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Fully Probabilistic Reliability and Resilience Analysis of Coastal Structures based on Damage-Evolution Simulation

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Abstract

An integrated methodology is presented to ensure coastal structures' resilience to extreme wave conditions, i.e. their ability to withstand further wave actions without reaching their ultimate limit state. Specifically, a fully probabilistic reliability analysis of a rubble mound breakwater is performed here, in combination with a prediction analysis of the expected time of the structure's maintenance. The methodology combines information on the joint probability density function of significant wave height, spectral peak wave period, and storms' duration and calculates the probability of occurrence of different levels of damage in the structure's lifetime via the use of a fully probabilistic reliability method. In this step, the breakwater's armor element's state is supposed to has a zero-damage level, before being exposed to each storm. Moreover, a resilience analysis is also described and applied by using the armor's damage evolution model by Melby and Kobayashi (1998), estimating the expected year that the breakwater's maintenance should take place. In this approach, the expected damage of the breakwater's armor at the end of a storm is a function of the storms' characteristics and the corresponding damage at the end of the previous storm. The methodology is applied to hindcast data at a location of the Barcelona coast in Spain. Thereby, an insight is gained for the extreme forcing on coastal structures induced by coastal storms, enabling the assessment of structures' probability of failure and repair.

Keywords: Coastal structures; Coastal storms; Failure probability; Reliability analysis; Cumulative damage

1 INTRODUCTION

Coastal zones and coastal structures are increasingly threatened by environmental forcing under a changing climate. Therefore, coastal structures, such as rubble mound breakwaters, should be properly designed to meet these requirements and ensure resilient coastlines.

Reliability analysis of these structures aims to estimate the probability that the structures meet predefined safety and performance levels, by considering uncertainties related to actions, resistance, and design tools. This assessment requires evaluation of the structures' response under a range of environmental actions. Different manuals (U.S. Army, 2003; Losada, 2001; Van der Meer, 2018, CIRIA et al., 2007, etc.) recommend the use of a variety of reliability methods for the estimation of failure and collapse probability of coastal structures. These are deterministic (Liu and Burcharth, 1996) and semi-probabilistic methods (PIANC, 1992), probabilistic methods with approximations, and fully probabilistic methods (U.S. Army, 2003). Fully probabilistic methods belong in the category of the advanced probabilistic reliability methods since they can consider the joint probability density function of all random variables involved, e.g. of significant wave height, peak wave period, wave direction, sea level etc. (Malliouri et al., 2021).

The aforementioned reliability methods can use a variety of stability design formulas (e.g. Hudson, 1958, Van der Meer, 1988, etc.) to produce the limit state function of the structure's elements for the assessment of the failure probability of the structure during its operating lifetime. Although crucial for selecting the design solution of the structure, this approach cannot evaluate the structure's maintenance requirements (Melby and Kobayashi, 1998). This is mainly because most existing stability formulas aim at ensuring a stable structure for a design storm, only if the structure has a zero-damage level before the structure is subject to this particular storm.

Nevertheless, coastal structures are not usually in their initial reliability level before they are exposed to each storm, as their damage level increases gradually from storm to storm when no maintenance takes place. Besides, these structures are commonly subject to severe environmental forcing, e.g. extreme storms, during which no maintenance may be safe to undertake, and, consequently, such structures can be exposed to additional risk of damage, or even total failure.

Thus, the consideration and prediction of a coastal structure's armor's damage progression for multiple storm events that are probable to occur during the structure's lifetime, called hereinafter as a resilience analysis of the structure, is vital for an advanced lifecycle cost analysis or the determination of maintenance requirements for a damaged coastal structure. Towards this direction, modern scientific developments have been made regarding a rubble mound breakwater's damage progression (Melby and Kobayashi, 1998; Sousa and Santos, 2006; Castillo et al., 2012; Lira-Loarca et al., 2020, among others). All these approaches highlight the importance to assess coastal structures' loss of functionality and damage evolution in the presence of storms.

Nevertheless, it is noted that the reliability and resilience analysis of coastal structures are rarely combined due to the different assumptions applied in the two approaches. However, since both of them use the information on all possible storms that the structure will be exposed to, we believe that they can be combined and hence useful recommendations could be derived and related for rubble mound breakwaters, regarding the selection of their design solution and maintenance strategy. The two approaches can well be applied and expanded for all coastal structures, but only by using the corresponding limit state functions and cumulative evolution models to each specific case.

