Measurement of Systemic Risk in Global Financial Markets and Its Application in Forecasting Trading Decisions

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Abstract: The global financial crisis in 2008 spurred the need to study systemic risk in financial markets, which is of interest to both academics and practitioners alike. We first aimed to measure and forecast systemic risk in global financial markets and then to construct a trade decision model for investors and financial institutions to assist them in forecasting risk and potential returns based on the results of the analysis of systemic risk. The factor copula-generalized autoregressive conditional heteroskedasticity (GARCH) models and component expected shortfall (CES) were combined for the first time in this study to measure systemic risk and the contribution of individual countries to global systemic risk in global financial markets. The use of factor copula-based models enabled the estimation of joint models in stages, thereby considerably reducing computational burden. A high-dimensional dataset of daily stock market indices of 43 countries covering the period 2003 to 2019 was used to represent global financial markets. The CES portfolios developed in this study, based on the forecasting results of systemic risk, not only allow spreading of systemic risk but may also enable investors and financial institutions to make profits. The main policy implication of our study is that forecasting systemic risk of global financial markets and developing portfolios can provide valuable insights for financial institutions and policy makers to diversify portfolios and spread risk for future investments and trade.

Keywords: stock markets; factor copula; dependence; forecasting risk; financial crisis

1. Introduction

With the development of the economy and the flourishing of the stock market, stock indexes have been identified as important social and economic indicators that can comprehensively reflect the overall trends and performance of the stock market. Based on the sheer size and breadth, global stock markets play a decisive role in global financial performance. Along with the increasingly closer economic ties amongst all countries, rapid capital flows are not rare in modern global stock markets, allowing for fast and frequent transactions, resulting in markets that tend to have higher dependencies on each other. Economic globalization has created accessibility and convenience for investors, managers, and relevant officers in financial markets, with the drawback being the acceleration of the risk contagions.

Global stock markets play a decisive role in global financial performance. Along with increasingly closer economic ties amongst all countries, rapid capital flows are not rare in modern global stock markets, which enable fast and frequent transactions, resulting in markets having higher dependencies amongst themselves. The financial crisis of 2008 is considered as the worst financial crisis since the Great...
Depression, and its consequences not only included production curtailment but also rapid destruction of the financial system in a short timespan. It has also triggered renewal of the notion of financial regulation (e.g., macro-prudential policy) and the concept of systemic risk. Systemic risk includes the potential fluctuations, losses, and crises of the whole financial system influenced by relevant financial activities, policies, transactions, and so on [1]. Influenced by the financial crisis, global stock markets faced a huge slump. Among all possible explanations for this catastrophe, mis-prediction and ignorance of potential risk are the main topics (see [2–4] and others). To reduce the risk of a financial crisis, measuring and monitoring the potential risk of global stock markets is important. Therefore, studies that focus on measurement of systemic risk in global stock markets are essential.

Systemic risk is defined as the risk of distress in a various group of institutions. Different from systematic risk, systemic risk represents a severe degree of loss and a large proportion of the institutions [5]. Measurements of systemic risk were examined from various perspectives. Many scholars focused on measuring or forecasting systemic risk. For example, Bartram et al. [6] estimated the risk of systemic failure in the global banking system. Song et al. [7] used stock markets in G20 nations to represent global financial market and estimated financial risk from 2007 to 2018. However, most other studies measured systemic risk in some countries or regions. For example, Acharya et al. [8], Browlees and Engle [9], and Banulescu and Dumitrescu [10] used a similar dataset of stock market to forecast systemic risk for the USA financial market. Reboredo and Ugolini [11] measured systemic risk in the European sovereign debt market. Bartels and Ziegelmann [12] measured systemic risk in Brazil Sao using 44 time series of financial assets from the Sao Paulo Stock Exchange. Shahzad et al. [13] analyzed spillover effect and systemic risk of Islamic equity markets. Other studies focused on the systemic risk of special financial markets, such as securities markets [14], banking system [15], and default swap [5]. Scholars have considerably contributed to regional financial markets in terms of examining systemic risk.

Scholars have also contributed by developing quantitative models to measure or forecast systemic risk. Studying the dependence between a large collection of assets is required for general analysis of systemic risk [5]. Thus, most scholars proposed methods that are a combination of dependence modelling and risk measuring methods to measure or forecast systemic risk. With regard to dependence modelling, mainstream models, such as Dynamic Conditional Correlation-GARCH and copula-GARCH, reduce the dimensions of data to two: Individual firm and stock market index, e.g., Browlees and Engle [9], Wu et al. [16], Yun and Moon [17], Banulescu and Dumitrescu [10], Calabrese and Osmetti [18], and Wei et al. [19]. These methods have the clear benefit of being parsimonious. To handle large collections of data, some researchers applied vine copula-GARCH approaches into dependence and risk analysis, such as Liu et al. [20], Reboredo and Ugolini [21], Pourkanali et al. [22], Shahzad et al. [13], and Song et al. [7]. In vine copula-GARCH approaches, the number of parameters exponentially increase with an increase in the number of variables, which restricts their application to high-dimensional data. Recently, factor copula-GARCH models have been proposed and extended to the application of systemic risk analysis [5,23]. Different types of factor copula-GARCH approaches were proposed by Oh and Patton [23] and Krupskii and Joe [24]. The factor copula of Krupskii and Joe [24] can be easily interpreted and is able to capture linear or nonlinear correlations between any asset and latent variables compared to the one proposed by Oh and Patton [23].

