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Form-finding of Tensegrity Structures Utilizing a Nonlinear Fletcher-Reeves Conjugate Gradient Method*

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Abstract— **In the domain of soft tensegrity robot, the self-equilibrium tensegrity structure is vital for the further analysis of robot's locomotion. Furthermore, form-finding is an important step for finding a self-equilibrium tensegrity structure. In this paper, a conjugate gradient form-finding (CGFF) algorithm is developed and investigated for the form-finding problems of tensegrity systems. Besides, a Fletcher-Reeves conjugate gradient method is employed to solve the nonlinear unconstrained optimization problems which transformed from the form-finding problems. Moreover, the initial conditions of the tensegrity structure such as the axial stiffness and rest lengths of the element have been utilized to explore the configuration details of the self-equilibrium tensegrity system. Eventually, several numerical simulations are provided to verify the accuracy and high-efficiency of the CGFF form-finding algorithm.**

I. INTRODUCTION

The name of tensegrity is presented by architect Fuller, because of its unique features it has been full developed in many fields such as the soft tensegrity robot and body bionics fields [1].

In the process of designing a tesegrity structure, the technique to find a self-equilibrium tensegrity structure which called form-finding is a crucial step in designing tensegrity structures. In the field of form-finding methods, the force destiny approach has been regarded as a valid approach for forming-finding process of a tensegrity structure. An algorithm which based on numerical analysis to find a self-equilibrium structure has been first presented, it can be regarded as the foundation of force destiny approach [2]. In order to find a group of force density coefficients which fit the rank deficiency conditions, an algorithm that utilized eigenvalue analysis and spectral decomposition has been developed in [3]. In [4], a form-finding method which utilized form-finding eigenvalue decomposition method (FDM) to dispose the form-finding problems has been presented. Moreover, in order to deal with complex tensegrity structures, the Monte Carlo iteration has been utilized to design large scale tensegrity structures with random initial configurations [5]. Besides, a numerical approach by calculating feasible force density vector of the structure has been proposed in [6]. Moreover, several approaches have been developed which utilizing genetic algorithm and two-time singular value decomposition (SVD) method have been proposed to solve the problems of force density coefficients [7]–[10]. Moreover, in [11]–[16], the form-finding problem is regarded as an optimization problem. Though these form-finding methods utilized few initial conditions, thus simplifying the form-finding process. However these form-finding algorithms don't involve configuration details of tensegrity systems. In general, the force destiny approach is not relate to configuration details of the tensegrity structure. Therefore, it is worth discussing the configuration details of the tensegrity structures in further research.

Conjugate gradient algorithm has advantages of fast convergence and simple algorithm structure. In [17]–[20], the authors have developed a series of modified conjugate gradient approaches, which show global convergence on dealing nonlinear optimization problems. In [21], a modified projective Dai-Yuan conjugate gradient approach has been proposed to solve the nonlinear optimization problem which converted robust control problem into nonlinear optimization problem by online solving exponential stabilizing robust controller. Besides, the authors also developed a zeroing neural network to solve the time varying nonlinear optimization problems [22]. Therefore, in this paper, a classical conjugate gradient algorithm has been utilized to establish a CGFF method. The CGFF approach has been utilized to solve the nonlinear unconstrained optimization problem which converted from the force destiny linear equilibrium equations. Not only that, the physical informations which include cross sectional area, Young's moduli and rest length are considered to explore the configuration details of the tensegrity system. This paper is organized as follows: In Section II, the nonlinear equilibrium equations modeling process is presented. Besides, the nonlinear unconstrained optimization problem which transformed from the form-finding problem is solved by CGFF method in section III. Furthermore, in section IV, numerical simulations which include two two-dimensional tensegrity structures and a three-dimensional tensegrity structure are provided to verify the accuracy of CGFF algorithm. Eventually, Section V describes the conclusion and prospects the future works.

