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6 Distortional buckling of perforated cold-formed steel beams subject to uniformly distributed
7 transverse loads

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13 **Abstract:** Thin-walled channel beams are easily punched with circular holes on the web to
14 allow the access for services such as plumbing pipes and electric wires. The presence of the
15 holes can alter the stress distribution in the member and reduce the cross-sectional property.
16 Consequently, it changes its buckling mode. Since perforated cold-formed steel beams are
17 usually placed between main structural frame and corrugated roof, the most common loading
18 case is the uniformly distributed transverse load. Recent work by Chen and Li has given the
19 solution for distortional buckling of channel-, zed- and sigma- sections subject to the uniformly
20 distributed transverse load. This paper is an extension of Chen and Li's research to explore the
21 distortional buckling behaviour of perforated cold-formed steel beams with holes. The effect
22 of perforations on the critical stress is evaluated. A new model is deduced to predict the critical
23 stress of distortional buckling by reducing the stiffness of the vertical spring. The Rayleigh-
24 Ritz method is used to solve eigenvalue problems. In order to validate the analytical model,
25 finite element analyses have been performed by using ANSYS. When the beam is longer than
26 3500 mm, the critical stress computed from the analytical model matches well with the critical
27 stress acquired from the finite element analyses.

28 **Keywords:** Distortional buckling; cold-formed steel; perforations; finite element analysis;
29 stress gradient.

30 31 1. Introduction

32 Cold-formed steel (CFS) members are considered as the important load carrying members in
33 building industries due to their unique material properties. The yield strength of CFS can be as
34 high as 550 MPa and its thickness can be less than 2 mm. Unlike hot-rolled steel sections, a
35 CFS section has low lateral and torsional stiffness due to its thin and open geometry. For this
36 reason, it is susceptible for CFS sections to buckle. To release the space of a structure,
37 perforated cold-formed steel (PCFS) beams tend to be used as secondary load carrying
38 members in buildings to let the plumbing pipes and electric wires pass through. However, the
39 restraint of the web to the compressed flange is reduced owing to the perforation patterns in
40 the web. This leads to PCFS beams being sensitive to distortional buckling.

41 Lau and Hancock [1-2] firstly presented the concept of distortional buckling, which can be
42 described as the buckling of compressed flange-lip component with respect to the web-flange

43 junction, meanwhile the web undergoes bending . In Lau and Hancock's model, the flange-lip
44 system can be isolated and considered alone, the influence of the web on the compressed flange
45 and lip was represented by the rotational and lateral spring. Since then, great efforts have been
46 made on developing analytical models to determine the critical stress of distortional buckling
47 of CFS members. Teng et al. [3] extended Lau and Hancock's approach for the CFS sections
48 subject to combined axial compression and biaxial bending. In Eurocode 3, the stiffener was
49 assumed to be supported at the elastic foundation with continuous spring along the length [4].
50 Later, Li and Chen [5] considered the bending behaviour of the stiffener by using the vertical
51 spring to replace the rotational spring. More recently, Zhu et al. [6-8] utilised a plate model
52 with an angle stiffener to replace the flange-lip system, the energy method was used to obtain
53 the critical stress. In addition, the numerical methods such as finite strip method (FSM), finite
54 element analysis (FEA) and generalised beam theory (GBT) [9] have also been regarded as the
55 suitable approaches to calculate the distortional buckling stress of CFS members.

56 Similar to CFS sections, the PCFS members may suffer three types of buckling, such as local
57 buckling, distortional buckling and lateral-torsional buckling. However, the presence of the
58 holes will not only alter the stress distribution in the member but also reduce the cross-sectional
59 property, and consequently change the buckling modes. The reductions of the critical buckling
60 stress and ultimate strength are mainly depending on the shape, arrangement and size of the
61 holes. Miller and Peköz [10] assumed that the web was idealised as two unstiffened element
62 and modify the unified effective width method for the perforated studs. Moen and Schafer [11]
63 presented simplified expressions for critical elastic buckling stress of perforated thin plates, for
64 both stiffened and unstiffened elements. Subsequently, they proposed analytical approaches for
65 calculating the global, distortional and local buckling stress of CFS members with holes [12].
66 Yu et al. [13-14] modified the EN 1993-1-3 model and Lau and Hancock's model to determine
67 the critical distortional buckling stress of PCFS beams subject to pure bending. They found
68 that the critical stress obtained from the modified Hancock's model is more accurate.

