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How do renewable and non-renewable co-move? Fresh evidence from the European energy market via ARJI-GARCH copula model*

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Abstract. The global energy crisis and rising climate concerns are encouraging the transition to a more sustainable economy. Monitoring energy prices has become increasingly important for analysts, policymakers and businesses to tackle the current situation by increasing the integration of renewable energy sources, strengthening the resilience of the energy system to price shocks and reducing its dependence on fossil fuel imports, as well as improving the affordability of energy for consumers. We set up a copula-based ARJI-GARCH model to investigate the time-varying and non-linear dependence between renewable and non-renewable asset prices in the European energy market. Our results show that the ARJI-GARCH specification is able to provide reliable forecasts and an effective tail risk assessment for the energy sector returns. We then use the ARJI-GARCH forecasts to analyse the co-movement structure of renewable and non-renewable energy asset prices by applying and comparing different copula specifications.

Keywords. ARJI-GARCH copula model, Non-linear dependence, Value at Risk, Energy markets, Renewable energy.

M.S.C. classification. 62M10, 62P05, 62H05.
J.E.L. classification. C40, P18, E44.

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1 Introduction

Several researches confirm the predictive power of Autoregressive Jump Intensity-GARCH (ARJI-GARCH) volatility forecasts within and between the main financial markets (covering stock, bond, cryptocurrency, and commodity markets). To better understand the consistency of these models to the features of different markets, only the most relevant papers will be quoted.
Jumps in asset pricing and their effect on volatility prediction are pointed out by an increasing number of studies, starting from the seminal work by [4], implemented by [14].
[4] employed the ARJI-GARCH model on daily returns of the Dow Jones Industrial Average price index, to take into account the persistence of conditional variance, the time variation in volatility and in conditional skewness and kurtosis related to the mean and variance of jump size, and jump intensity. The volatility process is affected by common return shocks, and by unexpected ones due to extreme price movements because of natural disasters, geopolitical developments, and strategic actions. Persistent innovation is assessed by the expected jump component captured by the autoregressive part in volatility, whereas unpredictable innovation is captured by the conditional variance and affects the higher-order moments of returns. [4] argued that the intensity of jumps is sensitive to economic states, rising during periods of extreme volatility; [14] improved the forecasts of volatility after relevant changes in stock returns and argued that the probability of jumps would increase before market crashes.
Due to the strategic impact and financialization of commodities in the global economy (see e.g. [15]), a number of studies have applied the ARJI-GARCH model to forecast the jump diffusion volatility within the energy market and between the energy and other markets.
Among others, [8] proposed asymmetric GARCH-Jump models that synthesize autoregressive jump intensities and volatility feedback in the jump component. Their results indicate that these models provide a better fit for the dynamics of the equity returns in the US and emerging Asian markets, irrespective of whether the volatility feedback is generated through a common GARCH multiplier or a separate measure of volatility in the jump intensity function. Moreover, referring to specific sample periods, some studies provide empirical evidence that time-varying jumps are able to capture several distinguishing features of the return dynamics, such as volatility persistence, leverage effects, fat tails.
Among these articles, [7] implemented the ARJI-GARCH model with structure changes on daily S&P 500 and West Texas Intermediate (WTI) oil prices, showing that high fluctuations in oil prices have asymmetric unexpected impacts on S&P 500 returns. Additionally, [12] found that the magnitude of oil price shocks varies with the fluctuating range of oil returns. [43] and [42] showed that the information on time-varying jumps in oil prices could predict changes in the price levels of agriculture. [41] applied the ARJI-GARCH model to three energy commodity futures prices (crude oil, natural gas, coal), stressing the importance of incorporating time-varying jump intensities. [33] analyzed the price movements in the oil market stimulated by extreme events (such as oil platform explosions,

geopolitical events, and financial crises) in order to understand the reaction and the persistence of these effects on the commodity prices. Based on a daily sample of closing prices of WTI from January 2010 to December 2017 obtained from NYMEX, the empirical results show that a time-varying conditional jump process can be specified, but it has little sensitivity to past shocks and very short-term persistence.

Dependence on fossil fuel for energy generation is a major driver of climate change and undermines the key objectives of the 2030 Agenda[1]. The effectiveness of the integration of renewable energy sources and the dependence among energy and other markets can be investigated by using copulas (see e.g. [1], [2], [3], [5], [9], [16],[17], [20], [21], [22],[23], [24], [25], [27], [26], [28], [29], [30], [31], [32], [38], [39], [34], [35]).

