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The Use of Concept Questions to Improve Student Understanding of Mechanics, and the Formulation of a Hierarchical Model of Student Understanding of Moments of Forces

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**The Use of Concept Questions to Improve Student Understanding of
Mechanics, and the Formulation of a Hierarchical Model of Student
Understanding of Moments of Forces**

STUART KENNETH ROWLANDS

A thesis submitted to the University of Plymouth for the degree of doctor of
philosophy

Centre for Teaching Mathematics
School of Mathematics and Statistics

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ABSTRACT

The use of Concept Questions to Improve Student Understanding of Mechanics and the Formulation of Hierarchical Models of Student Understanding of Moments of Forces

by

Stuart Kenneth Rowlands

The aims and objectives of the first part of the thesis are to create and evaluate a teaching package that would enable the teacher to facilitate student conceptual understanding of mechanics. The second part is to create a hierarchical model of student conceptual understanding of moments of forces.

The thesis reports on the various conflicting theories and recommended teaching strategies from the research into student misconceptions, and examines the various underlying philosophical trends (for example, radical and social constructivism, the Vygotskian perspective and positivism) that have influenced the research into misconceptions in mechanics. As a result, the philosophical perspective of the thesis is that the analysis of the structure of Newtonian mechanics ought to be before the consideration of challenging misconceptions. The question as to how students can construct for themselves the Newtonian system ought to be prior to the consideration of designing a strategy to tackle misconceptions. With prior consideration of the structure of mechanics, the thesis examines the formation of the *intuitive schema of force and motion*. The thesis proposes the Socratic method of strategic questioning as the most appropriate teaching method for constructing the Newtonian system and displacing the intuitive schema of force and motion. The thesis reports on the formation, and the evaluation, of the teaching package as an aid to facilitate the construction of the Newtonian system, and concludes that teachers have to be trained in the use of the Socratic method prior to any evaluation.

There has been little or no research in the area of misconceptions concerning moments. The thesis reports on the formation of a hierarchical model of understanding moments. From a sample of 417 students nation-wide, the thesis has identified 3 conceptual levels of understanding moments and discusses the teaching implications based on the responses from the sample.

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CHAPTER 1 - INTRODUCTION

1.1 THE ORIGINS, AIMS AND OBJECTIVES OF THE THESIS

Although related, there are two distinct parts to the thesis. The first part is the formation and evaluation of a teaching package (see appendix A, page 281). The aim of the package is to enable the teacher to facilitate the development of an improved conceptual understanding of Newtonian mechanics at A-Level. The second part of the thesis is the formation of a hierarchical model of student conceptual understanding of moments, similar to Graham's (1991) hierarchical model of student conceptual understandings of force and of momentum (see also Graham and Berry, 1996, 1997).

Graham and Berry (1990, 1992), Berry and Graham (1991) and Graham (1991) have identified many of the misconceptions that occur amongst A-level mechanics students in response to *concept questions* on force and motion. A misconception is a misunderstanding of the physical laws from a Newtonian standpoint. The classic student misconception is the Aristotelian view that where there is a net force on an object there is motion and *vice versa* (Berry, 1990). A concept question is a question that is designed to test student understanding of the basic concepts or principle upon which the models of mechanics are based. A concept question demands a qualitative approach to phenomena that can be explained according to the Newtonian system, compared with the traditional mechanics question that demands a quantitative approach to idealised examples. A correct answer to a concept question is a Newtonian explanation of the phenomena modelled. However, not only can concept questions be used diagnostically but they have implications for learning. According to Berry and Graham (1991), concept questions can be incorporated into a teaching strategy that will identify a misconception and develop an approach that

challenges the student to reach a correct view before the mathematical approach begins. The authors suggest that concept questions work the following way (Berry and Graham, 1991):

- a) first, a question often exposes student weaknesses which require a restructuring of their ideas;*
- b) then a group discussion, investigation or appropriate practical work can help restructure the student's ideas into new ones;*
- c) finally the new ideas need testing with the original problem or new similar problems.*

The importance of b) is the restructuring of student ideas into new ones. This raises three related and very pertinent questions: 1) What is it that requires restructuring? 2) What form does this restructuring take? 3) What is required so that restructuring does take place? Each question is dealt with in turn:

1) What is it that requires restructuring?

Student ideas that are formed and used to answer concept questions are sometimes referred to as misconceptions (or preconceptions, intuitive frameworks, etc.) One of the difficulties with analysing misconceptions is that they sometimes defy classification. Some misconceptions resemble the views of Aristotle (e.g. 'force is in the direction of motion'), and some resemble the medieval impetus theory ('the object contains the force'). Some misconceptions confuse force with notions of energy (Viennot's, 1985, *force-energy mixing*), and some mix force with momentum, or pressure. Boeha (1990), using transcripts of interviews from a sample of Papua New Guinea science students, has classified misconceptions of force and motion as:

- Designed forces: objects that are endowed with forces (medieval impetus theory)
- Motive force: forces act upon an object in the direction of the movement (Aristotelian theory).
- Operative forces: force is an action, similar to energy, that can be transformed or consumed at a rate proportional to the level of activity (e.g. 'force was needed to overcome gravity but it was all used up').

- Encounter forces: force as an interaction between bodies or other forces. Velocity is sometimes treated as a force (e.g. 'the force due to the object's velocity counters gravity and so the ball moves up').
- Impact force: a collision is seen as a force similar to elastic/inelastic collisions that involve momentum. The force is sometimes conceived as continuing after the impact. (This view is the most insidious. Sometimes we hear 'A spongy ball does not bounce much because it absorbs most of the force of impact.' Most people think of modern cars as having the ability to absorb impact rather than the ability to extend the impact over a longer duration).
- Configuration forces: forces used to account for the absence of movement. Forces are used to account for situations where nothing is happening but that something is expected to happen (e.g. 'When a ball reaches its maximum height a force is required to counterbalance gravity.' The corollary is 'When a ball reaches maximum height, no forces can be acting on it').

One of the difficulties involved in classifying a student response to a qualitative question is that the misconception may be classified under more than one of the categories above and the category will be dependent upon the reasoning given by the student. The consideration of misconceptions requires a specificity regarding the framework from which the misconceptions occurs, namely:-

- 1) What kind of problem prompts the misconception?
- 2) What formulations are used in holding the misconception?
- 3) How is the misconception linked to other forms of reasoning? (Viennot 1985).

The term 'misconception' implies a concept that is wrong. However, instead of the student holding a fairly well defined concept that is wrong (e.g. 'force is a property of the body'), it may be the case that the student brings together a set of vague notions that are formed when asked to account for motion in some way. [It is argued in chapter 5 that

misconceptions of force are spontaneously created frameworks (*schemata*) by which force is slotted into the framework according to the way the student conceives of the *dominant features* of motion].

2) What form does this restructuring take?

Are the intuitive frameworks of force and motion to be modified or revised in some way, or is the intuitive framework to be removed or displaced so that the appropriate (Newtonian) framework can be constructed? Graham and Berry (1993a) refer to the use of concept questions to promote *cognitive conflict* so that intuitive beliefs can be restructured. The authors also refer to the use of *parallel situations* in conjunction with concept questions. In a parallel situation *the same principles of mechanics apply in the same way, but due to the nature of the parallel situation the students will naturally respond correctly*. The authors cite three examples of the use of parallel situations:

Concept question: *A ball is thrown up into the air. What are the forces acting on the ball?*

Student response: *An upward force.*

Parallel situation: *What is the horizontal force acting on a car that is braking?*

Concept question: *A car is overtaking a lorry. At the instant the car is level with the lorry how do their speeds compare?*

Student response: *The speeds of the two vehicles are equal.*

Parallel situation: *A car is overtaking a parked lorry. At the instant the car is overtaking the lorry how do their speeds compare?*

Concept question: *A car is driven round a roundabout at a constant speed. Draw a diagram to show the forces acting on the car.*

Student response: (Force acting toward centre omitted).

Parallel situation: *Compare car with conker on a piece of string that is swung round in a circle. What are the forces acting on the conker, and is there a component acting towards the centre.*

The use of concept questions and parallel situations (*parallel questions*) seem to be geared towards the inducement of cognitive conflict, rather than the gradual modification of misconceptions.

3) What is required so that restructuring does take place?

Berry and Graham (1991) propose the use of concept questions, in conjunction with practical work, group discussion, investigation and video (similar to the approaches taken by Borghi *et al*, 1987, and Hake, 1987). Much reference in the literature has been made to the role of practical work; however, there has been a great deal of recent criticism of the role of practical work (e.g. Driver, 1994; Kirschner, 1992; Osborne, 1996). A critique of practical work begins in chapter 3 and forms a major part of chapter 4.

Many researchers argue that student misconceptions have to be modified, and to do this the teaching strategy has to focus on student's intuitive ideas. For example, Osborne (1985) states that children often hold the strongly held view that objects move forward because there is 'something' *in* them keeping them moving. What Osborne proposes is that the traditional order of instruction should be reversed so that momentum be taught before force. The idea is that force can be taught as the agent that acts on objects, changing the momentum that the object possesses. What is implicit is that intuitive ideas can be modified, in this case by redirecting the misconception of a moving body possessing force to the idea that it is the momentum, as a quantity of motion, that the body possesses. Force can then be taught as that which changes momentum. The major problem with this approach is that it begins with intuitive ideas and utilises those ideas to arrive at the target concept - the target concept may be assimilated idiosyncratically according to the misconceptions that are utilised (the title of Osborne's article is *Building on Children's Intuitive Ideas*). For example, there is no guarantee that the student will not develop the notion that a body has to have force in order for its momentum to change - if momentum is a quantity that a moving body possesses, then why shouldn't the body possess another quantity that changes the momentum (despite any assertion that forces act on bodies). From the perspective of chapter 3 - that the consideration of the logical structure of

Newtonian mechanics should be prior to the consideration of intuitive ideas - the momentum approach to understanding force might be an appropriate strategy but only because force may be defined in terms of that which is required to change momentum. Osborne is a constructivist, and his approach seems to be a typical one for many constructivists. Many constructivists emphasise subjectivity at the expense of the objective content of the subject to be constructed, and this approach is centrally criticised in chapter 2. The length of chapter 2 is because it deals in depth with the serious pedagogical consequences of this approach.

The aim of the teaching package is to enable the teacher to facilitate the development of a conceptual understanding of Newtonian mechanics. The objectives, however, are twofold. The first objective is to critically evaluate the literature so that a sound methodological approach can be adopted in the creation of the package. The second objective is to critically evaluate the package itself.

1.2 BACKGROUND TO THE THESIS

Mechanics is not an easy option for many students. Those with an 'aptitude' for mathematics may not necessarily find mechanics difficult if presented as the application of a set of formulae and techniques, as they are able to interpret the data according to the learnt underlying principle. For the rest it is a struggle to make sense of abstract ideas and how they relate to quantitative problems. For the majority of students, however, mechanics as a description of the physical world has little or no meaning. Many students who have successfully completed a traditional course in mechanics are quite often incapable of

explaining qualitatively¹ even the most simple of phenomenon in Newtonian terms (Berry, 1990). When asked to give a qualitative judgement on an unfamiliar problem, the student is most likely to resort to intuitive reasoning. Over twenty years of international research has revealed the inability of many students to give a qualitative scientific explanation due to the retention of alternative conceptions. These alternative conceptions have been acquired by the students from the world around them² and are persistent to the extent that they are found in graduates in the UK. (Jagger 1985) and in the U.S.A. (Peters 1985). These 'alternative understandings' of important mechanical concepts can hinder the development of further ideas (Clement 1982), despite the ability to provide a correct answer to the traditional text-book question that demands a quantitative response. This dual perspective view of mechanics, as a set of tricks for solving special simplified problems and as a model of the physical world, has been described, for example, by, Berry and Graham (1991).

Much of the vast research literature on student misconceptions suggest that all students develop intuitive ideas of force and motion prior to a mechanics course, and nearly all students to some extent retain those ideas alongside the ideas learnt in mechanics

¹ How is *understanding mechanics* to be defined? A distinction between *qualitative* and *quantitative* understanding can be made. The former describes the ability of the student to account for physical phenomenon in his or her words but which is in accord with scientific reasoning. The latter describes the ability to apply a series of remembered formulae to a question that either demands a calculation or lends itself to be answered by a *rule of thumb* procedure [similar to Halloun and Hestenes', 1987, *procedural knowledge* - the 'formula-centered' approach to problem solving that is not defective]. Arons (1973) made the distinction between *scientific knowledge*, which is the memorisation of concepts, and *understanding*, by which the student discovers the concept with the aid of the teacher as guide. Subsequently, Arons (1984b, 1990) has made the distinction between *declarative* and *operational* knowledge. The former is knowledge of factual information and the latter describes an understanding of why that information is true (Arons' notion of operational knowledge is criticised in chapter 4).

² We spend most of our lives in contact with objects, both static and moving; and so it would seem reasonable to suppose that people would 'naturally' gain a correct understanding of how forces act on objects. Not only is this not the case but most people develop their ideas of force contrary to physical laws (McCloskey *et al* 1980).

(according to Clement, 1982, those ideas can remain hidden and cope with simple situations, however it can also prevent the student from grasping an understanding of the concept at a higher level). The ideas of force and motion taught in mechanics are usually within the context of idealised examples and questions; for example, point particles sliding down smooth inclined planes connected by inextensible strings. The student learns the laws of motion as rules applied within a specific context, and the understanding of these laws is assessed by the teacher in terms of mathematical statements. The majority of students are unable to respond correctly to qualitative questioning concerning quantitative examples (Arons 1982, 1990; McDermott 1991), and are unable to provide a Newtonian explanation of physical phenomena (Berry and Graham 1991). This is hardly surprising given the traditional and typical mechanics question containing 3 or 4 marks for the mechanics and the rest out of twenty for the pure mathematics; for example:

A wedge of mass M is resting on a rough horizontal table with coefficient of friction μ . A particle of mass m is placed gently on a smooth face of the wedge which is inclined at an angle α to the horizontal, where $\tan\alpha > \mu$. Prove that, if the wedge moves, its acceleration is

$$(mg\sin\alpha\cos\alpha - \mu mg\cos^2\alpha - \mu Mg) / (M + m\sin^2\alpha - \mu m\sin\alpha\cos\alpha).$$

Find also (1) the normal reaction between the particle and the wedge and (2) the acceleration of the particle relative to the wedge.

(Oxford A-Level, 1984)

The mark scheme for this particular question carried 20% for the mechanics and 80% for the algebra (Berry 1990).

My thirteen years secondary teaching experience has shown that it may be possible to overcome misconceptions with a quantitative approach to the subject in the absence of any qualitative treatment of phenomena. Before leaving secondary education I administered Graham's (1991) Force Hierarchy Questionnaire to a class of six upper sixth mechanics students. Graham's thesis had established four levels of conceptual understanding of force

and motion: level 0 representing the lowest level and level 3 the highest. Four of the students achieved level 3 and two achieved level 2 (see appendix B, page 305). The method of teaching was the identification of the forces acting on the body and the application of the laws of motion. The mathematics followed from the laws of motion. For example, in *projectiles* the force identified was the force of gravity (ignoring air resistance) and it therefore followed that $d^2y/dt^2 = -g \Rightarrow$ vertical component of acceleration is due to gravity and is constant; and $d^2x/dt^2 = 0 \Rightarrow$ horizontal component of acceleration is zero because there is no horizontal force acting on the projectile. Had the approach been the application of the constant acceleration formulae (e.g. the application of $s = ut + \frac{1}{2}at^2$ with the assumption that $a = -g$ vertically upwards) then all the students concerned might have failed on the level 3 questions concerning force in two dimensions. Such an approach in the quantitative teaching of mechanics might enable the student to correctly answer a qualitative question **but perhaps only because the question is within the scope of the subject already taught!** The students could correctly identify the forces acting on a ball thrown vertically upward because that was the approach given in the teaching of motion under gravity. If the question is totally unfamiliar to the student, for example the identification of forces acting on a penny sliding on a revolving turntable, then the student will most likely resort to his or her own intuitive reasoning (one of the above six students argued that if the coin slipped then a force must be acting radially outward). The aim of the project is to develop a teaching strategy that will enable the student to model unfamiliar phenomena as a consequence of a qualitative understanding of force and motion, rather than the 'rule-of-thumb' application of quantitative techniques³. Chapter 5 of the thesis, in collaboration with chapter 3, argues that the ability to model the unfamiliar would require more than the class developing a qualitative understanding of a series of phenomena as

³ This is not to undermine the obvious importance of quantitative questions. The point is that quantitative questions are, by themselves, insufficient to enable the student to develop a Newtonian understanding of physical phenomena.

presented by the teacher. There is evidence to suggest that a piecemeal strategy to overcome misconceptions is ineffectual (Hestenes, 1992), and this implies that the ability to provide a Newtonian account for the unfamiliar requires the construction of the Newtonian system as a *unified-form* of description. The precise meaning of this term 'unified-form' will be one of the central features of chapter 3, and the relevance of such a term (especially in chapter 5) will be shown in the way intuitive ideas of force and motion lacks such a unified-form.

1.3 ON THE NATURE OF MISCONCEPTIONS

A variety of names such as *misconceptions*, *preconceptions*, *alternative frameworks*, *intuitive ideas* and *student beliefs* have been used to refer to ideas that have been assimilated in such a way that a true understanding of concepts in mechanics has become almost impossible. The most frequently used term is *misconceptions*, but for many researchers such a term is inappropriate. Arons (1990), for example, argues that the term is derogatory and misleading, leaving the impression that the misconception is to be removed through asserting the correct notion. Arons prefers *preconceptions* since student concepts were initially held by many of our predecessors. This is consistent with the work of Piaget, who had found a parallel between the development of ideas in the history of science and the development in children's thinking in psychological studies (Smith 1993). Research has revealed student ideas of force similar to Aristotle's (e.g. Whitaker 1983) and medieval impetus theory (e.g. Halloun and Hestenes 1985 b). However, the historical developments of certain areas in mathematics and physics were an essential prerequisite in the formation of Newton's laws and so it does not make sense to say that Aristotle's ideas of force were misconceptions. Although student ideas of force are strikingly similar to Aristotle's and may be classified as such, **nevertheless, they are not strictly Aristotelian!** Aristotle

argued that the force in the direction of motion was caused by air creating a turbulence behind the missile and air conveying the motion given by the prime mover⁴. Many students who give a 'force in direction of movement response' also state *ignoring air resistance!* Halloun and Hestenes (1985b) found that while many students use either Newtonian, Aristotelian or impetus forms of reasoning when solving problems, these do not reflect the complexity of the original conceptions⁵. The term *preconceptions* may seem applicable to student ideas prior to a mechanics course but not subsequently, despite the failings of the course. Either *preconceptions* or *misconceptions* might be used depending on the context and with reference to the researcher concerned⁶ (It is argued in chapter 5 that misconceptions are created as newly formed synthesis of spontaneous reasoning. At this stage it may be more appropriate to refer to 'misconceptions' rather than 'preconceptions').

⁴ See, for example, *Physics*, book VIII, 266b27 - 267a20; *De Caelo*, book III, 301b22-27; *De Anima*, III, 435a4; *Mechanics*, XXXII, 858a15 and *Mechanics*, XXXIII, 858a21.

⁵ According to Franco and Colinviaux-De-Dominguez (1992), Aristotle's explanation for the movement of projectiles, that air has a contradictory role of being responsible for propulsion and resistance (a contradiction which opened the way to the growth of impetus theory), was shaped by the need to deny the existence of vacuum. *Aristotle and other scientists and philosophers from the past must be considered formal thinkers* (Franco and Colinviaux-De-Dominguez, 1992), and many students who embark on an undergraduate physics course are concrete operational thinkers (Renner, 1976a). Although a parallel may be drawn between student conceptions and the history of science, nonetheless, to stretch the parallel too far may become untenable. Misconceptions may be regarded as *common sense* if we define common sense as *recognising the regularities with which we are familiar from everyday experience* (Toulmin, 1967). According to Hankinson (1995), one aspect of Aristotle's science is that *it is commonsensical. It tries to explain the general structure and functioning of the world in terms of processes whose operations are evident to all; it involves no theoretical arcana. Of course, in a sense, that is what hampers it and prevents it from fulfilling Aristotle's own aspirations of providing a complete, and completely intelligible, guide to reality.* It may be the notion of *common sense* that enables a comparison to be made between misconceptions and pre-Newtonian science, and compatible with such a comparison is the notion of a *Gestalt-shift*, i.e. a switch from 'seeing that' to 'seeing as' (see introduction to chapter 5).

⁶ There are many constructivists who argue that the terms 'misconceptions', and even 'preconceptions', should be replaced altogether with 'student conceptions' or 'alternative frameworks' (Goodchild, 1995). This is because of the view that knowledge is subjective rather than objective. This viewpoint has serious pedagogical implications and will be critically evaluated in chapter 2.

A misconception is an incorrect interpretation or explanation of a physical phenomenon with respect to the Newtonian system. They are found in a wide range of mechanics topics and occur across the range of age and abilities. Misconceptions have the appearance of being formed from personal experiences, and one of the most common misconceptions is that a force is required to maintain motion. Many students not trained in mechanics expect to find a force acting in the direction of motion - they find it difficult to visualise a situation whereby the net force is in direct opposition to the motion (Clement 1982). Viennot (1985) reported on the belief that the force acting on an object is proportional to its velocity (the 'V-F' response) and that if a body is at rest then no forces can be present. The irony is $F \propto V$ is within our experience, and in many situations is a good model since $F=R$ and $R \propto V$.

Various parts of a mechanics course might confirm to the student the 'validity' of his or her misconceptions. The mechanics course that does not take these ideas into consideration might serve to confirm these ideas. Consider, for example, the idea that objects move in the same direction as pushed. This idea is often true within that part of Newtonian mechanics covering horizontal linear motion. There is a consistency between the student's ideas and the Newtonian system at this juncture. As the course develops, various parts of the course will validate student intuitive reasoning. For example, the resistive force that is proportional to the velocity of a body might give credence to the idea that the velocity of an object is proportional to the force maintaining motion (Viennot's, 1985, 'V-F' response that is a common characteristic of many misconceptions), and to the idea that force is always in the direction of motion (Clement, 1982) confirmed by the application of the laws of motion in one dimension. The gravitational force acting on a dropped ball does not conflict with the idea of an upward thrust on a ball moving vertically upward (for example, see chapter 5, section 5.5) - the contradiction between the two under the Newtonian system

may not be apparent to the student if the only consideration of upward vertical motion is an exercise in kinematics e.g. the application of the constant acceleration formula with $a = -g$. The student may be able to cope with various parts of the course because they are according to his or her schema of force and motion. The downward force acting on a vertically downward moving ball satisfies both the ideas of the student and the Newtonian system. The parts of the course that would reveal an inconsistency between scientific and student reasoning (e.g. the force acting on a ball moving upward) may be overlooked by the student: many students do not spontaneously confront the conflicts between the accepted theory and their reasoning (Viennot 1985). All individuals have a reluctance to modify a well-established schema⁷ (see chapter 5), especially one that has developed from experience and may have been confirmed through various parts of a mechanics course. The major drawback of the student developing a quantitative understanding, without a qualitative one, is that a quantitative understanding would most likely be unable to challenge existing misconceptions. A quantitative understanding can coexist alongside misconceptions, and in some cases the coexistence can be encouraged by the design of the course and the need to pass the examination. The coexistence may not be a peaceful one as the course develops - resulting in a possible breakdown in the adaptation to new ideas and circumstances that are not consistent with the existing schema.

Other reasons for student misconceptions seem to include the fact that there are some teachers and authors of textbooks who themselves have misconceptions. Warren (1979) has compiled a list of quotations from textbooks as evidence of misconceptions. Helm (1980) argued that many textbooks have a misleading approach to concepts and concluded that there is no single cause of misconceptions. Hellingman (1992) found that even professional

⁷ *Preconceptions are amazingly tenacious and resistant to extinction* (Ausubel, quoted in Cosgrove and Osborne, 1985, and Driver, 1994).

physicists have misconceptions, and that it would be more worthwhile to do research amongst physicists rather than amongst students. However; the misconceptions of some teachers, authors and physicists may augment, rather than become one of, the causes of student misconceptions and demonstrate the extent and 'persuasive' power of misconceptions. Chapter 4 is a discussion of the influence of positivism and its pedagogical implications regarding mechanics. It discusses at length the misconception of the third law of motion regarding Arons (1990), a prominent physicist and leading exponent in the research of student preconceptions. The importance of the discussion is the identification of the author's misconception with his positivist approach to mechanics and the teaching of mechanics.

Some teachers tend to overlook, or are even unaware of, the existence of misconceptions. As a consequence, many students who have successfully completed a traditional mechanics course have only gained a quantitative understanding and a paper qualification. Implicit in some standard teaching approaches is the regard of students as having no knowledge of mechanics (Clement 1982) and that the teacher can impose his or her concepts in place of the student (Gilbert *et al* 1982)⁸. In empathy, many teachers want to save their students from having to undergo the same struggle to achieve the same insights as they had to undergo. Consequently many teachers apply generalisations to specific examples and expect students to then apply them to new situations. According to McDermott (1991) the teaching method may be summed-up as going from the general to the particular in which the reasoning is mostly deductive rather than inductive (However, there is a problem with regarding scientific law inductively. This problem will be discussed in chapter 4). Most researchers have stressed the importance of teaching strategies that acknowledge the

⁸ That students are 'blank-slates' (*tabula-rasa*) waiting to be impregnated with knowledge is a common implicit philosophy of teaching. Chapter 2 begins with the 'starting point' of constructivism: that students are **not** blank-slates.

existence of misconceptions as a *serious alternative hypothesis* (a phrase coined by Halloun and Hestenes 1985b).

1.4 REMEDIAL APPROACHES TO MISCONCEPTIONS

Some researchers have suggested remedial approaches to **particular** misconceptions found in kinematics and dynamics. For example, Hewson (1985) has shown that a misconception can be identified with a diagnostic micro-computer programme and acknowledged by the student. The misconception can then be replaced by the development of a new concept through the remedial stage of the computer programme. Some researchers, for example Hestenes (1992), have advocated a 'global approach' to the teaching of physics - that misconceptions ought to be tackled on a systematic basis with respect to the construction of Newtonian models structured by the axioms of mechanics. According to Renner (1976a), since the majority of students are concrete thinkers who find it impossible to assimilate formal abstract concepts given in instruction, an experimental 'hands-on' approach is required if formal reasoning of the concepts is to develop. Borghi *et al* (1987) suggested a four-stage strategy involving firstly, simple introductory experimental work, followed by films or videos that illustrate further practical situations that cannot be carried out in the laboratory (coupled with the opportunity for discussion), computer simulation, and lastly further laboratory work in order to reinforce the concepts under study. To promote Newtonian thinking, Hake (1987) designed a course that consisted of practical sessions to encourage concept formation, lectures that emphasised problem solving, qualitative analysis and video. Arons (1982, 1984a, 1984b, 1990) and McDermott (1991) have argued for the *phenomenological teaching approach* that continually encourages the student to explain qualitatively the phenomenon under consideration. According to the authors, such an approach not only reveals the preconceptions held by the student, but

develops a qualitative understanding through stages akin to the historical development of the understanding of the concept. Many students are being taught higher order concepts without an understanding of the principles on which they are built. Hence the importance that some time and thought be given to the history of how these concepts were developed and what considerations were involved in the process (Arons 1982, 1990). Halloun and Hestenes (1985a) devised a test that identified the presence of misconceptions in the fundamental topics of mechanics. Previous research has had a tendency to examine only single concepts and not the complete student understanding of mechanics, and so the research has tended to ignore the most fundamental characteristics of the force concept - the *coherence* of the Newtonian theory. What the authors propose (Halloun and Hestenes, 1987) is a general strategy for dealing with misconceptions (called *model-centered instruction* or *the method of paradigm problems*) - that the laws of motion are revealed by applying the laws to construct and validate models of specific physical phenomena, and that the strategy should elicit from the students explicit formulations of alternatives to Newtonian concepts to be analysed and evaluated.

What approach should the project adopt in the construction of the teaching package to improve student understanding of mechanics? Or, more specifically, how can student ideas be restructured into new ones? The problems with the various approaches that have been suggested in the vast literature on student misconceptions are 1), the conflict between some of the approaches, and 2), the way various trends have changed over the years. For example, compare the following two statements:

- *Our research has shown that 50% of Oklahoma's freshmen entering college and 66% of its high-school seniors still occupy the concrete operational stage of intellectual development (McKinnon and Renner).....If a secondary-school or college student is not a formal thinker, he is a concrete thinker. His mental operations are confined to utilizing information he has received from exploring concrete objects, events and/or situations.....There is, however, a necessity for concrete operational thinkers to interact with concrete objects, events and/or situations if they are to progress from the concrete to the formal thought stage (Renner, 1976b).*

- *Goodstein and Howe showed that students who are still in the concrete operational phase and thus are not capable of formal reasoning do not profit from the use of concrete models and exemplars. This aspect becomes all the more salient when one takes into account that more than half of those attending universities are still in the concrete operational phase (Chiappetta) and that one-third or less has achieved the formal operational level (Robbins, Toothacker). Woolnough and Allsop even go so far as to say that the logical solution to teaching abstract concepts to students who are still capable of thinking only in concrete terms is: not to attempt it. If one tries it, 'the effect is to reinforce or introduce misunderstandings in the students' minds which will take much unlearning later...So let us consciously remove the formal, abstract ideas of science from their practical base in order to learn how to handle them maturely' (Kirschner, 1992).*

How is the following statement:

- *Our experience indicates that it is much easier to engage students intellectually in a learning situation that is activity centered than in a traditional laboratory/lecture format.....We recognise that if we want teachers to be able to provide this kind of instruction, we must give them the opportunity to learn in a 'hands-on' manner (McDermott, 1991).*

to be reconciled with the following two statements?

- *One cannot discover what one cannot conceive. Likewise, students must become familiar with the Newtonian World before they can recognise reflections of the Physical World within it and use it as a conceptual tool for understanding the Physical World (Hestenes, 1992).*
- *As Hodson has commented, 'pupils need to spend more time interacting with ideas and less time interacting with apparatus' (Osborne, 1996).*

On the one hand, von Glasersfeld (1995) suggests that, by the process of 'reflection', students can 're-conceptualise' the relationship *force implies motion* to *force implies acceleration*. Yet, on the other hand, Champagne *et al* (1982) argued thirteen years previously that the change from *force implies motion* to *force implies acceleration* does not involve the simple modification of ideas but requires a dramatic shift to a new 'paradigm'.

After all that has been written on misconceptions and the various strategies to deal with them, the latest contribution to the literature (*American Journal of Physics*, February 1997) has this to say:

What should be our goals? Is the central purpose of physics instruction to uncover the students' misconceptions, to confront them with counterintuitive examples which many

physics teachers themselves misinterpret the first time they see them (after that, they're logical), and then to hone the students' skills in recognizing and avoiding such snares? This sounds suspiciously like electro-shock treatment, in which the temporary disorientation opens the otherwise resistant patient to the guidance of the therapist in constructing a better version of reality. Physics is a spiral subject and significant misconceptions tend to get corrected with further exposure and experience.....

If content must be reduced, truncate mechanics. It may be that the subtleties of Newton's laws of motion should be reserved for professional physicists (Geilker, 1997; my emphasis).

Of course, it's obvious! After twenty years of much research into misconceptions, the solution to the problem has been staring us in the face all along: do not bother to teach mechanics!!

Much of the literature either refers to *constructivism* or is based on '*constructivist principles*'. According to Hwang (1996):

Constructivism, at its essence, implies that human knowledge is constructed.....In recent years, the constructivist view has been one of the major influences in the field of instructional design and development. Yet, the term constructivism has been used differently by various scholars in instructional development (ID), and its meaning appears to derive from different constructivist positions.

Given the reference to constructivism by many researchers in science education, it would be appropriate to review constructivism before adopting a particular approach in the creation of a teaching package to improve student understanding in mechanics. It is for this reason that chapter 2 provides a critique of constructivism.

1.5 THE PHILOSOPHICAL AND PEDAGOGICAL APPROACH OF THE PROJECT

Consider the following suggestion of representing gravity with the use of the microworld:

Thus gravity comes to be represented by one of the more powerful modern computational ideas: an object inherently endowed with the capacity to have properties. As a first shot at specifying those properties I have found it useful to suggest that 'gravity eats vertical velocity.' Whenever another object has a vertical velocity, Gravity, now personalized, detects this and begins to nibble away at it. So our figure's vertical velocity will successively be 10, 9, 8 and eventually 0 and then -1, -2 and this means that it is coming

down. But there is still a bug. When the character hits the ground it should stop, but on the present definition of gravity it will just keep going into the ground. We next have to build into the definition of the gravity object a clause that turns it into: gravity eats vertical velocity of unsupported objects (Papert, 1996; author's emphasis).

As a teaching strategy, Papert suggests *personifying* an abstract scientific concept. This is an example of *anthropomorphism*, the endowment of scientific concepts with human-like qualities, that appears to be one of the prevalent trends in constructivism. Chapter 2 argues that this view is unacceptable because if a scientific concept is learned as an extension of an idiosyncratic interpretation, then the scientific concept will be understood idiosyncratically. Chapter 2 traces this pedagogical approach to its philosophical roots - that knowledge is not of an external world but is subjective (*idealism*). The first half of chapter 2 is a review of constructivism as expressed by researchers in the field of student misconceptions of scientific concepts. The second half of chapter 2 is essentially a review of the recent controversy in constructivism, and argues that it is the idealist content of constructivism that has generated the controversy. Chapter 2 concludes that, contrary to radical or social constructivism, **any considerations that lead to the formation of the teaching package must begin with the analysis of the logical structure of Newtonian mechanics before the analysis of student misconceptions.** That the question how misconceptions can be challenged, is subordinate to the question how the student can construct for him or her self the Newtonian system. Chapter 2 concludes that the consideration of the logical structure of Newtonian mechanics should be prior to the consideration of intuitive ideas. The conclusion is consistent with Chalmers (1978) point that the analyses of theories, problem situations, etc. as objective structures are prior to, and independent of, the analysis of the psychology of individual scientists and the sociology of scientific communities (to stress the point further, once we have an understanding of the nature of the products of scientific practice, namely scientific theories, then we are in a better position to understand the behaviour of scientists and the structure

of scientific communities). It is also consistent with Vygotsky's (1962) point that it is the learning of scientific concepts, divorced from the immediate concrete experiences, that structures are supplied (by the scientific concepts) for the upward development of the child's spontaneous concepts toward consciousness and deliberate use. This, in turn, is consistent with Hestenes (1992) argument that Newtonian theory rests on a constructivist epistemology because the theory makes a sharp distinction between the physical world and the concepts constructed to describe it (Hestenes is a constructivist because of his treatment of the logical structure of mechanics, despite not having a subjectivist approach to intuitive ideas. That Newtonian mechanics should rest on a constructivist epistemology is discussed in chapter 3).

Chapter 3 is an analysis of the logical structure of Newtonian mechanics as expressed by Wittgenstein and Hestenes. The analysis provides a description of the Newtonian system as a 'unified-form' and thus provides clues as to how the system can be constructed by individuals as a 'world-view' or paradigm to explain the unfamiliar (that the structure of Newtonian mechanics is determined by a set of axioms that implicitly well-define the concept of force, and that a massive cognitive reorganisation is required to construct a system that is structured by a set of axioms). Chapter 3 attempts to show that the Newtonian schema of force and motion is well defined and can account for all prior contingencies, whereas chapter 5 attempts to show that the student intuitive schema of force may not be well defined and is specific to the **dominant features** of the situation. How the student perceives the dominant feature of the situation triggers his response to the situation. Berry and Graham's (1991) point that concept questions are not only diagnostic tools but have implications for learning can now be elaborated into the question: *How can the specific examples of motion that elicit specific but discrepant responses be utilised to transform the existing schema?* Chapter 5 argues that it is because of the structure of the

Newtonian system that the Socratic method of strategic questioning may be employed, and may be the most appropriate teaching method. By raising anomalies to student misconceptions, it may be possible for the student to experience the lack of coherence of his or her schema if presented with two or more parallel situations that require the same explanation under the Newtonian system. The intellectual search for coherence is a search for a consistent explanation. The juxtaposition of an apparent diversity of instances can create cognitive conflict and the motivation to resolve the conflict. Minstrell (1982) observed the tendency for many students to express an interest in using consistent reasoning e.g. focusing on the similarity between the effects on bodies rather than the differences between the bodies causing the effects. Chapter 5 examines the Socratic method of strategic questioning as a means to induce cognitive conflict and to facilitate the construction of the Newtonian system.

1.6 THE FORMATION AND EVALUATION OF THE TEACHING PACKAGE

Chapter 5 also reports on a pilot-study to assess the effectiveness of the Socratic method of strategic questioning (see section 5.6), and chapter 7 examines the bank of student responses to concept and parallel questions that has emerged from the pilot-study. In the pilot-study, a list of student responses to concept questions was anticipated and a structure to the discussions was designed. The pilot-study consisted of a six hour introduction to force and motion involving an experimental group and a control group of foundation year university engineering students. The teaching method for the control group was the use of concept questions but without the method of strategic questioning that was used for the experimental group. Both groups were pre- and post-tested (see chapter 5) and the results of the experimental group were significantly higher than that of the control group. A video recording was made of the experimental group, and a transcript made of the recording. The

teaching package was constructed using the transcript (see chapter 7). The transcript enabled the examination of student responses to concept questions and the effectiveness of parallel questions, from which the teaching package could be constructed. The teaching package consists of 13 units, each unit containing a concept question, anticipated student responses that the teacher can expect, and parallel questions to tackle the responses.

Chapter 8 is a report of the evaluation of the teaching package. Eighteen schools and one university foundation year engineering group were involved in the project, forming nine control groups and eleven experimental groups. One of the criteria for choosing the control groups was their use of concept questions. Both the control and experimental groups were pre- and post-tested but the results yielded no significant difference between them, despite the reported success of the package by many of the teachers involved in the experimental groups. An evaluation was made on how the package was deployed, and it became apparent that the package was used by many teachers as a bank of concept questions rather than as a structured device for strategic questioning. The chapter concludes that teachers have to be trained in the art and structure of Socratic dialogue before any valid assessment of the effectiveness of the package can be made.

1.7 A CONCEPTUAL MODEL OF STUDENT UNDERSTANDING OF MOMENTS

Many students find aspects of the topic of moments of forces to be stumbling blocks, yet there has been little or no research in this area. Chapter 9 reports on a large scale investigation of student understanding of moments of forces that indicates the nature of intuitive ideas in this area, and offers some suggestions as to the appropriate teaching strategy. The investigation consisted of a moments questionnaire involving a sample of 417 students across the U.K. Using the approach taken by Hart (1981), Brown (1981), Graham

(1991) and Graham and Berry (1996, 1997), a hierarchical model of student understanding of moments was formed from the results of the questionnaire by grouping together questions that have similar degrees of difficulty, and measuring the association between each question with the product-moment correlation coefficient for dichotomous data, *phi*. Using this approach, it was possible to establish 3 conceptual levels of understanding moments. The results of the investigation indicate that, unlike the intuitive schemata of force and motion, misconceptions of moments may be formed from experiences prior to confronting concept questions. Chapter 5 argues that the intuitive schema of force and motion is not formed until a concept of force is slotted into the intuitive schema of motion to account for motion, as presented by a concept question, in some way. The results of the investigation into student understanding of moments indicate that many misconceptions in this area are formed from prior experiences of beams, seesaw and scale-balances returning to the horizontal as if the horizontal is a 'natural' orientation for equilibrium. Other misconceptions in this area reflect deficiencies in other areas of mechanics (e.g. the failure to recognise the line of action of a force and its point of application).

CHAPTER 2 - REVIEW OF CONSTRUCTIVISM

2.0 INTRODUCTION TO THE REVIEW

The review is divided into three major sections. The first section is a critique of the constructivist perspective as expressed by researchers of student misconceptions in science, and attempts to unravel the philosophical assumptions that underlie the perspective. The second section is a critique of the recent and ongoing debate between radical constructivism, social constructivism and the Vygotskian perspective; and the third section places the debate within the context of three major philosophical approaches towards revolutionary change in the history of science. The relevance and justification for the review will be to determine the approach taken, by the thesis, towards creating a teaching strategy to overcome misconceptions in mechanics.

2.1.0 INTRODUCTION TO THE FIRST SECTION

Many researchers of preconceptions in science have either given explicit references to *constructivism* (e.g. Arons, 1990; Hestenes, 1992; McDermott, 1991;) or their article is overtly based on a *constructivist epistemology* (e.g. Driver, 1988; Larochelle and Desautels, 1991; Marin *et al*, 1994). With the exception of the work by Piaget, constructivism has only emerged recently over the past decade or so. According to Lerman (1993), constructivism has become a very influential theory of learning (to date). Given the influence of constructivism and its adoption by researchers in science education, it would be appropriate to provide a critical review of constructivism, as interpreted by educational researchers, as a starting point in the development of a teaching strategy to overcome misconceptions in mechanics. The essence of the first section will therefore consist in

drawing out some of the philosophical and pedagogical implications of constructivism to see if it would be appropriate to build the teaching strategy on a constructivist epistemology. The section concludes that it would be appropriate to structure the teaching package according to a constructivist epistemology; however, this conclusion should not be stated without qualification. According to Bruner (1986), constructivism cannot have a universal criterion (one holistic approach to theory and observation) and must remain ambiguous in practice. Constructivism does appear to have an ambiguity both as a theory of learning and as a guide to teaching, and the ambiguity is reflected in the fact that there are several interpretations (stated below) offered by researchers (the ambiguity is also reflected in the present debate as discussed in section 2.2). This section (section 2.1) will argue that one interpretation in particular - that knowledge is *subjective* - is unacceptable, both as a theory of learning and especially as a teaching guide. There are many constructivists who would claim that this particular interpretation *is the most fundamental trait of constructivist epistemology* (von Glasersfeld, 1984), and for this reason the argument is the central feature of the section. It will be shown that this interpretation regards conceptual schemes (*schemata* - see chapter 5) as not reflecting the structure of the external world in any way. This section will argue that this interpretation can be best understood as an idealist philosophy and, as a consequence, has serious implications with respect to preconceptions. The critique of this particular interpretation can best be described as a sort of 'demolition job'! The critique removes the philosophical layers, so to speak, and reveals an idealist philosophy that some constructivists (e.g. Bruner and von Glasersfeld) claim is the basis for constructivism. This section on constructivism is not entirely negative as it attempts to build a particular constructivist perspective, and it must be noted that this perspective may not have been possible had it not been for the 'demolition job'. According to Garnham and Oakhill (1994):

Realists believe that the structure in the world is independent of human conceptual abilities, whereas anti-realists or constructivists find the realists claim inconsistent, at least in part because there is no way it can be made without using concepts from a human

conceptual scheme [i.e. the statement 'the world is independent of human conceptual abilities' is derived from a human conceptual scheme, which appears inconsistent. I would argue, however, that the world is independent of the way we think, but the way we understand the world is dependent on the way we think. E.g. a law doesn't state how nature behaves, but states how nature would behave under certain, *specified*, circumstances].

Section 2.1 concludes that the constructivist perspective can become seriously weakened if it does not include a *realist* component. The consideration of a realist component, however, is not to 'save' constructivism as such; but to show how the structure of the intuitive schema of force and motion (a cognitive structure that contains misconceptions - see chapter 5) is inadequate as a representation of the physical world, compared with the structure of Newtonian mechanics. To speak of the intuitive schema as 'inadequate' requires justification, as it may go 'against the grain' for many constructivist educationalists [e.g. Goodchild (1995), who suggests that the term 'student-conceptions' should be used instead of 'misconceptions' or even 'preconceptions'].

2.1.1 INTERPRETATIONS OF CONSTRUCTIVISM

Some researchers of preconceptions offer a description of the constructivist perspective:

1. *The 'Construction Principle': People tend to organise their experiences and observations into patterns or mental models* (Redish, 1994).
2. *A key feature in this (the constructivist) perspective is that human beings construct mental models of their environment and new experiences are interpreted and understood in relation to existing mental models or schemes* [page 133]. (Driver, 1988).
3. *During the past decade a constructivist view on the learning process has acquired increasing support. In this view conceptual development is considered to be a process by which students restructure their preconceptions and hopefully arrive at a conceptual structure which resembles the scientific framework a bit more* [page 311]. (Thijs and Bosch, 1995).
4. *The main axiom of the ACM ('Alternative Conceptions Movement') is that a child's alternative framework is analogous to a scientific theory, and will only be exchanged when it is challenged and fails to hold good in the light of new evidence* (Baxter, 1991).
5. *All individuals must construct their own concepts, and the knowledge they already have (or think they have) significantly affects what they can learn. The student is not viewed as a passive recipient of knowledge but rather as an active participant in its creation. Meaningful learning, which connotes the ability to interpret and use knowledge in situations not identical to those in which it was initially acquired, requires deep mental*

engagement by the learner. The student is not a blank slate on which new information can be written without regard to what is already there. If the instructor does not make a conscious effort to guide the student into making the modifications needed to incorporate new informations correctly, the student may do the rearranging. In that case, the message inscribed on the slate may not be the one the instructor intended to deliver (McDermott, 1991).

6. *Constructivism, which has its origins in a diversity of disciplinary fields, postulates that we only have access to what we designate as reality by the means of the representations we construct about it, and that the nature and maintenance of these representations are dependent on their viability (p.376).....As a matter of fact, in the constructivist perspective, the upholding of a representation proceeds from its viability and the degree to which a person sees the margin for manoeuvring necessary to organise his or her experiences [page 379]. (Larochelle and Desautels, 1991).*

The starting point, or *premise*, of constructivism is that individuals are not ‘blank-slates’ (McDermott, 1991; Redish, 1994), ‘virgin wax tablets’ (Larochelle and Desautels, 1991) or ‘empty vessels’ waiting to be imprinted or filled with knowledge and the understanding of that knowledge. *The idea that one may present new descriptions of the world that are to be correctly learned but do not relate to previous knowledge, is on reflection, a very odd idea; but it seems to be the model for a great deal of teaching nevertheless (Svensson and Hogfors, 1988).* Understanding is dependent upon what is already known and understood. A 16-year old maths student who is struggling with GCSE mathematics would have little hope of understanding *tensor-algebra*. Similarly, an Amazonian Indian taken out of the rain forest would have tremendous difficulty in trying to understand (or to hold) the concept of *furniture*. **It would at least be fair to say that the evidence of preconceptions makes the premise of constructivism empirically sound, and not a mere assumption.**

Because constructivism is a recently developed philosophy, and perhaps because of the complexity of human cognition, there has yet to be an **established** general theory of cognition or of cognitive development that can be labelled constructivist (and it may well be the case that constructivism can never be a ‘general theory’, particularly as mental representations are cultural). Piaget’s four-stage theory of cognitive development remains

controversial (see Howard, 1987; Garnham and Oakhill, 1994), and his theory of *reflective abstraction* has yet to be fully developed outside the domain of adolescent cognition (Dubinski, 1991). So far in the history of constructivism there has been no general theory as such, but only a premise that is empirically justified (together with observation and data based on the premise). The premise, however, suggests several approaches in the teaching of concepts:

a) *Anchoring approach.* This approach recommends that the teacher becomes aware of student ideas, and encourages the generalisation of correct ideas (*anchors*) so that they extend to the *target-concept* (Clement *et al*, 1989).

b) *Historical approach.* Because student preconceptions bear a striking resemblance to theories and concepts in the history of science (Piaget, see Smith, 1993), then children should be aware of the resemblance and the difference of the resemblance to current theory (Arons, 1990; Howard, 1987). Stinner (1994) has written an article that gives a brief history of force. The aim of the article is to enable the teacher to challenge students to articulate their preconceptions in response to Aristotle, Buridan, Galileo, Newton, Mach and Einstein.

c) *The Socratic approach.* The class is encouraged to discuss phenomenon qualitatively, and preconceptions are challenged with anomalies (e.g. Arons, 1990; Hake, 1987; Hestenes, 1992). That is to say, **a)** and **b)** can be connected to **c)** in the sense that **c)** can be the 'form of delivery' for **a)** [the Socratic dialogue raises 'bridging analogies' from the anchor to the target concept]; and **b)** can be an element in **c)** [a preconception that emerges in the classroom discussion can be placed into context with reference to its historical counterpart]. This will be discussed in more detail in chapter 5.

d) *Utilisation of student conceptions.* This approach recommends the utilisation of student idiosyncratic interpretations of scientific concepts (*preconceptions* of scientific terms as determined by the use of such terms in everyday life) as a way to understand the scientific

interpretation of those concepts (e.g. Driver, 1988; Trumper, 1990). This approach requires elaboration and a good deal of discussion!

The pedagogical implication of **d)** appears to elevate preconceptions as having some kind of validity, and hence appears to contradict **c)** and perspective **4** above. Given the possible contradiction, with the corresponding pedagogical consequences, we can begin our analysis with **d)**.

2.1.2 THE UTILISATION OF STUDENT CONCEPTIONS DECONSTRUCTED

The premise of constructivism is open to interpretation, and the interpretation could lead to serious pedagogical implications. Consider the following statement by Trumper (1990):

One implication of a constructivist view of learning is that it is neither possible or desirable to remove these intuitive ideas and to replace them with the accepted scientific concept.

This contradicts constructivism as a view of learning built on 'correct' or accepted intuitive ideas and skills (the anchor approach), and contradicts constructivism as a way of discarding incorrect or unaccepted concepts (the Socratic approach). The core of this section's review on constructivism will attempt to underpin the interpretations and implications of the constructivist premise with respect to **d)**. The method of the review will be to begin with the pedagogical implications, and to trace the philosophy that has given rise to such pedagogy.

The review will begin with the history 'text-book' *Medieval Women* (Leyser, 1988). The book is an exemplar in that it places **d)** in the context of teaching history. The review will then examine **d)** in the context of teaching energy. The review will follow through with a

critique of the philosophical interpretations of constructivism with respect to **d**). Although it might appear a diversion to consider subject areas other than science, nonetheless such a consideration might broaden the scope of the analysis and place constructivism in a wider context. *Medieval Women* was chosen because of its unique constructivist character, and that there does not appear (as yet) to be any physics text-book that may be described as overtly constructivist in quite the same way. Constructivism is not mentioned in the book, probably because it is a book designed to be read by school-children. However, its whole approach is one that puts Driver's (1988) constructivist perspective into practice [see perspective 2) above].

It seems as if *Medieval Women* is the only 'text-book' that puts **d**) into practice because of its unique character: the attempt at constructing 'knowledge' that is completely based on the utilisation of existing 'knowledge.' It does this by utilising the ideas we have of the present as a narrative device to understand the past. The publication of *Medieval Women* (by the Oxford University Press, and presented to many teachers by lunch-time school visiting sale-reps) provides an opportunity to spell-out the consequences of **d**) in terms of the use of such a book. [in keeping with chapter 5, **d**) would only allow for the *assimilation* of ideas (into the existing schema) and would not allow for the *accommodation* of the existing schema towards new ideas]. In section 2.3 it will be argued that science can best be understood as an objective autonomous practice - that it is a process without a subject. This is in sharp contrast to the subjective approach of *Medieval Women* that would undermine the study of history as an objective (and rational) endeavour.

2.1.3 REVIEW OF MEDIEVAL WOMEN

Medieval Women was written for lower secondary pupils either for individual use or as a classroom text. It presents the life of medieval women by comparison with the life of women today. Implicit throughout the book is that the pupil can construct a picture of women in medieval society by comparing or contrasting the position or role of women in modern Britain. However, for such a comparison to be possible, the book attempts to develop a pupil awareness of the role of women today. In the introduction it states: *First we must look at what people in the Middle Ages thought women were like. For this you will need to have a copy of the Bible, because people thought that the Bible was the most important book in the Middle Ages.* (page 1, author's emphasis.) It then asks the following:

1. *Do you think it is an advantage or a disadvantage to be a girl?*
2. *Look at this list of some of the major changes for women in this century. Choose the one you think is the most important and say why. Women today:*
 - a) *can use contraception;*
 - b) *can get divorced legally;*
 - c) *can vote;*
 - d) *can take a university degree;*
 - e) *have equal opportunities at work.*

The comparison between life now and then is constantly urged throughout the book with questions such as:

Were any members of your family involved in the Second World War or the Falklands War? What was it like? How did it affect the lives of women? (page 17.)

This is placing Driver's perspective (see constructivist perspective 2 above) in the context of history: ideas that we can have of the past can be constructed from the ideas that we have of today; or to put it another way, the past can be interpreted and understood in relation to the existing ideas that we have of today. It does appear reasonable to suggest that life then can be understood as a reflection of life today. However, in the book's treatment of history, life then can **only** be understood in the context of life today! The problem with this approach is that history becomes subjective, since it is the prevailing

attitudes of today that play a large part in the way we look at today - hence the prevailing attitudes of today that can play a large part in the way we look at the past (a radical constructivist might argue, however, that this is no problem since all knowledge is subjective, and that history is, in a sense, a fiction based on interpretation). This approach suggests that the past is determined by how the individual, or the class as a whole, sees today. Logically, the approach implies that if a view of history (irrespective of how idiosyncratic the view) is considered to be a valid form of description then history can be regarded as 'in accordance' with that view. This approach is in conflict with what may be considered, from a non-subjective standpoint, as historical fact. Consider the following:

England in 1066 was conquered by soldiers. The country was then organised to be run by these soldiers. No one in this arrangement was interested in 'women's rights'. And women lost out (page 16).

The whole of the Saxon population lost out, and 'women's rights' was never an issue until the Nineteenth Century! This quote is an idiosyncratic interpretation of the Norman Conquest. This is not looking at history from the standpoint of today, but history reconstructed as a form of *demagogy* (an appeal to the 'common prejudice' of the class, so to speak). Change the perception of today and history will again be rewritten. Consider the following extract:

Today, more and more women are going out to work: 'More than 9 in 10 childless women under the age of 30 expect to return to work after they have become mothers.' (Government survey, *The Times*, 31 May 1984.) (page 21).

Given the current rate of unemployment, it is most likely that the statement is no longer true (statistics that accurately reflect the current state of unemployment are hard to come by!) If the quotation is no longer true, would that mean that *Medieval Women* has to be updated?

Understanding the past as a reflection of life today reduces history to what life was like. Not only is the past idiosyncratically interpreted, but as a consequence no explanation can

really be offered as to **why** life was like it was (with the exception that it was **attitude**, mainly influenced by the Bible, that determined the position of medieval women). If an approach to a subject-matter is subjective, then any explanation within that approach will be subjective. *Medieval Women* lacks *objectivity* because no reference is made to the *objective conditions* that determined the way society was organised (it leaves you with the impression *if only people in certain periods of history viewed women differently!*). Hardly any reference is made to the economic and social relationships between peasant and lord, the working day, the effects of disease etc. It does, however, give a brief but important reference to the socialisation of domestic work:

Women's work was never-ending. But at least it was sociable.....Even cooking was a neighbourly affair (page 23).

However, the book overlooks the fact that domestic work (because of its socialisation) may have had the same status as other forms of work. Domestic work was still a drudgery, but it was essentially a different form of work to what it is today; and may have had the same status as ploughing the field.

The subjective nature of the book makes the book lack any *historic specificity*: it singles out events and views them within a hidden agenda (*feminism*). The 'inequality of women' is portrayed in the book by reference to particular events between noblemen and noblewomen, the book then generalises that inequality to all women in the medieval period (e.g. page 14). This is in stark contrast with explaining a sequence or period of events with respect to the conditions that gave rise to the events. The contrast is rather like the difference between explaining the *Hundred Years War* as if it were a timeless pathological dislike between the French and the English ('Darren, what do you think of the recent demand for a ban on French goods?'), and an explanation that begins with the famine and disease of the Fourteenth Century and includes the formation of brigands who needed to maintain the war as a way of earning a living. In the context of the book's treatment of

history, Driver's perspective (see 2 above) consists in the ahistorical projection of feminism (the existing mental model) as a way to interpret history.

The subjective character of the book is an implication of *idealism*: that the world (including its past) is primarily a construct of the mind and has (or had) no objective and independent existence. The book's method consists of how we **consider** today as opposed to attempting to portray what was actually the case. This is one pedagogical implication (or, at least, an *interpretation*) of constructivism's premise in the context of teaching history, and it will be shown that it finds its reflection in the teaching of science. If, subsequent to constructivism's premise, we lose sight as to the **nature** of the subject matter that is being taught, then the conceptual constructs of the student will be radically different to that of the subject matter. If these conceptual constructs become universal, then the subject itself will change: any objective content will be replaced with the dominant idiosyncratic interpretation (for example, if *Medieval Women* becomes the exemplar in the teaching of history, then history as a subject will be in danger of becoming in essence an expression of the attitudes of today).

If a teaching strategy attempts to build upon the spontaneous concepts of individuals, or of the idiosyncratic interpretation of concepts - if the objective content of history, including the terms and definitions required within that objectivity, is ignored - then the intellectual development of the class could be adversely affected:

Vygotsky suggested that concept development can be promoted by careful use of language. It is particularly significant for teachers of history that he said that concepts which are specially taught because they belong to a particular discipline and are not acquired spontaneously are learned more consciously and completely. The significant use of a new concept promotes intellectual growth. Shif found that in social studies, when given sentence fragments ending in 'because', more children were able to complete the sentence using a concept consciously learned than using a spontaneous concept related to family situations. They understood 'exploitation' better than 'cousin'. He concluded that this was because the teacher had encouraged them to use 'because' consciously and explained new concepts, supplied information, questioned and corrected, and so these concepts had been learned in the process of instruction in collaboration with an adult (Cooper, 1994).

2.1.4 THE PEDAGOGICAL IMPLICATIONS OF D) WITH RESPECT TO ENERGY

Driver (1988) proposes the following teaching strategy as one out of many depending on the nature of the preconception and the learning goal:

Broadening the range of application of a conception: Students' prior conceptions may be a resource which can be extended. For example, for younger children energy is attributed to human energeticness and motion. By inviting children to consider what happens to their energy the notion can be generalised to encompass the motion of inanimate objects leading on to an appreciation of energy being 'stored' in springs, etc. (page 143).

A ball that is rolling along the ground can be considered to be running out of energy because when I run I also run out of energy! The problem is, how is it I have more energy stored as a result of running faster? The everyday use of the term 'energy' is usually a metaphor to describe human behaviour and how one feels, e.g. *I have no energy today; She is a very energetic person*. Energy is sometimes associated with the notion of a 'life-force' (Kruger, 1990). Much teaching may rely on metaphor, but to generalise this metaphor onto inanimate objects (*anthropomorphism*) might create even more confusion as to the scientific meaning of *energy*. Because *energy* is an abstract idea appropriate only to those who can handle abstract ideas, Trumper (1990) attempts to overcome this problem by asserting that energy is not an abstract idea: energy as *the ability or capacity to do work* is denied as an invalid and outmoded definition. Trumper's argument simply consists in quoting the following statement (made at the turn of the century) by Planck: *the traditional approach to the introduction of energy via the concept of work would be superseded in approximately twenty years*. Trumper may not only be taking Planck out of context, he is committing the *argument from authority* (the fallacy that it is the prestige of an authority which makes a statement true or false). Trumper attempts to make energy a concrete concept by utilising the idea of energy as a 'substance-like' quantity. Trumper speaks of

'energy carriers', 'energy sources' and 'energy receivers' based on the way children see things as 'storsers' of energy and things as 'providers' of energy. However, as Beynon (1990) points out, the idea of stored energy is a misconception. The gravitational potential energy (mgh) of a stone is not **stored** in the stone - it is the amount of work that can be done whilst the stone falls a distance h due to the Earth's gravitational pull (it is our choice which level zero potential energy is assigned!) Similar to the approach of *Medieval Women*, Trumper's approach is based on a form of demagoguery! There is a conflict between constructivism, as interpreted by approach **d**), and the scientific approach. The starting point of a constructivist epistemology should also include the standpoint of scientific thought, and its aim must be to develop that thought within the mind of the pupil or student. As Beynon (1990) points out, you cannot build upon the misconception of energy in the hope that the misconception will change or 'go-away' - on the contrary, the misconception will only be strengthened. Additionally, the use of children's' energy metaphors will encourage an anthropomorphic outlook rather than a scientific one, making the assimilation of subsequent scientific concepts that much more difficult!

If a teacher attempts to develop an understanding of a scientific concept by comparison with a misconception as a metaphor, then the concept will be understood in the context of the misconception; and, if a newly taught scientific concept is assimilated into an idiosyncratic framework, then the concept may be assimilated as a misconception. The philosophical basis of approach **d**) seems to be that a physical phenomenon can be explained by assimilation into a number of schemata; and if any schema makes sense (i.e. can provide in some way an explanation of the phenomenon) then it is equally valid. To take an extreme example of this philosophical position: a thunderstorm can be understood by scientific meteorology or mythological schemata, but who is to say that one understanding is superior to another? (A constructivist might argue that a choice between

the two is not a problem since all knowledge is subjective. However, a scientific meteorological explanation might tell me something about thunderstorms; whereas a mythological explanation couldn't. If an explanation is to tell me something about the world, then the explanation must contain the **possibility** to be falsified, i.e. the **possibility** that evidence will show the explanation to be false. A mythological explanation cannot be falsified because it all depends on how Zeus is feeling at the time!) The view that all knowledge is subjective and hence all knowledge has equal validity is echoed by Trumper (1990, see page 29 above) when he states:

One implication of a constructivist view of learning is that it is neither possible or desirable to remove these intuitive ideas and replace them with the accepted scientific concept.

The philosophical basis of this position implies a form of idealism: that the world is according to how it is portrayed, rather than the world as having an existence independent of the way it is portrayed. This is not an academic point as this position places the emphasis mainly on our description of the world, rather than whether or not the world can be described by these means. This is an epistemological point concerning science, the world and our ideas; and this point needs to be resolved if a teaching strategy is to be built on a secure foundation.

2.1.5 THE PHILOSOPHICAL BASIS OF D)

Consider the following feature *which may be seen as characteristic of such a (constructivist) perspective* (Driver, 1988):

Knowledge is not 'out there' but is personally and socially constructed, its status is problematic. It may be evaluated by the individual in terms of the extent to which it 'fits' with their experience and is coherent with other aspects of their knowledge (page 138).

As it stands the passage appears reasonable enough! Knowledge is in the mind of the knower. However, there is also the known which includes the external world. Driver

(1988) may be a little hard pressed to agree! Consider the following passage (which is consistent with the constructivist perspective 6 above - see page 27):

There is an epistemological implication of this view of knowledge as constructed which has yet to be taken seriously by educators, and that is that to know something does not involve the correspondence between our conceptual schemes and what they represent 'out there'; we have no direct access to the 'real world'. The emphasis in learning is not on the correspondence with an external authority but the construction by the learner of schemes which are coherent and useful to them (page 135, my emphasis).

If we did have direct access to the *real world* then all science would be superfluous! Just because we do not have direct access to the real world does not necessarily mean we cannot have knowledge of it. Driver reduces science to coherent schemes because of the argument that there is no 'correspondence' between knowledge and the world. Von Glasersfeld, whom Driver quotes and is influenced by, is a little more forthright: he denies that we can even **speak** of the existence of an independent, or objective, real world¹:

To appreciate this (that the world seems relatively stable), it is necessary to keep in mind the most fundamental trait of constructivist epistemology, that is, that the world which is constructed is an experiential world that consists of experiences and makes no claim whatsoever about 'truth' in the sense of correspondence with an ontological reality (von Glasersfeld, 1984; page 29).

Von Glasersfeld's argument against realism is the denial of the traditional *correspondence theory of truth* (facts have structure which is 'copied' or 'pictured' by propositions). However, to undermine the correspondence theory of truth does not undermine the realist conception of science. Science describes the world, but that does not mean to say that the world is according to the way that it is described. On the one hand an electron is a mental model, but we can show that the world can be described in this way by changing the world according to the model (for example, setting up an experiment to measure the ratio of the electron's charge and mass). On the one hand the atom is a mental model, yet on the other

¹ It must be pointed out at this juncture that Driver raises this philosophical issue in an article that is mainly concerned with (to quote a sub-heading) 'curriculum development as action research'. Unfortunately, the pedagogical implications of Driver's epistemology cannot be evaluated from the article. Instead, this philosophical issue will be pursued in an endeavour to place constructivism on a secure foundation - not as an academic exercise but as a starting point to build a teaching strategy to overcome misconceptions in mechanics.

hand Neils Bohr managed to split the damn thing! On the one hand an object will move with uniform motion along a frictionless surface after being given a quick shove (despite the impossibility of a frictionless surface!), yet on the other hand we can show that the real, external, physical world can be described this way by making surfaces as smooth as possible (this is an example of the relation between *theory and practice*, see page 57 and 58).

Scientific knowledge is knowledge of the world, not of our experience! Trumper (1990) is being quite consistent with von Glasersfeld when he elevates pupils' preconceptions of energy as having their own validity. In much the same way, *Medieval Women* has to rely on the preconceptions of today as its approach denies the objective ('material') conditions that gave rise to the social relations under study - the preconceptions of today (e.g. *women have equal opportunities at work*, which is debatable!) is generalised in order to encompass the life of medieval women within the pupil's experience.

If the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience, then how is the teaching of scientific concepts related to his or her experience? It will be argued later in this chapter that scientific concepts do not constitute knowledge of individual experience but of an *objective, autonomous practice*. To argue that scientific concepts constitute knowledge of experience begs the question: *How, purely on the experience of the individual, can mechanics be taught?* The response might be that the role of the teacher is to make mechanics part of the student's experience. But to say that is to say nothing! Drawing a force diagram on the blackboard is to make mechanics part of the experience of the student, just as any teaching strategy is to make what is taught part of the individual's experience. **Vygotsky emphasised the need to teach decontextualised concepts (concepts that are defined outside their possible**

everyday context, such as scientific concepts) because such concepts initially lie outside the experience of the individual and consequently the learning of such concepts develop the ability to reason. This point will be taken-up later in the chapter.

Quoting proposition 2.223 of Wittgenstein's *Tractatus* (*In order to discover whether the picture is true or false we must compare it with reality*) von Glasersfeld states that the comparison is not possible: *In order to make it, one needed to have direct access to a reality that lay beyond one's experience and remained untouched by one's 'pictures' and their linguistic formulations* (page 4). However, Wittgenstein (1974) is not referring to an ontological reality that lies beyond our experience. In the following proposition (2.224) Wittgenstein states: *It is impossible to tell from the picture alone whether it is true or false*. Within the modelling process in mechanics there is a stage in which a model is compared with reality - the model is verified by empirical data prior to the refinement of the model. It will be argued in section 2.3.0 that all observation is theory-laden and hence fallible; but the point here is that what is verified is not a model of ontological reality but a model that is constructed within an objective *practice* (as opposed to within a subjective *experience*). Wittgenstein makes the point: *A picture depicts reality by representing a possibility of existence and non-existence of states of affairs* (proposition 2.201) and 'A state of affairs is thinkable': *what this means is that we can picture it to ourselves* (proposition 3.001, my emphasis). Section 2.2.2 will attempt to show that reality is only known to us through our interaction with the world, and by our interaction states of affairs are created. In other words, we do picture the world and we do compare our picture with the world itself - but only according to our own terms and from our own actions.

Von Glasersfeld's idealism is a result of his emphasis on the cognizing subject as the source of knowledge. His idealism is not overt but a consequence of his epistemology (von

Glaserfeld idealism is a form of subjectivism). Bruner, a very influential cognitive psychologist and a leading exponent of constructivism, on the other hand, expresses an explicit form of idealism with respect to constructivism. Consider the following two statements:

I have argued in earlier chapters for a constructivist view of reality: that we cannot know an aboriginal reality; that there is none; that any reality we create is based on a transmutation of some prior 'reality' that we have taken as given. We construct many realities, and do so from differing intentions [page 158]. (Bruner, 1986).

*Its central thesis (referring sympathetically to Nelson Goodman's *Of Mind and Other Matters*), 'constructivism', is that contrary to common sense there is no unique 'real world' that pre-exists and is independent of human mental activity and human symbolic language; that what we call the world is a product of some mind whose symbolic procedures construct the world [page 95]. (Bruner, 1986).*

For Bruner, the world does not comprise of matter but is instead a mental construct. According to this view, the world does not have an existence independent of human thought and experience; consequently, the world 'isn't there' as a yardstick to compare different theories and ideas of the world. The implication is that every theory has its own validity. Again, the implication in practice is to treat every student misconception as a valid description of the world. Bruner (1986) criticises those who argue that mental constructions are in some way representations of an autonomous 'aboriginal' real world (e.g. his reference to Piaget). According to Bruner (1986), what makes a 'world-version' right or wrong is the way in which we assign the version to a world where it is right. In this way conflicting versions can be both true (e.g. the Earth in a geocentric system or in a heliocentric system tells us nothing about how the Earth behaves but tells us something about what these versions say. Both versions belong to an irreducible multiplicity of worlds). Bruner argues that constructivism cannot have a universal criterion and must remain ambiguous in practice. Not surprising since there is an (admitted) ambiguity over 'world' and 'world-version'. Consider the following statement by N. Goodman, sympathised with and quoted by Bruner:

We make versions, and right versions make worlds. And however distinct worlds may be from right versions, making right versions is making worlds [page 99], (Bruner, 1986).

For Bruner (1986) *the constructivist view, that what exists is a product of what is thought, can be traced to Kant, who first fully developed it* [p.96, my emphasis]. This is a fairly common misconception regarding Kant. The statement: *Kant's view of a world 'out there' being made up of mental products is Goodman's starting point* [page 96, my emphasis] is incorrect! Kant (1785), in his *Critique of Pure Reason*, puts forward a proof of the external world:

THESIS: The mere, but empirically determined, consciousness of my own existence proves the existence of objects in space outside me (page 245).

Kant then attempts a proof of the thesis. Contrary to Bruner's interpretation, the *Critique of Pure Reason* insists that empirical judgements arise from perceptions of the **physical world**: that the unknown and unknowable *things-in-themselves* 'affect' our senses, but what is affected is transformed by the mind imposing *synthetic a priori* principles. The point here is that the physical world exists prior to our cognition of it, but our understanding of the world - from empirical judgements to scientific reasoning - is due to the imposition of logical categories.

2.2 THE RADICAL VERSUS SOCIAL CONSTRUCTIVISM CONTROVERSY PLACED WITHIN THE VYGOTSKIAN FRAMEWORK.

2.2.0 INTRODUCTION

Constructivism is certainly the dominant theory (of learning), but it is being subjected to much criticism (Lerman, 1994). Despite the widespread support given by educational researchers since the 1980's, the credibility of constructivism is now under scrutiny. The controversy that surrounds constructivism has raised many issues, but the **central** issue may be expressed as follows: *If knowledge is constructed by the individual in order to*

account for his or her experience (the radical constructivist position), then how is common knowledge or a shared meaning of language possible (the social constructivist perspective)? For example, how could I ever know that what I understand by what you say is the same as what you mean? For Ernest (1994), social constructivism in essence consists in the consideration of a) the way in which the social domain influences the development of the individual in a formative way, and b) how the individual constructs or appropriates his or her meanings in response to his or her experiences in social contexts. However, if knowledge is an individual construction that describes or refers to the individual's experience (*a la* von Glasersfeld), as opposed to the individual having knowledge of an objective real world, then the 'social domain' is reduced to the experience of the individual. Lerman (1994) regards radical constructivism as a coherent yet restricted view of learning and communication, but warns that any attempt to reconcile social constructivism with radical constructivism will inevitably lead to an incoherent position.

He states:

The major confusion that arises from the desire to claim that knowledge is constructed by the individual but that sometimes knowledge is absorbed from culture (Cobb, et al), or as social convention (Ernest) or through the role of the social dimension (Bauersfeld) is that as long as the individual is at the heart of the process, as the one who ascribes meaning, any social interaction is itself interpreted individually. The complementary role these writers desire for 'the social' has no 'bite'.....As long as there is a separation between the subject and the world, including other people, one has to go all the way with solipsism, or give it up.

However, at the beginning of his book *Radical Constructivism, A Way of Knowing and Understanding*, von Glasersfeld (1995) writes:

Taken seriously, this (radical constructivism) is a profoundly shocking view. Some critics say that the emphasis on subjectivity is tantamount to solipsism (the view that nothing exists outside peoples' heads), because, they seem to think, it implies that individuals are free to construct whatever realities they like; others claim that the constructivist approach is absurd, because it disregards the role of society and social interaction in the development of an individual's knowledge. Both objections are unwarranted, and the later sections of this book will present formal arguments to demonstrate it (page 1).

Subsequently, von Glasersfeld does give what appears to be a coherent reconciliation between radical and social constructivism. This reconciliation may be expressed as follows:

I don't actually know whether or not my interpretation of what is said is the same as what is meant; however, I can always assume that my interpretation is the same as what is meant because languages are learnt through similar experiences. I can always change my assumption if there is a perturbation between what is said and what I previously understood as being said. Understanding what is being said is dependent upon the way the individual constructs a meaning of what is being said, and if what is being said is unfamiliar then an idiosyncratic interpretation is possible.

Radical constructivism as a theory of learning appears to be coherent, but only because it is circular - in the final analysis everything is reduced to the subject's experience. However, the circularity is broken once it is accepted that science models an objective real world. Once the objective content of science is accepted then the question as to how the individual can construct (for him or her self) that objective content has meaning - or, more specifically, the consideration of Newtonian mechanics as a form of description of the physical world opens the question as to how the individual constructs that form of description for him or her self. The objective content of science and the question of its construction overcomes Lerman's contradiction between the individual and the social because the objective content of science already presupposes a common meaning. Although von Glasersfeld's idealism has been discussed in section 2.1.5, his insistence throughout his recent publication that 'ontological reality' can never be described will be pursued further (in Section 2.3.0) for two reasons. Firstly, it will throw light on how science describes ontological reality and the way the description can be constructed. Secondly, it will help place the question of the 'subjective' and the 'intersubjective' into

context. Unless the objective content of science is considered, then the debate concerning constructivism may never be resolved. Although he describes himself as a radical constructivist, Thomas (1994) sees the need to consider the objective content of science in order to settle the debate. He states at the very beginning:

Now that radical constructivism has been paid the high compliment of being debunked, but, not being bunk, will not go away, it may be useful to try to improve upon it; hence this attempt at constructive criticism, some of which is radical.

And later:

With a properly circumscribed claim, we can claim to know the world. What else? If we had only our own observations to go on, then we might reasonably be concerned whether our knowings were indeed of the world. But since our scientific knowledge is concordant with that of others within common understandings of the world, there is no practical room for doubt that it is the common world that we understand and have knowledge of. It is all very well for von Glasersfeld to be 'post-epistemological', but to insist that it is not the world that we know is more like the post-modernist cutting off the branch on which one sits. Amusing as creative writing, but not to be taken seriously or taught to children. The very discussion of constructivism relies on common understanding; it is important for consistency and for teaching that it does not lapse into a self-contradictory absurdity comparable to that of a proselytizing solipsism.

Unfortunately, von Glasersfeld has to be taken seriously (especially for the sake of the children!) because it is precisely his idealism (which isn't so unique as he claims - it is an epistemology common to positivism and phenomenology) that still sustains the incompatibility between the social and the individual.

Many social constructivists refer to Vygotsky as an authority in order to substantiate their position. According to Smith (1994), social constructivism is *a genre of constructivism which emphasises social/cultural factors and which is currently dominated by those working in the legacy of Vygotsky*. For example, consider the following passage from Ernest (1994):

This approach (based on 'Vygotskian roots') views individual subjects and the realm of the social as indissolubly interconnected, with human subjects formed through their interactions with each other (as well as by their internal processes) in social contexts..... It is important to distinguish Vygotskian from radical constructivist varieties of social constructivism, for progress to be made in theoretical aspects of the psychology of mathematics education.

