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Longitudinal study of 'retraining' non-maths specialist teachers to become capable, confident teachers of mathematics

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ABSTRACT

One of the key problems to be solved in mathematics education in England is that the demand for mathematics teachers is far in excess of the supply. Acknowledging that there are simply too few mathematics teachers, the UK government has invested significantly in retraining programmes. These programmes 'retrain' out-of-field teachers, that is, teachers of other subjects or from other phases, to teach secondary mathematics. The increased mathematical demand of the reformed GCSE courses coupled with the expectation that most post-16 students will engage with some mathematics (studying for A and AS levels, a Core Maths qualification or re-sitting GCSE) means many more teachers of mathematics will be needed. We consider the viability of a retraining course as an effective way of alleviating the problem of the lack of well qualified teachers for mathematics. In this four-year longitudinal study, we followed teachers during their year of 'retraining' and in the succeeding years. Once a participant completes their part-time one-year course, the teacher is considered 'retrained'. However, we conclude that without ongoing professional development involving collaborative support retraining courses alone can have little impact on the problem of the lack of competent and confident teachers of mathematics in the secondary sector.

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KEYWORDS
SKE+ (Subject Knowledge Enhancement Plus); Teacher Subject Specialism Training (TSST); Lesson Study; Core Maths; Pre Initial Teacher Training (Pre-ITT); Post Initial Teacher Training (Post-ITT); Department for Education (DfE); National College for Teaching and Leadership (NCTL)

1. Background

The provision of sufficient numbers of *effective* teachers of mathematics in schools and colleges has long been a problem in England; Smith identified the shortfall in the UK in his 2004 report, 'Making Mathematics Count', where he described the shortage of specialist mathematics teachers as the most serious obstacle to 'ensuring the future supply of sufficient young people with appropriate mathematical skills' (2004, p. 4). The UK government's Building our Industrial Strategy green paper (GOV.UK, 2017) highlights the continuing skills shortage and states that action – in particular in the Further Education (FE) sector – will need to be taken to 'improve basic mathematics provision'.

Howson (2002) points out there is a clear distinction between *qualified mathematics teachers* and *teachers who teach mathematics*; and these two distinct sets of teachers can

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be described as *specialist* and *non-specialist*, with specialist teachers better equipped to be, or become, more *effective* practitioners (Cockcroft, 1982). Defining effective teaching is no easy task, and with effective practice seemingly dependent on multifarious factors, it is perhaps not surprising that an agreed definition remains elusive (Coe et al., 2014) but a more detailed exploration of 'effective teaching', in relation to this research, follows later. In general, Howson (2002) argues for specialist teachers of mathematics, at secondary or tertiary level, to have degrees in mathematics or a least a mathematically-related subject. Howson (2002) acknowledges that teachers without the best qualifications can be effective and indeed outstanding; likewise, teachers with the highest qualifications may be ineffective and poor practitioners. However, when considering the teaching profession as a whole, *it remains true that the lack of good qualifications must seriously limit the efficiency of teaching* (Howson, 2002, p. 81).

In England teachers *do* have specialisms but these are not related to QTS (Qualified Teacher Status): QTS is non-subject specific (Howson et al., 2015). Senior leaders are therefore free to assign any teaching work within the school to any teacher (Howson et al., 2015). With the ever-increasing shortage of specialist mathematics teachers, growing numbers of non-specialist teachers are being directed to teach beyond their area of expertise, to mathematics. Acronyms for teachers teaching outside the subject in which they initially trained are now commonplace: for example TOOF (teaching out of field), OOF (out-of-field teaching) and TAS (teaching across specializations).

Similar shortages of specialist mathematics teachers are faced by many other countries. In Australia, for example, Prince and O'Connor (2018) state that between 21% and 38% of Years 7–10 (age 12–16) mathematics classes are taught by out-of-field teachers. They also state that 'the supply of new graduates alone cannot solve the out-of-field problem on any acceptable time scale'. They go on to suggest that retraining out-of-field teachers needs to be a major part of any approach.

There is similar evidence from New Zealand where a Staffing Survey Report (New Zealand Post Primary Teachers' Association, 2018) states that over 40% of schools had to use out-of-field teachers during the year as specialist mathematics and science teachers could not be found. This was in part due to increased retirement rates with about 40% of teachers retiring and 20% moving to non-teaching employment. The education minister, Hipkins (2018), announced a package of incentives in October 2018 to encourage more graduates into the profession as well as financial incentives for teachers to return from overseas.

European countries are having similar problems; for example, Ireland is also employing increasing numbers of teachers out-of-field (estimated at about 20%) and has seen a collapse in the numbers recruited to teacher training courses (O'Brien, 2019). The Netherlands has a similar problem, particularly in Amsterdam where 15,000 pupils were sent home at the beginning of the school year in August 2018 because of a shortage of teachers (Boffey, 2018).

As with the UK, the lack of specialist mathematics teachers has been a real issue in the US for the past two decades. The early work of Ingersoll (2001) highlights the issues of teacher shortage – and this continues despite many initiatives to increase the numbers of specialist mathematics teachers.

Several elements have now combined to exacerbate this shortage of teachers in England and deepen the dilemma, creating what could be termed a 'perfect storm':

- the improving economy: mathematics graduates now have many more options and teaching may appear less attractive, particularly from a financial perspective;
- student surge – numbers in UK secondary schools are rising;
- higher demands from teachers – in terms of teaching time and skill levels – for the new more rigorous mathematics GCSE and A level courses;
- the requirement to re-sit mathematics GCSE post-16 (or study Functional Skills) if not passed at the end of Year 11 (age 16);
- the increasing popularity for A level mathematics which peaked in 2018 (however recent changes to the AS and A level curriculum have seen a decline in uptake in 2019);
- the recent introduction of post-16 'Core Maths' courses designed for students who pass GCSE Maths but do not continue on to AS or A Level mathematics courses (with the long term plan for all students (about 200,000)) in this category to take Core Maths courses (Core maths qualifications: technical guidance, GOV.UK, 2014);
- the as-yet-unknown implications of Brexit on teacher numbers.

This 'storm' has been confirmed by the National Audit Office (NAO) who, in February 2016, confirmed that the Department for Education's (DfE) trainee recruitment targets has now not been met for four years and that the number of unfilled positions in schools more than doubled between 2011 and 2014. The shortages in recruiting mathematics teachers is compounded by the fact that mathematics teachers – although clearly needed for teaching (DfE Press Release, GOV.UK, 2016) – are amongst the group most likely to leave the profession (NAO, 2016).

The shortage of mathematics teachers is also obvious from the proportion of lessons taught by teachers without a relevant degree (around 20–30%, Jones HMI, 2016) from a survey of school leaders (NAO, 2016). To put the shortage in context, for the DfE to meet its recruitment target it would need to attract 1 in every 5 of all mathematics graduates.

Smith (2017a) confirms that the Treasury has now acknowledged that mathematics teacher supply will never now be resolved (by traditional routes) and that alternative approaches need to be sought, and unlike a storm, the current 'shortage' situation will not simply resolve itself. More recently the NFER (2018) highlights not only the current shortage of well-qualified teachers but forecasts that trends in the number of secondary pupils and the number of secondary teachers, within a few years, will ensure a major crisis unless actions are taken immediately. For mathematics, the Nuffield Foundation report (2018) shows equally alarming evidence, namely that:

- the most inexperienced or out-of-field teachers are most frequently allocated to Key Stage 3 classes (age 11–14), and
- schools in disadvantaged locations have the most difficulty in recruiting and retaining well-qualified teachers of mathematics.