69 FSM was regarded as an effective approach for predicting the elastic buckling stress of PCFS
70 members by some researchers. For instance, Tovar and Sposito [15-16] presented some
71 perforated web models to represent the influence of the holes by using FSM. Smith and Moen
72 [17] introduced the approximate FSM for elastic buckling analysis of PCFS columns, in which
73 the principle of reduced thickness was considered. Pham [18] provided the solutions to find out
74 the shear buckling loads of perforated thin plates and PCFS sections using spline FSM.

75 For the elastic buckling controlled failure, the direct strength method (DSM) can predict the
76 ultimate strength of the CFS members accurately [19]. It is worth noting that some
77 experimental work has been conducted to extend the existing DSM for PCFS members. Moen
78 and Schafer [20] observed the relation between buckling behaviour and tested load-
79 displacement response of PCFS columns with slotted holes. Zhao et al. [21] presented
80 experimental investigations on PCFS beams with square holes subject to four-point bending,
81 from which the modified DSM formulas were proposed. Wang and Young [22-23] performed
82 the beam tests of built-up PCFS sections, from which the relative extended DSM was carried
83 out.

84 PCFS beams are usually placed between the main structure and roof as the secondary load
85 carrying members. Thus, the most common loading case of the PCFS members is the uniformly
86 distributed transverse load rather than pure bending. If the member is subject to the uniformly

87 distributed transverse load, the stress distribution along the longitudinal direction will be
88 different. Most existing analytical approaches were developed for the beams with pure bending
89 or pure compression, which cannot be applied to the beams with stress gradient. Little
90 information about the buckling behaviour of PCFS sections under the uniformly distributed
91 transverse load can be discovered in the literature. Li [24] investigated the lateral-torsional
92 buckling behaviour of CFS zed-purlins under both down and uplift load and presented an
93 analytical model to predict the critical load. Chu et al. [25-26] utilised the semi-analytical FSM
94 to study the local and distortional buckling of CFS zed- and channel-sections under the
95 uniformly distributed transverse load, from which the influence of stress gradient on the
96 buckling behaviour of the members was highlighted. Chen and Li [27] evaluated the effect of
97 stress gradient on the distortional buckling of CFS beams. However, the sections they
98 investigated are plain sections with no holes.

99 This paper aims to develop a solution for the distortional buckling stress of PCFS beams under
100 a uniformly distributed transverse load. The analytical model for computing the distortional
101 critical stress is derived which is based on Chen and Li's work [27]. The effect of holes on the
102 critical stress of distortional buckling is reckoned. The Rayleigh-Ritz method is used to solve
103 the eigenvalue problem. In order to validate the analytical results, FEA is performed by using
104 commercial software ANSYS. Comparisons between the theoretical result and the FEA data
105 are presented in this paper.

106 2. Analytical model

107 The cross section of a PCFS beam is shown in Fig.1a, which is the same as that in the literature
108 [13] and [14]. The notations of the web height, flange width, lip length and section thickness
109 are defined as h , b , c and t , respectively. The circular holes are punched in the centreline of the
110 web evenly, with a diameter defined as d . The shaded part represents the solid area in the
111 perforated strip. To simplify the calculation, the opening area is assumed to be same as the
112 solid area in the perforated strip of the plate. Therefore, the length of the beam is expressed as
113 $n_h \pi d / 2$, where n_h is a random constant, representing the total number of the holes, as shown in
114 Fig.1b.