Under the measurement framework of systemic-risk-based dependence modelling, some scholars proposed some methods of risk measurement that can be widely applied to various financial markets. One prominent measurement method is the marginal expected shortfall (MES) proposed by Acharya et al. [25], which has been widely applied by many, such as Tiwari et al. [26], Kleinow et al. [27], and Benoit et al. [28]. The MES measures marginal contribution of one financial institution or firm to total loss, but it cannot accurately reflect total loss or systemic risk. The other systemic risk measure is the delta conditional value-at-risk (CoVaR), proposed by Adrian and Brunnenneier [29]. The delta CoVaR method can be used to measure the spillover effects of systemic risk, but lacks sub-additive calculations. For instance, Reboredo [21] investigated systemic risk and dependence between oil and renewable energy markets in the United States; Reboredo and Ugolini [30] examined systemic risk...
in European sovereign debt markets before and during the Greek debt crisis; Reboredo et al. [31] studied systemic risk spillovers between currency and stock markets in emerging economies. On the basis of MES and CoVaR, the component expected shortfall (CES) was developed by Banulescu and Dumitrescu [10]. The CES can be used to measure a system’s aggregate loss using expected shortfall (ES), and also allows us to measure the contribution of one institution to systemic risk [15,32,33]. Therefore, this method is conducive to identifying important institutions systematically, thereby providing guidelines for formulating more relevant policies for governments and financial institutions.

Through reviewing the aforementioned literature, we summarize the main points of the existing literature as follows: First, systemic risk has been a major concern and many scholars have conducted related research from various perspectives. No study to date has investigated the systemic risk of global financial markets using high-dimensional data. Second, a variety of rich analytical methods are available for analyzing systemic risk, and have solved the problems of high-dimensional distribution, spillover effects, contribution of individual institutions to total risk, and magnitude of systemic risk. However, most of the methods do not allow for nonlinear correlation of financial assets in high-dimensional data.

Given this backdrop, we investigated the systemic risk of the global financial markets using the factor copula-GARCH models of Krupskii and Joe [24] and CES method. The stock markets of 43 major countries (G20 and European Union (EU) nations) were used to represent the global financial markets. The factor copula-GARCH models allow for better flexibility in joint distributions than vine copula, and capture nonlinear dependence and tail dependence between financial assets and latent variables. The latent variables were aggregated from many exogenous variables, such as crude oil price, political factors, interest rates, debt-to-GDP ratio, etc. For example, the co-movement and the spillover effect between crude oil price and the stock market performance were demonstrated in different countries [34,35]; and government debt and debt-to-GDP ratios are also key factors that influence the stock returns [36–38]. A large collection of assets representing global financial markets would improve the accuracy of systemic risk measurement. Therefore, there are three main contributions in this paper. First, to the best of our knowledge, this is one of the earliest attempts to investigate the systemic risk of the global financial markets using high-dimensional data. Second, we combine the factor copula-GARCH models with the CES approach for the first time, and applied the method to measure the systemic risk of global financial markets. Third, the measurement of systemic risk of global financial markets was used to diversify portfolios. A deeper understanding of the systemic risk and Banulescu portfolios may help investors and policymakers to design sound investment and risk management strategies and efficient macroeconomic policies.

The remainder of the paper is structured as follows: Section 2 describes models for marginals, factor copulas, and component expected shortfall. Section 3 describes the data. Section 4 presents the results. Section 5 provides our conclusions and policy implications.

2. Methodology

We used factor copula-based Glosten Jagannathan Runkel (GJR)-GARCH models [39] to construct a high dimensional distribution of the global financial market to describe asymmetric volatility and dependence structures of each stock return and latent variable. In the context of flexible fitting of high-dimensional distribution, we used the CES method to measure global systemic risk and identify important financial risk countries. The advantages of factor copula-based models are that any high-dimensional data are easier to fit by reducing the number of parameters, and that the dependence and tail dependence can be measured between stock returns and latent variables, which is why the factor copula model of Krupskii and Joe is superior compared to other factor copulas. Due to leptokurtosis and the fat tail of stock returns, the marginal distributions of stock returns were selected from normal distribution, skewed-normal distribution, skewed-t distribution (SSTD), and skewed generalized error distribution (SGED) according to the Akaike information criterion (AIC) and Bayesian information criterion (BIC). The specifications of GJR-GARCH, factor copulas, and CES are introduced in the following subsections.
2.1. Model for Marginals

Because the indices we studied exhibit time-varying volatility clustering, we first employed autoregressive moving average (ARMA)-GJR-GARCH models for the log-return. The ARMA-GJR-GARCH model presented by Glosten et al. [39] has been widely adopted to filter time series data because it allows for leverage effects, which are an often-observed phenomenon [40]. During crisis periods, the leverage effects tend to be more significant [41]. To be specific, the ARMA \((r,m)\) process is defined as:

\[
y_t = c + \sum_{k=1}^{r} \phi_k y_{t-k} + \sum_{k=1}^{m} \rho_k \epsilon_{t-k} + \epsilon_t
\]  

where the conditional mean is denoted as \(y_t\) and the error term \(\epsilon_t\). The GJR-GARCH\((p,q)\) model is defined as:

\[
\sigma_t^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I_{t-i}) \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]

where:

\[I_{t-1} = \begin{cases} 
0 & \text{if } \epsilon_{t-1} \geq 0, \\
1 & \text{if } \epsilon_{t-1} < 0. 
\end{cases}\]

the leverage effect is denoted as \(\gamma\); the i.i.d standard innovation variables are denoted as \(\eta_t\). Then, the error term can be computed by \(\epsilon_t = \sigma_t \eta_t\).

