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The policy and practice of mathematics mastery: the effects of neoliberalism and neoconservatism on curriculum reform.

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Neoliberalism and neoconservatism; Analytics of Government; Teaching for Mastery; Mathematics.

Abstract
This paper explores how the twin processes of neoliberalism and neoconservatism work together on, and through, curricula and their associated pedagogies. It bridges the gap between policy and classroom practice, focusing on the particular example of the school subject of mathematics and the notion of mastery, operationalised in the English education system as Teaching for Mastery (TfM). From this context it develops a theoretical argument using Dean's analytics of government as part of a broader Foucauldian frame, to analyse how TfM is constructed as a particular policy truth. It then shifts the analysis from a wide, social one to the individual classroom level using a psychological argument to critique TfM in its own terms, examining the onto-epistemological nature of mathematics as a subject. In doing so, it explores ways in which mastery might be problematic in classrooms, even whilst appearing to offer a solution at policy level to long-standing problems in English schooling. The aim is not to suggest that TfM has nothing to offer, but to point to ways in which it draws on the psychology of teaching and learning in a very particular manner, inscribing pupils with very specific mathematical subjectivities. By providing this insight into how neoliberal policy positions play out at practitioner level via curricula and pedagogies, the paper raises questions which are philosophical, political and ethical, regarding the potential effect of TfM on teachers' and pupils' experiences of mathematics in schools, including implications for equity of this experience amongst the latter.

Introduction
Education in England is largely dominated by a culture of testing and accountability aimed at raising scores in national standardised examinations (for example, Connell, 2013; Keddie, 2016; Pratt, 2016). In this sense it is not unique since such systems are becoming commonplace through global education reform (see, for example, Sahlberg, 2007). Nonetheless, England leads the way in terms of adapting its education system to processes of neoliberalism – where this term 'broadly means the agenda of economic and social transformation under the sign of the free market' (Connell, 2013, p. 100) and where 'the role of the state is to create and preserve an institutional framework appropriate to such practices' (Harvey, 2007, p. 2). For education in England, such a transformation has been happening since at least the early 1990s. Under John Major's Conservative, and then Tony Blair's New Labour, governments schooling has progressively been organised around

competition and market forces; the implication being that a neoliberal market will equalise the spread of opportunity to all and 'close the gap' between the highest and lowest attaining students.

However, since pure neoliberalism is based on a belief in individual interests and market freedoms it tends to lead to an increase in inequality, as those who are 'successful' accumulate more and more. For this reason, neoliberalisation is often associated with controlling discourses: authoritarianism, in some cases, but more usually neoconservatism in democratic societies (Apple, 2004; Harvey, 2007). Neoconservatism acts alongside neoliberalism in two ways: first, 'in its concern for order as an answer to the chaos of individual interests' (Harvey, 2007, p. 82) – in this case the competitive interests of schools, teachers (see, for example, Pratt, 2016, 2018) and commercial educational suppliers (Ball, 2004); and second, in the way it 'seeks to restore a sense of moral purpose, some higher-order values that will form the stable centre of the body politic' (Harvey, 2007, p. 83). Such conservatism has been strongly apparent since the 2010 Conservative/Liberal Democratic alliance, and even more so in consecutive Conservative-majority governments thereafter. To give a flavour of this combination, we note an example: the introduction in 2019 of a 'tables check' for all 9 year-olds 'to help ensure children in primary school know their times tables up to 12 off by heart' (Department for Education, 2018). We do not dispute the need for young people to be able to fluently recall or calculate multiplication and division, but this has been enshrined in the English National Curriculum since its inception some 30 years ago. Several points strike us about the introduction of this 'check', therefore. To begin with, its neoliberal roots are illustrated in its competitive economic language; the claim that it will 'continue to improve academic standards in order to deliver a truly world-class education' and 'make a positive contribution to the government's commitment through the Industrial Strategy to drive up the study of maths' (*ibid.*). Moreover, the idea of academic standards is constructed through comparison with other countries (especially Singapore) and in order 'to close that gap and raise national standards in mathematics' (*ibid.*). However, its conservatism is illustrated in several ways too. First, it checks 'times tables up to 12 off by heart', rather than to 10x10 which is the mathematically more sensible, since beyond 10 partitioning allows tables to be combined. However, 12x12 reflects the historical use of the imperial system which has been a touchstone of conservative populism since UK decimalisation in 1972. Second, it is called a check, and the government acknowledged at the time that it 'is similar to the checks many schools use already [but will] enable teachers to monitor a child's progress in a consistent and reliable way'. In this sense it is clearly an additional, compulsory test; though rather bizarrely it is also claimed as part of a set of measures to 'reduce the burden of tests on teachers and children' (*ibid.*)! We therefore see this initiative as an illustration of neoconservative thinking acting alongside neoliberal policy; liberal in its economic market focus, yet conservative in the way it monitors and controls teaching and harks back to an imaginary age of 'basic mathematical knowledge'.