In particular, using several copula models with different conditional dependence structures and time-varying parameters, [20] examined the relationship between crude oil prices, showing significant symmetric upper and lower tail dependence. A mixture copula-based ARJI–GARCH model was properly applied by [5] to capture the asymmetric dependence between crude oil spot and futures returns. By applying the copula-GARCH approach, [1] studied the conditional dependence structure between crude oil prices and US dollar exchange rates, in both bearish and bullish market phases, finding that Student-t copulas best capture the extreme dependence. In [3], the copula-GARCH approach was applied to investigate the dependence and extreme dependence of crude oil and natural gas prices, with applications to portfolio risk management in extreme economic conditions. The crude oil and gas markets tend to co-move closely together during bullish periods, but not at all during bearish periods. Moreover, taking the extreme co-movement into account leads to an improvement in the accuracy of the out-of-sample Value-at-Risk forecasts.

Focusing on renewable markets, [25] analyzed the relationship between oil and renewable return movements, finding significant time-varying average and symmetric tail dependence between oil returns and several global and sectoral renewable energy indexes. [30] demonstrated that, although the short-run connection between energy and clean energy stock prices appears to be weak, such a relationship seems strong in the long run. Using bivariate copula functions, [31] described the interdependence in the European and USA stock markets, to evaluate price spillovers, under normal and extreme market scenarios. For the period 2010–2019, European renewable-energy and low-carbon stocks co-move; upward and downward movements in low-carbon asset prices have sizeable effects on renewable-energy asset prices, and vice versa. In contrast, no interdependence occurs for the USA, as no significant upward or downward price spillover effects between renewable-energy and low-carbon stocks are found (see also [16], [17], [26], [32], [34], [35], [36]).

This paper contributes to the existing literature in two main directions. First, while previous studies mainly focused on dependence within the energy market

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[1] https://sdgs.un.org/2030agenda, Transforming our world: the 2030 Agenda for Sustainable Development"

and between energy and other markets, we analyse the dynamic relationship between renewable and non-renewable energy asset price time series. Our modelling approach relies on the application of an ARJI-GARCH model, which defines an error correction mechanism between renewable and non-renewable energy prices. Thus, our findings provide useful insights for the implementation of policies that support the transition to a more sustainable economy. Then, by applying and comparing two different copula specifications to the obtained forecasts, we study the co-movement structure of renewable and non-renewable energy asset prices, with implications for portfolio risk management and investment strategies.

The paper proceeds as follows: Section 2 describes the model employed to analyse the dynamic relationship between renewable and non-renewable energy. Section 3 shows the results obtained by applying the proposed modelling approach to the returns of renewable (ERIX) and non-renewable (MSCI Europe) energy indexes. Section 4 concludes.

2 The model

2.1 ARJI-GARCH model

[4] introduced a discrete time jump model for stock returns which combines the GARCH parameterization of volatility with the specification of a time-varying conditional jump intensity and jump size distribution:

$$r_t = c + \sum_{m=1}^{M} \phi_m + r_{t-m} + \sqrt{h_t} z_t + \sum_{k=1}^{n_t} Y_{t,k} \tag{1}$$
$$z_t \sim NID(0,1),\ Y_{t,k} \sim N(\theta_t, \delta^2)$$

where the number of jumps between $t-1$ and t, conditional on the information set Ω_t, follows a Poisson distribution with time-varying λ_t intensity:

$$P(n_t = j|\Omega_{t-1}) = \frac{\exp - \lambda_t \lambda_t^j}{j!} \tag{2}$$

The intensity parameter evolves according to the following process:

$$\lambda_t = \lambda_0 + \sum_{i=1}^{r} \rho_i \lambda_{t-i} + \sum_{i=1}^{s} \gamma_i \xi_{t-i} \tag{3}$$

where ξ_{t-i} represents the unpredictable component affecting the inference of the econometrician about the conditional mean of the counting process, and is calculated as

$$\xi_{t-i} = \sum_{j=0}^{\infty} j P(n_{t-i} = j|\Phi_{t-1}) - \lambda_{t-i} \tag{4}$$

[4] show that the ARMA specification of the jump intensity parameter allows to parsimoniously capture many forms of autocorrelation. The ARJI-GARCH specification was shown to be suitable to model the dynamics of stock returns, which are empirically characterized by strong persistence in the jump intensity: days with high probability of many (few) jumps tend to be followed by other days with a high probability of many (few) jumps.

