

Supplemental Material for the Manuscript - “Type-1 OWA Operators in Aggregating Multiple Sources of Uncertain Information : Properties and Real World Applications in Integrated Diagnosis”

Shang-Ming Zhou, Member, IEEE, Francisco Chiclana, Robert I. John, Senior Member, IEEE, Jon M. Garibaldi, Senior Member, IEEE, and Lin Huo

I. MORE EXAMPLES OF TYPE-1 OWA AGGREGATIONS

Example 1 (A T1OWA Operator). *Figure 1 depicts the results of a T1OWA operator to aggregate four fuzzy sets. Figure 1a shows the linguistic weights. Figure 1b shows the aggregated fuzzy set objects and aggregation result.*

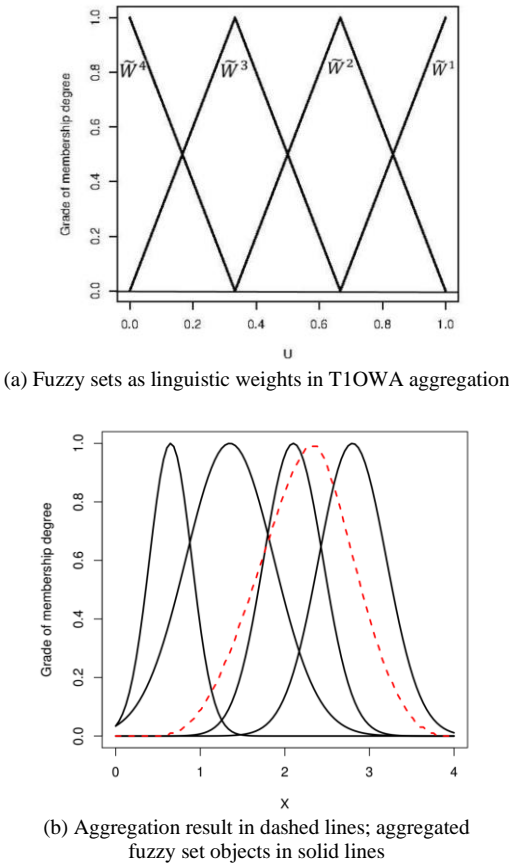


Fig. 1: A T1OWA operator

Example 2 (Join Operator). *Figure 2 illustrates the result of join operator to aggregate fuzzy sets as the extended maximum of fuzzy sets, a special T1OWA operator.*

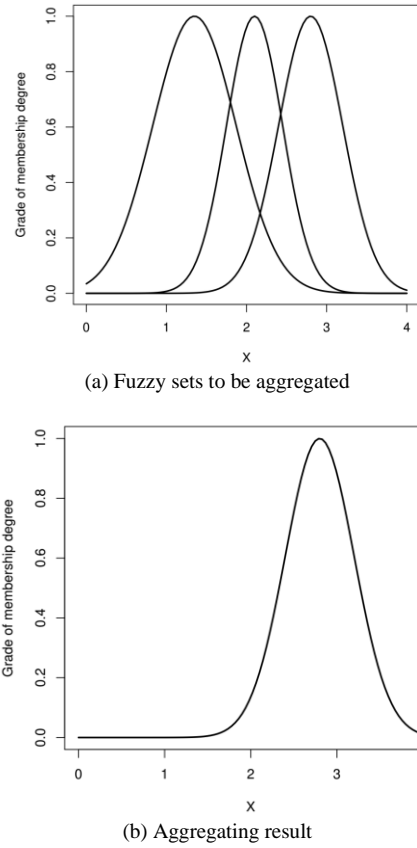


Fig. 2: Aggregation of T1OWA operator as join

Example 3 (Join-Like T1OWA Operator). *The join-like T1OWA operation requests that the first linguistic weight move towards the singleton fuzzy set, $\tilde{1}$, and all others towards the singleton fuzzy set $\tilde{0}$ (Figure 4a and Figure 4b). Using these weights to aggregate the fuzzy sets, Figure 3c illustrates how this T1OWA exhibits the join-like behaviour when aggregating three fuzzy sets. Clearly, the final aggregation fuzzy set is closer to the rightmost fuzzy set (i.e., the fuzzified maximum)*

