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## Characterisation of fatigue crack growth using the CJP model of crack tip fields or plastic CTOD

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### Abstract

This paper provides an update on the current status of work that has been done to validate the CJP model of crack tip stresses and also summarises some findings from linked work that has used DIC to determine the range of plastic crack tip opening displacement as a correlator of fatigue crack growth rate. The paper considers several ways of calculating an effective range of stress intensity factor that have been proposed in various papers, as the preferred option is not yet fully clear. It further highlights the potential value, in terms of elucidating the mechanisms involved in plasticity-induced crack tip shielding, arising from data obtained from using two different DIC techniques on the same specimens.

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### 1. Introduction

The CJP model is a meso-scale model of crack tip displacement and stress that was proposed a few years ago as an attempt to better characterise the elastic forces induced by the plastic enclave that surrounds a growing fatigue crack

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and hence enable direct prediction of the effective range of crack driving force. Considerable work has now been completed using this model, through a research grouping among Plymouth, Jaén (Spain), Gifu University (Japan), Southwest Jiaotong University and Xiamen University (China). This has convincingly demonstrated that the model has significant potential to provide a physically-based characterisation of fatigue crack growth (Yang, Vasco-Olmo et al. 2018) and hence also provide insight into the mechanisms underlying certain fatigue phenomena, e.g. overloads (Vasco-Olmo, Yang et al. 2018). Some of these conclusions have also been independently supported by other workers (Nowell, Dragnevski et al. 2018). However, the CJP stress intensity factors offer several ways of calculating an effective range of stress intensity factor and the preferred option is not yet fully clear. Equally, a significant body of work has successfully applied crack tip opening displacement (CTOD) to fatigue crack growth rate characterisation with the best results arising, unsurprisingly, from use of the plastic range of CTOD for this purpose. Some of the present authors have applied the DIC technique to the determination of plastic CTOD on Grade 2 titanium compact tension (CT) specimens at stress ratio values of 0.1 and 0.6 (Vasco-Olmo, Díaz et al. 2019); these specimens were also used to obtain CJP stress intensity parameters. This paper will summarise the results of this recent work on characterisation of fatigue crack growth rate in titanium and 2024-T6 aluminium CT specimens using the CJP stress intensity parameters and plastic CTOD. It will also highlight the value, in terms of elucidating the mechanisms involved in plasticity-induced crack tip shielding, arising from data obtained from two different DIC techniques on the same specimens.

### Nomenclature

CJP model	Christopher, James, Patterson model of crack tip stress and displacement fields
DIC	Digital image correlation technique
CTOD	Crack tip opening displacement
$\Delta\text{CTOD}_p$	Range of plastic CTOD
$K_F$	CJP stress intensity factor driving crack growth forwards
$K_R$	CJP stress intensity factor resisting crack growth
Q&T	Quenched and tempered

## 2. Progress in the development and application of the CJP model

The model was originally developed for Mode I crack tip stress fields and its innovation was to consider the elastic stresses induced through the fatigue process at the elastic-plastic interface between the plastic region surrounding a crack and the bulk elastic material in the specimen (Christopher, James et al. 2008). The rationale underlying development of the model was to obtain a modified set of stress intensity factors that take account of the influences on the forward elastic field driving crack growth that arise from the plastic enclave surrounding a growing fatigue crack. The model therefore takes account of forces potentially induced by plasticity-induced closure as well as shear stresses induced along the crack flanks through strain compatibility requirements between the different deformation processes in the plastic and elastic regions (Poisson's ratio is 0.5 in constant volume plastic deformation and perhaps 0.3 in elastic deformation).

As with the Irwin characterisation of crack tip stresses the CJP model defocuses attention from the near-tip process zone where cracking occurs, to consider the effects of the plasticity-based fatigue phenomenon on the elastic field ahead of the crack. The model provides modified stress intensity factors:  $K_F$  that combines the stress intensity factor of the applied load and force components arising from plasticity-induced shielding and compatibility of strains at the elastic-plastic boundary, and  $K_R$  that represents the forces resisting crack growth and that again includes components arising from plasticity-induced shielding and compatibility requirements. If there is no plasticity surrounding the crack,  $K_F$  is identical to  $K_I$ .

