remain constant. If these conditions apply then it is appropriate to value the debt interest 'tax shields' at the debt interest rate.

If these assumptions are relaxed, the benefit of debt interest deductibility is less certain. In an asymmetrical tax system which does not provide immediate and costless relief for tax losses, the effective tax value of the debt 'tax shield' will be reduced. Under the UK tax system, negative taxable capacity may occur as a result of substitute tax shields such as capital allowances, or losses brought forward or claimed from other companies in a group structure. These are all deducted before debt interest in determining taxable profit. In a corporate group structure it may be possible to transfer the excess debt interest to another group member with sufficient profits to obtain relief at the earliest tax payment date and at minimal cost. If this is not possible, the carry back and carry forward provisions will reduce the impact of tax shield substitutes on the financing decision. But it is not possible to carry back the interest on long term debt to set against previous years' profits.

Although the carry forward provisions allow excess debt tax shields to be set against future income if the firm has insufficient profits to utilise them fully, their present value will be reduced because of the time value of money and the uncertainty over the timing of the eventual setoff. The timing of setoff and the rate at which it occurs will be partly determined by the existence of other tax shields such as capital allowances, so that the investment and financing decisions interact. Where tax rates are not constant but are determined by the level of taxable profits and other tax deductible allowances, there is additional uncertainty over the eventual value of the debt interest tax shield in future periods. Future optimal financing decisions will also be affected by the carry forward of any tax shields, since it is less likely that the debt tax shields arising in those future periods will be set off immediately. The implication is that the amount of debt which the firm optimally issues will be reduced, and an optimal capital structure may therefore exist for each firm (DeAngelo and Masulis, 1980). In a UK context, Mayer (1986) also showed that an optimal level of gearing can occur when the firm has losses carried forward, which depends on the relationship between the firm's expected future profits and tax allowances.

The value of the debt interest tax shield therefore depends on the firm's present and future investment and financing decisions, but optimal investment and financing decisions can only be made with a knowledge of the tax effects of those decisions. A suitable technique for optimal investment and financing decisions under taxation should be capable of considering these decisions simultaneously. As Cooper and Franks (1983) commented:

'The effective tax rate for the firm with tax losses will be less than the full tax rate and is endogenous to the firm's current and future set of real assets and financial transactions; the latter include financial leasing, acquisitions and lending decisions. As a result, the value of any asset can only be calculated simultaneously with the firm's optimal choice of both real assets and financial transactions.'

5.3.2 Tax losses and options theory

The asymmetric tax provisions for losses arising as a result of debt or other tax shields have also been studied in an options framework. Green and Talmor (1985) show that the asymmetric nature of corporate tax influences a firm's investment choice. Because corporate tax payments are analogous to a call option on the firm's earnings, firm owners will sometimes choose less risky investments than they would under symmetric taxation.

Contingent claims analysis considers option-like cash flows that are non-linear functions of the values of some 'underlying' assets, and these values are assumed to be observable. Relative to those values, the value of a claim to the option-like cash flow depends on its total variability (Black and Scholes, 1973). The underlying asset could be regarded as the project or set of future cash flows of the firm as a whole, and the claim to be valued could be the tax claim on the company. Ball and Bowers (1982) applied the contingent claims approach to show the effect of imperfect loss offset under uncertainty on the value of a project and Majd and Myers (1987) applied it to firms in different tax positions. Cheung and Heaney (1993) examine the tax shield risk jointly with the tax asymmetry in an option pricing framework, and conclude that the tax shield risk (tax uncertainty) increases the disincentive of the tax asymmetry.

An asymmetrical tax claim is analogous to the value of a call option. For a price-taking firm, if profit is given by $(PQ-C)$ where P is an uncertain output price, Q is a certain quantity, and C is a certain cost, the tax payable at rate t (assuming t to be certain) is then

$$T = t\max[0, PQ - C] \tag{5.3}$$

This is analogous to the value of a call option on PQ, with exercise price C, at the date the option expires. At an earlier date, when P is seen as uncertain, option pricing theory gives the value of the claim to this non-linear function of P (Lund, 1992). Since the government holds a call option and is protected against downside risk, the value of its claim increases with the uncertainty about P. If the firm is not a price-taker, uncertainty about Q will give a similar result.

The complexity of real tax systems is, however, difficult to capture in this type of analysis. Where losses can be carried back or forwards the asymmetry is reduced, but not eliminated. If there is only one uncertain variable, the value of a linear tax claim is the sum of the values of the expected claims for all periods conditional on a risk-adjusted process for the uncertain variable, discounted at a risk free rate, but for non-linear tax claims the correct risk adjustment is specific to each tax claim and has no simple relationship to the uncertain variable (Lund, 1992). Numerical methods can be employed: for instance, Lund used Monte Carlo simulation to analyse the incentive effects of petroleum taxes under uncertainty. With more than one uncertain variable, the value of a complex tax claim would be complicated even further.

5.4 Debt financing and project appraisal

A number of methods may be used to incorporate the effects of debt financing into investment appraisal.

5.4.1 The weighted average cost of capital

The simplest method of incorporating the tax effects of debt finance into project appraisal is to discount the project's cash flows at the weighted average cost of capital (WACC). This is given by the sum of the after tax cost of debt, and the after tax cost of equity, multiplied by their respective proportions in the firm's capital structure:

$$WACC = r_d(1-T_c)D/V + r_e E/V \tag{5.4}$$

where

r_e is the cost of equity capital after all taxes

r_d is the pre-tax cost of debt

T_c is the corporation tax rate

D/V, E/V are the proportions of debt and equity by market value.

Market values should be used because a constant discount rate implies a constant capital structure related to the present market value of its assets (Miles and Ezzell, 1980). The existence of a WACC which is constant over time also requires the assumption that both the financial and the systematic risk of the firm remain constant. This condition will be met if the market value of a firm's debt is a fixed proportion of the total value of the firm. For project appraisal, the standard approach is that WACC may be used to evaluate an incremental project which is financed in the same proportions as the firm as a whole and has the same risk.

In a Miller world of certainty, the after tax cost of debt equals the cost of equity and there is no advantage to debt financing when interest charges are tax deductible. So the WACC should be the same as the ungeared cost of equity.

If the firm is permanently taxpaying at the full rate, and debt capacity is a constant proportion of the project's present value, and the project is of the same systematic risk as the firm's existing assets, then the weighted average cost of capital may be appropriate. But a constant discount rate is not appropriate where the firm is in a temporary non-taxpaying situation where relief for debt interest and other tax shields will be carried forward to a future period, since this will affect the cost of debt. Where ACT setoff restrictions lead to surplus ACT being carried forward, the cost of equity will also be affected. The use of the WACC in complex tax situations may therefore lead to suboptimal investment decisions.

5.4.2 Valuation of debt interest tax shields

There are two main approaches in the literature to valuing debt interest shields. The first involves evaluating the present value of the tax shields separately and adding it to the value of the firm or the incremental project being appraised. In perfect capital markets all the effects of the financing decision are captured in the tax shield created by debt financing, and under this assumption Modigliani and Miller show that the value of a geared cash flow stream is given by the sum of two present values, one representing the investment decision and the other the effects of the debt financing. This formed the basis of the 'Adjusted Present Value' approach of Myers (1974).

The second method is to derive the present value of a project by discounting the expected cash flows after corporation tax at an 'adjusted discount rate' which takes into account the effects of the debt interest tax shield. Much of the work in this area has been concerned with the incorporation of risk using the Capital Asset Pricing Model. Another aspect of the literature relaxes the MM assumption of perpetual debt levels to incorporate an active debt management policy.

The Adjusted Present Value method

The MM model requires the restriction that the firm is perpetually financed by a prespecified level of debt and that investors realise that this is so. Assuming an MM world of debt and taxes, Myers (1974) derived a formula for project evaluation in which the tax benefits of debt financing are valued separately from the operating cash flows. The relevant discount rate for the operating cash flows is then the ungeared cost of equity. Myers defines debt capacity as a proportion of market value, which in turn must reflect the benefits of debt financing. The increased market value as a result of the tax benefits of debt permits an increase in debt capacity, which increases the tax benefit of debt financing and in turn increases the firm's market value, until convergence is reached.

The main advantage of Myers' method is that the timing of the tax payments is treated explicitly in the cash flows rather than implicitly in the discount rate. A disadvantage is that the timing of the cash flows needs to be determined exogenously. In the context of the firm as a whole, the timing of the tax relief on debt interest payments, or any other tax shield such as capital allowances, will depend on the interaction between the project's cash flows and those of the firm's other activities.

The adjusted discount rate approach

If it is assumed that debt and equity income are subject to personal tax at the same rate, or zero, and that the firm is perpetually financed by a prespecified level of debt, then in a MM equilibrium the risk adjusted discount rate (RADR) is given by

$$RADR = r_u(1 - T_c L) \tag{5.5}$$

where L is the project's incremental debt capacity as a proportion of its present value. Miles and Ezzell (1980) made the same assumptions about taxation but assumed that the firm is financed with a constant proportion of debt and developed a discount rate for project valuation, adjusted for tax and risk:

$$RADR = r_u - T_c r_d L\left[\frac{1+r_u}{1+r_d}\right] \tag{5.6}$$

Each of the above approaches requires the specification of an appropriate cost of equity, and the literature in this area has focused on the application of the market-based Capital Asset Pricing Model to the MM and Miller equilibria. The observed beta will reflect the financial risk of the firm's capital structure, and unless the project will be funded in exactly the same proportions as the existing assets of the firm or proxy firm, an adjustment must be made to degear the equity beta for the financial risk component (for instance, Buckley, 1981; Schnabel, 1983). The ungeared beta must then be regeared to reflect the specific financing of the project.

In a Miller equilibrium, capital structure is irrelevant for the individual firm and so the values of the geared and equivalent ungeared firms are equal, as are their corresponding betas. The MM equilibrium is more complex. The MM assumption of a constant, perpetual tax shield whose value is certain is consistent with a 'passive debt management policy' or PDMP. The level of debt in all future periods, relative to the value of the future cash flows, is determined by the firm's expectations at the start and is not revised as expectations change.

An active debt management policy (ADMP) is the other extreme to a PDMP. The firm's debt is monitored and rebalanced each period as expectations of future cash flows are revised, so that the ratio of debt to the firm's opening realised market value is kept constant. This may be achieved by issuing debt, debt redemptions or share repurchases. In practice, firms must balance the issue costs incurred when they adjust their debt/equity ratios against the costs of deviating from a target capital structure.

Since under an ADMP the level of debt is known one period in advance, the one period discount rate for the interest tax shields will be the cost of debt. Under the assumption of an ADMP and an MM equilibrium, Lewellen and Emery (1986) valued the future stream of debt interest payments as two components: the certain tax shield for the first period, and the value of a the perpetual stream of subsequent future tax shields. Each of these is discounted at the cost of debt over the preceding period. But the risk of the future tax shields is given by the ungeared cost of equity, since Lewellen and Emery assumed that they have the same risk as the ungeared after-tax cash flows.

The logic of this approach is derived from the argument of Miles and Ezzell (1980) that the tax benefits of debt are related to the firm's earnings in the sense that a fall in earnings reduces the firm's debt capacity and the value of the associated debt interest tax shields. Therefore, the risk of the tax shields is the same as the risk of the operating cash flows. But this argument relies on the assumption that the firm expects to be profitable enough to pay tax in each period. If this assumption is relaxed then the alternative must be to assume that the tax system is symmetrical, with losses giving rise to rebates. If this is not so then the tax shields cannot have the same risk as the cash flows. This problem also applies to the subsequent extensions to Lewellen and Emery's work to incorporate personal taxes (Appleyard and Strong, 1989; Clubb and Doran, 1991, 1992).

5.4.3 Equivalence of methods of financing appraisal

Strong and Appleyard (1992) examine the extent to which the WACC subsumes all other approaches to investment appraisal under various capital structure equilibria.

They found that: in a Miller equilibrium, the WACC formula gives the same valuation as the APV or RADR approaches; in an intermediate or MM equilibrium, the APV and (MM) RADR are equivalent to the WACC if a passive debt management policy is assumed; and under an active debt management policy, use of the WACC, 'appropriately interpreted' gives an identical valuation to the APV or Miles-Ezzell RADR.

Strong and Appleyard argue that the WACC may therefore be applied regardless of the assumed capital structure equilibrium or corporate and personal tax regime:

'Observed equity and debt returns impound the ruling tax regime and consequent capital structure whatever this may be. The observed returns are already before personal tax rates, and in the case of debt, before corporate tax as well. They can therefore be substituted directly into the WACC as stated above. If financing side-effects other than the debt interest tax shields exist, these may be added on separately.'

However, in complex tax situations a discount rate derived from observed equity and debt returns may not be appropriate for project appraisal, because of the tax effects created by cash flow interactions between the incremental and existing activities. For instance, large capital allowances from new investments may create delays in the setoff of debt interest and/or ACT. This will affect the costs of the firm's sources of finance.

5.5 Taxation and finance leasing

In a leasing contract, the lessor buys the item of capital equipment and claims the tax benefits of the capital allowances available. Some of the tax benefits may be shared with the lessee by reducing the lease payments. The rental payments compensate the lessor for the capital cost, the time value of money and the risk of default by the lessee.

In the UK the normal situation is that the lessor buys the equipment which is then rented to the lessee over an initial 'primary' period. The rents paid in the primary period are calculated so as to repay the purchase price and finance costs incurred by the lessor and to give the lessor a profit after tax. The asset may then be sold and any sales proceeds will generally go to the lessee. Alternatively, there follows a 'secondary' period under which nominal rentals are paid. The overall length of the lease is normally no less than the useful life of the asset.

Prior to 1991 and the issuing of the Inland Revenue's Statement of Practice, the full rental payments made by the lessee were tax deductible. It was possible to accelerate the incidence of tax relief for the lessee by manipulation of the primary and secondary lease periods and the pattern of rental payments agreed between lessee and lessor. In April 1991, the Inland Revenue issued a statement of tax practice, SP 3/91, to the effect that the tax treatment for lessees would thereafter follow Statement of Standard Accounting Practice No. 21 (SSAP 21), *'Accounting for leases and hire purchase contracts'*. Under SSAP 21 assets are treated as if they are owned by the lessee. This approach is consistent with the view that a lease is essentially a financing arrangement, since the lessee carries both the risks and rewards of ownership and the operational responsibilities. The provisions of SSAP 21 are that assets leased for their useful life are capitalised on the balance sheet, and depreciation is charged. The present value of the future rentals, taking the implicit rate of interest in the lease as the discount rate, is shown as a liability in the balance sheet. The finance charge component of the lease payment is the interest on the present value of the future rentals, charged at the implicit rate. This is added to the liability account and charged to the profit and loss account as an expense of the period. The lessee's accounting profit is therefore reduced by the depreciation charge and finance charge, which together form the lessee's 'tax shield'. The lessor pays taxes on the full rentals, but claims the capital allowances on the leased asset.

Taxes and the value of lease contracts

Miller and Upton (1976) showed that in a perfect capital market where the lessor and lessee have symmetrical tax positions, the after tax present value to the lessor of the leasing rental income is the same as the discounted after tax cost to the lessee of the rental costs. In equilibrium, there is no advantage to leasing. Excess returns would be eliminated by competition between lessors, up to the point where the present value to the lessor of the asset's leasing rentals after tax would equal its discounted after tax purchase cost and disposal proceeds.

If, however, either of the critical assumptions are relaxed, there may be an advantage to leasing. The tax position of the lessee and lessor may differ: the lessee's profits, including the cash flows of the lease itself, may fall within the limit for the small companies' rate or into the band where the higher marginal rate applies, or there may be a tax loss so that the lessee is temporarily tax exhausted. Since the patterns of tax-deductible items are different for the lease or buy decision, the timing of the setoff of the firm's debt interest and ACT may be affected. Under these circumstances the cash flows from leasing and purchasing of an asset may be significantly different. Also, if lenders do not have perfect knowledge of a firm's leasing commitments, the lessee and lessor may face differing costs of debt so that leasing and purchasing are no longer financially equivalent.

5.6 Lease evaluation

The literature concerned with the analysis of the tax benefits of leasing, as compared to asset purchase, has focused mainly on the contractual nature of lease payments and therefore on their similarity to debt obligations. Leasing may therefore be regarded as displacing debt, and will affect the financial risk characteristics and debt capacity of the firm as a whole.

As a result of this, allocating funds to a leased project may affect the financing of subsequent projects leading to arbitrary decisions on project acceptance. For this reason, Drury (1989) argues that the firm's weighted average cost of capital should be used to evaluate specific projects before the financing stage is considered. But there are drawbacks to this approach. It is inappropriate to use the overall WACC where the project's risk differs significantly from the average risk of the firm's existing assets, or where the existing debt-equity ratio may not be constant before and after project acceptance. Also, the WACC cannot be used easily in a situation where the company is temporarily not paying taxes and tax relief on debt interest is delayed because the after-tax cost of debt will then be different in each period.

5.6.1 Leasing and debt finance

Assuming that the firm does not have limitless debt capacity and given the similarities in the contractual nature of both debt and leasing cash flows, leasing may be regarded as displacing debt. But if leasing displaces borrowing capacity, a further complication is that the amount of debt displaced depends on the value of the lease, but the value of the lease depends on the debt displaced. The need for an approach which can deal with these aspects simultaneously was recognised early in the development of the leasing literature. As Fawthrop and Terry (1975) pointed out:

'A decision to lease cannot be taken in isolation from the decision to acquire the asset. Given that a decision to acquire the asset must review, inter alia, taxation costs and benefits; and given that these differ when leasing is used as compared with ownership financing; then the investment appraisal becomes dependent on the financing decision. The decision to lease, therefore, is unquestionably a simultaneous investment and financing decision and, as such, any attendant technique must adequately recognise and engage this notoriously complex area of capital budgeting.'

A number of techniques for lease evaluation have been developed which attempt to meet this criterion.

Leasing and debt in the MM world

The appropriate discount rate to be applied to lease cash flows in a Modigliani-Miller world is the cost of debt. Debt financing is cheaper than equity financing if the firm has sufficient taxable income to get the benefit of the debt tax shields, even if the benefits are delayed because of tax exhaustion. Since the logical alternative to leasing is debt finance, the discount rates for the lease should represent the opportunity cost of debt.

Assuming an MM world of debt and taxes, Myers, Dill and Bautista (1976) (MDB) applied the 'adjusted present value' methodology developed by Myers (1974) to derive a lease valuation formula which solves simultaneously for the value of the lease contract and the value of the equivalent loan, that is, the value of the debt displaced by the lease. MDB modelled the situation where lease rental payments are allowable in full against tax, as in the US and formerly in the UK, as:

$$B_0 = \sum_{t=0}^{H} \frac{P_t(1-T)+b_tT}{(1+r^*)^t} \qquad (5.7)$$

where

P_t = lease payment in t per £1 of leased asset

T = the marginal corporate tax rate

b_t = depreciation allowance in t per £1 of the leased asset's value

r^* = the firm's after tax borrowing rate

H = the final period of the lease.

Since the cash flows in all future periods are assumed to be identical, the net advantage of leasing is the difference between the purchase cost, C, and the amount borrowed, B_0. The lost interest tax shields of displaced debt are implicitly recognised in the adjusted discount rate r^*:

$$r^* = r(1 - \lambda T) \tag{5.8}$$

where

λ = the proportion of the firm's debt capacity displaced by one unit of lease finance.

If it is assumed that the capital market is perfect in its information flows, there is no reason why lenders should regard the risks of leasing and debt finance differently. Investors will be aware of the firm's finance leases and so their method of disclosure in the financial statements will be irrelevant. λ will therefore be 1.

Franks and Hodges (1978) derived the MDB formula more simply, by focusing on the cash flows displaced rather than their present value. The debt displaced by the lease is the amount necessary to make the cash flows in each future period for buy and borrow exactly the same as those for leasing. This occurs where the loan repayment in each period (interest after tax plus repayment of principal) is equal to the sum of the corresponding after-tax lease payment and depreciation tax saving, that is, $P_t(1-T)+b_tT$.

The amount which must be borrowed initially to achieve this is the debt displaced by the lease, assuming that £1 of leasing displaces £1 of debt ($\lambda = 1$). The stream of repayment cash flows must repay a principal which is the present value of the payments stream discounted at the rate of interest on the loan after tax, $(1-T)r$, or r^* in Myers, Dill and Bautista's notation.

The amount borrowed is therefore B_0 as defined in (5.7).

Franks and Hodges argue that r^* must represent the cost of obtaining the amount and maturity of debt which equates the leasing and purchase cash flows. In their method, the schedule of cash flows for the loan makes its maturity structure explicit and the security offered for that debt should be equivalent to the security of the lease payments. Knowledge of the security and maturity for the loan enables the determination of the appropriate interest rate. Therefore, leasing should displace debt on a 1:1 basis. If the tax shields have risk characteristics similar to the firm's debt and lease obligations, then the only circumstances where $\lambda \neq 1$ would be if lenders took an irrational view of the lease. However, uncertainty about future tax rates would make the tax shields more risky than the contractual payments.

The MDB approach in the UK context

The MDB method of lease evaluation as applied under the UK tax system may be demonstrated by an example.

A firm is evaluating a project involving an investment in a machine costing £90,500. The project is expected to generate cash inflows of £49,000, £54,000 and £25,000 midway through years one, two and three respectively. The firm is in a full taxpaying position in each year and acceptance of the project would not alter this. The cash inflows and associated tax payments are discounted at a risk adjusted rate of 15 per cent. The tax savings from capital allowances have a similar risk to that of debt finance and are discounted at the after tax cost of debt r^* (Myers, Dill and Bautista, 1976) which is derived from the pre-tax cost of debt, r, from the formula

$$r^* = r - \left[rT / (1 + r^*) \right] \qquad (5.9)$$

Table 5.1

The lease/purchase decision

Year *	0	1	2	3	*Total*
The purchase alternative					
Purchase cost	(90,500)				(90,500)
Additional cash inflows		49,000	54,000	25,000	
Discounted value		45,693	43,787	17,628	107,108
Additional tax at 33%		16,170	17,820	8,250	
Discounted value		12,227	11,717	4,717	(28,661)
Additional capital allowances		22,625	16,969	50,906	
Tax savings		7,466	5,600	16,799	
Discounted value of tax savings		6,360	4,403	12,191	22,955
Net present value					10,901
The leasing alternative					
Cash inflows		49,000	54,000	25,000	
Discounted value		45,693	43,787	17,628	107,108
Tax on inflows		16,170	17,820	8,250	
Discounted value		12,227	11,717	4,717	(28,661)
Rental payments	33,210	33,210	33,210		
Discounted value	33,210	30,652	28,291		(92,153)
Finance charges		5,987	3,142	0	
Discounted value of tax savings		1,683	815	0	2,499
Depreciation		30,167	30,167	30,167	
Discounted value of tax savings		8,481	7,827	7,224	23,532
Net present value					12,324

* - see text for cash flow timing assumptions

Table 5.2

The lease/purchase decision for a temporary non-taxpayer

Year *	0	1	2	3	*Total*
Purchase alternative					
Purchase cost	(90,500)				(90,500)
Additional cash inflows		49,000	54,000	25,000	
Discounted value		45,693	43,787	17,628	107,108
Additional tax on inflows		16,170	17,820	8,250	
Discounted value					(24,151)
Capital allowances		22,625	16,969	50,906	
Capital allowances tax saving		7,466	5,600	16,799	
Discounted value					21,508
Net present value					13,965
Leasing alternative					
Cash inflows		49,000	54,000	25,000	
Discounted value		45,693	43,787	17,628	107,108
Tax on inflows		16,170	17,820	8,250	
Discounted value					(24,151)
Rental payments	33,210	33,210	33,210	0	
Discounted value	33,210	30,498	28,075	0	(91,783)
Finance charges		5,987	3,142	0	
Tax savings on finance charges		1,976	1,037	0	
Discounted value					2,170
Depreciation		30,167	30,167	30,167	
Tax savings on depreciation		9,955	9,955	9,955	
Discounted value					21,508
Net present value					14,852

* - see text for cash flow timing assumptions

(see Appendix 5.1)

The firm's pretax cost of debt is 12 per cent and so $r^*=$ 8.345 per cent approximately. A tax lag of one year is assumed here for the sake of simplicity.

The alternative to purchasing the asset is to lease it over the three years of the project's life at an annual rental of £33,210, with the payments falling due on the first day of each year. The implicit rate of interest in the lease, i, may be determined from

$$33{,}210+33{,}210/(1+i)+33{,}210/(1+i)^2=90{,}500$$

so that $i=$ 10.45 per cent approximately. The rental payments, the associated finance charge tax savings and depreciation tax savings are discounted at the after tax cost of debt. Table 5.1 shows that the NPV of leasing is higher than that of purchasing by £(12,324-10,901), so that £1,423 may be described as the net advantage of leasing.

Table 5.2 shows the situation where the firm has no taxable profits in years one and two but expects to return to a full taxpaying situation in year three. The taxes on the cash inflows in years one to three will be paid at the end of year four and are all discounted by a factor of $1/1.15^4$.

The present value of the tax shield on all the capital allowances, obtained in year four, is derived by discounting at an after tax cost of debt which takes into account the time lag (see Appendix 5.1 for derivation). This same discount rate is used for the relief on the financing charges and accounting depreciation. The rentals are discounted at the appropriate after tax cost of debt to reflect the timing of the payments (Appendix 5.1).

The net advantage of leasing in this situation is £887 (£14,852-13,965), a decrease of £536 compared to the full taxpaying case in Table 5.1, although still significant.

This analysis is, however, subject to a number of assumptions, which may not be valid in the context of the firm's overall tax position.

Assumptions and limitations of the MDB/Ashton/Franks and Hodges approach

(i) Debt is valuable, in the MM sense;

(ii) dividend policy is irrelevant;

(iii) in the absence of leasing, the firms borrows an 'optimal amount' which depends on the value and business risk characteristics of the firm's assets, and the value of tax shields generated by depreciation and interest;

(iv) the tax shields generated by the lease payments are treated in the same way as the shields generated by depreciation and interest.

These assumptions are questionable in the UK context, since the tax shields from leasing, depreciation and interest are not treated in the same way. Tax shields from leasing or depreciation may lead to a trading loss which may be carried back as well as forward, but excess charges on interest cannot. Under the imputation system the treatment of ACT may be affected by the level of debt interest payments, which reduces the mainstream corporation tax liability, and so dividend policy is not independent of the lease/purchase decision where the firm is ACT exhausted.

Also, in order to use the formula in project appraisal, it is necessary to know:

(i) the schedule of lease payments;

(ii) the asset's tax depreciation schedule;

(iii) the firm's cost of borrowing;

(iv) the firm's marginal tax rate.

The last three of these should be derived endogenously by the overall activities of the firm, including the leased project. If the project is appraised separately, as in the example above, incorrect assumptions may be made about the appropriate values of the cost of debt and the relevant marginal tax rate. The tax effects of the capital allowances available if the asset is purchased rather than leased will similarly depend on the interactions between the cash flows of the project and those of the firm's existing activities and debt commitments.

Although for the firm which is temporarily non taxpaying, the valuation method may be modified by adjusting the after tax cost of debt to take account of the time lag in obtaining tax relief, the future tax paying date is treated as certain and exogenous to the acceptance of the project, whereas the project's cash flows and tax effects and the firm's future tax situation will be interdependent. Acceptance of the project may even alter the firm's pattern of tax relief on the interest relating to existing debt commitments, so that the after tax cost of that debt will also be distorted.

Leasing and debt in the Miller world

Cooper and Franks (1983) examined the Miller (1977) equilibrium which would result if no costs were incurred in tax motivated financial transactions. If taxpaying firms are large in number, a competitive supply of them will be available. While aggregate corporate income is positive, the gains to tax paying firms from tax motivated transactions such as leasing will be zero. If there are no real costs to at least one mechanism for transferring tax between companies, firms with tax losses will have access to transactions with positive net present values until they resume taxpaying.

Following the assumptions of Miller, the after tax cost of debt for a firm in a full tax paying situation is the same as its cost of equity. If the firm is permanently or temporarily tax exhausted, equity finance is cheaper than debt. The logical alternative to leasing is to finance the purchase of the asset with equity and the cost of equity is therefore the appropriate discount rate.

5.6.2 Capital market imperfections and leasing

The differing tax treatment of leased and purchased assets may not be the only reason why leasing may be advantageous. If lenders have imperfect knowledge of a potential borrower's lease commitments, leasing may be attractive to both lessee and lessor. The interest rate which the lessee would pay on the displaced debt may be greater than that paid by the lessor on the funds used to support the debt, or the lessee debt displaced may be less than the debt supported for the lessor.

Also, Hull and Hubbard (1980) noted that some lessees may be prepared to pay a higher effective rate of interest for lease finance than for debt finance because of the the difficulty of arranging an 'equivalent loan' of the type suggested by Franks and Hodges. The unusual repayment pattern suggested in this type of analysis might be difficult to arrange with creditors. An alternative is to use short term borrowing to provide the equivalent loan, but this has the disadvantage of being repayable on demand. Short term debt interest payments will have a higher risk to those of longer term debt. However, in the UK, short term debt interest is allowed as a trading expense and can therefore form part of a trading loss which may be set against previous periods' profits, whereas an excess of longer term debt interest cannot. This reduces the risk of the short term debt interest flows.

If the imperfection of capital rationing is assumed, then a project's cash flows may have an additional value in terms of funding new capital investments with positive net present values. Fawthrop and Terry (1975) considered the impact in future periods of the cash flows generated over the life of the lease in terms of 'residual capital balances' whereby the residual amount of capital outstanding after the lease repayments have been made (compared to the purchase option) is available for reinvestment. They argued that this reinvestment opportunity should also be credited to the lease decision. If, however, there is no capital rationing then the firm would be able to fund projects with a positive NPV in any case without the need to rely on internally generated funds.

5.6.3 Leasing in an options framework

With incomplete setoff for tax losses, Heaton (1986) demonstrates that a leasing contract can be advantageous for lessees, even if they are in the same tax bracket as lessors in those states in which they have taxable income, or have the same probability of paying taxes. Firms may be able to decrease the net present value of their combined tax liabilities through leasing contracts, by either reducing state-contingent payments outright or by shifting taxes from high to low marginal utility states.

Smith and Wakeman (1985) examine leasing in a real option context. Lease contracts may include conditions under which the lessee may either extend the life of the lease or purchase the leased asset. These options are relevant when the residual value of the asset is significant. The value of the option is a positive function of the termnal value. A cancellation provision in a lease contract might allow the lessee the option of terminating the lease contract prematurely. The value of the cancellation option to the lessee is a positive function of the uncertainty about the asset's value in use, for instance if the asset may become technologically obsolete before the end of its useful life. A non-cancellable lease, however, precludes this option. McConnell and Schallheim (1983) developed a model for evaluating different types of leasing contract in an option pricing framework, and demonstrated its applicability in determining equilibrium rental payments.

If the earnings derived from the generation of residual capital balances by the use of a leasing contract are to be included as part of the lease decision, this can be done using an integrated mathematical programming model in which the current and future project options are specified.

5.7 Reasons for leasing: empirical evidence

Leasing theory indicates that taxation benefits and capital market imperfections such as information asymmetries are the only rational reasons to lease. Most empirical studies, however, have found that these factors did not explain much of the leasing behaviour observed in practice.

Tax factors

66 per cent of respondents to a survey by Fawthrop and Terry (1975), involving senior financial executives in major UK corporations, claimed that the tax factors were irrelevant to their leasing decision. Similarly, a survey by Sykes (1976) found that the availability of tax allowances was not regarded as an important factor by a

majority of respondents. However, Hull and Hubbard's study (1980) revealed a close relationship between leasing activity and a firm's tax status. More recently a survey by Drury and Braund (1990) found that corporate tax considerations were one of the most important factors in the lease/purchase decision, along with the implicit rate of interest in lease financing compared with borrowing and the conservation of working capital. Drury and Braund also found that many firms used incorrect valuation methods, such as the internal rate of return, with 40 per cent using the wrong discount rate.

Capital market imperfections

All of the studies mentioned above found that the 'off balance sheet' advantage of leasing was given little support. However, a number of other UK studies of the effects of leasing disclosures have found evidence that lenders' decisions regarding a firm's debt capacity are different if leasing is used, or that managers believe this is the case. Surveys by Sykes (1976) and Fawthrop and Terry (1975) show that managers believe leasing reduces borrowing capacity by a lesser amount than the equivalent loan. Over a quarter of the respondents to a survey by Hull and Hubbard (1980) believed that leasing provided an additional form of finance which did not affect other borrowing.

There is some evidence that full disclosure of finance leases creates an incentive to avoid capitalisation through the use of operating leases (for instance, Taylor and Turley, 1985). But although 41 per cent of Taylor and Turley's respondents believed that leasing would be less attractive under full disclosure, Drury and Braund's study (1990) showed that the introduction of SSAP 21 was not perceived to have a significant effect on leasing activity. Further, a field test by Ashton (1985) found that voluntary lease capitalisation did not affect financial performance indicators significantly.

5.8 Conclusion

The tax implications of the firm's optimal financing decision create complex cash flow interactions. Where the firm is in a restricted tax position, the present value benefit of debt interest deductibility may be far less than the statutory tax rate would imply because the debt interest tax shields may not be utilised until some uncertain future date. The tax rate at which the debt interest is set off will depend on the firm's other tax allowances, which will in turn depend on its investment decisions, so that investment and financing decisions interact. But methods of valuing debt interest tax shields in project appraisal normally require as input some assumptions about the relevant tax rates and cash flow timing. Also, the difficulty with approaches to project valuation which depend on the application of an exogenously determined discount rate to the estimated cash flows is that the firm's overall activities, including the projects being evaluated if these are accepted, determine the risk to the providers of debt and equity capital and the appropriate discount rate. The same criticism applies to methods of lease evaluation. The effect of any incremental project, and its financing, on the amount and timing of the firm's cash flows should ideally be assessed simultaneously in an overall tax context.

The optimal financing decision is complicated by the UK imputation system, which increases the relative attractiveness of equity financing as compared to the classical system of dividend taxation. However, the deductibility of debt interest, or of items relating to leasing contracts, may reduce the firm's taxable profits to the extent that ACT setoff may be restricted, thus reducing the effective imputation rate if the surplus ACT must be carried forward. This will increase the cost of paying a dividend and reduce the relative attractiveness of equity financing to the firm. On the other hand, additional debt finance might result in the firm becoming unable to utilise all of its debt tax shields, increasing the effective cost of debt. The determination of an optimal financing policy will therefore be particularly difficult for the firm with low taxable profits or for the temporary non-taxpayer. Again, a simultaneous consideration of the investment and financing decision is needed.

Chapter 6

Optimisation models in capital budgeting

6.1 Introduction

This chapter will review the optimisation models of financing and investment decisions that have been developed in the literature. A number of authors have considered the problem of project selection and financing under conditions of capital rationing, and developed mathematical programming models to evaluate the firm's decisions simultaneously. Lease evaluation models have also been developed using linear programming. A small number of models have incorporated aspects of the tax system, in differing levels of detail.

6.2 Early optimisation models for capital budgeting

The early capital budgeting models developed in the literature were concerned with the market imperfection of capital rationing, where firms face internally or externally imposed capital expenditure limits. In this situation the firm may not be able to fund all its capital projects which have a positive net present value, so that cash is regarded as a scarce resource. Lorie and Savage (1955) first formulated the capital budgeting problem under capital rationing as: given the net present value of a set of independent investment alternatives, and given the required outlays for the projects in each time period, find the subset of projects which maximises the total net present value of the accepted ones while simultaneously satisfying a constraint on the outlays in each of the periods. This constrained optimisation problem may then be represented in a linear programming formulation. The Lorie-Savage problem was originally formulated by Weingartner (1963) so that the discounted net present value of projects formed the basis of the objective function, that is, the expression which must be maximised (or minimised) in order to attain optimality.

6.3 Linear and integer programming approaches

If the assumption is made that project acceptance variables may be treated as continuous, dual prices associated with the cash flow constraints for each period emerge in the solution process. These dual prices represent the amount by which the objective function value would increase if an additional unit of cash were to be made available in the period and used optimally, and can be regarded as a measure of the opportunity cost.

In a capital rationing situation, the opportunity cost of funds can vary over time according to the scarcity of capital and the availability of borrowing facilities and hence projects with different cash flow patterns over time can have the same net present value. An LP analysis allows the present value of a project to be compared with the opportunity value associated with its requirement of the limited available funds, implying that a project may be accepted because of its favourable cash flow (valued at the opportunity cost of funds) even if its net present value is negative (Salkin and Kornbluth, 1973). Carsberg (1974) used the dual prices of cash to derive interest rates representing the maximum one-period rate which is worth paying for an additional marginal unit of capital.

But there was debate over the correct interpretation of the relationship between the dual prices and the interest rates that managers should be prepared to pay in order to obtain an additional unit of finance. For example, Baumol and Quandt (1965) analysed the implications of Hirschleifer's analysis (1958) on Weingartner's formulation of the Lorie-Savage problem and argued that if the objective function is the maximisation of net present value, then if the discount rate used is determined by the value of the dual on the rationing constraint, the only solution is a trivial one, although this finding was disputed by Atkins and Ashton (1976).

Also, the maximisation of net present value may imply the postponing of consumption in favour of reinvestment, ignoring the shareholder's utility in terms of a time preference pattern of consumption. Pope (1983) argued that if the objective function is the sum of the future payments to shareholders made possible by the set of accepted capital investment projects, discounted at the shareholder's rate of time preference, then the dual value associated with the funding constraint at any time t must represent the maximum amount *in present value terms* which is worth paying for a marginal £ at *t*.

However, this debate relates to a linear programming formulation in which variables are continuous and fractional acceptance of projects may occur in the optimal solution. In many practical applications this form of solution is unacceptable, since projects may not be divisible. The stipulation that certain variables may take only integer values changes the linear programming problem into a mixed integer problem, which in turn has implications for the interpretation of the solution output. There are major differences in the valuations of MIP constraints and shadow prices in LP. If a constraint in an LP model has slack capacity it represents a 'free good' and its shadow price is zero. If the constraint were omitted, the optimal LP solution would not change. By contrast, if a constraint in an IP model has positive slack it does not necessarily represent a free good and may therefore have a positive economic value. The shadow prices which appear in the solution do not therefore have a straightforward interpretation.

6.4 The objective function

Although Weingartner's net present value maximisation criterion was followed by a number of authors including Chambers (1971), Haley and Schall (1973) and Myers (1974), other authors disagreed, arguing that the utility of withdrawals was more relevant to the maximisation of shareholder wealth. Baumol and Quandt (1965) proposed the sum of withdrawals of cash made available by the projects undertaken in each of the future periods, each withdrawal weighted by its subjective utility. Myers (1972) suggested that the two approaches were equivalent under the assumptions of certainty and perfect markets. In Myers' model, the values of the dividends generated in the solution are those which maximise the value of the firm, whereas in the Weingartner model dividends are exogenous. Bhaskar (1976) proposed a model in which wealth is maximised as a surrogate for utility. Wealth is maximised when the market value of shares is maximised, using the traditional share valuation model in which the value of the firm is the present value of the firm's future dividend stream, discounted at the cost of capital.

A problem with models which aim to maximise net present value is that this aim is not always consistent with maximising shareholder wealth. Carsberg (1974) examined the impact of the UK imputation tax system on the share valuation model and demonstrated that a project with a positive NPV after corporation tax may not necessarily increase shareholder wealth in terms of the discounted additional dividends made possible by the project. This is a result of the impact of the mainstream corporation tax and ACT payments on the pattern of cash flows. Thus the dividend-based objective function would seem to be more appropriate, at least in the UK context. Carsberg (1974, chapter 10) also shows that in order to maximise shareholder wealth, no dividends will be issued during periods of capital rationing as long as there are worthwhile investment projects. Where a model is not constrained by capital rationing these restrictions are relaxed.

6.5 The financing problem

Later applications of the mathematical programming approach to project selection focused on the integration of the investment and financing decisions.

A linear programming model by Chambers (1971) was based on the objective of maximising the terminal net present value of the firm's projects, following Weingartner (1963). Dividend policy is taken to be exogenously imposed. The net present value of projects in existence at the horizon is derived by first calculating a weighted average cost of capital and the criterion is then extended to evaluate investments and financing strategies up to the horizon. An allowance is made for the effect of corporate taxes by treating the firm as a full tax-payer with the pre-tax project cash inflows and corporate debt payments being adjusted by a factor of $(1-T_C)$ to give the corresponding after tax flows. The model specifies optimal joint strategies for financing and investment and can accommodate complex projects which affect both financial structure and the supply of profitable opportunities, such as acquisitions. There is a certain range of choice of financing strategy, taking flotation costs into account, but a maximum debt/equity ratio is imposed and limits are placed on new borrowing. If debt finance is cheaper than equity finance after tax, the model would therefore presumably give solutions which utilise as much debt as possible. The quantity of new equity issued is restricted by a limit on the debt/equity ratio at the horizon. Since it is a linear model its dual prices and reduced costs may be interpreted, allowing the marginal cost of capital to be determined within the model in each year up to a planning horizon.

The Chambers model assumes shareholders will not put funds into the firm at less than the marginal returns they can obtain elsewhere, although the risk associated with these returns is not considered. The model includes market investment in other firms as one of the competing uses of funds, in order 'to ensure that managers will not hoard funds or invest them at returns lower than those prevailing in the market at large'. There appears to be an implicit assumption of market imperfections other than capital rationing, such as information asymmetry between managers and shareholders.

A model by Carsberg (1974) is similar to that of Chambers in its approach to the financing problem. Limited amounts of both new debt and new equity issues are permitted in Carsberg's model and a desired gearing ratio is defined exogenously. New equity issues are included either by deducting the present value of the proportion of the dividends going to new shareholders or by deducting the present value of the capital issued. Further constraints ensure that dividend policy complies with UK legal restrictions on the level of dividend payments. The objective function aims to maximise the present value of the equity shares in issue when the plan is formed. This is given by the present value of cash flows to shareholders, that is, dividends less subscriptions for new shares, plus the equity shareholders' interest in assets at the horizon date. This is given by the value at the horizon of the post horizon project cash flows, plus the value of machinery and current assets, less the debt capital outstanding and tax liability at the horizon. The discount rate is the cost of equity, estimated from the mix of debt and equity capital which is maintained at a constant level. The cash flows are adjusted for taxation, assuming an imputation system with simple proportional tax rates.

In defining the gearing ratio, the market value of equity is expressed as a proportion of the book value of borrowed funds, as a proxy for the market value of the debt. Although the issue costs of debt and equity are not taken into account, Carsberg acknowledges that small changes in debt capital from year to year would be impractical in the presence of transaction costs and suggests that small variations in gearing levels could be permitted provided that the firm keeps to the target ratio on average.

An advantage of assuming a fixed gearing ratio, as in the Carsberg and Chambers models, is that a cost of equity can be assumed by following a MM or Miller model of capital structure. Financial constraints ensure that the mix of debt and equity capital is maintained at a constant ratio, under the assumption that the equity shareholders' discount rate has been estimated using a specific level of financial risk. The disadvantage of constraining the model to a particular gearing ratio is that it restricts the range of solutions from which an optimal decision is selected.

An alternative approach to the financing problem was taken in the mixed integer programming model for financial planning by Myers and Pogue (1974), following Modigliani and Miller's (1963) result that the total market value of the firm is equal to its ungeared value plus the present value of taxes saved due to debt financing. The firm's proportion of debt is determined within the model rather than being specified in a given debt/equity ratio. The objective function maximises the overall market value of the firm in terms of the optimal combination of investment and financing options, which in the MM world are assumed to be perpetuities. Consistent with the assumption of an MM world, dividend policy is treated as irrelevant, but transaction costs associated with issuing new equity are included. However, Myers and Pogue assume an informational role for dividend payments, and allow for the 'smoothing' of dividend payments over time. The net present values of projects are calculated before the model is run, using as a discount rate the cost of capital based on all-equity financing. Analysis of the risk of the investment projects is also exogenous to the model, and it is assumed that the risk characteristics of a project can be evaluated independently of the risk characteristics of the firm's existing assets. This assumption appears reasonable under the model's treatment of tax cash flows: taxes are assumed to be simple and proportional, and the project cash flows are assumed to be net of corporation tax. If complex tax-induced interactions between cash flows were taken into account, the firm's overall risk characteristics would change and it would not be possible to evaluate project risk in isolation.

6.6 Borrowing and lending

Salkin and Kornbluth (1973) presented a linear programming model to maximise net present value under capital rationing, with revenues and costs discounted at the opportunity cost of funds. The model was extended to include short term borrowing and lending, with an upper bound on the amount of borrowing in each period. Where this constraint was binding, Salkin and Kornbluth showed that the acceptance of an additional project with a zero NPV but whose pattern of cash flows alleviated the shortage of cash in the critical periods could enable the firm to undertake more profitable investment opportunities.

Bhaskar (1974) considered the inclusion of the firm's borrowing and lending opportunities as possible 'projects', in a capital rationing situation where new debt may be incurred but not new equity issues. In a perfect capital market the firm should distribute excess funds to its shareholders, raising additional funds as and when worthwhile investment projects arise, rather than lend them at less than the cost of capital. But under capital rationing, lending can provide funds for future investments. The lending of funds, together with their subsequent reinvestment in a capital project, may have a positive net present value even if the initial lending is at an unattractive rate of interest. The objective function therefore includes terms which treat the firm's lending opportunities as investment projects, since they are similar to capital investment projects in having an initial cash outflow and subsequent cash inflows. The objective function is concerned with maximising the net present value of projects' cash flows, discounted at the weighted average cost of capital, which is assumed constant, following MM. These assumption cannot be valid where there are tax complexities such as temporary losses, unrelieved debt interest or surplus ACT, which would cause the weighted average cost of capital to vary over time. However, Bhaskar notes that his approach has the advantage of greater simplicity than the alternative of discounting the residual cash flows after debt payments and inflows at the cost of equity. If this latter technique is used then the cost of equity will depend on the level of debt financing, which is one of the decisions made by the model.

In Bhaskar's later (1978) model, capital rationing is interpreted differently as a situation where limited amounts of new debt and equity issues are both permitted, as in Carsberg (1974). The objective function maximises shareholder wealth in terms of the market value of the firm's shares, assuming that this is given by the sum of the horizon value and the dividend stream prior to the horizon, discounted at a cost of equity which takes into account the financial risk of the level of gearing employed. The horizon value represents an estimate of the post-horizon equity cash flows discounted back to the horizon date. An allowance is made for any cash flows of projects and lending opportunities extending beyond the horizon. The model includes opportunities for lending and short-term borrowing, noting that the cost of equity will be unaffected by the latter provided the amounts are comparatively small. For longer term debt finance, an increasing relationship is assumed between the cost of equity and the level of gearing. Bhaskar proposed an iterative method of solution to the derivation of the firm's cost of equity. If it is assumed that an optimal gearing ratio exists, then some function estimating the cost of equity as an increasing function of gearing may be used to derive a solution. The model may then be run using as the discount rate a cost of equity which assumes no gearing. The solution output would include the optimal financing arrangements at this discount rate. From the output a new cost of equity could be derived, outside the model, by reference to the assumed relationship between gearing and the cost of equity. The model would then be run again with this revised discount rate, and the process repeated until convergence was achieved. Bhaskar pointed out that a complication of this method is that a different cost of equity would apply to each period, since gearing levels would change over time according to the amounts of debt purchased and repaid, and the amounts of equity issued. A second difficulty anticipated by Bhaskar was that convergence might not be achieved under certain conditions, for instance, if the iterative process oscillated between two positions.

Constraints restrict the net cash flows from projects, dividends, lending less borrowing costs, less equity issues and tax payments, to the funds available. Lending may occur at the horizon if the lending rate is greater than the cost of equity. Taxable profits are assumed to be sufficient in each period so that project cash flows and debt interest are adjusted by the factor $(1-T_C)$, which Bhaskar acknowledges to be a 'simplistic and unsatisfactory' method.

6.7 Programming approaches to lease evaluation

In order to take complex tax effects fully into account, lease evaluation should take place in the context of the firm as a whole rather than in isolation. This may be achieved by the use of a mathematical programming model to incorporate all the relevant cash flows for both the leasing and purchasing alternatives.

Fawthrop and Terry (1976) recognised that a mathematical programming model could provide a suitable lease valuation technique, but did not attempt to develop a model of this type. An attempt to analyse the reasons for leasing in a mathematical programming framework was made by Ashton (1978). Ashton formulated the investment policy and financing policy of the firm as a problem in constrained optimisation, determining the contribution made by a lease using Kuhn-Tucker optimality conditions. The analysis assumes that any lease decision is made such that the subsequent rearrangement of debt and equity is optimal, with the appropriate discount rate arising from within the solution framework rather than being specified exogenously. By assuming that the debt capacity and the scale of the lease undertaken are continuous variables, the conditions for optimality when applied to the leasing variable may be derived. In this way values may be obtained for the marginal increase in the value of the firm associated with leasing and the opportunity cost of capital. A difficulty with this approach in the context of project appraisal is that a lease is generally a fairly large, discrete block of finance which is not continuously divisible. As a result, the linear program and the shadow prices which arise out of it cannot be applied to determine whether the lease or purchase alternative is preferable. Similarly, Cooper and Franks (1983) assessed the leasing and purchasing alternatives in a linear programming context, arguing that the timing of the tax effects of projects may be taken into account through the 'effective' tax rates which arise as shadow prices in the solution. This model allows for the possible carry forward of tax shields to future periods, so allowing for the impact of this on the effective tax rate. Franks and Hodges (1978) proposed a simple linear programming model which optimises the lessee's schedule of payments by setting the lessor's net present value at zero. The linear program maximises the initial cash flow for the buy and borrow alternative, subject to constraints which equate the future cash flows to those for leasing. The amounts borrowed and repaid are included as decision variables and additional equations calculate the loan balance each period. Franks and Hodges suggested that the approach could be extended from the simple situation where tax losses carried forward are subsequently absorbed in full in a single year, to a more complex scenario where absorption occurs over a number of years and the tax savings produced are limited by the company's stream of taxable income. However, there do not appear to have been any attempts in the literature to develop a mathematical programming model for lease evaluation in complex tax situations where a number of alternative tax strategies exist.

6.8 Project selection using the Capital Asset Pricing Model in a mathematical programming context

Thompson (1976) applied the analysis of Hamada (1969) to develop a mathematical programming model for capital budgeting of interrelated projects in a CAPM context.

The model deals with exogenous interdependencies between projects, such as mutual exclusivity or other conditions such as contingency. The after tax returns of the individual projects are also exogenously determined. Thompson assumes an MM valuation for the tax subsidy on debt finance, and assumes that the debt is risk free. The objective function maximises the risk-adjusted one period return, with the cash flows being adjusted for systematic risk and then discounted at the risk-free rate.

6.9 Optimisation models for taxation

The literature on optimisation models for corporate taxation in a mathematical programming context is sparse.

Ashton (1978) suggested that accept or reject decisions for capital projects may be performed fairly efficiently without the need for a complex mathematical programming model. But tax interactivities were not part of Ashton's analysis.

Kornbluth and Salkin (1986) describe an application of linear and dynamic programming techniques to the problem of optimal multinational tax planning, taking a general approach rather than incorporating specific legal requirements. Corporation taxes, withholding taxes and tax havens are considered, and one period models are developed to optimise the corporate structure for tax purposes and to derive an optimal loan structure. Kornbluth and Salkin comment that

'Whilst these models will not solve all the problems inherent in such a complicated field, they do show how analytical techniques can be used to structure such problems and to gain an insight into their solution.'

Other authors have taken a more specific approach, identifying features of the UK tax system which are particularly significant for project appraisal and incorporating these features into a mathematical programming model.

Berry and Dyson (1979) follow the approach of Buckley (1975) to show how a combination of two projects may take better advantage of the available capital allowances than either project does by itself, and concluded that the analysis implied that all possible combinations of economically independent projects must be generated and each of these combinations analysed on an after tax basis taking into account any cash flow from the firm's ongoing activities. Berry and Dyson's model has a mixed integer mathematical programming formulation in which certain decision variables are constrained to have integer values, to take account of indivisible projects. The objective function is the discounted net present value of the accepted projects after tax. The model concentrates on the tax effects of capital investment under the assumption of no capital rationing, and the optimal financing issue and the question of an appropriate discount rate are not included in the analysis.

The constraints of the model represent some of the main features of the UK system at that time when 100 per cent capital allowances were available for new capital investment. Marginal tax rates and the possible carry forward of unutilised capital allowances are included, although the carry back provisions are not. The imputation system is included in order to recognise the difference in timing between ACT and mainstream corporation tax, with the simplifying assumption that carrying back ACT is not possible. Berry and Dyson comment that 'our attempts to include this feature have resulted in models of vastly increased size or models involving non linearities'. Dividend payments are included for the same reason and taken as given.

The tax function is dealt with by an approach used to approximate a non-linear function by a set of connected linear segments, using a separable programming technique. The model includes marginal tax rates, but it has a fault in that instead of setting off unabsorbed capital allowances carried forward against taxable profits as soon as they become available, it allows them to be carried forward to be offset in more distant periods with higher tax rates. The UK tax system does not allow this. The model also excludes some important features such as the facility to carry back unabsorbed capital allowances, which limits the generality of the model and ignores a mechanism which may be significant in project appraisal.

Ashford, Berry and Dyson (1986) aimed to extend and develop Berry and Dyson's net present value model in a number of ways, in particular to reflect the changes made in the Finance Acts of 1984 and 1985 regarding the introduction of capital allowances on a reducing balance basis at a rate of 25 per cent per annum. The 'illegal' treatment of capital allowances carried forward was amended and the carry back provisions available at the time of writing were included. The model also incorporates a provision for short life assets. The non-convex structure of the tax function is dealt with as a piecewise linear function in a similar way to that used in the earlier Berry and Dyson paper. Unlike this earlier work, the imputation system is not incorporated in the model and so the interaction between trading losses as a result of excess capital allowances and the setoff of ACT is not addressed.

The model was tested on project data provided by Weingartner (1963), and the results showed evidence of tax induced interactions between projects: for instance, a particular project was not included in the optimum combination when the carry back provisions were excluded from the program, but included when these provisions were added. The effect of multiple tax rates was found to be small, but this related to the large scale of the cash flows involved. For a company with smaller profits, the effect would become more significant.

The two models described above aim to maximise net present value and so have the drawback, as previously discussed, that this may not lead to maximisation of shareholder wealth.

In a departure from mathematical programming techniques, a recent paper by Berry and Smith (1993) used a search paradigm based on the mechanics of natural population genetics (genetic algorithm) to examine taxation induced interactions in project selection. This model is based on net present values and is very similar in its scope to the mathematical programming model of Ashford, Berry and Dyson (1986), concentrating on the interactions between capital investment and tax. Multiple tax rates and the carry forward of losses are included, but not the carry back provisions or the mechanisms of ACT setoff. The procedure was tested on Weingartner's (1963) project data, and compared with the results obtained by Ashford, Berry and Dyson using the same data. These earlier results provided a knowledge of the optimal solutions which could be obtained. It was found that the genetic algorithm approach found the optimal solution in every case, with solution times of around one minute for the most 'complex' models. This is an interesting approach, although it is likely that a fuller treatment of the complexity of the tax rules would complicate the process considerably and extend the solution time. In chapter 8, it will be shown that a mixed integer programming model incorporating a large number of tax-induced interactions can be solved in a very short time, provided it is efficiently formulated.

Pointon (1981b, 1991) extended the work by Bhaskar (1978) to incorporate the main features of the UK imputation tax system in a model which provides for a joint solution to the firm's optimal investment and financing decisions. The model's objective function aims to maximise the returns to shareholders, taking into account dividend payments, equity issues and tax payments. The net dividend is adjusted by a factor of (l-m)/(l-b) to give the receipt to shareholders net of personal income taxes. Capital gains may be ignored since the model assumes full dividend payout.

The discount rate applied to the returns is the cost of equity for each period. The objective function follows the basic horizon valuation approach of Bhaskar (1978) but includes net mainstream corporation tax payments after the horizon and ACT payments after the horizon on dividends paid before the horizon. It is implicitly assumed that the cash flows after corporation tax arising after the horizon date are paid out as dividends. The possibility of ACT restrictions arising after the horizon is ignored.

The constraints cover the rules dealing with capital allowances carried backwards and forwards, ACT carried backwards and forwards, marginal tax rates, charges on income, restricted ACT setoff and the quarterly accounting system for ACT payments and debt interest. The liquidity constraints correspond with those of Bhaskar (1978), with adaptations for tax effects. The constituent parts of cash flows are decomposed to take account of debtors, creditors, accruals and prepayments, since the tax system is to some extent based on profit rather than cash flow and a tax may effectively arise on increases in working capital. Mainstream corporation tax liability is based on a weighted average of the basic profits of an accounting period which can straddle a fiscal year.

The cost of equity, used as the discount rate in Pointon's model, is assumed to be determined exogenously. The model does not examine the impact of debt financing on the risk of the returns to shareholders and the cost of equity, and short term and long term borrowing and lending are included primarily because of their tax significance: the tax deductibility of debt may be delayed if capital allowances on new investments exceed the firm's profits, and the tax treatment of debt interest received may be significant. Pointon follows the approach of Bhaskar (1978) of setting an upper bound on the amount of short-term and long-term borrowing in each period. As in Bhaskar's model, the interest rates are assumed to be exogenously imposed and their relationship to the firm's gearing level is not explored. New equity issues are similarly subject to an upper limit in each period. Limits on the cumulative levels of debt and equity (in terms of a gearing ratio) do not appear to be dealt with explicitly.

The tax interactivities that may arise between projects and financing policies as a result of ACT restrictions, debenture interest carried forward and the relevant marginal tax rates are accommodated implicitly by the constraints of the model. Because of the model's comprehensive coverage of the complex tax rules and the accommodation of quarterly accounting periods it is necessarily very large, and because of the method used to impose the logical conditions within the tax constraints, difficult to solve in practice. If the model were run for a five year period the number of variables would be approximately 1,600 and the number of constraints would be similar, with several hundred binary variables. But, as Pointon emphasises, the purpose of the model is to demonstrate that it is feasible to incorporate the tax complexities into a mathematical programming model that seeks an optimal solution to the selection of capital projects and the determination of an optimal financing mix. Further, it is unlikely that all the constraints would be required in any specific practical application.

A highly simplified version of Pointon's model was developed by Farahdel (1982) by omitting a large number of variables, specifically those relating to lending and borrowing and stock relief, and simplifying the treatment of debt interest. The number of binary (decision) variables was minimised by assuming that the firm was in a profitable situation with no losses to carry back or forwards to other periods. Operational use of the model, using a computerised version of the simplex method, clearly illustrated the impact of the firm's ongoing cash flows on the fractional acceptance of projects even in a relatively straightforward situation.

6.10 Conclusion

Although early applications of mathematical programming techniques to the joint decision problem of investment and financing concentrated on the problem of capital rationing, tax interactivities were not addressed. Linear programming models of lease evaluation have been developed, but the assumption of continuous variables is restrictive and the models do not incorporate potentially complex tax interactions. A number of authors have recognised that the mathematical programming technique is appropriate for determining the optimal tax strategy associated with the firm's investment and financing decisions. However, the models by Berry and Dyson (1979) and Ashford, Berry and Dyson (1986) include only certain features of the tax system that may be significant. Berry and Dyson assume that ACT cannot be carried back, while Ashford *et al.* concentrate on the capital investment decision in a net present value framework and do not consider ACT. Neither of these models incorporate the interactions which may result from the financing decision. The model by Pointon (1981, 1991) has detailed coverage of the tax rules, particularly those relating to the setoff of losses and ACT. This model is not operational, although a simplified version has been computerised.

The finance literature does not appear to contain an operational model which can evaluate investment and financing decisions in the context of complex tax situations. Situations of this type may include leasing possibilities and the potential benefits of group relief for losses and ACT. The determination of an appropriate discount rate is a problematic area, particularly in mixed integer programming formulations. In the absence of capital rationing, and with an objective function which aims to maximise shareholder wealth, the discount rate should reflect the shareholder's required rate of return. It should therefore be based on some measure of the risk of the returns to the shareholder. In the following chapter, a model will be developed which attempts to meet these criteria.

Chapter 7

The mathematical programming model

7.1 Introduction

In the first stage of the model building process (Appendix 7.1) the constraints were expressed in a format representing the logic of the relevant tax rules. These constraints were then presented in a mathematical programming format, which enforces the logical conditions using known upper bounds for the variables involved in conjunction with binary variables (Appendix 7.2). The final model was developed in a number of stages, with successive refinements and extensions being made at each stage.

The commercial mathematical programming package XPRESS-MP was selected to create an operational version of the model. Although it would be possible to build computerised optimisation models using a language such as FORTRAN, it was anticipated that there would be difficulties in solving them. Mixed integer programming models (MIPs) are much harder to solve than linear programs of similar size, and even relatively small problems may prove to be computationally insoluble. Commercial MIP codes make use of a system of strategies that have been developed over time and found to work effectively on a wide variety of real problems.

Model 1 was adapted to meet the requirements of the software to give Model 2 (Appendix 7.3). The subsequent extensions and refinements were made using the XPRESS-MP language directly, resulting in the final Model 3 (Appendix 7.4). A number of features were also developed and incorporated to assist the solution process and make it as efficient as possible. Some of these are implicit in the model structure, while others are concerned with control of the actual solution process.

7.2 Model 1 and software selection

A mathematical programming model for optimal capital investment selection after corporate taxes (Farrar, 1994) formed the basis of Model 1, the first stage of the model development. The main aim of the model is to incorporate the offset rules and the carry back and carry forward provisions relating to the treatment of trading losses, ACT and debenture interest, and the rules concerning finance leasing as an alternative to the purchase of capital assets. The model has a mixed integer format, with binary (zero-one) variables being used to represent project acceptance or rejection and to express the logic of the relevant tax rules and the choices they allow. The objective function aims to maximise the sum for all periods up to a horizon date of the net present value to the shareholder of the firm's portfolio of projects, including both ongoing activities and incremental projects selected. Since it represents the firm's cash flows which are available for distribution to shareholders, the discount rate is assumed to be the cost of equity before personal taxes.

Choice of software

The software package was selected on the basis of the following features:

- the maximum number of rows and columns allowed in the matrix;
- ability to deal with integer variables and Special Ordered Sets (see section 7.3.3);
- ease of generating the matrix;

- the facility to separate data from the structure of the model itself;
- user control over the search process; and
- provision of technical support and advice.

Many commercial software packages were unable to deal with matrices of the size generated by the author's models, while others had no integer programming capability, which limited the number of packages which were suitable for this application. The inclusion of a matrix generator was a highly desirable feature. Most packages use the MPS (Mathematical Programming System) format, which orders the model by columns in a fixed format based on the internal ordering of the matrix in the computer. This procedure is unwieldy, as a separate data statement is needed for every two coefficients in the model, and a source of clerical error. It is also inefficient where a large model arises partly from the repetition of a structure, as in this application. The MPS format needs the repetition of data as its input. A matrix generator has two major advantages: it enables the user to input the problem into the package in a 'natural' format of constraints, and it generates models with multiple repetition in an efficient way.

Of the packages that were able to deal with the size of the problem and with the mixed integer format, few offered automatic matrix generation. For instance, HYPERLINDO requires problems to be entered through an MPS or FORTRAN interface. With SAS/OR, it is possible to write a program to generate a matrix, given the necessary programming skills. HYPERLINDO cannot model Special Ordered Sets, which was a disadvantage computationally, and has quadratic programming capability, which was not required. XPRESS-MP has a matrix generator and can incorporate Special Ordered Sets and partial integer variables. All of these allow the user to control the branch and bound search process, although only to a limited extent in the case of SAS/OR.

The package XPRESS-MP was finally selected for the reasons above, and also for a number of other features. The data is kept separate from the structure of the model itself, which is convenient when the model is to be run many times with new data. The data may be taken from widely used spreadsheets and ASCII files. The package has a powerful optimiser which has been field tested on industrial problems. Dash Associates Ltd, who produced and supplied the software, expressed enthusiasm and support for the application and agreed to act as a collaborating establishment. Finally, a free trial and a fifty per cent discount on the purchase price were further incentives.

7.3 Model 2: Adaptation of Model 1 to the software requirements

The first model to be written for the XPRESS-MP software, Model 2, was based on Model 1 and is similar in its scope, but does not include the option of leasing capital assets. A number of adaptations were made in order to meet the requirements of the software. The most significant of these were the method of derivation of mutually exclusive variables, the formulation of the constraints and the use of Special Ordered Sets of variables.

7.3.1 Mutually exclusive variables

The application of the tax rules generates both positive and negative outcomes such as profit or loss. For example, the expression defining Schedule D trading profit or loss may be represented by z, where there will be a profit if $z > 0$ and a loss if $z < 0$. This may be written as

$$x = \max(z, 0)$$

$$y = \max(-z, 0)$$

where x and y represent the positive values of profit and loss respectively. This was the approach followed in Model 1 (for example, constraints 12 and 13 in Appendix 7.1).

The constraints of the mixed integer programming model may then be represented in a format using binary variables to enforce the logical conditions (for example, Williams, 1993). A constraint of the type

$$z - M\delta \leq 0$$

where M is a constant coefficient whose value is the known upper bound for z, forces δ to take the value 1 when $z > 0$. By combining this with the additional constraint

$$z\delta \geq 0$$

δ will take the value 1 if and only if z is non-negative.

Together with these constraints, $x - z\delta = 0$ and $y - z(\delta - 1) = 0$ enforce the conditions $x = \max(z,0)$ and $y = \max(-z,0)$ respectively (for example, constraints 12, 12a, 12b, 13 in Appendix 7.2).

However, it was not possible to express constraints in this way using the mathematical programming software XPRESS-MP which follows the convention that all variables are non-negative. An alternative formulation was therefore used to derive variables as mutually exclusive (non-negative) pairs x and y, where the values of x and y are determined by the sign and value of expression z:

$$x - y = z$$

For instance, if $z = (-7)$ then it is necessary to specify that $x = 0$ and $y = 7$. This requires the logical conditions

$$x > 0 \rightarrow y = 0$$

and $$y > 0 \rightarrow x = 0,$$

which are enforced by the additional constraints

$$x \leq M\delta$$

and $$y \leq M(1 - \delta)$$

where δ is a binary variable and M is the (unsigned) upper bound of z.

This technique was also used to force the model to choose between alternative courses of action, such as carrying a loss forward or back. In this case the constraint has the form

$$a + b = c$$

where a, b, c are non-negative variables, with

$$a \leq M\delta$$

and $$b \leq M(1 - \delta).$$

For example, if $\delta = 0$, $a = 0$ and so $c = b$, whereas if $\delta = 1$, $b = 0$ and $c = a$.

