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ORIGINAL ARTICLE

Optimising curve fitting techniques to look for standardisation of the analysis of defocus curves derived from multifocal intraocular lenses

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Abstract

Introduction: To establish the most appropriate curve fitting method to allow accurate comparison of defocus curves derived from intraocular lenses (IOLs).

Methods: Defocus curves were plotted in five IOL groups (monofocal, extended depth of focus, refractive bifocal, diffractive bifocal and trifocal). Polynomial curves from 2nd to 11th order and cubic splines were fitted. Goodness of fit (GOF) was assessed using five methods: least squares, coefficient of determination (R^2_{adj}), Akaike information criteria (AIC), visual inspection and Snedecor and Cochran. Additional defocus steps at −2.25 D and −2.75 D were measured and compared to the calculated visual acuity (VA) values. Area under the defocus curve and range of focus were also compared.

Results: Goodness of fit demonstrated variable results, with more lenient methods such as R^2_{adj} leading to overfitting and conservative methods such as AIC resulting in underfitting. Furthermore, conservative methods diminished the inflection points resulting in an underestimation of VA. Polynomial of at least 8th order was required for comparison of area methods, but overfitted the EDoF and monofocal groups; the spline curve was consistent for all IOLs and methods.

Conclusions: This study demonstrates the inherent difficulty of selecting a single polynomial function. The R^2 method can be used cautiously along with visual inspection to guard against overfitting. Spline curves are suitable for all IOLs, guarding against the issues of overfitting. Therefore, for analysis of the defocus profile of IOLs, the fitting of a spline curves is advocated and should be used wherever possible.

KEYWORDS
algorithms, cataract extraction, intraocular lenses

INTRODUCTION

Multifocal intraocular lenses (MIOLs) generate multiple focal points within the eye, extending the range of clear vision attained following cataract surgery. As MIOLs differ in both optical design and addition power, the achievable range of clear vision may differ post-implantation, resulting in varying levels of patient satisfaction. The preferred reading distance and lifestyle choices are important considerations in presbyopia correction to maximise patient satisfaction.[1] As such, clinicians must fully understand the optical characteristics of individual MIOLs to counsel patients considering implantation appropriately, and for informing the selection of the MIOL best suited to the patient's visual requirements.

The ability to compare directly the optical performance of different MIOLs can be challenging. A MIOL with a reading addition of +2.50 D in the spectacle plane will perform optimally if near visual acuity (VA) is assessed at 40 cm. However, this testing distance would be less favourable for a + 3.50 D addition power; hence, assessment of VA at varying distances is required. However, measuring VA at a range of distances is largely impractical due to the difficulties that arise in controlling target illuminance and angular size.[2] Therefore, defocus curves are often plotted to assess the functionality of a MIOL and its ability to provide clear vision over a range of distances.[3,4]

There are a diverse array of metrics that can be used to analyse defocus curves that may be categorised as either depth of focus or area of focus.[3,5–7] The relative and absolute depth of focus methods[5–7] specify the dioptric range over which participants can maintain a designated level of VA. With the relative depth of focus criterion, the cut-off VA is proportional to the best corrected VA, whereas the more commonly used absolute depth of focus method involves a VA cut-off, independent of best corrected VA. The level of acuity used is generally arbitrary, and a value of 0.30 LogMAR is often used as a cut-off value for MIOLs, as this is the visual driving standard in many countries.[8,9] The American Academy of Ophthalmology Task Force proposed the use of a more stringent value of 0.20 LogMAR as the cut-off value when evaluating Extended Depth of Focus (EDoF) IOLs.[10] Lapid-Gortzak and colleagues proposed a criteria where the cut-off value varies according to the level of defocus: The metric is designed to better represent the fluctuating visual demands that occur in the real world.[11]

The area of focus (AOF) metric proposed by Buckhurst et al.,[3] advocates dividing the defocus curve into three sections: Distance +0.50 D to −0.50 D, Intermediate −0.50 D to −2.00 D and Near −2.00 D to −4.00 D. This method considers the actual VA within the range, not just whether VA is better or worse than set criteria. In comparison with the relative and absolute depth of focus methods, this technique is able to differentiate between MIOL designs.[3]

Irrespective of the differences in how these metrics are calculated, they are all dependent on either optimal curve fitting or piecewise linear function fitting to the defocus curve data. However, no prior study has examined the optimal method for fitting a function to this form of data, and the majority of studies fail to report the type of curve fitted. Indeed, the American Academy of Ophthalmology Task Force consensus statement on EDoFs does not discuss the use of function fitting.[12] When comparing the few studies that reported this information, there are significant discrepancies. For example, Gupta[5] used 5th to 10th order polynomials, whilst Buckhurst[3] applied a 9th order polynomial and Wolffsohn[13] fitted a spline curve to the defocus curve data. Furthermore, Gupta[5] and Buckhurst[3] differed in their statistical assessment of goodness of fit (GOF). The former chose the polynomial order based on both the highest possible regression coefficient (R^2) achievable and visual inspection, whereas the latter selected a 9th order polynomial depending on when no noticeable further improvement to R^2 nor further decrease in standard error occured.[3,5] By their nature, adding a higher order term will always increase R^2, but an increase in R^2 is only relevant if it is significantly greater than that expected due to chance.[14] Polynomials are simple mathematical expressions that are clinically accessible given the ease with which they can be solved and integrated to facilitate the generation of the depth of focus and area of focus metric values. In comparison, a spline curve is guaranteed to pass through all data points, but requires more complex mathematical modelling to generate the desired metric values.