The CJP model was extended to cover mixed mode I and II loading and to deal with crack tip displacement fields (James, Christopher et al. 2013), and has been demonstrated (see Fig. 1) to provide an improved correlation of fatigue crack growth for Grade 2 titanium across several specimen geometries and stress ratio values, than obtained using the standard range of stress intensity factor  $\Delta K$  (Yang, Vasco-Olmo et al. 2018). Further work on a Q&T EA4T low alloy

railway axle steel (EN 13261) and on 2024-T3 aluminium alloy, where plasticity-induced shielding has a larger effect on crack growth rates than in Grade 2 titanium, demonstrated the same improved rationalisation of fatigue crack growth rates seen with the titanium (see Figs 2 and 3). It also appears to be the case that the rationalisation of crack growth data obtained using  $\Delta K_{CJP}$  applies over a wider range of stress intensity factor than the standard  $\Delta K$ , reflecting the incorporation in its derivation of crack tip plasticity influences on the elastic stress field. Thus the CJP model does appear to accurately predict the effective value of the stress intensity for growing fatigue cracks.

Other work has focused on the ability of the CJP model to characterise plastic zone size and shape, in comparison with the Williams and Westergaard models of elastic crack tip stress fields (Vasco-Olmo, James et al. 2016) and on the influence of overloads on plastic zone size (Vasco-Olmo, Diaz et al. 2018). The overall conclusions from all this work are that the CJP model provides a very useful tool for the study of fracture mechanics problems such as plasticity-induced crack shielding, the retardation effect induced by overloads on fatigue crack growth, and the mechanisms that may play a role in the crack growth transients attendant on an overload.

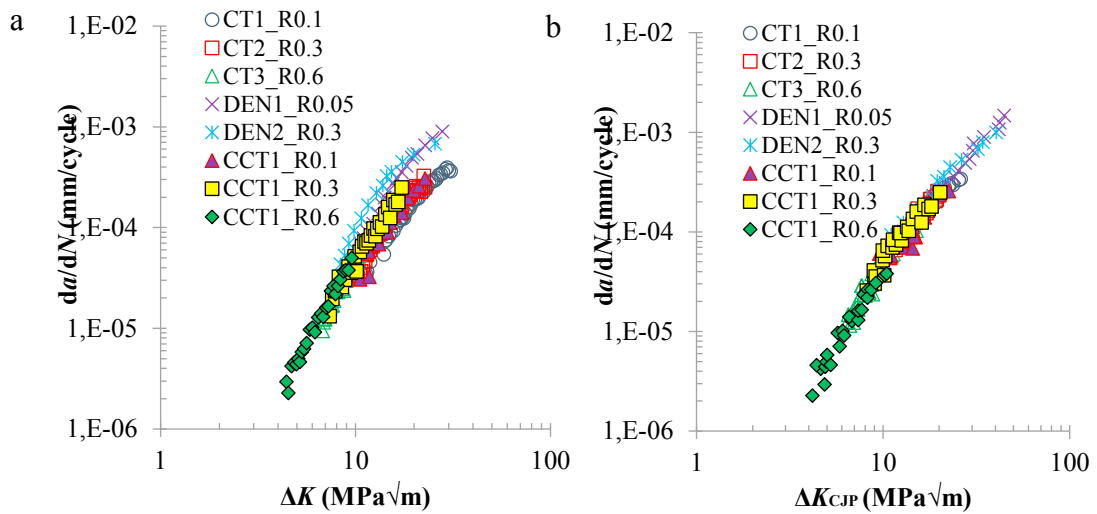


Fig. 1. Crack growth rate data for Grade 2 titanium: (a) as a function of the standard  $\Delta K$ ; (b) plotted against the CJP range of  $\Delta K$ .