Cockcroft (1982) clearly identified the shortage of specialist mathematics teachers and, a little over two decades later, the Smith (2004) report concluded likewise. As Cockcroft (1982) had done previously, the Smith (2004) report set out a series of recommendations to address these concerns, and three years later, the government funded Mathematics Development Programme for Teachers (MDPT) was launched. The MDPT was a fully funded

part-time course, eligibility for which depended on teachers (with no mathematics qualifications at degree level) having the support of their head teacher and being allocated a school-based mentor (Crisan & Rodd, 2011). In essence, MDPT was intended to enhance subject knowledge, but the associated pedagogy was inevitably interwoven (Stevenson, 2013). In 2011, the MDPT was replaced by a much contracted (in terms of both budget and time) programme for subject knowledge enhancement. This new programme, for qualified but non-specialist teachers of mathematics, was named Subject Knowledge Enhancement Plus (SKE+) to distinguish it from the existing pre-ITT SKE courses. The eligibility criteria for SKE+ was similar to that for its predecessor, the MDPT, but now also included 'returners' – teachers returning to the profession after an absence of 3 years or more (Crisan & Rodd, 2011, 2015).

In 2014, SKE+ was terminated by the NCTL and replaced with a 'teach and learn project'. One year later, having concluded this 'teach and learn project' to be successful, it was rebranded as Teacher Subject Specialism Training (TSST) and launched as such in 2015. TSST courses are funded through schools – not universities – and this is a significant departure from the previous retraining professional development models. The TSST programme is coordinated through Lead Schools, who have successfully bid for funding, originally from the National College for Teaching and Leadership (NCTL) and since 2017, from the DfE. For the purpose of these retraining models, non-specialists are defined as teachers 'who have not undertaken initial teacher training (ITT) in the target subject' (GOV.UK, 2018).

Retraining, by way of the professional development models described above, is one of the major initiatives taken by the government to alleviate the lack of specialist teachers of mathematics. With the acknowledgement that the government is seeking greater numbers of *effective* practitioners by way of retraining, here we consider what is considered to be *effective* mathematics teaching, this in the absence of a globally agreed definition.

The ongoing difficulty in defining effective teaching, may best be summarized by the much used quote from Gates (2010): 'Unfortunately, it seems the field doesn't have a clear view of what characterizes good teaching.' The report by Coe et al. (2014) for the Sutton Trust suggests effective practice is that which 'leads to improved student achievement using outcomes that matter to their future success' (2014, p. 2). This then poses further questions: What outcomes 'matter' and what do we mean by 'future success'? The current Chief Inspector of Ofsted, Spielman believes outcomes that matter are linked to society's regard concerning the body of knowledge we wish to impart to the next generation, so children can 'flourish in the future' (GOV.UK 2018, p. 2). It therefore may be helpful to define the term 'effective' using Campbell et al.'s (2004) definition, which is more closely aligned to 'efficacy': teacher effectiveness is 'the power to realise socially valued objectives agreed for teachers' work, especially, but not exclusively, the work concerned with enabling students to learn' (2004, p. 4).

The relationship between a teacher's subject knowledge and their effectiveness has faced much scrutiny (Davis & Simmt, 2006) and there is now 'widespread agreement that the quality of primary and secondary school mathematics teaching depends crucially on the subject-related knowledge that teachers are able to bring to bear on their work' (Rowland & Ruthven, 2011, p. 1).

Table 1. Framework to identify effective mathematics teaching (adapted from 'guidelines to identify effective teaching' (Burghes, 2005)).

Teacher is a good communicator, loves mathematics and likes teaching
Teacher orchestrates activities and can respond to unexpected outcomes
Teaching is aspirational and challenging
Teacher gives clear explanations; can select and instruct efficient and effective methods
Teacher can see the 'big picture' and promotes mathematical content connections
Teacher promotes deep thinking (For example: Why? How? What if? questioning technique)
Teacher encourages creativity and discovery
Teacher listens to learners
Teacher likes being with learners
All mathematics written by teacher clear, correct and precise; mathematical language embedded throughout
Considered interactive questioning techniques to involve all learners, and to reflect and evaluate progress
Teacher has control of the class
Non-confrontational ethos in the classroom
Learners keen, enthusiastic and motivated to learn
Ownership of ideas encouraged and active participation expected: including, for example, demonstrating and articulating at the board
All learners feel encouraged and are able to make progress
Learners cooperate and collaborate with peers
Learners on task

For the purposes of these retraining programmes, a working definition of effective mathematics teaching was required. Drawing on literature searches, effective mathematics teaching was defined to be that which promotes or encapsulates the attributes outlined in the 'Framework to identify effective mathematics teaching' (see Table 1); this is disseminated to *all* participants on our retraining programme.

In summary, for the purposes of this study, effective teaching is described as that which promotes active participation and deep thinking amongst learners, to promote deep understanding.

2. The retraining programme design

Since 2012, we have delivered one-year retraining courses, along with other providers, in a variety of locations around England. With a blend of face-to-face meetings and e-learning, our model has these aims:

- to enhance participants' mathematical knowledge to give confidence to teach up to and including Higher Level GCSE Mathematics,
- to inspire and enthuse teachers
- to provide motivating introductory activities, tasks and presentations for teaching mathematics.

Acknowledging the (2007) report, highlighting the need for teachers to observe and reflect and identify what makes for great teaching in their subject, demonstration lessons are delivered whenever feasible – providing a vehicle for discussion and debate as participants and tutor reflect on the lesson together. In practice this is only possible when the face-to-face sessions are being delivered within a school venue. The demonstration lessons give participants a rare opportunity to actually observe a mathematics lesson and to see how ideas discussed during the face-to-face retraining sessions can be put into practice.

In addition to (but quite separate from) the demonstration lessons, participants are provided with an overview of Lesson Study and how it is implemented in Japan where Lesson Study provides a meticulously managed method of collaborative practice. Typically for Lesson Study in Japan, a small group of teachers will meet regularly and spend many hours planning and reviewing a single lesson, building on what other teachers have previously done and what the research suggests regarding students' learning (Hiebert et al., 2002). The lesson will then be taught by one of the teachers from the group and will be observed by a large number of teachers and university academics or 'knowledgeable others' (with purpose-built classroom walls and doors being removed to allow access). The review or feedback session is chaired by the visiting academic (or 'knowledgeable other') with the intention of revising or 'polishing' the teacher's professional knowledge and pedagogy, and their theories of teaching and learning. From such a thorough review a revised lesson plan will emerge. The aim of such a review however, is not simply to revise one lesson plan but to build rigour and expertise within the profession. The whole process can thus be summed up as: collaboratively plan, implement, review and revise lesson.

This model has been adapted for Western cultures and sensitivities. Lesson Study in England may be summarized by the points outlined by Burghes and Robinson (2010, p. 7):

- Small group of teachers, ideally with an outside 'expert',
- Choose a suitable topic to study,
- Identify the goals of the unit to study,
- Jointly map out a series of lessons that will achieve these goals,
- Identify the key lesson in the series which then becomes the research lesson,
- Jointly plan the research lesson,
- One of the group delivers the lesson,
- The others observe the lesson,
- Review and reflect on the lesson,
- Revise the lesson plan and continue the cycle.

Lesson Study is introduced to our participants during the face-to-face sessions of the retraining programme, and include cycles of focus, observation and reflective review. Lesson Study sessions are based, but much condensed, on the principles outlined above, the focus being predetermined by the topics being studied in a particular session. Participants act as Lesson Study groups, planning, observing, reflecting and revising lessons. Gaps in knowledge and understanding can be collaboratively explored and teachers can receive guidance and support from the course tutors, thereby preventing the dissemination of poor practice (Burghes & Robinson, 2010). Collective effort and knowledge (along with other resources) create the micro-lessons; the 'efficacy of collaborative approaches to mathematics teacher education is well-established' (Hodgen, 2011, p. 38). Shared responsibility is assured as no-one knows at the outset who will teach the micro-lesson, the 'teacher' being randomly selected once planning is complete. All other participants (and the tutor) become both 'students' and 'observers'.

