Faculty of Science and Engineering

School of Engineering, Computing and Mathematics

2021-04-21

Anomalous wave statistics following sudden depth transitions: application of an alternative Boussinesq-type formulation

Bonar, PAJ

http://hdl.handle.net/10026.1/17645

10.1007/s40722-021-00192-0 Journal of Ocean Engineering and Marine Energy Springer Nature

All content in PEARL is protected by copyright law. Author manuscripts are made available in accordance with publisher policies. Please cite only the published version using the details provided on the item record or document. In the absence of an open licence (e.g. Creative Commons), permissions for further reuse of content should be sought from the publisher or author.

Anomalous wave statistics following sudden depth transitions:

² Application of an alternative Boussinesq-type formulation

- ³ Paul A. J. Bonar¹ · Colm J. Fitzgerald² ·
- ⁴ Zhiliang Lin³ \cdot Ton S. van den Bremer^{4,5} \cdot
- 5 Thomas A.A. Adcock^{4,*} ·
- 6 Alistair G. L. Borthwick^{1,6}
- 7 Received: date / Accepted: date

Abstract Recent studies of water waves propagating over sloping seabeds have shown that sudden transitions from deeper to shallower depths can produce significant increases in the 9 skewness and kurtosis of the free surface elevation and hence in the probability of rogue 10 wave occurrence. Gramstad et al. (2013, Phys. Fluids 25 (12): 122103) have shown that the 11 key physics underlying these increases can be captured by a weakly dispersive and weakly 12 nonlinear Boussinesq-type model. In the present paper, a numerical model based on an al-13 ternative Boussinesq-type formulation is used to repeat these earlier simulations. Although 14 qualitative agreement is achieved, the present model is found to be unable to reproduce ac-15 curately the findings of the earlier study. Model parameter tests are then used to demonstrate 16 that the present Boussinesq-type formulation is not well-suited to modelling the propagation 17 of waves over sudden depth transitions. The present study nonetheless provides useful in-18 sight into the complexity encountered when modelling this type of problem and outlines a 19 number of promising avenues for further research. 20

¹ School of Engineering, The University of Edinburgh, Mayfield Road, Edinburgh, EH9 3FB, UK

² Inland Fisheries Ireland, 3044 Lake Drive, Citywest Business Campus, Dublin, D24 Y265, Ireland

³ School of Naval Architecture, Ocean, and Civil Engineering, Shanghai Jiao Tong University, Shanghai, 200240, China

⁴ Department of Engineering Science, University of Oxford, Parks Road, Oxford, OX1 3PJ, UK

⁵ Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, 2628CD, The Netherlands

 $^{^6}$ School of Engineering, Computing, and Mathematics, University of Plymouth, Drake Circus, Plymouth, PL4 8AA, UK

^{*} Corresponding author: thomas.adcock@eng.ox.ac.uk

21 Keywords Rogue wave · freak wave · Boussinesq-type equations · skewness · kurtosis

22 1 Introduction

Long considered the stuff of legend, rogue waves are now recognised as a serious hazard 23 to ships and offshore structures. Historical reports of giant, powerful waves appearing first 24 without warning and then suddenly vanishing have since been supported by theory and ex-25 periment (Dysthe et al., 2008; Kharif et al., 2009). In recent decades, numerous studies have 26 explored both the physical mechanisms which might produce such waves and the statisti-27 cal parameters that may be used to estimate their occurrence probability. Comprehensive 28 reviews are provided by Dysthe et al. (2008), Kharif et al. (2009), Slunyaev et al. (2011), 29 Onorato et al. (2013), and Adcock & Taylor (2014), amongst others. 30

Rogue waves are typically defined as those having heights which are more than twice 31 the local significant wave height (e.g. Holthuijsen, 2007) but their study is complicated by 32 a limited number of real-world measurements (Kharif et al., 2009) and conflicting views as 33 to how much information can be inferred from these (Dysthe et al., 2008). The key ques-34 tion at present is whether such observations represent 'classical' extremes which can be 35 described by conventional models and statistics, or 'freak' waves requiring new theories 36 and approaches (Haver & Andersen, 2000; Dysthe et al., 2008; Kharif et al., 2009). Some 37 authors take the view that rogue waves are rare instances of random superposition in seas of 38 weakly nonlinear waves (Christou & Ewans, 2014; Fedele et al., 2016) whilst others hypoth-39 esise that certain waves, such as the well-known Draupner wave, must have been produced 40 by some other forcing mechanism (Adcock et al., 2011; Cavaleri et al., 2016). 41

Other possible rogue wave generating mechanisms include modulational instability; interactions with variable bathymetry, opposing currents, or between crossing seas; wind forcing; or some combination of these factors (Dysthe et al., 2008; Kharif et al., 2009; Onorato et al., 2013; Fedele et al., 2016). Attempts to derive a single, unifying theory are complicated by the facts that geometric focusing cannot explain the transient nature of rogue waves (Janssen & Herbers, 2009), that modulational instability requires an improbable set of initial conditions (deep-water waves with a narrow spectral bandwidth and narrow directional 49

50

spreading) (Dysthe et al., 2008), and that rogue waves can be produced even when several of the foregoing factors are absent (Mori & Janssen, 2006; Kharif et al., 2009).