2.2 Marginal density specification

The AutoRegressive Jump-Intensity-GARCH (ARJI-GARCH) specification by [5] extends the ARJI-GARCH model of [4] and defines the following process for the i asset return:

$$r_{i,t} = c_i + \sum_{m=1}^{bm} a_{im} r_{i,t-m} + \sum_{n=1}^{bn} a_{kn} r_{k,t-n} + a_{iEC} EC_{t-1} + \varepsilon_{1t}^i + \varepsilon_{2t}^i \qquad (5)$$

where c_i is a constant, $r_{i,t-m}$ is the m-order lagged return of i, $r_{k,t-n}$ is the m-order lagged return of another asset (k), a_{im} and a_{kn} are the associated coefficients.
In our application to the energy market, the two assets are a non-renewable and a renewable energy index, whose prices in t are denoted as $P_{tr,t}$ and $P_{re,t}$, with returns defined as $r_{tr,t}$ and $r_{re,t}$ respectively.
The error correction component reflects the deviation from the long-run equilibrium between traditional and renewable energy prices and is defined as

$$EC_{t-1} = \ln(P_{tr,t-1}) - \ln(P_{re,t-1}). \qquad (6)$$

The two residual components ε_{1t}^i and ε_{2t}^i are assumed to be independent. The first one captures the return innovation related to the persistence of shocks to volatility, while the second represents the shock arising from large and unexpected price changes.
The specification of the first error term ε_{1t}^i affects volatility through the GARCH variance factor h_t^i:

$$\varepsilon_{1t}^i = \sqrt{h_t^i} z_t \qquad z_t \sim NID(0,1) \qquad (7)$$

where z_t is an i.i.d standard normal innovation.
The conditional variance h_t^i has the following dynamics:

$$h_t^i = \alpha_0^i + \alpha_1^i \varepsilon_{t-1}^i + \alpha_2^i h_{t-1}^i + \alpha_{EC}^i EC_{t-1}^2, \qquad (8)$$

where α_0^i is a constant, α_1^i and α_2^i are the ARCH and GARCH coefficients respectively, while α_{EC}^i expresses the impact of the error correction term on return volatility.
So far, the model has an AR-GARCH specification. The ARJI-GARCH model is obtained by adding the second error term, which is defined as the difference

of the total jump size between $t-1$ and t and its expected value, given the information set Ω_{t-1}:

$$\varepsilon_{2t}^{i} = \sum_{j=1}^{n_t^i} Y_{t,j}^{i} - E\left(\sum_{j=1}^{n_t^i} Y_{t,j}^{i} \,|\Omega_{t-1}\right) \qquad Y_{t,j}^{i} \sim NID(\theta_{i,t}, \delta_{i,t}^{2}). \tag{9}$$

The jump size is assumed to follow a normal distribution with mean $\theta_{i,t}$ and variance $\delta_{i,t}^2$, whose dynamics is assumed to be related to the error correction process:

$$\theta_{i,t} = \theta_0^i + \theta_1^i EC_{t-1} \tag{10}$$

$$\delta_{i,t}^2 = \beta_0^i + \beta_1^i EC_{t-1}^2. \tag{11}$$

Given the filtration Ω_{t-1}, the jump number n_t^i follows instead a Poisson distribution, with density

$$P\left(n_t^i = j\,|\Omega_{t-1}\right) = \frac{\exp(-\lambda_t^i)(-\lambda_t^i)^m}{m!} \qquad m = 0, 1, 2, \ldots \tag{12}$$

where the jump intensity λ_t^i is the expected number of jumps from time $t-1$ to t and is assumed to follow a simil-ARMA process:

$$\lambda_t^i = \phi_0^i + \phi_1^i \lambda_{t-1}^i + \phi_2^i \zeta_{t-1}^i \tag{13}$$

where $\phi_0^i > 0$ and $\phi_1^i > \phi_2^i$ to ensure $\lambda_t^i > 0$.
The ζ_{t-1}^i component is the forecasting error associated with the updating in the information set:

$$\zeta_{t-i}^i = E\left[n_{t-1}^i\,|\Omega_{t-1}\right] - E\left[n_{t-1}^i\,|\Omega_{t-2}\ \ \right] = E\left[n_{t-1}^i\,|\Omega_{t-1}\ \ \right] - \lambda_{t-1}^i. \tag{14}$$