115 Fig.2 shows Li and Chen's method [5], the difference with the model developed by Lau and
116 Hancock [1] is that the vertical spring at the centroid of the compressed flange-lip component
117 is used to replace the rotational spring at the flange-web corner. It is very similar to the one
118 used in Eurocode 3 [4], which represents the restraint of web to the compressed flange-lip
119 component, however it considers the flexural behaviour of the flange-lip component itself. To
120 apply the Li and Chen's model for PCFS beams, the stiffness of the vertical spring should be
121 reduced to take account the influence of the web openings.

122 It is clear that the spring stiffness can be determined by means of the flexural stiffness of the
123 web and flange of the section. The unit load F at the centroid of the compressed flange and lip
124 is used to determine the modified stiffness of the vertical spring, see Fig. 3. Then, the strain
125 energy of the compressed flange and web due to bending can be determined by the following
126 formula,

$$U = \frac{1}{2EI_1} \int_0^{b-y_0} (Fy)^2 dy + \frac{1}{2EI_1} \int_0^h [F(b-y_0) - \frac{F(b-y_0)}{h} z]^2 dz$$

$$+ (\frac{1}{2EI_2} - \frac{1}{2EI_1}) \int_{\frac{h-d}{2}}^{\frac{h+d}{2}} [F(b-y_0) - \frac{F(b-y_0)}{h} z]^2 dz$$

where y_0 is the horizontal distance between the shear centre and centroid, E is the Young's modulus, I_1 and I_2 are the moment of inertia of the area with and without holes, respectively.

The deflection of the vertical spring is

$$\delta = \frac{\partial U}{\partial F}$$

$$= \frac{F(b-y_0)^3}{3EI_1} + \frac{F(b-y_0)^2 h}{3EI_1} + (\frac{1}{EI_2} - \frac{1}{EI_1}) F(b-y_0)^2 \frac{h}{3} [(\frac{h+d}{2h})^3 - (\frac{h-d}{2h})^3]$$

The modified stiffness of the vertical spring can be obtained according to Eq. (1) and Eq. (2), which is given as,

$$k'_z = \frac{F}{\delta} = \frac{3D}{(b-y_0)^2(b-y_0+h)} \times \frac{1}{1 + \frac{1}{4}(\frac{d}{h}) \left[\frac{3+(d/h)^2}{1+(b-y_0)/h} \right] (\frac{D}{D_d} - 1)}$$

where D is the flexural rigidity of the stiffened element and the web strip without holes, D_d is the flexural rigidity of the web strip with holes. In this case, $D_d=0.5D$ due to the concept of equal width. If there are no holes in the web ($d=0$), Eq. (3) could be expressed as follows,

$$k_{z0} = \frac{1}{\frac{(b-y_0)^3}{3D} + \frac{(b-y_0)^2}{\frac{3D}{h}}}$$

where $3D/h$ represents the unmodified rotational spring stiffness at the web-flange corner.

It should be pointed out herein that the stiffness of rotational stiffness in Hancock's model [2] is modified and multiplying the reduction factor which is obtained as follows,

$$k_\phi = \frac{2Et^3}{5.46[h + \frac{4}{3}(b-y_0)]} (1 - 1.11 \frac{\sigma_{od}}{Et^2} \frac{h^4 \lambda_{cr}^2}{12.56\lambda_{cr}^4 + 2.192h^4 + 13.39\lambda_{cr}^2 h^2} \frac{h}{h-2z_0})$$

where λ_{cr} is the half-wavelength and σ_{od} is the compressive stress of the web. Eq. (5) can be solved by the iterative procedure which could be found in Literature [14]. In the following calculation, Eq. (5) is used to replace $3D/h$ in Eq. (4).