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II. PROBLEM FORMATIONS

A. Equation Form of Force Density

As for a tensegrity structure, it has v elements and n free points, respectively. A topological matrix $\boldsymbol{\rho}$ could represent the topological structure of the presented tensegrity system. The v element, which connect point i and point j, can be defined by $d_{ii} = \sqrt{(x_i - x_i)^2 + (y_i - y_i)^2 + (z_i - z_i)^2}$

$$\boldsymbol{\rho} = \begin{cases} 1 & p = i \\ -1 & p = j \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

supposing i and j represent the beginning and end nodal points of the element v, $f_{i,j}$ and $l_{i,j}$ mean the internal force and present length of element v. Hence, the force destiny coefficient of element v could be defined as follows:

$$q_{i,j} = \frac{f_{i,j}}{l_{i,j}} = \frac{l_{i,j} - l^0_{i,j}}{l_{i,j}} \times E_{i,j} \times A_{i,j}\,. \tag{2}$$

Here, $\mathbf{A}$, $\mathbf{E}$ and $\mathbf{l^0}$ denote cross sectional area, Young's moduli and rest length, respectively. Assuming that there is a three-dimensional ($d = 3$) tensegrity structure, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ represent nodal coordinates along three directions and $\mathbf{q}$ is the force destiny vector. Therefore, the linear equilibrium equations along three directions can be defined by

$$\mathbf{Uq} = \begin{bmatrix} \boldsymbol{\rho}^{\mathrm{T}}\mathrm{diag}\,(\boldsymbol{\rho}\mathbf{x}) \\ \boldsymbol{\rho}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{\rho}\mathbf{y}) \\ \boldsymbol{\rho}^{\mathrm{T}}\mathrm{diag}(\boldsymbol{\rho}\mathbf{z}) \end{bmatrix} \mathbf{q} = \begin{bmatrix} \mathbf{F_x} \\ \mathbf{F_y} \\ \mathbf{F_z} \end{bmatrix}, \tag{3}$$

which $\mathbf{U}$ is the equilibrium matrix represents the connection between the nodal coordinates and force destiny coefficient matrix. $\mathbf{F}$ is the external force vector.

B. Requirement on Rank Deficiency Conditions

As for a self-equilibrium tensegrity structure, the force destiny matrix $\mathbf{D}$ can be defined by

$$\mathbf{D} = \boldsymbol{\rho}^{\mathrm{T}}\mathrm{diag}(\mathbf{q})\boldsymbol{\rho}, \tag{4}$$

where $\boldsymbol{\rho}$ is topological matrix. So, the null space of $\mathbf{D}$ is defined by

$$\mathrm{n_D} = \mathrm{n} - \mathrm{r_D}, \tag{5}$$

where $\mathrm{r_D}$ is the rank of matrix $\mathbf{D}$, the minimum value of $\mathrm{r_D}$ is lager than d. Meanwhile, the $\mathrm{n_U}$ means the null space of matrix $\mathbf{U}$, which defined as

$$\mathrm{n_U} = v - \mathrm{r_U}, \tag{6}$$

the minimum value of $\mathrm{n_U}$ is $\mathrm{n_U} \geq 1$.

C. Nonlinear Equilibrium Equations

In this subsection, the cross sectional area, Young's moduli, rest length and nodal coordinates are substituted into the linear equilibrium equations and transformed the linear equilibrium equations into nonlinear form which the coordinates vector $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are the variables.

The connection between the element length and nodal coordinates through topological matrix $\boldsymbol{\rho}$ can be expressed as follow:

$$\mathbf{l} = \sqrt{(\boldsymbol{\rho}\mathbf{x})^2 + (\boldsymbol{\rho}\mathbf{y})^2 + (\boldsymbol{\rho}\mathbf{z})^2}. \tag{7}$$