Key points

- Polynomials have previously been advocated for the fitting of curves to a defocus curve data set, but there is no standardisation over what statistical method to use to determine the goodness of fit.
- The results demonstrated that no single polynomial can be selected that will sufficiently fit multifocal, extended depth of focus and monofocal intraocular lenses.
- For best practice, a spline curve should be fitted wherever possible.

Alternatively, a piecewise linear function can be fitted to the defocus curve data which, like a spline curve, is guaranteed to connect to all data points on the curve. This also allows the simple calculation of area using the trapezoidal rule.[15]

Curvilinear regression (fitting a curve) finds a mathematical expression that produces a curved line to be the closest or exact fit to the measured data points, when the relationship between the variables is non-linear.[14] The validity of this curvilinear regression must be assessed statistically, and there are five main models that can be used to assess GOF. The aim of the present study is to use these models to determine if a single polynomial function can be utilised to fit a variety of MIOL and EDoF defocus curves to an equivalent standard as a spline curve.

METHODS

Participants and measurements

This retrospective cohort study recruited participants who had undergone bilateral phacoemulsification and IOL implantation either for cataract or elective lens replacement with one of five different IOLs (Table 1). All patients included had no other ocular pathology or corneal astigmatism

TABLE 1 Intraocular lens (IOL) characteristics

	Bi-flex 677AB	Bi-flex MY	Mplus	Symfony	AT LISA 839
Design	Monofocal	Diffractive bifocal	Refractive bifocal	EDoF	Diffractive trifocal
Asphericity (µm)	0.00	0.00	0.00	−0.27	−0.18
Add Power at the IOL plane	N/A	+3.50 D	+3.00 D	N/A	+3.33 D Near +1.66 intermediate

≥1.50 D. All participants gave informed consent, and the study adhered to the tenets of the Declaration of Helsinki and received ethical approval from the NHS South West Ethics Board and the University of Plymouth Faculty of Health Ethics Committee.

A total of 126 participants were recruited (Table 2). The five groups of patients had been bilaterally implanted with either the Bi-Flex 677AB (Medicontur Medical Engineering, medicontur.com) monofocal IOL (n = 28); Bi-Flex MY (Medicontur Medical Engineering, medicontour.com) diffractive bifocal IOL (n = 30); Mplus (Oculentis, oculentis.com) refractive bifocal (n = 25); Tecnis Symfony (Johnson & Johnson Vision, jjvision.com) extended depth of focus IOL (n = 18) or the AT LISA 839MP (Carl Zeiss, zeiss.com) diffractive trifocal IOL (n = 25). There was a significant difference in ages between the five groups ($F_5 = 15.49$, $p < 0.01$; Table 2).

All surgeries were performed by one of two experienced consultant ophthalmic surgeons (RA and HK) using small incision phacoemulsification. The same surgeon implanted both lenses for an individual subject. In each case, a 2.2 mm clear corneal incision was located according to the steepest corneal meridian. The pre- and post-operative medication regime was the same regardless of the surgeon.

At 3–6 months post-operatively, defocus curves were plotted for each subject binocularly, with defocus ranging from +1.50 D to −5.00 D in 0.50 D steps.[13] Prior to defocus curve assessment, a combination of objective and subjective techniques were used to determine the best distance correction. Retinoscopy was conducted using the Keeler Professional retinoscope (Keeler, keeler.co.uk). Standard subjective refraction was performed using the Thomson Test Chart 2000 (Thomson Software Solutions, thomson-software-solutions.com). The same chart was also used for assessing the defocus curve at 6 m. All measures were performed at a photopic illuminance of 120 cd/m^2 (luminance meter LS-150; Konica Minolta, konicaminolta.com) and luminance of 95 lux (Light meter LX1010BS; Dr Meter, drmeter.com). The letters and defocus lenses were randomised between measures and participants were prompted once by the phrase, 'Can you read any more letters on the line below?'[16]

Statistical analysis

Results were tested for normality using the Kolmogorov–Smirnov test. All defocus data were corrected for spectacle magnification (SM), assuming a thin lens calculation with back vertex distance (BVD) of 12 mm, that is

$$\mathrm{SM} = \frac{\text{Lens Power}}{(\text{BVD} \times \text{Lens Power})} \tag{1}$$

Defocus curves were plotted and both polynomial (2nd to 11th order) [MATLAB function Polyfit] and cubic spline curves [MATLAB function csapi] were fitted using MATLAB R2017b (MathWorks, mathworks.com).