Commonly there are some initial reservations in regard to both teaching and in offering anything but positive platitudes whilst observing, but over time more constructive feedback is appreciated and developmental input becomes the 'norm'. With the limitations of time

and of inauthentic teaching scenarios this experience is intended to be only an introduction to the ideas and principles of Lesson Study.

All participants are encouraged to ask for Lesson Study type support with mathematics colleagues in their own school, particularly in their 'induction' year after this part-time one-year training.

3. The research

The contextual background framing the research is the widely acknowledged shortage of specialist mathematics teachers in England. In conjunction with this are the theoretical considerations and complexities of defining the term 'effective' (discussed above). Retraining has been assumed to be a solution, at least in part, by consecutive governments (GOV.UK, 2016). There is however, very little research to support or to refute this strategy; the 'need is growing to learn more about the implications of the out-of-field phenomenon' (du Plessis, 2017). The intent of this research is to contribute to the knowledge in this field.

For this research, a global question such as *Is it possible for non-specialist teachers to be retrained to become effective mathematics teachers?*, is refined into:

- What factors most impact upon out-of-field teachers' ability to teach mathematics following their participation in an upskilling course?

These questions relate to the development of knowledge and skills for teaching mathematics amongst teachers who were not initially trained to do this and who are not mathematics graduates; some of them also have limited prior mathematical qualifications. In this regard, these teachers are referred to as non-specialist teachers and 'retraining' refers to the process of upskilling and reskilling qualified teachers to teach a different subject.

Our study follows the path of eight teachers enrolled on our 2013–2014 SKE+ retraining programme. In this four-year longitudinal study, we followed the teachers during their year of 'retraining' and in the succeeding years. All participants volunteered to take part in this research project. Being willing to volunteer is a significant factor to ventilate: teachers prepared to participate in an intrusive study *may* be more reflective and effective than those who choose not to do so (Browne, 2006).

The participants were recruited using a set of criteria approved by our University ethics committee. These criteria are shown in Table 2.

Table 2. The criteria for recruiting participants to the research study.

Criterion 1: Those currently teaching at least one mathematics class
Criterion 2: Two participants (meeting Criterion 1), selected randomly, from each of the three following groups∗ (the groups reflect different background experience):
1. Whole class teaching (i.e. teachers of: Science; Geography; History; R.E.; English; Languages)
2. Managing large groups in different environments (i.e. teachers of: P.E.; Drama; Dance; Music; Art; D&T)
3. Teaching small groups and/or involving a lot of one-to-one tuition (i.e. teachers of: ICT; Business Studies; Child care; Music instrument (individual/small groups))

∗If it is not possible to select a participant from a particular group, the participant will be randomly selected from subject specialisms not yet represented.

Teachers were required to be teaching at least one mathematics group to be eligible for government funding. (This government-stipulated criterion has subsequently been suspended for the current TSST.) The research participants attended one of two venues: Venue 1 was a university campus with 14 participants attending; Venue 2 (with 10 participants) at a school where demonstration lessons with pupils could be observed. The participants each experienced 100 h of face-to-face tuition over the course of 11 full training days (9am to 6pm), from December 2013 until November 2014, with an equivalent time commitment expected for e-learning. Funding was available to reimburse schools for the 11 days of teacher release time, but travel expenses were not included. Some schools paid their participants for travel, other participants had to self-fund travel expenditure.

4. Methodology

The focus for the research was to allow issues and ideas to emerge from the participants' perspective. Acknowledging that the researchers were at the centre of this research – and that any findings or discoveries would be mutually constructed between researchers and participants – the methodology design is rooted in constructivist grounded theory (Charmaz et al., 2012). With a working hypothesis that effective practitioners need to possess sufficient and fluent subject knowledge, an adjustment was made to the constructivist grounded theory approach to incorporate predetermined themes relating to subject knowledge and teacher effectiveness; this in line with Strauss' systematic grounded theory (Strauss & Corbin, 1990).

In summary, the methodological approach of constructivist grounded theory is blended with elements of systematic grounded theory (with a core phenomenon), whilst incorporating an appreciation of narrative analysis (Ezzy, 2002) and a connection with common interactive acts (Schatzman, 1991). These elements combine, and together facilitate the drawing together of deep, rich narratives to move beyond the descriptive towards understanding – with participant voices, and the layering of thick descriptions, allowing findings to emerge from the data.

Interpretative methodology is based on a naturalistic approach to data collection. Interviews and observations were therefore the predominant methods in this study.

Using an approach similar to the feminist communitarian researcher described by Fontana and Frey (2005), a framework of opening questions and prompts was used for the first set of interviews, and participants were invited to reply and then allowed scope to expand at will. This form of semi-structured interviews is typical for qualitative research (Wragg, 1994). Maintaining an open approach helped to bring to the surface significant issues and concerns. To avoid participants' relevant issues being missed or obscured, and to ensure findings are genuinely rooted in the participant's perspective, the second set (and subsequent) interviews commenced with broad, sweeping, exploratory and general questions (Glaser, 1998).

Lesson observations were another key source for data collection. The importance of observations has been highlighted by Ernest (1989); it has been shown that there can be 'a great disparity between a teacher's espoused and enacted models of teaching mathematics' (1989, p. 2).

Following the initial lesson observations, informal developmental verbal feedback was offered. Capitalizing on established rapport, each participant teacher was offered individualized points on which to focus. The second (and subsequent) observations were hence conducted as developmental lesson observations. With encouragement and consent from the participants a more robust form of both verbal and written feedback was then implemented.

The lesson observation notes were written in real time, and then used as a vehicle for discussion immediately following the lesson. At this juncture, the strengths and limitations of the lesson were discussed and co-constructed by the teacher and tutor, with reference to the high inference items from the 'Framework of effective mathematics teaching' (see above). The teacher was always invited to reflect and feed back first, reflecting the ethos of the Japanese Lesson Study model (APEC, 2013).

These focused mentoring and coaching sessions became a key feature of the lesson observation process. The difference between mentoring and coaching can be subtle and we used them virtually synonymously as a continuation to the support given during the retraining programme. Mutual trust and respect were essential, and comments from the participants highlight the significance of these considerations.

Alongside interviews and observations, various questionnaires were employed. These included the standard Mathematics Attitude Questionnaire and Subject Questionnaire for Mathematics SKE+ Evaluation administered to all participants on commencement and completion of the course. Other questionnaires included feedback-on-feedback to capture reflections from participants on their post-lesson feedback and reflective and reflexive questionnaires designed to pick up on previous comments and prime participants prior to a more in-depth interview. Participants own personal reflections, in the form of a journal or diary, along with regular dialogue and email communication added to the data collected. In some instances, this research was further triangulated by contributions from an external evaluator of the 2013–2014 Plymouth retraining course (Hedger, 2014). In addition, memos and field notes were used to document and collate varied and extensive data and ideas gathered along the way, channelling Glaser's (2001) dictum 'all is data' (2001, p. 145).

The schedule for the points of data collection and methods used is shown in Table 3.

Qualitative research studies, such as this, are by definition intrusive and probing. An ethical approach was clearly appropriate and essential. Examining personal lives poses significant risks for potential harm. Ethical considerations encompassing privacy, confidentiality, betrayal, deception, and harm (Cohen et al., 2011; Creswell, 2012; Yin, 2014) were examined. Potential tensions and dilemmas were clear: the aim of the research was to capture an honest representation of the observed 'truth' whilst simultaneously being ethically bound not to inflict harm. Strike et al. (2002) offer two principles for ethical guidance in this field: the principle of benefit maximization and the principle of equal respect. The principle of benefit maximization is pragmatic, in terms of promoting the best course of action to be the one which produces the greatest benefit for most people; being uninhibited to reveal full disclosure within the summary narratives may ultimately be seen to benefit more people, with our actions judged in the light of their consequences (Strike et al., 2002). The principle of equal respect 'demands that we respect the equal worth of all people' (Cohen et al., 2011, p. 98).