The simplest theory assumes that the dynamics of ocean surface waves are purely linear, 51 that the free surface elevation is a stationary, Gaussian process, and that the wave amplitudes 52 are well approximated by the Rayleigh distribution (Ochi, 2005; Holthuijsen, 2007). How-53 ever, because ocean waves are inherently (weakly) nonlinear (Trulsen, 2018), wave-wave 54 interactions or other mechanisms can result in considerable deviations from the Gaussian 55 model (Fedele et al., 2016). Some authors have suggested that rogue waves may be a re-56 sult of non-equilibrium dynamics: if waves are somehow forced into an unstable state, their 57 statistics can deviate in such a way as to suggest an increased likelihood of extreme events 58 (Janssen & Herbers, 2009; Viotti & Dias, 2014). The kurtosis of the free surface elevation 59 is a convenient metric by which to quantify such deviations: an increase in free surface kur-60 tosis signifies an increase in the probability of rogue wave occurrence (Onorato et al., 2004; 61 Mori & Janssen, 2006). 62

Waves propagating into shallower water are known to be transformed by shoaling and 63 nonlinear effects (Dean & Dalrymple, 1991; Dingemans, 1997) but recent studies have 64 shown that sudden transitions between deeper and shallower domains can also produce 65 strongly non-Gaussian wave statistics. Physical experiments by Trulsen et al. (2012), Zhang 66 et al. (2019), and Trulsen et al. (2020) showed significant increases in free surface skew-67 ness and kurtosis for irregular waves near the crest of an inclined seabed of 1-in-20 slope 68 connecting otherwise flat domains, and these findings have been supported by numerical 69 simulations due to Sergeeva et al. (2011), Gramstad et al. (2013), Viotti & Dias (2014), 70 Ducrozet & Gouin (2017), Zhang et al. (2019), and Zheng et al. (2020). Similar results 71 have also been obtained in experimental and numerical studies of waves propagating over 72 submerged bars (Ma et al., 2014, 2015), shoals (Janssen & Herbers, 2009; Raustøl, 2014; 73 Fallahi, 2016; Trulsen et al., 2020), compound slopes (Kashima et al., 2014), and vertical 74 steps (Zheng et al., 2020). 75

The foregoing local increases in skewness and kurtosis usually coincide with local enhancements of higher harmonic content related to the sudden decreases in depth and cor-

responding increases in nonlinearity (Gramstad et al., 2013; Zhang et al., 2019; Trulsen et 78 al., 2020). In fact, Zheng et al. (2020) have recently shown that second-order terms in wave 79 steepness are responsible for the change in the statistical properties near the depth transition 80 for the cases examined by Trulsen et al. (2012) and Gramstad et al. (2013). These deviations 81 are also expected to depend on the initial steepness, spectral bandwidth, and directionality 82 of the waves (Ducrozet & Gouin, 2017; Støle-Hentschel et al., 2018; Trulsen et al., 2020; 83 Zheng et al., 2020), the gradient of the seabed slope, and the depth beyond the slope: for 84 milder slopes and deeper depths beyond the slopes, there may be no local maxima, or per-85 haps even local minima, in skewness and kurtosis (Zeng & Trulsen, 2012; Gramstad et al., 86 2013; Raustøl, 2014; Fallahi, 2016; Trulsen et al., 2020). 87

In this paper, the phenomenon of increased free surface skewness and kurtosis following 88 a sudden depth transition is explored further using an accurate yet computationally efficient 89 Boussinesq-type model, following the work of Gramstad et al. (2013), whose model appears 90 to be the simplest of those describing such anomalous statistical deviations. The aim is to 91 first reproduce the findings of Trulsen et al. (2012) and Gramstad et al. (2013) and then 92 extend the parameter space in our numerical simulations to provide further insight into the 93 underlying physics. The paper is structured as follows: §2 provides a brief description of 94 the numerical model, set-up of the numerical simulations, and grid convergence and sponge 95 layer calibration tests; §3 compares the present findings with those of Trulsen et al. (2012) 96 and Gramstad et al. (2013) and summarises the results of a model parameter study; and §4 97 presents the discussion, conclusions, and recommendations for further work. 98

99 2 Model

100 2.1 Numerical model

The present simulations are performed using OXBOU, a depth-integrated hybrid numerical model designed to simulate the propagation in one horizontal dimension of ocean surface gravity waves from intermediate to shallow and zero water depth. A brief overview of the model features will suffice here; detailed descriptions of the numerical implementation and verification and validation tests are given by Orszaghova (2011), Orszaghova et al. (2012),
and Fitzgerald et al. (2016).

The OXBOU model uses two sets of governing equations and two numerical schemes: 107 unbroken waves are simulated using weakly dispersive, weakly non-linear Boussinesq-type 108 equations, which are solved using a fourth-order finite difference method, whilst broken 109 waves are modelled as bores using the non-dispersive, non-linear shallow water equations, 110 which are solved using a shock-capturing finite volume scheme (Orszaghova et al., 2012). 111 The model switches from the Boussinesq-type to shallow water equations when certain 112 depth or free surface slope criteria are met, but the present simulations involve non-breaking 113 waves solely and so employ only the Boussinesq-type model. The numerical scheme in-114 corporates a moving boundary piston paddle wavemaker, which is facilitated by a mapping 115 between stretching-compressing physical and fixed computational sub-domains, and is ca-116 pable of producing waves with approximately correct second-order bound harmonics (see 117 Orszaghova et al., 2012). The scheme also includes an absorbing-generating sponge layer 118 which allows incident waves to propagate freely inshore whilst simultaneously removing 119 offshore-travelling reflections (see Fitzgerald et al., 2016). 120

OXBOU solves the Boussinesq-type equations of Madsen & Sørensen (1992), which were selected for their enhanced linear dispersion characteristics and computational efficiency (Borthwick et al., 2006; Orszaghova et al., 2012). Following Orszaghova et al. (2012) and Fitzgerald et al. (2016), these equations are presented in a well-balanced, stagedischarge (η , q) form as

$$\eta_t + q_x = \psi(\eta_o - \eta), \tag{1}$$

$$q_{t} + \left(\frac{q^{2}}{d} + \frac{1}{2}g(\eta^{2} - 2\eta b)\right)_{x} = -g\eta b_{x} - \frac{\tau_{b}}{\rho} + \frac{1}{3}h^{2}q_{xxt} + \frac{1}{3}hh_{x}q_{xt} + B\left(h^{2}q_{xxt} + gh^{3}\eta_{xxx} + 2gh^{2}h_{x}\eta_{xx}\right) + \psi(q_{o} - q), \quad (2)$$