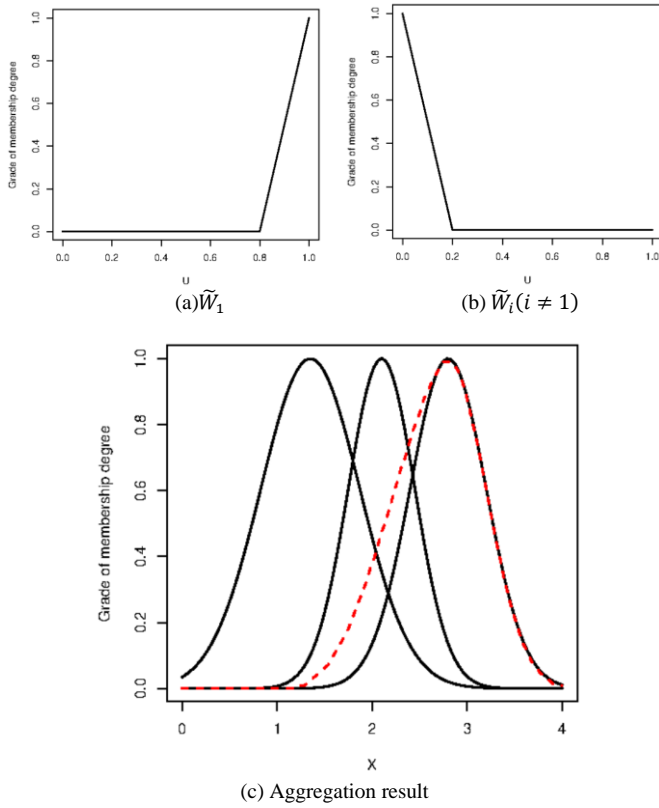


Fig. 3: A join-like T1OWA operator. (a)(b)-Linguistic weights;(c)-Aggregation result. Solid lines representing the aggregated objects; dashed line representing the aggregation result.

Example 4 (Joinness of Join-Like T1OWA Operator). The Figure 4c shows an example of the joinness of the join-like T1OWA operator defined by the fuzzy weights in Figure 4a and Figure 4b.

Example 5 (Meet Operator). Figure 5 illustrates the result of meet operator to aggregate fuzzy sets as the extended minimum of fuzzy sets, a special T1OWA operator.

Example 6 (Meet-Like T1OWA Operator). Meet-like T1OWA operation requests that the last linguistic weight to be close to $\bar{1}$ (Figure 4a), and all other weights close to $\bar{0}$ (Figure 4b). Figure 6 illustrates how this T1OWA exhibits the meet-like behaviour when aggregating three fuzzy sets. Clearly, the final aggregation fuzzy set is closer to the leftmost fuzzy set (i.e., the fuzzified minimum).

Example 7 (Counterexample of α -Equivalently Ordered Relation). Figure 7 shows the groups of fuzzy numbers $\{\tilde{B}^1, \tilde{B}^2, \tilde{B}^3\}$ are not α -equivalently ordered with another group $\{\tilde{A}^1, \tilde{A}^2, \tilde{A}^3\}$: Figure 7a shows that at the $\alpha = 0.2$ level: $\tilde{A}_{0.2+}^3 \geq \tilde{A}_{0.2+}^2 \geq \tilde{A}_{0.2+}^1$, i.e., the permutation operator $\sigma = (3; 2; 1)$, but $\tilde{B}_{0.2+}^3 \geq \tilde{B}_{0.2+}^1 \geq \tilde{B}_{0.2+}^2$; while Fig.7b shows that $\tilde{A}_{0.2-}^3 \geq \tilde{A}_{0.2-}^2 \geq \tilde{A}_{0.2-}^1$, i.e., the permutation operator $\eta = (3; 2; 1)$, but $\tilde{B}_{0.2-}^3 \geq \tilde{B}_{0.2-}^1 \geq \tilde{B}_{0.2-}^2$.