Let

$$\alpha = \frac{1}{1 + \frac{1}{4}(\frac{d}{h}) \left[\frac{3+(d/h)^2}{1+(b-y_0)/h} \right] (\frac{D}{D_d} - 1)}$$

148 The vertical spring stiffness of modified Li and Chen's model for PCFS beams is

$$149 \quad k_z = \alpha \cdot \frac{1}{\frac{(b-y_0)^3}{3D} + \frac{(b-y_0)^2}{k_\phi}} \quad (7)$$

150 **Fig. 4** shows that the moment curve is parabolic when the beam is subject to uniformly
 151 distributed transverse loads and the moment curve is straight when the beam is subject to pure
 152 bending. For this reason, the wave lengths of the distortional buckling mode under pure
 153 bending are equal, but the wave lengths of the distortional buckling mode under uniformly
 154 distributed transverse loads are unequal due to the moment gradient effect. Hence, the buckling
 155 modes of the PCFS beam not only depend on the cross-section dimension but also the length
 156 of the beam, as well as the perforations. For simplicity, the uniformly distributed transverse
 157 load is supposed to be applied at the shear centre so that the member will not twist.

158 If the PCFS beam is simply supported and subject to a uniformly distributed transverse load,
 159 the internal bending moment can be described as follows,

$$160 \quad M(x) = \frac{q_z l}{2} x - \frac{1}{2} q_z x^2 \quad (8)$$

161 Hence, the external force in the flange-lip component is

$$162 \quad P = \sigma_x A = \frac{M(x)}{M_{cr}} \sigma_{cr} A = \frac{4x(l-x)}{l^2} \sigma_{cr} A \quad (9)$$

163 However, when the simply supported beam is subject to the pure bending, the relative external
 164 force in the flange-lip component is

$$165 \quad P = \sigma_{cr0} A \quad (10)$$

166 where q_z is the uniformly distributed transverse load, $M_{cr} = q_z l^2 / 8$ is the largest internal moment,
 167 σ_{cr} is corresponding critical stress when it is subject to uniformly distributed transverse loads,
 168 $A = (b+c)t$ is the cross-section area of the compressed flange and lip component, σ_{cr0} is critical
 169 stress when it is subject to pure bending.

170 When the beam occurs a distortional buckling, the buckled flange-lip system will have
 171 translational and rotational displacements. The strain energy due to the buckling displacements
 172 stored in the flange-lip system and the loss of potential energy can be evaluated. According to
 173 Chen and Li's model [27], they can be defined as follows,

$$174 \quad U_0 = \frac{1}{2} \int_0^l [(EI_w + EI_y b^2) \left(\frac{d^2 \phi}{dx^2}\right)^2 + GJ \left(\frac{d\phi}{dx}\right)^2 + k_z (b-y_0)^2 \phi^2] dx \quad (11)$$

$$175 \quad W_p = \frac{z_0^2 + (b-y_0)^2 + r_c^2}{2} \int_0^l P \left(\frac{d\phi}{dx}\right)^2 dx \quad (12)$$

176 where l is length of the beam, I_w is the warping constant which is equal to zero in present study,
 177 I_y is the second moment of the section area about the y axis, ϕ is the rotation of the section
 178 about the shear centre, G is the shear modulus, J is the St. Venant torsional constant, k_z is the

179 stiffness of the vertical spring, z_0 is the vertical distance between the centroid and the shear
 180 centre and r_c is the polar radius of gyration of the cross section about the centroid. Some of the
 181 above parameters can be obtained as follows,

$$182 \quad \phi(x) = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{2\pi x}{l} + a_3 \sin \frac{3\pi x}{l} + \dots + a_n \sin \frac{n\pi x}{l} \quad (13)$$

$$183 \quad y_0 = b - \frac{b^2/2 + bc}{b+c} \quad (14)$$

$$184 \quad z_0 = \frac{c^2/2}{b+c} \quad (15)$$

$$185 \quad I_y = \frac{1}{12}bt^3 + btz_0^2 + \frac{1}{12}c^3t + ct\left(\frac{c}{2} - z_0\right)^2 \quad (16)$$

$$186 \quad I_z = \frac{1}{12}tb^3 + bt\left(y_0 - \frac{b}{2}\right)^2 + \frac{1}{12}ct^3 + cty_0^2 \quad (17)$$

$$187 \quad G = \frac{E}{2(1+\mu)} \quad (18)$$

$$188 \quad J = \frac{(b+c)t^3}{3} \quad (19)$$

$$189 \quad r_c^2 = \frac{1}{A}(I_y + I_z) \quad (20)$$

190 where μ is the Poisson's ratio, a_1, a_2, \dots, a_n are arbitrary constants, n is not equal to m and the
 191 sum $(m+n)$ is even.