All ARMA-GJR-GARCH\((1,1)\) parameters from Equations (1) and (2) can be obtained using the maximum likelihood estimation (MLE) method. The cumulative distribution functions can be obtained via simple transformations.

2.2. Factor Copula Models

One of the latest copula models, known as factor copulas, was adopted in this study due to the simplicity of its calculation process, its processing speed, practicality in estimation of the dependence of parameters, and ease of interpreting results. Briefly, all endogenous and exogenous variables that affect economic performance are treated as common latent variables by factor copulas. Therefore, we judged that the estimation results are better and easier to explain.

For factor copulas, we first transformed marginal distributions into uniform distributions, \(F_{X_j} = u_j \sim U(0, 1), \ j = 1, \cdots, d\). Compared with the other factor copula models, the factor copula model proposed by Krupskii and Joe [24] was adopted for its simplicity and traceability. Thus, the joint CDF can be obtained by \(U\) and \(C(u_1, \cdots, u_d)\). Without losing generality, given \(p\) common latent variables—each of which was i.i.d. and uniformly distributed, which is \(V_1, \cdots, V_p\)—the factor copula model is defined as:

\[
C(u_1, \cdots, u_d) = \int_{[0,1]^p} \prod_{j=1}^{d} F_{j|V_1, \cdots, V_p}(u_j|v_1, \cdots, v_p) dv_1 \cdots dv_p
\]

Consider when \(p = 1\), which is one-factor copula. Let \(C_{j,V_1}\) represent the joint CDF of \((U_j, V_1)\), \(c_{j,V_1}\) denotes the probability density function, and the conditional copula \(C_{j|V_1}(u_j|v_1) = F_{j|V_1}(u_j,v_1) = \partial C_{j|V_1}(u_j,v_1)/\partial v_1\). According to the algorithm of copula approach, Equation (3) can be simplified as:

\[
C(u_1, \cdots, u_d) = \int_0^1 \prod_{j=1}^{d} F_{j|V_1}(u_j|v_1) dv_1 = \int_0^1 \prod_{j=1}^{d} C_{j|V_1}(u_j|v_1) dv_1
\]
Since \( \frac{\partial}{\partial u} C_{ij}(u|v_1) = \frac{\partial^2}{\partial u \partial v_1} C_{ij}(u, v_1) = c_{ij}(u, v_1) \), the density function of one factor copula can be expressed as:

\[
c(u_1, \ldots, u_d) = \frac{\partial^d C(u_1, \ldots, u_d)}{\partial u_1 \cdots \partial u_d} = \int_0^1 \prod_{j=1}^d c_{ij}(u_{ij}, v_1) dv_1
\]  

(5)

We considered how to compute numerical integration and construct a likelihood function for one factor copula. Suppose that we have a uniform distribution dataset \((u_{i1}, u_{i2}, \ldots, u_{id})\), where \(i\) ranges from 1 to \(n\), and its joint density of one factor copula is given by \(c(u_{i1}, u_{i2}, \ldots, u_{id}; \theta)\), where \(\theta\) is a vector of dependence parameters. The joint density \(c(u_{i1}, u_{i2}, \ldots, u_{id}; \theta)\) can be estimated using the MLE method. Therefore, the likelihood \(L(u_{i1}, u_{i2}, \ldots, u_{id}; \theta)\) can be written as:

\[
L(u_1, \ldots, u_d; \theta) = \prod_{i=1}^n c(u_{i1}, \ldots, u_{id}; \theta)
\]  

(6)

According to Equation (6), the corresponding one-factor copula density function with parameter \(\theta\) can be written as:

\[
c(u_{i1}, \ldots, u_{id}; \theta) = \int_0^1 \prod_{j=1}^d c_{ij}(u_{ij}, v_1; \theta) dv_1
\]  

(7)

where vector \(\theta = (\theta_j)_{1 \leq j \leq d}\), and \(\theta_j\) represents the dependence between the stock market of the \(j\)th country and latent factors.

The Gauss–Legendre quadrature approach was applied in this study to obtain the approximation of the integral and is better than the Gauss–Hermite quadrature [24, 42]. Therefore, the approximation of \(c(u_1, \ldots, u_d; \theta)\) according to the Gauss–Legendre approach is:

\[
c(u_{i1}, \ldots, u_{id}; \theta) \approx \sum_{k=1}^{n_q} \omega_k \prod_{j=1}^d c_{ij}(u_{ij}, x_k; \theta) dv_1
\]  

(8)

The quadrature weights are represented by \(\omega_k\), the number of quadrature points by \(n_q\), and the nodes by \(x_k\).

