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Stabilization of physical systems via saturated controllers with partial state measurements

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***Abstract*— This paper provides a constructive passivity-based control approach to solve the set-point regulation problem for input-affine continuous nonlinear systems while considering bounded inputs. As customary in passivity-based control, the methodology consists of two steps: energy shaping and damping injection. In terms of applicability, the proposed controllers have two advantages concerning other passivity-based control techniques: (i) the energy shaping is carried out without solving partial differential equations, and (ii) the damping injection is performed without measuring the passive output. As a result, the proposed methodology is suitable to control a broad range of physical systems, e.g., mechanical, electrical, and electro-mechanical systems, with saturated control signals. We illustrate the applicability of the technique by designing controllers for systems in different physical domains, where we validate the analytical results via simulations and experiments.**

***Index Terms*— Passivity-based control, saturation, port-Hamiltonian systems, Brayton-Moser equations, dynamic extension, damping injection.**

I. Introduction

The behavior of a physical system is ruled by its energy, the interconnection pattern among its elements, its dissipation, and the interaction with its environment. These components are the main ingredients of passivity-based control (PBC). Hence, this control approach arises as a natural choice to control a wide variety of physical systems while taking into account conservation laws and other physical properties of the system under study see, for instance, [26], [27], [11], [31].

Due to its versatility, PBC has proven to be a powerful control approach to solve different problems, such as set-point regulation or trajectory tracking [26], [27], [30]. However, the implementation of these controllers may be hampered by physical limitations such as the operation ranges of the actuators or unavailable state measurements due to the lack of sensors. To address these issues, we propose a PBC approach suitable to stabilize a class of passive systems, where the controller is saturated and does not require full state measurements. These properties can be helpful to protect the actuators of the system, to avoid undesired oscillations, or to deal with the lack of sensors to measure specific elements of the state.

The injection of damping into the closed-loop is essential to guarantee that the system converges to the desired configuration. However, the signals involved in this process are not always measurable, e.g., velocities in mechanical systems. In this regard, observers offer a solution to this problem; we refer the reader to [33] for the port-Hamiltonian (pH) approach and [32] for a class of mechanical systems. Nevertheless, the implementation of observers in nonlinear systems can hinder the stability analysis of the closed-loop system. In this work, we avoid the use of observers by proposing a new state vector, the dynamics of which are designed to inject damping into the closed-loop system using only measurable signals. For mechanical systems, a similar method to inject damping while avoiding velocity measurements is adopted in [19], [24] for the Euler-Lagrange (EL) approach and in [8], [35] for the pH framework. In this paper, we generalize the results reported in [35] to passive systems in different physical domains, not necessarily in the pH approach. Some differences between the methodology proposed in this paper and the results reported in [19], [24], [8] are:

(i) The proposed controllers can handle input saturation.
(ii) The use of the open-loop dissipative terms to improve the transient response of the closed-loop system. In particular, in mechanical systems, we exploit the natural damping to modify the damping of the closed-loop system without measuring velocities.
(iii) The methodology encompasses, in addition to the EL and pH approaches, other representation of passive systems, such as systems presented by the Brayton-Moser equations.

Some examples of nonlinear control techniques that deal with the saturation problem for mechanical systems are [1], [6], [12], [18], [22], [23], [35]. Our approach differs from the mentioned references in the following aspects:

(i) We propose a PBC approach that is suitable to stabilize a broad class of physical (passive) systems. This contrasts with the mentioned references, where the controllers are designed for specific systems or particular physical domains.
(ii) The proposed controllers are suitable to consider the natural dissipation terms and exploit them to improve the performance of the closed-loop system. None of the mentioned references (except for [35]) study this problem.

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(iii) In contrast to [6], [22], [23], we consider underactuated systems.
(iv) The proposed methodology does not require any change of coordinates during the control design.

The main contributions of this paper are:

C1 We present a generalized framework for controlling passive systems, i.e., we consider input-affine nonlinear passive systems. This class of systems encompasses, but is not limited to, some popular modeling approaches, such as the pH framework or the EL formalism. Hence, we provide a method to stabilize nonlinear systems in different physical domains and whose models are not restricted to a particular modeling approach.

C2 We propose a method that considers input saturation without jeopardizing the stability of the closed-loop system nor increasing the stability analysis complexity. Consequently, the class of systems that can be stabilized is not reduced by considering saturated inputs.

C3 We propose an approach to exploit natural dissipation to improve the performance of the closed-loop system. Moreover, this approach does not destroy the saturation of the controllers.

C4 We provide the analysis of particular cases of interest, such as mechanical systems and electrical circuits, where the controllers are designed without solving partial differential equations (PDEs).

In addition, we stress that the proposed framework is intended for real-world applications. Hence, the resulting controllers can be tailored to address implementation issues without increasing the complexity of their design or understanding.

The remainder of this paper is organized as follows: we provide some preliminaries and the problem setting in Section II. Then, Sections III and IV are devoted to the control design, where we report our main results. Particular cases of interest are studied in Section V. Three relevant examples are provided in Section VI. Concluding remarks and future work are given in Section VII.

Caveat: to ease the readability and simplify the notation in the proofs contained in this paper, when clear from the context, we omit the arguments of the functions.

Notation: we denote the $n \times n$ identity matrix as I_n. The symbol $\mathbf{0}$ denotes a vector or matrix of appropriate dimensions whose entries are zeros. The symbols diag$\{\}$ and block$\{\}$ are used to denote diagonal and block diagonal matrices, respectively. Consider a vector $x \in \mathbb{R}^n$, a smooth function $f : \mathbb{R}^n \to \mathbb{R}$, and the mappings $F : \mathbb{R}^n \to \mathbb{R}^m$, $G : \mathbb{R}^n \to \mathbb{R}^{n\times m}$. We define the differential operator $\nabla_x f := \frac{\partial f}{\partial x}$ and $\nabla_x^2 f := \frac{\partial^2 f}{\partial x^2}$. The $ij-$th element of the $n \times m$ Jacobian matrix of $F(x)$ is given by $(\nabla_x F)_{ij} := \frac{\partial F_j}{\partial x_i}$. We omit the subindex in ∇ when it is clear from the context. Given the distinguished element $x_\star \in \mathbb{R}^n$, we define the constant vectors $F_\star := F(x_\star) \in \mathbb{R}^m$, $(\nabla f)_\star := \nabla_x f(x)|_{x=x_\star}$, and the constant matrices $G_\star := G(x_\star) \in \mathbb{R}^{n\times m}$, $(\nabla F)_\star := \nabla_x F(x)|_{x=x_\star}$. We denote the Euclidean norm as $\|x\|$, i.e., $\|x\| = \sqrt{x^\top x}$, and the weighted Euclidean norm as $\|x\|_A := \sqrt{x^\top A x}$, where $A \in \mathbb{R}^{n\times n}$ is positive (semi-)definite, i.e., $A \succ 0$ ($A \succeq 0$). The symbol e_i denotes the i^{th} element of the canonical basis of $\mathbb{R}^a$, where the context determines a, i.e., e_i is a column vector such that its i-th element is one and the rest are zero.

II. Preliminaries and problem setting

Consider the input-affine nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input, with $m \leq n$, $f : \mathcal{X} \to \mathbb{R}$ denotes the so-called *drift* vector field, $g : \mathcal{X} \to \mathbb{R}^{n\times m}$ is the input matrix, which satisfies rank$\{g(x)\} = m$ for all $x \in \mathcal{X}$.

A broad class of physical systems, in different domains, can be described by the dynamics given in (1). In this work, we are interested in the design of controllers that solve the set-point regulation problem for a class of nonlinear systems which admit the representation (1). Therefore, we aim to ensure that the closed-loop system has an asymptotically stable equilibrium at the desired point. Accordingly, the first step to formally formulate the control problem is to identify which points can be assigned as equilibria for the closed-loop system. Towards this end, we define the set that characterizes the assignable equilibria for the system (1), which is given by

$$\mathcal{E} := \left\{ x \in \mathcal{X} | g^\perp(x) f(x) = \mathbf{0} \right\}, \tag{2}$$

where $g^\perp : \mathcal{X} \to \mathbb{R}^{(n-m)\times n}$ is the left annihilator of $g(x)$, i.e., $g^\perp(x)g(x) = \mathbf{0}$.

There exist several nonlinear control design techniques that solve the set-point regulation problem. However, the implementation of these techniques is sometimes hampered by physical limitations, which are not considered by the controller. Two common problems that hinder the practical implementation of such controllers are:

- The lack of sensors to measure some relevant signals, for instance, the passive output, which is often necessary to inject damping into the closed-loop system.
- The necessity of saturated control signals to ensure the safety of the equipment or to avoid undesired transient behaviors due to the limited working range of the actuators.

The objective of this work is to propose controllers that regulate physical systems at the corresponding desired point while overcoming the issues mentioned above. Below we set the control problem.

Problem formulation. Given the system (1), propose a systematic control design approach such that:

- The closed-loop system has a locally asymptotically stable equilibrium at the desired equilibrium $x_\star \in \mathcal{E}$.

- The elements of the control law u are saturated, i.e., $u_i \in [\mathcal{U}_{\min}, \mathcal{U}_{\max}]$, where the limits of the interval are bounded and can be chosen.
- The controller injects damping into the closed-loop system without measuring the passive output.

III. Control design

Developing a general control design approach to stabilize systems that admit the representation given in (1) is, at best, a challenging task. Nonetheless, when dealing with physical systems, we can take advantage of some of their inherent properties. In particular, in this work, we restrict our attention to passive systems because: (i) an extensive variety of physical systems are passive, making these systems relevant in real-world applications, and (ii) passivity can be exploited for control purposes. Moreover, the resulting controllers have a physical interpretation in most cases. A thorough exposition of passive and cyclo-passive systems can be found in [14], [30]. Here, for the sake of completeness, we provide the following definition of passive and cyclo-passive systems.

Definition 1: The system (1) *is said to be passive if there exists a function* $S : \mathbb{R}^n \to \mathbb{R}_+$*, called the storage function, and a signal* $y \in \mathbb{R}^m$*, refer to as the passive output, such that for all initial conditions* $x(0) = x_0 \in \mathbb{R}^n$ *the following inequality holds*

$$S(x(t)) \leq S(x_0) + \int_0^t u^\top(s) y(s) ds. \tag{3}$$

Moreover, (1) *is said to be cyclo-dissipative if the storage function is not necessarily nonnegative, i.e.,* $S : \mathbb{R}^n \to \mathbb{R}$.

Energy and dissipation play an essential role in the behavior of (cyclo-)passive systems. Consequently, *energy-based* controllers, such as the ones derived from PBC techniques, represent a suitable choice to control physical passive systems while preserving some physical intuition during the control design process. In this section, we develop a PBC approach that complies with the requirements established in Section II. Hence, the first step consists in proving that the system to be controlled is (cyclo-)passive. To this end, we characterize the input-affine nonlinear systems that are cyclo-passive via the following assumption.

Assumption 1: Given the system (1), there exists $S : \mathcal{X} \to \mathbb{R}$ such that

$$[\nabla S(x)]^\top f(x) = -\|\ell(x)\|^2, \tag{4}$$

where $\ell : \mathcal{X} \to \mathbb{R}^r$ for some positive integer r.

Assumption 1 is closely related to Hill-Moylan's theorem [14], which provides necessary and sufficient conditions to determine whether (1) is cyclo-passive or not. However, at this point, the output of the plant has not been defined yet. Thus, it is not possible to establish the (cyclo)-passivity property of (1). The proposition below establishes that Assumption 1 guarantees that (1) is cyclo-passive and provides the structure of the passive output y corresponding to the storage function $S(x)$.

Proposition 1: Consider the system (1) *satisfying Assumption 1, and some mappings* $w : \mathcal{X} \to \mathbb{R}^{r \times m}$*,* $D : \mathcal{X} \to \mathbb{R}^{m \times m}$*. Then,* $\dot{S} \leq u^\top y$*, with*

$$y = g^\top(x)\nabla S(x) + 2w^\top(x)\ell(x) + [w^\top(x)w(x) + D(x)]u, \tag{5}$$

where $D(x) = -D^\top(x)$.

Proof: Compute the derivative of $S(x)$ along the trajectories of (1), that is,

$$\begin{aligned} \dot{S} &= (\nabla S)^\top (f + gu) \\ &= -\|\ell\|^2 + y^\top u - 2\ell^\top w u - u^\top w^\top w u \\ &= -\|\ell + wu\|^2 + y^\top u \leq y^\top u, \end{aligned} \tag{6}$$

where we used (5) and $u^\top D u = 0$. ■

Customarily, PBC techniques consist of two steps: first, the so-called *energy-shaping* where the new energy—storage—function is modified to have a minimum at the desired equilibrium. Second, the damping injection into the closed-loop system ensures that the trajectories converge to the desired point. We present the following assumption to characterize the class of systems for which the controllers devised in this section can assign the desired equilibrium to the closed-loop system and render it stable.