Substituted (2), (7) into linear equilibrium equations (3), such that the linear equilibrium equations (3) has been transformed into following form:

$$\mathbf{U}\frac{\left(\sqrt{(\boldsymbol{\rho}\mathbf{x})^2 + (\boldsymbol{\rho}\mathbf{y})^2 + (\boldsymbol{\rho}\mathbf{z})^2} - \mathbf{l_0}\right)\mathbf{EA}}{\sqrt{(\boldsymbol{\rho}\mathbf{x})^2 + (\boldsymbol{\rho}\mathbf{y})^2 + (\boldsymbol{\rho}\mathbf{z})^2}} = \begin{bmatrix} \mathbf{F_x} \\ \mathbf{F_y} \\ \mathbf{F_z} \end{bmatrix}. \tag{8}$$

In the general form-finding process, the tensegrity structure is always in a self-equilibrium situation, which means that there is no external force acting on the tensegrity system. Therefore, in the modeling process, external force vector is equal to zero. Thereby, nonlinear equilibrium equations with external force (8) could be transformed into following form:

$$-\mathbf{U}\frac{\left(\sqrt{(\boldsymbol{\rho}\mathbf{x})^2 + (\boldsymbol{\rho}\mathbf{y})^2 + (\boldsymbol{\rho}\mathbf{z})^2} - \mathbf{l_0}\right)\mathbf{EA}}{\sqrt{(\boldsymbol{\rho}\mathbf{x})^2 + (\boldsymbol{\rho}\mathbf{y})^2 + (\boldsymbol{\rho}\mathbf{z})^2}} = \boldsymbol{\eta}\,(\mathbf{X}), \tag{9}$$

where $\boldsymbol{\eta}\,(\mathbf{X}) \in \mathbb{R}^{3n}$ represents the external force minus internal force which is the residual force form of the nonlinear equilibrium equations, and $\mathbf{X} = [\mathbf{x}, \mathbf{y}, \mathbf{z}]^{\mathrm{T}} \in \mathbb{R}^{\mathbf{3n}}$ denotes the nodal coordinates vector. Thus, a relationship which connected the force destiny method and nonlinear equations has been build.

The nonlinear equilibrium equations (9) which transformed from the equilibrium matrix $\mathbf{U}$ and force vector $\mathbf{q}$ could be seen as a nonlinear unconstrained optimization problem. In next section, the Fletcher-Reeves conjugate gradient method is utilized to solve the proposed nonlinear unconstrained optimization problem.

III. NONLINEAR UNCONSTRAINED OPTIMIZATION PROBLEM

A. Nonlinear Optimization Problem

The nonlinear equilibrium equation can not be directly solved by utilizing traditional algorithm because of its highly nonlinear. Moreover, it exists the solution of the equation may cause zero lengths which may lead the whole tensegrity system to instability. Hence, the nonlinear equilibrium equations have been transformed into least-square form. With this approach, the conjugate gradient method has been utilized to solve the nonlinear unconstrained optimization problems which transformed from nonlinear equilibrium equations. In addition, after the nonlinear equilibrium equations are transformed into a nonlinear unconstrained optimization problem through the least square method, the objective function must be a convex function. Hence, the nonlinear equilibrium equations (9) are converted to following form:

$$\min \mathbf{f}(\mathbf{X}) = \frac{1}{2}\sum_{i=1}^{3n}(\boldsymbol{\eta}_i^2(\mathbf{X})). \tag{10}$$

Here, $\mathbf{f}(\mathbf{X})$ is equal to $\frac{1}{2}\|\boldsymbol{\eta}(\mathbf{X})\|_2^2$. The Jacobian matrix of $\boldsymbol{\eta}$ can be exploited to express the derivative of $\mathbf{f}(\mathbf{X})$. The

Jacobian matrix of $\boldsymbol{\eta}$ could be represented through a first-order derivative of $\mathbf{H}(\mathbf{X})$ which is defined as follows:

$$\mathbf{H}(\mathbf{X}) = \left[\frac{\partial \boldsymbol{\eta}_j}{\partial \mathbf{X}_i}\right]_{i=j=1\ldots 3n}. \tag{11}$$

Defining $\mathbf{H}(\mathbf{X})$ as $\nabla\boldsymbol{\eta}(\mathbf{X}_j)$, then

$$\nabla\mathbf{f}(\mathbf{X}) = \mathbf{H}(\mathbf{X})^{\mathrm{T}}\boldsymbol{\eta}(\mathbf{X}) = \sum_{j=1}^{3n}\nabla\boldsymbol{\eta}_j(\mathbf{X})\boldsymbol{\eta}(\mathbf{X}). \tag{12}$$

It's worth noting that $\nabla\mathbf{f}(\mathbf{X})$ is gradient of $\mathbf{f}(\mathbf{X})$, the conjugate gradient method doesn't need to calculate the Hessian matrix. Thereby, it is a more simpler approach for form-finding of tensegrity structure which could reduce the complexity of calculation.