192 The critical stress of torsional and flexural-torsional buckling of the flange-lip system with
 193 spring support can be determined by using energy method, that is,

$$194 \quad \partial^2 \Pi = \partial^2 (U_0 - W_p) = 0 \quad (21)$$

195 Substituting Eq. (9), Eq. (11) and Eq. (12) into Eq. (21) yields,

$$196 \quad \frac{z_0^2 + (b - y_0)^2 + r_c^2}{l^2} \left[\sum_{n=1}^{\infty} \left(\frac{n^2 \pi^2}{3} - 1 \right) a_n^2 - 4 \sum_n \sum_m \left(\frac{mn}{(m+n)^2} + \frac{mn}{(n-m)^2} \right) a_m a_n \right] 2\sigma_{cr} - \sum_{n=1}^{\infty} \left[\frac{EI_w + EI_y b^2}{A} \frac{n^4 \pi^4}{l^4} + \frac{GJ}{A} \frac{n^2 \pi^2}{l^2} + \frac{k_z (b - y_0)^2}{A} \right] a_n^2 = 0 \quad (22)$$

197 Eq. (22) can be used to predict the critical stress of distortional buckling of CFS beams without
 198 holes subject to uniform distributed transverse loads. However, to determine the critical stress
 199 of distortional buckling of PCFS beams, the stiffness of the vertical spring k_z must be modified.
 200 Therefore, Eq. (7) should be applied into Eq. (22). Based on the Rayleigh-Ritz method, deriving
 201 with respect to a_1, a_2, \dots, a_n is equal to zero. The following type of linear equations in $a_1, a_2, \dots,$
 202 a_n can be acquired.

$$203 \quad \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} - \sigma_{cr} \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} = 0 \quad (23)$$

204 in which,

$$205 \quad a_{ij} = 0 \quad (i \neq j) \quad (24a)$$

$$206 \quad a_{mm} = \frac{(EI_w + EI_y b^2) \left(\frac{n\pi}{l}\right)^4 + GJ \left(\frac{n\pi}{l}\right)^2 + k_z (b - y_0)^2}{A} \quad (24b)$$

$$207 \quad b_{pn} = 0 \quad (p \neq n; p + n = \text{odd}) \quad (24c)$$

$$208 \quad b_{mn} = \frac{z_0^2 + (b - y_0)^2 + r_c^2}{l^2} \left[-16 \times \frac{nm(n^2 + m^2)}{(n^2 - m^2)^2} \right] \quad (24d)$$

$$209 \quad b_{nn} = \frac{z_0^2 + (b - y_0)^2 + r_c^2}{l^2} \left[2 \times \left(\frac{n^2 \pi^2}{3} - 1 \right) \right] \quad (24e)$$

210 Eq. (23) is a matrix with infinite dimensions which represents an eigenvalue problem, the
 211 eigenvalues are computed by software MATLAB. Among all the eigenvalues, the smallest one
 212 is the critical stress of distortional buckling. It is well known that the half-wave length of
 213 distortional buckling of the CFS beam is about 500 mm although the exact length depends on
 214 the section dimensions. Table 1 shows the exact distortional half-wave lengths obtained using
 215 the free software package CUFSM [28] for the CFS channel-sections produced by Albion
 216 Section. Therefore the largest value of n can be evaluated by the beam length, e.g. for the 3000
 217 mm beam if its half-wave length is 500 mm, n should be at least taken as 6.