The AIC and BIC are computed by \(AIC = 2k - 2ln(L)\) and \(BIC = ln(n)k - 2ln(L)\), where \(k\) represents the number of parameters and \(n\) is the number of samples. According to Equations (5) and (6), the AIC and BIC for factor copula models are:

\[
AIC = 2k - 2ln(\prod_{i=1}^n c(u_{i1}, \ldots, u_{id}; \theta))
\]

\[
BIC = ln(n)k - 2ln(\prod_{i=1}^n c(u_{i1}, \ldots, u_{id}; \theta))
\]  

(9)

Krupskii and Harry Joe [24] explained the two and more factor copula models. However, constrained to codes and computer calculation capacity, the calculation time for two-factor copulas with 42(43)-dimensional data was more than nine hours for each CES estimation and 261 estimations were required for each period. Due to the time consumption and estimation practicality, we adopted one-factor copulas as our method.
2.3. Component Expected Shortfall

VaR, as a kind of risk measurement, is already widely used due to its simplicity. Denoting \( \alpha \in (0, 1) \) as the quantile and \( r_t \) as the real log-return, VaR can be explained as: Occurrences of the losses at the \( \alpha \) quantile. VaR is defined as:

\[
VaR_{\alpha}(X) = \inf \{ x \in \mathbb{R} : P(r_t < x) > \alpha \} \tag{10}
\]

After having adopted VaR as the standard risk measurement calibration in the few years previously, the Basel Committee on Banking Supervision decided to replace the VaR method by ES in 2016 \([43]\) because the VaR may provide incorrect estimations during turbulent periods. Currently, some multivariate models, such as vine copula and multivariate GARCH models, are combined with ES to forecast financial risk \([5,6,11]\). Let \( R_t \) denote the total weighted log-return at time \( t \), \( r_{it} \) and \( W_{it} \) are the real daily log-return of country \( i \) and the corresponding market capitalization at time \( t \), respectively. Then, \( R_t = \sum_{i=1}^{n} (r_{it}W_{it}) / \sum_{i=1}^{n} W_{it} \). In this paper, the integral form of ES at time \( t \) is defined as:

\[
ES_{\alpha}(X)_t = -E(R_t | R_t < VaR_{\alpha}) = -\frac{1}{\alpha} \int_{0}^{\alpha} VaR_{\gamma}(X) d\gamma \tag{11}
\]

where \( \alpha \in (0, 1) \) is the quantile, set to 1%.

In 2015, Banulescu and Dumitrescu \([10]\) proposed a hybrid measure called the component expected shortfall (CES), which combined the Too Interconnected To Fail and the Too Big To Fail logics. The CES allows the decomposition of the aggregate financial risk. Therefore, it is able to determine the contributions of each country to the total financial system. The CES for country \( i \) at time \( t \) is defined as:

\[
CES_{it} = w_{it} \frac{\partial ES_{it}}{\partial w_{it}} \tag{12}
\]

where summation of CES\(_{it}\) equals ES, and \( w_{it} = W_{it} / \sum_{i=1}^{n} W_{it} \). Actually, the ES was defined as average VaR(AVaR) in \([44,45]\). The CES encompasses the popular Marginal ES measure, and does not consider the liabilities, as the SRISK did. Moreover, the CES can reflect the percentual contribution to total loss of an institution. The larger the contribution, the more systemically important the institution. Therefore, the percentual contribution to systemic risk, CES\(_{\%t}\), is defined as

\[
CES_{\%it} = \frac{CES_{it}}{ES_{t}} \times 100 = \frac{w_{it} \frac{\partial ES_{it}}{\partial w_{it}}}{\sum_{i=1}^{n} w_{it} \frac{\partial ES_{it}}{\partial w_{it}}} \times 100 \tag{13}
\]

Here, we opted for the CES approach to analyze the contributions from different countries to systemic risk and applications to portfolios. The factor copula-GJR-GARCH models with CES was performed in R software. Some key packages, such as rugarch \([46]\), CopulaModel \([47]\), and PerformanceAnalytics \([48]\), have been applied to filter data, estimate the coefficients, and generate simulations. The factor copulas are limited by the CopulaModel package in R. We only applied four copula families in this study: Gumbel, Frank, Student-t, and Gaussian.

To forecast the systemic risk of the global financial market and countries’ contribution to systemic risk, we performed a one-period ahead forecasting. First, the dataset was divided into in-sample and out-of-sample. Second, GJR-GARCH models were used to filter the stock returns. We employed maximum likelihood estimation (MLE) to estimate GJR-GARCH models for in-sample data, thereby obtaining standardized residuals for each stock market. In this process, we estimated the GJR-GARCH models with different margins, including normal distribution, skewed normal distribution, skewed Student’s t distribution, and skewed generalized error distribution, for each stock.
index, and the best GJR-GARCH model with margin for each stock index was selected in terms of AIC and BIC. Second, we transformed the standardized residuals into uniform distribution using marginal conditional cumulative distribution functions and plugged them into factor copulas. Third, the maximum likelihood method was implemented to estimate the parameters of factor copulas (see inference function for margins in Joe [23] and Krupskii and Joe [24]). Similarly, the best factor copula was chosen at the one that had the lowest value for both AIC and BIC. Fourth, we drew 10,000 random numbers from the best factor copula, and used the inverse function of the corresponding marginal distribution of each variable to obtain the standardized residuals. Fifth, we forecast the value of each variable at the \( t + 1 \) period using the GJR-GARCH model; thus, 10,000 possible values were generated at the \( t + 1 \) period for each variable. Therefore, the total systemic risk of the global stock market, systemic risk, and contribution of each country to the total financial market risk were calculated by Equations (14)–(16) respectively.