Assumption 2: Consider (1) satisfying Assumption 1, and the desired equilibrium $x_\star \in \mathcal{E}$, with $\mathcal{E}$ defined in (2). There exists $\gamma : \mathcal{X} \to \mathbb{R}^m$ such that

$$\begin{aligned} &\dot{\gamma} = y \\ &(\nabla S)_\star + (\nabla \gamma)_\star \kappa = \mathbf{0} \\ &\left(\nabla^2 S\right)_\star + \sum_{i=1}^m (\nabla \gamma_i)_\star (\nabla \gamma_i)_\star^\top \alpha_i \beta_i + \left(\nabla^2 \left(\gamma^\top \kappa\right)\right)_\star \succ 0. \end{aligned} \tag{7}$$

where $(\cdot)_\star$ denotes the function $(\cdot)$ evaluated at $x = x_\star$, α_i, β_i are positive constants, and the constant vector $\kappa \in \mathbb{R}^m$ is defined as

$$\kappa := (g_\star^\top g_\star)^{-1} g_\star^\top f_\star. \tag{8}$$

The systems characterized by Assumptions 1 and 2 are relevant from a practical perspective because they encompass many physical systems. Theorem 1 provides a saturated controller that addresses the stabilization problem for these systems.

Theorem 1: Suppose the system (1) *and the desired equilibrium* $x_\star \in \mathcal{E}$*, with* $\mathcal{E}$ *defined in* (2)*, satisfy Assumptions 1 and 2. Consider the control law*

$$u = -\nabla_\gamma \Phi(\gamma(x)) - \kappa - \sum_{i=1}^m e_i k_{\mathtt{p}_i} \tanh(y_i), \tag{9}$$

where $k_{\mathtt{p}_i} > 0$*, for* $i \in \{1, \ldots, m\}$*, and*

$$\Phi(\gamma(x)) := \sum_{i=1}^m \frac{\alpha_i}{\beta_i} \ln\left(\cosh(\beta_i (\gamma_i(x) - \gamma_{i_\star}))\right). \tag{10}$$

(i) *The control signals satisfy*

$$u_i \in [-\kappa_i - k_{\mathtt{p}_i} - \alpha_i, -\kappa_i + k_{\mathtt{p}_i} + \alpha_i].$$

(ii) *The closed-loop system has a locally stable equilibrium point at $x_\star$ with Lyapunov function*

$$S_{\mathtt{d}}(x) := S(x) + \Phi(\gamma(x)) + \kappa^\top \gamma(x) + c \tag{11}$$

where the constant $c \in \mathbb{R}$ is defined as

$$c := -S_\star - \Phi_\star - \kappa^\top \gamma_\star.$$

(iii) *The equilibrium is locally asymptotically stable if, on a domain $\Omega \subseteq \mathcal{X}$ containing $x_\star$,*

$$\left.\begin{aligned} \ell &= w\left(\nabla_\gamma \Phi(\gamma(x)) + \kappa\right) \\ y &= \mathbf{0} \end{aligned}\right\} \implies x = x_\star \tag{12}$$

Proof: To prove (i) note that

$$\nabla_\gamma \Phi(\gamma(x)) = \sum_{i=1}^{m} e_i \alpha_i \tanh(\beta_i \left(\gamma_i(x) - \gamma_{i_\star}\right)). \tag{13}$$

Hence, the control law takes the form

$$u_i = -\kappa_i - \alpha_i \tanh(\beta_i \left(\gamma_i(x) - \gamma_{i_\star}\right)) - k_{\mathtt{p}_i} \tanh(y_i).$$

Because the function $\tanh(\cdot)$ is saturated, we have that

$$-\kappa_i - k_{\mathtt{p}_i} - \alpha_i \leq u_i \leq -\kappa_i + k_{\mathtt{p}_i} + \alpha_i. \tag{14}$$

To prove (ii), we compute

$$\begin{aligned} \dot{S}_{\mathtt{d}} &= -\|\ell + wu\|^2 + y^\top u + \dot{\gamma}^\top \nabla_\gamma \Phi + \dot{\gamma}^\top \kappa \\ &= -\|\ell + wu\|^2 + y^\top \left(u + \nabla_\gamma \Phi + \kappa\right) \\ &\leq y^\top \left(u + \nabla_\gamma \Phi + \kappa\right) \end{aligned} \tag{15}$$

which follows from (6) and (13). Moreover, substituting (9) in (15) yields

$$\dot{S}_{\mathtt{d}} \leq -y^\top \sum_{i=1}^{m} e_i k_{\mathtt{p}_i} \tanh(y_i) = -\sum_{i=1}^{m} k_{\mathtt{p}_i} y_i \tanh(y_i) \leq 0. \tag{16}$$

To prove that $S_{\mathtt{d}}(x)$ qualifies as a Lypunov function, note that, from Assumption 2, we have the following

$$\left(\nabla S_{\mathtt{d}}\right)_\star = \left(\nabla S\right)_\star + \left(\nabla \gamma\right)_\star \kappa = \mathbf{0}, \tag{17}$$

$$\begin{aligned} \left(\nabla^2 S_{\mathtt{d}}\right)_\star &= \sum_{i=1}^{m} \left(\nabla \gamma_i\right)_\star \left(\nabla \gamma_i\right)_\star^\top \alpha_i \beta_i + \left(\nabla^2 \left(\gamma^\top \kappa\right)\right)_\star \\ &\quad + \left(\nabla^2 S\right)_\star \succ 0. \end{aligned} \tag{18}$$

Hence, $x_\star$ is a strict minimum of $S_{\mathtt{d}}(x)$. Moreover, $S_{\mathtt{d}_\star} = 0$ and (16) implies that $S_{\mathtt{d}}(x)$ is non-increasing. Thus, $S_{\mathtt{d}}(x)$ is positive definite with respect to $x_\star$. Accordingly, $x_\star$ is stable with Lyapunov function $S_{\mathtt{d}}(x)$.

To prove (iii), note that (15) and (16) yield

$$\dot{S}_{\mathtt{d}} = 0 \iff \begin{cases} \ell + wu = \mathbf{0} \\ y = \mathbf{0}. \end{cases} \tag{19}$$

However,

$$y = \mathbf{0} \implies u = -\left(\nabla_\gamma \Phi(\gamma(x)) + \kappa\right).$$

Therefore,

$$\dot{S}_{\mathtt{d}} = 0 \iff \begin{cases} \ell = w\left(\nabla_\gamma \Phi(\gamma(x)) + \kappa\right) \\ y = \mathbf{0}. \end{cases}$$

Furthermore, $x = x_\star$ implies $\dot{S}_{\mathtt{d}} = 0$. Hence, (12) implies that, on the domain Ω,

$$\dot{S}_{\mathtt{d}} = 0 \iff x = x_\star.$$

Thus, the asymptotic stability of $x_\star$ follows by invoking LaSalle's invariance principle. See [21]. ■

Note that the saturation limits of the control law (9) can be adjusted by modifying the control parameters α_i and $k_{\mathtt{p}_i}$. Furthermore, we point out that the natural dissipation plays an important role in the stabilization of the system. In particular, we make the following remarks.

Remark 1: If $\ell_\star \neq \mathbf{0}$, then the desired equilibrium can be assigned only by shaping the energy using a γ derived from a passive output with *relative degree* zero. A proof of this fact can be found in [3]. This phenomenon is called the *dissipation obstacle*. We refer the reader to [25] for further details on this topic.

Remark 2: If

$$\ell = w\left(\nabla_\gamma \Phi(\gamma(x)) + \kappa\right) \implies x = x_\star, \tag{20}$$

then it is not necessary to inject damping into the closed-loop system to ensure the asymptotic stability of the desired equilibrium. Moreover,

$$u = -\kappa - \nabla_\gamma \Phi(\gamma(x))$$

solves the regulation problem. On the other hand, if $\ell = \mathbf{0}$, then (12) reduces to

$$y \equiv \mathbf{0} \implies x = x_\star. \tag{21}$$

In particular, when $x_\star = \mathbf{0}$ and $f(\mathbf{0}) = \mathbf{0}$, (21) becomes

$$y \equiv \mathbf{0}, u \equiv \mathbf{0} \implies x = x_\star,$$

which is referred to as *zero-state observability*, see [30]. Note that (21) is more conservative than (12).

The control law (9) addresses the regulation problem and ensures that the control signals are saturated, where the damping is injected through the passive output. However, the measurement of this signal is not always available, e.g., in mechanical systems without velocity sensors. To overcome this issue, we propose a modified control law such that the damping injection does not require the measurement of y. To this end, we introduce the controller state $x_{\mathsf{c}} \in \mathbb{R}^m$, and we define the following mappings

$$\begin{aligned} z_{\mathsf{c}}(\gamma(x), x_{\mathsf{c}}) &:= \gamma(x) - \gamma_\star + x_{\mathsf{c}} \\ \Phi_{\mathsf{c}}(z_{\mathsf{c}}(\gamma(x), x_{\mathsf{c}})) &:= \sum_{i=1}^m \frac{\alpha_{\mathsf{c}_i}}{\beta_{\mathsf{c}_i}} \ln\left(\cosh(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i}(\gamma_i(x), x_{\mathsf{c}_i}))\right), \end{aligned} \tag{22}$$

where $\alpha_{\mathsf{c}_i}, \beta_{\mathsf{c}_i}$ are positive constants. Without loss of generality, we consider $x_{\mathsf{c}_\star} = \mathbf{0}$. Moreover, to simplify the notation, we omit the argument of z_{c}.

The following theorem provides a saturated control law that shapes the energy of the closed-loop system and injects damping without measuring y.

Theorem 2: Suppose that the system (1) *and the desired equilibrium $x_\star \in \mathcal{E}$, with $\mathcal{E}$ defined in* (2)*, satisfy Assumptions 1 and 2. Consider the positive definite matrices $R_{\mathsf{c}}, K_{\mathsf{c}} \in \mathbb{R}^{m\times m}$, the dynamics*[1]

$$\dot{x}_{\mathsf{c}} = -R_{\mathsf{c}}\left[\sum_{i=1}^m e_i \alpha_{\mathsf{c}_i} \tanh(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i}) + K_{\mathsf{c}} x_{\mathsf{c}}\right], \tag{23}$$

and the control law

$$u = -\kappa - \sum_{i=1}^m e_i \alpha_{\mathsf{c}_i} \tanh(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i}). \tag{24}$$

(i) *The control signals satisfy $u_i \in [-\kappa_i - \alpha_{\mathsf{c}_i}, -\kappa_i + \alpha_{\mathsf{c}_i}]$.*

(ii) *There exists K_{c} such that the closed-loop system has a locally stable equilibrium point at $(x_\star, \mathbf{0})$ with Lyapunov function*

$$S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}}) := S(x) + \Phi_{\mathsf{c}}(z_{\mathsf{c}}) + \kappa^\top \gamma(x) + \frac{1}{2}\|x_{\mathsf{c}}\|^2_{K_{\mathsf{c}}} + c, \tag{25}$$

where $\Phi_{\mathsf{c}}(z_{\mathsf{c}})$ is given in (22) *and c is defined as in Theorem 1.*

(iii) *The equilibrium is locally asymptotically stable if, on a domain $\Omega_{\mathsf{c}} \subseteq \mathcal{X} \times \mathbb{R}^m$ containing $(x_\star, \mathbf{0})$, the following condition holds*

$$\left.\begin{aligned} \ell &= w\left(\kappa - K_{\mathsf{c}} x_{\mathsf{c}}\right) \\ y &= \mathbf{0} \end{aligned}\right\} \implies \left\{\begin{aligned} x &= x_\star \\ x_{\mathsf{c}} &= \mathbf{0} \end{aligned}\right. \tag{26}$$

Proof: To prove (i) note that

$$u_i = -\kappa_i - \alpha_{\mathsf{c}_i} \tanh(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i}).$$

Thus

$$-\kappa_i - \alpha_{\mathsf{c}_i} \leq u_i \leq -\kappa_i + \alpha_{\mathsf{c}_i}. \tag{27}$$

To prove (ii) note that

$$\nabla_\gamma \Phi_{\mathsf{c}} = \nabla_{x_c} \Phi_{\mathsf{c}} = \sum_{i=1}^m e_i \alpha_{\mathsf{c}_i} \tanh(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i}). \tag{28}$$