where $\eta = b + h + \zeta$ is the free surface elevation above a prescribed horizontal datum (with *b* the depth of the datum below the seabed, *h* the still water depth, and ζ the free surface elevation above still water level); q is depth-integrated velocity; ψ is the sponge layer damping strength; $d = h + \zeta$ is the total depth; g is acceleration due to gravity; τ_b is bed stress; ρ is the fluid density; the subscripts t and x denote partial derivatives with respect to time and horizontal distance, respectively; the subscript o refers to solutions imposed by the sponge layers; and B is a linear dispersion coefficient such that the wave celerity, c, is given by

$$\frac{c^2}{gh} = \frac{1 + Bk^2h^2}{1 + \left(B + \frac{1}{3}\right)k^2h^2},$$
(3)

where *k* is the wave number. Setting B = 1/15 embeds the [2,2] Padé approximant of the exact linear dispersion relation within the momentum equation, whereas setting B = 0 recovers the classical equation derived by Peregrine (1967) (Orszaghova et al., 2012).

136 2.2 Set-up of numerical simulations

Following Gramstad et al. (2013), the first set of simulations is designed to replicate the 137 physical experiments described by Trulsen et al. (2012), which were performed in the shal-138 low water basin at the Maritime Research Institute Netherlands (MARIN). These experi-139 ments considered three cases of long-crested irregular waves propagating from a piston-type 140 wavemaker (at x = 0 m) first over a deeper flat domain, then over a 1-in-20 inclined seabed 141 slope (from x = 143.41 m to 149.4 m), and finally over a shallower flat domain leading to 142 an absorbing beach (at x = 173.41 m). In all three experimental cases, the still water depths 143 before and after the slope were h = 0.6 and 0.3 m, respectively, and the nominal input sig-144 nificant wave height was $H_s = 0.06$ m. Cases 1, 2 and 3 were distinguished by the nominal 145 peak periods of their input wave spectra: $T_p = 1.27$, 1.70, and 2.12 s, respectively. Wave 146 records were obtained from eight gauges placed along the length of the basin, and the influ-147 ence of the depth transition on the probability of rogue wave occurrence was examined by 148 calculating the skewness and kurtosis of the free surface elevation and exceedance function 149 of the (Hilbert) wave envelope at each location. 150

In repeating these experiments, the present study follows closely the methodology described by Trulsen et al. (2012) but uses OXBOU to output results at 1 m spatial intervals,



Fig. 1: Schematic diagram showing a simulation performed using coupled (a) incident and (b) run-up domains. Identical irregular waves are produced by the moving boundary wavemakers (left), and absorbing (right) and absorbing-generating sponge layers (centre) are used to eliminate reflections from the ends of the tanks and submerged seabed slope.

and moves the seabed slope 0.01 m closer to the wavemaker to facilitate the use of uniform 153 (fixed) computational grids. The simulations for each case are performed as follows. The 154 wavemaker is used to generate identical irregular waves in both an incident domain and a 155 run-up domain. In the incident domain, the numerical wave tank (from x = 0 m to 200 m) is 156 assigned a flat seabed profile (h = 0.6 m), whilst in the run-up domain, the tank comprises 157 deeper (h = 0.6 m) and shallower (h = 0.3 m) sections connected by a 1-in-20 seabed slope 158 (from x = 143.4 m to 149.4 m). In both domains, the bed is frictionless and the waves prop-159 agate into an absorbing sponge layer (from x = 185.8 m to 200 m), which gradually reduces 160 ζ and q to zero to ensure that there are no reflections either from the end of the tank or 161 the absorbing layer itself. Meanwhile, in the run-up domain, reflections from the slope are 162 removed by an additional absorbing-generating sponge layer (from x = 92.9 m to 107.1 m), 163 which adjusts the free surface elevation, ζ_r , and depth-integrated velocity, q_r , to match those 164 in the incident domain, ζ_i and q_i (Fig. 1). 165

Irregular waves are produced as the sum of wave components obtained from a truncated JONSWAP spectrum with peak frequency $f_p = 1/T_p$ and upper and lower cut-off frequencies $f_{max} = 3f_p$ and $f_{min} = 0.5f_p$. The JONSWAP function is given by

$$S(f) = \alpha \frac{g^2}{(2\pi)^4} \frac{1}{f^5} \exp\{-1.25(f_p/f)^4\} \gamma^{\exp\{-(f-f_p)^2/2(\sigma f_p)^2\}},\tag{4}$$

where f is the component frequency, α is the energy scale parameter, $\gamma = 3.3$ is the peak 169 shape parameter, and σ is the peak width factor, which is assigned values of σ = 0.07 for f \leq 170 f_p and $\sigma = 0.09$ for $f > f_p$ (Ochi, 2005; Holthuijsen, 2007). Pseudo-random wave signals 171 are generated using the random-amplitude/random-phase approach of Tucker et al. (1984), 172 in which the amplitudes and phases of the linear components are determined, respectively, 173 from a Rayleigh distribution with scale parameter $\sqrt{S(f) \triangle f}$, where $\triangle f$ is the frequency 174 domain sampling interval, and a uniform distribution on $[0, 2\pi]$ (Fitzgerald et al., 2016). The 175 corresponding linear wavemaker signal is then calculated using the Biésel transfer function, 176 and a large number of harmonic components is chosen to ensure that the repeat period of the 177 signal is greater than the duration of the simulation. This linear signal can also be corrected 178 by applying a second-order transfer function approximated from the wavemaker theory of 179 Schäffer (1996) but, for ease of computation, only first-order accurate wavemaker signals 180 are considered initially. 181