The following five models for assessing GOF were then used to analyse each polynomial function: least squares; coefficient of determination (R^2); Akaike Information Criterion (AIC); Snedecor and Cochran method and visual inspection.

Least squares

The least squares method of curve fitting finds the curve which best represents a data set, such that the sum of square of the vertical distance from each data point to the line is a minimum.[17] This provides an F statistic, a ratio of the variance in the dependent variable as a function of the independent variable and the residual deviation from the curve where:

$$F = \frac{\text{Mean Square (model)}}{\text{Mean Square (residual)}} \tag{2}$$

A large F statistic, suggests a curve with a good fit to the data.

Coefficient of determination (R^2)

The coefficient of determination is the ratio of variance explained by the model (curve) to the total variance.[18,19] It considers the proportion of the variance in the dependent variable that is predictable from the independent variable. An R^2 equal to 1 suggests that the curve fits the data points perfectly. The coefficient of determination will always increase as more variables are added to a model and as such overfitting can occur, with a deceptively high R^2 being achieved.[19] As such, in this study the adjusted R^2 (R^2_{adj}) was used as it includes an additional calculation to adjust for the number of variables included in the curve fitting. Therefore, the R^2_{adj} will only increase if the rise in R^2 following the addition of an extra term is more than can be explained by chance.

TABLE 2 Demographics

	Bi-flex 677AB Monofocal	Bi-flex MY bifocal	Mplus	Symfony	AT LISA trifocal
Age (years) ± standard deviation	77.68 ± 5 20	76.53 ± 6.75	62.57 ± 8.24	62.67 ± 12.98	67.76 ± 7.50
Male/Female	10/18	6/24	9/16	7/11	7/18

TABLE 3 Comparison of curve fitting methods

	Least squares	R^2_{adj}	AIC	Snedecor and Cochran	Visual inspection
Bi-Flex 677AB	7th	5th	3rd	2nd	3rd
Bi-Flex MY	9th	8th	4th	2nd	7th
Mplus	9th	8th	2nd	2nd	7th
Symfony	10th	4th	2nd	2nd	3rd
AT LISA 839	11th	7th	2nd	2nd	7th

Abbreviation: AIC, Akaike Information Criterion value.

Akaike information criterion

Akaike Information Criterion (AIC) provides an estimate of the relative quality of statistical models for a given dataset. When various models are used, AIC estimates the quality of each model relative to the other by estimating the relative information lost by a given model[20]; the less information lost, the higher the quality. In order to estimate information lost, AIC assesses the trade-off between GOF and the simplicity of the model.

$$\mathrm{AIC} = 2\mathrm{K} - 2\ln(\hat{}\mathrm{L}) \tag{3}$$

where K = number of parameters, ln = natural logarithm and ^L = maximum value of the likelihood function; thus the lower the AIC, the better the model. When the sample size is small, as in this study, a correction factor is employed to give an AICc value.

Snedecor and Cochran method

Snedecor and Cochran[21] described a method to analyse curve fitting to minimise the risk of overfitting. It aims to find the minimum order (least complex equation) that can be fitted that gives a significant improvement to the sum of squares of the regression. An *F* statistic is calculated by assessing the change in the sum of squares between the higher order curve and the previous order, to see if the increased order has provided a significant improvement.

Visual inspection

All plotted curves were assessed visually and a determination of the best fit was made by the same observer. A fit was considered poor if certain issues were observed, for example 'overfitting' if there were additional inflection points observed at the extremes (Runge phenomena),[22] or 'underfitting' if there was no inflection observed in the intermediate section of the curve (defocus −0.50 to −2.00 D) as expected.

Visual acuity measurements for −2.25 D and − 2.75 D of defocus were obtained in the monofocal and bifocal group for the purpose of validation; these additional points were excluded from the initial curve fitting process. The fitted curves were used to interpolate the y value (VA) when χ (defocus) equalled −2.25 D and −2.75 D, and the results of these predictions were compared to the actual measured values. As part of this validation exercise, the same software was used to calculate the area under the curve and range of focus assuming a ceiling of $y = 0.30$ LogMAR using previously published methodology.[3]

Repeated measures analysis of variance (ANOVA) and a post-hoc Bonferroni test were used to compare the means for validation data points, and a two-way repeated measures ANOVA was used for area distance, area intermediate, area near, total range of focus and actual range of focus, with the level of significance set at $p < 0.05$. Pearson's correlation coefficients assessed the relationship between the actual and predicted metrics, whilst Bland and Altman analysis was used to test the limits of agreement.[23]

RESULTS

Table 3 outlines the best fit polynomial as determined by each of the statistical methods. Least squares and R^2_{adj} advocate higher order polynomial curve fitting, whereas AICc and Snedecor and Cochran methods are conservative and suggest lower order polynomials are sufficient to fit all the IOLs.

Only the Snedecor and Cochran method was consistent across all IOLs, which is likely due to its conservative nature. However, from visual inspection it was clear that this constituted underfitting in the MIOLs and important information in the intermediate and near sections of the defocus profile was lost. Visual inspection of the curves fitted for the bifocal/trifocal IOLs suggested that higher order curves were required to avoid missing the intermediate and near inflection points of the defocus profile in MIOLs, but lower order curves would suffice in the monofocal and EDoF.