\[
ES_{t+1}(\tilde{C}) = \sum_{i=1}^{n} CES_{i,t+1}(\tilde{C}) \tag{14}
\]

\[
CES_{i,t+1}(\tilde{C}) = w_{it} \frac{\partial ES_{t+1}(\tilde{C})}{\partial w_{it}} \tag{15}
\]

\[
CES_{\%i,t+1}(\tilde{C}) = \frac{CES_{i,t+1}(\tilde{C})}{\sum_{i=1}^{n} CES_{i,t+1}(\tilde{C})} \times 100 \tag{16}
\]

where \( \tilde{C} \) is a threshold that equals to the out-of-sample VaR value at time \( t + 1 \) for the 10,000 possible values at coverage rates of 1% [10].

Finally, we repeated the above-mentioned steps and used the rolling window method to forecast the systemic risk of the global financial market for all out-of-sample data.

3. The Data

For the purpose of financial risk analysis, the stock index data of 43 countries were obtained. Due to the high volatility, the large number of exogenous variables caused by terrorism events, the crash of the dotcom bubble, and other events, the data of the early 2000s were not adopted in this study. The data cover the period from 1 May 2003 to 31 May 2019. To forecast systemic risk and observe the performance of the global financial market in different periods, we separated the data into three periods: Pre-crisis period, crisis period, and post-crisis period. According to Tachibana [49] and Allen et al. [50], the pre-crisis period started on 1 May 2003 and ended on 31 July 2007. Following the pre-crisis period, the endpoint of the crisis period is considered as the last day of 2012 [51]. So, the post-crisis period started at 1 January 2013 and ended at 30 June 2019. In each period, the dataset was separated into two parts: In-sample and out-of-sample. The out-of-sample data were a one-year trading period, 261 days. The in-sample data ranged from 1 May 2003 to 30 December 2006 and the out-of-sample data from 31 December 2006 to 31 December 2007 for the pre-crisis period. The out-of-sample data from 31 December 2011 to 31 December 2012 for the crisis period, and from 30 June 2018 to 30 June 2019 for the post-crisis period. Daily log-return data were obtained by simple transformation from daily data, which are \( r_i = \ln(P_i/P_{i-1}) \), where \( P \) is the close price in the stock markets.

The daily data of stock indexes and their corresponding capitalization in 43 sovereign countries were collected from Bloomberg and Reuters. Since the establishment of the Cyprus Main Market (CYMAIN) in 2006, only 42 countries were considered during the pre-crisis period and 43 countries for the crisis and post-crisis periods. The names of the countries and their abbreviations of their corresponding indexes are presented in Table 1.
Table 1. The stock indexes used in the empirical research.

<table>
<thead>
<tr>
<th>Country</th>
<th>Name</th>
<th>Country</th>
<th>Name</th>
<th>Country</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>SPX</td>
<td>India</td>
<td>SENSEX</td>
<td>Hungary</td>
<td>BUX</td>
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<tr>
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<td>Saudi</td>
<td>TASI</td>
<td>Ireland</td>
<td>ISEQ</td>
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<td>N225</td>
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<td>JTOPI</td>
<td>Latvia</td>
<td>OMXGI</td>
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<td>Lithuania</td>
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<td>ATX</td>
<td>Luxembourg</td>
<td>LUXX</td>
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<tr>
<td>Canada</td>
<td>SandP/TSX</td>
<td>Belgium</td>
<td>BEL20</td>
<td>Malta</td>
<td>MSE</td>
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<tr>
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<td>FTSEMI</td>
<td>Bulgaria</td>
<td>SOFIX</td>
<td>Netherlands</td>
<td>AEX</td>
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<tr>
<td>Russian</td>
<td>IMOEX</td>
<td>Cyprus</td>
<td>CMAIN</td>
<td>Poland</td>
<td>WIG2O</td>
</tr>
<tr>
<td>Australia</td>
<td>SandP/ASX20</td>
<td>Croatia</td>
<td>CRBEX</td>
<td>Portugal</td>
<td>PSI20</td>
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<td>PX</td>
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<td>Denmark</td>
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<td>OMXGI</td>
<td>Slovenia</td>
<td>SBITOP</td>
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<td>SandP/BMV</td>
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<td>Spain</td>
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<td>Sweden</td>
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<td>Indonesia</td>
<td>JKSE</td>
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Note: For the pre-crisis period, Cyprus is not included as CMAIN was only issued from 2006.

4. Empirical Results

4.1. Estimation Results for Factor Copula

Table 2 provides the details of the AIC and BIC of the factor copulas for all three crisis time periods. Based on the AIC and BIC values, the Gumbel copula was adopted, which showed upper tail dependence for our sample data during the period before the crisis. For the other times, the Frank copula was the most suitable model for measuring tail dependence. Frank copula showed the tail as being located far from the extremes but not within the limits, which implied a lower tail concentration and high probability of obtaining extreme values.

Table 2. Akaike information criterion (AIC) and Bayesian information criterion (BIC) of factor copulas.

