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Type-1 OWA Operators in Aggregating Multiple Sources of Uncertain Information: Properties and Real-World Applications in Integrated Diagnosis

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Supplemental Material for the Manuscript - "*Type-1 OWA Operators in Aggregating Multiple Sources of Uncertain Information : Properties and Real World Applications in Integrated Diagnosis*"

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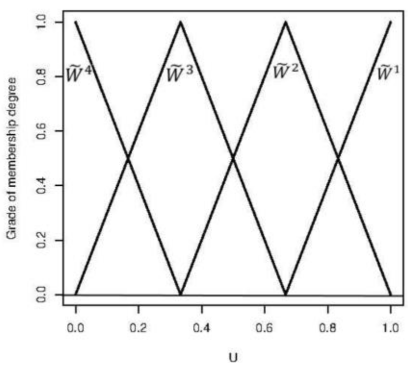
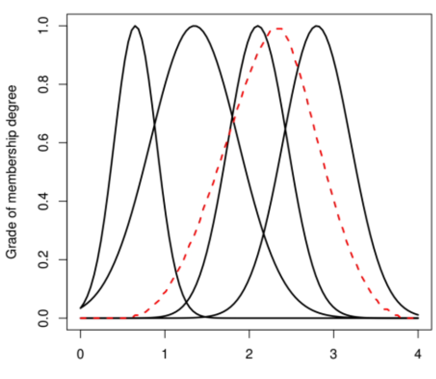
I. More Examples of Type-1 OWA Aggregations

Example 1 (A T1OWA Operator)**.** *Figure 1 depicts the results of a T1OWA operator to aggregate four fuzzy sets. Figure 1a shows the linguistic weights. Figure 1b shows the aggregated fuzzy set objects and aggregation result.*



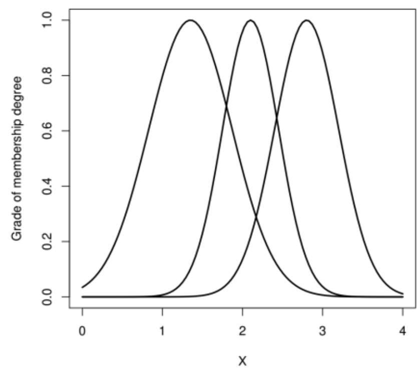
(a) Fuzzy sets as linguistic weights in T1OWA aggregation



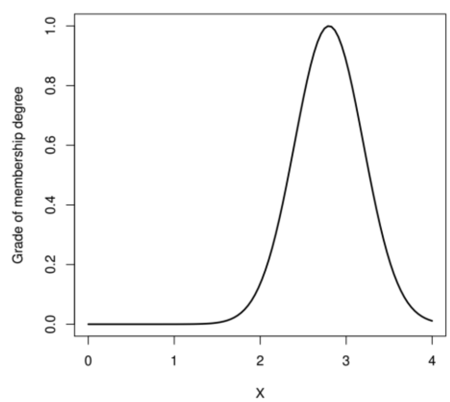
(b) Aggregation result in dashed lines; aggregated fuzzy set objects in solid lines

Fig. 1: A T1OWA operator

Example 2 (Join Operator). *Figure 2 illustrates the result of join operator to aggregate fuzzy sets as the extended maximum of fuzzy sets, a special T1OWA operator.*