Hence,

$$\begin{aligned} \dot{S}_{\mathsf{d}_{\mathsf{c}}} &= -\|\ell + wu\|^2 + y^\top(u + \kappa) + \dot{z}_{\mathsf{c}}^\top \nabla_{x_c} \Phi_{\mathsf{c}} + \dot{x}_{\mathsf{c}}^\top K_{\mathsf{c}} x_{\mathsf{c}} \\ &= -\|\ell + wu\|^2 - \|\dot{x}_{\mathsf{c}}\|^2_{R_{\mathsf{c}}^{-1}} \leq 0 \end{aligned} \tag{29}$$

which follows from (6), (23), (24), and (28). We recall that $\|\dot{x}_{\mathsf{c}}\|_{R_{\mathsf{c}}^{-1}}$ denotes a weighted Euclidean norm, i.e., $\|\dot{x}_{\mathsf{c}}\|^2_{R_{\mathsf{c}}^{-1}} = \dot{x}_{\mathsf{c}}^\top R_{\mathsf{c}}^{-1} \dot{x}_{\mathsf{c}}$. Moreover, we adopt the notation $(\cdot)_\star$ to denote the function $(\cdot)$ evaluated at the desired equilibrium $(x_\star, \mathbf{0})$. Hence, $z_{\mathsf{c}_\star} = \mathbf{0}$ and

$$(\nabla_x S_{\mathsf{d}_{\mathsf{c}}})_\star = (\nabla S)_\star + (\nabla \gamma)_\star \kappa. \tag{30}$$

Therefore, from Assumption 2, $(\nabla_x S_{\mathsf{d}_{\mathsf{c}}})_\star = \mathbf{0}$. Moreover,

$$\nabla_{x_c} S_{\mathsf{d}_{\mathsf{c}}} = \nabla_{x_c} \Phi_{\mathsf{c}} + K_{\mathsf{c}} x_{\mathsf{c}} \implies (\nabla_{x_c} S_{\mathsf{d}_{\mathsf{c}}})_\star = \mathbf{0}.$$

Consequently, $(x_\star, \mathbf{0})$ is a critical point of $S_{\mathsf{d}_{\mathsf{c}}}(x, x_c)$. Furthermore, some simple computations show that

$$\begin{aligned} \left(\nabla^2 S_{\mathsf{d}_{\mathsf{c}}}\right)_\star = &\begin{bmatrix} \left(\nabla^2 S\right)_\star + \left(\nabla^2\left(\gamma^\top \kappa\right)\right)_\star & \mathbf{0} \\ \mathbf{0} & K_{\mathsf{c}} \end{bmatrix} \\ &+ \begin{bmatrix} (\nabla \gamma)_\star \\ I_m \end{bmatrix} \sum_{i=1}^m e_i e_i^\top \alpha_{\mathsf{c}_i} \beta_{\mathsf{c}_i} \left[(\nabla\gamma)_\star^\top \;\; I_m\right]. \end{aligned} \tag{31}$$

Note that the block $(1,1)$ of $\left(\nabla^2 S_{\mathsf{d}_{\mathsf{c}}}\right)_\star$ can be expressed as $\left(\nabla^2 S_{\mathsf{d}}\right)_\star$; see (18). Thus, the blocks $(1,1)$ and $(2,2)$ of $\left(\nabla^2 S_{\mathsf{d}_{\mathsf{c}}}\right)_\star$ are positive definite. Moreover, K_{c} is a control parameter whose only restriction is to be positive definite. Therefore, a Schur complement analysis shows that a K_{c} large enough ensures that $\left(\nabla^2 S_{\mathsf{d}_{\mathsf{c}}}\right)_\star \succ 0$. Hence, an appropriate selection of K_{c} ensures that $S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}})$ has a strict minimum at $(x_\star, \mathbf{0})$. Note that $S_{\mathsf{d}_{\mathsf{c}}}(x_\star, \mathbf{0})$ is zero. Therefore, the fact that (x, x_{c}) is a strict minimum of $S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}})$, together with (29), implies that $S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}})$ is positive definite with respect to the equilibrium. Thus, $S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}})$ is a Lyapunov function and the desired equilibrium is locally stable.

To prove (iii) note that, from (29),

$$\dot{S}_{\mathsf{d}_{\mathsf{c}}} = 0 \iff \begin{cases} \ell + wu = \mathbf{0} \\ \dot{x}_{\mathsf{c}} = \mathbf{0}. \end{cases}$$

Note that, from (23) and (28),

$$\dot{x}_{\mathsf{c}} = \mathbf{0} \implies \nabla_{x_c} \Phi_{\mathsf{c}} + K_{\mathsf{c}} x_{\mathsf{c}} = \mathbf{0}. \tag{32}$$

Moreover, from (28), we compute

$$\frac{d}{dt}\left(\nabla_{x_c} \Phi_{\mathsf{c}} + K_{\mathsf{c}} x_{\mathsf{c}}\right) = \sum_{i=1}^m e_i \dot{z}_{\mathsf{c}_i} \alpha_{\mathsf{c}_i} \left[\operatorname{sech}(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i})\right]^2 + K_{\mathsf{c}} \dot{x}_{\mathsf{c}},$$

where

$$\dot{z}_{\mathsf{c}_i} = \dot{\gamma}_i + \dot{x}_{\mathsf{c}_i} = y_i + \dot{x}_{\mathsf{c}_i}.$$

Therefore, recalling that $\dot{x}_c = \mathbf{0}$, we have the following chain of implications

$$\begin{aligned} \nabla_{x_c} \Phi_{\mathsf{c}} + K_{\mathsf{c}} x_{\mathsf{c}} = \mathbf{0} &\implies \frac{d}{dt}\left(\nabla_{x_c} \Phi_{\mathsf{c}} + K_{\mathsf{c}} x_{\mathsf{c}}\right) = \mathbf{0} \\ \implies \sum_{i=1}^m e_i y_i \alpha_{\mathsf{c}_i} \left[\operatorname{sech}(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i})\right]^2 = \mathbf{0} &\implies y = \mathbf{0}, \end{aligned} \tag{33}$$

where the last implication is obtained noting that $[\operatorname{sech}(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i})]^2$ is always positive. Moreover, (24) takes the form $u = -\kappa + K_{\mathsf{c}} x_{\mathsf{c}}$. Hence, (26) implies that $\dot{S}_{\mathsf{d}_{\mathsf{c}}} = 0$ if and only if $(x, x_{\mathsf{c}}) = (x_\star, \mathbf{0})$. Accordingly, the asymptotic stability of the desired equilibrium point follows from LaSalle's invariance principle. ■

[1] where α_{c_i} and β_{c_i} satisfy (7).

In the control law (24), the damping is injected via the controller state x_{c}. In particular, we propose the specific dynamics given in (23). This can be interpreted as a filter; see [24] and [9]. In contrast to the mentioned references, we extend this approach to a more general class of systems, i.e., input-affine nonlinear systems, while considering saturation in the inputs. Note that, as a consequence of (33), the desired equilibrium is asymptotically stable if (21) holds.

The saturated controllers developed in this section address the regulation problem by shaping the energy of the system and injecting damping either through the passive output or the controller state x_{c}. In both cases, the damping injection is closely related to the output port. However, to improve the performance of the closed-loop system, it may be necessary to inject damping into coordinates that are not associated with y. In the following section, we provide an alternative to address this issue.

IV. On the role of dissipation

Dissipation is present in most physical systems. Nonetheless, the mathematical models that represent these systems commonly neglect the dissipation inherent to them. This section proposes a method to inject damping into the coordinates with natural dissipation without measuring them. This can be exploited to improve the convergence rate of the closed-loop system or to remove an undesired transient behavior, such as oscillations. Furthermore, this approach can be instrumental when the system under study exhibits poor damping propagation, resulting in a slow convergence rate.

We characterize the systems for which this new type of damping injection is suitable through the following assumption.

Assumption 3: Given the passive output $y \in \mathbb{R}^m$, there exists $\eta : \mathcal{X} \to \mathbb{R}^s$, with $1 \leq s \leq n - m$, such that the system (1) satisfies

$$\begin{aligned} [\nabla\gamma(x)]^\top \nabla\eta(x) &= \mathbf{0} \\ -\|\ell(x) + w(x)u\|^2 &= -\|\dot{\eta}\|^2_{\Lambda_\ell(x)} - \|y\|^2_{\Lambda_{\mathsf{c}}(x)} \end{aligned}$$

where $\dot{\gamma} = y$ and $\Lambda_\ell : \mathcal{X} \to \mathbb{R}^{s\times s}$, $\Lambda_{\mathsf{c}} : \mathcal{X} \to \mathbb{R}^{m\times m}$ are positive definite diagonal matrices.

Assumption 3 is related to the *natural* damping of the system to be controlled. Some examples of systems that satisfy this assumption are mechanical systems including friction between surfaces and electrical circuits containing resistors in series with inductors. To exploit the natural damping in the control design, we introduce the virtual state $x_\ell \in \mathbb{R}^m$, and the following mappings

$$\begin{aligned} z_\ell(\eta(x), x_\ell) &:= \Upsilon\left[\eta(x) - \eta_\star\right] + K_\ell x_\ell \\ \Phi_\ell(\eta(x), x_\ell) &:= \sum_{i=1}^{m} \frac{\alpha_{\ell_i}}{\beta_{\ell_i}} \ln\left(\cosh(\beta_{\ell_i} z_{\ell_i}(\eta(x), x_\ell))\right) \end{aligned} \quad (34)$$

where the constant matrix $\Upsilon \in \mathbb{R}^{m\times s}$ satisfies $\operatorname{rank}\{\Upsilon\} = \min\{m, s\}$, $K_\ell \in \mathbb{R}^{m\times m}$ is a diagonal positive definite matrix, and $\alpha_{\ell_i}, \beta_{\ell_i}$ are positive constant parameters. Without loss of generality, we consider $x_{\ell_\star} = \mathbf{0}$. To simplify the notation, we omit the argument from z_ℓ.

The following theorem provides a saturated control law that shapes the energy and modifies the damping of the coordinates that are naturally damped.

Theorem 3: Suppose that the system (1) *and the desired equilibrium* $x_\star \in \mathcal{E}$*, with* $\mathcal{E}$ *defined in* (2)*, satisfy Assumptions 1–3. Consider the control law*

$$u = -\kappa - \nabla_{z_\ell}\Phi_\ell(z_\ell) - \nabla_\gamma\Phi_{\mathsf{c}}(z_{\mathsf{c}}), \quad (35)$$

where $\Phi_{\mathsf{c}}(z_{\mathsf{c}})$ *is defined in* (22)*,* $\Phi_\ell(z_\ell)$ *is defined in* (34)*, the dynamics of* x_{c} *are given by* (23)*, and*

$$\dot{x}_\ell = -R_\ell \sum_{i=1}^{m} e_i \alpha_{\ell_i} \tanh(\beta_{\ell_i} z_{\ell_i}). \quad (36)$$

(i) *The control signals satisfy*

$$u_i \in \left[-\kappa_i - \alpha_{\ell_i} - \alpha_{\mathsf{c}_i}, -\kappa_i + \alpha_{\ell_i} + \alpha_{\mathsf{c}_i}\right].$$

(ii) *There exist* K_{c}*,* K_ℓ*, and* R_ℓ *such that the closed-loop system has a locally asymptotically stable equilibrium point at* $(x_\star, \mathbf{0}, \mathbf{0})$*, with Lyapunov function*

$$S_{\mathsf{d}_\ell}(x, x_\ell, x_{\mathsf{c}}) := S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}}) + \Phi_\ell(z_\ell),$$

with $S_{\mathsf{d}_{\mathsf{c}}}(x, x_{\mathsf{c}})$ *defined in* (25)*.*

(iii) *For an appropriate selection of* K_ℓ *and* R_ℓ*, the equilibrium is locally asymptotically stable if, on a domain* $\Omega_{\mathsf{d}} \subseteq \mathcal{X}\times\mathbb{R}^m\times\mathbb{R}^m$ *containing* $(x_\star, \mathbf{0}, \mathbf{0})$*, the following condition holds*

$$\left.\begin{aligned} \dot{\eta} &= \mathbf{0} \\ y &= \mathbf{0} \\ \dot{x}_\ell &= \mathbf{0} \\ \dot{x}_{\mathsf{c}} &= \mathbf{0} \end{aligned}\right\} \implies \left\{\begin{aligned} x &= x_\star \\ x_\ell &= \mathbf{0} \\ x_{\mathsf{c}} &= \mathbf{0}. \end{aligned}\right. \quad (37)$$

Proof: To prove (i) note that, from (28) and (34), the control law (35) can be rewritten as

$$u = -\kappa - \sum_{i=1}^{m} e_i \left\{\alpha_{\mathsf{c}_i} \tanh\left(\beta_{\mathsf{c}_i} z_{\mathsf{c}_i}\right) + \alpha_{\ell_i} \tanh\left(\beta_{\ell_i} z_{\ell_i}\right)\right\}.$$