In this paper we exemplify these twin processes of neoliberalism and neoconservatism working together on, and through, curriculum by focusing on a particular example: the school subject of *mathematics*, in the National Curriculum in England for primary pupils (5 – 11 years) (Department for Education, 2013); and the policy initiative of *mastery* which has appeared relatively recently, in various government policies and the curriculum's latest, accompanying non-statutory guidance (Department for Education, 2020), as well as in commercial educational products. We start by using the sociological lens of Foucault to analyse the particular political construction of mastery in the English context, in particular drawing on Dean's (2010) analytics of government to show how mastery is constructed as a 'truth'. Foucault's interest, like ours, was not in objective notions of what *is* true, but in how within any society some ideas become a 'regime of truth, its "general politics" of truth: that is, the types of discourse which it accepts and makes function as true' (Foucault, 1980, p.131). Thus, our focus is on mastery as a political construction which serves the purposes of stakeholders in different ways. We therefore also consider how this political construction might be problematic for teachers and pupils, turning to a psychological argument which moves the lens of our analysis from a wide, social one to an individual one at classroom level. The use of such different theoretical frames might seem incongruous, but, as we will argue, English teachers' theorisations are predominantly rooted in the 'self-evidence' of constructivist psychology. Our purpose is not to create our own truth by rejecting such ideas, but to illustrate how the kind of apparently self-evident logic that underpins mastery is contingent on 'the instruments required to discover it, the categories necessary to think it, and an adequate language for formulating it in proposition' (Foucault, 2006, p. 236). Whilst our discussion takes place in the English context, the constructivist psychological underpinning of learning theory and the individualised, neoliberal policy context of schooling are both common to other nations. We use the, rather clumsy, phrase 'Anglo-American' to describe this context, recognising, in doing so, that it is shorthand for a much wider cultural and geographical spread of education systems and assuming that readers will make connections to their own contexts.

Dean's analytics of government

Dean's (2010) analytics of government is a way of understanding 'regimes of practices', that is 'the organized practices through which we are governed and through which we govern ourselves' (p. 28), and is built on Foucault's notion of governmentality (Foucault, 2007, 2014); not, as we stated above, focused on what *is* true but on what is *taken to be* true and how this comes about. Dean proposes four 'dimensions' to a regime of practices, namely:

- Forms of *visibility* – how objects are illuminated and obscured through different forms of representation.
- *Technical* aspects of government – the procedures, tactics, techniques, technologies and vocabularies used to govern.
- Forms of *knowledge* used in, and created by, the process of governing.
- Individual and collective *identities* authorized by commonly accepted discourses used to judge and validate practice.

These are exemplified in our analysis below, but to give a general sense of them as an analytical tool it is through objects such as the curriculum, its surrounding statutory policy and non-statutory guidance, and associated professional discourses that what 'ought' to be taught is made visible to teachers, and this is operationalised technically through procedures mapped out in, say, inspection frameworks, websites providing curriculum support (work schemes for example, whether official or unofficial), and the accepted language of teaching (such as 'mastery'). All these objects and processes also create knowledge in particular forms – for example how school 'standards' come to mean test outcomes, despite never being officially defined as such. Importantly, these dimensions are not independent of each other and one cannot be understood without the others. Forms of visibility produce forms of knowledge; but the particularity of such forms then affects how knowledge can be made visible and how the technical aspects of governing are managed. And all these effects interact with the subjectivities of those involved, constructing, and being reconstructed by, them.

The case of Mastery

We now apply this framework to one important component of the operationalisation of reforms in the mathematics curriculum in England over the last ten years, namely the idea of *mastery*. This idea was introduced largely on the back of successive governments' interest in supposedly high performing countries such as Shanghai (actually a metropolitan area of China) and Singapore (Boylan et al., 2018), and one outcome of this interest was the Mathematics Teacher Exchange (MTE) in which teachers from England and Shanghai made reciprocal teaching visits. A government press release from 2016 boldly proclaimed:

> *South Asian method of teaching maths to be rolled out in schools. 8,000 primary schools in England will receive £41 million over 4 years to support the 'maths mastery' approach.* (Department for Education, 2016, np)