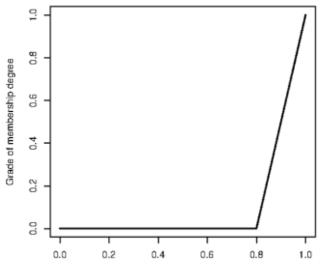
(a) Fuzzy sets to be aggregated



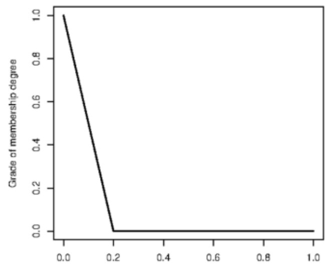
(b) Aggregating result

Fig. 2: Aggregation of T1OWA operator as join

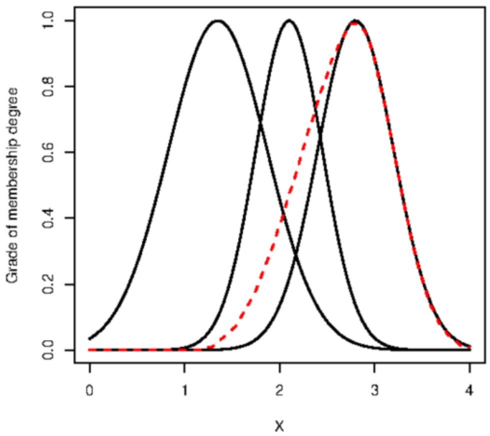
Example 3 (Join-Like T1OWA Operator)**.** *The join-like T1OWA operation requests that the first linguistic weight move towards the singleton fuzzy set,* $\tilde{1}$*, and all others towards the singleton fuzzy set* $\tilde{0}$ *(Figure 4a and Figure 4b). Using these weights to aggregate the fuzzy sets, Figure 3c illustrates how this T1OWA exhibits the join-like behaviour when aggregating three fuzzy sets. Clearly, the final aggregation fuzzy set is closer to the rightmost fuzzy set (i.e., the fuzzified maximum)*



(a)$\widetilde{W}_1$



(b) $\widetilde{W}_i(i \neq 1)$



(c) Aggregation result

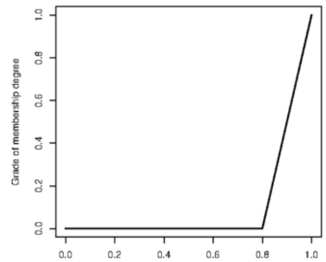
Fig. 3: A join-like T1OWA operator. (a)(b)-Linguistic weights;(c)-Aggregation result. Solid lines representing the aggregated objects; dashed line representing the aggregation result.

Example 4 (Joinness of Join-Like T1OWA Operator)**.** *The Figure 4c shows an example of the joinness of the join-like T1OWA operator defined by the fuzzy weights in Figure 4a and Figure 4b.*

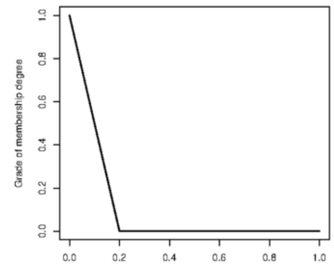
Example 5 (Meet Operator)**.** *Figure 5 illustrates the result of meet operator to aggregate fuzzy sets as the extended minimum of fuzzy sets, a special T1OWA operator.*

Example 6 (Meet-Like T1OWA Operator)**.** *Meet-like T1OWA operation requests that the last linguistic weight to be close to* $\tilde{1}$ *(Figure 4a), and all other weights close to* $\tilde{0}$ *(Figure 4b). Figure 6 illustrates how this T1OWA exhibits the meet-like behaviour when aggregating three fuzzy sets. Clearly, the final aggregation fuzzy set is closer to the leftmost fuzzy set (i.e., the fuzzified minimum).*

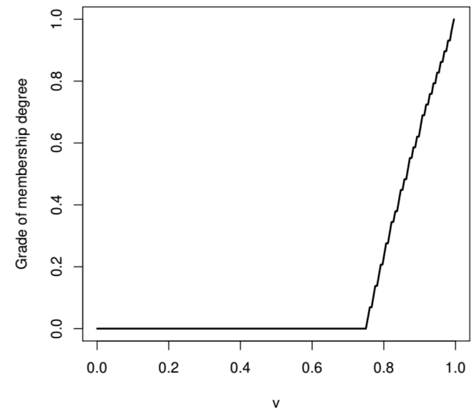
Example 7 (Counterexample of α-Equivalently Ordered Relation)**.** Figure 7 shows the groups of fuzzy numbers $\{\tilde{B}^1, \tilde{B}^2, \tilde{B}^3\}$ are not α-equivalently ordered with another group $\{\tilde{A}^1, \tilde{A}^2, \tilde{A}^3\}$*:* Figure 7a shows that at the α = 0*:*2 level: $\tilde{A}^3_{0.2+} \geq \tilde{A}^2_{0.2+} \geq \tilde{A}^1_{0.2+}$, i.e., the permutation operator σ = (3*;* 2*;* 1), but $\tilde{B}^3_{0.2+} \geq \tilde{B}^1_{0.2+} \geq \tilde{B}^2_{0.2+}$; while Fig.7b shows that $\tilde{A}^3_{0.2-} \geq \tilde{A}^2_{0.2-} \geq \tilde{A}^1_{0.2-}$, , i.e., the permutation operator η = (3*;* 2*;* 1), but $\tilde{B}^3_{0.2-} \geq \tilde{B}^1_{0.2-} \geq \tilde{B}^2_{0.2-}$.