Therefore,

$$-\kappa_i - \alpha_{\mathsf{c}_i} - \alpha_{\ell_i} \leq u_i \leq -\kappa_i + \alpha_{\mathsf{c}_i} + \alpha_{\ell_i}.$$

To prove (ii) note that (36) can be rewritten as

$$\dot{x}_\ell = -R_\ell \nabla_{z_\ell}\Phi_\ell. \quad (38)$$

Hence, from (29), (35), and Assumption 3, we have that[2]

$$\begin{aligned} \dot{S}_{\mathsf{d}_\ell} &= -\|\dot{\eta}\|^2_{\Lambda_\ell} - \|y\|^2_{\Lambda_{\mathsf{c}}} + (\dot{z}_\ell - y)^\top \nabla_{z_\ell}\Phi_\ell - \|\dot{x}_{\mathsf{c}}\|^2_{R_{\mathsf{c}}^{-1}} \\ &= -\|\dot{\eta}\|^2_{\Lambda_\ell} - \|y\|^2_{\Lambda_{\mathsf{c}}} - \|\dot{x}_{\mathsf{c}}\|^2_{R_{\mathsf{c}}^{-1}} - \|\dot{x}_\ell\|^2_{K_\ell R_\ell^{-1}} \\ &\quad + (\Upsilon\dot{\eta} - y)^\top \nabla_{z_\ell}\Phi_\ell \\ &= -\left[\dot{\eta}^\top \; y^\top \; \dot{x}_\ell^\top\right] \Theta \left[\dot{\eta}^\top \; y^\top \; \dot{x}_\ell^\top\right]^\top - \|\dot{x}_{\mathsf{c}}\|^2_{R_{\mathsf{c}}^{-1}}. \end{aligned} \quad (39)$$

[2] Note that R_ℓ and K_ℓ are diagonal. Thus, their product commutes.

where

$$\Theta := \begin{bmatrix} \Lambda_\ell & \mathbf{0} & \frac{1}{2}\Upsilon^\top R_\ell^{-1} \\ \mathbf{0} & \Lambda_{\mathsf{c}} & -\frac{1}{2}R_\ell^{-1} \\ \frac{1}{2}R_\ell^{-1}\Upsilon & -\frac{1}{2}R_\ell^{-1} & K_\ell R_\ell^{-1}. \end{bmatrix} \quad (40)$$

Thus, $\dot{S}_{\mathsf{d}_\ell} \leq 0$ if Θ is positive semi-definite. Moreover, via a Schur complement analysis, we get that $\Theta \succeq 0$ if and only if

$$\begin{bmatrix} \Lambda_\ell & \mathbf{0} \\ \mathbf{0} & \Lambda_{\mathsf{c}} \end{bmatrix} - \frac{1}{4}\begin{bmatrix} \Upsilon^\top \\ -I_m \end{bmatrix} K_\ell^{-1} R_\ell^{-1} \begin{bmatrix} \Upsilon & -I_m \end{bmatrix} \succeq 0. \quad (41)$$

Note that (41) holds for K_ℓ and R_ℓ large enough. Accordingly, an appropriate selection of these matrices ensures that $S_{\mathsf{d}_\ell}(x, x_\ell, x_{\mathsf{c}})$ is non-increasing. Similar to the proof of Theorem 2, we adopt the notation $(\cdot)_\star$ to denote the function $(\cdot)$ evaluated at the desired equilibrium $(x_\star, \mathbf{0}, \mathbf{0})$. Hence, $z_{\ell_\star} = \mathbf{0}$ and $(\nabla_{z_\ell}\Phi_\ell)_\star = \mathbf{0}$. This, together with $(\nabla S_{\mathsf{d}_{\mathsf{c}}})_\star = \mathbf{0}$—see the proof of Theorem 2—yields $(\nabla S_{\mathsf{d}_\ell})_\star = \mathbf{0}$. Furthermore,

$$\left(\nabla^2 S_{\mathsf{d}_\ell}\right)_\star = \left(\nabla^2 S_{\mathsf{d}_{\mathsf{c}}}\right)_\star + \left(\nabla^2 \Phi_\ell\right)_\star.$$

Some simple computations show that $\left(\nabla^2 \Phi_\ell\right)_\star \succeq 0$ and, using the arguments of the proof of Theorem 2, a K_{c} large enough guarantees that $\left(\nabla^2 S_{\mathsf{d}_{\mathsf{c}}}\right)_\star \succ 0$. Therefore, there exists K_{c} such that $(x_\star, \mathbf{0}, \mathbf{0})$ is a strict minimum for the closed-loop storage function. Note that $S_{\mathsf{d}_\ell}(x_\star, \mathbf{0}, \mathbf{0}) = 0$. Therefore, the closed-loop storage function is non-increasing and positive definite with respect to the desired equilibrium. Accordingly, $S_{\mathsf{d}_\ell}(x, x_\ell, x_{\mathsf{c}})$ qualifies as a Lyapunov function and the closed-loop system has a stable equilibrium at $(x_\star, \mathbf{0}, \mathbf{0})$.

To prove (iii), we consider K_ℓ and R_ℓ such that $\Theta \succ 0$. Then, from (39),

$$\dot{S}_{\mathsf{d}_\ell} = 0 \iff \begin{cases} \dot{\eta} &= \mathbf{0} \\ y &= \mathbf{0} \\ \dot{x}_\ell &= \mathbf{0} \\ \dot{x}_{\mathsf{c}} &= \mathbf{0} \end{cases}$$

Thus, (37) implies that, on Ω_{d}, the desired equilibrium is the only solution to $\dot{S}_{\mathsf{d}_\ell} = 0$. Consequently, the asymptotic stability of the equilibrium follows by invoking LaSalle's invariance principle. ■

Remark 3: The stability properties of the equilibrium points in Theorems 1–3 are *global* if the corresponding Lyapunov function is *radially unbounded*, see [21].

In general, to ensure the existence of $\gamma(x)$ in Assumption 2, it is necessary to solve a PDE. However, in some particular cases, $\gamma(x)$ can be found by satisfying some algebraic conditions. A thorough discussion on this topic is provided in [2]. Notably, the well-defined structure of some physical systems permits finding $\gamma(x)$ without solving PDEs, as is shown in Section V.

V. Particular cases

The controllers developed in Sections III and IV are devised to stabilize a rather general class of nonlinear systems characterized by Assumptions 1–3. In principle, such assumptions should be checked system by system. However, in some particular cases of interest, these assumptions always hold or can be straightforwardly verified. This section focuses on mechanical systems modeled in the pH framework and electrical circuits represented via the Brayton-Moser equations and how the mentioned assumptions translate to these systems. We stress that these modeling approaches encompass a broad range of systems. See, [4], [5], [11], [16], [31].

A. Mechanical systems in the pH representation

Consider a standard mechanical system represented by

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} \mathbf{0} & I_n \\ -I_n & -\mathcal{D}(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H(q,p) \\ \nabla_p H(q,p) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ G \end{bmatrix} u, \\ H(q,p) &:= \frac{1}{2} p^\top M^{-1}(q) p + V(q), \end{aligned} \quad (42)$$

where:[3]

- $q, p \in \mathbb{R}^n$ represent the generalized positions and momenta, respectively.
- $V : \mathbb{R}^n \to \mathbb{R}_+$ denotes the potential energy of the system.
- The so-called inertia matrix $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is positive definite. For further details on the computation and properties of this matrix we refer the reader to [20], [29].
- The Hamiltonian $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ is given by the total energy of the system.
- The input matrix $G \in \mathbb{R}^{n\times m}$, with $m \leq n$, is given by $G = \begin{bmatrix} \mathbf{0} & I_m \end{bmatrix}^\top$.
- $\mathcal{D} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is a diagonal positive semi-definite matrix that represents the system's dissipation (damping).

The set of assignable equilibria for (42) is

$$\mathcal{E}_{\mathcal{M}} := \left\{(q,p) \in \mathbb{R}^n \times \mathbb{R}^n | G^\perp \nabla V(q) = \mathbf{0}, p = \mathbf{0}\right\}. \quad (43)$$

We make the following observations about the system (42):

O1 It admits a representation of the form (1), with

$$f(x) = \begin{bmatrix} \mathbf{0} & I_n \\ -I_n & -\mathcal{D}(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H(q,p) \\ \nabla_p H(q,p) \end{bmatrix}, \; g(x) = \begin{bmatrix} \mathbf{0} \\ G \end{bmatrix}.$$

O2 Some simple computations show that

$$\dot{H} = -\|\dot{q}\|^2_{\mathcal{D}(q,p)} + \dot{q}^\top G u.$$

Hence, Assumption 1 holds for $S(x) = H(q,p)$, and $\|\ell(x)\|^2 = \|\dot{q}\|^2_{\mathcal{D}(q,p)}$. Moreover, the passive output is given by $y = G^\top \dot{q}$. Thus, $\gamma(x) = G^\top q$.

O3 In this case, $\kappa = -G^\top (\nabla V)_\star$. Then, from (43), we have $(\nabla H)_\star + (\nabla \gamma)_\star \kappa = \mathbf{0}$.

O4 Since $\ell_\star = \mathbf{0}$, there is no dissipation obstacle.

[3] Without loss of generality, we consider that mechanical systems have dimension $2n$.

O5 Since $\mathcal{D}(q,p)$ is diagonal, we can rewrite it as follows

$$\mathcal{D}(q,p) = \text{block}\left\{\mathcal{D}_{\mathtt{u}}(q,p), \mathcal{D}_{\mathtt{a}}(q,p)\right\},$$

where $\mathcal{D}_{\mathtt{u}} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{(n-m)\times(n-m)}$ and $\mathcal{D}_{\mathtt{a}} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{m\times m}$ are diagonal matrices. Accordingly, if $\mathcal{D}_{\mathtt{a}}(q,p)$ has full rank and $\mathcal{D}_{\mathtt{u}}(q,p)$ has at least one nonzero entry, Assumption 3 holds with $\Lambda_{\mathtt{c}}(x) = \mathcal{D}_{\mathtt{a}}(q,p)$, the diagonal matrix $\Lambda_\ell(x)$ consists of all the nonzero entries of $\mathcal{D}_{\mathtt{u}}(q,p)$, and $\eta(x)$ is given by the positions that satisfy $q_j \mathcal{D}_{\mathtt{u}_j}(q,p) \neq \mathbf{0}$, where

$$q_j := e_j^\top q, \quad \mathcal{D}_{\mathtt{u}_j}(q,p) := e_j^\top \mathcal{D}_{\mathtt{u}}(q,p) e_j,$$

for $j \in \{1,\ldots,n-m\}$.

From the observations listed above, we conclude the controllers developed in Sections III and IV stabilize (42) at the desired equilibrium if

$$\begin{aligned} &\left(\nabla^2 V\right)_\star + G\left(\text{diag}\{\alpha_1\beta_1,\ldots,\alpha_m\beta_m\}\right)G^\top \succ 0, \\ &\left(\mathcal{D} + GG^\top\right)\dot{q} = \mathbf{0} \implies \begin{cases} q = q_\star \\ p = \mathbf{0}. \end{cases} \end{aligned} \tag{44}$$

1) Fully actuated mechanical systems: A mechanical system such that $n = m$ is said to be *fully actuated*. This subclass of mechanical systems is of great interest in robotics as a broad range of robotic arms satisfies the aforementioned conditions.

When dealing with fully actuated mechanical systems, it is possible to modify Theorem 2 to provide a stronger result, i.e., a saturated controller that guarantees the global asymptotic stability of the desired equilibrium while avoiding velocity measurements. This controller is introduced in the following proposition.