B. Conjugate Gradient Method

The traditional way for conjugate gradient method is to update the nodal coordinates as follows:

$$\mathbf{X}_{k+1} = \mathbf{X}_k + \alpha\mathbf{d}_k, \tag{13}$$

where α denotes the search step length and $\mathbf{d}_k$ means search direction. In general, conjugate gradient method is an optimization algorithm that to generate conjugate direction of Hessian matrix about the minimum point of convex quadratic function by utilizing descent direction at the current point for each iteration step. Besides, evaluating whether the gradient of the objective function $\nabla\mathbf{f}(\mathbf{X}_k)$ is less than the setting error. Yet different from the traditional way, this algorithm works by estimating $\mathbf{d}_k$ which is set equal to $\alpha\mathbf{d}_k$ whether or not fulfill the setting error. From that, the step length can be updated at any moment during the iterative process. In CGFF algorithm, how to adjust search step α is an important step for the optimization algorithm. If α is too small, the iteration process may cost too much time. On the contrary, if α is too large, the current $\mathbf{d}_k$ may not the correct descent direction and may miss the optimal solution. In this paper, a Fletcher-Reeves conjugate gradient form-finding approach has been utilized to solve the nonlinear unconstrained optimization problem which transformed from form-finding problems.

Afterwards, calculating the Jacobian matrix $\mathbf{H}(\mathbf{X})$ and $\nabla\mathbf{f}(\mathbf{X})$ which can be regarded as the gradient of $\mathbf{f}(\mathbf{X})$. Then, $\mathbf{d}_k$ has been utilized to update the nodal coordinate $\mathbf{X}$ and the 2-norm of $\mathbf{d}_k$ is the cut-off condition of the CGFF method. The current nodal coordinates $\mathbf{X}$ could been seen as the suitable nodal coordinates when the 2-norm of $\mathbf{d}_k$ is less than setting error. Otherwise, updating the $\mathbf{f}(\mathbf{d}_k)$ by utilizing current nodal coordinates $\mathbf{X}$ when the 2-norm of $\mathbf{d}_k$ is greater than setting error. Ulteriorly, if $\mathbf{f}(\mathbf{X}_{k+1})$ is less than $\mathbf{f}(\mathbf{X}_k)$, set $\alpha = \alpha/\beta$ and $k = k+1$, the opposite of this circumstance is when $\mathbf{f}(\mathbf{X}_{k+1})$ is greater than $\mathbf{f}(\mathbf{X}_k)$, then setting $\alpha = \alpha\cdot\beta$. Via this approach, the search direction $\mathbf{d}_k$ could be adjusted. The algorithm details are shown in Table I and the flowchart of the CGFF approach is shown in Fig. 1.