(a) $\widetilde{W}_1$



(b) $\widetilde{W}_i(i \neq 1)$



(c) Joinness

Fig. 4: Joinness of a join-like T1OWA operator

Example 8. *Supposing the numerical domains U ={0.0, 0.5,1.0} and* ***X*** *= {0.0, 1.0, 2.0}. Let the given linguistic*

weights $\widetilde{W} = \begin{pmatrix} \omega_i \\ \mu_{\widetilde{W}}(\omega_i) \end{pmatrix}_{\omega_i \in U}$ *on* ***U*** *be*

$$\widetilde{W}^1 = \begin{pmatrix} 0.0 & 0.5 & 1.0 \\ 1.0 & 0.5 & 0.0 \end{pmatrix}; \widetilde{W}^2 = \begin{pmatrix} 0.0 & 0.5 & 1.0 \\ 0.0 & 1.0 & 0.0 \end{pmatrix};$$

$$\widetilde{W}^3 = \begin{pmatrix} 0.0 & 0.5 & 1.0 \\ 0.0 & 0.5 & 1.0 \end{pmatrix}$$

and the fuzzy sets to be aggregated be

$$\tilde{A}^1 = \begin{pmatrix} 0.0 & 1.0 & 2.0 \\ 0.0 & 0.5 & 1.0 \end{pmatrix}; \tilde{A}^2 = \begin{pmatrix} 0.0 & 1.0 & 2.0 \\ 1.0 & 0.5 & 0.0 \end{pmatrix};$$

$$\tilde{A}^1 = \begin{pmatrix} 0.0 & 1.0 & 2.0 \\ 0.0 & 1.0 & 0.0 \end{pmatrix}.$$

Let G be the aggregation result of this T1OWA defined by the above linguistic weights. The aggregation result by the join operator J can be calculated as follows.

Case I. *α = 0.0*
As demonstrated by the Example 1 in [5], *one can calculate G*$_\alpha$=*{0.0,1.0,2.0}*. But

$$J_\alpha(\tilde{A}^1_\alpha, \tilde{A}^2_\alpha, \tilde{A}^3_\alpha)_+ = max(\tilde{A}^1_{\alpha+}, \tilde{A}^2_{\alpha+}, \tilde{A}^3_{\alpha+})$$
$$=2.0$$

$$J_\alpha(\tilde{A}^1_\alpha, \tilde{A}^2_\alpha, \tilde{A}^3_\alpha)_- = max(\tilde{A}^1_{\alpha-}, \tilde{A}^2_{\alpha-}, \tilde{A}^3_{\alpha-})$$

$$=0.0$$

So $G_{\alpha-} \leq J_{\alpha-},\ G_{\alpha+} \leq J_{\alpha+}$.