Proposition 2: Consider the system (42)*, with* $G = I_n$*, the function* $\Phi_{\mathtt{c}}(\gamma(x), x_{\mathtt{c}})$ *given in* (22)*, with* $\gamma(x) = q$*, and the dynamics of* $x_{\mathtt{c}}$ *provided in* (23)*. Assume that* $\nabla V(q)$ *is bounded. Then, the saturated control law*

$$u = \nabla V(q) - \nabla_q \Phi_{\mathtt{c}}(q, x_{\mathtt{c}}), \tag{45}$$

ensures that $(q_\star, \mathbf{0}, \mathbf{0})$ *is a globally asymptotically stable equilibrium point of the closed-loop system with Lyapunov function*

$$H_{\mathtt{d}}(q,p,x_{\mathtt{c}}) = \Phi_{\mathtt{c}}(q,x_{\mathtt{c}}) + \frac{1}{2}p^\top M^{-1}(q)p + \frac{1}{2}x_{\mathtt{c}}^\top K_{\mathtt{c}} x_{\mathtt{c}}. \tag{46}$$

Proof: Note that the closed-loop system takes the form

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{x}_{\mathtt{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I_n & \mathbf{0} \\ -I_n & -\mathcal{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -R_{\mathtt{c}} \end{bmatrix} \begin{bmatrix} \nabla_q H_{\mathtt{d}} \\ \nabla_p H_{\mathtt{d}} \\ \nabla_{x_{\mathtt{c}}} H_{\mathtt{d}} \end{bmatrix}$$

with $H_{\mathtt{d}}$ defined in (46). Therefore

$$\dot{H}_{\mathtt{d}} = -\|\dot{q}\|^2_{\mathcal{D}} - \|\nabla_{x_{\mathtt{c}}} H_{\mathtt{d}}\|^2_{R_{\mathtt{c}}}.$$

Furthermore, some simple computations show that $(\nabla H_{\mathtt{d}})_\star = \mathbf{0}$ and $\left(\nabla^2 H_{\mathtt{d}}\right)_\star \succ 0$ for any $K_{\mathtt{c}} \succ 0$. Accordingly, $(q_\star, \mathbf{0}, \mathbf{0})$ is a stable equilibrium point for the closed-loop system.

To prove asymptotic stability, note that, following the arguments given in the proof of Theorem 2, we have that $\dot{H}_{\mathtt{d}} = 0$ implies $\dot{x}_{\mathtt{c}} = \mathbf{0}$ and $y = \dot{q} = \mathbf{0}$. In particular, the latter leads to $M^{-1}(q)p = \mathbf{0}$. Hence,

$$p = \dot{p} = \nabla_q \Phi_{\mathtt{c}} = \mathbf{0}.$$

Moreover, since $\nabla_{x_{\mathtt{c}}} \Phi_{\mathtt{c}} = \nabla_q \Phi_{\mathtt{c}}$ and $K_{\mathtt{c}}$ is full rank, we get that $\dot{x}_{\mathtt{c}} = \mathbf{0}$ implies $x_{\mathtt{c}} = \mathbf{0}$. Hence,

$$\nabla_q \Phi_{\mathtt{c}} = \sum_{i=1}^n e_i \alpha_{\mathtt{c}_i} \tanh(\beta_{\mathtt{c}_i}(q - q_\star + x_{\mathtt{c}})) = \mathbf{0}$$

implies $q = q_\star$. The proof is completed noting that $H_{\mathtt{d}}(q,p,x_{\mathtt{c}})$ is radially unbounded. ■

Remark 4: As a result of the comparison between the controllers (24) and (45), we note that in the latter, the term $-\kappa$ is replaced with $\nabla V(q)$. The physical interpretation of this is that the controller is canceling the effect of the open-loop potential energy while assigning a new potential energy function with a minimum at the desired position. An example of this is the gravity compensation in robotic arms.

2) Removing the steady-state error: Due to the complexity of their characterization, some nonlinear phenomena, e.g., static friction and asymmetry in the motors, are often neglected in the mathematical model of a mechanical system. Nevertheless, these phenomena may affect the behavior of the closed-loop system. In particular, steady-state errors may arise. A common practice to deal with this problem is adding an integrator of the position error or a filter. However, it is necessary to ensure that the integrator–or filter–does not jeopardize the stability of the closed-loop system. Some solutions to this problem involve a change of coordinates. See, for instance, [7], [10], [13]. However, this may lead to controllers that depend implicitly on the velocities. Here, we provide a condition that is sufficient to ensure that the addition of the filter that deals with the steady-state error does not affect the stability properties of the closed-loop system. To this end, we propose a direct application of the so-called Lyapunov's indirect method, see [21]. While this result is only local, it provides a simple way to ensure the stability of the closed-loop system after the addition of the integrator or filter without involving changes of coordinates nor velocity measurements.

Consider the system (42), let $\psi \in \mathbb{R}^m$ be the state of the filter, and $f_\psi : \mathbb{R}^m \to \mathbb{R}^m$, $\Psi : \mathbb{R}^m \to \mathbb{R}^{m\times m}$ be differentiable functions. Consider the error $\tilde{q} := q - q_\star$, a filter with dyamics

$$\dot{\psi} = f_\psi(\psi) + \Psi(\psi) G^\top \tilde{q}, \tag{47}$$

and the augmented state vector $\zeta := (q, p, x_{\mathtt{c}}, x_\ell, \psi)$. Hence,

$$\dot{\zeta} = f_\zeta(\zeta, u) := \begin{bmatrix} M^{-1}(q)p \\ -\nabla_q H(q,p) - D(q,p)M^{-1}(q)p + Gu \\ -R_{\mathtt{c}}\left(K_{\mathtt{c}} x_{\mathtt{c}} + \nabla_{x_{\mathtt{c}}} \Phi_{\mathtt{c}}(z_{\mathtt{c}})\right) \\ -R_\ell \nabla_{x_\ell} \Phi_\ell(z_\ell) \\ f_\psi(\psi) + \Psi(\psi) G^\top \tilde{q} \end{bmatrix} \tag{48}$$

where $H(q,p)$, $\Phi_{\mathtt{c}}(z_{\mathtt{c}})$, and $\Phi_\ell(z_\ell)$ are defined in (42), (22), and (34), respectively. Moreover, some simple computations show that $\zeta_\star = (q_\star, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$ is an assignable equilibrium for the augmented system (48) if $(q_\star, \mathbf{0}) \in \mathcal{E}_{\mathcal{M}}$. Let us modify the control law (35) as follows

$$\begin{aligned} u = &-\sum_{i=1}^{n} e_i \left[\alpha_{\mathtt{c}_i} \tanh\left(\beta_{\mathtt{c}_i}\left(\tilde{q} + x_{\mathtt{c}}\right)\right) + \alpha_{\ell_i} \tanh(\beta_{\ell_i} z_{\ell_i})\right] \\ &+ G^\top \left(\nabla_q V\right)_\star + u_\psi(\psi), \end{aligned} \tag{49}$$

where $u_\psi : \mathbb{R}^m \to \mathbb{R}^m$ is a new input related to the filter (47). Thus, (48) in closed-loop with (49) can be expressed as $\dot{\zeta} = f_{\zeta_{cl}}(\zeta)$, with $f_{\zeta_{cl}} : \mathbb{R}^{2n+3m} \to \mathbb{R}^{2n+3m}$. Therefore, it follows from Lyapunov's indirect method that $\zeta_\star$ is a locally asymptotically stable equilibrium for the closed-loop system if the linear system $\dot{\tilde{\zeta}} = \left(\nabla f_{\zeta_{cl}}\right)_\star \tilde{\zeta}$ is stable, where $\tilde{\zeta} := \zeta - \zeta_\star$. At this point we make the following observations:

O6 If $f_\psi = \mathbf{0}$ and $\Psi(\psi) = I_m$, then (47) is an integrator. Furthermore, by fixing $u_\psi(\psi) = -\psi$, we obtain a classical integrator of the position error.

O7 If $u_\psi(\psi)$ is saturated and $\left(\nabla_q V\right)_\star$ is bounded, then the controller (49) is saturated. Then, a suitable choice is

$$u_\psi(\psi) = -\sum_{i=1}^{m} e_i \alpha_{\psi_i} \tanh(\beta_{\psi_i} \psi_i),$$

where α_{ψ_i} and β_{ψ_i} are positive constants.

Note that the results presented in this section can be straightforwardly adapted to fully actuated mechanical systems, where $x_{\mathtt{c}} \in \mathbb{R}^n$, and x_ℓ is not necessary. Moreover, for fully actuated mechanical systems, the approaches proposed in [6], [22] remove the steady-state error via an adaptive term that does not rely on the linearization of the system.

B. Electrical circuits

In general, the pH approach is suitable to model electrical circuits composed of passive components. Nevertheless, the state variables of such models are fluxes and charges, which most of the time are not measurable signals. A solution to this problem is to represent the behavior of the electrical networks via the Brayton-Moser equations, where the state variables are voltages and currents. In this section, we study the Brayton-Moser equations that represent a broad class of electrical networks. Then, we provide sufficient conditions to ensure that the controllers developed in Sections III and IV are suitable for stabilizing these systems. In particular, we provide conditions to identify y and guarantee the existence of $\gamma(x)$ without solving PDEs. To this end, we restrict our attention to *topologically complete* networks, which are introduced below. For further discussion on this topic we refer the reader to [34].

Definition 2 ([34]): A topologically complete network of two-terminal voltage-controlled and current-controlled elements has a graph that possesses a tree containing all the capacitive branches and none of the inductive branches. Moreover, each resistive tree branch corresponds to a current-controlled resistor, and each resistive link corresponds to a voltage-controlled resistor. Finally, the location of the resistive branches is such that there is no fundamental loop in which resistive branches appear as both tree branches and as links.

Topologically complete networks can be split into two subnetworks: one containing all the inductors and current-controlled resistors and the other containing all the capacitors and voltage-controlled resistors. In particular, we consider a topologically complete electrical network consisting of ς linear inductors, ϖ linear capacitors, and none controlled nor constant source. Then, $i_{\mathtt{L}} := \left[i_{\mathtt{L}_1}, \cdots, i_{\mathtt{L}_\varsigma}\right]^\top \in \mathbb{R}^\varsigma$ represents the currents through the inductors, and $v_{\mathtt{C}} := \left[v_{\mathtt{C}_1}, \cdots, v_{\mathtt{C}_\varpi}\right]^\top \in \mathbb{R}^\varpi$ denotes the voltages across the capacitors, where $\varsigma + \varpi = n$. Hence, the network can be represented via the Brayton-Moser equations, see [4], [5], as

$$\begin{aligned} -L\frac{di_{\mathtt{L}}}{dt} &= \nabla_{i_{\mathtt{L}}} \hat{P}(i_{\mathtt{L}}, v_{\mathtt{C}}) + \hat{g}_{\mathtt{L}} u_{\mathtt{L}} \\ C\frac{dv_{\mathtt{C}}}{dt} &= \nabla_{v_{\mathtt{C}}} \hat{P}(i_{\mathtt{L}}, v_{\mathtt{C}}) + \hat{g}_{\mathtt{C}} u_{\mathtt{C}} \end{aligned} \tag{50}$$

where the positive definite matrices $L \in \mathbb{R}^{\varsigma\times\varsigma}$ and $C \in \mathbb{R}^{\varpi\times\varpi}$ denote the inductance and capacitance matrices, respectively; the constant matrices $\hat{g}_{\mathtt{L}} \in \mathbb{R}^{\varsigma\times m_\varsigma}$, $\hat{g}_{\mathtt{C}} \in \mathbb{R}^{\varpi\times m_\varpi}$ represent the *voltage-related* input matrix and the *current-related* input matrix, respectively; and $\hat{P} : \mathbb{R}^\varsigma \times \mathbb{R}^\varpi \to \mathbb{R}$ denotes the so-called *mixed-potential* function, which is given by

$$\hat{P}(i_{\mathtt{L}}, v_{\mathtt{C}}) := i_{\mathtt{L}}^\top \Gamma v_{\mathtt{C}} + \hat{P}_{\mathtt{R}}(i_{\mathtt{L}}) - \hat{P}_{\mathtt{G}}(v_{\mathtt{C}}), \tag{51}$$

where:

- The matrix $\Gamma \in \mathbb{R}^{\varsigma\times\varpi}$ determines the interconnection between the inductors and capacitors of the system, and all its entries are either 1, -1, or zero.
- The mappings $\hat{P}_{\mathtt{R}} : \mathbb{R}^\varsigma \to \mathbb{R}$ and $\hat{P}_{\mathtt{G}} : \mathbb{R}^\varpi \to \mathbb{R}$ are the *dissipative current-potential* and the *dissipative voltage-potential*, respectively, with[4]

$$\begin{aligned} \hat{P}_{\mathtt{R}}(i_{\mathtt{L}}) &:= \int_0^1 v_{\mathtt{R}}^\top(i_{\mathtt{L}} s) i_{\mathtt{L}} ds \\ \hat{P}_{\mathtt{G}}(v_{\mathtt{C}}) &:= \int_0^1 i_{\mathtt{G}}^\top(v_{\mathtt{C}} s) v_{\mathtt{C}} ds, \end{aligned} \tag{52}$$

  where $v_{\mathtt{R}} : \mathbb{R}^\varsigma \to \mathbb{R}^\varsigma$, $i_{\mathtt{G}} : \mathbb{R}^\varpi \to \mathbb{R}^\varpi$, $v_{\mathtt{R}}(0) = \mathbf{0}$, $i_{\mathtt{G}}(0) = \mathbf{0}$, and

$$\begin{aligned} i_{\mathtt{L}}^\top v_{\mathtt{R}}(i_{\mathtt{L}}) &\geq 0,\ \nabla v_{\mathtt{R}}(i_{\mathtt{L}}) \succeq 0\ \forall\ i_{\mathtt{L}} \in \mathbb{R}^\varsigma \\ v_{\mathtt{C}}^\top i_{\mathtt{G}}(v_{\mathtt{C}}) &\geq 0,\ \nabla i_{\mathtt{G}}(v_{\mathtt{C}}) \succeq 0\ \forall\ v_{\mathtt{C}} \in \mathbb{R}^\varpi. \end{aligned} \tag{53}$$

- The inputs $u_{\mathtt{L}} \in \mathbb{R}^{m_\varsigma}$, $u_{\mathtt{C}} \in \mathbb{R}^{m_\varpi}$, with $m_\varsigma + m_\varpi = m$, denote the external (not controlled nor constant) voltage sources in series with the inductors and the external current sources in parallel with the capacitors, respectively.