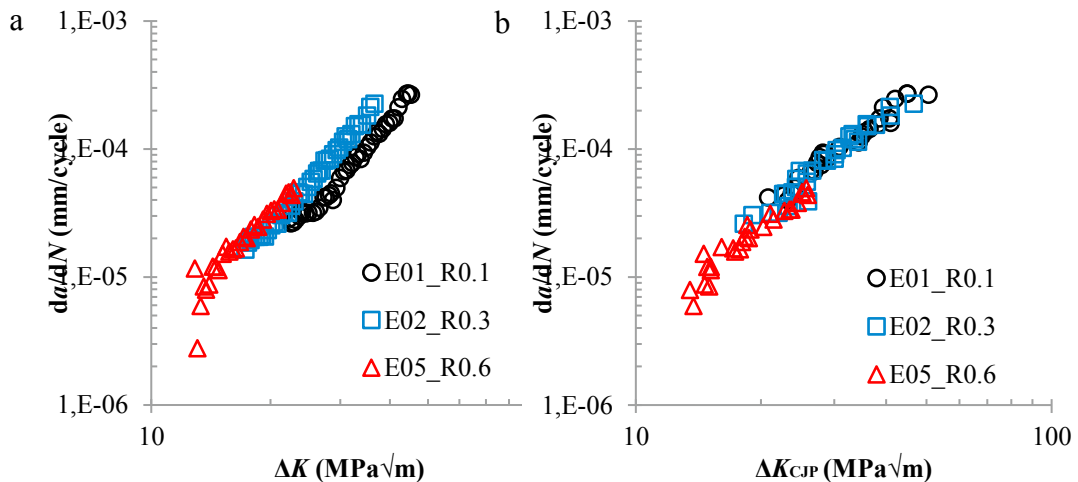


Fig. 2. Crack growth rate data for Q&T EA4T axle steel: (a) as a function of the standard  $\Delta K$ ; (b) plotted against the CJP range of  $\Delta K$ .

The work on overloads (Vasco-Olmo, Díaz et al. 2018, Vasco-Olmo, Yang et al. 2018) has shown that plasticity-induced shielding is not a complete explanation for the observed crack growth rate changes during and after an overload. The observed changes in shape and size of the plastic zone indicate that the effect of crack plasticity on crack growth rate during, and subsequent to, an overload may reflect influences from shielding (evidenced through the reduction observed in the CJP value of  $\Delta K_{eff}$ ), ratcheting, and  $K_{max}$ . These conclusions are perhaps not surprising, as a two-parameter characterisation of fatigue crack growth has been regularly advanced as providing a better description of the stress ratio effect, e.g. (Vasudeven, Sadananda et al. 1994, James and Wenfong 1999). Understanding fatigue phenomena like overloads has been made difficult because obtaining accurate values for the effective value of  $\Delta K$  is fraught with difficulties using traditional techniques such as offset compliance. These difficulties can be overcome using DIC techniques, as was reported by Nowell et al in their paper (Nowell, Dragnevski et al. 2018) but a theoretical underpinning to calculating the effective range of stress intensity factor, against which experimental data can be checked, has been lacking prior to the development of the CJP model