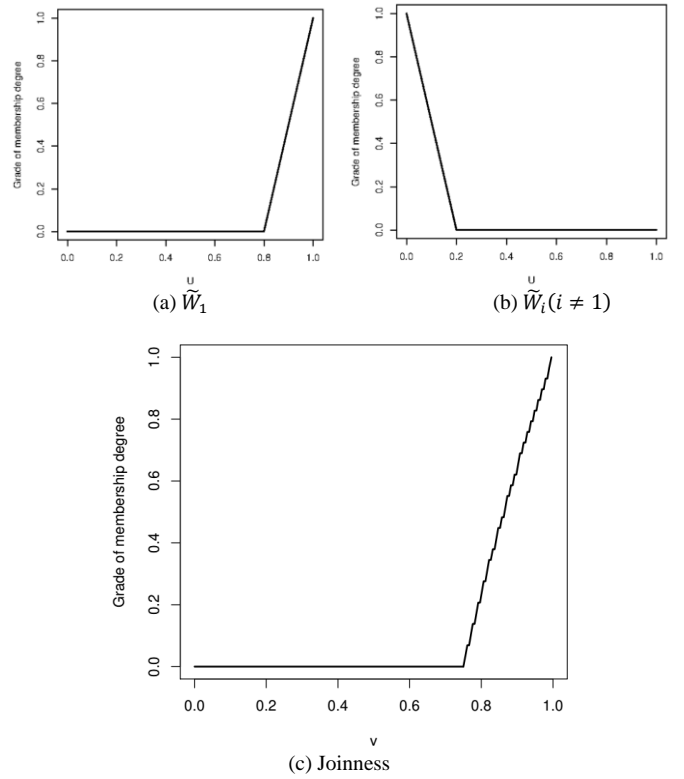


Fig. 4: Joinness of a join-like T1OWA operator

Example 8. Supposing the numerical domains $U = \{0.0, 0.5, 1.0\}$ and $X = \{0.0, 1.0, 2.0\}$. Let the given linguistic

weights $\tilde{W} = \left(\mu_{\tilde{W}}(\omega_i) \right)_{\omega_i \in U}$ on U be

$$\tilde{W}^1 = \begin{pmatrix} 0.0 & 0.5 & 1.0 \\ 1.0 & 0.5 & 0.0 \end{pmatrix}; \tilde{W}^2 = \begin{pmatrix} 0.0 & 0.5 & 1.0 \\ 0.0 & 1.0 & 0.0 \end{pmatrix};$$

$$\tilde{W}^3 = \begin{pmatrix} 0.0 & 0.5 & 1.0 \\ 0.0 & 0.5 & 1.0 \end{pmatrix}$$

and the fuzzy sets to be aggregated be

$$\tilde{A}^1 = \begin{pmatrix} 0.0 & 1.0 & 2.0 \\ 0.0 & 0.5 & 1.0 \end{pmatrix}; \tilde{A}^2 = \begin{pmatrix} 0.0 & 1.0 & 2.0 \\ 1.0 & 0.5 & 0.0 \end{pmatrix};$$

$$\tilde{A}^3 = \begin{pmatrix} 0.0 & 1.0 & 2.0 \\ 0.0 & 1.0 & 0.0 \end{pmatrix}.$$

Let G be the aggregation result of this T1OWA defined by the above linguistic weights. The aggregation result by the join operator J can be calculated as follows.

Case I. $\alpha = 0.0$

As demonstrated by the Example 1 in [5], one can calculate $G_\alpha = \{0.0, 1.0, 2.0\}$. But

$$J_\alpha(\tilde{A}_\alpha^1, \tilde{A}_\alpha^2, \tilde{A}_\alpha^3)_+ = \max(\tilde{A}_{\alpha+}^1, \tilde{A}_{\alpha+}^2, \tilde{A}_{\alpha+}^3) = 2.0$$

$$J_\alpha(\tilde{A}_\alpha^1, \tilde{A}_\alpha^2, \tilde{A}_\alpha^3)_- = \max(\tilde{A}_{\alpha-}^1, \tilde{A}_{\alpha-}^2, \tilde{A}_{\alpha-}^3)$$

$$=0.0$$

So $G_{\alpha-} \leq J_{\alpha-}$, $G_{\alpha+} \leq J_{\alpha+}$.