182 2.3 Grid convergence and sponge calibration tests

Model solutions converged for a uniform computational grid spacing of 0.02 m and a time 183 step of ~ 0.0066 s. Figure 2a shows the excellent agreement in free surface time series ob-184 tained when computational grids of resolution 0.018 m, 0.02 m, and 0.022 m (which repro-185 duce the tank using 11,000, 10,000, and 9,000 grid points, respectively) are used to simulate 186 an example focused wave group, which is created by bringing 128 harmonic wave com-187 ponents from the Case 2 spectrum to a linear focus amplitude of 0.03 m at the toe of the 188 seabed slope (x = 143.4 m). Wave records from a point just beyond the crest of the slope 189 (x = 150 m) show excellent agreement, with root mean square error (RMSE) values ranging 190 from $\sim 2.47 \times 10^{-5}$ m to $\sim 5.68 \times 10^{-5}$ m, as do the corresponding frequency-domain re-191 sults, which are not shown for brevity. Excellent results are also obtained in tests for mass 192



Fig. 2: Free surface elevation time histories at x = 150 m showing excellent agreement between (a) records of a crest-focused group simulated on computational grids of resolution 0.018 m (circles), 0.02 m (line), and 0.022 m (crosses), and (b) subsequent repeat periods (crosses, line) of a periodic irregular wave signal.

conservation, reversibility, and the accumulation of round-off error, with model errors typi cally much less than 1%.

The absorbing and absorbing-generating sponge layers are then calibrated to ensure that 195 they are able to damp effectively waves passing through without altering the incoming wave 196 field. The absorbing-generating layer, which is used only in the run-up domain and placed 197 such that its midpoint lies halfway along the one-dimensional tank (Fig. 1), is assigned a 198 triangular strength profile (such that ψ increases and decreases linearly and symmetrically 199 about the midpoint of the layer), whilst the identical absorbing layers, which are placed at 200 the ends of the tanks in both the incident and run-up domains, are given linearly increasing 201 strength profiles (Fitzgerald et al., 2016). 202

Calibration is undertaken by comparing, for different sponge layer lengths, L_s, and in-203 tegrated sponge layer strengths, $\overline{\psi}$, the wave records obtained from points upstream and 204 downstream of the sponge layers. With the absorbing-generating layer switched off, a crest-205 focused wave group is first propagated from left to right through the absorbing layers, which 206 are temporarily moved 20 m upstream so that measurements can be taken both upstream and 207 downstream of the layers, and measurements are taken in the run-up domain as the waves are 208 damped to zero. With the absorbing layers calibrated and moved back to the end of the tank, 209 the reflected wave group, which is obtained from an additional simulation with no sponge 210

layers, is then propagated from right to left through the absorbing-generating layer, which 211 is set to damp the waves to the conditions in the incident domain (in this case, still water). 212 Excellent absorption properties are achieved by setting, for all layers, $L_s = 4\lambda_p = 14.2$ m 213 and $\overline{\psi} = 4\omega_p = 14.8$ rad/s, where λ_p is the peak wavelength and ω_p is the peak angular 214 frequency of the Case 2 spectrum. Following Fitzgerald et al. (2016), a periodic irregular 215 wave signal with repeat period $\sim 2.17 \times 10^2$ s is then used to determine the efficacy of the 216 sponge layer absorption by testing for repeatability in the wave record at a given gauge. 217 Figure 2b shows the excellent agreement (RMSE $\approx 2.64 \times 10^{-4}$ m) in free surface time 218 series obtained between subsequent repeat periods in the wave record at x = 150 m in the 219 run-up domain, which confirms that the reflections from the end of the tank and submerged 220 seabed slope are negligible. 221

222 3 Results

223 3.1 Comparison with the results of Trulsen et al. (2012) and Gramstad et al. (2013)

The three experimental cases performed at MARIN are simulated by first discretising their 224 input spectra into 214 harmonic wave components to produce irregular wave signals and 225 corresponding linear paddle signals with repeat periods $\sim 1.67 \times 10^4\,\text{s},\, 1.11 \times 10^4\,\text{s},\, \text{and}$ 226 1.39×10^4 s, respectively (Figs. 3a, 3b). OXBOU is then used to run each simulation for a 227 duration of $T_d = 1.10 \times 10^4$ s with the linear dispersion coefficient tuned for optimal dis-228 persion: B = 1/15. With the three simulations complete, the wave records are compiled and 229 the first 200s of each is neglected, following Trulsen et al. (2012), which leaves, at each 230 grid point, records of duration $\sim 8.48 \times 10^3$, 6.36×10^3 , and 5.90×10^3 peak wave periods, 231 respectively. Figure 3c shows, for the Case 2 simulation, the convergence of the normalised 232 mean, standard deviation, skewness, and kurtosis of the free surface elevation with number 233 of time samples in the wave record at x = 150 m. Each statistic is normalised by the corre-234 sponding value obtained for the entire record, and it is clear that the $\sim 1.644 \times 10^6$ samples 235 are sufficient to provide robust estimates for each experimental case. 236



Fig. 3: Example plots from the present Case 2 simulation showing (a) the input JONSWAP spectrum, (b) the nominal input wave signal, and (c) the convergence of the statistical moments G with the number of time samples n in the wave record (which has a total of $N \approx 1.644 \times 10^6$ samples) at x = 150 m: mean (dotted line), standard deviation (dashed-dotted line), skewness (dashed line), and kurtosis (solid line).

Figure 4 then compares, for each case, the simulated variations in variance, skewness, 237 and kurtosis along the length of the tank with those obtained from the Boussinesq-type nu-238 merical simulations of Gramstad et al. (2013) and the physical experiments of Trulsen et 239 al. (2012). The results from the present Boussinesq-type simulations are shown with 95% 240 confidence intervals determined using histograms produced by calculating the same statis-241 tics for 1000 bootstrap samples, which are obtained by random sampling with replacement 242 of 5% of the available data. Although the trends for each statistic are qualitatively similar, 243 the present profiles do not match those reported by Trulsen et al. (2012) and Gramstad et al. 244 (2013): the skewness results are consistently lower and initially negative, and the kurtosis 245



Fig. 4: Profiles of free surface elevation statistics: variance (**a**, **b**, **c**), skewness (**d**, **e**, **f**), and kurtosis (**g**, **h**, **i**) for Cases 1 (left column), 2 (centre column), and 3 (right column). Results are obtained from the physical experiments of Trulsen et al. (2012) (crosses), the Boussinesq-type simulations of Gramstad et al. (2013) (solid lines), and the present Boussinesq-type simulations (dots with 95% confidence intervals shaded in grey). The vertical dotted lines mark the positions of the toe (left) and crest (right) of the submerged seabed slope.