7.3.2 Constraint formulation

Many of the constraints of Model 2 are formulated in a manner which is far more economical than the more conventional mathematical programming style of Model 1. This may be shown by a comparison of the constraint sets dealing with the changes in the unutilised ACT offset capacity (constraints 24 to 40 in Model 1, SA1 to SA33 in Model 2). For example, constraints 30 and 31 in Model 1 are:

$$U_{t-3}^{t} = \max\Big[\big(b\pi_{C_{t-3}} - b\,DIV_{t-3}/(1-b) - \min(SACT_{F,t-3}, U_{t-3}) - \sum_{i=1}^{2} SACT_{DIV_{t-i},t-3}\big), 0\Big] \quad (30)$$

$$SACT_{L_t,t-3} = \max\Big[-\big(b\pi_{C_{t-3}} - b\,DIV_{t-3}/(1-b) - \min(SACT_{F,t-3}, U_{t-3}) + SACT_{DIV_{t-3}} + \sum_{i=1}^{2} SACT_{L_{t-i},t-3} - \sum_{i=1}^{2} SACT_{DIV_{t-i},t-3}\big), 0\Big] \quad (31)$$

(30) defines the unutilised ACT offset capacity in t-3 which is available to absorb surplus ACT carried back from t, as:
[the basic rate of income tax times the adjusted profits in t-3 after the carry back of losses from t, minus the ACT on dividends in t-3, minus the minimum of (surplus ACT brought forward to t-3, unutilised ACT capacity in t-3 after offset of ACT on dividends paid in t-3) minus the sum of any surplus ACT from t-2 and t-1 already carried back to t-3], provided this is positive.

(31) defines the surplus ACT arising in t-3 due to the offset of losses from t, by adjusting the same expression to add back surplus ACT which had already arisen in t-3.

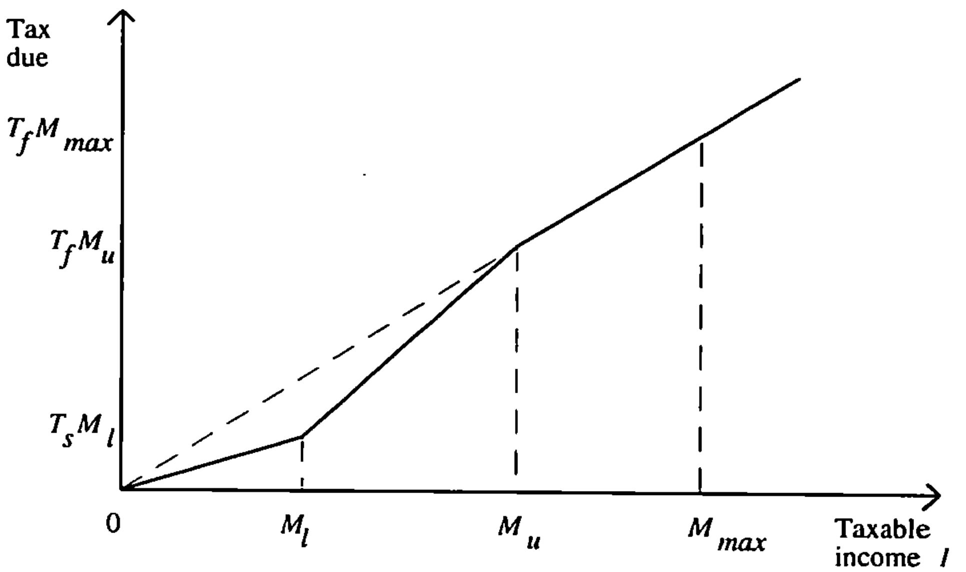
By contrast, Model 2 defines the same two variables in constraint SA19:

```
U5(t-3)-SL3(t-3)=BASR(t-3)*P6(t-3)-BASR(t-3)*P5(t-3)
```

where U5(t-3) is the unutilised ACT offset capacity in t-3 which is available to absorb surplus ACT carried back from t and SL3(t-3) is the surplus ACT arising in t-3 due to the offset of losses from t. Both are determined by the right hand side of the expression above, which is the difference between (the basic rate of income tax in t-3 times the profit in t-3 after the carry back of losses from t) and (the basic rate of income tax in t-3 times the profit in t-3 before the carry back of losses from t) in other words, it represents a comparison of the maximum ACT offset capacity before and after the carry back of losses from t. The simplicity of the formulation is the result of the method of derivation of pairs of mutually exclusive variables described above.

7.3.3 Special Ordered Sets of variables

The structure of the corporate tax function was shown in Fig. 2 and is reproduced below:



M_{max} represents the maximum possible income.

This may be modelled using binary variables to express the logical conditions:

$$I > M_u \rightarrow T = T_f I$$

$$M_l < I \leq M_u \rightarrow T = T_f I - \frac{(M_u - I)}{(M_u - M_l)}(T_f - T_s) M_l$$

$$0 < I \leq M_l \rightarrow T = T_f I - (T_f - T_s) I$$

$$= T_f I - (I/M_l)(T_f - T_s) M_l.$$

Combining these gives:

$$T = T_f I - \left\{ \left[(I/M_l)(1-\delta_m) + \frac{(M_u - I)}{(M_u - M_l)} \delta_m \right] (T_f - T_s) M_l \right\} (1 - \delta_f) \quad (7.1)$$

where

$\delta_m = 1$ if $I \geq M_l$, otherwise $\delta_m = 0$

$\delta_f = 1$ if $I \geq M_u$, otherwise $\delta_f = 0$.

(Appendix 7.1, constraint 17; Appendix 7.2, constraints 17 - 17d)

This method has the advantage that it is not necessary to specify the upper bound on income, M_{max}

However, a less cumbersome and more economical technique was used in Model 2 which represents the non-linear function by a piecewise linear approximation, or Special Ordered Set of type 2 (SOS2). This type of set may contain at most two non-zero variables, and these must be adjacent in the ordering given to the set. Although the conditions represented by the SOS2 may also be modelled by the use of binary variables, there is a great computational advantage in the set specification (Beale and Tomlin, 1969).

T is related to I in the SOS2 by the following relationships:

$$I = 0\lambda_1 + M_l\lambda_2 + M_u\lambda_3 + M_{max}\lambda_4 \tag{7.2}$$

$$T = 0\lambda_1 + T_s M_l\lambda_2 + T_f M_u\lambda_3 + T_f M_{max}\lambda_4 \tag{7.3}$$

$$\sum_{i=1}^{4}\lambda_i = 1 \tag{7.4}$$

$\lambda_i \geq 0$ for all i.

The λ_i may be interpreted as weights attached to the respective vertices.
Because of the non-convex structure the SOS2 condition that only two adjacent λ_i may be non-zero is necessary, to ensure that taxable income is correctly located between two of the vertices and that the amount of tax due is calculated correctly. If this stipulation is not made, the model generates an illegal optimal solution in the case where $M_l < I < M_u$, by specifying I and T in terms of λ_2 and λ_4 in order to avoid paying tax at the higher marginal rate.

Under the current (1994/95) income limits,

M_l =£300,000

M_u=£1,500,000

T_s =25 per cent

T_m =35 per cent

T_f =33 per cent

Under the current tax limits, if taxable income is assumed to be £500,000 and $M_{\max}$ is £10 million:

$$500{,}000 = 0\lambda_1 + 300{,}000\lambda_2 + 1{,}500{,}000\lambda_3 + 10{,}000{,}000\lambda_4$$

$$\lambda_1 = 0$$

$$\lambda_2 = 5/6$$

$$\lambda_3 = 1/6$$

$$\lambda_4 = 0$$

The amount of tax due is then calculated as:

$$T = 0\lambda_1 + 75{,}000\lambda_2 + 495{,}000\lambda_3 + 3{,}300{,}000\lambda_4$$

$$T = £145{,}000.$$

7.4 Model formulation

The formulation and solution of the models were performed in a number of stages:

(i) Models were assembled using MS-DOS Editor in files with the extension `.mod`, in terms of constraints or rows. The global entities were specified as such in the model file.

(ii) The model matrices were then transposed by the matrix generator MP-MODEL into the standard Mathematical Programming System (MPS) format which requires its input in terms of variables, or columns.

(iii) The transposed matrix formed the input for the optimisation module, MP-OPT. This performs a linear optimisation first, and if the linear problem is found to be feasible a search for integer solutions can proceed. The output was written to a solution file.

The models' data inputs were imported from tables generated from external sources, thus enabling the data to be changed easily without having to alter the actual model file. In each run, the current data in the specified sources were taken to create the model matrix. The external data sources used were ASCII (text) files and the spreadsheet package QUATTRO-PRO, which conforms to the Lotus 1-2-3 format.

7.4.1 Structure of Model 2

The LET section defines symbols to be used later on in the model and establishes the model parameters, such as the number of time periods. If a five year future horizon is assumed, the total number of time periods relevant to the model is eleven because the six previous years are relevant to the carry back provisions. The current period is denoted by `t=7` and the horizon by NT.

TABLES and DISKDATA

The TABLES section defines the data tables to be used in generating the model matrix, with their respective dimensions specified as (rows, columns). The DISKDATA section gives the source of the data input, with ASCII files having the extension .dat. DISKDATA -1 lists the data to be taken from specified blocks of spreadsheets of the Lotus 1-2-3 format, that is, from the package QUATTRO-PRO.

ASSIGN

The ASSIGN section assigns computed data values to scalars or tables with specified dimensions: for example, the proportion of a net dividend payable as ACT (ACTR) is computed from the relevant data in the table of the basic rate of income tax (BASR).

VARIABLES

This section defines the model variables. The models contain many similar variables which occur in a repetitive way, often relating to each time period. These are named using a root and a number of indices. For example, the statement

```
U1(t=7:NT)
```

where NT is the number of time periods in the model, generates five variables when NT=11. The range of the time period indices includes data input values where appropriate: for instance, the relevant taxable profits associated with the years before the model's start time.

CONSTRAINTS

This defines the constraints which act upon the decision variables of the model. Each constraint has the form

[name] [subscript range] : [linear expression 1] R [linear expression 2]

where R is one of

> greater than or equal to

< less than or equal to

= equal to

$ unconstrained.

Constraints were specified in the same way as variables, by their given name and indices. Summation expressions follow the 'Σ' notation conventionally used in mathematical programming. For instance, the expression for capital expenditure on projects accepted in time period t may be written as

```
SUM(p=1:NPROJ)PROJCOST(p,t)*accept(p)
```

where p represents an individual project and NPROJ the total number of projects. The variable accept is specified as binary in the BOUNDS section of the model.

Since the software assumes that variables are non-negative, it is not necessary to enforce this condition explicitly with constraints.

The objective function is included as an unconstrained expression within the CONSTRAINTS section.

SETS

This section defines the Special Ordered Sets used in the model. Associated with each set of variables there is a reference row whose function is to impose an order on the set members. The format of the SETS section is

[set name] : [membership expression] T [reference row]

where T is either .S1. or .S2.

BOUNDS

The BOUNDS section specifies simple bounds and integer and semi-continuous variables. The bounds specified in this model are the binary variables (.BV.) used to enforce logical conditions.

DIRECTIVES

This section creates a DIRECTIVES file which helps direct the solution process by assigning priorities to the non-linear entities such as binary variables. A greater priority (.PR.) is assigned to those entities that are regarded as having a relatively greater importance in determining a solution. The lower the priority number, the greater the entity's priority, with the default value being 500.

7.4.2 Constraints of Model 2

!PREVIOUS DATA

This first group of constraints defines the values of variables relating to the firm's tax position at the start time of the model, which are contained in the data table PREV.DAT.

!DETERMINATION OF PROFIT/LOSS, EXCESS CHARGES ON INCOME

The treatment of profits and losses and the associated model variables are summarised in Table 7.1.

The determination of the variables proceeds in a number of steps:

(i) Constraint TXN1 derives the trading profit P1(t) or loss L1(t) for the period as: the net operating cash flows from ongoing activities and from accepted projects, plus (minus) any increase (decrease) in working capital due to project acceptance. Constraints TXN2, TXN3 ensure that either P1(t)=0 or L1(t)=0 in any integer solution.

(ii) If there is a trading loss, the model decides whether to carry it forward at this stage or set it against other income of the period, if any (constraints OFS1 to OFS3). If the latter decision is made, the setoff of any trading loss against other income of the period is accommodated by constraints OFS4 to OFS6.

(iii) Any loss remaining after this step (L1b(t)) may then be either carried forward, or carried back and set against previous years' total profits, with any

Table 7.1

Relationships between model variables for profits and losses

Profit	*Model variable(s)*	£	*Losses c/f* £
Trading profit in t	P1(m,t)	x	
less: losses brought forward to t	LF(m,t-1)	(x)	
Excess of losses brought forward to t, if any	LFF(m,t)		x
		x	
Other income	IN1(m,t)+iL(t)*LNS(m,t)	x	
	P2(m,t)	x	
less: interest on long term debt	DBIN(m,t)	(x)	
	P3(m,t)	x	
Excess of charges on income in t, if any	XC(m,t)		x
less: group relief if applicable		(x)	
Profits on which Corporation Tax is payable	P3a(m,t)	x	
Loss			
Trading loss in t	L1(m,t)	x	
less: amount surrendered as group relief (if applicable)	L1g(m,t)	(x)	
		x	
less: loss (if chosen for carry forward)	LF1(m,t)	(x)	x
		x	
less: loss (if set against other income in t)	IN1(m,t)-IN2(m,t)	(x)	
		x	
less: loss (if chosen for carry forward)	LF2(m,t)	(x)	x
Amount of loss to be carried back	LB(m,t)	x	
less: set against profits in t-1		(x)	
	L2(m,t)	x	
less: set against profits in t-2		(x)	
	L3(m,t)	x	
less: set against profits in t-3		(x)	
	L4(m,t)		x
Loss carried forward to next period	LF(m,t)		x

remainder being carried forward. This decision again is made by the model (constraints CBF1 to CBF3).

(iv) Any trading profit for the period will be reduced by any losses brought forward but these may not be set against other (non-trading) income. Constraints LSS1 to LSS3 ensure that any losses brought forward to t are set only against trading income, with the outcome being either a remaining amount of profit, P2(t) or a remaining amount of loss LFF(t) to be carried forward and set against the next year's trading profit.

(v) Finally, the interest on long term debt, DBIN(t) is set against the total profits, if any, remaining after the setoff procedures described above, generating either an amount of profit P3(t) upon which that year's corporation tax payment will be assessed, or an amount of excess charges on income XC(t).

!SETOFF OF LOSSES CARRIED BACK

In any one time period t, the profits may be adjusted a number of times as a result of successive carry backs of losses from subsequent periods. The constraint indices and time subscripts reflect this perspective. Constraints LCB1 to LCB3 ensure that the setting of a loss from t+1, LB(t+1) against the profits of t, P3(t) will result in an outcome where there remains either a positive amount of the loss from t+1 (L2(t+1)) or a positive amount of profit in t, P4(t). Constraints LCB4 to LCB9 deal with any carry back of losses from t+2 and t+3 respectively in a similar way.

!TOTAL LOSS CARRIED FORWARD

The total amount of losses carried forward comprises the amounts which the model decided to carry forward in steps (ii) and (iii) of the profit/loss determination procedure above, plus the remainder of any losses carried back and set against previous years' profits, plus any excess charges on income.

!CALCULATION OF TAX PAYMENTS

The amounts of corporation tax payable in each period `t` will change each time a loss from a subsequent period is carried back and set against the profits in `t`. The constraints `TN5` to `LSM4` calculate the tax payments corresponding to each level of adjusted profits in `t`, so that the tax rebates arising from the carry back of losses may be calculated. The method corresponds to the SOS2 formulation discussed in section 7.3.3.

!DETERMINATION OF ACT SETOFF CAPACITY / SURPLUS ACT

The treatment of surplus ACT and the associated model variables are summarised in Table 7.2.

The constraint set relating to the treatment of ACT is based on the concept of an 'unutilised capacity' in each period for ACT setoff. This is determined by two criteria: the maximum setoff, given by the basic rate of income tax multiplied by the taxable profits for the period, and the amount of ACT already set off in that period. The unutilised capacity in any period `t` will be affected by two possible events: the reduction in taxable profits by the carry back of losses from `t+1, ..., t+3`, and the carry back of ACT from `t+1, ..., t+6`. If the maximum setoff in `t` is reduced

Table 7.2

Relationships between model variables for surplus ACT

	Model variable(s)		*Surplus c/f*
		£	£
Surplus ACT from distributions:	SD(m,t)	x	
less: Amount chosen for carry forward	SDF(m,t)	(x)	x
Amount chosen for carry back	SDB1(m,t)	x	
less: surplus set off in t-1		(x)	
	SDB2(m,t)	x	
less: surplus set off in t-2		(x)	
	SDB3(m,t)	x	
less: surplus set off in t-3		(x)	
	SDB4(m,t)	x	
less: surplus set off in t-4		(x)	
	SDB5(m,t)	x	
less: surplus set off in t-5		(x)	
	SDB6(m,t)	x	
less: surplus set off in t-6		(x)	
Remainder	SDB7(m,t)		x
ACT surrendered from parent and unutilised (if applicable)	ADs(m-1,t)		x
Surplus ACT brought forward to t and not offset in t	SFF(m,t)		x
Surplus ACT arising from the carry back of losses from t	SL1(m,t), SL2(m,t), SL3(m,t)		x
Total surplus ACT carried forward	SF(t)		x

by losses carried back from subsequent years, so that it is less than the amount of ACT already set off, surplus ACT will be created.

(i) Constraints SA1 to SA3 set the ACT payable on the net dividend DIV(t) against the initial offset capacity based on the profit P3(t). The outcome is either a surplus of ACT, SD(t), which may be freely split between an amount carried forward and an amount carried back (constraint SAD), or else an amount of unutilised capacity U1(t).

(ii) Constraints SA4 to SA6 set any surplus ACT brought forward to t against U1(t) to generate either an amount of surplus ACT which must be carried forward again to t+1 (SFF(t)) or an amount of unutilised capacity remaining, U2(t).

(iii) U2(t) is set against the reduction in the maximum ACT offset arising from any reduction in taxable profits due to the carry back of losses from t+1, generating either an amount SL1(t) of surplus ACT arising from the loss carry back, or an adjusted amount of unutilised capacity U3(t) (constraints SA7 to SA9).

(iv) In constraints SA10 to SA12 the amount of the surplus ACT from t+1 chosen for carry back, SDB1(t) is set against U3(t). The outcome is either an amount of untilised capacity remaining, U3A(t), or a remaining amount of ACT from t+1 which will then be carried back further to t-1.

(v) The constraints SA13 to SA24 deal with the carry back of losses and ACT from t+2 and t+3 in a similar manner to constraints SA7 to SA12.

(vi) No losses may be carried back to `t` from `t+4`, ..., `t+6` and so the relevant adjustments made to the unutilised capacity in `t` relate only to the carry back of ACT from those years (constraints `SA25` to `SA33`).

Constraints `ACTP`, `ACTA` relate to the ACT payment actually made and the amount which may be set against mainstream corporation tax respectively.

The total amount of ACT which is carried forward from any period `t` is given by the amount chosen to be carried forward by the model in step (i), plus the surplus generated in step (ii), plus any remainder of the amount selected for carry back in step (i), plus any surplus generated from the carry back of a loss from `t` (constraint `TSF`).

7.4.3 The objective function

The model's objective is to maximise: the cash flows arising from project acceptance, less the associated capital expenditure, less the relevant tax payments made and rebates received by the firm, all discounted at a rate which reflects the timing of the payments. Since only those cash flows which are affected by the model's decisions are included in the objective function, it excludes debt interest payments, ACT payments and the cash inflows from the firm's existing activities, which are independent of the project accept/reject decisions under the assumptions of the model. The incremental NPV due to the accepted projects may be determined from a comparison of the objective values of matrices run with and without a set of constraints to limit project acceptance. The case where the model is constrained to ensure that no projects are accepted, represents the optimal tax strategy that could be obtained from the firm's ongoing cash flows. This objective value should be deducted from that of the optimal project combination to determine the improvement in net present value which is attributable to that set of projects.

7.5 Assumptions of Model 1 and Model 2

(i) there is no capital rationing;

(ii) the projects are funded entirely from retentions;

(iii) acceptance of the projects will not change the firm's risk in any period;

(iv) the firm's cost of capital is constant;

(v) the horizon date is set beyond the completion of all the projects under consideration;

(vi) the firm expects to have sufficient profits in the year after the horizon date to absorb any amount of losses and ACT carried forward from the horizon;

(vii) the firm has no franked investment income;

(viii) the firm's non-trading income is unaffected by project acceptance;

(ix) assets are purchased at the start of the year;

(x) the net operating cash inflows, debt interest payments and ACT payments occur midway through the year;

(xi) other tax effects occur nine months after the year end;

(xii) any timing differences between the tax year and the firm's own accounting period are ignored;

(xiii) any interim dividend payments are ignored.

Although the projects are not funded by debt, the model takes into account the tax treatment of the firm's existing debt interest payments, which may be significant for project appraisal. The dividend payments are included for the same reason. The discount rate is assumed to be constant over time and unaffected by project acceptance, under the assumptions (ii) and (iii).

7.6 Development of Model 3

Model 3 was achieved by substantially developing and extending Model 2. The new areas incorporated into the model relate to the tax effects that may occur in a group structure; the treatment of the firm's financing decisions, including leasing; the formulation of a dividend-based objective function which aims to maximise shareholder wealth rather than net present value. The appropriate discount rate to be applied to the shareholders' returns is determined exogenously by an iterative process, using the Capital Asset Pricing Model.

7.6.1 Group tax effects

Model 3 incorporates the group relief provisions for trading losses, excess charges on income and ACT, and the group income election for the payment of intra-group dividends, assuming a simple group structure where there is a parent firm and one subsidiary which may be fully or partially owned by the parent. The parameter NM was introduced to define the number of member firms in the structure. The tables of data inputs relating to the tax situation of the firm(s) were then extended by the additional dimension NM, and the model variables applicable to both parent and subsidiary were assigned an additional index NM. All the modifications to deal with areas which do not apply to a single firm are enclosed within the statements `IF NM>1...(ELSE)...ENDIF` and are ignored if `NM=1`. The most significant of these relate to the treatment of losses, excess charges on income and ACT. The limitation of the availability of relief for losses and excess charges to 75 per cent group structures is enforced by the `PCENT` parameter. The model assumes that if `NM>1` there is at least a 51 per cent group structure, since if this is not the case there can be no group tax effects within the model.

The subsidiary's tax situation is treated similarly to that of the parent and the model allows the same range of tax strategies within the subsidiary, in addition to the potential tax interactions between parent and subsidiary such as group relief for losses and ACT. The model can therefore consider simultaneously all the tax options available, and will utilise the group relief provisions only to the extent that they provide an optimal solution. Since the main aim of including a group structure in the model is to incorporate the group tax effects that may change the cash flows of the group and hence the returns to its shareholders, the model assumes that the subsidiary's own investment and financing decisions are determined exogenously.

Group relief for losses

The availability of group relief for losses is determined by the constraints AVP1 to AVP3. Losses and excess charges surrendered may be set against the claimant's total profits including chargeable gains, after relief for the claimant's losses brought forward and of the current period, whether claimed or not, and after deducting charges on income. This determines the maximum amount of group relief that the claimant company can accept and it is possible for only a partial claim to be made. Each member company's trading loss and excess charges, if any, are therefore divided between an amount retained and an amount surrendered to another company for group relief (constraints XCG1, GRL), and the amounts retained by the company can be given other forms of loss relief such as carry forward or carry back. Following the legislation, the charges available for surrender are limited to the excess of charges over the total profits before deducting losses carried forward from earlier periods (constraint XCG2).

Advance Corporation Tax

The variables DIV(NM,NT) refer to net dividends paid by the parent and subsidiary companies on which ACT will be paid shortly afterwards. The subsidiary's dividend payment is apportioned between the parent company and minority interests (constraints DVP2, DVP3).

If a group income election has been made (GELEC=1) there is no ACT payment due on the subsidiary's dividend payment to the parent, EDIV. If the subsidiary is not wholly owned, then ACT will be paid on its distributions to minority interests. The parent pays ACT on the whole of its dividend payment.

If there is no group election (GELEC=0) the subsidiary pays ACT on the dividend payments it makes to the parent and outside the group. The parent's ACT payment will be based on the excess, if any, of its own gross dividend over the grossed up dividends received in the period (constraints GDP1 to GDP3). If the subsidiary's gross dividend to the parent exceeds the parent's own gross dividend, then surplus franked investment income may arise and this will be set against future periods' distributions on which ACT is payable. It is assumed that the subsidiary has no franked investment income.

The surrender of ACT from parent to subsidiary is dealt with in the constraints GSA to SA1. The parent's ACT on its dividends for the period is divided into an amount retained and an amount surrendered. The surrendered ACT is utilised first by the subsidiary and the maximum offset capacity is determined by the adjusted profits of the subsidiary after any group relief from another group member (constraint GA1). The surrendered ACT from the parent may therefore free the subsidiary's own ACT to be carried back.

7.6.2 Financing and the uses of funds

The treatment of the firm's financing includes the options of letting the model choose the levels of debt and equity funding, specifying an increasing cost of debt as the proportion of debt funding increases, and the inclusion of a target debt/equity ratio.

Long term borrowing

The firm's cost of debt may be determined by its debt/equity ratio, so that it faces an increasing interest rate on incremental debt as the ratio rises.

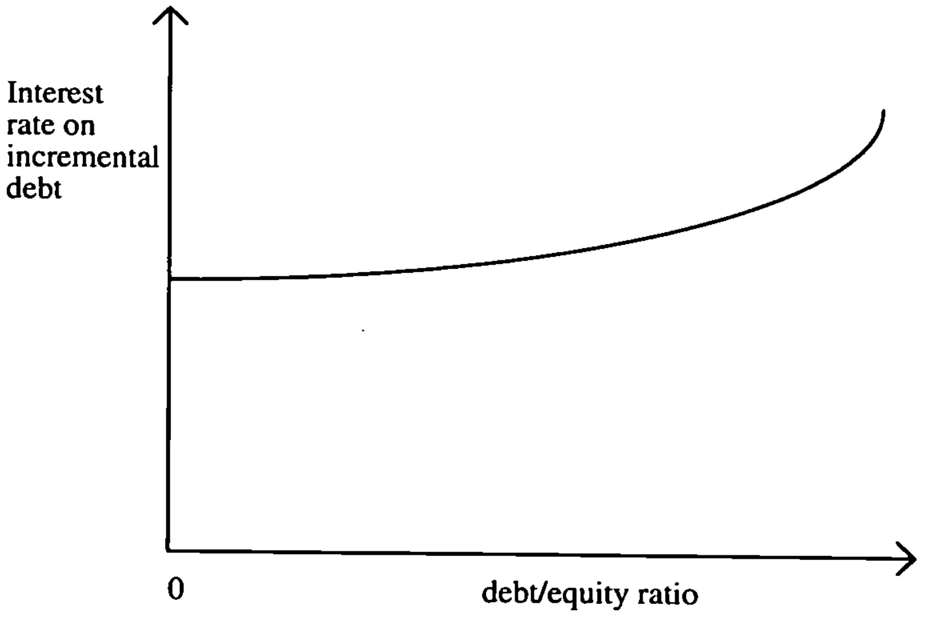


Fig. 11 Increasing cost of debt as debt/equity ratio increases

Unfortunately, this non-linearity cannot be modelled as a SOS2. In order to create the linear segments approximating the function, a number of values of the debt/equity ratio would need to be specified, and since the amount of equity is a model variable, these values could not then be multiplied by other variables representing the 'weights' given to the set members.

An alternative approach was taken, treating the debt as a series of steps defined in terms of the debt/equity ratio, each step having a higher rate of interest than the last.

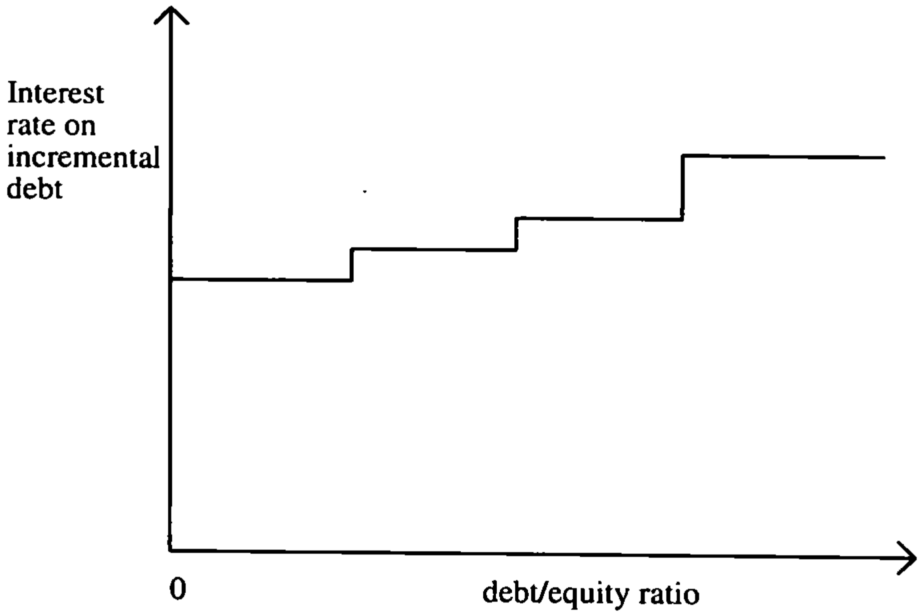


Fig. 12 A stepped approximation to an increasing cost of debt

The series of constraints `DF1` to `DF19` specifies the quantity of debt held during `t` in this manner. Constraint `DF1` tests whether the opening balance of debt, `DB(t-1)` plus new debt issued at the start of `t`, `DBI(t)` is greater than the upper limit for the first step. This upper limit is given by a specified proportion, `DLMT(t,1)` of the amount of equity for the period, `EQ(m,t)`. Constraints `DF2`, `DF3` force `x1(t)` to be zero if constraint `DF1` has RHS>0, and `D2(t)` to be zero if RHS<0.

Similarly, constraints `DF4` to `DF6` test whether the amount of debt which exceeds the upper limit of the first step also exceeds the upper limit of the second step, and the process is repeated for subsequent steps (`DF7` to `DF12`). The amounts of debt falling within each step are determined by the variables y1(t) to y4(t) and thus the debt interest for the period may be calculated (constraint DBNT).

The variable DBC(t) is included in order to allow the firm to issue debt whose function is not to fund specific capital projects but to maintain a target debt/equity ratio, if applicable. This debt is assumed to be free of issue costs.

Short term lending or borrowing, assumed to be mutually exclusive in any one period, are determined by constraints OB1 to OB4. In the initial period this is given by the opening cash balance, minus the issue costs of debt and equity to fund capital projects accepted in t, minus the minimum cash balance needed by the business. In future periods short term lending or borrowing also takes into account the amount of retentions in the previous period, minus any negative balance of funds available for distribution, and an adjustment for any changes in the minimum level of cash held. The subsidiary's short term lending and borrowing are similarly determined but without the variables relating to project acceptance by the parent. Short term borrowing is restricted to an upper bound DBSMAX(m, t).

Capital structure constraints UDL1 to LDL1 restrict the upper bound of debt held during a period to a proportion given by the parameter DLMT(5, t) of the opening value of equity for the period. If a target debt/equity ratio is specified by the parameters DBMAX and DBMIN, it is enforced by the constraints UDL5 and LDL1.

Uses of funds

The balance of funds which are available for distribution (by the parent company in a group structure) is determined by the constraints CLB1 to CLB3. In each year this is given by the net operating income from existing projects plus projects accepted by the model, plus the value of asset disposals, if any, plus income from other sources, plus the gross dividend received from the subsidiary, minus the tax payable on the previous period's taxable profits, plus the ACT from the previous period which may be set against MCT for that period, minus debt interest payments and loan repayments, minus the rental payments on leased projects accepted, plus the amount and interest of short term lending during the period, minus the amount including interest of any short term borrowing during the period. In the subsidiary company the funds balance is determined similarly, with the debt payments as an exogenous input.

Any positive amount of funds `CSHP(m,t)` is divided between dividends paid and the associated ACT payment, amounts retained and amounts invested in capital projects. In the subsidiary `CSHP(m,t)` is divided between dividends paid outside the group with the associated amount of ACT, dividends paid to the parent, retentions and amounts reinvested in capital equipment. Project funding is by debt and equity issues and retentions (constraint `FUND`).

The values of debt and equity for the period are given by the constraints `DBT1` to `EQTG`. The closing book value of debt is the closing balance of debt for the previous period, adjusted by the amounts issued to fund new investment or to maintain the debt/equity ratio, and by repayments made at the end of the period.

The value of equity funds is approximated by the opening market value, plus subsequent injections of equity capital from shareholders, adjusted for any change in retentions. In a group structure a proportion of the subsidiary's equity value is included, determined by the `PCENT` parameter.

The model assumes that markets are efficient, so that the firm's current market value must incorporate the present value of its expected future cash flows including those arising from future investment opportunities. This has a number of implications for the firm's investment and financing decisions:

- The firm's objective should be to maximise its current market value (not its future value, because this will be included in current value) assuming that shareholders' consumption opportunities are affected by the firm's investment and financing decisions only through wealth changes.

- There can be no gain from manipulating earnings per share, since the underlying cash flows which determine the market value are unchanged.

- If new equity issues are made at a market price that reflects future cash flows, there is no reason to consider dilution (the sharing of positive net present value projects with new shareholders).

Since it is unlikely that the firm would wish to issue small amounts of equity at frequent intervals, the model makes the assumption that equity finance is normally raised, if at all, in an issue of some minimum size.

This may be represented using a semicontinuous variable x, which is restricted to take either the value zero, or values at least as large as unity and no larger than some specified upper bound. As with the corporate tax calculation, the situation could be modelled using binary variables, but semi-continuous variables have a computational advantage (Beale and Tomlin, 1969).

One of the following two constraints must be satisfied:

$$x = 0 \qquad \text{or} \qquad x \geq a$$

where $a > 0$.

This condition is modelled by specifying the quantity x in relation to a semicontinuous variable y, which may take the value zero or any value between 1 and a finite upper bound M / a, where $x \leq M$. The necessary constraint is then $x = ay$, so that

$$x = 0 \rightarrow y = 0$$

$$x = a \rightarrow y = 1$$

$$a < x \leq M \rightarrow 1 < y \leq M / a$$

$$x = M \rightarrow y = M / a$$

This is enforced in the model by specifying a semicontinuous variable `issue` in the BOUNDS section:

```
issue(t=7:NT).SC. (EQIMAX/EQIMIN)
```

and including a constraint on equity issues `EQI`:

```
EQI(t)=EQIMIN*issue(t)
```

with the values of `EQIMIN` and `EQIMAX` being specified in the `LET` section.

7.6.3 Leased assets

The calculation of the finance charge component of lease payments utilises the data given in the tables of rental payments, implicit interest rates and the purchase cost of the asset. It is assumed that the first rental payment occurs at the beginning of the period in which the lease commences and that the payments are equal. The rental payments are calculated within the spreadsheet file from which the model takes its data inputs, so that for instance for a three year project the rental payment R is determined given the purchase cost C and the implicit rate of interest i from

$$C = R + R/(1+i) + R/(1+i)^2$$

$$R = C/\left[1 + 1/(1+i) + 1/(1+i)^2\right]$$

The finance charge may then be calculated as in Model 1 (constraints 9 and 10). The same approach is used in the `ASSIGN` section of the model to calculate the finance charge for a leased project in a particular time period, `FC(p,t)`. It is necessary for the model to be able to distinguish between the first, subsequent, and final periods of the lease. This is achieved by conditional generation of the appropriate statements, utilising the data given in the tables for lease rentals (`PROJRENT`), purchase cost of the asset (`PROJCOST`) and implicit rate of interest (`iIMP`).

If the rental payments are zero in the previous year but greater than zero in both the current and the next year, then the necessary conditions for the first year of the lease are present and the finance charge is calculated accordingly:

```
FC(p=1:NPROJ,t=7:NT-1|PROJRENT(p,t+1)>0.AND.
PROJRENT(p,t)>0.AND.PROJRENT(p,t-1)=0)
=iIMP(p)*PROJCOST(p,t)-iIMP(p)*PROJRENT(p,t)
```

where the symbol | has the meaning 'provided that the following conditions are true'.

If the rental payment is greater than zero in the previous year and in the next year, then the finance charge is calculated from

```
FC(p=1:NPROJ,t=8:NT-1|PROJRENT(p,t+1)>0.AND.
PROJRENT(p,t-1)>0)=FC(p,t-1)+iIMP(p)*FC(p,t-1)
-iIMP(p)*PROJRENT(p,t)
```

In all other conditions the finance charge will be zero.

7.6.4 The horizon

The horizon is set at the point where all potential projects currently being appraised will be completed.

Horizon valuation

The horizon value may be represented by the value of the firm's funds and assets at the horizon after its obligations to debt holders have been paid. This was the approach taken by, for example, Bhaskar (1978), Pointon (1981b). Following the approach of Pointon (1981b) the tax effects occurring after the horizon date have been included in the model. The gross corporation tax paid in respect of the final period's profits takes into account debt interest for the final period and is reduced by the ACT on the final period's distributions which may be set against it. Since this approach does not represent an actual expectation that the firm will cease to exist at the horizon and pay a final liquidating dividend to its shareholders, it would be irrational to write off the surplus ACT and losses carried forward at the horizon, since these may be set against future tax liabilities. The horizon value therefore includes these, assuming that they will be relieved in the year following the horizon date and that relief for losses will occur at the 'full' rate of corporate tax. Since the firm's overall tax situation beyond the horizon is not known, more specific assumptions cannot be made. The average cost of equity over the periods prior to the horizon (`Avke`) is used to discount tax effects back to the horizon date.

The 'book' value of fixed assets at the horizon is used as a proxy for their true value. Under the 'liquidation' assumption it would be consistent to assume that the assets' estimated sale proceeds would be the correct valuation.

In a group structure, the horizon value of the subsidiary is derived similarly, but without the terms relating to project acceptance in the parent firm and surplus franked investment income. The horizon value of the group is then given by that of the parent plus the specified proportion of the subsidiary's value (constraint MVA1).

The objective function is then to maximise the discounted sum of gross dividends paid to shareholders, less the discounted sum of new equity issues, plus the relevant discounted horizon value.

Post-horizon dividends

Empirical evidence (section 5.2.2) suggests that dividends act as a signal to shareholders of future cash flows, while it appears less likely that firms attract tax clienteles of investors in differing personal tax positions. For this reason the model allows the option of a constant growth pattern in the actual level of dividends, rather than a constant ratio of dividends to retentions.

If a stable dividend growth constraint is incorporated into the model, so that each year's dividend payment D_t is at least $(1+g)D_{t-1}$, and if it is assumed that the firm's activities will continue indefinitely beyond the horizon, then the value of assets at the horizon must be sufficient so that either

(i) the level of dividend payment at the horizon can be maintained in future years, or

(ii) the rate of dividend growth can be maintained indefinitely.

If (i) is assumed then A_H, the value at the horizon of the firm's post-dividend assets which are expected to generate net residual cash flows at a rate equal to the cost of equity k_e in each period after the horizon, must be at least equal to the value of a perpetual horizon level dividend capitalised at the rate k_e:

$$A_H \geq D_H / k_e \tag{7.5}$$

This was the approach taken by Weingartner (1966).

On the other hand, if (ii) is assumed, then A_H must be equal to the sum of a perpetual series of dividend payments growing at the rate g:

$$A_H = D_H(1+g)/(1+k_e) + D_H(1+g)^2/(1+k_e)^2 + D_H(1+g)^3/(1+k_e)^3 + \dots \quad \text{(i)}$$

If A_H is less than the sum of this stream of dividends, the firm will not be able to maintain the level of dividend growth.

Multiplying (i) by $(1+g)/(1+k_e)$:

$$A_H\,(1+g)/(1+k_e) = D_H(1+g)^2/(1+k_e)^2 + D_H(1+g)^3/(1+k_e)^3$$

$$+ D_H(1+g)^4/(1+k_e)^4 + \dots \quad \text{(ii)}$$

$$D_H(1+g)^n/(1+k_e)^n \to 0 \text{ as } n \to \infty$$

(i) - (ii) gives

$$A_H - A_H\,(1+g)/(1+k_e) = D_H\,(1+g)/(1+k_e)$$

or

$$A_H = \frac{D_H(1+g)/(1+k_e)}{1-(1+g)/(1+k_e)}$$

that is,

$$A_H = \frac{D_H(1+g)}{k_e - g}. \tag{7.6}$$

At the end of $H+1$, assuming retained funds earn the required rate of return k_e, the firm will have an amount available for distribution or retention of $(A_H - D_H)(1+k_e)$ and will pay a dividend of $D_H(1+g)$. The amount of retained funds at the end of $H+1$ will therefore be $(A_H - D_H)(1+k_e) - D_H(1+g)$.

At the end of $H+2$, the firm will have $(A_H - D_H)(1+k_e)^2 - D_H(1+g)(1+k_e)$ available for distribution or retention, and will pay out $D_H(1+g)^2$ as dividend. The retention will be $(A_H - D_H)(1+k_e)^2 - D_H(1+g)(1+k_e) - D_H(1+g)^2$, and so on.

The amount retained at the end of any period $H+t$ can be expressed as

$$(A_H - D_H)(1+k_e)^t - D_H \sum_{n=1}^{t} (1+g)^n (1+k_e)^{t-n}$$

or,

$$(A_H - D_H)(1+k_e)^t - D_H(1+g)\left[\frac{(1+k_e)^t - (1+g)^t}{k_e - g}\right] \tag{7.7}$$

The number of years at which the rate of dividend growth can be maintained is given by the highest integer value of t for which equation (7.7) is greater than zero.

The stipulation can therefore be made at the horizon that $A_H \geq \frac{D_H(1+g)}{k_e - g}$. Where the dividend growth assumption is made (constraint DMIN), the model ensures that the horizon value must be at least as great as the horizon dividend multiplied by the factor GFAC2 (constraints GDST, DST3). The net dividend is taken as the relevant payment, assuming that the firm can fully recover its ACT and ignoring the small difference in the amounts of ACT payable from one period to the next.

7.7 Derivation of the cost of equity

The returns to shareholders are discounted at the cost of equity, but the appropriate discount rate cannot be determined within the model using the mixed integer programming approach. This is because the objective function contains variables which are divided by terms including the discount rate, which therefore cannot itself be a variable as this would introduce an 'illegal' type of non-linearity into the objective function. Also, the MIP formulation means that there is no straightforward interpretation of the shadow prices in the solution output, which has been used in LP models (for example, Carsberg, 1974) to determine opportunity costs of capital.

Further, it was argued in Chapter 3 that the returns to shareholders should be discounted at a rate which incorporates a market-based measure of risk. The CAPM required rate of return for each period is estimated by running the model under each of three possible 'states of nature' X, Y and Z, with corresponding sets of cash flows for the firm's ongoing activities and for additional projects under consideration. The discount rate is initially taken as the expected rate of return on the market portfolio, implying that the firm has a beta of 1. The returns to shareholders in each state of nature for each period are taken as the residual cash flows which accrue to the ordinary shareholders, that is, the amounts arising within the period which are available for distribution, divided by the opening value of the firm's equity. The returns are gross of personal taxes, because the returns on the market portfolio are adjusted to be gross of personal tax. This output from the model forms the basis for the calculation of a CAPM required rate of return for each period, using a spreadsheet model (Table 7.3). The risk free rate is assumed to be 6 per cent, which is typical of the yield on British government securities at the time of writing (December 1993). The expected return on the market portfolio is derived from the historic average for the UK market risk premium, which has been estimated by Dimson and Brealey (1978) and more recently by Allen *et al* (1986) as 9 per cent and 9.15 per cent respectively.

The model is then re-run, taking the objective function value from the first run as the opening value of equity for the first period, and the required rates of return from the first run as the discount rates to be applied to the residual cash flows. The new discount rates may cause the model to make different decisions in the second run. This iterative process is repeated until a satisfactory convergence is achieved. The method will be discussed in more detail in the following chapter.

7.8 The solution process

A mixed integer programming problem is solved either when an optimal solution is produced, or when it is shown to be either unbounded or infeasible. A number of different methods for solving problems of this type have been developed and mathematically elegant approaches exist such as cutting planes methods, enumerative methods and pseudo-Boolean methods which exploit the analogy with Boolean algebra. In practice the most successful method for solving this type of problem is the relatively crude 'branch and bound' algorithm, which is used by almost all commercial packages with an integer programming facility including XPRESS-MP.

Table 7.3

Example of cost of equity derivation using the Capital Asset Pricing Model

State of nature		X	Y	Z
Probability		0.3	0.5	0.2
Return to shareholders for period, £'000		1,190	1,590	1,870
Opening value of equity funds, £'000	10,000			
Return		0.1190	0.1590	0.1870
Return on market portfolio		0.1150	0.1550	0.1900
Risk free rate		0.0600	0.0600	0.0600
Expected risk free return	0.06			

Covariance of returns with the market portfolio

Outcome	*Return*	*Expected return*	*Difference (i)*	*Rm*	*Expected Rm*	*Difference (ii)*
X	0.1190	0.1526	-0.0336	0.1150	0.1500	-0.0350
Y	0.1590	0.1526	0.0064	0.1550	0.1500	0.0050
Z	0.1870	0.1526	0.0344	0.1900	0.1500	0.0400

Outcome	*Cross product (i) x (ii)*	*Prob. (iii)*	*Average (i)x(ii)x(iii)*
X	0.001176	0.3000	0.000353
Y	0.000032	0.5000	0.000016
Z	0.001376	0.2000	0.000275

Variance of firm returns	0.000596
Variance of returns on market portfolio	0.000700
Covariance	0.000644
Correlation coefficient	0.997176

Beta	**0.920**
Required rate of return	**14.280**
Expected rate of return	**15.260**

Table 7.4

Required rate of return where Beta = 1

State of nature		X	Y	Z
Probability		0.3	0.5	0.2
Return to shareholders for period		0.1150	0.1550	0.1900
Return on market portfolio		0.1150	0.1550	0.1900
Risk free rate		0.0600	0.0600	0.0600
Expected risk free return	0.06			

Covariance of returns with the market portfolio

Outcome	*Return*	*Expected return*	*Difference (i)*	*Rm*	*Expected Rm*	*Difference (ii)*
X	0.1150	0.1500	-0.0350	0.1150	0.1500	-0.0350
Y	0.1550	0.1500	0.0050	0.1550	0.1500	0.0050
Z	0.1900	0.1500	0.0400	0.1900	0.1500	0.0400

Outcome	*Cross product (i) x (ii)*	*Prob. (iii)*	*Average (i)x(ii)x(iii)*
X	0.001225	0.3000	0.000368
Y	0.000025	0.5000	0.000012
Z	0.001600	0.2000	0.000320

Variance of firm returns	0.000700
Variance of returns on market portfolio	0.000700
Covariance	0.000700
Correlation coefficient	1.000000

Beta	**1.000**
Required rate of return	**15.000**
Expected rate of return	**15.000**

It was possible to guide the search for integer solutions, and so improve the efficiency with which models were solved, using a knowledge of the model's structure and of the mechanism of the branch and bound process. This is discussed in the following section.

Solution of MIP models by the branch and bound algorithm

The first step of the process involves the relaxation, or dropping, of the integrality constraints and the solution of the resulting linear programming (LP) problem. The outcome of this process may be that the LP is infeasible, or cannot be solved, in which case the IP must also be infeasible. This occurs when the constraints of the model cannot logically hold simultaneously. If the LP is feasible, all the integrality constraints may be satisfied in the LP solution, which is then also the IP solution. If the LP solution does not satisfy all the integrality constraints, the continuous optimum must be further constrained by the branch and bound process, which can be represented as a tree search in which the nodes of the tree each represent an LP problem. The search process, which is to some extent arbitrary, has the following steps:

1) One of the global entities (binary variables and special ordered sets) which has a fractional value in the optimal solution to the relaxed problem is selected as the branching variable. This forms the 'root' of the tree structure. If the entity is a binary variable it must take the value of either 0 or 1 in an integer solution.

2) The branching results in two new subproblems which are both solved in their relaxed form. One of the subproblems is selected for further development and a branching variable selected as in step 1.

Steps 1 and 2 are repeated, progressively constraining the problem further with an accompanying deterioration in the optimal objective values of the subproblems. When a point is reached where all the integer variables have integer values, an integer solution has been found which may or may not be the optimal one.

3) The objective value of this solution is a bound on the problem and provides a 'cut-off' which is applied to the undeveloped nodes: if the LP relaxation of a node gives a value that is no better than the integer solution already found, then that node need not be considered any further and is 'pruned'.

4) One of the remaining active nodes is selected and the procedure beginning at step 1 is reiterated.

A number of integer solutions may be found before the search process is complete.

The size of the problem depends critically on the number of nodes and therefore on the number of entities that give rise to nodes, such as binary variables and special ordered sets. These increase the size of the tree in a non-linear manner.

7.9 Efficient solution techniques

Since the model matrices often contained a large number (up to several hundred) global entities, it was necessary to use procedures to improve the efficiency of the search and therefore minimise the solution time. These were of two types: those concerned with aspects of the model's formulation, and those relating to the branch and bound search process itself. Examples of the impact of these techniques on solution times will be presented in the following chapter.

7.9.1 Efficient model formulation

A large number of binary variables and other global entities are generated in the final model, so that a huge number of potential combinations of these entities exist and there is the possibility of the branch and bound tree search 'fanning out' and becoming very difficult, or impossible, to solve within a realistic time. This difficulty was minimised by exploiting the logical relationships betweeen binary variables, and by tightening the bounds on continuous variables.

Exploitation of the logical structure

One method of minimising the difficulties created by large numbers of global entities was to make use of the model's logical structure to assign specific values to some entities when others are determined. For example, two of the model variables are:

`P4(m,t)` = taxable profit for firm `m` in year `t` after deduction of losses carried back from `t+1`

`P5(m,t)` = taxable profit for firm `m` in year `t` after further deduction of losses carried back from `t+2`

Indicator (binary) variables are linked to these variables by the conditions

$$P4(m,t)>0 \rightarrow d4(m,t)=1, \quad P5(m,t)>0 \rightarrow d5(m,t)=1$$

Since $P5(m,t)>P4(m,t)$ is a logical impossibility, it is possible to add the constraint

$$d5(m,t) \leq d4(m,t)$$

to enforce the condition

`d4(m,t)=0→d5(m,t)=0.`

A number of the binary variables were linked by constraints of this type in the model section `!LOGICAL STRUCTURE`, forming chains such that if one member of the chain takes the value zero, all subsequent members also take the value zero (Fig. 13). The result was to make large sections of the potential branch and bound tree search redundant, greatly improving the efficiency of the solution process.

Tightening bounds

The branch and bound algorithm solves the MIP formulation most efficiently if the optimal solution to the linear formulation is as close as possible to the optimal integer solution. This was achieved by the addition of constraints to limit the freedom of the LP solution to depart from the reality of the situation being modelled as represented by the integrality constraints, by ensuring that bounds on variables were as tight as possible.

A difficulty arose from the formulation $x - y = z$ where $x \leq M\delta$, $y \leq M(1-\delta)$. Provided that $\delta \in \{0,1\}$ then either x or y, depending on the sign of z, is forced to take the value 0. But when the integrality requirement for δ is relaxed, solutions are possible where $x > 0$, $y > 0$, $0 < \delta < 1$. The optimal solution to the relaxed problem was then potentially very different to any feasible integer solution, so that the solution process was less efficient and solution times became extended.

For example, the constraints `LSS1` to `LSS3` deal with the offset of trading losses brought forward from the previous year (`LF(m,t-1)`) against Schedule D Case I

Fig. 13

Logical relationships that exist between the model's binary variables

$$
\begin{array}{ccccccccccccccccccccc}
 & & & & & & d20 & \geq & d19 & \geq & d1 & \leq & d7 & & & & & & & & \\
 & & & & & & & & + & & & & & & & & & & & & \\
 & & & & & & & & d2 & & & & & & & & & & & & \\
 & & & & & & & & \geq & & & & & & & & & & & & \\
d6 & \leq & d5 & \leq & d4 & \leq & & & d3 & \geq & d21^{**} & & & & & & & & & & \\
 & & & & & & & & \geq & & & & & & & & & & & & \\
 & & & & & & & & d22^{*} & & & & & & & & & & & & \\
 & & & & & & & & \geq & & & & & & & & & & & & \\
 & & & & & & & & d8 & \geq & d9 & \geq & d10 & \geq & d11 & \geq & d12 & \geq & d13 & \geq & d14 \geq d15 \geq d16 \geq d17 \geq d18
\end{array}
$$

$$d25 \geq d26 \geq d27 \geq d28$$

* only in group structures

** only in 75 per cent group structures

trading profit of the current year (`P2(m,t)`). Since the loss must be set off as soon as possible, there can be only two possible outcomes: either there is a positive amount of loss remaining after offset (`LFF(m,t)`) and the profit remaining in the current year (`P2(m,t)`) is zero, or else `P2(m,t)` is positive and `LFF(m,t)` is zero. These conditions are enforced by the binary variable δ_2 (d2) in constraints `LSS2` and `LSS3`. In the optimal solution to the relaxed problem where $0 < \text{d2} < 1$, variables may take values such as

P2(m,t)	-	LFF(m,t)	=	P1(m,t)	-	LF (m,t-1)
230	-	210	=	60	-	40

The difficulty was minimised by the additional constraints in the model section `!UPPER BOUNDS`, which specify the variables' logical upper bounds explicitly so that, for example, `P2(m,t)` cannot be greater than `P1(m,t)`.

Similarly, the value of M was made as small as possible while remaining a true upper bound for x. There are a number of reasons for this:

(i) When M is made smaller, the size of the feasible region of the LP corresponding to the MIP problem is reduced. For instance, if M's value is specified in the model as 1000 when it is certain that $x \leq 100$, a wide range of values will satisfy the constraint $x \leq M\delta$. The LP solution could have

x=55, $0.055 \leq \delta \leq 1.0$

If M is specified as 100, then the LP solution would have to be

x=55, $0.55 \leq \delta \leq 1.0$.

(ii) As M becomes larger, it becomes more likely that LP solutions will be generated with values of δ small enough to fall below the tolerance which indicates whether a variable is integer. False integer values may result. Since the integer values were linked into 'chains' in many cases (section 7.9.1), the problem could have been significant.

(iii) A large disparity in coefficient sizes in a model increases the difficulty of the solution process. Although the software has an automatic scaling procedure, it is good practice to avoid unnecessary disparity.

7.9.2 Control of the branch and bound search

The efficiency of the branch and bound search is affected by the node selection strategy and by the selection of a branching variable once a node has been chosen.

Node selection

Initially the only waiting node is the node corresponding to the continuous optimum. As the tree search proceeds, there is at any time a list of active subproblems representing a partial tree of active nodes. The number of these depends partly on the cut-offs which have already been applied. The strategies used for node selection therefore aimed to achieve a good bound early in the solution process to minimise the likelihood of the tree spreading out into a large number of active nodes.

A number of possible strategies for node selection exist, depending on the extent to which a strict depth-first search is carried out, that is, considering the sub-problems and sub-sub-problems arising from a chosen node. One rule for node selection is a depth-first search with backtracking, or last in, first out (LIFO). In a depth first search, the next node considered is one of the two descendants of the current node. When a node is pruned, the branch is re-traced backwards to the first node (if any) with a descendant which has not yet been considered. This approach has the advantage of relatively straightforward computation since the parent and descendant nodes differ by only one integer constraint. It may therefore lead to an integer solution, although this may not necessarily be close to the optimum integer solution, at a relatively early stage in the search. This provides the cut-off needed to reduce the number of remaining nodes that must be explored.

The opposite strategy is breadth first, where all the nodes at a given 'distance' (the number of edges in the unique path between the node and the root) from the root are considered before any at the next level.

The default option (NDSEL1=1) in XPRESS-MP is a depth first search considering both descendant nodes, unless both of these are pruned, in which case all outstanding nodes are considered. The node with the best estimate is chosen on the basis of the estimated degradation in the objective value resulting from the additional integer constraint. Backtracking is not used to determine the next node to develop unless the option NDSEL1=3 is set, which forces a strict depth-first search. NDSEL1=2 forces all nodes to be considered at each selection stage.

For the simpler models with few potential tax strategies, it was frequently found that the default node selection strategy gave an optimal integer solution in the shortest time. However, more complex models, where a large number of strategies existed were often solved most efficiently by a period of breadth-first search, possibly with a later switch to a depth-first or backtracking strategy.

Branching variable selection

The selected active node has associated with it an optimal LP solution. A global entity with a fractional value in this solution must be chosen upon which to branch the problem, and one of the two resulting sub-problems must be considered first.

The selection of a branching variable was controlled by setting priorities in the `DIRECTIVES` section so that the probability of a variable being selected was determined by its perceived importance in the global solution. The most efficient ordering of priorities for branching variable selection was assessed from a knowledge of the model's structure and data inputs, and by experimentation and experience. The global entities in the tax models may be divided into two types:

- those which represent decisions, such as project acceptance/rejection or loss carry back/ carry forward

- those which represent logical conditions of the `IF...THEN` type, which are dependent on the decisions made by the model.

The first type was generally expected to be a relatively important determinant of the optimal solution and was therefore given a greater priority in the `DIRECTIVES` section. The lower the priority value, the greater was the likelihood that the variable would be selected for branching.

Having selected a branching variable, the direction in which branching occurs first may also be specified which in the case of a binary variable involves a forced choice of either zero or one. If a project appeared likely to be accepted in an optimal solution, the binary variable signalling its acceptance could be forced to take the value 1 in priority to 0 with an UP or DOWN entry in the directives file.

Summary

A computerised mixed integer programming model was developed using the mathematical programming software XPRESS-MP. The final model aims to maximise shareholder wealth, subject to a number of constraints representing the logic of the relevant tax rules and other externally or internally imposed restrictions on the firm's behaviour. Within the bounds imposed by the constraints, all possible combinations of investment, financing and tax strategies are assessed simultaneously in the solution process and the optimal solution is determined.

The main areas incorporated in the model are: the offset rules and the carry back/carry forward provisions relating to the treatment of trading losses, ACT and debenture interest, including the possibility of group relief, and the provisions relating to finance leasing as an alternative to capital asset purchase. Constraints ensure that the net cash flows from projects, dividend payments, lending less borrowing costs, equity issues and tax payments, are sufficient to maintain the firm's liquidity. Short term lending and borrowing are permitted. A target debt/equity ratio may be imposed, or alternatively the model may be given a free choice of funding policy. Two alternative forms of horizon valuation may be used, depending on the assumptions made about the fate of the firm and its dividend policy beyond the horizon.

The final version of the model aims to maximise the returns to shareholders after all taxes, taking into account dividend payments and new equity issues. The returns are derived as a residual, after deduction of all contractual payments such as debenture interest and other cash outflows. Dividends may be constrained according to some given stable growth requirement, or the model may be allowed to choose a pattern of dividend payments which maximises shareholder wealth. The returns to shareholders

are discounted at the cost of equity, which is determined by an iterative method using a CAPM spreadsheet model in conjunction with the optimisation model.

If the final model is run for a five year period for a single firm with no other group members, the matrix typically contains approximately 900 rows (constraints) and 700 columns (variables) of which approximately 160 are restricted to integer values. The model contains a number of features which were developed to improve the efficiency of the solution process.

Chapter 8

Operational use of the model

8.1 Introduction

This chapter presents the results of applications of the final model to a variety of situations, arising from different assumptions about the firm's structure, existing activities and tax situation, and policies regarding dividend payments and capital structure. Group situations and lease or purchase decisions are included. The values of incremental projects, as determined by the model, are reconciled with their values under conventional net present value analysis. The effect on solution times of the features designed to improve the model's efficiency is demonstrated.

8.2 Reconciliation of model output and net present value analysis

The values of a number of potential projects are shown under conventional net present value analysis in Table 8.1. For simplicity it is assumed in this example that debt finance is unavailable and that the cost of equity funding is 15 per cent, which is also the rate at which the leased project cash flows are discounted for comparison purposes. The tax rate is assumed to be 33 per cent and ACT payments and issue costs are ignored. The cash flows are discounted to reflect the timing, at the end of each year, of the increased dividend payments to shareholders made possible by the acceptance of the projects. For instance, where a leased project is accepted, it is assumed that the first lease rental payment will result in a reduction in the payment to shareholders at the end of year 1, and the tax on profits in year t is assumed to reduce the cash available for payments to shareholders in year $t+1$. Under these assumptions, projects 1a, 1b, 2a, 2b, 4a, and 5b have net present values greater than zero. Since 1a and 1b, 2a and 2b are mutually exclusive options the selection to maximise net present value would be 1a, 2b, 4a and 5b, and the increase in NPV would be £68,081.

The optimisation model was run initially with data inputs from Table 8.2 to reflect the assumptions above, with the imputation rate set to zero and the rate of tax set to 33 per cent for all bands of taxable profits. The firm's income from existing projects, its debt finance and the relevant data inputs relating to earlier periods are also shown in Table 8.2. Note that the model incorporates eleven time periods, of which the first six relate to the time prior to the current period, in order to accommodate the potential carry back of surplus ACT for six years. The cost of equity was set to 15 per cent. The model was run once with the addition of constraints to ensure that none of the additional projects were accepted (REC 01) and again with a free choice of project acceptance subject to constraints to ensure that at most one of the mutually exclusive options were accepted (REC 02). For instance,

```
EXC1:SUM(p=1:2)accept(p)<1
```

Tables 8.3, 8.4 show the values of the key variables in the model's output. Table 8.5 reconciles the model's optimal solutions with and without the additional projects, demonstrating that the choice of projects and the increase in the objective function value due to project acceptance are the same as under the NPV analysis. The model correctly calculates the taxable profits, tax and debt interest payments, and chooses to make a full dividend payout rather than retain profits or make loan capital repayments. This would be expected since retentions are assumed to earn the risk free rate of 6 per cent and the debt interest rates are less than the input cost of equity.

The model was then run again using the same project data, but with the marginal limits and rates of tax set to their current values (Table 8.6), and with the basic rate of tax set at 20 per cent. The firm has paid ACT on its dividend in period 6, which is divided between an amount of surplus ACT carried forward (variable SF) and an amount which may be set against mainstream corporation tax (variable A2). The change in taxable profits and cash inflow due to project acceptance are the same as in the previous run. The model's output with no additional projects, and with a free choice as before, is shown in Tables 8.7 and 8.8. Again the model chooses to make a full dividend payout. Table 8.9 reconciles the increase of £115,585 in the objective function value with the value obtained when no additional projects are accepted.

It may be shown that the increased incremental value of the projects in the second scenario arises because of the setoff of ACT payments against mainstream tax liabilities. With the imputation rate set to zero, but with the marginal limits and rates of tax at their current values, the increase in the objective function value would be only £59,242 (Table 8.10).

8.3 Valuation under differing outcomes

Although the analysis in the previous section assumes a constant discount rate of 15 per cent, the appropriate discount rate will be determined by the cash flows to shareholders and may be estimated using the Capital Asset Pricing Model as described in Chapter 7. In all of the following it is assumed that the firm faces three possible future scenarios or states of nature, referred to as Outcome X, Y or Z, with different cash flows in each. The iterative process to determine the appropriate discount rates will be described below.

8.3.1 Project selection

MDL 01 : *The 'base case' scenario*

This situation considers the ongoing activities only of a single firm (NM=1) without any possible selection of additional projects. The cash flows that will occur in each outcome, and their probabilities, are presented in Table 8.11 with the most likely outcome, Y, having the same cash flows as the example in the previous section. The optimisation model was run under each of the three possible outcomes with an initial required rate of return of 15 per cent, assuming a beta value of 1.0 (see Table 7.4). The returns to shareholders, in the form of gross dividends, and the expected values of equity in each period, formed the input to the spreadsheet CAPM model discussed in Chapter 7. The required rates of return determined by the spreadsheet model formed the input for the next iteration of the optimisation model under each of the three possible outcomes. The expected objective function value was also an input to the next iteration, as the opening value of equity. This input is needed by the model in order to calculate the debt interest payments, which vary according to the gearing ratio. The new output values from the next iteration again formed the input to the spreadsheet CAPM model and required rates of return were determined. The process was repeated until, on comparison of two successive iterations, all the following conditions were satisfied:

(i) the selected strategy of tax treatment remained stable;

(ii) the required rates of return determined were stable to four decimal places;

(ii) the change in the expected objective function value was less than £5,000 (approximately 0.055 per cent in this instance).

The most important of the model's output variables under each outcome are presented in Tables 8.12a to 8.12c. Table 8.12d shows the final objective function values, beta characteristics and rates of return determined by the optimisation model in conjunction with the spreadsheet model. In this case the expected rates of return are all greater than the CAPM required rate.

The expected value of equity at the start of the model's period 7, corresponding to the expected objective function value, was unaltered over the time span considered since the model chose to make no retentions. Although the constraints to enforce a target gearing ratio were not included in this model the debt/equity ratio also remained unaltered, since no debt repayments were made.

Under the 'pessimistic' outcome X, a trading loss of £100,000 arises which the model chooses to set against other profits of the same period. An excess of debt interest charges then results, which is carried forward until it may be set against future trading profits in periods 9 and 10.

The full dividend payout, combined with low taxable profits as a result of the firm's capital allowances, results in surplus ACT which is carried forward since there is no remaining ACT offset capacity in earlier years. A large amount of surplus ACT exists at the horizon period which is valued, according to the assumptions of the model, by discounting back to the horizon at the average required rate of return. This presents a difficulty, since the surplus ACT at the horizon may not be recoverable.

The model was therefore run again under each set of outcome assumptions, with the horizon value constraint adjusted so that surplus ACT at the horizon had no value. The decision to make a full dividend payout was unaltered, although the horizon value and therefore the objective function value were reduced by the appropriately discounted value of the surplus ACT at the horizon date. Also, the data input value of the firm's assets at the horizon date may be regarded as incorporating the expectation that some of the ACT may be irrecoverable. For these reasons the horizon valuation of surplus ACT was retained in the model.

MDL 02 : *Project selection under equity funding*

The potential projects may have different cash flows under the different states of nature X, Y and Z. The cash flows for the 'most likely' outcome, Y, are those in Table 8.1. The cash flows under the 'pessimistic' outcome X and the 'optimistic' outcome Z are shown in Tables 8.13 and 8.14 respectively. Projects 2 and 5 are assumed to have the same cash flows in each of the outcomes.

The model was run under each outcome with a free choice of project acceptance. The final output is shown in Tables 8.15a to 8.15d. The initial iteration was performed using the required rates of return derived from MDL 01, which provides a 'base case' with no incremental projects accepted. The model was constrained to use only equity funding to finance project acceptance. Projects 1a, 2b, 4a and 5b, were accepted in each outcome and this choice remained stable over successive iterations. The leased versions of the projects 2 and 5, as expected, are preferred by the model when the alternative is to purchase using equity finance. The model chooses to borrow the amount needed for the issue costs of the equity finance for the projects, repaying this short-term debt in the following period. The final converged solution shows required rates of return which are little different to those derived in MDL 01, with an increase in the objective function of approximately £91,000.

MDL 03, 04: *Project selection with debt/equity ratio maintained*

The model was run again as in MDL 02, but with a target ratio of debt to debt plus equity whose upper and lower limits were set at 0.175 and 0.225 respectively. On the first iteration the opening value of equity was taken as that of MDL 01, that is, £9.06m. On subsequent iterations the opening value of equity was set equal to the expected objective function value from the previous iteration, with a resulting change in the maximum amount of debt funding which the model can use without violating the upper bound of the target gearing ratio.

In this instance the model's choices were not the same across the three outcomes: in outcomes X and Y, projects 1a, 2b, 4a and 5a were selected, whereas in the 'optimistic' outcome Z the selection made was projects 1a, 2b, 3, 4a and 5b. In subsequent iterations, the choice for outcome Y oscillated between these two combinations and a stable solution was not obtained.

Constraints were added to force acceptance in all outcomes of each of these combinations in turn, and the iterative process was begun again. MDL 03 (Tables 8.16a to 8.16d) was run with a forced acceptance of combination 1a, 2b, 3, 4a and 5b and MDL 04 (Tables 8.17a to 8.17d) with the combination 1a, 2b, 4a and 5a. The second combination is superior, although the dividend payments are lower, because the required rates of return are slightly lower and because there is no initial equity funding cash flow.