The paper is structured as follows. In section 2, the methodology is presented, including the derivation of coastal storms, the fully probabilistic reliability assessment of a rubble mound breakwater's armor element with no consideration of damage evolution, and the fully probabilistic resilience assessment based on armor's damage evolution simulation. In section 3, the methodology is applied to a case study on the Mediterranean coast of Spain, Barcelona, and its results are also presented. Section 4 summarizes the highlights of this work.

1.

2 METHODOLOGY

2.1 Derivation of Coastal Storms

In this work, the coastal storm is defined as any meteorologically-induced disturbed sea state that causes changes and damages to the coastal zones, impinging the coastal morphology and the infrastructure, following the definitions of Harley (2017) and Ciavola et al. (2014). The identification of coastal storms is usually accomplished by applying specific thresholds for a) the significant wave height, b) the minimum duration, and c) the calm period between consecutive events (Sanuy et al., 2019, Duo et al., 2020). In this study, the threshold of the significant wave height is defined as the 95% of the significant wave height data at the examined location, the minimum duration is set at 9 hours and the calm period at 12 hours based on the local characteristics, as described in Martzikos et al. (2021). Consequently, an event that exceeds the threshold of the significant wave height for longer than the minimum duration is considered a storm event, while it can be unified with the consecutive one if the calm period between them is shorter than 12 h.

2.2 Fully Probabilistic Reliability Assessment with No Consideration of Damage Evolution

The reliability of an element depends on the safety margin between the strength (i.e. resistance R) and the load or the action A. The limit state function g describes the relation between resistance and action upon an element and is formulated as follows:

$$g = R - A \qquad [1]$$

Moreover, the probability of failure of a structural element P_f and the reliability level of that element R_e are defined by Eq. [2] and Eq. [3], respectively.

$$P_f = Prob(g < 0) = Prob(A > R) \qquad [2]$$

$$R_e = 1 - P_f \qquad [3]$$

When a fully probabilistic reliability method is applied, e.g. a Direct Integration Method (DIM), or a Monte Carlo Method (MCM), P_f can be calculated accurately based on the probabilistic framework of all random variables involved. The core problem of DIM and MCM is the exact estimation of the joint probability density function (jpdf) of these variables. Given that the marginal probability density functions (pdf) of the variables considered are known, the calculation of their joint pdf is necessary only if these variables are correlated; otherwise, their jpdf is equal to the product of the marginal pdfs of the variables.

In the case of correlated random variables, an efficient way to account for their correlation is to apply the conditional probability model in order to calculate their jpdf. The said model can be illustrated by the total probability law applied in the following Eq. [4] indicatively to two variables, but could be generalized to more than two variables, as well:

$$f_{X_1,X_2}(x_1,x_2) = f_{X_1|X_2}(x_1|x_2) f_{X_2}(x_2) \quad [4]$$

where $f_{X_1,X_2}(x_1,x_2)$ is the jpdf of the random variables X_1, X_2, $f_{X_1|X_2}(x_1|x_2)$ is the conditional pdf of X_2, given X_1, and $f_{X_2}(x_2)$ is the marginal pdf of X_2.

If $f_{\vec{X}}$ denotes the joint probability density function (jpdf) of all random variables involved, i.e. of the vector $\vec{X} = (X_1, X_2, \dots, X_n)$, the probability of failure can be calculated via the following integral (US Army, 2003):

$$P_f = \int_{g(\vec{x})<0} f_{\vec{X}}(\vec{x}) d\vec{x} \quad [5]$$

The MCM is based on a large number of simulations N, a part of which (N_f) leads to the element's failure. Thus, it is assumed that provided N is high enough, P_f attains acceptable convergence and is computed as:

$$P_f = \frac{N_f}{N} \quad [6]$$

One way (e.g. inverse cumulative distribution function (CDF) method) to generate the random sample for each variable is described here. Specifically, each simulation starts by drawing a random number from a uniform probability density function $U(0,1)$. Through those numbers (x_{u_i}), a sample of the variable (X) can be produced by the latter's inverse cumulative distribution function (cdf) $F_X^{-1}(x)$ as follows:

