Iterative learning control - moving from theory to applications

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Iterative Learning Control — Moving from Theory to Applications

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***Abstract*— Iterative learning control applies to systems performing the same finite duration task repeatedly. Each execution is known as a trial, and the finite duration of each trial is known as the trial length. A reference trajectory is specified, and then the design can proceed based on minimizing the error sequence with entries formed as the difference between the output on a trial and the reference vector. The starting point for ILC is widely accepted as research in the mid-1980s in the robotics area. Since then, ILC has remained an active area of research, starting with the underlying theory and proceeding through design algorithm development and onward to applications. Applications for ILC range across engineering and, more recently, healthcare. This mini-symposium gives an overview of this area, focusing on research conducted by the Southampton group and international partners.**

I. Analysis and Design

Iterative learning control (ILC) has been specially developed for the many systems that complete the same finite duration, termed a trial (or pass in some literature) task repeatedly, with resetting to the starting location once each pass is complete. The distinguishing feature of ILC is using information from previous trials to construct the input for the subsequent trial, including temporal information that would be non-causal in standard control systems.

The first research on ILC is widely credited to [1] and a significant application area is industrial robots executing a pick and place operation to, e.g., place a sequence of objects on a moving conveyor. Once each is placed, the robot returns to the starting location. All data generated on the completed trial is available to compute the control input for the subsequent trial. Since this work, ILC has been an established area of research and applications, including many outside robotics, where the survey papers [2], [3] and the monograph [4] are starting points for the literature.

Suppose also that a reference trajectory (or profile) has been specified, where, in the case of a pick and place robot, this would be the desired path to transfer the payload from the pick to the place location. Then the error on any trial is constructed as the difference between the reference trajectory and the output on that trial. The design problem is to find a control sequence that forces this error to converge with increasing trial number, either to zero or some acceptable tolerance.

Of course, the question of what form of control action to design is as fundamental to this area as any other. Given the structure of the underlying dynamics, one option is to form the control law for the subsequent trial as the sum of that used on the previous trial plus a correction term. Moreover, once a trial is complete, all information from it is, at the cost of storage, available for use in constructing the following trial input, e.g., at sample p on the current trial information from the previous trial at $p+\lambda$ where $\lambda \neq 0$ can be used. This feature, which is not possible in standard designs, is unique to ILC, and if it is not present, then an equivalent feedback controller exists, and ILC has no added value.

Consider discrete linear dynamics. Then since the trial length is finite, the values of a variable along a trial, e.g., the control input, can be assembled into a finite-dimensional column vector. Doing this for all variables results in a standard linear difference equation describing the updating of the error dynamics from trial to trial. Hence standard discrete linear systems theory can be used for analysis and design. This approach is often termed lifted ILC design. Again see [2]–[4] for the background.

Given that the trial length is finite, trial-to-trial error convergence will occur even if the state matrix is unstable. Moreover, a system with stable but lightly damped poles could have unacceptable dynamics along the trial. In the lifted design setting, the route is first to design a stabilizing feedback control loop and then apply ILC to the resulting dynamics, resulting in a two-step design procedure.

It is possible to formulate an ILC design in a 2D systems setting, i.e., systems where information is in two independent directions, i.e., from trial-to-trial, where the trial number is denoted by the nonnegative integer k and along the trial, with p denoting the sample number. Hence such systems evolve over $(k,p) \in [\infty,0] \times [0,\infty]$, but for ILC the variable p is finite and hence the dynamics evolve over $(k,p) \in [\infty,0] \times [0,\alpha]$. Options for the analysis and design of ILC laws in this setting include using the 2D Roesser and Fornasini-Marchesini state-space models (the original references for these models are given in [5]).

Repetitive processes make a series of sweeps, termed passes in the literature, through a set of dynamics defined over a finite duration known as the pass length. The output on each pass, termed the pass profile, acts as a forcing function and contributes to the dynamics produced on the next pass. As a result, oscillations that increase in amplitude from pass-to-pass can arise, and these cannot be regulated by standard control action. A stability theory and associated tests for this property are required to provide a setting for control law design.

A stability theory for linear repetitive processes has been developed [5], and given the finite pass length, these processes are a more natural match to ILC dynamics. In the literature on ILC design, the word pass is replaced by

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trial in a repetitive process setting. For linear dynamics, this stability theory has been used as a starting point for developing designs that have seen experimental validation, see, e.g., [6]. Compared to the lifted setting, this is a one-stage design. Hence, the possibility of a trade-off between regulation of the dynamics along the trials versus trial-to-trial error convergence. Also, a natural extension allows design for differential dynamics along with the trial, i.e., design by emulation as per standard linear systems.

In the case where ILC design must be based on a nonlinear model of the dynamics, there have been results on the use of feedback linearization. Also, a stability theory for nonlinear repetitive processes has begun to emerge, see, e.g., [7]. For discrete dynamics, the Newton method, where the first step is to rewrite the nonlinear dynamics as a set of static nonlinear multivariable equations provides a strong setting for analysis and design, as highlighted in the next section.

II. Applications

Over the past three decades, there has been much research reported on experimental validation using laboratory facilities across a diverse range of applications from robotics, manufacturing, and free-electron lasers—one starting point for the literature is [8]. Also, there has been substantial progress in the use of ILC in healthcare applications. The starting point for this area is robotic-assisted stroke rehabilitation. People who suffer a stroke lose functionality down one side of their body, which impacts daily living, e.g., the ability to reach out across a tabletop to an object or reach up to close an open drawer.

The recommended method for relearning lost functionality is repeated attempts at a task, where learning from the previous (or several previous) attempts is employed. The application of precisely controlled electrical stimulation enables muscle movement, and hence relearning is possible. However, stroke patients most often cannot move the affected limb, so no learning occurs.

The role of ILC is to adjust the stimulation applied on the next attempt, using information from previous attempts. Consult [9] for the control engineering development in the first case considered, reaching out in the plane, and [10] reporting the clinical trial results in the healthcare literature. Since this initial research, there has been substantial progress in the tasks considered, e.g., reaching out and lifting the arm or reaching out and opening the hand to grasp an object, together with developments to enable take-home technology. The ultimate aim would be that the patient practices at home and the results are logged and transmitted to a healthcare professional for evaluation — currently, patients have to attend hospitals and receive one-to-one attention. Consult [8] for coverage of all of this research, including Newton method ILC design with supporting clinical trial results and references to other (more recent) applications of ILC in healthcare.

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