MDL 05: *Project selection with free choice of financing*

MDL 05 was run under the same assumptions and inputs as MDL 02, but with no target debt structure and with a free choice of financing, with the cost of debt being subject to the gearing ratio. In all outcomes the model made the same choice, that is, projects 1a, 2a, 3, 4a and 5a, fully funded by debt. The purchase options for projects 2 and 5 were preferred to the leasing option, even though part of the debt interest in period 7 in outcome X cannot be set off and must be carried forward. (Tables 8.18a to 8.18d)

8.3.2 Dividend growth restrictions

`MDL 06`: *Base case, with target debt ratio*

This represents the case where no incremental projects are accepted, with the firm following a target debt to debt plus equity ratio between 0.175 and 0.225, and with dividend payments in each year being constrained to be at least 1.05 times as great as the previous year. Because of this stipulation, the minimum horizon value constraints described in chapter 7 were included. In the iterative process to estimate the required rate of return, the opening value of equity for each period was taken as the expected value of the model variable `EQ(t)` which takes into account increases in retained funds. The returns to shareholders in each period were taken to include the increase in retentions for the period.

Tables 8.19a to 8.19d show the solution output. The model is forced to use some funds to make retentions in order to smooth the dividend payments, according to the growth constraint. Retained funds are assumed to be invested at the risk free rate of 6 per cent and the interest is treated by the model in the same way as the firm's other non-trading income (constraint `OFS4`). Because there is not a full dividend payout, the amounts of surplus ACT carried forward to the horizon are reduced compared to the case of `MDL 01`. In outcomes Y and Z, debt issues must be made in order to maintain the target ratio within its limits.

MDL 07 : *Project selection*

This represents a free choice of project selection under the same conditions specified above for MDL 06, with the initial required rate of return and opening value of equity for the first iteration being taken from the solution to MDL 06. The model chooses in each outcome the combination 1a, 2b, 4b, 5b. The leased versions of projects 2 and 5 are selected instead of the purchased options, although the upper limit of the target debt constraint would allow sufficient debt funding to enable the firm to purchase with debt. Another interesting feature is that project 4b is selected in each outcome, although its present value under conventional NPV analysis is less than that of 4a because of the delay in implementation. The output is shown in Tables 8.20a to 8.20d. Internal funds are used for this project in outcomes X and Y, with a combination of internal funding and externally raised debt being chosen in outcome Z. However, comparison with the output from the base case MDL 06 indicates that acceptance of this project combination does not increase shareholder wealth when its risk characteristics are taken into account, since the overall cash flow pattern leads to a higher required rate of return in all periods after project acceptance. The required rate of return is actually less than the expected return in period 7.

MDL 08 : *Base case, passive debt management policy*

This is the equivalent to MDL 06 but with the level of debt maintained at its starting amount. The cost of the debt interest payments decreases, however, over time as the firm retains funds and the gearing ratio changes (Tables 8.21a to 8.21d)

MDL 09 : *Project selection*

This was run under the same conditions as MDL 08 but with a free project choice. The combination chosen initially was 1a, 2b, 3, 4a, and 5b in outcomes Y and Z, with combination 1a, 2b, 4b and 5b chosen in outcome X. On subsequent iterations the same selection was made in outcome X as in Y and Z. The solution output (Tables 8.22a to 8.22d) shows that the risk characteristics of the overall combination of ongoing and incremental activities again lead to an increase in the required rate of return as compared to the case MDL 09 with no incremental projects. However, some improvement still occurs in the expected objective value as a result of undertaking the projects. As with MDL 07, the expected return is less than the required return in period 7.

8.3.3 ACT offset capacity available in earlier periods

Making a new assumption that the firm has only begun paying dividends in period 6, there will be unutilised ACT offset capacity in earlier periods. The model will have the ability to carry back surplus ACT and/or to carry back losses without creating surplus ACT. The changed data inputs are shown as Variation 1 in Table 8.23.

MDL 10 : *The base case*

This shows that the model will choose to carry back as much surplus ACT as possible, exhausting the offset capacity in the earlier periods and leading to a tax rebate in period 7 which provides additional funds available for distribution. The model chooses to set the loss of £200,000 in period 7 of outcome X against other profits of the period, which leaves all of the debt interest to be carried forward as excess charges. (Tables 8.24a to 8.24d)

MDL 11 : *Acceptance of an additional large project, debt/equity ratio maintained*

This represents the situation where the firm as in MDL 10 is considering the acceptance of the large project (purchased version) shown in Table 8.25. The project's cash flows are expected to be the same in each outcome. The ratio of debt to debt plus equity is to be maintained at a target level between 0.175 and 0.225.

The final converged solution (Tables 8.26a to 8.26d) uses a funding ratio of approximately 49:51 debt:equity for the new project. In outcome X, there is a trading loss of £361,765 which is set first against other profits of the period, with the remainder of £161,765 being carried back to period 6. The effect of this carry back is to create surplus ACT relating to period 6 of £32,353. The surplus ACT from period 7 is carried back to the earlier periods and a tax rebate is obtained in all outcomes.

MDL 12 : *Acceptance of an additional large project, free choice of funding*

Here the conditions and inputs are as for MDL 11, but with no target debt ratio imposed. The model chooses to fund the project entirely with debt, and the expected objective function value is slightly higher than that of MDL 11. The same strategy is followed with the setoff of the trading loss in outcome X and the carry back of surplus ACT to previous periods. (Tables 8.27a to 8.27d)

8.3.4 The effect of changed ongoing capital allowances

The capital allowances from the firm's ongoing activities were assumed to be lower than in the previous examples, with values as shown in Table 8.23, Variation 2. This leads to a smaller amount of surplus ACT arising from the model's dividend payments, since the firm's taxable profits are greater than previously assumed.

MDL 13 : *The 'base case'*

The dividend payments create a large surplus of ACT in period 7, which may be carried back and set against earlier periods with unutilised ACT offset capacity. In later periods there is a relatively small amount of surplus ACT. In outcome X there is a trading loss of £100,000 in period 7 which the model chooses to set against the other available profits in that period. (Tables 8.28a to 8.28d)

`MDL 14`: *Acceptance of an additional large project, debt/equity ratio maintained*

The conditions and inputs are the same as for `MDL 13`, but with the choice of accepting the large project in Table 8.25. The target debt/equity ratio is imposed, and in the final solution the approximate funding ratio is 54:46 debt:equity. The loss in period 7 of outcome X is set against other profits of the period, with all of the debt interest of the period becoming excess. (Tables 8.29a to 8.29d) An identical result in terms of tax payments would be obtained by carrying forward the loss, setting off as much debt interest as possible, and carrying forward the remainder.

`MDL 15`: *Acceptance of an additional large project, free choice of funding*

This was run under the same conditions and inputs as `MDL 14`, but with a free choice of funding. The model again chooses to fund the project entirely with debt (Tables 8.30a to 8.30d).

8.3.5 Lease/purchase decisions

`MDL 16`: *The lease/purchase decision with maintenance of existing debt/equity ratio*

It is assumed that the firm's present activities are the same as in `MDL 01`, and that it is considering a single lease/purchase project shown in Table 8.25. The implicit rate of interest on the lease is 8.5 per cent, the same as the firm's lowest rate of interest on debt finance. A mutual exclusivity constraint was included in the model to ensure that either project option, or neither, may be accepted but not both.

In the first iteration the discount rates used were those derived from the 'base case' MDL 10 and the opening value of equity was taken as that of MDL 01, that is, £9.06m. The model chose to accept the purchased version of the project in each state of nature. The additional funding of £1m was selected by the model in the approximate ratio of 52:48 debt:equity, taking the debt level to its maximum limit of 22.5 per cent of total financing. However, the first iteration gave an expected objective function value of £9.369m. On the second iteration this value was used as the opening value of equity, which enabled the model to increase the amount of debt finance used to fund the project. The iterative process ultimately converged to give a funding ratio of approximately 63:37 debt:equity. The final solution therefore represents a value of equity which incorporates the new project, financed in such a way that the firm's target gearing ratio is maintained. The final funding ratio differs from the solution to MDL 11, since the expected value of equity is greater and so the upper bound on the debt limit is higher.

The solution output (Tables 8.31a to 8.31d) shows that in outcome X, there is a trading loss in period 7 of £264,330 which is set firstly against other profits of the period and then carried forward rather than back. Although the firm has profits in earlier periods against which the loss could be set, this strategy would result in the displacement of ACT set off in those earlier periods which would then be carried forward to the horizon since there is no capacity to set it off before the horizon.

MDL 17 : *The lease/purchase decision, free choice of funding*

This is the equivalent to MDL 16, but with debt and equity funding selected by the model. Again the purchased version of the project is chosen, with 100 per cent debt financing even in outcome X, where there is a loss in period 7 and the debt interest has to be carried forward. The required rates of return are similar, with the expected rates of return being slightly lower than in MDL 16. The expected value of the objective function is greater, since there is no negative outflow at time zero as with equity finance. (Tables 8.32a to 8.32d)

MDL 18 : *Leasing displaces debt*

Here no target gearing ratio is imposed, and the outstanding capital on the lease is assumed to have the same impact as other forms of debt on the firm's cost of incremental debt, so that leasing finance has the effect of displacing debt on a 1:1 basis. The model is constrained to accept only the leased project, ignoring the purchase option. The constraints relating to the bands of debt finance are modified to include the additional variables `LD(t=7:NT)` and `LDB(t=7:NT)` :

```
LDE1(t=7:NT):LD(t)=SUM(p=1:NPROJ)(PROJRENT(p,t)-
FC(p,t))*accept(p)
```

`LDB(t)` is the amount of debt commitment represented by the lease at the start of `t`, derived by additional constraints such as:

```
LDE2(t=7:7):LDB(t)= LD(t+1)+LD(t+2)+LD(t+3)+LD(t+4)
```

The constraints `UDL1` and `DF1` are modified to take into account the variable `LDB(t)`:

```
UDL1(m=1:1,t=7:NT):DB(t-1)+DBI(t)+DBC(t)+LDB(t)
      <DLMT(5,t)*EQ(m,t-1)
```

```
DF1(m=1:1,t=7:NT):D2(t)-x1(t)=LDB(t)+DB(t-1)+DBI(t)
      +DBC(t-DLMT(1,t)*EQ(m,t-1)
```

The cost of the firm's debt interest payments in the solution output reflects the lease finance commitment, which declines over the life of the lease. The returns to shareholders are lower than the corresponding payments under the debt financing option MDL 17, because of the enforced 'repayment' of the lease, whereas MDL 17 does not choose to make any debt repayments but instead carries the new level of debt finance through to the horizon. The effect on the objective function value is the same as if the model had been constrained to repay the additional debt at the end of the project's life of five years. A feature of the solution output (Tables 8.33a to 8.33d) is that the expected rate of return in period 7 is actually lower than the required rate, although acceptance of the leased project would significantly increase shareholder wealth.

MDL 19 : *Leasing does not displace debt*

By comparison, this represents the situation where leasing does not displace debt for the purposes of calculating the appropriate debt interest rate. The output (Tables 8.34a to 8.34d) shows that the debt interest charges are slightly less than in MDL 17 and the tax charges are correspondingly higher, because there is less debt interest relief. The closing balance of funds and the dividend payments differ only slightly from those in MDL 17. A feature of both models in that in outcome X, where there is a trading loss in period 7, the model chooses to carry the whole amount of the loss forward, so that full use is not made of the available setoff against other profits of the period. It would have been possible, for instance, in outcome X to set the loss of £265,147 against the other income for the period of £200,000, with the remainder of £65,147 being carried forward along with the £191,250 debt interest charge which could not then be set off. The total loss carried forward would then be only £256,397 and the tax for period 7 would be nil. However, the optimal chosen strategy maximises the relief at the 35 per cent marginal tax rate in period 9.

8.3.6 Group tax effects

MDL 20 : *A 75 per cent group, with group election for ACT*

This represents a simple group structure, with a parent company and a 75 per cent owned subsidiary. The relevant cash flows and input variable values are shown in Table 8.11. The subsidiary has unutilised ACT offset capacity in periods prior to the time scale of the model. The parent company's cost of debt is calculated by the model on the assumption that the value of the parent's equity is £9.06m, as in MDL 01, and the required rates of return for the first iteration were those of MDL 01. A group election for ACT payments is assumed to operate (GELEC=1).

The solution output is shown in Tables 8.35a to 8.35g. In outcome X, the model chooses to surrender the parent's trading loss of £100,000 to the subsidiary, reducing the subsidiary's taxable profits and hence its ability to set off its own ACT on its dividend payments outside the group. The parent surrenders a sufficient amount of its ACT to the subsidiary so that all of the subsidiary's own ACT becomes surplus, and is carried back to an earlier period, providing a tax rebate of £21,421. In periods 8, 9 and 10 in outcome X, and in periods 7, 8, 9, and 10 in outcomes Y and Z, the model again chooses to surrender just enough of the parent's ACT to the subsidiary to enable the subsidiary to carry back its own displaced ACT. However in the final period (NT) the parent carries forward ACT in preference to surrendering it to the subsidiary. This is probably because of the model's assumption that the parent's surplus ACT will be set off in the period NT+1, and because the subsidiary is not wholly owned: surrendered ACT is ultimately used by the subsidiary to make higher dividend payments outside the group, as well as to the parent.

MDL 21: *A 51 per cent group, with group election for ACT*

This model has the same inputs and assumptions as MDL 20 above, except that the subsidiary is only 51 per cent owned by the parent company (PCENT=51). For this reason, there can be no group relief for losses or excess debt interest. The subsidiary pays higher dividends outside the group, and makes a lower group payment to the parent, in comparison with MDL 20. There is therefore more potential for the subsidiary to carry back ACT and obtain a rebate, if the parent surrenders a sufficient amount of its ACT to the subsidiary.

The solution output (Tables 8.36a to 8.36g) shows that in all outcomes, the model chooses to surrender enough of the parent's ACT in period 7 to the subsidiary to displace all the subsidiary's own ACT, freeing it to be carried back. However, in subsequent periods the parent's ACT is not surrendered and is carried forward to the horizon instead, with the subsidiary's ACT offset capacity in earlier years remaining unutilised. The reason this strategy is optimal may be that the objective function aims to maximise only the wealth of the group's shareholders: the model aims to avoid surrendering the parent's asset, unutilised ACT, to a subsidiary which would ultimately pay almost half of the asset's value outside the group in the form of dividends.

MDL 22: *A 75 per cent group, no group election for ACT*

The assumptions and inputs are the same as for MDL 20, except that no group election is in force (GELEC=0). The subsidiary therefore has much more ACT of its own to carry back and set against previous years' unutilised offset capacity. The model chooses to surrender enough of the parent's ACT to enable the subsidiary to do this as quickly as possible, and once the subsidiary's unutilised ACT offset capacity in prior years is exhausted, no further ACT is surrendered from the parent. Although the subsidiary has sufficiently high taxable profits in periods 9, 10 and 11 to accept more surrendered ACT, the model chooses not to follow this strategy. Instead, the parent's surplus ACT is carried forward. (Tables 8.37a to 8.37g)

MDL 23 : *A 51 per cent group, no group election for ACT*

The assumptions and inputs are those of MDL 22, except that PCENT=51. The solution output is shown in Tables 8.38a to 8.38g. In outcome X, the model chooses to carry forward the £100,000 trading loss in period 7, although the alternative strategy of setting the loss against other profits of the period would have resulted in carrying forward a loss (excess charges) of only £91,250.

Again the model surrenders to the subsidiary enough ACT to take full advantage of the unutilised ACT offset capacity in earlier years. Also, just sufficient of the parent's ACT is surrendered in period 10 to bring the ACT offset capacity in the subsidiary to zero in that period, after setting off the subsidiary's own ACT on its dividends. This does not occur in the final period, presumably because of the assumption that surplus ACT at the horizon is set off in the following period.

A comparison of the objective function values of the group models, with and without a group election for ACT payments, indicates that the better choice for the firm in this situation is to avoid making a group election.

MDL 24 : *A 75 per cent group, no group election, with incremental project*

It is assumed that GELEC=0 and PCENT=75, and that the firm has the choice of accepting the large project in Table 8.25 with a free choice of funding. The initial required rates of return were those derived from MDL 22.

The model chooses to accept the project and to fund it entirely with debt finance. The calculation of the parent's debt interest payments by the model depends, as before, on the ratio of debt to equity employed by the firm. Because the objective function value includes an element of the subsidiary's value, it was not used directly as the input value of the parent's equity for the purpose of calculating the debt/equity ratio. Instead the objective function was adjusted by subtracting the value of the subsidiary's equity, taken as the difference between the 'base case' objective function values of `MDL 22` and `MDL 01`.

The solution output is shown in Tables 8.39a to 8.39g. In period 7 in outcome X, there is a trading loss of £272,000 which the model chooses to surrender as group relief from the parent to the subsidiary. Excess debt interest of £84,266 also arises, of which the model chooses to surrender only £6,032 as group relief, carrying the rest forward in the parent company. This strategy is selected in preference to the more obvious alternative, of setting off the whole amount of excess debt interest as group relief against the available profits of the subsidiary. The reason appears to be the availability of tax relief at 35 per cent in period 8, since the carry forward of most of the excess debt interest reduces the parent's taxable profits in period 8 to the lower marginal limit of £150,000. The amount of ACT chosen to be surrendered by the parent is just sufficient to make the whole of the subsidiary's own ACT for period 7 available for carry back. This chosen amount of ACT takes into account the reduction in the subsidiary's ACT setoff capacity, created by the parent's surrender of its trading loss and a small amount of its excess charges.

Excess debt interest arises again in the parent company in period 11 of outcome X. The model chooses this time to surrender the whole amount as group relief, with a resulting decrease in the subsidiary's taxable profits. This creates some surplus ACT in the subsidiary which is carried back to period 10, leading to a small tax rebate.

In all three outcomes, the model makes full use of the available ACT setoff capacity in the subsidiary prior to period 7. Once this is exhausted, the model surrenders only as much ACT as the subsidiary can set off immediately. The exception is the final period, where the parent's ACT is carried forward to the horizon.

MDL 25 : *A 51 per cent group, no group election, with incremental project*

The parameters are the same as those of MDL 24 above, except that PCENT=51 and so there is no possibility of group relief for losses or excess charges. The input value of the parent's equity, for the purpose of calculating the debt/equity ratio, was derived in an analogous way to that used for MDL 24. The value of the subsidiary's equity was taken as the difference between the 'base case' objective function values of MDL 23 and MDL 01.

The model again chooses to accept the incremental project and to fund it entirely with debt. Group relief is unavailable for the losses which occur in periods 7 and 11 in outcome X. The model chooses to set these against other profits where possible, and otherwise to carry them forward (tables 8.40a to 8.40g). Allowing for this difference, the optimal strategies are similar to those of MDL 24 where PCENT=75.

8.4 Efficiency of the solution process

In section 9 of Chapter 7 the features of the model that aim to improve the efficiency of the solution process, and hence reduce the time taken to complete the branch and bound search, were discussed. The effect of these on the efficiency of the solution process may be illustrated by a comparison of the search completion times for a particular matrix.

A single firm matrix

Table 8.41 shows the results obtained for a number of different combinations of priorities (`DIRECTIVES`) for outcome Y of `MDL 02`, assuming that all the `LS` type and `LP` type constraints were included in the formulation. The lower the numerical value given to the entity, the more likely it is to be selected for branching, with a default value of 500 being assigned to entities not given a specified value. The table shows that the optimal solution time for this matrix was obtained with a relatively simple set of Directives. Assigning priorities to those binary variables which are linked by constraints of the LS type appears to be redundant or counter-productive in terms of improving solution efficiency. Some improvement was gained by forcing the branching direction for the `accept` variables (relating to project acceptance), so that the branch and bound process assumed first a value of `accept(p)=1` for those projects with a positive net present value under conventional analysis.

The power of the LS and LP type constraints was also demonstrated by omitting both sets of constraints, and running the model again with the most efficient set of Directives as determined in Table 8.40. No solution was obtained after 900 seconds and the run was aborted. The model was run again with the LS type constraints included but the LP type constraints omitted, and the search was completed in 287 seconds. With the LS but not the LP type constraints, a search completion time of 108 seconds was obtained, indicating that the LP type constraints also have a significant impact on solution efficiency.

Little or no improvement in search completion time was obtained by varying the node selection strategy in the branch and bound process.

Group structures

Matrices where there was a group structure (MDL 20 to MDL 25) were much more difficult to solve than those with only a single firm structure. This was expected, since most of the constraints are replicated for the group members and the group relief provisions greatly increase the potential number of tax strategies. Multiple integer solutions were generally found before the search process was complete. The time taken to complete the search varied from several minutes for the less complex tax situations to approximately one hour. Again, it was found that the LP and LS type constraints generally had great value in reducing the search completion time, and that relatively basic sets of Directives were optimal in terms of improving the efficiency of the solution process.

In contrast to the single firm matrices, the group structure matrices were found to be easier to solve by a breadth-wise node selection strategy (NDSEL1=2) which forces all nodes at a particular distance from the 'root' of the branch and bound tree to be considered before those at more remote levels. The reason for this may be the parallel nature of the tax situation of the member firms in the group. A search strategy which frequently proved effective was a period of breadth-first search, with a later switch to a depth-first or backtracking strategy to complete the search process once a 'good' solution had been found during the search. Solutions were judged for this purpose initially on the basis of an estimated optimal solution, obtained from a preliminary inspection of the problem. In subsequent iterations of the same matrix, knowledge of the output from the previous iteration was used to judge the solutions obtained.

8.5 Summary

It has been demonstrated in this chapter that the model's output may be reconciled with a conventional NPV analysis under certain assumptions: specifically, assuming equity funding of incremental projects, a zero imputation rate, a single rate of corporate tax, and a constant discount rate.

If these assumptions are relaxed, the model's decisions and their values may be significantly different from those obtained by the NPV analysis of projects in isolation. The model is able to decide, for any set of input parameters, the optimal combination of investment decisions, financing decisions and tax strategy simultaneously. Via an iterative process using the CAPM model, a discount rate can be obtained which takes into account the effect of taxes on the returns to shareholders. The process also enables the model to accommodate a constant gearing ratio constraint. The firm's revised debt and equity financing includes the effect on the value of equity of incremental projects, which itself is determined on the assumption that the optimal tax strategy is followed at the same time.

The model can simultaneously evaluate leasing or purchasing decisions, taking into account the optimal tax strategy associated with each. It may be assumed that leasing displaces debt for the purpose of calculating debt interest payments, or alternatively that leasing finance does not affect the cost of debt.

Group tax effects may be accommodated, with or without a group election for the treatment of ACT. It was shown that this group election may not be optimal where the subsidiary has unutilised ACT offset capacity in earlier periods. The model is able to make an optimal choice of group relief strategy, which may not involve taking full advantage of the profits available in the group for setoff of losses or excess charges, while simultaneously determining whether or not to accept incremental investments.

Table 8.1

Expected net present values of projects, £'000

Discount rate = 0.15
Tax rate = 0.33

Time period	0	1	2	3	4	5
Project 1a						
Capital outlay	(60.000)					
Cash inflow		17.800	36.000	41.000	20.800	
Present value		15.478	27.221	26.958	11.892	
Tax on inflows		5.874	11.880	13.530	6.864	
Present value		(4.442)	(7.811)	(7.736)	(3.413)	
Capital allowances		15.000	11.250	8.438		
Balancing allowance					25.313	
Present value of tax benefits		3.743	2.441	1.592	4.153	
Working capital increase		5.000				
decrease					5.000	
Present value of tax effects		(1.248)			0.820	
Disposal proceeds	*n/a*					
Present value including tax effects	*n/a*					
Net present value 9.650						
Project 1b						
Capital outlay	(60.000)					
Cash inflow		17.800	36.000	41.000		
Present value		15.478	27.221	26.958		
Tax on inflows		5.874	11.880	13.530		
Present value		(4.442)	(7.811)	(7.736)		
Capital allowances		15.000	11.250			
Balancing allowance				33.750		
Present value of tax benefits		3.743	2.441	6.368		
Working capital increase		5.000				
decrease					5.000	
Present value of tax effects		(1.248)			0.820	
Disposal proceeds (at end of year)				12.000		
Present value including tax effects				7.440		
Net present value 9.234						

Table 8.1 continued

Time period		0	1	2	3	4	5
Project 2a							
Capital outlay		(100.000)					
Cash inflow			5.000	50.000	60.000	45.000	20.000
Present value			4.348	37.807	39.451	25.729	9.944
Tax on inflows			1.650	16.500	19.800	14.850	6.600
Present value			(1.248)	(10.849)	(11.321)	(7.383)	(2.853)
Capital allowances			25.000	18.750	14.063	10.547	
Balancing allowance							31.641
Present value of tax benefits			6.238	4.068	2.653	1.730	4.514
Working capital increase			5.000				
decrease							5.000
Present value of tax effects			(1.248)				0.713
Disposal proceeds		*n/a*					
Present value including tax effects		*n/a*					
Net present value	2.295						
Project 2b (leased)							
Implicit rate of interest	0.1055						
Asset cost, if purchased		(100.000)					
Rental payments			24.200	24.200	24.200	24.200	24.200
Present value			21.043	18.299	15.912	13.836	12.032
Accounting depreciation			20.000	20.000	20.000	20.000	20.000
Present value of tax benefits			4.991	4.340	3.774	3.281	2.853
Finance charge			8.000	6.290	4.400	2.310	0.000
Present value of tax benefits			1.996	1.365	0.830	0.379	0.000
Net present value	25.777						

Table 8.1 continued

Time period	0	1	2	3	4	5
Project 3						
Capital outlay	(250.000)					
Cash inflow		64.000	94.000	94.000	34.000	6.200
Present value		55.652	71.078	61.807	19.440	3.082
Tax on inflows		21.120	31.020	31.020	11.220	2.046
Present value		(15.970)	(20.396)	(17.736)	(5.578)	(0.885)
Capital allowances		62.500	46.875	35.156	26.367	
Balancing allowance						79.102
Present value of tax benefits		15.595	10.171	6.633	4.326	11.285
Working capital increase	*n/a*					
decrease	*n/a*					
Present value of tax effects	*n/a*					
Disposal proceeds						10.000
Present value including tax effects						6.200
Net present value (45.295)						
Project 4a						
Capital outlay	(200.000)					
Cash inflow		108.000	116.000	81.000	24.000	
Present value		93.913	87.713	53.259	13.722	
Tax on inflows		35.640	38.280	26.730	7.920	
Present value		(26.949)	(25.170)	(15.283)	(3.938)	
Capital allowances		50.000	37.500	28.125		
Balancing allowance					84.375	
Present value of tax benefits		12.476	8.137	5.307	13.843	
Working capital increase		20.000				
decrease					20.000	
Present value of tax effects		(4.991)			3.281	
Disposal proceeds	*n/a*					
Present value including tax effects	*n/a*					
Net present value 15.321						

Table 8.1 continued

Time period		0	1	2	3	4	5
Project 4b							
Capital outlay			(200.000)				
Cash inflow				108.000	116.000	81.000	24.000
Present value				81.664	76.272	46.312	11.932
Tax on inflows				35.640	38.280	26.730	7.920
Present value				(23.434)	(21.887)	(13.290)	(3.424)
Capital allowances				50.000	37.500	28.125	
Balancing allowance							84.375
Present value of tax benefits				10.849	7.075	4.614	12.038
Working capital increase				20.000			
decrease							20.000
Present value of tax effects				(4.340)			2.853
Disposal proceeds	*n/a*						
Present value including tax effects	*n/a*						
Net present value	(12.764)						
Project 5a							
Capital outlay		(90.500)					
Cash inflow			49.000	54.000	25.000		
Present value			42.609	40.832	16.438		
Tax on inflows			16.170	17.820	8.250		
Present value			(12.227)	(11.717)	(4.717)		
Capital allowances			22.750	17.063			
Balancing allowance					50.688		
Present value of tax benefits			5.677	3.702	9.564		
Working capital increase	*n/a*						
decrease	*n/a*						
Present value of tax effects	*n/a*						
Disposal proceeds	*n/a*						
Present value including tax effects	*n/a*						
Net present value	(0.340)						

Table 8.1 continued

Time period		0	1	2	3	4	5
Project 5b (leased)							
Implicit rate of interest	0.1045						
Asset cost, if purchased		(90.500)					
Rental payments			33.210	33.210	33.210		
Present value			28.878	25.112	21.836		
Accounting depreciation			30.167	30.167	30.167		
Present value of tax benefits			7.527	6.546	5.692		
Finance charge			5.987	3.142	(0.000)	(0.000)	(0.000)
Present value of tax benefits			1.494	0.682	(0.000)	(0.000)	(0.000)
Net present value	17.332						

Table 8.2

Data inputs to XPRESS-MP

I. External parameters

	Time period				
	7	8	9	10	11
Lower marginal income limit, £m	0.3	0.3	0.3	0.3	0.3
Upper marginal income limit, £m	1.5	1.5	1.5	1.5	1.5
Maximum income limit, £m	10	10	10	10	10
Tax due at lower limit, £m	0.099	0.099	0.099	0.099	0.099
Tax due at upper limit, £m	0.495	0.495	0.495	0.495	0.495
Tax due at maximum limit, £m	3.3	3.3	3.3	3.3	3.3
Basic rate of income tax	0	0	0	0	0
Full rate of corporation tax	0.33	0.33	0.33	0.33	0.33
II. Firm-specific parameters					
Short term lending rate	0.06	0.06	0.06	0.06	0.06
Short term borrowing rate	0.1	0.1	0.1	0.1	0.1
Short term borrowing limit, £m	0.1	0.1	0.1	0.1	0.1
Debt/equity ratios at which cost of debt changes:	0.2500	0.2500	0.2500	0.2500	0.2500
	0.3000	0.3000	0.3000	0.3000	0.3000
	0.3500	0.3500	0.3500	0.3500	0.3500
	0.4000	0.4000	0.4000	0.4000	0.4000
	0.4500	0.4500	0.4500	0.4500	0.4500
Interest rates applicable to bands of debt:	0.085	0.085	0.085	0.085	0.085
	0.09	0.09	0.09	0.09	0.09
	0.095	0.095	0.095	0.095	0.095
	0.1	0.1	0.1	0.1	0.1
	0.105	0.105	0.105	0.105	0.105
Minimum cash balance held, £m	0.05	0.05	0.05	0.05	0.05

Table 8.2 continued

	Time period				
	7	8	9	10	11
Ongoing activities, £m					
capital allowances	1.2	1.1	1.025	0.9688	0.9266
non-trading income	0.2	0.2	0.2	0.2	0.2
closing market value of fixed assets					9.3
net operating cash flow (positive)	1.5	1.5	1.5	1.5	1.5
net operating cash flow (negative)	0	0	0	0	0

Relevant data inputs, £m

LF(t=6)	0
P3/P3a (t=6)	0.2875
P4(t=5)	0.2875
P5(t=4)	0.2875
SF(t=6)	0.0675
U2(t=6)	0
U3A(t=5)	0
U4A(t=4)	0
U5A(t=3)	0
U6(t=2)	0
U7(t=1)	0
OCB(t=7)+CSHC(t=7)	0.05
DB(t=6)	2.25
EQ(t=6)	8.656
DIV(t=6)	0.5
A2(t=6)	0.0575
TR(t=6)	0

Table 8.3

Matrix REC 01

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	308,750	408,750	483,750	540,000	582,190
Gross corporation tax payable	94,875	101,887	134,887	159,637	178,200	192,123
Closing balance of funds	-	1,413,875	1,406,862	1,373,862	1,349,112	1,330,550
Gross dividends paid	-	1,413,875	1,406,862	1,373,862	1,349,112	1,330,550

Table 8.4

Matrix REC 02

Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	380,646	547,651	590,871	464,053	568,440
after losses carried forward	-	380,646	547,651	590,871	464,053	568,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	389,396	556,401	599,621	472,803	577,190
Gross corporation tax payable	94,875	128,501	183,612	197,875	156,025	190,473
New equity issued	-	260,000	0	0	0	0
Closing balance of funds	-	1,536,265	1,578,839	1,474,728	1,376,475	1,348,525
Gross dividends paid	-	1,536,265	1,578,839	1,474,728	1,376,475	1,348,525

Table 8.5

Reconciliation of matrices REC 01 and REC 02

	Time period 7	8	9	10	11	post-horizon
REC 01						
Taxable profits (P3)	308,750	408,750	483,750	540,000	582,190	
Tax at 33%	(101,888)	(134,888)	(159,638)	(178,200)	(192,123)	
Tax payment made		(101,888)	(134,888)	(159,638)	(178,200)	(192,123)
Present value*	0	(77,042)	(88,691)	(91,273)	(88,597)	(83,060)
Total present value (428,662)						
REC 02						
Changes in taxable profits due to project acceptance:						
Cashflow:						
Project 1a	17,800	36,000	41,000	20,800	0	
2b	5,000	50,000	60,000	45,000	20,000	
4a	108,000	116,000	81,000	24,000	0	
5b	49,000	54,000	25,000	0	0	
Working capital changes:						
Project 1a	5,000	0	0	(5,000)	0	
2b	5,000	0	0	0	(5,000)	
4a	20,000	0	0	(20,000)	0	
5b	0	0	0	0	0	
Capital allowances:						
Project 1a	(15,000)	(11,250)	(8,438)	(25,313)	0	
2b	0	0	0	0	0	
4a	(50,000)	(37,500)	(28,125)	(84,375)	0	
5b	0	0	0	0	0	
Accounting depreciation:						
Project 1a	0	0	0	0	0	
2b	(20,000)	(20,000)	(20,000)	(20,000)	(20,000)	
4a	0	0	0	0	0	
5b	(30,167)	(30,167)	(30,167)	0	0	
Finance charges:						
Project 1a	0	0	0	0	0	
2b	(8,000)	(6,290)	(4,400)	(2,310)	0	
4a	0	0	0	0	0	
5b	(5,987)	(3,142)	0	0	0	
Total change in taxable profits	80,647	147,651	115,871	(67,198)	(5,000)	
Change in tax charge (at 33%)	(26,613)	(48,725)	(38,237)	22,175	1,650	
Change in tax payments made		(26,613)	(48,725)	(38,237)	22,175	1,650
Present value*	0	(20,124)	(32,037)	(21,862)	11,025	713
Total present value (62,285) *(a)*						

Table 8.5 continued

Changes in cash flow due to project acceptance:

Increased cash inflow		179,800	256,000	207,000	89,800	20,000
less rental payments:						
Project 2b		(24,200)	(24,200)	(24,200)	(24,200)	(24,200)
5b		(33,210)	(33,210)	(33,210)	0	0
Increase/(decrease) in cash available for distribution		122,390	198,590	149,590	65,600	(4,200)
Present value*		106,426	150,163	98,358	37,507	(2,088)
Total present value	390,365 *(b)*					
Equity issue	(260,000)					
costs	0					
Total present value	(260,000) *(c)*					
Change in objective function value: *(a)+(b)+(c)*			**68,080**			

* discounted at 15 per cent

Table 8.6

Data inputs to XPRESS-MP, actual rates and limits

I. External parameters

	Time period				
	7	8	9	10	11
Lower marginal income limit, £m	0.3	0.3	0.3	0.3	0.3
Upper marginal income limit, £m	1.5	1.5	1.5	1.5	1.5
Maximum income limit, £m	10	10	10	10	10
Tax due at lower limit, £m	0.075	0.075	0.075	0.075	0.075
Tax due at upper limit, £m	0.495	0.495	0.495	0.495	0.495
Tax due at maximum limit, £m	3.3	3.3	3.3	3.3	3.3
Basic rate of income tax	0.20	0.20	0.20	0.20	0.20
Full rate of corporation tax	0.33	0.33	0.33	0.33	0.33

II. Firm-specific parameters

	7	8	9	10	11
Short term lending rate	0.06	0.06	0.06	0.06	0.06
Short term borrowing rate	0.1	0.1	0.1	0.1	0.1
Short term borrowing limit, £m	0.1	0.1	0.1	0.1	0.1
Debt/equity ratios at which cost of debt changes:	0.2500	0.2500	0.2500	0.2500	0.2500
	0.3000	0.3000	0.3000	0.3000	0.3000
	0.3500	0.3500	0.3500	0.3500	0.3500
	0.4000	0.4000	0.4000	0.4000	0.4000
	0.4500	0.4500	0.4500	0.4500	0.4500
Interest rates applicable to bands of debt:	0.085	0.085	0.085	0.085	0.085
	0.09	0.09	0.09	0.09	0.09
	0.095	0.095	0.095	0.095	0.095
	0.1	0.1	0.1	0.1	0.1
	0.105	0.105	0.105	0.105	0.105
Minimum cash balance held, £m	0.05	0.05	0.05	0.05	0.05

Table 8.7

Matrix REC 03

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	308,750	408,750	483,750	540,000	582,190
Gross corporation tax payable	71,875	113,062	139,312	159,000	173,766	192,123
ACT payable on dividends	-	298,875	298,487	295,487	293,237	291,550
ACT set against MCT liability	57,500	61,750	81,750	96,750	108,000	116,438
Surplus ACT on dividends paid	-	237,125	216,737	198,737	185,237	175,112
chosen to carry forward	-	237,125	216,737	198,737	185,237	175,112
carried back to previous periods	-	0	0	0	0	0
Surplus ACT carried forward	67,500	304,625	521,362	720,100	905,337	1,080,449
Closing balance of funds	-	1,494,375	1,492,435	1,477,437	1,466,187	1,457,750
Gross dividends paid	-	1,494,375	1,492,435	1,477,437	1,466,187	1,457,750

Table 8.8

Matrix REC 04

Time period	6	7	8	9	10	11
Trading profit	-	380,646	547,651	590,871	464,053	568,440
after losses carried forward	-	380,646	547,651	590,871	464,053	568,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	389,396	556,401	599,621	472,803	577,190
Gross corporation tax payable	71,875	106,107	164,740	179,867	135,481	172,016
ACT payable on dividends	-	322,209	335,802	320,976	302,881	292,726
ACT set against MCT liability	57,500	77,775	111,280	119,924	94,561	115,438
Surplus ACT on dividends paid	-	244,434	224,522	201,052	208,321	177,288
chosen to carry forward	-	244,434	224,522	201,052	208,321	177,288
carried back to previous periods	-	0	0	0	0	0
Surplus ACT carried forward	67,500	311,394	536,455	737,507	945,828	1,123,116
New equity issued	-	260,000	0	0	0	0
Closing balance of funds	-	1,611,045	1,679,009	1,604,880	1,514,407	1,463,630
Gross dividends paid	-	1,611,045	1,679,009	1,604,880	1,514,407	1,463,630

Table 8.9

Reconciliation of matrices REC 03 and REC 04

		Time period					
		7	8	9	10	11	post-horizon
REC 03							
Tax cash flows							
Gross tax liability, t-1			(78,062)	(113,062)	(139,312)	(159,000)	(173,766)
less: ACT setoff		0	61,750	81,750	96,750	108,000	116,438
Tax payment made in t		0	(16,312)	(31,312)	(42,562)	(51,000)	(57,328)
Surplus ACT horizon value							1,080,449
Increase/(decrease) in cash available for distribution		0	(16,312)	(31,312)	(42,562)	(51,000)	1,023,121
Present value*		0	(12,334)	(20,588)	(24,335)	(25,356)	442,323
Total present value	359,710 *(a)*						
REC 04							
Project cash flows							
Increased cash inflow from accepted projects		179,800	256,000	207,000	89,800	20,000	
less: lease rental payments		(57,410)	(57,410)	(57,410)	(24,200)	(24,200)	
Increase/(decrease) in cash available for distribution		122,390	198,590	149,590	65,600	(4,200)	
Present value*		106,426	150,163	98,358	37,507	(2,088)	
Total present value	390,365 *(b)*						
Tax cash flows							
Gross tax liability, t-1			(106,107)	(164,740)	(179,867)	(135,481)	(172,016)
less: ACT setoff		0	77,775	111,280	119,924	94,561	115,438
Tax payment made in t		0	(28,332)	(53,460)	(59,943)	(40,920)	(56,578)
Surplus ACT horizon value							1,123,116
Increase/(decrease) in cash available for distribution		0	(28,332)	(53,460)	(59,943)	(40,920)	1,066,538
Present value*		0	(21,423)	(35,151)	(34,273)	(20,344)	461,094
Total present value	349,903 *(c)*						
Equity issue	(260,000)						
costs	(5,200)						
Total present value	(265,200) *(d)*						
Change in objective function value: *(b)+(c)-(a)-(d)*			115,358				

* discounted at 15 per cent

Table 8.10

Reconciliation of matrices REC 03 and REC 04, with no ACT setoff

		Time period					
		7	8	9	10	11	post-horizon
REC 03							
Tax cash flows							
Gross tax liability, t-1			(78,062)	(113,062)	(139,312)	(159,000)	(173,766)
less: ACT setoff		0	0	0	0	0	0
Tax payment made in t		0	(78,062)	(113,062)	(139,312)	(159,000)	(173,766)
Surplus ACT horizon value							0
Increase/(decrease) in cash available for distribution		0	(78,062)	(113,062)	(139,312)	(159,000)	(173,766)
Present value*		0	(59,026)	(74,340)	(79,652)	(79,051)	(75,124)
Total present value	(367,193) *(a)*						
REC 04							
Project cash flows							
Increased cash inflow from accepted projects		179,800	256,000	207,000	89,800	20,000	
less: lease rental payments		(57,410)	(57,410)	(57,410)	(24,200)	(24,200)	
Increase/(decrease) in cash available for distribution		122,390	198,590	149,590	65,600	(4,200)	
Present value*		106,426	150,163	98,358	37,507	(2,088)	
Total present value	390,365 *(b)*						
Tax cash flows							
Gross tax liability, t-1			(106,107)	(164,740)	(179,867)	(135,481)	(172,016)
less: ACT setoff		0	0	0	0	0	0
Tax payment made in t		0	(106,107)	(164,740)	(179,867)	(135,481)	(172,016)
Surplus ACT horizon value							0
Increase/(decrease) in cash available for distribution		0	(106,107)	(164,740)	(179,867)	(135,481)	(172,016)
Present value*		0	(80,232)	(108,319)	(102,840)	(67,358)	(74,367)
Total present value	(433,116) *(c)*						
Equity issue	(260,000)						
costs	(5,200)						
Total present value	(265,200) *(d)*						
Change in objective function value: *(b)+(c)-(a)-(d)*			59,242				

* discounted at 15 per cent

Table 8.11

Spreadsheet containing data inputs to XPRESS-MP

I. External parameters

	Time period				
	7	8	9	10	11
Lower marginal income limit, £m	0.3	0.3	0.3	0.3	0.3
Upper marginal income limit, £m	1.5	1.5	1.5	1.5	1.5
Maximum income limit, £m	10	10	10	10	10
Tax due at lower limit, £m	0.075	0.075	0.075	0.075	0.075
Tax due at upper limit, £m	0.495	0.495	0.495	0.495	0.495
Tax due at maximum limit, £m	3.3	3.3	3.3	3.3	3.3
Basic rate of income tax	0.2	0.2	0.2	0.2	0.2
Full rate of corporation tax	0.33	0.33	0.33	0.33	0.33

II. Firm-specific parameters

	7	8	9	10	11
Short term lending rate	0.06	0.06	0.06	0.06	0.06
Short term borrowing rate	0.1	0.1	0.1	0.1	0.1
Short term borrowing limit, £m					
parent	0.1	0.1	0.1	0.1	0.1
subsidiary	0.1	0.1	0.1	0.1	0.1
Lower limit of target debt/equity ratio	0.175	0.175	0.175	0.175	0.175
Upper limit of target debt/equity ratio	0.225	0.225	0.225	0.225	0.225
Debt/equity ratios at which cost of debt changes:	0.2500	0.2500	0.2500	0.2500	0.2500
	0.3000	0.3000	0.3000	0.3000	0.3000
	0.3500	0.3500	0.3500	0.3500	0.3500
	0.4000	0.4000	0.4000	0.4000	0.4000
	0.4500	0.4500	0.4500	0.4500	0.4500
Interest rates applicable to bands of debt:	0.085	0.085	0.085	0.085	0.085
	0.09	0.09	0.09	0.09	0.09
	0.095	0.095	0.095	0.095	0.095
	0.1	0.1	0.1	0.1	0.1
	0.105	0.105	0.105	0.105	0.105
Debt issue costs (as a proportion)	0.02	0.02	0.02	0.02	0.02
Equity issue costs (as a proportion)	0.02	0.02	0.02	0.02	0.02
* Debt interest payments made by subsidiary, £m	0.12	0.12	0.12	0.12	0.12
Minimum growth rate of dividend payment (paren	0.05	0.05	0.05	0.05	0.05
Minimum cash balance held, £m					
parent	0.05	0.05	0.05	0.05	0.05
* subsidiary	0.05	0.05	0.05	0.05	0.05

Table 8.11 continued

Ongoing activities, £m	Time period 7	8	9	10	11
capital allowances					
parent	1.2	1.1	1.025	0.969	0.927
* subsidiary	0.2	0.2	0.2	0.2	0.2
non-trading income					
parent	0.2	0.2	0.2	0.2	0.2
* subsidiary	0	0	0	0	0
closing market value of fixed assets (parent)					9.3
Outcome X:					
Probability = 0.3					
net operating cash flow (positive)					
parent	1.1	1.1	1.1	1.1	1.1
* subsidiary	0.9	0.9	0.9	0.9	0.9
net operating cash flow (negative)					
parent	0	0	0	0	0
* subsidiary	0	0	0	0	0
Outcome Y:					
Probability = 0.5					
net operating cash flow (positive)					
parent	1.5	1.5	1.5	1.5	1.5
* subsidiary	1	1	1	1	1
net operating cash flow (negative)					
parent	0	0	0	0	0
* subsidiary	0	0	0	0	0
Outcome Z:					
Probability = 0.2					
net operating cash flow (positive)					
parent	1.7	1.7	1.7	1.7	1.7
* subsidiary	1.1	1.1	1.1	1.1	1.1
net operating cash flow (negative)					
parent	0	0	0	0	0
* subsidiary	0	0	0	0	0

* These inputs are ignored by the model if NM>1.

Table 8.11 continued

Relevant data inputs, £m

	parent	* subsidiary
LF(t=6)	0	0
P3/P3a (t=6)	0.288	0.68
P4(t=5)	0.288	0.68
P5(t=4)	0.288	0.68
SF(t=6)	0.068	0
U2(t=6)	0	0
U3A(t=5)	0	0
U4A(t=4)	0	0
U5A(t=3)	0	0
U6(t=2)	0	0
U7(t=1)	0	0
OCB(t=7)+CSHC(t=7)	0.05	-
DB(t=6)	2.25	-
EQ(t=6)	8.656	5
DIV(t=6)	0.5	0.2
A2(t=6)	0.058	0.05
TR(t=6)	0	0
SFII(t=6)	0	-
SBVAL(t=NT)	-	5.25
SDBT(t=NT)	-	1.5
OCB(t=7)	-	0.05

Table 8.12a

Matrix FPRP **MDL 01 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	0	115,000	173,440
Other income (after any set off of trading losses in t)	-	100,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	0	8,750	8,750	123,750	182,190
Trading loss	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	100,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	91,250	0	0	0	0
Total losses carried forward	0	91,250	91,250	16,250	0	0
Gross corporation tax payable	71,875	0	2,187	2,187	30,937	45,547
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	218,875	220,000	219,912	196,912	184,074
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	218,875	220,000	219,912	196,912	184,074
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	286,375	506,375	726,287	923,200	1,107,274
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,094,375	1,108,750	1,108,312	1,108,312	1,102,562
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,094,375	1,108,750	1,108,312	1,108,312	1,102,562
Increase in retentions	-	0	0	0	0	0

Table 8.12b

Matrix FPRE **MDL 01 Outcome Y**

Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	308,750	408,750	483,750	540,000	582,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	78,062	113,062	139,312	159,000	173,766
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	237,125	216,737	198,737	185,237	175,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	237,125	216,737	198,737	185,237	175,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	304,625	521,362	720,100	905,337	1,080,449
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,494,375	1,492,437	1,477,437	1,466,187	1,457,750
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,494,375	1,492,437	1,477,437	1,466,187	1,457,750
Increase in retentions	-	0	0	0	0	0

Table 8.12c

Matrix FPRO **MDL 01 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	508,750	608,750	683,750	740,000	782,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	148,062	183,062	209,312	229,000	243,766
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	237,125	210,737	192,737	179,237	169,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	237,125	210,737	192,737	179,237	169,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	304,625	515,362	708,100	887,337	1,056,449
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,694,375	1,662,437	1,647,437	1,636,187	1,627,750
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,694,375	1,662,437	1,647,437	1,636,187	1,627,750
Increase in retentions	-	0	0	0	0	0

Table 8.12d

MDL 01 **Risk and return**

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,058	8,058
Y	9,322	7,951
Z	9,911	7,903
Expected value	9,061	7,974

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.915	0.1423	0.1561
8	0.850	0.1365	0.1558
9	0.826	0.1343	0.1546
10	0.807	0.1326	0.1538
11	0.803	0.1322	0.1529

Table 8.13

Outcome X, cash flows and net present values of projects, £'000

Discount rate = 0.15
Tax rate = 0.33

Time period		0	1	2	3	4	5
Project 1a							
Capital outlay		(60.000)					
Cash inflow			15.000	30.000	35.000	18.000	
Present value			13.043	22.684	23.013	10.292	
Tax on inflows			4.950	9.900	11.550	5.940	
Present value			(3.743)	(6.509)	(6.604)	(2.953)	
Capital allowances			15.000	11.250	8.438		
Balancing allowance						25.313	
Present value of tax benefits			3.743	2.441	1.592	4.153	
Working capital increase			5.000				
decrease						5.000	
Present value of tax effects			(1.248)			0.820	
Disposal proceeds		*n/a*					
Present value including tax effects		*n/a*					
Net present value	0.725						
Project 1b							
Capital outlay		(60.000)					
Cash inflow			15.000	30.000	35.000		
Present value			13.043	22.684	23.013		
Tax on inflows			4.950	9.900	11.550		
Present value			(3.743)	(6.509)	(6.604)		
Capital allowances			15.000	11.250			
Balancing allowance					33.750		
Present value of tax benefits			3.743	2.441	6.368		
Working capital increase			5.000				
decrease						5.000	
Present value of tax effects			(1.248)			0.820	
Disposal proceeds (at end of year)					12.000		
Present value including tax effects					7.440		
Net present value	1.450						

Table 8.13 continued

Time period		0	1	2	3	4	5
Project 2a							
Capital outlay		(100.000)					
Cash inflow			5.000	50.000	60.000	45.000	20.000
Present value			4.348	37.807	39.451	25.729	9.944
Tax on inflows			1.650	16.500	19.800	14.850	6.600
Present value			(1.248)	(10.849)	(11.321)	(7.383)	(2.853)
Capital allowances			25.000	18.750	14.063	10.547	
Balancing allowance							31.641
Present value of tax benefits			6.238	4.068	2.653	1.730	4.514
Working capital increase			5.000				
decrease							5.000
Present value of tax effects			(1.248)				0.713
Disposal proceeds		*n/a*					
Present value including tax effects		*n/a*					
Net present value	2.295						
Project 2b (leased)							
Implicit rate of interest	0.1055						
Asset cost, if purchased		(100.000)					
Rental payments			24.200	24.200	24.200	24.200	24.200
Present value			21.043	18.299	15.912	13.836	12.032
Accounting depreciation			20.000	20.000	20.000	20.000	20.000
Present value of tax benefits			4.991	4.340	3.774	3.281	2.853
Finance charge			8.000	6.290	4.400	2.310	0.000
Present value of tax benefits			1.996	1.365	0.830	0.379	0.000
Net present value	25.777						

Table 8.13 continued

Time period	0	1	2	3	4	5
Project 3						
Capital outlay	(250.000)					
Cash inflow		60.000	90.000	90.000	30.000	5.000
Present value		52.174	68.053	59.176	17.153	2.486
Tax on inflows		19.800	29.700	29.700	9.900	1.650
Present value		(14.972)	(19.528)	(16.981)	(4.922)	(0.713)
Capital allowances		62.500	46.875	35.156	26.367	
Balancing allowance						79.102
Present value of tax benefits		15.595	10.171	6.633	4.326	11.285
Working capital increase	*n/a*					
decrease	*n/a*					
Present value of tax effects	*n/a*					
Disposal proceeds						10.000
Present value including tax effects						6.200
Net present value	(53.863)					
Project 4a						
Capital outlay	(200.000)					
Cash inflow		100.000	110.000	75.000	20.000	
Present value		86.957	83.176	49.314	11.435	
Tax on inflows		33.000	36.300	24.750	6.600	
Present value		(24.953)	(23.868)	(14.151)	(3.281)	
Capital allowances		50.000	37.500	28.125		
Balancing allowance					84.375	
Present value of tax benefits		12.476	8.137	5.307	13.843	
Working capital increase		20.000				
decrease					20.000	
Present value of tax effects		(4.991)			3.281	
Disposal proceeds	*n/a*					
Present value including tax effects	*n/a*					
Net present value	2.682					

Table 8.13 continued

Time period		0	1	2	3	4	5
Project 4b							
Capital outlay			(200.000)				
Cash inflow				100.000	110.000	75.000	20.000
Present value				75.614	72.327	42.881	9.944
Tax on inflows				33.000	36.300	24.750	6.600
Present value				(21.698)	(20.755)	(12.305)	(2.853)
Capital allowances				50.000	37.500	28.125	
Balancing allowance							84.375
Present value of tax benefits				10.849	7.075	4.614	12.038
Working capital increase				20.000			
decrease							20.000
Present value of tax effects				(4.340)			2.853
Disposal proceeds	n/a						
Present value including tax effects	n/a						
Net present value	(23.755)						

		0	1	2	3
Project 5a					
Capital outlay		(90.500)			
Cash inflow			49.000	54.000	25.000
Present value			42.609	40.832	16.438
Tax on inflows			16.170	17.820	8.250
Present value			(12.227)	(11.717)	(4.717)
Capital allowances			22.750	17.063	
Balancing allowance					50.688
Present value of tax benefits			5.677	3.702	9.564
Working capital increase	n/a				
decrease	n/a				
Present value of tax effects	n/a				
Disposal proceeds	n/a				
Present value including tax effects	n/a				
Net present value	(0.340)				

Table 8.13 continued

Time period		0	1	2	3	4	5
Project 5b (leased)							
Implicit rate of interest	0.1045						
Asset cost, if purchased		(90.500)					
Rental payments			33.210	33.210	33.210		
Present value			28.878	25.112	21.836		
Accounting depreciation			30.167	30.167	30.167		
Present value of tax benefits			7.527	6.546	5.692		
Finance charge			5.987	3.142	(0.000)	(0.000)	(0.000)
Present value of tax benefits			1.494	0.682	(0.000)	(0.000)	(0.000)
Net present value	17.332						

Table 8.14

Outcome Z, cash flows and net present values of projects, £'000

Discount rate = 0.15
Tax rate = 0.33

Time period		0	1	2	3	4	5
Project 1a							
Capital outlay		(60.000)					
Cash inflow			22.000	45.000	50.000	25.000	
Present value			19.130	34.026	32.876	14.294	
Tax on inflows			7.260	14.850	16.500	8.250	
Present value			(5.490)	(9.764)	(9.434)	(4.102)	
Capital allowances			15.000	11.250	8.438		
Balancing allowance						25.313	
Present value of tax benefits			3.743	2.441	1.592	4.153	
Working capital increase			5.000				
decrease						5.000	
Present value of tax effects			(1.248)			0.820	
Disposal proceeds		*n/a*					
Present value including tax effects		*n/a*					
Net present value	23.039						
Project 1b							
Capital outlay		(60.000)					
Cash inflow			22.000	45.000	50.000		
Present value			19.130	34.026	32.876		
Tax on inflows			7.260	14.850	16.500		
Present value			(5.490)	(9.764)	(9.434)		
Capital allowances			15.000	11.250			
Balancing allowance					33.750		
Present value of tax benefits			3.743	2.441	6.368		
Working capital increase			5.000				
decrease						5.000	
Present value of tax effects			(1.248)			0.820	
Disposal proceeds (at end of year)					12.000		
Present value including tax effects					7.440		
Net present value	20.910						

Table 8.14 continued

Time period		0	1	2	3	4	5
Project 2a							
Capital outlay		(100.000)					
Cash inflow			5.000	50.000	60.000	45.000	20.000
Present value			4.348	37.807	39.451	25.729	9.944
Tax on inflows			1.650	16.500	19.800	14.850	6.600
Present value			(1.248)	(10.849)	(11.321)	(7.383)	(2.853)
Capital allowances			25.000	18.750	14.063	10.547	
Balancing allowance							31.641
Present value of tax benefits			6.238	4.068	2.653	1.730	4.514
Working capital increase			5.000				
decrease							5.000
Present value of tax effects			(1.248)				0.713
Disposal proceeds		*n/a*					
Present value including tax effects		*n/a*					
Net present value	2.295						
Project 2b (leased)							
Implicit rate of interest	0.1055						
Asset cost, if purchased		(100.000)					
Rental payments			24.200	24.200	24.200	24.200	24.200
Present value			21.043	18.299	15.912	13.836	12.032
Accounting depreciation			20.000	20.000	20.000	20.000	20.000
Present value of tax benefits			4.991	4.340	3.774	3.281	2.853
Finance charge			8.000	6.290	4.400	2.310	0.000
Present value of tax benefits			1.996	1.365	0.830	0.379	0.000
Net present value	25.777						

Table 8.14 continued

Time period	0	1	2	3	4	5
Project 3						
Capital outlay	(250.000)					
Cash inflow		70.000	100.000	100.000	40.000	8.000
Present value		60.870	75.614	65.752	22.870	3.977
Tax on inflows		23.100	33.000	33.000	13.200	2.640
Present value		(17.467)	(21.698)	(18.868)	(6.563)	(1.141)
Capital allowances		62.500	46.875	35.156	26.367	
Balancing allowance						79.102
Present value of tax benefits		15.595	10.171	6.633	4.326	11.285
Working capital increase	*n/a*					
decrease	*n/a*					
Present value of tax effects	*n/a*					
Disposal proceeds						10.000
Present value including tax effects						6.200
Net present value (32.442)						
Project 4a						
Capital outlay	(200.000)					
Cash inflow		120.000	125.000	90.000	30.000	
Present value		104.348	94.518	59.176	17.153	
Tax on inflows		39.600	41.250	29.700	9.900	
Present value		(29.943)	(27.123)	(16.981)	(4.922)	
Capital allowances		50.000	37.500	28.125		
Balancing allowance					84.375	
Present value of tax benefits		12.476	8.137	5.307	13.843	
Working capital increase		20.000				
decrease					20.000	
Present value of tax effects		(4.991)			3.281	
Disposal proceeds	*n/a*					
Present value including tax effects	*n/a*					
Net present value 34.280						

Table 8.14 continued

Time period		0	1	2	3	4	5
Project 4b							
Capital outlay			(200.000)				
Cash inflow				120.000	125.000	90.000	30.000
Present value				90.737	82.190	51.458	14.915
Tax on inflows				39.600	41.250	29.700	9.900
Present value				(26.038)	(23.585)	(14.766)	(4.280)
Capital allowances				50.000	37.500	28.125	
Balancing allowance							84.375
Present value of tax benefits				10.849	7.075	4.614	12.038
Working capital increase				20.000			
decrease							20.000
Present value of tax effects				(4.340)			2.853
Disposal proceeds	*n/a*						
Present value including tax effects	*n/a*						
Net present value	3.721						
Project 5a							
Capital outlay		(90.500)					
Cash inflow			49.000	54.000	25.000		
Present value			42.609	40.832	16.438		
Tax on inflows			16.170	17.820	8.250		
Present value			(12.227)	(11.717)	(4.717)		
Capital allowances			22.750	17.063			
Balancing allowance					50.688		
Present value of tax benefits			5.677	3.702	9.564		
Working capital increase	*n/a*						
decrease	*n/a*						
Present value of tax effects	*n/a*						
Disposal proceeds	*n/a*						
Present value including tax effects	*n/a*						
Net present value	(0.340)						

Table 8.14 continued

Time period		0	1	2	3	4	5
Project 5b (leased)							
Implicit rate of interest	0.1045						
Asset cost, if purchased		(90.500)					
Rental payments			33.210	33.210	33.210		
Present value			28.878	25.112	21.836		
Accounting depreciation			30.167	30.167	30.167		
Present value of tax benefits			7.527	6.546	5.692		
Finance charge			5.987	3.142	(0.000)	(0.000)	(0.000)
Present value of tax benefits			1.494	0.682	(0.000)	(0.000)	(0.000)
Net present value	17.332						

Table 8.15a

Matrix PREP **MDL 02 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	135,651	178,871	57,253	168,440
after losses carried forward	-	0	113,727	178,871	57,253	168,440
Other income (after any set off of trading losses in t)	-	169,326	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	0	122,477	187,621	66,003	177,190
Trading loss	-	30,674	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	30,674	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	0	30,619	46,905	16,501	44,297
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	240,049	234,573	210,519	218,433	184,812
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	240,049	234,573	210,519	218,433	184,812
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	307,549	542,122	752,641	971,074	1,155,886
Short term lending	-	0	0	0	0	0
Short term borrowing	-	5,200	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	260,000	0	0	0	0
Closing balance of funds	-	1,200,245	1,295,340	1,240,216	1,158,169	1,101,250
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,200,245	1,295,340	1,240,216	1,158,169	1,101,250
Increase in retentions	-	0	0	0	0	0

Table 8.15b

Matrix PREE **MDL 02 Outcome Y**

Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	380,126	547,651	590,871	464,053	568,440
after losses carried forward	-	380,126	547,651	590,871	464,053	568,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	388,876	556,401	599,621	472,803	577,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	106,107	164,740	179,867	135,481	172,106
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	244,234	224,522	201,052	208,321	177,288
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	244,234	224,522	201,052	208,321	177,288
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	311,934	536,455	737,507	945,828	1,123,116
Short term lending	-	0	0	0	0	0
Short term borrowing	-	5,200	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	260,000	0	0	0	0
Closing balance of funds	-	1,611,045	1,679,009	1,604,880	1,514,407	1,463,630
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,611,045	1,679,009	1,604,880	1,514,407	1,463,630
Increase in retentions	-	0	0	0	0	0

Table 8.15c

Matrix PREO	MDL 02	Outcome Z				
Time period	6	7	8	9	10	11
Trading profit	-	596,326	765,651	808,871	674,253	768,440
after losses carried forward	-	596,326	765,651	808,871	674,253	768,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	605,076	774,401	817,621	683,003	777,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	181,777	241,040	256,167	209,051	242,016
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	244,434	218,036	194,512	201,781	170,982
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	244,434	218,036	194,512	201,781	170,982
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	300,394	529,969	724,481	926,262	1,097,244
Short term lending	-	0	0	0	0	0
Short term borrowing	-	5,200	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	260,000	0	0	0	0
Closing balance of funds	-	1,827,245	1,864,579	1,790,181	1,691,907	1,632,100
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,827,245	1,864,579	1,790,181	1,691,907	1,632,100
Increase in retentions	-	0	0	0	0	0

Table 8.15d

MDL 02: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,150	8,059
Y	9,417	7,989
Z	10,043	7,939
Expected value	9,162	8,000

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.943	0.1448	0.1673
8	0.861	0.1375	0.1749
9	0.829	0.1346	0.1675
10	0.806	0.1325	0.1577
11	0.804	0.1324	0.1517

Table 8.16a

Matrix PODP **MDL 03 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	178,776	233,715	60,886	96,838
after losses carried forward	-	0	105,202	233,715	60,886	96,838
Other income (after any set off of trading losses in t)	-	157,416	200,000	200,000	200,000	200,000
Debt interest charges	-	230,990	230,990	230,990	230,990	230,990
Taxable profits	287,500	0	74,213	202,725	29,896	65,848
Trading loss	-	42,584	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	42,584	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	73,574	0	0	0	0
Total losses carried forward	0	73,574	0	0	0	0
Gross corporation tax payable	71,875	0	18,553	50,681	7,474	16,462
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	243,001	254,278	218,033	223,556	202,493
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	243,001	254,278	218,033	223,556	202,493
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	310,501	564,779	782,811	1,006,367	1,208,860
Short term lending	-	0	0	0	0	0
Short term borrowing	-	10,200	0	0	0	0
New debt raised: to fund projects	-	445,503	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	64,497	0	0	0	0
Closing balance of funds	-	1,215,005	1,345,600	1,292,890	1,147,674	1,078,316
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,215,005	1,345,600	1,292,890	1,147,674	1,078,316
Increase in retentions	-	0	0	0	0	0

Table 8.16b

Matrix PODE **MDL 03 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	372,445	594,776	649,715	471,686	498,038
after losses carried forward	-	372,445	594,776	649,715	471,686	498,038
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	230,990	230,990	230,990	230,990	230,990
Taxable profits	287,500	341,226	563,786	618,725	440,696	467,048
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	89,429	167,325	186,554	124,244	133,467
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	257,716	235,326	207,861	213,021	195,572
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	257,716	235,326	207,861	213,021	195,572
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	325,216	560,542	768,403	981,424	1,176,996
Short term lending	-	0	0	0	0	0
Short term borrowing	-	10,200	0	0	0	0
New debt raised: to fund projects	-	445,503	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	64,497	0	0	0	0
Closing balance of funds	-	1,629,805	1,740,416	1,658,032	1,505,802	1,444,906
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,629,805	1,740,416	1,658,032	1,505,802	1,444,906
Increase in retentions	-	0	0	0	0	0

Table 8.16c

Matrix PODO **MDL 03 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	594,645	818,776	873,715	687,886	699,838
after losses carried forward	-	594,645	818,776	873,715	687,886	699,838
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	230,990	230,990	230,990	230,990	230,990
Taxable profits	287,500	563,426	787,786	842,725	656,896	668,848
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	167,199	245,725	264,954	199,914	204,097
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	257,716	228,660	201,141	206,301	189,086
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	257,716	228,660	201,141	206,301	189,086
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	325,216	553,876	755,017	961,318	1,150,404
Short term lending	-	0	0	0	0	0
Short term borrowing	-	10,200	0	0	0	0
New debt raised: to fund projects	-	445,503	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	64,497	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,852,005	1,931,086	1,848,432	1,688,402	1,614,276
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,852,005	1,931,086	1,848,432	1,688,402	1,614,276
Increase in retentions	-	0	0	0	0	0

Table 8.16d

MDL 03: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,193	7,675
Y	9,479	7,615
Z	10,120	7,565
Expected value	9,221	7,623

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.951	0.1456	0.1683
8	0.878	0.1390	0.1802
9	0.830	0.1347	0.1723
10	0.808	0.1328	0.1558
11	0.807	0.1326	0.1486

Table 8.17a

Matrix PIXP **MDL 04 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	151,898	158,351	57,253	168,440
after losses carried forward	-	0	104,929	158,351	57,253	168,440
Other income (after any set off of trading losses in t)	-	175,539	200,000	200,000	200,000	200,000
Debt interest charges	-	222,507	222,507	222,507	222,507	222,507
Taxable profits	287,500	0	82,422	135,843	34,745	145,932
Trading loss	-	24,461	0	0	0	0
carried forward (LF1)	-	24,461	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	46,968	0	0	0	0
Total losses carried forward	0	46,968	0	0	0	0
Gross corporation tax payable	71,875	0	20,605	33,961	8,686	36,483
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	240,041	242,974	221,666	218,951	185,125
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	240,041	242,974	221,666	218,951	185,125
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	307,541	550,515	772,181	991,132	1,176,256
Short term lending	-	0	0	0	0	0
Short term borrowing	-	7,010	0	0	0	0
New debt raised: to fund projects	-	350,500	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,200,206	1,297,292	1,244,171	1,129,500	1,071,555
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,200,206	1,297,292	1,244,171	1,129,500	1,071,555
Increase in retentions	-	0	0	0	0	0