We see, in this proclamation, an example of how mastery is made visible through a technical procedure of 'rolling out' and as a particular, legitimising form of knowledge through the use of 'the' maths mastery approach. Its meaning and how it relates to other practices from which it is derived are, in fact, far from clear to us (and see Boylan et al., 2018; Boylan et al., 2019). Despite several documents on the Gov.uk website which refer to it, none that we could find gives a clear, explicit definition. Rather, the term seems taken as read. For example, the recently published non-statutory guidance for mathematics at Key Stages 1 and 2 (5 – 11 years) (Department for Education, 2020) identifies (in both the colloquial and Foucauldian sense) seven 'Mastery Specialist Teachers' named in the list of 20 authors and 'has been produced to help teachers and schools make effective use of the National Curriculum to develop primary school pupils' mastery of mathematics' (p. 4). In this way it makes visible particular forms of mathematical knowledge and identifies itself with an authorised version of mathematics teaching. It also identifies as an authoritative voice (Bakhtin, 1981), telling readers that pupils need to 'achieve mastery' of one mathematical idea 'before moving on to' the next (p. 121) and that for pupils 'to meet [an assessment] criterion, they need to demonstrate mastery of the structures' of the mathematics it involves. Again, such language illuminates a common-sense vision of the idea of mastery and obscures any sense of its contestability, identifying pupils as being subject to its conditions. But nowhere is the word itself defined. Similarly, the recent research review in mathematics published by the government's inspection service, Ofsted (Office for Standards in Education, Children's Services and Skills, 2021), makes four mentions of mastery, again with no definition, simply noting that:

> *'Mastery' pedagogical approaches that have influenced English mathematics education tend to require pupils to demonstrate high levels of achievement before they are moved on to new content. Some mastery approaches place a greater emphasis on problem-solving and on deepening pupils' understanding.* (np)

For a clearer definition one must go to the government funded National Centre for Excellence in Teaching Mathematics (NCETM), which provides professional development for teachers and schools and whose director of primary mathematics, Debbie Morgan, is also the lead author on the non-statutory guidance mentioned above, along with three of her NCETM colleagues. Here one finds that:

> *Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject.*
>
> *The phrase 'teaching for mastery' describes the elements of classroom practice and school organisation that combine to give pupils the best chances of mastering maths.*
>
> *Achieving mastery means acquiring a solid enough understanding of the maths that's been taught to enable pupils to move on to more advanced material.* (National Centre for Excellence in Teaching Mathematics, nd)

In its authoritative voice, these paragraphs construct the idea of a particular form of knowledge that is 'solid', 'deep', 'long-term' and 'secure' and promises teachers the secret of how to attain this in their pupils through practices that are 'best'. In this sense a truth is proffered, one that is seductive for those whose responsibility it is to demonstrate, in the language of the inspection system, 'outstanding teaching'. We note, too, the neoconservative overtones of such statements. They appeal to common-sense logic – after all, who would not want pupils to master things? – a clear end-goal – mastery – and a sense of orderliness. It is in the professional logic of such ideas that a *regime of practices* starts to be generated in which 'some directions of thought, perhaps implicit and inductive, seem to flow more easily, affording, rather than constraining, certain actions over others' (Alderton & Pratt, 2021, p. 3). We now develop this analysis of how such a regime is constituted by examining the origins and associations of the word mastery itself as it appears in the English context.

Mastery – a brief overview

It is clear from seeing the word used in various ways, above, that mastery, in its current English school usage, echoes a range of ideas which each have their own history. Whilst the notion of 'a deep, long-term, secure and adaptable understanding of the subject' (National Centre for Excellence in Teaching Mathematics, nd) may reflect the constructivism of Piaget (e.g. 1952), Bruner (e.g. 1966) and Vygotsky (e.g. 1978), mastery actually has its theoretical roots in the behaviourism of Skinner (1938, 1948) and Thorndike (1898), leading in the 1960s, to *personalised systems of instruction* (PSI), 'an individually based, student-paced approach to mastery instruction wherein students typically learn independently' (Block & Burns, 1976, p. 9). In the early 1960s, however, another American psychologist, John Carroll (1963), took up mastery in a slightly different way focusing on the idea that instead of thinking about how much could be learnt by different children in a fixed time, one might ask how much time is needed to teach different children a fixed amount. Carroll's ideas were subsequently adopted by Bloom (1968) who emphasised the need for frequent and regular 'diagnostic-progress tests which can be used to determine whether or not the student has mastered the unit and what, if anything, the student must still do to master it' (p.9). Subsequently, teachers would be able to 'pace the learning of students and help motivate them to put forth the necessary effort at the appropriate time' (p.9) with specific teaching planned accordingly; the aim ultimately being to find 'the best match between individuals and alternative learning resources' (p.10).