Lerman (1994), who seems keen to replace constructivism altogether with a Vygotskian perspective, articulates the incoherence between radical and social constructivism with reference to a Vygotskian approach:

The 'problem' of the social is no problem at all if one accepts that social interactions are indeed on the same plane as physical interactions and both are separate from the autonomous meaning-maker.....The incoherence which I am attempting to describe arises when the social constructivists attempt to place much greater emphasis on the social, without any satisfactory mechanism whereby it impinges on individuals without their choice. Where meaning is carried in social practices and people are positioned by those practices one can begin to analyse and describe the nature of people as social beings. However this is to argue that one starts from the notion of a priority of the social plane over the individual, a Vygotskian idea rejected by the radical constructivists.

Some radical constructivists also appeal to the authority of Vygotsky, sometimes with reservation:

We appeal to Vygotsky's general genetic law of cultural development for insight into how he understood the influence of learning on development..... Our orientation is compatible but not identical with Vygotsky's ontogenetic orientation, and We believe Vygotsky would agree that.....(Steffe and Tzur, 1994).

According to von Glasersfeld (1995):

Social constructivists, who claim Vygotsky as their founding father and now and then argue quite vehemently against radical constructivism, may be surprised at this quotation ('To understand another's speech, it is not sufficient to understand his words - we must understand his thought. But even that is not enough - we must also know its motivation'). It is fully compatible with my view (page 141).

Many of the positions taken up in the controversy refer to Vygotsky's *zone of proximal development*, and the main interpretation seems to be that no separation can be made between the individual and the social context within which knowledge and understanding develops - whether it be the understanding of everyday concepts as facilitated by adults, or scientific knowledge as facilitated by the teacher within the culture of the classroom².

However, the next section will attempt to show that this interpretation is somewhat naive in

² E.g. By 'socio-cultural' I am referring to theories which argue that social and cultural forces are *constitutive* of human consciousness not merely *causative*.....students can perform differently in different situations. We need to take account of contexts, of transfer into the classroom and from school mathematics to outside practices, and of shifts of meanings, when researching mathematics teaching and learning (Lerman, 1996; author's emphasis).

that it overlooks the context (or domain) in which research into cognition and pedagogy would be most appropriate. Although learning takes place in social contexts, nevertheless, according to Vygotsky, there are higher psychological functions characterised by reflective control and deliberate awareness similar to metacognitive skills. What distinguishes higher mental functions is the shift of control from the environment to the individual, that is, the emergence of voluntary regulation; and the emergence of conscious realisation of mental processes. According to Vygotsky (1978):

An interpersonal process is transformed into an intrapersonal one. Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals (page 57, author's emphasis).

Vygotsky believed that it is through contact with scientific concepts that the child can develop deliberate control over everyday concepts. It is through learning concepts **separate** from the immediate and the concrete that structures are provided:

Scientific concepts in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use (Vygotsky, 1962).

This would appear to be consistent with Piaget's quest to answer the question as to how *necessary knowledge* - knowledge that transcends the circumstances and contexts from which it arises - is possible (according to Smith, 1993, the history of Piaget's research is mainly the attempt to answer this question).

The next sub-section (section 2.2.1) will give an outline of Vygotsky's zone of proximal development, from which the following section will attempt to show how constructivism is domain specific. Not domain specific in the *complementarist* sense of the separation between the social and individual domains (e.g. as in Ernest, 1991), but domain specific in the context as to whether the concept is acquired or in the process of becoming acquired - that the answer as to which domain is relevant is dependent on whether or not the concept

has been successfully learnt. The subsequent sub-section (section 2.2.2) will argue that, depending on the nature of the concept and the acquirement of the concept within the zone of proximal development, there are three domains:

1) *Subjective domain*: the domain of cognitive processes that have already matured in the individual as an end-product at any stage of the individual's cognitive development.

2) *Intersubjective domain*: the domain of the social, the consideration of which is essential if we are to understand the individual's cognitive development as a process rather than as end-product.

3) *Objective domain*: the domain that examines the objective content of what is taught and learnt or understood.

There will be an attempt to show that the boundary between 1) and 2) is determined by whether or not a task can be completed. In either case 1) or 2) overlaps with 3).

According to the complimentarist view, radical and social constructivism is domain specific because there is a boundary between the individual and the social. According to Ernest (1991), the boundary is negotiable but confusion and incoherence arise when they overlap. If one accepts the complimentarist position (e.g. Ernest, 1991), or the standpoint that no separation should be made between the individual and the social (e.g. Ernest, 1994; Lerman, 1994), then there is no place or relevance for the objective content of science. Science would then become a question of consensus, and the question as to how the objective content of science can be constructed would be overlooked! Given the importance of domain 3), von Glasersfeld's disagreement with objectivity will be pursued in section 2.3. The three domains will be illustrated with reference to the philosophy of science concerning revolutionary changes in science.

2.2.1 VYGOTSKY'S ZONE OF PROXIMAL DEVELOPMENT

Vygotsky defines the zone of proximal development as *the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers* (1978, page 86). According to Zeuli (1986), the zone represents a phase in development in which an individual is unable to perform a task alone but can eventually accomplish and internalise it with the help and supervision of someone more experienced. If a child can complete a task independently, then the functions for doing such a task (e.g. spatial awareness) have matured in the child. However, if the child cannot solve the task independently but requires assistance then the zone of proximal development defines those functions that have not yet matured but are in the process of maturation. A group of children may have equal levels of mental development, but their capability to learn under a teacher's guidance could vary considerably. Vygotsky (1978, page 86) illustrates this point with two children who have, supposedly, the same developmental levels - who are both capable of successfully completing the same tasks unaided - but differ in the completion of tasks that require facilitation. For Vygotsky (1978), the two students are equivalent in terms of their independent activity but differ in their immediate potential development and therefore do not have the same mental development. He states: *The zone of proximal development furnishes psychologists and educators with a tool through which the internal course of development can be understood* (1978, page 87); and according to Zeuli (1986), the zone of proximal development has provided researchers with a tool for understanding how the transition from social interaction to individual cognitive abilities occur.

The zone of proximal development may be described as *the ability of the child to recognise the value of hints and props even before he is conscious of their full significance (Bruner). For Vygotsky, a key ingredient for learning within the zone of proximal development was instruction that preceded maturing abilities. The more experienced person takes major responsibility for structuring the interaction, leading the other through the steps of a task, and providing the necessary support until the learner is able to do the task independently (Zeuli, 1986).* For Vygotsky, the school environment provides the context for learning decontextualised concepts within a discipline distinct from the everyday learning of everyday concepts. School learning is advanced by the teacher supporting the learning of decontextualised concepts, and it is by learning decontextualised concepts that the pupil develops metacognitive skills:

The child becomes conscious of his spontaneous concepts relatively late; the ability to define them in words, to operate with them at will, appears long after he has acquired the concepts. He has the concept (i.e., knows the object to which the concept refers), but is not conscious of his own act of thought. The development of a scientific concept, on the other hand, usually begins with its verbal definition and its use in non-spontaneous operations - with working on the concept itself. It starts its life in the child's mind at the level that his spontaneous concepts reach only later..... One might say that the development of the child's spontaneous concepts proceed upward, and the development of his scientific concepts downwards, to a more elementary and concrete level. This is a consequence of the different ways in which the two kinds of concepts emerge. The inception of a spontaneous concept can usually be traced to a face-to-face meeting with a concrete situation, while a scientific concept involves from the first a 'mediated' attitude toward its object (Vygotsky, 1962, page 108; author's emphasis).

Children's' everyday concepts are unsystemised and characterised by a lack of conscious awareness. Children may be able to talk spontaneously and correctly about everyday concepts, but they have a difficulty in focusing on the concepts: the child is not conscious of the concept because his or her attention is always centred on the object to which the concept refers (Vygotsky, 1962). Children find it difficult to answer correctly abstract questions about concepts that have been placed in contexts separate from their immediate concrete experiences. However, Vygotsky (1962) argued that deliberate control over everyday concepts is developed through contact with scientific concepts. It is by learning

scientific concepts divorced from immediate concrete experiences that structures are supplied - by the scientific concepts - *for the upward development of the child's spontaneous concepts toward consciousness and deliberate use* (Vygotsky, 1962; page 109). This is only possible because:

In the scientific concepts that the child acquires in school, the relationship to an object is mediated from the start by some other concept. Thus the very notion of scientific concept implies a certain position in relation to other concepts, i.e., a place within a system of concepts. It is our contention that the rudiments of systematization first enter the child's mind by way of his contact with scientific concepts and are then transferred to everyday concepts, changing their psychological structure from the top down (Vygotsky, 1962; page 93).

Vygotsky (1962, page 106) provides empirical evidence to show that a sample of students were more successful using reasoning skills involving decontextualised social studies concepts than reasoning skills involving familiar everyday concepts. This goes against expectation as one would have thought that students would have performed better on reasoning skills involving everyday experiences. Vygotsky states:

How are we to explain the fact that problems involving scientific concepts are solved correctly more often than similar problems involving everyday concepts? We can at once dismiss the notion that the child is helped by factual information acquired at school and lacks experience in everyday matters..... The child must find it hard to solve problems involving life situations because he lacks awareness of his concepts and therefore cannot operate with them at will as the task demands (page 106, my emphasis).

Although the learning of scientific concepts is transferred to everyday concepts, the pupils' everyday concepts must nevertheless already be at a certain level in order that scientific concepts can be learnt (Vygotsky, 1962). The understanding of causal (because) relationships in everyday speech is necessary if the notion of causation is to be understood within a scientific context. However, this does not mean to say that the pupil has deliberate control over the concept of causation within the context of everyday speech. Although the teacher can facilitate the notion of causation, Zeuli (1986) warns that Vygotsky makes no suggestion that teachers working within the zone of proximal development should make connections to what the learner already knows. This point will be considered in greater depth below. According to Zeuli (1986):

Bruner's examples of learning within the 'zone of proximal development' support the view that the primary focus is on the adult's assistance as the student tries to understand the relationships between concepts - not how connections are made to the student's everyday concepts. Bruner also points out the importance of schooling as 'joint culture-creating', and later compares the zone to the way 'Socrates guides the slaveboy through geometry in the Meno - a kind of negotiation in which the abler frames the questions, the less able replies and gains in insight'.

Bruner also claims that the more recent research of Collins and his colleagues on Socratic tutoring programs illustrates how the teacher supports learning within the zone of proximal development. Collins and Stevens analysed how expert teachers guide student learning in various disciplines. In geography, for example, a number of different factors could affect rice growing in a country, such as fresh water, a fault area, fertile soil, and warm temperature. As Collins and Stevens point out, the teacher can use various strategies to help students understand the relationships between concepts in a discipline: 'If a student says they do not grow rice in Oregon because it lacks a flat terrain (which is unnecessary), one can pick Japan which is also mountainous, but produces rice..... If a student thought rice could not be grown in Wyoming because it is too dry (which is insufficient because it is also too cold), the teacher could ask, Suppose that it rained a lot in Wyoming, do you think they could grow rice then?'..... It is this kind of collaborative support that Vygotsky considered integral to learning within the zone of proximal development. Initial and frequent connections to students' everyday concepts suggests that they may even interfere with learning scientific concepts. Current research in science education supports this view.

Zeuli (1986) has reported on the way some researchers have used Vygotsky's theory to emphasise how the teacher should make connections to the students' everyday familiar concepts, and argues that this is inconsistent with Vygotsky's characterisation of school learning. For example, Greenfield studied how adults provide 'scaffolds' to children learning within the zone of proximal development - the teacher builds on what the learner can do, thus closing the gap between task requirement and the skill level of the learner.

Zeuli (1986) notes that:

- 1) Vygotsky does not suggest that, within the zone of proximal development, the teacher should make immediate connections to what the learner already knows. On the contrary - for Vygotsky - the school environment is the creation of a special context for understanding deconceptualised concepts distinct from everyday learning.
- 2) That, according to Luria (a colleague of Vygotsky), students will initially fail to establish any connection between academic concepts and events in their everyday life.

3) Students' everyday concepts may interfere with learning unfamiliar scientific concepts: *Connections to students' existing knowledge may not foster their understanding but may instead reinforce their misconceptions* (Zeuli, 1986). This point has already been raised earlier in the chapter within the context of extending an idiosyncratic interpretation in order to assimilate the corresponding scientific concept. For Zeuli (1986):

Students learning about light believed that they saw objects 'because light shines on things and brightens them up' (Eaton, Anderson and Smith), rather than being reflected off the objects. Students resisted relinquishing these faulty notions even after further instruction in the concepts being studied. Thus, teachers cannot view students' everyday ideas as something that can be built on and refined. Although it is helpful that teachers take into consideration students' preconceptions during instruction (Roth, Driver), sharp breaks between school instruction and students' everyday concepts may more likely promote students' scientific understanding.

Zeuli further warns that while it is important for instruction to be sensitive to the cultural setting within which it occurs, nevertheless, it does not mean that school learning must be continual with these cultures. On the contrary, schools should provide students with educative breaks from their everyday experiences in order to further students' objective judgement and scientific understanding. According to Zeuli, there is much research that uses Vygotsky's theory to justify the notion that school learning should be compatible with native cultures; yet this 'compatibility' does not square with Vygotsky's position. Such research does not take into account Vygotsky's analysis of the limitation of students' everyday concepts or his emphasis on the discontinuity between everyday concepts and higher psychological functions.

2.2.2 THE SOCIAL VERSES THE RADICAL

The present controversy surrounding constructivism seems to be over the relationship between the following two statements:

(a) Cognitive processes develop in the individual as the individual attempts to make sense of either the external world (e.g. Thomas, 1994, who argues that it is **because** knowledge is

knowledge of the external world that makes common knowledge and understanding possible) or the experiential world (e.g. von Glasersfeld).

(b) Cognitive processes develop in the individual as the individual interacts with the social world.

The difficulty with the controversy is the attempt to resolve philosophical questions that have been controversial over the past two millennia (for example: *what is knowledge and how is it possible?*). Von Glasersfeld (1995) gives a potted but idiosyncratic history of philosophy to justify radical constructivism *as a model of rational knowing, not as a metaphysics that attempts to describe a real world* (page 24). He states that *there is nothing new about the ideas that make up radical constructivism. The only novelty may be the way they have been pulled together and separated from metaphysical embroidery* (page 24). For von Glasersfeld, you cannot begin to talk of knowledge as knowledge of an external world because you then enter the realm of metaphysical speculation (philosophically, his position is similar to the logical positivists [see chapter 4 of the thesis] who attempted to banish metaphysics from epistemology). In terms of the individual, the social world is part of the external world; so if you cannot account for knowledge with reference to an external world, then you cannot account for knowledge with respect to the social world. This raises the question as to how the public can be reconciled with the private; or to be more particular, how teaching can be reconciled with conceptual development. For Ernest (1994):

One approach to this problem is to propose a social constructivist theory (of learning mathematics). On the face of it, this is a theory which acknowledges that both social processes and individual sense making have central and essential parts to play in the learning of mathematics. Possibly as a consequence of this feature, social constructivism is gaining in popularity. However a problem that needs to be addressed is that of specifying more precisely the nature of this perspective (page 63).

Ernest (1994) gives references to many researchers who have offered perspectives for social constructivism, and provides an outline of the many interpretations; the problem

is.....there is a lack of consensus about what is meant by the term, and what its underpinning theoretical bases and assumptions are (page 63).

Much reference is made to Vygotsky in order to place the social and the individual in perspective. However, it would seem that Vygotsky is open to interpretation. Vygotsky's statement that all the higher functions originate as actual relations between human individuals seems to have been interpreted in a 'strong' form and in a 'weak' form.

According to Steffe and Tzur (1994):

'Social relations or relations among people genetically underlie all higher functions and their relationships' (Steffe and Tzur quoting Vygotsky, as quoted by Wertsch and Toma). Consequently, social interaction would seem to play a major role in Vygotsky's approach to learning. Interpreters of Vygotsky have made this clear:

'One of the basic tenets of the Vygotskian approach to education is the assumption that individual learning is dependent on social interaction. However, it should be clear from the outset that this is not merely a statement of correlation between individual learning and social context. This thesis should be interpreted in its strongest possible form, proposing that the qualities of thinking are actually generated by the organizational features of social interaction' (van Oers).

Wertsch and Toma also interpreted Vygotsky's general genetic law of cultural development in its strong form. 'In an essential sense, the same mental functions appear on the social and individual planes'.

The complementarist view, that there is a correlation between individual learning and social context, may be considered as a 'weak' interpretation of Vygotsky's statement. The 'strong' interpretation is that no separation can be made between the individual and the social - whether it be learning and its social context, knowledge and understanding as being 'collective' or knowledge as built from and reflecting social interaction within society (for example, to quote Wertsch, 1985 :

Saxe has demonstrated that the arithmetic system and its uses are not natural or universal but instead depend on sociohistorical context. Hence Saxe and his colleagues assume that a full understanding of the zone of proximal development is possible only if the historical specific context is taken into account (page 75).

It would seem that reference is made to Vygotsky in order to 'clear things up', but the problem is that Vygotsky is now open to interpretation. For Ernest (1994), there is a Vygotskian version of social constructivism. For Lerman (1993), social constructivism is

incoherent and he suggests that it should be replaced with a Vygotskian theory of mind. If much reference is made to Vygotsky as an authority, but that authority is open to interpretation, then it may help matters if an attempt is made to understand his epistemological perspective - the considerations that underlie his method of research. Vygotsky was a Marxist and attempted to build a Marxist psychology (Cole and Scribner, introduction to Vygotsky, see Vygotsky, 1978; Wertsch, 1985). The following is an attempt to understand Vygotsky but within the context that he himself constructed: a psychology structured by the Marxist epistemology of *theory and practice*, that in order to know the world it is necessary to interact with it.

According to Cole and Scribner (introduction to Vygotsky, see Vygotsky, 1978):

Contrary to the stereotype of Soviet scholars scurrying to make their theories conform to the Politburo's most recent interpretation of Marxism, Vygotsky clearly viewed Marxist thought as a valuable scientific resource from very early in his career. 'A psychologically relevant application of dialectical and historical materialism' would be one accurate summary of Vygotsky's sociocultural theory of higher mental processes. Vygotsky saw in the methods and principles of dialectical materialism a solution to key scientific paradoxes facing his contemporaries. A central tenet of this method is that all phenomena be studied as processes in motion and change (page 6).

According to Wertsch (1985):

The fundamental claim in Vygotsky's genetic or developmental analysis is that human mental processes can be understood only by considering how and where they occur in growth: 'We need to concentrate not on the product of development but on the very process by which higher forms are established.....Thus, the historical [that is, in the broadest sense of history] study of behaviour is not an auxiliary aspect of theoretical study, but rather forms its very base' (Vygotsky). Vygotsky contrasted his genetic approach with approaches that attempt to analyze psychological phenomena without regard for their place in development. He argued that such research can provide description but not explanation: 'Following Lewin, we can apply [the] distinction between the phenotypic [descriptive] and genotypic [explanatory] viewpoints to psychology. By a developmental study of a problem, I mean the disclosure of its genesis, its causal dynamic basis. By phenotypic I mean the analysis that begins directly with an object's current features and manifestations. It is possible to furnish many examples from psychology where serious errors have been committed because these viewpoints have been confused' (Vygotsky). The last sentence is particularly important because it reflects Vygotsky's concern with the problem of how assumptions about method influence the interpretation of psychological phenomena. He was arguing that misunderstandings often arise among researchers because they do not share assumptions about how a phenomenon should be investigated, and hence about what it is (page 17).

The Marxist epistemology is that although we can think about the world, nevertheless we can only know the world if no separation is made between theory and practice. According to Marx (1969):

The question whether objective [gegenstandliche] truth can be attributed to human thinking is not a question of theory, but is a practical question. In practice man must prove the truth, that is, the reality and power, the this-sidedness [Diesseitigkeit] of his thinking. The dispute over the reality or non-reality of thinking which is isolated from practice is a purely scholastic question. (Thesis no. II, author's emphasis).

To **know** the world I must interact with it, otherwise any theory I have about the world would be **pure** metaphysical assumption (Vygotsky's epistemology does have a Marxist metaphysical premise, but that need not concern us here). However, by interacting with the world I also change the world - just as the anthropologist changes the behaviour of the tribe that he is researching. *The philosophers have only interpreted the world, in various ways; the point, however, is to change it* (Marx, 1969, *thesis no. XI*, author's emphasis). This statement isn't making the ethical judgement that causing revolutions is a more worthwhile occupation than armchair theorising, it is making the epistemological point that I have to change the world in order to know it. For Lukacs (1974a): *Marx clearly defined the conditions in which a relation between theory and practice become possible. 'It is not enough that thought should seek to realise itself; reality must also strive toward thought'.* For example, to verify Snell's law for the refraction of light, a light-box must be physically constructed so that a parallel beam of light can be produced - a parallel beam of light did not exist in nature prior to its invention. A parallel beam of light is an invention, or creation, required by theory in order to explain refraction as a physical phenomenon. *It is precisely the alteration of nature by men, not nature as such, which is the most essential and immediate basis of human thought* (Engles, *Dialectics of Nature*; quoted in Vygotsky, 1978. Emphasis given). It is in the light of this Marxist epistemology that we can begin to understand Vygotsky. Vygotsky's approach was called the *experimental-developmental* method: *which calls for an experimenter to intervene in some developmental process in*

order to observe how such intervention changes it. Again the primary motivation for doing this is to observe genetic processes: 'Our method may be called experimental-developmental in the sense that it artificially provokes or creates a process of psychological development. This approach is equally appropriate to the basic aim of dynamic analysis. If we replace object analysis by process analysis, then the basic task of research obviously becomes a reconstruction of each stage in the development of the process: the process must be turned back to its initial stages (Vygotsky)' (Wertsch,1985).

In a child's zone of proximal development, if we are to understand the child's cognitive abilities as a **process** then we have to instigate that process by interaction with the child. It is in this sense that the 'social' cannot be separated from the 'individual'. If a child can successfully complete a task unaided, then prior knowledge of the abilities required to complete the task will enable us to say what abilities the child has. However, we would be looking at the child's abilities that have already matured in the child - we would be looking at a 'snap-shot' of the maturation process as an end-product. In order to understand the maturation process **as a process** then we would have to facilitate the child's completion of a task that the child cannot do unaided. How a child responds to the mediation in completing a task enables us to **explain** the abilities of the child as they mature, rather than simply **describe** the abilities that have already developed.

From a Vygotskian perspective, an explanation of the development of the higher mental functions involved in the learning of mechanics would involve an interaction with that development. Any explanation would be based on the interaction. However, any interaction, including instruction, would influence the development of the higher mental functions:

According to Vygotsky, instruction in the zone of proximal development 'calls to life in the child, awakens and puts in motion an entire series of internal processes of development. These processes are at the time possible only in the sphere of interaction with those surrounding the child and in collaboration with companions, but in the internal course of development they eventually become the internal property of the child'.....Instruction

'awakens and rouses to life an entire set of functions which are in the stage of maturing, which lie in the zone of proximal development' (Wertsch, 1985).

Although some researchers recommend that teachers ask questions in order to realise the initial knowledge state of uninstructed students (see chapter 5), by asking questions the initial knowledge state might itself change. Chapter 5 argues that many students would not have thought about force and motion until asked to do so for the first time by a teacher, and that the intuitive schema of force and motion isn't formed until the student uses *force* to account for what the student considers to be the *dominant features* of motion. Although questioning may reveal the understanding, or state of knowledge, of the class, the teacher should nevertheless realise at the outset that the very questioning itself could transform the cognitive state of the class. In chapter 7, details of a pilot-study illustrates how some students can develop a remarkable coherence to their misconceptions in their response to strategic questioning.

Lerman (1994) argues that the radical constructivist perspective is a limited view of learning. I would argue that, from a Vygotskian perspective, radical constructivism is limited because it can only **describe** the constructs of the student that have **already** developed - it can never **explain how** the constructs develop without reference to the interaction between the student and the 'mediator' or 'facilitator' (e.g. the strategies employed, the questions posed and the metaphors used). This is not to argue, however, that radical constructivism is invalid. It can provide us with a description of what misconceptions there are, but it cannot explain how misconceptions are constructed. Although in the constructivism controversy much reference is made to Vygotsky as an authoritative figure, the irony is the prophesy of his perspective. According to Wertsch (1985):

For Vygotsky an essential aspect of the definition of a psychological phenomenon is its position in genetic transition. He assumed that the form of a phenomenon reflects the transformations it has undergone and the various factors that have entered into its development. Vygotsky's point is not that psychological research which fails to use a

genetic method is invalid or useless. Elsewhere in his writings he explicitly stated that such research can make an important contribution to the overall picture of psychology. However, he believed that without genetic analysis one can only describe certain aspects of psychological phenomena and cannot understand inner workings and causal dynamics.he believed that the failure to recognise the impact of method on the interpretation and definition of psychological phenomena can lead to confusion (page 18, my emphasis).

The present controversy that surrounds constructivism appears to be a confusion between the 'weak' and 'strong' interpretations of Vygotsky, which in turn appears to be a confusion over Vygotsky's epistemology and its metaphysical assumption. According to Wertsch (1985):

The influence of Marx's claim on Vygotsky is manifested in the following statement: 'To paraphrase a well-known position of Marx's, we could say that humans' psychological nature represents the aggregate of internalized social relations that have become functions for the individual and forms of the individual structure'.....On the bases of this Marxian axiom Vygotsky argued that 'the social dimension of consciousness is primary in time and in fact. The individual dimension of consciousness is derivative and secondary' (page 58).

For Lerman (1994), this would appear to be Vygotsky's *priority of the social plane over the individual*. This seems to be the basis for the 'strong' interpretation of Vygotsky that characterises the social constructivist position that the individual cannot be separated from the social (e.g. Ernest, 1994). The complimentarist position within social constructivism (e.g. Ernest, 1991) seems to be the 'weak' interpretation that radical and social theories of constructivism are of different domains - one is a description of the individual domain, the other is the influence of the social on the individual (whether it be the prioritisation of the individual over the social, with the latter in terms of social interaction, e.g. Richards; or of socially constructed knowledge, e.g. Confrey; or the adoption of two complimentary frameworks of the intra-individual and the interpersonal, e.g. Bauersfield, Driver, Murray; or the emphasis placed on the social negotiation of classroom norms, e.g. Cobb, Wood and Yackel. [Ernest, 1991]). Although all forms of thought may be considered as originating from social relations and interactions, nonetheless it would seem unlikely that Vygotsky (or Marx) would have denied that forms of thought can also be evaluated in separation from

their social roots. The truth of the theorem of Pythagoras or the validity of the Newtonian system is independent of the social relations that existed at the time of Ancient Greece or the research programme of the Renaissance (just as art can be aesthetically judged independently of the considerations of the historic and social conditions that gave rise to the art):*the relations between origin and validity are much more complex here than in the case of the forms of the objective spirit. Marx saw the problem clearly: 'But the difficulty does not consist in realising that Greek art and epic are bound to certain social forms of development. The difficulty is that they still give us artistic pleasure and that, in a sense, they stand out as norms and as models that cannot be equalled.'*just as it is clear that Copernican astronomy was true before Copernicus but had not been recognised as such. (Lukacs, 1974b). In much the same way, the Vygotskian perspective considers all higher mental functions as originating from the interaction between human beings - but the functions themselves can transcend the context from which they originate. A student's understanding of mechanics may be evolving such that the understanding is still specific to the context in which it is taught (e.g. specific to the examples given and the way the subject is given). A fully evolved understanding of mechanics is independent of the specific examples used, and the approach taken, by the teacher.

2.3.0 THE CONSTRUCTIVISM CONTROVERSY PLACED WITHIN THE CONTEXT OF THREE APPROACHES IN THE PHILOSOPHY OF SCIENCE.

All forms of thought originate from social relations and interactions, but the products of thought can be evaluated independently of their social origins. A position adopted can be evaluated within the context of its adoption, but the logic or validity of the position can also be evaluated independently of its origins or social context. For Lerman (1994), however, no such separation should be made:

Positions adopted carry with them much more than 'ideas'; indeed it would contradict my support for the assertion that knowledge manifests in practices were I to claim that those ideas are independent of social practices. We are engaged in typical activities of academic communities including: defending the 'corners' one may have argued in meetings and conferences and in papers; identifying one's own view by comparison with another, which includes 'selective' presentation of that other view; claiming phrases of the moment as part of one's case, such as 'overcoming Cartesian dualism'; identifying with groups of people (page 48).

and, on the previous page:

How can individuals arrive at the mathematical concepts generally accepted by the mathematical community or at least the mathematics education community? It seems that the radical constructivists can only argue that their analysis results in common concepts by a right intuition, as the mathematical constructivists claimed, a Kantian view of necessary features of the mind (page 47).

Lerman's position seems to be a *consensus* one: that the best theories will be those that best meet the standards and needs of an academic community.

According to Chalmers (1978), there are three approaches to the question of the nature of scientific knowledge: the *subjective* approach, the *consensus* approach and the *objective* approach. The following is a summary of (and taken from) Chalmers' (1978) account of the three approaches: In the consensus view, the beliefs of individual scientists are subservient to those of the scientific community. Scientific knowledge consists of theories that have been accepted by the scientific community, which maintains the standards for the assessment of theories through rigorous educational and training programmes. The *consensus* approach raises such questions as: *What are the causes for bringing about a change in the theories of a community? What caused a change in consensus concerning the nature of the universe in the Copernican revolution (e.g. the need to reform the calendar)?* These are valid questions, but the consensus approach is a severely limited one because it considers the **social group as the primary notion** and the actual science itself, including its practice, as the secondary one. However, a particular community may or may not practice a legitimate science; and different social set-ups have created different legitimate contributions to a single physics (e.g. Galileo's Italy and Newton's England, or the United

States and the Soviet Union). For the **objectivist** approach, this implies the existence of physics as an autonomous practice, independent of individual or consensus opinion, that constitutes the activity involved in its development. Although the development of scientific theory is dependent on the participation of the individual scientist and of the community, nevertheless, scientific theories bear a relationship to each other and to available evidence independently of whether or not the individual or the community realise it. *Science is a process without a subject*, because its development creates objective problem situations whether or not scientists realise the problem situation. The objective approach would ask such questions as: *How does this theory relate to the available evidence? Is the theory coherent or consistent? What is the relation between Aristotelian and Newtonian mechanics? Is there a sense in which science progresses?* These questions cannot be answered by the consensus approach. In the **subjective** approach, scientific knowledge is held by individual scientists whether it be derived from sensory experience, intuition or reasoning. Scientific theory resides in individual minds; for example, positivism assumes that scientific knowledge consists of a set of statements the truth of which is guaranteed by private experience. The subjectivist asks such questions as: *What psychological changes have taken place when an individual abandons one theory for a rival one, and what are the causes in bringing about such a change? What convinced Galileo that Copernicus was right?*

Radical constructivism is a subjective approach because it places the **emphasis on individual cognition**. For von Glasersfeld (1995), scientific knowledge is a question of 'viability' and the world of our experience. According to von Glasersfeld (1995), science employs 'fictions', such as gravity, that are useful as a substitute for something that is inaccessible ('ontological reality'). Such fictions *can explain anything you want to explain* (page 46), and should be *recognised as tools for the rational organisation of experience*

and not mistaken for phenomenon that are real in the sense that they themselves could be experienced (page 46). This is an admitted Instrumentalist position (von Glasersfeld states that *Radical constructivism is uninhibitedly instrumentalist*. Page 22) that does not square with the history of science. According to Chalmers(1978):

The fact that theories can lead to novel predictions is an embarrassment for naive instrumentalists. It must seem a strange kind of accident to them that theories that are supposed to be mere calculating devices can lead to the discovery of new kinds of observational phenomena by way of concepts that are theoretical fictions. The development of theories concerning the molecular structure of organic chemical compounds provide a nice example. The idea that the molecular structure of some compounds, benzene for instance, should consist of closed rings of atoms was first proposed by Kekule. Kekule himself had a somewhat instrumentalist attitude towards his theory and regarded his ring structures as useful theoretical fictions. On this view, it must be regarded as a remarkable coincidence that these theoretical fictions can nowadays be seen almost 'directly' through electron microscopes. Likewise, instrumentalist defenders of the kinetic theory of gases should have been somewhat taken aback to observe the results of collisions of their theoretical fictions with smoke particles in the phenomenon of Brownian motion. Finally, Hertz himself reported that he had been able to produce the fields of Maxwell's electromagnetic theory in a 'visible and almost tangible form'. Episodes such as these undermine the naive instrumentalist claim that theoretical entities have a fictitious or unreal existence in a way that observable entities do not (page 117).

For von Glasersfeld (1995):

Actions, concepts, and conceptual operations are viable if they fit the purposive or descriptive contexts in which we use them. Thus, in the constructivist way of thinking, the concept of viability in the domain of experience, takes the place of the traditional philosopher's concept of Truth, that was to indicate a 'correct' representation of reality. This substitution, of course, does not affect the everyday concept of truth, which entails the faithful repetition or description of a prior experience (page 14, authors emphasis).

Section 2.1 argued that the constructivist perspective can become seriously weakened if it did not include a realist component. However, the appeal to realism must be qualified!

According to Chalmers (1978):

There is doubtless at least some rough, common sense in which the aim of science is truth. On that level, Newtonian astronomy is an attempt to explain what the solar system is really like, in the same way that a description of the stone buildings surrounding the main quadrangle at Sydney University is an attempt to explain what that particular portion of the world is really like. And as some wit once said, anyone inclined to doubt the latter should be invited to drive into one of the buildings 'at a speed proportional to his or her disbelief' (page 113).

However, as Chalmers (1978) explains, the problem of the common sense (*naive realist*) view is that if it is pursued in an attempt to make more precise sense of it, then difficulties

are encountered. Theories are structured systems of concepts that are the result of scientific practice. Consequently, theories are subject to either modification or revolutionary change. However, the character of the real world that the theories are designed to describe remains unchanged. The question remains, what is the relationship between the constantly changing world of conceptual theory and the unchanging character of the physical world (Chalmers, 1978)? As science progresses so it approaches closer to the truth, but if this statement is taken too literally, according to Chalmers (1978), then it becomes untenable. There is no obvious convergence to an end point (the true description) as science progresses. If theory A can explain more observational statements than theory B, then A is 'closer to the truth' than B; however, observational statements are theory-laden and fallible and so it may be that B is closer to the truth. If a theory clashes with some observational statement, then it may be the observational statement that is at fault (e.g. the modern description of the moon's trajectory versus the observational statements that refer to the size of the moon when it is near the horizon. That the moon appears larger near the horizon is regarded as an illusion, even though the cause of the illusion is not well understood) (Chalmers (1978). The problem of realism as a representation of 'ontic reality' can be overcome if we adopt the objectivist position of science as an objective practice - that through our interaction with the world we create real problems that require real solutions. Physics doesn't describe the world as it is, it describes our interaction with the world (so to speak).

According to Chalmers (1978), the product of scientific practice is the maze of theories that make up a science - at a particular historical juncture the maze of theories will constitute a *problem situation* that will have an objective, autonomous existence. For Chalmers (1978), scientific theories can and often do have consequences that were unforeseen and unintended by the original proponents of the theory, and that these consequences exist as properties of the theory that are there to be discovered by further scientific practice.

Chalmers (1978) illustrates with the example of Maxwell: when Maxwell introduced the concept of a displacement current to Faraday's concept of an electric field, he was unaware of the far reaching consequences of such a move, namely, that it predicted a new kind of phenomenon, radio waves. This consequence was not realised until two years after his death. Also unknown to Maxwell was that such a move was a first step towards undermining the Newtonian world view of the universe as a material system governed by Newton's laws - a world view supported by Maxwell and his school. Already present in the writings of Maxwell, in the form of an objectively existing opportunity, was a programme that was pursued by Lorenz, Hertz and others - an opportunity that was not fully grasped by the Maxwellians Oliver Lodge and Joseph Larmor or by Maxwell himself. The fact that problem situations provide objective opportunities helps to explain the many examples of simultaneous discoveries in science, such as the law of conservation of energy by several independent workers in the late 1840s (Chalmers, 1978).

Scientific theories or research programmes flourish or flounder as a result of scientific practice and not as a result of the decisions of individual scientists (Chalmers, 1978). The scientific practice will exist in a particular society as an autonomous practice provided it plays an appropriate role or function in that society. Secondly, given that a particular scientific practice has a role to play, the ideological practice of education will adjust in such a way that there will be sufficient individuals with the appropriate consciousness for carrying out the various roles necessary for the particular practice in question (Chalmers, 1978). The position argued for here is that the **analysis of theories, problem situations, etc. as objective structures is prior to and independent of the analysis of the psychology of individual scientists and the sociology of scientific communities**. This position is an objective one because it argues that it is the structure of theories themselves, and the programme that they offer, that represents an important causal factor in any

explanation of conformity among the members of scientific communities (Chalmers 1978). To stress the point further, once we have an understanding of the nature of the products of scientific practice, namely scientific theories, then we are in a better position to understand the behaviour of scientists and the structure of scientific communities (Chalmers, 1978). In very much the same way, once we understand the structure of Newtonian mechanics in the way it describes the world then we are in a better position to examine student intuitive ideas and the steps necessary to change those ideas. **Hence the need to examine the logical structure of the Newtonian system, in the next chapter, prior to the consideration of student cognition.**

The consideration of student cognition prior to the logical structure of Newtonian mechanics may result in serious pedagogical consequences. For example, consider Von Glasersfeld's subjectivist perspective that knowledge is not of the world but of our experiences of the world. His book *Radical Constructivism: A Way of Knowing and Learning* (1995), contains 10 chapters, the first 9 of which attempts to explain and justify his perspective. The final chapter, *To Encourage Students' Conceptual Constructing*, provides pedagogical advice based on the perspective argued in the previous 9 chapters. However, because of the perspective, the advice given in chapter 10 contains serious flaws. To encourage conceptual change, the teacher has to assess the students conceptual structures. There is a need for the teacher to 'infer student thinking', and von Glasersfeld suggests that *if one starts from the assumption that students generally try to make sense of their experience, it is usually possible to get some idea of how they think* (page 187). However, it does not necessarily follow from the assumption that conceptual change can occur as a result of presenting a new 'experience' that the student cannot account for. But this is what von Glasersfeld is stating: *It would seem more efficient* (as opposed to stating what is correct) *to present the students with situations where the lay theory they have been*

using does not work. The motive to look for a more successful theory may then arise from their own perspective (page 187, my emphasis). This begs the question as to how the student knows that his theory 'does not work' when confronted with an example that his theory cannot account for. If it is a question of experience, then the student can always instantiate an idiosyncratic framework to account for the 'experience'. Von Glasersfeld refers to Minstrell who spoke of preconceptions that serve the students quite well in their daily lives. Within the perspective of von Glasersfeld - that it is all a question of constructing sense from our experience - there is no clue as to **how** the student comes to understand scientific concepts as a way of understanding his or her experience. However, von Glasersfeld offers what he calls a 'down-to-earth' approach for teachers by outlining different types of conceptual change (with reference to a paper by Dyksta, Boyle and Monarch published in 1992):

- 1. Differentiation, wherein new concepts emerge from existing, more general concepts - for example, velocity and acceleration emerging from generic ideas of motion....*
- 2. Class extension, wherein existing concepts considered different are found to be cases of one subsuming concept - for example, rest and constant velocity coming to be viewed as equivalent from the Newtonian point of view.*
- 3. Re-conceptualisation, wherein a significant change in the nature of and relationship between concepts occur - for example, the change from 'force implies motion' to 'force implies acceleration'....*

A catalogue of such patterns may give the teacher a better chance to make an educated guess about the student's 'zone of proximal development'. What Dykstra calls 're-conceptualization' clearly involves reflection. I would suggest that mapping items such as motion, velocity, and acceleration in terms of attentional frames (see chapter 4) would induce students to reflect and help them to form new abstractions. (page 189).

However, compare 1 and 3 with the following statement made by Champagne *et al* (1982) a decade previously:

The typical uninstructed student has the motion schema: 'A push produces motion'. As a result of appropriate instructional experiences, the student's motion schema could become: 'A force produces acceleration'. The fixed portion, 'push' in the initial schema has been replaced by a more general variable, 'force' which can take on several values in addition to 'push'. Similarly, the general variable, 'acceleration', which can have different values, has replaced the initial schema's fixed portion, 'motion'. The modified schema is considerably more abstract and, hence, should have a much broader range of applicability....

Empirical evidence on mechanics learning demonstrates that this instructional strategy is not generally effective and suggests that, while the gradual modification of schemata doubtlessly involves generalization and specialization, in highly integrated schemata

more dramatic changes, amounting essentially to a shift to a new paradigm (in Kuhn's [1962] sense), must also take place (my emphasis).

Von Glasersfeld underestimates the nature of misconceptions. For example, the concept of velocity and acceleration may not emerge from generic ideas of motion if students hold the *position criteria of speed* (in response to the question *If a car travelling at 70 mph overtakes a lorry travelling at 40 mph, how do their speeds compare when the front of the car is level with the front of the lorry?* many students would reply that the speeds are equal. See Hewson, 1985). Also, von Glasersfeld seems to imply the scaffolding approach which is a misinterpretation of the Vygotskian perspective. Within the Vygotskian perspective, the teaching of scientific concepts develops cognitive growth. The starting point must therefore be the consideration of the structure of those concepts prior to the consideration of intuitive ideas, especially if the structure is so highly integrated that it requires a 'paradigm-shift'.

This chapter concludes that 1) the *subjective approach*, 2) the *consensus approach* and 3) the *objective approach* to the history of science are valid forms of inquiry, nonetheless 3) is prior to 2) and 2) is prior to 1). However, from a pedagogical point of view, the **development** of individual cognition cannot be separated from the interaction by the teacher. Consequently, 2) ought to be structured according to an *intersubjective approach* that examines the influence of teaching strategy in cognitive development.

2.4.0 SUMMARY: THE DISPUTE BETWEEN THE CONSTRUCTIVIST/RELATIVIST POSITION AND MY OWN POSITION WITH REGARD TO CHILDREN'S LEARNING AND THE STATUS OF KNOWLEDGE.

If knowledge is of our experience, and not of the external world, then the status of knowledge is undermined. Scientific concepts are not theoretical fictions that can explain

anything you want to explain. They enable us to model the physical world and hence explain the world according to the model. The ability to explain the world according to the model is a hallmark of intellectual achievement. If a relativist emphasis is placed on science as if it were merely a consensual or cultural product, then apart from raising the obvious question as to why bother teaching science, a whole generation may be robbed of a very important intellectual resource. Science is consensual, but it also creates objective problem situations and it is this aspect of science that promotes intellectual growth. If knowledge was merely a question of viability and experience, as von Glasersfeld and many constructivists maintain, then science lessons perhaps ought to be replaced with extra language lessons. It is **because science speaks of the external world that science has a greater potential to arouse higher mental functions.** We can see this in the distinction between a quantitative and qualitative understanding of mechanics. The former requires the ability to apply the appropriate rules to given idealised examples, whereas the latter also requires the ability to form the idealised example as a representation of the physical world. Downplaying the external world would probably not make that much of a difference if the aim was to improve understanding in mechanics as a traditional subject formerly known as 'applied mathematics'. It would make a difference, however, if mechanics models the external world - **as educators, we have to understand the way in which mechanics speaks of the world if we are to understand the way students can also model the external world according to the Newtonian system.** An obvious point, but one that may be lost on many science educators given the influence of constructivism. Without constructivism and the emphasis placed on relativism and subjectivism/idealism, this chapter would not have been necessary!

If the construction of knowledge is the construction of sense from our experience, as the radical constructivists maintain, then how can we construct abstract scientific concepts

such as *electrons*, *latent heat* and *force* from our experience? What experiences of the world does a child have such that abstract scientific concepts can be constructed? Of course, the teacher should always relate the abstract to the concrete whenever possible, but that is entirely different to constructing the abstract from the concrete which is rarely, if ever, possible! What experiences does a child have of latent heat such that the concept of latent heat can be constructed? The experiences of the world that a child has may include ice melting into water or water changing into steam, to which the teacher may refer to in the elaboration of latent heat, but the child does not have direct experience of molecular structure from which the concept of latent heat becomes apparent. The experiences a child may have of molecular structure would have been from previous lessons or from a book - but never from the everyday experience of living in the real world.

Of course, a teacher cannot teach tensor-algebra to a GCSE intermediate class. From the constructivist perspective, consideration has to be given to what the student knows and understands. However, prior experience in terms of prior knowledge and understanding is essentially very different to prior experience in terms of everyday experience - and constructivism appears to confuse this very distinction! By not specifying exactly what is meant by prior experience, constructivism begins with the (acceptable) initial premise that students are not blank-slates, but then confuses the issue by referring to prior experience as experience of the everyday world. Social constructivism is as culpable as the radical variation for clouding this very issue. For example, Davis *et al* (1993), advocating the relativism of social constructivism, complains that:

A Hispanic in downtown Miami is assumed to require the same science content and to learn in the same manner as a Caucasian in rural northern Florida, or an African-American from suburbia.

Is there such a thing as an Hispanic, Caucasian or African-American universal law of gravitation; and is there any reason why an Hispanic, as opposed to a middle-class

Caucasian, should not learn the universal law of gravitation? Surely the manner in which to teach science should be determined according to the best way that it can be understood, which has more to do with the concepts of the subject than it has to do with culture [and indeed, according an international survey by Thijs and Van Den Berg (1995), there is essentially little cultural variation in the ‘misconceptions’ that students have of science concepts, and that: *[s]tudents seem to know how to distinguish between the world of school physics and most possible cultural and religious connotations of science concepts*].

The relativism of social and radical constructivism downplays science as an objective body of knowledge that models the real (external) world and has the potential to promote cognitive growth. If scientific concepts are ‘fictions’ that can explain anything you want to explain [as von Glasersfeld (1995) maintains], and if everyday experience is the source of knowledge and understanding, **then the obvious question that constructivism must address is *why teach science, or indeed, why have schooling?***

The position taken in this thesis is that science, although consensual, is an objective body of knowledge that has the potential to arouse the students minds to life. Science has this potential because it raises objective problem situations that do not admit to a subjectivist or consensual/relativist approach. The objective problem situations created by science pertain to the relationship between the concepts, laws and theories (and available evidence) of the science independently of individual cognition or social acceptance. This thesis is constructivist in the sense that students are not blank-slates and that some form of construction process does take place. However, with respect to the teaching of mechanics (in a qualitative sense), this construction process is not the assimilation of what is taught into existing cognitive structures. Scientific concepts are decontextualised and distinct from everyday ones, so their construction becomes a question of facilitation by the teacher

rather than the student making sense of experience. Within the student's ZPD, the teacher has to realise what the student already knows, but that does not mean that the teacher has to make connections to existing knowledge. Connections made with misconceptions will foster misconceptions rather than an understanding of science. The constructivism of this thesis has the following characteristics:

- It is realist as opposed to subjectivist or idealist. It is realist in the sense that the world exists independently of me, and that knowledge pertains to an objective world as opposed to subjective experience.
- It is not realist in the sense of *naive realism* or the *correspondence theory of truth* (see pages 38, 64 and 65).
- It is realist in the sense of being *objectivist* (see pages 63 to 69). However, its objectivism includes, but is also prior to, the subjective and consensus approach (see page 69).
- It is relativist - **but not in the sense of denying science as an objective body of knowledge**. It is relativist because it sees the appropriateness of investigating the individual psychology of scientists, students and the behaviour of academic communities (see page 69). However, it is maintained that the analysis of theories and problem situations is prior to and independent of the analysis of the psychology of individual scientists and the sociology of scientific communities (see page 66). Although knowledge is historically conditional (a relativist position), nevertheless science consists of objective structures that exist within a *practice* independently of individual cognition or consensus (see pages 65 and 66).
- Pedagogically, it therefore follows that an understanding of Newtonian mechanics would put us in a better position to examine misconceptions with a view to changing them (see page 67).

This thesis is constructivist, but it denies the subjectivism/idealism of radical constructivism and the narrow relativism of social constructivism. Because of the emphasis placed on objectivism, this thesis may be described as *Objective Constructivism*.

CHAPTER 3 - THE LOGICAL STRUCTURE OF NEWTONIAN MECHANICS AND THE PEDAGOGICAL IMPLICATIONS

3.1 INTRODUCTION

It was argued in chapter 2 that an understanding of the structure of Newtonian mechanics would put us in a better position to examine student intuitive ideas and the steps necessary to change those ideas. This argument followed Chalmers' (1978) point that the analysis of scientific theory as objective structures is prior to and independent of the analysis of the psychology of individual scientists and the sociology of scientific communities, so-much-so that an understanding of scientific theory puts us in a better position to understand the behaviour of scientists and the structure of scientific communities. Contrary to the radical constructivist emphasis on subjectivity, chapter 3 will examine the logical structure of the Newtonian system before the consideration of student cognition (chapter 5). **This would appear consistent with the Vygotskian perspective - that it is the teaching of scientific concepts divorced from spontaneous everyday concepts that cognitive structures are supplied.** Pedagogically, the consideration of the Newtonian concept of force ought to be prior to the consideration of the various misconceptions of force precisely because of the former's decontextualised character - the role it plays in a structured system of concepts. That the consideration of the former ought to be before the latter can be illustrated by the following passage:

The concept 'force' as used in physics is precise because it acquires its meaning from the role it plays in a precise, relatively autonomous theory, Newtonian mechanics. The use of the same word in everyday language (the force of circumstances, gale-force winds, the force of an argument, etc.) is imprecise just because the corresponding theories are multifarious and imprecise (Chalmers, 1978).

The logical structure of the system will therefore be a major influence in the design of the teaching strategy. For example, if the laws of motion are considered to be *empirical propositions* like the assertion that ‘all ravens are black’ (for example, according to Black, 1964, Newton himself construed his principles as empirical generalisations of universal scope) then the appropriate teaching strategy might consist in a series of ‘hands-on’ experiments that verify the truth-value of those laws. If the laws are considered to be *analytic propositions* (for example, according to Black, 1964, Kant considered the laws of motion to be self-evident truths) then the appropriate teaching strategy might be the presentation of the three laws of motion at the outset and the application of those laws to phenomena as a deductive process (presumably the coherence of the system compared with intuitive ideas would be ‘highlighted’ - made apparent - in the application of the system as a deductive process). To illustrate this point further, consider the logical status of the first law of motion. If we define ‘impressed force’ to **mean** the same as ‘rate of change of momentum’, then the first law reduces to the triviality that a body continues in its state of rest or uniform motion unless its momentum is changing (i.e. unless it is not at rest or in uniform motion). According to Eddington, the first law may be expressed as: *Every body continues in its state of rest or uniform motion in a straight line, except in so far as it doesn’t* (Black, 1964). **If this happens to be the logical status of the first law, then the first law can only be asserted by the teacher (rather in the manner of stating Euclidean axioms as a set of premises in a geometrical proof) and not constructed by the class as such. The corollary is that how a teaching strategy considers the logical structure of Newtonian mechanics will have a consequence in the way that strategy is applied in the classroom¹.** For example, Arons (1990), despite his references to *constructivism*, considers

¹ This corollary would seem to be consistent with Koulaidis and Ogborn’s ,1995, point that there exists a large body of empirical research that indicates that the teaching of science is related in many ways to the teachers’ views on the philosophy of science. This point is discussed further at the beginning of chapter 4.

science (including mechanics) as a summary of sense experience (*positivism*). He stresses the notion of *operationalism* whereby definitions are given by the 'operations' which are used to measure it. Consequently, he proposes the *phenomenological teaching approach* that urges qualitative discussion of hands on experiences that lead to the operational definition of the concept to be taught. Throughout his book *An Introductory Guide to Physics Teaching*, Arons (1990) stresses the precept *idea first and definition afterwards* as if the operational definition should be given by the teacher after it has had a conceptual 'airing' as such. Arons refers to the *Socratic method* of teacher-led questioning, but within the context of the phenomenological approach. Consequently, Arons' Socratic method appears little more than a qualitative discussion of phenomena between teacher and class and not as a method of strategic questioning whereby the class itself arrives at the target-concept. Despite Arons' reference to constructivism, the class cannot itself construct the target-concept if, at the end of the day, the operational definition of the concept has to be 'given' with reference to an experiment (There are further problems with the positivist approach! For example, if the operational definition of mass can only be given with reference to a collision between two bodies, then how does that square with the definition of mass, as a measure of inertia, as given by the first law?) A critique of positivism and its pedagogical consequences will be discussed in chapter 4. The essential point here is that if the class is to construct for itself the Newtonian system, as opposed to the system being 'given' by the teacher via a series of experiments, then the logical structure of the system must provide the 'key' as to the possibility of its construction.

An awareness of the problems concerning the understanding of the concept of force was given by Heinrich Hertz in the introduction to his *The Principles of Mechanics* (published in 1894):

Weighty evidence seems to be furnished by the statements which one hears with wearisome frequency, that the nature of force is still a mystery, that one of the chief problems of

physics is the investigation of the nature of force, and so on. In the same way electricians are continually attacked as to the nature of electricity. Now, why is it that people never in this way ask what is the nature of gold, or what is the nature of velocity? I fancy the difference must lie in this. With the terms 'velocity' and 'gold' we connect a large number of relations to other terms; and between all these relations we find no contradictions which offend us. We are therefore satisfied and ask no further questions. But we have accumulated around the terms 'force' and 'electricity' more relations than can be completely reconciled amongst themselves. We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections existing between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions.... We are convinced,.....that the existing defects are only defects in form; and that all indistinctness and uncertainty can be avoided by suitable arrangement of definitions and notations, and by due care in the mode of expression.....(quoted by Fann, 1969).

It is not by finding out more and more connections between force and phenomena that will reduce the number of 'vexations' concerning force; rather, it is the removal of those 'painful' contradictions between the concept of force in mechanics and our understanding of phenomena. Hertz states that the removal of those contradictions will not answer as to the **nature** of force, the implication is that such a removal will only enable the ability to describe phenomena qualitatively according to the concept of force in mechanics (this is analogous to describing phenomena with reference to the electron as either a particle model or as a wave model. To ask whether matter is comprised of waves or particles is to ask an illegitimate question - the electron as a wave or particle is a form of description - the point here is *can the phenomenon under investigation be described in terms of electrons as waves or particles?*). In essence, the problem - that is, the 'vexation' concerning force - may be expressed as: *the propositions of mechanics that define force are not expressions of fact* [and if 'fact' were to be defined as a proposition that is true, as Bertram Russell did, then the defining propositions of force are themselves not facts]. Hooke's Law, or Coulomb's law for the coefficient of friction, are propositions the truth-value of which is dependent upon experiment. That the coefficient of friction between the surface of the road outside and the surface of the average tyre is 0.78, pictures a possible fact. Such

propositions express a relation between measurable variables that have a physical interpretation, and so it can be shown whether the relation holds or does not hold. The problem with 'force' is that the propositions that define it do precisely that - they define force and do not simply express a relation between variables (if all you knew about Newtonian mechanics was that the thrust of a spring may be proportional to its compression, then you would be none the wiser as to the Newtonian understanding of the concept of force. In much the same way, $F = ma$ is not enough to define force as a concept and it is not enough to understand phenomena qualitatively). It will be argued that the laws of motion are not facts but a system of axioms that together define *force*, that the axioms provide a way of speaking about the world, and that *cognitive reorganisation* is required to speak that way.

Wittgenstein was greatly influenced by Hertz, and Hertz's conception of the nature of the problems in the philosophy of science seem to be exactly like those of Wittgenstein (Fann, 1969). Wittgenstein (1961), in his *Tractatus Logico-Philosophicus* provides an analysis of the logical structure of Newtonian mechanics and the way the system speaks of the world [according to Bolton (1979): *there exists a great harmony between the 'Tractatus' and seventeenth-century physics.....the 'Tractatus' is the true philosophy of that physics*]. The analysis provides a description of the system as a 'unified-form', and thus provides a clue as to how the system can be constructed by the class as a 'world-view' or *paradigm* to explain the unfamiliar. [There is evidence to suggest that a piecemeal strategy to overcome misconceptions are ineffectual (Hestenes, 1992), and this implies that the ability to provide a Newtonian account for the unfamiliar requires the construction of the Newtonian system as a *unified-form* of description. The precise meaning of this term 'unified-form' will be one of the central features of this chapter, and the relevance of such a term (especially in chapter 5) will be shown in the way intuitive ideas of force and motion lacks such a

unified-form]. Wittgenstein's analysis and its relevance to the teaching strategy will be considered in the next section.

3.2 THE LOGICAL STRUCTURE OF NEWTONIAN MECHANICS

How are the laws of physics related to the laws of logic and yet still speak of the objects of the world? Wittgenstein's position is as follows: The laws of physics are stated in mathematical form, and mathematics is a method of logic. The propositions of logic are tautologies (they say nothing of the world and so 'lack-sense' - *sinnlos*) because they are true for all combinations of the truth-values of their component propositions (e.g. *Either it is raining or it is not raining* is always true and hence is a tautology - it therefore tells me nothing about the weather. Compare this with *it is raining* which is a proposition which represents a possible fact). However, Wittgenstein argues, the laws of physics are not tautologies. As he puts it:

The laws of physics, with all their logical apparatus, still speak, however indirectly, about the objects of the world (Tractatus, proposition 6.3431).

How is this possible? Wittgenstein explains with the use of a 'network' analogy together with a 'building of bricks' metaphor:

Newtonian mechanics, for example, imposes a unified form on the description of the world. Let us imagine a white surface with irregular black spots on it. We then say that whatever kind of picture these make, I can always approximate as closely as I wish to the description of it by covering the surface with a sufficiently fine square mesh, and then saying of every square whether it is black or white. In this way I shall have imposed a unified form on the description of the surface. The form is optional, since I could have achieved the same result by using a net with a triangular or hexagonal mesh. Possibly the use of a triangular mesh would have made the description simpler: that is to say, it might be that we could describe the surface more accurately with a coarse triangular mesh than with a fine square mesh (or conversely), and so on. The different nets correspond to different systems for describing the world. Mechanics determines one form of description of the world by saying that all propositions used in the description of the world must be obtained in a given way from a given set of propositions - the axioms of mechanics. It thus supplies the bricks for building the edifice of science, and it says, 'Any building that you want to erect, whatever it may be, must somehow be constructed with these bricks, and with these alone' (6.341, my emphasis).

The system of mechanics is to be understood as a co-ordinate system rather than as a set of assertions about reality such as 'water always flows downhill unless compelled to do otherwise' (Black, 1964). *Whatever kind of pictures these make*, the form of the description, that is, the shape of the net, remains the same. Similarly, mechanics imposes the same form of description on the motion of a ball thrown as it does the motion of a car braking. The laws of motion jointly provide a way of talking about nature - it is therefore misleading to call Newtonian mechanics either true or false (Black, 1964). The choice of one system of mechanics rather than another, of one theoretical language rather than another, is not wholly arbitrary:

Although no scientific language can fit the world perfectly, the extent of the adjustment needed to obtain empirical consequences of a predetermined accuracy are a measure of the theory's adequacy. The greater the number of observable phenomena that must be neglected, the less adequate is the system of description in question (Black, 1964).

Just as the shape of a suit tells us something about the contours of its wearer, though the fit is imperfect, so the choice of the most suitable system tells us something about the character of the actual world (Black, 1964). [Although this implies the *correspondence theory of truth* - that facts have structure that is 'copied' or 'pictured' by propositions - nevertheless, with regard to scientific propositions, that which is pictured are states of affairs that are structured according to scientific *practice*. Towards the end of each cycle of the modelling process we 'compare our results with reality' even though, according to Chalmers (1978), the observations and measurements involved are theory laden and fallible. The reality that scientific propositions picture, is not the 'ontological reality' of von Glasersfeld, but the physical reality that presents itself through scientific practice. This point regarding Wittgenstein's *Tractatus* is discussed in chapter 2, page 40]. This is a realist way of speaking, because it says that physics, no matter how **indirectly**, 'still speaks about the objects of the world':

The possibility of describing a picture like the one mentioned above with a net of a given form tells us nothing about the picture. (For that is true of all such pictures.) But what

*does characterise the picture is that it can be described **completely** by a particular net with a **particular** size of mesh.*

*Similarly the possibility of describing the world by means of Newtonian mechanics tells us nothing about the world: but what does tell us something about it is the **precise way** in which it is possible to describe it by these means. We are told something about the world by the fact that it can be described more simply with one system of mechanics than with another (6.342, author's emphasis).*

Newtonian mechanics imposes a *unified-form* on our description of the world in the sense that if we want to describe the world in terms of Newtonian mechanics, then we must describe the world in terms of the basic concepts of Newtonian mechanics - which, in turn, means accepting the basic axioms of Newtonian mechanics:

*Mechanics is an attempt to construct according to a single plan all the **true** propositions that we need for the description of the world (6.343, author's emphasis).*

The laws of motion are the possible forms of the propositions of physics, and *are about the net and not about what the net describes* (6.35). Physics is a theoretical sub-language (the 'net') complete with syntactical and semantical rules for its use. According to Black (1964), the syntactical rules of the sub-language are the correlate of the *form* of the network (the laws of motion express the syntactical connections of the basic theoretical terms - the laws state which is to count as 'body', 'mass', 'change of motion' and 'force'), and the semantical rules answer to the procedure for placing the net over the picture and obtaining a determinate verdict (the semantical rules link the theory with observation, conferring a physical meaning upon what would otherwise be an empty mathematical skeleton). **The *form* is 'unified' in the sense of being 'complete' in that it provides for all contingencies by prior stipulations about how they shall be treated in the theory** (Black, 1964). However, this is not to say that a theory can accommodate all empirical data. Research in physics will always produce anomalies that the prevailing theory cannot accommodate (Bernal, 1969) - the data that a theory does accommodate does so with one structure, plan or 'net'. The essential point is that if a student is to account for unfamiliar phenomena qualitatively according to the Newtonian system, then his or her understanding

of the system must be unified in exactly the same way as the system as a form of description is unified. It is in this sense that a student can be said to have a 'global-understanding' of mechanics: **the 'penny drops', so to speak, when the student comprehends the system as a unified form of description** [It is because the Newtonian system is a unified form of description that we can begin to understand the Newtonian system as a *paradigm* - and the notion of paradigm is one of the important concepts in chapter 5. According to Chalmers, 1978, it is because of the nature of a paradigm that it belies definition; however, there are typical components that make up a paradigm. Newton's laws form part of the Newtonian paradigm, and the paradigm includes methods of applying the laws to planetary motion, pendulums, billiard-ball collisions and so on. Also included in the paradigm will be instrumental techniques necessary in bringing the laws of the paradigm to bear on the real world. A further component of paradigms consists of some very general metaphysical principles that guide work within the paradigm.

According to Chalmers (1978):

Throughout the nineteenth century, the Newtonian paradigm was governed by an assumption something like, 'The whole of the physical world is to be explained as a mechanical system operating under the influence of various forces according to the dictates of Newton's laws of motion' (page 87).

Finally, all paradigms will contain general methodological prescriptions such as 'Make serious attempts to match your paradigm with nature' (Chalmers, 1978)].

The laws of motion are not facts in the sense that they are neither true or false as such. They are axioms that jointly provide a way of talking about nature. They are prescriptions for the formulation of the propositions of the system, and provide for the formulation of all propositions belonging to mechanics. What we call the laws of nature are the laws of our method of representing it: *the law of causality is not a law but the form of a law* (6.32).

This is not a denial of natural laws, but it is a denial that such laws are the **explanations** of natural phenomena (6.371) - natural laws do not enable us to say that B happened **because** of A. Natural laws describe, they do not 'explain' as such. Natural laws have the character of being 'logical models' from which all propositions of certain logical forms can be derived or formulated, and as such are possible forms in which the propositions of science can be stated (Proctor, 1966). This implies a distinction between a scientific theory and a law. A theory is a collection of propositions which are descriptive of the facts, and is a system which brings the descriptions into a unified form; a law says only how the facts are to be described and are only possible forms of the propositions [e.g. the kinetic theory of gases is structured according to the laws of motion: the pressure of a gas on the walls of a container, $(p, = 1/3\rho c^2$, where ρ is the density of the gas and c the mean square velocity of the molecules of the gas) is derived as a modelling procedure from the laws of motion]. Laws treat of the symbolism and not of what is symbolised (i.e. laws do not speak of the world as such, rather they structure the way we can speak of the world - the laws *are about the net and not about what the net describes*), and are in a sense licences to make inferences (Proctor, 1966). Any proposition that is in accordance with the laws will be a true description of any fact having that particular logical structure (just as the interference or diffraction effects of light exhibit a structure that does not admit to being described by the corpuscular theory of light) - and this is the realist 'component' of the Newtonian system. The Newtonian system is **both** a *constructivist* and a *realist epistemology* because **meaning is constructed** (the syntactical rules of the system) and **matched with reality** (the semantic rules of the system). Newtonian mechanics makes a distinction between the actual physical world and the concepts constructed to describe it. This is in stark contrast to positivism which holds that the meaning of physical concepts is **extracted** from physical experience. Contrary to the positivist position, the pedagogical implication of mechanics as a form of description is that the 'hands-on' experimental approach cannot be prior to the

construction of meaning required to describe scientifically the phenomenon under investigation. Like a game of chess, *beginners must know these rules before they can play Newtonian games with any assurance* (Hestenes, 1992). If a student is to conduct an experiment, then the student must understand the theory in order to make sense of the experiment (the design, construction, aims and objectives of the experiment) - and this is consistent with Toulmin's (1967) argument that the scientist does not perform an experiment without a point in mind². Pedagogically, scientific theory and meaning cannot be abstracted from the 'hands-on' approach - on the contrary, experiments can only provide a means by which theory can be applied to make sense of the phenomenon under investigation. This is consistent with Chalmers' (1978) point that *precise, clearly formulated theories are a prerequisite for precise observation statements. In this sense theories precede observation* (page 27).

Consider the following extract from an A-level physics text-book:

Events often seem to contradict the first law, for it is our natural experience that there are many familiar examples of motion in which moving objects come to rest when (apparently) left to their own devices. Closer examination of the circumstances, however, reveals that in every case there is some sort of retarding force acting. Such forces are often due to friction between solid surfaces or to air resistance (Muncaster, 1985).

It is not so much a 'closer examination of the circumstances,' more an **expectation** of there being a retarding force because of the first law of motion! Of course, we may experience resistance forces to appreciate why a moving object, 'left to it's own devices,' comes to a halt; but we must bear in mind that Aristotle and natural philosophers during the following millennia and a half overlooked this.

² Driver (1994) pedagogically illustrates this point: *The slogan 'I do and I understand' is commonly used in support of practical work on science teaching. We have classrooms where activity plays a central part. Pupils can spend a major portion of their time pushing trolleys up runways, gathering, cutting and sticking tangling metres of ticker tape.....To what end? In many classrooms, I suspect, 'I do and I am even more confused'.*

The Newtonian system imposes a unified form on our description of the world, and the laws of motion are possible forms of the propositions of the system - hence the axiomatic nature of the laws of motion. According to Hestenes (1987, 1992), concepts in mechanics are either *explicitly well defined* or *implicitly well defined*. A concept is *well defined* if its meaning is established in relation to other concepts. A concept is defined *explicitly* by expressing it directly with other concepts (e.g. *momentum* as the product of *mass* and *velocity*). A concept is defined *implicitly* by specifying a set of axioms that relate it to other concepts, and so force and mass are implicitly defined by Newton's axioms (Hestenes, 1987,1992). Hestenes argues that Newton's axioms must also include the zeroth law, *that every real object has a continuous history in space and time*, which specifies the primitive kinematical properties of position and motion thus defining the Newtonian concepts of space and time.

The Newtonian system is likened to a hierarchical system of bricks, with the axioms of the system supplying the bricks. This suggests that the understanding of force and motion is essential in understanding mechanics as a whole, and indeed physics in general! Galili (1995) reported on the way student conceptions in electromagnetism were influenced by prior conceptions of force and motion, e.g. the misconception that a particle's trajectory coincides with the line of force, parallels the Aristotelian understanding of the movement-force relationship. According to Galili (1995):

Research should not only discover the form of the misconception, but also the factors which could cause or influence the construction of this view by an individual. This could provide the necessary basis for the constructive intervention by the educator. Physics is known as being an especially 'fertile soil' for students' misconceptions. A huge edifice, which today we call physics, consists of various domains. The importance of mechanics is more than just being one of these domains. It determines the 'rules of the game', defines the main tools in physics, presents the most universal laws of nature. It actually describes the method of the discipline of physics which is then applied in all other domains in this discipline. This is why mechanics always opens any physics curriculum (my emphasis).

The importance of mechanics is reflected in the importance of its defining axioms - the laws of motion. The importance of the laws of motion implies that the misconceptions concerning momentum and energy are not so serious compared with the misconceptions concerning force and motion; and there is evidence to suggest that misconceptions of momentum and energy may reflect misconceptions lower down the hierarchy. For example, the misconception that the momentum of a body in horizontal circular motion is constant (see Graham, 1996) reflects a misconception concerning the vector nature of velocity, and an understanding of velocity is essential in the understanding of acceleration and force (as given by the laws of motion together with the zeroth law). That the failure to realise that the magnitude of the momentum of a puck increases if a force is exerted at right-angles to its path [see question 13(ii) of Graham's (1996) momentum hierarchy questionnaire] reflects the lack of understanding force as the agent that changes momentum (as defined by the second law). That the defining features of the system are determined by a set of axioms may suggest why many students find the system difficult to comprehend. According to Tall (1991), the ability to comprehend a system defined by a set of axioms requires a *massive cognitive reorganisation* (this is consistent with the point made by Champagne *et al*, 1982, see page 74 above, that understanding the Newtonian system would more than likely require a dramatic change in schemata as the system amounts to a shift to a new paradigm). The difficulty to comprehend the Newtonian system is further compounded by the existence of misconceptions - misconceptions that stand in the way of constructing force as an abstract and implicitly well-defined concept!

All the concepts in Newtonian mechanics are defined by concepts, but there is no infinite regress because *force* is defined by a set of axioms upon which the system is structured. According to Vygotsky, a child's reasoning ability can develop by learning scientific concepts because scientific concepts are de-contextualised - a de-contextualised concept

has meaning in relation to other concepts. This is not to imply, however, that the scientific meaning of a concept can be constructed by simply knowing the definition of the concept - in the student's zone of proximal development, the teacher facilitates the understanding of the concept, the concept should not be simply 'given' as such. However, because the concept of force is central to the Newtonian system, the understanding of force takes on a different character to the understanding of, say, momentum or kinetic energy. Momentum or kinetic energy is put into context by definition with other concepts. Force, on the other hand, is defined in such a way that structures the context within which momentum and kinetic energy has meaning within the Newtonian system (and this is highly suggestive of the teacher facilitating the understanding of the concept of energy as the ability or capacity to do work, with reference to work done in terms of force and the distance moved by the point of application of the force. This is consistent with the law of conservation of mechanical energy as derived from the laws of motion). The essential point here is that concepts acquire a precise meaning by way of a coherently structured theory³.

Mechanics is a description of the world that is consistent for many phenomena. The laws applied to a vertically thrown ball offer the same description as of a car braking: both the ball and car are slowing down due to an opposing force to the motion. Many misconceptions appear to be fragmentary in that they seem particular to the motion under consideration: a thrown ball has to overcome gravity, hence a force 'pushing' the ball. To

³ This would seem to be consistent with Chalmer's (1978) observation of the character of scientific theory in the history of science, that a theory should be so well structured that it contains clear clues and prescriptions as to how the theory should be developed and extended. The theory should be, in a sense, an open-ended structure that offers a *research programme*. According to Chalmers (1978), Newtonian mechanics offered such a programme for eighteenth- and nineteenth-century physics because it provided a system for explaining the entire physical world in terms of mechanical systems involving various forces governed by the laws of motion. However, the character of a theory is different to its history. For Chalmers (1978), the history of a concept involves the initial emergence of the concept as a vague idea followed by a gradual clarification as the theory within which the concept plays a part takes a more precise and coherent form.

understand the Newtonian system as a form of description of the world, is to understand *force* as an abstract concept defined axiomatically and applied according to the axioms. A student is more likely to build the concept of force as an abstract concept (as opposed to an intuitive and idiosyncratic one) if he or she is aware of the consistency of the Newtonian system as a form of description compared with the inconsistency of intuitive ideas. Consider the following two *concept questions*:

1) *A plane travelling with uniform horizontal motion releases a bomb. Draw a diagram showing the subsequent motion of the bomb.*

2) *The dotted line represents a ball travelling along the ground. The line marked F is a kick applied to the ball at right-angles to its motion. Draw the subsequent motion of the ball.*

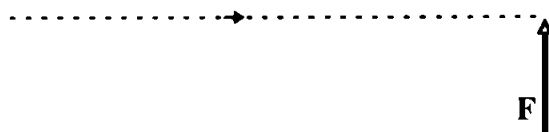
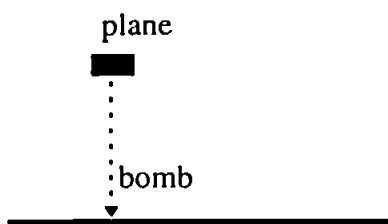


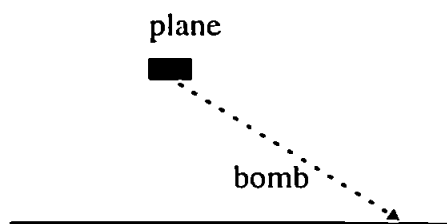
Figure 1 - Diagram Representing Ball Travelling Along the Ground (Question 2)

Both questions may illicit different student responses. Question 1 was included in the teaching package post-survey (see chapter 8) and question 2 was included in the pre-survey. Some of the various responses were (figure 2):

1)

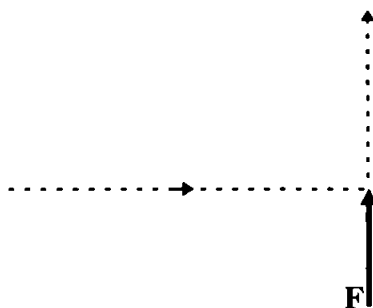


11 responses out of 232 (4.7%)

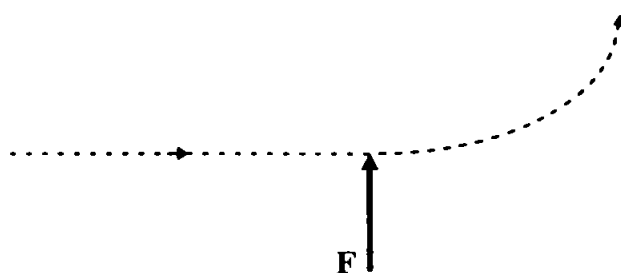


61 responses out of 232 (26.3%)

2)



40 responses out of 290 (13.8%)



9 responses out of 290 (3.1%)

Figure 2 - Some of the Responses Given to two Similar Concept Questions

Both questions exhibit the same logical form, hence both questions fall under the same Newtonian description: that the force under consideration (the weight of the bomb in question 1, and F in question 2) has no component in the previous direction of motion and so does not affect the motion in that direction⁴ (despite the force as constantly applied in one and instantaneous in the other).