$$X_i = F_X^{-1}(x_{u_i}), \quad i = 1,2,3, \dots, N \quad [7]$$

An element's failure probability during the structure's lifetime is estimated in the present paper by assuming independent failures from year to year, and by assuming stationary wave conditions among years (Malliouri et al., 2021). Following the above assumptions, the equations displayed below are applied:

$$P_{f,L,e} = 1 - \left(1 - P_{f,1yr,e}\right)^L \quad [8]$$

where:

$$P_{f,1yr,e} = \int_{g(\vec{x})<0} f_{\vec{X}}(\vec{x}) d\vec{x} \quad [9]$$

where $P_{f,L,e}$ is the estimated element's failure probability during the structure's lifetime L, and $P_{f,1yr,e}$ is the mean annual element's failure probability estimated in the sample of storm events. It is noted that $P_{f,L,e}$ is the union of the failure probabilities of all years in L, whereby the failure probability of each year is equal to $P_{f,1yr,e}$, assuming stationary wave conditions among years.

Furthermore, if we use only the information on the mean annual exceedance probability of one single design storm event, e.g. the maximum significant wave height of the design storm, as is usually the case of the deterministic design, then $P_{f,L,e}$ could be assessed as follows (U.S Army, 2003):

$$P_{f,L,e} = 1 - \left(1 - \frac{1}{\lambda_e T_r}\right)^{\lambda_e L} \quad [10]$$

where λ_e is the ratio of the number of independent storm events observed in Y years of observations towards Y, and T_r is the return period of the maximum significant wave height of the design storm. Eq. [10] implies that every time an extreme event occurs, the probability that this event may cause the element's failure is equal to the mean annual exceedance probability of the maximum significant wave height of the design storm, $\frac{1}{\lambda_e T_r}$. Thus, Eq. [10] assumes independent failures and equal failure probabilities among events.

Referring here to the reliability requirements according to PIANC (1992), the acceptable probability of failure $P_{f,L,e}$ can be equal to 0.01, 0.05, 0.10, 0.20, or 0.40, depending on the damage consequences of the structure.

2.3 Fully Probabilistic Resilience Assessment Based on Damage Evolution Simulation

In this section, a resilience analysis is described by using the armor's damage evolution model by Melby and Kobayashi (1998), estimating the expected year that maintenance of the breakwater should take place. In this approach, the expected cumulative damage $S_c(t)$ of the breakwater's armor layer at the end of a storm is a function of the storms' characteristics and its corresponding damage $S_c(t_n)$ at the end of a previous storm:

$$S_c(t) = S_c(t_n) + a_p \frac{(N_{mo})_n^5}{\left(T_p\right)_n^b}(t^b - t_n^b), \quad for\, t_n \le t \le t_{n+1} \qquad [11]$$

where a_p and b are empirical coefficients, which are functions of the structure slope, wave period, beach slope, structure permeability and armor gradation, and $N_{mo} = \frac{H_s}{\Delta D_{n50}}$, where H_s is the significant wave height, T_p is the spectral peak wave period, $\Delta = S_r - 1$, where S_r is the specific gravity.

The input data for Eq. [11] and the procedure for the estimation of the failure probability of the structure's armor layer, considering the cumulative damage of the armor in sequences of storms, are presented in Figure 1. Specifically, the $S_c(t)$ is calculated via Eq. [11] considering M different sequences of storms based on the joint probability distribution of duration, the maximum Hs of each storm (maxHs), and Tp. It is noteworthy that in each storms' sample, storms generated from the jpdf of duration, maxHs, and Tp by means of Eq. [6] are set to random order. Besides, the number of storm events per year is equal to the mean rate of storms per year, λ_e, as extracted from the hindcast data. For each timeseries, the cumulative damage of the element is calculated as a function of time. The number M of the sequences of storms is considered sufficient large in order that the role of the ordering of storm events to the ultimate damage level be properly investigated. The result of this procedure, which resembles to a MCM, is $Sc(t)$ and failure probability of the element $P_f(t)$ as a function of t, e.g. years, estimated as follows:

$$P_f(t) = \frac{n(t)_{Sc(t)>Sd}}{M} \qquad [12]$$

where $n(t)_{Sc(t)>Sd}$ is the number of storms' sequences where $Sc(t)$ of the element at time t is larger than the total damage level Sd towards the number of storms' sequences M. This methodology has been also applied by Sousa and Santos (2006), and Lira-Loarca et al. (2020), but here is presented as a MCM.