Historically, then, though growing from slightly different roots and variously termed *Mastery Learning* and *Learning for Mastery*, the essential theme of mastery has been to encourage teachers to undertake careful diagnostic assessment followed by individualised differentiation of materials, teaching and/or additional support. However, the influence of South Asia, especially the Mathematics Teacher Exchange outlined above, has encouraged a different emphasis, focused on teaching and specifically on classes taught as a whole with all pupils moving together from one topic to the next. The combination of this influence of South Asia and mastery's Anglo-American origins has resulted in the idea that, in England,

> *mastery learning should be distinguished from a related approach sometimes known as "teaching for mastery" [… which …] is characterised by teacher-led, whole-class teaching; common lesson content for all pupils; and use of manipulatives and representations. (Education Endowment Foundation, n.d.)*

This shift towards teaching has been most strongly captured in the NCETM's Teaching for Mastery (TfM) programme, funded by the DfE to the tune of £73M, focused on cascading professional development centred around Maths Hubs, and aiming to reach 9300 primary schools by 2023 (Boylan et al., 2019). Because of its reach in the English system we take the TfM programme as our focus in the rest of this paper; but recognise that mastery has also been made visible and constructed as a regime through other technologies, involving a market of providers in the English system who have operationalised it via various commercial schemes and packages of professional development. Indeed, even within TfM, and more so in the wider market, one can see how mastery, in its evolution from differing historical periods and cultural contexts, has come to represent an array of ideas variously including: a teaching approach (Teaching for Mastery); a particular organisation of a curriculum (the mastery curriculum); a perceived process of learning (mastery learning); and an end-state of learning (achieving mastery). These knowledge forms make visible a regime of teaching practices founded on two approaches that each try to address individuals but in ways that are fundamentally opposed to each other. In its early forms, mastery learning is divergent, as pupils are understood to learn different things at different rates; TfM is convergent, with the aim of providing pupils with extra support so they move through the curriculum together, 'acquiring a solid enough understanding of the maths that's been taught to enable pupils to move on to more advanced material' (National Centre for Excellence in Teaching Mathematics, nd). And moreover, all this still lies within a neoliberal system in which winners and losers are identified through testing of both pupils and schools. The convergent nature of TfM can therefore be understood as an example of Harvey's (2007) neoconservative resistance to, and the finding of a moral order in, the potential neoliberal chaos of individual psychology and marketised provision of educational opportunity.

Adaptive Teaching and Ready to Progress Criteria

Central to this plethora of ideas, and strongly featured in TfM, are two others playing a part in this neoconservatism: *adaptive teaching* and *ready to progress criteria*. To understand the former it is easiest to examine what adaptive teaching is *not*. According to Ofsted, which inspects education services in England:

> ***In-class differentiation**, through providing differentiated teaching, activities or resources, has generally not been shown to have much impact on pupils' attainment. …*
>
> *On the other hand, **adapting teaching** in a responsive way, for example by providing focused support to pupils who are not making progress, is likely to improve outcomes. However, this type of adaptive teaching should be clearly distinguished from forms of differentiation that cause teachers to artificially create distinct tasks for different groups of pupils or to set lower expectations for particular pupils.*
> *(Office for Standards in Education, Children's Services and Skills, 2019, p. 17, emphasis in original)*

Adaptive teaching is therefore conceived as an alternative to differentiation; a term which of course takes its root from 'different'. Differentiation reflects the individualising nature of the two original forms of mastery outlined above, both of which draw on psychological theories of individual child development which are still deeply rooted in English primary schools from the progressive movement of the 1950s – 1980s (for example Holt, 1964; Plowden, 1967) and the associated constructivism of Piaget, Vygotsky and Bruner. Indeed, the first set of non-statutory guidance which accompanied the original mathematics national curriculum in 1989 stated that:

> *The mathematical development of each pupil is different and is difficult to predict. Mathematical concepts form a network through which there are many different paths. Different pupils will need to take different paths through the network, and approach learning from a variety of perspectives.* (National Curriculum Council, 1989, p. B8)

As we noted above, if TfM is constructed as a regime of practices that focus on convergent outcomes, to fit this logic it must find forms of knowledge and teaching techniques that work towards convergence and make these visible in officially sanctioned representations. To do this, adaptive teaching for mastery is premised on particular forms of knowledge made visible in the guise of common lesson content for all pupils, and technical judgements about making progress using criteria which specify when children are 'ready to progress'. These are identified, and then specified, in the most recent non-statutory guidance accompanying the mathematics national curriculum as

> *the most important conceptual knowledge and understanding that pupils need as they progress from year 1 to year 6 … [which] provide a coherent, linked framework to support pupils' mastery of the primary mathematics curriculum.* (Department for Education, 2020, p. 5)

These curricular technologies, and their associated knowledge forms of readiness to progress and adaptive teaching, fit with the overall logic of mastery of the subject by all pupils. In this sense they are so central to the notion of TfM and its regime of practices that from this point on we simply use TfM to refer to the whole interacting network of ideas.