[4] For a more detailed explanation about the mixed-potential function (51)-(52), we refer the reader to [15], [16].

The system (50)-(51) admits a more compact representation of the form

$$\hat{Q}\dot{x} = \nabla\hat{P}(x) + \hat{g}u \tag{54}$$

with

$$x = \begin{bmatrix} i_{\mathtt{L}} \\ v_{\mathtt{C}} \end{bmatrix}, \; \hat{Q} := \text{block}\{-L, C\}, \; \hat{g} := \begin{bmatrix} \hat{g}_{\mathtt{L}} \\ \hat{g}_{\mathtt{C}} \end{bmatrix}. \tag{55}$$

Note that the system (54) can be expressed as in (1) with

$$f(x) = \hat{Q}^{-1}\nabla\hat{P}(x), \quad g = \hat{Q}^{-1}\hat{g}. \tag{56}$$

To ensure that Assumption 1 holds, we look for a new pair $(Q(x), P(x))$ such that

$$\hat{Q}^{-1}\nabla\hat{P}(x) = Q^{-1}(x)\nabla P(x). \tag{57}$$

For a detailed discussion on this topic we refer the reader to [4], [5], [17], and [3]. In particular, we have the following result.

Proposition 3: Suppose that the Hessian of $\hat{P}(x)$ has full rank. Define

$$\begin{aligned} P(x) &:= \frac{1}{2}\left[\nabla\hat{P}(x)\right]^\top \Xi\nabla\hat{P}(x) \\ Q(x) &:= \nabla^2\hat{P}(x)\Xi\hat{Q} \end{aligned} \tag{58}$$

where $\hat{P}(x)$ and $\hat{Q}$ are defined in (51) and (55), respectively, and $\Xi := \text{block}\left\{L^{-1}, C^{-1}\right\}$. Then, the system (54) can be rewritten as

$$\dot{x} = Q^{-1}(x)\nabla P(x) + gu. \tag{59}$$

Furthermore, the map $u \mapsto -g^\top Q^\top(x)\dot{x}$ is passive with storage function $P(x)$. *Proof:* Note that

$$\nabla P = \nabla^2\hat{P}\Xi\nabla\hat{P}.$$

Hence, (57) holds, and the expression (59) is obtained from (56). To prove passivity note that

$$Q = \begin{bmatrix} -\nabla v_{\mathtt{R}} & \Gamma \\ -\Gamma^\top & -\nabla i_{\mathtt{G}} \end{bmatrix}.$$

Thus, from (53), the symmetric part of $Q(x)$ is negative semi-definite. Hence, by premultiplying both sides of (59) by $\dot{x}^\top Q(x)$, we obtain

$$\dot{x}^\top Q\dot{x} = \dot{P} + \dot{x}^\top Qgu \implies \dot{P} \leq -\dot{x}^\top Qgu.$$

■

In light of Proposition 3, we make the following observations:

O8 Assumption 1 holds for $S(x) = P(x)$.

O9 The elements of y are given in terms of the derivatives of voltages and currents, which may be non-measurable signals. However, since $\dot{\gamma} = y$, $\gamma(x)$ can be expressed in terms of voltages and currents, which are, in general, the available measurements in an RLC network.

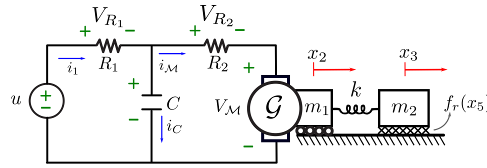

Fig. 1. Electromechanical actuator.

O10 The asymptotic stability of the equilibrium in Theorems 1–3 is ensured if

$$\text{diag}\{\nabla v_{\mathtt{R}}(i_{\mathtt{L}}), \nabla i_{\mathtt{G}}(v_{\mathtt{C}})\}\,\dot{x} = \mathbf{0} \implies x = x_\star.$$

We conclude this section with the following remark concerning the integrability of the passive output provided in Proposition 3.

Remark 5: As a result of Poincaré's lemma, there exists $\gamma(x)$ such that $\dot{\gamma} = y$ if $\nabla(Q(x)g) = [\nabla(Q(x)g)]^\top$.

VI. Examples

In this section, we present simulations and experimental results derived from the stabilization of three systems in different physical domains. In particular, the example provided in Section VI-A illustrates the results of Section IV, i.e., how to exploit the natural damping of the system. Then, the example given in Section VI-B shows how to implement the controller provided in Theorem 2 in a nonlinear circuit, illustrating the discussion exposed in Section V-B. Finally, in Section VI-C, we provide experimental results from the implementation of the controller given in Theorem 2 and the filter proposed in Section V-A. In all the examples, we propose large values for $\beta_{\mathtt{c}_i}$ and β_{ℓ_i} to illustrate the saturation of the controllers. However, these are extreme scenarios and the mentioned parameters should be tuned to obtain the desired performance in each application.

A. Electromechanical actuator

Consider the system depicted in Fig. 1, where u is the voltage provided by the source; R_1 and R_2 denote linear resistors; C represents a linear capacitor, the electrical part of the system is *coupled* with the mechanical one via the *gyrator* $\mathcal{G}$; the symbol k represents a linear spring; x_1 is the charge across the capacitor; x_2 and x_3 are the positions of the masses; x_4 and x_5 are the momenta; and the term $f_{\mathtt{R}}(x_5)$ is an approximation of the friction force present in the second mass, which is given by

$$f_{\mathtt{R}}(x_5) = \frac{a_1}{m_2}x_5 + a_2\tanh(a_3x_5),$$

where a_1, a_2, and a_3 are positive constant parameters. The dynamics of this system can be represented as in (1), with

$g = e_1 \frac{1}{R_1}$ and

$$f(x) = \begin{bmatrix} -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C}x_1 + \frac{1}{a_0 m_1 R_2}x_4 \\ \frac{1}{m_1}x_4 \\ \frac{1}{m_2}x_5 \\ -k\left(x_2 - x_3\right) + \frac{1}{a_0 R_2}\left(\frac{1}{C}x_1 - \frac{1}{a_0 m_1}x_4\right) \\ k\left(x_2 - x_3\right) - \frac{a_1}{m_2}x_5 - a_2\tanh(a_3 x_5) \end{bmatrix},$$

where a_0 is a positive constant parameter that characterizes the relation between the electrical and mechanical variables of the motor.

The set of assignable equilibria for this system is given by

$$\mathcal{E} = \left\{x \in \mathbb{R}^5 \mid x_1 = x_4 = x_5 = 0, x_2 = x_3\right\},$$

and the control objective is to stabilize the mass m_2 at the desired point $x_{3_\star}$ while considering that the voltage source has a limited operation range. Moreover, we consider that there are no velocity sensors. To solve the control problem, consider the total energy of the system, given by

$$S(x) = \frac{1}{2C}x_1^2 + \frac{1}{2}k\left(x_2 - x_3\right)^2 + \frac{1}{2m_1}x_4^2 + \frac{1}{2m_2}x_5^2.$$

Then, some simple computations show that

$$\begin{aligned} \left[\nabla S(x)\right]^\top f(x) &= -\tfrac{1}{C^2 R_1}x_1^2 - \tfrac{1}{R_2}\left(\tfrac{1}{C}x_1 - \tfrac{1}{a_0 m_1}x_4\right)^2 \\ &\quad - \tfrac{a_1}{m_2^2}x_5^2 - \tfrac{a_2}{m_2}x_5\tanh(a_3 x_5). \end{aligned}$$

Accordingly, Assumption 1 is satisfied. Moreover,

$$\begin{aligned} \dot{S} &= -\tfrac{R_1 R_2}{R_1 + R_2}\dot{x}_1^2 - \tfrac{1}{a_0^2(R_1+R_2)}\dot{x}_2^2 \\ &\quad - \left[a_1\dot{x}_3 + a_2\tanh(m_2 a_3 \dot{x}_3)\right]\dot{x}_3 + y^\top u \le y^\top u \end{aligned} \tag{60}$$

with

$$y = \frac{R_2}{R_1 + R_2}\dot{x}_1 + \frac{1}{a_0\left(R_1 + R_2\right)}\dot{x}_2.$$

Therefore,

$$\gamma(x) = \frac{R_2}{R_1 + R_2}x_1 + \frac{1}{a_0\left(R_1 + R_2\right)}x_2 \tag{61}$$

satisfies $\dot{\gamma} = y$. Furthermore, Assumption 2 holds for $\gamma(x)$ given in (61), $\kappa = 0$, and any $\alpha, \beta > 0$. Note that, given (60), Assumption 3 is satisfied for $\eta = x_3$, where Λ_c depends on the value of the resistors. For this example, we have $\Upsilon = 1$ and $x_\ell \in \mathbb{R}$. Hence, from Theorem 3, it follows that the controller (35) ensures that the closed-loop system has a stable equilibrium at $(0, x_{2_\star}, x_{3_\star}, 0, 0)$, with $x_{2_\star} = x_{3_\star}$. We remark that, given (61) and $\eta = x_3$, the control law does not depend on x_4 and x_5, which are the states related to the velocities of the masses.

To prove asymptotic stability of the equilibrium, we check if (37) holds. For this example, we have the following chain of implications

$$\begin{aligned} \dot{\eta} = 0 \;&\Longleftrightarrow\; \dot{x}_3 = 0 \;\Longleftrightarrow\; x_5 = 0 \\ \Longrightarrow\; \dot{x}_5 = 0 \;&\Longleftrightarrow\; x_2 = x_3 \;\Longrightarrow\; \dot{x}_2 = \dot{x}_3 \\ \Longrightarrow\; \dot{x}_2 = 0 \;&\Longleftrightarrow\; x_4 = 0 \;\Longrightarrow\; \dot{x}_4 = 0 \\ \Longleftrightarrow\; x_1 = 0 \;&\Longrightarrow\; \dot{x}_1 = 0 \;\Longrightarrow\; u = 0. \end{aligned} \tag{62}$$

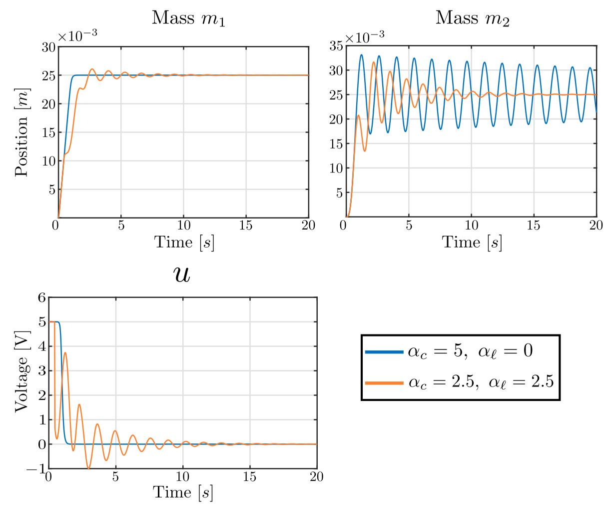

Fig. 2. Evolution of the masses and the control law with and without damping injection through the term $\nabla_{z_\ell}\Phi_\ell(z_\ell)$.

On the other hand, from (38),

$$\dot{x}_\ell = 0 \iff \nabla_{z_\ell}\Phi_\ell = 0 \iff z_\ell = 0. \tag{63}$$

Hence, from (35) and recalling that $\kappa = 0$, we get that

$$u = 0 \implies \nabla_\gamma \Phi_\mathsf{c} = 0 \iff z_\mathsf{c} = 0. \tag{64}$$

Therefore, from (23) and (64), we get that $\dot{x}_\mathsf{c} = 0$ if and only if $x_\mathsf{c} = 0$. Thus, $z_\mathsf{c} = 0$ if and only if $\gamma = \gamma_\star$, which implies that $x_2 = x_{2_\star}$. Note that, from (62), $x_2 = x_{2_\star}$ implies $x_3 = x_{3_\star}$. Hence, from the definition of z_ℓ and (63), we conclude that $x_\ell = 0$. Consequently, (37) holds, and the equilibrium point is asymptotically stable. Furthermore, in this case, $S_{\mathsf{d}_\ell}(x)$ is *radially unbounded*. Thus, the equilibrium is globally asymptotically stable.