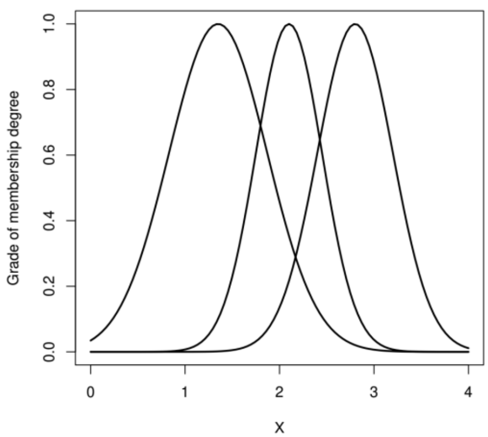
Case II. *α = 0.5*

As demonstrated by the Example 1 in [5], *one can calculate* G_α={*1.0*}. But

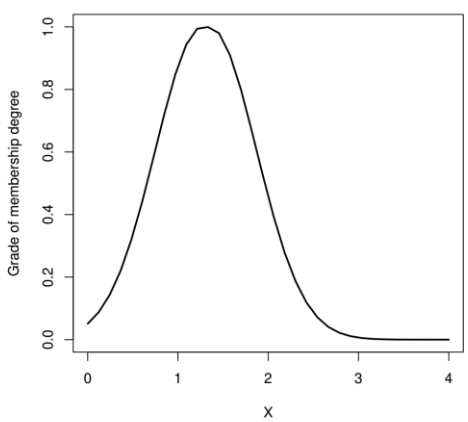
$$J_\alpha\left(\tilde{A}_\alpha^1, \tilde{A}_\alpha^2, \tilde{A}_\alpha^3\right)_+ = max\left(\tilde{A}_{\alpha+}^1, \tilde{A}_{\alpha+}^2, \tilde{A}_{\alpha+}^3\right)$$
$$= max\ (2.0,1.0,1.0)$$
$$=2.0$$

$$J_\alpha\left(\tilde{A}_\alpha^1, \tilde{A}_\alpha^2, \tilde{A}_\alpha^3\right)_- = max\left(\tilde{A}_{\alpha-}^1, \tilde{A}_{\alpha-}^2, \tilde{A}_{\alpha-}^3\right)$$
$$= max\ (1.0,0.0,1.0)$$
$$=1.0$$

So $G_{\alpha-} \leq J_{\alpha-},\ G_{\alpha+} \leq J_{\alpha+}$.



(a) Fuzzy sets to be aggregated



(b) Aggregating result

Fig. 5: Aggregation of T1OWA operator as meet

Case III. *α = 1.0*

As demonstrated by the Example 1 in [5], *one can calculate* $G_\alpha = \emptyset$. But

$$J_\alpha\left(\tilde{A}_\alpha^1, \tilde{A}_\alpha^2, \tilde{A}_\alpha^3\right)_+ = max\left(\tilde{A}_{\alpha+}^1, \tilde{A}_{\alpha+}^2, \tilde{A}_{\alpha+}^3\right)$$
$$= max\ (2.0,0.0,1.0)$$
$$=2.0$$

$$J_\alpha\left(\tilde{A}_\alpha^1, \tilde{A}_\alpha^2, \tilde{A}_\alpha^3\right)_- = max\left(\tilde{A}_{\alpha-}^1, \tilde{A}_{\alpha-}^2, \tilde{A}_{\alpha-}^3\right)$$
$$= max\ (2.0,0.0,1.0)$$
$$=2.0$$

So $G_{\alpha-} \leq J_{\alpha-},\ G_{\alpha+} \leq J_{\alpha+}$.

Then according to the Definition about partial order relation of fuzzy sets, we have J $\succcurlyeq$ *G*. *Similarly, G* $\succcurlyeq$ *M, the meet operator.*

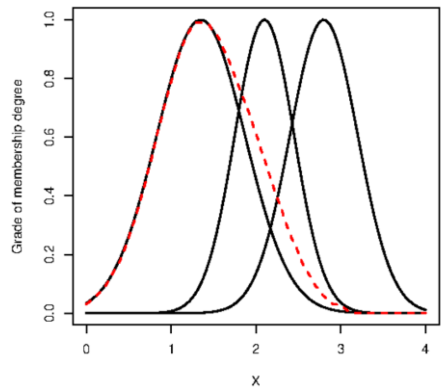
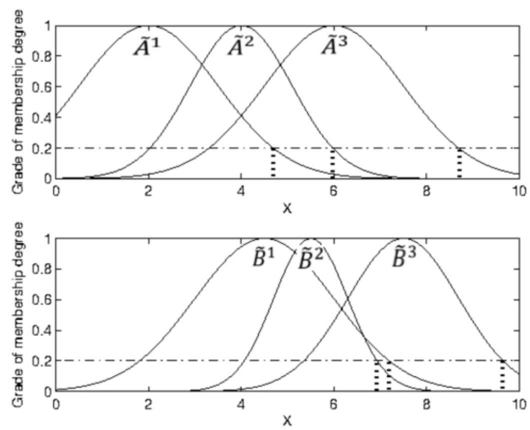
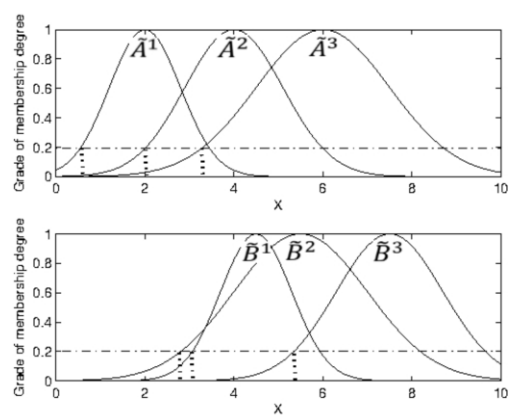