<table>
<thead>
<tr>
<th>Period</th>
<th>Gaussian</th>
<th>Frank</th>
<th>t-Copula</th>
<th>Gumbel</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre</td>
<td>AIC</td>
<td>4001.889</td>
<td>5219.750</td>
<td>5066.657</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>4158.405</td>
<td>5376.265</td>
<td>5223.172</td>
</tr>
<tr>
<td>in</td>
<td>AIC</td>
<td>27,238.640</td>
<td>30,307.380</td>
<td>28,853.510</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>27,410.311</td>
<td>30,472.891</td>
<td>29,025.182</td>
</tr>
<tr>
<td>post</td>
<td>AIC</td>
<td>19,457.490</td>
<td>22,915.350</td>
<td>21,266.656</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>19,632.780</td>
<td>23,090.630</td>
<td>21,441.930</td>
</tr>
</tbody>
</table>

Note: The bold denotes the lowest value of AIC and BIC.

The estimated results of $\theta$, which indicates the dependence of each country to the common latent variables, were computed by the previously selected factor copula models. Table 3 lists the 10 countries with the biggest $\theta$ and the 10 with the smallest $\theta$. The table shows that the coefficients $\theta$ of the crisis period, particularly the top 10 largest, are significantly larger than in the pre- and post-crisis periods, which proves that during the crisis period, most countries tended to be more vulnerable. For all three periods, the top 10 countries that were most sensitive to the common factors were European countries, whereas China and Malta remained the least sensitive countries. Three European countries, France, Germany, and Italy, demonstrated a lower dependence on the common latent variables during the pre-crisis period but then became three of the top 10 countries with the largest coefficient. The reasons behind this may be two-fold. Firstly, from previous research [7], these three countries have high correlation; therefore, after Italy was affected by the financial crisis, harmful consequences occurred in Germany and France sequentially. Secondly, for all three countries, the debt-to-GDP ratios increased...
during the pre-crisis period. In 2012, the ratios were 123.4%, 90.6%, and 80.7% for Italy, France, and Germany, respectively. The high ratios may have been the trigger for the crisis and the cause of public panic. This was harmful to the development of the economy and the international credibility of the country; therefore, countries were more sensitive to common latent variable. Even if Germany reduced its debt-to-GDP ratio, the ratios of France and Italy were still more than 90%. If Italy and France cannot reduce their high debt-to-GDP ratios, not only these two countries, but Germany as well, will retain the higher dependence on common latent variables compared with other countries. Notably, countries such as China and Malta have a relatively low dependence with common latent variables during all three periods. The explanations for the two countries may be different. For China, the government intervention, macroscopic control, and relatively lower market openness before the financial crisis may have affected the result. As for Malta, due to its stable economic growth and the strong performance during 2008, it was ranked 19 out 149 countries on the Legatum Prosperity Index. The reason for this might be the soundness of the banking system, effective supervision, high solvency ratio, and buffers [52]. In addition, the high rankings on safety and economic quality markers might be another reason for the lack of impact of the 2008 financial crisis [53]. In general, findings presented above are consistent with the data.

**Table 3. Estimated results of the factor copulas during each period.**

<table>
<thead>
<tr>
<th>Country</th>
<th>Pre-Crisis (Gumbel)</th>
<th>Crisis (Frank)</th>
<th>Post-Crisis (Frank)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 10</td>
<td>θ</td>
<td>Last 10</td>
<td>Top 10</td>
</tr>
<tr>
<td>Slovakia</td>
<td>2.863 ***</td>
<td></td>
<td>4.997 ***</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2.775 ***</td>
<td>Australia</td>
<td>1.033 ***</td>
</tr>
<tr>
<td>Sweden</td>
<td>2.732 ***</td>
<td>France</td>
<td>1.029 ***</td>
</tr>
<tr>
<td>Spain</td>
<td>2.498 ***</td>
<td>England</td>
<td>1.028 ***</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.345 ***</td>
<td>Saudi</td>
<td>1.026 ***</td>
</tr>
<tr>
<td>Ireland</td>
<td>1.749 ***</td>
<td>China</td>
<td>1.023 ***</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.740 ***</td>
<td>Poland</td>
<td>1.016 ***</td>
</tr>
<tr>
<td>Austria</td>
<td>1.669 ***</td>
<td>Latvia</td>
<td>1.014 ***</td>
</tr>
<tr>
<td>Portugal</td>
<td>1.560 ***</td>
<td>Malta</td>
<td>1.014 ***</td>
</tr>
<tr>
<td>Greece</td>
<td>1.483 ***</td>
<td>Japan</td>
<td>1.012 ***</td>
</tr>
</tbody>
</table>

Note: “Top 10” stands for the 10 countries with the biggest θ and “Last 10” stands for those with the smallest θ. ** and *** indicate confidence levels of 95% and 99%, respectively.