2.3 Copula function specification

Drawing now attention to the co-movement structure and possible tail dependence in the energy market, we consider two different copula specifications: the Normal and the Student's t one. We recall that Normal and Student's t copulas allow for a symmetric tail dependence. More specifically, the Normal copula is designed to model dependence in the center of the joint distribution, while the Student's t copula is suitable to model both lower and upper tail dependence (see e.g. [19]). As opposite to other types of copula families (e.g. Archimedean copulas), the dependence parameters of both the Normal and Student's t copulas can take positive as well as negative values, allowing them to accommodate for both positive and negative dependence.
Under the ARJI-GARCH model, the copula functions can be applied to the transformed series $u_{tr,t}$ and $u_{re,t}$, calculated as follows:

$$u_{i,t} = F_i(r_{i,t}|\Omega_{t-1}) \tag{15}$$

$$= \sum_{m=0}^{\infty} \int_{-\infty}^{r_{i,t}} \frac{1}{\sqrt{2\pi(h_t^i + m\delta_{i,t}^2)}}$$

$$\exp\left(-\frac{r_{i,t} - c_i - \sum_{m=1}^{bm} a_{im} r_{i,t-m} - \sum_{n=1}^{bn} a_{kn} r_{k,t-n} + a_{iEC} EC_{t-1} - m\theta_{i,t} + \theta_{i,t}\lambda_t^i}{2(h_t^i + m\delta_{i,t}^2)}\right). \tag{16}$$

The copula ARJI-GARCH log-likelihood function is then composed by the two marginal density functions and the copula density:

$$\ln L = \sum_{t=1}^{T} \ln f_{tr}(r_{re,t}\,|\Omega_{t-1}\,) + \ln f_{re}(r_{re,t}\,|\Omega_{t-1}\,) + \sum_{t=1}^{T} \ln c(u_{re,t}, u_{re,t}\,|\Omega_{t-1}\,) \tag{17}$$

where $f(\cdot)$ and $c(\cdot)$ denote the probability density function and the copula density function respectively.

From both the Normal and the Student's t copula we obtain, as a measure of dependence, the Kendall's tau:

$$\tau = Pr[(r_{tr,1} - r_{tr,2})(r_{re,1} - r_{re,2}) > 0] - Pr[(r_{tr,1} - r_{tr,2})(r_{re,1} - r_{re,2}) < 0]. \tag{18}$$

The sign of τ represents the direction of co-movements in the energy market: if τ is positive (negative), the probability that the two assets move in the same direction is larger (smaller) than the probability of moving in the opposite direction. The absolute value of τ measures the magnitude of the estimated dependence.
To take into account the time-changing nature of dependence between traditional and renewable energies, we assume that τ evolves according to the following process:

$$\tau_t^j = \Gamma(b_0 + b_1\tau_{t-1} + b_2\tau_{t-2} + b_3|u_{tr,t} - u_{re,t}|) \tag{19}$$

with $\Gamma(z) = 1/(1 + e^{-z})$.

3 Empirical study

3.1 Data

Focusing on the European energy market, our study investigates the dependence between renewables and non-renewable energy using data relative to daily closing prices of two European stock indexes: the European Renewable Energy index

(ERIX)[2] and the MSCI Europe Energy Index[3]. The first one tracks the performance of the largest companies in the areas of renewable energy such as wind, solar, biomass and water energy. The second one includes securities classified in the Energy sector as per the Global Industry Classification Standard (GICS) in the large and mid cap segments across 15 European countries. The sample period spans from 22/09/2003 to 04/04/2022, for a total of 4768 observations. Both series were retrieved from Bloomberg.
Figure 1 plots the ERIX and MSCI price time series analyzed in the paper.