The least squares technique indicated a minimum of a 7th order polynomial was required to fit the monofocal IOL group, and even higher orders were required for the remaining IOLs, despite all other methods suggesting the use of lower order polynomials. It appeared that the least squares method is the least conservative and most likely to lead to overfitting.

As there was no agreement on the most suitable polynomial between methods, nor between IOLs, we were unable to establish a definitive polynomial order that was appropriate to all IOLs included in the present study.

The validation analysis examined 58 participants (30 Bi-Flex MY and 28 Bi-Flex 677 AB) and compared the VA for −2.25 D and − 2.75 D of defocus to the VA interpolated from the curves fitted to the defocus data. Comparison of the mean VA by repeated measures ANOVA revealed significant differences in the bifocal IOL group for both −2.25 D (F_{10} = 13.65, $p < 0.01$) and − 2.75 D of defocus (F_{10} = 55.56, $p < 0.001$). Post-hoc pairwise comparison demonstrated significant differences between the actual VA and those generated with the 2nd to 8th order polynomials when defocus was −2.25 D and 2nd to 6th order for defocus of −2.75 D (Figure 1a,b). In the monofocal IOL group, a significant difference (Figure 2a) was found with −2.25 D defocus (F_8 = 9.15, $p < 0.01$); post-hoc testing found a pairwise significant difference only when using a 2nd order polynomial. In comparison, with −2.75 D of defocus the differences were not significant (F_8 = 1.95, p = 0.05) (Figure 2b). Amongst the bifocal IOL group, using a polynomial of insufficient order led to an underestimation of the VA. It was not possible to fit 10th and 11th order polynomials to the monofocal IOL data, as the majority of participants had VA worse than 1.00LogMAR at defocus of −4.00 D and above; hence, insufficient data points were recorded to facilitate these higher order polynomials.

Bland and Altman plots taken from the monofocal IOL group demonstrated comparable means, with the narrowest limits of agreement (LoA) occurring after the 3rd (Figure 3a) and 5th (Figure 3b) order polynomials for the −2.25 D and − 2.75 D defocus measurements, respectively.

In the MIOL group, comparable means and narrowest LoAs were only achieved when employing 9th order or higher (Figure 3c) and 8th order (Figure 3d) polynomials for the −2.25 D and −2.75 D defocus measurements, respectively.

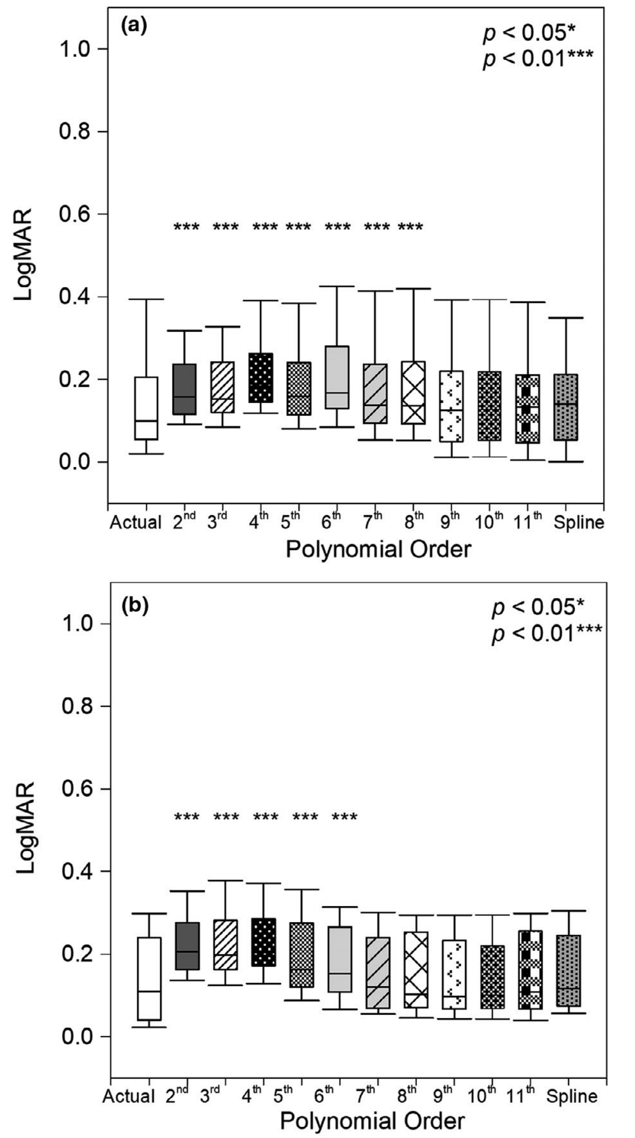

FIGURE 1 Comparison of means for additional defocus points with the bi-flex MY. (a) −2.25 D defocus, (b) −2.75 D defocus

Area of focus was calculated for each lens using methods previously described.[3] The areas derived from each of the polynomials were compared to those from the spline curve using repeated measures ANOVA and Bonferroni post-hoc pairwise comparisons, which revealed significant differences ($p < 0.05$) (Figure 4).