The axioms of mechanics exhibit possible states of affairs. If a ball is thrown deep in space, then it will travel with uniform motion in a straight line (until affected by the influence of

⁴In a maths 'masterclass' poster session, two groups of thirteen year old pupils displayed a poster of question 2 and the correct diagram; but with the explanation that the subsequent motion was due to two perpendicular forces: one being F and the other being 'due' to the original motion.

another body). If an actual ball is rolling in a straight line and is given an impulse perpendicular to its motion, then the state of affairs is represented by concept question 2 above. We may observe these possible states of affairs in nature, but there is a difficulty. For the ball thrown deep in space, the state of affairs can only be observed in (or from) space. For the rolling-ball, certain conditions may have to be considered. The ball may have to be represented by an ice-puck on a surface that has a very low coefficient of friction, and the spinning effects taken into consideration. The state of affairs exhibited by mechanics represent possible facts [and this is only possible because the axioms are not propositions but *propositional functions* (the idea of a propositional function is derived from Russell)]. We can state predicate expressions like 'is a frictionless surface', even though the class of frictionless surfaces is empty, because the expression is of the form: *x is a frictionless surface*. If *x* is not determined then the propositional function **does not say anything about the world!**). One may see the correspondence between a possible state of affairs as expressed by mechanics and an actual state of affairs depicted. However, the actual state of affairs may have to be constructed in some way (e.g. reducing the coefficient of friction, conducting an experiment in space etc.). The axioms of mechanics exhibit possible states of affairs according to the way the axioms **provide** a description of the world. The student does not become aware of the way mechanics describes the world through an abstraction from actual state of affairs; rather, the student describes the state of affairs with reference to mechanics - the student describes the state of affairs by reference to the **possible** state of affairs that the system pictures e.g. 'if we model the body as a particle, and if we ignore spinning effects and air-resistance, then according to the laws of motion the body will do *that*,' etc. The model of an actual state of affairs is only an approximation, but the approximation shows the correspondence between the possible state of affairs depicted by the system and the actual state of affairs described by the system. For a better approximation, either the model has to be refined by the inclusion of other

variables, or the state of affairs to be depicted is construed in a way that replicates the possible state of affairs given by the system, e.g. surfaces that are made almost smooth so that friction can be ignored (*'reality must also strive towards thought'* - Lukacs, see page 57 above). To understand how mechanics speaks of the world, the student must be able to construct the possible states of affairs as a consequence of the defining axioms and to picture actual states of affairs by the possible state of affairs constructed. How the student can develop the skill in representing actual state of affairs by the possible state of affairs pictured by mechanics is what Hestenes (1992) refers to as the *modelling-game*. It will be shown in chapter 5 how the Socratic dialogue can be employed in the modelling game; however, it is necessary to explain the logical structure of the model before the rules of the game can be worked out.

3.3 MODELLING IN MECHANICS

The form of description in Newtonian mechanics is already predetermined, which implies that the ability to model - forming a 'picture' of a possible state of affairs - is prior to an actual state of affairs to be modelled. A possible state of affairs pictured by mechanics may be considered to be a *schema* of the actual state of affairs modelled (a discussion of the definition and structure of *schema* is given in chapter 5). In the modelling process, the *referent* (the actual body referred to) is represented as a body or particle. With reference to the influence of other bodies (in terms of either *contact* or *action at a distance*), the referent is pictured in terms of forces acting on the model representing the referent. Next the picture may be simplified by the exclusion of one or more of the forces e.g. *ignoring air resistance*. The simplification is a reduction of an actual state of affairs to a possible state of affairs constructed from the defining axioms of the system. To refine the model, a new 'picture' is formed by the inclusion of a previously excluded force. The new picture formed

exhibits the same structure as the previous picture, but with an additional variable (both structures are the same, the additional variable is slotted in accordance with the structure). That structure is determined by the defining axioms of the system. This has two interrelated pedagogical implications:

- 1) the ability to model requires the ability to construct a possible state of affairs as a representation of an actual state of affairs to be investigated, and
- 2) that ability requires the construction of the defining axioms of the system that determine what state of affairs is possible.

The ability to model requires an understanding of the laws of motion as a form of description of the world; that is, it requires the ability to 'explain' phenomena qualitatively according to the Newtonian system. That ability is restricted by the presence of misconceptions concerning the phenomenon modelled. The ability to model within the modelling process is prior to the presentation of the state of affairs to be modelled; however, that ability to model is in itself determined by the ability to 'explain' phenomena qualitatively. A qualitative question in mechanics (*concept-question*) is structured according to the possible states of affairs as determined by the laws of motion, and given meaning with reference to the phenomenon presented. A qualitative question not only serves as a beginning in the modelling process, but may reveal the misconceptions present that would impede the development of that process.

The ability to model is the ability to construct a possible state of affairs as determined by defining axioms of the system. This possible state of affairs may be referred to as *idealised abstraction*. However, the term needs qualification! The term refers to the possible state of affairs as an abstraction that is idealised - *abstract* in the **strict** sense that it is 'far-removed' from the situation that it depicts, and *idealised* in that factors are removed so that the actual state of affairs can be simplified and hence modelled. The term is not to imply

that the model is in any way 'abstracted' from the phenomenon. The Newtonian system is a form of description - it states how phenomena are to be treated, and it does this through idealised abstraction! Idealised abstraction imposes the conditions that determine the parameters within which the actual state of affairs can be spoken of. For example, a book sliding on a smooth surface will move with uniform motion; despite the impossibility of a smooth surface. It will be shown in chapter 5 that students have a tremendous difficulty with idealised abstraction, and that this difficulty is compounded by the fact that intuitive ideas may be, in a certain sense, a 'closer-fit' compared with the idealised abstraction that represents the actual state of affairs. For example, there are some students who, in their first lesson, would represent the weight of a body as:

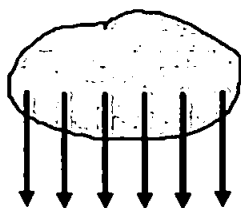


Figure 3 - The Representation of the Weight of a Body by Some Uninstructed Students

This is a closer 'fit' compared to the representation of weight by the one arrow. The weight of a body acts on the whole body, not at a point! However, the Newtonian system, as a Cartesian 'mesh', specifies a force as a line. Idealised abstraction, therefore, removes the arbitrariness of the many lines representing the one force, and allows for the quantification of that force (one student from the pilot study, see chapter 7, represented the normal reaction of a table on a book as a number of lines. I stated that the trouble with having a number of arrows from the table pointing up is that one person might put four arrows and another person five arrows. See page 207).

Although no reference is made to Wittgenstein; nevertheless, Hestenes (1987) extends Wittgenstein's ideas on mechanics as a unified form of description by stating:

A scientific theory can be regarded as a system of design principles for modeling real objects. The theory consists of:

- 1. A framework of generic and specific laws characterising the descriptive variables of the theory.*
- 2. A semantic base of corresponding rules relating the descriptive variables to properties of real objects.*
- 3. A superstructure of definitions, conventions and theorems to facilitate modeling in a variety of situations (author's emphasis).*

Because of the consistency between Hestenes and Wittgenstein, a brief description of Hestenes systems theory of modelling will be given.

The games of scientific explanation and prediction are two popular deployment games. A physical phenomenon can be explained by a theory only to the extent that it can be modelled within the theory. Thus the model is the explanation! Most explanations are only partial or qualitative. Often in a qualitative explanation no model is mentioned, though one is implied to someone who knows the theory. For example, an explanation like 'the tides are due to the gravitational attraction of the moon' makes no explicit mention of a model, but an explicit model is essential to explain such details as why the tidal period is half the period of the earth's rotation. In a prediction game, the model is usually more explicit, because it is needed to generate some trend in simulated data (Hestenes, 1992; author's emphasis).

A model in mechanics can be considered to be *qualitative* if it provides an explanation (in accord with the form of description of the system), and the model may also be *explicit* if the representations of the properties of the object are quantified in some way. Hestenes (1987) states four components of a mathematical model:

- (1) A set of names for the object and agents that interact with it, as well as for any parts of the object represented in the model.*
- (2) A set of descriptive variables (or descriptors) representing properties of the object.*
- (3) Equations of the model, describing its structure and time evolution.*
- (4) An interpretation relating the descriptive variables to properties of objects in the reference class of the model.*

The use of *concept-questions* in mechanics is an example of qualitative modelling. The proposed teaching strategy will mainly involve the use of concept questions as a way to overcome misconceptions; consequently it will not involve (3), or (4) in relation to (3).

Nevertheless, the following points made by Hestenes (1987) would provide valuable insights into the logical structure of a model:

- We assume that a model is a conceptual representation of a real object.
- The represented object is said to be the *referent* of the model.
- A model may have more than one referent.
- The set of all referents of a model is called its *reference class*.
- If its reference class is empty, a model is said to be *fictitious*.
- There are three types of descriptors:
 - 1) *object variables*, which represent intrinsic properties of the object, e.g. mass is an object variable for a material particle while moment of inertia and specification of size and shape are object variables for a rigid body.
 - 2) *state variables*, which represent intrinsic properties with values which may vary with time, e.g. position and velocity are state variables for a particle.
 - 3) *interaction variables*, which represent the interaction of external objects with the object being modelled (the basic interaction variable in mechanics is force).
- An assignment of a particular referent or reference class to a given model is called a *factual interpretation* of the model, or a *physical interpretation* if the model belongs to physics.
- A single model may be given many different factual interpretations. For example, the one-dimensional harmonic oscillator may be interpreted as a model for such diverse objects as an elastic solid, a pendulum, a diatomic molecule or an atom.
- The interpretation of a model is specified by a set of attribute functions for its properties (the set of objects with a given property is called the *scope* or reference class of that property). The attribute function for a property assigns particular values of the descriptive variable to objects in its reference class. When specific numerical values are assigned to certain variables, these variables are said to be *instantiated*. As examples of

instantiation in particle mechanics, we have the assignment of a particular mass to a particle or particular initial conditions for its trajectory.

How are experiments to be deployed in the teaching of mechanics? The MEI A-level structured mathematics modular scheme proposes that either an experimental approach or a modelling approach can be deployed in the teaching of the MEI mechanics syllabus. However, such a distinction between the two approaches may be superfluous! According to Hestenes (1992):

Experimental games can be classified as model deployment games. Deployment is the empirical component of modelling. Experimental deployment games differ from the theoretical deployment games just mentioned in that their objective is to test and validate models.

Experimentalists might object that they are engaged in exploring the physical world for new phenomena, not merely evaluating models proposed by theorists. Experimentalists often underestimate the influence of theory even on their own activities, and positivism reinforces this tendency by claiming that theory is subservient to experiment (author's emphasis).

(This is consistent with Chalmers', 1978, argument that experimentation is theory laden).

A critique of positivism and its pedagogical implications will be discussed in the next chapter.

CHAPTER 4 - A CRITIQUE OF POSITIVISM AND ITS PEDAGOGICAL IMPLICATIONS

4.1 - INTRODUCTION

According to Koulaidis and Ogborn (1995), there exists a large body of empirical research that has examined science teachers philosophical views on the nature of science. The research seems to indicate that the teaching of science is related in many ways to the teachers' views on the philosophy of science. However, the problem with the existing body of research, the authors argue, is the failure of the research instruments to make explicit the various philosophical positions. The authors have noted that the research has tended towards forming idiosyncratic collages from various philosophical positions; and that this tendency appears to stem from the fact that most of the studies fail to recognise the existence of conflicting philosophical models of science. An example of the problem of inexplicitness, quoted by the authors, is from TOUS (Test of Understanding Science). Item 43 of TOUS asks:

Which of the following is the best description of a scientific law:

- A. It is an exact report of the observation of scientists.*
- B. It is a generalised statement of relationships among natural phenomena.*
- C. It is a theoretical explanation of a natural phenomenon.*
- D. It is enforced by nature and cannot be violated.*

According to Koulaidis and Ogborn (1995):

This item is intended to have a 'correct' response, which is B. The alternative ('wrong') responses are not obviously chosen from competing philosophical positions, but appear to be chosen from a number of possible explications of the notion of scientific law, from various eclectically chosen standpoints.

Later, the authors ask: *On what basis is B correct?* B is a statement that reflects an *empirical-inductivist* view of science: that science begins with particular facts and generalises the regularities of these facts to general laws (Koulaidis and Ogborn, 1995).

That TOUS should regard B as correct is an indication of the influence of *logical-positivism* in science and the teaching of science (and it may well be the case that TOUS does not recognise alternative responses as specific to certain, explicit, philosophical positions **because** of the influence of positivism)! Koulaidis and Ogborn (1995) have noted a report showing teachers leaning more towards an empiricist-logical positivist position. This is consistent with the following statements that sum up a widely held common-sense view of science (Chalmers, 1978):

- Scientific knowledge is proven knowledge.
- Scientific theories are derived in some rigorous way from the facts of experience acquired by observation and experiment.
- Science is based on what we can see and hear and touch, etc.
- Personal opinion of preferences and speculative imaginings have no place in science.

Driver (1994) has spelt out the pedagogical consequences of this common-sense view of science:

*Through the eyes of those initiated in the **currently accepted theories of science**, common school demonstrations, such as trolleys and ticker tapes, experiments with batteries and bulbs, or work with ray boxes, mirrors and prisms, appear to offer self-sufficient support for the underlying principles they are designed to demonstrate, whether it is Newton's laws of motion or the laws of reflection of light. If children fail to abstract and understand these principles from their experiments, it may be seen as the children's error for either not observing accurately or not thinking logically about the pattern in the results. The constructivist view of science, on the other hand, indicates the fallacy here. If we wish children to develop an understanding of the conventional concepts and principles of science, more is required than simply providing practical experiences (page 47, my emphasis).*

Not only is positivism the prevalent view of science, it is also the 'dominant ideology' in education (Hwang, 1996). This chapter reviews positivism and the pedagogical implications in the teaching of mechanics; the importance of such a review is reflected in the widespread influence of positivism, despite the numerous references of researchers of misconceptions to constructivism.

Generally speaking, *empiricism* is the philosophical view that concepts are, wholly or partly, based on experience through the senses. *Positivism* is a form of empiricism which asserts that scientific propositions are generalisations of appearances. *Logical-positivism* asserts that something is meaningful if and only if it is either verifiable empirically, i.e. ultimately (not necessarily directly) by observation through the senses, or is a tautology of logic or mathematics¹ (Lacey, 1986). *Operationalism*, a doctrine of positivism, asserts that concepts must be defined in terms of the operations employed in applying them. For example, length can only be defined in the terms of techniques of measurement: length applied to a football pitch is a different concept to that applied to stellar diameters (Lacey, 1986). I would argue that the difference between positivism and constructivism is such that the two positions are pedagogically incompatible (despite Hwang's, 1996, claim that both positions can be applied in a complementary way in instructional development). If the laws of motion were empirical generalities, then the understanding of the laws of motion would be a question of assimilating those laws into the existing cognitive structures. Pedagogically, it would require the presentation of the laws of motion through the operations used to define them. In other words, a series of experiments would be a necessary and sufficient condition in the understanding of the laws of motion. If the laws of motion are not empirical generalities, but instead, axioms that implicitly define force, then the existing cognitive structures have to accommodate to the concept of force as

¹ Carnap (1934), one of the leading exponents of the influential Vienna Circle of logical positivists states: *...our position is related to that of Positivism which, like ourselves, rejects Metaphysics and requires that every scientific statement should be based on and reducible to statements of empirical observations. On this account many (and we ourselves at times) have given our position the name of Positivism (or New Positivism or Logical Positivism). The term may be employed, provided it is understood that we agree with Positivism only in its logical components, but make no assertions as to whether the Given is real and the Physical World appearance, or vice versa; for Logical Analysis shows that such assertions belong to the class of unverifiable pseudo-statements. Our views are related, in similar fashion, to those of Empiricism, since we follow that theory so far as to reject a priori judgements; Logical Analysis shows that every statement is either empirically verifiable (i.e. on the basis of protocol statements), analytic, or self contradictory. On this account, we have at times been classified, both by ourselves and by others, as Empiricists* (page 27, author's emphasis).

defined by the laws of motion. Accommodation is the adaptation of the cognitive structure to the concept - the cognitive structure has to undergo a transformation. In other words, the student has to **construct** rather than merely assimilate the Newtonian concept of force. Assimilation and accommodation will be taken up in the next chapter; however, given the influence of positivism it is necessary beforehand to provide a critique of the pedagogical implications of positivism with respect to mechanics.

4.2 THE INFLUENCE OF POSITIVISM IN THE PHILOSOPHY AND TEACHING OF SCIENCE

The concept of 'operational definition' has its origins in the positivist epistemology most powerfully espoused by Ernst Mach in his famous critique of Newtonian Mechanics ('The Science of Mechanics'). Mach held that physical laws are merely summaries of sensory experience and the meaning of physical concepts is determined only by specifying how they are related to experience. It is in this sense that 'operational definitions' are intended to give meanings to physical terms. As a prime example, Mach presented a procedure for measuring the masses of particles in terms of their accelerations which is often presented as a typical operational definition today. On the contrary, our constructivist epistemology invites us to regard this simply as an application of Newton's axioms to the design of an experiment for measuring mass, which is possible only because mass is already well defined by Newton's axioms (Hestenes, 1992).

Positivism in general regards science as constituting nothing more than a collection of facts, with the role of theory restricted to the organisation of these facts into a logically coherent system from which new facts can be deduced or predicted (Heather, 1976). For Mach, science aims at the most economical description of appearances, i.e. ultimately of our sense-experience, or sensations (Lacey, 1986). For Mach, 'unobservables' in science, such as atoms, should be treated as a mere manner of speaking (Lacey, 1986). Mach's critique of Newtonian mechanics may be considered *empiricist* (that our concepts are based upon experience through the senses). Consider the following passage:

A motion may, with respect to another motion, be uniform. But the question whether a motion is in itself uniform, is senseless. With just as little justice, also, may we speak of an 'absolute time' - of a time independent of change. This absolute time can be measured by comparison with no motion; it has therefore neither a practical nor a scientific value; and

no one is justified in saying that he knows aught about it. It is an idle metaphysical conception (Mach, 1959, page 273. Author's emphasis).

For Mach, experience can only tell us about the motion of a body relative to another body; that it is a 'senseless' or 'idle' metaphysical conception to consider a motion to be in itself uniform. Mach considered Newton's conception of absolute time and absolute motion as 'idle metaphysical' ideas, and this may have been a reaction to the notion that Newtonian mechanics rests on a *metaphysical foundation*.

The notion of absolute motion with respect to absolute space, that there is an infinite three-dimensional Euclidean space within which moving bodies satisfy the laws of motion, lies at the basis of Newtonian mechanics. **However**, in our actual experience of nature we are given neither a Newtonian absolute space nor cases of rectilinear inertial motion whereby bodies move with uniform motion along Euclidean straight lines in the absence of external forces (Friedman, 1992). All that we observe are cases of motion that are relative to some physically specified frames of reference such as the Earth or the 'fixed' stars, but these frames of reference are not fixed [as stated eloquently by Zukav (1991): *On the one hand, we have the laws of classical mechanics, which are indispensable to physics, and, on the other hand, these same laws are predicated upon a co-ordinate system which may not even exist* (page 149)]. How, then, is it possible to apply the notion of absolute motion with respect to absolute space to our actual experience of nature (Friedman, 1992)? According to Friedman (1992), Newton thought it was possible to **infer** the true or absolute motions from their observable effects - that we can begin with **apparent** (relative) motions from which we can determine the true or absolute motions such as the Earth truly rotating around the sun and not vice versa. However, in quite the reverse fashion, Newton (in book 111 of his *Principia*) **begins** with the ideas of absolute space and absolute motion, formulates his laws of motion with respect to this pre-existing spatio-temporal framework,

and finally uses the laws of motion to determine the true motions in the solar system from the observable - Kant, on the other hand, conceives this very argument as a **constructive procedure for first defining the concept of true motion** (Friedman, 1992). For Kant, this procedure does not find, discover or infer true motion but instead makes an objective concept of true motion possible - that the laws of motion are conditions under which alone the concept of true motion has meaning (that is, true motions are just those that **satisfy** the laws of motion). Newton's argument in book III of the *Principia* culminates in the determination of the centre of mass frame of the solar system as a privileged state of rest; however, as Kant points out, this empirical procedure cannot end here! The centre of mass of the solar system is rotating with respect to the Milky Way which in turn is rotating with respect to a common centre of galaxies, and so on (Friedman, 1992). For Kant, the Newtonian procedure for determining a privileged frame of reference is necessarily non-terminating; for it must ultimately aim at the 'common centre of all matter' which is forever beyond our reach (Friedman, 1992). Kant therefore replaces the Newtonian notion of absolute space with only a procedure for determining better and better approximations to a privileged frame of reference - 'absolute space', to quote Kant, is *a necessary concept of reason, thus nothing other than a mere idea* (quoted by Friedman, 1992) and Kant saw this as the *metaphysical foundation* of Newtonian science (Friedman, 1992).

That Newtonian mechanics should rest on a 'metaphysical foundation' is an abhorrence to Mach. However, Mach's positivism misses the point! To quote Friedman (1992):

For Kant, then, Newton's theory does not require the actual existence of such a privileged frame; rather, it specifies a constructive procedure for finding better and better approximations - a procedure which never actually fully attains its goal. Thus, if we think of Kant's 'absolute space' as the ideal end-point of this constructive procedure - the privileged frame of reference towards which it 'converges,' as it were - it becomes clear why 'absolute space' in this sense is characterised as an idea of reason (page 144, my emphasis. [Interestingly enough, Bruner, 1986, regarded Kant as the original constructivist!]).

This 'idea of reason' may be considered as a 'controlled use of the imagination': we can imagine a privileged frame of reference as a kind of *thought-experiment*. To quote Heather (1976):

It is worth noting at this point that what the Vienna Circle (a circle of logical-positivists who were influenced by Mach) set out to do was to prescribe how science should be done, not to describe how it is actually done. This introduces the question, what is the task of a philosophy of science. Is it to dictate to scientists how they should do their work, or is it, more modestly, to demonstrate the rules and methods by which successful science is accomplished? Harre and Secord have argued convincingly that real science, as opposed to science as prescribed by the positivists, cannot proceed without the controlled use of imagination. Moreover, this imagination is used to construct ideas of things and processes which are unobservable in order to explain what we do actually see. It will be noted how this account of science really does proceed differs markedly from the positivists' notions of what it should be doing (page 18, author's emphasis)².

The notion of a privileged frame of reference is an *idealised abstraction* as defined in chapter 3, and it is through the 'controlled use of the imagination' that such a notion can be *constructed!* Kant's constructive procedure that (asymptotically) terminates at the privileged frame of reference (the centre of mass of the universe) is an extension of Newton's procedure to establish the privileged frame of reference (the centre of mass of the solar system) relative to which true motion can be first empirically defined (it is only after establishing the centre of mass frame of the solar system in Proposition XI of Book III that Newton can settle the issue of heliocentrism in Proposition XIII) (Friedman, 1992). However, as Warren (1979) states:

It is sufficient to postulate that physical laws can be considered from the ideal standpoint of an imaginary 'inertial observer' who finds that Newton's first law is true as far as his measurements are concerned (page 7).

For all intents and purposes, we may regard ourselves as inertial observers of terrestrial phenomena. Although we are in a rotating frame (the earth), nonetheless the effects can be considered negligible (provided the phenomena does not cover a large portion of the earth)

² According to Chalmers (1978), many of Galileo's experiments were in fact not empirical experiments but thought experiments. That it was not the case that new theories were derived from the facts in some way, but that Galileo was in the process of making a contribution to the building of a new mechanics that would be capable of supporting detailed experimentation at a later stage. That process involved analogies and illustrative metaphors rather than detailed experimentation.

- and in much the same way as we sometimes ignore air resistance or consider strings to be light. We can apply the laws of motion to model real terrestrial phenomenon but within the context of idealised abstraction - **as axioms, rather than as propositions that are true relative to the privileged frame of reference.**

That Descartes developed a mechanical view of nature (Gillott and Kumar, 1995) which finds its reflection in the Newtonian 'atomistic' conception of matter (Zukav, 1991) should not override the fact that Newtonian mechanics rests on a constructivist epistemology. There are some constructivists who do not appreciate this fact! Consider the following passage:

Commentators, among them Reese and Overton, say the mechanist perspective 'reflects a model of man as a reactive organism..... a Newtonian perspective.' Constructivist theories, in contrast, are rooted in a model of man as 'spontaneously active,' an Einsteinian perspective. Kaplan says these positions are 'irreconcilable, and even prevent full communication,' so different criteria for determining the truth of propositions makes a reconciliation impossible (Marshall, 1993).

'Mechanist' is another term for 'behaviourist' and behaviourism is a narrowed version of positivism (Heather, 1976). It is ironic that some constructivists should view the 'Newtonian perspective' from a positivist point of view, but this merely reflects the pervasive influence of positivism in our culture. According to (the psychologist) Heather (1976):

The positivist attitude has become something we have inherited from earlier generations and, like other aspects of inherited culture, it forms and constrains the way we think. It has become.....part of the 'natural order of things' which we tend to accept with unquestioning compliance (page 13).

And according to the physicist J. D. Bernal (1969):

The influence of the positivism of Ernst Mach on the theoretical formulation of modern physical theories was a predominating one. Most physicists have so absorbed this positivism in their education that they think of it as an intrinsic part of science, instead of being an ingenious way of explaining away an objective world in terms of subjective ideas (page 746).

The following are examples of the influence of positivism in the research of instructional design in mechanics:

- *Understanding the form of scientific knowledge: Knowing the characteristics of a useful scientific law is an important aspect of understanding the form of scientific knowledge. The instructional technique we developed was to present students with alternative laws for each microworld, and have them select the best law. We observed that when students were evaluating these sets of laws, they spontaneously engaged in discussions concerning the simplicity of a law, the precision of its predictions, and its range of applicability. The set of laws was carefully constructed to elicit such discussions and this approach thus appears to have been highly successful (White and Horwitz, 1988).*

[For students to select the best law, using such criteria as simplicity and applicability, suggests selecting the proposition that best describes what appears in the microworld, i.e. selecting the best law as a summary of sense-perception].

- *Idealisation of the string: Since many students do not understand the consequences of the assumption that a string is massless, an operational definition for this term is made explicit. The students are guided through the line of reasoning necessary to replace a massive rope by an idealized string. A series of structured questions leads them to conclude that, under the conditions that $M_R \ll M_A$ and $M_R \ll M_B$ (M_R : mass of the rope, M_A : mass of body A and M_B : mass of body B) the mass of the rope can be neglected. Using Newton's second law, they find that $F_{AR} - F_{BR} = 0$ and thus $F_{AR} = F_{BR}$. From this equality, the students conclude that no net force is needed to accelerate a massless string (McDermott et al, 1994).*

[If a string is massless, then it is a contradiction to make explicit the operational definition of something that does not actually exist! A massless string is an *idealised abstraction*].

- *I would claim that 'weight' and 'weightlessness' have as much to do with physical sensations as they do with physics.....Our weight (let us not muddy the issue by calling it 'apparent weight') increases when we are in an elevator with an upward acceleration, and decreases when the elevator has a downward acceleration. By this definition, an astronaut in a spacecraft orbiting the Earth has zero weight - i.e. is weightless. But so also, I contend, is a penny or a human being in free fall at the Earth's surface, because their actual physical states are equivalent to that of the astronaut (French, 1995).*

[Physical states (last sentence) are equivalent to physical sensations (first sentence). The misconception that falling objects are weightless is taught in the video *Gravity, Weight and Weightlessness*, Bailey Film Associates, 1985; and Warren, 1979, reports on the popularity, many decades ago, of the misconception with many text book authors].

- The equals sign in $F_{net} = ma$ is not an ordinary functional equality. It conceals the combination of arbitrary definition and laws of nature lying behind either the Machian or Newtonian approach to the Second Law (Arons, 1990, page 82).
- To illustrate the preceding remark ('Description needed for interpreting Newton's law') consider Newton's mechanics law $ma = F_{tot}$. To interpret this law in any particular instance, one must determine the mass m and acceleration a of the particle of interest, and then relate these quantities to the total force F_{tot} (obtained by adding vectorially all the individual forces on the particle). This can only be done if one has first adequately described the mass of the particle, its motion, and the forces due to its interactions with all other objects (Reif, 1995).

[That ma should appear on the left hand side of the equation is no accident, as:these ideas are those summarized as motion described by velocity and acceleration, a few important interactions ultimately elaborated into the three mechanics laws (Reif, 1995). For Reif, the interpretation of Newton's laws begin kinematically. This implies that, 'in any particular instant', we should begin with the motion to elaborate the laws, rather than begin with the laws in order to elaborate the state or change of state of motion].

According to the A-Level book *Physics: Concepts and Models* (Wenham et al, 1972), where the authors assume that students using the text are familiar to some extent with the ideas and content of such courses as the Nuffield O-Level Physics course (page v):

At its simplest, a law is a summary of observed, measurable behaviour philosophers have discussed at length the impertinence of scientists who assume that what is true for a necessarily limited number of observations made in the past is true for all similar observations both now and in the future (page 5).

However, as Driver (1994) points out:

The most simplest view of the scientific enterprise is, perhaps, the empiricist's view, which holds that all knowledge is based on observation. Scientific laws are reached by a process of induction from the 'facts' of sense data.....This inductivist position was criticised when it was first suggested by Bacon nearly 400 years ago, yet it has reasserted itself early in this century in the heuristic movement and later in some of the more naive interpretations of the discovery method adopted by the Nuffield science schemes (page 43).

The following is *Physics: Concepts and Models* treatment of the first law of motion:

Experiment 1 demonstrates that, during a collision, a large mass undergoes a smaller change in velocity than the larger mass. (X_3 underwent a change of velocity of $0.45 - 0.70 = -0.25\text{ms}^{-1}$ whereas Y_1 suffered a change of 0.75ms^{-1} .) Again we can say that the larger

mass has more inertia as it tends to maintain its velocity. Mass can be defined as a measure of the inertia of a body, as well as being the quantity of matter in the body. Newton summed up these findings in his first law, (Newton 1) which states: 'Every body remains at rest, or moves with constant velocity, unless acted on by an unbalanced force' (page 33).

The book then explains the meaning of the term 'unbalanced force'. The book spends much detail and data on trolley collisions so that the data can be generalised into a 'rule' or law of collisions (essentially the law of conservation of momentum), then makes the leap to Newton's first law as if the first law summed up these findings. After having to assimilate all the data, the reader is expected to conceive Newton's first law as a summary of this data. The book's style is very much a 'theoretical', as well as a quantitative, treatment of physics; and attempts a conceptual and historical approach to the subject. However, any potential interest that the reader may have towards the subject may be dampened by virtue of the book's positivist treatment towards physics. From the following quote it would seem, ironically, as if the authors' interest in the subject has been dampened by their own treatment of the subject:

But, when one has marvelled at the skill of these scientists in reducing a mass of observed behaviour into a succinct law of behaviour; when one has admired their implicit belief in a physical world whose behaviour can be expressed in the regularities described by these laws, one may still be surprised at the remarkable dullness of so many of these laws (page 2).

4.3 THE PEDAGOGICAL IMPLICATIONS OF POSITIVISM IN MECHANICS

Based on twenty years of his own research into student misconceptions and the research of others, Arons has published the book *A Guide to Introductory Physics Teaching* (1990).

According to Gould and Gould (1991):

The goal of Aron's book is to make explicit the conceptual and reasoning difficulties that many students encounter and the aspects of logical structure and development that are not presented clearly in substantial segments of the textbook literature.

The book is unique, and may be the first of its kind based on the vast area of research in misconceptions.

Arons has done a major service by summarizing these developments and giving practical suggestions for their implementation. His book should be read by anyone who teaches physics at any level from high school through introductory college and university calculus-based courses. Parts of the book should be read by everyone concerned with the general state of education in the United States (Gould and Gould, 1991).

However, despite the twenty years of his own research, Arons himself is not immune from misconceptions. Consider the following passage, taken from the sub-heading *Logical Status of the Third Law* of his book:

If we push on one end of a long rod, the other end of which is in contact with a block, the block does not exert an equal and opposite force on the rod at the same instant we push. A finite time interval elapses between our push and an effect at the block, the time interval being determined by the velocity of the elastic wave that passes down the rod. Thus, Newton's Third Law does not hold, instant by instant, for the forces at either end of the rod; it holds only layer by layer of material along the length of the rod, and momentum and energy are both conserved only by virtue of propagation of the elastic wave (page 67, author's emphasis).

If the push at one end of a rod (with our hand, say) is the *action*, then under the third law of motion, the *reaction* is the push of the rod on our hand! According to Warren (1979):

The use of the terms 'action' and 'reaction' implies a sequence in time and the relation of cause and effect, whereas the forces referred to in Newton's third law both arise simultaneously from the same interaction and are of the same nature (page 12).

Hellingman (1992), in an article entitled *Newton's Third Law Revisited*, found that *professional physicists to quite a large extent do not have a full understanding of force. It would be worthwhile to do research among professional physicists instead of among students* (author's emphasis). Arons (and McDermott) often refer to 'the third-law pairs' which has the implication that the third law of motion refers to two different forces (*action* and *reaction*) rather than the one force acting between two different but mutually interacting bodies. However, Hellingman (1992) argues that the very idea of *action* and *reaction* as representing two forces acting on two mutually interacting bodies is not only confusing, but also reflects a deep misconception of force as a property:

For instance in the case of colliding bodies we learn about 'the forces two bodies exert on each other'. This formulation of course entails a strong suggestion of two forces, each belonging to 'its' body.....

If we stick to these terms we must know that action and reaction are related in the way the two ends of a rope are. It is even more to the point to compare a pulling force with a piece of a stretched elastic band, fixed to an object. It is impossible to imagine that there would be no other end, fixed to or held by another object, and that this object would not experience the same force in the opposite direction. The elastic band is the interaction agent pulling the objects to each other; one two-sided agent, acting in between the two objects.....

One single intermediate action! One can almost hear the word interaction, much in use nowadays. Interpreting forces as sides of a single interaction implies a very important shift of focus of attention. The attention is drawn away from the objects themselves to 'somewhere' between the objects. Failure to see the 'between' - like character of a force lies at the bottom of all misconceptions (author's emphasis).

Hellingman argues that it would be better to regard action and reaction as 'two sides of the same coin' (so to speak); or, as he puts it: 'both sides of the one stretched elastic band.' For Hellingman, not to conceive of action and reaction in this way would be to commit a misconception that two interacting bodies must have their own force (force as a property of the body)! Agreeing with Hellingman's argument may overcome the confusion of regarding the third law in terms of 'third law pairs' (which is sometimes responsible for people making the error that action and reaction are two forces acting on the one body). For example, Arons (page 65), as well as Warren (page 12), speaks of 'action' and 'reaction' as two different forces acting on two different bodies; and both correctly warn of the misconception of 'action' and 'reaction' as two different forces acting on the one body [as Arons advocates (page 66), two separate force diagrams should represent the one interaction between two bodies]. However, **in the context of the third law**, Arons speaks of 'passive' versus 'active' forces:

Active forces are exemplified by animate pushes and pulls, the gravitational force, electric and magnetic forces. Passive forces are defined as those that arise, and adjust themselves, in response to active ones, for example, in compression of a spring, deformation of the table or floor under the load of a block, frictional forces, and so on (page 65).

'Active' and 'passive' forces only make sense within the context of the first law and not the third, and Arons is in fact speaking within the context of two different forces acting on

the **one** body (Warren, 1979, refers to this particular misconception as the ‘reaction force law’). It may well be the case that action and reaction conceived in terms of ‘third law pairs’ may lead to the error of the ‘reaction force law’ (an error committed by Arons but fortunately not by Warren, despite Warren’s reference to the ‘third law pairs’). This point has been suggested by Hellingman (1992):

In the development of the system of mechanics ‘action’ and ‘reaction’ were considered as synonymous for ‘force’ in general, giving rise to the well known statement that forces ‘always occur in pairs.’ But the consequences of this point of view were probably never worked out; were, at any rate, not widely understood, probably because the law was only rarely applied as mentioned above.

That the compression of a spring or the deformation of a table can be spoken of as a ‘passive’ force is to speak of force but only within the context of its effect. To speak of force but only within the context of its effect may have its origins in the notion of operationalism, and it may be that notion that has confused Arons. According to Warren (1979):

Confusion between pairs of forces acting on different bodies (according to the third law) and pairs of forces acting on a single body that is in equilibrium (according to the first law) is sometimes explicitly taught (page 12).

Arons (1990) often refers to the need to establish the operational definition of the concept to be taught. It may well be the case that the notion of operational definition sustains Arons’ misconception concerning the third law. Consider the following passage:

The point is that the Third Law does not always hold, and this is why modern physics has given primacy to Conservation of Momentum in the hierarchy of physical law. Although one would not discuss all these aspects with students at the time of first introduction of the Third Law, it is well to start laying the groundwork for eventual perception of where the law fails. The rod pushing the block makes a good starting point (page 68).

It would seem that, for Arons, the rod pushing the block may serve both as an ‘operational definition’ of the third law, and (following the tradition of positivism’s rewriting of physics as noted by Heather) as a demonstration of why ‘the third law does not hold.’ However, there is a contradiction! **A failure of the third law would be a failure of momentum**

conservation [according to Warren (1979) the law of conservation of momentum is deduced from the second and third laws; and, according to Hestenes (1987), the third law may also be interpreted as *a law of momentum exchange*]. Warren (1979) suggests *that it would be very much better to state the meaning of the law and omit the slogan* (page 12).

Throughout his book, Arons (1990) advocates the slogan *Idea first and name afterwards* as a teaching strategy in the introduction of every new concept. E.g:

I suggest that an individual who has acquired some degree of scientific literacy will possess the ability to:

(no. 2) Recognise that to be understood and correctly used, such terms require careful operational definition, rooted in shared experience and in simpler words previously defined; to comprehend, in other words, that a scientific concept involves an idea first and a name afterwards, and that understanding does not reside in the technical terms themselves (page 289, author's emphasis).

Putting the idea first may provide the opportunity for Socratic dialogue (a *dialectic* of teacher initiated and led discussion through a process of questioning) and allowing misconceptions to emerge. Arons (1990) recognises that the description and analysis of student conceptual difficulties require the consideration of the logical structure of the laws of motion. However, the logical structure of mechanics has become, for Arons, a question of *operational definition* that is totally at odds with the analysis of the logical structure of mechanics given by Wittgenstein and Hestenes.

According to Arons (1990), there are two approaches to the 'operational definition' of force and mass: one labelled the 'Newtonian' sequence (although apparently, according to Arons, Newton never propounded a clear 'operational definition'), and the other given by Mach. To quote Arons (1990):

In Mach's sequence, inertial mass is defined first. This is done by invoking the reaction car experiment, accepting as a law of nature the empirical observation that the ratio of the accelerations (and hence of the velocity changes) of the two bodies is a fixed property of the bodies, and defining the ratio of the masses as the inverse ratio of the accelerations. The net force acting on one body is then defined as the 'ma' product for that body.....I find that a significant minority (of textbooks) use the Mach sequence. Since this sequence is

basically sound and internally consistent, I shall not discuss the pedagogy in detail except to say that most of these presentations are so cryptic and so abstract that few students have any real chance of forming a sound operational grasp of the concepts from text presentations. To induce such grasp, teachers would have to expand the development, give it far greater concreteness, and lead students to interpret, explain, and analyse in their own words (page 51).

This leaves you wondering as to the **utility** of such a definition, especially as it requires elucidation from the teacher and an interpretation by the student! Would a student explanation and analysis of this definition possibly overcome any misconceptions that the student might have concerning force? An affirmative answer would be dependent upon the precept *Idea first and operational definition afterwards*, but there is no hint of a guarantee of an affirmation. According to Zhaoyao (1993):

Many students often complain that it is easy to understand mechanics theory but difficult to do exercises. As is known to all, Newton's second law plays an important role in mechanics. This formula itself is not complex and has only three physical quantities. However, the solution of mechanics problems concerning the formula involves many concrete problems, such as how to identify the body for analysis, determine the forces acting on it, choose a suitable reference frame, and so on.

What Zhaoyao is describing is the modelling process which involves more than knowing the operational definition of Newton's second law.

Arons is rather partial to the 'Newtonian' sequence (starting with force rather than inertial mass) which is given in the majority of textbooks (Arons, 1990). Under the sub-heading *An Operational Interpretation of the First Law*, Arons states the following:

Newcomers to dynamics, burdened with common sense ideas and rules about the behaviour of moving bodies, have very great difficulty following this (historic) breakthrough (of the law of inertia), and the learning problems this entails will be discussed in later sections. Here I wish to consider only one facet of the First Law: how to interpret it operationally in the sequence of definitions of concepts.

Among the list of definitions at the beginning of the 'Principia,' we find the following Definition IV:

An impressed force is an action exerted upon a body in order to change its state, either of rest, or of uniform motion in a straight line.

Then, as Law I of three Laws of Motion, we find:

Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.

The circularity here is quite apparent, but it, in fact, does suggest how we might help a student interpret Law I in our modern sequence: Up to this point, we have generated only

operational definitions of the concepts of kinematics, and 'force' and 'mass' remain undefined. Once we begin to accept the view that rest or uniform rectilinear motion are natural states of objects and that interactions with other objects are necessary to produce 'changes' in such motion, we can interpret Law I as giving us a 'qualitative' operational definition of 'force,' namely that action, by an agent external to the moving body, that imparts a 'change' in velocity, and 'change' includes both magnitude and direction. This becomes a first step toward an operational definition of 'force.' The next steps come from construction of Law II (page 52, my emphasis).

It would seem that Arons regards the first law as 'circular' because the first law does not admit to an operational definition; and because the first law does not admit to an operational definition, Arons obviously finds it necessary to rewrite the first law (and this is in keeping with the positivist tradition of stating how physics ought to be rather than describing how it is!) Although the first law, as rewritten by Arons, is not in itself 'invalid,' nevertheless it is an attempt to state the first step in a sequence that leads to the operational definition of 'force.' Arons states that the re-interpretation of the first law provides us with a *qualitative* operational definition of force, *the next step* (to quote Arons, page 52) *is to refine the concept by making it quantitative* (the construction of law II). Arons suggests an elaborate experiment not dissimilar to (but appears more complicated than) the 'ticker-tape'/incline-plane procedure. Arons' sequence that leads to the operational definition of force is on the one hand cumbersome and complicated, and yet appears little more than as an excuse for an experimental 'hands-on' approach. The whole sequence begs the question: *will it overcome student misconceptions of force?* Arons' remedial treatment for student misconceptions is an experimental 'hands-on' approach in which qualitative questions can be asked in a Socratic context (see Arons, page 61) - this would appear as sound and reasonable advice, but it is advice that is divorced from his own initial presentation of the laws of motion. Consider the following Socratic questions based on a 'hands-on' performance involving a smoothed 50-lb block of dry ice placed on a large levelled glass plate (page 61):

1. How does the block behave once it is moving?

2. What action on our part is necessary to make the object move faster and faster, that is, accelerate continuously?
3. What is the difference in the behaviour of the block when the block is acted on by a steady push that keeps up with the block, and when the block is given a quick shove?
4. How large a force is necessary to impart any acceleration at all to the block, that is, is there a threshold effect?
5. Suppose the block of dry ice is already moving: what must be done to make it slow down very slowly without changing the direction of its motion?
6. Suppose the block is moving to begin with, and we exert a steady force, either speeding the block up or slowing it down. How does the block behave? Now suppose we make our steady force smaller and smaller. How does the block behave? How will it behave when the force we are exerting reaches zero?
7. Suppose we exert two steady forces on the block in opposite directions, one with each hand. How does the block behave when one force is larger than the other? When the forces are of equal magnitude?
8. Suppose the block is moving: what actions change the direction of its motion? What do you have to do to make the block move at right angles to its initial path? In some other specified direction? In an (approximate) circle?
9. What happens if you start the block spinning about a vertical axis? Without using any as yet undefined technical terminology, what are some implications of the observed behavior?

These questions are based on a (simple but ingenious) physical construction that represents an idealised abstraction. The questions demand a qualitative response, but the correct answer may require more than describing the experience of the phenomena. A student who understands the operational definition of force as outlined by Arons may not necessarily be able to correctly describe the phenomena according to the laws of motion.

The correct answer to each question presupposes the ability to model - that is, it presupposes the ability to represent each state of affairs by the possible state of affairs as constructed by the axioms of mechanics (see chapter 3). These questions can play a part in Hestenes' *modelling game*, but it would be most unlikely for a student to provide correct qualitative answers to these questions by simply understanding Arons' 'modern sequence'! In fact, Arons advises that the teacher asks questions and elicits responses - a modelling procedure despite the student interaction with the referent. The 'hands-on' approach in this example makes the modelling game *concrete*!

The notion of operationalism undermines Arons' own 'phenomenological' teaching approach (qualitative questioning based upon the way phenomena appears, in which the above nine questions serves as an example) since implicit is the definition of 'understanding' as the registering of the operational definition. In fact, through his book, Arons defines *operative knowledge* as the understanding as to where *declarative knowledge* (known 'facts') comes from or what underlies it (e.g. page 314). Arons often makes the point that some students will try to regurgitate the operational definition. This begs the question as to why have the notion of *operational definition* if in practice the student can learn the operational definition divorced from understanding the concept? By contrast and in addition, however, Arons leaves the impression that he would not be adverse to the notion that to understand a concept requires the construct of that concept [he does actually speak of the invention of concepts (page 24 and page 289)]. McDermott (1991) also juxtaposes operational definition with explicit references to constructivism. She speaks of the need for students to have the ability to define concepts operationally and the need to construct their own concepts. Arons' use of *operational definition* seems to be tied up with his notion of *operational understanding*, and it would appear on the surface that operationalism is not in conflict with the 'phenomenological' teaching approach. However,

if a concept is implicitly defined with reference to other concepts through a set of axioms, then the invention of that concept is dependent upon the accommodation of those other concepts. This contradicts the 'operational definition' of a concept (that the definition rests upon the operations used to establish its application) and contradicts the 'operational understanding' of a concept (the understanding of a concept is dependent upon the understanding of its operational definition).

It would seem that, for Arons and McDermott, the initial presentation of the laws of motion is a question of interpretation (as opposed to the rote memorisation of the 'operational definition') rather than the construction of the concepts involved. This is despite any reference to constructivism. Arons' acceptance of the Machian sequence (even though he proposes the 'Newtonian sequence') is merely because it is*basically sound and internally consistent* (page 51). This completely overlooks the **coherence** of the Newtonian system as a hierarchical structure resting on its axiomatic base and underestimates the **power** of such a structure in overcoming misconceptions through its re-invention. Because of the emphasis placed on phenomena and experience, the phenomenological teaching approach avoids the **cognitive issue** of construction and overlooks the 'controlled use of the imagination,' within the context of *idealised abstraction*, that provides a clue as to how the student will be able to construct *force* as an abstract and implicit but well defined concept. To overlook the Newtonian system as a hierarchy structured by an axiomatic base is to overlook the system as a paradigm. Consequently, any teaching strategy may underestimate the cognitive reorganisation required to embrace the system.

Arons discusses the 'absolute' frame of reference only in the context of the historical search for the electromagnetic ether (page 259), or the contrast between inertial frames and

noninertial frames of reference within the context of fictitious forces (pages 78 and 108),

e.g:

In order to understand what an inertial frame is, one must begin to understand what it is not, and situations such as those proposed above (e.g. a pendulum bob hanging from the roof of an accelerated car or a person sitting in a car that begins to accelerate) are a first opportunity to make this point in rectilinear dynamics (page 78, author's emphasis).

Arons does not discuss what an inertial frame is. This may not be surprising because, if he did, he would be placed in the contrary position of having to assert the operational definition of something that does not actually exist. To skate around this problem by asserting that an understanding of what an inertial frame *is* can be achieved by understanding what it is *not*, is rather like saying that you can understand complex numbers if you understand them as not being real numbers. What is Arons to do if a student asks him to define an inertial frame? (*Well, it certainly isn't a rotating frame! Next?*)!!

CHAPTER 5 - OVERCOMING THE INTUITIVE SCHEMATA OF FORCE AND MOTION

5.1 INTRODUCTION

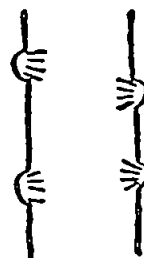
Consider the sixteenth-century controversy about the motion of the earth. Suppose that Tycho Brahe and Kepler stand on a hill facing east at dawn. According to Hanson (a philosopher of science who wrote 'Patterns of Discovery'), there is a sense in which Tycho and Kepler see the same thing. They both 'see' an orange disc between green and blue colour patches. But there also is a sense in which Tycho and Kepler do not see the same thing. Tycho 'sees' the sun rising from below the fixed horizon. Kepler 'sees' the horizon rolling beneath the stationary sun. To see the sun as Kepler sees it is to have effected a 'Gestalt'-shift (Losee, 1972, page 200).

According to Losee (1972), a 'gestalt-shift' is a conceptual revolution in science whereby relevant facts come to be viewed in a new way. A 'gestalt-shift' demands a reconstruction from 'seeing that' to 'seeing as'. The revolution 'demands' that the prevailing *paradigm* be 'overcome' and a new one put in its place. A paradigm is a corpus of theories (Gillott and Kumar, 1995) involving a 'disciplinary matrix' of beliefs, values and techniques (Losee, 1972). Basically, a paradigm is a way of looking at things - a 'world view' or 'outlook' - that governs an approach to ideas, problems or theory [up until the latter half of the nineteenth century, the Newtonian paradigm governed the approach of physics to nature; and the paradigm was structured according to the laws of motion (Chalmers, 1978)]. That a change in paradigm should be described as a 'revolution' is precisely because a paradigm is a way of interpreting and making sense of the world and is therefore reluctant to change.

That a way of looking at the world can be so intransigent to change was eloquently illustrated by Alan Watts (1961) in his *Psychotherapy East and West*:

If we go back in imagination to an India entirely uninfluenced by Western ideas, and especially those of Western science, it is easy to see that this cosmology (the law of karma and the cycle of reincarnation) would have been something much more than a belief. It would have seemed to be a matter of fact which everyone knew to be true. It was taken for granted, and was also vouched for by the authority of the most learned men of the time, an authority just as impressive then as scientific authority is today. Without the distraction of some persuasive alternative one can know that such a cosmology is true just as one can

know that the sun goes round the earth - or just as one can know that the following figure is a bear climbing a tree, without being able to see the bear:



Or is it simply a trunk with burls on it?

To the degree, then, that this cosmology was a matter of ingrained common sense, it would have been as difficult for the average Hindu to see the world otherwise as it is for us to imagine what a physicist means by curved space, or to believe him when he says that matter is not solid (page 48, author's emphasis).

The scientific meaning of 'curved space,' or 'matter isn't solid,' is determined within the particular paradigm of physics that describes the world in that particular way. This is only to say, however, that 'curved space' can only be understood within the context of the general theory of relativity; or that 'matter isn't solid' within the context of sub-atomic physics. In very much the same way, Newtonian mechanics is a paradigm - it is a way of describing the world; and the way it describes the world is determined by a set of axioms.

[For Watts (1961), interestingly enough, all paradigms would have an axiomatic base:

The difficulty is that man can hardly think or act at all without some kind of metaphysical premise, some basic axiom which he can neither verify nor fully define. Such axioms are like the rules of games (page 19).

This thesis applies Wittgenstein's network analogy to the way scientific theory speaks of the world, and specifically to Newtonian mechanics. Watts (1961), on the other hand, applies the metaphor of the grid to all our different ways of thinking and experiencing the world:

What happens when we touch and feel a rock? Speaking very crudely, the rock comes in touch with a multitude of nerve ends in our fingers, and any nerve in the whole pattern of ends which touches the rock 'lights up.' Imagine an enormous grid of electric light bulbs connected with a tightly packed grid of push buttons. If I open my hand and with its whole surface push down a group of buttons, the bulbs will light up in a pattern approximately resembling my hand. The shape of the hand is 'translated' into the pattern of buttons and bulbs. Similary, the feeling of a rock is what happens in the 'grid' of the nervous system when it translates a contact with the rock. But we have at our disposal 'grids' far more complex than this - not only optical and auditory but also linguistic and mathematical. These, too, are patterns into whose terms the world is translated in the same way as the rock is translated into nerve patterns. Such a grid, for example, is the system of

coordinates, three of space and one of time, in which we feel that the world is width, and depth filling all space, and though the earth does not go 'ticktock' when it revolves. Such a grid is also the whole system of classes, of verbal pigeonholes, into which we sort the world as things or events; still or moving; light or dark; animal, vegetable, or mineral; bird, beast, or flower; past, present, or future.

It is obvious, then, that when we are talking about the order and structure of the world, we are talking about the order of our grids. 'Laws, like the law of causation, etc., treat of the network and not of what the network describes' (reference to Tractatus, proposition 6.37, given). (page 30).

The thesis, however, will restrict Wittgenstein's grid metaphor to scientific theory, even though it may be possible to regard physics as an extension of the ways in which we perceive the world].

The ability to model phenomena according to the Newtonian system may be described as the ability to 'see the world the Newtonian way.' The ability to model - the understanding of the Newtonian system as a form of description - requires the understanding of force as an abstract concept. What often confounds the understanding of force as an abstract concept is the existence of misconceptions. Misconceptions exist because, similar to paradigms (and it may be argued that misconceptions are themselves fragmented or incoherent paradigms, although this chapter will argue that misconceptions tend to be more spontaneous rather than expressing 'world-views') they help make sense of the world. Many researchers of student misconceptions (e.g. Maloney, 1984; Viennot, 1979) have reported on the reluctance of intuitive ideas to change, so just how distractive a *persuasive alternative* (in Alan Watt's sense) has to be in order to instigate a 'Gestalt-shift' within the domain of Newtonian mechanics is in a sense the central question of this chapter.

According to the philosophers of science Thomas Kuhn and Stephen Toulmin, it was the occurrence of anomalies that provided the stimulus for the invention of alternative paradigms (Losee, 1972). This was not to say, however, that a few anomalies were sufficient to cause a revolutionary change. Nor does it imply that the presentation of a few anomalies to student misconceptions would be sufficient to cause conceptual change. In an

empirical study that examined how students restructure their scientific ideas in response to contradiction, Burbules and Linn (1988) concluded:

Thus students construct new understandings in the face of efforts to contradict their current ideas, but they do not rush to embrace new viewpoints. Rather, they cling to ideas that form part of their world view even when confronted by information that does not coincide with this view. Presumably, such selective weighting of new information helps students to deal with threatening or confusing data. This process also contributes to students' abilities to incorporate information into a cohesive perspective. Educators and curriculum designers need to exploit these features of the learner to encourage reflective construction of more complete and inclusive exploratory models.

Burbules and Linn's findings suggest that the presentation of a number of anomalies will not be sufficient to promote conceptual change. This appears consistent with Chalmers (1978) account of conceptual change in the history of physics:

...the Copernican Revolution did not take place at the drop of a hat or two from the Leaning Tower of Pisa.....New concepts of force and inertia did not come about as a result of careful observation and experiment. Nor did they come about through the falsification of bold conjectures and the continual replacement of one bold conjecture by another. Early formulations of the new theory, involving imperfectly formulated novel conceptions, were persevered with and developed in spite of apparent falsifications. It was only after a new system of physics had been devised, a process that involved the intellectual labour of many scientists over several centuries, that the new theory could be successfully matched with the results of observation and experiment in a detailed way. No account of science can be regarded as anywhere near adequate unless it can accommodate such factors (page 71).

The success of an emerging theory is dependent on whether it can accommodate discrepant data. A paradigm doesn't die because of a few falsifications; rather, it dies if there is already in existence a new paradigm that can accommodate both it's own anomalies and the anomalies of the old paradigm. According to Kuhn, a revolution in science occurs when there is in existence a new paradigm (Losee, 1972). If a paradigm is unable to explain **significant** observations, and there seems no way to resolve this, then a crisis will develop from which a new paradigm will hopefully emerge (Gillott and Kumar, 1995). Science progresses because of mismatches between paradigms and nature; and when a mismatch is serious, a crisis develops whereby a revolutionary step is taken by replacing the paradigm by another (Chalmers, 1978). This suggests that conceptual change can take place in the classroom, not by comparing misconceptions with reality via a set of anomalies but by

facilitating the construction of the Newtonian system that will account for the anomalies - it is not so much the presentation of anomalies that promotes a revolutionary change in science or a conceptual change in the classroom, rather, it is the structure of the emerging paradigm and its clash with the coherence (or rather, lack of coherence) of the prevailing paradigm¹.

A teacher can present a number of anomalies that the class cannot account for under the prevailing paradigm, but can be accounted for under the alternative paradigm. Howard (1987) offers the following advice:

Confront students' schemata with anomalies. Revolutionaries and union organisers often work by stirring up dissatisfaction with the way things are. They confront people with anomalies in their life situation, with contradictions that they may never even have thought about before.....The teacher also needs to be a revolutionary, or a troublemaker, making students dissatisfied with their current schemata by showing them anomalies the misconceptions cannot handle. The dissatisfaction may then motivate students to acquire a new schema that can resolve the contradictions. Learning some new schemata can only be considered as a revolution in everyday thinking. The notions of matter consisting of particles or the continents on floating, moving plates are indeed revolutionary. The teacher also must persuade and convince, not simply present facts (Howard, 1987; author's emphasis).

Newtonian mechanics rests upon a set of defining axioms; and according to Tall (1991), the understanding of a system of concepts that is defined by a set of axioms requires a massive cognitive reorganisation. This chapter argues that, in the teaching of mechanics, the most efficient way to induce a 'gestalt shift' is to induce cognitive conflict through the presentation of anomalies; and that the anomalies be presented according to the structure of Newtonian mechanics as a 'new' paradigm. The rationale of chapter 5 is based on the

¹ According to Chalmers (1978), the Copernican revolution lasted nearly a century and a half. In that time, there were more anomalies that the Copernican paradigm had to resolve than were ever raised against the old paradigm. For example, Copernicus had to employ the use of epicycles to sustain his heliocentric theory. Galileo (successfully) answered as to why a ball, dropped from a tall tower, did not land at a point displaced from the foot of the tower (if the earth spins at the rate that it does, then the ball 'should not' land at the foot of the tower). There were many factors contributing to the success of the Copernican paradigm. However, the difference between the Copernican revolution and the cognitive changes that can take place in the classroom is that the Copernican paradigm, or, more specifically, the Newtonian paradigm, is now complete and can answer to the anomalies.

rationale of chapter 3: that the logical structure of Newtonian mechanics pictures the world in a consistent manner, and this is only possible because the structure of mechanics corresponds in some way to the structure of the world. Through eliciting misconceptions from the class, the teacher raises anomalies that the Newtonian system can account for consistently. If the class is aware of the inconsistency of its own paradigm, or indeed the incoherence of having a number of (intuitive) paradigms, then there is a possibility of the class embracing Newtonian mechanics as a better description of the world. If the class can begin to become aware of the consistency between the anomalies, which is tantamount to becoming aware of the inconsistency of its own ideas, then there may be the possibility of the class constructing *force* as an abstract concept. By the raising of anomalies, the class may be able to 'lift' a pattern of meaning - a contextual schema - that is the consistency of the Newtonian system (or rather, the consistency of the anomalies in that they have the one explanation under the Newtonian system). This 'lift' is a shift from the context of the particular anomalies as presented by the teacher, to a higher level of abstraction: a consistency that is determined by the defining axioms of the Newtonian system as a form of description (however, this is not to imply that the laws of motion can be 'lifted' from concrete experience). This is what Piaget may have described as an example of *decalage*. According to Perry (1981): *Vertical decalage manifests itself in the 'lifting' of a pattern of meaning from a concrete experience and using it as an analogue for meaning at a level of greater abstraction* [as opposed to *horizontal decalage* which is the process of drawing an analogy between different areas of experience (Perry, 1981)]. Through the process of raising anomalies (Socratic dialogue, see chapter 2, page 28) it is hoped that the class will be able to lift the pattern of meaning that is the laws of motion.

According to Wildman (1981), at the heart of much research in human cognition is the sense that humans possess an intricate set of information-processing capabilities and that

these capabilities are used to learn about and to understand the environment. Wildman (1981) states that this research views learning as a constructive process whereby experience is organised into meaningful networks or schemas. For Wildman (1981), a significant problem for cognitive research is concept development; and that this problem is significant for many cognitive theorists because *the knowledge one currently possesses has an important influence on what will be learned and remembered from subsequent experience*. A student's understanding of mechanics will be influenced to some extent by his or her present state of knowledge, and so this chapter will begin with a definition of concept and schema and will subsequently attempt to outline the processes in the formation of intuitive ideas of force and motion. By also considering the logical structure of Newtonian mechanics as a schema, the chapter proposes the Socratic dialogue as the most efficient way of overcoming student misconceptions of force and motion. The chapter will argue that the Socratic dialogue is the efficient way for the class to construct the Newtonian system as a 'world-view' to account for the unfamiliar.

5.2 SOME PRELIMINARIES: CONCEPTS AND SCHEMATA

Many researchers of misconceptions speak of the *intuitive concept of force* without defining what is meant by *concept*. In psychology, the most widely used definition of *concept* is that it is a mental representation of a category (Howard, 1987). A category is a class of stimuli that has been lumped together on the basis of some similarities between them:

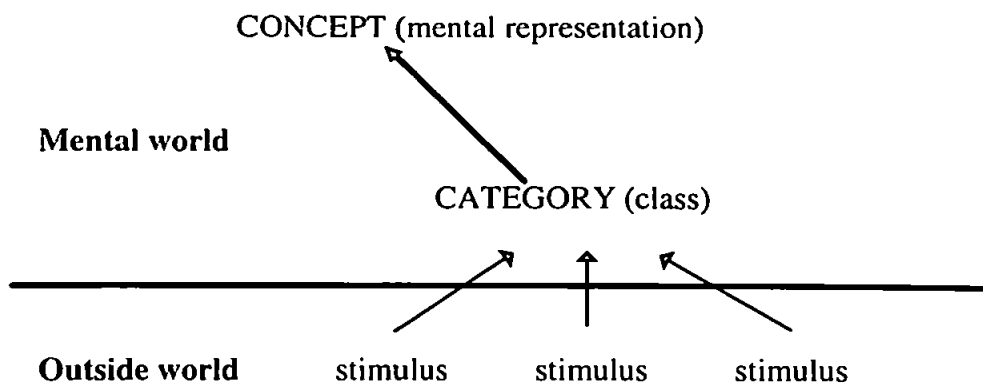


Figure 4 - Concept as a Mental Representation of a Class of Stimuli

We categorise our experience of the world to bring order to it, otherwise we would have to treat every stimulus encountered as unique. This is not to imply, however, that concepts are **formed** by the subject abstracting certain resemblances among otherwise dissimilar stimuli. The subject does not passively abstract², but utilises rules of relations such that stimuli are ordered by the same relation. **Concepts are the expression of the ways in which experience has become organised** (Bolton, 1977). A particular stimulus may be placed in many different categories. For example, a particular sparrow may be categorised as a bird, an animal, a life-form, a danger, a flying object, a nuisance, or a breeder. How we may classify on a given occasion largely depends on our objectives at the time (Howard, 1987).

A physicist would classify friction, reaction, weight, tension and thrust as instances of force. The defining features of the concept of force are the defining features of the classification. However, there is a contradiction! The ability to classify may be independent of knowing or understanding the defining features of the concept of force. A student who is able to classify instances of force may not necessarily understand the Newtonian concept of

² For example, the meaning of a concept is not acquired through observation. An inductivist will argue that a concept arises from discerning the common element from a set of perceptual experiences; but as Chalmers (1978) points out, such an account presupposes the very concept the acquisition of which it is meant to explain.

force. Chapter 3 has attempted to show that force is implicitly well-defined by the axioms of mechanics; however, the ability to classify may be independent of how force is defined. The point here is that a taxonomy of force is not a sufficient condition for the understanding of force as a Newtonian concept. Additionally, the Newtonian understanding of force lies beyond simply **knowing** the three statements. The ability to classify presupposes the formation of a concept, but not necessarily the understanding of the Newtonian concept or indeed the formation of an intuitive concept that is well-defined. A first-year foundation engineering mechanics class in their first lesson were asked to define force. Some responses were:

Force is that which stops or starts motion.

Force is that which controls motion.

Force is that which maintains motion.

Er...force is all around you.

It would be very unlikely that a student would classify friction as a force as a consequence of regarding force as that which maintains motion. **The ability to classify under a concept does not necessarily imply an understanding of the concept with respect to its defining features.** It is rather like knowing your 'times-table' but not understanding multiplication as the *addition of additions*. We may speak of the intuitive concept of force as a concept that is not well-defined; however, student intuitive reasoning may be better understood from a cognitive-psychological point of view if we speak of the *intuitive schema of force and motion* rather than the intuitive concept of force. The next section will attempt to show that how force is intuitively conceived is dependent on how the motion of the body under consideration is conceived. Also, since the defining features of the Newtonian concept of force is a set of propositions rather than a single proposition, it would, at times, be more appropriate to speak of the Newtonian schema, rather than concept, of force.

A schema may be defined as a mental representation of a set of related categories i.e. as a **cluster of related concepts** (Howard, 1987). A schema is an organised body of knowledge, a mental structure, that helps us to make sense of the world. A schema establishes the relation between concepts. For example, *eye*, *mouth*, *ear* and *nose* are all concepts; yet *face* is a schema which organises how these concepts are arranged (Howard, 1987). A schema consists of a set of expectations about how parts of the world are organised. For example, if we walk into a dark room and see a pair of eyes then we would instantiate our *face* schema (Howard, 1987). A schema has *slots* or *variables* that are filled in with concepts and are organised in a certain way (Howard, 1987).

The problem with the distinction between concepts and schemata is that a concept can be a schema and vice-versa. *Face* can be a concept as well as a schema. However, a mental representation may be considered either a concept or a schema depending on what you want to do with it (Skemp, 1971). Skemp (1971) gives an example of *vegetable*: as a concept it has such instances as beans and potatoes, and as a schema its relationship to your meat (relative times to cook, etc). Words are symbols for concepts ('force' can therefore name a number of different concepts). A good working model is to regard the single concept as a basic unit of the concept system, next is the proposition and a higher level unit is the schema (Howard, 1987). This should not be taken as absolute since any concept can be seen as a schema.

Modelling in mechanics involves identifying the forces acting on a body. By identifying the forces we can explain the state of motion of the body, quantify, and make predictions. Force is defined in such a way that the definition itself determines how the modelling procedure is to take place (identifying the forces acting on the body, making assumptions,

applying the equations of motion in two chosen directions etc). It is most **unlikely** that the fresh mechanics student would be able to explain the state of motion of a body with respect to his or her intuitive definition of force. The student may not have a definition of force. On the contrary, it is most likely that the student would define force with respect to the motion (e.g. for a thrown ball, a force is required to push the ball in order to overcome gravity. To maintain circular motion, there must be a force that acts radially outwards). 'Force' may be utilised by the student as a metaphor to describe motion.

Hearing the same name applied to various stimuli may induce a person to try and form a concept. We have experienced many forms of motion most of our lives, and most of us would have associated the term 'force' with motion. However, the term in itself is not the concept (the term *labels* the concept); and the concept that is formed would be personal, idiosyncratic and subject to much variation. Although we associate 'force' with motion, it may well be the case that the intuitive concept of force is not formed until the subject is asked to describe a particular motion in terms of force. If the subject is never given the opportunity to describe a motion in terms of force, then it may well be the case that the subject will hold, form or develop a group of poorly differentiated concepts rather than form a concept (intuitive or otherwise) of force. According to Champagne *et al* (1982), the following is characteristic of the contents of the cognitive state of uninstructed physics students:

Concepts are poorly differentiated. For example, students use the terms speed, velocity, and acceleration interchangeably; thus, the typical student does not perceive any difference between two propositions such as these - (a) the speed of an object is proportional to the [net] force on the object; (b) The acceleration of an object is proportional to the [net] force on the object.

An 'intuitive concept of force' seems to suggest a concept that is fairly well formed; whereas it might well be the case that instead of a concept of force, we have a schema of loosely related and poorly differentiated concepts.

Spontaneous reasoning may be defined as reasoning applied to an unfamiliar situation. Spontaneous reasoning is usually the 'first thing that comes to mind'. Consider the following question:

A rocket is travelling in a straight line deep in space, where it is not affected by air resistance or gravity. If its motors are turned down halving the force that they exert, then what would happen to the motion of the rocket?

It would be most unlikely that the student would respond correctly unless the student was already familiar with $F = ma$. What is remarkable about spontaneous reasoning is that students may give their intuitive response **despite** their knowing $F = ma$. Viennot (1979) reported that even *teachers tend to make similar mistakes when they answer in a hurry and that the intuitive scheme.....reappears even in the expert when he or she lacks time to reflect*. In spontaneous reasoning, *students are not usually conscious of the 'notion' they use and may call it, sometimes indifferently, 'force', 'impetus', 'energy', 'momentum', and so on* (Viennot, 1985).

It is not as though fresh students bring with them into the classroom an intuitive, but coherent and explicit, concept-map of force and motion, ready to be applied to any phenomenon. Many students will have to think about a qualitative question, concerning force and motion, for the first time. Asked to account for a particular motion in terms of force, the student will slot 'force' into his or her schema instantiated to account for the motion. 'Force' will be used to account for the motion in some way, and the slotting in of 'force' will depend on the way the student conceives of the motion (a ball thrown up, circular motion etc). Spontaneous reasoning may be considered as the instantiation of preconceptions that are arranged, rearranged or modified to account for a phenomena for the first time.

If a schema of motion is instantiated with force as one of the slots, then the definition of the student's intuitive concept of force is dependent upon its relation to the other slots in the schema. If your schema of a thrown ball includes a force pushing the ball, then the definition of your concept of force has to be consistent with that which overcomes gravity to maintain motion. The intuitive concept of force is so ill-defined, and so dependent on the way the student conceives of the various forms of motion, that it would be more appropriate to speak of the intuitive schema of force and motion rather than the intuitive concept of force.

Not only would it be more appropriate to speak of the intuitive schema of force and motion, it would also be more useful in our understanding of how intuitive ideas are formed. A schema acts like a filter - we only take in information that is relevant to the schema, otherwise the information is either ignored or interpreted idiosyncratically (Howard, 1987). A good example is reading a text-book for the first time. A few parts of the book may be vaguely familiar, and pulling the parts together according to what we already know may yield an interpretation different to what the author intended (Howard, 1987). Perkins and Simmons (1988) refer to *problems of garbled knowledge* whereby newly acquired knowledge is mixed up in various ways, and the authors refer to uninstructed students who recognize friction as a force operative when one object is moving against another but not when both objects are stationary. Discrepant data in mechanics may be interpreted by the intuitive schema in a way that confirms mistaken ideas. According to Champagne *et al* (1982):

Uninstructed students apply propositions that link force with motion, whereas Newtonian mechanics links force with change in motion. Moreover, the meaning uninstructed students attribute to technical terminology is different from the technical meaning. For example, the technical meaning of acceleration is a change in the magnitude of velocity or direction of velocity of an object, while the meaning uninstructed students attribute to acceleration is speeding up.

According to Warren (1979), confusion over the concept of *acceleration* (and *velocity*) is sometimes caused by poor definition:

In non-scientific use the word acceleration means rate of increase of speed; misunderstanding can therefore arise from confusion between the scientific and the common meanings. Such confusion is sometimes caused by the incorrect definition of acceleration in terms of increase of velocity, and by the invalid use of the separate term deceleration to mean certain cases of acceleration (page 2; author's emphasis).

The specific problem at this juncture is that uninstructed students do not understand or consider the importance of 'rates of change'. Firstly, many uninstructed students do not distinguish between motion and a change in motion. Secondly, they do not understand the relationship between speed, velocity and acceleration. If you do not focus in on the change in motion, then there appears to be only the motion left, hence 'force' to explain the motion. This is perhaps the similarity between the uninstructed student and Aristotle: neither held the concept of change in motion (a change in motion requires quantification, and Aristotle had no measuring device).

Without understanding the 'subtle differences' between force as that which maintains motion and that which changes motion, or acceleration as going faster or as a change in velocity, or (to quote Perkins and Simmons, 1988) *the point that zero velocity does not imply zero acceleration*, we have the non-scientific usage of scientific terms as *naive theories* (Perkins and Simmons, 1988) which reflect 'phenomenological primitives' (diSessa's *p-primes*) that have a salience in everyday life but which mask the explanatory breadth of other concepts.

5.3 THE PSYCHOLOGY OF LEARNING MECHANICS.

There is much experimental and observational research to suggest that the student's intuitive schema of force and motion may have defining-features. Student responses to a

question may be varied, but there may also be similarities between many of the responses. Those similarities can be said to be the defining features of the intuitive schema of force and motion.

These similarities, or defining-features, have been identified in empirical research internationally. One very common defining-feature is that *force is a property of the body* (e.g. Bocha, 1990; Viennot, 1985; Whitaker, 1983). e.g. the force from the thrower is transferred to the ball and stored in its mass, and it is that force which pushes the ball. The intuitive schema of force and motion may not be explicit enough for the student to conceive of the proposition *force is a property of the body*; the proposition may only be an implication of the kind of response given to a question [sometimes children use grammatical constructions such as *the marble has no more force* (Leboutet-Barrell, 1976)]. The intuitive schema of force and motion is built from the experience of bodies in motion and the association of the term *force* with that experience. *Force* as a notionallows a simple and synthetic description of spontaneous reasoning (Viennot, 1985). We perceive bodies in motion, we do not perceive the forces acting on the bodies (Scaife, 1990). In other words, we experience effects; we do not experience that which causes the effect apart from the action of other bodies. Force is not perceived, therefore it is not abstracted from experience. What is abstracted from experience is bodies in various forms of motion (bodies that go up, come down, increasing in speed horizontally etc), and the use of the term *force* may be particular to how the student conceives (thinks of) the various forms of motion. According to Champagne *et al* (1982):

Motion-of-objects schemata of uninstructed students are situation-specific, thus suggesting that no naive abstract representation is extant in the schemata to make them appear to be applicable to a large number of physical situations.....For example, students do not recognise that the same physical laws apply to objects in free fall and to objects sliding down an inclined plane.

From an interview with a student, Marton (1986) found that *the conception the student shows initially (the force in the direction of the movement exceeds the sum of the forces in the opposite direction) is always linked with focusing on the fact the body is moving (instead of being at rest).*

A ball rolling horizontally is slowing down (hence a force opposing the motion), where as a ball thrown up is going up despite gravity (hence a force pushing the ball). We tend to focus on the *dominant features* of motion ('up', 'down', 'moving horizontally', 'large body' etc), and force as a concept is assimilated into a number of different ways according to the various schemata of motion. Whether it is a young child assimilating the **term** *force*, or a mature student assimilating the Newtonian **concept** of force, *force* is assimilated in a way that makes it compatible with the schema. Student reasoning tends to focus on the body in the context of the motion, rather than the motion in the context of forces acting on the body [For example, Viennot (1985) found that 'V-F' reasoning (see chapter 1, page 12) occurs mainly when motion is a striking feature of the proposed physical situation]. The conceived dominant features of motion, the type of motion or body that is conceived by the student, determines the meaning of force for that student. If a body is in constant motion, then force is conceived as that which maintains motion. If a pulley system contains two unequal weights, the vertical portion of the string holding the larger mass may be considered to have a greater tension than the vertical portion holding the smaller mass:

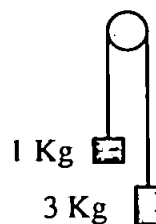


Figure 5 - Pulley with two Unequal Weights

By 'dominant features' I do not wish to mean the striking perceptual features of the given configuration. Montanero *et al* (1995), has evidence to show that many students have a high level of internal consistency to their answers (albeit right or wrong) based on conceptual factors (which will be considered as the dominant features rather than perceptual elements).