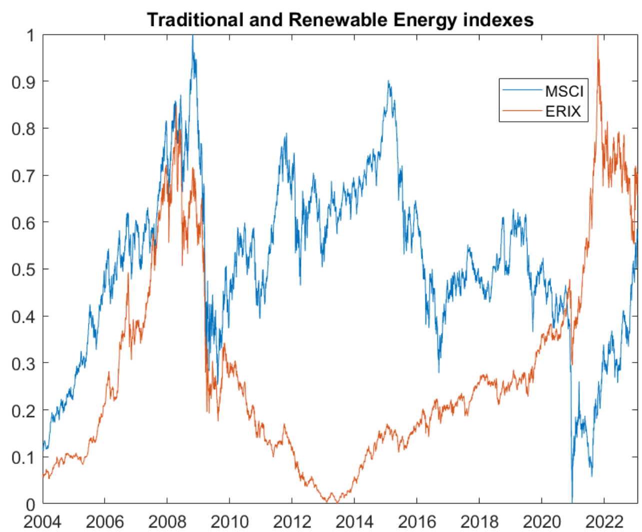

Fig. 1. MSCI and ERIX price time series (scaled values)

The International Paris Agreement adopted at the Paris climate conference (COP21) in December 2015 underlines the need to convert the EU into a fair, competitive and sustainable economy reducing greenhouse gas emissions by at least 40 per cent (compared to 1990 levels), with a renewable energy share of at least 32 per cent, with at least 32.5 per cent energy efficiency. Based on the Regulation on the Governance of the Energy Union and Climate Action (September 2020), the EU has adopted integrated rules to ensure planning, monitoring and reporting of progress towards its 2030 climate and energy target and its international commitments under the Paris Agreement[4]. From 2003 to 2008, both

[2] https://sgi.sgmarkets.com/en/index-details/TICKER:ERIX/
[3] https://www.msci.com/www/fact-sheet/msci-europe-energy-index/08601638
[4] https://ec.europa.eu/clima/eu-action/international-action-climate-change/climate-negotiations/paris-agreement$_i t$.

energy indices tripled in value, essentially following the surge in the price of oil, which rose from about 33/bar to a peak of 145/bar (as measured by WTI). In addition, the renewable energy sector continued to break records year after year from 2004 to 2008, until the impact of the 2008-2009 Global Financial crisis (GFC). Subsequently, the European sovereign debt crisis (ESDC) from 2010 to 2012 also had a significant impact on new investments in renewable energy sectors negatively affecting the performance of the ERIX until 2012. In the same period fossil fuel showed a mirror performance, due to the substitution effects between the ERIX and the MSCI Europe Energy Index. At the end of 2019, the EU launched the European Green Deal by planning climate-relevant policies. The sharp drop in the oil prices in the early 2020 stemmed mainly from the negative impact of the Covid pandemic and the price war crisis between Saudi Arabia and Russia. The turmoil in the oil markets, demand uncertainty and the Covid pandemic have prompted a global sector-wide downturn in the oil and gas industry that has left the oil-dependent economies vulnerable. In addition the volatility of oil and gas prices was amplified by the Russian invasion of Ukraine in February 2022 and the related EU policy responses, increasing uncertainty over oil-sector supply, which made prices more sensitive to changes in the outlook for energy supply, while the renewable sector has received increased interest due to regulatory incentives.

3.2 Results for the marginal density specification

Table 1 reports maximum likelihood estimates of parameters for the ARJI-GARCH, comparing them to their values in the AR-GARCH (with no jumps) model. The comparison of the proposed specification with the nested AR-GARCH one allows to better highlight the contribution of the jump innovation part in improving prediction of returns and tail risk assessment for the energy market. We use a number of jumps equal to 21, that means we consider, at each time t, the jump behaviour of energy asset returns in the last trading month. For renewable energy, the estimated a_{EC} is negative and statistically significant, while the same parameter is not significant for the traditional energy index. This means that returns of renewable energy sector do respond to the error correction term to adjust to the long-run equilibrium, while returns of traditional energy do not react, though they are significantly related with lagged returns of the renewable index. Both the a_{EC} parameters are instead non-significant in the AR-GARCH model.

The ARCH and GARCH coefficients are positive and statistically significant and their sum is close to 1 for both markets, indicating high persistence of shocks to conditional variance in the energy market. The error correction term does not affect conditional variance in the estimated ARJI-GARCH model, while the α_{EC} coefficients are significant in the AR-GARCH specification. In the ARJI-GARCH model, significant error correction terms are rather found in the jump component, with a positive θ_1 parameter for renewable energy and both positive β_1's, meaning that a significant reaction to disequilibrium is found in the mean and variance jump size of renewable asset returns and in the variance jump size