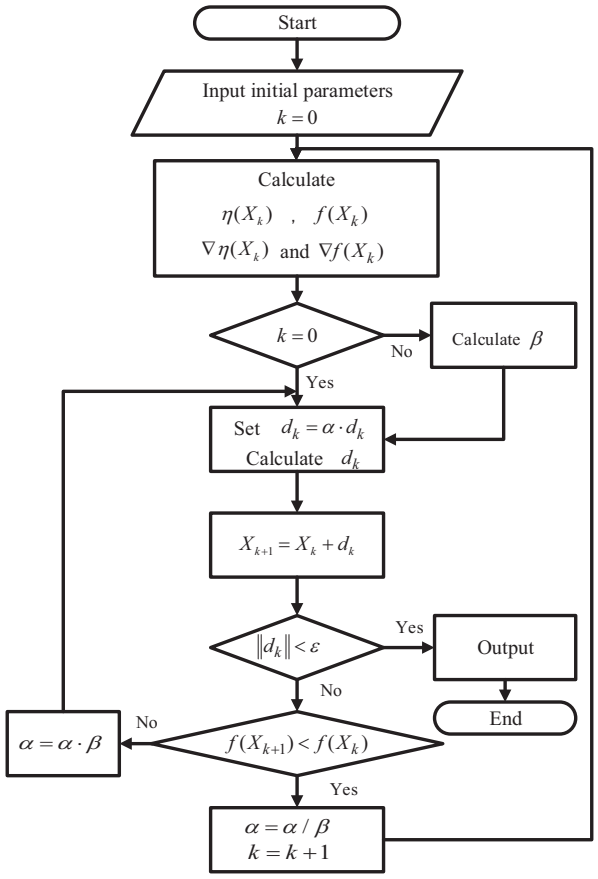

Fig. 1. Flowchart of the CGFF method

IV. NUMERICAL EXPERIMENT

In this section, the accuracy and high-efficiency of CGFF approach has been proved through three examples including two two-dimensional polygons and a three-dimensional prism.

A. Two-dimensional Tensegrity Structures

Firstly, a two-dimensional two-strut tensegrity structure which is consisted of 4 cables and 2 struts has been selected to find the self-equilibrium configuration in this subsection. Fig.2 shows the obtained topological structure of self-equilibrium tensegrity structure. The obtained nodes in Fig.3 show the different iteration speed under different setting error. As shown in Fig.3, with the improvement of the design error accuracy, the iteration steps are increased. The chosen error is $\varepsilon = 1\times10^{-8}$ which corresponding to the eighth point and form-finding procedure converges in ten iterations. The obtained nodal coordinates which utilizing CGFF method lying in Table II are as consistent as the obtained nodal coordinates by using FDM approach. The comparison between two series of coordinates can prove the accuracy of the CGFF algorithm. Secondly, in order to verify the CGFF approach still valid in the two-dimensional space, a

TABLE I

CGFF APPROACH PROCESS

Steps	CGFF approach details.
1.	Input initial conditions, set k=0. Calculate $\boldsymbol{\eta}(\mathbf{X}_k)$, $\mathbf{f}(\mathbf{X}_k)$, $\nabla\boldsymbol{\eta}(\mathbf{X}_k)$, $\nabla\mathbf{f}(\mathbf{X}_k)$.

2.	**If** k is equal to 0
	Calculate $\mathbf{d}_k$, $\mathbf{X}_{k+1}$ and $2-\text{norm}(\mathbf{d}_k)$.
	While $\|\mathbf{d}_k\|_2 > \varepsilon$, calculate $\mathbf{f}(\mathbf{X}_{k+1})$,
	otherwise output $\mathbf{X}$.
	If $\mathbf{f}(\mathbf{X}_{k+1}) < \mathbf{f}(\mathbf{X}_k)$, set $\alpha = \alpha/\beta$
	and k=k+1.
	Back to calculate $\boldsymbol{\eta}(\mathbf{X}_k)$, $\mathbf{f}(\mathbf{X}_k)$, $\nabla\boldsymbol{\eta}(\mathbf{X}_k)$
	and $\nabla\mathbf{f}(\mathbf{X}_k)$.
	Else set $\alpha = \alpha \cdot \beta$ then back to calculate
	$\mathbf{d}_k$, $\mathbf{X}_{k+1}$.
	End if
	End while
	End if