Table 3. Schedule of Data Collection.

2013–2014:	
December 2013	Mathematics Attitude Questionnaire1
	Subject Questionnaire for Mathematics SKE+ Evaluation1
January–March 2014	Interviews 1
January–July 2014	Personal reflections, kept by way of log or diary and emailed from participants to researchers
	Lesson Observations 1 using framework
2014–2015:	
September–December 2014	Personal reflections, kept by way of log or diary and emailed from participants to researchers
October 2014	External course evaluator visit; interview with some participants; evaluation notes made
December 2014	Mathematics Attitude Questionnaire 2
	Subject Questionnaire for Mathematics SKE+ Evaluation2
	Reflections on Course Questionnaire
	Pre-Interview 2 Questionnaire
January–March 2015	Interview 2
	Lesson Observation(s) 2 and 3
2015–2016:	
September 2015–July 2016	Lesson Observations 4, 5, 6, 7
	Feedback-on-feedback questionnaires
	Personal reflections, kept by way of log or diary and emailed from participants to researchers
October 2015	Interview with Senior Staff
2016–2017:	
September 2016–January 2017	Lesson Observations 8, 9
January 2017	Final Questionnaire on Progress in becoming a teacher of mathematics
	Interview 3
July 2016–January 2018	Personal reflections, kept by way of log or diary and emailed from participants to researchers

5. Results

Close and comparative attention to the emerging data was the first analytical step. Being immersed in the data, reading and re-reading the interviews and dialogue transcripts, the observation notes and questionnaires – and getting a sense of the whole – preempted the beginning of the coding process.

Coding the data, separating it into pieces by segmenting and labelling sections of the text, initially using in-vivo codes (derived from the participants' words) and later introducing in-vitro codes (constructed by the researcher to refine terminology), provided a means to organize and make sense of the data.

Continuous coding data throughout the research informed decisions in terms of theoretical sampling – helping to suggest areas to explore in subsequent interviews, and to extend, refine or confirm coding categories. The interview questions gradually became more probing; this was possible as the relationships with the participants strengthened and thrived, enabling more open and intimate conversations. Later the initial codes were examined for overlap or, in the case where they had been barely populated, for redundancy. Codes were subsequently aggregated into broad themes.

Conforming with constructivist grounded theory principles, extensive narratives for each participant were woven together, with depth being added over time, from the data collected, using the various research instruments described above. An overview of all eight participants, with relevant information, is shown in Table 4 below.

Table 4. An Overview of Each Participant.

Participant and venue	Original subject	Highest previous mathematics qualification	Number of years teaching in total (as of Sept 2017)	Number of years teaching maths (as of Sept 2017)	SKE+ Course Award
Harvey Venue 1	PE	GCSE, grade A	10	4	Merit (82%)
Darcy Venue 2	PE	A level, grade D	14	5	Distinction (100%)
Bea Venue 1	Science	A level, grade C	28	4, no longer teaching maths, as of Sept 2017	Distinction (94%)
Anna Venue 1	Science	A level, grade C	7	3, no longer teaching maths, as of Sept 2016	Merit (84%)
Katy Venue 1	Childcare	GCSE, grade C, after resit		5	Merit (86%)
Cath Venue 1	Science	A level Statistics, grade B	15	8	Distinction (96%)
Euan Venue 2	Computing	A level, grade B	14, resigned from teaching July 2015	2, no longer teaching maths, as of Sept 2015	Distinction (100%)
Janet Venue 2	Music; English; Business Studies	GCSE, grade A	6	1, no longer teaching maths, as of Sept 2014	Merit (86%)

6. Brief overview of participants

6.1. Details of progress

Harvey:

Prior to retraining	In 2013, Harvey's school was struggling to recruit mathematics teachers. As a consequence, Harvey was offered the opportunity to transfer from the PE department to the mathematics department and he keenly embraced this opportunity, enthusiastic to retrain.
During retraining	A committed and engaged participant throughout the whole year of the course (with 100% attendance), Harvey positively embraced lesson observations with the associated mentoring and coaching; his motivation was clear: *I want to develop; I want to get better* (Lesson Observation 5).
Post retraining	Over the course of several lesson observations it was possible to see the development of Harvey as a teacher of mathematics. Lesson Observations 3 through to 7 showed a clear trajectory of a teacher developing and honing their practice. Harvey regularly described the period of his retraining as a *transformation* – from PE teacher to mathematics teacher – and believes that this would not have been possible without significant support from the school and department. In September 2016, Harvey changed schools for a promotion to Assistant Head of House. His support network dropped away. Evidence (Lesson Observation 8 and Lesson Observation 9) suggests a backward step in the development of his mathematics teaching with students being taught to rely solely on procedures and memorize formula, no whole class teaching and obvious teaching opportunities missed. In a school with more emphasis on marking and testing, Harvey's time to plan lessons has been reduced. This is proving to be especially problematic for Harvey as, whilst his own subject knowledge is not secure or comprehensive, he believes: *I think my time would be better spent planning good lessons than marking the work*. (Interview 3)

Darcy:

Prior to retraining	Darcy was a qualified PE teacher with 14 years teaching experience when, in September 2012, Darcy was appointed to teach mathematics full time. Darcy presents as a very driven, ambitious and hard-working teacher.
During retraining	A desire to be outstanding and to be the best appeared to be evident during the retraining course. Darcy really enjoyed doing the on-line tests and was determined to score 100%. Acknowledging the distinction between being able to do maths and being able to teach maths, Darcy said: *my maths knowledge was very – I could do it but I couldn't necessarily teach it* (Interview 3). Allegedly keen to develop her mathematical pedagogy, Darcy commented: *the course was a bit of a must for me* (Interview 3).
Post retraining	A didactic and superficial approach to teaching was usually demonstrated. Often there appeared very little challenge or context to the lessons. There was no big picture awareness and no evidence of the style of teaching advocated on the course. Active pupil participation throughout lessons was limited, as too were opportunities to develop deeper understanding. Opportunities for discovery or creativity were not encouraged, as evident from one boy who deserved (but did not receive) credit for an ingenious (but unanticipated) approach to a problem. Darcy operates within an 'exam factory' type of school and the driver for Darcy's approach to teaching appears to be pressure for exam grades.

Bea:

Prior to retraining	Bea started teaching in 1989 and has taught in several co-educational state comprehensive schools. In 2004, Bea became an Assistant Head teacher at her current school; prior to this she was a Head of Science. In her role as an Assistant Head, Bea continued to teach Science and line manage several departments including mathematics. In 2013 the school failed to find a Head of Mathematics or indeed a mathematics teacher at all. However, according to Bea, there was *an embarrassment of riches* (Interview 1) in the science department. So the decision was made to merge the mathematics and science departments: five science teachers *volunteered* to each teach one mathematics group, thereby negating the need to recruit a mathematics teacher. Two of these science teachers (Bea and Anna) enrolled on the SKE+ course.
During retraining	Bea was keen to *address students' misconceptions in mathematics, link mathematical topics and boost student confidence* (Mathematics Attitude Questionnaire 1). Bea believes the course addressed these needs and now feels able to teach mathematics as she would science: deliberately challenging students' misconceptions to provoke deeper thinking.
Post retraining	Evidence from lesson observations and interviews suggests Bea successfully achieved her above aims. But participating on the course was, for Bea, more than being able to improve her own mathematics teaching. Bea wanted to have the confidence to transform the whole department, in the *image* championed on the SKE+ course, believing without the course *the department wouldn't be as it is* (Interview 4); transforming a stagnating department was the objective for Bea.