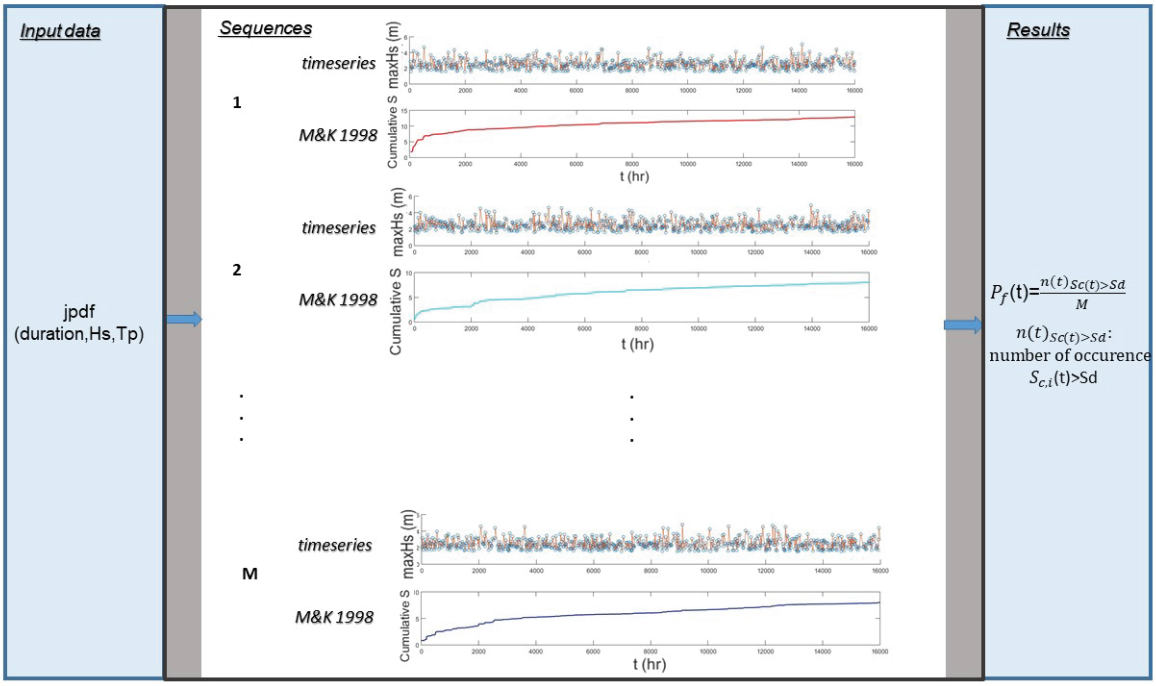

Figure 1. Flowchart of the MCM procedure adopted for the estimation of the failure probability of the structure's armor layer considering its cumulative damage.

3 APPLICATION EXAMPLE

3.1 Case Study Description

Data of wave recordings from the buoy of Barcelona are taken as a case study. The data are provided by the Puertos del Estado (www.puertos.es) and were obtained at a location where the sea depth is 68 m and the distance from the coast is almost 2500 m. The specific buoy is selected among others in the Mediterranean Sea based on the longest dataset during its operation, covering the period of 2004 to 2021 with a sampling interval of 1 h.

An ideal rubble mound breakwater with rock armor units is considered in the present paper, located at a water depth equal to 20 m in the coast of Barcelona (Spain), with slope 1:2, rock density equal to 2650 kg/m3, and water density 1025 kg/m3. The working lifetime of the coastal structure is supposed to be 50 years, which is a common lifetime for coastal structures of this type.