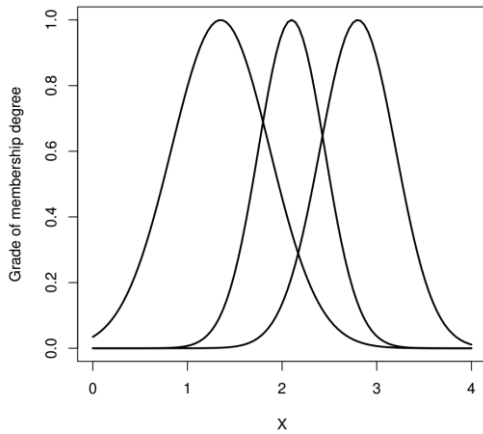
Case II. $\alpha = 0.5$

As demonstrated by the Example 1 in [5], one can calculate $G_{\alpha} = \{1.0\}$. But

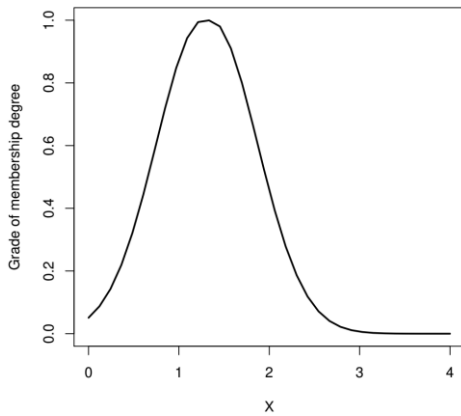
$$\begin{aligned} J_{\alpha}(\tilde{A}_{\alpha}^1, \tilde{A}_{\alpha}^2, \tilde{A}_{\alpha}^3)_{+} &= \max(\tilde{A}_{\alpha+}^1, \tilde{A}_{\alpha+}^2, \tilde{A}_{\alpha+}^3) \\ &= \max(2.0, 1.0, 1.0) \\ &= 2.0 \end{aligned}$$

$$\begin{aligned} J_{\alpha}(\tilde{A}_{\alpha}^1, \tilde{A}_{\alpha}^2, \tilde{A}_{\alpha}^3)_{-} &= \max(\tilde{A}_{\alpha-}^1, \tilde{A}_{\alpha-}^2, \tilde{A}_{\alpha-}^3) \\ &= \max(1.0, 0.0, 1.0) \\ &= 1.0 \end{aligned}$$

So $G_{\alpha-} \leq J_{\alpha-}$, $G_{\alpha+} \leq J_{\alpha+}$.



(a) Fuzzy sets to be aggregated



(b) Aggregating result

Fig. 5: Aggregation of TIOWA operator as meet

Case III. $\alpha = 1.0$

As demonstrated by the Example 1 in [5], one can calculate $G_{\alpha} = \emptyset$. But

$$\begin{aligned} J_{\alpha}(\tilde{A}_{\alpha}^1, \tilde{A}_{\alpha}^2, \tilde{A}_{\alpha}^3)_{+} &= \max(\tilde{A}_{\alpha+}^1, \tilde{A}_{\alpha+}^2, \tilde{A}_{\alpha+}^3) \\ &= \max(2.0, 0.0, 1.0) \\ &= 2.0 \end{aligned}$$

$$\begin{aligned} J_{\alpha}(\tilde{A}_{\alpha}^1, \tilde{A}_{\alpha}^2, \tilde{A}_{\alpha}^3)_{-} &= \max(\tilde{A}_{\alpha-}^1, \tilde{A}_{\alpha-}^2, \tilde{A}_{\alpha-}^3) \\ &= \max(2.0, 0.0, 1.0) \\ &= 2.0 \end{aligned}$$

So $G_{\alpha-} \leq J_{\alpha-}$, $G_{\alpha+} \leq J_{\alpha+}$.

Then according to the Definition about partial order relation of fuzzy sets, we have $J \succcurlyeq G$. Similarly, $G \succcurlyeq M$, the meet operator.