Anna:

Prior to retraining	Anna has taught in one school, an 11–16 city comprehensive; she had been a science teacher, alongside Bea, for 3 years when the SKE+ course commenced. Anna was apprehensive about being roped in to teach mathematics and about being retrained to do so: *it has been nearly 10 years since I had done any maths and the thought of teaching it was scary* (Reflections on Course).
During retraining	During the year of the retraining programme, Anna taught one mathematics group. Hoping the course would help develop her pedagogy for teaching mathematics – as she *didn't have the knowledge of how to teach things* (Interview 1) – Anna felt it fulfilled this objective: *I am much more confident in my own ability, the new skills that I have gained … I feel my students are getting a much better deal from me having been on the course* (Reflections on Course)
Post retraining	The Key Stage 3 lessons observed shortly after the completion of retraining, showed signs of developing along the lines advocated by the retraining course. But Anna found this ethos much more difficult to enact in later years with Key Stage 4 lessons (with lower attainers); this in part because for Anna, the retraining now seemed somewhat distant. This 'distance' from the course was compounded by Anna's relatively little mathematics teaching experience. Anna felt wedded to the idea of teaching with the ethos advocated on the course, and of that being promoted by her then head of department, Bea. In practice, Anna had yet to master teaching in this way before she was promoted to Assistant Head and, with a much reduced teaching commitment, no longer taught any mathematics.

Katy:

Prior to retraining	Katy was a child-education teacher in an FE college when she first started teaching some mathematics. She had discovered that her students had very poor numeracy skills so offered to teach a little basic maths to support her child-care students (Functional Skills, level 1).
During retraining	Katy decided to retrain to upskill her own mathematical skills; she was nevertheless anxious her own skills would be inadequate. She was encouraged to continue as she was *so* keen and *so* determined to improve the life chances of her students by improving her own mathematical skills.
	Katy found the face-to-face sessions *the best thing* because she 'struggled' when working on her own (Interview 2).
	Katy is a reflective, articulate and insightful teacher. During the year of retraining she kept detailed recordings of her own self development and emotional state.
Post retraining	Katy has spent significant time immersing herself in the subject, reading related books, watching YouTube, doing the students' activities ahead of them, networking and attending any professional development she could find, whilst encouraging colleagues to do likewise. And although there remain significant subject knowledge weaknesses, Katy is continuing to develop. Katy is aware she struggles to see the *bigger picture* and link concepts together. These concerns most likely stem from subject knowledge limitations, and in the final interview with Katy, she perceptively said the problem that underlies it all *is my lack of subject knowledge*
	By 2016, Katy was *solely* teaching maths (to GCSE resit) for 18 hrs/week.

Cath:

Prior to retraining	Cath is a qualified Science teacher, with Biology as her specialism; she had 12 years teaching experience at the start of the SKE+ course, five of which were teaching maths. Cath teaches at a school for teenage mums; previously she has also taught in a Secondary Modern and in a Catholic Comprehensive. Cath, having previously been very *comfortable* teaching science at the school (Interview 1) was abruptly asked to teach maths (to fill the vacant position). Cath recalls finding this a daunting prospect.
During retraining	With strong personal subject knowledge, Cath found the collaborative practice one of the most valuable aspects of the training: *I wanted more of that actually … the bits I was learning the most* (Interview 3).
	Although Cath believed the SKE+ course was allowing her to grow in confidence as a mathematics teacher, she felt it had yet to translate to an increase in student performance.
Post retraining	Cath believes that since completing the course, her confidence with delivering the subject has increased (Interview 3).
	In order to survive the school merged with hospital education in 2017. Cath's cohort of students is becoming increasingly diverse and vulnerable, with a number of students *in care, they have had a dark life, there is so much hostility they are not ready to learn* (Interview 3).
	Cath is a reflective and caring practitioner with a keen sense of wanting to 'give' of herself, to help improve the life chances of others. She desperately wants to do a good job but is unsure she is doing so. Undoubtedly the retraining programme gave her a much needed opportunity to share experiences and collaborate with others. Knowing her subject knowledge was secure and experiencing new approaches and activities, gave Cath a huge boost to her professional self-esteem.
	She is still wracked with confidence issues and frequently worries that: *I am doing something wrong and letting them [the students] down*

Euan:

Prior to retraining	Euan has worked in just one school – for a period of 14 years – a mixed, 11–18 comprehensive school. An IT teacher, Euan taught some mathematics during most of his time teaching. Having a degree in computing, Euan had worked in the IT industry for about 15 years prior to teaching.
During retraining	During the period of retraining, Euan taught one mathematics class – a lower-attaining Year 11 group. Euan had to work hard to maintain the perception of a relaxed and calm environment; the students mostly uninterested and disengaged with mathematics. A really good introduction to a lesson (imitated from that seen on the SKE+ course) was set up with video, music and a challenging introductory problem. But in spite of Euan's efforts, it seemed evident that the students were *not really up for it* (discussion following Lesson Observation 1). The prepared lesson was swiftly discarded in favour of very mundane activities; the tasks 'allowed' to be very undemanding. Euan believed his teaching style needed to improve suggesting he had *fallen into some bad habits* and was *afraid the default mode* he used with his Year 11's was *to get didactic* (Interview 1). Euan, clearly a competent mathematician, enjoyed the retraining course and in particular enjoyed the demonstration lessons and the sharing of resources. By the end of the course, Euan had become much more enthused about teaching mathematics and, in spite of any apparent interest or involvement from the Senior Leadership, Euan expressed his determination to carry on teaching in the *new* interactive and engaging style: *It will last definitely* (Interview 2).
Post retraining	In the subsequent year Euan also taught one group only – a top set Year 8. A more amenable group – Euan felt able to teach in a more *adventurous* and engaging way. Euan made links and connections to other areas of mathematics and to material from previous lessons – clearly remembered by the students (Lesson Observation 2). Euan was keen to challenge the students and was clear in his belief of *doing it this way* pointing out that *if the pupils are challenged they will be engrossed* (discussion following Lesson Observation 2). This was an interesting echo of the previous (post Year 11 lesson observation) 'mentoring' discussion. Feeling unsupported by his senior teachers, Euan felt particularly let down and *knocked back* following an in-house lesson observation (email, March 2015). At the end of that year, Euan left teaching altogether.

Janet:

Prior to retraining	Janet had been working as a supply teacher at her current school when the head teacher approached her and offered a permanent position from the September of 2013. At the time it was not clear what subject she would be required to teach; the head teacher was simply keen to *fill holes*. Later the head teacher asked her how good she was at mathematics and Janet thought she *could do Key Stage 3* (Interview 1).
During retraining	Janet taught Year 7 and Year 8 (for 12 h a week) throughout the 2013–2014 academic year. Self-motivated, Janet enrolled for the SKE+ course, as in her words: *now that I've landed in it [maths teaching] I want to do it as well as I can* (Interview 1). The school had recently lost two mathematics teachers and was *struggling to recruit* (Interview 1), so Janet believed this would be an investment into a long-term commitment to the subject switch. In fact, Janet only taught mathematics for that one year. Janet described her teaching as poor *as the rest of the department to be honest* (Interview 1). Janet believed this to be *because I don't know my subject knowledge well enough to start creating exciting stuff which is why I'm really enjoying this [course]* (Interview 1). Janet suggested that until *you know your subject knowledge really well* the only option is to teach by *sticking to the tried and tested, speaking at the front, chalk and talk, work out of the text book kind of thing*. Janet concluded that this *is not the way I want to teach. It is not where I want to end up* (Interview 1).
Post retraining	Towards the end of the retraining, Janet discovered the decision had been made (my senior management) she would no longer be teaching any mathematics. Janet reached the conclusion that a degree in mathematics isn't necessary to teach the subject but that a good A level is essential to be able to teach GCSE and hence *why it is unlikely I will ever feel comfortable doing this* (Personal Reflections).