Table 8.17b

Matrix PIXE **MDL 04 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	386,339	563,898	570,351	464,053	568,440
after losses carried forward	-	386,339	563,898	570,351	464,053	568,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	222,507	222,507	222,507	222,507	222,507
Taxable profits	287,500	363,831	541,390	547,843	441,545	545,932
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	97,341	159,487	161,745	124,541	161,076
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	249,435	228,665	212,248	209,874	178,226
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	249,435	228,665	212,248	209,874	178,226
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	316,935	545,600	757,849	967,723	1,145,948
Short term lending	-	0	0	0	0	0
Short term borrowing	-	7,010	0	0	0	0
New debt raised: to fund projects	-	350,500	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,611,006	1,684,718	1,609,084	1,490,916	1,437,061
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,611,006	1,684,718	1,609,084	1,490,916	1,437,061
Increase in retentions	-	0	0	0	0	0

Table 8.17c

Matrix PIXO	MDL 04 Outcome Z					
Time period	6	7	8	9	10	11
Trading profit	-	602,539	781,898	788,351	674,253	768,440
after losses carried forward	-	602,539	781,898	788,351	674,253	768,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	222,507	222,507	222,507	222,507	222,507
Taxable profits	287,500	580,031	759,390	765,843	651,745	745,932
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	173,011	235,787	238,045	198,111	231,076
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	249,435	222,179	205,708	203,334	171,920
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	249,435	222,179	205,708	203,334	171,920
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	316,935	539,114	744,823	948,157	1,120,076
Short term lending	-	0	0	0	0	0
Short term borrowing	-	7,010	0	0	0	0
New debt raised: to fund projects	-	350,500	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,827,206	1,870,288	1,794,384	1,668,416	1,605,531
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,827,206	1,870,288	1,794,384	1,668,416	1,605,531
Increase in retentions	-	0	0	0	0	0

Table 8.17d

MDL 04: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,216	7,729
Y	9,494	7,663
Z	10,121	7,613
Expected value	9,236	7,673

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.935	0.1441	0.1658
8	0.859	0.1373	0.1739
9	0.822	0.1340	0.1664
10	0.807	0.1326	0.1536
11	0.802	0.1322	0.1474

Table 8.18a

Matrix PDEP **MDL 05 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	202,563	223,533	72,649	85,197
after losses carried forward	-	0	115,882	223,533	72,649	85,197
Other income (after any set off of trading losses in t)	-	168,339	200,000	200,000	200,000	200,000
Debt interest charges	-	254,860	254,860	254,860	254,860	254,860
Taxable profits	287,500	0	61,182	168,673	17,789	30,337
Trading loss	-	31,661	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	31,661	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	54,860	0	0	0	0
Total losses carried forward	0	86,521	0	0	0	0
Gross corporation tax payable	71,875	0	15,295	42,168	4,447	7,584
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	248,871	263,592	226,383	226,383	209,783
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	248,871	263,592	226,383	226,383	209,783
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	316,371	579,962	811,644	1,038,027	1,247,810
Short term lending	-	0	0	0	0	0
Short term borrowing	-	14,010	0	0	0	0
New debt raised: to fund projects	-	700,500	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,244,354	1,379,140	1,327,081	1,149,706	1,079,251
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,244,354	1,379,140	1,327,081	1,149,706	1,079,251
Increase in retentions	-	0	0	0	0	0

Table 8.18b

Matrix PDEE	**MDL 05**	**Outcome Y**				
Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	383,139	618,653	639,533	483,449	486,937
after losses carried forward	-	383,139	618,653	639,533	483,449	486,937
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	254,860	254,860	254,860	254,860	254,860
Taxable profits	287,500	328,279	563,703	584,673	428,589	431,537
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	84,898	167,296	174,636	120,006	121,038
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	266,175	242,439	221,382	216,530	203,103
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	266,175	242,439	221,382	216,530	203,103
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	333,675	576,114	797,496	1,014,026	1,217,129
Short term lending	-	0	0	0	0	0
Short term borrowing	-	14,010	0	0	0	0
New debt raised: to fund projects	-	700,500	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,659,154	1,775,898	1,691,585	1,511,239	1,447,052
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,659,154	1,775,898	1,691,585	1,511,239	1,447,052
Increase in retentions	-	0	0	0	0	0

Table 8.18c

Matrix PDEO **MDL 05 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	605,339	842,563	863,533	699,649	688,197
after losses carried forward	-	605,339	842,563	863,533	699,649	688,197
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	254,860	254,860	254,860	254,860	254,860
Taxable profits	287,500	550,479	787,703	808,673	644,789	633,337
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	162,668	245,696	253,036	195,676	191,668
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	266,175	235,773	214,662	209,810	196,617
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	266,175	235,773	214,662	209,810	196,617
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	333,675	569,448	784,110	993,920	1,190,537
Short term lending	-	0	0	0	0	0
Short term borrowing	-	14,010	0	0	0	0
New debt raised: to fund projects	-	700,500	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,881,354	1,966,568	1,881,985	1,693,839	1,616,422
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,881,354	1,966,568	1,881,985	1,693,839	1,616,422
Increase in retentions	-	0	0	0	0	0

Table 8.18d

MDL 05: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,221	7,456
Y	9,514	7,400
Z	10,154	7,350
Expected value	9,254	7,407

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.947	0.1452	0.1707
8	0.878	0.1391	0.1832
9	0.826	0.1344	0.1751
10	0.811	0.1330	0.1556
11	0.806	0.1325	0.1481

Table 8.19a

Matrix GROP **MDL 06 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	0	114,475	173,440
Other income (after any set off of trading losses in t)	-	100,000	218,598	236,846	253,647	268,807
Debt interest charges	-	191,775	191,388	191,250	191,250	191,250
Taxable profits	287,500	0	27,211	45,596	176,872	250,997
Trading loss	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	100,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	91,775	0	0	0	0
Total losses carried forward	0	91,775	91,775	16,775	0	0
Gross corporation tax payable	71,875	0	6,803	11,399	44,218	62,749
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	156,776	159,173	163,727	146,114	140,363
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	156,776	159,173	163,727	146,114	140,363
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	224,276	383,449	547,176	693,289	833,653
Short term lending	-	0	309,969	614,104	894,111	1,146,787
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,093,850	1,437,179	1,758,340	2,054,228	2,315,501
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	783,881	823,075	864,229	907,440	952,812
Increase in retentions	-	309,969	304,135	280,007	252,676	215,901

Table 8.19b

Matrix GROE **MDL 06 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	236,442	272,331	306,348	338,476
Debt interest charges	-	191,775	191,250	191,250	191,250	191,250
Taxable profits	287,500	308,225	445,192	556,081	646,348	715,603
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	77,879	125,817	164,628	196,222	220,461
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	115,652	97,124	84,254	75,974	72,385
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	115,652	97,124	84,254	75,974	72,385
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	183,152	280,276	364,530	440,504	512,889
Short term lending	-	0	607,364	1,205,512	1,772,463	2,307,931
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	59,559
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,493,850	2,136,322	2,749,814	3,334,149	3,883,142
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	886,486	930,810	977,351	1,026,218	1,077,529
Increase in retentions	-	607,364	598,148	566,951	535,467	497,682

Table 8.19c

Matrix GROO **MDL 06 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off						
of trading losses in t)	-	200,000	245,900	289,888	332,272	372,863
Debt interest charges	-	191,775	191,250	191,250	194,448	206,646
Taxable profits	287,500	508,225	654,650	773,638	869,074	939,657
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	147,879	199,128	240,773	274,176	298,880
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	84,124	64,127	50,082	41,236	37,872
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	84,124	64,127	50,082	41,236	37,872
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	151,624	215,751	265,833	307,069	344,940
Short term lending	-	0	765,007	1,498,138	2,204,528	2,881,054
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	37,625	143,505
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,693,850	2,473,423	3,228,578	3,956,306	4,646,910
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	928,843	975,286	1,024,050	1,075,252	1,129,015
Increase in retentions	-	765,007	733,131	706,391	676,525	636,841

Table 8.19d

MDL 06: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,780	9,205
Y	8,800	10,410
Z	9,220	10,907
Expected value	8,578	10,148

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.966	0.1469	0.1648
8	0.885	0.1396	0.1583
9	0.841	0.1356	0.1512
10	0.806	0.1325	0.1454
11	0.777	0.1300	0.1401

Table 8.20a

Matrix PRUP **MDL 07 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	133,151	204,496	188,503	84,065
after losses carried forward	-	0	31,667	204,496	188,503	84,065
Other income (after any set off						
of trading losses in t)	-	200,000	204,372	229,367	252,943	271,094
Debt interest charges	-	197,192	197,101	196,581	196,350	196,350
Taxable profits	287,500	2,807	38,947	237,282	245,096	158,809
Trading loss	-	101,474	0	0	0	0
carried forward (LF1)	-	101,474	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	101,474	0	0	0	0
Gross corporation tax payable	71,875	0	9,737	59,321	61,274	39,702
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	164,607	165,637	134,641	142,184	169,001
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	164,607	165,637	134,641	142,184	169,001
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	232,107	397,744	532,385	674,569	843,570
Short term lending	-	0	72,862	489,449	882,388	1,184,903
Short term borrowing	-	1,200	0	0	0	0
New debt raised: to fund projects	-	60,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,098,702	1,356,582	1,792,877	2,140,917	2,363,192
Internal funding of new projects	-	0	200,000	0	0	0
Gross dividends paid	-	825,841	867,133	910,490	956,014	1,003,815
Increase in retentions	-	72,862	416,587	392,939	302,515	174,475

Table 8.20b

Matrix PRUE	MDL 07	Outcome Y				
Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	301,326	547,151	616,496	597,303	488,065
after losses carried forward	-	301,326	547,151	616,496	597,303	488,065
Other income (after any set off of trading losses in t)	-	200,000	221,723	264,676	304,498	338,875
Debt interest charges	-	197,192	196,560	196,180	196,180	196,180
Taxable profits	287,500	304,133	572,314	684,992	705,621	630,760
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	76,447	170,310	209,747	216,967	190,766
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	126,664	82,402	69,710	75,919	101,744
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	126,664	82,402	69,710	75,919	101,744
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	194,164	276,566	346,275	422,194	523,938
Short term lending	-	0	362,051	1,077,933	1,741,632	2,314,584
Short term borrowing	-	1,200	0	0	0	0
New debt raised: to fund projects	-	60,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	2,000	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,499,503	2,062,257	2,775,172	3,399,801	3,901,236
Internal funding of new projects	-	0	200,000	0	0	0
Gross dividends paid	-	937,451	984,324	1,033,540	1,085,217	1,139,478
Increase in retentions	-	362,051	715,882	663,699	572,952	447,174

Table 8.20c

Matrix PRUO	MDL 07	Outcome Z				
Time period	6	7	8	9	10	11
Trading profit	-	505,526	765,180	834,496	810,503	694,065
after losses carried forward	-	505,526	765,180	834,496	810,503	694,065
Other income (after any set off of trading losses in t)	-	200,000	239,560	290,777	338,867	381,463
Debt interest charges	-	197,192	209,735	208,978	208,978	208,978
Taxable profits	287,500	508,333	795,005	916,295	940,392	866,550
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	147,917	248,252	290,703	299,137	273,292
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	96,725	49,310	35,468	41,585	67,836
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	96,725	49,310	35,468	41,585	67,836
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	164,225	213,535	249,003	290,588	358,424
Short term lending	-	0	659,336	1,512,945	2,314,449	3,024,379
Short term borrowing	-	1,200	0	0	0	0
New debt raised: to fund projects	-	60,000	148,563	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,705,702	2,554,501	3,408,083	4,172,694	4,811,605
Internal funding of new projects	-	0	51,437	0	0	0
Gross dividends paid	-	991,958	1,041,556	1,093,634	1,148,315	1,205,731
Increase in retentions	-	662,307	850,638	801,504	709,928	581,495

Table 8.20d

MDL 07: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,732	9,138
Y	8,778	10,373
Z	9,288	10,976
Expected value	8,566	10,123

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	1.163	0.1647	0.1456
8	0.949	0.1454	0.1814
9	0.856	0.1370	0.1695
10	0.815	0.1334	0.1553
11	0.788	0.1309	0.1408

Table 8.21a

Matrix ERNP **MDL 08 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	0	114,531	173,440
Other income (after any set off of trading losses in t)	-	100,000	218,654	236,965	253,830	269,063
Debt interest charges	-	191,719	191,330	191,250	191,250	191,250
Taxable profits	287,500	0	27,324	45,715	177,111	251,253
Trading loss	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	100,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	91,719	0	0	0	0
Total losses carried forward	0	91,719	91,719	16,719	0	0
Gross corporation tax payable	71,875	0	6,831	11,429	44,278	62,813
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	156,600	158,965	163,509	145,862	140,098
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	156,600	158,965	163,509	145,862	140,098
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	224,100	383,065	546,574	692,436	832,534
Short term lending	-	0	310,905	616,079	897,169	1,151,042
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,093,906	1,438,230	1,760,427	2,057,463	2,319,999
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	783,001	822,151	863,258	906,421	951,742
Increase in retentions	-	310,905	305,173	281,090	253,873	217,215

Table 8.21b

Matrix ERNE **MDL 08 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	236,285	271,995	305,817	337,730
Debt interest charges	-	191,719	191,250	191,250	191,250	191,250
Taxable profits	287,500	308,281	445,035	555,745	645,817	719,920
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	77,898	125,762	164,511	196,036	221,972
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	116,176	97,717	84,911	76,699	72,172
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	116,176	97,717	84,911	76,699	72,172
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	183,676	281,392	366,303	443,002	515,174
Short term lending	-	0	604,747	1,199,922	1,763,614	2,295,506
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,493,906	2,133,539	2,743,912	3,324,819	3,875,114
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	889,159	933,617	980,298	1,029,313	1,080,779
Increase in retentions	-	604,747	595,175	563,692	531,892	498,829

Table 8.21c

Matrix ERNO **MDL 08 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	245,286	288,588	330,218	370,175
Debt interest charges	-	191,719	191,250	191,250	191,250	191,250
Taxable profits	287,500	508,281	654,036	772,338	870,218	952,365
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	147,898	198,913	240,318	274,576	303,328
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	86,172	66,412	52,613	43,391	37,833
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	86,172	66,412	52,613	43,391	37,833
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	153,672	220,084	272,696	316,087	353,920
Short term lending	-	0	754,767	1,476,465	2,170,296	2,836,243
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,693,906	2,462,561	3,205,697	3,923,414	4,614,635
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	939,139	986,096	1,035,401	1,087,171	1,141,529
Increase in retentions	-	754,767	721,698	693,832	665,946	636,862

Table 8.21d

MDL 08: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,790	9,210
Y	8,846	10,459
Z	9,343	11,047
Expected value	8,629	10,202

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.960	0.1464	0.1639
8	0.879	0.1391	0.1574
9	0.835	0.1352	0.1505
10	0.804	0.1323	0.1448
11	0.789	0.1310	0.1399

Table 8.22a

Matrix PORP **MDL 09 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	178,776	233,715	60,886	96,838
after losses carried forward	-	0	96,505	233,715	60,886	96,838
Other income (after any set off						
of trading losses in t)	-	156,126	200,000	224,206	251,713	269,333
Debt interest charges	-	238,396	202,152	191,250	191,250	191,250
Taxable profits	287,500	0	94,354	266,671	121,349	174,921
Trading loss	-	43,874	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	43,874	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	82,270	0	0	0	0
Total losses carried forward	0	82,270	0	0	0	0
Gross corporation tax payable	71,875	0	23,589	66,668	30,337	43,730
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	162,791	152,059	126,143	164,181	162,889
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	162,791	152,059	126,143	164,181	162,889
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	230,291	382,350	508,493	672,674	835,562
Short term lending	-	0	0	403,432	861,877	1,155,553
Short term borrowing	-	10,200	0	0	0	0
New debt raised: to fund projects	-	510,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	393,646	116,354	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	813,953	1,258,084	1,759,261	2,097,806	2,338,369
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	813,953	854,651	897,384	942,253	989,366
Increase in retentions	-	0	403,432	458,445	293,676	193,450

Table 8.22b

Matrix PORE **MDL 09 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	370,926	594,776	649,715	471,686	498,038
after losses carried forward	-	370,926	594,776	649,715	471,686	498,038
Other income (after any set off of trading losses in t)	-	200,000	211,235	260,500	304,341	338,173
Debt interest charges	-	238,396	191,446	191,250	191,250	191,250
Taxable profits	287,500	332,530	614,565	718,965	584,777	644,961
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	86,385	185,098	221,638	174,672	195,736
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	118,524	71,369	60,202	97,240	95,913
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	118,524	71,369	60,202	97,240	95,913
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	186,024	257,393	317,595	414,835	510,748
Short term lending	-	0	187,249	1,008,341	1,739,019	2,302,888
Short term borrowing	-	10,200	0	0	0	0
New debt raised: to fund projects	-	510,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	510,000	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,112,399	1,979,748	2,758,997	3,373,865	3,904,095
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	925,150	971,408	1,019,978	1,070,977	1,124,526
Increase in retentions	-	187,249	821,092	730,678	563,869	476,681

Table 8.22c

Matrix PORO **MDL 09 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	593,126	818,776	873,715	687,886	699,838
after losses carried forward	-	593,126	818,776	873,715	687,886	699,838
Other income (after any set off of trading losses in t)	-	200,000	221,204	278,988	331,562	373,924
Debt interest charges	-	238,396	191,250	191,250	191,250	191,250
Taxable profits	287,500	554,730	848,730	961,453	828,198	882,512
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	164,155	267,055	306,508	259,869	278,879
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	85,295	36,307	24,065	61,534	62,030
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	85,295	36,307	24,065	61,534	62,030
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	152,795	189,102	213,167	274,701	336,731
Short term lending	-	0	353,394	1,316,463	2,192,702	2,898,729
Short term borrowing	-	10,200	0	0	0	0
New debt raised: to fund projects	-	510,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	510,000	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,334,599	2,346,728	3,274,481	4,034,596	4,700,973
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	981,205	1,030,265	1,081,779	1,135,868	1,192,661
Increase in retentions	-	353,394	963,069	876,240	706,027	609,583

Table 8.22d

MDL 09: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,803	9,197
Y	8,869	10,454
Z	9,406	11,087
Expected value	8,657	10,203

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.812	0.1331	0.1233
8	1.168	0.1651	0.1896
9	0.869	0.1382	0.1753
10	0.817	0.1335	0.1522
11	0.800	0.1320	0.1414

Table 8.23

Data inputs to XPRESS-MP: variation 1

Capital allowances	*Time period* 7	8	9	10	11
parent	1.2	0.2	0.2	0.2	0.2
subsidiary	0.2	0.2	0.2	0.2	0.2

Relevant data inputs, £'000

variable	*parent*	*subsidiary*
LF(t=6)	0	0
P3/P3a (t=6)	0.2875	0.68
P4(t=5)	0.2875	0.68
P5(t=4)	0.2875	0.68
SF(t=6)	0.0675	0
U2(t=6)	0	0
U3A(t=5)	0.0575	0
U4A(t=4)	0.0575	0
U5A(t=3)	0.0575	0
U6(t=2)	0.0575	0
U7(t=1)	0.0575	0
OCB(t=7)+CSHC(t=7)	0.05	-
DB(t=6)	2.25	-
EQ(t=6)	9.138	5
DIV(t=6)	0.5	0.2
A2(t=6)	0.0575	0.05
TR(t=6)	0	0
SFII(t=6)	0	-
SBVAL(t=NT)	-	5.48
SDBT(t=NT)	-	1.5
OCB(t=7)	-	0.05

Table 8.23 continued

Data inputs to XPRESS-MP: variation 2

Capital allowances	*Time period*				
	7	8	9	10	11
parent	1.2	0.2	0.2	0.2	0.2
subsidiary	0.2	0.2	0.2	0.2	0.2

Relevant data inputs, £'000

variable	*parent*	*subsidiary*
LF(t=6)	0	0
P3/P3a (t=6)	0.2875	0.68
P4(t=5)	0.2875	0.68
P5(t=4)	0.2875	0.68
SF(t=6)	0.0675	0
U2(t=6)	0	0
U3A(t=5)	0.0575	0
U4A(t=4)	0.0575	0
U5A(t=3)	0.0575	0
U6(t=2)	0.0575	0
U7(t=1)	0.0575	0
OCB(t=7)+CSHC(t=7)	0.05	-
DB(t=6)	2.25	-
EQ(t=6)	9.138	5
DIV(t=6)	0.5	0.2
A2(t=6)	0.0575	0.05
TR(t=6)	0	0
SFII(t=6)	0	-
SBVAL(t=NT)	-	5.48
SDBT(t=NT)	-	1.5
OCB(t=7)	-	0.05

Table 8.24a

Matrix NOVP **MDL 10 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	0	31,250	73,440
after losses carried forward	-	0	0	0	0	0
Other income (after any set off of trading losses in t)	-	0	100,000	175,000	200,000	200,000
Debt interest charges	-	191,966	191,966	191,966	191,966	191,966
Taxable profits	287,500	0	0	0	8,034	8,034
Trading loss	-	200,000	100,000	25,000	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	200,000	100,000	25,000	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	191,966	91,966	16,966	0	0
Total losses carried forward	0	191,966	283,932	300,899	269,649	196,209
Gross corporation tax payable	71,875	0	0	0	2,008	2,008
Tax rebate	0	248,415	0	0	0	0
Surplus ACT on dividends paid	-	248,415	201,607	201,607	200,000	199,920
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	201,607	201,607	200,000	199,920
carried back to previous periods	-	248,415				
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	18,415	0	0	0	0
Surplus ACT carried forward	67,500	67,500	269,107	470,713	670,713	870,633
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,242,073	1,008,034	1,008,034	1,008,034	1,007,632
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,242,073	1,008,034	1,008,034	1,008,034	1,007,632
Increase in retentions	-	0	0	0	0	0

Table 8.24b

Matrix NOVE **MDL 10 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,966	191,966	191,966	191,966	191,966
Taxable profits	284,500	308,034	408,034	483,034	539,284	581,474
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	77,812	112,812	139,062	158,749	173,516
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	294,625	216,759	198,759	185,259	175,133
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	7,125	216,759	198,759	185,259	175,133
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	74,625	291,384	490,143	675,402	850,535
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,781,159	1,491,829	1,476,829	1,465,579	1,457,141
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,781,159	1,491,829	1,476,829	1,465,579	1,457,141
Increase in retentions	-	0	0	0	0	0

Table 8.24c

Matrix NOVO **MDL 10 Outcome Z**

Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,966	191,966	191,966	191,966	191,966
Taxable profits	287,500	508,034	608,034	683,034	739,284	781,474
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	147,812	182,812	209,062	228,749	246,516
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	294,625	210,759	192,759	179,259	169,133
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	7,125	210,759	192,759	179,259	169,133
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	74,625	285,384	478,143	657,402	826,535
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,981,159	1,661,829	1,646,829	1,635,579	1,627,141
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,981,159	1,661,829	1,646,829	1,635,579	1,627,141
Increase in retentions	-	0	0	0	0	0

Table 8.24d

MDL 10: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,229	7,688
Y	8,782	7,735
Z	9,340	7,801
Expected value	8,428	7,734

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	1.231	0.1708	0.1969
8	1.092	0.1583	0.1638
9	1.065	0.1559	0.1626
10	1.045	0.1541	0.1616
11	1.031	0.1528	0.1609

Table 8.25

Cash flows for a single large project, £'000

Discount rate = 0.15
Tax rate = 0.33

(a) Purchased project

Time period		0	1	2	3	4	5
Capital outlay		(1000.000)					
Cash inflow			50.000	500.000	600.000	450.000	200.000
Present value			43.478	378.072	394.510	257.289	99.435
Tax on inflows			16.500	165.000	198.000	148.500	66.000
Present value			(12.476)	(108.490)	(113.207)	(73.831)	(28.534)
Capital allowances			250.000	187.500	140.625	105.469	
Balancing allowance							316.406
Present value of tax benefits			62.382	40.684	26.533	17.304	45.141
Working capital increase			50.000				
decrease							50.000
Present value of tax effects			(12.476)				7.133
Disposal proceeds		*n/a*					
Present value including tax effects		*n/a*					
Net present value	22.947						

(b) Leased project

		0	1	2	3	4	5
Implicit rate of interest	0.0850						
Asset cost, if purchased		(1000.000)					
Rental payments			233.900	233.900	233.900	233.900	233.900
Present value			233.900	203.391	176.862	153.793	133.733
Accounting depreciation			200.000	200.000	200.000	200.000	200.000
Present value of tax benefits			49.905	43.396	37.736	32.814	28.534
Finance charge			65.147	50.796	35.226	18.331	(0.000)
Present value of tax benefits			16.256	11.022	6.646	3.008	(0.000)
Net present value	158.540						

Table 8.26a

Matrix NEDP **MDL 11 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	212,500	434,380	375,780	0
after losses carried forward	-	0	0	377,574	375,780	0
Other income (after any set off of trading losses in t)	-	0	200,000	200,000	200,000	200,000
Debt interest charges	-	234,653	234,653	234,653	234,653	234,653
Taxable profits	287,500	0	0	342,921	341,127	0
Trading loss	-	361,765	0	0	0	92,970
carried forward (LF1)	-	0	0	0	0	92,970
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	161,765	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	234,653	34,653	0	0	34,653
Total losses carried forward	0	234,653	56,806	0	0	127,623
Gross corporation tax payable	71,875	0	0	90,022	89,394	0
Tax rebate	0	254,853	0	0	0	0
Surplus ACT on dividends paid	-	246,765	293,069	244,485	210,556	228,836
arising from losses carried back	-	32,353	0	0	0	0
carried back to previous periods	-	246,765	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	16,765	0	0	0	0
Surplus ACT carried forward	67,500	99,853	392,922	637,408	847,964	1,076,800
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	488,253	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	511,747	0	0	0	0
Closing balance of funds	-	1,233,825	1,465,347	1,565,347	1,393,909	1,144,178
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,233,825	1,465,347	1,565,347	1,393,909	1,144,178
Increase in retentions	-	0	0	0	0	0

Table 8.26b

Matrix NEDE **MDL 11 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	138,388	712,500	934,380	875,780	407,030
after losses carried forward	-	138,388	712,500	934,380	875,780	407,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	234,653	234,653	234,653	234,653	234,653
Taxable profits	287,500	103,582	677,847	899,727	841,127	372,377
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	25,895	207,246	284,904	264,394	100,332
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	332,578	256,464	218,789	193,852	239,360
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	45,078	256,464	218,789	193,852	239,360
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	112,578	369,042	587,831	781,683	1,021,043
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	488,253	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	511,747	0	0	0	0
Closing balance of funds	-	1,766,472	1,960,168	1,993,670	1,810,388	1,569,178
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,766,472	1,960,168	1,993,670	1,810,388	1,569,178
Increase in retentions	-	0	0	0	0	0

Table 8.26c

Matrix NEDO **MDL 11 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	338,388	912,500	1,134,380	1,075,780	607,030
after losses carried forward	-	338,388	912,500	1,134,380	1,075,780	607,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	234,653	234,653	234,653	234,653	234,653
Taxable profits	287,500	303,582	877,847	1,099,727	1,041,127	572,377
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	76,191	277,246	354,904	334,393	170,332
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	332,578	254,405	212,789	187,852	233,360
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	45,047	254,405	212,789	187,852	233,360
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	112,578	366,983	579,772	767,624	1,000,984
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	488,253	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	511,747	0	0	0	0
Closing balance of funds	-	1,966,472	2,149,872	2,163,670	1,980,388	1,739,178
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,966,472	2,149,872	2,163,670	1,980,388	1,739,178
Increase in retentions	-	0	0	0	0	0

Table 8.26d

MDL 11: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,754	7,534
Y	9,252	7,427
Z	9,834	7,383
Expected value	8,919	7,450

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	1.152	0.1637	0.1848
8	1.076	0.1568	0.2075
9	0.938	0.1445	0.2131
10	0.919	0.1427	0.1929
11	0.933	0.1440	0.1656

Table 8.27a

Matrix BENP **MDL 12 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	212,500	434,380	375,780	0
after losses carried forward	-	0	0	277,970	0	0
Other income (after any set off of trading losses in t)	-	0	200,000	200,000	200,000	200,000
Debt interest charges	-	284,297	284,297	284,297	284,297	284,297
Taxable profits	287,500	0	0	193,987	291,482	0
Trading loss	-	372,000	0	0	0	92,970
carried forward (LF1)	-	0	0	0	0	92,970
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	172,000	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	284,297	84,297	0	0	84,297
Total losses carried forward	0	284,297	156,095	0	0	177,267
Gross corporation tax payable	71,875	0	0	48,497	72,871	0
Tax rebate	0	243,082	0	0	0	0
Surplus ACT on dividends paid	-	234,482	283,140	264,343	212,904	220,226
arising from losses carried back	-	34,400	0	0	0	0
chosen to carry forward	-	0	283,140	264,343	212,904	220,226
carried back to previous periods	-	234,482	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	4,482	0	0	0	0
Surplus ACT carried forward	67,500	101,900	385,040	649,383	862,288	1,082,513
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,172,409	1,415,702	1,515,702	1,356,003	1,101,128
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,172,409	1,415,702	1,515,702	1,356,003	1,101,128
Increase in retentions	-	0	0	0	0	0

Table 8.27b

Matrix BENE **MDL 12 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	128,000	712,500	934,380	875,780	407,030
after losses carried forward	-	128,000	712,500	934,380	875,780	407,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,297	284,297	284,297	284,297	284,297
Taxable profits	287,500	43,702	628,202	850,082	791,482	322,732
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	10,296	189,871	267,529	247,019	82,956
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	334,625	257,063	220,278	195,342	240,850
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	47,125	257,063	220,278	195,342	240,850
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	114,625	371,688	591,966	787,307	1,028,157
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,716,670	1,913,517	1,951,472	1,768,190	1,526,980
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,716,670	1,913,517	1,951,472	1,768,190	1,526,980
Increase in retentions	-	0	0	0	0	0

Table 8.27c

Matrix BENO **MDL 12 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	328,000	912,500	1,134,380	1,075,780	607,030
after losses carried forward	-	328,000	912,500	1,134,380	1,075,780	607,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,297	284,297	284,297	284,297	284,297
Taxable profits	287,500	243,702	828,202	1,050,082	991,482	522,732
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	60,926	259,871	337,529	317,019	152,956
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	334,625	255,063	214,278	189,342	234,850
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	47,125	255,063	214,278	189,342	234,850
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	114,625	369,688	583,966	773,307	1,008,157
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,916,827	2,103,517	2,121,472	1,938,190	1,696,980
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,916,827	2,103,517	2,121,472	1,938,190	1,696,980
Increase in retentions	-	0	0	0	0	0

Table 8.27d

MDL 12: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,872	7,043
Y	9,382	6,928
Z	9,965	6,885
Expected value	9,046	6,954

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	1.156	0.1640	0.1762
8	1.066	0.1559	0.1993
9	0.938	0.1444	0.2051
10	0.899	0.1409	0.1856
11	0.921	0.1429	0.1585

Table 8.28a

Matrix NEXP **MDL 13 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	900,000	900,000	900,000	900,000
after losses carried forward	-	0	800,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,862	191,862	191,862	191,862	191,862
Taxable profits	287,500	0	816,275	908,137	908,137	908,137
Trading loss	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	100,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	91,862	0	0	0	0
Total losses carried forward	0	91,862	0	0	0	0
Gross corporation tax payable	71,875	0	255,696	287,848	287,848	287,848
Tax rebate	0	273,441	0	0	0	0
Surplus ACT on dividends paid	-	273,441	58,372	21,512	18,756	18,756
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	58,372	21,512	18,756	18,756
carried back to previous periods	-	273,441	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	43,441	0	0	0	0
Surplus ACT carried forward	67,500	67,500	125,872	147,384	166,140	184,896
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,367,203	1,108,137	1,015,696	1,001,917	1,001,917
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,367,203	1,108,137	1,015,696	1,001,917	1,001,917
Increase in retentions	-	0	0	0	0	0

Table 8.28b

Matrix NEXE **MDL 13 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	1,300,000	1,300,000	1,300,000	1,300,000
after losses carried forward	-	300,000	1,300,000	1,300,000	1,300,000	1,300,000
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,862	191,862	191,862	191,862	191,862
Taxable profits	287,500	308,137	1,308,137	1,308,137	1,308,137	1,308,137
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	77,848	427,848	427,848	427,848	427,848
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	294,625	36,756	6,756	6,756	6,756
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	7,125	36,756	6,756	6,756	6,756
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	74,625	111,381	118,137	124,893	131,648
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,781,262	1,491,917	1,341,917	1,341,917	1,341,917
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,781,262	1,491,917	1,341,917	1,341,917	1,341,917
Increase in retentions	-	0	0	0	0	0

Table 8.28c

Matrix NEXO MDL 13 Outcome Z

Time period	6	7	8	9	10	11
Trading profit	-	500,000	1,500,000	1,500,000	1,500,000	1,500,000
after losses carried forward	-	500,000	1,500,000	1,500,000	1,500,000	1,500,000
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,862	191,862	191,862	191,862	191,862
Taxable profits	287,500	508,137	1,508,137	1,508,137	1,508,137	1,508,137
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	147,848	497,685	497,685	497,685	497,685
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	294,625	30,756	0	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	7,125	30,756	0	0	0
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	74,625	105,381	106,169	106,958	107,746
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,981,262	1,661,917	1,512,080	1,512,080	1,512,080
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,981,262	1,661,917	1,512,080	1,512,080	1,512,080
Increase in retentions	-	0	0	0	0	0

Table 8.28d

MDL 13: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,560	7,119
Y	8,760	7,020
Z	9,343	6,972
Expected value	8,516	7,040

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.997	0.1497	0.1992
8	0.904	0.1414	0.1656
9	0.800	0.1320	0.1501
10	0.827	0.1344	0.1495
11	0.827	0.1344	0.1495

Table 8.29a

Matrix NICP

MDL 14 Outcome X

Time period	6	7	8	9	10	11
Trading profit	-	0	1,212,500	1,359,380	1,244,530	733,590
after losses carried forward	-	0	916,557	1,359,380	1,244,530	733,590
Other income (after any set off of trading losses in t)	-	0	200,000	200,000	200,000	200,000
Debt interest charges	-	234,267	234,267	234,267	234,267	234,267
Taxable profits	287,500	0	882,290	1,325,113	1,210,263	699,323
Trading loss	-	261,675	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	61,675	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	234,267	0	0	0	0
Total losses carried forward	0	295,943	0	0	0	0
Gross corporation tax payable	71,875	0	278,802	433,789	393,592	214,763
Tax rebate	0	269,839	0	0	0	0
Surplus ACT on dividends paid	-	269,839	136,689	47,655	27,341	82,974
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	136,689	47,655	27,341	82,974
carried back to previous periods	-	269,839	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	39,839	0	0	0	0
Surplus ACT carried forward	67,500	67,500	204,189	251,844	279,184	362,159
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	483,753	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	516,247	0	0	0	0
Closing balance of funds	-	1,349,197	1,565,733	1,563,389	1,346,966	1,114,193
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,349,197	1,565,733	1,563,389	1,346,966	1,114,193
Increase in retentions	-	0	0	0	0	0

Table 8.29b

Matrix NICE **MDL 14 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	137,254	1,612,500	1,759,380	1,644,530	1,133,590
after losses carried forward	-	137,254	1,612,500	1,759,380	1,644,530	1,133,590
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	234,267	234,267	234,267	234,267	234,267
Taxable profits	287,500	104,057	1,578,233	1,725,113	1,610,263	1,099,323
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	26,014	520,817	569,287	531,387	354,763
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	332,560	76,459	27,090	16,241	71,415
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	45,060	76,459	27,090	16,241	71,415
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	112,560	189,019	216,109	232,350	303,766
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	483,753	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	516,247	0	0	0	0
Closing balance of funds	-	1,766,858	1,960,530	1,860,562	1,691,468	1,456,398
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,766,858	1,960,530	1,860,562	1,691,468	1,456,398
Increase in retentions	-	0	0	0	0	0

Table 8.29c

Matrix NICO	MDL 14 Outcome Z					
Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	337,254	1,812,500	1,959,380	1,844,530	1,333,590
after losses carried forward	-	337,254	1,812,500	1,959,380	1,844,530	1,333,590
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	234,267	234,267	234,267	234,267	234,267
Taxable profits	287,500	304,057	1,778,233	1,925,113	1,810,263	1,299,323
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	76,420	586,817	635,287	597,387	424,763
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	332,560	74,392	21,890	11,041	66,215
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	45,060	74,392	21,890	11,041	66,215
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	483,753	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	516,247	0	0	0	0
Closing balance of funds	-	1,966,858	2,150,195	2,034,562	1,865,468	1,630,398
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,966,858	2,150,195	2,034,562	1,865,468	1,630,398
Increase in retentions	-	0	0	0	0	0

Table 8.29d

MDL 14: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	7,993	6,768
Y	9,199	6,663
Z	9,814	6,621
Expected value	8,960	6,686

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.962	0.1465	0.1891
8	0.910	0.1419	0.2114
9	0.725	0.1252	0.2031
10	0.805	0.1324	0.1825
11	0.801	0.1321	0.1561

Table 8.30a

Matrix GINP **MDL 15 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	1,212,500	1,359,380	1,244,530	733,590
after losses carried forward	-	0	857,008	1,359,380	1,244,530	733,590
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,500	284,500	284,500	284,500	284,500
Taxable profits	287,500	0	773,516	1,275,888	1,161,038	650,098
Trading loss	-	272,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	72,000	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	284,500	0	0	0	0
Total losses carried forward	0	356,500	0	0	0	0
Gross corporation tax payable	71,875	0	240,025	416,208	376,010	197,181
Tax rebate	0	257,281	0	0	0	0
Surplus ACT on dividends paid	-	257,281	148,800	50,979	28,848	84,481
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	148,800	50,979	28,848	84,481
carried back to previous periods	-	257,281	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	27,281	0	0	0	0
Surplus ACT carried forward	67,500	67,500	216,300	267,279	296,127	380,608
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,286,406	1,515,500	1,529,775	1,304,268	1,071,495
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,286,406	1,515,500	1,529,775	1,304,268	1,071,495
Increase in retentions	-	0	0	0	0	0

Table 8.30b

Matrix GINE **MDL 15 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	128,000	1,612,500	1,759,380	1,644,530	1,133,590
after losses carried forward	-	128,000	1,612,500	1,759,380	1,644,530	1,133,590
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,500	284,500	284,500	284,500	284,500
Taxable profits	287,500	43,500	1,528,000	1,674,880	1,560,030	1,049,090
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	10,875	504,240	552,710	514,810	337,181
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	334,625	77,065	28,396	17,547	72,721
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	47,125	77,065	28,396	17,547	72,721
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	114,625	191,690	220,086	237,633	310,354
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,716,625	1,913,325	1,816,860	1,647,766	1,412,696
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,716,625	1,913,325	1,816,860	1,647,766	1,412,696
Increase in retentions	-	0	0	0	0	0

Table 8.30c

Matrix GINO **MDL 15 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	328,000	1,812,500	1,959,380	1,844,530	1,333,590
after losses carried forward	-	328,000	1,812,500	1,959,380	1,844,530	1,333,590
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,500	284,500	284,500	284,500	284,500
Taxable profits	287,500	243,500	1,728,000	1,874,880	1,760,030	1,249,090
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	60,875	570,240	618,710	580,810	407,181
Tax rebate	0	287,500	0	0	0	0
Surplus ACT on dividends paid	-	334,625	75,065	23,196	12,347	67,521
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	47,125	75,065	23,196	12,347	67,521
carried back to previous periods	-	287,500	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	57,500	0	0	0	0
carried back to t-3	-	57,500	0	0	0	0
carried back to t-4	-	57,500	0	0	0	0
carried back to t-5	-	57,500	0	0	0	0
carried back to t-6	-	57,500	0	0	0	0
Surplus ACT carried forward	67,500	114,625	189,690	212,886	225,233	292,754
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,916,625	2,103,325	1,990,860	1,821,766	1,586,696
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,916,625	2,103,325	1,990,860	1,821,766	1,586,696
Increase in retentions	-	0	0	0	0	0

Table 8.30d

MDL 15: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,084	6,326
Y	9,235	6,211
Z	9,847	6,169
Expected value	9,012	6,237

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.970	0.1473	0.1809
8	0.904	0.1414	0.2036
9	0.700	0.1230	0.1962
10	0.793	0.1314	0.1756
11	0.790	0.1311	0.1495

Table 8.31a

Matrix CHEP **MDL 16 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	312,500	534,380	475,780	7,030
after losses carried forward	-	0	1,100	488,679	475,780	7,030
Other income (after any set off of trading losses in t)	-	0	200,000	200,000	200,000	200,000
Debt interest charges	-	246,800	246,800	246,800	246,800	246,800
Taxable profits	287,500	0	0	441,879	428,980	0
Trading loss	-	264,600	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	64,600	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	246,800	45,701	0	0	39,770
Total losses carried forward	0	311,400	45,701	0	0	39,770
Gross corporation tax payable	71,875	0	0	124,658	120,143	0
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	213,365	310,640	242,264	207,588	243,771
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	213,365	310,640	242,264	207,588	243,771
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	281,865	591,505	833,769	1,041,357	1,285,127
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	630,003	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	369,997	0	0	0	0
Closing balance of funds	-	1,066,825	1,553,200	1,653,200	1,466,918	1,218,853
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,066,825	1,553,200	1,653,200	1,466,918	1,218,853
Increase in retentions	-	0	0	0	0	0

Table 8.31b

Matrix CHEE **MDL 16 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	135,683	712,500	934,380	875,780	407,030
after losses carried forward	-	135,683	712,500	934,380	875,780	407,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	246,800	246,800	246,800	246,800	246,800
Taxable profits	287,500	88,600	665,700	887,580	828,980	360,230
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	22,150	202,995	280,653	260,143	96,080
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	275,645	256,614	219,153	194,217	239,725
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	275,645	256,614	219,153	194,217	239,725
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	343,145	599,759	818,912	1,013,129	1,252,853
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	630,003	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	369,997	0	0	0	0
Closing balance of funds	-	1,466,825	1,948,770	1,983,345	1,800,063	1,558,853
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,466,825	1,948,770	1,983,345	1,800,063	1,558,853
Increase in retentions	-	0	0	0	0	0

Table 8.31c

Matrix CHEO **MDL 16 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	335,683	912,500	1,134,380	1,075,780	607,030
after losses carried forward	-	335,683	912,500	1,134,380	1,075,780	607,030
Other income (after any set off						
of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	246,800	246,800	246,800	246,800	246,800
Taxable profits	287,500	288,600	865,700	1,087,580	1,028,980	560,230
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	72,150	272,995	350,653	330,143	166,080
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	275,645	254,614	213,153	188,217	233,725
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	343,145	597,759	810,912	999,124	1,232,853
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	630,003	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	369,997	0	0	0	0
Closing balance of funds	-	1,666,825	2,138,770	2,153,345	1,970,063	1,728,853
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,666,825	2,138,770	2,153,345	1,970,063	1,728,853
Increase in retentions	-	0	0	0	0	0

Table 8.31d

MDL 16: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,562	7,569
Y	9,800	7,507
Z	10,411	7,463
Expected value	9,551	7,517

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.868	0.1381	0.1452
8	0.849	0.1364	0.1956
9	0.722	0.1250	0.2008
10	0.726	0.1254	0.1254
11	0.737	0.1264	0.1561

Table 8.32a

Matrix DEBP **MDL 17 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	312,500	534,380	475,780	7,030
after losses carried forward	-	0	0	410,031	475,780	7,030
Other income (after any set off of trading losses in t)	-	0	200,000	200,000	200,000	200,000
Debt interest charges	-	282,240	282,240	282,240	282,240	282,240
Taxable profits	287,500	0	0	328,160	393,540	0
Trading loss	-	272,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	72,000	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	282,240	82,240	0	0	75,210
Total losses carried forward	0	354,240	123,980	0	0	75,210
Gross corporation tax payable	71,875	0	0	84,856	107,739	0
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	206,277	303,552	257,920	210,999	237,746
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	206,277	303,552	257,920	210,999	237,746
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	273,777	577,329	835,249	1,046,248	1,283,994
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,031,385	1,517,760	1,617,760	1,448,536	1,188,729
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,031,385	1,517,760	1,617,760	1,448,536	1,188,729
Increase in retentions	-	0	0	0	0	0

Table 8.32b

Matrix DEBE **MDL 17 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	128,000	712,500	934,380	875,780	407,030
after losses carried forward	-	128,000	712,500	934,380	875,780	407,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	282,240	282,240	282,240	282,240	282,240
Taxable profits	287,500	45,760	630,260	852,140	793,540	324,790
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	11,440	190,591	268,249	247,739	83,676
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	277,125	257,042	220,216	195,280	240,788
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	277,125	257,042	220,216	195,280	240,788
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	344,625	601,667	821,884	1,017,163	1,257,951
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,431,385	1,915,472	1,953,221	1,769,939	1,528,729
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,431,385	1,915,472	1,953,221	1,769,939	1,528,729
Increase in retentions	-	0	0	0	0	0

Table 8.32c

Matrix DEBO	MDL 17	Outcome Z				
Time period	6	7	8	9	10	11
Trading profit	-	328,000	912,500	1,134,380	1,075,780	607,030
after losses carried forward	-	328,000	912,500	1,134,380	1,075,780	607,030
Other income (after any set off						
of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	282,240	282,240	282,240	282,240	282,240
Taxable profits	287,500	245,760	830,260	1,052,140	993,540	524,790
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	61,440	260,591	338,249	317,739	153,676
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	277,125	255,042	214,216	189,280	234,788
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	277,125	255,042	214,216	189,280	234,788
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	344,625	599,667	813,884	1,003,163	1,237,951
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,631,385	2,105,472	2,123,221	1,939,939	1,698,729
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,631,385	2,105,472	2,123,221	1,939,939	1,698,729
Increase in retentions	-	0	0	0	0	0

Table 8.32d

MDL 17: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,650	7,209
Y	9,888	7,147
Z	10,500	7,102
Expected value	9,639	7,157

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.860	0.1374	0.1402
8	0.844	0.1360	0.1903
9	0.724	0.1252	0.1958
10	0.702	0.1232	0.1772
11	0.731	0.1258	0.1516

Table 8.33a

Matrix LSEP **MDL 18 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	249,203	439,774	362,919	123,440
after losses carried forward	-	0	0	423,830	362,919	123,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	197,424	195,737	193,906	192,781	191,703
Taxable profits	287,500	2,576	4,263	429,924	370,138	131,737
Trading loss	-	265,147	0	0	0	0
carried forward (LF1)	-	265,147	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	265,147	15,944	0	0	0
Gross corporation tax payable	71,875	644	1,066	120,474	99,548	32,934
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	180,345	273,194	208,411	183,738	183,428
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	180,345	273,194	208,411	183,738	183,428
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	247,845	521,039	729,451	913,189	1,096,617
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Lease 'debt' commitment (LDB)	-	1,000,000	831,246	648,143	449,469	233,900
Closing balance of funds	-	904,301	1,370,235	1,471,981	1,288,830	1,048,876
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	904,301	1,370,235	1,471,981	1,288,830	1,048,876
Increase in retentions	-	0	0	0	0	0

Table 8.33b

Matrix LSEE **MDL 18 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	134,853	649,203	839,774	762,919	523,440
after losses carried forward	-	134,853	649,203	839,774	762,919	523,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	197,479	195,792	193,961	192,806	191,728
Taxable profits	287,500	137,374	653,411	845,813	770,113	531,712
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	34,343	198,694	266,035	239,540	156,099
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	233,374	222,006	191,663	171,262	171,429
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	233,374	222,006	191,663	171,262	171,429
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	300,874	522,880	714,543	885,805	1,056,907
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Lease 'debt' commitment (LDB)	-	1,000,000	831,246	648,143	449,469	233,900
Closing balance of funds	-	1,304,246	1,763,440	1,804,128	1,626,422	1,388,855
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,304,246	1,763,440	1,804,128	1,626,422	1,388,855
Increase in retentions	-	0	0	0	0	0

Table 8.33c

Matrix LSEO MDL 18 Outcome Z

Time period	6	7	8	9	10	11
Trading profit	-	334,853	849,203	1,039,774	962,919	723,440
after losses carried forward	-	334,853	849,203	1,039,774	962,919	723,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	197,479	195,792	193,961	192,806	191,728
Taxable profits	287,500	337,374	853,411	1,045,813	970,113	731,712
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	88,081	268,694	336,035	309,540	226,099
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	233,374	219,258	185,663	165,262	165,429
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	233,374	219,258	185,663	165,262	165,429
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	300,874	520,133	705,797	871,057	1,036,486
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Lease 'debt' commitment (LDB)	-	1,000,000	831,246	648,143	449,469	233,900
Closing balance of funds	-	1,504,246	1,949,702	1,974,128	1,796,422	1,558,855
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,504,246	1,949,702	1,974,128	1,796,422	1,558,855
Increase in retentions	-	0	0	0	0	0

Table 8.33d

MDL 18: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,567	8,014
Y	9,802	7,941
Z	10,410	7,897
Expected value	9,553	7,954

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.867	0.1381	0.1282
8	0.840	0.1356	0.1762
9	0.725	0.1253	0.1820
10	0.734	0.1260	0.1632
11	0.737	0.1264	0.1383

Table 8.34a

Matrix LSRP **MDL 19 Outcome X**

Time period	6	7	8	9	10	11
Trading profit	-	0	249,203	439,774	362,919	123,440
after losses carried forward	-	0	0	423,830	362,919	123,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	8,750	8,750	432,580	371,669	132,190
Trading loss	-	265,147	0	0	0	0
carried forward (LF1)	-	265,147	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	265,147	15,944	0	0	0
Gross corporation tax payable	71,875	2,187	2,187	121,403	100,084	33,047
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	180,345	273,132	208,366	183,659	183,382
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	180,345	273,132	208,366	183,659	183,382
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	247,845	520,977	729,344	913,003	1,096,385
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	910,475	1,374,412	1,474,412	1,289,963	1,049,100
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	910,475	1,374,412	1,474,412	1,289,963	1,049,100
Increase in retentions	-	0	0	0	0	0

Table 8.34b

Matrix LSRE **MDL 19 Outcome Y**

Time period	6	7	8	9	10	11
Trading profit	-	134,853	649,203	839,774	762,919	523,440
after losses carried forward	-	134,853	649,203	839,774	762,919	523,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	143,603	657,953	848,524	771,669	532,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	35,901	200,284	266,983	240,084	156,266
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	233,374	221,943	191,527	171,180	171,382
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	233,374	221,943	191,527	171,180	171,382
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	300,874	522,818	714,344	885,525	1,056,907
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,310,475	1,767,670	1,806,157	1,627,571	1,389,100
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,310,475	1,767,670	1,806,157	1,627,571	1,389,100
Increase in retentions	-	0	0	0	0	0

Table 8.34c

Matrix LSRO **MDL 19 Outcome Z**

Time period	6	7	8	9	10	11
Trading profit	-	334,853	849,203	1,039,774	962,919	723,440
after losses carried forward	-	334,853	849,203	1,039,774	962,919	723,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	343,603	857,953	1,048,524	971,669	732,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	71,875	90,261	270,284	336,983	310,084	226,266
Tax rebate	0	0	0	0	0	0
Surplus ACT on dividends paid	-	233,374	219,071	185,527	165,180	165,382
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	233,374	219,071	185,527	165,180	165,382
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67500	300,874	519,946	705,472	870,653	1,036,035
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,510,475	1,953,310	1,976,157	1,797,571	1,559,100
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,510,475	1,953,310	1,976,157	1,797,571	1,559,100
Increase in retentions	-	0	0	0	0	0

Table 8.34d

MDL 19: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	8,584	8,014
Y	9,819	7,941
Z	10,427	7,896
Expected value	9,570	7,954

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.866	0.1379	0.1286
8	0.838	0.1354	0.1763
9	0.723	0.1251	0.1819
10	0.732	0.1259	0.1631
11	0.736	0.1262	0.1380

Table 8.35a

Matrix INDP (p) **MDL 20 Outcome X (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	0	131,250	173,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	8,750	8,750	83,750	140,000	182,190
available for group relief	-	0	8,750	8,750	140,000	182,190
after any group relief for losses	287,500	8,750	8,750	8,750	140,000	182,190
Trading loss	-	100,000	0	0	0	0
surrendered as group relief	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	2,187	2,187	20,937	35,000	48,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	96,000	116,000	116,000	116,000	0
retained by parent	-	184,388	188,241	185,873	185,123	296,550
Surplus ACT on dividends paid	-	182,638	186,491	169,123	157,123	260,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	182,638	186,491	169,123	157,123	260,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	250,138	436,630	605,753	762,876	1,022,988
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,401,941	1,521,207	1,509,365	1,505,615	1,482,750
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,401,941	1,521,207	1,509,365	1,505,615	1,482,750
Increase in retentions	-	0	0	0	0	0

Table 8.35b

Matrix INDP (s) MDL 20 Outcome X (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	700,000	700,000	700,000	700,000	700,000
after losses carried forward	-	700,000	700,000	700,000	700,000	700,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	580,000	580,000	580,000	580,000	580,000
available for group relief	-	580,000	580,000	580,000	580,000	580,000
after any group relief for losses	680,000	480,000	580,000	580,000	580,000	580,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	153,000	188,000	188,000	188,000	188,000
Tax rebate	0	21,421	27,526	26,737	26,737	0
ACT surrendered by parent	-	96,000	116,000	116,000	116,000	0
Surplus ACT on dividends paid	-	21,421	27,526	26,737	26,737	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	21,421	0	0	0	0
carried back to t-2	-	0	27,526	0	0	0
carried back to t-3	-	0	0	26,737	0	0
carried back to t-4	-	0	0	0	10,316	0
carried back to t-5	-	0	0	0	16,421	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	628,421	750,526	734,737	734,737	708,000
Dividend paid (gross) to parent	-	321,316	412,895	401,053	401,053	381,000
Dividend paid (net) outside group	-	85,684	110,105	106,947	106,947	101,600
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.35c

Matrix INDE (p) **MDL 20 Outcome Y (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	308,750	408,750	483,750	540,000	582,190
available for group relief	-	308,750	408,750	483,750	540,000	582,190
after any group relief for losses	287,500	308,750	408,750	483,750	540,000	582,190
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	93,062	128,062	154,312	174,000	188,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	136,000	136,000	136,000	136,000	0
retained by parent	-	240,178	253,119	250,119	247,869	377,500
Surplus ACT on dividends paid	-	178,428	171,369	153,369	139,869	261,062
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	178,428	171,369	153,369	139,869	261,062
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	245,928	417,297	570,666	710,533	971,597
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,880,888	1,945,595	1,930,595	1,919,345	1,887,500
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,880,888	1,945,595	1,930,595	1,919,345	1,887,500
Increase in retentions	-	0	0	0	0	0

Table 8.35d

Matrix INDE (s) **MDL 20 Outcome Y (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	800,000	800,000	800,000	800,000	800,000
after losses carried forward	-	800,000	800,000	800,000	800,000	800,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	680,000	680,000	680,000	680,000	680,000
available for group relief	-	680,000	680,000	680,000	680,000	680,000
after any group relief for losses	680,000	680,000	680,000	680,000	680,000	680,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	223,000	223,000	223,000	223,000	223,000
Tax rebate	0	26,684	31,211	31,211	31,211	0
ACT surrendered by parent	-	136,000	136,000	136,000	136,000	0
Surplus ACT on dividends paid	-	26,684	31,211	31,211	31,211	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	26,684	0	0	0	0
carried back to t-2	-	0	31,211	0	0	0
carried back to t-3	-	0	0	28,106	0	0
carried back to t-4	-	0	0	3,105	0	0
carried back to t-5	-	0	0	0	31,211	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	733,684	824,211	824,211	824,211	793,000
Dividend paid (gross) to parent	-	400,263	468,158	468,158	468,158	444,750
Dividend paid (net) outside group	-	106,737	124,842	124,842	124,842	118,600
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.35e

Matrix INDO (p) **MDL 20 Outcome Z (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	508,750	608,750	683,750	740,000	782,190
available for group relief	-	508,750	608,750	683,750	740,000	782,190
after any group relief for losses	287,500	508,750	608,750	683,750	740,000	782,190
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	163,062	198,062	224,312	244,000	258,123
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	156,000	156,000	156,000	156,000	0
retained by parent	-	275,967	280,635	277,635	275,385	424,340
Surplus ACT on dividends paid	-	174,217	158,885	140,885	127,385	267,902
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	174,217	158,885	140,885	127,385	267,902
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	241,717	400,602	541,487	668,872	936,774
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	2,159,836	2,183,174	2,168,174	2,156,924	2,121,700
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	2,159,836	2,183,174	2,168,174	2,156,924	2,121,700
Increase in retentions	-	0	0	0	0	0

Table 8.35f

Matrix INDO (s) **MDL 20 Outcome Z (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	900,000	900,000	900,000	900,000	900,000
after losses carried forward	-	900,000	900,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	780,000	780,000	780,000	780,000	780,000
available for group relief	-	780,000	780,000	780,000	780,000	780,000
after any group relief for losses	680,000	780,000	780,000	780,000	780,000	780,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	257,400	257,400	257,400	257,400	257,400
Tax rebate	0	31,947	35,716	35,716	35,716	0
ACT surrendered by parent	-	156,000	156,000	156,000	156,000	0
Surplus ACT on dividends paid	-	31,947	35,716	35,716	35,716	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	31,947	0	0	0	0
carried back to t-2	-	0	35,716	0	0	0
carried back to t-3	-	0	0	18,337	0	0
carried back to t-4	-	0	0	17,379	0	0
carried back to t-5	-	0	0	0	35,716	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	838,947	914,316	914,316	914,316	878,600
Dividend paid (gross) to parent	-	479,211	535,737	535,737	535,737	508,950
Dividend paid (net) outside group	-	127,789	142,863	142,863	142,863	135,720
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.35g

MDL 20: Risk and return

	Objective function value, £'000	Horizon value, £'000
Outcome		
X	11,173	10,824
Y	12,633	10,704
Z	13,500	10,695
Expected value	12,368	10,738

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.836	0.1353	0.1446
8	0.732	0.1259	0.1505
9	0.728	0.1256	0.1493
10	0.719	0.1247	0.1486
11	0.705	0.1235	0.1462

Table 8.36a

Matrix DANP (p) **MDL 21 Outcome X (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	0	115,000	173,440
Other income (after any set off of trading losses in t)	-	100,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	0	8,750	8,750	123,750	182,190
Trading loss	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	100,000	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	91,250	0	0	0	0
Total losses carried forward	0	91,250	91,250	16,250	0	0
Gross corporation tax payable	85,625	0	2,187	2,187	30,937	48,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	116,000	0	0	0	0
retained by parent	-	146,149	273,566	266,724	266,063	264,848
Surplus ACT on distributions	-	146,149	271,816	264,974	241,313	228,410
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	146,149	271,816	264,974	241,313	228,410
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	213,649	485,465	750,440	991,752	1,220,162
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,310,747	1,367,830	1,333,622	1,330,313	1,324,239
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,310,747	1,367,830	1,333,622	1,330,313	1,324,239
Increase in retentions	-	0	0	0	0	0

Table 8.36b

Matrix DANP (s)	MDL 21 Outcome X (subsidiary)					
Time period	6	7	8	9	10	11
Trading profit	-	700,000	700,000	700,000	700,000	700,000
after losses carried forward	-	700,000	700,000	700,000	700,000	700,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	580,000	580,000	580,000	580,000	580,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	188,000	188,000	188,000	188,000	188,000
Tax rebate	0	44,220	0	0	0	0
ACT surrendered by parent	-	116,000	116,000	116,000	116,000	0
Surplus ACT on dividends paid	-	44,220	0	0	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	44,220	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	651,220	708,000	641,784	635,295	634,659
Dividend paid (gross) to parent	-	230,122	259,080	225,310	222,000	221,676
Dividend paid (net) outside group	-	176,878	199,136	173,179	170,636	170,386
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.36c

Matrix DANE (p) **MDL 21 Outcome Y (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	308,750	408,750	483,750	540,000	582,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	93,062	128,062	154,312	174,000	188,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	136,000	0	0	0	0
retained by parent	-	217,458	355,973	345,029	342,001	340,237
Surplus ACT on dividends paid	-	155,708	144,795	126,795	113,295	232,598
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	155,708	274,223	248,279	234,001	223,799
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	223,208	497,431	745,710	979,711	1,203,510
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,767,288	1,779,867	1,725,146	1,710,003	1,701,184
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,767,288	1,779,867	1,725,146	1,710,003	1,701,184
Increase in retentions	-	0	0	0	0	0

Table 8.36d

Matrix DANE (s) MDL 21 Outcome Y (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	800,000	800,000	800,000	800,000	800,000
after losses carried forward	-	800,000	800,000	800,000	800,000	800,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	680,000	680,000	680,000	680,000	680,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	223,000	223,000	223,000	223,000	223,000
Tax rebate	0	55,084	0	0	0	0
ACT surrendered by parent	-	136,000	0	0	0	0
Surplus ACT on dividends paid	-	55,084	0	0	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	55,084	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	762,084	793,000	715,114	707,481	706,733
Dividend paid (gross) to parent	-	286,663	302,430	262,708	258,815	258,434
Dividend paid (net) outside group	-	220,337	232,456	201,925	198,933	198,639
Fixed asset investment		200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.36e

Matrix DANO (p) **MDL 21 Outcome Z (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	508,750	608,750	683,750	740,000	782,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	163,062	198,062	224,312	244,000	258,123
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	156,000	0	0	0	0
retained by parent	-	248,766	398,705	386,576	383,431	381,656
Surplus ACT on dividends paid	-	147,016	276,955	249,826	235,431	225,218
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	147,016	276,955	249,826	235,431	225,218
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	214,516	491,470	741,296	976,728	1,201,946
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	2,023,829	1,993,523	1,932,880	1,917,157	1,908,281
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	2,023,829	1,993,523	1,932,880	1,917,157	1,908,281
Increase in retentions	-	0	0	0	0	0

Table 8.36f

Matrix DANO (s) **MDL 21 Outcome Z (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	900,000	900,000	900,000	900,000	900,000
after losses carried forward	-	900,000	900,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	780,000	780,000	780,000	780,000	780,000	780,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	257,400	257,400	257,400	257,400	257,400
Tax rebate	0	65,949	73,728	73,728	44,595	0
ACT surrendered by parent	-	156,000	0	0	0	0
Surplus ACT on dividends paid	-	65,949	0	0	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	65,949	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	872,949	878,600	789,103	780,332	779,473
Dividend paid (gross) to parent	-	343,204	346,086	300,442	295,969	295,531
Dividend paid (net) outside group	-	263,796	266,011	230,929	227,490	227,154
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.36g

MDL 21: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	10,242	10,130
Y	11,632	10,049
Z	12,388	10,010
Expected value	11,366	10,066

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.861	0.1375	0.1480
8	0.759	0.1283	0.1495
9	0.726	0.1253	0.1451
10	0.710	0.1239	0.1441
11	0.706	0.1235	0.1434

Table 8.37a

Matrix NGEP (p) **MDL 22 Outcome X (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	75,000	131,250	173,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	8,750	8,750	83,750	140,000	182,190
available for group relief	-	8,750	8,750	83,750	140,000	182,190
after any group relief for losses	287,500	8,750	8,750	83,750	140,000	182,190
Trading loss	-	100,000	0	0	0	0
surrendered as group relief	-	100,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	2,187	2,187	20,937	35,000	48,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	96,000	116,000	103,600	0	0
retained by parent	-	120,125	105,662	118,062	220,912	220,350
Surplus ACT on dividends paid	-	118,375	103,912	101,312	192,912	183,012
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	118,375	103,912	101,312	192,912	183,012
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	185,875	289,787	391,100	584,012	767,924
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,385,875	1,500,562	1,480,012	1,409,362	1,397,910
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,462,187	1,598,625	1,572,937	1,485,562	1,471,950
Increase in retentions	-	0	0	0	0	0

Table 8.37b

Matrix NGEP (s) **MDL 22 Outcome X (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	700,000	700,000	700,000	700,000	700,000
after losses carried forward	-	700,000	700,000	700,000	700,000	700,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	580,000	580,000	580,000	580,000	580,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	188,000	188,000	188,000	188,000	188,000
Tax rebate	0	101,750	130,750	111,500	0	0
ACT surrendered by parent	-	96,000	116,000	103,600	0	0
Surplus ACT on dividends paid	-	101,750	130,750	111,500	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	15,750	0	0	0	0
carried back to t-3	-	0	70,250	0	0	0
carried back to t-4	-	0	60,500	0	0	0
carried back to t-5	-	0	0	25,500	0	0
carried back to t-6	-	0	0	86,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	708,750	853,750	819,500	708,000	693,600
Dividend paid (net) to parent	-	305,250	392,250	371,700	304,800	296,160
Dividend paid (net) outside group	-	101,750	130,750	123,900	101,600	98,720
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.37c

Matrix NGEE (p) **MDL 22 Outcome Y (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	308,750	408,750	483,750	540,000	582,190
available for group relief	-	308,750	408,750	483,750	540,000	582,190
after any group relief for losses	680,000	308,750	408,750	483,750	540,000	582,190
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	93,062	128,062	154,312	174,000	188,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	136,000	136,000	72,600	0	0
retained by parent	-	160,125	159,487	219,887	290,237	288,550
Surplus ACT on dividends paid	-	98,375	77,737	123,137	182,237	172,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	98,375	77,737	123,137	182,237	172,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	165,875	243,612	366,750	548,987	721,099
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,860,875	1,922,187	1,859,637	1,806,987	178,810
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,955,937	2,033,375	1,958,937	1,895,937	1,874,450
Increase in retentions	-	0	0	0	0	0

Table 8.37d

Matrix NGEE (s) MDL 22 Outcome Y (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	800,000	800,000	800,000	800,000	800,000
after losses carried forward	-	800,000	800,000	800,000	800,000	800,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	680,000	680,000	680,000	680,000	680,000
available for group relief	-	680,000	680,000	680,000	680,000	680,000
after any group relief for losses	680,000	680,000	680,000	680,000	680,000	680,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	223,000	223,000	223,000	223,000	223,000
Tax rebate	0	126,750	148,250	69,000	0	0
ACT surrendered by parent	-	136,000	136,000	72,600	0	0
Surplus ACT on dividends paid	-	126,750	148,250	69,000	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	40,750	0	0	0	0
carried back to t-3	-	0	45,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	17,000	0	0	0
carried back to t-6	-	0	0	69,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	833,750	941,250	862,000	793,000	775,600
Dividend paid (net) to parent	-	380,250	444,750	397,200	355,800	345,360
Dividend paid (net) outside group	-	126,750	148,250	132,400	118,600	115,120
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.37e