- ²⁴⁶ profiles exhibit greater reductions along the tank and much less prominent spikes near the
- ²⁴⁷ crest of the submerged seabed slope.
- 248 3.2 Case 2 parameter study
- ²⁴⁹ To investigate these discrepancies, a parameter study based on the Case 2 simulation is used
- ²⁵⁰ to examine the effects of various model inputs on the kurtosis profiles obtained for irreg-



Fig. 5: Kurtosis profiles from the Case 2 parameter study. (a) Flat domain: still water depth, h = 0.6 m (solid line); narrower input spectrum (dashed line); lower input kurtosis (dashed-dotted line); and h = 0.3 m (dotted line). (b) Submerged seabed slope: single realisation (solid line); quasi-ensemble average of the single realisation divided into fifths (dashed line); ensemble average of five alternate, independent realisations (dashed-dotted line); reduced sponge layer strengths (dotted line); and shorter simulations using first- (circles) and second-order (crosses) accurate wavemaker signals.

ular waves propagating over a flat, horizontal bed (Fig. 5a) as well as over the submerged 251 seabed slope (Fig. 5b). For a flat domain with still water depth h = 0.6 m, the kurtosis pro-252 file obtained for x < 143.4 m (Fig. 5a: solid line) is practically identical to that obtained 253 in the Case 2 simulation (Fig. 5b: solid line), which confirms that the upstream kurtosis 254 profile is unaffected by the reflections from the submerged slope. This flat-bed simulation 255 also demonstrates a reduction in kurtosis along the length of the tank: the kurtosis decreases 256 from the input value of \sim 3 and appears to stabilise at a value of \sim 2.9 towards the end of the 257 domain. Repeating this simulation with a lower input value of kurtosis (which is done by re-258 placing the original wavemaker signal with the negatively skewed wave record subsequently 259 obtained at x = 160 m) yields a more uniform profile, which further suggests an equilibrium 260 kurtosis value of ~ 2.9 for this case. However, this equilibrium value is found to depend, as 261 in earlier studies (see Janssen, 2003; Zeng & Trulsen, 2012), on both the still water depth 262 (Fig. 5a: dotted line) and the bandwidth of the input wave spectrum (Fig. 5a: dashed line). 263 For simulations including the submerged seabed slope, the kurtosis profiles appear in-264 sensitive to the location of the generating-absorbing sponge layer and the end-of-tank bound-265

²⁶⁶ ary condition. A similar profile is also obtained when the strengths of the absorbing and

²⁶⁷ absorbing-generating layers are reduced by 90% (Fig. 5b: dotted line), which implies that

the observed reduction in kurtosis is not the result of excess numerical damping. Dividing 268 each wave record from the Case 2 simulation into five equal sections and taking the quasi-269 ensemble average of these fifths yields a similar profile (Fig. 5b: dashed line), as does taking 270 the ensemble average across five alternate, independent realisations (Fig. 5b: dashed-dotted 271 line). This demonstrates that the present results do not depend on the type of measurement 272 taken. Moreover, the kurtosis profiles obtained from shorter-duration (for ease of computa-273 tion) simulations using first- and second-order accurate wavemaker signals are very similar 274 (Fig. 5b: circles; crosses), which implies that neither are the observed trends due to error 275 waves produced by the first-order accurate wavemaker (see Orszaghova et al., 2014). 276

277 4 Discussion and conclusions

The kurtosis profiles obtained in each experimental case agree qualitatively with those of 278 Trulsen et al. (2012) and Gramstad et al. (2013) but the present numerical model is clearly 279 unable to capture accurately the spikes near the crests of the submerged seabed slopes (Figs. 280 4g, 4h, 4i). A parameter study has confirmed that the present results do not depend on the 281 type of measurement taken, the position or damping strengths of the sponge layers, or the 282 order of accuracy of the wavemaker signal (Fig. 5b). Further discrepancies are also evi-283 dent: for the depths considered here, second-order bound harmonics are expected to posi-284 tively skew the probability distribution function for the free surface elevation (Onorato et al., 285 2005) but the present skewness results are initially negative (Figs. 4d, 4e, 4f). Replication 286 of an example irregular wave simulation with the 'fully nonlinear' OceanWave3D model 287 (see Engsig-Karup et al., 2009) (comparison not shown for brevity) confirms that OXBOU 288 produces consistently lower values of free surface elevation skewness and kurtosis. 289

The discrepancies between the present results and those of Gramstad et al. (2013) most likely stem from differences in the underlying momentum equations. The exact source of these discrepancies, however, is difficult to determine. When examining the propagation of irregular waves over a compound slope, Kashima et al. (2014) found that the present equation set returned values of skewness and kurtosis which were considerably lower than those obtained in the corresponding physical experiment. These lower values were explained as

being the result of insufficient nonlinearity in the numerical simulations, but Gramstad et 296 al. (2013) were able to use a similar weakly nonlinear model to reproduce the results of 297 Trulsen et al. (2012). Further, in deriving the present equation set, Madsen & Sørensen 298 (1992) adopted a mild slope assumption which retained only the lowest-order spatial deriva-299 tives of the water depth. This means that the present model is unable to capture the effects 300 of the sudden depth transition as well as that of Gramstad et al. (2013), which retains these 301 high-order terms. It is also worth noting that two of the present three experimental cases con-302 sider water depths which exceed the depth limit ($k_p h < 1$, where k_p is the peak wavenumber 303 of the input spectrum) recommended to ensure the accuracy of the present equation set (see 304 Madsen & Sørensen, 1992, 1993). 305