7. Key emerging themes

Four of the most significant themes arising from this longitudinal study, and threaded throughout the participants' narratives, are:

(1) Subject knowledge issues, including: the impact of the lack of mathematical subject knowledge; the benefits of being immersed in mathematics; and confidence issues related to teaching mathematics.
(2) Mentoring and coaching: the impact of these for the participants.
(3) Collaborative practice, including opportunities (and the lack of opportunities) for: collaborative planning; networking; sharing of good practice; peer and developmental lesson observations; and Lesson Study.
(4) Senior management influence (support/pressure) pre, during and post the retraining programme.

7.1. Subject knowledge

All participants, at various junctures, mirrored Katy's reflection that to be effective *it is essential to know your subject well* (Personal reflections), a point definitively made by Rowland and Ruthven (2011). The issue of being restricted by subject knowledge weaknesses does in fact appear to be one of the most significant themes arising from this research.

All participants believed their subject knowledge improved as a result of retraining. However, concerns surrounding issues with subject knowledge remained evident for all but one of the participants (the exception, Bea, having previously been a physics teacher).

A criterion for applicants for the government funded retraining SKE+ course (GOV.UK, 2016) was that they had no prior formal mathematics qualifications or training at degree level. With no consensus in the research community on what, or if, mathematical qualifications are important (Davis & Simmt, 2006; Rowland & Ruthven, 2011), Janet suggests: *a good mathematics A-Level is required* to be able to teach mathematics at GCSE; failing this *exceptional subject knowledge at GCSE* could suffice if one was skilled enough *in order to "look back" at these concepts*; Janet believes teachers *learning up to A∗ GCSE material is not a viable option* (Personal Reflections, September 2014). Janet's thoughts may resonate with the idea of teaching knowledge involving the explicit 'unpacking' of mathematical ideas (Ball et al., 2000); Janet was still focussed on *doing mathematics* for herself – with no 'unpacking' possible.

Janet is clear that her subject knowledge is insufficient, and so arrives at the realization she would never become a proficient practitioner; she had no confidence she could be *good* (Personal Reflections). As Janet clearly articulated, confidence to teach mathematics appears to be closely linked with subject knowledge: as subject knowledge increased so too did general confidence and self-efficacy. Being immersed in teaching the subject had a similar effect: as immersion increased (through involvement with the retraining course and teaching several mathematics classes) confidence increased; as it waned (as the course became more distant) confidence and self-efficacy deteriorated. The significance of subject immersion, to develop high levels of subject knowledge, is recognized by Garet et al. (2001).

Increased confidence can be seen to have enabled all the participants to start to develop their mathematical teaching pedagogy, moving away from a predominantly *chalk and talk* and *just telling* (Euan, Interview 2) approach to a style more rooted in students developing understanding for themselves. Key to this is of course the teacher's own subject knowledge and understanding, so as this develops so too does their scope to develop understanding in their students.

Comments from participants illustrate a growing appreciation of the need to make connections, to be able see what is coming next ('horizon knowledge' (Ball et al., 2000)), to use and unpick misconceptions in their teaching and to be able to respond to students' real-time needs (Mason & Spence, 1999). Much of this is captured in the ideas of different forms of knowledge, suggested by the 'Knowledge Quartet' (Turner & Rowland, 2011). Having further developed their 'foundation knowledge', all the participants could be seen attempting to use 'transformation knowledge' in terms of the choice of examples and explanations offered (Turner & Rowland, 2011). 'Connection knowledge' and 'contingency knowledge' (Turner & Rowland, 2011) proved more difficult: having wide enough knowledge to make connections to promote conceptual understanding and being able to view students' interjections as teaching opportunities and to use them in real-time, proved challenging for most of the participants. Lack of this 'web of interconnections' (Davis & Simmt, 2006, p. 301) may have limited the participants' effectiveness of teaching. Anna's comments reflect this:

> My only experience of maths teaching was stand at the front – here is a method and get going. Now when I started teaching maths last year, last September, I knew that is not how I wanted to teach – it's not how I teach Science. But I didn't know how to teach maths differently. (Interview 2)

For all participants, opportunities to use student misconceptions to promote deeper thinking and questioning were frequently missed or avoided, as too were students' challenging questions or unexpected responses. Similarly, not seeing how to connect or link lessons was very common. Participants also expressed concerns about not knowing how to extend or differentiate for higher attaining students – and most struggled with the level at which to pitch the mathematics, usually erring on the side of too easy. Berliner (2001, p. 470) refers to the 'degree of challenge' as the feature which most discriminates between the expert and non-expert teacher, this being dependent on deep subject knowledge.

The retraining (and associated mentoring) enabled the teachers to identify where they could begin to develop their pedagogical practice. Euan noted he needed to relinquish some control; Bea and Cath referred to the idea of doing more *real-time* (Bea, Interview 3) or *live* teaching (Cath, Interview 3); Darcy acknowledged the need to assimilate the Higher GCSE content and to improve her knowledge of terminology; Anna wished she could employ a more *guided-rediscovery* approach (Interview 2); Katy wanted to introduce more *problem solving* (Interview 3); Harvey was keen to teach less didactically and explore using different tasks and activities; Janet wanted to move away from *teaching to the text* (Interview 1).

Some of the participants were immediately immersed in teaching mathematics, while for others this was not the case. In general, the experiences of the participants suggest the more mathematics teaching they undertake, the better their progress; in particular, having the opportunity to teach parallel groups in the same school year (allowing for instant adjustment, polishing and refinement of the subject content – to the benefit of students in

the second group). Berliner (2001) highlights a similar finding, with student teachers preferring to 'teach the same thing twice' giving them an opportunity to improve and 'polish' their practice (2001, p. 474).

Immersion in teaching mathematics does suggest opportunity for substantial subject knowledge growth (Ball et al., 2000; Berliner, 2001; Jacob & Rockoff, 2011). There does however seem to be one caveat: for teachers with the least subject knowledge and the least confidence, teaching full time mathematics may be too stressful for the teacher and too *damaging* for the students (Bea: Interview 4). Katy believed she would have been *overwhelmed* if she had more than one mathematics group at the outset, believing she simply did not have enough subject knowledge to plan for, and engage with, more than one group (Interview 3); similarly so, for Janet. Both Anna and Euan initially noted they appreciated having only one mathematics group on which to focus but later believed being immersed in teaching mathematics would have been a far more productive experience otherwise as Euan articulated, it would take 5 or 6 years to accrue the same experience as a fulltime mathematics teacher would accumulate in a single year. Anna did express her concern that the increased demands of being immersed in teaching mathematics would result in a deterioration of her lesson plans; support and additional time for planning could provide a solution. Bea was a strong advocate for immersion, believing you learn best by teaching. She did however temper this stance with her concerns for students; if a teacher is *not very good at it* a lot of students could be badly affected (Interview 3).

Without exception, all the participants appeared to embrace the ethos of teaching for understanding and to promote deep thinking – the retraining providing the catalyst to provoke their interest. Some teachers were able to enact this style more successfully than others. All agreed secure subject knowledge to be essential for this approach but for some this was a tricky obstacle and for Janet, an insurmountable barrier. Providing sustained subject support after a retraining course seems an essential step. Such support could be provided by mentoring and coaching.

7.2. Mentoring and coaching

All the participants reported a strong positive response to the mentoring and coaching experience and all considered it to be very helpful for the development of their practice.