Assumptions in TfM

Thus far, what we have done is to recount our version of the history of mastery and analyse some of its associated ideas from a Foucauldian perspective to show how they form a regime of practices. In the rest of this paper, we analyse some of the assumptions that underpin these ideas from an onto-epistemological point of view and critique them in terms of common-sense understandings of how mathematics is learnt. Our critique is built on first making visible a series of assumptions which we argue have underpinned mathematics teaching in English schools since the start of compulsory schooling, but which have been made more visible and brought into sharper focus by TfM. We then focus on the role of these assumptions in the associated regime of practices. In doing so we want to emphasise that we are not critical *of* TfM but are offering a critique that we hope exposes the way in which it is constructed *as* a regime.

In making the assumptions visible we are still following Dean's analytical framework, however the subsequent critique is rooted in psychology because psychology (of individual cognition), and particularly the notion of constructivism articulated through Piaget, Vygotsky and Bruner, is the dominant theoretical language within which Anglo-American learning is *interpreted* in schools, albeit often tacitly. Note that this is not to claim that, historically, TfM has *arisen* only from Anglo-American psychological ideas, since its South Asian origins would imply a complex mix of social and cultural understandings. Our point is about the filter through which it is interpreted in practice, rather than where it came from.

An assumption of objectification

If one is to instruct teachers on what to teach and when, one must objectify it; since one cannot plan it, measure it, decide what comes before and after it, if it is not an 'it' in the first place. This, then, is an onto-epistemological assumption, a form(ation) of knowledge; that mathematics is constituted of a series of objects that are distinct in the sense that they can become the objectives of lessons, can be related to each other, measured in assessment, and hence used as a commodity in the economy of neoliberalisation (Pratt, 2016). The assumption is evident throughout the documents referred to in this paper, for example in phrases such as 'pupils must be able to write and solve *addition problems* with 3 or more *addends* before they can connect *repeated addition* to *multiplication'* (Department for Education, 2020, p. 70, emphasis added). It is important to note that we are not suggesting that objectification is, in itself, an issue. In fact it cannot be avoided if we are to think and talk about anything, mathematics being no exception. Our point, though, is that the choice of objects is a cultural arbitrary. The addition problems mentioned above are chosen as addition problems, only in as far as addition is separated, epistemologically, from subtraction. At the same time, addition and subtraction are seen as related, by their inverse nature, and anyone who has taught children will know that they, like most adults, often solve one 'kind' of problem using the other 'kind' of operation – indeed, this is something they are meant to be taught. Another example is in the focus in the non-statutory guidance on 'numbers with up to 2 decimal places' – which occurs 19 times in the document. Specifying that children should operate with '2 decimal places' separates such numbers from those with 1 decimal place, 3 decimal places, 4 etc., objectifying '2 decimal places' as something with pedagogical significance. Again, this may be helpful, but may also discourage the generalisation that any mathematician would seek in, say, rounding numbers. If you can do it with 2dp, you can do with 3dp, and 4dp etc. and hence the selection of two is, again, arbitrary.

In TfM the notion of readiness to progress accentuates this idea, assuming that there are mathematical objects to master which are definitively, not arbitrarily, 'the most important conceptual knowledge and understanding that pupils need as they progress from year 1 to year 6' (Department for Education, 2020, p. 5 – as above). This formation of knowledge affords the kinds of teaching techniques, vocabulary, assessment technologies (Alderton & Pratt, 2021; Pratt & Alderton, 2019) and so on used to govern schooling and acts as a commodity in the economy of educational success.

An assumption of linearity and readiness

If objects form the first assumption, the second is that these objects have different importance in terms of mathematical thinking and can be defined in a linear sequence that is 'based on logical progression' (Department for Education, 2020, p. 7). Again, this notion is not new in TfM, but is emphasised in such regular reference to what 'must' or 'should' be learnt/done 'before' something else. Once objects are defined and sequenced there is a related assumption that children's learning can be made to follow this 'logical progression' of mathematical ideas. Thus comes readiness; the notion that we can decide when a learner is prepared appropriately to move onto the 'next' thing, based on criteria which specify what is 'needed' and 'most important' for progression.

An assumption of control

Finally, then, through the preceding assumptions TfM assumes the idea that teachers can control pupils' learning. Linearity follows objectification and both are necessary if teachers are to manage and control understanding. In the modern classroom this might seem taken as read, but we note again the quote above from the non-statutory guidance for the original 1989 curriculum, just 33 years ago, suggesting directly the opposite – that learning 'was difficult to predict' (National Curriculum Council, 1989, p. B8). Elsewhere we have argued that the shift to a professional discourse of control of learning has relied on discourses of willing participation in, and an acceptance of, responsibility for pupils' progress, affording a belief in the kind of meritocracy that underpins neoliberalism (Pratt, 2018). The logic of practice in TfM is that opportunity is equalised by the system of teaching, so it will be those who strive hardest (and are most talented, perhaps) who are successful. Thus, we see the control of learning as one aspect of the regime of practices underpinned by the assumptions made in TfM.