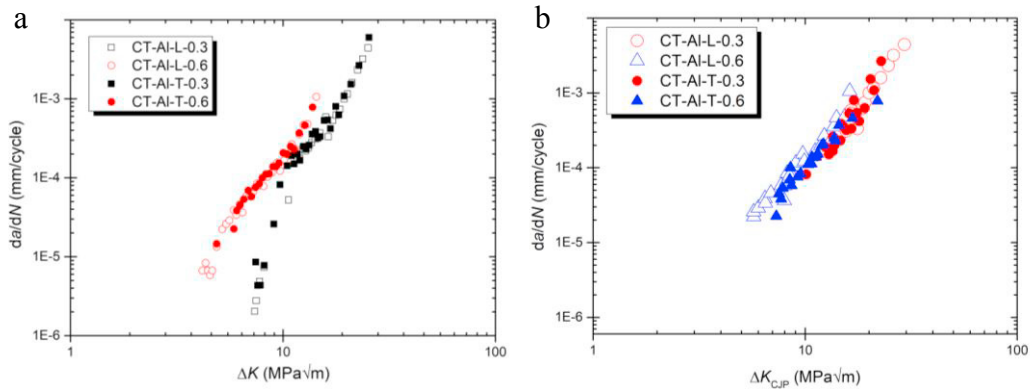


Fig. 3. Crack growth rate data for 2024-T3 aluminium: (a) as a function of the standard  $\Delta K$ ; (b) plotted against the CJP range of  $\Delta K$ .

### 3. Fatigue crack growth rate characterisation using plastic CTOD

A recent paper by Ritchie and co-workers (Hosseini, Dadfarnia et al. 2018) provides a very useful summary of the various more analytical attempts at modelling fatigue crack growth rate and analyses the stress and strain fields near a propagating crack subject to constant amplitude loading by incorporating a proper constitutive model for cyclic loading. The work by Hosseini, Dadfarnia et al (Hosseini, Dadfarnia et al. 2018) hence provides an underpinning constitutive model for the use of crack tip opening displacement (CTOD) to characterise fatigue crack growth rate. In contrast to their work, the CJP model is a crack tip field model that incorporates elastic stresses induced by the plastic enclave and does not consider the overall constitutive relationships. There is therefore considerable benefit in combining the CJP model with plastic CTOD studies on the same specimens so as to advance understanding and interpretation of fatigue phenomena. Vasco-Olmo et al (Vasco-Olmo, Díaz et al. 2019) have recently presented work that measured CTOD using DIC and then resolved the data into elastic and plastic components via an offset compliance technique. Additionally, a sensitivity analysis was performed to determine the optimum position in the crack wake to make CTOD measurements.

Fig. 4 shows the correlation of fatigue crack growth rate data achieved for two stress ratio values, 0.1 and 0.6 in a Grade 2 titanium compact tension specimen. The position behind the crack tip of the points where the CTOD is measured was found to be important in the horizontal plane along the crack, but less restrictive in terms of vertical distance from the crack plane. The conclusion of the work by Vasco-Olmo et al was that  $\Delta CTOD_p$  is a viable alternative technique to stress intensity factor in characterising fatigue crack growth rate, since the range of CTOD should intrinsically take into account both the fatigue threshold and crack shielding. However, the plastic CTOD approach is unlikely to shed light on the physical mechanisms underlying such phenomena as plasticity-induced crack tip shielding or overload growth rate transients, and a combination of approaches will be required to advance understanding, e.g. the use of  $\Delta CTOD_p$  and the CJP model of crack tip fields.