A safe and non-judgmental approach to the mentoring was something considered by the participants to be key to the success of the training. They believed with Harvey that there *are not many opportunities to get feedback that is non-judgmental – this is an opportunity to get [that] feedback* (discussion following Lesson Observation 6). The teachers actively wanted to engage with the process,and willingly relinquished significant periods of their free time to do so, this indicative of the high regard in which it was held; Katy:

> Thank YOU! I wish you could come more often. That really was one of the most useful pedagogic discussions I've been able to have in years! I wish lesson observations were more like that (constructive). This is the best observational process – it is developmental and non-judgmental. (email: October 2015)

The written and verbal feedback were valued equally by participants. Engaging in a verbal discussion immediately after the lesson enabled a two-way, inclusive, reflective process;

the detailed written feedback, received a few days later, was valued for its longevity and as a point of reference. Both formats fulfilled Wragg's (2012) directive that if lessons are 'worth observing' then they are also worthy of proper critique and feedback (2012, p. 2).

Bea and Anna, both strong advocates for sustained mentoring and coaching, shared the concern of who *in-house* could fulfil the role, believing experience and subject specialist knowledge to be essential (Anna: Interview 3). Katy, Cath, Janet and Euan all expressed a similar concern believing a mentor should be an expert practitioner; *It is important to be observed by someone who is a good teacher. The mentor needs to be credible* (Janet: discussion following Lesson Observation 1). The Carter (2015) review also recommends that mentors be excellent teachers who can demonstrate outstanding practice as well as disseminate valuable ideas and concepts; Wragg (2002) and Evans et al. (2014) concluded likewise.

Mentoring and coaching could be seen to have the most immediate and direct impact when a teacher had two parallel (or similar) teaching groups, the mentoring session directly influencing an improved experience for the students in the 'second' group. This was most clearly articulated by Katy but also commented upon by Bea, Anna, Cath and Janet, resonating with the idea that reaching an understanding of subject matter for teaching in a way that makes the subject 'comprehensible to others' (Shulman, 1986, p. 9) takes time as teachers reiterate the teaching of topics to different groups (Ball et al., 2000).

A powerful influence for change (Boyle et al., 2005), mentoring and coaching have enabled the teachers to realize for themselves what could be improved and enhanced (Hafford-Letchfield et al., 2008). This self-determination to change is critical (Mason, 2002). The impact of this tailored input for each teacher may now be being seen in the classroom. Later lesson observations highlighted more teaching demanding deeper thinking from learners. To maintain this momentum and to avoid stagnation or a backward slide, continued and sustained support appears necessary. A real problem arises if there appears to be no one *in house* who can adequately fulfil this role, this being the scenario described by several participants. Professional development by way of Lesson Study could provide a viable alternative.

7.3. Collaborative practice

The introduction to Lesson Study – encountered on the course – was appreciated by all participants and for some was the most valuable component of the retraining programme. Being able to practice in a *safe* and *supportive* environment (Cath, Interview 2) with *well-structured* (Janet, Personal Reflections), *constructive* (Euan, Feedback-on-feedback1) and *useful* (Harvey, Interview 2) feedback was considered invaluable. The independent evaluator, Hedger, was present during one such Lesson Study session and wrote in his report: *An excellent session that will doubtless lead to improvements in the practice of course members and a deeper understanding of higher level topics.* (Hedger, 2014). As the participants grew in confidence with these mini-Lesson Study sessions, the value of their feedback contributions to colleagues grew in breadth and depth; Hedger: *there were many supportive and creative suggestions from the group about how to improve aspects of each presentation* (2014).

Appreciating what Lesson Study actually is and defining it in culturally relevant terms is not easy; much may have been lost in translation (Wiliam, 2016). Fernandez et al. (2003) argue 'substantial challenges … must be overcome to make this practice purposeful and

powerful' in a western context (2009, p. 181); simply explaining to teachers the research nature of the process does not necessarily equip teachers to conduct and sustain such a practice (Fernandez et al., 2003). Teachers must 'learn how to generate powerful questions about their practice, skilfully design lessons that can answer the questions, and look for concrete evidence in a lesson to shed light on their questions' (Fernandez et al., 2003, p. 182). Initiating and sustaining such a process takes time, often a scarce resource (Bowland, 2014; Wiliam, 2016). Bea's comments, referring to Lesson Study as potentially *an enormous fuss for a drop in the ocean* (Interview 4), may be typical of the British mindset. And Wiliam (2016) concurs, concluding that the costs may outweigh the benefits. Katy and her department *were* in the enviable position of *having* significant synchronous planning time and yet this was never used for anything other than organizing administrative activities. Katy's lack of opportunity to collaborate with colleagues for teaching and learning purposes was typical for the participants. Katy experienced a very *closed door* approach and lamented the lack of opportunity to observe others (Interview 3). Katy, as others, believed greater senior management involvement and support to be essential.

7.4. Senior management influence

Harvey, Anna, Bea, Cath, Euan and Janet were all *roped in* to teach mathematics by their senior managements, some more willingly than others. For all the teachers – other than Harvey – the sanctioning of their attendance on the retraining course appears to be the extent of the support offered by their senior management teams. No other support during or post the course has been evident, this summed up by Katy: *there is nothing, no support, nothing… there has to be more support after the course* (Interview 3).

Guskey (2002) points out that it is the level of senior management support, or lack of, that determines the success, or otherwise, of a professional development initiative.

Cockcroft (1982) cites lack of interest and lack of support from Senior Management, following professional development, as reasons for 'training courses to result in no long-term improvement' (Cockcroft, 1982, p. 226). This may be especially pertinent when teachers have been directed, by their senior managements, to step outside their comfort zones to retrain to teach mathematics. Euan, Janet and Anna have already resigned from teaching mathematics. Cath and Katy regularly spoke about feelings of isolation stemming from lack of support, and Cath broke down discussing these issues during the final interview. A teacher's sense of well-being, involving their professional self-esteem and self-efficacy, is intertwined with their confidence in the job they are doing (Hobbs, 2015). Senior management may need to accept responsibility for this and safeguard their teachers by providing further and on-going professional development.

All the participants were initially directed to teach younger students or lower attaining mathematics groups – a decision endorsed, or even encouraged, by senior leadership. Younger and lower-attaining students, who require the most help, stand to lose significantly more than their older or higher-attaining counterparts if placed with less experienced or less effective teachers (Cockcroft, 1982 Marshall, 2013; Ofsted, 2012; Wiliam, 2016;). The loss these students can experience is often irreversible (Marshall, 2013).

The Sutton Trust (2015) suggests that professional development should be a priority for all teachers and a responsibility schools should not evade. Senior management teams

must decide how best to provide post-retraining support, be it by way of mentoring and coaching, or by collaborative practice such as Lesson Study, or by other means.

8. Conclusions

Back in 1982, Cockcroft identified the inadequate supply of competent mathematics teachers. Acknowledging the limitations of graduate recruitment, Cockcroft concluded 'any improvement in the standards of mathematics in schools must come largely as a result of the efforts of those teachers who are already in post' (1982, p. 217). The recommendation for these teachers to 'receive all possible support' (Cockcroft, 1982, p. 217) to improve the effectiveness of their teaching, can be seen to be the catalyst for government retraining initiatives such as the MDPT, SKE+ and latterly TSST each version of which has progressively become more contracted in scope, time and budget.

From this study, a particular finding is that the participants, all of whom are experienced and in some instances senior teachers, appeared to exhibit some patterns of behaviours very similar to those of student and novice teachers, described by Berliner (2001), so supporting Ingersoll's (2002) proposition that 'Highly qualified teachers, may actually become highly unqualified if they are assigned to teach subjects for which they have few qualifications or training'. Common behaviours observed, and which reflect Berliner's (2001) findings, include: the participants being 'inflexible', and sometimes ignoring or restricting interesting points made by students thereby 'letting teachable moments go' (2001, p. 475), 'fear and inadequate cognitive resources' (2001, p. 476) preventing the teachers from doing otherwise. The participants were often incapable of 'in flight' decisions' (Berliner, 2001, p. 475) and usually 'stayed close to lesson plans' (2001, p. 475), static teaching, rather than dynamic, being the norm. Many described not being able to 'understand all that was happening in a classroom while it was happening' and felt 'cognitively overloaded' (2001, p. 475). And it was common for the participants to struggle to 'accommodate a range of learner skills and abilities' (Berliner, 2001, p. 474). Apart from Bea, all, at times referred to simply not knowing 'enough of the topic to discuss it freely' (Berliner, 2001, p. 475). Only Bea regularly demonstrated the flexibility to regularly *live teach* (Bea, Interview 3), a kind of 'plan independence' demonstrated by experts when teaching in areas of their pedagogical strength (Berliner, 2001, p. 475; Schempp et al., 1998).