3.2 Derivation of Storms at the Buoy Location

Following the methodology described in section 2.1, 313 storm events are identified (see Figure 2). It is noted that the Barcelona buoy gives also directional information. However, for reasons of simplicity, and due to the fact that wave data show relatively low variability with respect to mean wave direction, a unidirectional analysis has been applied in the present study, i.e. by considering normal incidence for all storms attacking the structure.

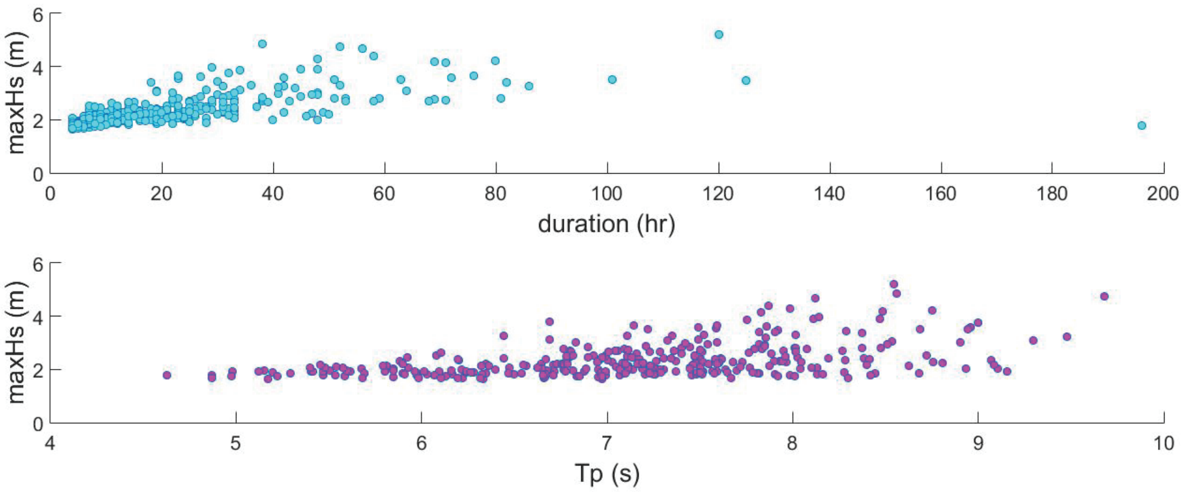

Figure 2. Scatter diagram of storms' duration and maximum Hs of each storm (upper) and the scatter diagram of Tp and maximum Hs at the buoy location.

3.3 Probabilistic Representation of Storms at the Coastal Structure's Location

The model selected in the present paper to transfer storms from deeper waters to the structure's location is the one developed by Malliouri et al. (2019). This is a statistical wave propagation model that uses the long-term wave statistics in deep waters as input data and by using the short-term wave statistics for each sea state or storm event in deep waters, it estimates the long-term wave statistics in shallower waters. Specifically, the short-term joint distribution of individual wave height (H), period (T), and direction (θ) for every sea state or storm event is produced in deep waters by using: i) data/measurements of significant wave height (Hs), mean wave period (Tm), and mean wave direction (θm) in deep waters, ii) the dimensionless short-term jpdf of T/Tm and H/Hm by Longuet-Higgins (1983) in deep waters, and a theoretical expression for wave directionality adjusted in an individual wave statistical analysis by Malliouri et al. (2019). In Figure 3, the scatter diagrams of duration and maximum Hs (maxHs) of each storm events at the buoy's and at the structure's location are presented, while in Figure 4, the multivariate kernel jpdf of Tp and maxHs and the scatter diagram of the transferred data are indicatively depicted for two duration intervals