Similar results were found when the absolute range of focus was analysed using a cut-off value of 0.30 LogMAR (Figure 5). Significant differences between the range of focus derived from polynomials and spline curves were found statistically ($p < 0.05$), but no clear pattern was established.

The minimum polynomial order required for each metric to provide similar areas to the spline curve is outlined in Table 4 following repeated measures ANOVA and assessment of Bland and Altman plots.

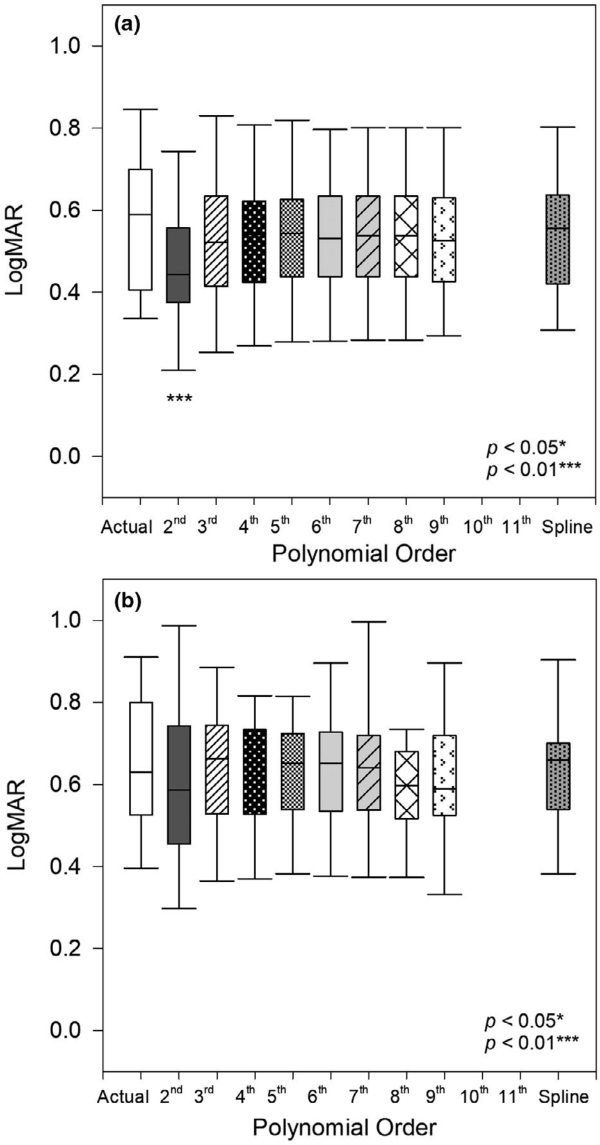

FIGURE 2 Comparison of means for additional defocus points with the bi-flex 677AB. (a) −2.25 D defocus, (b) −2.75 D defocus

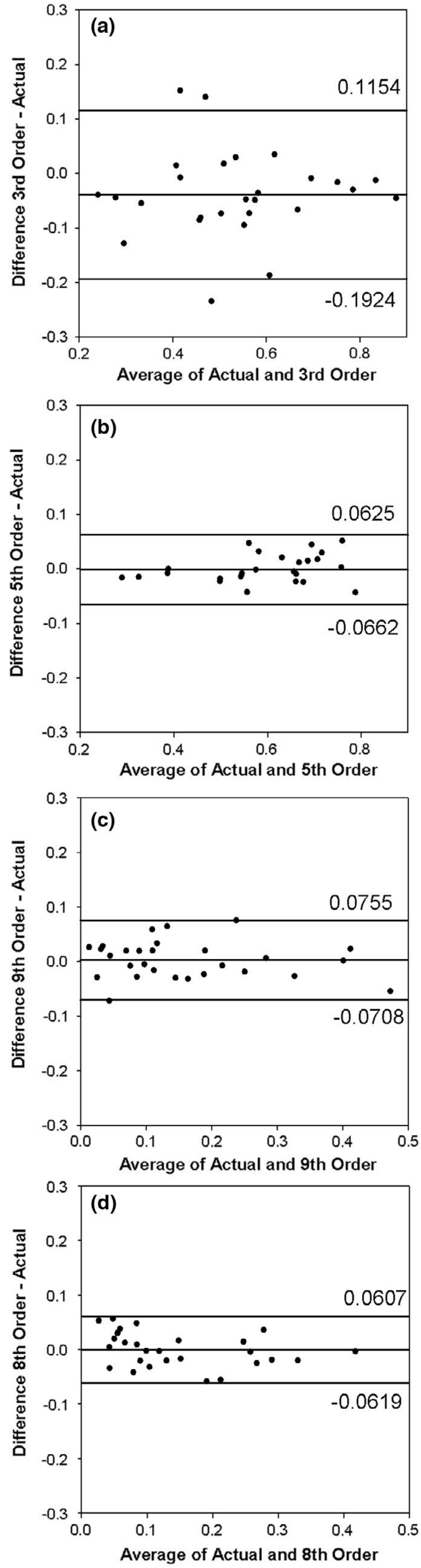

FIGURE 3 Bland and Altman comparisons. (a) Bi-Flex 677AB Monofocal, −2.25 D defocus with 3rd order polynomial; (b) Bi-Flex 677AB Monofocal, −2.75 D defocus with 5rd order polynomial; (c) Bi-Flex MY, −2.25 defocus with 9th order polynomial; (d) Bi-Flex MY, −2.75 defocus with 8th order polynomial