Matrix NGEO (p) **MDL 22 Outcome Z (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	508,750	608,750	683,750	740,000	782,190
available for group relief	-	508,750	608,750	683,750	740,000	782,190
after any group relief for losses	287,500	508,750	608,750	683,750	740,000	782,190
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	163,062	198,062	224,312	244,000	258,123
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	156,000	156,000	38,360	0	0
retained by parent	-	180,125	173,487	288,127	324,237	322,550
Surplus ACT on dividends paid	-	78,375	51,737	151,377	176,237	166,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	78,375	51,737	151,377	176,237	166,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	145,875	197,612	348,990	525,226	691,338
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	2,135,875	2,156,387	2,053,157	2,028,347	2,007,742
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	2,249,687	2,283,625	2,158,337	2,130,137	2,106,490
Increase in retentions	-	0	0	0	0	0

Table 8.37f

Matrix NGEO (s) **MDL 22 Outcome Z (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	900,000	900,000	900,000	900,000	900,000
after losses carried forward	-	900,000	900,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	780,000	780,000	780,000	780,000	780,000
available for group relief	-	780,000	780,000	780,000	780,000	780,000
after any group relief for losses	680,000	780,000	780,000	780,000	780,000	780,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	257,400	257,400	257,400	257,400	257,400
Tax rebate	0	151,750	169,650	22,600	0	0
ACT surrendered by parent	-	156,000	156,000	38,360	0	0
Surplus ACT on dividends paid	-	151,750	169,650	22,600	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	65,750	0	0	0	0
carried back to t-3	-	0	20,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	63,400	0	0	0
carried back to t-6	-	0	0	22,600	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	958,750	1,048,250	901,200	878,600	878,600
Dividend paid (net) to parent	-	455,250	508,950	420,720	407,160	394,992
Dividend paid (net) outside group	-	151,750	169,650	140,240	135,720	131,664
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.37g

MDL 22: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	11,282	10,714
Y	12,742	10,607
Z	13,572	10,543
Expected value	12,470	10,626

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.863	0.1377	0.1497
8	0.752	0.1277	0.1566
9	0.647	0.1182	0.1510
10	0.708	0.1237	0.1459
11	0.697	0.1227	0.1443

Table 8.38a

Matrix BANP (p)	MDL 23	Outcome X (parent)				
Time period	6	7	8	9	10	11
Trading profit	-	0	0	75,000	131,250	173,440
after losses carried forward	-	0	0	0	106,250	173,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	287,500	8,750	8,750	8,750	115,000	182,190
Trading loss	-	100,000	0	0	0	0
carried forward (LF1)	-	100,000	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	100,000	100,000	25,000	0	0
Gross corporation tax payable	85,625	2,187	2,187	2,187	28,750	48,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	116,000	116,000	106,600	14,400	0
retained by parent	-	100,125	105,662	115,062	207,262	220,600
Surplus ACT on dividends paid	-	98,375	103,912	113,312	184,262	184,262
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	98,375	103,912	113,312	184,262	184,262
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	165,875	269,787	383,099	567,361	751,623
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,288,195	1,367,392	1,362,598	1,315,576	1,310,264
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,340,087	1,432,162	1,426,170	1,367,392	1,362,080
Increase in retentions	-	0	0	0	0	0

Table 8.38b

Matrix BANP (s) **MDL 23 Outcome X (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	700,000	700,000	700,000	700,000	700,000
after losses carried forward	-	700,000	700,000	700,000	700,000	700,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	580,000	580,000	580,000	580,000	580,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	188,000	188,000	188,000	188,000	188,000
Tax rebate	0	101,750	127,000	115,250	0	0
ACT surrendered by parent	-	116,000	116,000	106,600	14,400	0
Surplus ACT on dividends paid	-	101,750	127,000	115,250	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	15,750	0	0	0	0
carried back to t-3	-	0	70,250	0	0	0
carried back to t-4	-	0	56,750	0	0	0
carried back to t-5	-	0	0	29,250	0	0
carried back to t-6	-	0	0	86,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	708,750	835,000	823,250	708,000	708,000
Dividend paid (gross) to parent	-	259,462	323,850	317,857	259,080	259,080
Dividend paid (net) outside group	-	199,430	248,920	244,314	199,136	199,136
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.38c

Matrix BANE (p)	MDL 23 Outcome Y (parent)					
Time period	6	7	8	9	10	11
Trading profit	-	300,000	400,000	475,000	531,250	573,440
after losses carried forward	-	300,000	400,000	475,000	531,250	573,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	308,750	408,750	483,750	540,000	582,190
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	93,062	128,062	154,312	174,000	188,766
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	136,000	136,000	72,600	17,400	0
retained by parent	-	160,125	159,487	219,887	272,837	288,550
Surplus ACT on dividends paid	-	98,375	77,737	123,137	164,837	172,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	98,375	77,737	123,137	164,837	172,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	165,875	243,612	366,749	531,586	703,698
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,739,195	1,779,867	1,732,533	1,693,131	1,684,694
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,803,837	1,855,475	1,800,057	1,753,617	1,745,180
Increase in retentions	-	0	0	0	0	0

Table 8.38d

Matrix BANE (subsidiary) **MDL 23 Outcome Y (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	800,000	800,000	800,000	800,000	800,000
after losses carried forward	-	800,000	800,000	800,000	800,000	800,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	680,000	680,000	680,000	680,000	680,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	223,000	223,000	223,000	223,000	223,000
Tax rebate	0	126,750	148,250	69,000	0	0
ACT surrendered by parent	-	136,000	136,000	72,600	17,400	0
Surplus ACT on dividends paid	-	126,750	148,250	69,000	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	40,750	0	0	0	0
carried back to t-3	-	0	45,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	17,000	0	0	0
carried back to t-6	-	0	0	69,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	833,750	941,250	862,000	793,000	793,000
Dividend paid (gross) to parent	-	323,212	378,037	337,620	302,430	302,430
Dividend paid (net) outside group	-	248,430	290,570	259,504	232,456	232,456
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.38e

Matrix BANO (p) **MDL 23 Outcome Z (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	500,000	600,000	675,000	731,250	773,440
after losses carried forward	-	500,000	600,000	675,000	731,250	773,440
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	191,250	191,250	191,250	191,250	191,250
Taxable profits	-	508,750	608,750	683,750	740,000	782,190
available for group relief	-	508,750	608,750	683,750	740,000	782,190
after any group relief for losses	287,500	508,750	608,750	683,750	740,000	782,190
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	163,062	198,062	224,312	244,000	258,123
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	180,125	173,488	287,128	303,958	322,550
retained by parent	-	195,603	190,792	302,432	317,801	336,393
Surplus ACT on dividends paid	-	78,375	51,698	150,378	155,958	166,112
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	78,375	51,698	150,378	155,958	166,112
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	145,875	197,573	347,951	503,909	670,021
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
New debt raised: to fund projects	-	0	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,990,195	1,993,524	1,918,527	1,898,056	1,889,619
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	2,067,587	2,080,045	1,990,049	1,967,273	1,958,836
Increase in retentions	-	0	0	0	0	0

Table 8.38f

Matrix BANO (s) MDL 23 Outcome Z (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	900,000	900,000	900,000	900,000	900,000
after losses carried forward	-	900,000	900,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	780,000	780,000	780,000	780,000	780,000	780,000
available for group relief	-	0	0	0	0	0
after any group relief for losses	0	0	0	0	0	0
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	257,400	257,400	257,400	257,400	257,400
Tax rebate	0	151,750	169,650	22,600	0	0
ACT surrendered by parent	-	156,000	156,000	38,360	20,280	0
Surplus ACT on dividends paid	-	151,750	169,650	22,600	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	65,750	0	0	0	0
carried back to t-3	-	0	20,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	63,400	0	0	0
carried back to t-6	-	0	0	22,600	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	958,750	1,048,250	901,200	878,600	878,600
Dividend paid (gross) to parent	-	386,962	432,607	357,612	346,086	346,086
Dividend paid (net) outside group	-	297,430	332,514	274,870	266,011	266,011
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.38g

MDL 23: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	10,178	9,742
Y	11,562	9,638
Z	12,313	9,575
Expected value	11,297	9,657

Time period	*Beta*	*Rate of return* required	*Rate of return* expected
7	0.882	0.1394	0.1520
8	0.789	0.1310	0.1569
9	0.688	0.1220	0.1527
10	0.729	0.1256	0.1487
11	0.725	0.1252	0.1480

Table 8.39a

Matrix ACCP (p) **MDL 24 Outcome X (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	0	312,500	534,380	475,780	7,030
after losses carried forward	-	0	234,266	534,380	475,780	7,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,266	284,266	284,266	284,266	284,266
Taxable profits	-	0	150,000	450,114	391,514	0
available for group relief	-	0	150,000	450,114	391,514	0
after any group relief for losses	287,500	0	150,000	450,114	391,514	0
Trading loss	-	272,000	0	0	0	0
surrendered as group relief	-	272,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	84,266	0	0	0	77,236
surrendered as group relief	-	6,032	0	0	0	77,236
Total losses carried forward	0	78,234	0	0	0	0
Gross corporation tax payable	85,625	0	37,500	142,540	122,030	0
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	60,394	116,000	98,259	13,353	0
retained by parent	-	142,728	187,147	222,388	269,290	234,401
Surplus ACT on dividends paid	-	142,728	157,147	133,366	190,987	234,401
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	142,728	157,147	133,366	190,987	234,401
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	210,228	367,375	500,741	691,728	926,129
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,320,859	1,928,013	1,975,928	1,718,017	1,476,087
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,397,171	2,031,082	2,067,852	1,794,217	1,553,007
Increase in retentions	-	0	0	0	0	0

Table 8.39b

Matrix ACCP (s)	MDL 24 Outcome X (subsidiary)					
Time period	6	7	8	9	10	11
Trading profit	-	700,000	700,000	700,000	700,000	700,000
after losses carried forward	-	700,000	700,000	700,000	700,000	700,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	580,000	580,000	580,000	580,000	580,000
available for group relief	-	580,000	580,000	580,000	580,000	580,000
after any group relief for losses	680,000	301,968	580,000	580,000	580,000	580,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	90,689	188,000	188,000	188,000	160,967
Tax rebate	0	101,750	137,426	104,824	0	1,047
ACT surrendered by parent	-	60,394	116,000	98,259	13,353	0
Surplus ACT on dividends paid	-	101,750	137,426	104,824	0	1,047
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	1,047
carried back to t-2	-	15,750	0	0	0	0
carried back to t-3	-	0	70,250	0	0	0
carried back to t-4	-	0	67,176	0	0	0
carried back to t-5	-	0	0	18,824	0	0
carried back to t-6	-	0	0	86,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	708,750	887,131	812,824	708,000	708,000
Dividend paid (gross) to parent	-	381,562	515,348	459,618	381,000	381,000
Dividend paid (net) outside group	-	101,750	137,426	122,565	101,600	101,600
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.39c

Matrix ACCE (p)	MDL 24	Outcome Y (parent)				
Time period	6	7	8	9	10	11
Trading profit	-	128,000	712,500	934,380	875,780	407,030
after losses carried forward	-	128,000	712,500	934,380	875,780	407,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,266	284,266	284,266	284,266	284,266
Taxable profits	-	43,734	628,234	850,114	791,514	322,764
available for group relief	-	43,734	628,234	850,114	791,514	322,764
after any group relief for losses	287,500	43,734	628,234	850,114	791,514	322,764
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	10,933	204,882	280,538	261,200	97,967
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	136,000	136,000	72,600	17,400	0
retained by parent	-	147,122	246,709	314,700	333,644	302,567
Surplus ACT on dividends paid	-	138,375	121,063	144,677	175,341	238,015
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	138,375	121,063	144,677	175,341	238,015
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	205,875	326,938	471,615	646,956	884,971
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,795,859	2,358,297	2,333,699	2,111,019	1,868,637
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,890,921	2,469,485	2,432,999	2,199,969	1,957,587
Increase in retentions	-	0	0	0	0	0

Table 8.39d

Matrix ACCE (s) MDL 24 Outcome Y (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	800,000	800,000	800,000	800,000	800,000
after losses carried forward	-	800,000	800,000	800,000	800,000	800,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	680,000	680,000	680,000	680,000	680,000
available for group relief	-	680,000	680,000	680,000	680,000	680,000
after any group relief for losses	680,000	680,000	680,000	680,000	680,000	680,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	223,000	223,000	223,000	223,000	223,000
Tax rebate	0	126,750	148,250	69,000	0	0
ACT surrendered by parent	-	136,000	136,000	72,600	17,400	0
Surplus ACT on dividends paid	-	126,750	148,250	69,000	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	40,750	0	0	0	0
carried back to t-3	-	0	45,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	17,000	0	0	0
carried back to t-6	-	0	0	69,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	833,750	941,250	862,000	793,000	793,000
Dividend paid (gross) to parent	-	475,312	555,937	496,500	444,750	444,750
Dividend paid (net) outside group	-	126,750	148,250	132,400	118,600	118,600
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.39e

Matrix ACCO (p) **MDL 24 Outcome Z (parent)**

Time period	**6**	**7**	**8**	**9**	**10**	**11**
Trading profit	-	328,000	912,500	1,134,380	1,075,780	607,030
after losses carried forward	-	328,000	912,500	1,134,380	1,075,780	607,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,266	284,266	284,266	284,266	284,266
Taxable profits	-	243,734	828,234	1,050,114	991,514	522,764
available for group relief	-	243,734	828,234	1,050,114	991,514	522,764
after any group relief for losses	287,500	243,734	828,234	1,050,114	991,514	522,764
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	70,307	273,317	346,538	327,200	167,967
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	156,000	156,000	38,360	20,280	0
retained by parent	-	167,122	262,835	383,253	365,564	337,367
Surplus ACT on dividends paid	-	118,375	97,188	173,230	167,261	232,815
chosen to carry forward	-	118,375	97,188	173,230	167,261	232,815
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	185,875	283,063	456,293	623,554	856,369
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	2,070,859	2,603,124	252,884	2,336,379	2,093,997
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	2,184,671	2,730,361	2,633,964	2,438,169	2,195,787
Increase in retentions	-	0	0	0	0	0

Table 8.39f

Matrix ACCO (s) MDL 24 Outcome Z (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	900,000	900,000	900,000	900,000	900,000
after losses carried forward	-	900,000	900,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	680,000	780,000	780,000	780,000	780,000	780,000
available for group relief	-	780,000	780,000	780,000	780,000	780,000
after any group relief for losses	680,000	780,000	780,000	780,000	780,000	780,000
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	257,400	257,400	257,400	257,400	257,400
Tax rebate	0	151,750	169,650	22,600	0	0
ACT surrendered by parent	-	156,000	156,000	38,360	20,280	0
Surplus ACT on dividends paid	-	151,750	169,650	22,600	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	65,750	0	0	0	0
carried back to t-3	-	0	20,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	63,400	0	0	0
carried back to t-6	-	0	0	22,600	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	958,750	1,048,250	901,200	878,600	878,600
Dividend paid (gross) to parent	-	569,062	636,187	525,900	508,950	508,950
Dividend paid (net) outside group	-	151,750	169,650	140,240	135,720	135,720
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.39g

MDL 24: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	11,804	9,886
Y	13,266	9,804
Z	14,117	9,727
Expected value	12,998	9,813

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.828	0.1345	0.1387
8	0.736	0.1262	0.1840
9	0.598	0.1138	0.1820
10	0.678	0.1210	0.1637
11	0.673	0.1206	0.1446

Table 8.40a

Matrix HLFP (p) **MDL 25 Outcome X (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	0	312,500	534,380	475,780	7,030
after losses carried forward	-	0	0	406,348	475,780	7,030
Other income (after any set off of trading losses in t)	-	0	200,000	200,000	200,000	200,000
Debt interest charges	-	284,266	284,266	284,266	284,266	284,266
Taxable profits	-	0	0	322,082	391,514	0
Trading loss	-	272,000	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	200,000	0	0	0	0
carried forward (LF2)	-	72,000	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	284,266	84,266	0	0	77,236
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	-	356,266	128,032	0	0	77,236
Gross corporation tax payable	85,625	0	0	97,729	122,030	0
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	116,000	116,000	106,600	14,400	0
retained by parent	-	87,122	187,147	216,547	272,084	234,401
Surplus ACT on dividends paid	-	87,122	187,147	152,131	193,781	234,401
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	87,122	187,147	152,131	193,781	234,401
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	154,622	341,769	493,900	687,681	922,082
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,223,179	1,774,814	1,870,020	1,639,686	1,379,271
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,275,071	1,839,584	1,933,591	1,691,502	1,431,087
Increase in retentions	-	0	0	0	0	0

Table 8.40b

Matrix HLFP (s) **MDL 25 Outcome X (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	700,000	700,000	700,000	700,000	700,000
after losses carried forward	-	700,000	700,000	700,000	700,000	700,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	580,000	580,000	580,000	580,000	580,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	188,000	188,000	188,000	188,000	188,000
Tax rebate	0	101,750	127,000	115,250	0	0
ACT surrendered by parent	-	116,000	116,000	106,600	14,400	0
Surplus ACT on dividends paid	-	101,750	127,000	115,250	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	15,750	0	0	0	0
carried back to t-3	-	0	70,250	0	0	0
carried back to t-4	-	0	56,750	0	0	0
carried back to t-5	-	0	0	29,250	0	0
carried back to t-6	-	0	0	86,000	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	708,750	835,000	823,250	708,000	708,000
Dividend paid (gross) to parent	-	259,462	323,850	317,857	259,080	259,080
Dividend paid (net) outside group	-	199,430	248,920	244,314	199,136	199,136
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.40c

Matrix HLFE (p) **MDL 25 Outcome Y (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	128,000	712,500	934,380	875,780	407,030
after losses carried forward	-	128,000	712,500	934,380	875,780	407,030
Other income (after any set off						
of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,266	284,266	284,266	284,266	284,266
Taxable profits	-	43,784	628,234	850,114	791,514	322,764
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	10,933	204,882	280,538	261,200	97,967
Tax rebate	0	126,750	148,250	69,000	0	0
ACT surrendered to subsidiary	-	136,000	136,000	72,600	17,400	0
retained by parent	-	147,122	246,710	314,700	333,644	302,567
Surplus ACT on dividends paid	-	138,365	121,063	144,677	175,341	238,014
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	138,365	121,063	144,677	175,341	238,014
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	205,865	326,928	471,605	646,946	884,960
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,674,179	2,215,977	2,206,595	1,997,163	1,754,781
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	1,738,821	2,291,585	2,274,119	2,057,649	1,815,267
Increase in retentions	-	0	0	0	0	0

Table 8.40d

Matrix HLFE (s) **MDL 25 Outcome Y (subsidiary)**

Time period	6	7	8	9	10	11
Trading profit	-	800,000	800,000	800,000	800,000	800,000
after losses carried forward	-	800,000	800,000	800,000	800,000	800,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	0	0	0	0	0	0
available for group relief	-	0	0	0	0	0
after any group relief for losses	0	0	0	0	0	0
Trading loss	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	223,000	223,000	223,000	223,000	223,000
Tax rebate	0	126,750	148,250	69,000	0	0
ACT surrendered by parent	-	136,000	136,000	72,600	17,400	0
Surplus ACT on dividends paid	-	126,750	148,250	69,000	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	40,750	0	0	0	0
carried back to t-3	-	0	45,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	17,000	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	833,750	941,250	862,000	793,000	793,000
Dividend paid (gross) to parent	-	323,212	378,037	337,620	302,430	302,430
Dividend paid (net) outside group	-	248,430	290,570	259,504	232,456	232,456
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.40e

Matrix HLFO (p) **MDL 25 Outcome Z (parent)**

Time period	6	7	8	9	10	11
Trading profit	-	328,000	912,500	1,134,380	1,075,780	607,030
after losses carried forward	-	328,000	912,500	1,134,380	1,075,780	607,030
Other income (after any set off of trading losses in t)	-	200,000	200,000	200,000	200,000	200,000
Debt interest charges	-	284,266	284,266	284,266	284,266	284,266
Taxable profits	-	243,734	828,234	1,050,114	991,514	522,764
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	85,625	70,307	273,317	346,538	327,200	167,967
Tax rebate	0	0	0	0	0	0
ACT surrendered to subsidiary	-	156,000	156,000	38,360	20,280	0
retained by parent	-	167,122	262,835	383,253	365,564	337,367
Surplus ACT on dividends paid	-	118,375	97,188	173,230	167,261	232,815
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	118,375	97,188	173,230	167,261	232,815
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
carried back to t-4	-	0	0	0	0	0
carried back to t-5	-	0	0	0	0	0
carried back to t-6	-	0	0	0	0	0
Surplus ACT carried forward	67,500	185,875	283,063	456,293	623,554	856,369
Short term lending	-	0	0	0	0	0
Short term borrowing	-	20,000	0	0	0	0
New debt raised: to fund projects	-	1,000,000	0	0	0	0
to maintain D/E ratio	-	0	0	0	0	0
Loan capital repaid	-	0	0	0	0	0
New equity issued	-	0	0	0	0	0
Closing balance of funds	-	1,925,179	2,440,260	2,394,153	2,206,088	1,963,706
Internal funding of new projects	-	0	0	0	0	0
Gross dividends paid	-	2,002,571	2,526,781	2,465,676	2,275,305	2,032,923
Increase in retentions	-	0	0	0	0	0

Table 8.40f

Matrix HLFO (s) MDL 25 Outcome Z (subsidiary)

Time period	6	7	8	9	10	11
Trading profit	-	900,000	900,000	900,000	900,000	900,000
after losses carried forward	-	900,000	900,000	900,000	900,000	900,000
Other income (after any set off of trading losses in t)	-	0	0	0	0	0
Debt interest charges	-	120,000	120,000	120,000	120,000	120,000
Taxable profits	-	780,000	780,000	780,000	780,000	780,000
Trading loss	-	0	0	0	0	0
carried forward (LF1)	-	0	0	0	0	0
set against other profits in t	-	0	0	0	0	0
carried forward (LF2)	-	0	0	0	0	0
carried back to t-1	-	0	0	0	0	0
carried back to t-2	-	0	0	0	0	0
carried back to t-3	-	0	0	0	0	0
Excess debt interest	-	0	0	0	0	0
surrendered as group relief	-	0	0	0	0	0
Total losses carried forward	0	0	0	0	0	0
Gross corporation tax payable	223,000	257,400	257,400	257,400	257,400	257,400
Tax rebate	0	151,750	169,650	22,600	0	0
ACT surrendered by parent	-	156,000	156,000	38,360	20,280	0
Surplus ACT on dividends paid	-	151,750	169,650	22,600	0	0
arising from losses carried back	-	0	0	0	0	0
chosen to carry forward	-	0	0	0	0	0
carried back to previous periods	-	0	0	0	0	0
carried back to t-1	-	86,000	0	0	0	0
carried back to t-2	-	65,750	0	0	0	0
carried back to t-3	-	0	20,250	0	0	0
carried back to t-4	-	0	86,000	0	0	0
carried back to t-5	-	0	63,400	0	0	0
carried back to t-6	-	0	0	22,600	0	0
Surplus ACT carried forward	0	0	0	0	0	0
Short term lending	-	0	0	0	0	0
Short term borrowing	-	0	0	0	0	0
Closing balance of funds	-	958,750	1,048,250	901,200	878,600	878,600
Dividend paid (gross) to parent	-	386,962	432,607	357,612	346,086	346,086
Dividend paid (net) outside group	-	297,430	332,514	274,870	266,011	266,011
Fixed asset investment	-	200,000	200,000	200,000	200,000	200,000
Increase in retentions	-	0	0	0	0	0

Table 8.40g

MDL 25: Risk and return

	Objective function value, £'000	*Horizon value, £'000*
Outcome		
X	10,677	8,906
Y	12,075	8,835
Z	12,847	8,775
Expected value	11,810	8,844

		Rate of return	
Time period	*Beta*	*required*	*expected*
7	0.844	0.1360	0.1399
8	0.811	0.1322	0.1865
9	0.618	0.1156	0.1871
10	0.676	0.1208	0.1686
11	0.699	0.1229	0.1476

Table 8.41

The effect of global entity priorities on solution times:

MDL 02 (Outcome Y)

Global entity	Values							
d1	70	70*	50	70	70	70	-	-
d2	150	150	100	150	150	150	-	-
d3	250	-	-	-	250	-	-	-
d4	200	-	-	-	-	250	-	-
d5	230	-	-	-	-	400	-	-
d6	260	-	-	-	-	450	-	-
d7	150	-	-	-	150	-	-	-
d8	100	100	300	-	100	100	-	-
d9	290	-	-	-	-	-	-	-
d10	320	-	-	-	-	-	-	-
d11	350	-	-	-	-	-	-	-
d12	380	-	-	-	-	-	-	-
d13	410	-	-	-	-	-	-	-
d14	430	-	-	-	-	-	-	-
d15	450	-	-	-	-	-	-	-
d16	470	-	-	-	-	-	-	-
d17	490	-	-	-	-	-	-	-
d18	500	-	-	-	-	-	-	-
d19	150	-	-	-	150	-	-	-
d20	200	-	-	-	200	-	-	-
d24	-	-	-	-	450	-	-	-
d25	-	-	-	-	300	-	-	-
d26	-	-	-	-	350	-	-	-
d27	-	-	-	-	400	-	-	-
d28	-	-	-	-	450	-	-	-
d29	-	-	-	-	100	-	-	-
d30	-	-	-	-	450	-	-	-
S	150	150	150	150	150	-	-	-
Sa	200	-	-	-	200	-	-	-
Sb	230	-	-	-	230	-	-	-
Sc	260	-	-	-	260	-	-	-
accept01	50	10	10*	10*	50	50	-	10*
accept02	50	10	10	10	50	50	-	10
accept03	50	10	10	10	50	50	-	10
accept04	50	10	10*	10*	50	50	-	10*
accept05	50	10	10	10	50	50	-	10
accept06	50	10	10*	10*	50	50	-	10*
accept07	50	10	10	10	50	50	-	10
accept08	50	10	10	10	50	50	-	10
accept09	50	10	10*	10*	50	50	-	10*
Solution time, seconds	61	68	60	57	38	23	23	17

* indicates an entity on which the first branching direction must be upwards (.UP.)

Chapter 9

Conclusion

The system of corporate taxation in the UK can create interdependencies between capital investment projects and the associated financing decision, and between ongoing and additional projects being considered; and the tax rules themselves interact, creating particularly complex situations where the firm has no taxable profits or where there is surplus ACT.

In this thesis the features of the UK corporate tax system which are particularly significant for project appraisal were discussed. It was shown that a project's value may change in the presence of imperfect loss relief, marginal tax rates and restricted setoff of advance corporation tax. Tax deductible items such as relief for losses, capital allowances and debt interest may reduce the firm's taxable profits so that its marginal tax rate may be higher or lower than the statutory 'full' rate, affecting the value of incremental projects after tax. But the acceptance of incremental projects can in itself change the firm's tax situation as a result of additional capital allowances, or the interest on debt payments associated with the financing of the project.

The investment decision was considered in detail and it was shown that the system of tax allowances for depreciation of capital investment has interesting implications for the investment decision. It was shown by the author that where there are tax lags and accelerated depreciation allowances, an optimal cost of capital exists at which the present value of the capital expenditure relief is maximised. In the case of capital expenditure on plant and machinery, this optimal cost of capital is independent of the proportion of the asset cost allowable against tax in the first year. Instead, it is determined by the writing down allowance on the declining balance in subsequent years, and by the lag which elapses between the purchase of the asset and the incidence of tax relief, which may occur after some years of tax exhaustion. In the case of buildings, where the writing down allowances are equal after the first year, it was shown that the optimal cost of capital is determined by the values of both the initial allowance and subsequent writing down allowances and by the time lag between asset purchase and the obtaining of tax relief.

It was demonstrated that ACT setoff restrictions can create an interaction between dividend policy and capital investment decisions: the changes in taxable profit that may occur as a result of new capital investments will change the amount of ACT which the firm can set off. Surplus ACT must be carried forward, reducing its present value, unless it can be set against previous years' tax liabilities. Since a number of different methods of setoff of losses and ACT are possible, the firm's optimal tax strategy may be difficult to determine, particularly in a group structure.

The financing decision also has complex tax implications. Unless a firm is permanently paying tax at the full rate its debt interest may not be set off until some time in the future, reducing its value as a tax shield, and the tax rate at which it is finally relieved will be determined partly by the level of the firm's capital allowances, which can act as substitute tax shields. Investment and financing decisions are therefore linked by the tax system. Conventional methods of valuing debt interest tax shields require assumptions to be made about the relevant tax rates and timing of cash flows, whereas these will be endogenous to the firm's overall set of activities including any accepted additional projects. Lease evaluation techniques suffer from the same disadvantage. The UK imputation system complicates matters still further, since debt interest deductibility may result in restricted setoff of ACT so that the cost of debt and the cost of equity interact. Optimal financing policy is therefore particularly difficult for the firm in a restricted tax situation, and again, a simultaneous consideration of the investment and financing decision is needed.

To be consistent with an objective of maximising shareholder wealth, project appraisal should incorporate all the relevant cash flows, including taxes, and the tax cash flows used should take into account the optimal tax strategy of the firm as a whole if the project is accepted. The outcome of this appraisal should be compared with the firm's optimal situation after tax without the additional investment. If more than one incremental project is being considered, an optimal tax strategy will exist for every possible project combination, which will take into account the overall cash flows of the combination of new projects and the firm's existing activities and any interactions that occur between them. However, conventional project appraisal techniques assume that cash flows are independent and therefore that the value of the firm as a whole is the same as the sum of its projects' values when evaluated separately, which is not so where tax induced interactions between cash flows exist.

Also, assuming a firm objective of maximising shareholder wealth, the appropriate risk measure should be based on a consideration of the residual cash flows to the diversified shareholder. These cash flows should incorporate any tax-induced cash flow interactions.

A firm wishing to make optimal decisions regarding capital investments, financing decisions and tax strategy may be unable to do so because of the vast number of possible decision combinations that may arise. It was argued that there is a need for an operational optimisation model, capable of appraising investment and financing choices in the context of the firm's overall tax situation and simultaneously determining optimal investment, financing and tax decisions. Mathematical programming techniques can be used to develop a model to determine the optimal combination of capital investments, financing mix and tax strategy, so as to maximise a given objective subject to constraints enforcing the tax rules and other externally or internally imposed restrictions.

The programming models of financing and investment decisions in the literature were reviewed. Early applications of the technique were concerned with the problem of project selection under capital rationing constraints, while a number of authors have modelled features of the corporation tax system which can be important in capital investment appraisal and financing decisions. It was concluded that the existing finance literature does not contain an operational model which can evaluate investment and financing decisions in the context of complex tax situations. Situations of this type may include the group relief provisions for losses and ACT, the differing tax treatment of finance leasing as opposed to asset purchase, and the need to include procedures to estimate the risk of the shareholders' returns. In the absence of capital rationing, and with an objective function which aims to maximise shareholder wealth, the discount rate should reflect the shareholder's required rate of return. It was argued that a suitable risk measurement procedure should therefore be based on the risk of the shareholder's returns.

A mixed integer programming model was developed in stages and computerised using commercially available mathematical programming software. The model can choose simultaneously the optimal combination of investment decisions, financing decisions and tax strategy, given a set of input data taken from a spreadsheet. The final version of the model includes the tax provisions relating to losses, ACT and interest on long term debt; short term lending and borrowing; group relief, and finance leasing. Lease or purchase decisions may be evaluated with the optimal tax strategies being compared simultaneously for both options. An optimal choice of group relief strategy may be determined, which maximises the wealth of the group's shareholders. The model also includes gearing restrictions, including an increasing cost of debt as the proportion of debt financing increases, and dividend growth constraints. The objective of the model is to maximise shareholder wealth after corporate taxes, in terms of the discounted future stream of dividends less new equity issues and the discounted horizon value. The discount rate, obtained iteratively using the CAPM model, incorporates directly the impact of the company's tax situation on the returns to its shareholders. Operational experience of the model revealed that its decisions may be significantly different from those indicated by NPV analysis of individual projects. Application to a variety of complex tax scenarios showed that it can make decisions within reasonable solution times in this type of situation.

With regard to practical applications, the firm would need to identify the likely investment opportunities that will arise over the period between the present time and the horizon, and estimate their values at the horizon. If the model is 'unaware' of these opportunities it will choose to make a full dividend payout, and this choice will determine its optimal tax strategy. Since the model's output and the required rate of return are sensitive to the input horizon value of the firm's assets, exclusive of tax effects, the appropriate value of this input would need to be considered carefully in any practical situation. The model could be run under varying assumptions about the likely future investment projects and the relevant horizon value, to assist the decision making process.

One drawback of the model in its present form is that although it is capable of making optimal decisions assuming a given discount rate, this is determined iteratively outside the model itself. The discount rate cannot be a variable within a mixed integer programming model of this type, since this technique cannot accommodate the type of non-linearity where a variable is divided by another variable, as would occur in the objective function. It might be possible to use a form of non-linear programming to develop a model capable of deciding endogenously on the optimal strategy which would maximise the value of the firm, taking into account the discount rate which would apply under all possible strategies. A model of this type would, however, be considerably more difficult to solve than the mixed integer programming model developed in this thesis.

Finally, this type of approach may be adopted to model the broad features of other tax systems, for example, other European systems. This could provide insights into the variations in investment incentives between countries that may occur when their specific tax provisions are used in an optimal way.

Appendix 5.1

Lease evaluation example: the after-tax cost of debt

If there is no delay in tax payment, the after tax cost of debt $r*$ is given by:

$$r* = r - rT$$

where r is the pre-tax borrowing rate and T is the corporation tax rate, both assumed constant over the life of the lease.

Where there is a delay of one time period this becomes:

$$r* = r - rT/(1 + r*)$$

so that if $r = 0.12$ and $T= 0.33$, $r*$can be found by successive approximations to be 0.08345.

In the example in the text, the firm returns to a position of taxable profits in year three. The after tax cost of debt for year three, k_3, is 0.08345 because the tax deductibility of year three's debt interest occurs in year four. For year two, the tax saving is discounted for two years to reflect the delay between years two and four:

$$k_2 = 0.12\left[1 - 0.33/(1 + k_3)^2\right]$$

$$= 0.12\left[1 - 0.33/(1.08345)^2\right]$$

$$= 0.08627.$$

Similarly,

$$k_1 = 0.12\left[1 - 0.33 / \{(1+k_2)(1+k_3)\}\right]$$

$$= 0.12\left[1 - 0.33 / \{(1.08627)(1.08345)\}\right]$$

$$= 0.08894.$$

The rental payments are made at the start of each year and so the first payment is not discounted. The rental paid at the start of year two is discounted by the after tax cost of debt for year one, so the discount factor is given by $1/\left[(1+k_1)\right]$. Similarly the third rental payment, at the start of year 3, is discounted by $1/\left[(1+k_1)(1+k_2)\right]$ to reflect the two-year delay before it is paid. The discount factor which reflects the delay of four years before capital allowances, etc are offset is given by $1/\left[(1+k_1)(1+k_2)(1+k_3)(1+k_4)\right]$ where k_4 is 0.08345, assuming a full taxpaying position in year five.

Appendix 7.1

A mathematical programming approach to trading losses and surplus ACT

Purchased assets

Assuming that an asset is purchased at the start of the year and capital allowances are obtained one year later, then the written down value of the asset at the end of year t will be:

$$WDV_{jt} = C_j(1-\alpha'_j)(1-\alpha)^{t-1} \tag{1}$$

where C_j is the asset cost at time 0, α'_j is the proportion of that cost which is the capital allowance available in the first year and α is the writing down allowance available on a reducing balance basis in the second and subsequent years of the asset's life.

The writing down allowance (WDA) in the first year will be:

$$WDA_{j_{t=1}} = \alpha' C_j \tag{2}$$

and in any subsequent year t:

$$WDA_{j_{t>1}} = \alpha C_j(1-\alpha'_j)(1-\alpha)^{t-2}. \tag{3}$$

Representing incremental projects being considered at t by $j = 1,\dots,J$, then capital outlay at the start of t will be:

$$\sum_{j=1}^{J} C_{j_t} \delta_j$$

where δ_j is a binary variable which has the value 1 if project j is accepted and 0 if it is rejected.

The closing balance on the capital allowance pool at the end of year t, CP_t, will be:

$$CP_t = (1-\alpha)(CP_{t-1} - S_t) + (1-\alpha')\sum_{j=1}^{J} C_{j_t} \delta_j \tag{4}$$

where S_t represents asset disposals made in t.

The total of capital allowances obtained at the end of the year will be:

$$WDA_t = \alpha\left[\max(CP_{t-1} - S_t), 0\right] + \alpha' \sum_{j=1}^{J} C_{j_t} \delta_j \tag{5}$$

If the disposal value in t exceeds the 'qualifying expenditure' then a balancing charge BC_t will arise. Assuming that the disposal proceeds are less than the cost of the assets, then:

$$BC_t = \max\left[(S_t - CP_{t-1} - \sum_{j=1}^{J} C_{j_t} \delta_j), 0\right] \tag{6}$$

Where an asset is treated as a short life asset (SLA) then a balancing allowance may arise on disposal:

$$BA_{j_t} = \max\left[(WDV_{j_t} - S_{j_t}), 0\right]$$

that is, the excess if any of the written down value of asset j at the start of the year t in which it is sold, over its sale price. The balancing allowance for year t will be:

$$BA_t = \sum_{j=0}^{J} \max\left[(WDV_{j_t} - S_{j_t}), 0\right] \delta_{j*} \tag{7}$$

where $j = 0$ represents the firm's ongoing projects at the start of t and δ_{j*} is a binary variable which takes the value 1 if the asset is 'depooled' and 0 if it is not.

Leased assets

Lease rentals R_{j_L} will result in a cash outflow at the start of each year of the term of the lease on asset j. Assuming a straight line basis of depreciation, the accounting depreciation in year t on a leased asset j_L will be:

$$D_{j_{L_t}} = C_{j_L} / n_j \tag{8}$$

where n is the number of years of the asset's life.

The finance charge in year 1 of the leased asset's life will be:

$$F_{j_{L_{t=1}}} = i_{j_L}(C_{j_L} - R_{j_L}) \tag{9}$$

where i_{j_L} is the rate of interest implicit in the lease.

In year 2:

$$F_{j_{L_{t=2}}} = i_{j_L}\left[(C_{j_L} - R_{j_L})(1 + i_{j_L}) - R_{j_L}\right]$$

and in year 3:

$$F_{jL_{t=3}} = i_{jL}\left\{\left[(C_{jL} - R_{jL})(1+i_{jL}) - R_{jL}\right](1+i_{jL}) - R_{jL}\right\}$$

so that in general, the finance charge in each year after year 1 will be:

$$F_{jL_t} = \left[(F_{jL_{t-1}} / i_{jL})(1+i_{jL}) - R_{jL}\right] i_{jL}$$

$$= F_{jL_{t-1}}(1+i_{jL}) - R_{jL} i_{jL}. \quad (10)$$

For any asset j which may be either leased or purchased, a decision to accept at most one of these alternatives in the optimal solution may be forced by imposing the constraint:

$$\delta_j + \delta_{jL} \leq 1. \quad (11)$$

(Where 'depooling' the asset would be considered if the asset were purchased, the constraint must be $\delta_j + \delta_{j*} + \delta_{jL} \leq 1$).

Taxable profit

Schedule D trading profit in t is given by

$$P_t = \max\left\{\left[(N_{j_t} - D_{jL_t} - F_{jL_t})_{j=0} + \sum_{j=1}^{J} N_{j_t}\delta_j - \sum_{j=1}^{J} D_{jL_t}\delta_{jL_t}\right.\right.$$

$$\left.\left. - \sum_{j=1}^{J} F_{jL_t}\delta_{jL} - WDA_t + BC_t - BA_t\right], 0\right\} \qquad (12)$$

where N_{j_t} = net operating cash flows from project j in t, ignoring any changes in working capital.

Conversely the trading loss in t will be:

$$L_t = \max\left\{-\left[(N_{j_t} - D_{jL_t} - F_{jL_t})_{j=0} + \sum_{j=1}^{J} N_{j_t}\delta_j - \sum_{j=1}^{J} D_{jL_t}\delta_{jL_t}\right.\right.$$

$$\left.\left. - \sum_{j=1}^{J} F_{jL_t}\delta_{jL} - WDA_t + BC_t - BA_t\right], 0\right\}. \qquad (13)$$

If a trading loss is made in t then provided the firm has no other income, the interest on long term borrowing carried forward from t, X_t, will be:

$$\sum_{j=0}^{J} I_{j_t}$$

representing the total debt interest payments made in t.

But if a trading profit is made in t, it may be reduced by any losses carried forward from previous years, $L_{F,t}$:

$$P_t' = \max\left[(P_t - L_{F,t}),0\right].$$

The assessable profits after debt interest is deducted, π_t, will be

$$\pi_t = \max\left[(P_t' - \sum_{j=0}^{J} I_{j_t}),0\right] \tag{14}$$

and the debt interest carried forward will be:

$$\max\left[-(P_t' - \sum_{j=0}^{J} I_{j_t}),0\right]$$

$$\text{So, } X_t = \max\left[-(P_t' - \sum_{j=0}^{J} I_{j_t}),0\right]\delta_{x_t} + \sum_{j=0}^{J} I_{j_t}(1-\delta_{x_t}) \tag{15}$$

where $\delta_{x_t} = 1$ if $P_t' > 0$, otherwise $\delta_{x_t} = 0$.

Any remainder of the losses carried forward to t which have not been offset will be carried forward again to $t+1$:

$$L_{F_{t,t+1}} = \max\left[-(P_t - L_{F,t}),0\right]. \tag{16}$$

The corporation tax payment in respect of year t will be calculated on the basis of π_t. The structure of the tax function has been described in Chapter 2. Following the notation in Chapter 2, the tax payment in any year t may be calculated as follows:

If $\pi_t > M_{u_t}$, $\qquad T_{\pi_t} = T_{f_t}\pi_t$.

If $M_{l_t} < \pi_t \leq M_{u_t}$, $\quad T_{\pi_t} = T_{f_t}\pi_t - \dfrac{(M_{u_t} - \pi_t)}{(M_{u_t} - M_{l_t})}(T_{f_t} - T_{s_t})M_{l_t}$.

If $0 < \pi_t \leq M_{l_t}$, $\quad T_{\pi_t} = T_{f_t}\pi_t - (T_{f_t} - T_{f_t})\pi_t$

$$= T_{f_t}\pi_t - (\pi_t / M_{l_t})(T_{f_t} - T_{s_t})M_{l_t}.$$

Combining these gives:

$$T_{\pi_t} = T_{f_t}\pi_t - \left\{ \left[(\pi_t / M_{l_t})(1 - \delta_{m_t}) + \frac{(M_{u_t} - \pi_t)}{(M_{u_t} - M_{l_t})}\delta_{m_t}\right](T_{f_t} - T_{s_t})M_{l_t} \right\}(1 - \delta_{f_t}) \tag{17}$$

where

$\delta_{m_t} = 1$ if $\pi_t \geq M_{l_t}$, otherwise $\delta_{m_t} = 0$

$\delta_{f_t} = 1$ if $\pi_t \geq M_{u_t}$, otherwise $\delta_{f_t} = 0$

Losses may be carried back or carried forward. The profits of $t-1$, $t-2$ and $t-3$ will be adjusted to absorb L_{t_B}, the amount carried back. Adjusted profit figures will then arise for those years and these may be denoted by $\pi_{A_{t-1}}$, $\pi_{B_{t-2}}$ and $\pi_{C_{t-3}}$ respectively.

Any one year's profits will be adjusted three times in total. The adjusted profit for $t-1$ after carrying back a loss from t is:

$$\pi_{A_{t-1}} = \pi_{t-1} - L_{t,t-1}$$

where $L_{t,t-1} \geq 0$.

Similarly:

$$\pi_{B_{t-2}} = \pi_{A_{t-2}} - L_{t,t-2}$$

$$\pi_{C_{t-3}} = \pi_{B_{t-3}} - L_{t,t-3}.$$

The following constraints ensure that the carry back is made, as far as possible, against more recent periods first:

$$L_{t,t-1} = \min\left[L_t, \pi_{t-1}\right] \tag{18}$$

$$L_{t,t-2} = \min\left[(L_t - L_{t,t-1}), \pi_{A_{t-2}}\right] \tag{19}$$

$$L_{t,t-3} = \min\left[(L_t - \sum_{i=1}^{2} L_{t,t-i}), \pi_{B_{t-3}}\right] \tag{20}$$

The amount of the loss in t which is carried forward to $t+1$ will be:

$$L_{t_{F,t+1}} = L_t - \sum_{i=1}^{3} L_{t,t-i} \tag{21}$$

Combining (14), (16) and (21) gives the total amount of losses carried forward to $t+1$:

$$L_{F,t+1} = L_{F_{t},t+1} + L_{t_{F,t+1}} + X_t \tag{22}$$

This includes debt interest carried forward, which is treated in the same way as trading losses carried forward.

A rebate in respect of tax paid in earlier years may arise as a result of the offset of losses. The tax rebate at t, TR_t, will be:

$$TR_t = \left(T\pi_{t-1} - T\pi_{A_{t-1}}\right) + \left(T\pi_{A_{t-2}} - T\pi_{B_{t-2}}\right) + \left(T\pi_{B_{t-3}} - T\pi_{C_{t-3}}\right) - \sum_{i=1}^{3} SACT_{L_{t},t-i} \tag{23}$$

The final term is an adjustment to take into account ACT restrictions, which will be dealt with in detail in the next section.

Setoff of ACT

The initial ACT offset capacity in t is $b\pi_t$, where b is taken as the basic rate of income tax. (At the time of writing, the imputation rate was set at 20 per cent, the rate applicable to the first £2,000 of personal tax liability, rather than the 25 per cent 'basic' rate. It was envisaged that this 20 per cent rate would later be applied to the whole of the basic rate tax band.) After taking into account the ACT relating to DIV_t, the net dividend distributions in t, any remaining 'unused' capacity will be given by:

$$U_t = \max\left[\left(b\pi_t - b\,DIV_t/(1-b)\right), 0\right]. \tag{24}$$

If U_t is positive then any surplus ACT brought forward to t from previous years, $SACT_{F,t}$, may be offset against U_t. The amount of this surplus which is offset in t will be:

$$\min\left(SACT_{F,t}, U_t\right).$$

Any $SACT_{F,t}$ remaining after this offset will be carried forward to $t+1$. The amount carried forward will be:

$$SACT_{F,t} - \min(SACT_{F,t}, U_t).$$

If $b\,DIV_t/(1-b)$ exceeds $b\pi_t$, then U_t will be zero and surplus ACT will arise in t as a result of the dividend distributions made in t. This surplus, $SACT_{DIV_t}$, is given by:

$$SACT_{DIV_t} = \max\left[-(b\pi_t - b\,DIV_t/(1-b)), 0\right] \tag{25}$$

$SACT_{DIV_t}$ may be carried back and offset against previous years' unutilized ACT capacity. The capacity in $t-1$ which is available to absorb $SACT_{DIV_t}$, U^t_{t-1}, will be determined by the adjusted profit figure resulting from the possible carry back of losses from t:

$$U^t_{t-1} = \max\left[\left(b\pi_{A_{t-1}} - b\,DIV_{t-1}/(1-b) - \min(SACT_{F,t-1}, U_{t-1})\right), 0\right] \tag{26}$$

Since $\pi_{A_{t-1}}$ will be smaller than π_{t-1} if losses from t have been carried back to $t-1$, surplus ACT may be created.

$$SACT_{L_t,t-1} = \max\left[-\left(b\pi_{A_{t-1}} - b\,DIV_{t-1}/(1-b) - \min(SACT_{F,t-1}, U_{t-1}) + SACT_{DIV_{t-1}}\right), 0\right] \tag{27}$$

This surplus will be carried forward from t and may be offset in subsequent years.

Similarly, the unutilized offset capacity in $t-2$ which is available for absorption of $SACT_{DIV_t}$ will be:

$$U^t_{t-2} = \max\left[\left(b\pi_{B_{t-2}} - b\,DIV_{t-2}/(1-b) - \min(SACT_{F,t-2}, U_{t-2}) - SACT_{DIV_{t-1},t-2}\right), 0\right] \tag{28}$$

taking into account the effect of losses carried back from t to $t-2$, and any surplus ACT carried back from $t-1$ to $t-2$.

Surplus ACT created as a result of the loss carried back from t to $t-2$ will be:

$$SACT_{L_{t},t-2} = \max\left[-\left(b\pi_{B_{t-2}} - b\,DIV_{t-2}/(1-b) - \min(SACT_{F,t-2}, U_{t-2}) + SACT_{DIV_{t-2}} + SACT_{L_{t-1},t-2} - SACT_{DIV_{t-1},t-2}\right), 0\right] \quad (29)$$

Similarly in $t-3$:

$$U_{t-3}^{t} = \max\left[\left(b\pi_{C_{t-3}} - b\,DIV_{t-3}/(1-b) - \min(SACT_{F,t-3}, U_{t-3}) - \sum_{i=1}^{2} SACT_{DIV_{t-i},t-3}\right), 0\right] \quad (30)$$

$$SACT_{L_{t},t-3} = \max\left[-\left(b\pi_{C_{t-3}} - b\,DIV_{t-3}/(1-b) - \min(SACT_{F,t-3}, U_{t-3}) + SACT_{DIV_{t-3}} + \sum_{i=1}^{2} SACT_{L_{t-i},t-3} - \sum_{i=1}^{2} SACT_{DIV_{t-i},t-3}\right), 0\right] \quad (31)$$

At $t-4$, $t-5$ and $t-6$ there is no further possibility of carrying back losses from t, and so no further surplus ACT will be created in those years.

$$U_{t-4}^{t} = \max\left[\left(b\pi_{C_{t-4}} - b\,DIV_{t-4}/(1-b) - \min(SACT_{F,t-4}, U_{t-4}) - \sum_{i=1}^{3} SACT_{DIV_{t-i},t-4}\right), 0\right]$$

or,

$$U_{t-4}^{t} = \max\left[\left(U_{t-4}^{t-1} - SACT_{DIV_{t-1},t-4}\right), 0\right] \quad (32)$$

Similarly:

$$U_{t-5}^{t} = \max\left[(U_{t-5}^{t-1} - SACT_{DIV_{t-1},\,t-5}),0\right] \tag{33}$$

$$U_{t-6}^{t} = \max\left[(U_{t-6}^{t-1} - SACT_{DIV_{t-1},\,t-6}),0\right]. \tag{34}$$

$SACT_{DIV_t}$ may be carried forward or carried back. A carry back claim may be restricted to part of the surplus ACT, but of the amount carried back, offset must be made in more recent years in priority to earlier ones. The following constraints ensure this:

$$SACT_{DIV_t,\,t-1} = \min\left[SACT_{DIV_t}, U_{t-1}^{t}\right] \tag{35}$$

$$SACT_{DIV_t,\,t-2} = \min\left[(SACT_{DIV_t} - SACT_{DIV_t,\,t-1}), U_{t-2}^{t}\right] \tag{36}$$

$$SACT_{DIV_t,\,t-3} = \min\left[(SACT_{DIV_t} - \sum_{i=1}^{2} SACT_{DIV_t,\,t-i}), U_{t-3}^{t}\right] \tag{37}$$

$$SACT_{DIV_t,\,t-4} = \min\left[(SACT_{DIV_t} - \sum_{i=1}^{3} SACT_{DIV_t,\,t-i}), U_{t-4}^{t}\right] \tag{38}$$

$$SACT_{DIV_t,\,t-5} = \min\left[(SACT_{DIV_t} - \sum_{i=1}^{4} SACT_{DIV_t,\,t-i}), U_{t-5}^{t}\right] \tag{39}$$

$$SACT_{DIV_t,\,t-6} = \min\left[(SACT_{DIV_t} - \sum_{i=1}^{5} SACT_{DIV_t,\,t-i}), U_{t-6}^{t}\right]. \tag{40}$$

The total amount of $SACT_{DIV_t}$ carried back will be claimed as recovered ACT:

$$RACT_t = \sum_{i=1}^{6} SACT_{DIV_t,\,t-i} \tag{41}$$

Any remainder will be carried forward to future years. The total surplus ACT which is carried forward from t to $t+1$ will be:

$$SACT_{F,t+1} = SACT_{F,t} - \min(SACT_{F,t}, U_t) + SACT_{DIV_t} - RACT_t + \sum_{i=1}^{3} SACT_{L_{t,t-i}} \quad (42)$$

Finally, mainstream corporation tax, MCT_t, will be:

$$MCT_t = T\pi_t - \min\left[\left(b\,DIV_t/(1-b), b\pi_t\right)\right] \quad (43)$$

The objective function

The aim of this model is to maximise the post-tax net present value of the firm's portfolio of projects, including both ongoing activities and incremental projects, for all periods from $t = 1$ to the horizon time, H.

Maximise:

$$\sum_{t=1}^{H} \Big[\sum_{j=1}^{J} N_{j_t}\delta_j / (1+k_t)^{t-0.5} + N_{j=0,t} / (1+k_t)^{t-0.5} - \sum_{j=1}^{J} C_{j_t}\delta_j / (1+k_t)^{t-1}$$

$$-\sum_{j=1}^{J} R_{jL_t}\delta_{jL} / (1+k_t)^{t-1} - R_{j=0,t} / (1+k_t)^{t-1} - \sum_{j=0,t}^{J} I_{j_t} / (1+k_t)^{t-1}$$

$$-\left(b\,DIV_t/(1-b)\right) / (1+k_t)^{t-0.5} - MCT_t / (1+k_t)^{t+0.75}$$

$$+(TR_t + RACT_t) / (1+k_t)^{t+0.75}\Big]$$

The assumptions made are:

(i) the horizon is sufficiently far away that the discounted value of the firm's activities at the horizon is negligible;

(ii) assets are purchased or leased at the start of the year;

(iii) the net operating cash inflows, debt interest payments and ACT payments occur midway through the year;

(iv) other tax effects occur nine months after the year end.

The derivation of an appropriate discount rate is a complex issue which is not considered here in any depth. k_t will vary in each period as a result of the changes in the investment and financing mix over time and the associated level of risk, while the optimal investment and financing decisions will in turn depend on the values of k_t, so that an iterative approach may be necessary.

Appendix 7.2

Model 1: the mathematical program

The constraints of the mixed integer programming model in Appendix 1 may be represented in a mathematical program, using binary variables to enforce the logical conditions (see, for example, Williams (1993). For instance, a constraint of the type

$$x - M\delta \leq 0$$

where M is a constant coefficient whose value is the known upper bound for x, forces δ to take the value 1 when $x > 0$. By combining this with the additional constraint

$$x\delta \geq 0$$

δ will take the value 1 if and only if x is non-negative.

The constraints of the model may be re-expressed in this format. In the following, the binary variables used to enforce these logical conditions apply in each of the time periods. The t subscripts are omitted for convenience.

1) $$WDV_{j_t} - C_j(1-\alpha'_j)(1-\alpha)^{t-1} = 0$$

2) $$WDA_{j_{t=1}} - \alpha' C_j = 0$$

3) $$WDA_{j_{t>1}} - \alpha C_j(1-\alpha'_j)(1-\alpha)^{t-2} = 0$$

4) $$CP_t - (1-\alpha)(CP_{t-1} - S_t) - (1-\alpha')\sum_{j=1}^{J} C_{j_t}\delta_j = 0$$

5) $$WDA_t - \alpha(CP_{t-1} - S_t)\,\delta_1 - \alpha' \sum_{j=1}^{J} C_{j_t}\delta_j = 0$$

5a) $$(CP_{t-1} - S_t) - M\,\delta_1 \leq 0$$

5b) $$(CP_{t-1} - S_t)\,\delta_1 \geq 0$$

6) $$BC_t - (S_t - CP_{t-1} - \sum_{j=1}^{J} C_{j_t}\delta_j)\delta_2 = 0$$

6a) $$(S_t - CP_{t-1} - \sum_{j=1}^{J} C_{j_t}\delta_j) - M\,\delta_2 \leq 0$$

6b) $$(S_t - CP_{t-1} - \sum_{j=1}^{J} C_{j_t}\delta_j)\delta_2 \geq 0$$

7) $$BA_t - \sum_{j=0}^{J}(WDV_{j_t} - S_{j_t})\,\delta_3\delta_j = 0$$

7a) $$\sum_{j=0}^{J}(WDV_{j_t} - S_{j_t}) - M\,\delta_3 \leq 0$$

7b) $$\sum_{j=0}^{J}(WDV_{j_t} - S_{j_t})\,\delta_3 \geq 0$$

8) $$D_{j_{L_t}} - C_{j_L} / n_j = 0$$

9) $$F_{j_{L_{t=1}}} - i_{j_L}(C_{j_L} - R_{j_L}) = 0$$

10) $$F_{j_{L_t}} - F_{j_{L_{t-1}}}(1 + i_{j_L}) + R_{j_L} i_{j_L} = 0$$

11) $$\delta_j + \delta_{j_L} \leq 1$$

12) $$P_t - \Big[(N_{j_t} - D_{jL_t} - F_{jL_t})_{j=0} + \sum_{j=1}^{J} N_{j_t}\delta_j - \sum_{j=1}^{J} D_{jL_t}\delta_{jL_t} - \sum_{j=1}^{J} F_{jL_t}\delta_{jL} - WDA_t + BC_t - BA_t\Big]\delta_4 = 0$$

12a) $$\Big[(N_{j_t} - D_{jL_t} - F_{jL_t})_{j=0} + \sum_{j=1}^{J} N_{j_t}\delta_j - \sum_{j=1}^{J} D_{jL_t}\delta_{jL_t} - \sum_{j=1}^{J} F_{jL_t}\delta_{jL} - WDA_t + BC_t - BA_t\Big] - M\,\delta_4 \leq 0$$

12b) $$\Big[(N_{j_t} - D_{jL_t} - F_{jL_t})_{j=0} + \sum_{j=1}^{J} N_{j_t}\delta_j - \sum_{j=1}^{J} D_{jL_t}\delta_{jL_t} - \sum_{j=1}^{J} F_{jL_t}\delta_{jL} - WDA_t + BC_t - BA_t\Big]\delta_4 \geq 0$$

13) $$L_t - \Big[(N_{j_t} - D_{jL_t} - F_{jL_t})_{j=0} + \sum_{j=1}^{J} N_{j_t}\delta_j - \sum_{j=1}^{J} D_{jL_t}\delta_{jL_t} - \sum_{j=1}^{J} F_{jL_t}\delta_{jL} - WDA_t + BC_t - BA_t\Big](\delta_4 - 1) = 0$$

14) $$\pi_t - (P'_t - \sum_{j=0}^{J} I_{j_t})\delta_6 = 0$$

14a) $$(P'_t - \sum_{j=0}^{J} I_{j_t}) - M\,\delta_6 \leq 0$$

14b) $$(P'_t - \sum_{j=0}^{J} I_{j_t})\delta_6 \geq 0$$

15) $$X_t - (P'_t - \sum_{j=0}^{J} I_{j_t})\delta_5\delta_4 - \sum_{j=0}^{J} I_{j_t}(1-\delta_4) = 0$$

15a) $$(P'_t - \sum_{j=0}^{J} I_{j_t}) - M\,\delta_5 \leq 0$$

15b) $(P_t' - \sum_{j=0}^{J} I_{j_t})\delta_5 \geq 0$

16) $L_{F_{t,t+1}} - (-P_t' + L_{F,t})\delta_7 = 0$

16a) $(-P_t' + L_{F,t}) - M\delta_7 \leq 0$

16b) $(-P_t' + L_{F,t})\delta_7 \geq 0$

17) $T_{\pi_t} - T_{f_t}\pi_t + \left\{ \left[(\pi_t / M_{l_t})(1-\delta_8) + \frac{(M_{u_t} - \pi_t)}{(M_{u_t} - M_{l_t})}\delta_8 \right] (T_{f_t} - T_{s_t}) M_{l_t} \right\} (1-\delta_9) = 0$

17a) $(\pi_t - M_{l_t}) - M\delta_8 \leq 0$

17b) $(\pi_t - M_{l_t})\delta_8 \geq 0$

17c) $(\pi_t - M_{u_t}) - M\delta_9 \leq 0$

17d) $(\pi_t - M_{u_t})\delta_9 \geq 0$

18) $L_{t,t-1} - \left[L_t(1-\delta_{10}) + \pi_{t-1}\delta_{10} \right] = 0$

18a) $(L_t - \pi_{t-1}) - M\delta_{10} \leq 0$

18b) $(L_t - \pi_{t-1})\delta_{10} \geq 0$

19) $L_{t,t-2} - \left[(L_t - L_{t,t-1})(1-\delta_{11}) + \pi_{A_{t-2}}\delta_{11} \right] = 0$

19a) $(L_t - L_{t,t-1} - \pi_{A_{t-2}}) - M\delta_{11} \leq 0$

19b) $(L_t - L_{t,t-1} - \pi_{A_{t-2}})\delta_{11} \geq 0$

20) $L_{t,t-3} - \left[\pi_{B_{t-3}}\delta_{12} + (L_t - \sum_{i=1}^{2} L_{t,t-i})(1-\delta_{12})\right] = 0$

20a) $(L_t - \sum_{i=1}^{2} L_{t,t-i} - \pi_{B_{t-3}}) - M\delta_{12} \leq 0$

20b) $(L_t - \sum_{i=1}^{2} L_{t,t-i} - \pi_{B_{t-3}})\delta_{12} \geq 0$

21) $L_{t_{F,t+1}} - L_t - \sum_{i=1}^{3} L_{t,t-i} = 0$

22) $L_{F,t+1} - L_{F_{t,t+1}} - L_{t_{F,t+1}} - X_t = 0$

23) $$TR_t - (T\pi_{t-1} - T\pi_{A_{t-1}}) - (T\pi_{A_{t-2}} - T\pi_{B_{t-2}}) - (T\pi_{B_{t-3}} - T\pi_{C_{t-3}}) + \sum_{i=1}^{3} SACT_{L_{t,t-i}} = 0$$

24) $U_t - \left[b\,\pi_t - b\,DIV_t / (1-b)\right]\delta_{13} = 0$

24a) $\left[b\,\pi'_t - b\,DIV_t / (1-b)\right] - M\,\delta_{13} \leq 0$

24b) $\left[b\,\pi_t - b\,DIV_t / (1-b)\right]\delta_{13} \geq 0$

25) $SACT_{DIV_t} - \left[-b\pi_t + b\,DIV_t / (1-b)\right]\delta_{14} = 0$

25a) $\left[-b\pi_t + b\,DIV_t / (1-b)\right] - M\delta_{14} \leq 0$

26) $U_{t-1}^{t} - \left[b\pi_{A_{t-1}} - b\,DIV_{t-1} / (1-b) - U_{t-1}\delta_{15} - SACT_{F,t-1}(1-\delta_{15})\right]\delta_{16} = 0$

26a) $\left[b\pi_{A_{t-1}} - b\,DIV_{t-1} / (1-b) - U_{t-1}\delta_{15} - SACT_{F,t-1}(1-\delta_{15})\right] - M\delta_{16} \leq 0$

26b) $\left[b\pi_{A_{t-1}} - b\,DIV_{t-1} / (1-b) - U_{t-1}\delta_{15} - SACT_{F,t-1}(1-\delta_{15})\right]\delta_{16} \geq 0$

26c) $(SACT_{F_{t-1}} - U_{t-1}) - M\delta_{15} \leq 0$

26d) $(SACT_{F_{t-1}} - U_{t-1})\delta_{15} \geq 0$

27) $$SACT_{L_t,t-1} - \left[b\pi_{A_{t-1}} - b\,DIV_{t-1} / (1-b) + SACT_{DIV_{t-1}} - U_{t-1}\delta_{15} - SACT_{F,t-1}(1-\delta_{15})\right](\delta_{16} - 1) = 0$$

28) $$U^{t}_{t-2} - \left[b\pi_{B_{t-2}} - b\,DIV_{t-2} / (1-b) - U_{t-2}\delta_{17} - SACT_{F,t-2}(1-\delta_{17}) - SACT_{DIV_{t-1,t-2}}\right]\delta_{18} = 0$$

28a) $$\left[b\pi_{B_{t-2}} - bDIV_{t-2} / (1-b) - U_{t-2}\delta_{17} - SACT_{F,t-2}(1-\delta_{17}) - SACT_{DIV_{t-1,t-2}}\right] - M\delta_{18} \leq 0$$

28b) $$\left[b\pi_{B_{t-2}} - b\,DIV_{t-2} / (1-b) - U_{t-2}\delta_{17} - SACT_{F,t-2}(1-\delta_{17}) - SACT_{DIV_{t-1,t-2}}\right]\delta_{18} \geq 0$$

28c) $(SACT_{F,t-2} - U_{t-2}) - M\delta_{17} \leq 0$

28d) $(SACT_{F,t-2} - U_{t-2})\delta_{17} \geq 0$

29) $$SACT_{L_t,t-2} - \left[b\pi_{B_{t-2}} - bDIV_{t-2} / (1-b) -, U_{t-2}\delta_{17} - SACT_{F,t-2}(1-\delta_{17}) + SACT_{DIV_{t-2}} + SACT_{L_{t-1},t-2} - SACT_{DIV_{t-1,t-2}}\right](\delta_{18} - 1) = 0$$

30) $U_{t-3}^{t}-\left[b\pi_{C_{t-3}}-b\,DIV_{t-3}/(1-b)-U_{t-3}\delta_{19}-SACT_{F,t-3}(1-\delta_{19})-\sum_{i=1}^{2}SACT_{DIV_{t-i},t-3}\right]\delta_{20}=0$

30a) $\left[b\pi_{C_{t-3}}-b\,DIV_{t-3}/(1-b)-U_{t-3}\delta_{19}-SACT_{F,t-3}(1-\delta_{19})-\sum_{i=1}^{2}SACT_{DIV_{t-i},t-3}\right]-M\,\delta_{20}\leq 0$

30b) $\left[b\pi_{C_{t-3}}-b\,DIV_{t-3}/(1-b)-U_{t-3}\delta_{19}-SACT_{F,t-3}(1-\delta_{19})-\sum_{i=1}^{2}SACT_{DIV_{t-i},t-3}\right]\delta_{20}\geq 0$

30c) $(SACT_{F,t-3}-U_{t-3})-\delta_{19}\leq 0$

30d) $(SACT_{F,t-3}-U_{t-3})\delta_{19}\geq 0$

31) $SACT_{L_{t},t-3}-\left[b\pi_{C_{t-3}}-b\,DIV_{t-3}/(1-b)-U_{t-3}\delta_{19}-SACT_{F,t-3}(1-\delta_{19})-\sum_{i=1}^{2}SACT_{DIV_{t-i},t-3}+SACT_{DIV_{t-3}}+\sum_{i=1}^{2}SACT_{L_{t-i},t-3}\right](\delta_{20}-1)=0$

32) $U_{t-4}^{t}-(U_{t-4}^{t-1}-SACT_{DIV_{t-1},t-4})\delta_{21}=0$

32a) $(U_{t-4}^{t-1}-SACT_{DIV_{t-1},t-4})-M\delta_{21}\leq 0$

32b) $(U_{t-4}^{t-1}-SACT_{DIV_{t-1},t-4})\delta_{21}\geq 0$

33) $U_{t-5}^{t}-(U_{t-5}^{t-1}-SACT_{DIV_{t-1},t-5})\delta_{22}=0$

33a) $(U_{t-5}^{t-1}-SACT_{DIV_{t-1},t-5})-M\delta_{22}\leq 0$

33b) $(U_{t-5}^{t-1}-SACT_{DIV_{t-1},t-5})\delta_{22}\geq 0$

34) $U_{t-6}^{t}-(U_{t-6}^{t-1}-SACT_{DIV_{t-1,t-6}})\delta_{23}=0$

34a) $(U_{t-6}^{t-1}-SACT_{DIV_{t-1,t-6}})-M\delta_{23}\leq 0$

34b) $(U_{t-6}^{t-1}-SACT_{DIV_{t-1,t-6}})\delta_{23}\geq 0$

35) $SACT_{DIV_{t,t-1}}-SACT_{DIV_t}(1-\delta_{24})-U_{t-1}^{t}\delta_{24}=0$

35a) $(SACT_{DIV_t}-U_{t-1}^{t})-M\delta_{24}\leq 0$

35b) $(SACT_{DIV_t}-U_{t-1}^{t})\delta_{24}\geq 0$

36) $SACT_{DIV_{t,t-2}}-(SACT_{DIV_t}-SACT_{DIV_{t,t-1}})(1-\delta_{25})-U_{t-2}^{t}\delta_{25}=0$

36a) $(SACT_{DIV_t}-SACT_{DIV_{t,t-1}}-U_{t-2}^{t})-M\delta_{25}\leq 0$

36b) $(SACT_{DIV_t}-SACT_{DIV_{t,t-1}}-U_{t-2}^{t})\delta_{25}\geq 0$

37) $SACT_{DIV_{t,t-3}}-(SACT_{DIV_t}-\sum_{i=1}^{2}SACT_{DIV_{t,t-i}})(1-\delta_{26})-U_{t-3}^{t}\delta_{26}=0$

37a) $(SACT_{DIV_t}-\sum_{i=1}^{2}SACT_{DIV_{t,t-i}}-U_{t-3}^{t})-M\delta_{26}\leq 0$

37b) $(SACT_{DIV_t}-\sum_{i=1}^{2}SACT_{DIV_{t,t-i}}-U_{t-3}^{t})\delta_{26}\geq 0$

38) $SACT_{DIV_{t,t-4}}-(SACT_{DIV_t}-\sum_{i=1}^{3}SACT_{DIV_{t,t-i}})(1-\delta_{27})-U_{t-4}^{t}\delta_{27}=0$

38a) $$(SACT_{DIV_t} - \sum_{i=1}^{3} SACT_{DIV_{t,t-i}} - U_{t-4}^{t}) - M\delta_{27} \leq 0$$

38b) $$(SACT_{DIV_t} - \sum_{i=1}^{3} SACT_{DIV_{t,t-i}} - U_{t-4}^{t})\delta_{27} \geq 0$$

39) $$SACT_{DIV_{t,t-5}} - (SACT_{DIV_t} - \sum_{i=1}^{4} SACT_{DIV_{t,t-i}})(1-\delta_{28}) - U_{t-5}^{t}\delta_{28} = 0$$

39a) $$(SACT_{DIV_t} - \sum_{i=1}^{4} SACT_{DIV_{t,t-i}} - U_{t-5}^{t}) - M\delta_{28} \leq 0$$

39b) $$(SACT_{DIV_t} - \sum_{i=1}^{4} SACT_{DIV_{t,t-i}} - U_{t-5}^{t})\delta_{28} \geq 0$$

40) $$SACT_{DIV_{t,t-6}} - (SACT_{DIV_t} - \sum_{i=1}^{5} SACT_{DIV_{t,t-i}})(1-\delta_{29}) - U_{t-6}^{t}\delta_{29} = 0$$

40a) $$(SACT_{DIV_t} - \sum_{i=1}^{5} SACT_{DIV_{t,t-i}} - U_{t-6}^{t}) - M\delta_{29} \leq 0$$

40b) $$(SACT_{DIV_t} - \sum_{i=1}^{5} SACT_{DIV_{t,t-i}} - U_{t-6}^{t})\delta_{29} \geq 0$$

41) $$RACT_{t} = \sum_{i=1}^{6} SACT_{DIV_{t,t-i}}$$

42) $$SACT_{F,t+1} - SACT_{F,t} + SACT_{F,t}(1-\delta_{30}) + U_t\delta_{30} - SACT_{DIV_t} + \sum_{i=1}^{6} SACT_{DIV_{t,t-i}} - \sum_{i=1}^{3} SACT_{L_{t,t-i}} = 0$$

42a) $$SACT_{F,t} - U_t - M\delta_{30} \leq 0$$

42b) $$(SACT_{F,t} - U_t)\delta_{30} \geq 0$$

43) $MCT_t - T\pi_t + (1-\delta_{31}) b\, DIV_t / (1-b) + b\pi_t \delta_{31} = 0$

43a) $b\, DIV_t / (1-b) - b\pi_t - M\delta_{31} \leq 0$

43b) $(b\, DIV_t / (1-b) - b\pi_t)\delta_{31} \geq 0$

The objective function of the model will be unchanged.