Critiquing Mastery: a psychological perspective

It is important to emphasise that our analysis is examining a logic of practice and, by definition, this implies a set of discourses that makes sense to teachers in their daily work. Teaching is a practical affair and anyone who has had responsibility for 30 pupils appreciates the need for structure and direction over various periods of time. Indeed, as ex-schoolteachers ourselves, we recognise much of this logic; but at the same time our job as academics interested in Foucault is to subject it to a critique. As we have said, we use a psychological argument to deconstruct TfM, for the reasons articulated above, but we emphasise that the whole thing is undertaken within a Foucauldian framework. It is one thing to show that there have been other truths available over a 30-year historical period; but another to analyse whether the current regime of truth in England's schools is consistent in its own terms. Our claim, therefore, is not that the critique we undertake represents 'the truth', but that it offers an alternative perspective that challenges the assumptions implicit in TfM. Of course, other readings are possible, and our warrant is only that ours is a reasonable one; this should be judged by the extent to which it resonates with those who work in or around schools and their associated policy frameworks.

Overview of our argument

Whilst teaching is practical it also a subtle affair and what is learned goes far beyond what is planned and taught. However, the fundamental requirement for control, which is entirely necessary to support the competitive, performance-driven nature of schooling and neoconservative demands of orderliness, is founded on the assumptions that precede it above. Importantly, if one falls then the whole edifice collapses. As we have shown, TfM comes out of several historical arenas, but affords the idea that learning is an individualised process of 'logical' construction of mathematical ideas – albeit in a social arena – controlled by what 'must' be in place 'before' moving on to the next idea. We now examine the first and second assumptions articulated above, turning to the third – control – in our discussion, so as to offer a critique which we hope is of interest to those involved in teaching, policy development and research.

Objectification problematised

As we have argued, in English schools learning is interpreted through a largely psychological lens, founded in constructivism. Whatever its origins, it is through this lens that teachers in English schools interpret and plan learning – as individual cognition rooted in building new understanding on existing knowledge through interaction with social and material artefacts. Our critique draws particularly on the work of Sfard (1991) and Grey and Tall (Gray et al., 1999; Gray & Tall, 1994), all of whom build on this fundamentally constructivist position. Sfard's paper, which references Piaget, is entitled 'On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin'. This speaks well to the issue because as she points out, mathematical concepts can be both *structural* – that is, referring to an abstract, static, and apparently real object – and *operational* – when understood as processes of some sort, a 'potential rather than actual entity, which comes into existence upon request in a sequence of actions' (Sfard, 1991, p. 5). Though incompatible (an object cannot also be a procedure), Sfard claims that they are complementary, and it is this idea that Gray and Tall (1994) pick up on in naming, what the non-statutory guidance would call a mathematical idea, a 'procept'; that is, something that can be understood (and therefore learnt) as both a *pro*cedure and a con*cept*. Thus, addition, for example, may be understood as the property of two sets combined and as the procedure of combining two sets and finding their total. As Sfard notes, this distinction was well rehearsed even before 1991, for example in such ideas as Skemp's (1976) *relational* and *instrumental* understanding and, we would add, also in Bruner's (1966) enactive and symbolic representations. However, Sfard's insight is that far from being in opposition with each other, she focuses attention on their complementary nature, 'in much the same sense as in physics, where entities at subatomic level must be regarded both as particles and as waves to enable full description and explanation' (1991, p. 5). Sfard thus stresses their 'unity'; an idea which has been developed in theories of situated cognition in which thinking about the way procedures are shaped by their context has led to a focus on how 'knowledge' is situated in, or distributed across, these forms of experience (Brown et al., 1989; Pea, 1993). The common theme in all these theorisations is that procedure does not lead, subsequently, to concept, but that the two mutually constitute each other. We see, therefore, that the idea of objectification, and therefore of readiness to progress, is problematic in two respects. Firstly, onto-epistemologically, in speaking of mathematical 'its', the guidance is failing to recognise the dual nature of mathematical ideas themselves. And secondly, since ideas can only be fully understood in the context of other ideas, 'they' must be to some extent arbitrary and hence any ordering of them must be contestable – the problematisation that now follows.