of both markets.
Shocks to the number of jumps are highly persistent (both ϕ_1 parameters are close to 1) and both series react considerably to the innovation in jump intensity, being the ϕ_2 parameters highly significant.
Overall, our results reveal that there is a long-run equilibrium relationship between renewable and non-renewable energy asset prices, sustained by the significant negative error correction parameter expressing the reaction of the mean of renewable energy returns to changes in the non-renewable ones. This is in line with the findings of [37] and [30], amongst the others. The reaction of renewable energy indexes to changes in the non-renewable ones can be an indication for policymakers, when defining incentives to green energy consumption. In addition, a significant and bidirectional positive association is found in the mean and variance of the jump size: this indicates the presence of tail co-movements in the renewable and non-renewable energy asset returns, which should be taken into account by investors when assessing their risks. Table 1 also reports the values of the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), which agree in choosing the ARJI-GARCH model as the preferable specification.
By considering the root mean squared error (RMSE) associated to mean and volatility predictions, it can be seen from Table 2 that including the jump component improves accuracy of both mean and volatility forecasts: in only one case - the prediction of ERIX volatility - the AR-GARCH specification leads to a lower error. We then evaluate the ARJI-GARCH model performance from a risk management point of view, by calculating the failure rate of the in-sample Value-At-Risk (VaR). Specifically, for a given confidence level (α=95%, 97% and 99%), VaR_α is the α-th percentile of losses predicted by the model[5]. As it can be seen from Table 2, the frequency of VaR violations is lower with the ARJI-GARCH model at all considered confidence levels. The VaR failure rate of the AR-GARCH specification nearly always exceeds the nominal level, meaning that the model without jumps underestimates the true risk. According to these results, the ARJI-GARCH specification is more capable of assessing the tail risk of the energy market.
Figure 2 shows the time series of observed and fitted returns (95% confidence bands) for the ARJI-GARCH model, which is shown to capture the energy market dynamics well.

3.3 Results for the copula specification

We now show the results obtained by applying time-varying Normal and Student's t copulas to the time series of energy market returns estimated through the ARJI-GARCH model.
As a preliminary analysis, we perform a maximum likelihood-based selection using the VineCopula R package [18], which chooses the Student's t bivariate

[5] The percentile values are obtained from Monte Carlo simulations. Details are available upon request.

Table 1. Maximum likelihood estimates of parameters for the estimated ARJI-GARCH and AR-GARCH models (standard errors in brackets).

	ARJI-GARCH		AR-GARCH	
Parameter	Renewables	Traditional	Renewables	Traditional
a_0	-0.0117 (0.0924)	0.1153*** (0.0492)	0.0282 (0.0605)	0.0360 (0.0390)
a_{re1}	-0.0107 (0.0157)	-0.0403*** (0.0111)	0.0007 (0.0168)	0.1095*** (0.0166)
a_{re2}	0.0348** (0.0158)	0.0190* (0.0112)	0.0408*** (0.0167)	-0.0255 (0.0164)
a_{re3}	-0.0438*** (0.0155)	-0.0023 (0.0112)	-0.0339*** (0.0165)	-0.0326** (0.0165)
a_{re4}	-0.0146 (0.0154)	0.0019 (0.0158)	0.0030 (0.0164)	-0.0308* (0.0162)
a_{tr1}	0.1494*** (0.0182)	0.0777*** (0.0177)	0.1526*** (0.0191)	-0.0455*** (0.0112)
a_{tr2}	-0.0324* (0.0182)	-0.0317* (0.0165)	-0.0233 (0.0190)	0.0283*** (0.0113)
a_{tr3}	0.0197 (0.0178)	-0.0416*** (0.0160)	0.0071 (0.0189)	-0.0076 (0.0113)
a_{tr4}	-0.0112 (0.0181)	-0.0105 (0.0112)	-0.0224 (0.0187)	-0.0089 (0.0114)
a_{EC}	-0.1034* (0.0562)	-0.0192 (0.0306)	-0.0420 (0.0388)	-0.0273 (0.0268)
α_0	0.0033*** (0.0133)	0.0068*** (0.0027)	0.0712*** (0.0143)	0.0191*** (0.0049)
α_1	0.0030** (0.0014)	0.0425*** (0.0064)	0.1042*** (0.0098)	0.1047*** (0.0087)
α_2	0.9893*** (0.0038)	0.9369*** (0.0078)	0.8673*** (0.0124)	0.8783*** (0.0094)
α_{EC}	0.0001 (0.0005)	0.0018 (0.0012)	0.0076*** (0.0032)	0.0085*** (0.0022)
θ_0	0.0547 (0.0927)	-0.6284 (0.3103)		
θ_1	0.1207*** (0.0581)	0.0586 (0.1603)		
β_0	1.0306*** (0.2941)	0.4594* (0.2421)		
β_1	0.4098*** (0.1088)	0.3716*** (0.1340)		
ϕ_0	0.0344*** (0.0079)	0.0248* (0.0137)		
ϕ_1	0.9615*** (0.0088)	0.8826*** (0.0570)		
ϕ_2	0.6368*** (0.0858)	0.5074*** (0.1521)		
Log-likelihood	-8802.7	-7690.8	-8939.4	-7808.0
AIC	17647	15424	17907	15644
BIC	17783	15560	17997	15735