The participants exhibited typical behaviours to those of novices yet were expected to perform at a commitment level commensurate with their years of teaching experience, adding to an already heavy work-load burden.

Professional development programmes to retrain teachers, *may* help mitigate some of the issues faced by teachers teaching-out-of-field (TOOF). The research question in this study is designed to explore these issues and probe the impact of retraining and to determine:

- What factors most impact upon out-of-field teachers' ability to teach mathematics following their participation in an upskilling course?

Retraining teachers can clearly be seen to affect teachers' practices – but the extent and longevity of these developments depend on a multitude of factors, many beyond the control of an individual teacher. All participants could be seen to develop subject knowledge,

evident from the online assessments and from participant comments. The reach and depth of this new knowledge appeared to be dependent on the participant's starting point. All participants described a developmental process in terms of pedagogical practice with teaching practices demanding deeper thinking and understanding from students. Other successes of retraining may have been many, including motivating, inspiring and retaining teachers within the profession and the impact on student experience. Harvey, Darcy, Bea and Katy would now favour teaching mathematics, as they feel they are now making more of a difference to young lives, with *much more impact, much more impact* (Darcy, Interview 2). But there have been barriers and limitations too and the development of teachers' practices has been stymied by short-comings in in-depth subject knowledge and a complete absence of ongoing support. The factors affecting whether any changes are sustainable in terms of the embedding of practice, reflect these limitations. The suggestion from Katy that: *it was harder to teach maths the further away from the course we got* (discussion following Lesson Observation 6) may imply that her environment was limiting her ability to sustain any changes in practice previously developed (McKinsey 2007). Bea also believes the course would have more sustainable impact if professional support continued in subsequent years. The solution suggested by both Katy and Bea and endorsed by others – is for professional support to be maintained post retraining, to provide opportunities to continue to develop as a teacher of mathematics. It appears more time, space and opportunities for further professional development may be needed for changes in practice to be sustained (Boyle et al., 2005) and so embedded in everyday practice. Continuous professional development models could include mentoring and coaching, and collaborative practice such as Lesson Study (Boyle et al., 2005; Doig & Groves, 2012). Mentoring and coaching were considered hugely valuable by all the participants; there was consensus of opinion in terms of questioning where this expertise could come from, given the shortage of specialist mathematics teachers. Collaborative practice, along the lines of Lesson Study, was equally valued by all but with very limited opportunities to emulate in schools. Without on-going support, progress for teachers may be impeded – or indeed reversed. Follow-up sessions might quite simply be essential for development to be sustained in the long term (Cockcroft, 1982). Watterson (NCTL) appeared to reflect this sentiment when she acknowledged that a range of professional learning opportunities should be promoted post retraining (Watterson, 2016) – yet none were observed in practice.

9. Implications

We *could* question whether we do in fact need greater numbers of mathematics teachers. A different solution – and one at least worthy of future research – could be the clever use of technology.

Smith (2017a) encouraged this consideration, suggesting we need to be more imaginative in understanding how technology could be employed and citing social media as one example. Smith (2017a): 'the whole world is unimaginative to the use of technology; this needs to change; we need to think more creatively; teachers teaching is a tiny bit of it.' Smith (2017a) also suggested: 'If there is a star lecturer – video it and share.' The popularity of YouTube videos amongst teenagers is undeniable and these are generally easily accessible on their phones or tablets. Popular science YouTube videos are already grabbing teenagers'

attention. Harnessing this power of dissemination for mathematics could contribute to a sea-change in the styles of teaching and learning we wish to promote.

Advanced technology already exists and according to Promethean, technology for Hologram teachers has already been developed. Curtis (2017), the European head of Promethean, suggests technology could provide 'remote teaching' where there is a shortage of specialist teachers, but that technology is a tool to support teaching rather than a replacement for teachers. In essence, schools 'have not changed that much since Roman times, because at heart it is the personal contact that really matters' (Smithers & Robinson, 2013: iii). So it seems that whatever 'the technological advances, there will still be a substantial requirement' for effective mathematics teachers (Smithers & Robinson, 2013, p. iii).

Online teaching and learning opportunities do of course already exist, with more now becoming synchronous, such as the Advanced Mathematics Support Programme (AMSP) Live Online Tuition (2016). HowCloud and FutureLearn also offer online learning platforms and are clearly growing in popularity. With a shortage of specialist teachers, free online resources, such as the Wolfram Math World (2018), *may* provide more effective direction than inadequately trained non-specialist teachers.

There are also other related strategies that could be considered and piloted; for example:

- Expert teachers taking larger classes (for example, double class size or maybe, whole year groups in the hall) with other teachers and teaching assistants working alongside the main teacher.
- Retraining teaching assistants who have mathematical experience, as full teachers; this though would require a part-time degree route that builds on their subject knowledge and increases their scope for teaching through dedicated lesson study.
- Recruiting more teachers from overseas but this could be morally questionable: 'Attempting to solve the current recruitment crisis with teachers from abroad helps no one' (Jones, 2017).
- Reducing the number of normal teaching sessions and utilizing online reinforcement and extension (see above).
- Accepting more students onto mathematics degree courses or designing new mathematics courses not purely focused on academic mathematics but on using and applying mathematics, with education being one of the applications available.

Recruitment to mathematics degree courses may be negatively impacted by the recent changes made to the syllabus and assessment of A Level mathematics. The more demanding, linear only model has led to a dramatic drop in pass marks for many exam boards (for example, students need only attain 55% to achieve Grade A), this to keep parity with previous years. This development, together with tougher new GCSE mathematics courses, may further fuel negative attitudes towards the study of mathematics in a society which already chooses to demonize the subject (Smith, 2017b). With a drop in mathematics A level participation already apparent (in 2019), numbers are also likely to decline at degree level suggesting an even smaller pool of graduates from which to recruit new mathematics teachers.

Handscombe (2018) suggests that we should encourage the best mathematicians at school to become teachers but much of the public evidence paints a negative picture of

teaching conditions in state secondary schools, these perceptions provoked by the Ofsted inspection service. Indeed, the NFER report (2018) gives evidence that Ofsted may contribute to the retention issue, with teachers in schools rated 'inadequate' by Ofsted more likely than most to leave the profession.

A country that does not fit the international trend of difficulties with recruitment and retention of effective mathematics teachers is Finland (Aristorena, 2018). Here there is essentially no competition between schools (learners attend their local school), there is no inspection service and no national tests until age 18. Headteachers and teachers are trusted to provide their children with an effective and rounded education. The teaching profession is held in high regard and there is real competition to become a teacher (The Guardian, 2015) and, crucially, the country is a very high performer in international tests (including those for mathematics, science and native language (Education GPS OECD, 2020)).

The Finland model may seem a step too far for many but as is clear from the data in two recent reports (NFER, 2018), (Parliament. House of Commons Library, 2018) the situation surrounding the shortage of skilled teachers of mathematics in England appears to be deteriorating; there is a major crisis with the supply and demand for secondary teachers, particularly mathematics. Whilst retraining provision needs to be enhanced along the lines suggested above, we also need creative initiatives, researched, piloted and developed, to ensure that future generations are given every opportunity to become confident and capable mathematicians with transferable skills for the economic wellbeing of the country.

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