Linearity and readiness problematised

In analysing how mathematical objects are understood by learners, Sfard suggests that there are essentially three psychological phases: interiorization; condensation; and reification. The first involves experimenting with the procedures associated with the new idea – perhaps, for example, using counting as the procedure for beginning to understand addition. These procedures are then 'condensed' into shorter ones – counting on from the higher number, for example – before, lastly, being reified into an object. Counting all, then counting on, and finally realising that the sum and the addends, are interchangeable, so that rather than six plus three 'making' nine, six plus three 'is' nine, and vice versa. Reification is reflected in the language used above, requiring us to speak of 'it' and 'these' and in the etymology of 'realising', whose stem is 'real'. Sfard (1991) also notes that reification is an ontological shift because pupils must 'recognise' these new objects; and 're-cognise' hints at the ontological nature of the task. However, the most important observation she makes is about the order in which interiorization, and reification happen. The main motive to condense a procedure is to use it, as an object, in a higher procedure. But one can only engage in the higher procedure by using the objects … that have not yet been reified because to do so means using them in the higher procedure. Thus, as Sfard describes it (*ibid.*, p. 31) 'the lower-level reification and the higher-level interiorization are prerequisite for each other!'

It should be apparent from this analysis that the ideas of linearity and readiness are both problematic. Sfard (*ibid.*, p. 32) goes on to suggest that:

> *According to our model of concept development, however, no clear order of abilities can be established. The thesis of the "vicious circle" implies that one ability cannot be fully developed without the other: on one hand, a person must be quite skillful at performing algorithms in order to attain a good idea of the "objects" involved in these algorithms; on the other hand, to gain full technical mastery, one must already have these objects, since without them the processes would seem meaningless and thus difficult to perform and to remember.*

Does this therefore imply that mathematics cannot be learned? Of course not; and hence why we were keen to point out above that structure and organisation are important for teachers. However, it does imply several transgressions of the regime of truth that learning is a smooth, step-by-step process of acquiring mathematical knowledge.

First, Sfard's argument implies that learners must be willing to suspend the need to understand immediately, and to experiment with the way ideas work. Indeed, those 'who are not prepared to actively struggle for meaning (for reification) would soon resign themselves to never understanding mathematics' (Sfard, 1991, p. 33). Being comfortable with the struggle of not-understanding may be difficult anyway, but is made harder in a competitive environment in which 'understanding' is a market commodity (Keddie, 2016). Moreover, Popkewitz (2018, p.85) has argued that,

> *'[one] meaning of subject is how the curriculum creates the reason and "reasonable people" by governing the "soul". The soul here, as discussed earlier, refers to an interior of the child that is observed and administered by pedagogical practices and its sciences that mark the "good", "productive", and "right" kind of child.*

The nature of TfM, with its 'pedagogical practices and its sciences' as a result of which 'all children learn together', identifies pupils as being not only in a constant state of needing to understand, but to do so immediately since being seen to understand becomes a prerequisite for moving on; for being the 'right kind of child'. We return to this point more fully in our discussion below.

Second, the process/object duality implies, epistemologically, that 'understanding' is not simply located 'in' the mathematical 'object' itself, but in its relationships with other ideas. In contemporary psychological terms, mathematical knowing is situated in the context of its use and distributed across various people and materials (Boaler, 2002; Lave, 1988; Nunes et al., 1993; Waite & Pratt, 2015).

In summary, rather than progressing in a linear fashion, mathematics moves in ontological leaps, albeit in a vague general direction. Moreover, what one knows mathematically is intrinsically linked to the context of coming to know it. It is therefore impossible, despite the persuasive claims of TfM, to say definitively that someone is 'ready' to move on, except in as far as they seem able to hold the epistemological tension implied by Sfard – and of course since teachers *must* move on because teaching takes place in and over finite time periods.

Discussion

In this paper we have shown how TfM, articulated through various curriculum guidance documents and recommended forms of teaching, produces a particular logic of practices. In particular, we have focused on two key aspects of TfM: that teaching can be adapted to ensure all children can access mathematical ideas if they are carefully sequenced and organised; and that children can also be assessed closely enough to therefore know when they are ready to progress to the 'next' idea. However, as noted in the last section, from the psychological framework within which Anglo-American schools generally interpret teaching and learning, there are some important questions to ask about the veracity of these logics, given Sfard's argument about the ontological dilemmas of concept development. In this final section we want to expand our argument back to the wider social plane to consider the kinds of effects that TfM might be having on teachers and pupils in English schools.