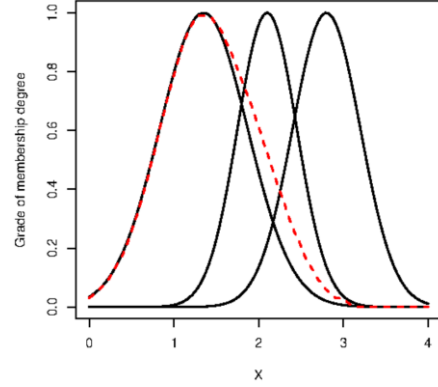
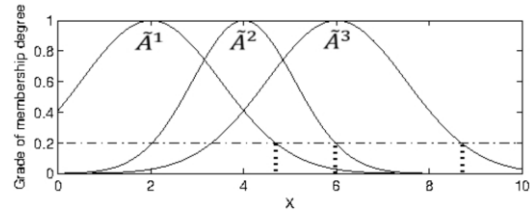
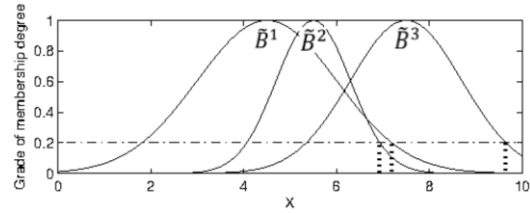


Fig. 6: Aggregation result by a meet-like TIOWA operator. Solid lines representing the aggregated objects; dashed line representing the aggregation result.



(a) $\tilde{A}_{0.2+}^3 \geq \tilde{A}_{0.2+}^2 \geq \tilde{A}_{0.2+}^1$ and $\tilde{B}_{0.2+}^3 \geq \tilde{B}_{0.2+}^1 \geq \tilde{B}_{0.2+}^2$



(b) $\tilde{A}_{0.2-}^3 \geq \tilde{A}_{0.2-}^2 \geq \tilde{A}_{0.2-}^1$ and $\tilde{B}_{0.2-}^3 \geq \tilde{B}_{0.2-}^1 \geq \tilde{B}_{0.2-}^2$

Fig. 7: Fuzzy numbers $\tilde{B}^1, \tilde{B}^2, \tilde{B}^3$ (bottom) not α -equivalently ordered with $\tilde{A}^1, \tilde{A}^2, \tilde{A}^3$ (up) separately

II. CASE STUDY

A. The TIOWA based non-stationary fuzzy system

Figure 8 illustrates the structure of TIOWA based nonstationary fuzzy inference system, the TIOWA operator is used to aggregate the multiple fuzzy decisions from non-stationary fuzzy inference engine.

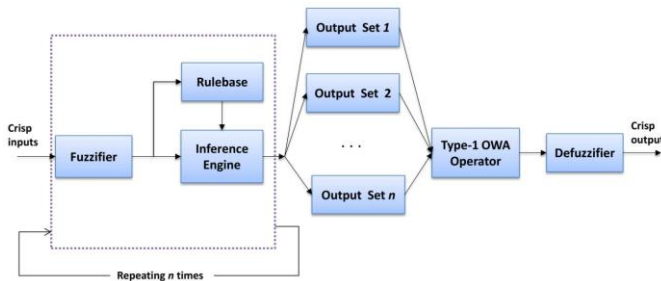


Fig. 8: TIOWA based non-stationary fuzzy inference system.