Using a boundary element method with fast multipole acceleration to solve Laplace's 306 equation for potential flow with fully nonlinear boundary conditions, Zheng et al. (2020) 307 have recently predicted the local changes in the statistical properties of irregular waves 308 propagating over a range of submerged slopes in close agreement with the experiments 309 by Trulsen et al. (2012). In doing so, Zheng et al. (2020) have demonstrated that these lo-310 cal changes are driven by second-order terms, which may help to explain why the peaks in 311 skewness and kurtosis cannot be accurately captured by the present Boussinesq-type model. 312 The present equation set includes a linear dispersion coefficient, B, which may be tuned to 313 produce either enhanced dispersion characteristics or approximately correct second-order 314 bound harmonics (Yao, 2007). Herein, B is assigned a value of 1/15 for optimal dispersion. 315 It is reasonable to assume that if the bound waves are inaccurate, significant errors in skew-316 ness and kurtosis will arise near the sudden depth transition, because the peaks in skewness 317 and kurtosis at this location are likely a consequence of the release of second-order bound 318 waves by the depth transition (Zheng et al., 2020). Although there is no value of B which 319 can make the present equation set equivalent to that of Gramstad et al. (2013), it is possible 320 to match the linear dispersion relations by setting B = 0.057. However, this is found to make 321 no appreciable difference to the present results and does not address the need to correct the 322 bound waves. Frequency domain comparisons between OceanWave3D and OXBOU (again 323

not shown for brevity) demonstrate that there is also no value of *B* which gives satisfactory agreement on sub-harmonic and super-harmonic content.

Modelling this sudden depth transition problem is challenging because it requires an 326 accurate yet computationally efficient numerical code which is able to incorporate the ef-327 fects of both dispersion and nonlinearity on the evolution of the wave field. The work of 328 Gramstad et al. (2013) has shown that the key physics underlying this localised increase in 329 the probability of rogue wave occurrence can be captured by a weakly dispersive, weakly 330 nonlinear Boussinesq-type model. There are, however, many different sets of Boussinesq-331 type equations and the present study demonstrates the importance of making an appropriate 332 selection. Although OXBOU is a very useful tool for modelling nearshore wave propaga-333 tion, run-up, and overtopping, it is clear that the underlying equation set is not well-suited to 334 modelling the propagation of waves over a sudden depth transition. It is thus recommended 335 that this problem be revisited using a revised version of OXBOU based on an improved set 336 of Boussinesq-type equations. The equations of Schäffer & Madsen (1995), for instance, 337 provide the same enhanced linear dispersion characteristics as those of Madsen & Sørensen 338 (1992) but are not limited to mildly sloping seabeds. It should also be noted, however, that 339 the accuracy of any numerical model will depend on the means by which the spatial and tem-340 poral derivatives are calculated (Borthwick et al., 2006), and that sudden depth transitions 341 invariably prove challenging for any low-order finite difference scheme. Shock-capturing 342 schemes offer an alternative approach but are generally less accurate and may introduce 343 further complications. 344

In future studies, it would prove valuable to compare statistical results not only between 345 different Boussinesq-type formulations but also between weakly and highly nonlinear mod-346 els, following Viotti & Dias (2014), Ducrozet & Gouin (2017), and Zheng et al. (2020), as 347 well as with physical experiments, following Zhang et al. (2019) and Trulsen et al. (2020). It 348 would also be interesting to explore whether idealised, multi-layer numerical models, such 349 as SWASH (Zijlema et al., 2011), can provide additional insight. Future work should exam-350 ine not only the extreme amplitudes but also the shapes and periods of these rogue waves, 351 which are crucial in understanding the strength of the wave impact and the resilience of ships 352

and offshore structures (Kharif et al., 2009; Adcock & Taylor, 2014). The effects of direc-353 tionality must also be considered because large waves evolve differently in unidirectional 354 and directionally spread seas (Adcock & Taylor, 2014), and studies have shown that even 355 a small amount of counter-propagating wave energy can result in a significant reduction in 356 free surface kurtosis (Ducrozet & Gouin, 2017; Støle-Hentschel et al., 2018). Finally, real-357 world observations should be included wherever possible in studies of rogue wave formation 358 and occurrence probability (Slunyaev et al., 2011) because it is the ocean that provides the 359 most representative conditions with which to test and revise new theories. 360

Acknowledgements The authors gratefully acknowledge support from the UK's Engineering and Physical 361 Sciences Research Council (EPSRC) and Natural Environment Research Council (NERC), which sponsored 362 this research under grant number EP/R007632/1. The authors wish to thank Dr Jana Orszaghova and Prof. 363 Paul H. Taylor, who contributed greatly to the development of the OXBOU model; Tianning Tang, who 364 carried out OceanWave3D simulations to compare with the present results; Prof. Vengatesan Venugopal, for 365 his support during the later stages of the project; and three anonymous reviewers for their helpful comments. 366 PAJB also wishes to thank Drs Tim Bunnik, Jacob Dobson, Samuel Draycott, Frances M. Judge, Yan Li, 367 James N. Steer, and James Young for providing much valuable information and many helpful discussions. 368 TSvdB was supported by a Royal Academy of Engineering Research Fellowship. 369

370 **References**

Adcock, TAA, Taylor, PH (2014) The physics of anomalous ('rogue') ocean waves. Rep.

Prog. Phys. 77: 105901, https://doi.org/10.1088/0034-4885/77/10/105901.