Table 2. Predictive accuracy measures and VaR failure rate for the estimated ARJI-GARCH and AR-GARCH models.

	ARJI-GARCH		AR-GARCH	
	Renewables	Traditional	Renewables	Traditional
RMSE (mean)	1.8402	1.5943	1.8416	1.5973
RMSE (volatility)	9.1655	10.1141	9.1082	10.4912
$\text{VaR}_{95\%}$ failure rate	0.0386	0.0516	0.0493	0.0589
$\text{VaR}_{97\%}$ failure rate	0.0233	0.0279	0.0371	0.0405
$\text{VaR}_{99\%}$ failure rate	0.0071	0.0101	0.0216	0.0201

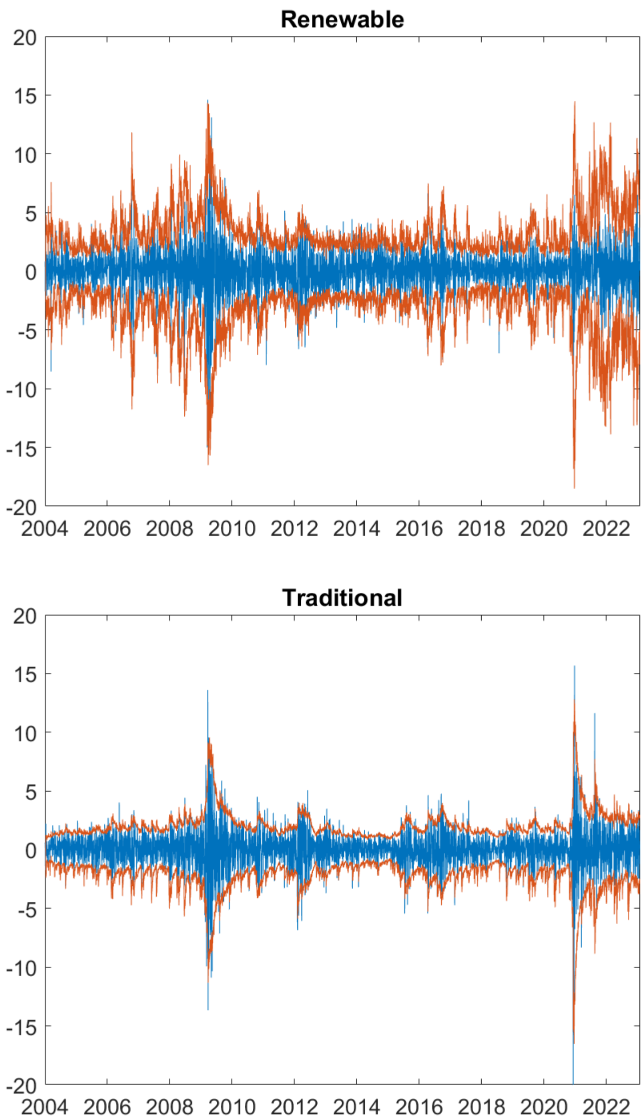

Fig. 2. Observed and fitted values (95% confidence bands) for the ARJI-GARCH model applied to the ERIX (top) and MSCI (bottom) series (percentage returns).

distribution with 10 degrees of freedom as the best one among many well-known copula specifications. The Student's t distribution turns out to be preferable to the Normal one according to both AIC and BIC criteria, as shown in Table 3.

	Normal copula	Student's t copula
AIC	-839.4	-940.1
BIC	-832.9	-927.1

Table 3. Maximum likelihood-based comparison between the Normal and the Student's t copulas with jump process.