Appendix 7.3

```
!Model 2

LET NPROJ=6                    !NO. OF PROJECTS BEING CONSIDERED
LET NT=11                      !NO.OF TIME PERIODS
LET DRATE=0.10                 !DISCOUNT RATE
LET TAXRATE=0.33               !POST-HORIZON TAX RATE
LET M=10000                    !UPPER BOUND ON MUTUALLY EXCLUSIVE
                                                     VARIABLES

TABLES

ILMT(3,NT)                     !BREAK POINTS FOR SMALL COMPANY INCOME
TLMT(3,NT)                     !TAX DUE AT BREAK POINTS
BASR(NT)                       !BASIC RATE OF INCOME TAX, EACH PERIOD
ACTR(NT)                       !PROPORTION OF NET DIVIDEND PAID AS ACT
DFTAX(NT)                      !DISCOUNT FACTORS FOR TAX PAYMENTS
DFCASH(NT)                     !DISCOUNT FACTORS FOR CASH INFLOWS
DFOUT(NT)                      !DISCOUNT FACTORS FOR PROJECTS' CAPITAL
                                                       OUTLAY

INPT(11)                       !RELEVANT DATA ON PREVIOUS PERIODS
IN1(NT)                        !NON TRADING INCOME
DIV(NT)                        !DIVIDEND PAYMENTS (EXOGENOUS)
DINT(NT)                       !DEBT INTEREST PAYMENTS
EXPRINC(NT)                    !POSITIVE N.O.C.F. FROM EXISTING
                                                 ACTIVITIES
EXPRLSS(NT)                    !NEGATIVE N.O.C.F. FROM EXISTING
                                                 ACTIVITIES
EXCAPALL(NT)                   !CAPITAL ALLOWANCES FROM EXISTING
                                                 ACTIVITIES
PROJINC(NPROJ,NT)              !INCOME FROM EACH PROJECT
CAPALL(NPROJ,NT)               !CAPITAL ALLOWANCES FROM PROJECTS
PROJCOST(NPROJ,NT)             !INITIAL CAPITAL OUTLAY ON PROJECTS
WCAPI(NPROJ,NT)                !WORKING CAPITAL INCREASE DUE TO PROJECTS
WCAPD(NPROJ,NT)                !WORKING CAPITAL DECREASE DUE TO PROJECTS
```

```
DISKDATA -1

ILMT=C:\XPRESSMP\QPRO\DATA01.WK1(C8..M10)
TLMT=C:\XPRESSMP\QPRO\DATA01.WK1(C12..M14)
BASR=C:\XPRESSMP\QPRO\DATA01.WK1(C16..M16)
INPT=C:\XPRESSMP\QPRO\DATA01.WK1(B33..B43)
IN1=C:\XPRESSMP\QPRO\DATA01.WK1(C20..M20)
DIV=C:\XPRESSMP\QPRO\DATA01.WK1(C21..M21)
EXPRINC=C:\XPRESSMP\QPRO\DATA01.WK1(C23..M23)
EXPRLSS=C:\XPRESSMP\QPRO\DATA01.WK1(C24..M24)
EXCAPALL=C:\XPRESSMP\QPRO\DATA01.WK1(C25..M25)
DINT=C:\XPRESSMP\QPRO\DATA01.WK1(C26..M26)

PROJCOST=C:\XPRESSMP\QPRO\NPV01.WK1(J5..T10)
PROJINC=C:\XPRESSMP\QPRO\NPV01.WK1(J15..T20)
CAPALL=C:\XPRESSMP\QPRO\NPV01.WK1(J25..T30)
WCAPI=C:\XPRESSMP\QPRO\NPV01.WK1(J35..T40)
WCAPD=C:\XPRESSMP\QPRO\NPV01.WK1(J45..T50)

ASSIGN

ILM1(t=4:NT)=ILMT(1,t)
ILM2(t=4:NT)=ILMT(2,t)
ILM3(t=4:NT)=ILMT(3,t)
TLM1(t=4:NT)=TLMT(1,t)
TLM2(t=4:NT)=TLMT(2,t)
TLM3(t=4:NT)=TLMT(3,t)

ACTR(t=7:NT)=BASR(t)/(1-BASR(t))

DFTAX(t=1:6)=1
DFTAX(t=7:7)=1/(1+DRATE)^1.75
DFTAX(t=8:NT)=DFTAX(t-1)/(1+DRATE)
DFCASH(t=1:6)=1
DFCASH(t=7:7)=1/(1+DRATE)^0.5
DFCASH(t=8:NT)=DFCASH(t-1)/(1+DRATE)
DFOUT(t=1:7)=1
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DFOUT(t=8:8)=1/(1+DRATE)
DFOUT(t=9:NT)=DFOUT(t-1)/(1+DRATE)

VARIABLES

accept(NPROJ)              !BINARY VARIABLE FOR PROJECT ACCEPTANCE
P1(t=7:NT)                 !TAX PROFIT (SCH D CASE 1)
P2(t=7:NT)                 !            AFTER DEBT INTEREST
P3(t=6:NT)                 !            AFTER LOSSES C/F
P4(t=5:NT-1)               !            AFTER LOSSES C/B FROM t+1
P5(t=4:NT-2)               !                                  t+2
P6(t=4:NT-3)               !                                  t+3
L1(t=7:NT)                 !TAX LOSS (SCH D CASE 1)
LF1(t=7:NT)                !DITTO: CHOSEN TO BE C/F
L1a(t=7:NT)                !DITTO: CHOSEN TO BE OFFSET AGAINST TOTAL
                             PROFITS
L1b(t=7:NT)                !       REMAINING AFTER OFFSET
IN2(t=7:NT)                !NON-TRADING INCOME REMAINING
                                   AFTER TRADING LOSS OFFSET

LB(t=7:NT)                 !L1b CHOSEN FOR CARRY BACK
LF2(t=7:NT)                !                CARRY FORWARD
L2(t=7:NT)                 !LOSS REMAINING AFTER OFFSET IN t-1
L3(t=7:NT)                 !                               t-2
L4(t=7:NT)                 !                               t-3
T1(t=6:NT)                 !TAX PAYABLE ON P3
T2(t=5:NT-1)               !               P4
T3(t=4:NT-2)               !               P5
T4(t=4:NT-3)               !               P6
LM1(t=6:NT)                !VARIABLES USED IN TAX FUNCTION
LM2(t=6:NT)
LM3(t=6:NT)
LM4(t=6:NT)
LM5(t=5:NT-1)
LM6(t=5:NT-1)
LM7(t=5:NT-1)
LM8(t=5:NT-1)
LM9(t=4:NT-2)
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```
LM10(t=4:NT-2)
LM11(t=4:NT-2)
LM12(t=4:NT-2)
LM13(t=4:NT-3)
LM14(t=4:NT-3)
LM15(t=4:NT-3)
LM16(t=4:NT-3)

OUT(t=7:NT)             !PROJECT CAPITAL EXPENDITURE
DCF(t=7:NT)             !DISCOUNTED PROJECT CASH INFLOW
XC(t=7:NT)              !EXCESS CHARGES
LF(t=6:NT)              !LOSSES CARRIED FORWARD FROM t
LFF(t=7:NT)             !LOSSES C/F TO t AND NOT OFFSET IN t
TR(t=7:NT)              !TAX REBATE FROM LOSSES CARRIED BACK

                        !SURPLUS ACT:
SD(t=7:NT)              !  ARISING DUE TO DIVIDEND PAYMENT
SF(t=6:NT)              !  CARRIED FORWARD FROM t
SFF(t=7:NT)             !  C/F TO t AND AGAIN FROM t TO t+1
SL1(t=6:NT-1)           !  ARISING FROM C/B OF LOSSES FROM t+1
SL2(t=5:NT-2)           !  ARISING FROM C/B OF LOSSES FROM t+2
SL3(t=4:NT-3)           !  ARISING FROM C/B OF LOSSES FROM t+3

                        !UNUTILISED ACT SETOFF CAPACITY:
U1(t=7:NT)              !  AFTER DIV(t) TAKEN INTO ACCOUNT
U2(t=6:NT)              !  AFTER OFFSET OF SURPLUS b/f
U3(t=6:NT-1)            !  AFTER C/B OF LOSSES FROM t+1
U4(t=5:NT-2)            !                           t+2
U5(t=4:NT-3)            !                           t+3

                        !SETOFF OF SURPLUS ACT:
SDF(t=7:NT)             !  AMOUNT SELECTED FOR CARRY FORWARD
SDB1(t=7:NT)            !  AMOUNT SELECTED FOR CARRY BACK
SDB2(t=7:NT)            !  REMAINDER AFTER C/B TO t-1
SDB3(t=7:NT)            !                         t-2
SDB4(t=7:NT)            !                         t-3
SDB5(t=7:NT)            !                         t-4
SDB6(t=7:NT)            !                         t-5
```

```
SDB7(t=7:NT)               !                    t-6

                           !UNUTILISED ACT SETOFF CAPACITY
U3A(t=5:NT-1)              !  AFTER C-B OF SURPLUS ACT FROM t+1
U4A(t=4:NT-2)              !                                t+2
U5A(t=3:NT-3)              !                                t+3
U6(t=2:NT-4)               !                                t+4
U7(t=1:NT-5)               !                                t+5
U8(t=1:NT-6)               !                                t+6

A1(t=7:NT)                 !ACT ACTUALLY PAID ON DIVIDEND
A2(t=7:NT)                 !    ALLOWED AGAINST MCT

                           !BINARY VARIABLES TO DETERMINE:
d1(t=7:NT)           !                         P1, L1
d2(t=7:NT)           !                         P2, LFF
d3(t=7:NT)           !                         P3, XC
d4(t=6:NT-1)         !                         P4, L2
d5(t=5:NT-2)         !                         P5, L3
d6(t=4:NT-3)         !                         P6, L4
d7(t=7:NT)           !                         L1a, LF1
d8(t=7:NT)           !                         U1, SD
d9(t=7:NT)           !                         U2, SFF
d10(t=6:NT-1)        !                         U3, SL1
d11(t=6:NT-1)        !                         U3a, SDB2
d12(t=5:NT-2)        !                         U4, SL2
d13(t=5:NT-2)        !                         U4a, SDB3
d14(t=4:NT-3)        !                         U5, SL3
d15(t=4:NT-3)        !                         U5a, SDB4
d16(t=3:NT-4)        !                         U6, SDB5
d17(t=2:NT-5)        !                         U7, SDB6
d18(t=1:NT-6)        !                         U8, SDB7
d19(t=7:NT)          !                         IN2, L1b
d20(t=7:NT)          !                         LB, LF2
```

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CONSTRAINTS

!PREVIOUS DATA

DA1(t=6:6):LF(t)=INPT(1)
DA2(t=6:6):P3(t)=INPT(2)
DA3(t=5:5):P4(t)=INPT(3)
DA4(t=4:4):P5(t)=INPT(4)
DA5(t=6:6):SF(t)=INPT(5)
DA6(t=4:4):U4A(t)=INPT(6)
DA7(t=5:5):U3A(t)=INPT(7)
DA8(t=6:6):U2(t)=INPT(8)
DA9(t=3:3):U5A(t)=INPT(9)
DA10(t=2:2):U6(t)=INPT(10)
DA11(t=1:1):U7(t)=INPT(11)

!DETERMINATION OF PROFIT/LOSS/EXCESS CHARGES ON INCOME

TXN1(t=7:NT):P1(t)-L1(t)=EXPRINC(t)-EXPRLSS(t)  &
                 -EXCAPALL(t)+SUM(p=1:NPROJ)(PROJINC(p,t)-CAPALL(p,t) &
                  +WCAPI(p,t)-WCAPD(p,t))*accept(p)

TXN2(t=7:NT):P1(t)<M*d1(t)
TXN3(t=7:NT):L1(t)<M-M*d1(t)

OFS1(t=7:NT):L1a(t)+LF1(t)=L1(t)
OFS2(t=7:NT):L1a(t)<M*d7(t)
OFS3(t=7:NT):LF1(t)<M-M*d7(t)

OFS4(t=7:NT):IN2(t)-L1b(t)=IN1(t)-L1a(t)
OFS5(t=7:NT):IN2(t)<M*d19(t)
OFS6(t=7:NT):L1b(t)<M-M*d19(t)

CBF1(t=7:NT):LB(t)+LF2(t)=L1b(t)
CBF2(t=7:NT):LB(t)<M*d20(t)
CBF3(t=7:NT):LF2(t)<M-M*d20(t)

LSS1(t=7:NT):P2(t)-LFF(t)=P1(t)-LF(t-1)
```

```
LSS2(t=7:NT):P2(t)<M*d2(t)
LSS3(t=7:NT):LFF(t)<M-M*d2(t)

DB1(t=7:NT):P3(t)-XC(t)=P2(t)+IN2(t)-DINT(t)
DB2(t=7:NT):P3(t)<M*d3(t)
DB3(t=7:NT):XC(t)<M-M*d3(t)

!SETOFF OF LOSSES CARRIED BACK

LCB1(t=6:NT-1):P4(t)-L2(t+1)=P3(t)-LB(t+1)
LCB2(t=6:NT-1):P4(t)<M*d4(t)
LCB3(t=6:NT-1):L2(t+1)<M-M*d4(t)

LCB4(t=5:NT-2):P5(t)-L3(t+2)=P4(t)-L2(t+2)
LCB5(t=5:NT-2):P5(t)<M*d5(t)
LCB6(t=5:NT-2):L3(t+2)<M-M*d5(t)

LCB7(t=4:NT-3):P6(t)-L4(t+3)=P5(t)-L3(t+3)
LCB8(t=4:NT-3):P6(t)<M*d6(t)
LCB9(t=4:NT-3):L4(t+3)<M-M*d6(t)

TLF(t=7:NT):LF(t)=LFF(t)+L4(t)+LF1(t)+LF2(t)+XC(t)

!CALCULATION OF TAX PAYMENTS

TN5(t=6:NT):P3(t)=0*LM1(t)+ILM1(t)*LM2(t)+ILM2(t)*LM3(t)   &
                                                  +ILM3(t)*LM4(t)
TF1(t=6:NT):T1(t)=0*LM1(t)+TLM1(t)*LM2(t)+TLM2(t)*LM3(t)   &
                                                  +TLM3(t)*LM4(t)
LSM1(t=6:NT):LM1(t)+LM2(t)+LM3(t)+LM4(t)=1

TN6(t=5:NT-1):P4(t)=0*LM5(t)+ILM1(t)*LM6(t)+ILM2(t)*LM7(t)   &
                                                  +ILM3(t)*LM8(t)
TF2(t=5:NT-1):T2(t)=0*LM5(t)+TLM1(t)*LM6(t)+TLM2(t)*LM7(t)   &
                                                  +TLM3(t)*LM8(t)
LSM2(t=5:NT-1):LM5(t)+LM6(t)+LM7(t)+LM8(t)=1
```

```
TN7(t=4:NT-2):P5(t)=0*LM9(t)+ILM1(t)*LM10(t)+ILM2(t)*LM11(t)    &
                                                   +ILM3(t)*LM12(t)
TF3(t=4:NT-2):T3(t)=0*LM9(t)+TLM1(t)*LM10(t)+TLM2(t)*LM11(t)    &
                                                   +TLM3(t)*LM12(t)
LSM3(t=4:NT-2):LM9(t)+LM10(t)+LM11(t)+LM12(t)=1

TN8(t=4:NT-3):P6(t)=0*LM13(t)+ILM1(t)*LM14(t)+ILM2(t)*LM15(t)  &
                                                   +ILM3(t)*LM16(t)
TF4(t=4:NT-3):T4(t)=0*LM13(t)+TLM1(t)*LM14(t)+TLM2(t)*LM15(t)   &
                                                   +TLM3(t)*LM16(t)
LSM4(t=4:NT-3):LM13(t)+LM14(t)+LM15(t)+LM16(t)=1

!TAX REBATE
TREB(t=7:NT):TR(t)=T1(t-1)-T2(t-1)+T2(t-2)-T3(t-2) &
                        T3(t-3)-T4(t-3)-SL1(t-1)-SL2(t-2)   &
                        -SL3(t-3)+SDB1(t)-SDB7(t)

!DETERMINATION OF ACT PAYMENTS/SETOFF CAPACITY/SURPLUS ACT

ACTP(t=7:NT):A1(t)=ACTR(t)*DIV(t)

ACTA(t=7:NT):A2(t)=A1(t)-SD(t)

SA1(t=7:NT):U1(t)-SD(t)=BASR(t)*P3(t)-ACTR(t)*DIV(t)
SA2(t=7:NT):U1(t)<M*d8(t)
SA3(t=7:NT):SD(t)<M-M*d8(t)

SAD(t=7:NT):SD(t)=SDB1(t)+SDF(t)

SA4(t=7:NT):U2(t)-SFF(t)=U1(t)-SF(t-1)
SA5(t=7:NT):U2(t)<M*d9(t)
SA6(t=7:NT):SFF(t)<M-M*d9(t)

SA7(t=6:NT-1):U3(t)-SL1(t)=BASR(t)*P4(t)-BASR(t)*P3(t)+U2(t)
SA8(t=6:NT-1):U3(t)<M*d10(t)
SA9(t=6:NT-1):SL1(t)<M-M*d10(t)

SA10(t=6:NT-1):U3A(t)-SDB2(t+1)=U3(t)-SDB1(t+1)
```

```
SA11(t=6:NT-1):U3A(t)<M*d11(t)
SA12(t=6:NT-1):SDB2(t+1)<M-M*d11(t)

SA13(t=5:NT-2):U4(t)-SL2(t)=BASR(t)*P5(t)-BASR(t)*P4(t)+U3A(t)
SA14(t=5:NT-2):U4(t)<M*d12(t)
SA15(t=5:NT-2):SL2(t)<M-M*d12(t)

SA16(t=5:NT-2):U4A(t)-SDB3(t+2)=U4(t)-SDB2(t+2)
SA17(t=5:NT-2):U4A(t)<M*d13(t)
SA18(t=5:NT-2):SDB3(t+2)<M-M*d13(t)

SA19(t=4:NT-3):U5(t)-SL3(t)=BASR(t)*P6(t)-BASR(t)*P5(t)+U4A(t)
SA20(t=4:NT-3):U5(t)<M*d14(t)
SA21(t=4:NT-3):SL3(t)<M-M*d14(t)

SA22(t=4:NT-3):U5A(t)-SDB4(t+3)=U5(t)-SDB3(t+3)
SA23(t=4:NT-3):U5A(t)<M*d15(t)
SA24(t=4:NT-3):SDB4(t+3)<M-M*d15(t)

SA25(t=3:NT-4):U6(t)-SDB5(t+4)=U5A(t)-SDB4(t+4)
SA26(t=3:NT-4):U6(t)<M*d16(t)
SA27(t=3:NT-4):SDB5(t+4)<M-M*d16(t)

SA28(t=2:NT-5):U7(t)-SDB6(t+5)=U6(t)-SDB5(t+5)
SA29(t=2:NT-5):U7(t)<M*d17(t)
SA30(t=2:NT-5):SDB6(t+5)<M-M*d17(t)

SA31(t=1:NT-6):U8(t)-SDB7(t+6)=U7(t)-SDB6(t+6)
SA32(t=1:NT-6):U8(t)<M*d18(t)
SA33(t=1:NT-6):SDB7(t+6)<M-M*d18(t)

TSF(t=7:NT):SF(t)=SFF(t)+SDB7(t)+SDF(t)+SL1(t-1)+SL2(t-2)+SL3(t-3)

!CAPITAL PROJECT EXPENDITURE/INCOME

EXP(t=7:NT):OUT(t)=SUM(p=1:NPROJ)DFOUT(t)*PROJCOST(p,t)*accept(p)
DFLO(t=7:NT):DCF(t)=SUM(p=1:NPROJ)PROJINC(p,t)*DFCASH(t)*accept(p)
```

```
!OBJECTIVE FUNCTION (MAXIMISE)
MAX:SUM(t=7:NT)DCF(t)-SUM(t=7:NT)OUT(t)     &
                 -SUM(t=7:NT)DFTAX(t)*T1(t)   &
                 +SUM(t=7:NT)DFTAX(t)*A2(t)     &
                 +SUM(t=7:NT)DFTAX(t)*TR(t)     &
                 +SUM(t=NT:NT) (DFTAX(t)/(1+DRATE))*TAXRATE*LF(t) &
                 +SUM(t=NT:NT) (DFTAX(t)/(1+DRATE))*SF(t)    $

SETS
!SPECIAL ORDERED SET TYPE 2
S(t=6:NT):LM1(t)+LM2(t)+LM3(t)+LM4(t)  .S2. TN5(t)
Sa(t=5:NT-1):LM5(t)+LM6(t)+LM7(t)+LM8(t)  .S2. TN6(t)
Sb(t=4:NT-2):LM9(t)+LM10(t)+LM11(t)+LM12(t)  .S2. TN7(t)
Sc(t=4:NT-3):LM13(t)+LM14(t)+LM15(t)+LM16(t)  .S2. TN8(t)

BOUNDS
accept(p=1:NPROJ)      .BV.
d1(t=7:NT)             .BV.
d2(t=7:NT)             .BV.
d3(t=7:NT)             .BV.
d4(t=6:NT-1)           .BV.
d5(t=5:NT-2)           .BV.
d6(t=4:NT-3)           .BV.
d7(t=7:NT)             .BV.
d8(t=7:NT)             .BV.
d9(t=7:NT)             .BV.
d10(t=6:NT-1)           .BV.
d11(t=6:NT-1)           .BV.
d12(t=5:NT-2)           .BV.
d13(t=5:NT-2)           .BV.
d14(t=4:NT-3)           .BV.
d15(t=4:NT-3)           .BV.
d16(t=3:NT-4)           .BV.
d17(t=2:NT-5)           .BV.
d18(t=1:NT-6)           .BV.
d19(t=7:NT)             .BV.
d20(t=7:NT)             .BV.
```

```
DIRECTIVES
accept(p=1:NPROJ)  .PR.    20
d1(t=7:NT)         .PR.    60
d2(t=7:NT)         .PR.    100
d7(t=7:NT)         .PR.    100
d19(t=7:NT)        .PR.    100
d3(t=7:NT)         .PR.    145
d20(t=7:NT)        .PR.    150
d4(t=6:NT-1)       .PR.    200
d8(t=7:NT)         .PR.    200
d5(t=5:NT-2)       .PR.    250
d1(t=7:NT)         .UP.

GENERATE
```

Appendix 7.4

```
!Model 3


LET NPROJ=11            !NO. OF PROJECTS BEING CONSIDERED
LET NT=11               !NO.OF TIME PERIODS
LET WDARATE=0.25        !WRITING DOWN RATE FOR FIXED ASSETS
LET TAXRATE=0.33        !POST-HORIZON TAX RATE
LET M=2                 !UPPER BOUND ON MUTUALLY EXCLUSIVE
                                                      VARIABLES
LET NM=1                !NO.OF MEMBER FIRMS IN GROUP STRUCTURE
LET PCENT=0             !OWNERSHIP OF SUBSIDIARY BY PARENT
LET GELEC=0             !GROUP ELECTION MADE FOR ACT PAYMENTS:
                          1=YES       0=NO
LET EQIMIN=0.05
LET EQIMAX=10

TABLES

ILMT(3,NT)              !BREAK POINTS FOR SMALL COMPANY INCOME
TLMT(3,NT)              !TAX DUE AT BREAK POINTS
BASR(NT)                !BASIC RATE OF INCOME TAX, EACH PERIOD
FULL(NT)                !FULL RATE OF CORPORATION TAX
ACTR(NT)                !PROPORTION OF NET DIVIDEND PAID AS ACT
GRDIV(NT)               !GROSSING UP FACTOR FOR DIVIDENDS

IN1(NM,NT)              !NON-TRADING INCOME (EXOGENOUS)
EXPRINC(NM,NT)          !POSITIVE N.O.C.F. FROM EXISTING
                                                    ACTIVITIES
EXPRLSS(NM,NT)          !NEGATIVE N.O.C.F. FROM EXISTING
                                                    ACTIVITIES
EXCAPALL(NM,NT)         !CAPITAL ALLOWANCES FROM EXISTING
                                                    ACTIVITIES
EXBVAL1(NT)             !MARKET VALUES OF EXISTING ASSETS OF
                                             PARENT AT START OF t
```

```
EXBVAL2(NT)                  !MARKET VALUES OF EXISTING ASSETS OF
                                                  PARENT AT END OF t

DINT(NT)                     !DEBT INTEREST PAID BY SUBSIDIARY
NDIV(NT)                     !TOTAL NET DIVIDEND PAYMENT BY SUBSIDIARY

IF NM>1
INPT(21,NM)                   !RELEVANT DATA INPUTS
ELSE
INPT(17,NM)                   !RELEVANT DATA INPUTS
ENDIF

PROJINC(NPROJ,NT)             !N.O.C.F. FROM EACH PROJECT
CAPALL(NPROJ,NT)              !CAPITAL ALLOWANCES FROM PROJECTS
BAL(NPROJ,NT)                 !BALANCING ALLOWANCE ON SHORT LIFE ASSETS
PROJCOST(NPROJ,NT)            !INITIAL CAPITAL OUTLAY ON PROJECTS
PROJRENT(NPROJ,NT)            !PROJECT LEASE RENTALS
iIMP(NPROJ)                   !RATE OF INTEREST IMPLICIT IN LEASE
FC(NPROJ,NT)                  !FINANCE CHARGE COMPONENT OF LEASE
PAYMENTS
DEPN(NPROJ,NT)                !ACCOUNTING DEPRECIATION ON LEASED ASSET
WCAPI(NPROJ,NT)               !WORKING CAPITAL INCREASE DUE TO PROJECTS
WCAPD(NPROJ,NT)               !WORKING CAPITAL DECREASE DUE TO PROJECTS
DISP(NPROJ,NT)                !DISPOSAL PROCEEDS OF ASSETS (END OF t)
PRBVAL1(NPROJ,NT)             !MARKET VALUES OF PROJECTS CONSIDERED FOR
                              !  ACCEPTANCE, AT START OF t
PRBVAL2(NPROJ,NT)             !MARKET VALUES OF PROJECTS CONSIDERED FOR
                              !  ACCEPTANCE, AT END OF t


                              !PARENT COMPANY:
g(NT)                         !MINIMUM RATE OF INCREASE IN DIVIDEND
GFAC1(1)                      !REQUIRED DIVIDEND GROWTH IN LAST
                              !                          PERIOD OF MODEL
GFAC2(1)                      !FACTOR TO ENSURE CONTINUAL DIVIDEND
                                                 GROWTH AFTER HORIZON
Ke(NT)                        !COST OF EQUITY (ESTIMATE)
AVKe(1)                       !AVERAGE COST OF EQUITY
DFACT(NT)                     !DISCOUNT FACTORS
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DFTERM(1)                 !DISCOUNT FACTOR FOR CASH FLOWS AT NT+1
DFVAL(NT)                 !DISCOUNT FACTORS FOR VALUE OF EQUITY
DLMT(5,NT)                !DEBT/EQUITY BREAK POINTS
IDLMT(5,NT)               !INTEREST DUE ON BANDS OF DEBT
iD(NM,NT)                 !INTEREST RATE ON SHORT TERM BORROWING
iL(NT)                    !INTEREST RATE ON SHORT TERM LENDING
DBMIN(NT)                 !MINIMUM PROPORTION OF DEBT IN CAPITAL
                                                         STRUCTURE
DBMAX(NT)                 !MAXIMUM PROPORTION OF DEBT IN CAPITAL
                                                         STRUCTURE
ISCD(NT)                  !ISSUE COSTS OF DEBT (AS PROPORTION)
ISCE(NT)                  !               EQUITY

DBSMAX(NM,NT)             !MAXIMUM AMOUNT OF SHORT TERM BORROWING
CSHMIN(NM,NT)             !MINIMUM CASH BALANCE


DISKDATA -1

IF NM>1

ILMT=C:\XPRESSMP\QPRO\DGRO.WK1(D8..N10)
TLMT=C:\XPRESSMP\QPRO\DGRO.WK1(D12..N14)
BASR=C:\XPRESSMP\QPRO\DGRO.WK1(D16..N16)
Ke=C:\XPRESSMP\QPRO\TRIAL.WK1(A3..K3)

IN1=C:\XPRESSMP\QPRO\DGRO.WK1(D81..N82)
EXPRINC=C:\XPRESSMP\QPRO\DGRO.WK1(D65..N66)
EXPRLSS=C:\XPRESSMP\QPRO\DGRO.WK1(D69..N70)
EXCAPALL=C:\XPRESSMP\QPRO\DGRO.WK1(D77..N78)
EXBVAL1=C:\XPRESSMP\QPRO\DGRO.WK1(D72..N72)
EXBVAL2=C:\XPRESSMP\QPRO\DGRO.WK1(D74..N74)
DINT=C:\XPRESSMP\QPRO\DGRO.WK1(D52..N52)
NDIV=C:\XPRESSMP\QPRO\DGRO.WK1(D56..N56)
INPT=C:\XPRESSMP\QPRO\DGRO.WK1(B87..C107)
g=C:\XPRESSMP\QPRO\DGRO.WK1(D54..N54)

DLMT=C:\XPRESSMP\QPRO\DGRO.WK1(D37..N41)
```

```
IDLMT=C:\XPRESSMP\QPRO\DGRO.WK1(D43..N47)
iD=C:\XPRESSMP\QPRO\DGRO.WK1(D27..N27)
iL=C:\XPRESSMP\QPRO\DGRO.WK1(D25..N25)
DBSMAX=C:\XPRESSMP\QPRO\DGRO.WK1(D30..N31)
DBMIN=C:\XPRESSMP\QPRO\DGRO.WK1(D33..N33)
DBMAX=C:\XPRESSMP\QPRO\DGRO.WK1(D35..N35)

ISCD=C:\XPRESSMP\QPRO\DGRO.WK1(D49..N49)
ISCE=C:\XPRESSMP\QPRO\DGRO.WK1(D50..N50)
CSHMIN=C:\XPRESSMP\QPRO\DGRO.WK1(D59..N60)


ELSE

ILMT=C:\XPRESSMP\QPRO\DGRO.WK1(D8..N10)
TLMT=C:\XPRESSMP\QPRO\DGRO.WK1(D12..N14)
BASR=C:\XPRESSMP\QPRO\DGRO.WK1(D16..N16)
IN1=C:\XPRESSMP\QPRO\DGRO.WK1(D81..N81)
EXPRINC=C:\XPRESSMP\QPRO\DGRO.WK1(D65..N65)
EXPRLSS=C:\XPRESSMP\QPRO\DGRO.WK1(D69..N69)
EXCAPALL=C:\XPRESSMP\QPRO\DGRO.WK1(D77..N77)
EXBVAL1=C:\XPRESSMP\QPRO\DGRO.WK1(D72..N72)
EXBVAL2=C:\XPRESSMP\QPRO\DGRO.WK1(D74..N74)
INPT=C:\XPRESSMP\QPRO\DGRO.WK1(B87..B103)
DLMT=C:\XPRESSMP\QPRO\DGRO.WK1(D37..N41)
IDLMT=C:\XPRESSMP\QPRO\DGRO.WK1(D43..N47)
iD=C:\XPRESSMP\QPRO\DGRO.WK1(D27..N27)
iL=C:\XPRESSMP\QPRO\DGRO.WK1(D25..N25)
DBSMAX=C:\XPRESSMP\QPRO\DGRO.WK1(D30..N30)
DBMIN=C:\XPRESSMP\QPRO\DGRO.WK1(D33..N33)
DBMAX=C:\XPRESSMP\QPRO\DGRO.WK1(D35..N35)
ISCD=C:\XPRESSMP\QPRO\DGRO.WK1(D49..N49)
ISCE=C:\XPRESSMP\QPRO\DGRO.WK1(D50..N50)
CSHMIN=C:\XPRESSMP\QPRO\DGRO.WK1(D59..N59)
g=C:\XPRESSMP\QPRO\DGRO.WK1(D54..N54)
Ke=C:\XPRESSMP\QPRO\TRIAL.WK1(A3..K3)
```

```
ENDIF

PROJINC=C:\XPRESSMP\QPRO\NPVOPC.WK1(k21..u31)
CAPALL=C:\XPRESSMP\QPRO\NPVOPC.WK1(k36..u46)
BAL=C:\XPRESSMP\QPRO\NPVOPC.WK1(k51..u61)
PROJCOST=C:\XPRESSMP\QPRO\NPVOPC.WK1(k6..u16)
PROJRENT=C:\XPRESSMP\QPRO\NPVOPC.WK1(k114..u124)
iIMP=C:\XPRESSMP\QPRO\NPVOPC.WK1(q146..q156)
DEPN=C:\XPRESSMP\QPRO\NPVOPC.WK1(k130..u140)
WCAPI=C:\XPRESSMP\QPRO\NPVOPC.WK1(k66..u76)
WCAPD=C:\XPRESSMP\QPRO\NPVOPC.WK1(k82..u92)
DISP=C:\XPRESSMP\QPRO\NPVOPC.WK1(k98..u108)
PRBVAL1=C:\XPRESSMP\QPRO\NPVOPC.WK1(k162..u172)
PRBVAL2=C:\XPRESSMP\QPRO\NPVOPC.WK1(k178..u188)

ASSIGN

ILM1(t=4:NT)=ILMT(1,t)/NM
ILM2(t=4:NT)=ILMT(2,t)/NM
ILM3(t=4:NT)=ILMT(3,t)/NM
TLM1(t=4:NT)=TLMT(1,t)/NM
TLM2(t=4:NT)=TLMT(2,t)/NM
TLM3(t=4:NT)=TLMT(3,t)/NM

ACTR(t=7:NT)=BASR(t)/(1-BASR(t))

AVKe(1)=SUM(t=7:NT)Ke(t)/(NT-6)
GFAC1(1)=SUM(t=NT:NT)g(t)
GFAC2(1)=(1+GFAC1(1))/(AVKe(1)-GFAC1(1))

DFACT(t=6:6)=1
DFACT(t=7:7)=1/(1+Ke(t))
DFACT(t=8:NT)=DFACT(t-1)/(1+Ke(t))
DFTERM(1)=1/(1+AVKe(1))
DFVAL(t=7:NT)=1/(1+Ke(t))

FC(p=1:NPROJ,t=7:NT-1|PROJRENT(p,t+1)>0.AND.PROJRENT(p,t)>0.AND. &
      PROJRENT(p,t-1)=0)=iIMP(p)*PROJCOST(p,t)-iIMP(p)*PROJRENT(p,t)
```

```
FC(p=1:NPROJ,t=8:NT-1|PROJRENT(p,t+1)>0.AND.PROJRENT(p,t-1)>0) &
                    =FC(p,t-1)+iIMP(p)*FC(p,t-1)-iIMP(p)*PROJRENT(p,t)

FC(p=1:NPROJ,t=NT:NT)=0


VARIABLES

accept(NPROJ)           !BINARY VARIABLE FOR PROJECT ACCEPTANCE
P1(NM,t=7:NT)           !TAX PROFIT (SCH D CASE 1)
P2(NM,t=7:NT)           !            AFTER DEBT INTEREST
P3(NM,t=6:NT)           !            AFTER LOSSES C/F
P4(NM,t=5:NT-1)         !            AFTER LOSSES C/B FROM t+1
P5(NM,t=4:NT-2)         !                                  t+2
P6(NM,t=4:NT-3)         !                                  t+3
L1(NM,t=7:NT)           !TAX LOSS (SCH D CASE 1)
LF1(NM,t=7:NT)          !DITTO: CHOSEN TO BE C/F
L1a(NM,t=7:NT)          !DITTO: CHOSEN TO BE OFFSET AGAINST TOTAL
                                                        PROFITS
L1b(NM,t=7:NT)          !        REMAINING AFTER OFFSET
IN2(NM,t=7:NT)          !NON-TRADING INCOME REMAINING
                                   AFTER TRADING LOSS OFFSET

LB(NM,t=7:NT)           !L1b CHOSEN FOR CARRY BACK
LF2(NM,t=7:NT)          !                 CARRY FORWARD
L2(NM,t=7:NT)           !LOSS REMAINING AFTER OFFSET IN t-1
L3(NM,t=7:NT)           !                               t-2
L4(NM,t=7:NT)           !                               t-3
T1(NM,t=6:NT)           !TAX PAYABLE ON P3
T2(NM,t=5:NT-1)         !               P4
T3(NM,t=4:NT-2)         !               P5
T4(NM,t=4:NT-3)         !               P6
LM1(NM,t=6:NT)          !VARIABLES USED IN TAX FUNCTION
LM2(NM,t=6:NT)
LM3(NM,t=6:NT)
LM4(NM,t=6:NT)
LM5(NM,t=5:NT-1)
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LM6(NM,t=5:NT-1)
LM7(NM,t=5:NT-1)
LM8(NM,t=5:NT-1)
LM9(NM,t=4:NT-2)
LM10(NM,t=4:NT-2)
LM11(NM,t=4:NT-2)
LM12(NM,t=4:NT-2)
LM13(NM,t=4:NT-3)
LM14(NM,t=4:NT-3)
LM15(NM,t=4:NT-3)
LM16(NM,t=4:NT-3)

XC(NM,t=7:NT)          !EXCESS CHARGES ON INCOME
LF(NM,t=6:NT)          !LOSSES CARRIED FORWARD FROM t
LFF(NM,t=7:NT)         !LOSSES C/F TO t AND NOT OFFSET IN t
TR(NM,t=6:NT)          !TAX REBATE FROM LOSSES CARRIED BACK

                       !SURPLUS ACT:
SD(NM,t=7:NT)          !  ARISING DUE TO DIVIDEND PAYMENT
SF(NM,t=6:NT)          !  CARRIED FORWARD FROM t
SFF(NM,t=7:NT)         !  C/F TO t AND AGAIN FROM t TO t+1
SL1(NM,t=6:NT-1)       !  ARISING FROM C/B OF LOSSES FROM t+1
SL2(NM,t=5:NT-2)       !  ARISING FROM C/B OF LOSSES FROM t+2
SL3(NM,t=4:NT-3)       !  ARISING FROM C/B OF LOSSES FROM t+3

                       !UNUTILISED ACT SETOFF CAPACITY:
U1(NM,t=7:NT)          !  AFTER DIV(t) TAKEN INTO ACCOUNT
U2(NM,t=6:NT)          !  AFTER OFFSET OF SURPLUS b/f
U3(NM,t=6:NT-1)        !  AFTER C/B OF LOSSES FROM t+1
U4(NM,t=5:NT-2)        !                            t+2
U5(NM,t=4:NT-3)        !                            t+3

                       !SETOFF OF SURPLUS ACT:
SDF(NM,t=7:NT)         !  AMOUNT SELECTED FOR CARRY FORWARD
SDB1(NM,t=7:NT)        !  AMOUNT SELECTED FOR CARRY BACK
SDB2(NM,t=7:NT)        !  REMAINDER AFTER C/B TO t-1
SDB3(NM,t=7:NT)        !                         t-2
SDB4(NM,t=7:NT)        !                         t-3
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SDB5(NM,t=7:NT)              !                    t-4
SDB6(NM,t=7:NT)              !                    t-5
SDB7(NM,t=7:NT)              !                    t-6

                             !UNUTILISED ACT SETOFF CAPACITY
U3A(NM,t=5:NT-1)             !  AFTER C-B OF SURPLUS ACT FROM t+1
U4A(NM,t=4:NT-2)             !                                t+2
U5A(NM,t=3:NT-3)             !                                t+3
U6(NM,t=2:NT-4)              !                                t+4
U7(NM,t=1:NT-5)              !                                t+5
U8(NM,t=1:NT-6)              !                                t+6

A1(NM,t=7:NT)                !ACT ACTUALLY PAID ON DIVIDEND
A2(NM,t=6:NT)                !    ALLOWED AGAINST MCT

IF NM>1
IF PCENT>=75
L1g(NM,t=7:NT)              !PORTION OF L1 SURRENDERED AS GROUP RELIEF
L1m(NM,t=7:NT)              !               RETAINED
XCg(NM,t=7:NT)              !PORTION OF EXCESS CHARGES SURRENDERED
XCm(NM,t=7:NT)              !                          RETAINED
AP(NM,t=7:NT)               !AVAILABLE PROFITS TO SET AGAINST GROUP
                                                                RELIEF
x(NM,t=7:NT)                !DUMMY VARIABLE
XMAX(NM,t=7:NT)             !EXCESS CHARGES WHICH MAY BE SURRENDERED
                              AS GROUP RELIEF
xx(NM,t=7:NT)               !DUMMY VARIABLE
ENDIF

P3a(NM,t=6:NT)              !PROFITS AFTER GROUP RELIEF IF APPLICABLE
ADIV(t=7:NT)                !GROSS DIVIDEND ON WHICH ACT IS PAYABLE
                            !                   UNDER A GROUP ELECTION
SFII(t=6:NT)                !SURPLUS FRANKED INVESTMENT INCOME
EDIV(m=2:NM,t=7:NT)         !DIVIDEND PAID FROM SUBSIDIARY TO PARENT
                            !                   UNDER A GROUP ELECTION
ADm(t=7:NT)                 !ACT RETAINED BY PARENT
ADg(t=7:NT)                 !ACT SURRENDERED TO SUBSIDIARY
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Ug(m=2:NM,t=7:NT)          !UNUTILISED ACT CAPACITY IN SUBSIDIARY
                              AFTER OFFSET OF ACT SURRENDERED BY
                                                            PARENT
ADs(t=7:NT)                !SURRENDERED ACT C/F IN  SUBSIDIARY
ENDIF

OCB(m=1:NM,t=7:7)          !OPENING CASH BALANCE
LNS(m=1:NM,t=7:NT)         !SHORT TERM LENDING, FROM START OF t
DBS(m=1:NM,t=7:NT)         !SHORT TERM BORROWING, FROM START OF t
DBI(t=7:NT)                !NEW DEBT ISSUED AT START OF t TO FUND
                                                          PROJECTS
DBC(t=7:NT)                !NEW DEBT ISSUED TO MAINTAIN CAPITAL
                                                         STRUCTURE
LREP(t=7:NT)               !LOAN CAPITAL REPAYMENT, END OF t
DB(t=6:NT)                 !AMOUNT OF LONG TERM DEBT AT END OF t
DBIN(t=7:NT)               !LONG TERM DEBT INTEREST PAID AT END OF t
EQI(t=7:NT)                !NEW EQUITY ISSUED AT START OF t
issue(t=7:NT)              !SEMICONTINUOUS VARIABLE FOR EQUITY ISSUE
EQ(m=1:NM,t=6:NT)          !VALUE OF ORDINARY SHARE CAPITAL
                           !  DURING t

D2(t=7:NT)                 !VARIABLES TO CALCULATE DEBT INTEREST
D3(t=7:NT)
D4(t=7:NT)
D5(t=7:NT)

x1(t=7:NT)
x2(t=7:NT)
x3(t=7:NT)
x4(t=7:NT)

y1(t=7:NT)
y2(t=7:NT)
y3(t=7:NT)
y4(t=7:NT)

CSHC(t=7:NT)               !INTERNALLY GENERATED FUNDS REINVESTED IN
                           !  CAPITAL PROJECTS AT START OF t
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RET(m=1:NM,t=7:NT)              !INTERNALLY GENERATED FUNDS NOT REINVESTED
                                !  IN CAPITAL PROJECTS IN NEXT PERIOD

RTN(m=1:NM,t=7:NT)              !INCREASE IN RETAINED FUNDS
RTD(m=1:NM,t=7:NT)              !DECREASE IN RETAINED FUNDS

CSHP(m=1:NM,t=7:NT)             !POSITIVE CLOSING CASH BALANCE BEFORE
                                !  DISTRIBUTIONS
CSHN(m=1:NM,t=7:NT)             !NEGATIVE CLOSING CASH BALANCE BEFORE
                                !  DISTRIBUTIONS
DIV(m=1:NM,t=6:NT)              !NET DIVIDEND PAYMENT OUTSIDE GROUP, END
                                                                   OF t
DIVG(t=7:NT)                    !GROSS DIVIDEND PAYMENT, END OF t
HV(m=1:NM,t=NT:NT)              !HORIZON VALUATION

IF NM>1
HVAL(t=NT:NT)                   !HORIZON VALUATION OF GROUP
SBVAL(m=2:NM,t=NT:NT)           !HORIZON VALUE OF SUBSIDIARY'S ASSETS
SDBT(m=2:NM,t=NT:NT)            !HORIZON VALUE OF SUBSIDIARY'S DEBT
ENDIF
                                !BINARY VARIABLES TO DETERMINE:
d1(NM,t=7:NT)                   !                        P1, L1
d2(m=1:NM,t=7:NT)               !                        P2, LFF
d3(m=1:NM,t=7:NT)               !                        P3, XC
d4(m=1:NM,t=6:NT-1)             !                        P4, L2
d5(m=1:NM,t=5:NT-2)             !                        P5, L3
d6(m=1:NM,t=4:NT-3)             !                        P6, L4
d7(m=1:NM,t=7:NT)               !                        L1a, LF1
d8(m=1:NM,t=7:NT)               !                        U1, SD
d9(m=1:NM,t=7:NT)               !                        U2, SFF
d10(m=1:NM,t=6:NT-1)            !                        U3, SL1
d11(m=1:NM,t=6:NT-1)            !                        U3a, SDB2
d12(m=1:NM,t=5:NT-2)            !                        U4, SL2
d13(m=1:NM,t=5:NT-2)            !                        U4a, SDB3
d14(m=1:NM,t=4:NT-3)            !                        U5, SL3
d15(m=1:NM,t=4:NT-3)            !                        U5a, SDB4
d16(m=1:NM,t=3:NT-4)            !                        U6, SDB5
d17(m=1:NM,t=2:NT-5)            !                        U7, SDB6
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d18(m=1:NM,t=1:NT-6)          !                U8, SDB7
d19(m=1:NM,t=7:NT)            !                IN2, L1b
d20(m=1:NM,t=7:NT)            !                LB, LF2
IF NM>1
IF PCENT>=75
d21(m=1:NM,t=7:NT)            !                AP, x
ENDIF
d22(m=2:NM,t=7:NT)            !                Ug, ADs
d23(t=7:NT)                   !                ADIV, SFII
ENDIF

d24(m=1:NM,t=7:NT)            !                LNS, DBS
d25(t=7:NT)                   !                 D2, x1
d26(t=7:NT)                   !                 D3, x2
d27(t=7:NT)                   !                 D4, x3
d28(t=7:NT)                   !                 D5, x4
d29(m=1:NM,t=7:NT)            !                CSHP, CSHN
d30(m=1:NM,t=8:NT)            !                RTN, RTD
IF NM>1.AND.PCENT>75
d31(m=1:NM,t=7:NT)            !                XMAX, xx
ENDIF

CONSTRAINTS

!PREVIOUS DATA

DA1(m=1:NM,t=6:6):LF(m,t)=INPT(1,m)
IF NM>1
DA2(m=1:NM,t=6:6):P3a(m,t)=INPT(2,m)
ELSE
DA2(m=1:NM,t=6:6):P3(m,t)=INPT(2,m)
ENDIF
DA3(m=1:NM,t=5:5):P4(m,t)=INPT(3,m)
DA4(m=1:NM,t=4:4):P5(m,t)=INPT(4,m)
DA5(m=1:NM,t=6:6):SF(m,t)=INPT(5,m)
DA6(m=1:NM,t=6:6):U2(m,t)=INPT(6,m)
DA7(m=1:NM,t=5:5):U3A(m,t)=INPT(7,m)
DA8(m=1:NM,t=4:4):U4A(m,t)=INPT(8,m)
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DA9(m=1:NM,t=3:3):U5A(m,t)=INPT(9,m)
DA10(m=1:NM,t=2:2):U6(m,t)=INPT(10,m)
DA11(m=1:NM,t=1:1):U7(m,t)=INPT(11,m)
DA12(m=1:1,t=7:7):OCB(m,t)+CSHC(t)=INPT(12,m)
DA13(m=1:1,t=6:6):DB(t)=INPT(13,m)
DA14(m=1:NM,t=6:6):EQ(m,t)=INPT(14,m)
DA15(m=1:1,t=6:6):DIV(m,t)=INPT(15,m)
DA16(m=1:NM,t=6:6):A2(m,t)=INPT(16,m)
DA17(m=1:NM,t=6:6):TR(m,t)=INPT(17,m)
IF NM>1
DA18(m=1:1,t=6:6):SFII(t)=INPT(18,m)
DA19(m=2:NM,t=NT:NT):SBVAL(m,t)=INPT(19,m)
DA20(m=2:NM,t=NT:NT):SDBT(m,t)=INPT(20,m)
DA21(m=2:NM,t=7:7):OCB(m,t)=INPT(21,m)
ENDIF

!TAXABLE INCOME/LOSS/EXCESS CHARGES ON INCOME

TXN1(m=1:1,t=7:NT):P1(m,t)-L1(m,t)=EXPRINC(m,t)-EXPRLSS(m,t)   &
              -EXCAPALL(m,t)+SUM(p=1:NPROJ)(PROJINC(p,t)-CAPALL(p,t)&
              -BAL(p,t)+WDARATE*DISP(p,t)+WCAPI(p,t)-WCAPD(p,t) &
              -FC(p,t)-DEPN(p,t))*accept(p)-ISCD(t)*DBI(t) &
              -iD(m,t)*DBS(m,t)

IF NM>1
TXNG(m=2:NM,t=7:NT):P1(m,t)-L1(m,t)=EXPRINC(m,t)-EXPRLSS(m,t)   &
               -EXCAPALL(m,t)
ENDIF
TXN3(m=1:NM,t=7:NT):P1(m,t)<M*d1(m,t)
TXN4(m=1:NM,t=7:NT):L1(m,t)<M-M*d1(m,t)

IF NM>1.AND.PCENT>=75
GRL(m=1:NM,t=7:NT):L1m(m,t)+L1g(m,t)=L1(m,t)
OFS1(m=1:NM,t=7:NT):L1a(m,t)+LF1(m,t)=L1m(m,t)
ELSE
OFS1(m=1:NM,t=7:NT):L1a(m,t)+LF1(m,t)=L1(m,t)
ENDIF
OFS2(m=1:NM,t=7:NT):L1a(m,t)<M*d7(m,t)
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OFS3(m=1:NM,t=7:NT):LF1(m,t)<M-M*d7(m,t)

OFS4(m=1:NM,t=7:NT):IN2(m,t)-L1b(m,t)=IN1(m,t)-L1a(m,t) &
                                                    +iL(t)*LNS(m,t)
OFS5(m=1:NM,t=7:NT):IN2(m,t)<M*d19(m,t)
OFS6(m=1:NM,t=7:NT):L1b(m,t)<M-M*d19(m,t)

CBF4(m=1:NM,t=7:NT):LB(m,t)+LF2(m,t)=L1b(m,t)
CBF5(m=1:NM,t=7:NT):LB(m,t)<M*d20(m,t)
CBF6(m=1:NM,t=7:NT):LF2(m,t)<M-M*d20(m,t)

LSS1(m=1:NM,t=7:NT):P2(m,t)-LFF(m,t)=P1(m,t)-LF(m,t-1)
LSS2(m=1:NM,t=7:NT):P2(m,t)<M*d2(m,t)
LSS3(m=1:NM,t=7:NT):LFF(m,t)<M-M*d2(m,t)

DB1(m=1:1,t=7:NT):P3(m,t)-XC(m,t)=P2(m,t)+IN2(m,t)-DBIN(t)
IF NM>1
DBG(m=2:NM,t=7:NT):P3(m,t)-XC(m,t)=P2(m,t)+IN2(m,t)-DINT(t)
ENDIF
DB2(m=1:NM,t=7:NT):P3(m,t)<M*d3(m,t)
DB3(m=1:NM,t=7:NT):XC(m,t)<M-M*d3(m,t)

IF NM>1
IF PCENT>=75
AVP1(m=1:1,t=7:NT):AP(m,t)-x(m,t)=P1(m,t)+IN1(m,t)+iL(t)*LNS(m,t) &
                                          -LF(m,t-1)-L1(m,t)-DBIN(t)

AVPG(m=2:NM,t=7:NT):AP(m,t)-x(m,t)=P1(m,t)+IN1(m,t)+iL(t)*LNS(m,t) &
                                          -LF(m,t-1)-L1(m,t)-DINT(t)

AVP2(m=1:NM,t=7:NT):AP(m,t)<M*d21(m,t)
AVP3(m=1:NM,t=7:NT):x(m,t)<M-M*d21(m,t)

XCG1(m=1:NM,t=7:NT):XC(m,t)=XCm(m,t)+XCg(m,t)
XCD1(m=1:1,t=7:NT):XMAX(m,t)-xx(m,t)=DBIN(t)-P1(m,t)-IN1(m,t)
XCD2(m=2:NM,t=7:NT):XMAX(m,t)-xx(m,t)=DINT(t)-P1(m,t)-IN1(m,t)
XCD3(m=1:NM,t=7:NT):XMAX(m,t)<M*d31(m,t)
XCD4(m=1:NM,t=7:NT):xx(m,t)<M-M*d31(m,t)
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XCG2(m=1:NM,t=7:NT):XCg(m,t)<XMAX(m,t)

TGR1(m=1:1,t=7:NT):L1g(m+1,t)+XCg(m+1,t)<AP(m,t)
TGR2(m=2:2,t=7:NT):L1g(m-1,t)+XCg(m-1,t)<AP(m,t)

PGR1(m=1:1,t=7:NT):P3a(m,t)=P3(m,t)-L1g(m+1,t)-XCg(m+1,t)
PGR2(m=2:2,t=7:NT):P3a(m,t)=P3(m,t)-L1g(m-1,t)-XCg(m-1,t)


ELSE
PGR1(m=1:NM,t=7:NT):P3a(m,t)=P3(m,t)
ENDIF

ENDIF

!SETOFF OF LOSSES CARRIED BACK

IF NM>1
LCB1(m=1:NM,t=6:NT-1):P4(m,t)-L2(m,t+1)=P3a(m,t)-LB(m,t+1)
ELSE
LCB1(m=1:NM,t=6:NT-1):P4(m,t)-L2(m,t+1)=P3(m,t)-LB(m,t+1)
ENDIF
LCB2(m=1:NM,t=6:NT-1):P4(m,t)<M*d4(m,t)
LCB3(m=1:NM,t=6:NT-1):L2(m,t+1)<M-M*d4(m,t)

LCB4(m=1:NM,t=5:NT-2):P5(m,t)-L3(m,t+2)=P4(m,t)-L2(m,t+2)
LCB5(m=1:NM,t=5:NT-2):P5(m,t)<M*d5(m,t)
LCB6(m=1:NM,t=5:NT-2):L3(m,t+2)<M-M*d5(m,t)

LCB7(m=1:NM,t=4:NT-3):P6(m,t)-L4(m,t+3)=P5(m,t)-L3(m,t+3)
LCB8(m=1:NM,t=4:NT-3):P6(m,t)<M*d6(m,t)
LCB9(m=1:NM,t=4:NT-3):L4(m,t+3)<M-M*d6(m,t)

IF NM>1.AND.PCENT>=75

TLF(m=1:NM,t=7:NT):LF(m,t)=LFF(m,t)+L4(m,t)+LF1(m,t)+LF2(m,t)  &
                                                        +XCm(m,t)
ELSE
TLF(m=1:NM,t=7:NT):LF(m,t)=LFF(m,t)+L4(m,t)+LF1(m,t)+LF2(m,t)+XC(m,t)
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ENDIF

!CALCULATION OF TAX PAYMENTS

IF NM>1
TN5(m=1:NM,t=6:NT):P3a(m,t)=0*LM1(m,t)+ILM1(t)*LM2(m,t)  &
                              +ILM2(t)*LM3(m,t)+ILM3(t)*LM4(m,t)
ELSE
TN5(m=1:NM,t=6:NT):P3(m,t)=0*LM1(m,t)+ILM1(t)*LM2(m,t)   &
                              +ILM2(t)*LM3(m,t)+ILM3(t)*LM4(m,t)
ENDIF
TF1(m=1:NM,t=6:NT):T1(m,t)=0*LM1(m,t)+TLM1(t)*LM2(m,t)   &
                              +TLM2(t)*LM3(m,t)+TLM3(t)*LM4(m,t)
LSM1(m=1:NM,t=6:NT):LM1(m,t)+LM2(m,t)+LM3(m,t)+LM4(m,t)=1

TN6(m=1:NM,t=5:NT-1):P4(m,t)=0*LM5(m,t)+ILM1(t)*LM6(m,t)  &
                              +ILM2(t)*LM7(m,t)+ILM3(t)*LM8(m,t)
TF2(m=1:NM,t=5:NT-1):T2(m,t)=0*LM5(m,t)+TLM1(t)*LM6(m,t)  &
                              +TLM2(t)*LM7(m,t)+TLM3(t)*LM8(m,t)
LSM2(m=1:NM,t=5:NT-1):LM5(m,t)+LM6(m,t)+LM7(m,t)+LM8(m,t)=1

TN7(m=1:NM,t=4:NT-2):P5(m,t)=0*LM9(m,t)+ILM1(t)*LM10(m,t)  &
                            +ILM2(t)*LM11(m,t)+ILM3(t)*LM12(m,t)
TF3(m=1:NM,t=4:NT-2):T3(m,t)=0*LM9(m,t)+TLM1(t)*LM10(m,t)  &
                            +TLM2(t)*LM11(m,t)+TLM3(t)*LM12(m,t)
LSM3(m=1:NM,t=4:NT-2):LM9(m,t)+LM10(m,t)+LM11(m,t)+LM12(m,t)=1

TN8(m=1:NM,t=4:NT-3):P6(m,t)=0*LM13(m,t)+ILM1(t)*LM14(m,t)  &
                            +ILM2(t)*LM15(m,t)+ILM3(t)*LM16(m,t)
TF4(m=1:NM,t=4:NT-3):T4(m,t)=0*LM13(m,t)+TLM1(t)*LM14(m,t)  &
                            +TLM2(t)*LM15(m,t)+TLM3(t)*LM16(m,t)
LSM4(m=1:NM,t=4:NT-3):LM13(m,t)+LM14(m,t)+LM15(m,t)+LM16(m,t)=1

!TAX REBATE
TREB(m=1:NM,t=7:NT):TR(m,t)=T1(m,t-1)-T2(m,t-1)+T2(m,t-2)-T3(m,t-2)  &
                    +T3(m,t-3)-T4(m,t-3)-SL1(m,t-1)-SL2(m,t-2)    &
                    -SL3(m,t-3)+SDB1(m,t)-SDB7(m,t)
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IF NM>1
!DIVIDENDS PAID OUTSIDE AND WITHIN GROUP

DVP2(m=2:NM,t=7:NT):DIV(m,t)=((100-PCENT)/100)*NDIV(t)
DVP3(m=2:NM,t=7:NT):EDIV(m,t)=(PCENT/100)*GRDIV(t)*NDIV(t)
ENDIF

!DETERMINATION OF ACT PAYMENTS/SETOFF CAPACITY/SURPLUS ACT

IF NM>1.AND.GELEC=0
GDP1(m=1:1,t=7:NT):ADIV(t)-SFII(t)=GRDIV(t)*DIV(m,t)-EDIV(m+1,t) &
                                                            -SFII(t-1)
GDP2(m=1:1,t=7:NT):ADIV(t)<M*d23(t)
GDP3(m=1:1,t=7:NT):SFII(t)<M-M*d23(t)

ACTP(m=1:1,t=7:NT):A1(m,t)=BASR(t)*ADIV(t)
ACTS(m=2:NM,t=7:NT):A1(m,t)=ACTR(t)*NDIV(m,t)

ELSE
ACTP(m=1:NM,t=7:NT):A1(m,t)=ACTR(t)*DIV(m,t)

ENDIF

IF NM>1
ACTA(m=1:1,t=7:NT):A2(m,t)=ADm(t)-SD(m,t)
ACTG(m=2:2,t=7:NT):A2(m,t)=ADg(t)-ADs(t)+ACTR(t)*DIV(m,t)-SD(m,t)

ELSE
ACTA(m=1:1,t=7:NT):A2(m,t)=A1(m,t)-SD(m,t)
ENDIF

IF NM>1
GSA(t=7:NT):BASR(t)*ADIV(t)=ADm(t)+ADg(t)

GA1(m=2:2,t=7:NT):Ug(m,t)-ADs(t)=BASR(t)*P3a(m,t)-ADg(t)
GA2(m=2:2,t=7:NT):Ug(m,t)<M*d22(m,t)
GA3(m=2:2,t=7:NT):ADs(t)<M-M*d22(m,t)
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GA4(m=2:2,t=7:NT):U1(m,t)-SD(m,t)=Ug(m,t)-ACTR(t)*DIV(m,t)

SA1(m=1:1,t=7:NT):U1(m,t)-SD(m,t)=BASR(t)*P3a(m,t)-ADm(t)
ELSE
SA1(m=1:NM,t=7:NT):U1(m,t)-SD(m,t)=BASR(t)*P3(m,t)-ACTR(t)*DIV(m,t)

ENDIF
SA2(m=1:NM,t=7:NT):U1(m,t)<M*d8(m,t)
SA3(m=1:NM,t=7:NT):SD(m,t)<M-M*d8(m,t)

SAD(m=1:NM,t=7:NT):SD(m,t)=SDB1(m,t)+SDF(m,t)

SA4(m=1:NM,t=7:NT):U2(m,t)-SFF(m,t)=U1(m,t)-SF(m,t-1)
SA5(m=1:NM,t=7:NT):U2(m,t)<M*d9(m,t)
SA6(m=1:NM,t=7:NT):SFF(m,t)<M-M*d9(m,t)


IF NM>1
SA7(m=1:NM,t=6:NT-1):U3(m,t)-SL1(m,t)=BASR(t)*P4(m,t)&
                                        -BASR(t)*P3a(m,t)+U2(m,t)
ELSE
SA7(m=1:NM,t=6:NT-1):U3(m,t)-SL1(m,t)=BASR(t)*P4(m,t) &
                                        -BASR(t)*P3(m,t)+U2(m,t)
ENDIF
SA8(m=1:NM,t=6:NT-1):U3(m,t)<M*d10(m,t)
SA9(m=1:NM,t=6:NT-1):SL1(m,t)<M-M*d10(m,t)

SA10(m=1:NM,t=6:NT-1):U3A(m,t)-SDB2(m,t+1)=U3(m,t)-SDB1(m,t+1)
SA11(m=1:NM,t=6:NT-1):U3A(m,t)<M*d11(m,t)
SA12(m=1:NM,t=6:NT-1):SDB2(m,t+1)<M-M*d11(m,t)

SA13(m=1:NM,t=5:NT-2):U4(m,t)-SL2(m,t)=BASR(t)*P5(m,t) &
                                        -BASR(t)*P4(m,t)+U3A(m,t)
SA14(m=1:NM,t=5:NT-2):U4(m,t)<M*d12(m,t)
SA15(m=1:NM,t=5:NT-2):SL2(m,t)<M-M*d12(m,t)