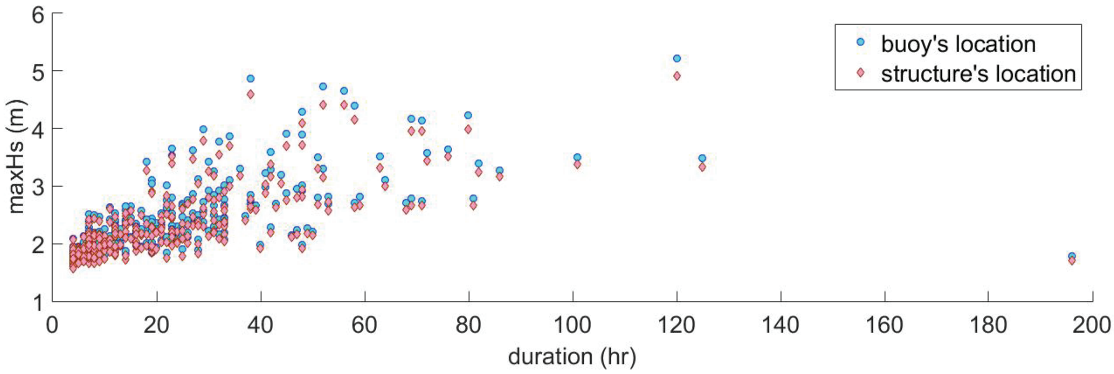

Figure 3. Scatter diagrams of duration and maximum Hs (maxHs) of each storm event at the buoy's location (depth of 68 m) and at the structure's location (depth of 20 m).

It is noted that, since the purpose of the present analysis is to estimate a theoretical jpdf of maxHs, Tp, and duration that fits well the transferred data at the structure's location, and since a stationary statistical analysis will be performed, a non-parametric joint probability distribution is chosen here to represent the data, i.e. a multivariate kernel one. This joint distribution has been selected to be applied after checking that this jpdf of maxHs, Tp, and duration represents the data better than other theoretical parametric distributions, e.g. the Generalized Pareto, Weibull, Gumbel for maxHs, lognormal for Tp.

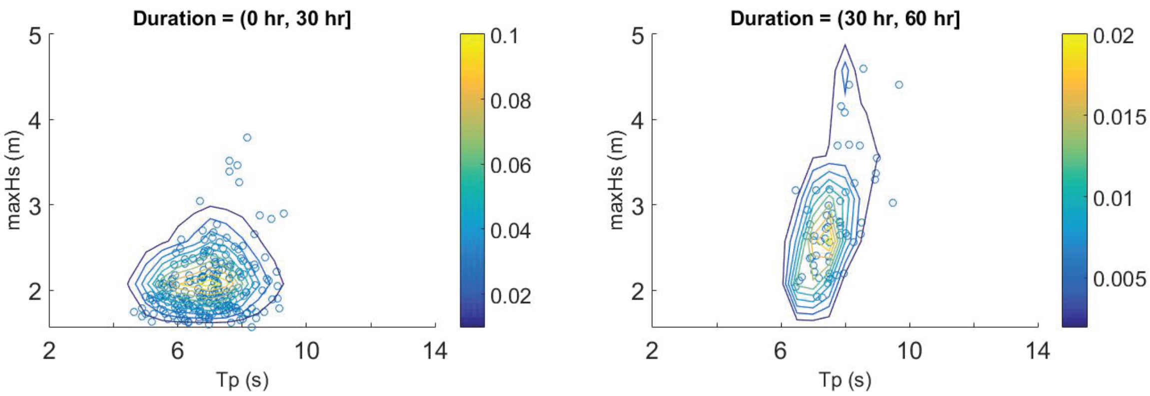

Figure 4. Kernel jpdf of Tp and maxHs and the scatter diagram of the transferred data for the duration interval of (0 hr, 30 hr] (left) and of (30 hr, 60 hr] (right).

3.4 Fully Probabilistic Reliability Assessment with No Consideration of Damage Evolution

In this section, a fully probabilistic reliability assessment is performed of the ideal rubble mound breakwater under design, with no consideration of damage progression among storms. For this purpose, Eq. 8 and 9 are utilized, whereby the limit state function is formed based on Van der Meer (1988) formulas and Melby and Kobayashi (1998) formula. The latter has also been based on the formulas by Van der Meer (1988). The formula by Melby and Kobayashi can be rewritten to apply for this case, i.e. assuming zero damage level of the armor layer before the structure is exposed to each storm, as below:

$$S(t) = a_p \frac{(N_{mo})_n^5}{\left(T_p\right)_n^b} (t^b) \qquad [13]$$

where t here is the storm duration. As expected, the annual failure probabilities of the armor layer as a function of the median mass of rock armor units, estimated via Van der Meer (1988) and Melby and Kobayashi (1998) formulas, are close to each other. However, since the first one is smoother, we have chosen $P_{f,1yr,e}$ estimated by Van der Meer to be used in Eq. [9] that calculates $P_{f,1yr,e}$.