DISCUSSION

The study demonstrates the inherent difficulties faced when selecting a single polynomial function to best fit a combination of monofocal, bifocal and trifocal IOLs. The statistical methods used to assess goodness of fit demonstrated variable results. More lenient methods such as adjusted R^2 or the least squares methods can easily lead to overfitting,[24] and as such, are insufficient when used in isolation to select a polynomial for defocus analysis. To some extent, visually inspecting the fit can help prevent overfitting, and will help exclude curves compromised by Runge phenomena and other anomalies. Buckhurst[3] found a 9th order polynomial was the universal best fit for the MIOLs tested; however, the findings of the present study indicates that this would potentially overfit a monofocal or EDoF IOL. Gupta[5] used

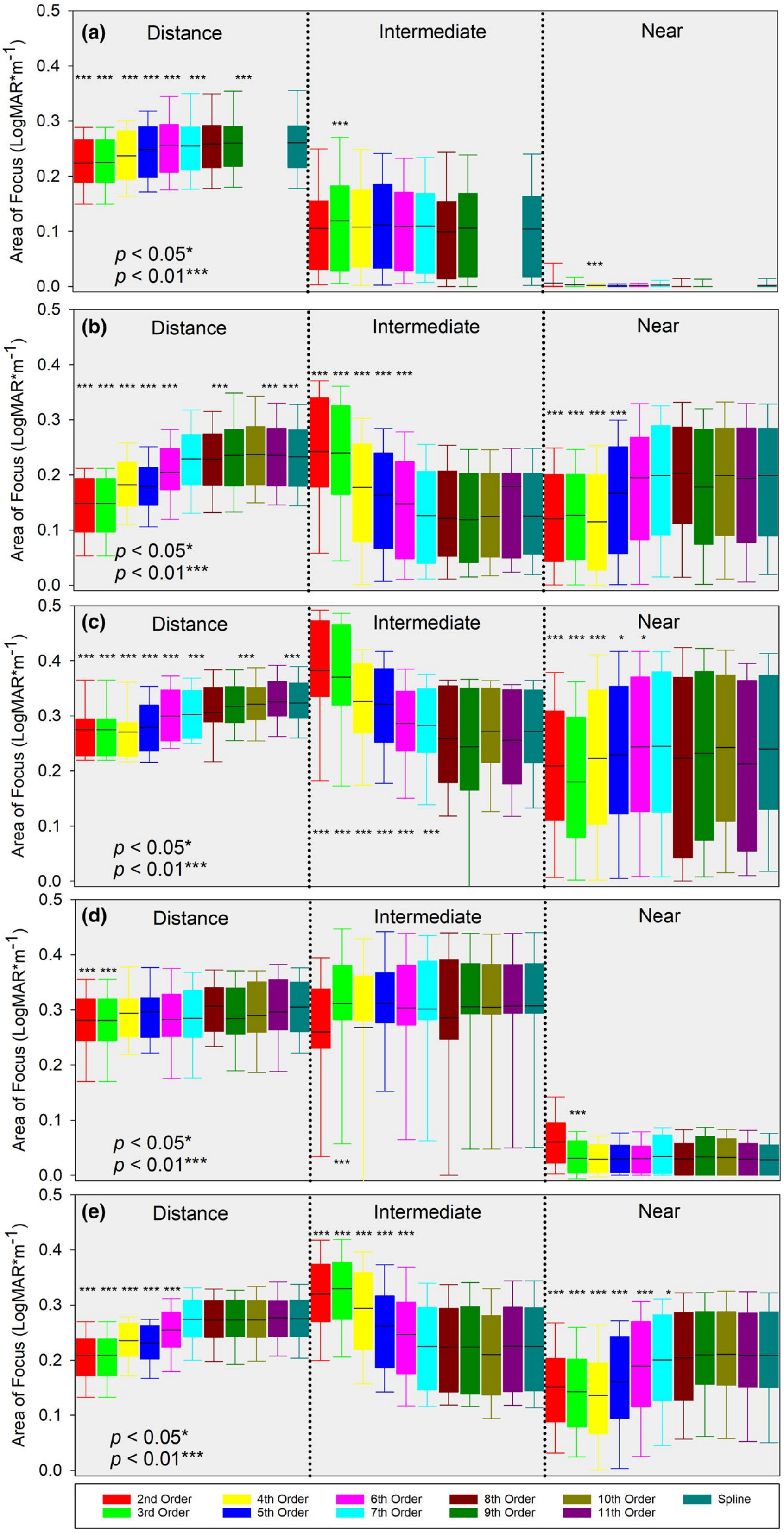

FIGURE 4 Area of focus results for the distance, intermediate and near areas. (a) Bi-flex 677AB; (b) Bi-flex MY; (c) Mplus; (d) Symfony; (e) AT Lisa

a variety of 5th to 10th order polynomials in their work, based on achieving $R > 0.99$. It is known that deceptively high R values can be achieved when data are overfitted; thus measures such as R^2_{adj} are more commonly used to ensure that the increase in R is greater than would be expected by chance.[19]