SA16(m=1:NM,t=5:NT-2):U4A(m,t)-SDB3(m,t+2)=U4(m,t)-SDB2(m,t+2)
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SA17(m=1:NM,t=5:NT-2):U4A(m,t)<M*d13(m,t)
SA18(m=1:NM,t=5:NT-2):SDB3(m,t+2)<M-M*d13(m,t)

SA19(m=1:NM,t=4:NT-3):U5(m,t)-SL3(m,t)=BASR(t)*P6(m,t)   &
                              -BASR(t)*P5(m,t)+U4A(m,t)
SA20(m=1:NM,t=4:NT-3):U5(m,t)<M*d14(m,t)
SA21(m=1:NM,t=4:NT-3):SL3(m,t)<M-M*d14(m,t)

SA22(m=1:NM,t=4:NT-3):U5A(m,t)-SDB4(m,t+3)=U5(m,t)-SDB3(m,t+3)
SA23(m=1:NM,t=4:NT-3):U5A(m,t)<M*d15(m,t)
SA24(m=1:NM,t=4:NT-3):SDB4(m,t+3)<M-M*d15(m,t)

SA25(m=1:NM,t=3:NT-4):U6(m,t)-SDB5(m,t+4)=U5A(m,t)-SDB4(m,t+4)
SA26(m=1:NM,t=3:NT-4):U6(m,t)<M*d16(m,t)
SA27(m=1:NM,t=3:NT-4):SDB5(m,t+4)<M-M*d16(m,t)

SA28(m=1:NM,t=2:NT-5):U7(m,t)-SDB6(m,t+5)=U6(m,t)-SDB5(m,t+5)
SA29(m=1:NM,t=2:NT-5):U7(m,t)<M*d17(m,t)
SA30(m=1:NM,t=2:NT-5):SDB6(m,t+5)<M-M*d17(m,t)

SA31(m=1:NM,t=1:NT-6):U8(m,t)-SDB7(m,t+6)=U7(m,t)-SDB6(m,t+6)
SA32(m=1:NM,t=1:NT-6):U8(m,t)<M*d18(m,t)
SA33(m=1:NM,t=1:NT-6):SDB7(m,t+6)<M-M*d18(m,t)

TSF(m=1:1,t=7:NT):SF(m,t)=SFF(m,t)+SDB7(m,t)+SDF(m,t)+SL1(m,t-1)   &
                  +SL2(m,t-2)+SL3(m,t-3)
IF NM>1
TSFG(m=2:2,t=7:NT):SF(m,t)=SFF(m,t)+SDB7(m,t)+SDF(m,t)+SL1(m,t-1)   &
                  +SL2(m,t-2)+SL3(m,t-3)+ADs(t)
ENDIF

!FUNDS BALANCE

IF NM>1
IF GELEC=0
CLB1(m=1:1,t=7:NT):CSHP(m,t)-CSHN(m,t)=(1+iL(t))*LNS(m,t)  &
   +EXPRINC(m,t)-EXPRLSS(m,t)+SUM(p=1:NPROJ)(PROJINC(p,t)   &
   -PROJRENT(p,t)+DISP(p,t))*accept(p)+(PCENT/100)*NDIV(t)&
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    +IN1(m,t)-T1(m,t-1)+A2(m,t-1)+TR(m,t)-DBIN(t)-LREP(t)&
    -(1+iD(m,t))*DBS(m,t)

ELSE
CLB1(m=1:1,t=7:NT):CSHP(m,t)-CSHN(m,t)=(1+iL(t))*LNS(m,t)  &
    +EXPRINC(m,t)-EXPRLSS(m,t)+SUM(p=1:NPROJ)(PROJINC(p,t)   &
    -PROJRENT(p,t)+DISP(p,t))*accept(p)+EDIV(m+1,t)&
    +IN1(m,t)-T1(m,t-1)+A2(m,t-1)+TR(m,t)-DBIN(t)-LREP(t)&
    -(1+iD(m,t))*DBS(m,t)

ENDIF

CLBG(m=2:NM,t=7:NT):CSHP(m,t)-CSHN(m,t)=(1+iL(t))*LNS(m,t)  &
      +EXPRINC(m,t)-EXPRLSS(m,t)+IN1(m,t)-T1(m,t-1)+A2(m,t-1)   &
      +TR(m,t)-DINT(t)-(1+iD(m,t))*DBS(m,t)

ELSE

CLB1(m=1:NM,t=7:NT):CSHP(m,t)-CSHN(m,t)=(1+iL(t))*LNS(m,t)   &
               +EXPRINC(m,t)-EXPRLSS(m,t)+SUM(p=1:NPROJ)(PROJINC(p,t) &
               -PROJRENT(p,t)+DISP(p,t))*accept(p)    &
               +IN1(m,t)-T1(m,t-1)+A2(m,t-1)+TR(m,t)-DBIN(t)-LREP(t)   &
               -(1+iD(m,t))*DBS(m,t)

ENDIF

CLB2(m=1:NM,t=7:NT):CSHP(m,t)<M*d29(m,t)
CLB3(m=1:NM,t=7:NT):CSHN(m,t)<M-M*d29(m,t)

!SHORT TERM LENDING/BORROWING

OB1(m=1:1,t=7:7):LNS(m,t)-DBS(m,t)=OCB(m,t)-ISCD(t)*DBI(t)     &
      -ISCE(t)*EQI(t)-CSHMIN(m,t)

OB2(m=1:1,t=8:NT):LNS(m,t)-DBS(m,t)=RET(m,t-1)+CSHMIN(m,t-1)  &
                  -CSHN(m,t-1)-ISCD(t)*DBI(t)-ISCE(t)*EQI(t)-CSHMIN(m,t)

OBX(m=1:1,t=7:7):OCB(m,t)>CSHMIN(m,t)
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IF NM>1
OBG1(m=2:NM,t=7:7):LNS(m,t)-DBS(m,t)=OCB(m,t)-CSHMIN(m,t)
OBG2(m=2:NM,t=8:NT):LNS(m,t)-DBS(m,t)=RET(m,t-1)+CSHMIN(m,t-1)   &
                                        -CSHN(m,t-1)-CSHMIN(m,t)
ENDIF

OB3(m=1:NM,t=7:NT):LNS(m,t)<M*d24(m,t)
OB4(m=1:NM,t=7:NT):DBS(m,t)<M-M*d24(m,t)

MXDS(m=1:NM,t=7:NT):DBS(m,t)<DBSMAX(m,t)

!USES OF FUNDS

DST1(m=1:1,t=7:NT-1):CSHP(m,t)=DIV(m,t)+A1(m,t)+RET(m,t)+CSHC(t+1)

DST2(m=1:1,t=NT:NT):CSHP(m,t)=DIV(m,t)+A1(m,t)+RET(m,t)

IF NM>1
DSTG(m=2:NM,t=7:NT):CSHP(m,t)=DIV(m,t)+A1(m,t)+EDIV(m,t)+RET(m,t)

ENDIF

FUND(t=7:NT-1):SUM(p=1:NPROJ|PROJRENT(p,t)=0)PROJCOST(p,t)   &
                               *accept(p)=CSHC(t)+DBI(t)+EQI(t)

HZN1(t=NT:NT):CSHC(t)+DBI(t)+EQI(t)=0

!BOOK VALUES OF DEBT AND EQUITY

DBT1(t=7:NT):DB(t)=DB(t-1)+DBI(t)+DBC(t)-LREP(t)
EQT(m=1:1,t=7:NT):EQ(m,t)=EQ(m,t-1)+EQI(t)+RTN(m,t)-RTD(m,t)

IF NM>1
EQTG(m=2:NM,t=7:NT):EQ(m,t)=EQ(m,t-1)+RTN(m,t)-RTD(m,t)
ENDIF
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!INCREASE IN RETENTIONS
CGN1(m=1:NM,t=7:7):RTN(m,t)=RET(m,t)
CGN2(m=1:NM,t=8:NT):RTN(m,t)-RTD(m,t)=RET(m,t)-RET(m,t-1)
CGN3(m=1:NM,t=8:NT):RTN(m,t)<M*d30(m,t)
CGN4(m=1:NM,t=8:NT):RTD(m,t)<M-M*d30(m,t)

!MINIMUM EQUITY ISSUE
ISSU(t=7:NT):EQI(t)=EQIMIN*issue(t)

!CAPITAL STRUCTURE CONSTRAINTS

UDL1(m=1:1,t=7:NT):DB(t-1)+DBI(t)+DBC(t)<DLMT(5,t)*EQ(m,t-1) &
                                        +DLMT(5,t)*EQI(t)

UDL5(m=1:1,t=7:NT):DB(t-1)+DBI(t)+DBC(t)>(DBMIN(t)/(1-DBMIN(t))) &
                         *EQ(m,t-1)+(DBMIN(t)/(1-DBMIN(t)))*EQI(t)

LDL1(m=1:1,t=7:NT):DB(t-1)+DBI(t)+DBC(t)<(DBMAX(t)/(1-DBMAX(t))) &
                         *EQ(m,t-1)+(DBMAX(t)/(1-DBMAX(t)))*EQI(t)

!DEBT INTEREST BANDS
DF1(m=1:1,t=7:NT):D2(t)-x1(t)=DB(t-1)+DBI(t)+DBC(t) &
                           -DLMT(1,t)*EQ(m,t-1)-DLMT(1,t)*EQI(t)

DF2(t=7:NT):D2(t)<M*d25(t)
DF3(t=7:NT):x1(t)<M-M*d25(t)

DF4(m=1:1,t=7:NT):D3(t)-x2(t)=D2(t)+(DLMT(1,t)-DLMT(2,t))*EQ(m,t-1) &
                                  +(DLMT(1,t)-DLMT(2,t))*EQI(t)

DF5(t=7:NT):D3(t)<M*d26(t)
DF6(t=7:NT):x2(t)<M-M*d26(t)

DF7(m=1:1,t=7:NT):D4(t)-x3(t)=D3(t)+(DLMT(2,t)-DLMT(3,t))*EQ(m,t-1) &
                                  +(DLMT(2,t)-DLMT(3,t))*EQI(t)
DF8(t=7:NT):D4(t)<M*d27(t)
DF9(t=7:NT):x3(t)<M-M*d27(t)
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DF10(m=1:1,t=7:NT):D5(t)-x4(t)=D4(t)+(DLMT(3,t)-DLMT(4,t))*EQ(m,t-1)&
                                        +(DLMT(3,t)-DLMT(4,t))*EQI(t)

DF11(t=7:NT):D5(t)<M*d28(t)
DF12(t=7:NT):x4(t)<M-M*d28(t)

DF13(t=7:NT):DB(t-1)+LDB(t)+DBI(t)+DBC(t)-D2(t)=y1(t)
DF14(t=7:NT):D2(t)-D3(t)=y2(t)
DF15(t=7:NT):D3(t)-D4(t)=y3(t)
DF16(t=7:NT):D4(t)-D5(t)=y4(t)

DBNT(t=7:NT):DBIN(t)=IDLMT(1,t)*y1(t)+IDLMT(2,t)*y2(t) &
                 +IDLMT(3,t)*y3(t)+IDLMT(4,t)*y4(t)+IDLMT(5,t)*D5(t)

!GROSS DIVIDEND
GRDV(m=1:1,t=7:NT):DIVG(t)=GRDIV(t)*DIV(m,t)

!MINIMUM GROWTH IN DIVIDENDS
DMIN(m=1:1,t=7:NT):DIV(m,t)>(1+g(t))*DIV(m,t-1)

!FUTURE DIVIDEND STREAM

IF NM>1
GDST(m=1:1,t=NT:NT):HVAL(t)>GFAC2(1)*DIV(m,t)
ELSE
DST3(m=1:1,t=NT:NT):HV(m,t)>GFAC2(1)*DIV(m,t)
ENDIF

!HORIZON VALUATION

IF NM>1
HRZ1(m=1:1,t=NT:NT):HV(m,t)=RET(m,t)+EXBVAL2(t)+SUM(p=1:NPROJ) &
        DISP(p,t)*accept(p)-CSHN(m,t)-DB(t)&
        +DFTERM(1)*T1(m,t)+DFTERM(1)*A2(m,t)+DFTERM(1)*FULL(t) &
        *LF(m,t)+DFTERM(1)*BASR(t)*SFII(t)+DFTERM(1)*SF(m,t)
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HRZ2(m=2:NM,t=NT:NT):HV(m,t)=RET(m,t)+SBVAL(m,t)-CSHN(m,t) &
       -SDBT(m,t)+DFTERM(1)*T1(m,t)
       +DFTERM(1)*A2(m,t)+DFTERM(1)*SF(m,t)+DFTERM(1)*FULL(t)*LF(m,t)

ELSE
HRZ1(m=1:1,t=NT:NT):HV(m,t)=RET(m,t)+EXBVAL2(t)+SUM(p=1:NPROJ) &
        DISP(p,t)*accept(p)-CSHN(m,t)-DB(t) &
        +DFTERM(1)*T1(m,t)+DFTERM(1)*A2(m,t)+DFTERM(1)*SF(m,t) &
        +DFTERM(1)*FULL(t)*LF(m,t)

ENDIF

IF NM>1
MVA1(m=1:1,t=NT:NT):HVAL(t)=HV(m,t)+(PCENT/100)*HV(m+1,t)

!OBJECTIVE FUNCTION

MAX:SUM(t=7:NT)DFACT(t)*DIVG(t)-SUM(t=7:NT)DFACT(t-1) &
                              *EQI(t)+SUM(t=NT:NT)DFACT(t)*HVAL(t) $
ELSE

MAX:SUM(t=7:NT)DFACT(t)*DIVG(t)-SUM(t=7:NT)DFACT(t-1) &
                             *EQI(t)+SUM(m=1:1,t=NT:NT)DFACT(t)*HV(m,t)     $
ENDIF


!LOGICAL STRUCTURE OF MODEL

LS1(m=1:NM,t=7:NT):d2(m,t)<d1(m,t)

LS2(m=1:NM,t=7:NT-1):d4(m,t)<d3(m,t)
LS3(m=1:NM,t=6:NT-2):d5(m,t)<d4(m,t)
LS4(m=1:NM,t=5:NT-3):d6(m,t)<d5(m,t)

LS5(m=1:NM,t=7:NT):d9(m,t)<d8(m,t)
LS6(m=1:NM,t=7:NT-1):d10(m,t)<d9(m,t)
LS7(m=1:NM,t=6:NT-1):d11(m,t)<d10(m,t)
LS8(m=1:NM,t=6:NT-2):d12(m,t)<d11(m,t)
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LS9(m=1:NM,t=5:NT-2):d13(m,t)<d12(m,t)
LS10(m=1:NM,t=5:NT-3):d14(m,t)<d13(m,t)
LS11(m=1:NM,t=4:NT-3):d15(m,t)<d14(m,t)
LS12(m=1:NM,t=4:NT-4):d16(m,t)<d15(m,t)
LS13(m=1:NM,t=3:NT-5):d17(m,t)<d16(m,t)
LS14(m=1:NM,t=2:NT-6):d18(m,t)<d17(m,t)

LS15(m=1:NM,t=7:NT):d19(m,t)>d1(m,t)
LS16(m=1:NM,t=7:NT):d7(m,t)>d1(m,t)
LS17(m=1:NM,t=7:NT):d20(m,t)>d19(m,t)
LS18(m=1:NM,t=7:NT):d3(m,t)<d2(m,t)+d19(m,t)

IF NM>1
IF PCENT>=75
LSG1(m=1:NM,t=7:NT):d21(m,t)<d3(m,t)
ENDIF
LSG1(m=2:NM,t=7:NT):d8(m,t)<d22(m,t)
LSG3(m=2:NM,t=7:NT):d22(m,t)<d3(m,t)
ENDIF

LS19(m=1:1,t=7:NT):d8(m,t)<d3(m,t)

LS20(t=7:NT):d26(t)<d25(t)
LS21(t=7:NT):d27(t)<d26(t)
LS22(t=7:NT):d28(t)<d27(t)

!CONSTRAINTS TO LIMIT LP SOLUTION

LP1(m=1:1,t=7:NT):P1(m,t)<EXPRINC(m,t)+SUM(p=1:NPROJ)(PROJINC(p,t)   &
                                  +WDARATE*DISP(p,t)+WCAPI(p,t))*accept(p)

LP2(m=1:1,t=7:NT):L1(m,t)<EXPRLSS(m,t)+EXCAPALL(m,t)  &
                      +SUM(p=1:NPROJ)(CAPALL(p,t)+BAL(p,t)+WCAPD(p,t)  &
                      +FC(p,t)+DEPN(p,t))*accept(p)+ISCD(t)*DBI(t)  &
                      +iD(m,t)*DBS(m,t)

IF NM>1
LG11(m=2:NM,t=7:NT):P1(m,t)<EXPRINC(m,t)
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LG12(m=2:NM,t=7:NT):L1(m,t)<EXPRLSS(m,t)+EXCAPALL(m,t)
ENDIF

LP3(m=1:NM,t=7:NT):IN2(m,t)<IN1(m,t)+iL(t)*LNS(m,t)
LP4(m=1:NM,t=7:NT):P2(m,t)<P1(m,t)
LP5(m=1:NM,t=7:NT):P3(m,t)<P2(m,t)+IN2(m,t)

IF NM>1
LPG1(m=1:NM,t=7:NT):P3a(m,t)<P3(m,t)
LPG2(m=1:NM,t=6:NT-1):P4(m,t)<P3a(m,t)
ELSE
LP6(m=1:NM,t=6:NT-1):P4(m,t)<P3(m,t)
ENDIF

LP7(m=1:NM,t=5:NT-2):P5(m,t)<P4(m,t)
LP8(m=1:NM,t=4:NT-3):P6(m,t)<P5(m,t)
LP9(m=1:NM,t=7:NT):LB(m,t)<L1b(m,t)
LP10(m=1:NM,t=7:NT):L1b(m,t)<L1a(m,t)

IF NM>1.AND.PCENT>=75
LPG5(m=1:NM,t=7:NT):L1a(m,t)<L1m(m,t)
LPG6(m=1:NM,t=7:NT):LF1(m,t)<L1m(m,t)
LPG7(m=1:NM,t=7:NT):L1m(m,t)<L1(m,t)
ELSE
LP11(m=1:1,t=7:NT):L1a(m,t)<L1(m,t)
LP12(m=1:1,t=7:NT):LF1(m,t)<L1(m,t)
ENDIF

IF NM>1.AND.PCENT>=75
LPG3(m=1:NM,t=7:NT):AP(m,t)<P1(m,t)+IN1(m,t)+iL(t)*LNS(m,t)
LPG4(m=1:1,t=7:NT):x(m,t)<LF(m,t-1)+L1(m,t)+DBIN(t)
ENDIF

LP13(m=1:1,t=7:NT):XC(m,t)<DBIN(t)

LP14(m=1:NM,t=7:NT):L2(m,t)<LB(m,t)
LP15(m=1:NM,t=7:NT):L3(m,t)<L2(m,t)
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LP16(m=1:NM,t=7:NT):L4(m,t)<L3(m,t)
LP17(m=1:NM,t=7:NT):LFF(m,t)<LF(m,t-1)

IF NM>1
LPG8(m=2:NM,t=7:NT):Ug(m,t)<BASR(t)*P3a(m,t)
LPG9(m=2:NM,t=7:NT):U1(m,t)<Ug(m,t)
LG10(m=1:1,t=7:NT):U1(m,t)<BASR(t)*P3a(m,t)
LG13(m=1:1,t=7:NT):ADIV(t)<GRDIV(t)*DIV(m,t)
LG14(m=1:1,t=7:NT):SFII(t)<EDIV(m+1,t)+SFII(t-1)
ELSE
LP18(m=1:1,t=7:NT):U1(m,t)<BASR(t)*P3(m,t)
ENDIF

LP19(m=1:NM,t=7:NT):U2(m,t)<U1(m,t)
LP20(m=1:NM,t=6:NT-1):U3(m,t)<U2(m,t)
LP21(m=1:NM,t=6:NT-1):U3A(m,t)<U3(m,t)
LP22(m=1:NM,t=5:NT-2):U4(m,t)<U3A(m,t)
LP23(m=1:NM,t=5:NT-2):U4A(m,t)<U4(m,t)
LP24(m=1:NM,t=4:NT-3):U5(m,t)<U4A(m,t)
LP25(m=1:NM,t=4:NT-3):U5A(m,t)<U5(m,t)
LP26(m=1:NM,t=3:NT-4):U6(m,t)<U5A(m,t)
LP27(m=1:NM,t=2:NT-5):U7(m,t)<U6(m,t)
LP28(m=1:NM,t=1:NT-6):U8(m,t)<U7(m,t)

LP29(m=1:NM,t=7:NT):SDB2(m,t)<SDB1(m,t)
LP30(m=1:NM,t=7:NT):SDB3(m,t)<SDB2(m,t)
LP31(m=1:NM,t=7:NT):SDB4(m,t)<SDB3(m,t)
LP32(m=1:NM,t=7:NT):SDB5(m,t)<SDB4(m,t)
LP33(m=1:NM,t=7:NT):SDB6(m,t)<SDB5(m,t)
LP34(m=1:NM,t=7:NT):SDB7(m,t)<SDB6(m,t)
LP35(m=1:NM,t=7:NT):SFF(m,t)<SF(m,t-1)

IF NM>1

LP36(m=1:1,t=7:NT):CSHP(m,t)<(1+iL(t))*LNS(m,t)+EXPRINC(m,t)   &
        +SUM(p=1:NPROJ)(PROJINC(p,t)+DISP(p,t))*accept(p)+EDIV(m+1,t)  &
        +IN1(m,t)+A2(m,t-1)+TR(m,t)
```

```
LP37(m=1:1,t=7:NT):CSHN(m,t)<EXPRLSS(m,t)+T1(m,t-1)+DBIN(t)+LREP(t)  &
        +SUM(p=1:NPROJ)PROJRENT(p,t)*accept(p)+(1+iD(m,t))*DBS(m,t)  &
        +CSHMIN(m,t)

LG15(m=2:NM,t=7:NT):CSHP(m,t)<(1+iL(t))*LNS(m,t)+EXPRINC(m,t)    &
                                        +A2(m,t-1)+IN1(m,t)+TR(m,t)

LG16(m=2:NM,t=7:NT):CSHN(m,t)<EXPRLSS(m,t)+T1(m,t-1)+DINT(t)  &
                                   +(1+iD(m,t))*DBS(m,t)+CSHMIN(m,t)

ELSE

LP36(m=1:NM,t=7:NT):CSHP(m,t)<(1+iL(t))*LNS(m,t)+EXPRINC(m,t)    &
                 +SUM(p=1:NPROJ)(PROJINC(p,t)+DISP(p,t))*accept(p)  &
                 +A2(m,t-1)+IN1(m,t)+TR(m,t)

LP37(m=1:NM,t=7:NT):CSHN(m,t)<EXPRLSS(m,t)+T1(m,t-1)+DBIN(t)+LREP(t)&
        +SUM(p=1:NPROJ)PROJRENT(p,t)*accept(p)+(1+iD(m,t))*DBS(m,t)  &
        +CSHMIN(m,t)

ENDIF

LP38(m=1:1,t=7:7):LNS(m,t)<OCB(m,t)

LP39(m=1:1,t=7:7):DBS(m,t)<ISCD(t)*DBI(t)+ISCE(t)*EQI(t)+CSHMIN(m,t)

LP40(m=1:1,t=8:NT):LNS(m,t)<RET(m,t-1)+CSHMIN(m,t-1)

LP41(m=1:1,t=8:NT):DBS(m,t)<CSHN(m,t-1)+ISCD(t)*DBI(t)  &
                                        +ISCE(t)*EQI(t)+CSHMIN(m,t)

IF NM>1
LP42(m=2:NM,t=7:7):LNS(m,t)<OCB(m,t)
LP43(m=2:NM,t=7:7):DBS(m,t)<CSHMIN(m,t)

LP44(m=2:NM,t=8:NT):LNS(m,t)<RET(m,t-1)+CSHMIN(m,t-1)
LP45(m=2:NM,t=8:NT):DBS(m,t)<CSHN(m,t-1)+CSHMIN(m,t)
ENDIF
```

```
SETS
!SPECIAL ORDERED SET TYPE 2
S(m=1:NM,t=6:NT):LM1(m,t)+LM2(m,t)+LM3(m,t)+LM4(m,t)  .S2. TN5(m,t)
Sa(m=1:NM,t=5:NT-1):LM5(m,t)+LM6(m,t)+LM7(m,t)+LM8(m,t)  .S2. TN6(m,t)
Sb(m=1:NM,t=4:NT-1):LM9(m,t)+LM10(m,t)+LM11(m,t) &
                                              +LM12(m,t)  . S2.TN7(m,t)
Sc(m=1:NM,t=4:NT-3):LM13(m,t)+LM14(m,t)+LM15(m,t) &
                                              +LM16(m,t)  . S2.TN8(m,t)

BOUNDS
accept(p=1:NPROJ)          .BV.
d1(m=1:NM,t=7:NT)          .BV.
d2(m=1:NM,t=7:NT)          .BV.
d3(m=1:NM,t=7:NT)          .BV.
d4(m=1:NM,t=6:NT-1)        .BV.
d5(m=1:NM,t=5:NT-2)        .BV.
d6(m=1:NM,t=4:NT-3)        .BV.
d7(m=1:NM,t=7:NT)          .BV.
d8(m=1:NM,t=7:NT)          .BV.
d9(m=1:NM,t=7:NT)          .BV.
d10(m=1:NM,t=6:NT-1)       .BV.
d11(m=1:NM,t=6:NT-1)       .BV.
d12(m=1:NM,t=5:NT-2)       .BV.
d13(m=1:NM,t=5:NT-2)       .BV.
d14(m=1:NM,t=4:NT-3)       .BV.
d15(m=1:NM,t=4:NT-3)       .BV.
d16(m=1:NM,t=3:NT-4)       .BV.
d17(m=1:NM,t=2:NT-5)       .BV.
d18(m=1:NM,t=1:NT-6)       .BV.
d19(m=1:NM,t=7:NT)         .BV.
d20(m=1:NM,t=7:NT)         .BV.
IF NM>1
IF PCENT>=75
d21(m=1:NM,t=7:NT)         .BV.
ENDIF
d22(m=2:NM,t=7:NT)         .BV.
d23(t=7:NT)                .BV.
```

```
ENDIF
issue(t=7:NT)       .SC.    (EQIMAX/EQIMIN)
d24(m=1:NM,t=7:NT)              .BV.
d25(t=7:NT)                     .BV.
d26(t=7:NT)                     .BV.
d27(t=7:NT)                     .BV.
d28(t=7:NT)                     .BV.
d29(m=1:NM,t=7:NT)              .BV.
d30(m=1:NM,t=8:NT)              .BV.
IF NM>1.AND.PCENT>75
d31(m=1:NM,t=7:NT)              .BV.
ENDIF

DIRECTIVES
accept(p=1:NPROJ)           .PR.     10
d1(m=1:NM,t=7:NT)           .PR.     50
d2(m=1:NM,t=7:NT)           .PR.     100

d4(m=1:NM,t=6:NT-1)         .PR.     200
d5(m=1:NM,t=5:NT-2)         .PR.     300
d6(m=1:NM,t=4:NT-3)         .PR.     400

d7(m=1:NM,t=7:NT)           .PR.     150
d19(m=1:NM,t=7:NT)          .PR.     100
d20(m=1:NM,t=7:NT)          .PR.     150

IF NM>1
IF PCENT>=75
d21(m=1:NM,t=7:NT)          .PR.     50
ENDIF
d22(m=2:NM,t=7:NT)          .PR.     50
ENDIF

d8(m=1:NM,t=7:NT)           .PR.     300

GENERATE
```

Appendix 8.1

Sample XPRESS-MP printout

This solution output corresponds to the data presented in Table 8.26a (Outcome X of MDL 11).

```
Problem Statistics
Matrix NEDP
Objective MAX
RHS RHS00001
Problem has     896 rows and    663 structural columns