4.2. Estimation Results for CES

According to the discussion of the estimation of CES in Section 2.3, the best factor copula based on GJR-GARCH models and weights was applied. Therefore, 10,000 simulations were generated and the global financial risk, represented by CES, was computed. Figure 1 describes all estimations of systemic risk for all periods, and each period is depicted by a different color. A larger absolute value of the CES during the crisis period indicates higher potential systemic risk compared to the other two periods. During the crisis period, there was a lower probability of having steady predictions and a higher probability of having huge fluctuations. The lack of financial stability and the frequent extreme fluctuations within a short time period are also characteristic of a financial crisis. Not only did the ES predictions perform well in the long run, but also for the extreme values before the slumps. On 27 February 2007, more than half of the main indexes suffered losses. Indexes, such as SPX (United States), S&P/TSX (Canada), DAX (Germany), UKX (England), and SHCOMP (China) dropped by 3.53%, 2.72%, 2.96%, 2.31%, and 8.84%, respectively. Extraordinary slumps for all countries rarely occurred simultaneously. Therefore, the CES predictions for the pre-crisis period increased from 27 February 2007 and reached their peak on 7 March 2007. After a few interventions from each country, potential risk first decreased and then surged at the end of the pre-crisis period. During the crisis period, the highlighted point, 1 June 2012, only 38 of all 42 countries (Saudi’s market was not open) experienced a decrease and all indexes from the G7 group countries declined by at least 1% (SPX 2.5%, S&P/TSX 2.2%, and DAX 3.5%), which led to a large ES. For the post-crisis period, huge fluctuation between 18 September and 28 December 2018 may have been driven by the poor performance of the SPX (United States) and SHCOMP (China). As the two largest economies, by Christmas Eve of 2018, SPX dropped around 19.73% from the peak point on 20 September, while SHCOMP slumped
to a four-year low. Another reason may be the slowdown in global economic growth and political instability, such as increasing the tariffs imposed by the American president and rising interest rates by the Federal Reserve. To summarize, the estimation of systemic risk has played the role as an early warning of possible loss in global financial markets.

From the previous results and discussion, some of the countries seem to have provided a higher contribution to systemic risk. The CES% was computed using Equation (16). Table 4 provides the descriptive statistics of the top five CES% in each period. As can be seen in Table 4, the United States and Japan were in the top five countries in terms of their contribution to systemic risk in all three periods, which indicates their strong influence on the financial market. China and Russia were the only two developing countries on the list, meaning that most of the potential financial risk was from the developed countries. Due to its rapid economic development, China possessed the second biggest economy and CES contribution, which indicated that even if with relatively low dependence on common latent variables, a large degree of capitalization is still related to high potential risk. Lastly, the maximum CES% of the United States was more than 80% during the crisis period and 90% in the pre- and post-crisis periods, which implies the dominance and influence of the United States in the financial market.

As the United States contributed more than half of the systemic risk in each period, we separated the remaining 42 countries (41 countries in the pre-crisis periods) into three groups, which included G7 (excluding the USA), G20 (excluding G7 members), and others (remaining countries with modified G7 and G20 group as above). Figure 2 describes the average CES% for different groups or countries. Figure 2 shows that the United States accounted for more than 60% of total risk, but the proportion was decreasing. For the G7 group, members of G7 countries had a greater risk level during the crisis period than during the pre- and post-crisis periods. As for the G20 group, the potential risk increased from the pre-crisis period to the post-crisis period. For other countries, they tended to be riskier in the post-crisis period than in the pre-crisis period. Possible explanations for this phenomenon may be due to several aspects. First, the slowdown in the USA economic growth, the decline in the market capitalization gap between the USA and other countries, and the deleveraging process might be the reasons for the decreasing CES ratio of the USA. Following the crisis in the subprime mortgage market of the United States in 2007, European countries, in particular, England, Germany, and France, faced huge losses. Therefore, the G7 group was more vulnerable during the crisis period than at other times. The main factor affecting the rising CES% of the G20 group may be the rapid economic growth of China and the potential bubble in Chinese stock and real estate markets. For the remaining European Union (EU)
countries, the unstable political environment including Britain exiting from the EU and bankruptcy of Greece may have driven the increase in the CES ratio.

Table 4. Descriptive statistic of the top five component expected shortfall (CES)% in each period.

<table>
<thead>
<tr>
<th>Period</th>
<th>Country</th>
<th>Minimum</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre</td>
<td>United States</td>
<td>0.439</td>
<td>0.792</td>
<td>0.805</td>
<td>0.931</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>0.001</td>
<td>0.035</td>
<td>0.029</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>England</td>
<td>0.000</td>
<td>0.029</td>
<td>0.028</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td>Canada</td>
<td>0.003</td>
<td>0.024</td>
<td>0.022</td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td>Russia</td>
<td>0.000</td>
<td>0.014</td>
<td>0.010</td>
<td>0.105</td>
</tr>
<tr>
<td>in</td>
<td>United States</td>
<td>0.473</td>
<td>0.655</td>
<td>0.645</td>
<td>0.811</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>0.037</td>
<td>0.081</td>
<td>0.082</td>
<td>0.148</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>0.021</td>
<td>0.055</td>
<td>0.056</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>0.012</td>
<td>0.034</td>
<td>0.033</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>China</td>
<td>0.003</td>
<td>0.031</td>
<td>0.026</td>
<td>0.105</td>
</tr>
<tr>
<td>post</td>
<td>United States</td>
<td>0.468</td>
<td>0.652</td>
<td>0.639</td>
<td>0.929</td>
</tr>
<tr>
<td></td>
<td>China</td>
<td>0.004</td>
<td>0.060</td>
<td>0.056</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>England</td>
<td>0.008</td>
<td>0.049</td>
<td>0.050</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>0.001</td>
<td>0.047</td>
<td>0.047</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>0.008</td>
<td>0.039</td>
<td>0.039</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Figure 2. Average CES% for different groups or countries during different periods (from left to right is the pre-crisis, crisis, and post-crisis periods).