3.	**If** k is not equal to 0
	Calculate β, $\mathbf{d}_k$, $\mathbf{X}_{k+1}$ and $2-\text{norm}(\mathbf{d}_k)$.
	While $\|\mathbf{d}_k\|_2 > \varepsilon$, calculate $\mathbf{f}(\mathbf{X}_{k+1})$,
	otherwise output $\mathbf{X}$.
	If $\mathbf{f}(\mathbf{X}_{k+1}) < \mathbf{f}(\mathbf{X}_k)$, set $\alpha = \alpha/\beta$
	and k=k+1.
	Back to calculate $\boldsymbol{\eta}(\mathbf{X}_k)$, $\mathbf{f}(\mathbf{X}_k)$, $\nabla\boldsymbol{\eta}(\mathbf{X}_k)$
	and $\nabla\mathbf{f}(\mathbf{X}_k)$.
	Else set $\alpha = \alpha \cdot \beta$ then back to calculate
	$\mathbf{d}_k$, $\mathbf{X}_{k+1}$.
	End if
	End while
	End if

4.	Output the nodal coordinates $\mathbf{X}$

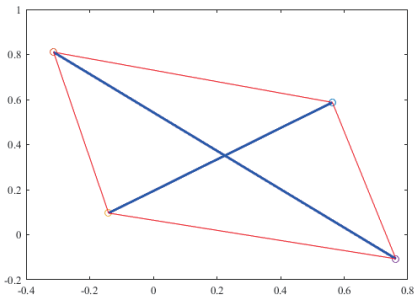

Fig. 2. The obtained geometry of the two-dimensional two-strut tensegrity structure

more complicated self-equilibrium structure is found through the CGFF method for a hexagon tensegrity structure which is formed by three struts and six cables. The self-equilibrium configuration of this tensegrity structure are obtained in Fig.4. The CGFF algorithm converges in ten iterations, and design error is $\varepsilon = 1 \times 10^{-8}$ which corresponding to eighth point in Fig.5. The nodal coordinates of hexagon tensegrity structure have shown in Table III. Analogously, the nodal coordinates which obtained by CGFF method are compared with the coordinates obtained by FDM approach, the results are very similar which could also prove the accuracy of CGFF algorithm.

B. Three-dimensional Triangular Prism Tensegrity Structure

To prove CGFF method is still effective in three-dimensional space, a three-dimensional triangular prism

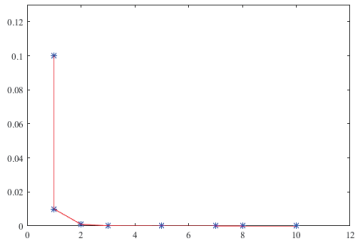

Fig. 3. The convergence of CGFF method for two-dimensional two-strut tensegrity structure

TABLE II

SELF-EQUILIBRIUM NODAL COORDINATES

Form-finding method	Nodal coordinates			
CGFF method	**x**: 0.5345	-0.2941	-0.0842	0.7737
	y: 0.5355	0.8044	0.1856	-0.0635
FDM method	**x**: 0.5366	-0.2974	-0.0821	0.7716
	y: 0.5499	0.8094	0.1959	-0.0636

tensegrity structure is presented in this subsection. A triangular prism tensegrity structure is consisted of 3 struts and 9 cables. The self-equilibrium geometry of triangular prism tensegrity structure is shown in Fig.6. The form-finding approach converges in twelve iterations and the design error is $\varepsilon = 1 \times 10^{-8}$ which is the eighth point in Fig.7. The nodal coordinates of self equilibrium for triangular prism tensegrity structure have shown in Table IV. Besides, the obtained coordinates by utilizing CGFF algorithm are largely consistent with the nodal coordinates which is obtained by the FDM method. These nodal coordinates which obtained by utilizing two different form-finding methods can prove the accuracy of the CGFF approach in three dimension.