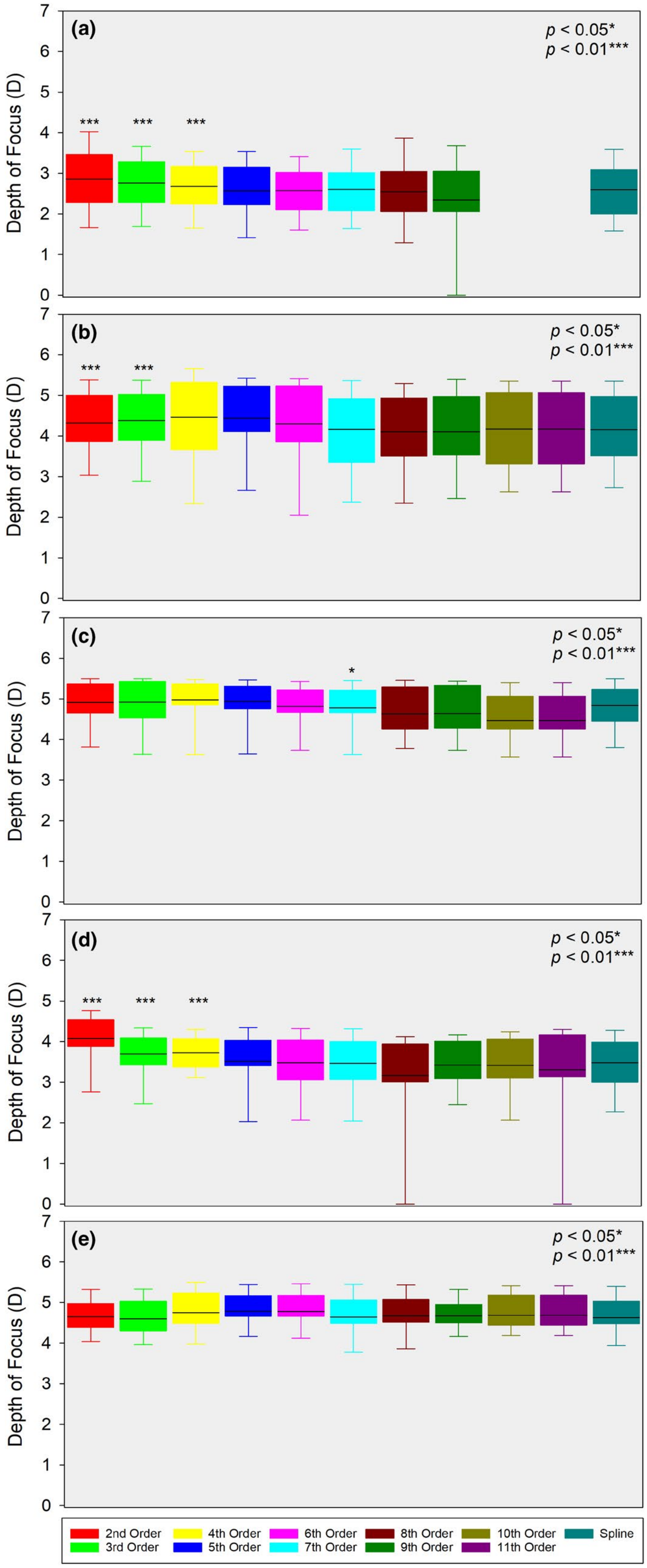

FIGURE 5 Range of focus. (a) Bi-flex 677AB; (b) Bi-flex MY; (c) Mplus; (d) Symfony; (e) AT Lisa

More stringent model fitting methods such as AICc and Snedecor and Cochran are specifically designed to guard against over fitting, and as such, show lower orders to be sufficient. Again, visual inspection will reveal when expected inflections in the defocus curve are omitted by the curve in favour of smoothing of the data, and can guard

TABLE 4 Minimum polynomial similar to spline

	Area distance	Area intermediate	Area near	Range of focus (Total)	Range of focus (actual)
Bi-Flex 677AB	8th	2nd	3rd	5th	5th
Bi-Flex MY	7th	7th	6th	4th	6th
Mplus	8th	8th	6th	2nd	2nd
Symfony	4th	3rd	3rd	5th	5th
AT LISA 839	7th	7th	7th	2nd	2nd

against underfitting. These inflection points typically demarcate intermediate and near focus and as such are crucial for accurate defocus analysis. Spline curves, which by nature will pass through each raw data point, will not omit these crucial peaks and troughs in the defocus profile. According to all methods of analysis with the exception of the least squares method, an 8th order polynomial was the minimum required fit for these curves, but there was considerable disagreement between methods.