In Figure 5, the annual failure probability of the armor layer, estimated via Van der Meer (1988) and Melby and Kobayashi (1998) formulas and Eq. [9], and the failure probability in the lifetime of the structure estimated using Van der Meer formula and Eq. [8], as a function of the median mass of rock armor units are presented. The failure probability is considered here to be the probability of the damage level exceeding the total failure level, which is equal to 12 according to Melby and Kobayashi (1998). As observed in Figure 5, if the reliability requirement determined a failure probability of the structure during its lifetime equal to 0.2, then the associated median mass M50 of the armor units to this failure probability should at least equal to 4500 kg.

Thus, the resilience analysis that follows in the next paragraph corresponds to M50 being equal to 4500 kg, as derived from the reliability analysis and the supposed reliability requirement.

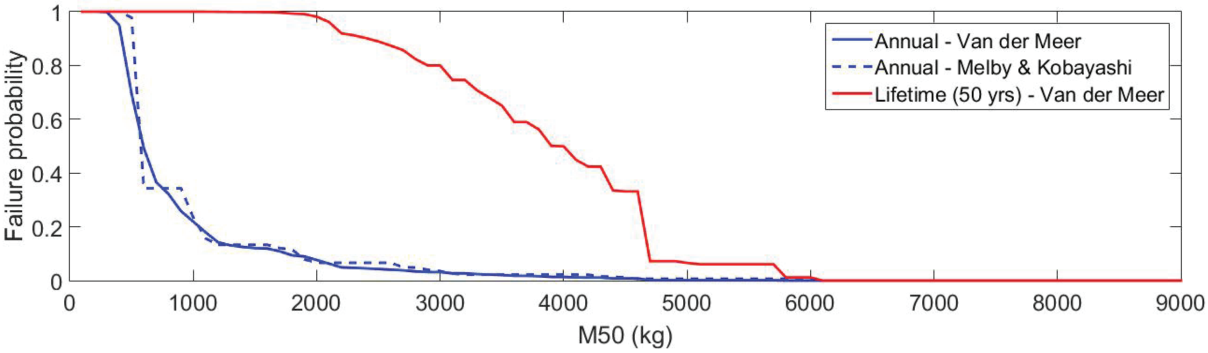

Figure 5. Annual failure probability of the armor layer, estimated via Van der Meer (1988) (solid blue line) and Melby and Kobayashi (1998) formulas (dotted blue line), and the failure probability in the lifetime (50 yrs) (red solid line) of the structure estimated via Van der Meer formula as a function of the median mass of rock armor units.

3.5 Fully Probabilistic Resilience Assessment Based on Damage Evolution Simulation

The cumulative damage of the armor layer is calculated via the methodology presented in Figure 1 considering M (=200) different sequences of storms (see Figure 6). Based on the empirical results by Melby and Kobayashi (1998), and following Lira-Loarca et al. (2020), the admissible level of failure (S_{adm}) and that of total failure (S_d: destruction level) are defined as 6 and 12, respectively. Hence, in Figure 7, the probability that the admissible and total failure level is exceeded among years is presented, as well as the expected year of the armor's maintenance. The latter is defined here as the year when the exceedance probability of the admissible damage level is very high, e.g. equal to 0.95, following Lira-Loarca et al. (2020).

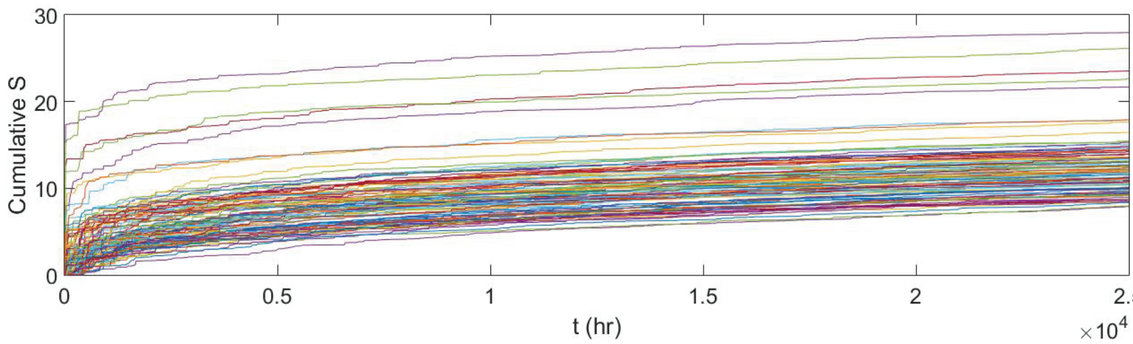

Figure 6. Cumulative damage level of the armor layer in M different sequences of storms (for M50=4500 kg).