- ³⁷³ Adcock, TAA, Taylor, PH, Yan, S, Ma, QW, Janssen, PAEM (2011) Did the ³⁷⁴ Draupner wave occur in a crossing sea? Proc. R. Soc. A 467: 3004–3021,
- ³⁷⁵ https://doi.org/10.1098/rspa.2011.0049.
- Borthwick, AGL, Ford, M, Weston, BP, Taylor, PH, Stansby, PK (2006) Solitary wave trans-
- formation, breaking, and run-up at a beach. Proc ICE Marit. Engng 159(3): 97–105, https://doi.org/10.1680/maen.2006.159.3.97.
- ³⁷⁹ Cavaleri, L, Barbariol, F, Benetazzo, A, Bertotti, L, Bidlot, J-R, Janssen, P, Wedi, N (2016)
- The Draupner wave: A fresh look and the emerging view. J. Geophys. Res. Oceans 121:
- ³⁸¹ 6061–6075, https://doi.org/10.1002/2016JC011649.

- ³⁸² Christou, M, Ewans, K (2014) Field measurements of rogue water waves. J. Phys. Oceanogr.
 ³⁸³ 44(9): 2317–2335, https://doi.org/10.1175/JPO-D-13-0199.1.
- ³⁸⁴ Dean, RG, Dalrymple, RA (1991) Water wave mechanics for engineers and scientists. World
 ³⁸⁵ Scientific.
- ³⁸⁶ Dingemans, MW (1997) Water wave propagation over uneven bottoms. World Scientific.
- ³⁸⁷ Ducrozet, G, Gouin, M (2017) Influence of varying bathymetry in rogue wave occurrence
- within unidirectional and directional sea-states. J. Ocean Engng Mar. Energy 3: 309–324,
- 389 https://doi.org/10.1007/s40722-017-0086-6.
- ³⁹⁰ Dysthe, K, Krogstad, HE, Müller, P (2008) Oceanic rogue waves. Annu. Rev. Fluid Mech.
 ^{40:} 287–310, https://doi.org/10.1146/annurev.fluid.40.111406.102203.
- Engsig-Karup, AP, Bingham, HB, Lindberg, O (2009) An efficient flexible-order
 model for 3D nonlinear water waves. J. Comp. Phys. 228(6): 2100–2118,
- ³⁹⁴ https://doi.org/10.1016/j.jcp.2008.11.028.
- Fallahi, S (2016) Freak waves over nonuniform depth with different slopes. Master's thesis,
 University of Oslo, Norway.
- ³⁹⁷ Fedele, F, Brennan, J, Ponce De León, S, Dudley, J, Dias, F (2016) Real world
- ocean rogue waves explained without the modulational instability. Sci. Rep. 6: 27715,
- ³⁹⁹ https://doi.org/10.1038/srep27715.
- ⁴⁰⁰ Fitzgerald, CJ, Taylor, PH, Orszaghova, J, Borthwick, AGL, Whittaker, C, Raby, AC (2016)
- ⁴⁰¹ Irregular wave runup statistics on plane beaches: Application of a Boussinesq-type model
- ⁴⁰² incorporating a generating-absorbing sponge layer and second-order wave generation.
- 403 Coast. Engng 114: 309–324, http://dx.doi.org/10.1016/j.coastaleng.2016.04.019.
- 404 Gramstad, O, Zeng, H, Trulsen, K, Pedersen, GK (2013) Freak waves in weakly nonlinear
- unidirectional wave trains over a sloping bottom in shallow water. Phys. Fluids 25(12):
- 406 122103, https://doi.org/10.1063/1.4847035.
- 407 Haver, S, Andersen, OJ (2000) Freak waves: Rare realizations of a typical population or typ-
- ical realizations of a rare population? In: Proceedings of the 10th International Offshore
- and Polar Engineering Conference, Seattle, WA, USA.
- 410 Holthuijsen, LH (2007) Waves in oceanic and coastal waters. Cambridge University Press.

- PAEM (2003)Janssen, Nonlinear four-wave interactions and freak 411 863-884, https://doi.org/10.1175/1520waves. J. Phys. Oceanogr. 33(4): 412 0485(2003)33<863:NFIAFW>2.0.CO;2. 413
- Janssen, TT, Herbers, THC (2009) Nonlinear wave statistics in a focal zone. J. Phys. Oceanogr. 39(8): 1948–1964, https://doi.org/10.1175/2009JPO4124.1.
- Kashima, H, Hirayama, K, Mori, N (2014) Estimation of freak wave occurrence from deep to shallow water regions. Coast. Engng Proc. 1(34): 36,
 https://doi.org/10.9753/icce.v34.waves.36.
- ⁴¹⁹ Kharif, C, Pelinovsky, E, Slunyaev, A (2009) Rogue waves in the ocean. Springer Science
 ⁴²⁰ & Business Media.
- 421 Ma, Y, Dong, G, Ma, X (2014) Experimental study of statistics of random waves propagating
- 422 over a bar. Coast. Engng Proc. 1(34): 30, https://doi.org/10.9753/icce.v34.waves.30.
- 423 Ma, Y, Ma, X, Dong, G (2015) Variations of statistics for random waves propagating over a
- 424 bar. J. Mar. Sci. Tech. 23(6): 864–869, https://doi.org/10.6119/JMST-015-0610-3.
- 425 Madsen, PA, Sørensen, OR (1992) A new form of the Boussinesq equations with im-
- ⁴²⁶ proved linear dispersion characteristics. Part 2. A slowly-varying bathymetry. Coast. En-
- 427 gng 18(3-4): 183-204, https://doi.org/10.1016/0378-3839(92)90019-Q.
- 428 Madsen, PA, Sørensen, OR (1993) Bound waves and triad interactions in shallow water.
- 429 Ocean Engng 20(4): 359–388, https://doi.org/10.1016/0029-8018(93)90002-Y.
- 430 Mori, N, Janssen, PAEM (2006) On kurtosis and occurrence probability of freak waves. J.
- ⁴³¹ Phys. Oceanogr. 36(7): 1471–1483, https://doi.org/10.1175/JPO2922.1.
- 432 Ochi, MK (2005) Ocean waves: The stochastic approach. Cambridge University Press.
- 433 Onorato, M, Osborne, AR, Serio, M (2005) On deviations from Gaussian statistics for sur-
- face gravity waves. arXiv preprint nlin/0503071, https://arxiv.org/pdf/nlin/0503071.pdf.
- Onorato, M, Osborne, AR, Serio, M, Cavaleri, L, Brandani, C, Stansberg, CT
 (2004) Observation of strongly non-Gaussian statistics for random sea surface gravity waves in wave flume experiments. Phys. Rev. E 70: 067302,
 https://doi.org/10.1103/PhysRevE.70.067302.