Following [5], we then compare the performance of the two copulas using two portfolio hedging-based criteria: the minimum variance hedging ratio and the expected utility of the hedged portfolio.
The first one is related to the common strategy of minimizing the variance of the hedging ratio

$$HR_t = \frac{Cov(r_{tr,t}, r_{re,t}|\Omega_{t-1})}{Var(r_{tr,t}|\Omega_{t-1})} \tag{20}$$

where, under the ARJI-GARCH model assumptions, $Var(r_{tr,t}|\Omega_{t-1})$ can be calculated using the following formula:

$$Var(r_{tr,t}|\Omega_{t-1}) = h_{tr,t} + (\theta^2_{tr,t} + \delta^2_{tr,t})\lambda^t_{tr}. \tag{21}$$

For the covariance calculation, we refer instead to [10] and [11].
The variance of the hedging strategy is then given by

$$Var(r_{re,t} - HR_t \times r_{tr,t}). \tag{22}$$

Following [10] and [11], the expected utility of the hedging portfolio is calculated as

$$EU = E(r_{re,t} - HR_t \times r_{tr,t}) - \kappa Var(r_{re,t} - HR_t \times r_{tr,t}) \tag{23}$$

where κ is the risk-aversion coefficient. The higher κ, the more the portfolio return variability is penalized, for a given expected value.
It can be seen from Table 4 that the Normal and Student's t copulas have similar hedge ratio mean and standard deviation, but the Student's t generates a lower hedging portfolio variance.
Looking then at expected utility, the preferable copula specification is again the Student's t one for all considered κ values.
These results show that the traditional Normal copula approach is not flexible enough to model the dependence between renewable and non-renewable energy indexes, which is, apparently, stronger in the tails of the return distribution. It is interesting to note from Table 5, which reports the estimated parameters for the Kendall's tau dynamics (19), that the coefficient associated to the difference

between u_{re} and u_{tr} is significant and negative-signed in both cases, meaning that a negative difference between renewable and non-renewable energy returns leads to an increase in dependence between the two indexes.

Table 4. Hedging Performance for the Normal and Student's t copula specifications with jump process.

	Normal copula	Student's t copula
Mean of hedge ratio	0.5249	0.5018
Standard deviation of hedge ratio	0.5343	0.5538
Variance of hedged portfolio	1.6919	1.6799
Utility of hedged portfolio(κ=3)	-5.0254	-4.9877
Utility of hedged portfolio(κ=5)	-8.4092	-8.3477
Utility of hedged portfolio(κ=10)	-16.8689	-16.8679

Table 5. Maximum likelihood estimates of parameters for the time-varying copula functions (standard errors in brackets).

Parameter	Normal	Student's t
b_0	3.5172*** (0.0521)	3.5267*** (0.0559)
b_1	0.0237 (0.0481)	0.0201 (0.0491)
b_2	-0.0187 (0.0485)	-0.0085 (0.0498)
b_3	-7.5739*** (0.0993)	-7.8869*** (0.1171)

The Kendall's tau values estimated by the two copulas are quite similar, ranging from -0.8072 and 0.7839 in the Normal case, and from -0.8332 to 0.7860 with the Student's t. The implied average linear correlation coefficients are 0.5248 and 0.5019 (closer to the sample correlation coefficient of 0.4860) respectively. Overall, the Student's t copula turns out to be preferable to the Normal one in driving investment choices in the energy market.

4 Conclusions

Focusing on the European energy market, in this paper we set up a copula-based ARJI-GARCH model to investigate the dependence between renewable and non-renewable energy returns.

Our results show that including a jump-type innovation component in an AR-GARCH model allows to well capture the energy asset dynamics and leads to an improvement in predictive accuracy and downside risk assessment with respect

to the only autoregressive specification.
Based on the ARJI-GARCH forecasts, we investigate the co-movement structure between renewable (ERIX) and non-renewable (MSCI) energy indexes.
Our findings highlight the ability of the Student's t copula to capture dependence in the tails, making it a more appropriate tool to model the European energy system in the current situation, characterized by the need for increasing the integration of renewable energy sources and strengthening the resilience to future price shocks. As suggested by the range of the estimated Kendall's tau, taking both positive and negative values, other choices of copula families allowing parameters which are exclusively positive (e.g. Archimedean copulas) would be unsuitable and rotations would need to be considered. This will be the object of future research. In addition, the Student's t copula turns out to be preferable to the Normal one in driving investment choices in the energy market.
Investing in clean energy companies is valuable not only for its contribution to a sustainable energy transition to renewable sources, but also for their attractiveness from a financial point of view, with both profitability and risk implications.

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