B. Fuzzy sets of the variables - "plaGlu", "BMI" and "Outcome"

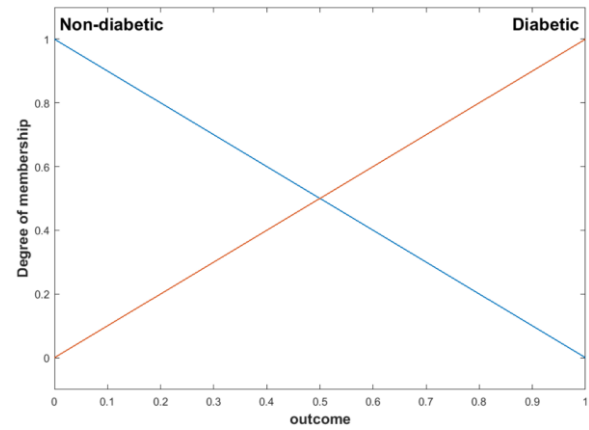
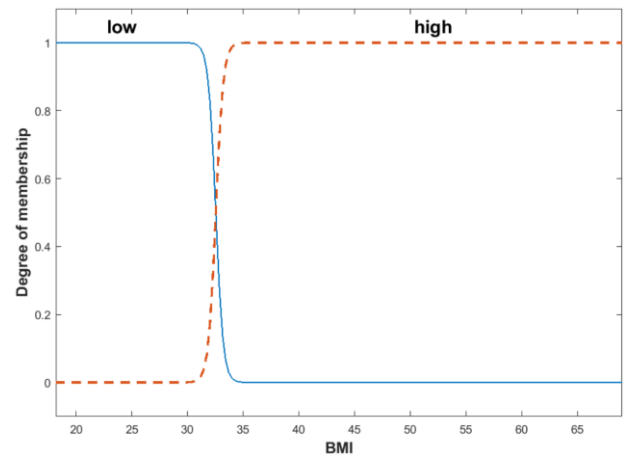
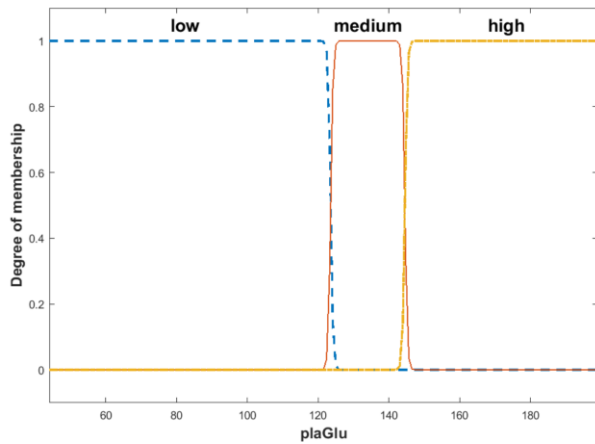


Fig. 9: Membership functions of fuzzy sets for the attributes of *plaGlu*, *BMI* and *outcome*.

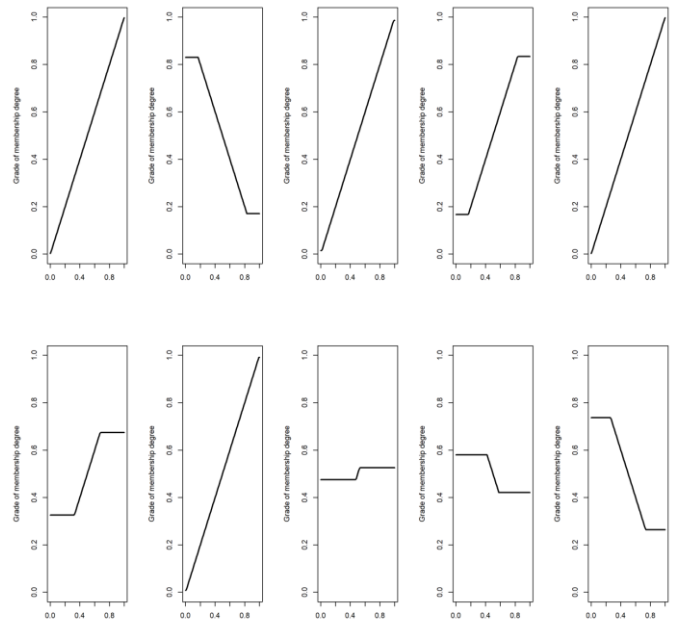


Fig. 10: Example of 10 fuzzy output decisions by the nonstationary fuzzy system for a patient.

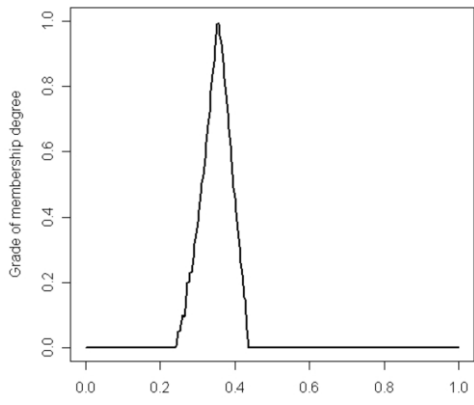


Fig. 11: Joinness of the type-2 quantifier “*most*” guided T1OWA operator.