Solution Statistics
Maximisation performed
Optimal solution found after      1 iterations
Objective function value is     7.753795
```

Rows Section

Number		Row	At	Value	Slack Value	Dual Value	RHS
E	1	DA1 0106	EQ	.000000	.000000	.184473	.000000
E	2	DA2 0106	EQ	.287500	.000000	-.268540	.287500
E	3	DA3 0105	EQ	.287500	.000000	.000000	.287500
E	4	DA4 0104	EQ	.287500	.000000	.000000	.287500
E	5	DA5 0106	EQ	.067500	.000000	.431618	.067500
E	6	DA6 0106	EQ	.000000	.000000	.642542	.000000
E	7	DA7 0105	EQ	.057500	.000000	.000000	.057500
E	8	DA8 0104	EQ	.057500	.000000	.000000	.057500
E	9	DA9 0103	EQ	.057500	.000000	.000000	.057500
E	10	DA100102	EQ	.057500	.000000	.000000	.057500
E	11	DA110101	EQ	.057500	.000000	.000000	.057500
E	12	DA120107	EQ	.050000	.000000	1.167573	.050000
E	13	DA130106	EQ	2.250000	.000000	-.946849	2.250000
E	14	DA140106	EQ	8.920000	.000000	.055951	8.920000
E	15	DA150106	EQ	.500000	.000000	.000000	.500000
E	16	DA160106	EQ	.057500	.000000	1.074160	.057500
E	17	DA170106	EQ	.000000	.000000	.000000	.000000
E	18	TXN10107	EQ	-.200000	.000000	-.140032	-.200000
E	19	TXN10108	EQ	-.100000	.000000	-.184473	-.100000
E	20	TXN10109	EQ	-.025000	.000000	-.184473	-.025000
E	21	TXN10110	EQ	.031250	.000000	-.173748	.031250
E	22	TXN10111	EQ	.073440	.000000	-.142434	.073440
L	23	TXN30107	UL	.000000	.000000	.012249	.000000
L	24	TXN30108	BS	-2.287500	2.287500	.000000	.000000
L	25	TXN30109	BS	-2.065620	2.065620	.000000	.000000
L	26	TXN30110	BS	-2.124220	2.124220	.000000	.000000
L	27	TXN30111	BS	.000000	.000000	.000000	.000000
L	28	TXN40107	BS	.361765	2.138235	.000000	2.500000
L	29	TXN40108	BS	2.500000	.000000	.000000	2.500000
L	30	TXN40109	UL	2.500000	.000000	.000000	2.500000
L	31	TXN40110	UL	2.500000	.000000	.010725	2.500000
L	32	TXN40111	BS	.092970	2.407030	.000000	2.500000
E	33	OFS10107	EQ	.000000	.000000	.140032	.000000
E	34	OFS10108	EQ	.000000	.000000	.184473	.000000
E	35	OFS10109	EQ	.000000	.000000	.184473	.000000
E	36	OFS10110	EQ	.000000	.000000	.184473	.000000
E	37	OFS10111	EQ	.000000	.000000	.142434	.000000
L	38	OFS20107	BS	-2.138235	2.138235	.000000	.000000
L	39	OFS20108	BS	-2.500000	2.500000	.000000	.000000
L	40	OFS20109	BS	-2.500000	2.500000	.000000	.000000
L	41	OFS20110	BS	-2.500000	2.500000	.000000	.000000
L	42	OFS20111	UL	.000000	.000000	.000000	.000000
L	43	OFS30107	UL	2.500000	.000000	.000000	.000000
L	44	OFS30108	BS	2.500000	.000000	.044441	2.500000
L	45	OFS30109	BS	2.500000	.000000	.000000	2.500000

L	46	OFS30110	BS	2.500000	.000000	.000000	2.500000
L	47	OFS30111	BS	.092970	2.407030	.000000	2.500000
E	48	OFS40107	EQ	.200000	.000000	-.140032	.200000
E	49	OFS40108	EQ	.200000	.000000	-.184473	.200000
E	50	OFS40109	EQ	.200000	.000000	-.184473	.200000
E	51	OFS40110	EQ	.200000	.000000	-.184473	.200000
E	52	OFS40111	EQ	.200000	.000000	-.142434	.200000
L	53	OFS50107	UL	.000000	.000000	.012249	.000000
L	54	OFS50108	BS	-2.300000	2.300000	.000000	.000000
L	55	OFS50109	BS	-2.300000	2.300000	.000000	.000000
L	56	OFS50110	BS	-2.300000	2.300000	.000000	.000000
L	57	OFS50111	BS	-2.300000	2.300000	.000000	.000000
L	58	OFS60107	BS	.161765	2.338235	.000000	2.500000
L	59	OFS60108	BS	2.500000	.000000	.000000	2.500000
L	60	OFS60109	BS	2.500000	.000000	.000000	2.500000
L	61	OFS60110	BS	2.500000	.000000	.000000	2.500000
L	62	OFS60111	UL	2.500000	.000000	.031314	2.500000
E	63	CBF40107	EQ	.000000	.000000	.140032	.000000
E	64	CBF40108	EQ	.000000	.000000	.184473	.000000
E	65	CBF40109	EQ	.000000	.000000	.184473	.000000
E	66	CBF40110	EQ	.000000	.000000	.184473	.000000
E	67	CBF40111	EQ	.000000	.000000	.142434	.000000
L	68	CBF50107	BS	-2.338235	2.338235	.000000	.000000
L	69	CBF50108	BS	-2.500000	2.500000	.000000	.000000
L	70	CBF50109	BS	-2.500000	2.500000	.000000	.000000
L	71	CBF50110	BS	-2.500000	2.500000	.000000	.000000
L	72	CBF50111	BS	-2.500000	2.500000	.000000	.000000
L	73	CBF60107	UL	2.500000	.000000	.044441	2.500000
L	74	CBF60108	BS	2.500000	.000000	.000000	2.500000
L	75	CBF60109	BS	2.500000	.000000	.000000	2.500000
L	76	CBF60110	BS	2.500000	.000000	.000000	2.500000
L	77	CBF60111	BS	2.500000	.000000	.000000	2.500000
E	78	LSS10107	EQ	.000000	.000000	-.127782	.000000
E	79	LSS10108	EQ	.000000	.000000	-.184473	.000000
E	80	LSS10109	EQ	.000000	.000000	-.184473	.000000
E	81	LSS10110	EQ	.000000	.000000	-.173748	.000000
E	82	LSS10111	EQ	.000000	.000000	-.142434	.000000
L	83	LSS20107	UL	.000000	.000000	.000000	.000000
L	84	LSS20108	BS	.000000	.000000	.000000	.000000
L	85	LSS20109	BS	-2.122426	2.122426	.000000	.000000
L	86	LSS20110	BS	-2.124220	2.124220	.000000	.000000
L	87	LSS20111	UL	.000000	.000000	.000000	.000000
L	88	LSS30107	BS	.000000	2.500000	.000000	2.500000
L	89	LSS30108	BS	.022153	2.477847	.000000	2.500000
L	90	LSS30109	UL	2.500000	.000000	.000000	2.500000
L	91	LSS30110	BS	2.500000	.000000	.000000	2.500000
L	92	LSS30111	BS	.000000	2.500000	.000000	2.500000
E	93	DB1 0107	EQ	.000000	.000000	-.184473	.000000
E	94	DB1 0108	EQ	.000000	.000000	-.184473	.000000
E	95	DB1 0109	EQ	.000000	.000000	-.184473	.000000
E	96	DB1 0110	EQ	.000000	.000000	-.173748	.000000
E	97	DB1 0111	EQ	.000000	.000000	-.142434	.000000
L	98	DB2 0107	UL	.000000	.000000	.000000	.000000
L	99	DB2 0108	UL	.000000	.000000	.000000	.000000
L	100	DB2 0109	BS	-2.157079	2.157079	.000000	.000000
L	101	DB2 0110	BS	-2.158873	2.158873	.000000	.000000
L	102	DB2 0111	UL	.000000	.000000	.034530	.000000
L	103	DB3 0107	BS	.234653	2.265347	.000000	2.500000
L	104	DB3 0108	BS	.034653	2.465347	.000000	2.500000
L	105	DB3 0109	UL	2.500000	.000000	.000000	2.500000
L	106	DB3 0110	UL	2.500000	.000000	.000000	2.500000
L	107	DB3 0111	BS	.034653	2.465347	.000000	2.500000
E	108	LCB10106	EQ	.000000	.000000	-.140032	.000000
E	109	LCB10107	EQ	.000000	.000000	-.184473	.000000
E	110	LCB10108	EQ	.000000	.000000	-.184473	.000000
E	111	LCB10109	EQ	.000000	.000000	-.184473	.000000
E	112	LCB10110	EQ	.000000	.000000	-.173748	.000000
L	113	LCB20106	BS	-2.374265	2.374265	.000000	.000000
L	114	LCB20107	UL	.000000	.000000	.000000	.000000
L	115	LCB20108	UL	.000000	.000000	.061380	.000000
L	116	LCB20109	BS	-2.157079	2.157079	.000000	.000000
L	117	LCB20110	BS	-2.158873	2.158873	.000000	.000000

L	118	LCB30106	UL	2.500000	.000000	.128508	2.500000
L	119	LCB30107	BS	.000000	2.500000	.000000	2.500000
L	120	LCB30108	BS	.000000	2.500000	.000000	2.500000
L	121	LCB30109	BS	2.500000	.000000	.000000	2.500000
L	122	LCB30110	BS	2.500000	.000000	.000000	2.500000
E	123	LCB40105	EQ	.000000	.000000	-.268540	.000000
E	124	LCB40106	EQ	.000000	.000000	-.184473	.000000
E	125	LCB40107	EQ	.000000	.000000	-.184473	.000000
E	126	LCB40108	EQ	.000000	.000000	-.142434	.000000
E	127	LCB40109	EQ	.000000	.000000	-.173748	.000000
L	128	LCB50105	BS	-2.212500	2.212500	.000000	.000000
L	129	LCB50106	BS	-2.374265	2.374265	.000000	.000000
L	130	LCB50107	UL	.000000	.000000	.000000	.000000
L	131	LCB50108	UL	.000000	.000000	.000000	.000000
L	132	LCB50109	BS	-2.157079	2.157079	.000000	.000000
L	133	LCB60105	BS	2.500000	.000000	.000000	2.500000
L	134	LCB60106	BS	2.500000	.000000	.000000	2.500000
L	135	LCB60107	BS	.000000	2.500000	.000000	2.500000
L	136	LCB60108	BS	.000000	2.500000	.000000	2.500000
L	137	LCB60109	BS	2.500000	.000000	.000000	2.500000
E	138	LCB70104	EQ	.000000	.000000	-.268540	.000000
E	139	LCB70105	EQ	.000000	.000000	-.184473	.000000
E	140	LCB70106	EQ	.000000	.000000	-.184473	.000000
E	141	LCB70107	EQ	.000000	.000000	-.119040	.000000
E	142	LCB70108	EQ	.000000	.000000	-.115465	.000000
L	143	LCB80104	BS	-2.212500	2.212500	.000000	.000000
L	144	LCB80105	BS	-2.212500	2.212500	.000000	.000000
L	145	LCB80106	BS	-2.374265	2.374265	.000000	.000000
L	146	LCB80107	UL	.000000	.000000	.000000	.000000
L	147	LCB80108	UL	.000000	.000000	.000000	.000000
L	148	LCB90104	BS	2.500000	.000000	.000000	2.500000
L	149	LCB90105	BS	2.500000	.000000	.000000	2.500000
L	150	LCB90106	BS	2.500000	.000000	.000000	2.500000
L	151	LCB90107	BS	.000000	2.500000	.000000	2.500000
L	152	LCB90108	BS	.000000	2.500000	.000000	2.500000
E	153	TLF 0107	EQ	.000000	.000000	.184473	.000000
E	154	TLF 0108	EQ	.000000	.000000	.184473	.000000
E	155	TLF 0109	EQ	.000000	.000000	.184473	.000000
E	156	TLF 0110	EQ	.000000	.000000	.142434	.000000
E	157	TLF 0111	EQ	.000000	.000000	.142434	.000000
E	158	TN5 0106	EQ	.000000	.000000	.000000	.000000
E	159	TN5 0107	EQ	.000000	.000000	.000000	.000000
E	160	TN5 0108	EQ	.000000	.000000	.000000	.000000
E	161	TN5 0109	EQ	.000000	.000000	.000000	.000000
E	162	TN5 0110	EQ	.000000	.000000	.000000	.000000
E	163	TN5 0111	EQ	.000000	.000000	.107905	.000000
E	164	TF1 0106	EQ	.000000	.000000	.000000	.000000
E	165	TF1 0107	EQ	.000000	.000000	.000000	.000000
E	166	TF1 0108	EQ	.000000	.000000	.000000	.000000
E	167	TF1 0109	EQ	.000000	.000000	.000000	.000000
E	168	TF1 0110	EQ	.000000	.000000	.000000	.000000
E	169	TF1 0111	EQ	.000000	.000000	-.431618	.000000
E	170	LSM10106	EQ	1.000000	.000000	.000000	1.000000
E	171	LSM10107	EQ	1.000000	.000000	.000000	1.000000
E	172	LSM10108	EQ	1.000000	.000000	.000000	1.000000
E	173	LSM10109	EQ	1.000000	.000000	.000000	1.000000
E	174	LSM10110	EQ	1.000000	.000000	.000000	1.000000
E	175	LSM10111	EQ	1.000000	.000000	.000000	1.000000
E	176	TN6 0105	EQ	.000000	.000000	-.268540	.000000
E	177	TN6 0106	EQ	.000000	.000000	.061247	.000000
E	178	TN6 0107	EQ	.000000	.000000	.023447	.000000
E	179	TN6 0108	EQ	.000000	.000000	.020263	.000000
E	180	TN6 0109	EQ	.000000	.000000	.025024	.000000
E	181	TN6 0110	EQ	.000000	.000000	.203991	.000000
E	182	TF2 0105	EQ	.000000	.000000	1.074160	.000000
E	183	TF2 0106	EQ	.000000	.000000	-.244987	.000000
E	184	TF2 0107	EQ	.000000	.000000	-.093789	.000000
E	185	TF2 0108	EQ	.000000	.000000	-.081054	.000000
E	186	TF2 0109	EQ	.000000	.000000	-.071498	.000000
E	187	TF2 0110	EQ	.000000	.000000	-.582832	.000000
E	188	LSM20105	EQ	1.000000	.000000	.000000	1.000000
E	189	LSM20106	EQ	1.000000	.000000	.000000	1.000000

E	190	LSM20107	EQ	1.000000	.000000	.000000	1.000000
E	191	LSM20108	EQ	1.000000	.000000	.000000	1.000000
E	192	LSM20109	EQ	1.000000	.000000	.002145	1.000000
E	193	LSM20110	EQ	1.000000	.000000	.017485	1.000000
E	194	TN7 0104	EQ	.000000	.000000	-.268540	.000000
E	195	TN7 0105	EQ	.000000	.000000	.061247	.000000
E	196	TN7 0106	EQ	.000000	.000000	.023447	.000000
E	197	TN7 0107	EQ	.000000	.000000	.020263	.000000
E	198	TN7 0108	EQ	.000000	.000000	.017874	.000000
E	199	TN7 0109	EQ	.000000	.000000	.203991	.000000
E	200	TF3 0104	EQ	.000000	.000000	1.074160	.000000
E	201	TF3 0105	EQ	.000000	.000000	-.244987	.000000
E	202	TF3 0106	EQ	.000000	.000000	-.093789	.000000
E	203	TF3 0107	EQ	.000000	.000000	-.081054	.000000
E	204	TF3 0108	EQ	.000000	.000000	-.071498	.000000
E	205	TF3 0109	EQ	.000000	.000000	-.582832	.000000
E	206	LSM30104	EQ	1.000000	.000000	.000000	1.000000
E	207	LSM30105	EQ	1.000000	.000000	.000000	1.000000
E	208	LSM30106	EQ	1.000000	.000000	.000000	1.000000
E	209	LSM30107	EQ	1.000000	.000000	.000000	1.000000
E	210	LSM30108	EQ	1.000000	.000000	.000000	1.000000
E	211	LSM30109	EQ	1.000000	.000000	.017485	1.000000
E	212	TN8 0104	EQ	.000000	.000000	.268540	.000000
E	213	TN8 0105	EQ	.000000	.000000	.207293	.000000
E	214	TN8 0106	EQ	.000000	.000000	.183846	.000000
E	215	TN8 0107	EQ	.000000	.000000	.163582	.000000
E	216	TN8 0108	EQ	.000000	.000000	.145708	.000000
E	217	TF4 0104	EQ	.000000	.000000	-1.074160	.000000
E	218	TF4 0105	EQ	.000000	.000000	-.829173	.000000
E	219	TF4 0106	EQ	.000000	.000000	-.735384	.000000
E	220	TF4 0107	EQ	.000000	.000000	-.654330	.000000
E	221	TF4 0108	EQ	.000000	.000000	-.582832	.000000
E	222	LSM40104	EQ	1.000000	.000000	.000000	1.000000
E	223	LSM40105	EQ	1.000000	.000000	.000000	1.000000
E	224	LSM40106	EQ	1.000000	.000000	.000000	1.000000
E	225	LSM40107	EQ	1.000000	.000000	.000000	1.000000
E	226	LSM40108	EQ	1.000000	.000000	.000000	1.000000
E	227	TREB0107	EQ	.000000	.000000	1.074160	.000000
E	228	TREB0108	EQ	.000000	.000000	.829173	.000000
E	229	TREB0109	EQ	.000000	.000000	.735384	.000000
E	230	TREB0110	EQ	.000000	.000000	.654330	.000000
E	231	TREB0111	EQ	.000000	.000000	.582832	.000000
E	232	ACTP0107	EQ	.000000	.000000	.397554	.000000
E	233	ACTP0108	EQ	.000000	.000000	-.093789	.000000
E	234	ACTP0109	EQ	.000000	.000000	-.081054	.000000
E	235	ACTP0110	EQ	.000000	.000000	-.071498	.000000
E	236	ACTP0111	EQ	.000000	.000000	-.151213	.000000
E	237	ACTA0107	EQ	.000000	.000000	1.471714	.000000
E	238	ACTA0108	EQ	.000000	.000000	.735384	.000000
E	239	ACTA0109	EQ	.000000	.000000	.654330	.000000
E	240	ACTA0110	EQ	.000000	.000000	.582832	.000000
E	241	ACTA0111	EQ	.000000	.000000	.431618	.000000
E	242	SA1 0107	EQ	.000000	.000000	.397554	.000000
E	243	SA1 0108	EQ	.000000	.000000	.303765	.000000
E	244	SA1 0109	EQ	.000000	.000000	.222711	.000000
E	245	SA1 0110	EQ	.000000	.000000	.151213	.000000
E	246	SA1 0111	EQ	.000000	.000000	.000000	.000000
L	247	SA2 0107	UL	.000000	.000000	.000000	.000000
L	248	SA2 0108	BS	.000000	.000000	.000000	.000000
L	249	SA2 0109	UL	.000000	.000000	.000000	.000000
L	250	SA2 0110	UL	.000000	.000000	.000000	.000000
L	251	SA2 0111	BS	.000000	.000000	.000000	.000000
L	252	SA3 0107	BS	.246765	2.253235	.000000	2.500000
L	253	SA3 0108	BS	.293069	2.206931	.000000	2.500000
L	254	SA3 0109	BS	.244485	2.255515	.000000	2.500000
L	255	SA3 0110	BS	.210556	2.289444	.000000	2.500000
L	256	SA3 0111	BS	.228836	2.271164	.000000	2.500000
E	257	SAD 0107	EQ	.000000	.000000	-1.074160	.000000
E	258	SAD 0108	EQ	.000000	.000000	-.431618	.000000
E	259	SAD 0109	EQ	.000000	.000000	-.431618	.000000
E	260	SAD 0110	EQ	.000000	.000000	-.431618	.000000
E	261	SAD 0111	EQ	.000000	.000000	-.431618	.000000

E	262	SA4 0107	EQ	.000000	.000000	.397554	.000000
E	263	SA4 0108	EQ	.000000	.000000	.303765	.000000
E	264	SA4 0109	EQ	.000000	.000000	.222711	.000000
E	265	SA4 0110	EQ	.000000	.000000	.151213	.000000
E	266	SA4 0111	EQ	.000000	.000000	.000000	.000000
L	267	SA5 0107	UL	.000000	.000000	.000000	.000000
L	268	SA5 0108	BS	.000000	.000000	.000000	.000000
L	269	SA5 0109	UL	.000000	.000000	.000000	.000000
L	270	SA5 0110	UL	.000000	.000000	.000000	.000000
L	271	SA5 0111	BS	.000000	.000000	.000000	.000000
L	272	SA6 0107	BS	.067500	2.432500	.000000	2.500000
L	273	SA6 0108	BS	.099853	2.400147	.000000	2.500000
L	274	SA6 0109	BS	.392922	2.107078	.000000	2.500000
L	275	SA6 0110	BS	.637408	1.862592	.000000	2.500000
L	276	SA6 0111	BS	.847964	1.652036	.000000	2.500000
E	277	SA7 0106	EQ	.000000	.000000	.642542	.000000
E	278	SA7 0107	EQ	.000000	.000000	.397554	.000000
E	279	SA7 0108	EQ	.000000	.000000	.303765	.000000
E	280	SA7 0109	EQ	.000000	.000000	.222711	.000000
E	281	SA7 0110	EQ	.000000	.000000	.151213	.000000
L	282	SA8 0106	BS	.000000	.000000	.000000	.000000
L	283	SA8 0107	UL	.000000	.000000	.000000	.000000
L	284	SA8 0108	BS	.000000	.000000	.000000	.000000
L	285	SA8 0109	UL	.000000	.000000	.000000	.000000
L	286	SA8 0110	UL	.000000	.000000	.000000	.000000
L	287	SA9 0106	BS	.032353	2.467647	.000000	2.500000
L	288	SA9 0107	BS	.000000	2.500000	.000000	2.500000
L	289	SA9 0108	BS	.000000	2.500000	.000000	2.500000
L	290	SA9 0109	BS	.000000	2.500000	.000000	2.500000
L	291	SA9 0110	BS	.000000	2.500000	.000000	2.500000
E	292	SA100106	EQ	.000000	.000000	.000000	.000000
E	293	SA100107	EQ	.000000	.000000	.397554	.000000
E	294	SA100108	EQ	.000000	.000000	.303765	.000000
E	295	SA100109	EQ	.000000	.000000	.222711	.000000
E	296	SA100110	EQ	.000000	.000000	.151213	.000000
L	297	SA110106	BS	.000000	.000000	.000000	.000000
L	298	SA110107	UL	.000000	.000000	.000000	.000000
L	299	SA110108	BS	.000000	.000000	.000000	.000000
L	300	SA110109	BS	.000000	.000000	.000000	.000000
L	301	SA110110	BS	.000000	.000000	.000000	.000000
L	302	SA120106	BS	.246765	2.253235	.000000	2.500000
L	303	SA120107	BS	.000000	2.500000	.000000	2.500000
L	304	SA120108	BS	.000000	2.500000	.000000	2.500000
L	305	SA120109	BS	.000000	2.500000	.000000	2.500000
L	306	SA120110	BS	.000000	2.500000	.000000	2.500000
E	307	SA130105	EQ	.000000	.000000	.000000	.000000
E	308	SA130106	EQ	.000000	.000000	.397554	.000000
E	309	SA130107	EQ	.000000	.000000	.303765	.000000
E	310	SA130108	EQ	.000000	.000000	.222711	.000000
E	311	SA130109	EQ	.000000	.000000	.151213	.000000
L	312	SA140105	BS	-2.442500	2.442500	.000000	.000000
L	313	SA140106	BS	.000000	.000000	.000000	.000000
L	314	SA140107	BS	.000000	.000000	.000000	.000000
L	315	SA140108	BS	.000000	.000000	.000000	.000000
L	316	SA140109	BS	.000000	.000000	.000000	.000000
L	317	SA150105	BS	2.500000	.000000	.000000	2.500000
L	318	SA150106	BS	.000000	2.500000	.000000	2.500000
L	319	SA150107	BS	.000000	2.500000	.000000	2.500000
L	320	SA150108	BS	.000000	2.500000	.000000	2.500000
L	321	SA150109	BS	.000000	2.500000	.000000	2.500000
E	322	SA160105	EQ	.000000	.000000	.000000	.000000
E	323	SA160106	EQ	.000000	.000000	.397554	.000000
E	324	SA160107	EQ	.000000	.000000	.397554	.000000
E	325	SA160108	EQ	.000000	.000000	.303765	.000000
E	326	SA160109	EQ	.000000	.000000	.151213	.000000
L	327	SA170105	UL	.000000	.000000	.397554	.000000
L	328	SA170106	BS	.000000	.000000	.000000	.000000
L	329	SA170107	BS	.000000	.000000	.000000	.000000
L	330	SA170108	BS	.000000	.000000	.000000	.000000
L	331	SA170109	BS	.000000	.000000	.000000	.000000
L	332	SA180105	BS	.189265	2.310735	.000000	2.500000
L	333	SA180106	BS	.000000	2.500000	.000000	2.500000

L	334	SA180107	BS	.000000	2.500000	.000000	2.500000
L	335	SA180108	BS	.000000	2.500000	.000000	2.500000
L	336	SA180109	BS	.000000	2.500000	.000000	2.500000
E	337	SA190104	EQ	.000000	.000000	.000000	.000000
E	338	SA190105	EQ	.000000	.000000	.397554	.000000
E	339	SA190106	EQ	.000000	.000000	.303765	.000000
E	340	SA190107	EQ	.000000	.000000	.222711	.000000
E	341	SA190108	EQ	.000000	.000000	.151213	.000000
L	342	SA200104	BS	-2.442500	2.442500	.000000	.000000
L	343	SA200105	UL	.000000	.000000	.000000	.000000
L	344	SA200106	BS	.000000	.000000	.000000	.000000
L	345	SA200107	BS	.000000	.000000	.000000	.000000
L	346	SA200108	BS	.000000	.000000	.000000	.000000
L	347	SA210104	BS	2.500000	.000000	.000000	2.500000
L	348	SA210105	BS	.000000	2.500000	.000000	2.500000
L	349	SA210106	BS	.000000	2.500000	.000000	2.500000
L	350	SA210107	BS	.000000	2.500000	.000000	2.500000
L	351	SA210108	BS	.000000	2.500000	.000000	2.500000
E	352	SA220104	EQ	.000000	.000000	.000000	.000000
E	353	SA220105	EQ	.000000	.000000	.397554	.000000
E	354	SA220106	EQ	.000000	.000000	.303765	.000000
E	355	SA220107	EQ	.000000	.000000	.397554	.000000
E	356	SA220108	EQ	.000000	.000000	.151213	.000000
L	357	SA230104	UL	.000000	.000000	.397554	.000000
L	358	SA230105	UL	.000000	.000000	.000000	.000000
L	359	SA230106	BS	.000000	.000000	.000000	.000000
L	360	SA230107	BS	.000000	.000000	.000000	.000000
L	361	SA230108	BS	.000000	.000000	.000000	.000000
L	362	SA240104	BS	.131765	2.368235	.000000	2.500000
L	363	SA240105	BS	.000000	2.500000	.000000	2.500000
L	364	SA240106	BS	.000000	2.500000	.000000	2.500000
L	365	SA240107	BS	.000000	2.500000	.000000	2.500000
L	366	SA240108	BS	.000000	2.500000	.000000	2.500000
E	367	SA250103	EQ	.000000	.000000	.000000	.000000
E	368	SA250104	EQ	.000000	.000000	.397554	.000000
E	369	SA250105	EQ	.000000	.000000	.397554	.000000
E	370	SA250106	EQ	.000000	.000000	.222711	.000000
E	371	SA250107	EQ	.000000	.000000	.397554	.000000
L	372	SA260103	UL	.000000	.000000	.397554	.000000
L	373	SA260104	BS	.000000	.000000	.000000	.000000
L	374	SA260105	BS	.000000	.000000	.000000	.000000
L	375	SA260106	BS	.000000	.000000	.000000	.000000
L	376	SA260107	BS	.000000	.000000	.000000	.000000
L	377	SA270103	BS	.074265	2.425735	.000000	2.500000
L	378	SA270104	BS	.000000	2.500000	.000000	2.500000
L	379	SA270105	BS	.000000	2.500000	.000000	2.500000
L	380	SA270106	BS	.000000	2.500000	.000000	2.500000
L	381	SA270107	BS	.000000	2.500000	.000000	2.500000
E	382	SA280102	EQ	.000000	.000000	.000000	.000000
E	383	SA280103	EQ	.000000	.000000	.397554	.000000
E	384	SA280104	EQ	.000000	.000000	.397554	.000000
E	385	SA280105	EQ	.000000	.000000	.222711	.000000
E	386	SA280106	EQ	.000000	.000000	.222711	.000000
L	387	SA290102	UL	.000000	.000000	.397554	.000000
L	388	SA290103	UL	.000000	.000000	.000000	.000000
L	389	SA290104	BS	.000000	.000000	.000000	.000000
L	390	SA290105	BS	.000000	.000000	.000000	.000000
L	391	SA290106	BS	.000000	.000000	.000000	.000000
L	392	SA300102	BS	.016765	2.483235	.000000	2.500000
L	393	SA300103	BS	.000000	2.500000	.000000	2.500000
L	394	SA300104	BS	.000000	2.500000	.000000	2.500000
L	395	SA300105	BS	.000000	2.500000	.000000	2.500000
L	396	SA300106	BS	.000000	2.500000	.000000	2.500000
E	397	SA310101	EQ	.000000	.000000	.000000	.000000
E	398	SA310102	EQ	.000000	.000000	.397554	.000000
E	399	SA310103	EQ	.000000	.000000	.397554	.000000
E	400	SA310104	EQ	.000000	.000000	.222711	.000000
E	401	SA310105	EQ	.000000	.000000	.222711	.000000
L	402	SA320101	BS	-2.459265	2.459265	.000000	.000000
L	403	SA320102	UL	.000000	.000000	.000000	.000000
L	404	SA320103	BS	.000000	.000000	.000000	.000000
L	405	SA320104	BS	.000000	.000000	.000000	.000000

L	406	SA320105	BS	.000000	.000000	.000000	.000000
L	407	SA330101	BS	2.500000	.000000	.000000	2.500000
L	408	SA330102	BS	.000000	2.500000	.000000	2.500000
L	409	SA330103	BS	.000000	2.500000	.000000	2.500000
L	410	SA330104	BS	.000000	2.500000	.000000	2.500000
L	411	SA330105	BS	.000000	2.500000	.000000	2.500000
E	412	TSF 0107	EQ	.000000	.000000	.431618	.000000
E	413	TSF 0108	EQ	.000000	.000000	.431618	.000000
E	414	TSF 0109	EQ	.000000	.000000	.431618	.000000
E	415	TSF 0110	EQ	.000000	.000000	.431618	.000000
E	416	TSF 0111	EQ	.000000	.000000	.431618	.000000
E	417	CLB10107	EQ	1.200000	.000000	1.074160	1.200000
E	418	CLB10108	EQ	1.200000	.000000	.829173	1.200000
E	419	CLB10109	EQ	1.200000	.000000	.735384	1.200000
E	420	CLB10110	EQ	1.200000	.000000	.654330	1.200000
E	421	CLB10111	EQ	1.200000	.000000	.582832	1.200000
L	422	CLB20107	BS	-1.266175	1.266175	.000000	.000000
L	423	CLB20108	BS	-1.034653	1.034653	.000000	.000000
L	424	CLB20109	BS	-.934653	.934653	.000000	.000000
L	425	CLB20110	BS	-1.106091	1.106091	.000000	.000000
L	426	CLB20111	BS	-1.355822	1.355822	.000000	.000000
L	427	CLB30107	UL	2.500000	.000000	.180517	2.500000
L	428	CLB30108	UL	2.500000	.000000	.038698	2.500000
L	429	CLB30109	UL	2.500000	.000000	.032996	2.500000
L	430	CLB30110	UL	2.500000	.000000	.027458	2.500000
L	431	CLB30111	UL	2.500000	.000000	.086324	2.500000
E	432	OB1 0107	EQ	-.050000	.000000	1.167573	-.050000
E	433	OB2 0108	EQ	.000000	.000000	.893643	.000000
E	434	OB2 0109	EQ	.000000	.000000	.790475	.000000
E	435	OB2 0110	EQ	.000000	.000000	.702388	.000000
E	436	OB2 0111	EQ	.000000	.000000	.626871	.000000
G	437	OBX 0107	BS	.050000	.000000	.000000	.050000
L	438	OB3 0107	BS	.000000	.000000	.000000	.000000
L	439	OB3 0108	BS	.000000	.000000	.000000	.000000
L	440	OB3 0109	UL	.000000	.000000	.000000	.000000
L	441	OB3 0110	BS	.000000	.000000	.000000	.000000
L	442	OB3 0111	BS	.000000	.000000	.000000	.000000
L	443	OB4 0107	BS	.020000	2.480000	.000000	2.500000
L	444	OB4 0108	BS	.000000	2.500000	.000000	2.500000
L	445	OB4 0109	BS	.000000	2.500000	.000000	2.500000
L	446	OB4 0110	BS	.000000	2.500000	.000000	2.500000
L	447	OB4 0111	BS	.000000	2.500000	.000000	2.500000
L	448	MXDS0107	BS	.020000	.080000	.000000	.100000
L	449	MXDS0108	BS	.000000	.100000	.000000	.100000
L	450	MXDS0109	BS	.000000	.100000	.000000	.100000
L	451	MXDS0110	BS	.000000	.100000	.000000	.100000
L	452	MXDS0111	BS	.000000	.100000	.000000	.100000
E	453	DST10107	EQ	.000000	.000000	-1.074160	.000000
E	454	DST10108	EQ	.000000	.000000	-.829173	.000000
E	455	DST10109	EQ	.000000	.000000	-.735384	.000000
E	456	DST10110	EQ	.000000	.000000	-.654330	.000000
E	457	DST20111	EQ	.000000	.000000	-.582832	.000000
E	458	FUND 07	EQ	.000000	.000000	.967400	.000000
E	459	FUND 08	EQ	.000000	.000000	.701184	.000000
E	460	FUND 09	EQ	.000000	.000000	.641098	.000000
E	461	FUND 10	EQ	.000000	.000000	.589969	.000000
E	462	HZN1 11	EQ	.000000	.000000	-.545833	.000000
E	463	DBT1 07	EQ	.000000	.000000	-.687001	.000000
E	464	DBT1 08	EQ	.000000	.000000	-.628978	.000000
E	465	DBT1 09	EQ	.000000	.000000	-.579396	.000000
E	466	DBT1 10	EQ	.000000	.000000	-.536144	.000000
E	467	DBT1 11	EQ	.000000	.000000	-.496508	.000000
E	468	EQT 0107	EQ	.000000	.000000	.002646	.000000
E	469	EQT 0108	EQ	.000000	.000000	.001840	.000000
E	470	EQT 0109	EQ	.000000	.000000	.001151	.000000
E	471	EQT 0110	EQ	.000000	.000000	5.50497E-04	.000000
E	472	EQT 0111	EQ	.000000	.000000	.000000	.000000
E	473	CGN10107	EQ	.000000	.000000	.002646	.000000
E	474	CGNx0107	EQ	.000000	.000000	-.002646	.000000
E	475	CGN20108	EQ	.000000	.000000	.001840	.000000
E	476	CGN20109	EQ	.000000	.000000	.001151	.000000
E	477	CGN20110	EQ	.000000	.000000	5.50497E-04	.000000

E	478	CGN20111	EQ	.000000	.000000	.000000	.000000
L	479	CGN30108	UL	.000000	.000000	.000000	.000000
L	480	CGN30109	UL	.000000	.000000	.000000	.000000
L	481	CGN30110	UL	.000000	.000000	.000000	.000000
L	482	CGN30111	BS	.000000	.000000	.000000	.000000
L	483	CGN40108	BS	.000000	2.500000	.000000	2.500000
L	484	CGN40109	BS	.000000	2.500000	.000000	2.500000
L	485	CGN40110	BS	.000000	2.500000	.000000	2.500000
L	486	CGN40111	BS	.000000	2.500000	.000000	2.500000
E	487	ISSU 07	EQ	.000000	.000000	.000000	.000000
E	488	ISSU 08	EQ	.000000	.000000	.000000	.000000
E	489	ISSU 09	EQ	.000000	.000000	.000000	.000000
E	490	ISSU 10	EQ	.000000	.000000	-.071988	.000000
E	491	ISSU 11	EQ	.000000	.000000	-.034160	.000000
L	492	UDL10107	BS	-1.506033	1.506033	.000000	.000000
L	493	UDL10108	BS	-1.506033	1.506033	.000000	.000000
L	494	UDL10109	BS	-1.506033	1.506033	.000000	.000000
L	495	UDL10110	BS	-1.506033	1.506033	.000000	.000000
L	496	UDL10111	BS	-1.506033	1.506033	.000000	.000000
G	497	UDL50107	BS	.737581	.737581	.000000	.000000
G	498	UDL50108	BS	.737581	.737581	.000000	.000000
G	499	UDL50109	BS	.737581	.737581	.000000	.000000
G	500	UDL50110	BS	.737581	.737581	.000000	.000000
G	501	UDL50111	BS	.737581	.737581	.000000	.000000
L	502	LDL10107	UL	.000000	.000000	.179777	.000000
L	503	LDL10108	BS	.000000	.000000	.000000	.000000
L	504	LDL10109	BS	.000000	.000000	.000000	.000000
L	505	LDL10110	BS	.000000	.000000	.000000	.000000
L	506	LDL10111	BS	.000000	.000000	.000000	.000000
E	507	DF1 0107	EQ	.000000	.000000	-.004448	.000000
E	508	DF1 0108	EQ	.000000	.000000	-.003223	.000000
E	509	DF1 0109	EQ	.000000	.000000	-.002755	.000000
E	510	DF1 0110	EQ	.000000	.000000	-.002403	.000000
E	511	DF1 0111	EQ	.000000	.000000	-.002202	.000000
L	512	DF2 07	BS	-2.119684	2.119684	.000000	.000000
L	513	DF2 08	BS	-2.119684	2.119684	.000000	.000000
L	514	DF2 09	BS	-2.119684	2.119684	.000000	.000000
L	515	DF2 10	BS	-2.119684	2.119684	.000000	.000000
L	516	DF2 11	BS	-2.119684	2.119684	.000000	.000000
L	517	DF3 07	BS	2.500000	.000000	.000000	2.500000
L	518	DF3 08	BS	2.500000	.000000	.000000	2.500000
L	519	DF3 09	BS	2.500000	.000000	.000000	2.500000
L	520	DF3 10	BS	2.500000	.000000	.000000	2.500000
L	521	DF3 11	BS	2.500000	.000000	.000000	2.500000
E	522	DF4 0107	EQ	.000000	.000000	.000000	.000000
E	523	DF4 0108	EQ	.000000	.000000	.000000	.000000
E	524	DF4 0109	EQ	.000000	.000000	.000000	.000000
E	525	DF4 0110	EQ	.000000	.000000	.000000	.000000
E	526	DF4 0111	EQ	.000000	.000000	.000000	.000000
L	527	DF5 07	BS	.000000	.000000	.000000	.000000
L	528	DF5 08	BS	.000000	.000000	.000000	.000000
L	529	DF5 09	UL	.000000	.000000	.000000	.000000
L	530	DF5 10	UL	.000000	.000000	.000000	.000000
L	531	DF5 11	UL	.000000	.000000	.000000	.000000
L	532	DF6 07	BS	.091271	2.408729	.000000	2.500000
L	533	DF6 08	BS	.091271	2.408729	.000000	2.500000
L	534	DF6 09	BS	.091271	2.408729	.000000	2.500000
L	535	DF6 10	BS	.091271	2.408729	.000000	2.500000
L	536	DF6 11	BS	.091271	2.408729	.000000	2.500000
E	537	DF7 0107	EQ	.000000	.000000	.000000	.000000
E	538	DF7 0108	EQ	.000000	.000000	.000000	.000000
E	539	DF7 0109	EQ	.000000	.000000	.000000	.000000
E	540	DF7 0110	EQ	.000000	.000000	.000000	.000000
E	541	DF7 0111	EQ	.000000	.000000	.000000	.000000
L	542	DF8 07	BS	.000000	.000000	.000000	.000000
L	543	DF8 08	BS	.000000	.000000	.000000	.000000
L	544	DF8 09	BS	.000000	.000000	.000000	.000000
L	545	DF8 10	BS	.000000	.000000	.000000	.000000
L	546	DF8 11	BS	.000000	.000000	.000000	.000000
L	547	DF9 07	BS	.471587	2.028413	.000000	2.500000
L	548	DF9 08	BS	.471587	2.028413	.000000	2.500000
L	549	DF9 09	BS	.471587	2.028413	.000000	2.500000

L	550	DF9	10	BS	.471587	2.028413	.000000	2.500000
L	551	DF9	11	BS	.471587	2.028413	.000000	2.500000
E	552	DF100107		EQ	.000000	.000000	.000000	.000000
E	553	DF100108		EQ	.000000	.000000	.000000	.000000
E	554	DF100109		EQ	.000000	.000000	.000000	.000000
E	555	DF100110		EQ	.000000	.000000	.000000	.000000
E	556	DF100111		EQ	.000000	.000000	.000000	.000000
L	557	DF11	07	BS	.000000	.000000	.000000	.000000
L	558	DF11	08	BS	.000000	.000000	.000000	.000000
L	559	DF11	09	BS	.000000	.000000	.000000	.000000
L	560	DF11	10	BS	.000000	.000000	.000000	.000000
L	561	DF11	11	BS	.000000	.000000	.000000	.000000
L	562	DF12	07	BS	.471587	2.028413	.000000	2.500000
L	563	DF12	08	BS	.471587	2.028413	.000000	2.500000
L	564	DF12	09	BS	.471587	2.028413	.000000	2.500000
L	565	DF12	10	BS	.471587	2.028413	.000000	2.500000
L	566	DF12	11	BS	.471587	2.028413	.000000	2.500000
E	567	DF13	07	EQ	.000000	.000000	.075623	.000000
E	568	DF13	08	EQ	.000000	.000000	.054799	.000000
E	569	DF13	09	EQ	.000000	.000000	.046827	.000000
E	570	DF13	10	EQ	.000000	.000000	.040849	.000000
E	571	DF13	11	EQ	.000000	.000000	.037434	.000000
E	572	DF14	07	EQ	.000000	.000000	.080072	.000000
E	573	DF14	08	EQ	.000000	.000000	.058023	.000000
E	574	DF14	09	EQ	.000000	.000000	.049582	.000000
E	575	DF14	10	EQ	.000000	.000000	.043252	.000000
E	576	DF14	11	EQ	.000000	.000000	.039636	.000000
E	577	DF15	07	EQ	.000000	.000000	.084520	.000000
E	578	DF15	08	EQ	.000000	.000000	.061246	.000000
E	579	DF15	09	EQ	.000000	.000000	.049582	.000000
E	580	DF15	10	EQ	.000000	.000000	.045655	.000000
E	581	DF15	11	EQ	.000000	.000000	.039636	.000000
E	582	DF16	07	EQ	.000000	.000000	.088969	.000000
E	583	DF16	08	EQ	.000000	.000000	.064470	.000000
E	584	DF16	09	EQ	.000000	.000000	.055091	.000000
E	585	DF16	10	EQ	.000000	.000000	.048058	.000000
E	586	DF16	11	EQ	.000000	.000000	.044040	.000000
E	587	DBNT	07	EQ	.000000	.000000	-.889687	.000000
E	588	DBNT	08	EQ	.000000	.000000	-.644700	.000000
E	589	DBNT	09	EQ	.000000	.000000	-.550911	.000000
E	590	DBNT	10	EQ	.000000	.000000	-.480581	.000000
E	591	DBNT	11	EQ	.000000	.000000	-.440397	.000000
E	592	GRDV0107		EQ	.000000	.000000	.859328	.000000
E	593	GRDV0108		EQ	.000000	.000000	.742849	.000000
E	594	GRDV0109		EQ	.000000	.000000	.649060	.000000
E	595	GRDV0110		EQ	.000000	.000000	.568006	.000000
E	596	GRDV0111		EQ	.000000	.000000	.496508	.000000
E	597	HRZ10111		EQ	9.300000	.000000	.496508	9.300000
N	598	MAX		BS	7.753795	-7.753795	.000000	.000000
E	599	EXC1	01	EQ	1.000000	.000000	-.423306	1.000000
E	600	EXC2	01	EQ	1.000000	.000000	.618900	1.000000
L	601	LS1	0107	BS	.000000	.000000	.000000	.000000
L	602	LS1	0108	BS	-1.000000	1.000000	.000000	.000000
L	603	LS1	0109	BS	.000000	.000000	.000000	.000000
L	604	LS1	0110	BS	.000000	.000000	.000000	.000000
L	605	LS1	0111	BS	.000000	.000000	.000000	.000000
L	606	LS2	0107	BS	.000000	.000000	.000000	.000000
L	607	LS2	0108	BS	.000000	.000000	.000000	.000000
L	608	LS2	0109	UL	.000000	.000000	.000000	.000000
L	609	LS2	0110	BS	.000000	.000000	.000000	.000000
L	610	LS3	0106	BS	.000000	.000000	.000000	.000000
L	611	LS3	0107	BS	.000000	.000000	.000000	.000000
L	612	LS3	0108	BS	.000000	.000000	.000000	.000000
L	613	LS3	0109	BS	.000000	.000000	.000000	.000000
L	614	LS4	0105	BS	.000000	.000000	.000000	.000000
L	615	LS4	0106	BS	.000000	.000000	.000000	.000000
L	616	LS4	0107	BS	.000000	.000000	.000000	.000000
L	617	LS4	0108	BS	.000000	.000000	.000000	.000000
L	618	LS5	0107	BS	.000000	.000000	.000000	.000000
L	619	LS5	0108	BS	.000000	.000000	.000000	.000000
L	620	LS5	0109	BS	.000000	.000000	.000000	.000000
L	621	LS5	0110	BS	.000000	.000000	.000000	.000000

L	622	LS5 0111	BS	.000000	.000000	.000000	.000000
L	623	LS6 0107	BS	.000000	.000000	.000000	.000000
L	624	LS6 0108	BS	.000000	.000000	.000000	.000000
L	625	LS6 0109	BS	.000000	.000000	.000000	.000000
L	626	LS6 0110	BS	.000000	.000000	.000000	.000000
L	627	LS7 0106	BS	.000000	.000000	.000000	.000000
L	628	LS7 0107	BS	.000000	.000000	.000000	.000000
L	629	LS7 0108	BS	.000000	.000000	.000000	.000000
L	630	LS7 0109	BS	.000000	.000000	.000000	.000000
L	631	LS7 0110	BS	.000000	.000000	.000000	.000000
L	632	LS8 0106	BS	.000000	.000000	.000000	.000000
L	633	LS8 0107	BS	.000000	.000000	.000000	.000000
L	634	LS8 0108	BS	.000000	.000000	.000000	.000000
L	635	LS8 0109	BS	.000000	.000000	.000000	.000000
L	636	LS9 0105	BS	-1.000000	1.000000	.000000	.000000
L	637	LS9 0106	BS	.000000	.000000	.000000	.000000
L	638	LS9 0107	BS	.000000	.000000	.000000	.000000
L	639	LS9 0108	BS	.000000	.000000	.000000	.000000
L	640	LS9 0109	BS	.000000	.000000	.000000	.000000
L	641	LS100105	BS	.000000	.000000	.000000	.000000
L	642	LS100106	BS	.000000	.000000	.000000	.000000
L	643	LS100107	BS	.000000	.000000	.000000	.000000
L	644	LS100108	BS	.000000	.000000	.000000	.000000
L	645	LS110104	BS	-1.000000	1.000000	.000000	.000000
L	646	LS110105	BS	.000000	.000000	.000000	.000000
L	647	LS110106	BS	.000000	.000000	.000000	.000000
L	648	LS110107	BS	.000000	.000000	.000000	.000000
L	649	LS110108	BS	.000000	.000000	.000000	.000000
L	650	LS120104	BS	.000000	.000000	.000000	.000000
L	651	LS120105	BS	.000000	.000000	.000000	.000000
L	652	LS120106	BS	.000000	.000000	.000000	.000000
L	653	LS120107	BS	.000000	.000000	.000000	.000000
L	654	LS130103	BS	.000000	.000000	.000000	.000000
L	655	LS130104	BS	.000000	.000000	.000000	.000000
L	656	LS130105	BS	.000000	.000000	.000000	.000000
L	657	LS130106	BS	.000000	.000000	.000000	.000000
L	658	LS140102	BS	.000000	.000000	.000000	.000000
L	659	LS140103	BS	.000000	.000000	.000000	.000000
L	660	LS140104	BS	.000000	.000000	.000000	.000000
L	661	LS140105	BS	.000000	.000000	.000000	.000000
G	662	LS150107	BS	.000000	.000000	.000000	.000000
G	663	LS150108	LL	.000000	.000000	.000000	.000000
G	664	LS150109	LL	.000000	.000000	.000000	.000000
G	665	LS150110	LL	.000000	.000000	.000000	.000000
G	666	LS150111	BS	1.000000	1.000000	.000000	.000000
G	667	LS160107	BS	1.000000	1.000000	.000000	.000000
G	668	LS160108	LL	.000000	.000000	.000000	.000000
G	669	LS160109	LL	.000000	.000000	.000000	.000000
G	670	LS160110	LL	.000000	.000000	.000000	.000000
G	671	LS160111	BS	.000000	.000000	.000000	.000000
G	672	LS170107	BS	1.000000	1.000000	.000000	.000000
G	673	LS170108	LL	.000000	.000000	.000000	.000000
G	674	LS170109	LL	.000000	.000000	.000000	.000000
G	675	LS170110	LL	.000000	.000000	.000000	.000000
G	676	LS170111	LL	.000000	.000000	.000000	.000000
L	677	LS180107	BS	.000000	.000000	.000000	.000000
L	678	LS180108	BS	-1.000000	1.000000	.000000	.000000
L	679	LS180109	BS	-1.000000	1.000000	.000000	.000000
L	680	LS180110	BS	-1.000000	1.000000	.000000	.000000
L	681	LS180111	BS	-1.000000	1.000000	.000000	.000000
L	682	LS190107	BS	.000000	.000000	.000000	.000000
L	683	LS190108	BS	.000000	.000000	.000000	.000000
L	684	LS190109	BS	-1.000000	1.000000	.000000	.000000
L	685	LS190110	BS	-1.000000	1.000000	.000000	.000000
L	686	LS190111	BS	.000000	.000000	.000000	.000000
L	687	LS20 07	BS	-1.000000	1.000000	.000000	.000000
L	688	LS20 08	BS	-1.000000	1.000000	.000000	.000000
L	689	LS20 09	BS	-1.000000	1.000000	.000000	.000000
L	690	LS20 10	BS	-1.000000	1.000000	.000000	.000000
L	691	LS20 11	BS	-1.000000	1.000000	.000000	.000000
L	692	LS21 07	BS	.000000	.000000	.000000	.000000
L	693	LS21 08	BS	.000000	.000000	.000000	.000000

L	694	LS21 09	BS	.000000	.000000	.000000	.000000
L	695	LS21 10	BS	.000000	.000000	.000000	.000000
L	696	LS21 11	BS	.000000	.000000	.000000	.000000
L	697	LS22 07	BS	.000000	.000000	.000000	.000000
L	698	LS22 08	BS	.000000	.000000	.000000	.000000
L	699	LS22 09	BS	.000000	.000000	.000000	.000000
L	700	LS22 10	BS	.000000	.000000	.000000	.000000
L	701	LS22 11	BS	.000000	.000000	.000000	.000000
L	702	LP1 0107	BS	-.100000	1.100000	.000000	1.000000
L	703	LP1 0108	BS	-.287500	1.287500	.000000	1.000000
L	704	LP1 0109	BS	-.165620	1.165620	.000000	1.000000
L	705	LP1 0110	BS	-.074220	1.074220	.000000	1.000000
L	706	LP1 0111	BS	-.200000	1.200000	.000000	1.000000
L	707	LP2 0107	BS	.100000	1.100000	.000000	1.200000
L	708	LP2 0108	BS	-.187500	1.287500	.000000	1.100000
L	709	LP2 0109	BS	-.140620	1.165620	.000000	1.025000
L	710	LP2 0110	BS	-.105470	1.074220	.000000	.968750
L	711	LP2 0111	BS	-.273440	1.200000	.000000	.926560
L	712	LP3 0107	BS	.000000	.200000	.000000	.200000
L	713	LP3 0108	BS	.200000	.000000	.000000	.200000
L	714	LP3 0109	UL	.200000	.000000	.000000	.200000
L	715	LP3 0110	UL	.200000	.000000	.010725	.200000
L	716	LP3 0111	BS	.200000	.000000	.000000	.200000
L	717	LP4 0107	BS	.000000	.000000	.000000	.000000
L	718	LP4 0108	BS	-.212500	.212500	.000000	.000000
L	719	LP4 0109	BS	-.056806	.056806	.000000	.000000
L	720	LP4 0110	BS	.000000	.000000	.000000	.000000
L	721	LP4 0111	BS	.000000	.000000	.000000	.000000
L	722	LP5 0107	UL	.000000	.000000	.056691	.000000
L	723	LP5 0108	BS	-.200000	.200000	.000000	.000000
L	724	LP5 0109	BS	-.234653	.234653	.000000	.000000
L	725	LP5 0110	BS	-.234653	.234653	.000000	.000000
L	726	LP5 0111	BS	-.200000	.200000	.000000	.000000
L	727	LP6 0106	BS	-.161765	.161765	.000000	.000000
L	728	LP6 0107	UL	.000000	.000000	.056691	.000000
L	729	LP6 0108	BS	.000000	.000000	.000000	.000000
L	730	LP6 0109	BS	.000000	.000000	.000000	.000000
L	731	LP6 0110	UL	.000000	.000000	.000000	.000000
L	732	LP7 0105	UL	.000000	.000000	.000000	.000000
L	733	LP7 0106	UL	.000000	.000000	.056691	.000000
L	734	LP7 0107	UL	.000000	.000000	.061380	.000000
L	735	LP7 0108	UL	.000000	.000000	.023394	.000000
L	736	LP7 0109	BS	.000000	.000000	.000000	.000000
L	737	LP8 0104	BS	.000000	.000000	.000000	.000000
L	738	LP8 0105	UL	.000000	.000000	.056691	.000000
L	739	LP8 0106	UL	.000000	.000000	.061380	.000000
L	740	LP8 0107	BS	.000000	.000000	.000000	.000000
L	741	LP8 0108	BS	.000000	.000000	.000000	.000000
L	742	LP9 0107	BS	.000000	.000000	.000000	.000000
L	743	LP9 0108	BS	.000000	.000000	.000000	.000000
L	744	LP9 0109	BS	.000000	.000000	.000000	.000000
L	745	LP9 0110	BS	.000000	.000000	.000000	.000000
L	746	LP9 0111	UL	.000000	.000000	.031314	.000000
L	747	LP100107	BS	-.200000	.200000	.000000	.000000
L	748	LP100108	BS	.000000	.000000	.000000	.000000
L	749	LP100109	BS	.000000	.000000	.000000	.000000
L	750	LP100110	BS	.000000	.000000	.000000	.000000
L	751	LP100111	BS	.000000	.000000	.000000	.000000
L	752	LP110107	BS	.000000	.000000	.000000	.000000
L	753	LP110108	BS	.000000	.000000	.000000	.000000
L	754	LP110109	BS	.000000	.000000	.000000	.000000
L	755	LP110110	BS	.000000	.000000	.000000	.000000
L	756	LP110111	BS	-.092970	.092970	.000000	.000000
L	757	LP120107	BS	-.361765	.361765	.000000	.000000
L	758	LP120108	BS	.000000	.000000	.000000	.000000
L	759	LP120109	BS	.000000	.000000	.000000	.000000
L	760	LP120110	BS	.000000	.000000	.000000	.000000
L	761	LP120111	BS	.000000	.000000	.000000	.000000
L	762	LP130107	BS	.000000	.000000	.000000	.000000
L	763	LP130108	BS	-.200000	.200000	.000000	.000000
L	764	LP130109	BS	-.234653	.234653	.000000	.000000
L	765	LP130110	BS	-.234653	.234653	.000000	.000000

L	766	LP130111	BS	-.200000	.200000	.000000	.000000
L	767	LP140107	BS	-.161765	.161765	.000000	.000000
L	768	LP140108	BS	.000000	.000000	.000000	.000000
L	769	LP140109	BS	.000000	.000000	.000000	.000000
L	770	LP140110	BS	.000000	.000000	.000000	.000000
L	771	LP140111	BS	.000000	.000000	.000000	.000000
L	772	LP150107	BS	.000000	.000000	.000000	.000000
L	773	LP150108	BS	.000000	.000000	.000000	.000000
L	774	LP150109	BS	.000000	.000000	.000000	.000000
L	775	LP150110	BS	.000000	.000000	.000000	.000000
L	776	LP150111	BS	.000000	.000000	.000000	.000000
L	777	LP160107	BS	.000000	.000000	.000000	.000000
L	778	LP160108	BS	.000000	.000000	.000000	.000000
L	779	LP160109	BS	.000000	.000000	.000000	.000000
L	780	LP160110	UL	.000000	.000000	.023394	.000000
L	781	LP160111	UL	.000000	.000000	.026969	.000000
L	782	LP170107	UL	.000000	.000000	.056691	.000000
L	783	LP170108	BS	-.212500	.212500	.000000	.000000
L	784	LP170109	BS	-.056806	.056806	.000000	.000000
L	785	LP170110	BS	.000000	.000000	.000000	.000000
L	786	LP170111	UL	.000000	.000000	.000000	.000000
L	787	LP180107	BS	.000000	.000000	.000000	.000000
L	788	LP180108	BS	.000000	.000000	.000000	.000000
L	789	LP180109	BS	-.068584	.068584	.000000	.000000
L	790	LP180110	BS	-.068225	.068225	.000000	.000000
L	791	LP180111	BS	.000000	.000000	.000000	.000000
L	792	LP190107	BS	.000000	.000000	.000000	.000000
L	793	LP190108	BS	.000000	.000000	.000000	.000000
L	794	LP190109	BS	.000000	.000000	.000000	.000000
L	795	LP190110	BS	.000000	.000000	.000000	.000000
L	796	LP190111	BS	.000000	.000000	.000000	.000000
L	797	LP200106	BS	.000000	.000000	.000000	.000000
L	798	LP200107	BS	.000000	.000000	.000000	.000000
L	799	LP200108	UL	.000000	.000000	.000000	.000000
L	800	LP200109	BS	.000000	.000000	.000000	.000000
L	801	LP200110	UL	.000000	.000000	.000000	.000000
L	802	LP210106	UL	.000000	.000000	.397554	.000000
L	803	LP210107	BS	.000000	.000000	.000000	.000000
L	804	LP210108	UL	.000000	.000000	.000000	.000000
L	805	LP210109	BS	.000000	.000000	.000000	.000000
L	806	LP210110	BS	.000000	.000000	.000000	.000000
L	807	LP220105	BS	.000000	.000000	.000000	.000000
L	808	LP220106	BS	.000000	.000000	.000000	.000000
L	809	LP220107	UL	.000000	.000000	.093789	.000000
L	810	LP220108	UL	.000000	.000000	.081054	.000000
L	811	LP220109	UL	.000000	.000000	.071498	.000000
L	812	LP230105	BS	-.057500	.057500	.000000	.000000
L	813	LP230106	BS	.000000	.000000	.000000	.000000
L	814	LP230107	BS	.000000	.000000	.000000	.000000
L	815	LP230108	BS	.000000	.000000	.000000	.000000
L	816	LP230109	BS	.000000	.000000	.000000	.000000
L	817	LP240104	BS	.000000	.000000	.000000	.000000
L	818	LP240105	BS	.000000	.000000	.000000	.000000
L	819	LP240106	BS	.000000	.000000	.000000	.000000
L	820	LP240107	UL	.000000	.000000	.174843	.000000
L	821	LP240108	UL	.000000	.000000	.152552	.000000
L	822	LP250104	BS	-.057500	.057500	.000000	.000000
L	823	LP250105	BS	.000000	.000000	.000000	.000000
L	824	LP250106	BS	.000000	.000000	.000000	.000000
L	825	LP250107	BS	.000000	.000000	.000000	.000000
L	826	LP250108	BS	.000000	.000000	.000000	.000000
L	827	LP260103	BS	-.057500	.057500	.000000	.000000
L	828	LP260104	BS	.000000	.000000	.000000	.000000
L	829	LP260105	BS	.000000	.000000	.000000	.000000
L	830	LP260106	BS	.000000	.000000	.000000	.000000
L	831	LP260107	BS	.000000	.000000	.000000	.000000
L	832	LP270102	BS	-.057500	.057500	.000000	.000000
L	833	LP270103	BS	.000000	.000000	.000000	.000000
L	834	LP270104	BS	.000000	.000000	.000000	.000000
L	835	LP270105	BS	.000000	.000000	.000000	.000000
L	836	LP270106	BS	.000000	.000000	.000000	.000000
L	837	LP280101	BS	-.016765	.016765	.000000	.000000

L	838	LP280102	BS	.000000	.000000	.000000	.000000
L	839	LP280103	BS	.000000	.000000	.000000	.000000
L	840	LP280104	BS	.000000	.000000	.000000	.000000
L	841	LP280105	BS	.000000	.000000	.000000	.000000
L	842	LP290107	BS	.000000	.000000	.000000	.000000
L	843	LP290108	BS	.000000	.000000	.000000	.000000
L	844	LP290109	BS	.000000	.000000	.000000	.000000
L	845	LP290110	UL	.000000	.000000	.000000	.000000
L	846	LP290111	BS	.000000	.000000	.000000	.000000
L	847	LP300107	BS	-.057500	.057500	.000000	.000000
L	848	LP300108	BS	.000000	.000000	.000000	.000000
L	849	LP300109	UL	.000000	.000000	.093789	.000000
L	850	LP300110	UL	.000000	.000000	.081054	.000000
L	851	LP300111	BS	.000000	.000000	.000000	.000000
L	852	LP310107	BS	-.057500	.057500	.000000	.000000
L	853	LP310108	BS	.000000	.000000	.000000	.000000
L	854	LP310109	BS	.000000	.000000	.000000	.000000
L	855	LP310110	UL	.000000	.000000	.174843	.000000
L	856	LP310111	BS	.000000	.000000	.000000	.000000
L	857	LP320107	BS	-.057500	.057500	.000000	.000000
L	858	LP320108	BS	.000000	.000000	.000000	.000000
L	859	LP320109	UL	.000000	.000000	.093789	.000000
L	860	LP320110	BS	.000000	.000000	.000000	.000000
L	861	LP320111	UL	.000000	.000000	.246341	.000000
L	862	LP330107	BS	-.057500	.057500	.000000	.000000
L	863	LP330108	BS	.000000	.000000	.000000	.000000
L	864	LP330109	UL	.000000	.000000	.093789	.000000
L	865	LP330110	BS	.000000	.000000	.000000	.000000
L	866	LP330111	UL	.000000	.000000	.071498	.000000
L	867	LP340107	BS	-.016765	.016765	.000000	.000000
L	868	LP340108	BS	.000000	.000000	.000000	.000000
L	869	LP340109	UL	.000000	.000000	.093789	.000000
L	870	LP340110	BS	.000000	.000000	.000000	.000000
L	871	LP340111	UL	.000000	.000000	.071498	.000000
L	872	LP350107	UL	.000000	.000000	.829173	.000000
L	873	LP350108	UL	.000000	.000000	.735384	.000000
L	874	LP350109	UL	.000000	.000000	.654330	.000000
L	875	LP350110	UL	.000000	.000000	.582832	.000000
L	876	LP350111	UL	.000000	.000000	.431618	.000000
L	877	LP360107	BS	.871472	.328528	.000000	1.200000
L	878	LP360108	BS	.965347	.234653	.000000	1.200000
L	879	LP360109	BS	.965347	.234653	.000000	1.200000
L	880	LP360110	BS	.875325	.324675	.000000	1.200000
L	881	LP360111	BS	.875952	.324048	.000000	1.200000
L	882	LP370107	BS	-.328528	.378528	.000000	.050000
L	883	LP370108	BS	-.234653	.284653	.000000	.050000
L	884	LP370109	BS	-.234653	.284653	.000000	.050000
L	885	LP370110	BS	-.324675	.374675	.000000	.050000
L	886	LP370111	BS	-.324048	.374048	.000000	.050000
L	887	LP380107	BS	-.050000	.050000	.000000	.000000
L	888	LP390107	BS	.000000	.050000	.000000	.050000
L	889	LP400108	BS	.000000	.050000	.000000	.050000
L	890	LP400109	BS	.000000	.050000	.000000	.050000
L	891	LP400110	BS	.000000	.050000	.000000	.050000
L	892	LP400111	BS	.000000	.050000	.000000	.050000
L	893	LP410108	BS	.000000	.050000	.000000	.050000
L	894	LP410109	BS	.000000	.050000	.000000	.050000
L	895	LP410110	BS	.000000	.050000	.000000	.050000
L	896	LP410111	BS	.000000	.050000	.000000	.050000

Columns Section

	Number	Column	At	Value	Input Cost	Reduced Cost
C	897	accept01	LL	.000000	.000000	.483500
C	898	accept02	LL	.000000	.000000	.484577
C	899	accept03	BS	1.000000	.000000	.000000
C	900	accept04	LL	.000000	.000000	.328195
C	901	accept05	LL	.000000	.000000	.732981
C	902	accept06	LL	.000000	.000000	.056683
C	903	accept07	BS	.000000	.000000	.000000
C	904	accept08	LL	.000000	.000000	.393973
C	905	accept09	LL	.000000	.000000	.383906
C	906	accept10	LL	.000000	.000000	1.133525

C	907	accept11		LL	.000000	.000000	1.029655
C	908	P1	0107	BS	.000000	.000000	.000000
C	909	P1	0108	BS	.212500	.000000	.000000
C	910	P1	0109	BS	.434380	.000000	.000000
C	911	P1	0110	BS	.375780	.000000	.000000
C	912	P1	0111	LL	.000000	.000000	.000000
C	913	P2	0107	BS	.000000	.000000	.000000
C	914	P2	0108	LL	.000000	.000000	.000000
C	915	P2	0109	BS	.377574	.000000	.000000
C	916	P2	0110	BS	.375780	.000000	.000000
C	917	P2	0111	BS	.000000	.000000	.000000
C	918	P3	0106	BS	.287500	.000000	.000000
C	919	P3	0107	BS	.000000	.000000	.000000
C	920	P3	0108	BS	.000000	.000000	.000000
C	921	P3	0109	BS	.342921	.000000	.000000
C	922	P3	0110	BS	.341127	.000000	.000000
C	923	P3	0111	BS	.000000	.000000	.000000
C	924	P4	0105	BS	.287500	.000000	.000000
C	925	P4	0106	BS	.125735	.000000	.000000
C	926	P4	0107	BS	.000000	.000000	.000000
C	927	P4	0108	BS	.000000	.000000	.000000
C	928	P4	0109	BS	.342921	.000000	.000000
C	929	P4	0110	BS	.341127	.000000	.000000
C	930	P5	0104	BS	.287500	.000000	.000000
C	931	P5	0105	BS	.287500	.000000	.000000
C	932	P5	0106	BS	.125735	.000000	.000000
C	933	P5	0107	BS	.000000	.000000	.000000
C	934	P5	0108	BS	.000000	.000000	.000000
C	935	P5	0109	BS	.342921	.000000	.000000
C	936	P6	0104	BS	.287500	.000000	.000000
C	937	P6	0105	BS	.287500	.000000	.000000
C	938	P6	0106	BS	.125735	.000000	.000000
C	939	P6	0107	BS	.000000	.000000	.000000
C	940	P6	0108	BS	.000000	.000000	.000000
C	941	L1	0107	BS	.361765	.000000	.000000
C	942	L1	0108	LL	.000000	.000000	.000000
C	943	L1	0109	BS	.000000	.000000	.000000
C	944	L1	0110	BS	.000000	.000000	.000000
C	945	L1	0111	BS	.092970	.000000	.000000
C	946	LF1	0107	BS	.000000	.000000	.000000
C	947	LF1	0108	LL	.000000	.000000	.000000
C	948	LF1	0109	LL	.000000	.000000	.000000
C	949	LF1	0110	LL	.000000	.000000	.042039
C	950	LF1	0111	BS	.092970	.000000	.000000
C	951	L1a	0107	BS	.361765	.000000	.000000
C	952	L1a	0108	BS	.000000	.000000	.000000
C	953	L1a	0109	BS	.000000	.000000	.000000
C	954	L1a	0110	BS	.000000	.000000	.000000
C	955	L1a	0111	BS	.000000	.000000	.000000
C	956	L1b	0107	BS	.161765	.000000	.000000
C	957	L1b	0108	LL	.000000	.000000	.000000
C	958	L1b	0109	BS	.000000	.000000	.000000
C	959	L1b	0110	BS	.000000	.000000	.000000
C	960	L1b	0111	BS	.000000	.000000	.000000
C	961	IN2	0107	BS	.000000	.000000	.000000
C	962	IN2	0108	BS	.200000	.000000	.000000
C	963	IN2	0109	BS	.200000	.000000	.000000
C	964	IN2	0110	BS	.200000	.000000	.000000
C	965	IN2	0111	BS	.200000	.000000	.000000
C	966	LB	0107	BS	.161765	.000000	.000000
C	967	LB	0108	LL	.000000	.000000	.000000
C	968	LB	0109	BS	.000000	.000000	.000000
C	969	LB	0110	BS	.000000	.000000	.000000
C	970	LB	0111	BS	.000000	.000000	.000000
C	971	LF2	0107	BS	.000000	.000000	.000000
C	972	LF2	0108	BS	.000000	.000000	.000000
C	973	LF2	0109	LL	.000000	.000000	.000000
C	974	LF2	0110	LL	.000000	.000000	.042039
C	975	LF2	0111	BS	.000000	.000000	.000000
C	976	L2	0107	BS	.000000	.000000	.000000
C	977	L2	0108	BS	.000000	.000000	.000000
C	978	L2	0109	BS	.000000	.000000	.000000

C	979	L2	0110	LL	.000000	.000000	.042039
C	980	L2	0111	BS	.000000	.000000	.000000
C	981	L3	0107	BS	.000000	.000000	.000000
C	982	L3	0108	BS	.000000	.000000	.000000
C	983	L3	0109	BS	.000000	.000000	.000000
C	984	L3	0110	BS	.000000	.000000	.000000
C	985	L3	0111	LL	.000000	.000000	.031314
C	986	L4	0107	LL	.000000	.000000	.084067
C	987	L4	0108	BS	.000000	.000000	.000000
C	988	L4	0109	BS	.000000	.000000	.000000
C	989	L4	0110	BS	.000000	.000000	.000000
C	990	L4	0111	BS	.000000	.000000	.000000
C	991	T1	0106	BS	.071875	.000000	.000000
C	992	T1	0107	BS	.000000	.000000	.000000
C	993	T1	0108	BS	.000000	.000000	.000000
C	994	T1	0109	BS	.090022	.000000	.000000
C	995	T1	0110	BS	.089394	.000000	.000000
C	996	T1	0111	BS	.000000	.000000	.000000
C	997	T2	0105	BS	.071875	.000000	.000000
C	998	T2	0106	BS	.031434	.000000	.000000
C	999	T2	0107	BS	.000000	.000000	.000000
C	1000	T2	0108	BS	.000000	.000000	.000000
C	1001	T2	0109	BS	.090022	.000000	.000000
C	1002	T2	0110	BS	.089394	.000000	.000000
C	1003	T3	0104	BS	.071875	.000000	.000000
C	1004	T3	0105	BS	.071875	.000000	.000000
C	1005	T3	0106	BS	.031434	.000000	.000000
C	1006	T3	0107	BS	.000000	.000000	.000000
C	1007	T3	0108	BS	.000000	.000000	.000000
C	1008	T3	0109	BS	.090022	.000000	.000000
C	1009	T4	0104	BS	.071875	.000000	.000000
C	1010	T4	0105	BS	.071875	.000000	.000000
C	1011	T4	0106	BS	.031434	.000000	.000000
C	1012	T4	0107	BS	.000000	.000000	.000000
C	1013	T4	0108	BS	.000000	.000000	.000000
C	1014	LM1	0106	BS	.041667	.000000	.000000
C	1015	LM1	0107	BS	1.000000	.000000	.000000
C	1016	LM1	0108	BS	1.000000	.000000	.000000
C	1017	LM1	0109	UL	.000000	.000000	.000000
C	1018	LM1	0110	UL	.000000	.000000	.000000
C	1019	LM1	0111	BS	1.000000	.000000	.000000
C	1020	LM2	0106	BS	.958333	.000000	.000000
C	1021	LM2	0107	LL	.000000	.000000	.000000
C	1022	LM2	0108	LL	.000000	.000000	.000000
C	1023	LM2	0109	BS	.964233	.000000	.000000
C	1024	LM2	0110	BS	.965728	.000000	.000000
C	1025	LM2	0111	BS	.000000	.000000	.000000
C	1026	LM3	0106	UL	.000000	.000000	.000000
C	1027	LM3	0107	LL	.000000	.000000	.000000
C	1028	LM3	0108	LL	.000000	.000000	.000000
C	1029	LM3	0109	BS	.035767	.000000	.000000
C	1030	LM3	0110	BS	.034272	.000000	.000000
C	1031	LM3	0111	LL	.000000	.000000	.051794
C	1032	LM4	0106	UL	.000000	.000000	.000000
C	1033	LM4	0107	BS	.000000	.000000	.000000
C	1034	LM4	0108	BS	.000000	.000000	.000000
C	1035	LM4	0109	UL	.000000	.000000	.000000
C	1036	LM4	0110	UL	.000000	.000000	.000000
C	1037	LM4	0111	LL	.000000	.000000	.345295
C	1038	LM5	0105	BS	.041667	.000000	.000000
C	1039	LM5	0106	BS	.580884	.000000	.000000
C	1040	LM5	0107	BS	1.000000	.000000	.000000
C	1041	LM5	0108	BS	1.000000	.000000	.000000
C	1042	LM5	0109	LL	.000000	.000000	.002145
C	1043	LM5	0110	LL	.000000	.000000	.017485
C	1044	LM6	0105	BS	.958333	.000000	.000000
C	1045	LM6	0106	BS	.419116	.000000	.000000
C	1046	LM6	0107	BS	.000000	.000000	.000000
C	1047	LM6	0108	BS	.000000	.000000	.000000
C	1048	LM6	0109	BS	.964233	.000000	.000000
C	1049	LM6	0110	BS	.965728	.000000	.000000
C	1050	LM7	0105	UL	.000000	.000000	-.128899

C	1051	LM7	0106	LL	.000000	.000000	.029398
C	1052	LM7	0107	LL	.000000	.000000	.011255
C	1053	LM7	0108	LL	.000000	.000000	.009726
C	1054	LM7	0109	BS	.035767	.000000	.000000
C	1055	LM7	0110	BS	.034272	.000000	.000000
C	1056	LM8	0105	UL	.000000	.000000	-.859328
C	1057	LM8	0106	LL	.000000	.000000	.195990
C	1058	LM8	0107	LL	.000000	.000000	.075031
C	1059	LM8	0108	LL	.000000	.000000	.064843
C	1060	LM8	0109	UL	.000000	.000000	-.012155
C	1061	LM8	0110	UL	.000000	.000000	-.099081
C	1062	LM9	0104	BS	.041667	.000000	.000000
C	1063	LM9	0105	BS	.041667	.000000	.000000
C	1064	LM9	0106	BS	.580884	.000000	.000000
C	1065	LM9	0107	BS	1.000000	.000000	.000000
C	1066	LM9	0108	BS	1.000000	.000000	.000000
C	1067	LM9	0109	LL	.000000	.000000	.017485
C	1068	LM100104		BS	.958333	.000000	.000000
C	1069	LM100105		BS	.958333	.000000	.000000
C	1070	LM100106		BS	.419116	.000000	.000000
C	1071	LM100107		BS	.000000	.000000	.000000
C	1072	LM100108		BS	.000000	.000000	.000000
C	1073	LM100109		BS	.964233	.000000	.000000
C	1074	LM110104		UL	.000000	.000000	-.128899
C	1075	LM110105		LL	.000000	.000000	.029398
C	1076	LM110106		LL	.000000	.000000	.011255
C	1077	LM110107		LL	.000000	.000000	.009726
C	1078	LM110108		LL	.000000	.000000	.008580
C	1079	LM110109		BS	.035767	.000000	.000000
C	1080	LM120104		UL	.000000	.000000	-.859328
C	1081	LM120105		LL	.000000	.000000	.195990
C	1082	LM120106		LL	.000000	.000000	.075031
C	1083	LM120107		LL	.000000	.000000	.064843
C	1084	LM120108		LL	.000000	.000000	.057198
C	1085	LM120109		UL	.000000	.000000	-.099081
C	1086	LM130104		BS	.041667	.000000	.000000
C	1087	LM130105		BS	.041667	.000000	.000000
C	1088	LM130106		BS	.580884	.000000	.000000
C	1089	LM130107		BS	1.000000	.000000	.000000
C	1090	LM130108		BS	1.000000	.000000	.000000
C	1091	LM140104		BS	.958333	.000000	.000000
C	1092	LM140105		BS	.958333	.000000	.000000
C	1093	LM140106		BS	.419116	.000000	.000000
C	1094	LM140107		BS	.000000	.000000	.000000
C	1095	LM140108		BS	.000000	.000000	.000000
C	1096	LM150104		LL	.000000	.000000	.128899
C	1097	LM150105		LL	.000000	.000000	.099501
C	1098	LM150106		LL	.000000	.000000	.088246
C	1099	LM150107		LL	.000000	.000000	.078520
C	1100	LM150108		LL	.000000	.000000	.069940
C	1101	LM160104		LL	.000000	.000000	.859328
C	1102	LM160105		LL	.000000	.000000	.663338
C	1103	LM160106		LL	.000000	.000000	.588307
C	1104	LM160107		LL	.000000	.000000	.523464
C	1105	LM160108		LL	.000000	.000000	.466265
C	1106	XC	0107	BS	.234653	.000000	.000000
C	1107	XC	0108	BS	.034653	.000000	.000000
C	1108	XC	0109	BS	.000000	.000000	.000000
C	1109	XC	0110	LL	.000000	.000000	.031314
C	1110	XC	0111	BS	.034653	.000000	.000000
C	1111	LF	0106	BS	.000000	.000000	.000000
C	1112	LF	0107	BS	.234653	.000000	.000000
C	1113	LF	0108	BS	.056806	.000000	.000000
C	1114	LF	0109	LL	.000000	.000000	.010725
C	1115	LF	0110	BS	.000000	.000000	.000000
C	1116	LF	0111	BS	.127623	.000000	.000000
C	1117	LFF	0107	BS	.000000	.000000	.000000
C	1118	LFF	0108	BS	.022153	.000000	.000000
C	1119	LFF	0109	BS	.000000	.000000	.000000
C	1120	LFF	0110	LL	.000000	.000000	.031314
C	1121	LFF	0111	BS	.000000	.000000	.000000
C	1122	TR	0106	BS	.000000	.000000	.000000

C	1123	TR	0107	BS	.254853	.000000	.000000
C	1124	TR	0108	BS	.000000	.000000	.000000
C	1125	TR	0109	BS	.000000	.000000	.000000
C	1126	TR	0110	BS	.000000	.000000	.000000
C	1127	TR	0111	LL	.000000	.000000	.000000
C	1128	SD	0107	BS	.246765	.000000	.000000
C	1129	SD	0108	BS	.293069	.000000	.000000
C	1130	SD	0109	BS	.244485	.000000	.000000
C	1131	SD	0110	BS	.210556	.000000	.000000
C	1132	SD	0111	BS	.228836	.000000	.000000
C	1133	SF	0106	BS	.067500	.000000	.000000
C	1134	SF	0107	BS	.099853	.000000	.000000
C	1135	SF	0108	BS	.392922	.000000	.000000
C	1136	SF	0109	BS	.637408	.000000	.000000
C	1137	SF	0110	BS	.847964	.000000	.000000
C	1138	SF	0111	BS	1.076800	.000000	.000000
C	1139	SFF	0107	BS	.067500	.000000	.000000
C	1140	SFF	0108	BS	.099853	.000000	.000000
C	1141	SFF	0109	BS	.392922	.000000	.000000
C	1142	SFF	0110	BS	.637408	.000000	.000000
C	1143	SFF	0111	BS	.847964	.000000	.000000
C	1144	SL1	0106	BS	.032353	.000000	.000000
C	1145	SL1	0107	LL	.000000	.000000	.000000
C	1146	SL1	0108	BS	.000000	.000000	.000000
C	1147	SL1	0109	LL	.000000	.000000	.000000
C	1148	SL1	0110	BS	.000000	.000000	.000000
C	1149	SL2	0105	LL	.000000	.000000	.642542
C	1150	SL2	0106	LL	.000000	.000000	.000000
C	1151	SL2	0107	BS	.000000	.000000	.000000
C	1152	SL2	0108	BS	.000000	.000000	.000000
C	1153	SL2	0109	BS	.000000	.000000	.000000
C	1154	SL3	0104	LL	.000000	.000000	.642542
C	1155	SL3	0105	LL	.000000	.000000	.000000
C	1156	SL3	0106	LL	.000000	.000000	.000000
C	1157	SL3	0107	BS	.000000	.000000	.000000
C	1158	SL3	0108	BS	.000000	.000000	.000000
C	1159	U1	0107	BS	.000000	.000000	.000000
C	1160	U1	0108	BS	.000000	.000000	.000000
C	1161	U1	0109	BS	.000000	.000000	.000000
C	1162	U1	0110	BS	.000000	.000000	.000000
C	1163	U1	0111	LL	.000000	.000000	.000000
C	1164	U2	0106	BS	.000000	.000000	.000000
C	1165	U2	0107	BS	.000000	.000000	.000000
C	1166	U2	0108	BS	.000000	.000000	.000000
C	1167	U2	0109	BS	.000000	.000000	.000000
C	1168	U2	0110	BS	.000000	.000000	.000000
C	1169	U2	0111	BS	.000000	.000000	.000000
C	1170	U3	0106	LL	.000000	.000000	.244987
C	1171	U3	0107	BS	.000000	.000000	.000000
C	1172	U3	0108	BS	.000000	.000000	.000000
C	1173	U3	0109	BS	.000000	.000000	.000000
C	1174	U3	0110	BS	.000000	.000000	.000000
C	1175	U4	0105	BS	.057500	.000000	.000000
C	1176	U4	0106	BS	.000000	.000000	.000000
C	1177	U4	0107	BS	.000000	.000000	.000000
C	1178	U4	0108	BS	.000000	.000000	.000000
C	1179	U4	0109	LL	.000000	.000000	.071498
C	1180	U5	0104	BS	.057500	.000000	.000000
C	1181	U5	0105	BS	.000000	.000000	.000000
C	1182	U5	0106	BS	.000000	.000000	.000000
C	1183	U5	0107	BS	.000000	.000000	.000000
C	1184	U5	0108	LL	.000000	.000000	.152552
C	1185	SDF	0107	LL	.000000	.000000	.642542
C	1186	SDF	0108	BS	.293069	.000000	.000000
C	1187	SDF	0109	BS	.244485	.000000	.000000
C	1188	SDF	0110	BS	.210556	.000000	.000000
C	1189	SDF	0111	BS	.228836	.000000	.000000
C	1190	SDB1	0107	BS	.246765	.000000	.000000
C	1191	SDB1	0108	BS	.000000	.000000	.000000
C	1192	SDB1	0109	BS	.000000	.000000	.000000
C	1193	SDB1	0110	BS	.000000	.000000	.000000
C	1194	SDB1	0111	BS	.000000	.000000	.000000

C	1195	SDB20107		BS	.246765	.000000	.000000
C	1196	SDB20108		BS	.000000	.000000	.000000
C	1197	SDB20109		BS	.000000	.000000	.000000
C	1198	SDB20110		BS	.000000	.000000	.000000
C	1199	SDB20111		BS	.000000	.000000	.000000
C	1200	SDB30107		BS	.189265	.000000	.000000
C	1201	SDB30108		BS	.000000	.000000	.000000
C	1202	SDB30109		BS	.000000	.000000	.000000
C	1203	SDB30110		BS	.000000	.000000	.000000
C	1204	SDB30111		BS	.000000	.000000	.000000
C	1205	SDB40107		BS	.131765	.000000	.000000
C	1206	SDB40108		BS	.000000	.000000	.000000
C	1207	SDB40109		BS	.000000	.000000	.000000
C	1208	SDB40110		BS	.000000	.000000	.000000
C	1209	SDB40111		LL	.000000	.000000	.000000
C	1210	SDB50107		BS	.074265	.000000	.000000
C	1211	SDB50108		BS	.000000	.000000	.000000
C	1212	SDB50109		BS	.000000	.000000	.000000
C	1213	SDB50110		BS	.000000	.000000	.000000
C	1214	SDB50111		BS	.000000	.000000	.000000
C	1215	SDB60107		BS	.016765	.000000	.000000
C	1216	SDB60108		BS	.000000	.000000	.000000
C	1217	SDB60109		LL	.000000	.000000	.000000
C	1218	SDB60110		BS	.000000	.000000	.000000
C	1219	SDB60111		BS	.000000	.000000	.000000
C	1220	SDB70107		LL	.000000	.000000	.642542
C	1221	SDB70108		LL	.000000	.000000	.000000
C	1222	SDB70109		BS	.000000	.000000	.000000
C	1223	SDB70110		LL	.000000	.000000	.000000
C	1224	SDB70111		BS	.000000	.000000	.000000
C	1225	U3A	0105	BS	.057500	.000000	.000000
C	1226	U3A	0106	BS	.000000	.000000	.000000
C	1227	U3A	0107	BS	.000000	.000000	.000000
C	1228	U3A	0108	BS	.000000	.000000	.000000
C	1229	U3A	0109	BS	.000000	.000000	.000000
C	1230	U3A	0110	LL	.000000	.000000	.151213
C	1231	U4A	0104	BS	.057500	.000000	.000000
C	1232	U4A	0105	BS	.000000	.000000	.000000
C	1233	U4A	0106	LL	.000000	.000000	.093789
C	1234	U4A	0107	BS	.000000	.000000	.000000
C	1235	U4A	0108	BS	.000000	.000000	.000000
C	1236	U4A	0109	LL	.000000	.000000	.151213
C	1237	U5A	0103	BS	.057500	.000000	.000000
C	1238	U5A	0104	BS	.000000	.000000	.000000
C	1239	U5A	0105	BS	.000000	.000000	.000000
C	1240	U5A	0106	LL	.000000	.000000	.081054
C	1241	U5A	0107	BS	.000000	.000000	.000000
C	1242	U5A	0108	LL	.000000	.000000	.151213
C	1243	U6	0102	BS	.057500	.000000	.000000
C	1244	U6	0103	BS	.000000	.000000	.000000
C	1245	U6	0104	BS	.000000	.000000	.000000
C	1246	U6	0105	LL	.000000	.000000	.174843
C	1247	U6	0106	BS	.000000	.000000	.000000
C	1248	U6	0107	LL	.000000	.000000	.397554
C	1249	U7	0101	BS	.057500	.000000	.000000
C	1250	U7	0102	BS	.000000	.000000	.000000
C	1251	U7	0103	BS	.000000	.000000	.000000
C	1252	U7	0104	LL	.000000	.000000	.174843
C	1253	U7	0105	BS	.000000	.000000	.000000
C	1254	U7	0106	LL	.000000	.000000	.222711
C	1255	U8	0101	BS	.040735	.000000	.000000
C	1256	U8	0102	LL	.000000	.000000	.397554
C	1257	U8	0103	LL	.000000	.000000	.397554
C	1258	U8	0104	LL	.000000	.000000	.222711
C	1259	U8	0105	LL	.000000	.000000	.222711
C	1260	A1	0107	BS	.246765	.000000	.000000
C	1261	A1	0108	BS	.293069	.000000	.000000
C	1262	A1	0109	BS	.313069	.000000	.000000
C	1263	A1	0110	BS	.278782	.000000	.000000
C	1264	A1	0111	BS	.228836	.000000	.000000
C	1265	A2	0106	BS	.057500	.000000	.000000
C	1266	A2	0107	LL	.000000	.000000	.642542

C	1267	A2	0108	BS	.000000	.000000	.000000
C	1268	A2	0109	BS	.068584	.000000	.000000
C	1269	A2	0110	BS	.068225	.000000	.000000
C	1270	A2	0111	BS	.000000	.000000	.000000
C	1271	OCB	0107	BS	.050000	.000000	.000000
C	1272	LNS	0107	LL	.000000	.000000	.060718
C	1273	LNS	0108	LL	.000000	.000000	.041905
C	1274	LNS	0109	LL	.000000	.000000	.035809
C	1275	LNS	0110	LL	.000000	.000000	.031238
C	1276	LNS	0111	LL	.000000	.000000	.028626
C	1277	DBS	0107	BS	.020000	.000000	.000000
C	1278	DBS	0108	BS	.000000	.000000	.000000
C	1279	DBS	0109	BS	.000000	.000000	.000000
C	1280	DBS	0110	BS	.000000	.000000	.000000
C	1281	DBS	0111	BS	.000000	.000000	.000000
C	1282	DBI	07	BS	.488253	.000000	.000000
C	1283	DBI	08	BS	.000000	.000000	.000000
C	1284	DBI	09	BS	.000000	.000000	.000000
C	1285	DBI	10	BS	.000000	.000000	.000000
C	1286	DBI	11	BS	.000000	.000000	.000000
C	1287	DBC	07	LL	.000000	.000000	.946849
C	1288	DBC	08	LL	.000000	.000000	.687001
C	1289	DBC	09	LL	.000000	.000000	.628978
C	1290	DBC	10	LL	.000000	.000000	.579396
C	1291	DBC	11	LL	.000000	.000000	.536144
C	1292	LREP	07	LL	.000000	.000000	.387159
C	1293	LREP	08	LL	.000000	.000000	.200195
C	1294	LREP	09	LL	.000000	.000000	.155988
C	1295	LREP	10	LL	.000000	.000000	.118186
C	1296	LREP	11	LL	.000000	.000000	.086324
C	1297	DB	06	BS	2.250000	.000000	.000000
C	1298	DB	07	BS	2.738253	.000000	.000000
C	1299	DB	08	BS	2.738253	.000000	.000000
C	1300	DB	09	BS	2.738253	.000000	.000000
C	1301	DB	10	BS	2.738253	.000000	.000000
C	1302	DB	11	BS	2.738253	.000000	.000000
C	1303	DBIN	07	BS	.234653	.000000	.000000
C	1304	DBIN	08	BS	.234653	.000000	.000000
C	1305	DBIN	09	BS	.234653	.000000	.000000
C	1306	DBIN	10	BS	.234653	.000000	.000000
C	1307	DBIN	11	BS	.234653	.000000	.000000
C	1308	EQI	07	BS	.511747	-1.000000	.000000
C	1309	EQI	08	LL	.000000	-.859328	.173371
C	1310	EQI	09	LL	.000000	-.742849	.115721
C	1311	EQI	10	BS	.000000	-.649060	.000000
C	1312	EQI	11	BS	.000000	-.568006	.000000
C	1313	issue	07	BS	10.234939	.000000	.000000
C	1314	issue	08	BS	.000000	.000000	.000000
C	1315	issue	09	BS	.000000	.000000	.000000
C	1316	issue	10	LL	.000000	.000000	.003599
C	1317	issue	11	LL	.000000	.000000	.001708
C	1318	EQ	0106	BS	8.920000	.000000	.000000
C	1319	EQ	0107	BS	9.431747	.000000	.000000
C	1320	EQ	0108	BS	9.431747	.000000	.000000
C	1321	EQ	0109	BS	9.431747	.000000	.000000
C	1322	EQ	0110	BS	9.431747	.000000	.000000
C	1323	EQ	0111	BS	9.431747	.000000	.000000
C	1324	D2	07	BS	.380316	.000000	.000000
C	1325	D2	08	BS	.380316	.000000	.000000
C	1326	D2	09	BS	.380316	.000000	.000000
C	1327	D2	10	BS	.380316	.000000	.000000
C	1328	D2	11	BS	.380316	.000000	.000000
C	1329	D3	07	LL	.000000	.000000	.004448
C	1330	D3	08	LL	.000000	.000000	.003223
C	1331	D3	09	BS	.000000	.000000	.000000
C	1332	D3	10	LL	.000000	.000000	.002403
C	1333	D3	11	BS	.000000	.000000	.000000
C	1334	D4	07	LL	.000000	.000000	.004448
C	1335	D4	08	LL	.000000	.000000	.003223
C	1336	D4	09	LL	.000000	.000000	.005509
C	1337	D4	10	LL	.000000	.000000	.002403
C	1338	D4	11	LL	.000000	.000000	.004404

C	1339	D5	07	LL	.000000	.000000	.004448
C	1340	D5	08	LL	.000000	.000000	.003223
C	1341	D5	09	LL	.000000	.000000	.002755
C	1342	D5	10	LL	.000000	.000000	.002403
C	1343	D5	11	LL	.000000	.000000	.002202
C	1344	x1	07	LL	.000000	.000000	.004448
C	1345	x1	08	LL	.000000	.000000	.003223
C	1346	x1	09	LL	.000000	.000000	.002755
C	1347	x1	10	LL	.000000	.000000	.002403
C	1348	x1	11	LL	.000000	.000000	.002202
C	1349	x2	07	BS	.091271	.000000	.000000
C	1350	x2	08	BS	.091271	.000000	.000000
C	1351	x2	09	BS	.091271	.000000	.000000
C	1352	x2	10	BS	.091271	.000000	.000000
C	1353	x2	11	BS	.091271	.000000	.000000
C	1354	x3	07	BS	.471587	.000000	.000000
C	1355	x3	08	BS	.471587	.000000	.000000
C	1356	x3	09	BS	.471587	.000000	.000000
C	1357	x3	10	BS	.471587	.000000	.000000
C	1358	x3	11	BS	.471587	.000000	.000000
C	1359	x4	07	BS	.471587	.000000	.000000
C	1360	x4	08	BS	.471587	.000000	.000000
C	1361	x4	09	BS	.471587	.000000	.000000
C	1362	x4	10	BS	.471587	.000000	.000000
C	1363	x4	11	BS	.471587	.000000	.000000
C	1364	y1	07	BS	2.357937	.000000	.000000
C	1365	y1	08	BS	2.357937	.000000	.000000
C	1366	y1	09	BS	2.357937	.000000	.000000
C	1367	y1	10	BS	2.357937	.000000	.000000
C	1368	y1	11	BS	2.357937	.000000	.000000
C	1369	y2	07	BS	.380316	.000000	.000000
C	1370	y2	08	BS	.380316	.000000	.000000
C	1371	y2	09	BS	.380316	.000000	.000000
C	1372	y2	10	BS	.380316	.000000	.000000
C	1373	y2	11	BS	.380316	.000000	.000000
C	1374	y3	07	BS	.000000	.000000	.000000
C	1375	y3	08	BS	.000000	.000000	.000000
C	1376	y3	09	LL	.000000	.000000	.002755
C	1377	y3	10	BS	.000000	.000000	.000000
C	1378	y3	11	LL	.000000	.000000	.002202
C	1379	y4	07	BS	.000000	.000000	.000000
C	1380	y4	08	BS	.000000	.000000	.000000
C	1381	y4	09	BS	.000000	.000000	.000000
C	1382	y4	10	BS	.000000	.000000	.000000
C	1383	y4	11	BS	.000000	.000000	.000000
C	1384	CSHC	07	LL	.000000	.000000	.200173
C	1385	CSHC	08	LL	.000000	.000000	.372976
C	1386	CSHC	09	LL	.000000	.000000	.188075
C	1387	CSHC	10	LL	.000000	.000000	.145415
C	1388	CSHC	11	LL	.000000	.000000	.108497
C	1389	RET	0107	LL	.000000	.000000	.179711
C	1390	RET	0108	LL	.000000	.000000	.038009
C	1391	RET	0109	LL	.000000	.000000	.032395
C	1392	RET	0110	LL	.000000	.000000	.026908
C	1393	RET	0111	LL	.000000	.000000	.086324
C	1394	RTN	0107	BS	.000000	.000000	.000000
C	1395	RTN	0108	BS	.000000	.000000	.000000
C	1396	RTN	0109	BS	.000000	.000000	.000000
C	1397	RTN	0110	BS	.000000	.000000	.000000
C	1398	RTN	0111	LL	.000000	.000000	.000000
C	1399	RTD	0107	BS	.000000	.000000	.000000
C	1400	RTD	0108	LL	.000000	.000000	.000000
C	1401	RTD	0109	LL	.000000	.000000	.000000
C	1402	RTD	0110	LL	.000000	.000000	.000000
C	1403	RTD	0111	BS	.000000	.000000	.000000
C	1404	CSHP0107		BS	1.233825	.000000	.000000
C	1405	CSHP0108		BS	1.465347	.000000	.000000
C	1406	CSHP0109		BS	1.565347	.000000	.000000
C	1407	CSHP0110		BS	1.393909	.000000	.000000
C	1408	CSHP0111		BS	1.144178	.000000	.000000
C	1409	CSHN0107		BS	.000000	.000000	.000000
C	1410	CSHN0108		BS	.000000	.000000	.000000

C	1411	CSHN0109	BS	.000000	.000000	.000000
C	1412	CSHN0110	BS	.000000	.000000	.000000
C	1413	CSHN0111	BS	.000000	.000000	.000000
C	1414	DIV 0106	BS	.500000	.000000	.000000
C	1415	DIV 0107	BS	.987060	.000000	.000000
C	1416	DIV 0108	BS	1.172278	.000000	.000000
C	1417	DIV 0109	BS	1.252278	.000000	.000000
C	1418	DIV 0110	BS	1.115127	.000000	.000000
C	1419	DIV 0111	BS	.915342	.000000	.000000
C	1420	DIVG 07	BS	1.233825	.859328	.000000
C	1421	DIVG 08	BS	1.465347	.742849	.000000
C	1422	DIVG 09	BS	1.565347	.649060	.000000
C	1423	DIVG 10	BS	1.393909	.568006	.000000
C	1424	DIVG 11	BS	1.144178	.496508	.000000
C	1425	HV 0111	BS	7.534429	.496508	.000000
C	1426	d1 0107	UL	.000000	.000000	-.030623
C	1427	d1 0108	LL	1.000000	.000000	.000000
C	1428	d1 0109	LL	1.000000	.000000	.000000
C	1429	d1 0110	LL	1.000000	.000000	.026812
C	1430	d1 0111	UL	.000000	.000000	.000000
C	1431	d2 0107	BS	.000000	.000000	.000000
C	1432	d2 0108	UL	.000000	.000000	.000000
C	1433	d2 0109	LL	1.000000	.000000	.000000
C	1434	d2 0110	LL	1.000000	.000000	.000000
C	1435	d2 0111	BS	.000000	.000000	.000000
C	1436	d3 0107	BS	.000000	.000000	.000000
C	1437	d3 0108	BS	.000000	.000000	.000000
C	1438	d3 0109	BS	1.000000	.000000	.000000
C	1439	d3 0110	BS	1.000000	.000000	.000000
C	1440	d3 0111	UL	.000000	.000000	-.086324
C	1441	d4 0106	LL	1.000000	.000000	.321271
C	1442	d4 0107	BS	.000000	.000000	.000000
C	1443	d4 0108	UL	.000000	.000000	-.153451
C	1444	d4 0109	LL	1.000000	.000000	.000000
C	1445	d4 0110	LL	1.000000	.000000	.000000
C	1446	d5 0105	LL	1.000000	.000000	.000000
C	1447	d5 0106	LL	1.000000	.000000	.000000
C	1448	d5 0107	BS	.000000	.000000	.000000
C	1449	d5 0108	BS	.000000	.000000	.000000
C	1450	d5 0109	LL	1.000000	.000000	.000000
C	1451	d6 0104	LL	1.000000	.000000	.000000
C	1452	d6 0105	LL	1.000000	.000000	.000000
C	1453	d6 0106	LL	1.000000	.000000	.000000
C	1454	d6 0107	BS	.000000	.000000	.000000
C	1455	d6 0108	BS	.000000	.000000	.000000
C	1456	d7 0107	LL	1.000000	.000000	.111104
C	1457	d7 0108	BS	1.000000	.000000	.000000
C	1458	d7 0109	BS	1.000000	.000000	.000000
C	1459	d7 0110	BS	1.000000	.000000	.000000
C	1460	d7 0111	UL	.000000	.000000	.000000
C	1461	d8 0107	BS	.000000	.000000	.000000
C	1462	d8 0108	LL	.000000	.000000	.000000
C	1463	d8 0109	BS	.000000	.000000	.000000
C	1464	d8 0110	BS	.000000	.000000	.000000
C	1465	d8 0111	LL	.000000	.000000	.000000
C	1466	d9 0107	BS	.000000	.000000	.000000
C	1467	d9 0108	LL	.000000	.000000	.000000
C	1468	d9 0109	BS	.000000	.000000	.000000
C	1469	d9 0110	BS	.000000	.000000	.000000
C	1470	d9 0111	LL	.000000	.000000	.000000
C	1471	d10 0106	LL	.000000	.000000	.000000
C	1472	d10 0107	BS	.000000	.000000	.000000
C	1473	d10 0108	LL	.000000	.000000	.000000
C	1474	d10 0109	BS	.000000	.000000	.000000
C	1475	d10 0110	BS	.000000	.000000	.000000
C	1476	d11 0106	LL	.000000	.000000	.000000
C	1477	d11 0107	BS	.000000	.000000	.000000
C	1478	d11 0108	LL	.000000	.000000	.000000
C	1479	d11 0109	LL	.000000	.000000	.000000
C	1480	d11 0110	LL	.000000	.000000	.000000
C	1481	d12 0105	LL	1.000000	.000000	.000000
C	1482	d12 0106	LL	.000000	.000000	.000000

C	1483	d12	0107	LL	.000000	.000000	.000000
C	1484	d12	0108	LL	.000000	.000000	.000000
C	1485	d12	0109	LL	.000000	.000000	.000000
C	1486	d13	0105	UL	.000000	.000000	-.993886
C	1487	d13	0106	LL	.000000	.000000	.000000
C	1488	d13	0107	LL	.000000	.000000	.000000
C	1489	d13	0108	LL	.000000	.000000	.000000
C	1490	d13	0109	LL	.000000	.000000	.000000
C	1491	d14	0104	LL	1.000000	.000000	.000000
C	1492	d14	0105	BS	.000000	.000000	.000000
C	1493	d14	0106	LL	.000000	.000000	.000000
C	1494	d14	0107	LL	.000000	.000000	.000000
C	1495	d14	0108	LL	.000000	.000000	.000000
C	1496	d15	0104	UL	.000000	.000000	-.993886
C	1497	d15	0105	BS	.000000	.000000	.000000
C	1498	d15	0106	LL	.000000	.000000	.000000
C	1499	d15	0107	LL	.000000	.000000	.000000
C	1500	d15	0108	LL	.000000	.000000	.000000
C	1501	d16	0103	UL	.000000	.000000	-.993886
C	1502	d16	0104	LL	.000000	.000000	.000000
C	1503	d16	0105	LL	.000000	.000000	.000000
C	1504	d16	0106	LL	.000000	.000000	.000000
C	1505	d16	0107	LL	.000000	.000000	.000000
C	1506	d17	0102	UL	.000000	.000000	-.993886
C	1507	d17	0103	BS	.000000	.000000	.000000
C	1508	d17	0104	LL	.000000	.000000	.000000
C	1509	d17	0105	LL	.000000	.000000	.000000
C	1510	d17	0106	LL	.000000	.000000	.000000
C	1511	d18	0101	LL	1.000000	.000000	.000000
C	1512	d18	0102	BS	.000000	.000000	.000000
C	1513	d18	0103	LL	.000000	.000000	.000000
C	1514	d18	0104	LL	.000000	.000000	.000000
C	1515	d18	0105	LL	.000000	.000000	.000000
C	1516	d19	0107	UL	.000000	.000000	-.030623
C	1517	d19	0108	BS	1.000000	.000000	.000000
C	1518	d19	0109	BS	1.000000	.000000	.000000
C	1519	d19	0110	BS	1.000000	.000000	.000000
C	1520	d19	0111	LL	1.000000	.000000	.078285
C	1521	d20	0107	LL	1.000000	.000000	.111104
C	1522	d20	0108	BS	1.000000	.000000	.000000
C	1523	d20	0109	BS	1.000000	.000000	.000000
C	1524	d20	0110	BS	1.000000	.000000	.000000
C	1525	d20	0111	BS	1.000000	.000000	.000000
C	1526	d24	0107	LL	.000000	.000000	.000000
C	1527	d24	0108	LL	.000000	.000000	.000000
C	1528	d24	0109	BS	.000000	.000000	.000000
C	1529	d24	0110	LL	.000000	.000000	.000000
C	1530	d24	0111	LL	.000000	.000000	.000000
C	1531	d25	07	LL	1.000000	.000000	.000000
C	1532	d25	08	LL	1.000000	.000000	.000000
C	1533	d25	09	LL	1.000000	.000000	.000000
C	1534	d25	10	LL	1.000000	.000000	.000000
C	1535	d25	11	LL	1.000000	.000000	.000000
C	1536	d26	07	LL	.000000	.000000	.000000
C	1537	d26	08	LL	.000000	.000000	.000000
C	1538	d26	09	BS	.000000	.000000	.000000
C	1539	d26	10	BS	.000000	.000000	.000000
C	1540	d26	11	BS	.000000	.000000	.000000
C	1541	d27	07	LL	.000000	.000000	.000000
C	1542	d27	08	LL	.000000	.000000	.000000
C	1543	d27	09	LL	.000000	.000000	.000000
C	1544	d27	10	LL	.000000	.000000	.000000
C	1545	d27	11	LL	.000000	.000000	.000000
C	1546	d28	07	LL	.000000	.000000	.000000
C	1547	d28	08	LL	.000000	.000000	.000000
C	1548	d28	09	LL	.000000	.000000	.000000
C	1549	d28	10	LL	.000000	.000000	.000000
C	1550	d28	11	LL	.000000	.000000	.000000
C	1551	d29	0107	LL	1.000000	.000000	.451293
C	1552	d29	0108	LL	1.000000	.000000	.096745
C	1553	d29	0109	LL	1.000000	.000000	.082490
C	1554	d29	0110	LL	1.000000	.000000	.068646

C	1555	d29	0111	LL	1.000000	.000000	.215809
C	1556	d30	0108	BS	.000000	.000000	.000000
C	1557	d30	0109	BS	.000000	.000000	.000000
C	1558	d30	0110	BS	.000000	.000000	.000000
C	1559	d30	0111	LL	.000000	.000000	.000000

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