Fig. 6: Aggregation result by a meet-like T1OWA operator. Solid lines representing the aggregated objects; dashed line representing the aggregation result.



(a) $\tilde{A}_{0.2+}^3 \geq \tilde{A}_{0.2+}^2 \geq \tilde{A}_{0.2+}^1$ *and* $\tilde{B}_{0.2+}^3 \geq \tilde{B}_{0.2+}^1 \geq \tilde{B}_{0.2+}^2$



(b) $\tilde{A}_{0.2-}^3 \geq \tilde{A}_{0.2-}^2 \geq \tilde{A}_{0.2-}^1$ *and* $\tilde{B}_{0.2-}^3 \geq \tilde{B}_{0.2-}^1 \geq \tilde{B}_{0.2-}^2$

Fig. 7: Fuzzy numbers $\widetilde{B}^1, \widetilde{B}^2, \widetilde{B}^3$ (bottom) not α-*equivalently ordered* with $\widetilde{A}^1, \widetilde{A}^2, \widetilde{A}^3$ (up) separately

II. CASE STUDY

A. The T1OWA based non-stationary fuzzy system

Figure 8 illustrates the structure of T1OWA based nonstationary fuzzy inference system, the T1OWA operator is used to aggregate the multiple fuzzy decisions from non-stationary fuzzy inference engine.

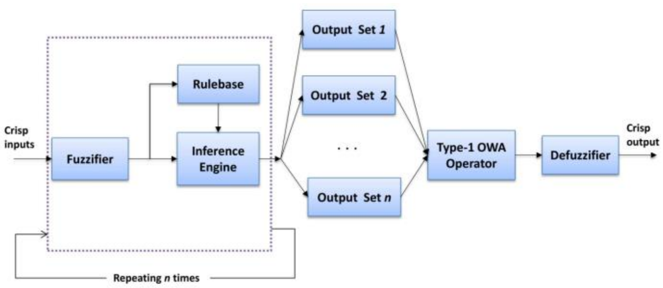
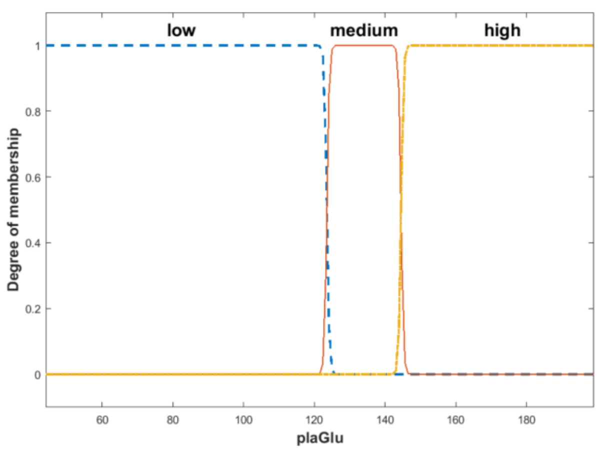


Fig. 8: T1OWA based non-stationary fuzzy inference system.