The intuitive schema of force and motion may have defining features, but those features are *not well-defined* and may only be determined by the dominant features of motion i.e. the defining features of the intuitive schema of force does not account for all different types of motion but is instead dependent on the dominant features of each type of motion. The Newtonian schema of force and motion is **well-defined**: *force* is defined by the laws of motion and can be applied to different types of motion without having to change the definition or meaning of force with respect to each type. The intuitive schema of force and motion cannot assimilate the Newtonian concept (or schema) of force, as defined by the laws of motion, without changing the meaning of force: the laws cannot be understood within the context of the intuitive schema. If the laws are given *didactically*, then they would most likely be interpreted *idiosyncratically* according to the intuitive schema instantiated, and learnt separately as a strategy to tackle quantitative questions - the student could be said to have a *dual perspective* in mechanics (Gilbert *et al*, 1982; Berry and Graham, 1991). This is synonymous with Perkins and Simmons' (1988) idea of *ritual concepts* whereby students appear to have a respectable understanding yet further analysis reveals that in fact the student applies knowledge in a somewhat ritualistic fashion, proving unable to deal with novel situations even when the knowledge base should be more than adequate for the task. Perkins and Simmons (1988) refer to the example of students who apply the non-existent 'impetus' force to a vertically thrown ball; despite their ability to cast the problem algebraically, identifying the only force acting on the ball as gravity. As

well as unsound intuitions overriding technical knowledge, another feature of ritualized concepts is the overgeneralisation of technical knowledge dominating a situation (Perkins and Simmons, 1988).

If the Newtonian concept of force is to be understood in a way that would enable the student to account for unfamiliar phenomenon qualitatively, rather than as a rule-centred approach to quantitative questions, then the intuitive schema has to undergo a massive transformation in order to adapt to the new concept (a 'gestalt-shift'). The intuitive schema has already made sense of the world for the student, and is therefore reluctant to change (Howard, 1989; Maloney, 1984; Redish, 1994; Skemp, 1971; Viennot, 1979;). To change the schema, the student has to construct the defining features of the Newtonian concept - and this will not happen unless the student sees the need to do so. If a student realises that his or her schema is inadequate in accounting for a phenomenon, then change is possible.

For the teacher to implement change, he or she must realise the schemata that is currently held by the class. According to Bowden *et al* (1992): *teachers can better assist conceptual change in students if they are clearer about what the current student conceptions are and in which direction they intend student understanding to develop*. The class should be allowed to express their views without ridicule or fear of admonishment (Arons, 1990; Howard, 1987). The teacher should then present a question that the schemata, currently held by the class, cannot account for. The *anomaly* presented would hopefully create the cognitive conflict necessary to implement change in the current schemata.

The presentation of anomalies may not necessarily be restricted to mechanics, but to many subjects including statistics [e.g. *How big does the obtained difference between the experimental and the control group have to be before deciding whether or not the drug had*

an effect? (Howard, 1987)] and plate-tectonics [e.g. *How can we explain the distribution of certain fossils in the Antarctic?* (Howard, 1987)]. In mathematics, however, the use of anomalies to induce change can be a little more straightforward than compared with mechanics. If a class of 11 year-olds can solve equations by guessing the value of x (e.g. $2x + 1 = 7$), then the teacher could present a question such as 'Can you solve $56x - 113 = 479$?' The question is within the scope or ability of the class, yet it is outside their present means for solving equations (a situation that would be described by Piaget as a state of *disequilibrium*). The class cannot extend their present method for solving equations analogically in order to handle $56x - 113 = 479$. Equations of the type $56x - 113 = 479$ cannot be assimilated into their present schema of solving equations, which means to say that the class cannot *generalise* their currently held method. By using a scale-balance as a *metaphor* for equations of the form $ax - b = c$ (e.g. *If I add 113 to one side, what must I do to the other side to restore balance?* etc.) it is possible for the class to construct a more general method for solving linear equations. The anomaly, if presented at the right time, can induce cognitive conflict and the motivation to resolve the conflict. The timing is important, because the class has to be at the stage where they see $56x - 113 = 479$ as being within their 'grasp' (yet outside their present strategy). The struggle to construct a more general method for solving equations is the equation-solving schema having to *accommodate* to the new input (anomaly) - the struggle is the schema transforming itself, which is often referred to as *cognitive (or mental) reorganisation* (e.g. Tall, 1991).

Anomalies presented in mechanics are not quite so straight-forward! It isn't so much the case of presenting an anomaly so that a target-concept may be reached - the student may **already** have formed an alternative concept to that of the target-concept. With the possible exception of developing technique in the handling of quantitative questions, the anomaly in mechanics is a device that aims to render the alternative concept impotent in its account of

the anomaly. It is more the attempt to *displace* the intuitive schema than developing, or transforming, a schema that was adequate before the introduction of the anomaly. Many students have a tendency towards consistent reasoning (Minstrell, 1982), and the anomaly may induce cognitive conflict by exposing the inconsistency of the intuitive schema of force and motion. Some intuitive ideas are correct. For example, many students would argue that the horizontal force acting on a car that is braking opposes the motion. These correct ideas may be used as anomalies to such ideas as a force pushing up the thrown ball. The juxtaposition of two or more different phenomena that have the same explanation under the Newtonian system may induce the necessary cognitive conflict. The intuitive schema of force and motion may be displaced by the need for consistent reasoning - the defining features of the intuitive schema may be seen to be inadequate to more consistently account for the anomalies - the student searches for *coherence*. If the student realises that both the car **and** the ball are slowing down, then the student may well be on the way of conceiving force as that which changes motion - replacing what he or she considers to be the dominant features of motion.

Given the intransigence of the intuitive schema, would it be that easy for a class to become aware of it's own inconsistencies and tend towards consistent reasoning? According to Perkins and Simmons (1988):

Intuitions have priority over internal coherence. The notion that objects of different masses fall at different speeds lacks coherence, as Galileo's famous argument established. The notion that a book on a table pushes on the table, but the table does not push back on the book, also does not yield a coherent analysis.....Such examples suggest that people commonly fail to notice the incoherencies in their intuitive mental models, and often, when incoherencies are brought to their attention, the incoherencies simply do not appear very important. The robust intuitive model seems worth preserving in the face of a few minor discrepancies (authors' emphasis).

However, Hake (1987), reporting on the effectiveness of the Socratic dialogue, observed that:

Student behavior was consistent with Piagetian ideas that people feel uneasy about disequilibrating experiences and spontaneously seek to reorganise their understanding.

On eliciting different student responses on a question concerning blowing on a stationary ball, and giving a similar blow in the opposite direction whilst the ball is in motion, White and Horwitz (1988) found that:

Since not everyone can be right, students are motivated to find out who has correct explanations and who has misconceptions.

However, for Perkins and Simmons (1988), conceptual change is not simply a question of presenting a few anomalies. Physics is not a question of empirical inquiry, and the intransigent schema can always ignore or idiosyncratically assimilate discrepant data - the epistemological foundation of physics is one of logical coherence and it is that coherence that has to be constructed:

For instance, Galileo's argument that objects of any mass fall at the same speed depends basically on logic and on our intuition that snipping a string between two objects is not going to make that much of a difference in their rates of fall. The proportionality of F with m in $F = ma$ can be justified in the same way.....To be sure, such justifications await for final verification on empirical evidence, but they can easily precede it and, in many cases at least, the world would be a very strange place if the principles did not hold up. They also frequently have the advantage of providing more compelling understandings of the phenomena concerned than does mere evidence. They show not that something happens empirically to be the case but why it almost has to be the case. Accordingly, the notion that the epistemological foundations of physics and other hard sciences are empirical through and through, requiring mountains of data, does mischief by depriving students of an important intellectual resource (my underline).

The last sentence in itself justifies the writing of chapter 4.

To merely provide anomalies is rather like providing evidence as to why something cannot be the case. Revolutions in science do not occur from the presentation of a few anomalies, but from the emergence of a new paradigm that can account for the anomalies - the new paradigm has to show **why** the anomalies have to be the case, hence there is an element of *argumentation*. According to Phillips (1977):

Of course, scientists do employ observations, experiments and various elements of the scientific method in their work. But these are not sufficient to account for scientific change and for the acceptance by the scientific community of some possibilities rather than others. In fact the scientist may use a variety of devices in trying to persuade the scientific community to accept his way of looking at things. One of these, I claim, is argumentation, which the scientist uses to create or increase the adherence of other minds to the theses or

world-views he presents. At the very least, he must create in his readers (or listeners) a willingness to pay attention to what he is saying. In short, he always has the task of gaining the adherence of the audience to whom he directs his claims (page 183, my emphasis).

In the confrontation of students' schemata with anomalies, the teacher must persuade and convince, not simply present facts (Howard, 1987). In addition to maintaining control of the class, the teacher must also **inspire** the class to learn - and this is perhaps true for any teaching method. The Socratic dialogue, perhaps more than any other teaching method, provides the means for the teacher to hold attention and to inspire through the process of constant interaction. The Socratic dialogue is therefore not only a question of logical structure and cognition, there is also a 'social dimension' as well. As a method of teaching, the Socratic dialogue has to be a *dialectic* between logical reason and persuasion. This point will be taken up under the sub-heading *Further Thoughts on the Socratic Dialogue* (page 149).

By the presentation of a number of anomalies, the class may be able to construct the defining-features of the Newtonian system through its search for coherence. Many concepts in mechanics are *explicitly* well-defined with reference to other concepts (e.g. *momentum* as the product of *mass* and *velocity*), but the difficulty with the concept of force is that it is *implicitly* well-defined by a set of axioms - the laws of motion (Hestenes, 1992; see chapter 3 above). Some explicitly defined concepts may be easily assimilated, but implicitly defined concepts require a **massive** cognitive reorganisation; especially if there is a set of preconceptions in the way of building the set of axioms that define *force*. The greater the cognitive conflict, the greater the motivation to search for coherence - and the greater the chance for the Newtonian system to be constructed by the student.

5.4 PROMOTING COGNITIVE CONFLICT

The introduction of anomalies encourages the student to search for a comparison between his or her own system of thought/interpretation with other systems of thought/interpretations. The comparison between such systems introduces *meta-thinking*: the capacity to examine thought including one's own. To quote Perry (1981):

The person, previously a holder of meaning, has become a maker of meaning. For most students, the event seems to be conscious and explicit; that is, the initial discovery of meta-thought occurs vividly in foreground, as figure, against the background of previous ways of thinking, and usually as an assimilation to the old paradigm - that is, as an item in the context of 'what They want' (author's emphasis).

This passage is written in the context of the student 'weighing more than one factor' or 'considering more than one approach' to a problem. Consequently meta-thinking, thinking about thinking, might occur *as an assimilation to the old paradigm - that is, as an item in the context of 'what they want' (to think)*. However, if the class is to construct *force* as an abstract concept then meta-thinking cannot occur as an assimilation to the existing cognitive structures. The existing cognitive structure has to adapt to the new concept (accommodation). Comparisons between different systems of thought have to show that some systems are better than others, and to regard one system as superior to your own requires cognitive conflict!. To quote Viennot (1979):

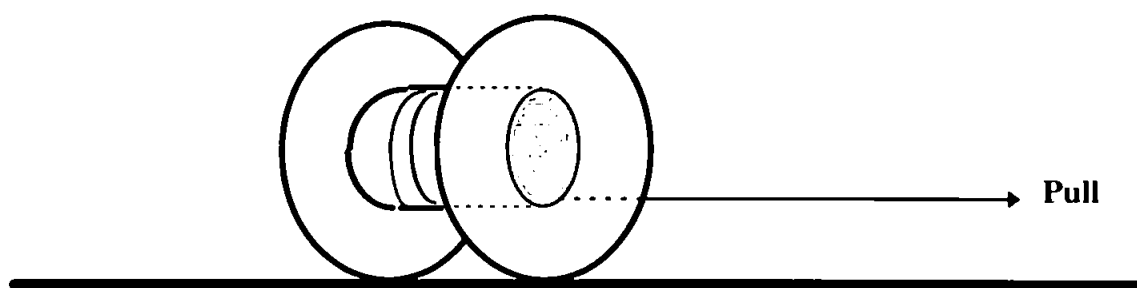
If the spontaneous scheme is to be replaced or overcome, a major teaching effort is needed which goes beyond the conventional teaching of the Newtonian scheme alone. As we have seen, the latter results merely in juxtaposing academic knowledge and the intuitive system, laying one on the other without conflict between the two. Teaching of the Newtonian scheme will only be fully effective when students are led to look at the discrepancies between it and their spontaneous ideas.

5.5 COGNITIVE CONFLICT AND IDEALIZED ABSTRACTION

Force as that which changes motion is not abstracted from experience; for one thing force is not perceived, but nor has uniform motion in the absence of force been perceived. A

teacher may demonstrate $F = ma$ using a ticker-tape experiment, but that is only to make experience compatible with the second law of motion. **Experiment cannot demonstrate directly the first and third laws of motion**; rather, it is the application of the laws of motion that make sense of the experiment - despite the design of the experiment so the laws can be applied. The laws of motion can only be attained through the process of *reasoning*. The role of experiment in developing an understanding of the laws of motion can either be that of model deployment (Hestenes, 1992; see chapter 3, page 97 above) or that of presenting anomalies. For example, take the following problem:

A string is partially wound around a drum:



If I pull the string, in which direction will the drum move?

Figure 6 - String Partially Wound Around Drum

The majority of students in three separate mechanics classes stated that the drum would unwind and hence move in the opposite direction to the pull. Moving the drum by actually pulling the string created facial expressions of cognitive-conflict³.

³ A few students from each class did state that the drum would move in the direction of the pull. However, a student reasoned that that was because motion was always in the direction of the applied force. I raised the anomaly of the force acting on a projectile, and stressed that the drum underwent a **change** in motion rather than **underwent** motion in the direction of the applied force.

Mechanics is a way of understanding the physical world, but ironically it is built on *idealised abstraction* (see chapter 3, page 93). It is most likely that two balls of unequal weight will not land at exactly the same time after being dropped simultaneously from a tower. Ice pucks do not move with uniform motion after being given a push. The weight of a body does not act through the centre of gravity (the centre of gravity may be defined as a point where the weight of the body is **imagined** to act). There is no such thing as a *light* framework or even a lamina. The terms *light* and *smooth* in mechanics may appear to be easily understood in the way students respond to quantitative questions, but students appear to have tremendous difficulties in coming to terms with idealised abstraction with respect to qualitative modelling [Whitaker (1983) raised the question: *If students are specifically instructed to ignore air resistance, will they?*]. Idealised abstraction is a hypothetical situation whereby conditions are imposed in order to ‘simplify.’ It asks of students: ‘Given such and such a situation, what happens and why?’ Students have to think according to the parameters set by the conditions imposed. The difficulty of having to think abstractly in mechanics is augmented by the experience of the physical world behaving differently. Consider the following question (Mildenhall, 1995):

The 1 tonne block rests on a frictionless table. The situation shown is released from rest, what happens?

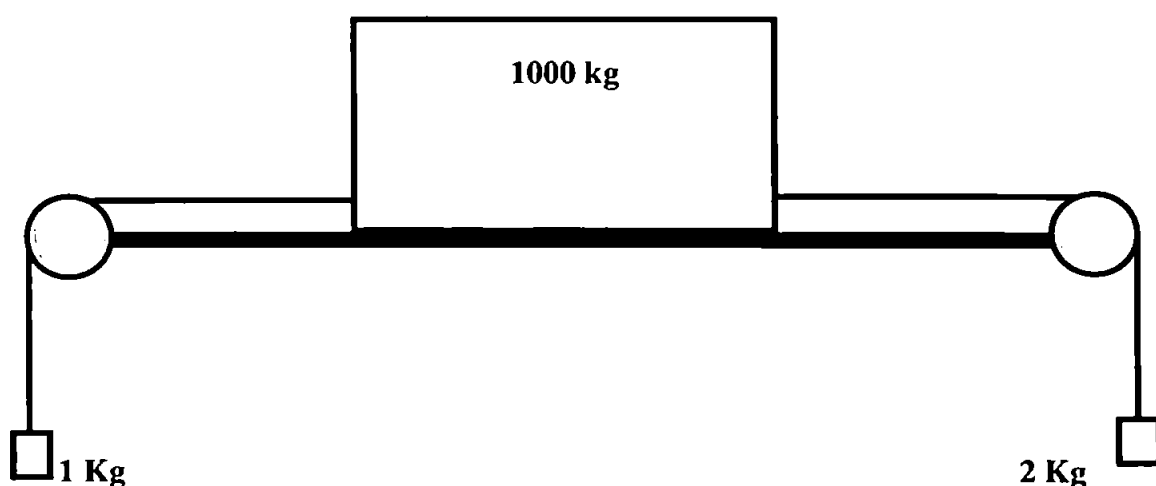


Figure 7 - Pulley System with a 1 Tonne Block Resting on a Smooth Table

28% in a sample of mechanics students stated that the 1 tonne block would remain at rest. Such a response may be considered 'common-sense' since in reality the 1 tonne block would most likely remain at rest. Unfortunately, the correct response to the question cannot be verified experimentally; the truth-value of the response can only be considered through reasoning - that is, thinking by following through the parameters set.

The ability to reason within the parameters set by idealised abstraction can be developed by the consideration of *possible-world* scenarios e.g. *Describe the motion of a ball thrown in an imaginary world of no gravity*. A possible-world is a situation that is conceivable and in a sense 'plausible,' but may not be actually realisable in the physical world [according to the *mental models theory*, when we think, we think about parts of either the real world or an imaginary world - the structure of the world that is represented by the mental model is manipulated to reflect possible changes in the world that we are thinking about (Garnham and Oakhill, 1994). Mental models of concrete situations are easier to work with than abstract models, and to find a conclusion that is true in each of a set of models is difficult! (Garnham and Oakhill, 1994)]. A *thought-experiment* is a possible-world that enables the consideration of scientific concepts divorced from any real situation.

Anomalies can be presented in the context of a possible-world. For example, three students in a university foundation engineering mechanics class articulated that the force imparted by the thrower onto a ball thrown vertically upward gradually weakens until overcome by the force of gravity:

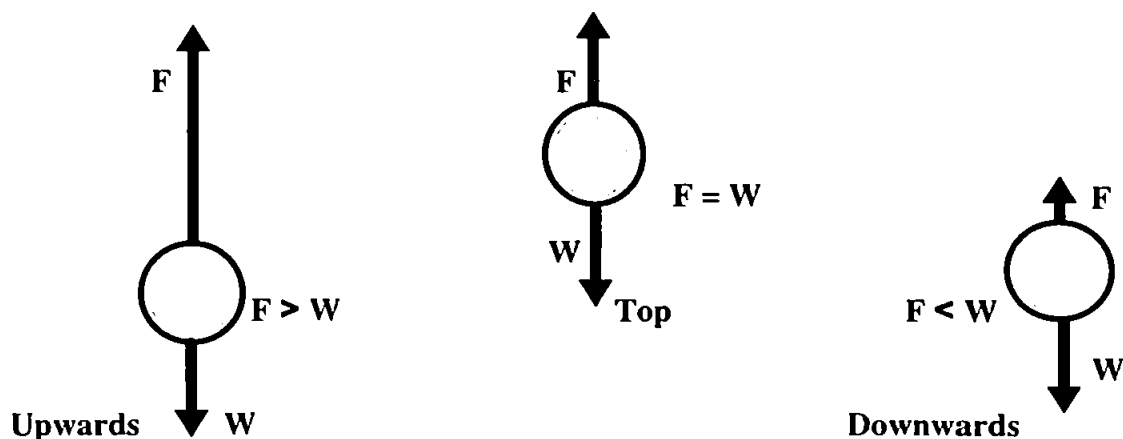


Figure 8 - Forces Acting on a Vertically Thrown Ball as Articulated by Three Students

The anomaly presented was a ball thrown deep in space outside the influence of gravity. One of the three students stated that the ball in space would travel with uniform motion, hence contradicting the original claim that the force impressed on the ball gradually weakens. The other two students, however, argued that the ball would slow down until coming to rest. Another anomaly had to be presented.

5.6 A PILOT-STUDY ON THE EFFECTIVENESS OF STRATEGIC QUESTIONING.

The above foundation engineering mechanics class (of approximately 30 students) took part in a pilot study to evaluate the effect of strategic questioning and to establish a bank of student responses to the questions (see chapter 7). The objective was to use these responses to produce a teaching package that could be evaluated in a large-scale survey (see chapter 8). A video-recording was taken of the class and a transcription was made of the recording. The aim of the recording was to monitor the effect of strategic questioning and to enable the compiling of student responses. Before each lesson a list of *concept-questions* (questions that demand a qualitative response to phenomena, such as *What force acts on a*

ball that is thrown vertically upwards? Graham and Berry, 1991) was drawn up. To each concept-question a list of possible responses was anticipated. To each possible response was a *parallel-question* that demanded the same **Newtonian** response as the original concept-question but to a different phenomenon, either in the form of a real situation or of a possible-world. The parallel-question is an anomaly to the intuitive response that would hopefully induce the necessary cognitive-conflict. The aim of presenting a series of parallel-questions would be to displace the intuitive schema of force and motion and to enable the student to embrace the Newtonian schema through the need for consistent reasoning in accounting for the anomalies.

For the benefit of the two remaining students who had insisted on a force pushing the upward moving ball, the class were invited to consider the force acting on a car when braking (both ball and car are slowing down, hence demanding the same Newtonian response). The class, including the two students, argued for a braking force acting in the opposite direction to the motion. In what appeared to be the dismay of the class, the two students still insisted on a force pushing the thrown ball. The following question was then put to the class:

I am sitting in a train that is travelling with a constant velocity of 100 m.p.h. Between my two fingers is a ball.

1) *What is the velocity of the ball relative to an observer, who is sitting on the station platform, watching the train go by?*

2) *Is there any horizontal force acting on the ball?*

Both the two students in question argued that the ball would be travelling at a uniform 100 m.p.h. with no horizontal force acting on the ball. It would appear, from the expressions on their faces, that the two students had realised that they had contradicted their previous claim - that a force is required to maintain motion. In a subsequent discussion, the two

students were asked to reconsider whether or not a force was acting on the ball that was thrown deep in space. They stated that they had changed their mind and that the ball would travel with uniform motion in the absence of any force. When they were then asked to compare the ball thrown on Earth with the car braking, they responded that both situations were similar and that the force acting on the ball moving upward was the force of gravity slowing the ball down. Could this be acceptance for peace? Their response to subsequent concept-questions revealed an understanding of the Newtonian concept. For example, they reasoned correctly in answering the following question:

A spaceship in deep space, where no forces are acting on it, drifts sideways from X to Y. When it reaches Y it fires its engines for a very short time (e.g. 0.5 seconds). Draw the path of the spaceship after it has fired its engines.

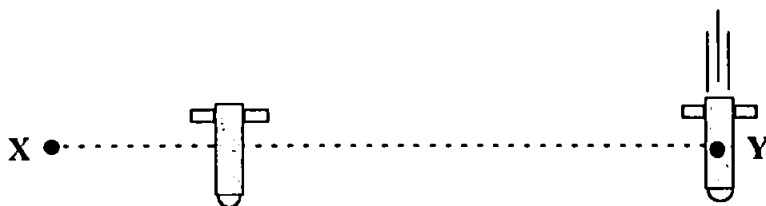


Figure 9 - Spaceship Drifting Sideways Until Engines are Fired

[Ironically, a different student, a quiet person who did not participate much in the discussions concerning force and motion, responded 'How can a spaceship drift sideways in the absence of any force?']

The strategies dealing with the idea of a force pushing the ball up may be summed-up as follows:

- 1) Possible world of no gravity.
- 2) Parallel situation of a car braking.
- 3) Force and the ball in the train.

It is tempting to regard 3) as the major catalyst such that 1) and 2) aren't really necessary. However, the impact and significance of 3) may be dependent on how students respond to 1) and 2). Subsequent interviews with other students suggest that 3) is the major catalyst, but that its magnitude is in a sense proportional to the intransigence over 1) and 2).

The anticipated student responses to concept questions have been revised in the light of the pilot-study, from which a teaching-package has been designed (chapter 8). The teaching-package is an aid to implementing the Socratic-method, which has been evaluated in a survey involving twenty schools (chapter 9).

The pilot-study consisted of the above experimental class and a control class that ran in parallel. The lecturer in the control class used concept questions but without the structure that was used in the experimental class. Six hours of lesson time were used for both classes. Both classes were pre- and post-tested. The pre-tests revealed a mean difference of 1.3% (experimental class: 38.3% - control class: 37.0%). The post-test revealed an adjusted mean difference of 7.9% (experimental class: 68.7% - control class: 60.8%). An ANCOVA was performed on the pre- and post-test results (for an explanation, see page 180), hence the adjusted means for the post-test results. For details of the calculation, see page 307. It must be noted that the results cannot be considered conclusive since other factors could be significant (e.g. setting of homework, personality of teacher etc). However, the aim of the pilot-study was to establish a bank of possible responses to student reasoning (the teaching package), which would build a framework of strategies to enable the teacher to draw out the contradictions and to create the cognitive conflict necessary for the modification of ideas. The pilot-study has enabled the development of the package, and has demonstrated the effectiveness of strategic questioning. According to Howard (1987), the Socratic dialogue has yet to be empirically evaluated (to date). **The aim of chapter 8 is to**

empirically evaluate the teaching package as a means to implement the Socratic dialogue.

5.7 FURTHER THOUGHTS ON THE SOCRATIC DIALOGUE

In this method the instructor is not a source of information but the moderator of discussion among the students who stimulates the discussion with probing questions to induce students to articulate, clarify, criticize, and justify their beliefs. The method has at least two major strengths which should be incorporated into any instructional program: (1) It shifts the locus of control from teacher to student, making students responsible for their own beliefs and judgements. It is student centered rather than teacher centered. (2) It encourages reflective thinking, leading students to insights into their own thinking processes. In short, it promotes intellectual independence. The pure Socratic method, however, has serious weaknesses: It is not systematically directed at specific objectives, and it lacks a mechanism for introducing new ideas and conceptual tools to improve the quality of discourse.

For this reason a modified Socratic method is to be preferred, in which the instructor introduces ideas and evidence to enhance and guide the discourse. But this must be done carefully, lest it interfere with the nurturing of student independence. To be optimally effective, Socratic sessions must have coherence over an entire course. Early on, the students must come to an understanding on criteria for posing questions, formulating answers, and evaluating evidence. Model-centered instruction provides a coherent framework for all this which includes an explicit formulation of the scientific method (Hestenes, 1992; author's emphasis).

By way of reference to the 'pure Socratic method,' it would appear that Hestenes is implicitly criticizing the phenomenological teaching approach of Arons and McDermott. In Arons' (1990) *A Guide to Introductory Physics Teaching*, the Socratic dialogue appears to be little more than a qualitative discussion of phenomena between teacher and class. The slogan *Idea first and name afterwards* is stated throughout Arons' book as if the target concept may not necessarily be constructed by the class but may have to be given by the teacher after it has had a good 'airing' (so to speak!). The 'pure Socratic method' may be rendered 'aimless' if there is no structure - the classroom discussion may 'meander' off the point such that the target concept is difficult to reach. What Hestenes proposes is a 'modified Socratic method' whereby each question is structured with the target concept in mind.

In fact, what Hestenes proposes is the Socratic 'dialogue' (or rather 'method' as it is a discourse between more than two people) in its original form. In the original Socratic dialogues, the majority of questions are asked with not only a point in mind but with a target concept to be reached. In the *Meno* for example, Socrates appears to give no information, yet the slave-boy deductively reaches the theorem of Pythagoras through a series of questions. In all of Plato's writings there appears to be very little evidence of the discussion 'meandering.'

In the original Socratic dialogues, however, it isn't really the case that the target concept is reached through pure deduction with no or very little information given - information is given through a series of 'loaded questions.' Consider the following extract taken from the *Meno*:

Socrates: *But does it contain these four squares, each equal to the original four-foot one?*

Boy: *Yes.*

Socrates: *How big is it then? Won't it be four times as big?*

Boy: *Of course.*

Socrates: *And is four times the same as twice?*

Boy: *Of course not.*

Socrates: *So doubling the side has given us not a double but a fourfold figure?*

Boy: *True.*

Socrates: *And four times four are sixteen, are they not?*

Boy: *Yes.*

.....and so on! (*Meno*, 83 - 84).

In Hestenes' Socratic method, the objective is to reach the target concept; the aim, however, is to illicit student reasoning. It is not so much the giving of information but to 'ease-out' the contradictions in intuitive ideas. The difference between the Socratic method of Plato and Hestenes is that for Plato the method is used as a deductive procedure; whereas for Hestenes, it is to be used as a 'model-centred instruction.' To quote Hestenes (1992):

All (six!) of Newton's laws are required to define the force concept. Thus the significance of Newton's Third law cannot be understood apart from its relation to the other laws. That relation is revealed only by applying the laws to construct and validate models of specific physical phenomena. This suggests a general strategy for dealing with misconceptions,

elsewhere called 'model-centered instruction': Concentrate on teaching explicitly the principles and techniques of modeling with the Newtonian rules; this includes model validation for specific situations. In other words, teach the Newtonian modeling games. The instruction should be designed to elicit from the students explicit formulations of alternatives to Newtonian concepts to be analyzed and evaluated. In this way student misconceptions are confronted in specific contexts where a superior alternative is available. That is one of the primary conditions for conceptual change, a condition that is rarely met in conventional instruction. It should be recognized also that a comparison of plausible alternatives is an essential part of the validation process. Instruction in a physical theory without due consideration of alternatives is hardly more than indoctrination (author's emphasis).

By the Socratic method of strategic questioning, the class may construct the Newtonian paradigm through the process of *reasoning*. However, as Hestenes intimated, the explicit teaching of the rules and techniques of the Newtonian modelling game also requires trying to *convince*! The aim of the Socratic dialogue as a teaching method is not for the teacher and the class to arrive at the truth, but for the teacher to induce a change in the current schemata of the class. Howard (1987) offers some advice:

Such anomalies may not induce schema change unless presented under certain conditions. These are as follows.

(a) The students must see that the datum is an anomaly. They must see that their existing schema cannot handle it. Careful explanation is often needed and the anomaly must be directly related to the schema. Often much background knowledge is needed to see that an anomaly is indeed an anomaly. For instance, Posner et al found that students often had trouble seeing that certain phenomena were inconsistent with Newtonian mechanics. Provide such knowledge if necessary.

(b) The anomalies must be strongly presented. People who dearly hold a certain schema may not be encouraged to alter it unless the anomalies are so strongly presented that they cannot be ignored or dodged....

(c) Avoid possible dodges by students. Ensure that students do not escape conceptual change by one of several routes. Firstly, do not allow them to compartmentalise the anomalies. Use many examples to get them to apply what they have learned to the real world (similar to Hestenes 'modelling games' - see chapter 3). Secondly, do not allow them to simply accept the anomalies as mysteries that do not need to be explained. Thirdly, guard against a partial accommodation, as in Nussbaum's study (that 'the earth is round' was interpreted by many pupils as the earth as a flat disc).

Present a new schema that accounts for the anomalies. The first two phases are mainly concerned with convincing students of the need for a new schema. Once they see that need, present a new schema, just as the revolutionary presents a revolution and a new order of society as a means of resolving the contradictions in society. Present the new schema and carefully explain why it is better than the old ones, demonstrating how it can account for the anomalies (page 195, author's emphasis).

However, because of the logical structure of Newtonian mechanics, it should not be necessary for the teacher to present the laws of motion subsequent to the class seeing the need for a new schema. It is more the class constructing for themselves the laws of motion with little information given by the teacher. To quote Champagne et al (1982):

By participating in the dialogues which occur in Socratic teaching, the student is forced to deal with counterexamples to proposals and to face contradictions in his or her ideas. To overcome the attacks of adversaries in the dialogues, the student must construct a new framework of ideas that will stand up to criticism. The newly constructed framework is, of course, a new schema, so it may be said that schema change has occurred as a result of the student's participation in the dialogues (my emphasis).

If the Socratic method *encourages reflective thinking, leading students to insights into their own thinking processes* (Hestenes, 1992; see point 2 in the quote above, page 149) then there might be a greater chance for the class to construct the new framework rather than the framework being 'given.' If the class is aware of its own inconsistencies (meta-cognition), then there might be a good chance that the class would become aware of the consistency between the anomalies presented and hence attempt to build a new framework in the search for coherence. It should be the case that the teacher states the laws of motion only after the laws have been articulated by the class. The teacher stating the laws should only be as a formality (as opposed to Aron's sense of a qualitative discussion leading to the teacher giving the 'operational definition').

For Hestenes (1992), particular misconceptions (for example, those concerning the third law) are especially recalcitrant not so much because they are deeply rooted in experience, but because most attempts to eliminate them are piecemeal: to concentrate on individual misconceptions, separated from the others, is to ignore *the most fundamental characteristics of the force concept, the coherence of the Newtonian theory* (author's emphasis). For Champagne *et al* (1982), it is not a question of modifying intuitive schemata to an appropriate scientific one; such as, for example, transforming the motion

schema *A push produces motion* to *A force produces acceleration* by replacing the variables *push* and *motion* with *force* and *acceleration*. To quote Champagne *et al* (1982):

Empirical evidence on mechanics learning demonstrates that this instructional strategy is not generally effective and suggests that, while the gradual modification of schemata doubtlessly involves generalization and specialization, in highly integrated schemata more dramatic changes, amounting essentially to a shift to a new paradigm (in Kuhn's [1962] sense), must also take place.

Hence the construction of a new framework/paradigm/schema and the displacement of the old!

The Socratic dialogue helps to enable the construction of the Newtonian paradigm through the process of logical reasoning. However, as already stressed earlier, there is also an element of *persuasion* and *argumentation*. The teacher has to **persuade** and **convince** (Howard, 1987), and this is in parallel with the scientist who is trying to implement conceptual change (Phillips, 1977). However, this does not mean that the teacher (or the scientist) should attempt to persuade and convince 'by any means necessary' - there has to be an element of reasoning and demonstration. Nevertheless, a concern with argumentation necessarily involves us with *rhetoric* (Phillips, 1977). For Plato, the Socratic dialogues were solely concerned with reason; and many of the dialogues of Plato were in fact used as an argument against rhetoric. Consider the following extract from Plato's *Gorgias* (taken from Phillips, 1977):

Socrates: *You claim you can make a rhetorician of any man who wishes to learn from you?*

Gorgias: *Yes.*

Socrates: *With the result that he would be convincing about any subject before a crowd, not through instruction but by persuasion?*

Gorgias: *Certainly.*

Socrates: *Well, you said just now that a rhetorician will be more persuasive than a doctor regarding health.*

Gorgias: *Yes, I said so, before a crowd.*

Socrates: *And before a crowd means among the ignorant, for surely, among those who know, he will not be more convincing than the doctor.*

Gorgias: *That is quite true.*

Socrates: *Then if he is more persuasive than the doctor, he is more persuasive than the man who knows?*

Gorgias: *Certainly.*

Socrates: *Though not himself a doctor.*

Gorgias: *Yes.*

Socrates: *And he who is not a doctor is surely ignorant of what a doctor knows.*

Gorgias: *Obviously.*

Socrates: *Therefore when the rhetorician is more convincing than the doctor, the ignorant is more convincing among the ignorant than the expert. Is that our conclusion, or is it something else?*

Gorgias: *That is the conclusion, in this instance.*

Socrates: *Is not the position of the rhetorician and of the rhetoric the same with respect to other arts also? It has no need to know the truth about things but merely to discover a technique of persuasion, so as to appear among the ignorant to have more knowledge than the expert? (Gorgias, 458e-459c. Quoted by Phillips, 1977; page 186).*

Phillips (1977) Makes the point:

Despite Socrates' dismissal of rhetoric, it seems to me that he himself is using emotionally persuasive language for the purpose of undermining the position of those emphasising the power of rhetoric.....But I see no compelling reason to accept a view of rhetoric as something involving a shrewd rhetorician and the common herd. While the rhetoric of the ancients was specifically the art of speaking persuasively in public, today argumentation (including of course, rhetoric) involves the use of printed texts: books, monographs, articles, essays. What is common to the ancient and contemporary use of rhetoric is the idea of an audience, those to whom the argument is directed (page 188, author's emphasis).

In the context of the classroom, the teacher also uses rhetoric; but (hopefully) it isn't the case of the ignorant trying to persuade the ignorant - the teacher is rather like the scientist (as opposed to Gorgias' rhetorician) in this sense! The teacher has to convince through reasoning (and demonstration) as well as using rhetoric.

The idea of the scientist (and more importantly, of the teacher) trying to persuade and to convince is contrary to positivism (Phillips, 1977):

Demonstrative proof, which is characteristic of positivistic science, is impersonal; it is binding on any normal mind, and its correctness is not dependent on the assent of any particular persons. In the practice of science, demonstration involves the application of rules enumerated beforehand. Demonstrations are intended to guarantee the truth of a science's affirmations. But demonstration, as Perelman observes, is based on a theory of knowledge which is not human, but divine; of knowledge as acquired by a unique and perfect Being.

Argumentation, on the other hand, grants the status of knowledge to viewpoints (opinions, formulations) which have survived the criticisms and objections of the particular audience to whom they have been directed. Thus, knowledge - including scientific knowledge - is human and social (Phillips, 1977, page 184; author's emphasis⁴).

⁴ Although knowledge is human and social, it must not be conflated above validity. See chapter 2 above.

As a teaching method, the Socratic method uses a mix of reasoning, demonstration and rhetoric. Nevertheless, the emphasis is on logical reasoning as the method (in Hestenes' sense) is employed to displace the intuitive schemata of force and motion with the construction of the Newtonian system in accordance with its logical structure. However, there are many students who think in a 'visual' sense rather than in a 'logical' sense of using words and language (Dreyfus, 1991). Some students may use visualizing as a process more than any other by which mental representations can come into being (Dreyfus, 1991). To accommodate this, the Socratic method should be designed as an interplay between visual and logical representation. Concept and parallel questions should be presented diagrammatically, and students allowed to present their responses diagrammatically. Logical reasoning can then 'surround' the pictorial representations of what is discussed. Computer simulation can play a role in this interplay between logical and pictorial representation, especially within the context of *idealised abstraction* which appears to be a difficult area for many students. This is discussed in the next section.

5.8 THE USE OF COMPUTER SIMULATION IN THE SOCRATIC DIALOGUE

It has to be stated at the outset that computer simulation has not been considered in the design of the teaching package - computer simulation would have had to have been an additional factor in the evaluation of the package, and not every teacher involved in the evaluation of the package would have access to a computer network. Nonetheless, computer simulation can play a part in the Socratic method of teaching mechanics because it enables the interaction between student and idealised abstraction.

The difficulty with idealised abstraction is that it cannot be represented physically - e.g. frictionless surfaces, possible worlds of no gravity, etc. Computer simulation can be used as a way of developing the ability to think of 'possible worlds' - e.g. *Interactive Physics*.

Interactive Physics not only presents possible worlds, but the student can interact with them as if the possible world presented is an extension of the real world. Computer simulation enables the analysis of ideal experimental situations which most appropriately **illustrates** a theory (Borghi *et al*, 1984), and enables a 'feeling' for reality while at the same time offering a *natural way to look for assumptions and simplifications which make it possible to explore reality* (Borghi *et al*, 1987). Grayson and McDermott (1996) reported on a computer program that combines the realism of animated motion with the idealizations of a textbook, and stated that the program could provide a useful bridge between *static, simplified textbooks and the dynamic, complicated real world*.

There could be an element of perceived 'contrivance' with the computer simulation of idealised abstraction. Hennessy and O'Shea (1993) have observed the tendency in some secondary school children to attribute 'magic properties' to the computer or deviousness to the programmer if there is a conflict between their expectation and what is observed. The authors hypothesise that attributions of magic are most prevalent when the simulated world is far removed from the student's own experience of the physical world. They argue that further empirical research is needed to establish the circumstances under which idealisation promotes constructive conflict and when it merely ruptures belief in that world. Their paper refers to 12-13 year olds of whom the authors regard as having the ability to distinguish those features of a simulation that map onto reality from those which would be nonsensical in the real world. However, the use of *Interactive Physics* is not to provide 'evidence' as to what would happen under certain circumstances within idealised abstraction, but to encourage students to reason what would happen given such-and-such

circumstances. For example, it is not to show that two stones of unequal weight dropped simultaneously from the same height land at the same time, but to encourage the student to explain *why* that would be the case if air resistance is ignored. If a student were to express disbelief at a simulation of idealised abstraction, then the student has not developed an appropriate schema to explain what is happening - and that would be a cue for the teacher to ask an appropriate concept-question.

5.9 CONCLUSION

The defining-features of the Newtonian concept of force are the laws of motion. The formula-centred approach, required by answering quantitative questions, is mainly a *deductive* process of applying the laws of motion to particular examples. It is mainly a procedure of 'translating' the second law in terms of the question so that the equations of motion can be established for that question. The remainder of what is required is pure-mathematics. This is not to undermine the importance of quantitative questions, but the ability to tackle quantitative questions by no means guarantees the ability to explain phenomena qualitatively in accordance with the laws of motion. Consider a pulley system whereby two unequal masses are connected by an inextensible light string. Many students who have the ability to form and solve two simultaneous equations involving the acceleration of the masses and the tension in the string, will, at the same time, intuitively regard the tension in that portion of the string holding the heavier mass as greater than the tension in the string holding the lighter mass. It appears bizarre that students should hold this intuitive idea at the same time as treating the tension as a constant in the quantitative approach (especially when forming the equations requires one tension), but this merely demonstrates the persistence of intuitive ideas. The dominant-features of motion take precedence over the laws of motion if the laws of motion are treated as a deductive process.

Understanding phenomena qualitatively, according to the laws of motion, is not a deduction process but a *construction* process whereby the student has to build the properties of force as an abstract concept. The student cannot simply generalise the defining-features of force (the laws of motion) to qualitative examples because of the intransigence of the intuitive schema - cognitive reorganisation is required to displace the intuitive schema, hence the need for anomalies that the intuitive schema cannot account for.

It might well be the case that out of all the anomalies presented, the anomaly that becomes the catalyst (the one that produces the facial expression of cognitive conflict) becomes the *exemplar* of one of the laws of motion for that student. The student may begin to understand the Newtonian concept of force, as a way of understanding the physical world, in the context of the exemplar (the exemplar may become one of a few powerful ideas, in the sense given by Wildman, 1981, and Redish, 1994, by which other ideas become organised around a holistic structure). The anomaly of the ball in the train became the exemplar of the first law of motion for the two intransigent students above, and the preceding anomalies that led to the exemplar were subsequently referred to with respect to the exemplar. The exemplar can be employed to build the defining-features of the Newtonian concept of force. The defining-features then become, for the student, common-features that all instances share. The student is able to build the *universality* of the Newtonian concept upon the exemplar as an instance.

For those students who think in terms of a pictorial representation may 'picture' the exemplar, and mentally 'superimpose' the pictorial representation on to the unfamiliar. In whichever way the student thinks of unfamiliar phenomena, either 'pictorially' or

‘logically’ or whatever, the student has to represent the state of affairs by the possible state of affairs as depicted by the axioms of mechanics. The possible state of affairs has a structure that is determined by the axioms of mechanics, and if a student is to successfully model an unfamiliar situation then the student has to construct the model in accordance with the axioms of mechanics. The construction process itself reveals a scaffolding, a framework, that is structured by the axioms of mechanics (we can always judge whether or not someone has correctly modelled a phenomenon); but the constructive process as a cognitive process in-itself may be ‘pictorial’, ‘logical’, ‘verbal’ or whatever the way we ‘think’ as individuals. For example, in consideration of the bomb-and-plane concept-question on page 90, I may reason to myself that since no force is acting horizontally (ignoring air resistance) then the horizontal motion of the bomb remains unaltered. I may, on the other hand, ‘picture’ the situation as:

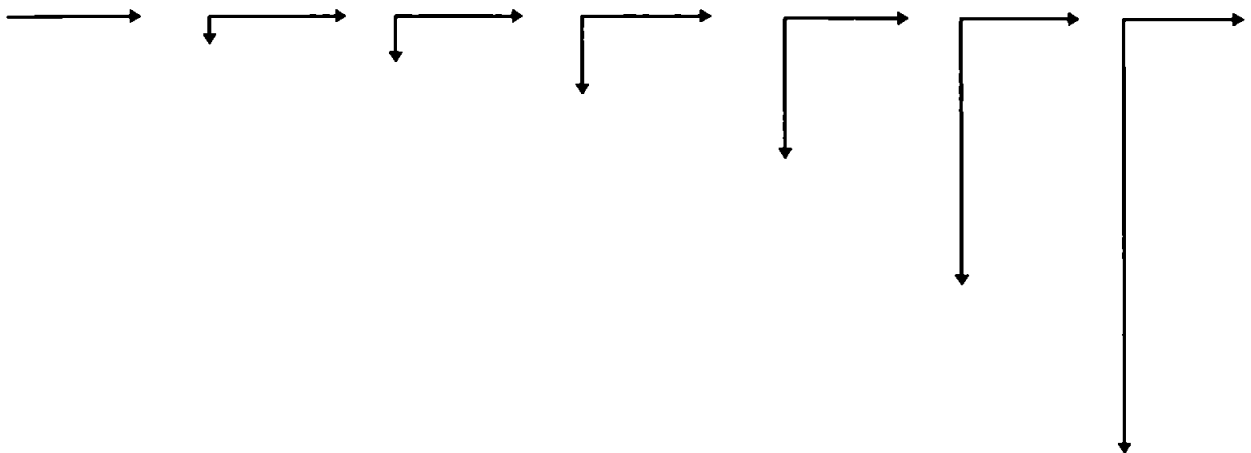


Figure 10 - ‘Picturing’ the Bomb-and-Plane Concept Question

However, in whatever ‘mode’ I cognitise the exemplar that has significance (for me) due to cognitive conflict, it’s very use in my successfully modelling phenomena *universalises* for me the defining features of the laws of motion. A separation is being made here between the **presentation** of the laws of motion, as stated by many text-books, and the idiosyncratic

ways in which we as individuals cognitise these laws. Hestenes (1992), acknowledging Popper, puts forward a tripartite model of three worlds: the *physical world* of things and processes, the *mental world* of thought processes (for Popper, the world of *mental states*) and the *conceptual world* of objective scientific knowledge (for Popper, the world of *objective content of thoughts*). Popper claims that each of the three worlds is autonomous and strictly independent of each other; he further claims that psychological claims are simply irrelevant to the conceptual world (Smith, 1993). So-much-so that psychological claims about the biography of Newton have no purchase on epistemological questions about the explanatory adequacy of the theory (Smith, 1993). For Hestenes (1992), however, there is a constant two-way interaction between the three worlds and states that the following diagram is a constructivist model:

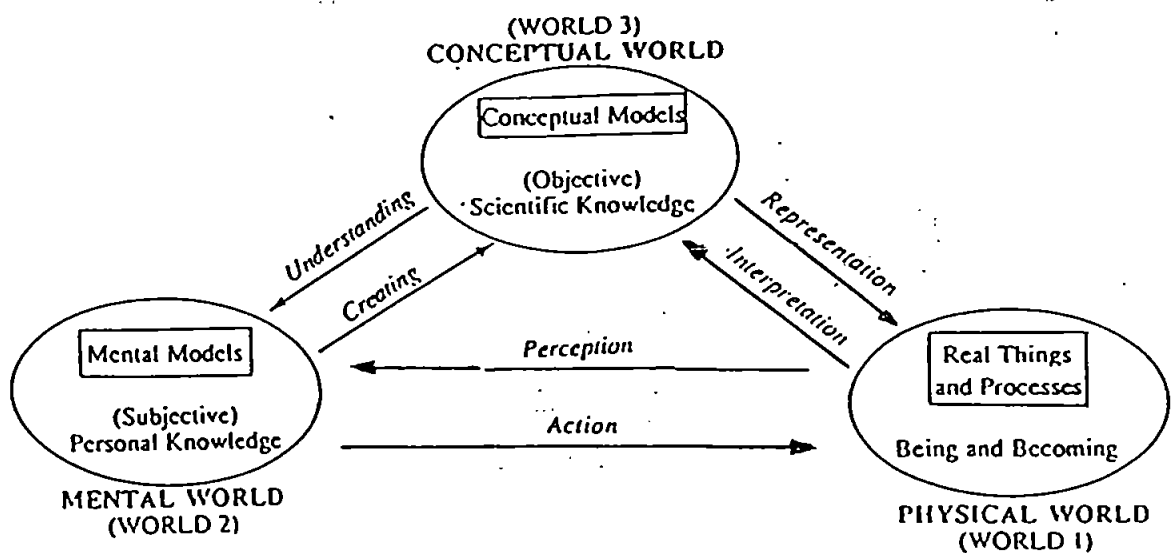


Figure 11 - Hestenes Constructivist Model

Hestenes (1992) states:

To drive the creative process, the student has the advantage of more powerful tools and stronger hints than Newton did. But Newton also had to draw on hints and tools from his predecessors and contemporaries.

Hestenes diagram may be considered constructivist because a part of it represents *subjective personal knowledge* creating and understanding *objective scientific knowledge*. Contrary to Popper's claim that the three worlds are mutually independent, Hestenes (1992) explains the problems Newton faced in overcoming his own misconceptions in the struggle to develop mechanics: **if Newtonian mechanics is a constructivist epistemology then the problems Newton faced must throw light on the way the theory is re-invented in the mind of the student.** The history of science is littered with cases of the same concepts attached to discrepant and sometimes contradictory beliefs, and Piaget claims that children's development is similar in this respect (Smith, 1993). Marton (1986) discovered reasonably clearcut parallels between conceptions found among Swedish teenagers and those in the history of science e.g. Euclid's idea of the 'beam of sight', and of course the Aristotelean idea of force. Stinner's (1994) paper on the brief history of force from Aristotle to Einstein is not meant as a teaching topic in the history of science. The paper *argues that a history-based exposure to the conceptual development of Newtonian mechanics is superior to a conventional textbook-centered approach, because it is contextual, shows the intellectual struggle involved in scientific thinking and relates better to students' knowledge and experience.*

It must be stressed, however, that although a separation can be made between mechanics as stated in text-books and our idiosyncratic way of understanding mechanics (the separation between the conceptual world and the mental world); nevertheless, it may well be the case that what Hestenes labels the *conceptual world* does in fact refer to the 'logical' interpretation of mechanics, and that the *mental world* actually refers to the other forms of interpretation (for example, the pictorial representation). Despite the illustration of any interaction between the two worlds, a separation made between the two worlds still retains

the Platonism inherent in Popper's model⁵. **The conceptual world has been separated from the mental world, but the conceptual world is only the mental world that has become *social*.** If someone is to understand someone else's model, then the model has to be coherent to both. It is the coherence that may leave the impression that the Socratic dialogue is only geared towards the 'logical' thinking; nevertheless, it does leave open the various ways in which we think. What it demands, however, is **coherent** thought!

5.10 SUMMARY - HOW THE THEORETICAL POSITION ADOPTED IN CHAPTERS 2 TO 5 LEADS TO THE ADOPTION OF THE SOCRATIC METHOD OF STRATEGIC QUESTIONING

The Socratic method of strategic questioning should involve discussions that are structured by the teacher according to the target-concept to be reached and the initial knowledge state of the class. The Socratic method should enable the teacher to facilitate the construction of the target-concept by first considering the initial knowledge state and then contradicting that state if it proves necessary. In mechanics the *concept question* asked may elicit an *intuitive schema of force and motion* (determined by how the student perceives the dominant features of the motion presented) that may be challenged by a series of *parallel questions* in which the same principles of mechanics apply. Consistent with the Vygotskian perspective outlined in Section 2.2.2, parallel questions may be seen as exemplifying anomalies that serve a contradictory role of becoming 'hurdles' to overcome in order to develop cognitive growth, yet which also serve as props or hints to facilitate the process.

⁵ According to Chalmers (1978), there is a distinction between our conceptual systems, whether they be scientific theories or those presupposed in everyday language, and the real world to which those real conceptual systems bear some relation. The external world and the world of theories are both real, but they are distinct. They are linked by a third real world of scientific practice. Scientific theories do not inhabit a 'third world of ideas' as Popper or Plato suggest, but are constantly produced and modified as a result of scientific practice. Scientific practice is a result of human interaction with the world, and scientific theory is an expression of that interaction. Scientific theories are linked with and are intended to cope with or accommodate the real world in some sense in a scientific practice. While there is a sharp distinction between real scientific theories on the one hand and the real external world on the other, there is no such sharp distinction between real theories and real scientific practice.

An anomaly is both a diagnostic tool and at the same time a tool of remediation. A teacher frames a question that challenges students to think according to the constraints of the question (e.g., *A ball is at rest on a frictionless table. How would you make the ball move across the table in uniform motion?*). The student response (e.g., *Give the ball a continual push* - revealing, in this example, an 'Aristotelian' response) is then challenged through an anomaly (e.g., *What happens to the motion of a rocket in deep space when its engines exert a continual push?*)⁶. By the asking of such questions, the teacher is asking the class: *Given such and such a situation, what happens and why?* Students have to think according to the parameters set by the conditions imposed, a condition that was described in chapters 3 and 5 as *idealised abstraction*. Idealised abstraction is similar to Nersessian's (1992) understanding of *thought experiment*:

What we do is construct a simulation of a prototypical situation and 'run' it. It is the simulative aspect of the experiment that gives it its empirical force. Although it is not well understood yet just how the simulation process takes place in thought, it does seem that it is the construction and developmental aspects of a thought experiment that gives it applicability to the real world.

An example of a 'prototypical situation' is 'running' (in thought) the scenario of the ball on the train in 'Galileo's example' (see page 146), or the 'running' of a ball on a frictionless surface. By the asking of such questions, a demand is being made on the use of scientific concepts. That use is structured according to idealised abstraction, but may reveal the spontaneous concept (the intuitive counterpart of the scientific one) as the student attempts to answer the question. This is consistent with Vygotsky's emphasis on the need to teach decontextualised concepts because such concepts initially lie outside the experience of the individual and consequently the learning of such concepts develop the ability to reason (see

⁶ If the student doesn't recognise the parallel question as an anomaly, then the student may respond in a way that is consistent with the original response (e.g. *The rocket will move in uniform motion*). An appropriate parallel question is left open for the reader to consider. An example of the way students can develop a consistency and coherence in the defence of their intuitive schema is the three 'Aristotelians' mentioned in section 5.6 (page 145). The transcript that traces the development of their consistency is given in section 7.3 (page 192).

page 50 - 53). To have to think according to the parameters set by the conditions imposed requires a move from thinking in the concrete to thinking in the abstract.

Idealised abstraction, structured by concept questions referring to the Newtonian system, together with the initial knowledge state of the student, as revealed by the response to the concept question, **comprises the student's *zone of proximal development*** as outlined in Chapter 2. In the context of strategic questioning in the teaching of mechanics, the student's ZPD may be defined as the difference between the Newtonian explanation of a concept question presented by the teacher (the 'target concept') and the intuitive response to the question. Parallel questions serve as props and hints to facilitate the construction of the Newtonian system in its abstract form (e.g. the first law of motion as applied to ice pucks moving on frictionless surfaces - a situation that could never exist in the real world) - a form that requires thinking **in the abstract**.

To summarise: the theoretical position developed in chapters 2 to 5 has led to the adoption of the Socratic method of strategic questioning because:

- A qualitative understanding of the world that is in accord with the Newtonian system cannot be constructed by just making sense of experience. The Newtonian system is a way of describing the physical world that is structured according to a set of axioms that define force. Newtonian mechanics constitutes knowledge of the world, not of subjective experience.
- To explain the world qualitatively, the student has to construct a possible state of affairs as a representation of a physical state of affairs to be modelled. This possible state of affairs was described in chapters 3 and 5 as *idealised abstraction*.

- Contrary to the positivist position, the laws of motion are not empirical generalities. Consequently, students cannot construct *force* as an abstract Newtonian concept from a practical 'hands-on' experience.
- To facilitate the construction of the Newtonian system, the teacher has to realise the initial knowledge state of students (as well as the target concept to be reached). Concept and parallel questions are not only diagnostic tools but also tools of remediation.

If the Newtonian concept of force is to be understood in a way that would enable the student to account for unfamiliar phenomena qualitatively, rather than as a rule-centred approach to quantitative questions, then the intuitive schema has to undergo a massive transformation in order to adapt to the new concept (a 'gestalt-shift'). However, this transformation or gestalt-shift is not so much a change of 'world-view' or some form of 'paradigmatic' change. It isn't so much the case that the intuitive schema is formed by the student so that the student can make sense of the world as a general 'outlook': Sections 5.2 and 5.3 argued that it is more the case that the intuitive schema is formed as the student tries to make sense of the separate physical phenomena under consideration. Once formed, the schema is therefore reluctant to change. To change the schema, the student has to construct the defining features of the Newtonian concept and this will not happen unless the student sees the need to do so. If a student realises that his or her schema is inadequate in accounting for a phenomenon, then change is possible. If a teacher presents an anomaly to student reasoning in mechanics, then the anomaly may contain the possibility of rendering the alternative concept impotent in its account of the anomaly and thus allow for the facilitation of the construction of the Newtonian system: *Learning a new conceptual structure involves more than creating dissatisfaction with existing representations. It includes this and active construction of new representations* (Nersessian, 1992; author's

emphasis). This dual aspect of anomalies - as a challenge to misconceptions and as cues, props or hints in the facilitation of learning the Newtonian system, is what Hestenes (1992) refers to as the *modelling game*.

The student's response to a concept question is often determined by the perceived dominant features of the example presented (e.g. for a ball to overcome gravity, then there must be a upward force to overcome the force of gravity). The parallel question is an anomaly to the student response. It is not the drowning of preconceptions in a sea of anomalies, nor is it the transformation of the motion schema 'a force produces motion' to 'a force produces acceleration' with the re-conceptualisation of the motion slot (as argued by von Glasersfeld, see page 68). It is:

- 1) the raising of a qualitative example that the student has to consider within the context of idealised abstraction,
- 2) challenging the student's spontaneous and intuitive response with a parallel question - a question that is specific to the student's response yet has the same explanation as the original concept question within the Newtonian system.

A series of concept and parallel questions facilitates the construction of the Newtonian system within the student's zone of proximal development because each question demands consistent reasoning: that demand challenges the student's cognitive state as revealed by his or her previous response. The target concept (as expressed by a concept question) stands at one end of the ZPD, and spontaneous concepts stand at the other. Parallel questions stand in between. Concept and parallel questions must not be seen as using and working with spontaneous concepts. They should, instead, be seen as a challenge to spontaneous concepts, and this would be consistent with Vygotsky's point that scientific concepts start their life in the student's mind at the level that his or her spontaneous concepts reach only later (see page 50).

CHAPTER 6 - THE RESEARCH METHODOLOGY THAT WAS ADOPTED BY THE THESIS

6.1 INTRODUCTION

The aim of this thesis is to create a teaching package that would enable the teacher to facilitate a qualitative understanding of force and motion. The explicit aim of the research proposal was to investigate how student misconceptions regarding force and motion can be challenged and overcome through the use of concept questions. The research proposal's plan of work included the development of a bank of concept questions, small scale trials of the concept questions followed by a larger scale evaluation. A bank of concept questions was formed, and the 'small scale trials' of the concept questions consisted in the pilot study and the follow up of some of the results of the study with informal interviews with sixth formers. The larger scale evaluation consisted in the construction and evaluation of the teaching package.

To evaluate the effectiveness of strategic questioning, the method comprised of:

- Drawing up a plan of concept questions, anticipated responses and parallel questions.
- Putting the plan into action with the pilot-study.
- Following through some of the results of the pilot study with interviews with sixth formers.
- Constructing the teaching package, and asking schools to implement the package, so that the package can be empirically evaluated as a quasi-experiment by pre- and post-testing a set of control and a set of experimental groups.

The first 6 hours of the pilot study was videotaped and a transcript made. Not all the discussions in the pilot study went according to plan (a certain amount of 'meandering' was allowed - see the details of the pilot study in chapter 7). However, what were considered to be the 'highlights' of the 6 hours were put into a schematic form (the teaching package) - the 'essence' of the highlights was reconstructed into a teaching package.

What I considered to be the 'highlights' of the 6 hours comprised of discussions whereby I attempted to contradict incorrect student reasoning. The purpose of the pilot study was not so much to provide evidence for the existence of cognitive conflict (nor was it to provide evidence for schema displacement¹), but rather to evaluate the effectiveness of certain questions. However, it is by observing body language (for example, facial expressions of cognitive conflict), together with noting the intuitive response, that the effectiveness of certain questions can be evaluated. Interviewing sixth formers and noting their body language (*intra-observer triangulation*), for example, suggested that the sequences that eventually led to the construction of unit 3 of the teaching package could induce cognitive conflict. By noting body language, it is possible to ascertain whether cognitive conflict has taken place. However, the displacement of the intuitive schema may not occur at a particular time but may take place over a period of some duration. This raises the following important question: how do we know that the intuitive schema has been displaced?

The aim of the teaching package was to discover if there were any *forms of reasoning* that may have been evoked by asking a series of questions. The problem is the lack of generalisability of the pilot study - the effect of one concept question on one class may be different on another class. However, the aim of the pilot study was not only to take note of the *kind* of response but also to see if there were patterns of reasoning in defence of

¹ There can be no 'evidence' of schema displacement, apart from what can be inferred from student reasoning.

intuitive schemata. Some discussions were allowed to meander just to see where the discussion lead and, subsequently, to take note of any possible pattern that may be of relevance. The idea is that if 'misconceptions' are resilient to change, then the pilot study might reveal ways in which misconceptions are defended (the ways in which anomalies can be incorporated into the intuitive schema instantiated) and what questions promote cognitive conflict. The next section discusses the findings of the pilot study; the subsequent section explains the research methodology employed.

The pilot study did reveal a pattern of reasoning:

- *Some students can develop their personal and intuitive schemata of force and motion with remarkable consistency and coherence, until an anomaly contradicts the ontological status of force.*
- *The greater the entrenchment, the greater the cognitive conflict necessary.*

Both patterns of reasoning will be fully elaborated in the next section, suffice to say that the consideration of both patterns of reasoning helped form unit 3 of the teaching package. Unit 3 is the most important unit, and is the one designed specifically to promote cognitive conflict. An occurrence of cognitive conflict does not necessarily mean that the intuitive schema has been displaced (there may follow a period of incubation). However, the package was designed in such a way that by the time the last unit, unit 13, was completed by the class, the intuitive schema would have been hopefully displaced. Units 1 and 2 were 'preparatory' units for unit 3. The subsequent units followed on from, and were structured by, unit 3. An incubation period could, of course, exceed a two to three week coverage of the package, but that also equally applies to a control group that uses concept questions. The objective of the package was to show that within a restricted time period, strategic questioning is more effective than a less structured use of concept questions. According to Hestenes (1992), when critical misconceptions are dealt with, most other misconceptions

tend to fade away without further intervention. The aim of the package is to enable the teacher to develop a qualitative understanding of force and motion sufficient to enable the students to understand unfamiliar situations as presented in quantitative modelling.

6.2 WHAT WAS LOOKED FOR IN THE PILOT STUDY

Nersessian (1992) gives an outline of what precisely is required in the conceptual change from 'seeing that' to 'seeing as' - that changing existing representations requires the construction of new concepts (that bear the same label, i.e. 'force') that are worked into a quite different representation of the phenomena:

*In Newtonian mechanics 'motion' is a state in which bodies remain unless acted upon by a force. 'Rest' and 'motion' have the same ontological status: they are both states. Like rest, motion per se does not need to be explained, only changes of motion. 'Force' is a functional quantity that explains changes in motion. Newtonian forces are relations between two or more bodies. Students, however, conceive of 'motion' as a process that bodies undergo and believe that all motion needs an explanation. They conceive of 'force' as some kind of power imparted to a body by an agent or another body. This makes 'force' ontologically a property or perhaps even an entity, but not a relationship.....We can see from the example that changing a student's beliefs will take more than just introducing them to the new relations among existing concepts. Teachers need to be aware that when they use scientific language, students may already have different representations associated with the same words, as we saw for 'motion' and 'force'. Changing existing representations requires that they be taught how to construct the new concepts and work these into a quite different representation of the phenomena. We can also see that calling this process 'restructuring' is potentially quite misleading. It makes it seem like the elements of the conceptual structure are fixed and all that is required is to rearrange these elements, as one would the furniture in a room. However, **learning a scientific conceptual structure requires more than rearranging existing elements and also more than fitting new facts into an existing framework: As discussed above, it requires constructing new concepts and working them into a new framework. I, thus, prefer to refer to this process as 'conceptual change'** [Nersessian (1992); author's bold emphasis, my underline].*

Conceptual change in force and motion consists in the change from conceiving 'force' as an ontological property of the body (characterised as the Aristotelian / medieval conception) to *force* as a relation between two bodies. The Gestalt switch from 'seeing that' to 'seeing as' is a switch from the ontological status of force (force as a property that maintains motion), to force as a functional quantity that explains changes in motion. What the transcripts of the pilot-study reveal is that students who are able successfully to defend

their arguments, that is, who respond to parallel questioning with a consistency to their original response, are in fact able to defend with consistent reasoning the ontological status of force. The displacement of the intuitive schema of force and motion occurs when the ontological status of force is exchanged for force as a functional relation between two bodies that changes the state of motion of the two bodies. What this implies, and has in fact been verified by the transcripts, is that a parallel question has the potential to create cognitive conflict if the question undermines the ontological status of force. If the student doesn't recognise the parallel question as an anomaly and is able to respond in a consistent manner, then the chances are that the parallel question has not undermined the student's ontological status of force. For example, 'Galileo's strategy' of the ball on the train (see page 145) undermined the 'three Aristotelians' developing argument that force is a property of the body because it revealed the prototypical situation of uniform motion with the total absence of any component of force in the direction of motion. To ask what force acts on a ball that has been projected in deep space does not challenge the initial response that a force is required to push a vertically thrown ball on earth. Similarly, to ask to throw a ball and then to place your hands in your pockets does not challenge the ontological status of force - many students often reply that the force from the hand is given to the ball. 'Galileo's strategy' does conflict with the ontological status of force because the concept question challenges the student to construct a simulation of a prototypical situation and 'run' it - **contrary to any possible expectation that a force is required to maintain the horizontal uniform motion of the ball on the train.** Through idealised abstraction (Nersessian's *thought experiment*), the student is asked to construct a 'scenario' in his mind such that 'running-it' will be contrary to the student's ontological status of force. Cognitive conflict seems to be the psychological correlate to the conflict between the ontological status of force and force as a relation.

Not all of the Socratic dialogue in the six hours of the pilot study consisted of strategic questioning. The following discussion took place in the pilot study (as reported in the transcript) but was not considered in the construction of the teaching package:

Student 1 *In space, a projectile's motion is a straight line. So why does it follow the path that it does on earth?*

Student 2 *Because of gravity! Because of force!*

Teacher *The problem with projectiles is of motion in two dimensions. Last week we considered a ball thrown vertically upwards.....*

Student 3 (one of the 'Aristotelian' students) *Doesn't that prove the point! In both cases (ball thrown in space and ball thrown on earth) the mass is holding the force.*

Teacher *You are saying this with such conviction.*

Student 3 *Doesn't it prove the point that uniform motion deep in space means that the mass is holding the force (this was the student who stated, only moments previously: But I would argue that it would eventually slow down).*

[Large show of hands]

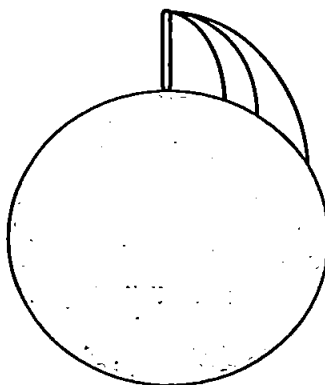
Student 4 (the diver) *But, if that is the case, if you are high enough and you throw a projectile hard enough, it will follow a circular orbit around the earth. It will just keep going round.....*

Student 3 (Same Aristotelian student, sitting in the front seat and turning round to the diver at the back, interrupts defensively in a slightly raised voice) *The satellite is held in its orbit, it is held in its trajectory.....*

Student 4 (the diver) *How is it held?*

Student 3 *I can't give you an explanation, but it is!*

Teacher *We will come onto circular motion at a later date. But consider these three cases (following diagram on board):*



[Large show of hands]

Student 5 (a student who has not spoken before) *If you throw a ball and the force stops.....*

Teacher *You mean the force 'pushing the ball'?*

Student 5 *Yes! If there is no such force then what makes it go to the top of the arc before gravity takes over?*

[I explain the Aristotelian position, with nods of agreement from the three 'Aristotelian' students]

Student 6 (a student who has not spoken before) *Surely you can test if there is a force in the ball, because if you throw it, then if there is a force in the ball, then it would take longer to land if you were to just drop it.*

Teacher *This would be a good experiment. If you throw a ball horizontally and dropped a ball at the same time, which ball would land first?*

The whole point of the discussion was to ascertain what forces act on a vertically thrown ball! This is an example of a discussion 'meandering': forces acting on a vertically thrown ball → satellite motion → projectiles etc. There are two main reasons as to why this part of the discussion was not considered for the teaching package:

- Although the discussion went the way that it did, it does not necessarily follow that the discussion will always go this way with different classes with different teachers.
- With reference to the above diagram drawn on the board, my reasoning consisted in asking what would happen if I increased the speed of projection each time, including the question as to what would happen if there was no gravity. This form of reasoning is similar to the Socratic approach in the *Meno*, and may be a successful strategy in modelling circular motion, but it was not successful in promoting the cognitive conflict necessary in displacing the intuitive schema of force and motion.

What was the criteria for the judgement that this part of the discussion did not promote cognitive conflict? The form of the discussion seemed to suggest that students do not readily connect satellite motion with a change in linear motion² (nor did they readily connect the horizontal throwing / dropping concept question with the forces acting on a vertically thrown ball. In fact, student 5 referred back to the forces acting on a vertically thrown ball without reference to the subsequent examples). After the lesson, some students asked me to go over the example again (without reference to the forces acting on a vertically thrown ball - they seemed more interested in the notion that a satellite is in free-fall).

The reasoning that I used in the example was: *If I projected a number of stones horizontally from a top of a tower, each time with increasing speed, what will eventually*

² Force as the agent that is central to both

happen? This form of reasoning does not challenge the ontological status of force! Consequently, students do not readily connect the example with the issue at hand (the forces acting on a vertically thrown ball) and hence cognitive conflict did not seem to be evident. The example could have been included in the package, for example, concept questions involving force as that which changes the direction of motion. It was excluded because it was felt that if students understood force as a *relation*, as representing the action of another body that changes the state of motion of the body under question, for example unit 3, then the motion of a satellite could be more readily assimilated under that relation. This is consistent with Graham's (1991) *Force Hierarchy* which places force in one dimension prior to force in two dimensions.

Many more concept questions could have been included in the teaching package. However, there was a danger that with so many concept questions the package would be too cumbersome and the teacher 'put-off' the project. The package is, in one sense, a 'skeletal-form' or 'rational essence' of the transcript; with additional questions included so as to 'complete the picture' in the introduction to force and motion (equilibrium and changes in motion). The major criterion for choosing the experimental groups (as well as the control groups) was the teacher's use of concept questions - it was assumed that prior familiarity with concept questions meant that an exhaustive list of concept and parallel questions was not necessary. The decision as to what concept and parallel questions to include in the package was based upon:

- 1 those questions that led to cognitive conflict and seemed central in understanding force and motion (namely, unit 3),
- 2 those questions that could lead onto, or follow from, unit 3, and
- 3, those questions that could be included with the rest so as to 'complete the picture' of equilibrium and changes in motion. For further details see chapter 7.

Those parts of the transcript involved in the making of unit 3 suggests that a student's ZPD may regress from the target-concept as a result of the response to mediation. With each parallel question, a student may attempt to construct a coherence to the intuitive schema instantiated in response to the initial concept question: *students construct new understandings in the face of efforts to contradict their current ideas, but they do not rush to embrace new viewpoints* (Burbules and Linn, 1988). What the 'ideal' suggests that if no regression takes place, then the Gestalt-shift would be a fairly 'smooth' and straightforward process: if regression did occur then a 'catalyst' would be required to untablise the intuitive schema. The scenario of the 'three Aristotelians' in section 5.6 and 7.3 suggests the law: *The greater the entrenchment, the greater the cognitive conflict necessary* (further research could verify the law. For example, the use of *inter-observer agreement triangulation* - see page 178). *Regression* is sometimes referred to in the context of peer collaboration (e.g. Tudge, 1990): a less capable peer can direct a more capable peer towards regression. However, regression in the context of 'entrenchment' may still develop reasoning skills that are not defective. For example, a group of students who agree upon an answer to a concept question and subsequently manage to be consistent in their responses to the parallel questions. Such an example can be found on pages 196 and 197. **I would contend, however, that there will always be an appropriate parallel question that would manifest a contradiction in the reasoning that has developed up to the point where an appropriate parallel question is required.** To argue otherwise is to contend that intuitive ideas could be more consistent and coherent than the Newtonian system, the point being that, at the end of the day, it is the teacher's experience rather than the superiority of the Newtonian system alone that will determine the outcome of a discussion. If, after a period of consistent reasoning by the students, the teacher manages to raise an anomaly that is in direct contradiction to their reasoning (for example: *If, as you say, the*

reaction to the bottle's weight is the normal reaction of the table as according to the third law of motion, then what is the reaction to the bottle's weight under free-fall?) and presses the point home, then at least the students would be aware that the anomaly exists. The greater the entrenchment suggests the greater the cognitive conflict necessary: but if reasoning skills have also developed, then the anomaly may have a greater significance if the reasoning skills develop along a path that has become (or is in the process of becoming) separated from the intent to win the argument. Classroom observation has revealed what seem to be facial expressions of surprise or perplexity as students immediately recognise that there is a contradiction between their argument and the example raised by the teacher (a *critical event*). This does not mean, however, that all students will immediately and automatically resolve the contradiction: given that students do not rush to embrace new viewpoints, there may follow an 'incubation' period whereby the contradiction is resolved by either embracing the Newtonian explanation or becoming further entrenched. The transcript of the pilot study involved in the making of unit 3 of the teaching package (see page 192), reveals that the concept question of the force acting on a vertically thrown ball was not resolved within one lesson.

No triangulation method took place in the classroom. A video recording was made of the class and a transcript made; however, the field of view of the camcorder was fixed (there was no 'cameraman') which did restrict subsequent discussions concerning 'classroom dynamics' or body language. However, there was inter-agreement between the supervisors involved and myself as to the critical events that seemed to have promoted cognitive conflict. These critical events of the pilot-study (for example, the drum problem of unit 2, and the law of entrenchment (see page 215) with 'Galileo's example' (unit 3) were verified by informal interviews with sixth-formers. However, these interviews were not conducted under clinical conditions but merely served to verify what we assumed to be the case. A

future research programme could employ clinical interviews to establish, in the form of a law, those conditions that prompt critical events in the classroom. It may be possible, for example, to create the conditions that prompts the 'misconception' so that we can begin to understand more clearly the formulations that are used in holding the misconception and how the misconception is linked to other forms of reasoning. It should then be possible to replicate the conditions so that generalisability can be established. However, the aim of the research project was not to formulate laws of reasoning in mechanics but to create a teaching package and to evaluate the effectiveness of the package (as outlined in the research proposal).

Students seem to have a 'natural' tendency to give an ontological response. Viennot's *force energy mixing* may be due to students thinking that the point of the question is to see if the motion can be accounted for in some way (i.e. to account for the motion rather than the force as actually demanded by the question) - if force is an ontological property, then why not re-label this property 'energy', or 'speed', 'pressure', 'momentum' etc. The problem is, we cannot get inside the student's head (which also makes it difficult to ascertain whether the intuitive schema has been displaced)! Nor do we know how we can run a prototypical situation in a way that gives it its empirical force (Nersessian, 1992). What thought processes are involved is an area of further research. However, it is by noting the responses to a series of parallel questions that we can begin to see, not so much a series or classification of what responses there are, **but how the responses change according to the questions**. The purpose of the pilot study was not to observe behavioural characteristics of the students, but to record their reasoning (and to see the effect of some of the questions). The teaching package was constructed according to the logic of their reasoning (and the mechanics constructed according to the laws of motion), and was verified by informal interview. It must be stressed, however, that **inter-observer**

triangulation³ is important in verifying critical events such as cognitive conflict, but it was felt that the emphasis at this stage was to get the teaching package underway rather than formulating laws of student reasoning.