- 439 Onorato, M, Residori, S, Bortolozzo, U, Montina, A, Arecchi, FT (2013) Rogue waves and
- their generating mechanisms in different physical contexts. Phys. Rep. 528(2): 47–89,
 https://doi.org/10.1016/j.physrep.2013.03.001.
- ⁴⁴² Orszaghova, J (2011) Solitary waves and wave groups at the shore. DPhil thesis, University
 ⁴⁴³ of Oxford, UK.
- 444 Orszaghova, J, Borthwick, AGL, Taylor, PH (2012) From the paddle to the beach A
- Boussinesq shallow water numerical wave tank based on Madsen and Sørensen's equa-
- tions. J. Comp. Phys. 231(2): 328–344, https://doi.org/10.1016/j.jcp.2011.08.028.
- 447 Orszaghova, J, Taylor, PH, Borthwick, AGL, Raby, AC (2014) Importance of second-order
- wave generation for focused wave group run-up and overtopping. Coast. Engng 94(2):
 63–79, http://doi.org/10.1016/j.coastaleng.2014.08.007.
- ⁴⁵⁰ Peregrine, DH (1967) Long waves on a beach. J. Fluid Mech. 27(4): 815–827,
 ⁴⁵¹ https://doi.org/10.1017/S0022112067002605.
- Raustøl, A (2014) Freake bølger over variabelt dyp. Master's thesis (in Norwegian), University of Oslo, Norway.
- 454 Schäffer, HA (1996) Second-order wavemaker theory for irregular waves. Ocean Engng
 455 23(1): 47–88, https://doi.org/10.1016/0029-8018(95)00013-B.
- 456 Schäffer, HA, Madsen, PA (1995) Further enhancements of Boussinesq-type equations.
- 457 Coast. Engng 26(1–2): 1–14, https://doi.org/10.1016/0378-3839(95)00017-2.
- 458 Sergeeva, A, Pelinovsky, E, Talipova, T (2011) Nonlinear random wave field in shallow
- 459 water: Variable Korteweg-de Vries framework. Nat. Hazards Earth Syst. Sci. 11: 323-
- 460 330, https://doi.org/10.5194/nhess-11-323-2011.
- ⁴⁶¹ Slunyaev, A, Didenkulova, I, Pelinovsky, E (2011) Rogue waters. Contemp. Phys. 52(6):
 ⁴⁶² 571–590, https://doi.org/10.1080/00107514.2011.613256.
- 463 Støle-Hentschel, S, Trulsen, K, Rye, LB, Raustøl, A (2018) Extreme wave statistics
 464 of counter-propagating, irregular, long-crested sea states. Phys. Fluids 30: 067102,
 465 https://doi.org/10.1063/1.5034212.
- ⁴⁶⁶ Trulsen, K (2018) Rogue waves in the ocean, the role of modulational instability, and abrupt
- ⁴⁶⁷ changes of environmental conditions that can provoke non equilibrium wave dynamics.

- ⁴⁶⁸ In: The ocean in motion. Springer Oceanography.
- Trulsen, K, Raustøl, A, Jorde, S, Rye, LB (2020) Extreme wave statistics of long-crested irregular waves over a shoal. J. Fluid Mech. 882: R2,
 http://dx.doi.org/10.1017/jfm.2019.861.
- 472 Trulsen, K, Zeng, H, Gramstad, O (2012) Laboratory evidence of freak
- waves provoked by non-uniform bathymetry. Phys. Fluids 24(9): 097101,
 http://dx.doi.org/10.1063/1.4748346.
- 475 Tucker, MJ, Challenor, PG, Carter, DJT (1984) Numerical simulation of a random sea: A
- ⁴⁷⁶ common error and its effect upon wave group statistics. Appl. Ocean Res. 6(2): 118–122,
- 477 https://doi.org/10.1016/0141-1187(84)90050-6.
- ⁴⁷⁸ Viotti, C, Dias, F (2014) Extreme waves induced by strong depth transitions: Fully nonlinear
- results. Phys. Fluids 26(5): 051705, https://doi.org/10.1063/1.4880659.
- Yao, Y (2007) Boussinesq-type modelling of gently shoaling extreme ocean waves. DPhil
 thesis, University of Oxford, UK.
- 482 Zeng, H, Trulsen, K (2012) Evolution of skewness and kurtosis of weakly nonlinear uni-
- directional waves over a sloping bottom. Nat. Hazards Earth Syst. Sci. 12(3): 631–638,
 https://doi.org/10.5194/nhess-12-631-2012.
- ⁴⁸⁵ Zhang, J, Benoit, M, Kimmoun, O, Chabchoub, A, Hsu, H-C (2019) Statistics of extreme
- 486 waves in coastal waters: Large scale experiments and advanced numerical simulations.
- ⁴⁸⁷ Fluids 4(2): 99, https://doi.org/10.3390/fluids4020099.
- ⁴⁸⁸ Zheng, Y, Lin, Z, Li, Y, Adcock, TAA, Li, Y, van den Bremer, TS (2020) Fully nonlinear
- simulations of extreme waves provoked by strong depth transitions: The effect of slope.
- ⁴⁹⁰ Phys. Rev. Fluids 5: 064804, https://doi.org/10.1103/PhysRevFluids.5.064804.
- ⁴⁹¹ Zijlema, M, Stelling, G, Smit, P (2011) SWASH: An operational public domain code for
- simulating wave fields and rapidly varied flows in coastal waters. Coast. Engng 58(10):
- ⁴⁹³ 992–1012, https://doi.org/10.1016/j.coastaleng.2011.05.015.