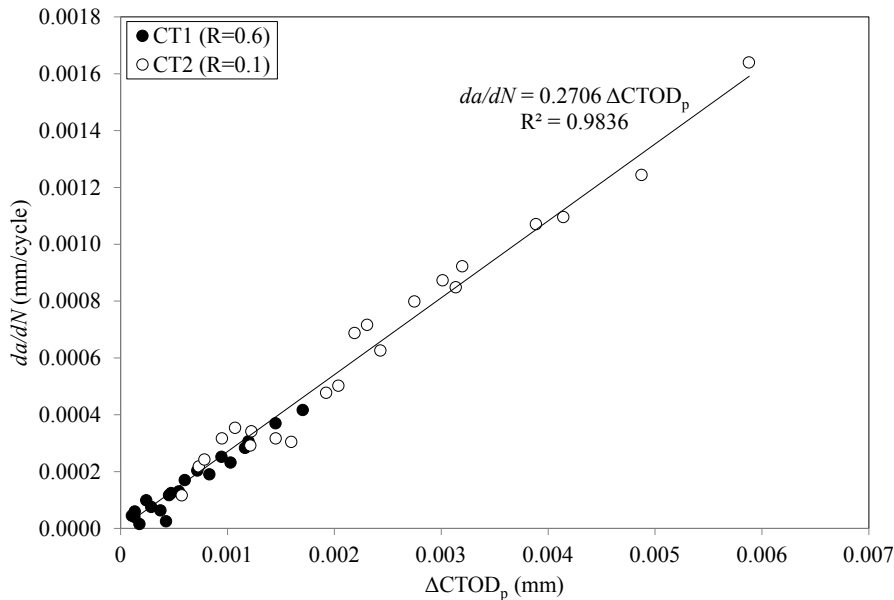


Fig. 4. Plot of crack growth rate,  $da/dN$ , versus the plastic range of CTOD,  $\Delta CTOD_p$ , for tests on Grade 2 titanium at low ( $R = 0.1$ ) and high ( $R = 0.6$ ) stress ratio values.

#### 4. Options to calculate the effective range of $\Delta K_{CJP}$

There are several possibilities to characterise the driving force for crack growth, or the effective range of stress intensity factor using the CJP model of crack tip stresses and the sensible way to decide which is the most appropriate is to compare their ability to characterise data from fatigue crack growth rate tests at several stress ratios using a statistical regression analysis. The equations proposed for the calculation of  $\Delta K_{CJP}$  by, for example, Yang et al - equation 1 (Yang, Vasco-Olmo et al. 2018) and Nowell et al - equation 2 (Nowell, Dragnevski et al. 2018), as well as simply using the range of  $K_F$ , if  $K_R$  is deemed to be a secondary influence in alloys which do not exhibit substantial amounts of plasticity-induced shielding.

$$\Delta K_{CJP} = (K_{F,max} - K_{R,max}) - (K_{F,min} - K_{R,min}) \quad (1)$$

Equation 1 is based on the assumption that a positive value of  $K_R$  acts to retard growth, based on the original assumption in its derivation that the positive direction for  $K_R$  is opposite to that of crack growth. When  $K_R$  is negative, it will therefore act to accelerate crack growth.

$$\text{Effective driving force} = (K_F + K_R) \quad (2)$$

$$\Delta K_F = K_{F,max} - K_{F,min} \quad (3)$$

The data in Fig. 5 show the crack growth rate data, obtained at two stress ratio values of 0.1 and 0.6 for Grade 2 titanium alloy, plotted as a function of the effective driving force obtained using equations 1, 2 and 3. It is immediately clear that Equation 2 leads to a discontinuous fitting, reflecting the much higher value of  $K_F$  necessary to drive crack growth rates at the higher  $R$  value. Thus equation 2 does not provide the required single correlation line for crack growth rate at different stress ratio values. A regression analysis was then performed on the data obtained using equations 1 and 3. This analysis was performed in SigmaPlot software using a cubic order polynomial. The regression coefficient  $r$  obtained with equation 1 was 0.9950 with a standard error of estimate 4.74E-8, while for equation 3 the

value of  $r$  was 0.9937 with a standard error of estimate of  $5.31E-8$ . As would be expected, even with this titanium alloy that exhibits low levels of plasticity-induced shielding, the incorporation of  $K_R$  into the definition of the effective driving force gives an improved indication of the effective driving force for crack growth. Future work will consider a more extensive set of results obtained using 2024-T3 aluminium alloy that shows a higher level of plasticity-induced shielding.