Similarly, comparison of the means found significant differences in both groups. Amongst the monofocal IOL group there was no significant difference when a 3rd order polynomial or higher was used. In the bifocal IOL group, at least an 8th order polynomial was required to achieve comparable results with the actual measured values of VA. Using a polynomial of insufficient order in the monofocal IOL group resulted in an overestimation of VA, whereas in the bifocal IOL group, using a polynomial of insufficient order resulted in an underestimation of VA (Figure 1). However, it must be considered that some variation between predicted *y* values and actual measured values will always exist, as actual measurements are limited to 0.02 LogMAR steps (1 letter). These results are also limited as they were not tested for the trifocal, refractive bifocal or EDoF IOLs.

Good correlation was achieved when validating the additional points measured in both groups with all curves fitted; however, the correlation coefficient improved in the monofocal IOL group when either a spline curve or a lower order polynomial was fitted. Conversely, in the bifocal group, higher order polynomials or spline curves improved correlation as shown by the Bland and Altman analysis.

Assuming the spline curve data to be most accurate for assessing the area of focus as it passes through each data point, Table 4 details the minimum polynomial required to reveal no significant difference in the area metrics when compared to the spline curve data. The order required varied depending on the specific measure, and thus there was no consistency even within an IOL group, nor was there any agreement between lenses.

A limitation is that this was a cohort study rather than a randomised trial, and as such, the demographical data of the groups were not similar. However, the purpose of the study was to examine the use of curve fitting to aid the development of a standardised method of analysis, rather than the comparison of IOL groups. As such, it was not necessary to either randomise or stratify the groups according to any demographic characteristics. A further limitation was that ocular biometry data were not collected, which is known to influence the defocus curve.

Defocus curves are widely used in the literature, and with the advent of EDoF lenses and an increasing range of MIOLs, it is likely that defocus analysis will remain prominent. Simple VA testing at arbitrary distances may be insufficient to differentiate between MIOLs, or conversely, it may bias results unfairly in favour of a particular lens. To allow fair comparison of defocus metrics in IOLs, it is essential that their defocus curves can be analysed using a standardised approach. There is a paucity of literature using curve fitting in defocus metrics, with most authors preferring to use only direct analysis, despite the benefits of range of focus and area metrics having been established.[3,5] The sluggish adoption of area metrics may be due to the inherent complexities with polynomial curve fitting. Fernandez and associates redressed this barrier by using the simpler trapezoidal method for calculating area and by developing specific software (Multifocal Lens Analyser) to allow clinicians to calculate such metrics easily.[25]

This study specifically examined the non-linear piecewise regression (spline curves) Fernandez and associates used for linear piecewise regression,[25] which connects straight lines between each data point. The mathematics involved in linear regression are simpler relative to non-linear regression, but both piecewise methods fit the data points exactly. The main differences between the two methods are that each segment for the linear model runs as a straight line between each data point, and are uninfluenced by data points following or preceding those two values, whereas spline curves are influenced by the positions of the surrounding data points. Future studies need to establish whether linear or non-linear function fitting is more appropriate for defocus curve assessment.

This study could not establish a conclusive method for choosing a polynomial fit, nor could it establish an order that suited all the IOL types tested sufficiently. Given the variability of results that this study found in relation to polynomial fitting, we advocate selecting the polynomial order for each individual subject according to the adjusted R^2, as other goodness of fit strategies are simply too conservative in these circumstances. Even with this individualised approach, particular care and attention must be paid to guard against over/underfitting. The results demonstrated that the use of spline curves is preferred over polynomial curve fitting as it avoids the limitations of polynomial fits.

CONFLICT OF INTEREST

The authors report grants from Bausch+Lomb and ZEISS outside the scope of the submitted work.

AUTHOR CONTRIBUTIONS

Elizabeth M Law: Conceptualization (equal); data curation (lead); formal analysis (lead); investigation (lead); methodology (lead); project administration (equal); resources (equal); software (equal); writing – original draft (lead); writing – review and editing (equal). **Hetal D Buckhurst:** Conceptualization (equal); methodology (supporting); supervision (equal); writing – review and editing (equal). **Rajesh K Aggarwal:** Conceptualization (supporting); investigation (supporting); methodology (supporting); project administration (supporting); writing – review and editing (equal). **Hosam Kasaby:** Methodology (supporting); project administration (supporting); writing – review and editing (equal). **Jonathan Marsden:** Project administration (supporting); supervision (supporting); writing – review and editing (equal). **Gary L Shum:** Project administration (equal); supervision (equal); writing – review and editing (equal). **Phillip J Buckhurst:** Conceptualization (equal); data curation (supporting); formal analysis (supporting); investigation (supporting); methodology (supporting); project administration (equal); supervision (equal); writing – review and editing (equal).

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