4.3. Computing Portfolios CES

After forecasting systemic risk, some studies concentrated on determinants of systemic risk, thereby expanding application values of systemic risk, such as Qin and Zhou [54], Qin and Zhu [55], and Sedunov [56]. Systemic risk measurement can play a key role in risk aversion, risk supervision, and investment decisions. To strengthen guidance of systemic risk measurement of portfolios, we applied a portfolio of a stock market based on CES prediction. The purpose of a CES portfolio is to make trading decisions by finding appropriate weights for each index in portfolios, therefore diversifying the risk.

Stoyanov et al. [44] and Rachev et al. [45] developed a link between the percentage contribution to ES and the minimum ES portfolio, and proposed a new portfolio that is marginally improved prediction by identifying risk contributors and risk diversifiers in long-only portfolios. Following approaches of Stoyanov et al. [44] and Rachev et al. [45], we introduce CES portfolios based on the results of CES
prediction in global financial markets. The proposed CES portfolio is constructed by minimizing the total risk with long-only portfolios, and can be expressed as follows.

$$\min_{w'} \sum CES_{it}$$

subject to $w'e = 1$

where $e$ is a vector of ones, $w'$ are the weights that minimize the total risk with the condition that the product $w'e = 1$, and $\sum CES_{it} = ES_t$. On the basis of $w'$ and CES%, risk contributors and risk diversifiers can be diagnosed [45]. The new CES portfolio strategies can be obtained by the following process:

1. Calculate the partial derivatives of $\sum CES$ to get marginal ES(MES).
2. Sort the $w'$ in descending order and the MES in ascending order.
3. Allocate the $i$th ranking country in MES with the $i$th ranking weight in $w'$.

Figure 3 describes the average percentage of top 10 weights for portfolios during pre-crisis, crisis, and post-crisis periods. Compared with the pre-crisis and post-crisis periods, the portfolios during the crisis periods are more scattered because there is no huge gap between each top 10 country. Moreover, countries with the highest weights during each periods all performed well in their stock markets. To be specific, AS51 index (Australia) surged 23.23% during the 261 days of the pre-crisis periods (from 31 July 2006 to 31 July 2007), and WIG20 index (Poland) increased 18.85% and 8.21% respectively during the 261 days of the crisis and post-crisis periods (from 28 December 2011 to 28 December 2012 and from 21 June 2018 to 21 June 2019), which indicates the practicality of this method.

Based on previous discussion, an application of this aforementioned model was performed assuming that all the transactions were intra-day trading, which means investors do not hold portfolios overnight. Investors calculate the weights of portfolios and buy them at the beginning of the day, and investors can sell all portfolios at the last price on the first night and then buy them at the same price the next morning. In this study, to deal with the exchange rate fluctuations, we assume that all trading was settled in U.S. dollars and the start-up capital of an investor is USD $1 million. In order to show the functionality of this method, a comparison group is added. Based on the idea of risk diversification, average weights are assigned to each index. Figure 4 represents the comparison of cumulative returns comparison between the CES portfolio method and average weight method. It is obvious from Figure 4 that the trend of cumulative returns based on the two method are similar. However, the newly proposed CES portfolio method was able to obtain relatively more returns overall. For the pre-crisis and post-crisis periods, the proposed cumulative returns of the CES portfolio weight method outperforms the average weight method. As for the crisis period, even though the average weight method result in more profit, the difference are negligible. However, our proposed risk-based CES portfolio method provides net profit during all crisis periods (i.e., all values are greater than 1). This indicates that the CES portfolio method provides stable returns and prevent the investors from the potential loss. Moreover, for most of the trading days, the profit of the CES portfolio is larger than the average weight portfolio, which indicates superiority of the proposed CES portfolio.
5. Conclusions

The main objectives of this study were to measure and forecast systemic risk in the global financial markets and then construct a trade decision model based on the results of CES predictions for investors and financial institutions to assist them in forecasting risk and potential returns. To this end, the GJR-GARCH (1,1), factor copula-GARCH models, and component expected shortfall (CES) were combined for the first time. A high-dimensional dataset of daily stock market indices of 43 countries covering the period 2003 to 2019 were used to represent the global financial markets.

The results revealed the practicality of the factor copula models and their ability to catch tail dependence even during the financial crisis period. Firstly, the parameters used to measure sensitivity to the latent variables are consistent with common knowledge, which indicates the accuracy of the estimations. Secondly, ES predictions are in line with the graph showing a higher level of risk during the crisis period than the other two periods. Thirdly, the functionality of the predictions with regard to high-dimensional data was proven. The CES% varied from the pre-crisis to the post-crisis period, which indicated structural changes in systemic risk, that is, the CES% of the USA continued to decrease, but it still contributed more than 60% to total systemic risk. Generated from the minimization of the CES notion, CES portfolios were examined, which performed well for all crisis periods for our dataset. This illustrated the importance of risk diversification and proved that even during the crisis period, there are still opportunities to gain a profit.

Author Contributions: J.L. and S.S. contributed to the conception and methodology of the study; Q.S. and J.L. contributed to the draft writing and data analysis; Y.Q. and S.R. helped to review and editing. All authors have read and agreed to the published version of the manuscript.

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