In taking this step back we use Popkewitz's (1987, 2009, 2018) argument (based, like ours, in Foucault) that it is through curricula and forms of teaching that schools make, or fabricate, certain kinds of people, 'inscribe[ing] cultural norms that simultaneously create[d] social stability and progress' (Popkewitz, 2018, p. 79). Popkewitz carefully delineates the role of psychology in this process and the way in which, across the 19th and 20th centuries, 'the language of psychology created a way to reason about social conduct as defined tasks to be evaluated in relation to universal attributes of individuals and notions of efficiency' (1987, p.16). Crucially, he argues that modelling childhood as a developmental process – one theorised largely through constructivism in Anglo-American schools – has made it possible to analyse children's work as a representation of the student's progress against a hypothetical norm. What this means is that the analysis, through assessment, of children's mathematical work is not about mathematics itself, but about a way of making sense of that child's development in the subject. Note again, for example, the arbitrary choice of two decimal places as the required mathematical idea for pupils at a particular age, despite this being illogical from the point of view of mathematical structure. Thus, TfM is a not a way of representing mathematics, but a way of representing a particular mathematical fabrication of pupils. And, we would argue, of teachers and schools too, since they are similarly fabricated by the demands on them to take up TfM, within a wider culture of neoliberal individualism and its attendant forms of neoconservatism which 'restore a sense of moral purpose' (Harvey, 2007, p. 83). Even though, as we have illustrated, there may be tensions in this approach to teaching, it appears that the logic of practices is enough to sustain it; TfM provides a convincing science of mathematical development, offering teachers and pupils a controllable, 'best practice' pathway to personal success (Pratt, 2018).

At this stage we re-emphasise that our critique is a particular one focused on the translation (Popkewitz, 2018) of subject (mathematics) to pedagogy (TfM), and an evaluation of something as complex as schooling must acknowledge the limitations of *any* policy and practice; one imperfect system might be very much better than another, or none at all. In all these senses we are certainly not suggesting that TfM has nothing to offer and our anecdotal evidence is that many teachers feel very supported by its introduction. Readers will, however, probably pick up a sense of scepticism. We have tried to illustrate how neoliberalism, at the macro level, might play out in classrooms and our concern is for pupils and what their experience will be like. England's statutory mathematics curriculum states that:

> *A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (Department for Education, 2013, p. 99)*

At the same time, 'the expectation is that most pupils will move through the programmes of study at broadly the same pace' (Department for Education, 2013, p. 99), as TfM is designed to do; the theory being that those who would previously have been left behind will keep up. However, the National Curriculum also suggests that:

> *Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through*

> *new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on. (Ibid.)*

What we have illustrated is that there is a two-fold tension for pupils in these curricular statements. First, if only those who 'grasp the concepts rapidly' get to be 'offered rich and sophisticated problems', in what sense is this reflecting an equal spread of educational opportunity? A basic grasp of numeracy for most perhaps; though, as Farquharson et al. (2022, p. 102) demonstrate, despite decades of policy changes there appears to be 'little if any shift in the gaps in educational attainment between children from different backgrounds'. The psychological argument about learning mathematics that we have presented suggests that the strategy of focused, step-by-step teaching is unlikely even to 'close the attainment gap' in this narrow sense. Nor, secondly, is it likely, for many pupils, to create a version of the subject involving 'an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject' (Department for Education, 2013, p. 99) referred to above. Indeed, as Popkewitz (2018, p.86) points out, words such as these

> *are not about some general and universal properties of the mind. They are bound to the particular logic and theories of learning and communication that order differences in the pedagogical properties of teaching.*

In other words, what curiosity and beauty mean must be understood in relation to the pedagogical practices and the forms of mathematical inscription that are considered 'right'. The English government's neoliberal stance demands that it organises education in a way that allows a free market to work, and hence inevitably leads to a gap between success and failure. Whilst for the system as a whole there may seem to be a sense of meritocratic rationality – those that work hard will rise to the top, deserving their success – for individuals this may well play out as a frustrating experience that, far from offering more equal opportunities, only serves to reflect the economic marketplace in terms of a widening gap between the haves and have-nots. Meanwhile, because government must also be seen to promote success for all, neoconservatism acts to put the brakes on too much innovation, through curriculum choices and pedagogical idealism, policed by testing and inspection.

What our analysis offers is not a ready-made solution to any of these dilemmas, but the idea that other regimes of practices are possible. Ultimately, a change has taken place in English mathematics classrooms under the banner of Teaching for Mastery. Central to our critique is the observation that the neoconservative pull to manage and control teaching makes use of a particular version of learning to construct a regime of truth around TfM. Quite apart from the questionable assumption that culturally specific pedagogies from South Asia can be transferred to new systems – a point that Boylan et al. (2019) make in their evaluation of TfM, and see also, for example, Alexander (2000) – such truths are on shaky ground in their own psychological terms given Sfard's ontological dilemma. As Popkewitz (1987, p.16) argues more generally, 'we need to consider that psychology is not "natural" to the selection and organization of school knowledge' and that 'the choice of curriculum involves philosophical, political and ethical questions'. Our analysis has highlighted the need to consider such questions, and in doing so sets a future research agenda, one that emphasises the need to consider the mathematical experiences of pupils, and of teachers, alongside any psychological translation of the discipline of mathematics into a curriculum.

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