The main research method involved in the pilot study (and in the moments pilot study), was *intra-observer triangulation*⁴ involving informal interviews with sixth-formers. Advantage was taken of the many sixth-form mechanics workshops held at the university or presented at the home school. The interviews with sixth-formers verified the law of entrenchment: if 'Galileo's strategy' is asked first, then nearly all the (fresh mechanics) students interviewed would answer incorrectly the concept question of the ball thrown vertically upward. If however, the first question asked was of the ball thrown vertically upward, and the series of parallel questions lead to 'Galileo's strategy' (as outlined in unit 3), then there were expressions of cognitive conflict (or what seemed to be expressions of cognitive conflict).⁵ In other words, the effectiveness of Galileo's strategy is dependent on when it is asked in the discussion in relation to the other concept questions. Through the use of intra-observer triangulation in the moments pilot-study, it became evident that many students expect a beam-balance that is in equilibrium to be horizontal or returning to the horizontal (hence suggesting that many students see the horizontal as the natural orientation for equilibrium).

³ Triangulation in classroom research involves checking the observations of more than one observer (Wragg, 1994). An example of inter-observer triangulation in verifying a critical event such as cognitive conflict would be two observers simultaneously observing a mechanics class. The two observers would record instances of cognitive conflict and compare notes afterwards

⁴ The intra-observer triangulation involved myself evoking (and hence observing) the same event at different occasions.

⁵ In many cases there would be an 'immediate silence,' the facial expression would look perplexed, but the correct answer would subsequently be given. Once it was established that force is not required to maintain motion, only to change it, then I would then follow up with asking the original concept question of unit 3 for the second time. If a prompt was required at this stage, then I would ask to describe the motion of the ball and where the forces acting on the ball were coming from.

6.3 REVIEW OF QUANTITATIVE METHODS WITH RESPECT TO THE TEACHING PACKAGE

The use of quantitative methods in the evaluation of the teaching package was to verify that the use of the package would significantly improve student understanding of force and motion. The teaching groups involved in the evaluation of the package were not randomly selected; consequently, the methodology was a *quasi-experiment* whereby variables that were inadequately controlled were, hopefully, accounted for. (For details of the quasi-experiment see chapter 8.) If there were a significant difference between the experimental groups compared with the control groups, then the question of *internal validity* and *external validity* may have to be scrutinised [although, for Hammersley (1992), to talk of the distinction between internal and external validity is misleading as the findings of a study are either valid or they are not]. Of the threats to internal validity [as listed by, for example, Cohen and Manion (1994)], *statistical regression* and *testing* seem to be the most serious.

According to Dolan (1978), statistical regression can occur if a number of subjects perform on an 'off-day', or if subjects are assigned to a wrong group. However, the former may be one of many extraneous factors unique to each control or experimental group. The latter is not applicable and the former would be applicable if the pre- and post-tests pertained to some skill rather than spontaneous and intuitive responses. For Cohen and Manion (1994):

Regression means, simply, that subjects scoring highest on a pre-test are likely to score relatively lower on a post-test; conversely, those scoring lowest on a pre-test are likely to score relatively higher on a post-test. In short, in pre-test - post-test situations, there is regression to the mean. Regression effects can lead the educational researcher mistakenly to attribute post-test gains and losses to low scoring and high scoring respectively.

According to Wragg (1994), the usual attempt to take a measure so that the initial differences, such as pre-testing, can be 'evened up' is almost always controversial.

The pre-tests may not only have caused statistical regression, but also the *testing* itself could be a threat to internal validity. *Testing* is a threat because the pre-tests may have sensitised groups to produce higher scores in the post-test measures: this was certainly possible given that each question in the pre-test had its counterpart, with the same question number, in the post-test. In other words, the pre-tests may have become a *concomitant variable* (sometimes known as a *covariate*). The pre-tests may therefore have been an extraneous factor operating in the study. An *analysis of covariance* was used to try and even out the effects of the initial differences of the pre-tests.

Initially, an ANOVA was performed on the post-test results, revealing no significant difference in the results. Subsequently, however, an ANCOVA (see page 307) was performed to see if there were any effects of the pre-tests on the post-tests (if an ANOVA alone was performed, then there would have been the possibility of committing a *type II error* since there was no significant difference in the post-test scores).

Threats to external validity apparently threaten the degree to which generalisations can be made from the settings of the quasi-experiment to other mechanics classes. One possible threat, on two counts, is the *Hawthorn effect*. According to Wragg (1994):

A great deal of the research which has compared two methods of teaching has revealed no general difference between the two, because different teachers are able to make various methods work effectively if they believe in them, or fail miserably if they do not (my emphasis).

The enthusiasm of the experimenters and the greater attention paid to the subjects is referred to by Wragg (1994) as the Hawthorn effect. Another example of the Hawthorn effect that may be relevant to the project, given by Cohen and Manion (1994) and Dolan

(1978), is the effect of subjects being aware that they are participating in an experiment and behaving accordingly. However, even if the results were significant, then the Hawthorn effect would not have been applicable as, on the one hand, the objective of the quasi-experiment was to determine any significant difference, but on the other hand, **the aim should have been to create a difference.** The role of the teachers participating in the experimental groups was to create a difference such that their enthusiasm would have been expected (ironically, Wragg reports that the major 'difficulty' with action research is that the control groups rarely ever 'win' due to the enthusiasm of the teachers involved with the experimental groups. This is not to imply, however, that the teachers involved in the experimental groups were not enthusiastic - see Section 8.3).

6.4 THE REASON AND METHODOLOGY BEHIND THE HIERARCHICAL MODEL OF STUDENT UNDERSTANDING OF MOMENTS OF FORCES

In contrast with the many hundreds of research papers that have reported student misconceptions of force and motion, there has been nothing in the literature that has identified misconceptions of moments. The first part of this thesis is the formation and evaluation of the teaching package. However, the second part of the thesis is the formation of a hierarchical model of student conceptual understanding of moments (see page 22): an essential prerequisite for any consideration as to the appropriate teaching strategy involving moments. One of the aims of the second part of the thesis was to see whether the Socratic method of strategic questioning would be an appropriate strategy. One of the characteristics of the hierarchical model is that many students regard the horizontal as a natural orientation for equilibrium, and informal interviews with sixth-formers suggested that the presentation of anomalies (for example, beams in equilibrium that do not return to the horizontal) may

not be enough to challenge misconceptions⁶. However, the results of the hierarchical model do suggest the use of the Socratic method as an appropriate teaching strategy, but in quite a different way compared to its use in the teaching of force and motion (see page 270). The possible criticisms of this method are [adapted from Noss *et al*'s (1989) criticism of Hart's (1981) CSMS project]:

- *A hierarchical model implies the Piagetian notion of a series of stages which represent some kind of 'natural' pattern of progress in learning* [However, although a hierarchical model does imply the subject as an ordered hierarchy, nevertheless such a hierarchy exists on a 'local level' - within the topic of moments - reflecting a structure which makes up that particular domain of mechanics. **This is not to imply that students have to necessarily progress through the hierarchy in order to achieve an understanding of that domain.** Such a hierarchy reflects more the structure of the domain than it does the psychological processes involved in understanding the domain, although we can expect a more complex level in the hierarchy to be more difficult to understand].
- *The hierarchy may reflect topic exposure, or rather the differentiation in topic coverage between schools, than it does the conceptual difficulties ('stumbling blocks') that students may have with the various aspects of the subject* [The majority of classes had yielded a large spread of results within each class. Additionally, many of the questions were conceptual rather than quantitative, so it is most likely that many of the questions were unfamiliar to the students].

For the purpose of developing a Socratic approach in the teaching of moments, the methodology can make several important contributions. For example, understanding level

⁶ On one occasion, a student recognised that a given centrally pivoted beam that was not horizontal was in equilibrium, and reasoned so using the principle of moments - **yet at the same time was 'convinced' that the beam should nevertheless still return to the horizontal.**

3 questions of the hierarchy can be facilitated by the teacher referring to the question's counterpart in level 2. Details are given in Section 8.9.

CHAPTER 7 - THE FORMATION OF THE TEACHING PACKAGE: THE PILOT-STUDY

7.1 INTRODUCTION

Chapter 6 examines the student responses to concept and parallel questions as recorded in the video transcripts of the pilot-study. The pilot-study is an evaluation of strategic questioning. The objective is to assess the effects of parallel questions by considering student responses. The aim is to create the teaching package based on the kind of responses students gave to concept questions and the parallel questions that were successful in promoting cognitive conflict. The structure of the teaching package is determined by the responses of the students (with consideration given to misconceptions that have been reported in the literature, for example: Graham and Berry, 1990, 1992) together with a critical evaluation of the concept and parallel questions that were involved with the responses. The teaching package consists of 13 units and will be referred to throughout chapter 6, and appears in appendix A, page 281.

According to chapter 5, it would appear that, for most students, the intuitive schema of force and motion will not be formed **until** the student instantiates a schema of motion to qualitatively explain motion in terms of force. The schema of motion is situation specific (Champagne *et al*, 1982; Viennot, 1985) whereby the student will account for the motion with reference to the motion's *dominant features* (see chapter 5). If a student answers a force concept question with reference to a phenomenon, then a schema of motion may be instantiated with 'force' as one of the slots: the slotting in of force will largely depend on the way the student conceives of the dominant features of the motion. Once formed, the intuitive schema of force and motion may become resilient to change - the newly formed

schema has made 'sense' of the world, and has required the mental effort in explaining the phenomenon. Consequently, an anomaly may not be enough for a student to have to abandon something that has made sense in order to understand something that doesn't. An anomaly may be 'compartmentalised' or assimilated idiosyncratically into the intuitive schema. A series of well-chosen anomalies, on the other hand, may become a threat to the stability of the existing schema. However, the existing schema may be so intransigent that the student actually develops the schema's coherence in it's account of the anomalies (the anomalies fail to become anomalies, so to speak!). If an intuitive schema manages to resolve a contradiction, then the schema has become *reinforced*! Resolving a contradiction means that there is no longer a contradiction for the student - the schema becomes more coherent and 'stronger' and hence more resilient. This is consistent with Vygotsky's findings that higher mental functions develop through contact with scientific concepts (see chapter 2, section 2.2.1). By considering a concept question, a student may have formed an intuitive schema of force and motion; but in the process of considering a series of related concept questions, the student may form the reasoning skills necessary to maintain the intuitive schema intact and free from apparent contradiction - the student may form the ability to use the intuitive schema divorced from the immediate example that led to the creation of the schema in the first place. The aim of the Socratic method (in the sense of Hestenes' *modified Socratic dialogue*) is for the class to arrive at the target concept via the process of answering a series of questions, but the process involves answering questions as anomalies to the intuitive schema instantiated to answer the questions. Strategic questioning requires consideration, by the teacher, of the schemata currently held and the target concept to be reached. However, the structure of the questioning should be such that the anomalies cause the intuitive schema to become unstable rather than creating the opposite effect of reinforcing the schema.

In the pilot-study, a list of student responses to the concept questions was anticipated. A structure to the discussions was designed, but the anticipation of how the discussion would actually develop was a matter of speculation. The consideration of student responses whilst actually teaching required much 'thinking on one's feet' (in fact, at one stage, I was at a loss to contradict the coherent reasoning of two students. See page 196), and consequently the discussions did have some tendency to 'meander.' The dialogues appearing later in this chapter show how 'meandering' can lead to the reinforcement of intuitive ideas. However, much insight into student reasoning was gained. In order to reveal the actual structure of the discussions, large sections rather than small passages of the discussions are quoted. Small passages may have been taken out of the full context in which it is placed, and so the context of the questions raised may not be appreciated and any coherence of intuitive schemata could be overlooked. Since not all student responses were anticipated, a choice always had to be made between keeping to the structure as planned or to pursue student reasoning in the hope of obtaining further insight into intuitive reasoning. Hence a certain degree of 'meandering' was allowed.

Despite the references made in chapter 5 to the class arriving at the target-concept through logical reasoning, I had found myself giving definitions - such as 'force is that which changes motion'. This was due to my own inexperience of the Socratic dialogue and my habit of didactic teaching. I had also illustrated the definitions with examples - such as my hand, throwing a bag of rubbish, changing the motion of the rubbish whilst it is still in contact with my hand. In fact, on many occasions, I have unwittingly followed Howard's advice of supplying the appropriate schema to account for the anomaly (see page 151) - advice which is criticised on page 152. Nevertheless, the intransigence of intuitive ideas were such that they still remained intact despite the definitions that were given. I had used rhetoric in the dialogues; for example:

Student 2. *Why does the book feel heavier when you push it up?*

Teacher. *Good question!.....* (page 188),

and although the use of rhetoric in Phillips' (1979) sense was permitted (see chapter 5, page 153), nevertheless a guard was made against 'arguing from authority'.

The following example is a framework of the first lesson plan:

Class discussion: Define force.

Aristotle's idea of force given: Aristotle's 'three laws of motion' as stated in the video *Mechanics in Action* (Mechanics in Action Project, University of Leeds. The video was not shown to the class).

Question: *If I stop pushing a car that has broken down, what happens?*

Newton's idea of force given - *force changes motion rather than maintains motion.*

Question: *A ball is thrown from a spaceship in deep space. Describe the motion of the ball.*

Question: *The same ball is thrown up in the air on Earth. What are the forces acting on the ball?*

Question: Parallel question of the car braking.

Question: *What is meant by a change in motion?*

Examples of force (taxonomy).

Warren (1979) warns of defining acceleration as a change in speed. However, it was necessary to define a change in motion as a change in speed because the students had to firstly appreciate force as that which changes speed as opposed to maintaining speed. The vector nature of acceleration had to be given later. Ideally, kinematics should be taught prior to dynamics; however, the timing of the pilot-study was such that this was not possible.

7.2 WHAT IS FORCE? THE MAKING OF UNIT 1 (AND 2).

At the beginning of the first session, the students were given five minutes to discuss amongst themselves in small groups how they would define force. The responses (written on the board) were given in the following order:

1. *A positive or negative reaction that causes motion to stop, start, fast or slow.*
2. *An external or internal influence that would have a variety of effects on or between two bodies; such as propulsion, resistance and direction of movement.*
3. *An invisible force which causes pressure.*
4. *An energy or potential energy in one vector, usually increasing in magnitude.*
5. *That which creates energy.*
6. *A change or transfer of energy.*
7. *Transferable energy.*
8. *A vector quantity - meaning force acts in many different ways as in many different directions.*

1 is an obscure response that relates force to motion in some way. The response is partly Newtonian because if force causes motion to 'stop' or 'start' then the implication is force causes a change in motion. 2 regards force as having an effect on bodies, as opposed to the common idea that force is in some way stored in the body. Although force is seen as acting on bodies, it is not seen in terms of the influence of one body on another. The vagueness of the response makes it difficult to ascertain exactly what is meant by 'force is that which affects the direction of movement'. On the one hand, force in two dimensions may affect the direction of movement, but on the other hand the statement is 'close' to the Aristotelian idea of force as acting in the direction of movement. 3 states that force causes pressure, and so is not an example of force-pressure mixing. 8 reflects some text-book experience. 4, 5, 6, and 7 are *force-energy mixing* that has been reported by many researchers (for example, Bocha, 1990; Champagne *et al*, 1982; Viennot, 1979). It is noteworthy that once the force-energy mixing had been raised, it was then referred to three times subsequently. The following discussion then took place (please note that the numbering of the

students only applies to that particular section of the discussion at the time, and does not necessarily refer to the same students at different sections of the discussion):

Teacher. *If you were asked to define force in a 'nutshell', using only one or two sentences, how would you do it?*

Student 1. *That which governs the transfer of energy, making a mass stable or unstable etc.*

Teacher. *The difficulty with such a definition is that you would need 10 minutes to explain what you would mean. Could you give a definition, using one or two sentences, that would 'sum-up' what force is - that would 'tie-up' what you mean by force.*

Student 2. *Force is applied that would make an object move.*

Student 3. *Force is that which causes motion.*

Student 4. *But doesn't force also stop motion as well?*

Teacher. *Could we say that force is that which creates and stops motion?*

Student 4. *Yeah! And controls motion as well.*

The idea that force causes a **change** in motion, albeit in a rather 'fuzzy' or vague form (force 'creates' and 'stops' motion), is beginning to emerge. To see what has been gained from session one, and to allow for an 'incubation' period (it was hoped that the class would think about what was discussed before session two), the class were again asked to define force at the start of session two:

Teacher. *To kick off, can anyone state what they understand by 'force'.*

Student 1. *Force is the pressure needed to make or cause a mass to move in a straight line. It is also the pressure needed to make or cause a mass to change direction. Force is also equal to mass times acceleration.*

Teacher. *You use the word 'pressure', are you saying that 'force' is synonymous with 'pressure'? Is force identical to pressure? Is it related?*

Student 1. *Its related.*

Teacher. *I asked this question because many students use 'pressure' instead of 'force'. You say they are related. What is the relation?*

Student 1. *It goes back to gravity....Er, you would require a pressure to push something along.....*

Teacher. *Rather than a force?*

Student 1. *I am calling force 'pressure'.*

Teacher. *Would it be fair for me to say: instead of the word 'pressure' we will use the word 'force'?*

Student 1. *Yes...OK*

Teacher. *So force is that which.....*

Student 1. *Force is that which makes a mass or body move.*

Teacher. *Makes it move?*

Student 1. *Yes.*

Student 2. *Force acts on an object in uniform motion and creates a reaction to the state of that motion.*

Student 3. *It is an invisible, unavoidable force which can be used and manipulated. It is all around us. It sometimes helps us or works against us. It is an undefinable force of energy.*

Student 1. *Can I ask a question? If a ball explodes from the inside, would that be pressure or would that be force?*

It should be noted that *force is equal to mass times acceleration* was not mentioned by anyone in session one. Because the intuitive schema of force and motion contains a set of poorly differentiated concepts, it would be expected that some students would mix 'force' with 'pressure'. However, it was not anticipated just how much students would insist on using 'pressure' instead of 'force'. It would be tempting to think that force-pressure mixing (or force-energy or force-momentum mixing) is a 'dodge' in Howard's (1987) sense (to avoid using the concept of force to explain something with a concept that is considered to be more familiar). However, one particular student - a diver by profession - was adamant in using 'pressure' instead of 'force' during both sessions. His persistence for using 'pressure' instead of 'force' during the first two sessions may have had some influence on the force-pressure mixing of other students. The diver by profession had argued that gravity is something that pushes you down and not something that pulls you down. Consider the following dialogue (session one):

Teacher. *When scientists speak of gravity, what they mean is 'gravitational attraction'. What is the connection between gravitational attraction and weight?*

Diver. *Why does it have to be an attraction? Why can't there be something out pushing in? That would be more logical.*

Teacher. *You are sitting in your seat. If there were no gravity, what would happen?*

Diver. *I would float off [To float off implies an upward force. However, I felt it necessary at this juncture to deal with 'one point at a time'].*

Teacher. *So you are saying that there is something above pushing you down?*

Diver. *Yes.*

Teacher. *Please go on.*

Diver. *The Earth is a ball with a very large mass under intense pressure. It is therefore more logical for something outside exerting force pushing in, holding it all in creating pressure in the middle.*

Teacher. *Let us look at the gravitational influence on you. Where and how is it acting?*

Diver. *Gravity, as I understand it, acts on all of you. It is a constant thing that, if you like, pushes you towards a central point - being, if you like, the centre of the Earth. So, therefore, where ever you go on the Earth you are maintained on the surface by a constant push towards a central point.*

Another student. *Surely gravity pulls rather than pushes!*

It would appear that the diver's experience of underwater pressure has instantiated a 'pressure' and 'push' schema to account for gravitational attraction, and that his force-pressure mixing (voluntarily raised by him on several occasions) was not a dodge. In retrospect, the obvious question to have given the diver would have been: *If gravity pushes from above, then from where above does it push from?* and to have raised the parallel situation of having to be behind a car in order to push it forward. As it happened, I referred to the statement *Surely gravity pulls rather than pushes!* and defined gravity as a force of attraction. I subsequently insisted that the term 'force' be used instead of 'pressure', 'energy' etc., when the concept of force is discussed.

Some of the above student responses, provided they had some form of coherence, were included in the package. Because the question *What is force?* is so open ended, *force-energy* or *force-pressure* mixing might be a distraction to any attempt to structure a dialogue (as my own experience has shown). I therefore did not include any such response in the package. However, in appendix 1 of the package (see appendix A, page 281), the teacher is advised to either insist on the term 'force' or to ask what is meant by such terms as 'energy' .

Vague notions of force that may possibly lead to the correct definition (e.g. force stops motion/starts motion/controls motion) were included in unit 1, with the strategy of asking to be more explicit.

The string partly wound around the drum problem (*if the string is pulled horizontally, in which direction will the drum move?* See chapter 5, section 5.5, page 142) was presented to the class (after the pilot-study). Every response stated that the drum would unwind. I presented a parallel situation: *if the string was wound around a student and the string was*

pulled, in which direction would the student move? Although many students answered correctly, there seemed to be a difficulty with relating this with the drum problem. However, some students seemed to appreciate that if the wound string was slack or loose then pulling the string would tighten the string before movement of the drum takes place - the tightening of the string would move the drum towards the pull rather than the expected unwinding of the drum. The drum problem became unit 2 of the package, and the strategy of 'tightening the string' included in the unit. It was decided to place the drum problem after unit 1 because many sixth-form students who answered this problem correctly (see chapter 5, section 5.5, page 142) said that the pull of the string moved the drum rather than the drum undergoing a change of motion. It was therefore decided to place this problem directly after unit 1 as the problem would not only go against expectation at an early stage, but it would also provide the opportunity to describe the drum as undergoing a change in motion (following unit 1 on defining force).

7.3 THE FORCE ACTING ON A VERTICALLY THROWN BALL. THE MAKING OF UNIT 3.

After giving a brief reference to Aristotle's 'laws of motion' (session 1) I instigated the following discussion:

Teacher. *For Aristotle, if his car breaks down he can always get out and push. The car moves because of the push. However, if he stopped pushing the car what would happen?*

Student 1. *The car carries on moving.*

Teacher. *So there is something not quite right with Aristotle's ideas. The car keeps going, but what actually happens to the motion of the car?*

Student 2. *It has momentum to keep on going.*

Teacher. *But what eventually happens?*

Student 3. *It slows down until it stops.*

Teacher. *Why does it slow down?*

Student 3. *Negative reaction or friction.*

Student 4. *Yeah, friction!*

Student 5. *Yeah, a force acting in the opposite direction.*

Teacher. *Yes, a force acting in the opposite direction.*

Student 6. *It has used up its stored energy.*

Teacher. *Er. thank you.... Newton, on the other hand, argued something quite radical. Force doesn't create and maintain motion - that if a body is in motion then it doesn't necessarily mean that a force is acting on it. What force does is to change the state of motion - it changes motion rather than maintains motion. Imagine this: I am in a spaceship deep in space where there is no gravity or air-resistance. I throw out a bag of rubbish. Describe what happens to the motion of the bag of rubbish.*

Student 1. *Constant velocity, it just keeps going.*

Student 2. *Constant acceleration.*

Teacher. *Constant acceleration, when?*

Student 2. *It is continuous, nothing is going to stop it because no force is acting on it and so it is just going to go on for ever and ever.*

Teacher. *So it just goes faster and faster? [class becomes a little unsettled, until:]*

Student 3. *It carries on at the same speed.*

Teacher. *Yes. Otherwise, if it kept on accelerating what would happen?*

Student 4. *It would get hot and...*

Teacher. *Probably reach the speed of light [laughter from class]. The bag accelerates in my hand when I push, but when it leaves my hand it carries on at the same speed. The acceleration is a change in the state of motion. The bag has gone from a state of rest to a state of motion. If the bag leaves my hand at 5 m.s^{-1} then it has gone from 0 m.s^{-1} to 5 m.s^{-1} - it has accelerated from 0 to 5 m.s^{-1} due to the force applied by my hand. So that force applied by my hand has created a change in the state of motion of the bag of rubbish. This bag leaves my hand with constant velocity for evermore unless it comes under the influence of a gravitational field. Now this bag of rubbish is on Earth. I throw it up into the air. What are the forces acting on it?*

Student 1. *Gravity, air resistance....and friction.*

Student 2. *Potential weight of the rubbish itself.*

Teacher. *Er...what is the connection between gravity and weight?*

Discussion of gravity. I then refer to the force acting on a thrown ball that is going up.

Student 1. *It is the pressure you have applied to it underneath.*

Teacher. *If I throw a ball up, and do you mind if I use the word force instead of pressure, I apply a force to the ball by my hand and the ball is on its way up. So you would say that the force acting on the ball is that one there [referring to diagram showing a force pushing ball up]?*

Student 1. *Yes.*

Teacher. *Many students argue this, internationally. Does this force wear out at all?*

Student 1. *Yes, when it gets to its...*

Student 2. *Peak.*

Teacher. *So when the ball becomes momentarily stationary, what force is acting then?*

Student 3. *None, both forces are in equilibrium.*

Teacher. *So the force of gravitational attraction counter-balances the force given by the hand?*

Student 4. *The force of momentum out-balances the force of attraction.*

Teacher. *Aristotle argued that the force given by the thrower is transmitted to the ball, as the ball moves up in the air, by the air flow. Do you agree with that? [A general consensus of no!]*

Teacher. *This is not what you argued, but it is consistent with what you argued. There is a force pushing the ball up, and that force is there even though the hands have been placed in the pocket.*

Student 1. *No, its mass moves it....*

Student 2. *Its mass gives it the momentum....*

[Even though I had made reference to Aristotle, it would seem as though students 1 and 2 would have preferred the medieval impetus theory of force as a property of the body].

Student 3. *Its mass gives it momentary energy until the larger mass takes over and brings it back down again.*

Student 4. *It is the same thing as the car slowing down, where there is a force opposing the motion* [although reference had been made earlier to a car slowing down, it was a student who had raised the example as a parallel situation].

Teacher. *I push the car, I then stop pushing the car and the car slows down. What force is acting on the car?*

Student 5. *Friction.*

Student 6. *Friction.*

Teacher. *Which direction is it acting?*

Student 7. *Straight down.*

Student 8. *Acting on the tyres.*

Teacher. *Which direction is it acting?*

Student 4. *Against the direction of motion.*

Teacher. *Against the direction of motion - that is what slows the car down. I throw a ball up, in terms of its motion, what does it do?*

Student 9. *De-accelerates.*

Teacher. *De-accelerates - it slows down. There is something slowing it down.*

Student 9. *Gravity.*

Teacher. *Thank-you! Which direction is the gravitational attraction? Which direction is the weight?*

Student 10. *Opposite to the motion.*

Teacher. *Opposite to the motion. If I threw the ball from a spaceship, away from gravity, would it slow down? [A general consensus of no!]*

Feeling confident that the parallel of the car slowing down had 'done the trick' (so to speak) for the original concept question of the thrown ball, I had moved the discussion to the concept of acceleration, the comparison between friction and gravity, and the comparison between equilibrium and disequilibrium with reference to a 'tug-of-war' match. To test whether the class as a whole had undergone a gestalt-shift from force as that which maintains motion to force as that which changes motion - to see whether the parallel of the car slowing down had worked - I had decided to begin session two by asking the class what they understood by 'force'. It was obvious from their response that, at the time when the video recording was made of the dialogue above, a gestalt-shift had not occurred within the class as a whole. In planning for the second session I had assumed that the class had displaced their intuitive schema of force and motion and that it had constructed in its

place the schema of a change of motion- car and thrown ball slowing down - with force as one of the slots. Because of the intransigence of intuitive ideas, it became necessary to cover the same ground concerning the force acting on a thrown ball. Unfortunately, because I had presumed a gestalt-shift had taken place, I had not prepared a fresh strategy and therefore I had to 'think on my feet' so to speak!

Teacher. *If I throw a ball up in the air, what would be the force acting on it?*

Student 1. *Force of gravity acting down.*

Teacher. *Now suppose I throw the ball at an angle. What is the force acting on the ball?*

Student 2. *Gravity.*

Student 3. *Mass and gravity.*

Teacher. *You are saying that mass is a force?*

Student 3. *No! I am saying that the mass of the ball affects how the ball is pulled down by gravity.*

Teacher. *So on a diagram how would you represent the force acting on the ball?*

Student 3. *Gravity downwards and a force... [gestures with hand a force tangential to trajectory].*

Student 4 [Friend of student 3, sitting in the next seat] *An acceleration force greater than gravity.*

An 'acceleration force' implies an 'imaginary force' that may explain why so many students have an intuitive difficulty with circular motion (see Savage, 1988; Twells, 1990; Warren, 1979; Williams, 1988).

Teacher. *An acceleration force? What force is that?*

Student 4. *The acceleration force which is greater than the gravitational force - it is the force which threw the projectile across.*

Teacher. *So there are two forces?*

Student 4. *Yes, one down and one across.*

Student 5 [friend of student 3 and 4, sitting next to them] *Yeah, one on its trajectory.*

Teacher. *But when I throw a ball and place my hand in my pocket.....*

Student 5. *Yeah, but the force has come from you but is decreasing.*

Teacher. *It's decreasing?*

Student 5. *Yeah, as its losing velocity, its going to slow down and the force of gravity is going to be greater which is forcing it downwards.*

Teacher. *But if I am on a hull of a spaceship and I throw the ball, how is it going to go?*

Student 5. *It is going to go straight.*

Teacher. *But what is its motion like when it is going straight?*

Student 5. *Uniform.*

Teacher. *Is there a force pushing it?*

Student 5. *Not after it has left you*

There appears to be a sigh of relief from the class.

Teacher [looking at students 3 and 4] *So where is this force coming from then?*

Student 3. *Its own mass.*

Student 4. *I'd say it is from you, but decreasing.*

Class appears to be a little unsettled at this point.

Student 3 appears to have a medieval impetus viewpoint, while student 4 appears to have an Aristotelian one. It was tempting to abandon the discussion since the whole class with the exception of the two students appeared to have grasped the point. However, this may have been a consensus within the class to 'keep me happy' so as to move on. Moving on was a temptation because subsequent concept-questions could always be related to the previous phenomena considered. Nevertheless, I had attempted to 'crack this nut' as it might have revealed a successful strategy.

Teacher. *But why is the force from me, from the hull of the spaceship.....*

Student 4. *There is no resistance...*

Student 3. *Oh, I know, its the force stored in the mass.*

Teacher. *Force stored in the mass? In fact that is quite often given, even by Aristotle [my mistake, I should have said the medieval impetus school]. Aristotle argued that force is a property of the object.*

Student 3. *The force given by the thrower goes into the ball - into the mass - and in that situation, stored in the mass, and because nothing is stopping it - it is in a vacuum - it just carries on. But if another force is applied to it, it will change direction [hand gesture of a projectile's trajectory].*

Student 6 [who has been waiting a long time to respond]. *The only force acting on a thrown ball is gravity. If a ball is in uniform motion then no force is acting on it.*

Student 7. *The force is gone.*

It had appeared at this point that students 3 and 4 had set themselves apart from the rest of the class. It is interesting to note that students 3, 4 and 5 (whom I used to refer to as the 'three Aristotelians') sat next to each other for the first time in session two. Given the general agreement of the class that force changes the state of motion, I felt it necessary to move on. However, students 3 and 4 were so entrenched in their position that they attempted to move the discussion back onto the idea that force maintains motion. The discussion did move back for a while, but it became very unstructured as I had ran out of strategies to deal with the responses of students 3 and 4. Consequently, the two students had become very coherent and consistent in their own arguments. For example:

If there is no such force [pushing a projectile] then why does it go to the top of the arc before gravity takes over?

If a ball hits a wall, then the dent in the wall is caused by the force in the ball.

There is a force opposing the motion of the car braking, namely friction; but the force pushing the car gradually weakens - and that is why the car slows down until it stops. When the car stops, the frictional force and the push are in equilibrium.

Because the first two sessions were unstructured to some degree, and because I had no appropriate strategy to overcome the intuitive ideas of students 3 and 4, the two students had become entrenched in their own position and managed to argue with their own consistency across the parallels. The major difficulty is that no one has experienced motion in the absence of force. One of the two students stated: *Doesn't it prove the point that uniform motion deep in space means that the mass is holding the force.* This is an example of 'hypothetical-deductive' reasoning (see Renner, 1976a) characteristic of formal operational thinking: a student conception has been given its own internal coherence because it has been applied or modified, without self-contradiction, across the parallels involving idealised abstraction (the Popperian argument that the validity of a theory depends upon its ability to be falsified seems inappropriate in this context). The three 'Aristotelians' were not concrete operational thinkers, yet at one stage they appeared to have drawn support from students who appeared to be so. On the bag of rubbish thrown deep in space:

Student 1. *How can it [the bag] slow down, there is no friction.*

Student 2. *But space has no total vacuum.*

Teacher. *Are we really concerned whether or not space is a total vacuum?*

Student 2. *So we have to lie in order to prove a theory?*

The same point was made concerning gravity deep in space. In fact, one student approached me after session two and stated that it was confusing for many students to consider a region deep in space where no gravitational influences exist, when we are told by scientists that gravity permeates all of space. Possible world scenarios and idealised abstraction may prove to be a difficulty for concrete thinkers.

Satisfied that the majority of the class had a Newtonian understanding of what forces are acting on a thrown ball, I decided the following plan for session three:

Equilibrium - example of a book at rest on a table.

Broken equilibrium - example of raising a book in the air with my hand.

Return to the 'tug-of-war' example - equilibrium and broken equilibrium.

Example of a ball thrown vertically upward - identifying the forces acting on the ball when it is momentarily stationary.

Taxonomy of force - discussion of the various contact forces and weight.

Just before the end of the session, after the class had completed exercise 5C from *Mechanics 1* (Berry *et al*, 1993), a student raised a point concerning projectiles (question 2 of exercise 5C). I raised the example of the ball held in my hand whilst sitting on a train travelling in uniform motion (see chapter 5, section 5.6, page 146), and asked what was the speed of the ball relative to a stationary observer and if there were any horizontal forces acting on the ball. There were facial expressions of cognitive conflict from the two remaining 'Aristotelian' students. I had raised the question almost by accident, as I had just remembered the example raised by Galileo of the sailor in the crow's nest who drops a ball - does the ball land in front of the mast, at the foot of the mast or behind the mast? (I had named the concept question of the ball in the train as *Galileo's Strategy* in the teaching package). Throughout the rest of the course the 'Aristotelians' were able to provide a Newtonian account of complicated qualitative models.

No one has observed or experienced uniform motion in the absence of force; nonetheless, 'Galileo's strategy' is in a sense the closest that one can consider uniform motion in the absence of force. The concept question of the bomb-and-plane, or of the hockey-ball (see chapter 3, section 3.2, page 89), both have the same logical structure as 'Galileo's

strategy': the horizontal motion of the bomb, or the previous motion of the hockey-ball prior to being struck, or the horizontal motion of the ball on the train, all remain unaltered due to the absence of force in the direction of motion under consideration. However, 'Galileo's strategy' is less complicated in that it represents force in one-dimension whereas the other two examples represent force in two-dimensions. For the bomb-and-plane, vertical forces acting on the bomb are important in explaining the trajectory of the bomb, whereas the vertical forces acting on the ball in the train can be initially ignored in the consideration of 'Galileo's strategy.' 'Galileo's strategy' is more apparent than Galileo's own example of the sailor dropping the ball from the mast of a ship travelling with constant speed (see Arons, 1990; Chalmers, 1978). It is apparent that the ball in the train has uniform motion with no force maintaining that motion. The law of inertia has to be applied to the ball dropped from the mast of a ship in order to explain why the ball lands at the foot of the mast. Pedagogically, Galileo's strategy is logically prior to the bomb-and-plane, hockey-ball and the ball dropped from the mast because the understanding of Galileo's strategy is required in understanding the other three.

I had decided to assign the concept question of the ball thrown vertically upward to unit 3 of the teaching package. Because 'Galileo's Strategy' is the major catalyst (see chapter 5, pages 147/148), it is placed last in the unit. It is hoped that the order of strategies in unit 3 is such that the teacher will not have to suffer a discussion that meanders to a point whereby students become internally coherent due to their own intransigence.

Unit 8 is a logical continuation of unit 3, but is placed further on in the package (to enable a subsequent lesson to 'loop-back' onto the idea that force changes rather than maintains motion). Because unit 8 is a logical continuation of unit 3, it will be considered next.

7.4 UNIFORM MOTION. THE MAKING OF UNIT 8.

The concept question of the dog in a car (unit 8) is similar to Galileo's strategy, and it's inclusion is to test and reinforce the notion that force is not required to maintain motion. The following concept question of the motorbike introduces uniform motion but with horizontal forces acting:

Horizontal uniform motion requires no force



Horizontal uniform motion and horizontal forces $\Rightarrow$ net horizontal force = 0

The horizontal forces acting on a motorbike raises the question of resistance and tractive force. The parachute question (unit 8) is similar to the motorbike question, but poses uniform motion and forces acting in a vertical direction. The vertical forces raises the question of air resistance and gravity. All three questions from unit 8 were borrowed from *Exploring Mechanics* (MEW group, 1989). The three concept questions were used with the class as an exercise in session 4. All the students in the class answered correctly.

7.5 THE FORCE ACTING ON A THROWN BALL AT ITS MAXIMUM HEIGHT. THE MAKING OF UNIT 4.

The following discussion took place half-way through the third session:

Teacher. *When a thrown ball is going up, what is happening to it's state of motion?*

Student 1. *It's slowing down.*

Teacher. *For Newton, what must be the force acting on the ball for it to slow down, ignoring air resistance?*

Student 2. *Gravity.*

Student 3. *Gravity.*

Student 4. *Gravity.*

Student 5. *Gravity, W, downwards.*

Teacher. *The ball now reaches it's maximum height. Is the ball in equilibrium, and if so what would happen.....?*

The ball has reached it's maximum height, what happens in the next moment?

Student 1. *It's going to go down.*

Teacher. *How would you describe the state of affairs whereby the ball is instantaneously at rest and about to go down.....?*

Is it uniform motion, is it at rest full stop?

Student 1. *It's accelerating as it goes down.*

Teacher. *The body is not in equilibrium. If it were in equilibrium, what would happen?*

(Consensus) *It will stay there.*

Teacher. *It will stay there, suspended. It is not suspended, it is about to go down. So what force is acting on the ball at B [maximum height]?*

A student. *Gravity.*

Teacher. *Gravity, W, acting down. How long is an instant?*

Student 6. *As long as you want it to be.*

Teacher. *Is it? Can I say a second?*

Student 6. *Yeah!*

Two other students. *Reasonably small. The blink of an eye.*

Teacher. *The blink of an eye could be something like point-four of a second. Could the ball be at B in point-four of a second?*

Student 6. *Could be.*

Teacher. *How about point-two of a second?*

Student 6. *Why not?*

Teacher. *Do you think it is arbitrary, do you think it depends on the ball?*

Student 6. *Yes.*

Teacher. *So some balls might stay there for point-two of a second, some balls might stay there for point-four of a second?*

Student 6. *No, it depends on....*

Another student. *It's a constant.*

Teacher. *What is this constant then?*

(Consensus). *Zero.*

Teacher. *Zero! [a ball is thrown upwards and the class asked to observe] If you are in a car travelling on the horizontal, slowing down and about to stop, and you put on the brakes, fine - you do not feel any jerk of the car. You have waited for the car to stop and you put on the brakes - no jerk! Now you try that on an incline! The car goes up an incline, and you decide to disengage the engine. What happens? It slows down until it eventually stops. But when it stops it then....*

Student 6. *Goes backwards.*

Teacher. *Goes backwards. Now how long does it stop for?*

Student 6. *Instantly.*

Teacher. *Instantly! Zero seconds! As soon as it stops, and try this with your car, as soon as it stops apply the hand brake or foot brake.....*

Another student. *You can't do it.*

Teacher. *It jerks! It jerks because it takes a certain amount of time to put the brakes on. It stops in no time at all, and you are putting the brakes on when in fact it is still in motion. This is why the car jerks. Try it as accurately as you can, but the car always jerks. This doesn't happen on the horizontal plane because when the car stops it remains at rest; wait half an hour, put on the brakes - no problem! Up an incline, when it comes to stop, it is not at rest. It is not in equilibrium. It is still in the process of a change in motion. Just as the ball is in B.*

The discussion then changed from strategic questioning to a formal definition of acceleration as the rate of change of velocity. In this phase of the discussion, a difference was made between acceleration/retardation and positive/negative acceleration. The following student then raised the following point:

Student 1. *The ball is not in equilibrium at the top because it is at an instant we can't measure, so surely it is all three? It is whatever you want it to be: going up, coming down, staying the same.*

Teacher. *All three is really the same thing. Look at it this way: Fred throws up a ball with a speed V , say 10 m/s. Now you are in a helicopter, rising in the air with a constant speed V . You travel upwards in uniform motion with a speed V , the ball starts off with a speed V but of course slows down until it comes to instantaneous rest and then goes down. You are on the helicopter looking at the ball. How will the motion of the ball appear to you.....? Imagine! You are going up with a speed V , but the ball doesn't maintain speed V . If the ball was in uniform motion then it would appear to be still.*

Student 2. *It would appear as if the ball was dropping, going the other way [his hands illustrate the relative motion].*

Teacher. *It would appear as if the ball was going the other way. You can imagine that you are stationary even though you are not - it is hard to visualise.*

Student 3. *It is the same with the parachutist then? One guy has jumped and he hasn't opened his parachute yet. The other guy has jumped and opens his parachute. The second guy will appear to move upwards.*

Teacher. *Have you seen the recent advert whereby two people are in free-fall? One opens his parachute but the camera is relative to the other one. The person who opens his parachute appears to be going up into the air.*

Student 3. *Same effect.*

Teacher. *Same effect. Now, imagine this! On the helicopter the ball appears to be going away from you, but when it reaches B and then goes down, C, how will it appear?*

Three students, consecutively. *It will seem to accelerate. Accelerate away from you. Accelerating really fast.*

Teacher. *How will the three accelerations downwards compare? That is in A, B and C? How will the three parts appear to you?*

One of the three students. *It will appear to be going down.*

Teacher. *It will appear to be accelerating downwards. And in fact you will be very hard pressed to point out position B from the helicopter - you could not do it! It will look as if the ball is just accelerating away from you. There would be no distinct three stages of the ball's motion relative to you on the helicopter. This is hard to visualise, so I will leave it for you to digest.*

The car on an incline example, and the helicopter example, were both borrowed from Arons' (1990) *A Guide to Elementary Physics Teaching*. The helicopter example is difficult to imagine, but it did seem to 'work' in the 'dialogue.' An *Interactive Physics* simulation might overcome the difficulty of having to 'imagine' the scenario. A portion of the class appeared to successfully imagine the scenario and to understand that to be

stationary does not necessarily mean to be at rest - that the ball is momentarily stationary (with respect to a stationary observer) and accelerating. It is hoped that a diagram showing the weight acting on a thrown ball that is momentarily stationary would no longer be an anomaly to 'picturing' the ball as stationary. Given that it met with some success, the helicopter scenario was included in the package; but only as an 'option' given the difficulty of 'picturing' the event. The aim of the unit is to construct the idea that 'stationary' does not imply 'equilibrium'.

7.6 THE FORCES ACTING ON A BOOK THAT IS AT REST ON A TABLE - THE MAKING OF UNIT 5

In raising the question to a class 'What keeps the book at rest on the table?', Minstrell (1982) found that many students had difficulty with the idea of an inanimate and inactive solid exerting a push. In the following dialogue with the foundation year mechanics class, however, the class appeared to have little difficulty in appreciating that there is a force exerted by the table (the acceptance of a force appeared at the outset). This may have been due to the fact that the foundation year mechanics class mainly consisted of mature students, whereas Minstrell's class consisted of high school students. However, Minstrell (1982) refers to a report by Clement that many University engineering students have the alternative conception that the table is simply 'in the way.' It is most probably the case that the acceptance of a table exerting a push was due to the dialogue appearing at the start of session 3, by which time the class was used to the idea of force (the class may have learnt to **expect** a force from the table!). Later in the dialogue I had decided to prompt any scepticism regarding the table exerting a force. Minstrell went to great lengths to try and convince his class (e.g. holding a book in an outstretched hand, placing the book on a spring and noting the compression, the deflection of a light beam from the table), and so to

cover any eventuality of a class not appreciating intuitively that an inanimate and stationary object can exert a push, many of Minstrell's strategies are included in unit 5. As it turns out, the following dialogue is mainly concerned with the equilibrium of forces of an object at rest.

Teacher. *If I drop a book it falls to the ground! What prevents the book falling to the ground if I place the book on the table? Quite an obvious question really, but what keeps the book at rest?*

Student 1. *The fact that the table is in equilibrium, and gravity, or more than gravity.....(?)....*

Teacher. *You mention gravity, what is the relation between the book.....*

Student 1. *The table is preventing the book from going down because it is a solid barrier.*

Teacher. *What is the relation between that side of the barrier [pointing to the top of the table] and the weight of the book?*

Student 1. *The barrier is opposing the action of gravity.*

Teacher. *Here is one book on the table [referring to diagram], and here is one force that we know [I draw the line of action representing the weight]. The table acts as a support, but how can we describe what the table does....?*

Student 1. *Arrows underneath the table pointing upwards.*

Teacher. *So an arrow here say?*

Student 1. *No, from underneath the table, and more than one.*

Teacher. *More than one? If we have more than one, what would that show?*

Student 1. *That the table is uniform. That the table is not opposing at one point but across.*

Teacher. *Ah! So if I represent the force from the table, and presumably you mean a force, with one arrow then it looks as if the force acts through a point. But the force does not really act through a point, it acts through the whole area of contact between the book and the table, yes?*

Student 2 [The diver]. *For every action there is an equal and opposite reaction, so for the weight of the book.....*

Teacher. *Where did you get that from, for every action there is an equal and opposite reaction?*

Student 2. *Er, you have the weight of the book acting down.....*

Teacher. *A couple of points: The trouble with having a number of arrows from the table pointing up is that one person might put four arrows and another person five arrows. You don't know the extent of the force. So what we do is, we represent..... how many forces acting from the table?*

Student 3. *One.*

Teacher. *We represent just the one force acting from the table with just the one arrow - but we don't wish to imply that the one force acts necessarily through the one point. Most text books and people who teach mechanics will represent the force from the table as acting from the surface of the table rather than from below the table as you suggested.*

Student 4. *To me that suggests that as you take the book away the table will go up.*

Teacher. *This is a source of confusion and does raise many problems. But before I come back to you I shall return to you [student 2]. This 'action being equal and opposite to the reaction' - if we call this force N [referring to force diagram of book on table] - and ' N ' is used to stand for 'normal reaction', 'normal' meaning 'perpendicular', i.e. at right angles to, not 'normal' as in 'natural' - then the relation between N and W is...?*

Student 2. *One cancels the other out.*

Teacher. *One cancels the other out, so the net force - the resultant force is...?*

Student 2. *Zero.*

Teacher. *So what can you say about the relation between N and W ?*

Student 5. *They are equal.*

Teacher. *They are equal, so N equals W . But most text books will say that N equals W as a consequence of the Third Law of Motion. N is in fact equal and opposite to W so, aha!, the Third Law of Motion! But when we come to the Laws of Motion [the students were aware that the laws of motion would be subsequently covered as part of the syllabus] we will find that this is not to do with the Third Law of Motion - even though the majority of text-books say it is, it is not the case! We will see that N equals W is a consequence of the First Law of Motion. Now going back to the first point: if we take W away, then N , which is a force.....what will happen?*

Student 6. *It will not, but it looks as if the table will go up because the weight isn't holding it down anymore.*

Teacher. *It looks as if the table will go up, but what are we representing here? What are these forces acting on?*

Student 6. *The arrow pointing up is a resistance, a normal force.....*

Teacher. *A normal force. From where to where?*

Student 6. *Between the table and the book.*

Teacher. *Is it from a body to a body? Between two bodies? What is it exactly, this N ?*

Student 6. *It's a barrier.*

Teacher. *In terms of force! The table does act as a barrier, but does N act from one body to another or does it act between two bodies?*

Student 7. *Between two bodies.*

Student 2. *In simplistic terms it is just a force of motion that is stopping the book from dropping.....*

Teacher. *It is a force that is stopping the book from dropping. I am making an issue of this because from the beginning of a mechanics course we accept that N equals W and we take this to the exam. However, do we really understand what is meant by a force from the table on the book? A force from the table - pushing up? When we actually begin to think about N then it doesn't seem so common-sensible. How can there be a force from the table - pushing the book up?*

Student 2. *It's not, the table itself is in uniform motion - isn't it?!*

Teacher. *Table in uniform motion?*

Student 2. *The table is at rest. The only force resulting there is the weight of the book - which is it's own force - on top of the table. The weight of the book is equal to the force pushing down onto the table.*

Teacher. *But you are not saying that N is W , are you?!*

Student 2. *It is an equal and opposite reaction.*

Teacher. *But what does this reaction mean? The reaction is from where?*

Student 2. *It's one solid surface against the other. So the weight of the book is its own force, resting against the table...*

Teacher. *Yes.....?*

Student 2. *The table itself.....er.....It's like you said the other day about two bodies attracting each other, and repelling one another, similar thing.*

Teacher. *Thanks! [student 7 puts up his hand]. Yes..?*

Student 7. *You cannot categorise N as a force of motion because it is not in motion.*

Teacher. *What are we looking at - the forces acting on what? We have to be clear in our minds. N and W are forces acting on what? Which bodies are they acting on?*

Student 8 [with nods of agreement from student 2]. *They are only acting when the book is on top. The only force that is applied upwards is due to the weight of the book acting down. So therefore N is only acting when the book is on the table.*

Teacher. *So take the book away, then there is no N acting on what?*

Student 8. *On W .*

Student 9. *On the book.*

Teacher. *On the book! Suppose we take the table away, and God comes about with a mysterious force that suspends the book in mid-air. We have the weight of the book acting down. What does God need to apply to make the book suspend in mid-air?*

Student 9. *N .*

Teacher. *N ! That force is acting on the book. But where is it acting from?*

Student 10. *Is it the resistance of the table, resisting being compressed by the book?*

Teacher. *That sounds pretty good! You can experience this yourselves. Hold out the book in your hand and keep it at rest. You have W acting down so there must be a force from your hand, like N , equal and opposite to W , holding the book at rest. Now, can you feel that force? You can feel the muscular contraction. If you cannot feel it then I'll get you to pick up the table - you will feel it then! [Laughter].*

Teacher. *That is the force exerted by me on the book [I hold the book in my outstretched hand]. It must be a force that is equal to the force exerted on me by the book.*

Student 1. *But the table hasn't got any muscles, but my arm has.....*

Student 2. *Yeah, but it's got mass; and if the book has a larger mass than the table then the book would go through.*

Student 3. *The table would break.*

Teacher. *So if the book was many tons, then the table would crumble? But this would depend on the internal structure of the table.*

Student 4. *The book has a mass, and if this mass was greater than the mass of the support then.....*

Teacher. *Not necessarily so! It is possible to have a light frame structure holding a large mass. But I understand what you mean - if you keep piling on the mass then there has to be a point where the support eventually breaks. But it is not necessarily the case that the body that is supporting has to be of a greater mass than the body supported. For equilibrium, for the book to be at rest, then there has to be forces that 'cancel' each other out - the resultant force must be zero. For the resultant force to be zero, N must equal W .*

It was decided to place the concept question *What keeps the book at rest on the table?* as unit 5, after establishing force as that which changes the state of motion (units 1 to 4). The main objective of unit 5 is for a class to construct the conditions for equilibrium, but restricted at this stage to a body which is at rest (rather than in uniform motion). Although a body at rest should not be treated as unique compared with a body in uniform motion (Warren, 1979), nevertheless it was decided to treat the 'at rest' conditions first. It was decided that the next unit, unit 6, should prompt the discussion for the conditions of broken equilibrium: $N > W$. The dialogue in 7.7 below is a continuation from the above dialogue over the 'at rest' conditions of the book.

Force as a concept is represented schematically through its line of action. Its representation appears so obvious that we as teachers quite overlook the fact that the representation is a human construct. Consequently, we underestimate the difficulty students might have in understanding the representation. As quoted in the dialogue on page 204, one student suggested representing the normal reaction of a table on a book at rest, as a number of arrows pointing upward from underneath the table. He suggested that the force acts across the area of contact rather than from one point. Normal reaction represented by the one line is an abstraction that does require cognitive reorganisation, similar to the centre of gravity *as the one point where the weight of a body is imagined to act* (see chapter 3, section 3.3, page 94; and chapter 5, section 5.5, page 143). It was evident in the marking of homework that students do not readily associate the line of action of a contact force with the point of contact between two bodies. The force of impact of a wall on a car would be drawn by most students in the correct direction but from behind the car rather than from the front:

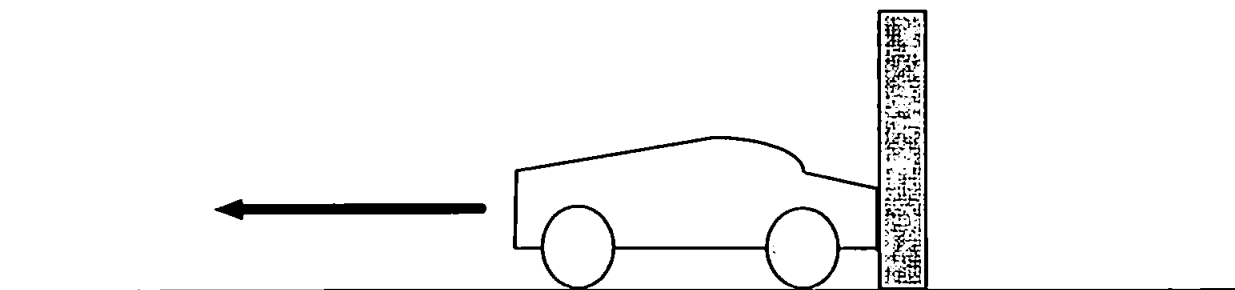


Figure 12 - A Typical Representation of the Force of Impact of a Wall on a Car

A normal reaction from a table on a book would sometimes be drawn from below the table as shown or even from above the book:

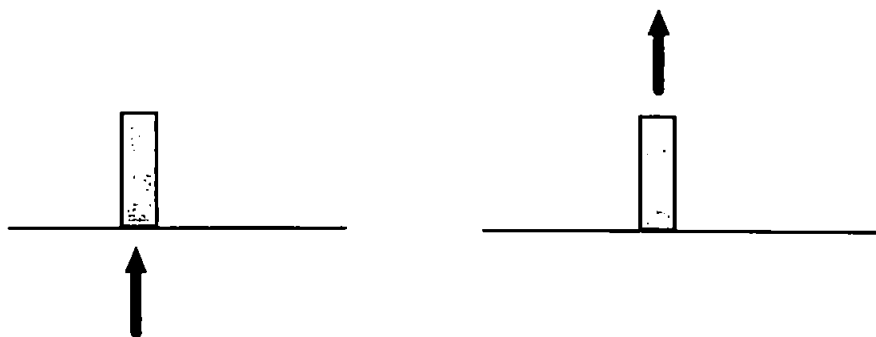


Figure 13 - Typical Representations of a Normal Reaction

It is of little wonder that some students draw imaginary forces. It is also evident from the homework that many students are confused as to which force acts on which body. A teaching strategy must include careful consideration as to how we represent forces, and must draw out the distinction between the forces acting on a body and the system as a whole. To simply state the distinction between an internal and external force may lead to further confusion. In the advice given to teachers towards the end of the package, the correct way in representing the forces acting on a body was stressed in unit 7.

The inability to consider the line of action of a contact force and its point of application can augment misconceptions concerning moments. This will be discussed in chapter 9.

7.7 BROKEN EQUILIBRIUM - THE MAKING OF UNIT 6

Unit 6 is a logical continuation from unit 5. Unit 5 refers to the forces acting on a book that is in equilibrium, unit 6 refers to the forces acting on a book that is not in equilibrium.

Teacher. *Now, if you can remember a statement that I made this time last week - that force causes a change in the state of motion - but the book in my hand is at rest. I am now going to raise the book. The book has gone upwards from a state of rest with a change in velocity back to a state of rest. Forget the 'coming back to rest.' When I raise the book, how does N compare with W ?*

Several students together. *N is greater than W !*

[Diagram on board: State of rest: $N = W$: Equilibrium. Change in motion: $N > W$]

Student 1. So what you are saying then, is one force has to be greater than the other for there to be motion?

Teacher. For there to be motion? This is the trouble: yes, there is motion! But I have emphasised a change in motion because it is not uniform. It is not going up with a speed of 5m/s all the time. It has gone up from a speed of 0m/s to 5m/s , or whatever, back to 0m/s again. We are not looking at a uniform motion but a change in motion. For a change in motion to take place then there has to be a resultant, non-zero, force acting.

Student 2. Why does the book feel heavier when you push it up?

Teacher. Good question! Suppose the weight of the book is one Newton. What must be the muscular strength of my arm to hold the book at rest?

Student 3. One Newton.

Teacher. Now I am going to raise the book. The muscular force from my arm, is that one Newton or more?

Student 3. It's more.

Teacher. It's more! I've got to exert more muscular strength to change the motion of the book. If I exert more muscular strength....

Student 4. The faster it is the more force.....

Teacher. Not faster! If I wanted to change it's motion quickly - instead of going like this [raises book slowly] I went like this [raises book at a faster rate] then I must have more muscular strength in the arms.

Student 5. So it is directly related to acceleration.

Teacher. Yes, a good point and I will come to that. The point here is that I exert more muscular strength, more effort is required, so some people might say 'that feels like a greater weight'. When I weigh myself on weighing scales, what do they actually measure?

Student 6 [the diver]. They measure the force acting downwards.

Student 7. The force down acting on you.

Teacher. Think of yourself on a weighing machine rather like the book on the table [refers to diagram on the board of a person on a weighing machine].

Student 6. It gives you a resistance between....

Teacher. What are we calling this resistance?

Student 6. Well, the spring....

Teacher. What are we calling here [refers to force on diagram]?

Student 6. N.

Teacher. N! Lets be consistent! So we have the weight acting on which body, the body being weighed or the scale-pan?

Student 8. The pan.

Teacher. So the weight is acting on the pan? I know the pan has it's own weight, but I am talking about the 'weight' it measures.

Student 6. Your weight.

Teacher. Your weight, the weight that belongs to you, not the pan's. [Referring to diagram] There is W , acting on you, not on the pan. The pan doesn't actually measure W , what it does measure is what you referred to as resistance - but we now shall refer to, for consistency, as normal reaction. As N equals W , then the reading it gives for N we take as the reading for W . It is a reading of the force required to keep the body at rest on the scale pan. Now, you mentioned acceleration? [looking at student 5] If I have gone from rest to 5m/s what have I done?

Student 5. Accelerated.

Teacher. Accelerated! A change in motion is technically speaking an acceleration. So force causes an acceleration.

The discussion continued into a 'tug-of-war' example. From the dialogue it was found that the example was quite complex, as it involved modelling the forces acting on each member of each team. It was therefore decided that unit 7 should involve another example of the 'at rest' condition, this time involving tension and weight. The student response to the extension of unit 7:

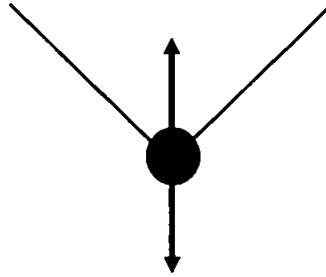


Figure 14 - The Student Response to the Extension of Unit 7

comes from a response to the homework given to the class.

7.8 GRAVITATIONAL ATTRACTION. THE MAKING OF UNIT 13.

In session 5 I handed back to the class their answers to exercise 5A of Mechanics 1 (Berry *et al*, 1993). The following discussion was of question 6, which asked for an estimation of the forces acting on a table tennis ball, a car and an apple.

Teacher. *A number of you in question number 6 stated that the force of gravity on the table tennis ball is $9.8m.s^{-1}$.*

Student (the diver). *OOPS!*

Teacher. *First of all, $9.8m.s^{-2}$ is the unit of what?*

Student 1. *Velocity.*

Teacher. *Pardon?*

Student 2. *Acceleration.*

Student 3. *Acceleration.*

Teacher. *$9.8m.s^{-2}$, what is that the acceleration of?*

Student 1. *Gravity.*

Student 2. *Acceleration due to gravity.*

Teacher. *Acceleration due to gravity. A 3kg. mass and a 1 kg. mass will have the same acceleration, $9.8m.s^{-2}$, towards the surface of the earth. Is the force of attraction on the 3kg. mass the same as on the 1kg. mass?*

Students 1 and 2. *No.*

Teacher. *Can we say therefore, in question 6, that the force of gravity is the same in each of the three cases?*

Student 1. *No.*

Student 2. *It's proportional.*

Student 3. *It's proportional to the weight.*

The diver. *It is proportional to the mass.*

Teacher. *Double the mass, double the force of gravity on it! So the weight of the table tennis ball is considerably lighter than the weight of a car. If you don't believe me, then pick up the ball and then pick up a car.*

Student (the diver). *So you are saying that the force of gravity is the same, but is proportional to the mass?*

Teacher. *The force of gravity is not the same! What is another way of saying 'the gravitational attraction of the earth on a body'?*

Same student. *The weight.*

Teacher. *The weight is the force of attraction of the earth on the body.*

Prior to the above discussion on gravity, the majority of the class had already accepted that objects of unequal weight would land simultaneously when dropped from the same height at the same time. The above discussion was prompted by the responses to the homework, and I wanted to be sure that the class understood weight as the force of gravity. Subsequent discussions revealed that the majority in the class realised that the weight acting on an object is proportional to the mass of the object. I wanted to design unit 13 in a way that would encourage the understanding that it takes 3 times the force of gravity to create the same acceleration for 3 times the mass. To hopefully achieve that understanding I designed the parallel-question of pushing a 1kg and 3kg air-puck (see unit 13). I had decided to place this unit at the end of the package because it entails the explanation of an 'empirical' fact (within the confines of idealised abstraction) involving Newton's second law of motion.

The pre-survey of both the experimental group and the control group revealed that 8 out of the 56 students thought that there was no gravity on the moon. This number is less than the 23% reported by Graham and Berry (1993b), (8% of Graham and Berry's sample denied the existence of gravity on the moon, and 15% stated that because both balls are weightless on the moon then both balls would fall at the same rate) but this may be due to the larger

sample size taken by Graham and Berry. It may also be due to the fact that Graham and Berry's sample were 16-18 year-olds, rather than mature students many of whom might have seen the absurdity of weightlessness on the moon. Given that the teaching package was designed for sixth-formers, it was decided to include the concept-question of dropping the two stones on the moon. It was a fairly simple task to get the foundation engineering class as a whole to see the absurdity of weightlessness on the moon. For example:

If there is no gravity on the moon, then how come nothing on the moon floats?

If there is no gravity on the moon, which means that nothing can fall, then how can the 3kg mass hits the moon's surface first?

One student in the group stated that he had seen film footage of Neil Armstrong floating on the moon. I responded with the question: *If Neil Armstrong was weightless and jumped vertically upwards from the moon's surface, what would subsequently happen to his motion?* and *What did actually happen when Armstrong jumped?* The student recanted!

7.9 NEWTON'S SECOND LAW OF MOTION. THE MAKING OF UNITS 9, 10 AND 11.

Because of the constraints of the syllabus and the time-table, Newton's laws of motion as a topic was not covered until after the six-hour introduction to force and motion. Consequently, no video recording was made of the teaching of this topic. However, the concept-questions that appear in units 9, 10 and 11 were used in the Socratic dialogues with the class. Student responses to the concept-questions were noted at the time, and used as student responses in the teaching package. Although some of the parallel-questions were designed in anticipation prior to teaching the topic, many of the parallel-questions were considered in hindsight. Consequently, the effectiveness of the parallel-questions were not evaluated in the pilot-study (apart from a subjective appraisal at the time of teaching).

The concept question of unit 9 appeared, word-for-word, in both the pre- and post-survey of the pilot study. In the pre-survey, 4/32 answered correctly; and in the post-survey, 11/22 answered correctly (there were 21 students who answered the question in both the pre- and post-surveys. Of these 21 students, 4 answered correctly in the pre-survey and 10 answered correctly in the post-survey). However, it must be noted that the teaching of the laws of motion as a formal topic, and the raising of the concept question, was subsequent to the class completing the post-surveys. It would appear that an understanding of the second law of motion had developed during the six hour introduction to force and motion.

It was decided to include units 9, 10 and 11 in the teaching package because it was felt that the laws of motion should not be treated as if it were a topic to be covered in mechanics. The laws of motion are the very foundations of the Newtonian paradigm (see chapter 3) and, as such, underpin all the units in the teaching package. The laws of motion are implicit in any introduction to force and motion, and should not be treated as if it were chapter 3 of a textbook.

Unit 9 involves the relation between constant force and acceleration. Unit 10, part a, follows on from unit 9 because it involves the relation between variable force and acceleration. Part b of unit 10, and part a of unit 11, loops back onto unit 9. Part b of unit 11 loops back onto part c of unit 10. Many of the parallel-questions are very similar to previous concept-questions. For example, a parallel-question in part a of unit 10 is similar to the concept-question in unit 9; and a parallel-question in part b of unit 11 is similar to the concept-question in part c of unit 10. That a parallel-question should be similar to a previous concept-question presupposes that the class has understood the previous concept-question. This reflects the strategy of developing consistent reasoning across different phenomena that require the same explanation under the Newtonian system.

No student in the class gave the second response to part a of unit 11. However, such a response was included in the package because of the reporting of such a response by McDermott (1991).

7.10 FORCE IN TWO DIMENSIONS. THE MAKING OF UNIT 12.

Similar to the making of units 9, 10 and 11 (Newton's second law of motion), force in two dimensions (unit 12) was not discussed with the class until after the six hour introduction to force and motion. However, the concept-question in part b of unit 12 was included in the pre-survey; and a similar question (see question 3 of the teaching package post-survey) was included in the post-survey. In the pre-survey, 10/32 answered correctly; and in the post-survey, 11/22 answered correctly (Out of the 21 students who answered both questions in the pre- and post-surveys, 7 answered correctly in the pre-survey and 11 answered correctly in the post-survey). About half the class could answer correctly a question involving force in two dimensions prior to the introduction of the topic. This seems to suggest that students are able to construct for themselves the first law of motion as a representation of two dimensional motion: i.e. that if no force acts in the original direction of motion, then the motion in that direction remains unchanged (see chapter 3, page 90; chapter 5, page 147 and chapter 7, page 198). In fact, most of the students had little difficulty with force in two dimensions when it was formally introduced. The parallel-questions of unit 12 were the ones used with the students that did have some difficulty with the topic.

Force in two dimensions is more complex than force in one dimension (as reflected in Graham's force hierarchy questionnaire; see Graham, 1991. An understanding of force in two dimensions consists of level 3, the highest level, in Grahams hierarchical model of

conceptual understanding of force and motion). Because of this it was decided to place force in two dimensions towards the end of the teaching package.

7.11 SUMMARY: INSTANCES WHEN ANOMALIES CAUSED THE INTUITIVE SCHEMA TO BECOME UNSTABLE

The making of Unit 3 revealed two patterns of reasoning:

- Some students can develop their personal and intuitive schemata of force and motion with remarkable consistency and coherence, until an anomaly contradicts the ontological status of force.
- The greater the entrenchment, the greater the cognitive conflict necessary to displace the intuitive schema.

The discussions that led to the formation of unit 3 spanned three sessions. After the first session I had felt confident that the parallel question of the car slowing down had countered the intuitive response that a force is required to overcome gravity (see page 194). During the second session I became aware that many students still regarded force as a property of the body (see pages 194 and 195). This suggests that a discussion with the class that leads to a number of correct responses from different students is not a guarantee that the intuitive schemata developed has become unstable or that the target concept has indeed been reached. However, it was by pursuing the three 'Aristotelians' that the two patterns of reasoning became apparent. In pursuing the three 'Aristotelians' the class became unsettled at one point (page 196), which suggested that it was only the 'Aristotelians' that were having problems with cognising force as that which changes the state of motion. Consequently, I was tempted at one stage to abandon the discussion for the interests of the class as a whole. However, I had decided to continue in the hope of revealing some pattern in student reasoning (ironically, the class as a whole may have benefited from the

continuation of the discussion). As the discussion continued, so the Aristotelians became more and more entrenched (see pages 196/197) - until Galileo's strategy was used, unstablising their ontological status of force. It was apparent that their intuitive schema had become unstable because Galileo's strategy not only 'stopped them dead in their tracks', but also created facial expressions of cognitive conflict. That Galileo's strategy had 'done the trick', so to speak, was verified by their correct response to subsequent concept questions (see page 147).

Galileo's strategy may be considered as the major catalyst in unstablising the intuitive schema of force and motion (provided it is used in conjunction with the possible world of no gravity and the parallel situation of a car braking. See pages 147/148). However, there are other examples, although perhaps not so pronounced, of intuitive schemata becoming unstable as a result of strategic questioning. For example, the schema of a thrown ball being at rest at the top of its flight (page 201), and the absurdity of weightlessness on the moon (page 212).

Future research could establish both patterns of reasoning in the form of a law. Unit 3, and groups of strategic questions that are similar to unit 3, could be tested in clinical interviews and then put into classroom practice. There would be a need at some stage for inter-observer triangulation if the effectiveness of such laws in practice are to be verified.

CHAPTER 8 - THE EMPIRICAL EVALUATION OF THE TEACHING PACKAGE

8.1 INTRODUCTION

Over one hundred schools throughout the United Kingdom were invited to participate in the evaluation of the teaching package. In the introductory letter and response form (appendix D, page 307), the schools were asked whether they would like to participate in the evaluation of the package, either as a control and/or experimental group, and/or to review and comment on the package. Initially, eighty-three schools agreed to participate in some way. The following table shows the level of involvement that the eighty-three schools were initially willing to commit:

Comment only	12
Experimental only	0
Control only	4
Comment and/or experimental	6
Comment and/or control	8
Experimental and/or control	2
Comment and/or experimental and/or control	51

Table 1 - The Initial Number of Schools Willing to Participate in the Experiment

Of the 4 schools that were willing to participate as a control group, and the remaining 61 schools that would perhaps participate as a control group, 30 schools had used concept

questions together with text-books that had a qualitative approach to mechanics (e.g. *Mechanics 1*, MEI Structured Mathematics; Berry *et al*, 1993). Given the teaching approach of the sample of 30 schools, it was decided to select the control groups from the sample. Similarly, the experimental groups were chosen on the basis as to whether the schools used concept-questions and the appropriate text-book. Similar to the pilot-study, the object was to see whether there was a significant difference between the use of concept-questions incorporated into the teaching of mechanics and the structured use of concept-questions within strategic questioning.

After reviewing the package, some schools had decided to drop-out of the experiment. It was also decided that some of the schools that were willing to participate in some way should participate in the moments survey (chapter 9) instead. Most schools that had shown an interest in the project were given a copy of the package. A preview of the package might have created a bias if the school became a control group. However, it was felt necessary to allow schools to preview the package if they were to decide whether to participate in the project and to what level (whether to review the package and/or to become a control group and/or an experimental group). The results of the experiment shown on page 220 indicate that a preview of the package made little difference to the use of concept-questions, whether the school became a control group or an experimental group. After the negotiations and decisions, 8 schools became the control group; 9 schools became the experimental group and 1 school involved two classes, 1 experimental and 1 control (3 other schools were initially involved in the project, but 2 failed to complete the project by not returning the required post-tests and 1 completed the pre-test after the introduction to force and motion. These 3 schools are excluded from the results of the project). In addition, a university foundation engineering class in the year following the pilot-study became an experimental group. 90 students were involved in the control group, and 130 students were

involved in the experimental group (out of the 9 control groups and 11 experimental groups, there were 298 students involved in the project, but only 220 students managed to complete both the pre- and post-tests).

Similar to the experimental groups, the control groups were introduced to force and motion, qualitatively, with the use of concept questions. The discussions of force and motion with the experimental groups, however, were to be structured according to the teaching-package. Not necessarily structured according to the order of student response and strategy as exactly laid out in the package, but structured according to the original concept question as stated at the beginning of each unit followed by asking parallel questions to student responses. Which parallel question to ask would be determined by the teacher according to the response of the class. The student response and corresponding strategy as stated in the package served merely as a guide. It was stated in the introduction to the teaching package (inside front cover, see appendix A , page 283):

For each concept question is a list of anticipated student responses. For each response is an appropriate reply that will (hopefully) forearm the teacher.

It was also stated at the back of the package (under the heading *Appendix 2: The Rationale of the Package*. See page 302):

The 'delivery' of the package should take the form of a structured discussion. The teacher asks concept questions, but gives no information regarding how the questions can be answered according to the Newtonian system (with the possible exceptions detailed in appendix 1).....The activity must be structured. In discussion, many students raise points or ask questions that they consider to be important (at least for themselves personally!). If all the points and questions raised are pursued then the discussion will at best 'meander'. The discussion may completely stray from the idea to be established, with disastrous results (a restless class who feels it has learnt nothing). Structure to the dialogue consists in maintaining the issue at hand, and requires a certain skill.

Both experimental and control groups were pre-tested and post-tested (see appendix E, page 318). The analysis of covariance on the results of the pre- and post- tests (see

appendix C, page 312) reveal no significant difference between the experimental and control groups:

GROUP	PRE-TEST		POST-TEST		
	Mean	S.D.	Mean	S.D.	Adj. Mean
Control	10.48	3.58	14.59	3.60	14.31
Experimental	9.73	4.07	13.96	3.33	14.15

$$F = 0.22 \quad p = 3.89$$

Table 2 - Results of the Pre- and post-test

Do the results imply that the method of strategic questioning makes no significant difference in the use of concept questions as a teaching strategy? Research into how the experimental groups implemented the package would throw light onto the failure of the empirical evaluation of the package to show any significant difference between the control and experimental groups; unfortunately, schools were not visited and no classroom observations were made. However, we can answer the question by examining the responses of teachers to the package. The next section will examine teacher reactions to the package by analysing the responses to the preview questionnaire (*Questionnaire on Teaching Package*, see appendix F, page 327) and, more importantly, the subsequent section will examine the response and deployment of the package as revealed by the *Log-Sheet and Questionnaire* (see appendix F, page 330) completed by the teachers of the experimental groups.

8.2 TEACHERS RESPONSE IN THE PREVIEW TO THE PACKAGE

Many teachers who previewed the package did not seem to appreciate the package as a guide to strategic questioning. Consider the following responses to question 1 and 2 of the preview questionnaire (1: *Would you be prepared to use the package?* 2: *If yes, how much time would you be prepared to allocate to use it?*):

- *No, not as a complete package. But I will attempt to use some of the units as and when appropriate.*
- *Perhaps parts of it, dealing with areas in which I already use concept questions.*
- *I'm not sure whether I would use it as a whole but parts are very good as introductions to particular lessons.*
- *Only selectively and at appropriate times in the course.*
- *Yes, but not in the way intended.....Would use as 'starter' ideas to classes, say 5-10 mins (might not include all).*
- *YES, although it doesn't really match with SMP 16-19 brilliantly.....3 or 4 lessons.*
- *Yes, I use this approach to teaching mechanics already and there are a few good additional ideas.....I would not use it in 'one go' but would pick out sections as I came to it in my usual course.*

Most teachers who previewed the package regarded the package as a possible aid to their teaching method rather than as a new strategy. The following was a letter received in lieu of the preview questionnaire:

I apologise for not writing to you earlier but, as you will have gathered from the non-return of the enclosed, we have decided not to go ahead with the trial of your package. We have discussed the matter a great deal in the department and have come to the conclusion that although your approach is probably a good one for teachers who are, perhaps, less familiar with the subject matter involved, for those of us with a strong background in Mechanics (which all members of my department have) the lack of flexibility implied by such a rigid framework would lead to inefficient teaching. Here we constantly use all sorts of practical illustrations during our teaching of Mechanics - to suit the teacher and the particular class involved. I have therefore reluctantly decided not to participate, although I am sure that the package will be of great value in schools where the teachers may have had less experience of practical teaching methods.