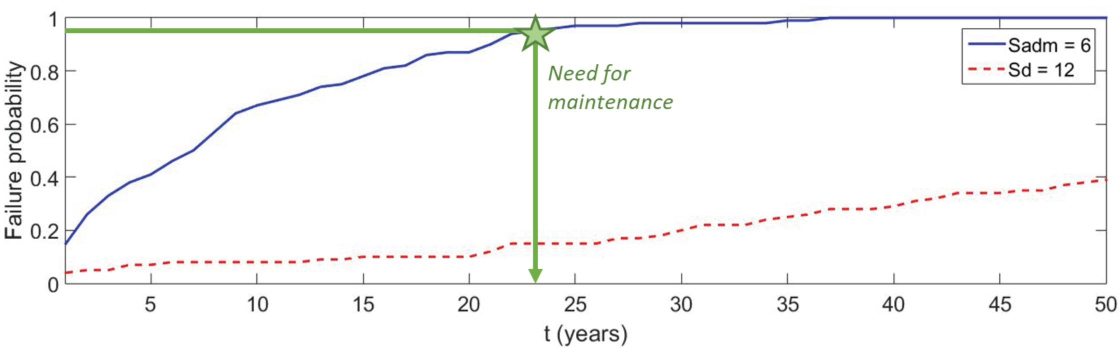

Figure 7. Probability of the admissible level (solid line) and total failure level (dotted line) being exceeded among years, and the expected year of maintenance (for M50=4500 kg).

3.6 Relating between Reliability and Resilience Analysis

In this section, the reliability and resilience analyses are related to each other. In Table 1, the results of the two approaches are displayed, as well as the expected year of repair. The results are presented for seven different cases regarding M50, although some of them might not be selected for the armor design in any cases, e.g. the case of M50 equal to 3000 kg. As seen in Table 1, by increasing M50, all failure probabilities are being decreased and, the expected year of repair is increased. Besides, the total failure probability Pf,L (Sc>12) derived from the resilience analysis is higher that the corresponding one Pf,L (S>12) derived from the reliability analysis for all values of M50 examined, although the two probabilities are of different kind. From all the above, it can be concluded that due to their inherent differences and assumptions, different requirements and acceptable failure probabilities should be used for the two probabilistic approaches.

Table 1. Results by the reliability analysis and resilience analysis for the ideal breakwater (L= 50 years).

	Reliability		**Resilience**	
M50 (kg)	Pf,L (S>12)	Pf,L (Sc>6)	Pf,L (Sc>12)	Year of Repair
3000	0.70	1.00	1.00	4
3500	0.58	1.00	1.00	7
4000	0.49	1.00	0.79	13
4500	0.20	1.00	0.40	23
5000	0.09	1.00	0.16	39
5500	0.07	0.83	0.10	72
6000	0.01	0.52	0.03	105

4 CONCLUSIONS

In summary, in the present paper, the fully probabilistic reliability and resilience analysis of rubble mound breakwaters, are combined to highlight the significance of both of them in the design and repair of those structures. In particular, reliability analysis assesses the failure probability of the structure's element in its lifetime considering no cumulative damage from storm to storm. On the other hand, resilience analysis assesses the failure probability in the samples of different possible sequences of storms considering cumulative damage among storms. Therefore, due to their inherent differences and assumptions, different requirements and acceptable failure probabilities should be used for the two probabilistic approaches. Conclusively, the present paper provides tools that can be used efficiently to check different design solutions and maintenance strategies for coastal structures and improve decision-making by coastal engineers.

5 ACKNOWLEDGEMENTS

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