TABLE I
PARAMETERS OF THE ELECTROMECHANICAL COUPLING DEVICE

Parameter	Value	Parameter	Value
R_1	100	R_2	100
C	2.2×10^{-4}	m_1	0.01
m_2	0.015	a_0	0.005
a_1	6×10^{-4}	a_2	8×10^{-5}
a_3	40	k	0.3

Simulations: To corroborate the effectiveness of the saturated controller, we perform simulations considering the parameters provided in Table I, where we are particularly interested in showing that the control signal is saturated and how the term $\nabla_{z_\ell}\Phi_\ell(z_\ell)$ can be used to reduce the oscillations in x_3, i.e., the position of m_2. To this end, we consider $x_{2_\star} = x_{3_\star} = 0.025$[m] and the control parameters

$$\begin{aligned} &K_\mathsf{c} = 10^6, &&R_\mathsf{c} = 0.3,\ \beta_\mathsf{c} = 450, \\ &K_\ell = 5.5 \times 10^{-4}, &&R_\ell = 33,\ \ \beta_\ell = 2 \times 10^6. \end{aligned} \tag{65}$$

To illustrate how the term $\nabla_{z_\ell}\Phi_\ell(z_\ell)$ affects the closed-loop behavior, we consider that the voltage source operates in the range of $\pm 5[V]$. Fig. 2 shows the results of simulating two different scenarios: (i) $\alpha_\mathsf{c} = 5,\ \alpha_\ell = 0$, which is plotted

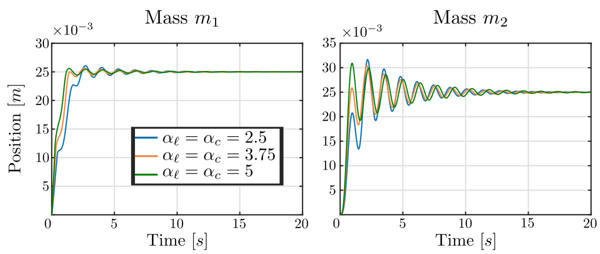

Fig. 3. Behavior of the masses for different saturation limits.

in blue, and (ii) $\alpha_{\mathsf{c}} = 2.5$, $\alpha_\ell = 2.5$ plotted in orange. In both cases the initial conditions are $\mathbf{0}$. From the plots, we observe that in the scenario (i), the first mass converges towards the desired position without oscillations. In contrast, the second mass exhibits an oscillatory behavior as the natural damping terms are relatively small. On the other hand, in the second scenario, it is evident that the term $\nabla_{z_\ell}\Phi_\ell(z_\ell)$ injects damping into the second mass, reducing notoriously the oscillations in x_3.

To show the saturation of the controller, we consider the control parameters (65) and three different set of values for α_{c} and α_ℓ, namely,

$$\alpha_{\mathsf{c}} = \alpha_\ell = 2.5,\ \alpha_{\mathsf{c}} = \alpha_\ell = 3.75,\ \alpha_{\mathsf{c}} = \alpha_\ell = 5.$$

Accordingly, the corresponding control laws must saturate at ± 5, ± 7.5, and ± 10, respectively. In all cases, we consider initial conditions equal to zero. The simulation results are depicted in Figs. 3 and 4. In the former, we note that the behavior of the masses is not drastically affected by the saturation limits. On the other hand, the saturation of the control signals is appreciated in Fig. 4, where the plot at the right-hand shows the first two seconds of simulation when the saturation takes place.

B. Nonlinear RLC circuit

Consider the circuit depicted in Fig. 5, which admits a representation of the form (54) with $\tilde{g}_{\mathsf{c}} = 0$, $\tilde{g}_{\mathsf{L}} = -e_1$, and

$$\begin{aligned} \hat{Q} &= \mathrm{diag}\{-L_1, -L_2, C\}, & \Gamma &= \begin{bmatrix} 1 & -1 \end{bmatrix}^\top, \\ i_{r_D}(x) &= a\left(e^{\frac{x_3}{b}} - 1\right), & v_{\mathtt{R}}(x) &= \begin{bmatrix} 0 & rx_2 \end{bmatrix}^\top, \end{aligned} \quad (66)$$

where x_1, x_2 denote the currents through the inductors, x_3 represents the voltage across the capacitor, the con-

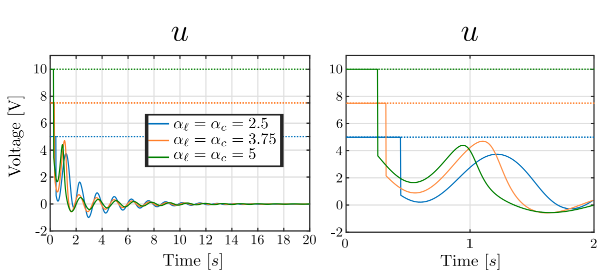

Fig. 4. Control signals evolution during the 20 seconds of simulation (left-hand), and only the first 2 seconds (right-hand). The saturation limits for each control signal are plotted with a dotted line.

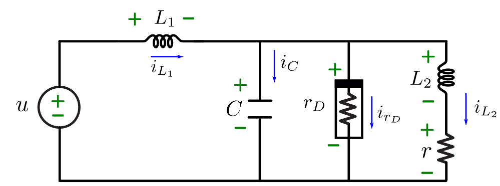

Fig. 5. Nonlinear RLC circuit.

stant parameters L_1, L_2, C denote the inductances and the capacitance, respectively, a and b are positive constant parameters, r_D is a nonlinear load, and r denotes the resistance of the linear resistor.

The control objective is to regulate the current through r_D at the desired value while keeping the supplied voltage bounded. Moreover, we consider that only the voltage across the capacitor can be measured. To solve this problem, we first define the set of assignable equilibria for this system, which is given by

$$\mathcal{E} = \left\{ x \in \mathbb{R}^3 \mid x_1 = \frac{1}{r}x_3 + a\left(e^{\frac{x_3}{b}} - 1\right), x_2 = \frac{1}{r}x_3 \right\}. \quad (67)$$

According to Proposition 3, this system can be represented as in (59) with $g = e_1 \frac{1}{L_1}$ and

$$Q(x) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -r & -1 \\ -1 & 1 & -\frac{a}{b}e^{\frac{x_3}{b}} \end{bmatrix},$$

$$P(x) = \frac{1}{2L_1}x_3^2 + \frac{1}{2L_2}(rx_2 - x_3)^2 + \frac{1}{2C}\left[x_1 - x_3 + i_{r_D}(x)\right].$$

Moreover, a passive output for this system is given by

$$y = -g^\top Q^\top \dot{x} = \frac{1}{L_1}\dot{x}_3,$$

with storage function $S(x) = P(x)$. Hence, Assumption 1 is satisfied. Furthermore, $\gamma(x)$ can be chosen as

$$\gamma(x) = \frac{1}{L_1}x_3,$$

and some simple computations show that Assumption 2 holds for every α, $\beta > 0$, and $\kappa = -x_{3_\star}$. Accordingly, from Theorem 2 it follows that the control law (24) renders the equilibrium point $(x_\star, 0)$ stable. Notice that, since $\gamma(x)$ depends exlcusively on x_3, the controller only requires to measure the voltage across the capacitor.

To prove the asymptotic stability of the equilibrium, consider $S_{\mathsf{d}_{\mathsf{c}}}$ defined in (25) and note that

$$-\|\ell(x) + w(x)u\| = \dot{x}^\top Q(x)\dot{x} = -r\dot{x}_2^2 - \frac{a}{b}e^{\frac{x_3}{b}}\dot{x}_3^2.$$

Hence, we have the following chain of implications

$$\begin{aligned} \dot{S}_{\mathsf{d}_{\mathsf{c}}} = 0 \iff \left\{ \begin{matrix} \dot{x}_3 = 0 \\ \dot{x}_2 = 0 \end{matrix} \right\} &\implies \dot{x}_1 = 0 \\ &\implies x_3 - x_{3_\star} = -\alpha_{\mathsf{c}}\tanh(\beta_{\mathsf{c}} z_{\mathsf{c}}). \end{aligned} \quad (68)$$

On the other hand,

$$\dot{x}_{\mathsf{c}} = 0 \iff K_{\mathsf{c}} x_{\mathsf{c}} = -\alpha_{\mathsf{c}}\tanh(\beta_{\mathsf{c}} z_{\mathsf{c}}). \quad (69)$$

Therefore, combining (68) and (69), we conclude that $K_{\mathtt{c}} x_{\mathtt{c}} = x_3 - x_{3_\star}$. Accordingly, we have

$$x_{\mathtt{c}} = \frac{L_1}{K_{\mathtt{c}} + L_1} z_{\mathtt{c}}.$$

Then, (69) can be rewritten as

$$\dot{x}_{\mathtt{c}} = 0 \iff \frac{K_{\mathtt{c}} L_1}{K_{\mathtt{c}} + L_1} z_{\mathtt{c}} = -\alpha_{\mathtt{c}} \tanh(\beta_{\mathtt{c}} z_{\mathtt{c}}),$$

which holds only if $z_{\mathtt{c}} = x_{\mathtt{c}} = 0$. Moreover, from (68), we conclude that

$$x_3 = x_{3_\star} \implies \begin{cases} x_1 = x_{1_\star} \\ x_2 = x_{2_\star}. \end{cases}$$

TABLE II
PARAMETERS OF THE NONLINEAR RLC CIRCUIT

Parameter	Value	Parameter	Value
r	100	L_1	0.01
L_2	0.02	C	2×10^{-4}
a	10^{-7}	b	0.25

Simulations: For illustration purposes, we consider the parameters given in Table II and the following scenario: the voltage source must operate in the range from 0 to 3.1[V], and the load demands a current of 20[mA]. Then, $u_\star = x_{3_\star} = 3.0515$. Thus, the load drives the voltage source near to its operation limit. Accordingly, we need to ensure that the control signal saturates to protect the source. To this end, we consider the control parameters $K_{\mathtt{c}} = 10$, $R_{\mathtt{c}} = 10$, and $\alpha_{\mathtt{c}} = 0.0485$. Note that the selected value for $\alpha_{\mathtt{c}}$ ensures that the control signal saturates at 3.003 and 3.1 volts. Fig. 6 depicts the simulation results considering initial conditions zero and different values for $\beta_{\mathtt{c}}$. We observe that larger values for $\beta_{\mathtt{c}}$ provoke that the control signal reaches the saturation limits more times—exhibiting an oscillatory behavior—because the control law becomes more sensitive to errors between the actual current and the desired one.

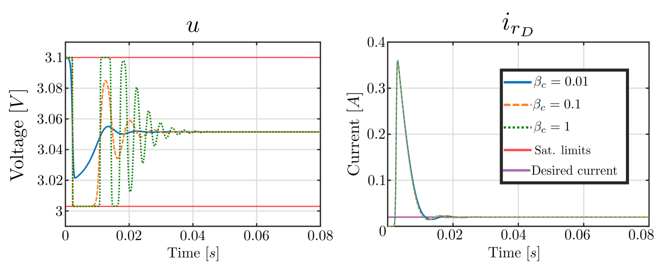

Fig. 6. Control signals for different values of $\beta_{\mathtt{c}}$ (left-hand), and current through the load (right-hand).

C. Philips Experimental Robot Arm

Consider the Philips Experimental Robot Arm (PERA) shown in Fig. 7, a robotic arm designed to mimic the human arm motion [28]. This setup is not equipped with velocity sensors. Hence, we only consider positions measurements.

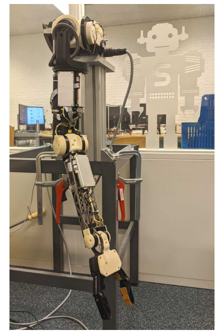

Fig. 7. PERA setup.