There was a mixed response to questions 3 (*Are the instructions/guidelines clear?*) and 4 (*Can the package be used in its present form?*):

- *The general instructions are good, but the students responses are a little simplistic and naive.....More structure and guidance are needed, especially for N.Q.T.'s who might like to use some or all of these units.*
- *Yes, but the package is as much about teaching style and the teacher's interactions with a group as it is about mechanics, therefore some inset on its use seems very desirable.....Yes, but see above. Until a teacher has become very familiar with each unit the constant references which she/he is likely to have to make to the package (to determine appropriate responses, etc.) will lead to disjointed lessons and a lowered quality of learning.*
- *Yes.....Yes, perhaps by a non-specialist, particularly.*
- *These could be better.....Not for me. I would not be able to find the responses quickly enough. I am a slow reader.*

Finally, many of the responses to question 7 (*Please make any suggestions for improvements*) revealed an uncertainty in the ability to use the package. For example:

- *More in the way of conclusions needs to be given at the ends of each unit. Many 'A' level maths teachers are pure mathematicians not physicists.*
- *I am uncertain as to who would use this package. A non-physicist would find lists of student responses and possible answers confusing I think.*
- *The teacher needs to prepare the lesson, then deliver it. For preparation, comprehensive notes are required, but during the lesson some easily-read, main-point prompt sheets are needed. Ideally the teacher would prepare these her/himself, but time! The pack falls between these two requirements, and perhaps could be improved by recognition of and provision for these two needs.*

Despite some of the comments, it was nevertheless decided not to alter the package. It was felt that the form of the strategic questioning was made fairly clear in the package: concept question → student response → parallel question → student response → parallel question, etc. A decision had to be made as to how much instruction in the use of the package should be given. It was decided not to alter the package for two reasons:

1) The teachers who were kind enough to use the package as part of the experiment would have to devote much time in the preparation required for its implementation. Much of the preparation would involve familiarity with the instructions given in the appendix as well as familiarity with the units. Any further details as to the use of the package, for example,

what pitfalls to expect in challenging student responses and how to overcome them (further advice that could have been based on the details of the pilot-study), would have made the package more cumbersome than it already was. To take an extreme example, if every possible and conceivable student response was listed, corresponding to which was every possible strategy, then the package might be unworkable in practice (on hindsight, however, it might have been appropriate to provide, from the transcripts of the pilot-study, examples of what to expect).

2) Although I am a qualified teacher, nevertheless my role in the project was that of researcher. Ethically and professionally, it was not my role to tell teachers how to teach. I gave details on how the package should be used, but ultimately the implementation of the package had to be left to the skill of the teacher. Although the teachers chosen were already familiar with concept questions as a teaching-aid, nevertheless the Socratic dialogue may have required a change in teaching style that the teacher would have had to accommodate. Once it was understood that the implementation of the package required strategic questioning, then it was left to the professionalism of the teacher to implement the package in the best way that he or she could (phone calls were made to the schools concerned to see if any difficulties were encountered, and to answer any queries).

8.3 THE RESPONSE TO THE QUESTIONNAIRE FOR TEACHERS INVOLVED IN THE 9 EXPERIMENTAL GROUPS.

Based on the introduction to force and motion in the pilot-study, the introduction to the teaching package (inside front cover) recommended 6 hours implementation. The following table shows the amount of time (in minutes) actually spent on each unit (taken from the *Log Sheet and Questionnaire*, appendix F, page 330):

Unit School	1	2	3	4	5	6	7	8	9	10	11	12	13	Tot al
1	5	2	2	2	1	0	2	3	1	1	0	2	5	26
2	15	10	5	5	5	3	2	5	4	5	4	6	2	71
3	9	14	7	9	1	1	3	2	4	3	9	2	2	66
4	10	4	6	2	2	5	2	8	3	8	3	5	3	61
5	5	10	10	3	5	5	5	15	5	10	5	10	10	98
6	13	15	10	3	5	8	5	10	5	3	5	5	2	89
7	10	10	15	5	10	?	?	?	?	?	?	8	1	59
8	10	10	10	?	?	?	?	?	?	?	?	?	?	30
Average (median)	10	10	9	3	5	4	3	8	4	4	5	5	2	

NOTE: No log sheet returned by school 9 (see following page)

Table 3 - The Time Taken for the Experimental Groups to Complete Each Unit

An average(median) total time of 63.5 minutes was spent implementing the package (mean: 62.5minutes). Unit 3, probably the most important unit of the whole package (as stated in appendix 1 of the package), and which took approximately 1 hour in total to implement in the pilot-study, took only an average of 9 minutes in the experiment. The time taken for each unit, especially the time taken for unit 3, may reflect a lack of understanding of what was involved in the strategy of strategic questioning. [The school 1 teacher who spent 2 minutes on unit 3 (and a total of 26 minutes on the package) stated on the *Log sheet and questionnaire* question 9 (*Any improvements you can suggest*):

Solutions for each unit would have been helpful. Some units were tricky for those more used to syllabus/exam question teaching, and confidence is needed that you are giving students correct information.

The style of such a package is the best way to start a mechanics course, although it was a pity that in this year all my new students were very much on the ball!! - on the other hand it should make my job easier!!

The pre-test result for school 1 was 9.2/20 and the post-test result was 13.2/20, giving an increase of 4.0/20].

The following table shows the responses of schools 2 - 8 to question 9 which sought suggestions for improvements:

School	Response
2	<i>A video to show the situation under discussion - sometimes it is difficult/impossible to set up a practical situation. On a video the actual happening would be shown after discussions - especially where slow motion would show what was happening more clearly.</i>
3	<i>Less repetition. More challenging questions. Structured in difficulty. Would have liked more practical questions (with equipment) - unit 2 was very successful.</i>
4	<i>Perhaps less questions and a video to set up the situation needing discussion - teaching an evening class in school I was stuck for things to use to help them understand the situation!</i>
5	(No response given)
6	<i>These type of concept questions have an important role in the understanding of mechanics. I often use this type of question to lead to discussion but I found that <u>all</u> discussion for several lessons was a bit too much. We broke up the activities by bringing in a roll of string etc., and other aids to the lessons. We also experimented by looking at our 'weight change' whilst moving in a lift. It was sometimes difficult to respond in the appropriate or <u>directed</u> manner as discussion is spontaneous and therefore some responses and examples I used were not identical to those in your package.</i>
7	<i>A computerised version, mapping the pupil through the various responses - would have to be given as choice a), b), c), d).</i>
8	<i>Some change in the layout of the package - to make it look more user friendly.</i>

Table 4 - Response to Question 9 of the *Log-Book and Questionnaire*.

The remaining school (referred to as school 9) wrote the following letter in lieu of the Log-sheet and Questionnaire:

The units promoted interesting discussions and there were indeed many similar responses to those suggested as likely ones in the teaching booklet - thus making us as teachers perhaps more aware than previously of the misconceptions that students have at the outset of a mechanics course.

In the third double period the students were becoming impatient - we therefore summarised the 'ideas' and terms used in note form and started our own scheme.

My colleague and I found the package very interesting but felt there were too many units to be considered all at once, but that they would be very useful at 'relevant' points in the course.

The package is good as a teaching aid for teachers - particularly for those 'new' to teaching mechanics and refreshing too for experienced ones. Trying to 'guide' constructively without giving the answers was a challenge!

The new text-books are much more user friendly for students and most of the 'questions' raised in the units appear at various points in the books.

Therefore in many respects the information in the booklet about how to 'use' the student responses is more important than the actual questions.

The publication of a book based on this package as a teaching aid would be a very worthwhile exercise.

One response suggested more challenging questions (school 3), yet another response stated that solutions to the units should be given as *some units were tricky for those more used to syllabus/exam question teaching and confidence is needed that you are giving students correct information* (school 1). It seems as if the response to the teaching package is a reflection of its use within the teaching style of the teacher rather than the effectiveness of its use in strategic questioning. Many responses (schools 2, 3, 4, and 6) suggested the use of practical activities and video, which may reflect teacher preference of practical work and video rather than strategic questioning (and may further reflect the difficulty of sustaining a Socratic dialogue for any length of time). The following table lists the responses to the package:

1) In your estimation, how effective is the package in enabling you to overcome student misconceptions?	
{a} Very effective	1
{b} Effective	7
{c} Not very effective	0
2) What proportion of your students appear to have benefited from the use of the package?	
{a} A large majority.	1
{b} A majority.	6
{c} A minority.	1
{d} A small minority.	0
3) Is the package very cumbersome when having to anticipate student responses?	
{a} Very cumbersome.	1
{b} Cumbersome.	5
{c} Not cumbersome.	1
{d} Not at all cumbersome.	1
4) Is the package very time-consuming when having to anticipate student responses?	
{a} Very time consuming.	0
{b} Time consuming.	7
{c} Not time consuming.	1
{d} Not at all time consuming.	0

5) - Refer to following text	
6) Please circle one of the following:	
{a} The package has been very useful and easy to implement.	1
{b} The package has been very useful but very time consuming in the preparation of the lesson.	6
{c} The usefulness of the package has been outweighed by the time and effort required for its implementation	0
{d} Although the package requires little time and effort to implement, it nevertheless has not been successful.	1
{e} The package has been a waste of time with regard to the effort required in its implementation.	0
7) In your opinion, would a more 'direct approach' to challenging misconceptions (e.g. providing the right answers to qualitative questions) be as effective as the package?	
{a} More effective.	0
{b} Less effective.	8
{c} Not at all effective.	0
8) In the light of your experience, do misconceptions warrant the need for such a package?	
{a} Yes, a lot.	4
{b} Yes, a little.	3
{c} Yes, occasionally.	0
{d} No.	0
No response	1

Table 5 - Teacher Responses to the Teaching Package Questionnaire (*Log Sheet and Questionnaire*):

The positive response to the package (e.g. the effectiveness of the package, that students have benefited from the use of the package etc.) may be more to do with the use of concept questions rather than the package as an aid to implementing strategic questioning.

The following are the responses to question 5 (*Were all your student responses predicted in the package?* - not given in table 5):

- *No.* (For unit 3, the teacher stated that there had not been too much difficulty with the unit as most students were doing A-Level physics).
- *No.* (For most of the units, the teacher stated that the students had little difficulty).
- *No - but mostly I did my best to 'stick to the script' but, especially in the latter modules, I kept to its spirit.*
- *Mostly.*
- *Yes. I found the layout of the package made it difficult to use.*

That some teachers found it difficult to sustain a Socratic dialogue for any length of time may find its reflection in the student response to the use of the package. Strategic questioning may have gone against student expectation, and the class may have become bored if the discussions lasted the whole period of the lesson. The next section will analyse the student response to the introduction to force and motion with the use of the package.

SECTION 8.4 THE RESPONSE TO THE *STUDENT QUESTIONNAIRE ON THE INTRODUCTION TO MECHANICS*.

The following table shows the results of the student questionnaire (see appendix F, page 334). 51 students (from 5 schools) completed the questionnaire:

1) Were you able to follow the discussions?	
{a} All of the time?	11
{b} Most of the time?	35
{c} Some of the time?	5
{d} Very little of the time?	0
{e} Not at all?	0
2) Did you find the discussions helpful in your understanding of force and motion?	
{a} Yes, a lot.	19
{b} Yes, a little.	29
{c} Not at all.	3
3) Have you changed the way that you think about mechanics?	
{a} A lot.	11
{b} A little.	36
{c} Not at all.	4
4) Have you been made to really think hard?	
{a} Yes, a lot.	24
{b} Yes, a little.	26
{c} Not at all.	1

Table 6 - Responses to the Student Questionnaire on the Introduction to Mechanics.

It must be stated that two students from one school crossed out 'discussions' in question 1 and wrote 'lessons', implying that the package was used rather in the manner of a

traditional lesson. Although overall there was a positive student response to the package, nevertheless the response has to be judged relative to **how** the package may have been used. It was evident from the following student responses to question 5 of the student questionnaire (table 7) that many 'traditional' topics (examples given were projectiles, vectors, resolving forces, equations of motion) were taught by some schools within the introduction to force and motion. This would account for why many post-test returns were delayed for up to six months. The following table shows some of the student responses to the introduction to force and motion, and may throw light onto how the package was implemented:

5) Please comment on those parts of the discussions that you {a} particularly liked or found interesting, {b} particularly disliked or did not find interesting.
{a} <i>I liked talking about equilibrium because it meant I understood it better afterwards.</i> {b} <i>I didn't like talking about tension and thrust because I didn't really understand it.</i>
{b} <i>Repeated questions. Questions which are concerning different aspects of mechanics should have different ideas or models; as it gets rather tedious with the same ball hanging from two pieces of string, or the book which stays at rest for ever on a slope.</i>
{b} <i>Boring things like a ball being suspended from a ceiling - this doesn't happen in real life. [It is likely that the student has had no experience of ballrooms!]</i>
{b} <i>The simpler examples were slightly tedious!</i>
{b} <i>Nothing was unlikable but there were a large number of questions and interest wanes after a while. Also, many of the questions were just repeating questions about Newton's laws or similar physics laws so you were just repeating your answer in a different form for a large percentage of the time.</i>
{a} <i>Talking about everyday occurrences in terms of forces certainly made mechanics seem more relevant to life.</i> {b} <i>Sometimes there was too much repetition from example to example and I did not find it stimulating at these points.</i>
{a} <i>I particularly liked the talk which concerned stationary objects in moving objects such as the fly on the train.</i> {b} <i>With the opening topic there is a lot of repetitive work.</i>
{a} <i>I generally felt discussing questions from the textbook was very useful in understanding what to do.</i> {b} <i>I found discussions about 'crossing a river' uninteresting as, part of the time, I did not understand why something must be done, or what you should do. Even on asking, I found I would not be able to fully cope with such a question when confronted with one in an exam.</i>
{a} <i>Clarification of physics work from previous years.</i> {b} <i>Repetition through several similar examples. Reasonably simple assumptions of modelling complicated by a lot of new vocab.</i>
{a} <i>Found demonstrations and diagrams easier to understand than just discussions.</i> {b} <i>Did not like assumptions which had to be made as I found it confusing as to when certain assumptions were made.</i>

<i>{a} Real life examples and class discussions.</i>
<i>{b} Not having physical examples of some of the discussions.</i>
<i>{b} The discussions were all mainly just theory and no numbers could be used so sometimes they seemed not to be leading anywhere and repeating themselves.</i>
<i>{b} Modelling as in the flow diagrams of a real world problem → model → mathematical problem → conclusion, seemed pointless.</i>
<i>{b} Modelling a particle.</i>
<i>{b} Mathematical modelling.</i>
<i>{b} I did not really find this part of the course interesting, as it did not involve any maths, it was based on physics.</i>
<i>{a} Space and vacuum. I found mechanics and forces much easier to understand if it is in space.</i>
<i>{b} Cylinder with a rope rapped around it. I found that difficult to understand.</i>
<i>{a} The concept of how a yo-yo acted when pulled horizontally on a table which taught me that if there was only one force acting on an object the object couldn't move in the opposite direction to that force.</i>
<i>{a} Forces in air and when something is pushed.</i>
<i>{b} Talking about why something would e.g. be at a constant speed and not accelerating etc.</i>
<i>{a} The examples that backed up the theories.</i>
<i>{b} The tests were hard to understand. The questions were difficult due to the wording.</i>

Table 7 - Some Student Responses to the Introduction to Force and Motion (*Student Questionnaire on the Introduction to Mechanics*).

Many students complained of the repetition of questions that involved Newton's laws. The raising of different examples that required the same explanation under the Newtonian system was perceived as tedious! If there were a significant improvement in the understanding of force and motion compared with the control group, then the 'tedium' of having to consider the many examples could be accounted for in terms of the rapid increase in understanding. However, the 'tedium' experienced, and the lack of any significant improvement in the understanding of force and motion, seems to suggest that the examples raised were not used as a constructive process in the building of force as an abstract concept. Instead, the examples raised may have been used in an 'illustrative' capacity rather in the manner of Aron's Socratic approach - **and not in the manner of strategic questioning whereby the examples would have (hopefully) evoked cognitive conflict.**

The response to question 6 of the student questionnaire (*Has it made you want to ask*

questions about other similar situations?) was mainly positive, but it depended on which school. The following comments are particularly interesting:

- *Yes as you have to rely on things you know more than what I would usually call 'common sense'.*
- *Yes, because I sometimes get confused between what I know is correct from my knowledge of physics and what my common sense tells me.*
- *Yes - it has helped me to think about these situations in a more logical way and to apply my knowledge (sometimes successfully) to other situations.*
- *There were a few occasions when I felt that a similar situation did not fit with certain ideas and did want to query them, but on thinking about it on my own I came to same kind of explanation as to why it was so.*
- *Yes, but it would be better to work with numbers rather than theory that cannot be tested.*

7.5 TEACHER COMMENTS ON THE UNITS OF THE PACKAGE.

The following comments are selected from the *Log Sheet and Questionnaire* (see appendix F, page 330). Given the way the package may have been implemented, the teacher comments on the units in the package (given below) may not be considered in any revision of the package. The majority of comments were either 'good responses' or that there was not much of a problem 'using this one'. However, some of the comments reveal the difficulty the teacher may have had in implementing the unit or the difficulty the teacher had in the physical ideas involved in the unit.

Unit 3 (ball thrown vertically upward)

- *Force from hand of thrower got in the way for a while.*
- *Some good ideas and ones I have used before - students had some good discussions.*
- *Not too much bother with this question - most of them are doing A level physics or have done physics at GCSE Some interesting discussions about the ball in the train.*
- *Initially some thought there must be an upwards force but we discussed horizontal motion on an air cushion or ice to show that motion does not need an applied force. Train problem led to good discussion.*

Unit 4 (ball reaches maximum height)

- *Again not too many problems.*
- *Students didn't have much problem using this one.*

Unit 6 (raising the book by my hand)

- *Discussion about initial motion, motion at constant speed and motion with acceleration.*
- *Interesting - students seem to appreciate the idea behind this.*
- *Not done.*

- *Tricky this one as it depends on whether it is accelerating or not - I didn't spend much time on this as I could see it could cause confusion [if the book is raised by my hand, then the book is accelerating. The doubt of the teacher might have been a contributory factor of any confusion].*
- *I am not sure I understood the purpose of this unit. Do I accelerate the book or move at constant velocity? I presume the latter from the appendix but then am confused by some responses [ditto].*

Unit 8 (consideration of the equilibrium of forces and uniform motion)

- *Good responses on the whole - they seem to be getting the message.*

Unit 9 (rocket halving the thrust of its motors)

- *There was a bit of confusion between velocity and acceleration.*
- *Initial wrong comments. Left to think for a few moments. They quickly established that acceleration is halved.*
- *Unit 9 not done.*
- *All done enough physics to be familiar with the law.*
- *One student got very confused with this - the one with little experience of physics.*

Unit 10 (spacecraft exerting a constant force, then a variable force and then switching off engines)

- *This helped to clarify some of the confusion in the last unit. Conceptions are improving.*
- *Seem to be getting their ideas sorted out now (no hint of any cognitive conflict given!)*

Unit 11 (air puck on smooth glass table)

- *Good responses - earlier discussions paying off.*
- *Some useful discussion. Free fall scenario was put forward: *
- *Not done.*
- *There was a bit of confusion as to whether there was any friction between the air puck and the table (no indication as to how the confusion was resolved).*
- *They found the idea of an air puck and the glass table difficult to visualise.*

Unit 12 (force in two dimensions).

- *(b) is the answer isn't it? They had fun with the bomb being dropped - a lot of heated argument.*
- *A lot of heated discussion about where the pistol would land. They were a bit suspicious about the bomb and where it would land in relation to the aeroplane - but I think they agreed in the end (that the class might have agreed suggests that either cognitive conflict was not induced or it was not recognised or sought after).*

Unit 13 (dropping two stones of unequal mass)

- *Both girls realised the bodies would land simultaneously (but did they realise why?)*
- *I am not sure they are convinced that they would land together.*
- *Some are not completely convinced that they will hit the ground together though they ought to.....!*

7.6 CONCLUSION

How the package was implemented can only be conjectural. A major weakness of the project was that no observation was made on the implementation of the package. A series

of observations might have thrown more light on the way the package was implemented. For example, the illustrative use of concept and parallel questions to emphasise the target-concept rather than the inducement of cognitive conflict; the formation of misconceptions that are too resilient, and perhaps made resilient, as the class responds to the questions of the teacher. However, although observing the implementation of the package might have been insightful, nevertheless, no inductive generalisation could have been made unless most of the teachers involved were observed at least once (including those of the control group, some of whom might have employed strategic questioning - although this is doubtful!) The naiveté of the project consists in underestimating the effort required in strategic questioning and failing to take into account what would have been required in accommodating a shift in teaching style. Much time and effort of the teaching profession has been expended in the implementation of GCSE, the needs of the National Curriculum, the anxiety caused by OFSTED and league tables, etc. The co-operation of teachers in such research projects, especially under the present conditions, bears testament to their commitment and professionalism.

According to Howard (1987), the Socratic method has yet (to date) to be empirically evaluated. Howard does not give any advice as to how this may be possible, nor does there appear to be any literature on the empirical evaluation of the Socratic method. The researchers who have reported the success of the method in mechanics teaching (e.g. Hake, 1987; Halloun and Hestenes, 1987), have evaluated the method within the context of their own classes. **The results of the project seem to imply that the Socratic method cannot be empirically evaluated unless the teachers who constitute the experimental groups are trained beforehand in the Socratic method.** To make the point more forcefully, the teaching package, by itself, is little more than a fragmented taxonomy of concept questions, anticipated student responses and parallel questions. By itself, it is a teaching-aid rather

than a teaching method. For the package to become a method the teachers involved would have to use the package as a *practice*. INSET (rather than the teacher's own time) may have to be provided so that teachers can train to use such a package. What form such training would take would probably involve role-play.

The Socratic method as a *practice* would involve certain skills. For example:

- The teacher asking questions that would enable the class to answer freely without the anxiety of ridicule.
- The teacher having to cope with possible student expectation of how mechanics 'ought' be taught.
- The teacher having some expectations of what the responses to the questions might be (which was one of the aims of the package).
- The teacher maintaining a certain *fluidity* between questions and responses.
- The teacher bearing in mind, at all times, the aim of the discussion; namely, the target concept.
- The teacher gauging how the discussion is going, how to maintain interest in prolonging a discussion, when to conclude and what to follow-up, when to interject with other forms of activity, etc.
- How to recognise cognitive conflict.
- To be aware of the possibility that a number of students might 'dodge', or compartmentalise, anomalies.
- To be aware that students might 'play the game' so that peace might be maintained.
- To be aware that students could develop a coherence to their argument.

This would seem to suggest that the Socratic method can only be empirically evaluated as a practice, whereby teachers who are adequately trained are compared with a control group.

Would some form of action research provide an adequate evaluation of the Socratic method? According to Stenhouse (1975), action research is done by, rather than on, teachers. Although a pre-condition of action research is that practitioner-researchers have 'a felt need....to initiate change' (Elliott, quoted by Somekh, 1994), nonetheless the starting point of action research seems to be the practical questions that arise from concerns in everyday practice (Somekh, 1994). Although action research regards what is researched as a practice; the emphasis, however, seems to be on the improvement of existing practice (e.g. McNiff, 1988) rather than the implementation of a radical change in the actual practice itself. For Kemmis (1988), action research is a form of research carried out by practitioners into their own practices - understood not as *..expressions of practitioners' perspectives, but rather as praxis:*

Since only the practitioner has access to the commitments and practical theories which inform praxis, only the practitioner can study praxis. Action research, as the study of praxis, must thus be research into one's own practice. The action researcher will embark on a course of action strategically (deliberately experimenting with practice while aiming simultaneously for improvement in the practice, understanding of the practice and the situation in which the practice occurs); monitor the action, the circumstances under which it occurs, and its consequences; and then retrospectively reconstruct an interpretation of the action in context as a basis for future action. Knowledge achieved in this way informs and refines both specific planning in relation to the practice being considered and the practitioner's general practical theory.

The Socratic method requires training, and such a requirement might fly in the face of ethical considerations regarding the autonomy of the 'teacher-researcher' with respect to the 'outside-researcher'. In other words, the Socratic method would require the teacher as 'teacher-student' rather than the teacher as 'teacher-researcher'. According to Howard (1987), the Socratic method requires the teacher to be a 'trouble-maker' so that the current schemata of the class can accommodate ideas that are 'indeed revolutionary'. The Socratic method may therefore require a revolutionary change in the practice of teaching itself rather than the modification of existing practice.

The failings of the project include the initial assumption that the package could have been implemented without the consideration of some form of training. According to Osborne and Freyberg (1985):

Unfortunately it is all too easy for the role of teacher-as-researcher to be confused with that of teacher-as-marketeer. A teacher with a 'new' teaching package or idea can be so keen to 'sell' it to others that a cold and dispassionate analysis of the context in which it has been used, its weaknesses as well as its strengths, and its limitations as well as its potential, are not given sufficient attention. On the other hand, where teachers are able to become effective researchers sharing their findings with other teachers, the level of professionalism is very high. Despite the frequent reporting of failure in some, if not many, aspects of their aspirations for a teaching sequence, morale can remain buoyant (Freyberg and Osborne). We are convinced that many teachers have this potential for obtaining and sharing findings of a research nature - not world shattering findings perhaps but by no means insignificant - which would add spice to their teaching and make it more creative and satisfying.

The 'world shattering findings' of research into a new teaching method presupposes more than the implementation of a teaching package or the development of existing practice as action research - it presupposes the research of the very teaching method itself as a practice. This would imply that the empirical evaluation of the Socratic method is only possible within the context of the method as a practice, i.e. the empirical evaluation of the method after competence has been gained in the method.

The training required to implement the Socratic method suggest two possibilities:

- 1) Teachers are given the opportunity to train in the Socratic method through the provision of INSET.
- 2) Novice teachers (student-teachers) are trained in the Socratic method.

There may be a possible advantage of 2) over 1): novice teachers can be trained 'afresh', whereas experienced teachers may have to undergo a radical departure from their current teaching style. However, it would be an advantage if those who are training in the Socratic method were already experienced in classroom management.

CHAPTER 9 - A HIERARCHICAL MODEL OF STUDENT

UNDERSTANDING OF MOMENTS OF FORCES

9.1 INTRODUCTION

There has been much research into student misconceptions and their reasoning in mechanics. The majority of this work has been based on particle mechanics and in particular the relationships between force and motion. Other topics of mechanics have been subjected to far less attention. One of these topics is moments, an important part of mechanics that presents a stumbling block to many students. The Mechanics Diagnostic Test of Halloun and Hestenes (1985a), for example, did not contain any question related to moments and the whole area seems to have been overlooked, unlike the area of basic particle dynamics. The reason why student misconceptions of moments has been overlooked may be because moments, although an important topic in mechanics, is simply one aspect of mechanics - unlike the Newtonian schema of force and motion that structures the very way the Newtonian paradigm speaks of the world (see chapter 3). Misconceptions of force and motion might effect a qualitative understanding of the subject as a whole. Any topic that is based on the direct application of the laws of motion (e.g. projectiles, circular motion, simple harmonic motion) might be affected by misconceptions concerning force and motion. Misconceptions of force and motion might even affect those topics that are a few steps removed from the laws of motion in the mechanics hierarchy (e.g. conservation laws). *Force and motion* is more 'fundamental' to mechanics than moments, and this might explain the lack of research in student misconceptions of moments. However, this is not to undermine the importance of moments in mechanics or to underestimate the importance of misconceptions in this area. The topic of moments forms an essential part of other areas in mechanics such as rigid bodies in equilibrium, rotations of rigid bodies, sliding and

toppling, jointed rods, frameworks and centres of gravity. Particle dynamics determines the very structure of mechanics, but a large area of mechanics is affected by moments. The first aim of the thesis is to create a teaching package that would enable the challenging of misconceptions in force and motion. The second aim is to create a conceptual model of student understanding of moments that would enable a structured approach to removing misconceptions in this area and to develop a deeper understanding. Part of that aim is to see if the principle of parallel questions can be used with students when a substantial reorganisation of their natural reasoning is required. A small-scale investigation of student understanding of moments provided some indication as to the nature of the mental models constructed by students. From the details of the small-scale investigation it was possible to create a moments questionnaire (see appendix G, page 336) for use in a much larger investigation involving a national survey of 417 mechanics students. The aim of the large investigation was to form a set of levels, each of which would contain questions that demanded a similar level of understanding. Each level was to consist of a set of test questions from which, in conjunction with the student responses, a detailed description of student understanding could be derived. If a description of their understanding was simply a matter of grouping together questions that had a similar degree of difficulty, then there would be no certainty that such a model would be scaleable, i.e. that success on a higher level required success on all lower levels.

Based on the work by Hart (1981), Brown (1981), Graham (1991) and Graham and Berry (1996), the following is a set of criteria which should be satisfied by the questions forming the hierarchical model of understanding:

- 1) The test questions forming each level should have similar degrees of difficulty, i.e. the questions on any one level ought to be answered correctly by a similar proportion of students.

- 2) There should be an association between each question within each level. This association is measured with the product-moment correlation coefficient for dichotomous data, *phi*. Details of *phi* are given in appendix H, page 345.
- 3) No level should be reached without passing all the lower levels. This may not be possible in practice, but the number of students that do not comply to this criteria ought to be very small.
- 4) Although each level is determined statistically (i.e. the percentage correct for each question and the association between each question measured by *phi*), each level should also make sense from a mechanics viewpoint.

The particular aims of the project are to identify correct reasoning that could be used as a basis for assimilation, areas where accommodation would be necessary, and especially to identify frequently occurring mental constructs that are either incorrect or not appropriate foundations for the assimilation of further knowledge (and act as obstacles to the development of student understanding).

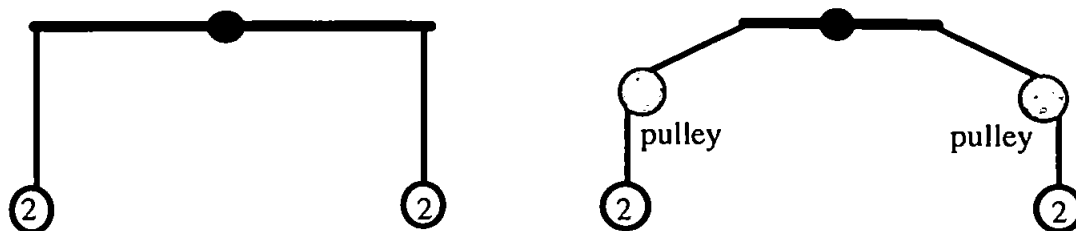
9.2 THE SMALL-SCALE INVESTIGATION

The aim of the small-scale investigation of student understanding of moments was to provide some indication as to the nature of intuitive ideas in this area. The approach taken has been to create a questionnaire to give to students in order to gain an insight into their natural reasoning. As there was little previous work to guide the investigation, a broad range of problems associated with moments were developed and from these a selection was made to form a questionnaire that could be presented to the students. A single group of students was used as the aim was to find a starting point to promote further research and to formulate initial ideas, **rather than to provide a definitive result about the state of**

student understanding of moments. The questionnaire was given to 18 university foundation year engineering students. They were a group that had followed an introductory course in mechanics, covering basic particle mechanics and the topic of moments in the context of statics problems. They completed the questionnaire some time after they had been taught the topic, so that the ideas that they would have come across would not be fresh in their minds and so that any 'natural reasoning' would have had a chance to re-establish itself. However, these students would have been exposed to a treatment of moments that tackled the topic 'conceptually' or qualitatively so they could not be considered to have their natural reasoning totally intact (just as we can expect a significant proportion of the sample in the large-scale survey to have covered moments in their mechanics lessons). It became clear that some questions were found to be very straightforward, answered correctly by almost all of the students while others were found to be much more difficult. From the results of the small-scale investigation it was possible to identify three stumbling blocks that made sense from a mechanics viewpoint and illustrated some of the cognitive issues associated with the development of an understanding of moments.

The questions that were answered correctly by all the students were concerned with situations where either forces act vertically and where the points of application were on the same horizontal level, or there was a strong degree of symmetry to the system. Some of these questions involved simple horizontal beams where the forces were either vertical, considered vertical or there was a strong degree of symmetry to the system. See figure 15:

In each case the rod is smoothly pivoted at its centre and the strings are light. The rod is held in the position shown and released. Describe what happens:



If forces are applied to a roundabout as shown, what will happen?

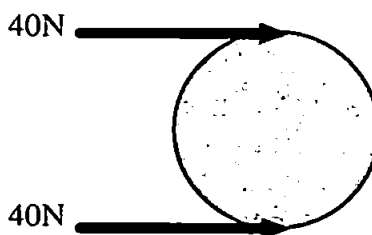
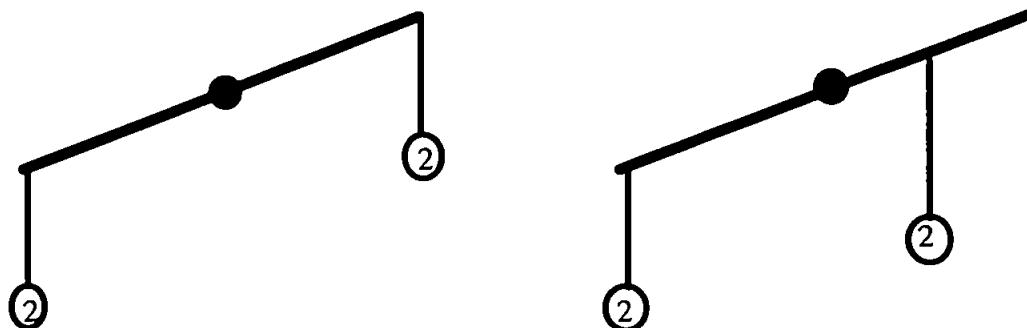


Figure 15 - Some of the Questions That Have a Strong Degree of Symmetry

:

These questions were all answered very well, demonstrating that the students had developed strategies for dealing with this type of problem. The first stumbling block seems to contain problems where the forces applied are still acting vertically, but the points of application of the forces are not at the same horizontal level. For example, see figure 16:

In each case the rod is smoothly pivoted at its centre and the strings are light. The rod is held in the position shown and released. Describe what happens:



Equal forces, F , are applied to a rod smoothly pivoted at its centre. Describe what will happen:

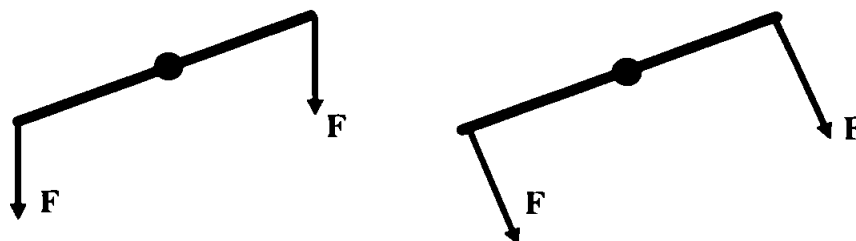
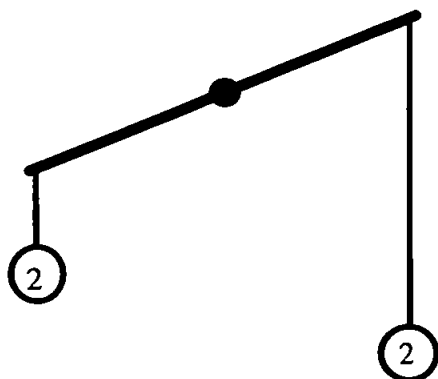


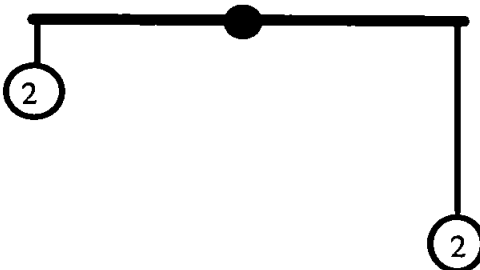
Figure 16 - Some of the Questions Where the Forces are not Acting on the Same Level

In each question the rod is in equilibrium but not in a horizontal position. For each question there were either one or two students who responded that the rod would return to the horizontal. For these students there was the association of equilibrium with the horizontal. Where the understanding of many of the 18 students begins to break down is the next stumbling block which involved questions that had no simple symmetry or appeared to have no symmetry. For example:

In each case the rod is smoothly pivoted at its centre and the strings are light. The rod is held in the position shown and released. Describe what happens.



[4/18 students incorrect]



[Same 4 students incorrect (5/18 students incorrect)]

The forces are applied to the roundabout as shown. What happens?

[3 of the same students incorrect
(4/18 students incorrect)]

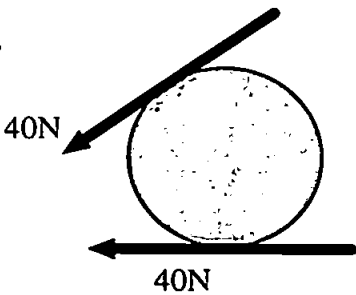


Figure 17 - Questions That Either Have no, or do not Portray any, Simple Symmetry

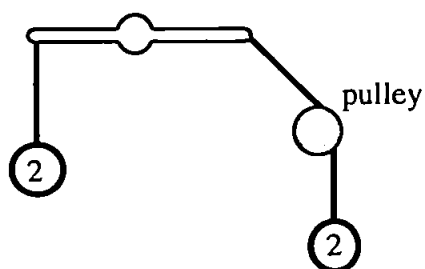
There appears to be difficulties when masses are attached by strings of different lengths, despite the stipulation that the strings are light. Several of the students expected that the rods would rotate clockwise, and subsequent discussions revealed that the students appeared to be confused between the magnitude of the force and the length of the string. Some students expected the rods to rotate anti-clockwise, and the reason given was because the left-hand mass had a greater gravitational potential energy than the right-hand mass.

The configuration representing the roundabout does not appear to be symmetrical, and so for 4 students the roundabout is not in equilibrium.

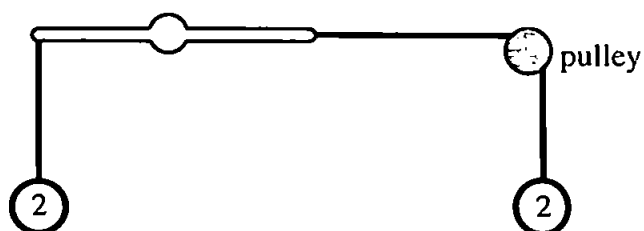
The third stumbling block contained questions that required the ability to recognise the line of action of the appropriate force and to take note of the point of application of the force.

Also included were questions that linked moments with rotation. See figure 18:

In each case the rod is smoothly pivoted at its centre and the strings are light. The rod is held in the position shown and released. Describe what happens in each case:

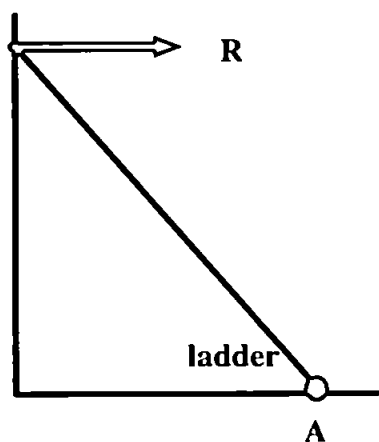


[14/18 students incorrect]



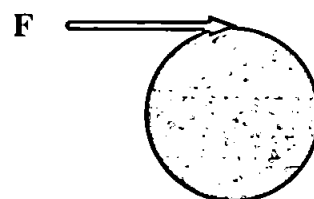
[17/18 students incorrect]

Draw the shortest distance between force R and point A :



[10/18 students incorrect]

Force F is a constant force. What happens to the roundabout?



[13/18 students correct]

Figure 18 - Some of the Questions That form the Third Stumbling Block

Consider the following responses to the two top questions in figure 18:

First Question		Second Question	
<i>Rotates clockwise</i>	11	<i>Rotates clockwise</i>	4
<i>Stays where it is</i>	1	<i>Stays where it is</i>	12
<i>Oscillates until returning to equilibrium</i>	2	<i>Don't know</i>	1
<i>Rotates anti-clockwise</i>	4	<i>Rotates anti-clockwise</i>	1

Table 8 - Responses to two Questions Representing the Third Stumbling Block

Failure to recognise the line of action of the appropriate force inhibits the ability to find the perpendicular distance of the force about a point as in the ladder question. Of the top right-hand question in figure 18, one student wrote *System turns clockwise due to right-hand mass being further from pivot than left-hand mass*. If you were to consider the line of action of the right-hand force acting on the rod as the vertical portion of the string, then the clockwise moment would appear greater than the appearance of the 'anti-clockwise moment'. One student subsequently said that she regarded the horizontal portion of the string as an extension of the rod. Reflecting on the responses to the first question, perhaps those 11 students who thought that a clockwise rotation was possible did so because the pulley was not in the way as in the second question. This may account for the 12 students thinking that in the second question the rod would stay where it was. All the students thought that the roundabout would rotate clockwise, but only 5 students thought that the rate of rotation would increase.

From the small-scale investigation there would appear to be a number of factors that inhibit the development of student understanding of moments. These are:

- 1) the expectation that a beam must be horizontal to be in equilibrium;
- 2) situations that do not exhibit any symmetry;
- 3) the reluctance to consider the line of action of a force rather than its point of application;
- 4) the failure to recognise the appropriate force acting on a beam;
- 5) the expectation that a constant force would maintain a constant rate of rotation.

The sample of students that answered the questionnaire was very small and therefore the results cannot be considered conclusive. However, it was a pilot-study and served as a starting point in identifying the possible hierarchical levels of understanding moments and provided a glimpse into the cognitive processes needed to reach an understanding of moments. The small-scale investigation questionnaire was redesigned and expanded in the light of the possible hierarchical levels of understanding moments. The redesigned questionnaire (see appendix G, page 336) was used in the large-scale investigation involving 417 students across the UK who are studying A-level or equivalent in mechanics (as a subject in mathematics).

9.3 THE LARGE-SCALE INVESTIGATION

A total of 417 student questionnaires were completed and returned for use in the analysis of the results. The participating students were all studying A-level (or equivalent) mathematics and drawn from a wide range of backgrounds including public, comprehensive and grammar schools as well as sixth form colleges and colleges of further education. Also included were foundation year engineering students.

The analysis of the results led to the formulation of a model of the development of student conceptual understanding. This model consists of three levels through which student understanding passes as it develops. The students were allocated to one of these four levels, with those not attaining the lowest level being assigned to a level 0. The model fits the survey well with only 9 out of the 417 students (2.2%) not conforming to it. The next section begins with a description of the evidence used in the formation of the levels. For each of the three levels the questions that were used to form the level are then given together with a description of the student abilities and the misconceptions and strategies that the students adopted at each level.

9.4 THE FORMATION OF THE MODEL OF CONCEPTUAL DEVELOPMENT

In practice it was necessary to develop a procedure for the analysis of the data that would allow the four criteria given in section 8.1 to be met. The four criteria are presented with evidence to support the formulation of the levels constituting the model of conceptual development:

1) The questions forming each level should have similar degrees of difficulty.

Not all the questions on a level would have exactly the same proportion of students giving correct responses, so for each level there would be a range of difficulty rather than a single value:

LEVEL	INTERVAL	RANGE
1	88-99%	11%
2	63-86%	23%
3	23-63%	40%

Table 9 - Correct Response Intervals for Each Level

The sizes of the intervals are not similar, ranging from 11% to 40%. However, there is no overlap; and since there are no anomalies the first criterion is satisfied.

2) There should be an association between each question within each level. This association is measured with the product-moment correlation coefficient for dichotomous data, *phi*.

The following steps describe the procedure that was used to meet this criteria:

(a) A scatter plot of percentage of students answering correctly against question number was prepared.

(b) Values of the phi coefficient were computed and tabulated for every pair of test questions.

(c) Using the data from step (b) lines were added to the scatter plot of step (a), joining pairs of questions that had phi values greater than or equal to 0.3. This value was chosen because it provided a good number of links but did not overcrowd the plot. Hart (1981), Brown (1981), Graham (1991) and Graham and Berry (1996) also used a value of phi close to 0.3. Brown (1981) found that with a criterion value of 0.4, many links between questions were formed but each link contains only a few items - if the criterion is close to 0.3 then the links span a wider range. This was also found to be the case in the moments survey. A phi value of 0.3 was chosen as the criterion for the moments survey, and the groups of questions formed by this criterion was found to be scalable, i.e. only 2.2% of the 417 students did not conform to this criterion.

(d) Each question that had a similar degree of difficulty, and a phi value of 0.3 or above, were extracted to form a group. Questions that had a similar degree of difficulty, but had values of phi lower than 0.3 (but greater than 0.2) were also included provided they were conceptually similar to the questions forming the group. Some groups that were formed were similar in difficulty yet the questions of one group had no significant phi links with

the questions of the other group. It was decided to place such groups within the same conceptual level **provided** the hierarchical model was scalable as a result. The groups of questions were tested for their scalability, that is that the students progressed through the levels of the hierarchy in order, without missing out a level. Only 2.2% of the students passed a level without passing all the easier or lower levels.

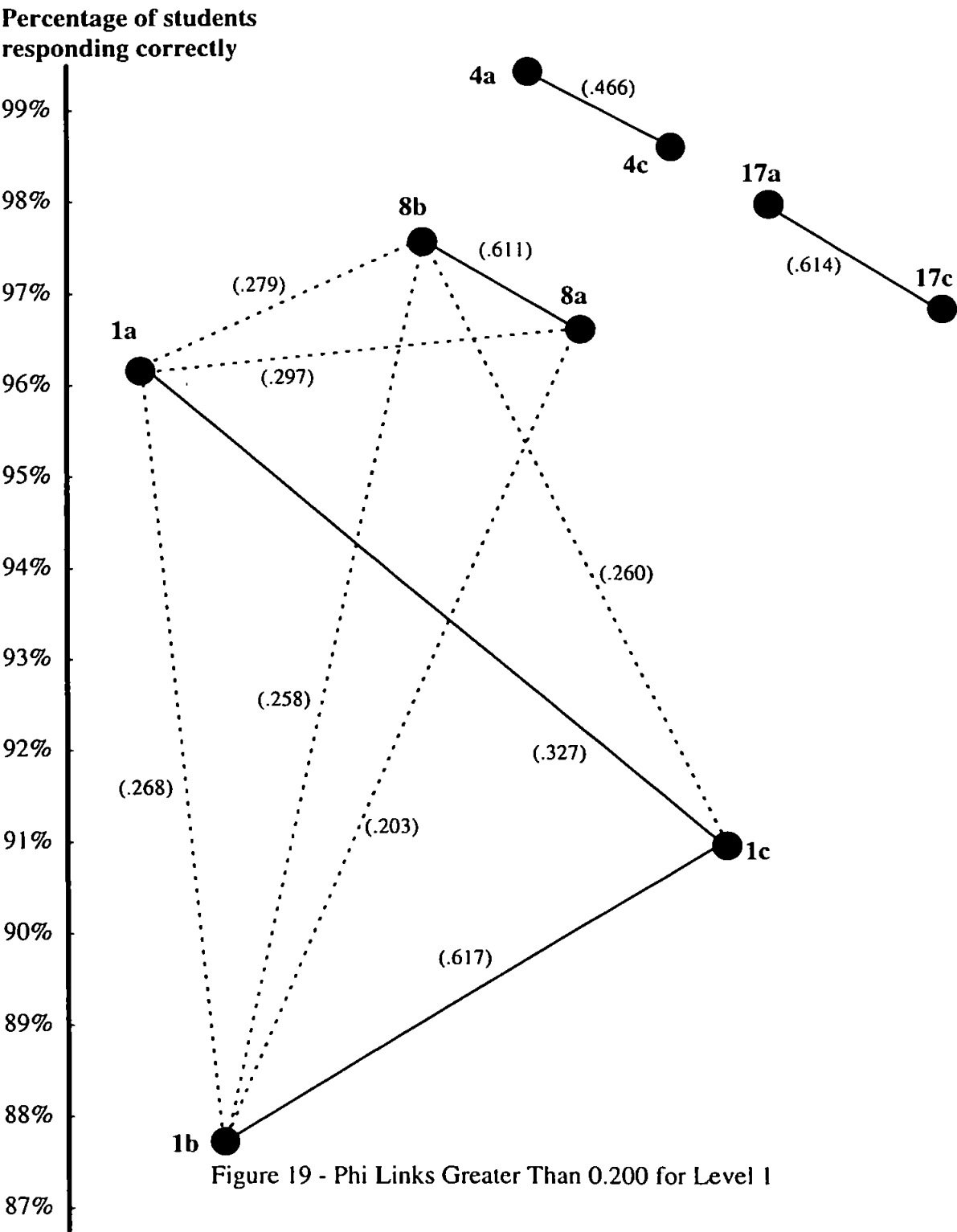
(e) A pass mark for each level had to be defined. The pass mark should be high enough to ensure that the students who attained that level are able to answer most of the questions at that level, but not so high as to exclude students who have answered incorrectly two or three questions yet still have an understanding appropriate to that level. Similar to Hart (1981) and Graham (1991), it was decided to establish the pass mark at approximately 70%. The percentage of students not conforming to the model was then determined. As this proved to be low (2.2%), the model was considered to be acceptable in this respect.

(f) The coherence of the levels with respect to mechanics was then examined. If any readjustment took place then the model was again tested for scalability.

Tables 10, 11, and 12 give the questions and the values of the phi coefficients for each of the levels, and figures 19, 20 and 21 illustrate these links. These tables and figures illustrate that there is a high degree of association between the questions that form each group for each level, thus satisfying criterion 2.

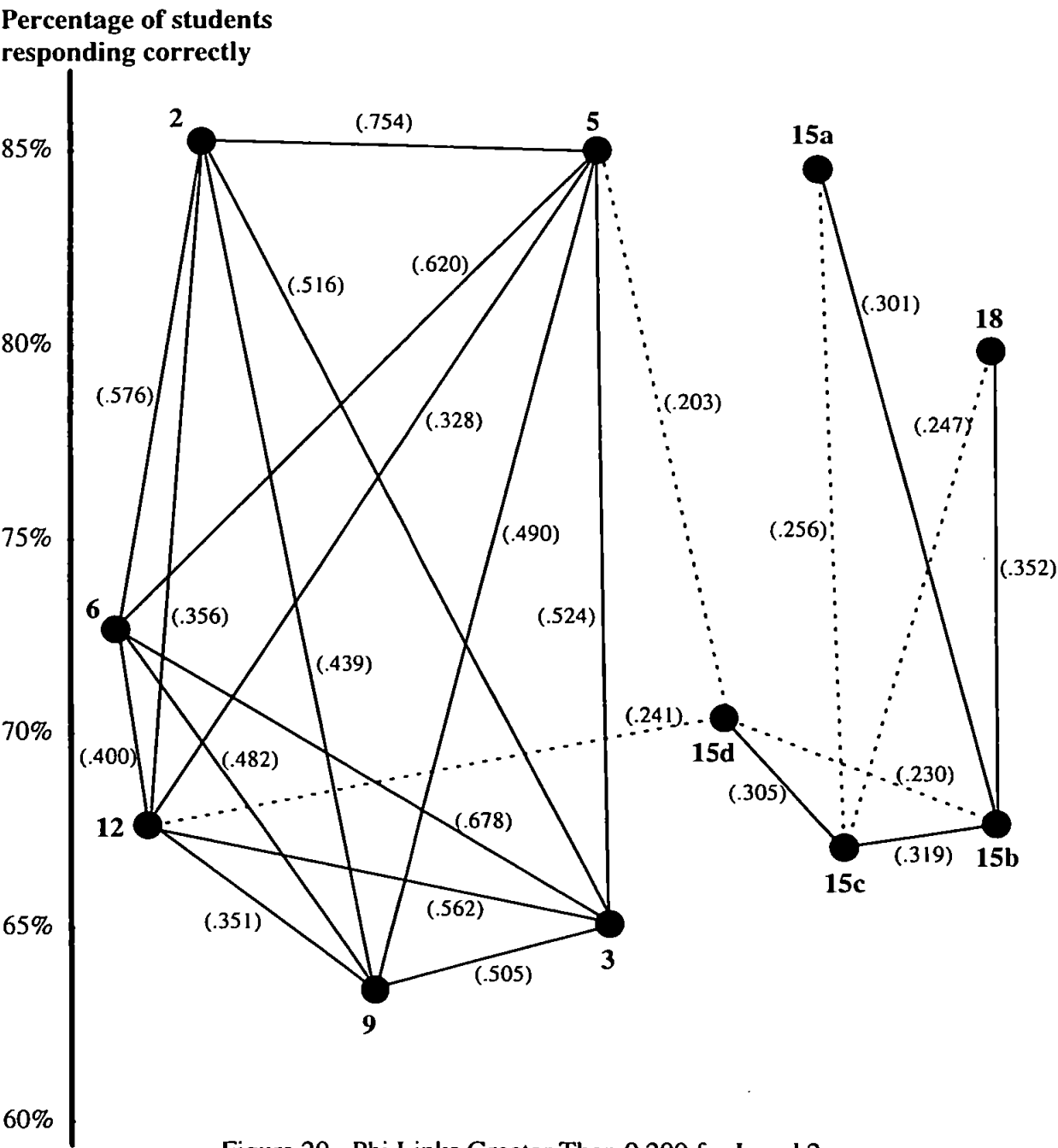
	1b	1c	4a	4c	8a	8b	17a	17c
1a	.268	.327	.131	.081	.297	.279	-0.030	.171
1b	-	.691	.141	.079	.203	.258	.096	.093
1c	-	-	.170	.172	.163	.260	.068	.080
4a	-	-	-	.466	.136	.163	-0.013	-0.159
4c	-	-	-	-	.085	.106	-0.018	-0.023
8a	-	-	-	-	-	.611	-0.029	.035
8b	-	-	-	-	-	-	.079	.135
17c	-	-	-	-	-	-	-	.614

Table 10 - Phi Values for the Level 1 Questions



	3	5	6	9	12	15a	15b	15c	15d	18
2	.516	.754	.576	.439	.356	.084	.137	.135	.152	.101
3	-	.524	.678	.505	.562	.109	.154	.140	.171	.108
5	-	-	.620	.490	.328	.107	.186	.198	.203	.168
6	-	-	-	.482	.400	.117	.182	.155	.154	.135
9	-	-	-	-	.351	.133	.192	.177	.147	.126
12	-	-	-	-	-	.147	.182	.080	.241	.097
15a	-	-	-	-	-	-	.301	.256	.163	.192
15b	-	-	-	-	-	-	-	.319	.230	.352
15c	-	-	-	-	-	-	-	-	.305	.247
15d	-	-	-	-	-	-	-	-	-	.196

Table 11 - Phi Values for the Level 2 Questions



	7a	7b	7c	10a	10b	10d	10e	11	14
4b	.259	.329	.149	.059	.120	.273	.267	.204	.493
7a	-	.874	.837	.212	.119	.171	.463	.083	.237
7b	-	-	.733	.221	.159	.210	.467	.107	.296
7c	-	-	-	.204	.143	.155	.396	.069	.175
10a	-	-	-	-	.598	.363	.285	.175	.130
10b	-	-	-	-	-	.488	.296	.158	.180
10d	-	-	-	-	-	-	.212	.323	.301
10e	-	-	-	-	-	-	-	.063	.248
11	-	-	-	-	-	-	-	-	.310

Table 12 - Phi Values for the Level 3 Questions

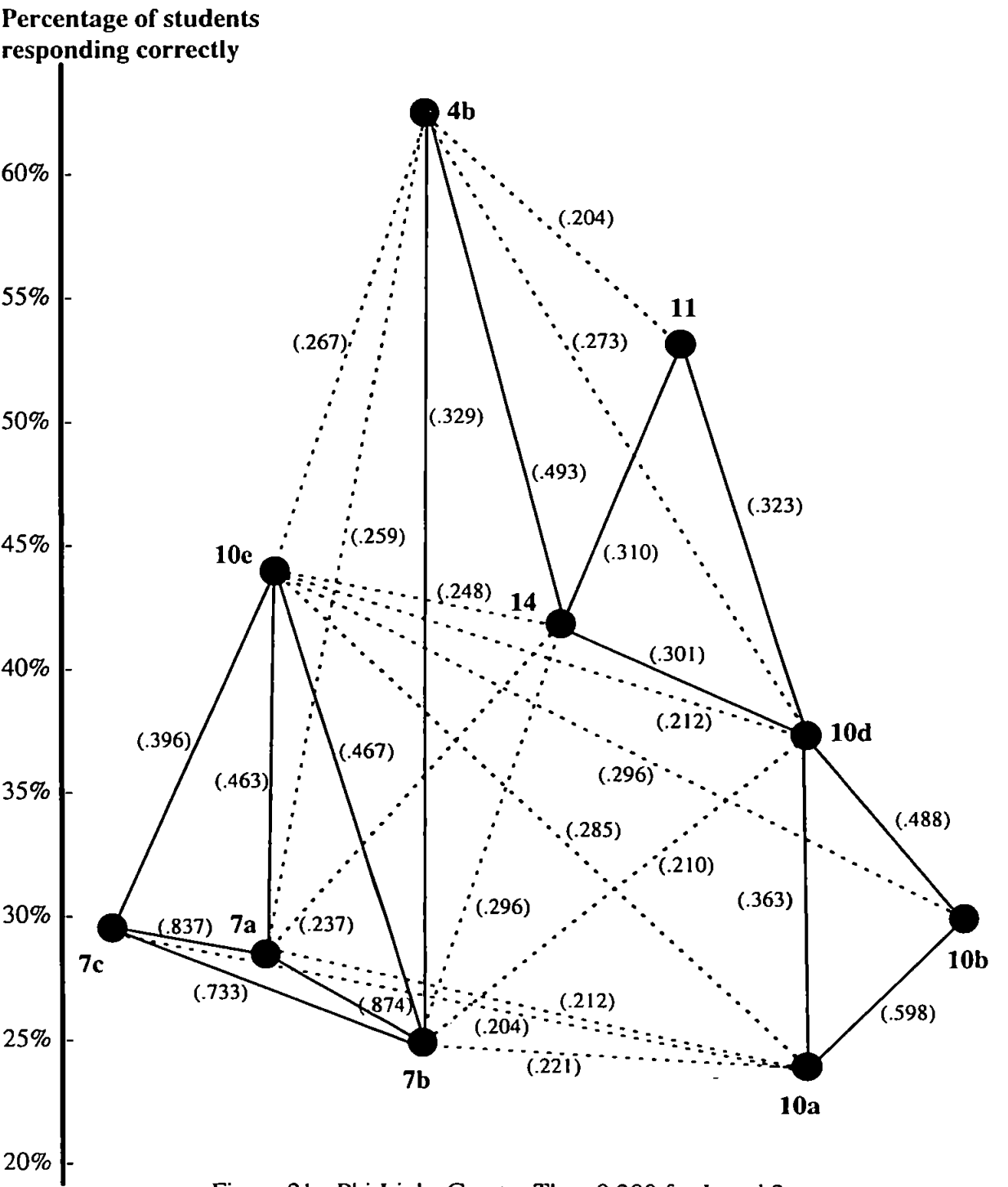


Figure 21 - Phi Links Greater Than 0.200 for Level 3

3) No level should be reached without passing all the lower levels. This may not be possible in practice, but the number of students that do not comply to this criteria ought to be very small.

The analysis leads to the formation of three levels, with those students not attaining any of the levels being classified as level 0. Of the 417 students, 9, (2.2%) attained a level without attaining all the lower levels. Thus the vast majority of the students conformed to the model.

A fourth level, consisting of 4 questions, each of which involved moments and rotation, may have been possible:

	17e (19.7% correct)	17f (18.0% correct)	17h (19.4% correct)
17b (17.5% correct)	.677	.622	.635
17e (19.7% correct)	-	.727	.733
17f (18.0% correct)	-	-	.670

Table 13 - Phi Values and Percentage Correct for Questions That Possibly Could have Formed Level 4.

If these four questions formed level 4, then 35 students would have passed this level without passing all the lower levels (all 35 students answered either 3 or 4 of the questions correctly). This would have raised the number of anomalies to a total of 44 out of 417 (10.6% compared with 2.2%), which would have been unacceptable. Although the 4 questions were conceptually linked to each other (moments and rotation), nevertheless this link would have consisted of only 4 questions. It was therefore decided to omit this level from the hierarchy. [The majority of responses to the 4 questions were either correct

statements of the sense of rotation and nothing else, or stated that the rate of rotation would be constant. The responses to the ‘roundabout’ questions involving moments and rotation will be discussed in section 9.8, page 266].

Table 14 shows the distribution of students to levels and the pass mark assigned to each level (excluding the 9 anomalies):

Level	Number and Percentage of Students		Pass mark	
0	14/408	3.4%		
1	132/408	32.3%	7/9	78%
2	195/408	47.8%	8/11	73%
3	67/408	16.4%	7/10	70%

Table 14 - Distribution of Students to the Levels of the Hierarchy

4) Although each level is determined statistically (i.e. the percentage correct for each question and the association between each question measured by phi), each level should also make sense from a mechanics viewpoint.

The questions that have been grouped together to form a level should make sense from a mechanics standpoint, and should therefore give an indication of the abilities that are required to reach that level. The phi links between each question grouped together implies the existence of conceptual links between the questions (if two questions have a similar number of incorrect responses, then the greater the number of incorrect responses that are from the same people answering the two questions, the greater the possibility that the reasons for the incorrect responses to one question are the same as for the other). A description of the questions forming each level is described below.

Level 1

Because only 3.4% of the sample (14/408) failed to attain this level, it would therefore be expected that the vast majority of mechanics students would be able to:

- 1) perform simple calculations involving horizontal beams;
- 2) recognise that a system is in equilibrium because of its strong degree of symmetry.

Level 2

To achieve an understanding at level 2 the students should be able to

- 1) recognise the perpendicular distance between a force and a point in a number of different schematic but simple configurations (such as the force and point given in the context of a ladder leaning against a wall, or when the force and point is simply given),
- 2) recognise whether a simple schematic representation of two forces acting on a rod, pivoted at its centre, is in equilibrium.

Level 3

To achieve an understanding at level 3 the student should be able to

- 1) recognise the line of action of the appropriate force and take note of the point of application of the force,
- 2) not confuse the tension of a string with the length of the string,
- 3) not confuse the horizontal as the 'natural' orientation with equilibrium.

This summary shows that there is a meaningful development through the levels of the hierarchy, which satisfies criterion 4). The levels are described in more detail in the

sections that follow. These sections also illustrate a meaningful progression in the development of the students understanding, and offer possible teaching strategies.

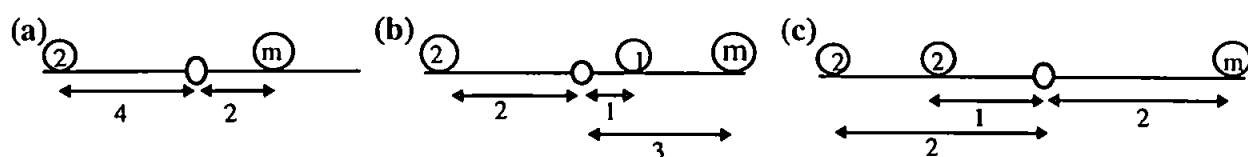
9.5 DESCRIPTION OF LEVEL 1

(Percentage correct for each question, calculated from the whole sample of 417, shown in brackets).

Questions Forming Level 1:

Each rod is light and in equilibrium, smoothly pivoted. The masses are in kg and the lengths are in metres.

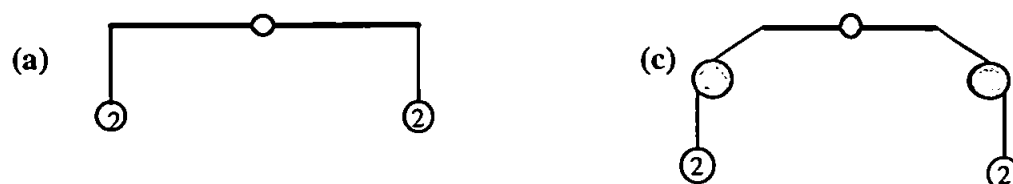
1. What is the value of m . [96.2%, 87.8%, 90.9%]



8. What is the value of x ? [96.4%, 97.4%]



4. In each case the rod is smoothly pivoted at its centre. The masses are in kg and the strings are light. The rod is held in the position shown and released. Describe what happens in each case. [99.3%, 98.6%]



17. Forces are applied to a playground roundabout as shown. Describe fully the motion of the roundabout in each case. [97.8%, 96.6%]

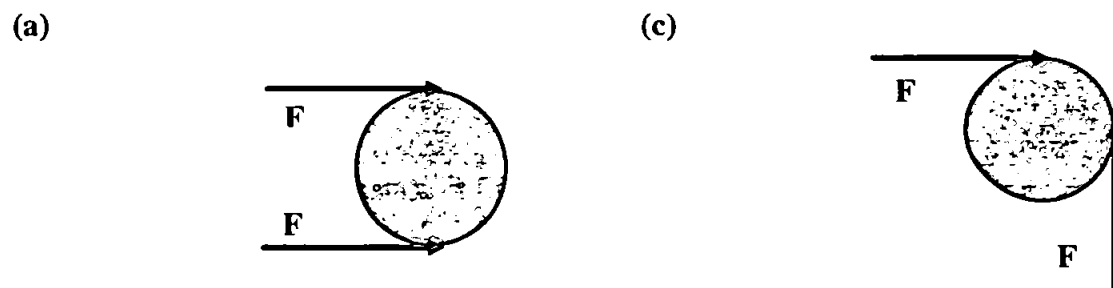


Figure 22 - Questions Forming Level 1

Nearly all the students have the ability to calculate unknown masses or unknown lengths in the context of horizontal beams where the forces act vertically and the points of application are on the same horizontal level. Nearly all the students could also recognise systems that have a strong degree of symmetry. For example, questions that involve horizontal beams where the forces were either vertical as in 4(a), considered vertical as in 4(c) or questions that involve forces applied to a disc in such a way that the configuration had a very obvious symmetry, as in 17(a) and 17(c). These nine questions were the only questions that lay within the 88-100% interval.

These questions were all answered very well, demonstrating that the students had developed strategies for dealing with this type of problem. It is very easy for students to assimilate the principle of applying moments in these situations of symmetry because it relates so closely to their previous experience. For example, the single beam balances used in primary schools, or a see-saw in a park, that all move to the horizontal when 'balanced' and move away from the horizontal when 'unbalanced'. Figure 23 shows the type of balance often used.

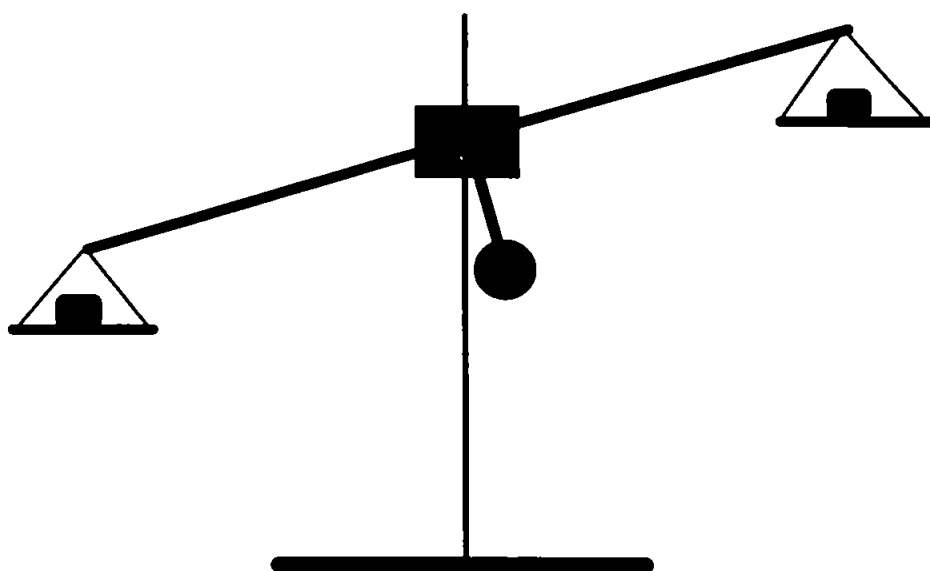


Figure 23 - Type of Scale Balance That is Often Used

While student reasoning may be dominated by the pans and their contents, the additional arm at the centre of the balance is of considerable importance and is often overlooked. While these students have developed effective strategies for dealing with a restrictive class of problems, the results of the remainder of the questionnaire suggests that they have made it more difficult for themselves to progress to a deeper understanding of moments and equilibrium. For some students the assimilation of the concept of moments has been so strongly based on an association with equilibrium in a horizontal situation that they will not be able to extend the principle easily to situations where beams are, for example, in equilibrium but not in a horizontal position. As the set of results in level 3 will show, there are students whose understanding of moments in the context of horizontal beams is developed in such a way that it is difficult, if not impossible, to extend those ideas to a beam or context that is not horizontal. To progress beyond this stage of development, these students will need to go through the process of accommodation where they have to be encouraged to reorganise their conceptual worlds to make room for a new model.

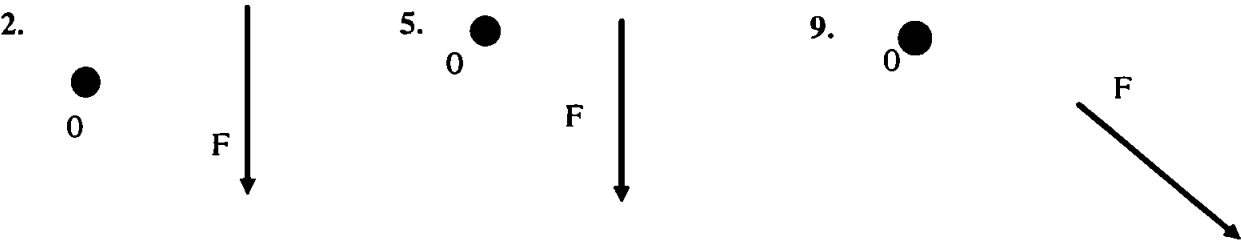
Many (and perhaps nearly all) of the students in the sample would have covered moments in the context of horizontal beams in equilibrium. Many of the students would have covered the topic in secondary school science. Prior experience of the topic, and the experience of systems that are in equilibrium when horizontal, would account for why these questions in level 1 were answered very well. Level 2 represents the first major stumbling block for many students (14 students remained at level 0 and 132 students remained at level 1 - see table 14. 35.8% of the sample, 146/408, experienced difficulties with level 2).

9.6 DESCRIPTION OF LEVEL 2

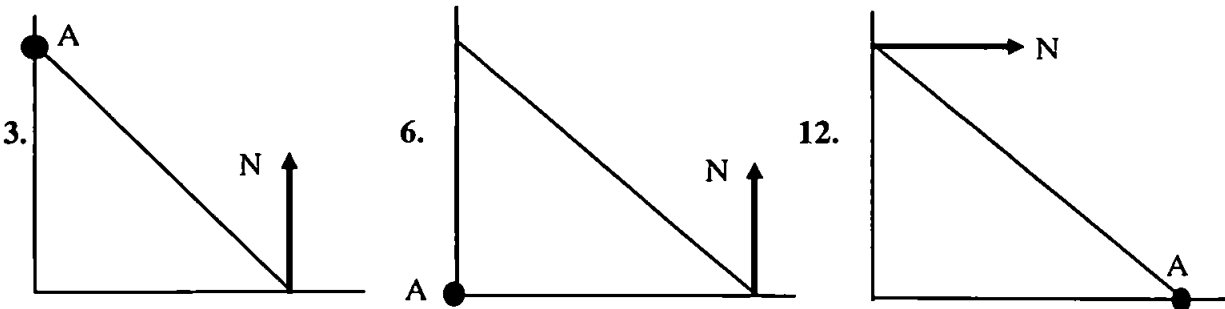
(Percentage correct for each question, calculated from the whole sample of 417, shown in brackets).

Questions Forming Level 2:

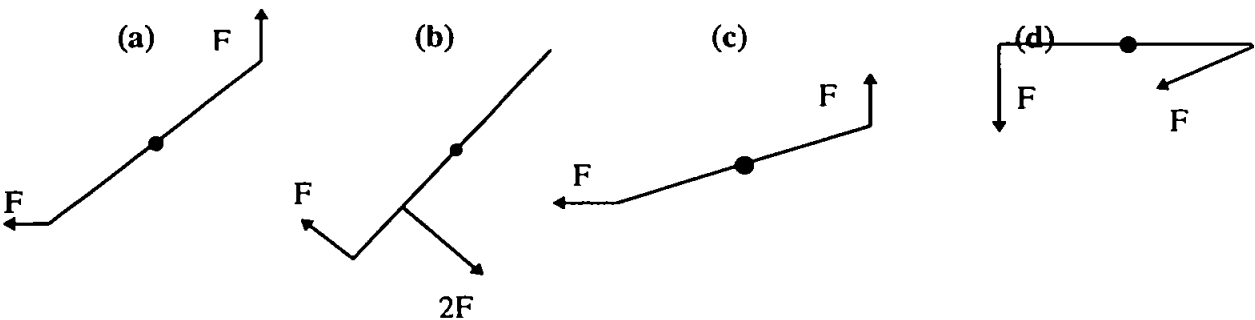
Draw the perpendicular distance of force F from point O . [85.6%, 84.7%, 63.3%]



The diagram shows a ladder leaning against a wall. N is the normal reaction. Draw the perpendicular distance of the force N about the point A . [65.0%, 72.7%, 67.4%]



15. *In each case the rod is smoothly pivoted at its centre and forces are applied as shown. If the rod is held in the position shown and released, what happens?* [84.2%, 67.4%, 67.1%, 70.3%]



18. *What happens to the lock gate?* [79.9%]

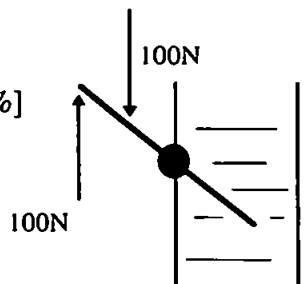


Figure 24 - Questions Forming Level 2

There were 18 types of response to question 9 (see figure 24), with 10.6% not answering the question. Those types of response above 2.0% were :

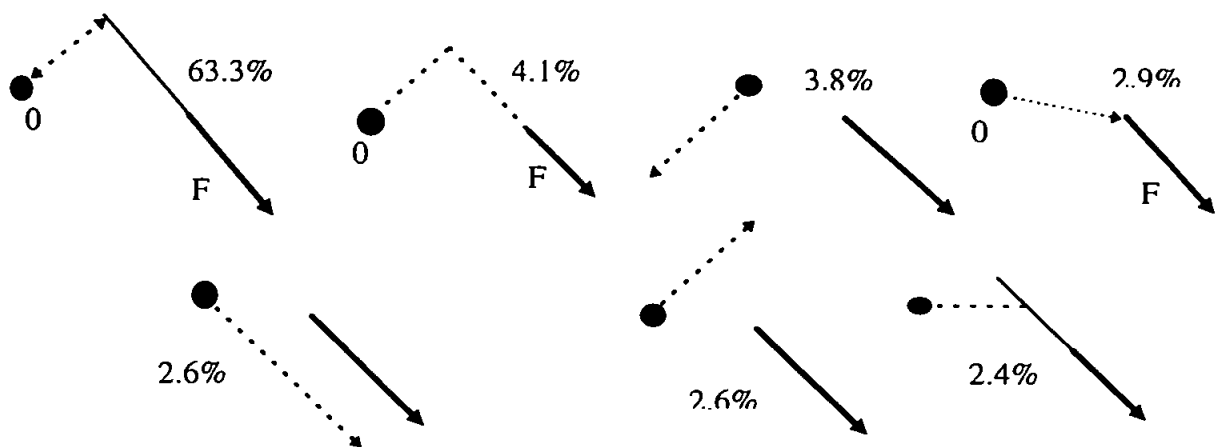


Figure 25 - Responses Above 2.0% for Question 9

The difference between the correct responses to question 9 and either question 2 or 5 is over 20%. Questions 3, 6 and 12 are very similar to questions 2, 5 and 9 (they are the same questions from a mechanics point of view, only questions 3, 6 and 12 involve a ladder); yet there is a disparity between the two sets of questions:

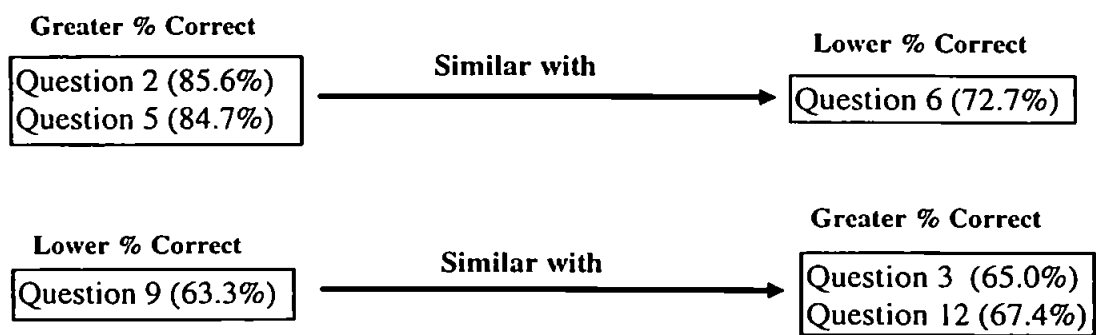


Figure 26 - Mapping Questions 2,5 and 9 to Questions 3, 6 and 12

The results imply that the perpendicular between a point and a line is fairly obvious if presented as in question 2 or 5, but not so obvious if presented within the context of a

ladder as in question 6. Ironically, however, the disparity in the percentage correct between question 9 and either questions 2 or 5 becomes significantly less when similar questions are presented within the context of the ladder (questions 3 and 12 are similar to question 9 from a mechanics point of view, yet the disparity in percentage correct of questions 3 and 12 with question 6 is much less than the disparity in percentage correct of question 9 with 2 and 5). One possible explanation is that questions 2 and 5 may be fairly obvious to a mechanics student with a school background in geometry, but question 9 may not be so obvious. Question 6 is very similar to question 9 from a mechanics point of view, but the difficulty that students have with question 6 no longer becomes significant when moments are considered in the context of ladders. The results suggest that students find the concept of the perpendicular distance between a point and a line easier to understand if the concept is simply presented within the context of a point and a line rather than with the context of a ladder. However, the difficulties that students have, when the perpendicular from the point does not intercept with the line, becomes less when presented in the context of a ladder. In other words, the teaching of moments within the context of leaning ladders may reduce the difficulties that students have with perpendiculars that do not pass through the required line. The problem then becomes the more general one of how to improve the understanding of moments as applied to non-horizontal beams or rods in equilibrium. In teaching of moments, questions 2 and 5 should be prior to 3, 6, and 12; yet questions 3, 6, and 12 should be prior to 9.