B. Fuzzy sets of the variables - "plaGlu", "BMI" and "Outcome



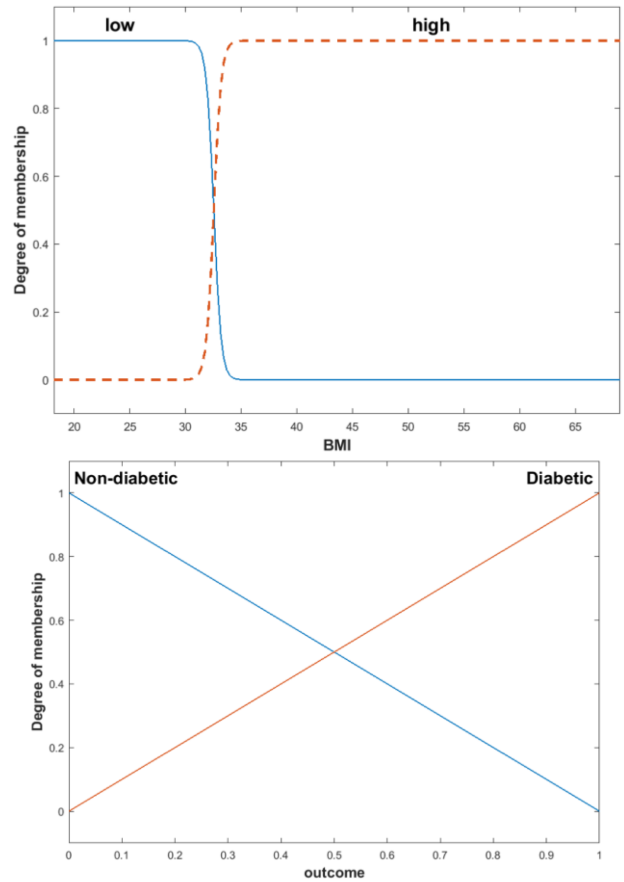


Fig. 9: Membership functions of fuzzy sets for the attributes of *plaGlu, BMI* and *outcome.*

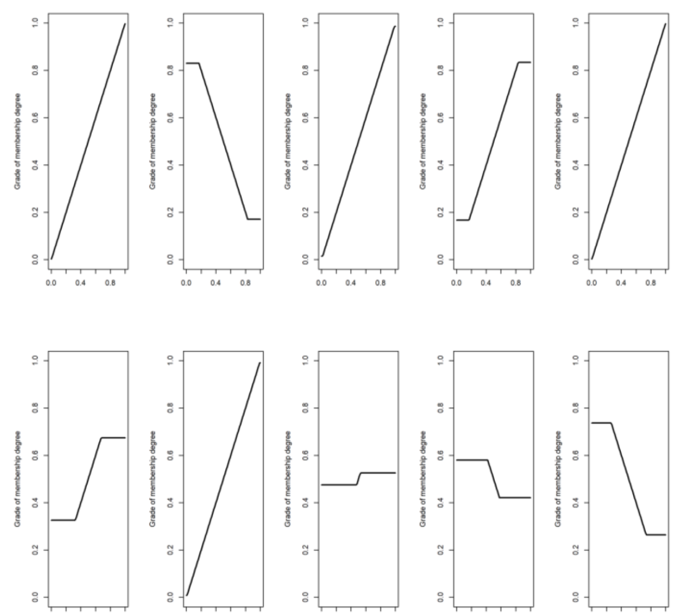


Fig. 10: Example of 10 fuzzy output decisions by the nonstationary fuzzy system for a patient.

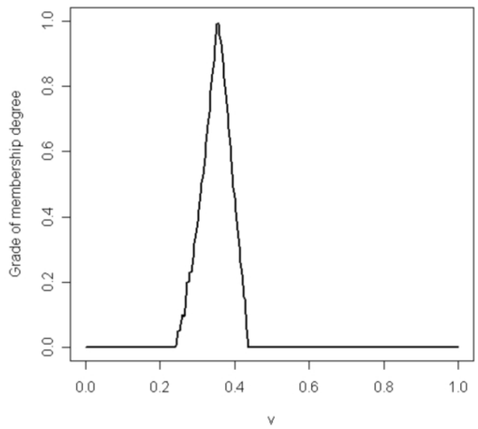


Fig. 11: Joinness of the type-2 quantifier “*most*” guided T1OWA operator.