To illustrate the applicability of the results reported in Sections III and V, we carry out experiments with the PERA system considering only three degrees-of-freedom, namely, the shoulder roll q_1, the elbow pitch q_2, and the elbow roll q_3. Note that the system admits a representation of the form (42) with $G = I_3$, $\mathcal{D} = \mathbf{0}$, and

$$\begin{aligned} M(q) &= \begin{bmatrix} m_1(q_2) & 0 & \mathcal{I}_3 \cos(q_2) \\ 0 & \mathcal{I}_2 + \mathcal{I}_3 + m_3 d_{\mathtt{c}3}^2 & 0 \\ \mathcal{I}_3 \cos(q_2) & 0 & \mathcal{I}_3 \end{bmatrix} \\ m_1(q_2) &:= \sum_{i=1}^{3} \mathcal{I}_i + m_3 d_{\mathtt{c}3}^2 \sin^2(q_2), \\ V(q) &= m_3 g_{\mathtt{r}} d_{\mathtt{c}3} \left(1 - \cos(q_2)\right) \end{aligned}$$

where $\mathcal{I}_i$ denotes the moment of inertia of the i^{th} link with $i \in \{1,2,3\}$; m_3 and $d_{\mathtt{c}3}$ denote the mass and the distance to the center of mass of the third link, respectively; and $g_{\mathtt{r}}$ represents the gravitational acceleration. The parameters of the system are provided in Table III.[5]

TABLE III
MODEL PARAMETERS

Parameter	Value	Parameter	Value
$g_{\mathtt{r}}$	9.81	$\mathcal{I}_1$	0.0054
$d_{\mathtt{c}3}$	0.16	$\mathcal{I}_2$	0.0768
m_3	1	$\mathcal{I}_3$	0.00211

The shoulder roll q_1 is controlled directly by the rotation of a motor with corresponding torque u_1. On the other hand, the other two angles are controlled via a differential drive, which consists of two motors with corresponding torque denoted by u_2 and u_3. Hence, the elbow pitch q_2 and the elbow roll q_3 are controlled by the combination of the torques from these two motors, i.e., by $u_2 + u_3$ and $u_2 - u_3$, respectively. Moreover, the maximum allowable input for each motor corresponds to $\pm 16A$. We refer the reader to [28] for further details on the conversion constants from current to torque.

The control objective is to stabilize the system at a desired

[5] A MATLAB® script for generating the PERA model can be found in `https://github.com/cachanzheng/PERA`.

TABLE IV
PARAMETERS FOR CONTROL LAW (45)

Parameter	Value	Parameter	Value
α	$(17, 3, 3.3)^\top$	K_c	$\text{diag}\{1, 1, 1\}$
β	$(80, 100, 80)^\top$	R_c	$\text{diag}\{0.1, 0.005, 0.05\}$

configuration $q_\star$ while ensuring that

$$\begin{aligned} |u_1| &\le 17.1007\ Nm \\ |u_2 + u_3|, |u_2 - u_3| &\le 7.901\ Nm \end{aligned} \tag{70}$$

to protect the motors.

1) Implementation of the control law (45): Following the results of Proposition 2, the saturated control law (45) renders $(q_\star, \mathbf{0}, \mathbf{0})$ globally asymptotically stable. To corroborate the effectiveness of the control approach, we perform an experiment under the initial conditions $q(0) = (-2.257, -0.206, 0.044)$, where the PERA is stabilized at the desired configuration $q_\star = (-1.81, \pi/2, 0.78)$ with the control parameters given in Table IV.

Note that the control law (45) is bounded by α and $\nabla_{q_2} V(q_2)$, where the latter reach its extrema when $\sin(q_2) = \pm 1$. Therefore, α is selected such that the controller satisfies the limits given in (70), i.e.,

$$\begin{aligned} |u_1| &\le \alpha_1 \le 17.1007 \\ |u_2 + u_3| &\le |\alpha_2 \pm \nabla_{q_2} V(q_2) + \alpha_3| \le 7.901 \\ |u_2 - u_3| &\le |\alpha_2 \pm \nabla_{q_2} V(q_2) - \alpha_3| \le 7.901, \end{aligned}$$

where the extrema of $\nabla_{q_2} V(q_2)$ are given by ± 1.57, and α_i is the i^{th} element of α with $i \in \{1, 2, 3\}$. On the other hand, we observe from Proposition 2 that β corresponds to the slope of $\Phi_c(q, x_c)$, and it is related to the sensitivity of the controller. Hence, we choose a high value for β to illustrate the control saturation. Finally, K_c and R_c are selected to be positive definite satisfying the conditions proposed in Proposition 2.

The results of the experiments are depicted in Fig. 8, where we show the evolution of the angular position of each joint. Furthermore, the corresponding control inputs are shown in Fig. 9, where the saturation for each joint is evident. Particularly, in Fig. 8, we note steady-state errors in q_1 and q_3. These errors may be caused by several factors, such as the neglected damping, the asymmetry of the motors, or their dead zones. Hence, to remove these errors, we implement an integral-like term as it is explained in Section V-A.2.

2) Implementation of the control law (49): To remove the steady-state error from the results of Section VI-C.1, we implement a filter of the form (47) with

$$\begin{aligned} f_\psi(\psi) &= -R_\psi \psi, \\ \Psi(\psi) &= \sum_{i=1}^{3} e_i e_i^\top \alpha_{\psi_i} \beta_{\psi_i} \,\text{sech}\,(\beta_{\psi_i} \psi_i), \\ u_{\psi_i}(\psi) &= -\sum_{i=1}^{3} e_i \alpha_{\psi_i} \tanh(\beta_{\psi_i} \psi_i), \end{aligned}$$

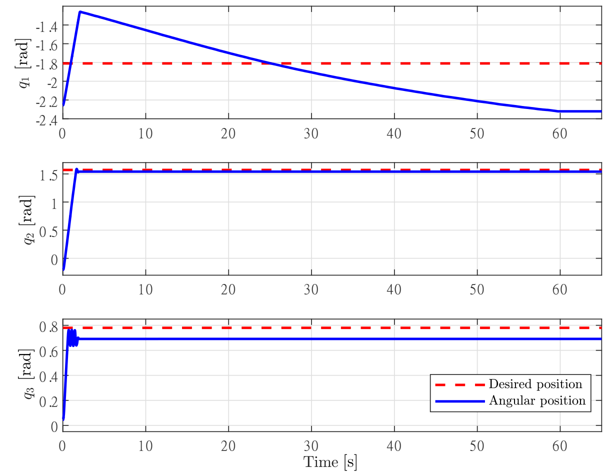

Fig. 8. Top: shoulder roll angular trajectory. Middle: elbow pitch angular trajectory. Bottom: elbow roll angular trajectory.

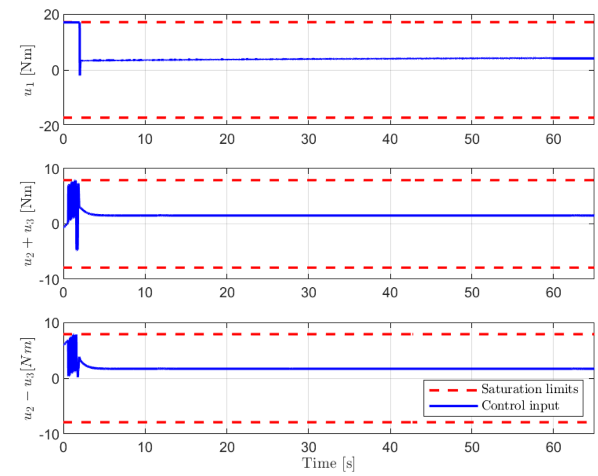

Fig. 9. Top: shoulder roll control input. Middle: elbow pitch control input. Bottom: elbow roll control input.

where R_ψ is a diagonal matrix with positive entries. Accordingly,

$$(\nabla f_{\zeta_{cl}})_\star = \begin{bmatrix} \mathbf{0} & M_\star^{-1} & \mathbf{0} & \mathbf{0} \\ -D_c & \mathbf{0} & -D_c & -D_\psi \\ -R_c D_c & \mathbf{0} & -R_c K_c - R_c D_c & \mathbf{0} \\ D_\psi & \mathbf{0} & \mathbf{0} & -R_\psi \end{bmatrix}, \tag{71}$$

where

$$\begin{aligned} D_c &:= \text{diag}\{\alpha_{c_1}\beta_{c_1}, \alpha_{c_2}\beta_{c_2}, \alpha_{c_3}\beta_{c_3}\} \\ D_\psi &:= \text{diag}\{\alpha_{\psi_1}\beta_{\psi_1}, \alpha_{\psi_2}\beta_{\psi_2}, \alpha_{\psi_3}\beta_{\psi_3}\}. \end{aligned}$$

Note that the controller (49) is bounded by α_c, α_ψ, and $\nabla_{q_2} V(q_2)$. Therefore, α_c and α_ψ are chosen such that they satisfy the limits provided in (70). Based on the discussion provided in Section V-A.2, the rest of parameters are selected to guarantee that the augmented system has a globally asymptotically equilibrium at $(q_\star, \mathbf{0}, \mathbf{0}, \mathbf{0})$. To

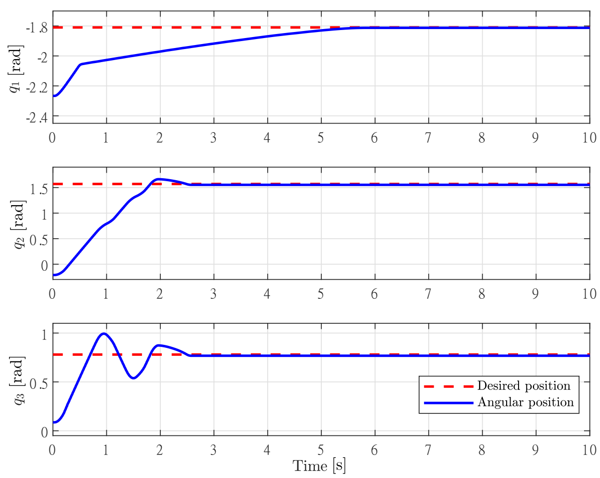

Fig. 10. Top: shoulder roll angular trajectory. Middle: elbow pitch angular trajectory. Bottom: elbow roll angular trajectory.

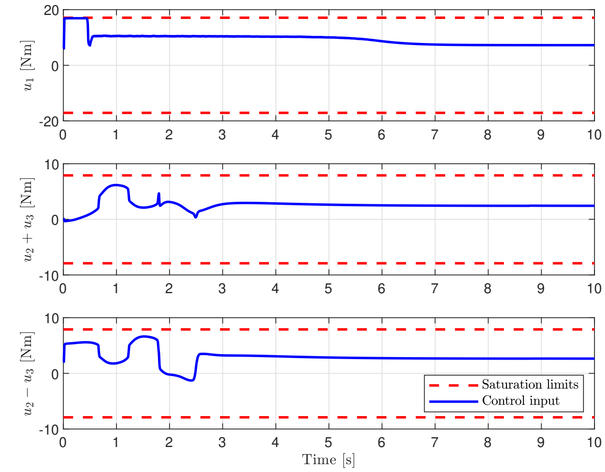

Fig. 11. Top: shoulder roll control input. Middle: elbow pitch control input. Bottom: elbow roll control input.

this end, we use the values provided in Table V for the parameters of the control law (49), where u_ψ is proposed as in Section VI-C.1. Hence, the matrix (71) is stable, i.e., the real part of its spectrum is negative.

TABLE V
PARAMETERS FOR THE CONTROL LAW (49)

Parameter	Value	Parameter	Value
α_c	$(6, 1.4, 1)^\top$	K_c	$\mathrm{diag}\{1, 1, 1\}$
β_c	$(120, 120, 120)^\top$	R_c	$\mathrm{diag}\{0.1, 0.005, 0.05\}$
α_ψ	$(11, 1.5, 2.4)^\top$	R_ψ	$\mathrm{diag}\{1, 1, 35\}$
β_ψ	$(7, 7, 7)^\top$		

To corroborate that the steady-state errors are removed, we carry out experiments under the initial conditions $q(0) = (-2.23, -0.212, 0.086)$, considering the same desired configuration as in Section VI-C.1. The results are shown in Figs. 10 and 11. We remark that by adding the filter, the rate of convergence of the angular trajectories has improved with respect to Section VI-C.1. Moreover, the results of Figs. 10 and 11 exhibit less energy consumption and fewer oscillations than the ones depicted in Figs. 8 and 9. We also underscore the absence of steady-state errors in the trajectories depicted in Fig. 10, where the improvement with respect to the results of VI-C.1 is particularly notorious in q_1 and q_3. Moreover, the saturation of u_1 is evident in Fig. 11. The video of this experiment can be watched at:

`https://www.youtube.com/watch?v=l-9DbTZvyD0`

VII. Concluding remarks and future work

We have presented a PBC approach to design saturated controllers suitable for stabilizing a broad class of physical systems. The proposed controllers do not require measuring the passive output to inject damping into the closed-loop system. Additionally, we have introduced a method to exploit the natural dissipation of the system to improve the performance of the controllers for systems with poor damping propagation. We have illustrated the applicability of the technique by controlling three systems in different physical domains, where the effectiveness of the methodology has been validated through simulations and experiments

As future work, we aim to propose a constructive approach to tune the gains of the controllers to guarantee appropriate performance of the closed-loop system and enlarge the class of systems that can be controlled with the proposed methodology.

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