The majority of students can cope with reasoning based on the distance of the forces from the pivot, whereas the first major stumbling block appears in level 2 which requires students to begin to use the line of action of a force rather than simply the force itself - and to use the idea of perpendicular distance rather than any other distance, such as the distance between a pivot and the point of application of a force. Students may find the principle of

moments easy if presented within the context of vertical forces acting on horizontal beams (level 1). Difficulties emerge when the principle of moments is applied to non-horizontal beams, ladders or rods; especially when the line of action of the force has to be considered rather than the force itself.

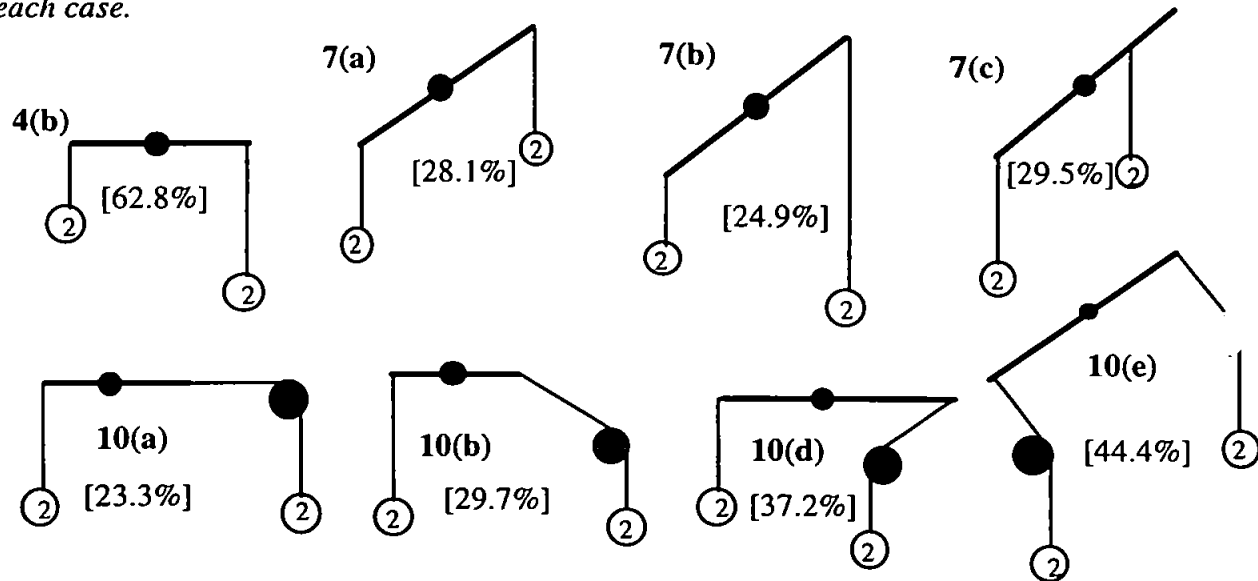
6.9% of the students thought that the rod in question 15(a) would return to the horizontal. However, only 0.5% stated that the rod in question 15(b) would return to the horizontal, with 22.1% stating that the rod would rotate anti-clockwise. Question 15(a) appears in equilibrium because of its symmetry, whereas, for nearly a quarter of the sample, question 15(b) is not in equilibrium because it does not appear symmetrical. Ironically, however, 16.5% thought that the rod in question 15(c) would remain where it is (with 5.5% stating that the rod would go clockwise and 0.7% stating that the rod would go clockwise 'until horizontal'). This may be because the configuration of forces in question 15(c) is the same as in question 15(a), despite the lack of symmetry in 15(c).

9.7 DESCRIPTION OF LEVEL 3

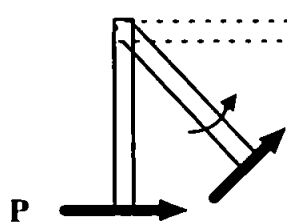
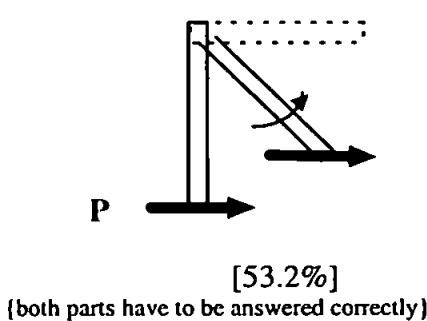
(Percentage correct for each question, calculated from the whole sample of 417, shown in brackets).

Questions Forming Level 3:

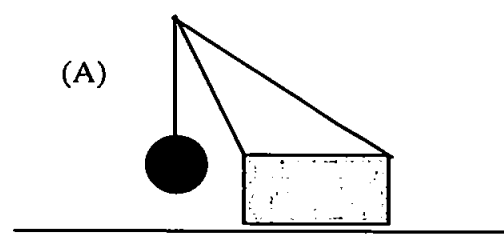
In each case the rod is smoothly pivoted at its centre. The masses are in kg and the strings are light. The rod is held in the position shown and released. Describe what happens in each case.



11. A force P is applied to a door. what happens to the moment about O if the force P always
(a) in the direction shown,
(b) at 90 degrees to the door?



14. Which crane (if any) is most likely to topple (consider the chain holding the mass to be light).



[42.0%]

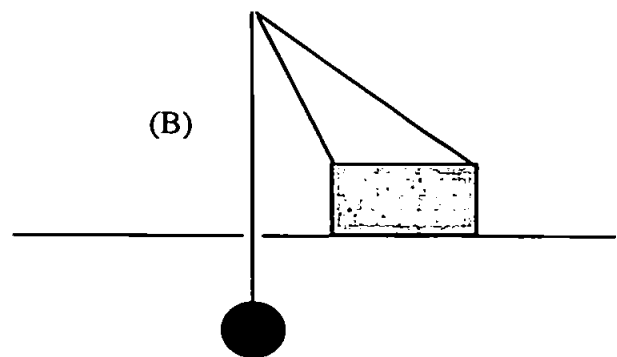


Figure 27 - Questions That Form Level 3

47.1% of the students remained at level 2, and only 16.4% passed level 3. Level 2 seems to represent a large portion of the sample who have had text-book experience of the application of the principle of moments to 'idealised' examples such as rods and ladders in equilibrium. From a mechanics viewpoint, there is no essential difference between levels 2 and 3. All the questions in level 3 involve the consideration of the principle of moments. However, the questions that form level 3 are not as 'schematic' as the questions in level 2. It would be fair to say that level 3 questions are more difficult than level 2 questions precisely because they are more complex and less schematic than those in level 2. Level 3 questions are complex because they demand the consideration of other factors, such as the line of action of the force, the point of application of the force, the height of the mass, the length of string, etc., prior to the consideration of the principle of moments. This would seem to imply that if the questions in level 3 were presented in a schematic form similar to the level 2 questions, then there would be no essential difference between the two levels. Level 3 questions, because of the additional factors involved, are 'unfamiliar' to many students, consequently their strategies for tackling level 2 questions break down and they resort to their intuitive reasoning.

There were 12 different responses to question 7(a), (see figure 27), and only 1.7% of the sample did not give an answer. **60.7% thought that the rod would return to the horizontal.** There was a similar response to question 7(b), (36.0% thought that the rod would return to the horizontal), and question 7(c), (51.8% thought that the rod would return to the horizontal). In question 7(a), 31.9% stated that the rod would rotate clockwise until horizontal, and a further 3.1% stated that the rod would turn clockwise. Similarly, in question 7(c), 25.9% stated that the rod would rotate clockwise until horizontal, and a further 11.5% stated that the rod would turn clockwise. In question 7(b) however, 13.9% stated that the rod would rotate clockwise until **vertical**, and a further 15.6% stated that the

rod would simply turn clockwise (3.1% stated that the rod would turn anti-clockwise). Many students, if not the majority, expected the horizontal to be the natural orientation for equilibrium. However, this expectation may conflict with the idea that the length of a light string would upset equilibrium. 33.5% of the students thought that the rod in question 4(b) would rotate clockwise, which might account for the mixed response to question 7(b).

From a mechanics point of view, question 4(b) is very similar to question 14, and the phi link between them is high (0.493). However, 62.8% answered 4(b) correctly and only 42.0% answered 14 correctly. In question 14, 29.5% thought that (B) would topple, 21.6% thought that (A) would topple with a further 2.2% stating that (A) would topple because of its higher centre of gravity. The phi links that connect 4(b) with 7(b) and 14 are high, which suggests that the confusion between tension and length of string is a dominant factor, along with the expectation that the horizontal is the natural orientation of equilibrium.

The responses to questions 10(a), 10(b), 10(d) and 10(e) are given in table 15:

Question 10(a)	No answer Correct answer 'Stays where it is' 'Clockwise' Other	4.1% 23.3% 64.3% 6.3% 2.2%
Question 10(b)	No answer Correct answer 'Stays where it is' 'Clockwise' 'Clockwise until in line with the string' Other	7.0% 29.7% 28.5% 25.2% 7.2% 2.3%
Question 10(d)	No answer Correct answer 'Stays where it is' 'Clockwise' Other	7.9% 37.2% 26.4% 24.9% 3.6%
Question 10(e)	No answer Correct answer 'Clockwise' 'Clockwise until horizontal' 'Horizontal' 'Anti-clockwise' Other	7.4% 44.4% 14.6% 15.3% 6.5% 8.9% 2.4%

Table 15 - Responses to Questions 10(a), 10(b), 10(d) and 10(e).

The responses to questions 10(b) and 10(d) are very similar in many respects. However, over twice as many students thought that the rod in question 10(a) would 'stay where it is' and only 6.3%, as opposed to a quarter of the sample, thought that the rod would turn clockwise. It would appear as if many of the students regarded the vertical portion of the right-hand string in each of the questions 10(a), 10(b) and 10(d) to be the force that is creating the clockwise moment of the rod: it is as if the system would turn clockwise due to the right-hand mass being further from the pivot than the left-hand mass [although it is obvious that the right-hand mass in question 10(d) is in fact closer to the pivot than is the case for questions 10(a) and 10(b)]. If you were to consider the line of action of the right-hand force acting on the rod as the vertical portion of the string, then the clockwise moment would appear greater than the left-hand moment [one student said that she

regarded the horizontal portion of the string in question 10(a) as an extension of the rod - it is possible that one of the reasons why nearly two thirds of the sample thought that the rod in question 10(a) would 'stay where it is' is because the pulley is in the way, preventing a clockwise turn]. Failure to recognise the line of action of the appropriate force acting on the rod means that the perpendicular distance of that force about a point cannot be found. Failure to recognise the line of action inhibits the ability to find the perpendicular distance of the force about a point.

9.8 THE QUESTIONS THAT LIE OUTSIDE THE HIERARCHY

Questions 17(b), 17(e), 17(f) and 17(h) have strong phi links between them, yet their inclusion would have weakened the hierarchy (see page 252). They have strong phi links with question 10(a), but that seems an anomaly. The majority of the students answered these four questions incorrectly, not because they could not recognise the sense of rotation, but because of the misconception that force maintains constant rotation. This seems to imply that the appropriate teaching strategy for force and rotation has more to do with force and motion than with moments.

Questions 8(c), 10(c), 13 and 17(d) lie in the 63%-86% interval for level 2, but have either none or very little phi links above 0.200 with any other question. Question 16 also lies within the same interval, and has many phi links with many questions (15 phi links between 0.2 and 0.3, and 2 phi links above 0.3). However, the links are formed with many different types of conceptual questions and across both levels 2 and 3. Questions 13 and 19 both lie within the 23%-63% interval for level 3, but question 13 has no significant phi link, and question 19 is similar to question 16 in that it has many phi links but none that has made any sense from a mechanics viewpoint.

9.9 IMPLICATIONS FOR TEACHING

It is very easy for students to assimilate the principle of applying moments in situations of 'symmetry', or of horizontal beams and vertical forces with their points of application on the same level as the beam. For some students the assimilation of the concept of moments has been so strongly based on an association with equilibrium in a horizontal situation that they will not be able to extend the principle easily to situations where beams are, for example, in equilibrium but not in a horizontal position. Clement *et al* (1989) have discussed the role of anchoring conceptions (intuitive yet valid preconceptions held by the student) in developing student understanding of mechanics. They have evidence to show that anchors based on symmetry tend to be 'brittle' i.e. the anchor cannot be extended analogically to the target concept. The relevance of this to the topic of moments means that the understanding of the principle in a horizontal context cannot be used as an anchoring conception by which the principle in general can be attained. Unlike the intuitive schemata of force and motion, whereby misconceptions of force are formed around what is considered to be the dominant features of motion, misconceptions of moments of forces seem to be created from concrete experiences of horizontal beams that are in equilibrium or beams that return to the horizontal. This would imply that for many students to progress beyond this stage of development, they would need to go through the process of accommodation where they have to be encouraged to reorganise their conceptual worlds to make room for a new model. The student may easily assimilate the principle of moments within the context of horizontal beams, but we cannot assume that this will serve as an anchoring conception that would extend to the principle in general. The student would find it difficult to accommodate the principle in general if his or her intuitive ideas of turning effects and equilibrium remains intact. In fact, the numerous examples of the horizontal

beam may tend to confirm the intuitive idea that the beam has to be horizontal to be in equilibrium.

In mechanics, moments may be taught in two very distinct stages. The first stage is represented by horizontal beams as in level 1. The second stage is represented by idealised non-horizontal configurations such as ladders leaning against walls. The results of the moments survey seems to suggest that students accommodate the second stage through the construction of schematic representations such as the questions that form level 2. However, these schematic representations break down when the student is confronted with similar questions that are made more complex (i.e. are less schematic). The problems that students have with level 2 seem geometrical, whereas the problems that students have with level 3 involve their expectations that a body in equilibrium will return to the horizontal, and the lack of recognition of the appropriate line of action and the point of application.

The difficulty in the transition from level 2 to level 3 in the teaching of moments may lie in the way we model bodies as particles, which overlooks the point of application of the forces involved and their lines of action. If this is the case, then we may have to seriously reconsider the way in which we represent forces that act on a body. For example, consider the forces acting on a car on an incline:

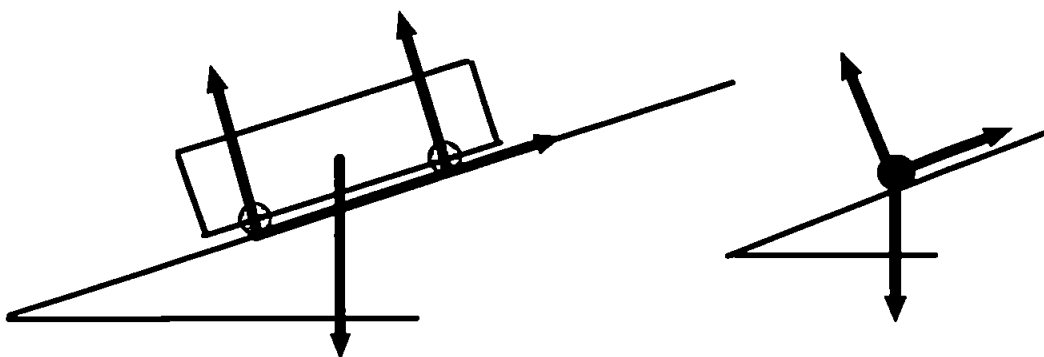


Figure 28 - Representation of Forces Acting on a Car (a) as a Body, and (b) as a Particle.

Figure 28 shows two possible approaches, one of which takes account of the positions of the forces acting. It may be important that teachers show or explain that the particle model is a simpler representation of a more complex situation. The pedagogical implications may be twofold:

- 1). Modelling bodies as particles, although essential in simplifying a problem and appropriate in many cases, can overlook the point of application of the forces involved and the resulting line of action. The particle model as a help or hindrance has already been discussed by Medley (1982).
- 2). Many textbook force diagrams are incorrect. In fact, it is from such diagrams as (b) in figure 28 that many students think of the normal reaction as the 'reaction' to the weight in the sense of the third law of motion (Warren, 1979). How can students begin to understand how to apply the principle of moments if confronted with diagrams such as (b) in figure 28. Put in another way, how can students accommodate the principle in general if their present schemas are flawed? This point has already been discussed in chapter 6 (see page 207).

Because level 2 contains schematic representations of moments, then it *seems* necessary to teach the schematic representation of moments prior to tackling level 3. However, this is not to suggest that the hierarchical model of conceptual understanding of moments requires the teaching of level 2 prior to the teaching of level 3. As a teaching method it might well prove to be difficult to extend the methods involved in answering correctly level 2 questions to answering level 3 questions. What this suggests is the opposite: the teacher directs the class or student to reduce the complexity of level 3 down to level 2 (rather than extend the simplicity of level 2 to the complexity of level 3). Level 3 presents a number of stumbling blocks for many students, one of which is the failure to recognise the line of action and the point of application of the applied forces. By asking the appropriate questions, it may be possible for the teacher to facilitate the construction of level 2 out of

what is given in level 3 so that level 3 can be understood in schematic form. For example, the configuration given in question 10(d) in level 3 'stripped' down to its 'simplified' version in question 15(d) of level 2. If a student were to argue that the rod in question 10(d) would rotate clockwise, then the teacher could ask the student to represent the forces acting on the rod as arrows [as shown in question 15(d) of level 2], and then to ask what moments act on the rod. If the student argues that the rod in question 7(a), level 3, will return to the horizontal, then the student may be invited to consider a schematic representation similar to the questions given in level 2. If the student regards the rod to be in equilibrium, but that its 'natural' resting place is horizontal, then the teacher could ask questions as to why the scale-balance on page 256 would also return to the horizontal and what forces are acting on the scale-balance.

Several issues in the teaching of moments have been considered in this section. In the next section the focus will be the influence of the results of this research on the implementation of the Socratic dialogue.

9.10 THE SOCRATIC DIALOGUE IN THE TEACHING OF MOMENTS

With the exception of question 18, all the questions in level 2 involve schematic representations of moments and ask either to identify perpendicular distances or to compare the left-hand with the right-hand turning effect¹. All the questions in level 3, on the other hand, are more the schematic representations of physical states of affairs than they are of moments. Question 14 could be demonstrated using a crane and a chain of negligible weight. Question 11 could be demonstrated phenomenologically, for example by pushing

¹ Question 18 is more a schematic representation of a lock, and it is conceivable that many students would attempt the question by imagining two people of equal strength pushing at the point of application of the forces shown.

the door. The remaining questions could all be demonstrated using pulleys, (almost) light strings, and weights. What this suggests is that students find it more difficult to answer questions on moments that involve physical states of affairs than it is to answer the same questions that involve the schematic representation of moments: it is obvious what the direction, magnitude and point of application of the forces are in level 2, whereas it is not so obvious in level 3. At level 3 many students confuse force with the length of the string and fail to recognise which portion of the string is appropriate. The consideration of the possible states of affairs presented in level 3 might instantiate ideas of what is 'natural.' For example, beams in equilibrium returning to the horizontal. What this suggests is a form of strategic questioning whereby the teacher frames a question that challenges the intuitive response of the student. The intuitive response reveals the misconception whereby the teacher raises the appropriate anomaly. For example, if the answer to question 10(d) is an initial clockwise rotation, then the teacher may ask questions involving parallel states of affairs such as whether the following situations would make any difference:

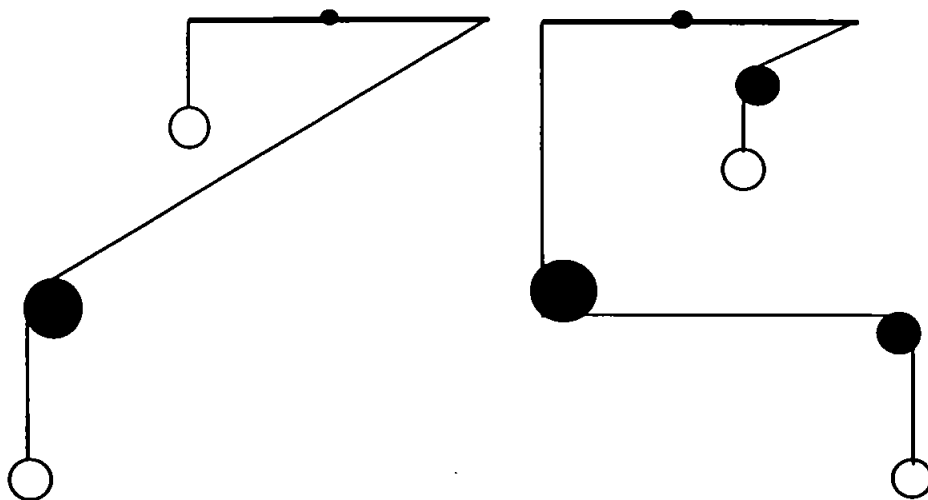


Figure 29 - Parallel Situations With Question 10(d)

Once the correct forces have been identified, then the state of affairs can be represented schematically as in level 2. If a student thinks that the rod in question 7(a), of level 3, will return to the horizontal, then the teacher could ask as to how the perpendicular distances of

the strings to the pivot compare. However, many sixth-formers have stated, in informal interviews, that they know the rod to be in equilibrium yet they would still expect the rod to return to the horizontal. This particular misconception has its origins in the experience of beam-balances and see-saws returning to the horizontal, and its nature is entirely different to the kind of misconception that appears in force and motion. No one has seen a force pushing a ball in flight. The misconception that a rod in equilibrium must return to the horizontal has more to do with memory than it has with making sense of a body in motion. What this suggests is that the teacher places into context, for the student, where the misconception comes from with respect to see-saws etc. returning to the horizontal. The teacher can then ask questions concerning what forces act on a see-saw compared to what forces act on the rod in question 7(a).

CHAPTER 10 - CONCLUSION: IMPLICATIONS FOR THE TEACHING OF MECHANICS AFTER THE INTRODUCTION OF FORCE AND MOTION

A bank of concept and parallel questions of force and motion would enable the teacher to challenge student misconceptions. However, the use of such a resource may not necessarily induce the cognitive conflict required to displace the intuitive schema and facilitate the construction of the Newtonian system. If a teacher were to use a bank of concept and parallel questions as a resource, then there would always be the possibility that the correct answer to concept and parallel questions would be learnt and remembered alongside the retention of intuitive ideas - the response by the student to concept and parallel questions may depend on the familiarity with the question¹. There is also the possibility that the use of a bank of concept and parallel questions as a resource would treat misconceptions in a piecemeal fashion. If a bank of concept and parallel questions were used **strategically**, however, then there would be a far greater possibility that the student would construct the Newtonian system in the search for coherence. According to Hestenes (1992):

The instruction should be designed to elicit from the students explicit formulations of alternatives to Newtonian concepts to be analyzed and evaluated. In this way student misconceptions are confronted in specific contexts where a superior alternative is available.....(W)hen instruction deals effectively with the most critical misconceptions, including the Impetus and Dominance principles², most of the other misconceptions tend to fade away without instructional intervention. This result is to be expected as students grasp the general force concept and integrate it into their thinking.

¹ There is a possibility that the difference between the correct answers of the post- and pre-tests may have been influenced by the familiarity of the questions in the post-test.

² *The Impetus Principle: Force is an inherent or acquired property of objects that makes them move.*

The Dominance Principle: In an interaction between two objects, the larger or more active object exerts the greater force (Hestenes, 1992).

Hestenes (1992) *Newtonian modelling games* is an holistic approach whereby qualitative questions challenges misconceptions and enables the student to construct the Newtonian system (similar to Berry and Graham's, 1991, suggestion that concept questions can be used diagnostically and have implications for learning). The concept and parallel questions that lead specifically to the inducement of cognitive conflict may be utilised as the exemplar of the Newtonian concept of force and motion for the student, from which the Newtonian system can be constructed.

The results of the empirical evaluation of the package (chapter 8) have indicated that teachers cannot adopt the Socratic method alongside their own particular teaching style simply for the process of evaluation. The successful implementation of the method requires an extensive training programme that perhaps would need to go beyond a few days of inservice training. Any evaluation of the Socratic method would first require the development of the method as a *practice*: an initial training period that would involve role play and the acquaintance with the theory of the method, followed by the implementation of the method in the classroom under observation and guidance. A research programme that investigated the Socratic method would have to first develop the method as a practice, involving the development of skills as outlined on page 234. Such a research programme would need to create a training programme that would enable the implementation of the package.

Graham and Berry (1997) suggests the teaching of statics before dynamics. He argues that a qualitative treatment of force would initially place more emphasis on the identification of forces, and this would benefit from being carried out first in a statics environment where there will be no motion to confuse the students. However, although statics may be taught

before dynamics, an understanding of systems in equilibrium cannot be separated from systems that aren't. This suggests the following teaching strategy:

Introduction to force and motion
(Similar to the approach given by the *teaching package*)



CONCEPT QUESTION:

A ball is moving in the direction of the arrows shown. Ignoring friction, what is the resultant force on the ball at the points A and B?



Figure 30 - Possible Teaching Strategy in the Introduction to Mechanics

The correct answer to the concept question requires the ability to find the resultant of two forces (weight and normal reaction). If the class can identify the weight and the normal reaction on the ball, then the concept question might create the disequilibrium necessary to find a method for combining the two forces. The teacher may then resolve the disequilibrium by introducing the *parallelogram of forces*, which may be achieved with the help of an experimental set-up similar to the one given in the *Leeds Mechanics Kit*:

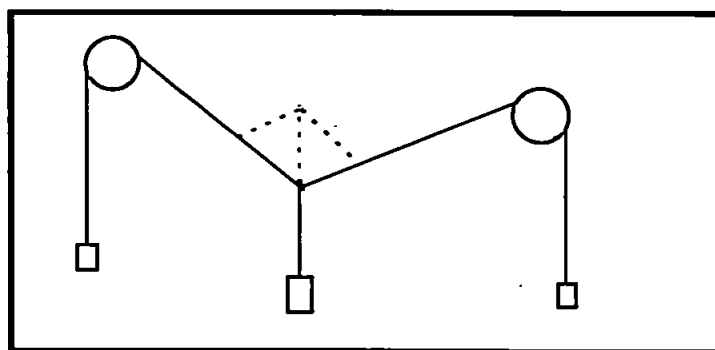


Figure 31 - Experiment to Demonstrate the Parallelogram of Forces

Subsequent to the introduction of force and motion, and the parallelogram of forces, two alternative routes suggest themselves:

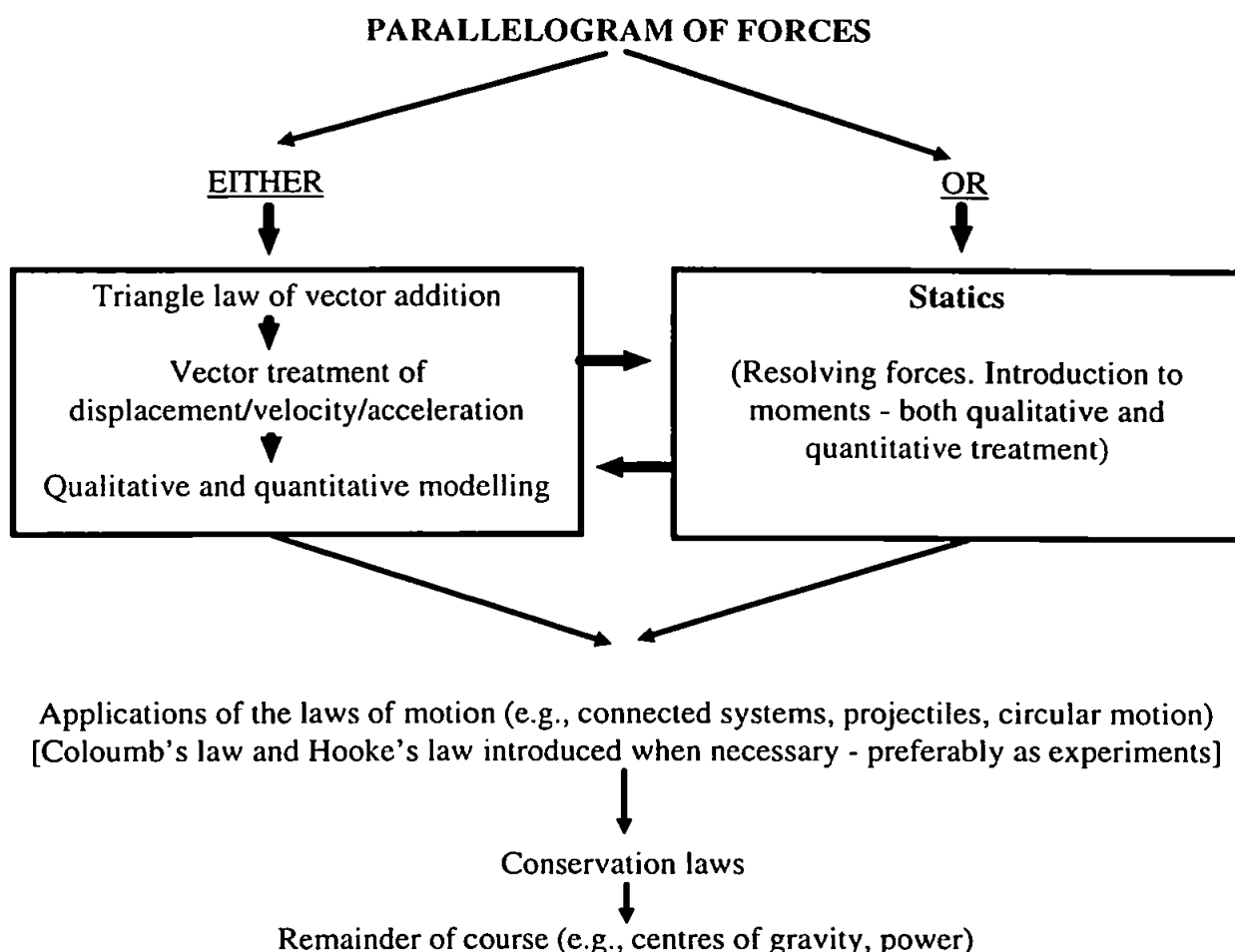


Figure 32 - Two Possible Routes After the Introduction to Force and Motion

Each topic in mechanics that involves establishing the equations of motion (from the second law of motion), for example, *rectilinear motion*, *projectiles*, *circular motion*, *simple harmonic motion*, may be introduced with concept questions that are 'looped-back' to earlier parts of the course according to the logical structure of Newtonian mechanics. For example, the concept question:

What is the resultant force acting on a dropped ball when it is in contact with the ground for the first time?

may serve as a parallel question to the following concept question that is given at a later part of the course:

An object hangs by a spring from a fixed point. The object is pulled down from its equilibrium position and released. Represent the resultant force on the object when it is

stationary at it's lowest point. How does the acceleration at this point compare with the acceleration at the mid-point of the oscillation?

Both concept questions, although introduced at different parts of the course, have the same logical structure: *that an object that is stationary does not necessarily have zero acceleration.*

The following concept question:

Two stones of unequal mass are dropped simultaneously from the same height. How do their speeds compare when they strike a level surface?

may become a parallel question to the concept question:

In a conical pendulum, the string makes an angle θ with the vertical. What happens to the angle θ if the mass of the bob is increased?

The answer: θ is unaffected because m does not appear in the equation $\theta = \tan^{-1}(v^2/rg)$

exhibits only a quantitative understanding, rather than a qualitative one. Both questions have the same logical structure: the two stones will strike the level surface at the same time because the heavier stone requires a greater gravitational force in order for it to have the same acceleration. If the mass of the bob is increased, then the increase in the mass would require a greater centripetal force to maintain the same circular path (an increase in the mass would increase the tension in the string, which would mean that the component of the tension acting towards the centre would also increase).

Although quantitative questions that appear in traditional mechanics textbooks have an important role to play in the teaching of mechanics, nevertheless they only form a part in the modelling procedure. They are presented to the student as if all the other components of modelling (identifying the object to be modelled, making assumptions etc.) do not exist.

However, quantitative questions can be based on a qualitative approach. For example:

A 1.1 kg puck, travelling along a horizontal ground with a uniform 5.0ms^{-1} , is struck at right-angles to it's path by a hockey-stick. If the force of impact can be averaged at 100N over a 0.3 second period, calculate θ , the angle the path of the puck makes with it's original direction of motion.

Projects in mechanics could include modelling **real** situations as opposed to 'real' situations that are in fact imaginary ones. For example:

After a motorway accident, a car left a 30m skid mark. The police want to establish whether or not the car exceeded the 70mph speed limit before the accident. Set up a model of the situation, stating all the assumptions you have made, and refine your model, to investigate whether the speed limit had been broken (adapted from Graham, 1994).

Such projects, however, should not be presented until the student is already familiar with the Newtonian world and can see the real world reflected in it (Hestenes, 1992).

Mechanics teaching has come a long way during the 1980's. Many A-level mathematics teachers have incorporated practical work and realistic modelling problems in the classroom (Berry, 1990). Modelling has now become an important element for many of the A-level examination boards, and many A-level text-books now include modelling and a qualitative treatment of force and motion. For example:

Essential Mechanics for A-level, Simmons and Nunn, 1984 (perhaps the first A-level textbook of it's kind!).

SMP 16-19 Mathematics (1991): Newton's Laws of Motion.

AEB Mechanics (1992).

MEI Mechanics 1 (Berry et al, 1993).

Mechanics (Graham, 1995).

However, despite the inclusion of the modelling component and the qualitative treatment of force and motion, many of the new textbooks suffer from an inductivist philosophy of science (perhaps because of the dominance of positivism in our culture). Consider, for example, the following extract from *AEB Mechanics (1992)*:

One of the simplest methods of finding a mathematical model for a given situation is to carry out an appropriate experiment to collect relevant data and from this data a formula relating the variables is found. This can be done using a graphic calculator, function graph plotter, spreadsheet or by simply drawing a graph....This is a common method of mathematical modelling for scientific situations and most basic models in mechanics are

found in this way. In this section you will have the opportunity to apply this method of approach to several physical situations (page 2, my emphasis).

That ‘this is a common method for scientific situations’ simply isn’t true (see chapter 3 and 4). Modelling in mechanics cannot be reduced to empirical modelling. On the contrary, modelling precedes experiment such that experimentation is model deployment (Hestenes, 1992), and observations are theory laden (Chalmers, 1978). The book’s philosophical approach affects its pedagogical approach. For example:

The period of a simple pendulum: A simple pendulum consists of a small object attached to a fixed point by a string of length l . As the object swings back and forth the period of the pendulum is the time taken for one complete cycle. Investigate the relationship between the period and the length of the pendulum. Does the result depend on the mass of the object? (page 3).

If the student manages to establish $T \propto l^{1/2}$ then the student has empirically established a relationship between two variables, but consequently the student will be none the wiser as to the forces acting on the bob of the pendulum or have a qualitative understanding of the pendulum’s motion. The student may realise that the result does not depend on the mass of the object, but the student still has to realise **why!** Empirical modelling may have a place in mechanics, but it does not constitute its starting point and does not play a part in forming its structure. Understanding mechanics will not result from establishing connections from experimental data, and to argue that it will, might result in confusion and bad practice. The MEI A-level mechanics syllabus offers a choice between a modelling approach and an experimental approach for problem solving in their coursework component (see, for example, Edsall, 1992). However, the experimental approach suffers from an ambiguity/ flaw as expressed in their ‘experimental cycle.’ Consider the following part of their ‘experimental cycle’ (Edsall, 1992):

Design an experiment → Conduct the experiment and derive practical results → Give a theoretical interpretation of results → Determine accuracy of solution of problem.

To 'give a theoretical interpretation of results' presupposes the ability to model prior to the 'experimental cycle'. The MEI's experimental approach, as an alternative to the modelling approach, therefore becomes superfluous, and would probably lead to confusion in mechanics teaching.

Mechanics is not the study of the connection between variables. It is a paradigm that has meaning, and the meaning can be constructed by the student, facilitated by the teacher through strategic questioning. The mathematics that follow is structured according to the meaning that is constructed. For example, differential equations that are derived from the equations of motion of a model. If a student is unable to model the real world, then the mechanics course has become empty. The traditional mechanics course treats mechanics as if it were a branch of pure mathematics. Modelling in mechanics is, in a sense, prior to mathematics. This point is expressed eloquently (if not, rather 'flowerily') by Zukav (1979):

A Master teaches essence. When the essence is perceived, he teaches what is necessary to expand the perception. The Wu Li Master does not speak of gravity until the student stands in wonder at the flower petal falling to the ground. He does not speak of laws until the student, of his own says, 'How strange! I drop two stones simultaneously, one heavy and one light, and both of them reach the earth at the same moment!' He does not speak of mathematics until the student says, 'There must be a way to express this more simply.'

However, we cannot wait until the student asks of such questions, we have to ask **them!**

APPENDIX A - THE TEACHING PACKAGE

Overcoming Preconceptions in Mechanics:

An Introductory Teaching Package



$F \propto v \Rightarrow F = kv.$

Equation of motion ($\uparrow$):

$$-mg + (-kv) = m \frac{dv}{dt}$$

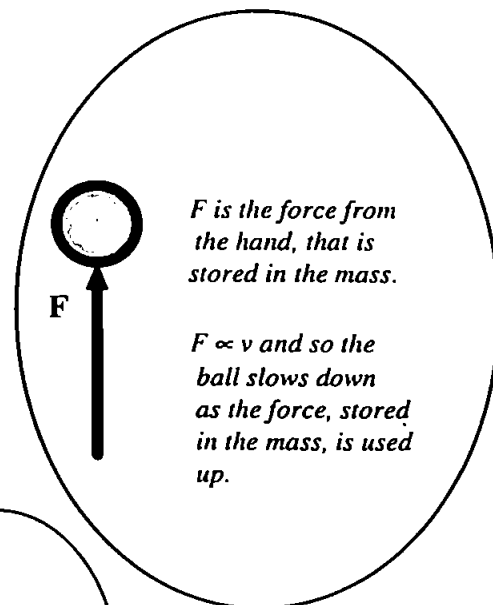
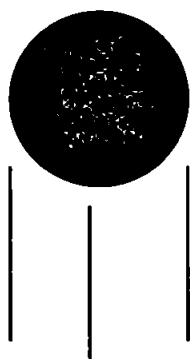
$$\Rightarrow \int \left(\frac{1}{(mg+kv)} \right) dv = -\left(\frac{1}{m} \right) \int dt.$$

$$\Rightarrow \left(\frac{1}{k} \right) \ln(mg+kv) = -\left(\frac{1}{m} \right) t + C$$

I.C: when $t = 0$, $v = 20\text{ms}^{-1}$ and $x = 0$.

Imposing I.C:-

$mg + F$



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A Teaching Package to Challenge Misconceptions in Mechanics

Introduction.

Many students begin a mechanics course with their own intuitive ideas of force ('misconceptions' or 'preconceptions'). These ideas can be so 'ingrained' that they cannot be replaced or modified through traditional teaching approaches. In many cases these ideas have existed alongside the ability to tackle quantitative A-Level examination questions. The recent changes in many of the A-Level syllabuses (e.g. the inclusion of a modelling approach) require a serious consideration of intuitive ideas. The following package will hopefully enable the teacher to challenge these ideas through a series of structured discussions with the class.

The package should be used at the beginning of the course, and the duration of its employment could be up to about 6 hours. The time taken will depend on the students and the time allocated for discussion. The package contains a set of 'concept questions' (questions that demand a qualitative response). For each concept question is a list of anticipated student responses. For each response is an appropriate reply that will (hopefully) forearm the teacher.

Appendix 1 gives a brief explanation on how to use each unit of the package. Appendix 2 provides a rationale for the package.

If you are willing to look and comment on the package, please would you complete the enclosed questionnaire.

If you are willing to use the package, please would you complete the enclosed log sheet as you use the package and the questionnaire after the package has been implemented. Please use the pre-survey enclosed prior to the implementation of the package. A post-survey will be forwarded after implementation. This will help me to evaluate the effectiveness of the package, which will be compared with control classes that run parallel.

If you have any recommendations with regards to the package, or with respect to any of the units, please would you forward the details with the questionnaire.

If the package proves to be successful, then it will be extended to provide an introduction to each major topic involving force and motion.

Many thanks

UNIT 1

BEGIN BY ASKING THIS QUESTION:

WHAT IS FORCE?

If you were asked to define force in a 'nutshell', using only one or two sentences, how would you do it?

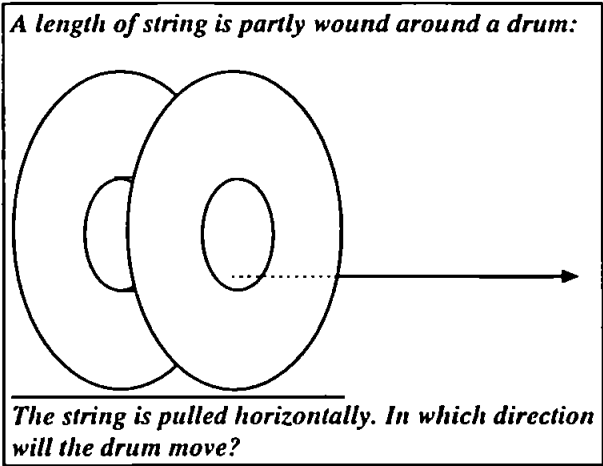
(SUGGESTION: Allow a few minutes peer group discussion. Then react to your student responses using the table below)

STUDENT RESPONSE	STRATEGY
Examples of force given, instead of a definition. E.g. Force is friction, weight... ect.	There are many different examples of force. Can you give a general definition of force? Prompting: How does force affect motion? What effect does force have? Or: Try using the example given by the student E.g. How does friction affect the motion of a billiard ball?
Force is that which causes a change in motion. Or Force is that which causes a change in velocity.	Ask to give examples, and draw out definition.
Force is that which causes a change in speed.	Illustrate with an example in one-dimensional motion (e.g. unit 2). Then ask: What if the speed is constant but changing direction? (E.g. Circular motion).
Force is that which causes a mass to accelerate.	What is meant by acceleration? And: If a body is not accelerating, then what can you say about the forces acting on the body?
Force equals mass times acceleration.	This is Newton's second law of motion. Can you answer the following with reference to the second law of motion? (Refer to unit 2).
Force is that which stops or starts motion.	Ask for examples (e.g. pushing a shopping trolley from rest, catching a ball). If force stops or starts motion, then what can we say that force does to motion in general?
Force is that which controls motion.	What do you mean by control?
Force is that which maintains motion.	Refer to unit 3.

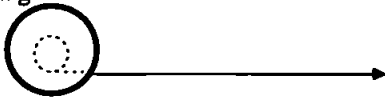
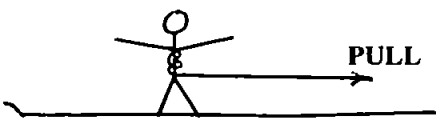
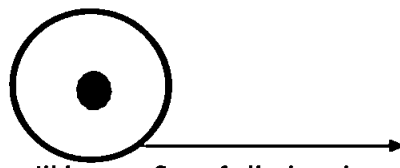
NOTE: If the Newtonian definition of force has not been reached by the class, then it may be necessary to define force as that which changes motion. Unit 2 is a good example.

CONCLUSION: Force is that which causes a change in motion.

UNIT 2



{SUGGESTION: Do not pull the string until later.}

STUDENT RESPONSE	STRATEGY
The drum will unwind, so the direction of movement of the drum is opposite the direction of pull.	What is the horizontal force acting on the drum? Draw diagram:  Using your definition of force, what would be the direction of movement of the drum? If this draws a blank, then: You are standing on ice with a rope wound around you. If the other end was pulled:  would you move away from the pull or towards the pull? Finally: Imagine the string is very loosely wound around the drum:  What will happen first of all when the string is pulled? Will this continue to happen when the drum moves? Pull the string.

STUDENT RESPONSE	STRATEGY
<i>The drum will move in the direction of the pull.</i> <i>YES: Motion is always in the direction of the applied force.</i> <i>NO: Motion is not always in the direction of the applied force.</i>	Verify by pulling the string. Then ask whether motion is always in the direction of the applied force. Use projectiles as a counter-example to disprove that force is always in the direction of motion (see unit 12, part c). Define force as that which changes motion, with reference to the change of motion of the drum. Ask for examples, then _____

UNIT 3

A ball is thrown vertically upward. Ignoring air resistance, what is the force acting on the ball whilst the ball is going up?



STUDENT RESPONSE	STRATEGY
Gravity. Gravitational attraction. Weight.	Write reponse on the board, and ask the class if they agree. Ask for any other responses.
The force acting on the ball is the force given to the ball by the thrower.	Throw a ball up in the air. Whilst the ball is going-up, place your hands in your pockets. What is the force acting on the ball?
Still the force given by my hand, only it is stored in the mass.	Does double the force mean double the mass?
The force given to it by its velocity.	Does velocity cause, or create, force? A car travels at a constant speed. Does the speed create all the forces acting on the car?
The force is the energy (or momentum, or pressure) given to the ball from the hand.	What do you mean by energy (or momentum / pressure). Please don't confuse force with energy (or momentum / pressure). Draw a line to represent the force acting on the ball.
There must be a force pushing the ball up to overcome gravity.	If gravity did not exist, suppose it was suddenly switched off, would you need such a force to push the ball up?

POSSIBLE WORLD STRATEGY:

A ball is thrown from the hull of a spaceship deep in space, where there is no significant gravitational attraction. Describe the motion of the ball.

The ball will move with uniform motion.	What force is acting on the ball?
None.	Can you account for the motion of a ball thrown on Earth in terms of force.
Must be the force given to the mass from the thrower (otherwise there would be no uniform motion).	Refer to Galileo's Strategy below (unit 3 continued).
The ball will accelerate.	If it travelled for ever, could it reach infinite velocity?
The ball would slow down until it stopped.	Why would it slow down? Is there any friction, air resistance or gravity?
It would slow down because the force given to it gradually weakens.	Refer to both strategies below (unit 3 continued).

UNIT 3 CONTINUED

PARALLEL QUESTION STRATEGY:

*A driver puts on the brakes and dis-engages the engine.
Which direction is the resultant force acting on the car?*

<i>Forwards.</i>	<i>Why then does the car slow down?</i>
<i>Backwards.</i>	<i>A force in the opposite direction to the motion opposes the motion. The car slows down! Why does the ball slow down after it has been thrown?</i>

GALILEO'S STRATEGY:

I am sitting on a train that is travelling at a constant 100 m.p.h. Between my two fingers is a ball. What is the speed of the ball relative to an outside stationary observer? Are there any horizontal forces acting on the ball?

<i>100 m.p.h. There are no horizontal forces.</i>	<i>Does uniform motion require a force to maintain it? What can you conclude about the ball thrown in space? What is the motion of the ball thrown on Earth, and how can we explain the motion in terms of force?</i>
<i>The force given by the train.</i>	<i>Is there anything in contact with the ball horizontally? Put the ball on a table in a train. What happens?</i>
<i>Friction between the ball and your fingers. Your fingers are connected to your body which is also travelling at 100 m.p.h.</i>	<i>Suppose I dropped the ball. Where would it land? (Refer to unit 12, part a).</i>

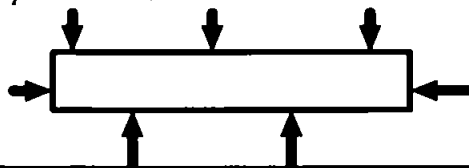
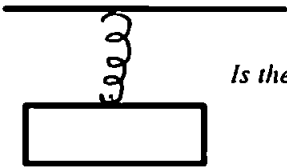
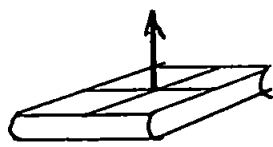
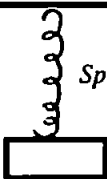
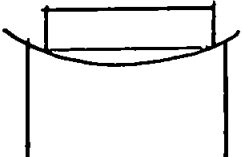
UNIT 4

A ball is thrown vertically upwards. When it reaches maximum height it is stationary for an instant. What is the force acting on the ball when it is stationary?

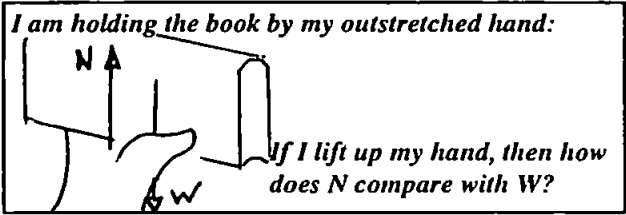
STUDENT RESPONSE	STRATEGY
<i>No force whatsoever.</i>	<i>What has happened to the force of gravity?</i>
<i>For the ball to be stationary, the net force acting on the ball must be zero.</i>	<i>What happens after the ball was stationary for an instant?</i> <i>What causes the change in motion?</i> <i>When the ball is stationary, does that mean it is also at rest?</i> <i>For how long is the ball stationary: 1 second, 0.5 seconds, 0.1 seconds?</i>
A non-zero period of time given.	<i>A car is travelling on a horizontal road. It slows down until it stops. When it stops the brakes are applied: no jerk is felt! However, if the car stops on an incline and then the brakes are applied - the car jerks! In fact, the greater the incline the greater the jerk! Why?</i> DISCUSSION (optional): <i>A ball is thrown up with an initial speed V. At the same time a helicopter rises vertically with a <u>constant</u> speed V. How would the ('3 stages' of) motion of the ball appear to the helicopter? Would the ball ever appear to be stationary relative to the helicopter?</i> <i>With respect to force, how can the overall motion of the ball be explained relative to the helicopter?</i> If students have a problem visualising the motion of the ball relative to the helicopter, then invite them to answer the following: <i>Two parachutists, A and B, are in free fall. A opens his chute. Describe the motion of A relative to B.</i>

UNIT 5

A book is at rest on a table. What keeps the book at rest on the table?

STUDENT RESPONSE	STRATEGY
Air pressure acts on all sides. 	A book is at rest on a table in a vacuum. What keeps the book at rest on the table?
The table is simply in the way.	 Is the spring simply in the way? What are the forces acting on the book?
Strength of the legs of the table / structure of table.	A book is at rest on the floor. What keeps the book at rest on the floor? What are the forces acting on the book?
There are no forces acting because the book is almost at the surface of the Earth. The book is therefore weightless.	 Lift the book by the string.
The table's molecules are tightly bound together, preventing the book passing through the table.	 Spring.  Beam. What can you say about the sum of all the upward forces of the molecules in the spring / beam compared to the weight of the book.
The only force is the weight of the book, otherwise it would float away.	Hold the book in your outstretched hand. Is your hand exerting a force?
Because 'action and reaction' is equal and opposite, there must be a force from the table equal and opposite to the weight of the book.	'Action and reaction are both equal and opposite' is Newton's third law of motion. However, suppose you dropped the book so that it is in freefall. If the weight of the book is the action, then according to the third law, what is the reaction? (Define the weight of the bottle as the gravitational attraction of the Earth on the bottle).
The table exerts a force on the book.	How can we best represent the force from the table on a diagram?
Arrows from the table acting upwards.	How many arrows? How can we be consistent in our representation of the force from the table?
There should not be just the one arrow. One arrow implies that the force acts through one point.	There is just the one force from the table, so by convention we draw just the one arrow, even though the force acts accross the area of contact.

UNIT 6



STUDENT RESPONSE	STRATEGY
Lifting my hand makes no difference to N (I still exert the same force). In fact , when I lift the book I don't feel as if I am exerting any extra effort. Therefore, $N = W$.	Hold 3 such books stationary in your hand. Now lift up your hand. How does N compare with W ? (Bearing in mind the weight of the three books, does N change between holding the three books and lifting your hand?)
In lifting my hand I feel no change in N . Therefore W has become less ($N > W$). W has become less due to the book being moved further away from the surface of the Earth.	Weigh yourself on a weighing machine placed on the floor. Now weigh yourself on the weighing machine placed on a chair. How do the results compare? OR: Place 3 such books on a sensitive weighing machine. Does the reading change whilst the weighing machine is raised upwards?
Yes, the reading would be less because the gravity is less.	DO IT.. What does the weighing machine actually measure?
N represents the strengh of my arm in holding the book. When I lift the book, N remains the same - only I have given an additional force by raising the hand.	How does the <u>sum</u> of N and the additional force compare with W ? What name can we give to the sum of N and the additional force?
Why does the book feel heavier when you push it up?	The weight of the book is what the book experiences. What you experience is muscular contraction. How does your muscular contraction compare between holding the book at rest and raising the book.

UNIT 7

A ball is suspended from the ceiling by a piece of string:

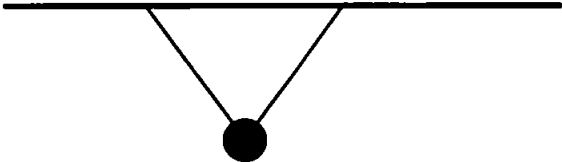


What are the forces acting on the ball?

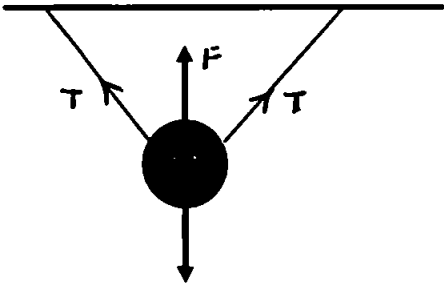
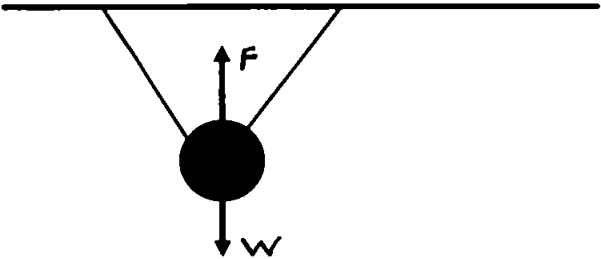
STUDENT RESPONSE	STRATEGY
Weight.	A ball <u>rests</u> on a table. What are the forces acting on the ball? How do the two forces compare? You are holding the ball with a piece of string. What forces act on the ball?
Tension in the string.	If no gravity, would there be a tension?

EXTENSION:

A ball is suspended from the ceiling by two pieces of string:



Copy the diagram and show the forces acting on the ball.

STUDENT RESPONSE	STRATEGY
	In mechanics we are concerned with two types of forces: contact forces (e.g. friction, normal reaction) and weight ('force at a distance'). What type of force is the one you have drawn vertically upwards?
	Is there a relationship between the force you have drawn vertically upward with the tension in the string?

UNIT 8

My friend says:

'My dog does not require bars to keep it in the back of the car. It stays pinned to the back door if I go fast enough!'

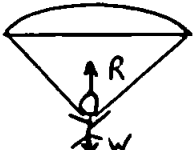
Do you agree?

STUDENT RESPONSE	STRATEGY
Yes.	What force is pinning the dog to the back door? If a ball is dropped out of the window of the car (whilst the car is travelling at constant speed), where would the ball land in relation to the car? What horizontal forces (ignoring air resistance) acts on the ball?

A motorbike is travelling at a steady speed along a straight road. What is the resultant force acting on the motorbike?

STUDENT RESPONSE	STRATEGY
For the car to be moving, the force of the engine has to be greater than the force of resistance. The resultant force has to be in the direction of motion.	Consider the following two questions: 1). A car that was stationary moves away from a set of traffic lights. What is the resultant force acting on the car? How does this situation compare with a car travelling at constant speed? 2). A frictionless ice puck moves with constant speed. What is the resultant force acting on the puck?
If the car is travelling at constant speed then there are no forces acting on the car, except weight and normal reaction.	The car is travelling at constant speed. Suddenly the engine is switched off. What happens and why?

Due to air resistance, a parachutist eventually falls at a constant rate. When falling at a constant rate, what is the resultant force acting on the parachutist?

STUDENT RESPONSE	STRATEGY
For the parachutist to travel downwards, there must be a resultant force downwards.	Initially the parachutist accelerates downwards from rest and eventually reaches constant speed. When the parachutist accelerates, what is the resultant force? How does this compare with uniform motion?
The weight of the parachutist acts downwards. Air resistance opposes the weight:  The resultant force is therefore $W - R$ downwards.	If the parachutist is travelling at a constant speed, then what can be said about the magnitude of $W - R$?

UNIT 9

A rocket travels in a straight line deep in space, where it is not affected by air resistance or gravity. It's motors are turned down halving the force that they exert, what happens to the motion of the rocket?

STUDENT RESPONSE	STRATEGY
<i>The speed of the rocket is halved.</i>	<i>A force of one Newton is applied to a 1kg. ball deep in space. What happens to the motion of the ball?</i>
<i>Before the motors were turned down, the rocket accelerated. At the time the motors were turned down the rocket had reached a certain speed. Turning the motors down means changing the speed to a lower value. The rocket therefore retards.</i>	<i>If the motors were not turned down, what would happen to the speed of the rocket?</i> <i>If the motors were turned off, what would happen to the speed of the rocket?</i> <i>If the motors were turned up, what would happen to the speed of the rocket?</i>

NOTE: If Newton's Second Law of Motion has not been established by the class, then it may be necessary to define force in terms of mass and acceleration.

UNIT 10

Part a

Deep in outer space a spacecraft is stationary. It fires its engines in such a way that they exert a force that increases uniformly in magnitude from zero to 5000 Newtons over a five minute period. What happens to the magnitude of the acceleration of the spacecraft during this time?

STUDENT RESPONSE	STRATEGY
<i>The acceleration is constant.</i>	<i>The spacecraft engines exert a <u>constant</u> force of 5000 Newtons. What happens to the acceleration whilst this constant force is exerted?</i> <i>If the engines exert a <u>variable</u> force that increases uniformly with time, what happens to the acceleration?</i>

Part b

After the initial period of exerting a variable force (part a), the engines now exert a constant force of magnitude 5000 Newtons for a further five minute period. What happens to the magnitude of the acceleration AND the velocity during this period?

STUDENT RESPONSE	STRATEGY
<i>The magnitude of the acceleration increases uniformly. The velocity increases at a variable rate.</i>	<i>An air puck is given a <u>constant</u> push of 5 Newtons for a 30 second period. What happens to the magnitude of the acceleration and the velocity over this period?</i>
<i>The acceleration becomes zero. The velocity is constant.</i>	<i>”</i>

Part c

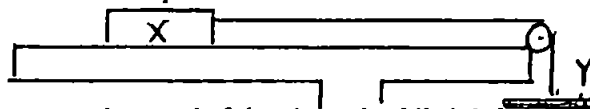
After the period of exerting a variable force (part a) and exerting a constant force (part b) the engines are switched off. What happens to the magnitude of the acceleration and the velocity now?

STUDENT RESPONSE	STRATEGY
<i>The acceleration becomes zero. The velocity becomes zero.</i>	<i>I push an air puck at a constant rate. After I finished pushing, what happens to the acceleration and velocity of the puck? (ignore friction and air resistance)</i>
<i>The acceleration continues at a constant rate due to the constant force that was initially exerted by the engine. The velocity increases uniformly.</i>	<i>”</i>

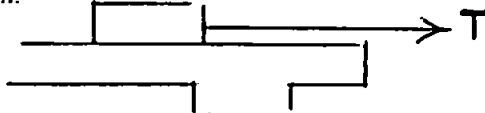
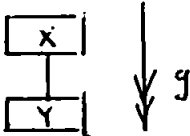
UNIT 11

Part a

The diagram shows an air puck, X, hovering on a smooth glass table and connected by a string running over a frictionless pulley to a mass, Y. When Y is released it pulls X across the table.

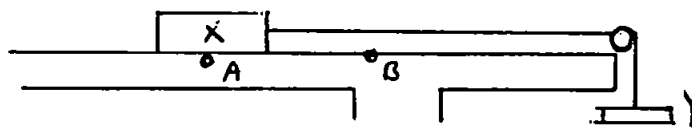


What happens to the speed of the air puck while it is being pulled by Y?

STUDENT RESPONSE	STRATEGY
Y doesn't travel in free-fall because it is connected (and hence restrained) to X. Y moves with constant speed, so X moves with constant speed as well.	Supposing you pulled the string with a constant force, T, as shown:  What happens to the speed of the air puck X?
There is no friction between the table and the air puck X. Therefore there is no force opposing the motion of X. Y is therefore free to accelerate at a value of g. X is connected to Y so X's speed increases uniformly at a rate of g. The situation is similar to X and Y connected in free-fall: 	Y pulls X. Is there a tension in the string? Is there a force opposing the motion of Y?

Part b

As in part a; only this time, when X reaches the point B, the string breaks:



What happens to the speed of X after it passes point B?

STUDENT RESPONSE	STRATEGY
X continues to accelerate because of the force given by Y (via. the string) in the first place.	A spaceship is stationary in deep space. It fires its engines so that they exert a constant force for a two minute period, and are then turned off. What happens to the speed of the spaceship after the engines are turned off?
When the string is cut, the puck X stops dead.	„
After point B, X slows down until it eventually stops.	After point B, what force opposes the motion of X?

Part c

A stationary air puck is hovering on a glass table, where it experiences no friction when it moves. What would you do to it, to make it move with a constant non-zero velocity

STUDENT RESPONSE	STRATEGY
Apply a constant force, such as a continual push with your hand/finger.	What happens to the motion of a spacecraft if its engines exert a continual force?

UNIT 12

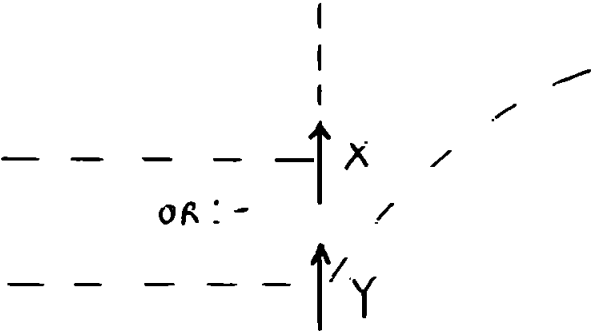
Part a

A sailor on top of a mast of a sailing ship (the 'crows-nest') drops his pistol. The speed of the ship is constant. Will the pistol land:
a) in front of the mast,
b) at the foot of the mast,
c) behind the mast?
(Ignore air resistance, wind etc)

STUDENT RESPONSE	STRATEGY
a) or b).	Supposing the ship is travelling at 20 knots. Relative to a stationary observer, what is the horizontal speed of the pistol immediately before and after release? Are there any horizontal forces acting on the pistol? Does the horizontal speed of the pistol change on the way down?

Part b

A football is rolling along the dotted line as shown. When it gets to the point X it is kicked by the point of a boot that moves in the direction shown by the solid arrow. Draw the path of the ball after it has been kicked, if the diagram shows the motion of the ball viewed from above.

STUDENT RESPONSE	STRATEGY
	A plane, travelling at a constant 500 m.p.h., releases a bomb. What is the speed of the bomb before release relative to a stationary observer on the ground? What is the horizontal speed of the bomb relative to the observer immediately after release (ignore air resistance)? Are there any horizontal forces acting on the bomb? Describe the path of the bomb relative to the observer. What is the force acting on the bomb?

Part c

A basketball player throws a ball towards the net.

What is the force acting on the ball?

STUDENT RESPONSE	STRATEGY
A force pushing the ball, given by the player.	The player throws the ball vertically upward. What is the force acting on the ball as it goes up?
There are two forces. The weight of the ball acting downwards; and a force acting horizontally, giving the ball a horizontal component of displacement.	A spaceman throws a ball deep in space. Describe the motion of the ball. What are the forces acting on the ball?

UNIT 13

A mass of 1kg and a mass of 3kg are dropped simultaneously from the same height. Ignoring air resistance, which mass reaches the ground first?

STUDENT RESPONSE	STRATEGY
<i>The 3kg mass.</i>	<i>A 1kg air-puck is given a horizontal push of 1 Newton. What will be it's acceleration?</i> <i>A 3kg air-puck is given a horizontal push of 3 Newtons. What will be it's acceleration?</i> <i>What is the weight (gravitational force of attraction) of a 1kg mass?</i> <i>What is the weight of a 3kg mass?</i> <i>Calculate the acceleration due to gravity of a 1kg and a 3kg mass.</i> <i>Drop two stones of unequal mass from the same height at the same time. Observe which stone hits the ground first.</i>

A 1kg mass and a 3kg mass are dropped simultaneously from the same height on the moon. Which mass hits the ground first?

STUDENT RESPONSE	STRATEGY
<i>Neither hits the ground. On the moon both masses are weightless.</i>	<i>Would you regard the rocks and boulders on the moon to float?</i> <i>It is an empirical fact that the gravitational attraction between 2 bodies is proportional to the product of their masses. Calculate the gravitational attraction of the moon on a mass of 1kg (the mass of the moon is one-sixth of the mass of the Earth).</i> <i>Calculate the acceleration due to gravity of an object on the moon.</i>

APPENDIX 1: HOW TO USE THE PACKAGE

UNIT 1

The package begins with the question: *What is force?* The student definitions should be written on the board. If the terms *energy*, *momentum* or *pressure* is used then the student should be asked what she or he means by such terms. If the student is incoherent then the student should be asked whether or not such terms are being used synonymously with *force*. Although unlikely, a student may be quite coherent; demanding, for example, that such-and-such phenomena can only be explained using energy and not force. Whether the ideas are coherent or incoherent, the teacher should insist on the term *force*. A student may give a learnt 'text-book' definition. Rather than proceed from the definition, it would be better to encourage further discussion by asking if anyone disagrees. The student who gave the 'text-book' definition can always be challenged by referring to a similar example.

UNIT 2

Many (fresh) students do not distinguish between motion and a change in motion. Pull the string rapped around the drum and the majority of students will state that the drum will unwind. However, some might argue that the drum will move in the direction of the applied force. Here it might be necessary for the teacher to point out that the drum is undergoing a **change** in motion in the direction of the applied force. At this stage it may be necessary to define force as that which changes motion.

UNIT 3

Unit 3 considers the force acting on a ball thrown vertically upward. Unit 3 is probably the most important unit with respect to understanding force and motion. It is certainly a very complex unit as many student conceptions of force will emerge. For this reason, three very important strategies have been singled out: a possible world of no gravity (so as to encourage abstract reasoning); a parallel situation of a car braking (so as to develop consistent reasoning); and an example involving the Galilean transformation (horizontal uniform motion in the absence of any horizontal forces). The last strategy is probably the most important, as no one has experienced uniform motion in the absence of force. The impact of this strategy will be felt after the other strategies have been used.

UNIT 4

Unit 4 may be considered as an extension of Unit 3. Many students regard all stationary bodies as being in a state of rest - hence in a state of equilibrium. A body that is **instantaneously** stationary is also accelerating, therefore it is not in equilibrium. It is important for students to understand the state of motion of a body in order to appreciate the forces involved.

UNIT 5

For many students, it is not obvious that a table will exert an upward force on a book which is at rest on the table. A student who responds with a text-book answer ('For the book to be at rest, $N = W$ ') may not necessarily appreciate that the table actually exerts a force on the book ('How can it, the table isn't doing anything!'). Text-book familiarity may not necessarily constitute understanding. At some stage, the student should be invited to hold the book at rest by an outstretched hand - experiencing the upward force. A student may quote Newton's Third Law of Motion, which is a common misconception regarding weight and reaction. The teacher may either refer to the Third Law, or delay discussion on the Third Law by using the example of $N = W$ as a consequence of the First Law of Motion.

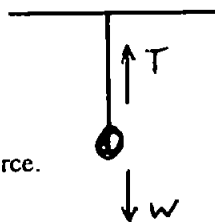
The teacher should point out that 'normal' reaction means a reaction perpendicular to the surface of contact - and not 'normal' as in 'natural'.

UNIT 6

Unit 6 may be considered as an extension of unit 5. It is the consideration of broken equilibrium as an extension of the 'at rest condition' of the same phenomenon. Many students should not have difficulty with this unit.

UNIT 7

Most students can identify the forces acting on a body at rest. However, many students will represent the forces as if they act outside of the body. E.g:

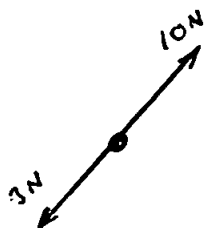


It is important to stress the point of application of a force.

The extension is important as some students might include an imaginary force.

UNIT 8

Unit 8 is the consideration of the equilibrium of forces and uniform motion. Some students may react to this, despite their understanding of force as that which changes the state of motion. Here it may be necessary to define 'resultant/net force'. A good 'beginners' example of the calculation of a resultant force is a ball in space:



What is the magnitude and direction of the resultant force acting on the ball?

UNITS 9, 10 and 11

Unit 9 leads onto the Second Law of Motion; with units 10 and 11 as an extension of 9, combining both the Second and First Law.

UNIT 12

Part a extends the ideas given in unit 3 ('Galileo's strategy'). Part a extends to part b which is the consideration of force in two dimensions.

UNIT 13

That the two masses reaches the ground at the same time is not only an empirical fact, but the phenomenon can be explained according to Newton's second law. However, many students who respond correctly to the first question give the wrong reason e.g. the masses reach the ground at the same time because the force of gravity is the same for both masses. Students must realise that it takes 3 times the force of gravity to create the same acceleration for 3 times the mass.

APPENDIX 2: THE RATIONALE OF THE PACKAGE

Understanding force is central to the understanding of mechanics. The majority of students, however, have developed their own ideas of force and motion (many examples are given in the package). These ideas, or *alternative conceptions/frameworks*, are mainly intuitive; formed from their experience of force in the real world. The ability to tackle the traditional quantitative mechanics question can exist alongside an alternative conception of force, but the ability to *model* requires an understanding of how the Newtonian concept of force is applied to the real world - in other words: the ability to model presupposes the ability to explain phenomenon *qualitatively* according to the Newtonian system, and this ability cannot really develop if alternative conceptions exist. Most students will always resort to their intuitive reasoning when accounting for the unfamiliar. For example, ask a student with some experience of mechanics to identify the forces acting on a penny sliding on a revolving turntable. It is likely that the student will include a force acting radially outwards.

To understand phenomenon according to the Newtonian concept of force, the student must realise the inadequacy of her or his own alternative framework of force. To modify, or to replace, a personally well-established and cherished framework requires a great deal of convincing. To be convinced is to be aware of the inadequacy, the incoherence, of the framework. A student might catch a glimpse of the coherence of the Newtonian system through numerous examples and demonstrations. Such a glimpse, however, may not be enough to modify the existing framework and to grasp the Newtonian system. The modification of a framework requires somewhat of a psychological 'shock' - a *cognitive conflict* within the existing framework, and the aim of the package is to promote *dis-equilibrium* within the existing framework.

For a student to be aware of the inadequacy of his or her ideas, and for the student to gain a 'global' understanding of mechanics, is a great leap forward in the personal intellectual development of the student. The teacher can create the conditions for such a development by allowing the class to express their ideas freely, without fear of ridicule or admonishment. A hallmark of intellectual development is the ability to reason consistently, and many students tend to search for consistency in any explanation that covers many examples. The aim of this package is to reveal to a class of mechanics students the inconsistency of their frameworks. The objective of the package is the construction of the Newtonian system in the way it describes the world; a construction by the members of the class, but with the aid of the teacher.

Hopefully, the package will enable the teacher to anticipate possible student responses to important qualitative questions (*concept questions*). The package contains a set of units, each unit contains a concept question and a list of possible responses. To each response is a strategy, or a number of strategies, that exposes the inadequacy or limitation of the response. The strategy may be a *parallel question* - a similar question concerning a different phenomenon that requires the same explanation under the Newtonian system (an appeal to consistent reasoning). The strategy might present a concrete scenario which contradicts the response; or a *possible-world* scenario (e.g. a *thought-experiment*) that invites the reconsideration of the original concept question adapted to an abstract or idealised situation (e.g. a world of no gravity: it would make sense to ask, in one form or another, whether the laws of motion are invariant in such a world.)

The 'delivery' of the package should take the form of a structured Discussion. The teacher asks concept questions, but gives no information regarding how the questions can be answered in accordance with the Newtonian system (with the possible exceptions detailed in appendix 1). The acceptance of the Newtonian answer will depend on reasoning rather than faith or authority, especially if the reasoning revolves around concrete rather than abstract examples (abstract examples require *hypothetical-deductive reasoning*, but this ability can develop by first considering concrete examples). The student should feel free to answer according to his or her own ideas; and should be discouraged in giving the 'expected', standard, text-book response (no 'stars' or 'merits' to be given!!).