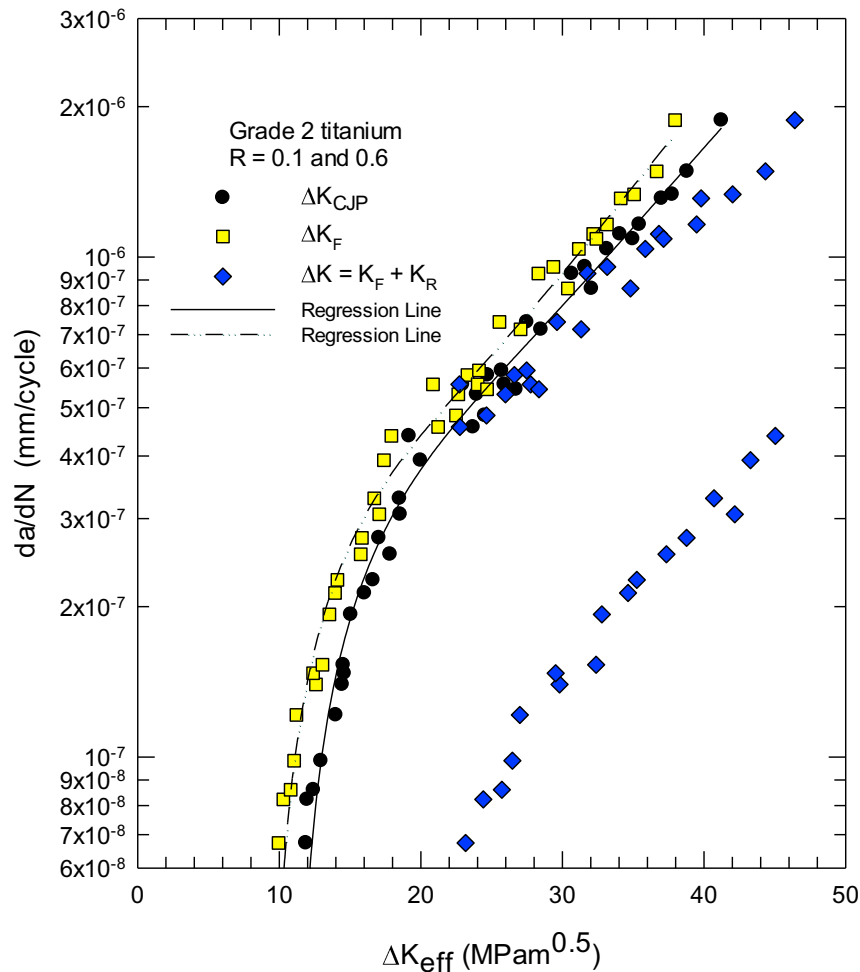


Fig. 5 Fatigue crack growth rate versus the effective range of stress intensity factor obtained using equations 1, 2 and 3. Regression lines are also shown for the data obtained using equations 1 and 3.

## 5. Conclusions

A significant body of research data has now been obtained using the CJP model of crack tip stresses (James, Christopher et al. 2013) and, thus far, the results all indicate that the model shows the following advantages compared with the use of the standard Irwin value of stress intensity factor in fatigue crack growth rate studies;

- i. The model provides a full-field analysis of stress or displacement fields at the tip of a growing fatigue crack.
- ii. It provides a more accurate estimate of plastic zone size and shape compared with those obtained using either the Westergaard or Williams solutions for crack tip stress fields.
- iii. It directly provides a value for the effective driving force for fatigue crack growth in the presence of plasticity-

induced shielding.

- iv. This effective value, termed  $\Delta K_{CJP}$ , is defined in equation 1 and correlates crack growth rate over a wider range of stress intensity factor than the Paris 'law'. The equation does not require a geometry correction (compliance) factor in the calculation of stress intensity factor.
- v. It can assist in determining the operative mechanisms underlying the transient growth rate changes observed following the application of overload cycles.

Other conclusions are that the range of plastic CTOD is a useful alternative technique to stress intensity factor in characterising fatigue crack growth rate because it should intrinsically take into account both the fatigue threshold and crack shielding. However, the plastic CTOD approach is unlikely to shed light on the physical mechanisms underlying such phenomena as plasticity-induced crack tip shielding or overload growth rate transients, and a combination of approaches is necessary to advance understanding of these issues. There should be a relationship between  $\Delta K_{CJP}$  and  $\Delta CTOD_P$  and the present authors will explore this issue.

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