From aforementioned three tensegrity systems form-finding examples, through comparing the nodal coordinates between the CGFF method and FDM approach, the efficient of the CGFF method could be proved. Although the error between two methods is existed, however, the error can be reduced through adjusting α and β. The current error $\varepsilon = 1 \times 10^{-8}$ could satisfy the need of form-finding process for tensegrity structures. If calibrating the parameters α and β further, the design error can not change drastically and it

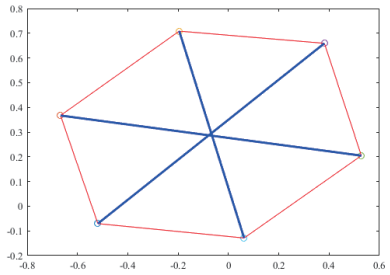

Fig. 4. The obtained geometry of the two-dimensional hexagon tensegrity structure

TABLE III

SELF-EQUILIBRIUM NODAL COORDINATES

Form-finding method	Nodal coordinates						
CGFF method	**x**:	-0.4916	-0.610	-0.1886	0.3527	0.4719	0.0497
	y:	-0.0270	0.3463	0.6659	0.6111	0.2378	-0.0812
FDM method	**x**:	-0.5189	-0.6673	-0.1955	0.381	0.5281	0.0622
	y:	-0.0705	0.3670	0.7086	0.6582	0.2046	-0.1288

TABLE IV

SELF-EQUILIBRIUM NODAL COORDINATES

Form-finding method	Nodal coordinates						
CGFF method	**x**:	1.7321	-0.866	-0.866	-1.5	0	1.5
	y:	0	1.5	-1.5	0.866	-1.7321	0.866
	z:	0	0	0	6.1485	6.1485	6.1485
FDM method	**x**:	1.6327	-0.855	-0.789	-1.399	-0.031	1.437
	y:	0.038	1.3994	-1.437	0.8558	-1.6358	0.789
	z:	0.095	0.095	0.095	6.0548	6.0548	6.0548

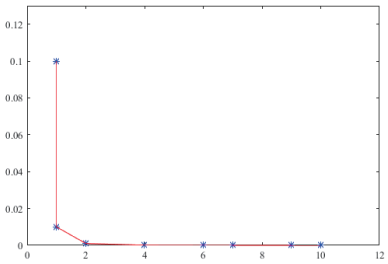

Fig. 5. The convergence of CGFF method for two-dimensional hexagon tensegrity structure

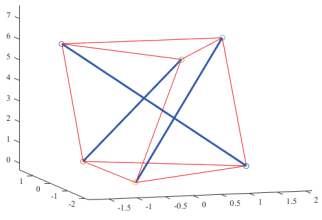

Fig. 6. The obtained geometry of three-dimensional triangular prism tensegrity structure

may cause the non-convergence of form-finding process.

Eventually, as an important index to test the efficiency of the CGFF algorithm, the comparison of computation time between different methods is taken into account. By comparing the computation time of finding the self-equilibrium configuration of two-dimensional hexagon tensegrity structure, the efficiency of the CGFF algorithm could be verified. Utilizing the CGFF approach and the built-in function "fsolve" of MATLAB which could be utilized to solve the nonlinear optimization problems for comparison. The form-finding process of hexagon tensegrity structure which using CGFF method takes 49.76 seconds to find the self-equilibrium structure with the design error $\varepsilon = 1 \times 10^{-8}$. In contrast, the built-in function "fsolve" takes 255.08 seconds to find the

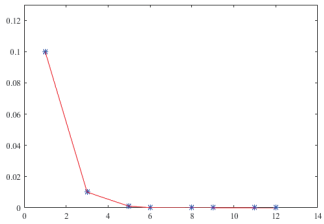

Fig. 7. The convergence of CGFF method for three-dimensional triangular prism tensegrity structure

self-equilibrium configuration of the tensegrity system with the same design error. By means of these results, the high-efficiency of the proposed CGFF method could be proved.

V. CONCLUSIONS

In this paper, from an optimization algorithm viewpoint, a CGFF approach has been proposed to be utilized during the form-finding process of tensegrity systems. By involving the physical informations of the structure in the nonlinear equilibrium equations, the configuration details of the tensegrity structure could be obtained. Utilizing conjugate gradient approach to solve the nonlinear unconstrained optimization problem which transformed from the linear equilibrium equations. Numerical results have ascertained the accuracy and high-efficiency of the presented CGFF approach for tensegrity structure form-finding solving. Future work will consider the application of neural network algorithms for the form-finding problems of tensegrity structures.

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