The activity must be **structured**. In discussion, many students raise points or ask questions that they consider to be important (at least for themselves personally!). If all the points and questions raised are pursued then the discussion will at best 'meander'. The discussion may completely stray from the idea to be established, with disastrous results (a restless class who feels it has learnt nothing). Structure to the dialogue consists in maintaining the issue at hand, and that requires a certain skill. This skill develops with the interaction between the teacher and the class. However, the following suggestions may be helpful:

- 1) A student may be totally incoherent (e.g. *The potential energy of this affects the velocity of that..... etc.*) Rather than be dismissive (an understandable reaction, especially when there is precious little time), the teacher could state that what was said was not understood. The student could then be invited to re-answer the question but in the context of, for example, the forces acting on the body.
- 2) A point, question, or answer given by a student may only resemble a relevance to the issue at hand. Rather than the discussion 'meandering', the teacher could stress the issue at hand; or better still, attempt to show **why** the point, question, or answer raised is irrelevant.

3) At some stage the teacher may find himself or herself playing the role of chairperson at a meeting. A debate arises and sides are taken (and there may be more than two sides). A teacher may wish to continue the discussion as a debate. If so, the teacher should not miss the opportunity to challenge some of the points raised - developing the level of the debate.

Many teachers begin mechanics by teaching statics first, thus enabling the class to reach an understanding of force prior to the complications of motion. However, the concept of equilibrium cannot really be separated from the concept of dis-equilibrium and changes in motion. The package therefore begins with force in relation to motion, from which the ideas of equilibrium can be established. The package could cover up to about the first six hours of a mechanics course.

APPENDIX B - GRAHAM'S HIERARCHICAL MODEL OF THE DEVELOPMENT OF STUDENT UNDERSTANDING OF FORCE

Level 1

To achieve an understanding at level 1 the student should be able to;

- (a) identify that the forces acting on a particular body are in equilibrium if it is at rest or moving with constant velocity,
- (b) predict the motion of an object when it is subjected to a constant force for a specified period of time,
- (c) understand that if all the forces acting on a body are removed it will move with constant velocity,
- (d) understand the effects of gravity on objects that are initially at rest.

Level 2

To achieve an understanding at level 2 the students should be able to;

- (a) predict the motion of an object when it is subjected to a total non-zero constant force for an undefined period of time,
- (b) describe what happens to the velocity and acceleration of a moving body when a constant force is applied to it,
- (c) state that the acceleration of a body is zero when no forces are acting on it,
- (d) understand the conditions under which a body moves at constant speed.

Level 3

To achieve an understanding at level 3 the students should be able to;

- (a) identify the forces acting in a wide variety of situations,
- (b) identify the resultant force on an object, when it is not in the direction of motion,
- (c) demonstrate a complete understanding of the idea of equilibrium,
- (d) understand the effects of gravity on an object in any initial situation.

**APPENDIX C - ANALYSIS OF COVARIANCE ON THE
RESULTS OF THE PRE- AND POST-TESTS RE THE PILOT
STUDY AND THE QUASI-EXPERIMENTAL DESIGN OF
THE TEACHING PACKAGE**

ANALYSIS OF COVARIANCE ON THE RESULTS OF THE PRE- AND POST-TESTS RE THE PILOT STUDY AND THE QUASI-EXPERIMENTAL DESIGN OF THE TEACHING PACKAGE

An analysis of covariance requires paired observations on k groups. Group 1: *control*; group 2: *experimental*.

	Group 1		Group 2
	$Y_{11} \quad X_{11}$		$Y_{12} \quad X_{12}$
	$Y_{21} \quad X_{21}$		$Y_{22} \quad X_{22}$
	-----		-----
	$Y_{n_1 1} \quad X_{n_1 1}$		$Y_{n_2 2} \quad X_{n_2 2}$
	-----		-----
Means:	$\overline{Y}_1 \quad \overline{X}_1$		$\overline{Y}_2 \quad \overline{X}_2$

X is the variable under study (the post-test as a dependent variable of the teaching package), and Y is the uncontrolled variable (the pre-test as a covariate).

To test the significance of the difference between k ($k = 2$) adjusted means on X , using the analysis of covariance, the following steps [taken from Ferguson (1981)] were involved:

1. Partition the total sum of squares on both Y and X into two components, a within-groups and a between-groups sum of squares, using the usual analysis-of-variance formulas.

Denote the sums of all the observations in the j^{th} group (in this case, $j = 1, 2$) for X and Y by T_{xj} and T_{yj} respectively. Thus

$$\sum_{i=1}^{n_j} X_{ij} = T_{xj} \qquad \sum_{i=1}^{n_j} Y_{ij} = T_{yj}$$

Denote the sums of X and Y for all the observations together in the k groups by T_x and T_y . Thus

$$\sum_{j=1}^k \sum_{i=1}^{n_j} X_{ij} = T_x \qquad \sum_{j=1}^k \sum_{i=1}^{n_j} Y_{ij} = T_y$$

The formula for the *total* sum of squares is

$$\sum_{j=1}^k \sum_{i=1}^{n_j} (X_{ij} - \overline{X})^2 = \sum_{j=1}^k \sum_{i=1}^{n_j} X_{ij}^2 - T_x^2 / N \qquad \sum_{j=1}^k \sum_{i=1}^{n_j} (Y_{ij} - \overline{Y})^2 = \sum_{j=1}^k \sum_{i=1}^{n_j} Y_{ij}^2 - T_y^2 / N$$

The *within-groups* sum of squares is

$$\sum_{j=1}^k \sum_{i=1}^{n_j} (X_{ij} - \overline{X}_j)^2 = \sum_{j=1}^k \sum_{i=1}^{n_j} X_{ij}^2 - \sum_{j=1}^k (T_{xj}^2 / n_j)$$

$$\sum_{j=1}^k \sum_{i=1}^{n_j} (Y_{ij} - \overline{Y}_j)^2 = \sum_{j=1}^k \sum_{i=1}^{n_j} Y_{ij}^2 - \sum_{j=1}^k (T_{yj}^2 / n_j)$$

The *between-groups* sum of squares is

$$\sum_{j=1}^k n_j (\bar{X}_j - \bar{X})^2 = \sum_{j=1}^k (T_{x_j}^2 / n_j) - T_x^2 / N$$

$$\sum_{j=1}^k n_j (\bar{Y}_j - \bar{Y})^2 = \sum_{j=1}^k (T_{y_j}^2 / n_j) - T_y^2 / N$$

2. Partition the total sum of products into two components, a within-groups and a between-groups sums of products.

The sum of products for the j th group may be represented as
$$\sum_{i=1}^{n_j} X_{ij} Y_{ij} = T_{x_j y_j}$$

and the sum of products for all observations in the k groups by
$$\sum_{j=1}^k \sum_{i=1}^{n_j} X_{ij} Y_{ij} = T_{xy}$$

The computation formula for the *total* sum of products is

$$\sum_{i=1}^k \sum_{i=1}^{n_j} (X_{ij} - \bar{X})(Y_{ij} - \bar{Y}) = T_{xy} - T_x T_y / N$$

The *within-groups* sums of products may be obtained by

$$\sum_{j=1}^k \sum_{i=1}^{n_j} (X_{ij} - \bar{X}_j)(Y_{ij} - \bar{Y}_j) = T_{xy} - \sum_{j=1}^k T_{x_j} T_{y_j} / n_j$$

The *between-groups* sums of products is

$$\sum_{j=1}^k n_j (\bar{X}_j - \bar{X})(\bar{Y}_j - \bar{Y}) = \sum_{j=1}^k T_{x_j} T_{y_j} / n_j - T_x T_y / N$$

The following five steps are illustrated within the calculations involved in the analysis of covariance for the pilot-study and the quasi-experiment for the teaching package (see over).

3. Calculate an adjusted total sum of squares on X to remove the linear effects of the covariate Y .

4. Calculate an adjusted within-groups sum of squares on X , using the within-groups regression of X on Y .

5. Calculate an adjusted between-groups sum of squares by subtraction; that is, subtract the adjusted within-groups sum of squares from the adjusted total sum of squares.

6. Obtain the variance estimates s_w^2 and s_b^2 by dividing the adjusted within-groups sum of squares on X by $df = N - k - 1$, and the between-groups by $df = k - 1$.

7. Test the significance of the adjusted means on X by referring $F = s_b^2/s_w^2$ to a table of F .

Computation for the Analysis of Covariance: Paired Observations on Y (the pre-test covariate) and X (the post-test) for the Pilot-study:

GROUP I (CONTROL)		GROUP II (EXPERIMENTAL)	
Y	X	Y	X
56	72	75	94
50	64	56	81
50	72	50	78
47	86	47	89
44	89	42	69
44	67	39	75
44	58	39	69
42	72	39	69
42	67	39	64
39	61	39	72
36	56	36	39
33	39	36	67
33	64	36	75
33	64	33	69
31	53	33	83
31	58	33	61
28	47	33	50
28	58	31	56
22	44	31	69
22	31	31	78
22	42	25	69
		19	47
n_j	21	22	
$T_{y_j} T_{x_j}$	777	1264	842 1523
$\sum_{i=1}^{n_j} Y_{ij}^2$	30707	3495	
$\sum_{i=1}^{n_j} X_{ij}^2$		80284	109047
$T_{x_j y_j}$	48989	60284	
$T_{y_j}^2 / n_j$	28749	32225.64	
$T_{x_j}^2 / n_j$	76080.76	105433.14	

$N = 43$

$T_y = 1619$

$\bar{Y} = 37.65$

$T_y^2 / N = 60957.23$

$\sum_{j=1}^k \sum_{i=1}^{n_j} Y_{ij}^2 = 65659$

$\sum_{j=1}^k T_{y_j}^2 / n_j = 60974.64$

$T_x = 2787$

$T_x^2 / N = 180636.49$

$\sum_{j=1}^k \sum_{i=1}^{n_j} X_{ij}^2 = 189331$

$\sum_{j=1}^k T_{x_j}^2 / n_j = 181513.9$

$T_{xy} = 109273$

$\sum_{j=1}^k T_{x_j} T_{y_j} / n_j = 105057.36$

$T_x T_y / N = 104933.79$

Sum of squares		
	Y	X
Between	60974.64 - 60957.23 = 17.41	181513.9 - 180636.49 = 877.41
Within	65659.00 - 60974.64 = 4684.36	189331 - 181513.9 = 7817.10
Total	65659.00 - 60957.23 = 4701.77	189331 - 180636.49 = 8694.51
Sum of products		
Between	105057.36 - 104933.79 = 123.57	
Within	109273.00 - 105057.36 = 4215.64	
Total	109273.00 - 104933.79 = 4339.21	

Table 16 - Computation for the Analysis of Covariance for the Pilot-Study.

Analysis of covariance for data of table 16:

	Source of variation		Total
	Between	Within	
Sum of squares:Y	17.41	4684.36	4701.77
Sum of squares:X	877.41	7817.10	8694.51
Sum of products	123.57	4215.64	4339.21
Adjusted sum of squares:X	666.62	4023.28	4689.90
df for adjusted sum of squares	1	40	41
Variance estimates	$s_b^2 = 666.62$	$s_w^2 = 100.58$	

$$F = 666.62 / 100.58 = 6.63 \quad (\text{critical value } 4.08) \quad p < 0.05$$

Table 17: Analysis of Covariance for Data of Table 16

The means of the post test are: (control) $\bar{X}_1 = 60.19$; (experimental) $\bar{X}_2 = 69.23$. The means for the covariate Y, however, do not differ so appreciably: (control) $\bar{Y}_1 = 37.00$; (experimental) $\bar{Y}_2 = 38.27$. This suggests that a substantial part of the variation in the post test means is not a result of the difference in the pre test means. All the terms necessary for the calculation of the sums of squares on X and Y and sums of products are given in Table - 16

Table 17 summarises the analysis of covariance. The adjusted total sum of squares for X is $8694.51 - 4339.21^2/4701.77 = 4689.21$. The adjusted within-groups sum of squares is $7817.10 - 4215.64^2/4684.36 = 4023.28$. The adjusted between-groups sum of squares is $4689.21 - 4023.28 = 666.62$. The variance estimates are $s_b^2 = 666.62$ and $s_w^2 = 100.58$, and $F = 6.63$. This ratio is significant as it is substantially greater than the critical value of F, i.e. almost all the variation in the two post-test means cannot be attributed to the influence of the pre-test scores. What this suggests (but does not necessarily imply) is that the difference in the post-test mean scores may be attributed to the teaching method in the experimental group.

It will be of interest to calculate the adjusted means on the post-test scores given by

$$\bar{X}_j^{''} = b_w(\bar{Y} - \bar{Y}_j) + \bar{X}_j$$

$$b_w = 4215.64/4684.36 = 0.900$$

$$(\text{control}) \quad \bar{X}_1^{''} = 0.900(37.65 - 37.00) + 60.19 = 60.78$$

$$(\text{experimental}) \quad \bar{X}_2^{''} = 0.900(37.65 - 38.27) + 69.23 = 68.67$$

Computation for the Analysis of Covariance: Paired Observations on Y (the pre-test covariate) and X (the post-test) for the Teaching Package Quasi-Experiment.

CONTROL GROUPS

EXPERIMENTAL GROUPS

SCH OOL	PRE- TEST	POST- TEST		SCH OOL	PRE- TEST	POST- TEST		SCH OOL	PRE- TEST	POST- TEST		SCH OOL	PRE- TEST	POST- TEST
1	12	15		6	11	19		10	12	16		16	9	15
1	10	19		6	17.5	18		10	8	11		16	13	16
1	13	13		6	15.5	20		10	10	12		16	11	16
1	14	18		7	13	17		10	13	17		16	10	16
1	15	15		7	12	18		10	7	15		16	6.5	11
1	13	18		7	12	16		10	9	14		17	12	14
1	16	16		7	14	19		10	6	12		17	20	19
1	6	13		7	8.5	15		10	9	12		17	14	18
1	15	19		8	3	8		10	13	19		17	18	19
1	9	7		8	3	5		10	11	13		17	12	15
2	10.5	18		8	6	13		10	10	15		18	16	17
2	14.5	18		8	6	16		11	11.5	12		18	15	16
2	10.5	18		8	4	7		11	6	7		18	12	17
2	15	17		8	11	18		11	6.5	14		18	11	18
2	10	19		8	2	9		11	13	17		18	19	17
2	10	16		8	8	17		11	7.5	9		18	14	14
2	15	19		9	9	10		11	8	10		18	11	12
2	13	19		9	12	14		11	7	13		18	11	10
2	13	17		9	6	14		11	4	11.5		18	18	18
2	12	12		9	9	10		11	4	14		18	13	16
2	13	17		9	4	7		11	6	12		18	8	12
3	9	14		9	12	11		11	6	10		18	14	18
3	4.5	15		9	8	16		11	5.5	11.5		18	17	20
3	8	10		9	6	9		11	6	12		18	8	11
3	12	16		9	12	14		11	5	11		18	15	16
3	11	14						11	8.5	13		18	16	19
3	8	12						11	8.5	16		18	19	19
3	5	10						11	3	15		18	19	19
3	11	18						11	5	15.5		18	17	19
3	13	15						11	8.5	14		18	16	19
3	10	15						12	5	9		18	18	18
3	12.5	17						12	13	16		18	16	18
3	12	17						12	11.5	17		19	7	13
3	8	17						12	5.5	13		19	5	17
3	10	12						12	4.5	14		19	6	12
3	15	19						12	4.5	11		19	8	12
4	9	16						12	7.5	13		19	6	9
4	12.5	13						13	7.5	18		19	5	8
4	9.5	14						13	7.5	10		19	6	7
4	7.5	8						13	8	12		19	8	11
4	14	18						13	12	17		19	4	5
4	12.5	13						13	18	17		19	5	12
4	9.5	12						13	12.5	18		20	8	15
4	10	14						13	13	11		20	13	18
4	11	13						13	13	19		20	6.5	9.5
4	9	15						13	6	14		20	12	18
5	12.5	17						14	5	7		20	7.5	17
5	6.5	14						14	3	10		20	12	16.5
5	10.5	14						14	12.5	16		20	5.5	6
5	6.5	8						14	12.5	17.5		20	7	15
5	11.5	16						14	12	18		20	10.5	15
5	8	15						14	12	15		20	9.5	16
5	11.5	10						14	9.5	14.5		20	5	14
5	7	13						14	9	14		20	9	12
5	7.5	8						15	5.5	13		20	9	13
5	5	8						15	10	15		20	6	9
5	6.5	14						15	8	15		20	8.5	15
6	10	18						15	14	17		20	12	12
6	18	19						15	16	18		20	9.5	16
6	13	16						15	7	13		20	9	10.5
6	14	18						15	9.5	12		20	10.5	14.5
6	9	13						16	6	11		20	5.5	10
6	19	18						16	9	14		20	7	12.5
6	17	17						16	9.5	9		20	6	12.5
6	13	17						16	9.5	11		20	1.5	7

Table 18 - Table of Pre- and Post-Test Results for Each Student

CONTROL GROUPS	
Pre-test mean, $\bar{Y}_1 = 10.48$	
Post-test mean, $\bar{X}_1 = 14.59$	
$n_1 = 90$	
EXPERIMENTAL GROUPS	
Pre-test mean, $\bar{Y}_2 = 9.73$	
Post-test mean, $\bar{X}_2 = 13.96$	
$n_2 = 130$	

$N = 220$
$T_y = 2208$
$\bar{Y} = 10.036$
$\overline{T_y}^2 / N = 22160.30$
$\sum_{j=1}^k \sum_{i=1}^{n_j} Y_{ij}^2 = 25473.50$
$\sum_{j=1}^k T_{y_j}^2 / n_j = 22190.72$
$T_x = 3127.5$
$T_x^2 / N = 44460.26$
$\sum_{j=1}^k \sum_{i=1}^{n_j} X_{ij}^2 = 47063.75$
$\sum_{j=1}^k T_{x_j}^2 / n_j = 44481.44$
$T_{xy} = 33470.25$
$\sum_{j=1}^k T_{x_j} T_{y_j} / n_j = 31414.12$
$T_x T_y / N = 31388.73$

Sum of squares			
	Y		X
Between	22190.72 - 22160.30 = 30.42		44481.44 - 44460.26 = 21.18
Within	25473.50 - 22190.72 = 3282.78		47063.75 - 44481.44 = 2582.31
Total	25473.50 - 22160.30 = 3313.20		47063.75 - 44460.26 = 2603.49
Sum of products			
Between	31414.12 - 31388.73 = 25.39		
Within	33470.25 - 31414.12 = 2056.13		
Total	33470.25 - 31388.73 = 2081.52		

Table 19 Computation for the Analysis of Covariance for the Teaching Package Quasi-Experiment

	Source of variation		Total
	Between	Within	
Sum of squares: Y	30.42	3282.78	3313.20
Sum of squares: X	21.18	2582.31	2603.49
Sum of products	25.39	2056.13	2081.52
Adjusted sum of squares: X	1.3	1294.48	1295.78
df for adjusted sum of squares	1	217	218
Variance estimates	$s_b^2 = 1.3$	$s_w^2 = 5.97$	
$F = 1.3 / 5.97 = 0.22$ (critical value 3.89) $p < 0.05$			

Table 20 - Analysis of Covariance for Data of Table 19

Table 20 summarises the analysis of covariance for the quasi-experiment involving the implementation of the teaching package. The adjusted total sum of squares for X is $2603.49 - 2081.52^2/3313.20 = 1295.78$. The adjusted within-groups sum of squares is $2582.31 - 2056.13^2/3282.78 = 1294.48$. The adjusted between-groups sum of squares is $1295.78 - 1294.48 = 1.3$. The variance estimates are $s_b^2 = 1.3$ and $s_w^2 = 5.97$, and $F = 1.3/5.97 = 0.22$. This ratio is not significant, and, indeed, it falls substantially short of the value of F of unity expected under the null hypothesis (that no correlation of any large magnitude exists between Y and X). This implies that almost all the variation in the X means can be attributed to the influence of the uncontrolled variable Y .

Examination of the pre-test scores from the raw data in table 18 shows how the pre-test scores may have influenced the post-test scores:

Number of control pre-test scores greater than or equal to 10 (out of 20) = 55 out of 90 (61.1%).

Number of experimental pre-test scores greater than or equal to 10 (out of 20) = 56 out of 130 (43.1%).

On the one hand, 61.1% of control pre-test scores ≥ 10 implies a greater familiarity with concept questions prior to the experiment; yet on the other hand, with such a high modal value we can expect a greater regression with the control group. However, if we compare the adjusted post-test means given by:

$$\overline{X}_j^{''} = b_w(\bar{Y} - \bar{Y}_j) + \bar{X}_j$$

where $b_w = 2056.13/3282.78 = 0.626$. The two adjusted post-test means are:

$$\text{(control)} \quad \overline{X}_1^{''} = 0.626(10.036 - 10.483) + 14.589 = 14.31$$

$$\text{(experimental)} \quad \overline{X}_2^{''} = 0.626(10.036 - 9.727) + 13.958 = 14.15$$

These adjusted means vary very little from one another, which is reflected in the small F ratio. We may therefore conclude that there is no significant difference between the post-test scores due to the effects of the pre-test.

**APPENDIX D - INTRODUCTORY LETTER AND
RESPONSE FORM RE INVOLVEMENT IN THE
EMPIRICAL EVALUATION OF THE TEACHING
PACKAGE**

15 February 1995

Dear

Research in the use of Concept Questions in Mechanics

We are working on a research project on the use of concept questions, specifically in the introduction to the concept of force and motion. An example of the type of question is given below:

A ball is thrown vertically, up in the air, as shown in the diagram. At A it is on its way up, at B it is at its highest point and at C it is on its way down. Draw arrows to show the direction of the force at each of these three points.

How does the magnitude of the force acting on the ball compare at each of these three points?

O B

↓ O C

↑ O A

We are looking for teachers and schools who can help us in one of three ways:

- 1) To comment on an introductory teaching package that has been prepared as a result of our initial research. If you are happy to do this then we will forward a copy as soon as you reply.
- 2) To act as a control group for our experiment. We would need to ask you to give a pre-test and a post-test to your mechanics group.
- 3) To act as an experimental group using our proposed package of concept questions.

If you are able to consider helping in one of the three ways then please return the response form in the enclosed post-paid envelope.

Thank you for reading this letter and responding to this initial enquiry. If you are interested in helping we will send you more details in the near future.

Yours sincerely

Ted Graham and Stuart Rowlands.

RESPONSE FORM

NAME (Mr. Mrs. Miss. Ms. Dr. Fr.)

SCHOOL

ADDRESS

Involvement in the Experiment

- A) I am willing to look and comment on the proposed teaching package, in the next few weeks. Yes / No
- B) I am willing to use the teaching package when I next begin to teach mechanics to new A-Level students. Yes / No
- C) I am willing to act as a control group in an experiment, and give a pre-test and a post-test to my classes (I also understand that the results will be treated in the strictest confidence). Yes / No

Details of the School / College

Approximate class size? _____

Number of classes studying mechanics? _____

A-Level syllabus (e.g. A.E.B., M.E.I., S.M.P., London)? _____

Principle Mechanics text-book used? _____

When will you next teach mechanics to a group for the first time? _____

Do you use concept questions in teaching mechanics? Yes / No

If yes:

(a) When did you start to use them? _____

(b) How were you introduced to them? _____

(c) What is your main source of concept questions? _____

THANK YOU FOR YOUR TIME AND EFFORT!

APPENDIX E - THE PRE- AND POST-TEST

SURVEY **NAME**..... **SCHOOL**.....

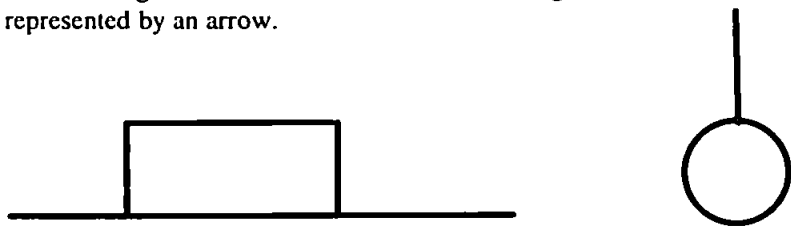
This survey is part of a research project and is designed to reveal what you understand by motion and force. This is not a test, so you need not worry about your score.

Please answer ALL questions in both sections A and B. Please write all your answers on the question paper.

If you do not know the answer you may guess, but please write (guess) alongside your answer.

You may justify your answer with a sentence or two if you wish.

1. The two diagrams below show a book resting on a table and a ball hanging from a length of string. On each diagram mark in the forces that are acting on the book and the ball. Each force should be represented by an arrow.



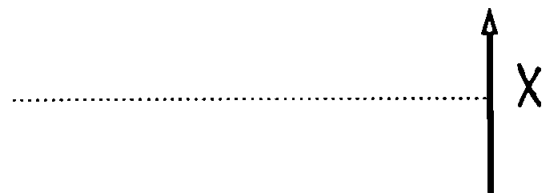
2. Two balls are exactly the same size and shape, but have masses of 1kg and 3kg. They are dropped at the same time from the same height. How does the time that they take to fall to the ground compare?

Answer:

If this experiment were repeated on the moon, how would the results compare?

Answer:

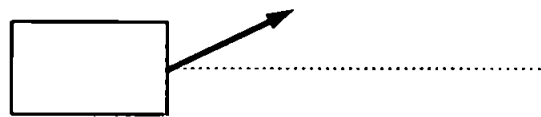
3. A football is rolling along the dotted line shown in the diagram. When it gets to the point X it is kicked by the point of a boot that exerts a force in the direction shown by the solid arrow. Draw the path of the ball after it has been kicked, if the diagram shows the motion of the ball viewed from above.



4. A small cube of ice rests on a smooth polished glass table. When it moves, the friction is so small it can be ignored. What would you do to the ice cube, to make it move with a constant non-zero velocity if it is initially stationary?

Answer:

5. In the diagram below you are looking down on a trolley at rest on a smooth horizontal surface. A constant force F will act in the direction shown by the solid arrow. A second additional force is needed to drive the trolley in the direction of the dotted line, draw an arrow to represent this force on the diagram.

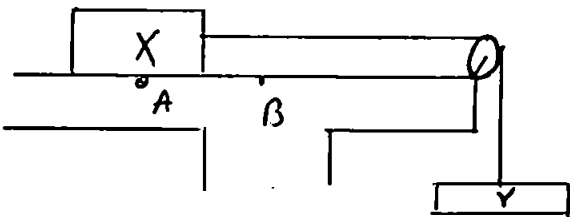


6. A heavy atomic particle, X, exerts an attractive force, F , on a much lighter particle, Y, as shown in the diagram. Compare the force, if any, that the small particle, Y, exerts on the bigger one, X, with the force that X exerts on Y.



7. The diagram shows an air puck, X, hovering on a smooth glass table and connected by a string running over a frictionless pulley to a mass, Y. When Y is released it pulls X across the table. What happens to the speed of the air puck X while it is being pulled by Y?

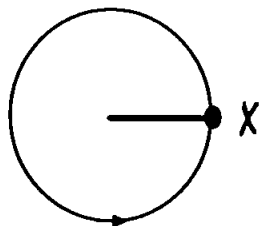
Answer:



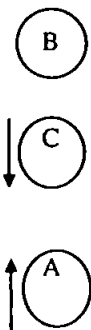
When the air puck, X, reaches B the string breaks, what happens to the speed of the puck after it passes point B?

Answer:

8. You are looking down on a ball on the end of a piece of string that is being swung round in a horizontal circle at fairly high speed. On the diagram draw the path of the ball if the string breaks at the point marked X.



9. A ball is thrown vertically, up in the air, as shown in the diagram. At A it is on its way up, at B it is at its highest point and at C it is on its way down. Draw arrows to show the direction of the force at each of these three points, the length of the arrow should represent the relative size of the force acting on the ball at each of the three points.

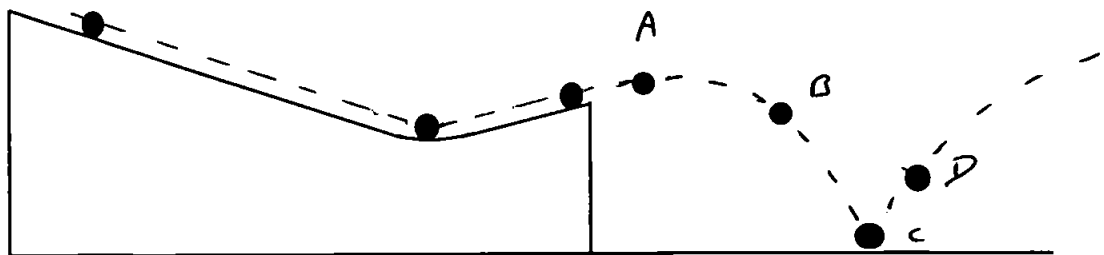


10. The diagram shows a rocket travelling in a straight line deep in space, where it is not affected by air resistance or gravity. Both of its motors are turned down halving the force that they exert, what happens to the motion of the rocket?

Answer:



11. The diagram shows the path of a ball as it slides along a track and then moves through the air. Use one arrow to indicate the resultant force acting on the ball at each of the positions marked with a letter.

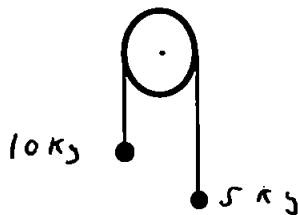


12. You and a friend, who is the same size and mass as you, are standing still facing each other on rolling skates. If you push your friend what happens
(a) to you? Answer:

(b) to your friend? Answer:

13. Two masses are connected by a string that passes over a smooth pulley as in the diagram. They are of mass 5kg and 10kg and released from the positions shown. How do the forces that the string exerts on these masses compare?

Answer:



14. A small ball is fired into the tube shown at high speed. The tube has very smooth sides and is fixed on a horizontal, smooth table. The diagram shows the tube viewed from above as it lays flat on the table. Draw the path of the ball after it leaves the tube.

POST - SURVEY NAME..... SCHOOL.....

Similar to the survey you completed before the start of the mechanics course, you need not worry about your score.

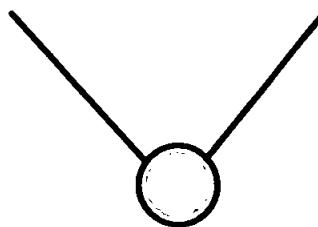
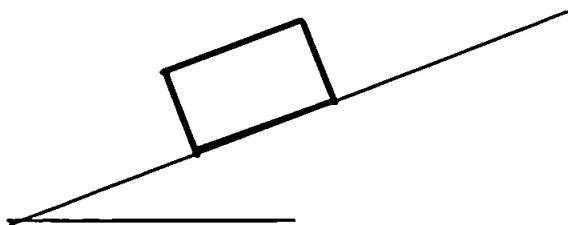
Please answer ALL questions in both sections A and B. **Please write all your answers on the question paper.**

If you do not know the answer you may guess, but please write *guess* alongside your answer.

Please justify your answer with a sentence or two if you can and if it is appropriate.

Please state, giving reasons, if you know the 'text-book' answer but you do not agree.

1. One diagram shows a book at rest on a rough inclined plane, and the other diagram shows a ball at rest hanging from two lengths of string. On each diagram mark in the forces that are acting on the book and the ball. Each force should be represented by an arrow.



2. When a ball is thrown in the air, would the force of gravitational attraction be the same regardless of the mass of the ball?

Answer:

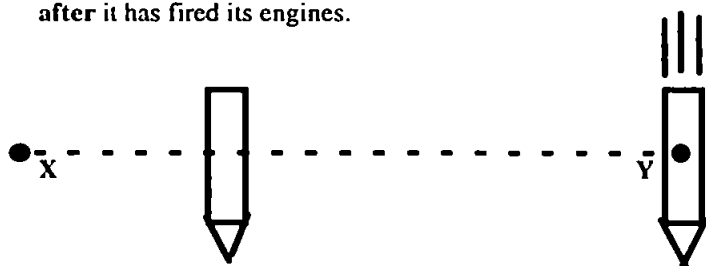
When a ball is thrown in the air, would the effects of gravity (namely, the acceleration) be the same regardless of the mass?

Answer:

If the ball were thrown vertically upwards from the surface of the moon, would the effects of gravity be the same regardless of the mass of the ball?

Answer:

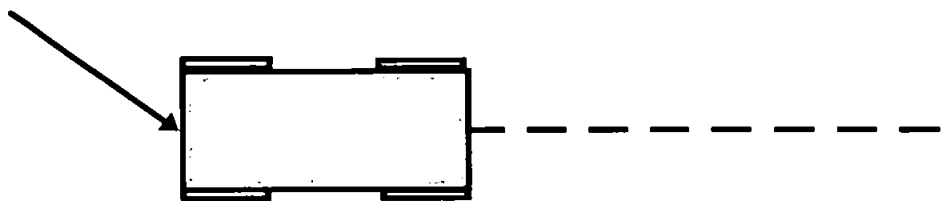
3. A spaceship in deep space, where no forces are acting on it, drifts sideways from X to Y. When it reaches Y it fires its engines for a very short time (eg. 0.5 seconds). Draw the **path** of the spaceship after it has fired its engines.



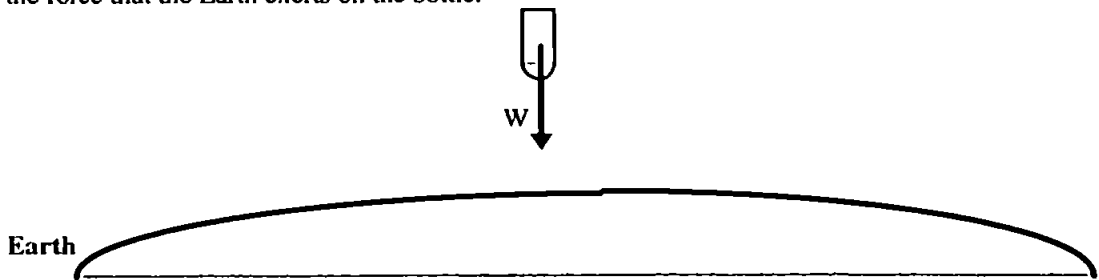
4. A supermarket trolley is at rest on a smooth polished floor. What would you do to the trolley to make it move with a constant velocity (assume that you are lucky enough to find a trolley with no tendency to veer off to one side, and that the friction between the floor and the trolley is so small that it can be ignored)?

Answer:

5. In the diagram below you are looking down on a trolley at rest on a smooth horizontal surface. A constant force F will act in the direction shown by the solid arrow. A second additional force is needed to drive the trolley in the direction of the dotted line, draw an arrow to represent this force on the diagram.



6. A bottle is in free fall, dropped from a bridge. W is the gravitational attractive force of the Earth on the bottle, as shown in the diagram. By drawing an arrow on the diagram, compare the force, if any, that the bottle exerts on the Earth, with the force that the Earth exerts on the bottle.



7. The diagram shows an air puck hovering on a smooth frictionless glass table. I push the air puck with my finger, applying a constant horizontal force. What happens to the speed of the air puck while it is being pushed?

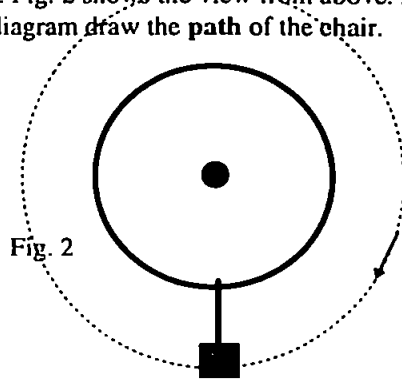
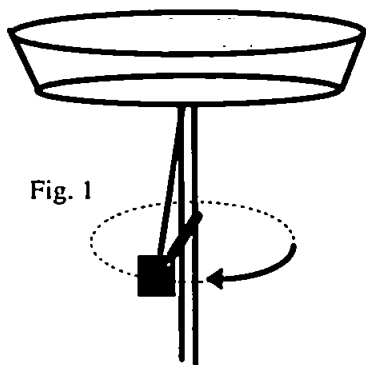
Answer:



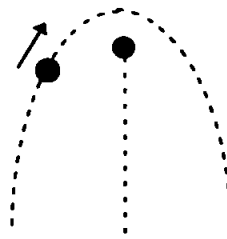
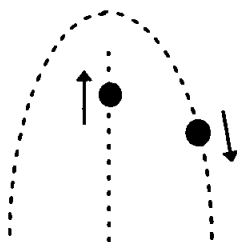
When I stop pushing the puck, what happens to its speed?

Answer:

8. Fig. 1 shows the side view of a 'chair-o-plane' funfair ride. Fig. 2 shows the view from above. At the position shown on each diagram the chain breaks. On each diagram draw the path of the chair.



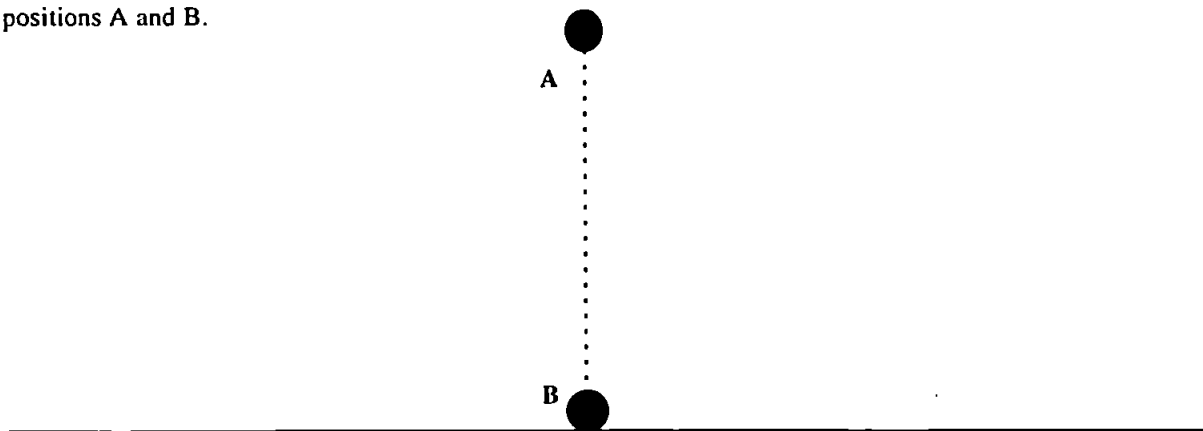
9. A juggler throws two balls, one vertically and the other in a parabolic path. The dotted lines in the diagram show the paths of each ball and the arrows show the direction of motion at points on the path. On the diagrams draw arrows to indicate the direction and the size of the force on each ball at the positions shown (ignore air resistance).



10. A rocket is travelling in a straight line deep in space, where it is not affected by air resistance or gravity. If its motors are turned down halving the force that they exert, then what would happen to the motion of the rocket?

Answer:

11. The diagram shows the path of a ball. At A it has just been released, and at B it has just made contact with the ground. Use one arrow to indicate the resultant force acting on the ball at each of the positions A and B.



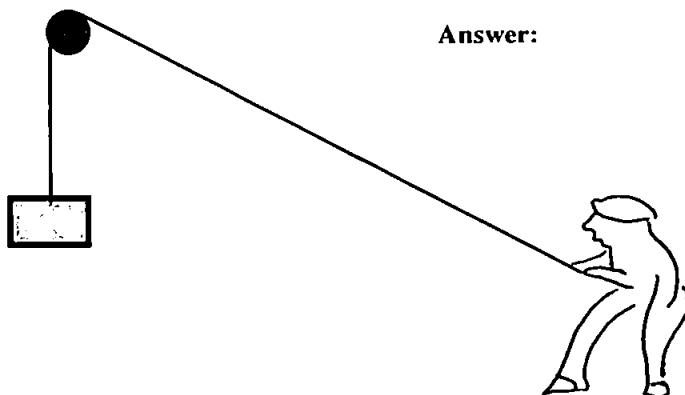
12. Two astronauts, of equal size and mass, are floating in space. One astronaut pushes the other. Describe what subsequently happens to each astronaut.

Answer:

13. The diagram shows a man holding the end of a rope that passes over a smooth pulley. Attached to the other end of the rope is a crate.

The crate is at rest. How do the forces that the rope exerts on the man and the crate compare?

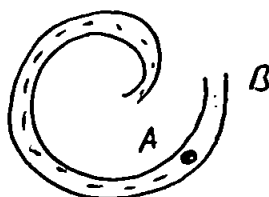
Answer:



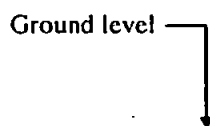
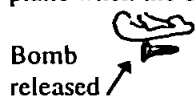
If the crate is being hauled up by the man, then how do the forces that the rope exerts on the crate and the man compare?

Answer:

14. A channel in the shape of a spiral lies fixed on a smooth horizontal table. A small ball is projected along the spiral from the centre. The ball leaves the spiral at point B. Sketch the path of the ball from point A on the channel through point B and its path on the table.



15. On a bombing run a plane approaches its target along a straight horizontal line at a constant speed. The plane releases the bomb and after 5 seconds the bomb hits the target. The plane maintains its course after the bomb has been released. Sketch the path of the bomb **from the moment of release to hitting the target**, and mark with a * the position of the plane after 5 seconds (i.e. the position of the plane when the bomb hits the target).



APPENDIX F:

- QUESTIONNAIRE ON TEACHING PACKAGE
- LOG-SHEET AND QUESTIONNAIRE
- STUDENT QUESTIONNAIRE ON THE INTRODUCTION TO MECHANICS

QUESTIONNAIRE ON TEACHING PACKAGE

Name _____ College/School _____

Thank you for looking at the package. Please would you answer the following:

- 1) Would you be prepared to use the package?
- 2) If yes, how much time would you be prepared to allocate to use it?
- 3) Are the instructions / guidelines clear?
- 4) Can the package be used in its present form?
- 5) Do you think your students would benefit from this type of approach?
- 6) If you think your students would not benefit from this type of approach, could you explain why.
- 7) Please make any suggestions for improvements.

8) Approximately how long did it take you to examine the package?

9) Which of the concept problems, if any, have you used before?

10) Comments on specific units:

Unit 1

Unit 2

Unit 3

Unit 4

Unit 5

Unit 6

Unit 7

Unit 8

Unit 9

Unit 10

Unit 11

Unit 12

Unit 13

THANK YOU!

NAME _____ SCHOOL/COLLEGE _____

LOG SHEET AND QUESTIONNAIRE

Please complete the log sheet on a unit by unit basis, and the questionnaire at the end of the sessions.

LOG SHEET

As you use the package please complete the log sheet below; giving the date, number of students and time spent on unit. Please add any specific comments on each unit, in particular any unresolved difficulties and student responses not anticipated by the package.

Unit 1 (Date _____ No. of students _____ Time spent _____)
Comments:

Unit 2 (Date _____ No. of students _____ Time spent _____)
Comments:

Unit 3 (Date _____ No. of students _____ Time spent _____)
Comments:

Unit 4 (Date _____ No. of students _____ Time spent _____)
Comments:

Unit 5 (Date _____ No. of students _____ Time spent _____)

Comments:

Unit 6 (Date_____No. of students_____Time spent_____)
Comments:

Unit 7 (Date_____No. of students_____Time spent_____)
Comments:

Unit 8 (Date_____No. of students_____Time spent_____)
Comments:

Unit 9 (Date_____No. of students_____Time spent_____)
Comments:

Unit 10 (Date_____No. of students_____Time spent_____)
Comments:

Unit 11 (Date_____No. of students_____Time spent_____)
Comments:

Unit 12 (Date_____No. of students_____Time spent_____)

Comments:

Unit 13 (Date_____No. of students_____Time spent_____)

Comments:

QUESTIONNAIRE

Please would you answer the following shortly after the package has been used.

FOR EACH QUESTION CIRCLE THE APPROPRIATE RESPONSE.

1) In your estimation, how effective is the package in enabling you to overcome student misconceptions?

- {a} Very effective.
- {b} Effective.
- {c} Not very effective.

2) What proportion of your students appear to have benefited from the use of the package?

- {a} A large majority.
- {b} A majority.
- {c} A minority.
- {d} A small minority.

3) Is the package very cumbersome when having to anticipate student responses?

- {a} Very cumbersome.
- {b} Cumbersome.
- {c} Not cumbersome.
- {d} Not at all cumbersome.

4) Is the package very time-consuming when having to anticipate student responses?

- {a} Very time consuming.
- {b} Time consuming.
- {c} Not time consuming.
- {d} Not at all time consuming.

5) Were all your student responses predicted in the package?

6) Please circle one of the following:

- {a} The package has been very useful and easy to implement.
- {b} The package has been very useful but very time consuming in the preparation of the lesson.
- {c} The usefulness of the package has been outweighed by the time and effort required for its implementation.
- {d} Although the package requires little time and effort to implement, it nevertheless has not been successful.
- {e} The package has been a waste of time with regard to the effort required in its implementation.

7) In your opinion, would a more 'direct approach' to challenging misconceptions (e.g., providing the right answer to qualitative questions) be as effective as the package?

- {a} More effective.
- {b} Less effective.
- {c} Not at all effective.

8) In the light of your experience, do misconceptions warrant the need for such a package?

- {a} Yes, a lot.
- {b} Yes, a little.
- {c} Yes occasionally.
- {d} No.

9) Any improvements you can suggest?

Thank you for your time and effort in answering the log sheet and questionnaire!

STUDENT QUESTIONNAIRE ON THE INTRODUCTION TO MECHANICS

Please would you answer the following questions:

For questions 1) to 4) please circle the appropriate response.

1) Were you able to follow the discussions

{a} All of the time?

{b} Most of the time?

{c} Some of the time?

{d} Very little of the time?

{e} Not at all?

2) Did you find the discussions helpful in your understanding of force and motion?

{a} Yes, a lot.

{b} Yes, a little.

{c} Not at all.

3) Have you changed the way that you think about mechanics?

{a} A lot.

{b} A little.

{c} Not at all.

4) Have you been made to really think hard?

{a} Yes, a lot.

{b} Yes, a little.

{c} Not at all.

P.T.O.

5) Please comment on those parts of the discussions that you
{a} particularly liked or found interesting,

{b} particularly disliked or did not find interesting.

6) Has it made you want to ask questions about other similar situations?

THANK YOU!

APPENDIX G - MOMENTS SURVEY QUESTIONNAIRE

SURVEY ON MOMENTS

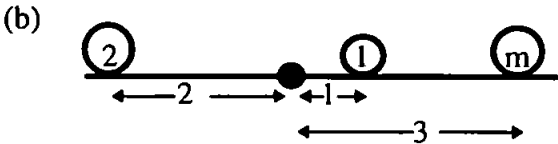
NAME..... SCHOOL.....

The survey is part of a research project. The project is to see how students understand moments. This is not a test, so do not worry about how well you do. Please answer the following. If you can, please give a reason for your answer in one or two sentences.

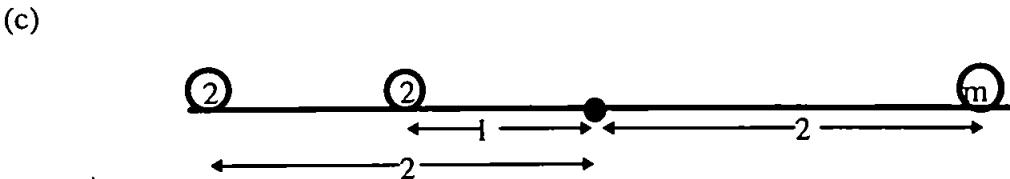
1. Each rod is light and in equilibrium, smoothly pivoted at O. The masses are in Kg and the lengths are in metres. What is the value of m?



m =

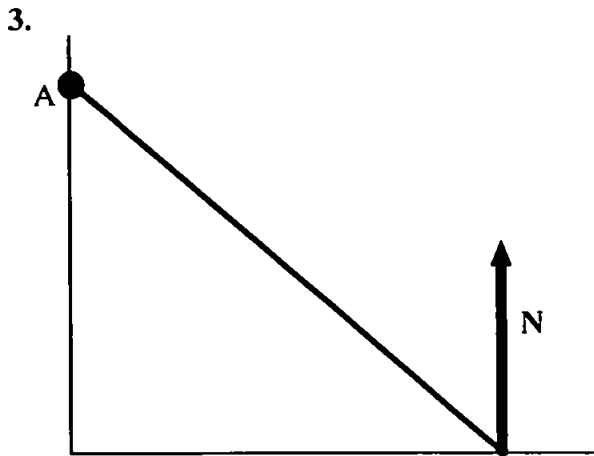
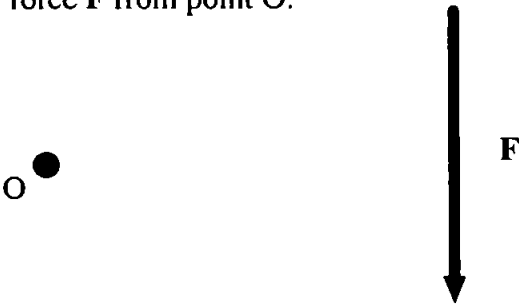


m =



m =

2. Draw the perpendicular distance of force F from point O:



The diagram shows a ladder leaning against a wall, and the normal reaction of the ground on the ladder: N. Draw the perpendicular distance of the force N about the point A.

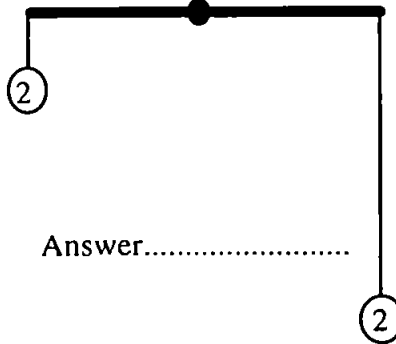
4. In each case the rod is smoothly pivoted at its centre. The masses are in Kg and the strings are light. The rod is held in the position shown and released. Describe what happens in each case (e.g. 'Stays where it is', 'swings clockwise until coming to rest vertically' etc.)

(a)



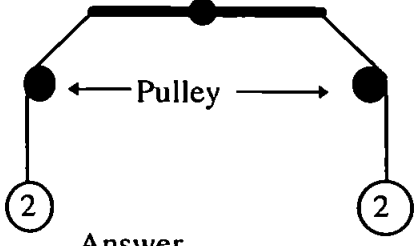
Answer.....

(b)



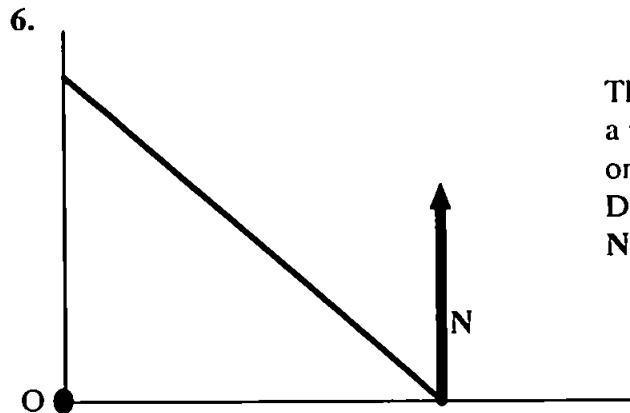
Answer.....

(c)



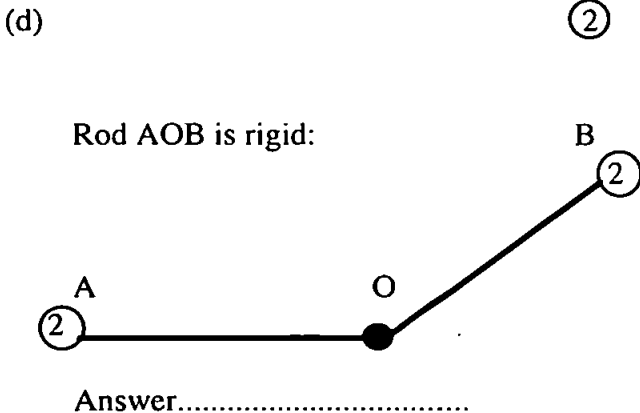
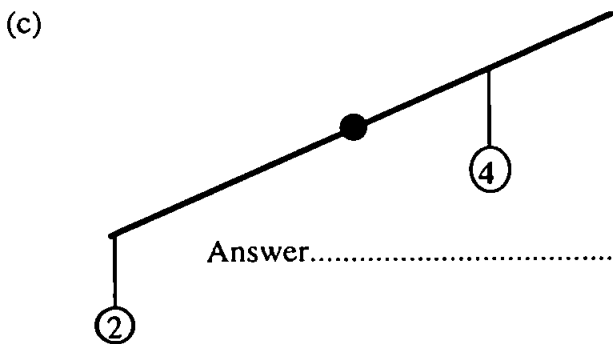
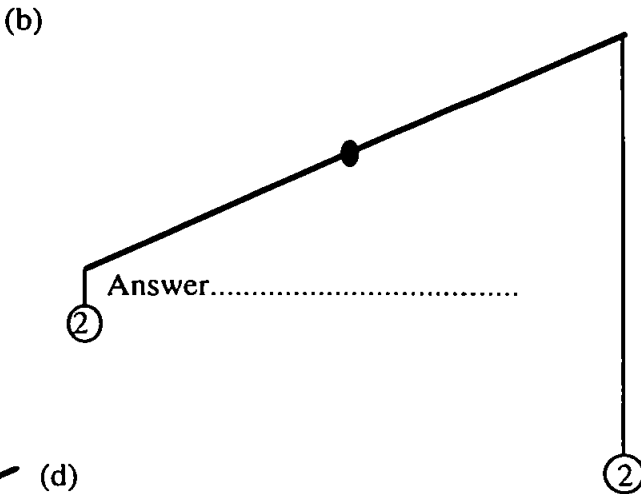
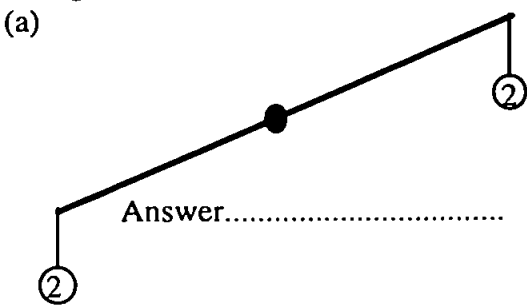
Answer.....

5. Draw the perpendicular distance of force F from point O :

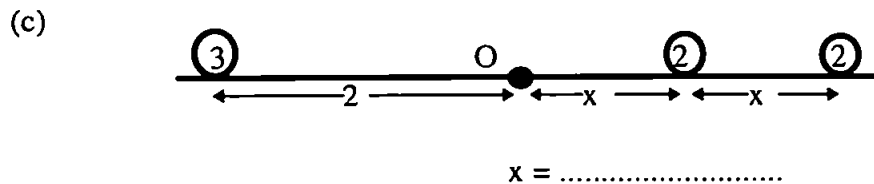
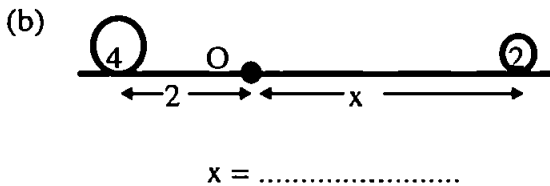
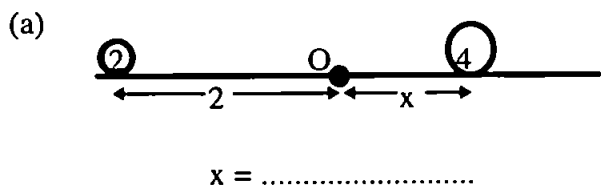


The diagram shows a ladder leaning against a wall, and the normal reaction of the ground on the ladder: N . Draw the perpendicular distance of the force N about the point O .

7. In each case the rod is smoothly pivoted at its centre. The masses are in Kg and the strings are light. The rod is held in the position shown and released. Describe what happens.



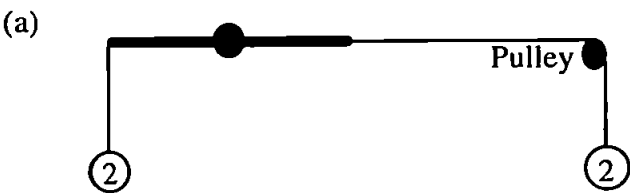
8. Each rod is in equilibrium, smoothly pivoted at O. The masses are in Kg and the lengths in m. What is the value of x?



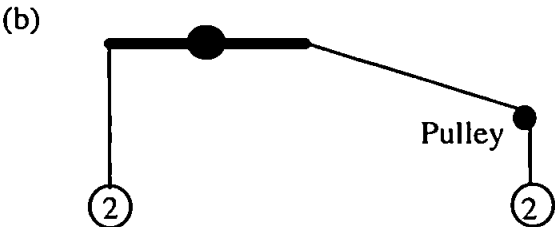
9. Draw the perpendicular distance of force F from point O:



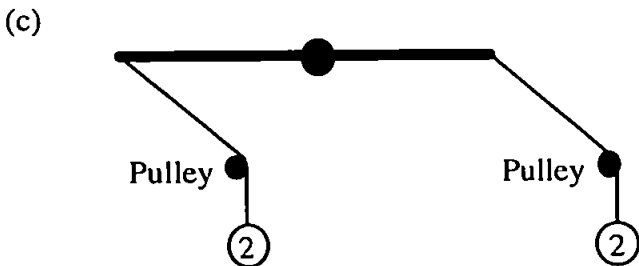
10. In each case the rod is smoothly pivoted at its centre. The masses are in Kg and the strings are light. The rod is held in the position shown and released. Describe what happens in each case.



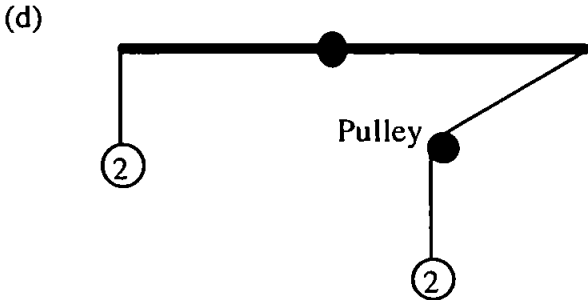
Answer.....



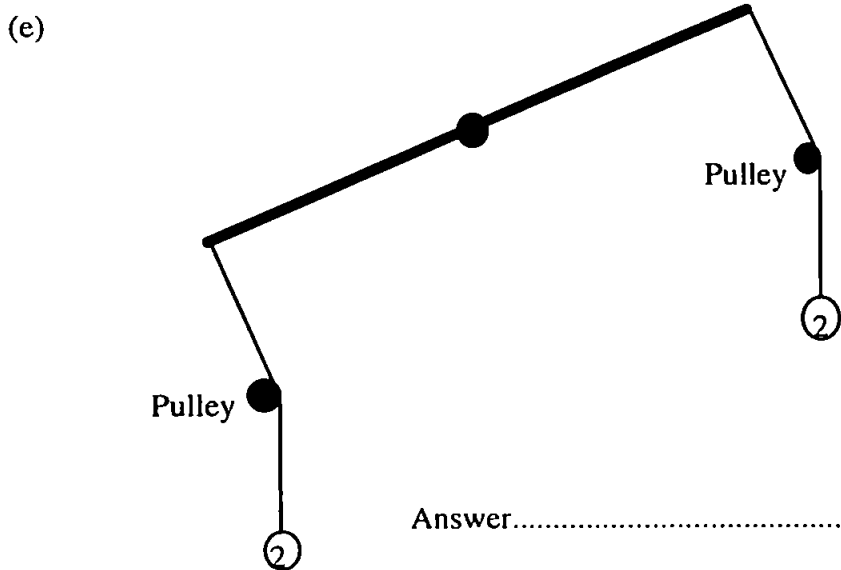
Answer.....



Answer.....



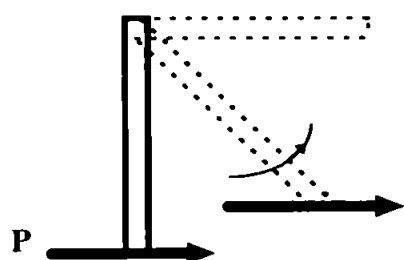
Answer.....



Answer.....

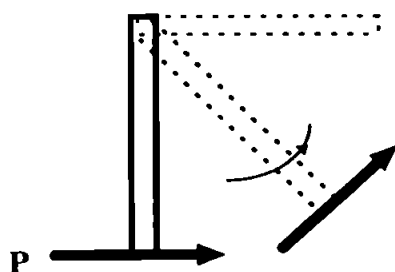
11. A force P is applied to a door. What happens to the moment about O if the force P always remains

(a) in the direction shown:

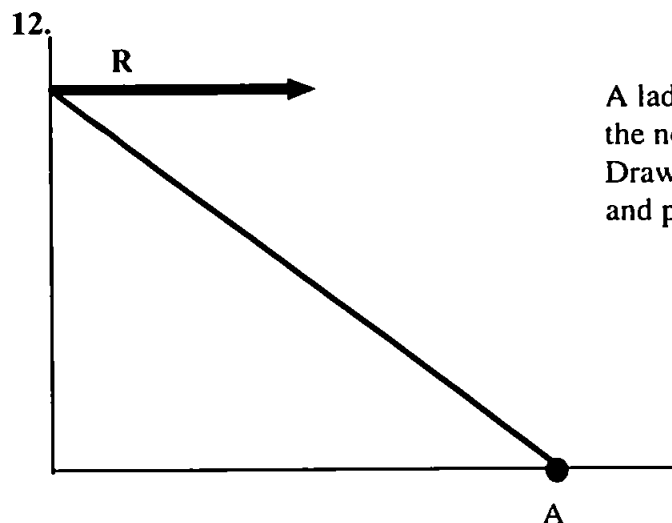


Answer.....

(b) at 90 degrees to the door?

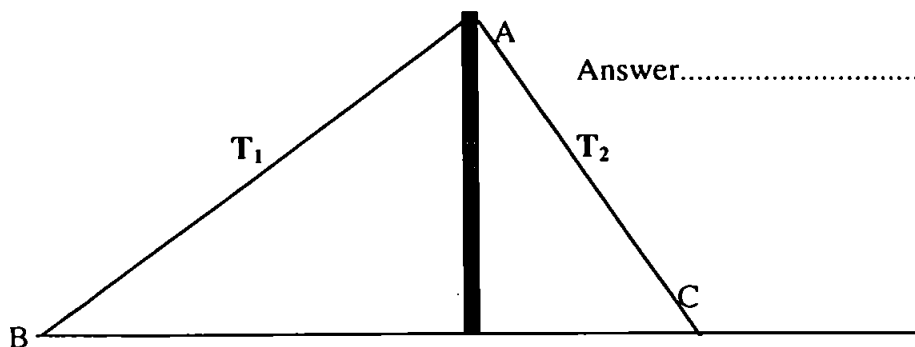


Answer.....



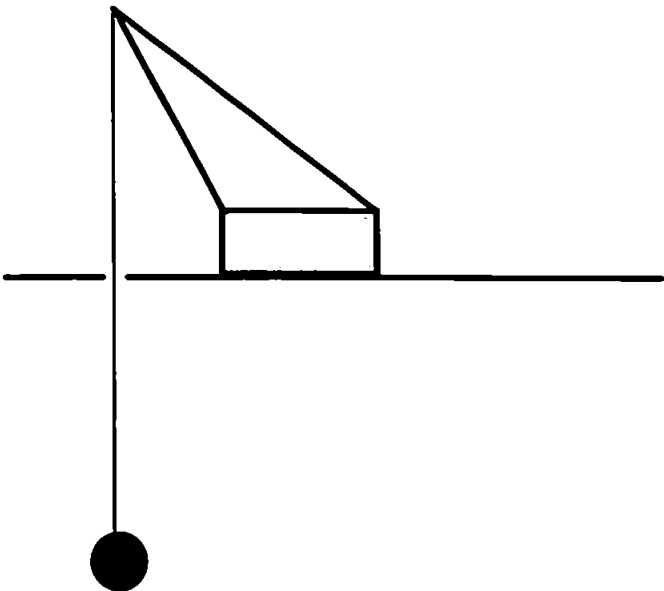
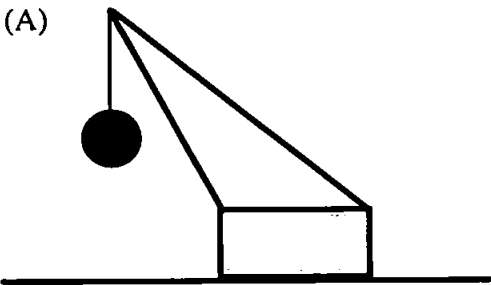
A ladder leans against a wall as shown. R is the normal reaction of the wall on the ladder. Draw the perpendicular distance between R and point A .

13. A pole is placed on the ground as shown, and held in position by two cables AB and AC . Which tension is the greater, T_1 or T_2 or are both tensions the same in magnitude?

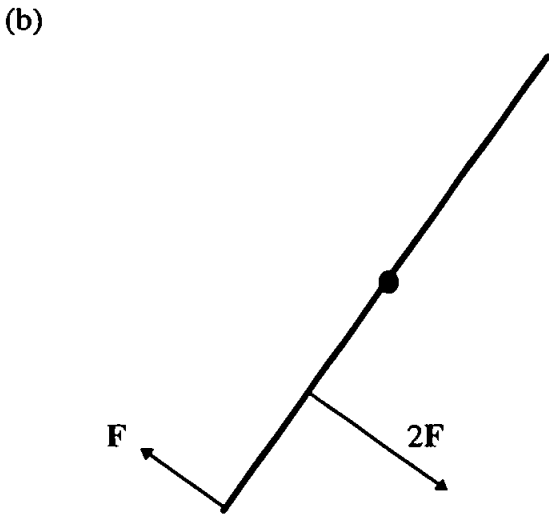
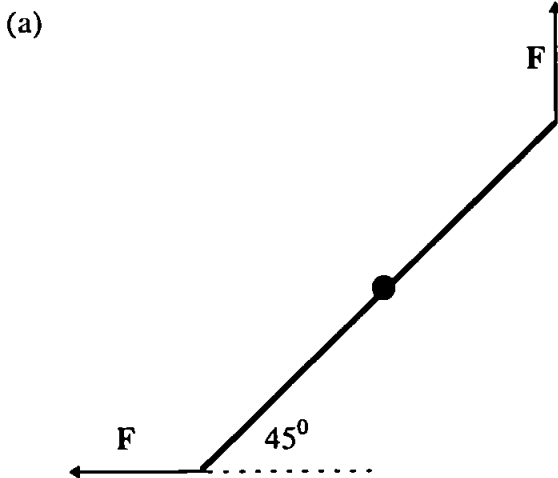


Answer.....

14. Which crane (if any) is most likely to topple (consider the chain holding the mass to be light)?

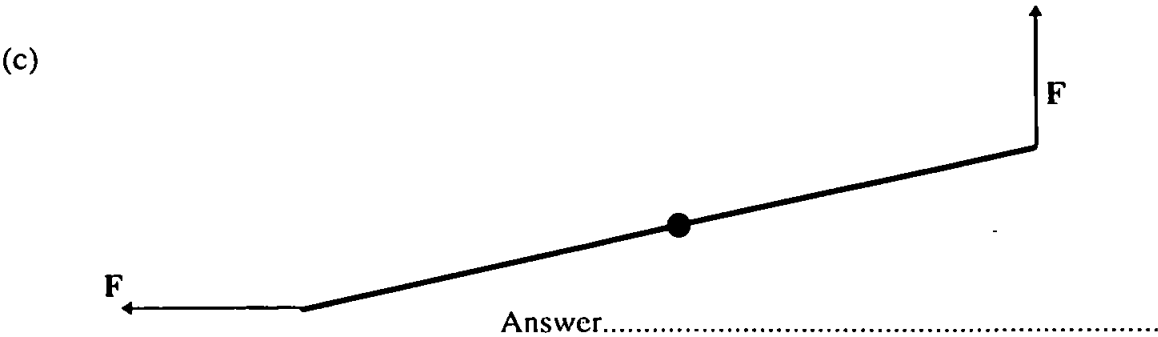


15. In each case the rod is smoothly pivoted at its centre and forces are applied as shown. If the rod is held in the position shown and released, what happens?

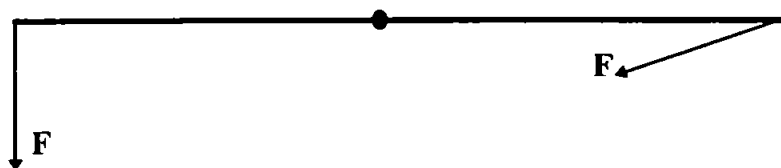


Answer.....

Answer.....

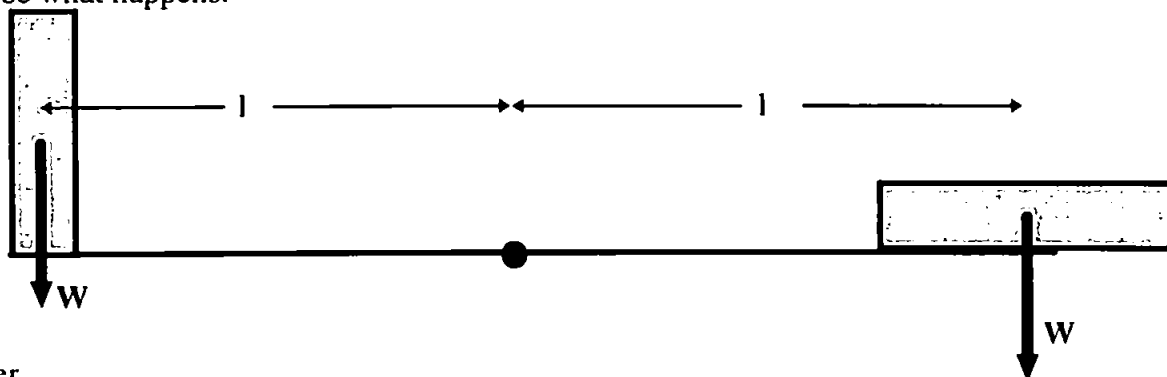


(d)



Answer.....

16. The rod is smoothly pivoted at its centre. It is held in the position shown and released. Describe what happens.

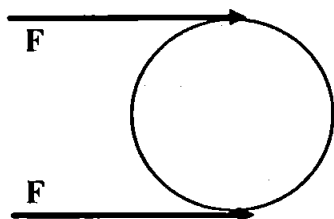


Answer.....

17. Forces are applied to a playground roundabout as shown. Describe fully the motion of the roundabout in each case (e.g. 'continues in a state of rest', 'rotates clockwise at a constant rate', 'rotates anticlockwise at an increasing rate' etc).

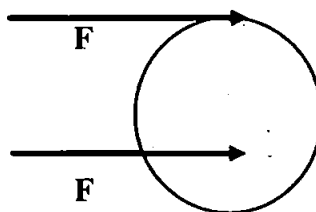
Neglect friction at the pivot and air resistance.

(a)



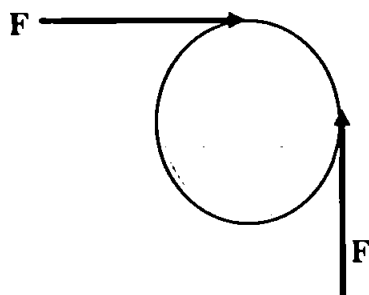
Answer.....

(b)



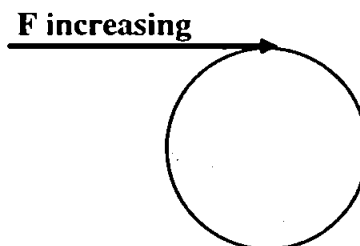
Answer.....

(c)

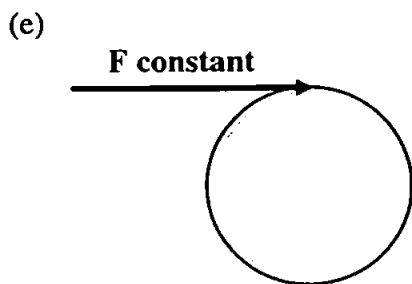


Answer.....

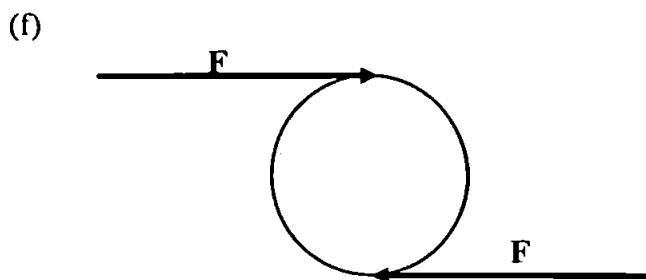
(d)



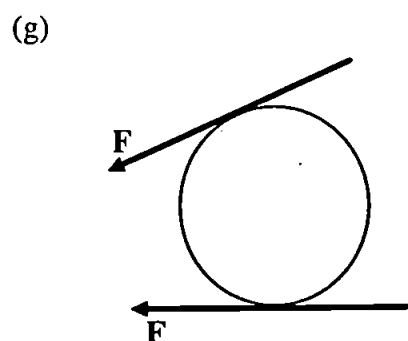
Answer.....



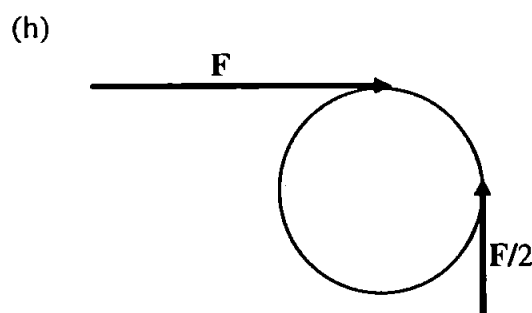
Answer.....



Answer.....

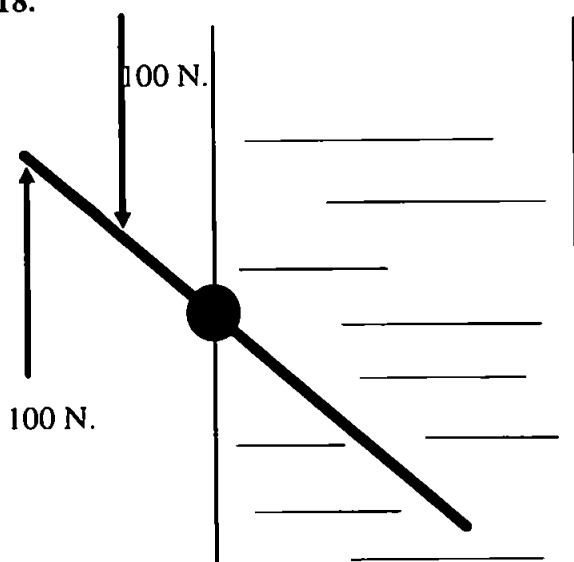


Answer.....



Answer.....

18.



What happens to the lock gate?

Answer.....

19. A disc rotates at a constant speed. What is the resultant moment on the disc?

Answer.....

APPENDIX H - THE PRODUCT-MOMENT CORRELATION COEFFICIENT FOR DICOTOMOUS DATA (*PHI* OR $\emptyset$)

THE PRODUCT-MOMENT CORRELATION COEFFICIENT FOR DICHOTOMOUS DATA

The product-moment correlation coefficient for dichotomous data, ϕ , is used extensively by psychologists to determine whether there exists any association between the responses given by the members of a sample under investigation (Ferguson, 1981). This method is commonly spoken of as *psychological test*, or *mental test*, theory (Ferguson, 1981).

If two test items are to be compared, then the procedure for calculating ϕ is as follows:

The responses of the students to the two test items are cross-tabulated to give the following table:

		ITEM 2	
		Fail	Pass
ITEM 1	Pass	a	b
	Fail	c	d

Table 16 - Fourfold Table Classifying Individuals According to Their Performance on Two
Test Items

The coefficient is then calculated using the formula:

$$\phi = \frac{bc - ad}{[(a+b)(a+c)(b+d)(c+d)]^{1/2}}$$

If the majority of the students either got both questions correct or both incorrect, then there is a high association between the questions. If a and d are both equal to 0, then there is a perfect association whereby $\phi = 1$. In the extreme case where $a = b = c = d$, then $\phi = 0$ and there is no association whatsoever between the two questions. If there is no association between the questions then the students would be randomly distributed to the four categories a , b , c and d .

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