218 When the beams are subject to pure bending, Substituting Eq. (10), Eq. (11) and Eq. (12) into
 219 Eq. (21) yields,

$$220 \quad \sigma_{cr0} = \frac{(EI_w + EI_y b^2) \left(\frac{n\pi}{l}\right)^2 + GJ + k_z (b - y_0)^2 \left(\frac{l}{n\pi}\right)^2}{A(z_0^2 + (b - y_0)^2 + r_c^2)} \quad (25)$$

221 It should be mentioned that the rotational stiffness of spring also needs to be modified to
 222 calculate the critical stress of distortional buckling of PCFS beams which had been derived in
 223 Ref. [13].

224 **Fig.5** shows the critical stresses of the distortional buckling of PCFS beams under uniformly
 225 distributed transverse loads and pure bending, which are calculated from Eq. (23) and Eq. (25).
 226 It can be observed that the critical stress curve for uniformly distributed transverse loads is
 227 highly depending on the beam length. It decreases dramatically with the increase of the length
 228 when the beam length is less than 3000 mm. The decrease diminishes when the beam length is
 229 longer than 4500 mm. The dash line is for the case subject to pure bending which is obtained
 230 from Eq. (25). As shown in **Fig. 5**, the critical stress of uniformly distributed transverse loads
 231 is continually higher than that of pure bending and when the beam is long enough these two

232 critical stresses tend to be the close. In most practical cases, these differences should be taken
233 into account.

234 3. Finite Element Analysis and Validation

235 In order to verify the proposed analytical approach, finite element analysis was performed by
236 using a commercial software program ANSYS. The cross-sectional dimension of the beam was
237 the same as C20625 ($h=200$ mm, $b=65$ mm, $c=20$ mm and $t=2.5$ mm) which was chosen from
238 Albion sections. Various lengths and hole sizes of the PCFS beam were analysed in which the
239 PCFS beams were modelled by using four-node shell 181. The fine element sizes were
240 controlled not to exceed 10 mm, Fig.6 shows a typical element mesh used in the analysis. The
241 material properties of the beam analysed are assumed to have Young's modulus of 205 GPa
242 and Poisson's ratio of 0.3.

243 The boundary conditions were considered to have zero lateral and transverse displacements
244 ($UX=UY=0$) and zero rotation about the longitudinal axis ($ROTZ=0$) for the web, flange and
245 lip lines at the two ends. To avoid the rigid movement, one node located in the middle of the
246 web at one end was set as zero longitudinal displacement ($UZ=0$). It should be noted that when
247 the length of the beam increases, the distortional buckling mode will be gradually coupled with
248 lateral-torsional mode, and hence the relevant critical stress will reduce. In this present study,
249 only the distortional buckling mode was examined. Hence, the two corner line between the web
250 and flange were both restrained laterally ($UX=0$) to prevent the lateral-torsional buckling
251 occurred.

252 The uniformly distributed transverse load q_z was assumed to be applied at the shear centre of
253 the beam, so that it can undergo bending without twist. There are two ways to simulate this
254 situation. One is to apply the distributed transverse load on the plate which has infinite rigidity.
255 The length of the plate is equal to the beam length and its width is same as the distance between
256 the web line and the shear centre, as shown in Fig.7 (a). The other way is to apply distributed
257 transverse loads at the web directly combined with an equivalent distributed twist moment. The
258 value of the twist moment is equal to the value of transverse load times the distance between
259 the web and the shear centre (see Fig.7 (b)). For simplicity, the second loading condition (Fig.7
260 (b)) was utilised in this FEA.

261 Fig.8 shows the typical distortional buckling modes of PCFS beams with different hole sizes
262 under a uniformly distributed transverse load. As can be seen in the Fig.8, the CFS channel
263 beams with and without holes have the analogous distortional buckling modes which were
264 controlled by the rotation of the compressive flange-lip component with respect to the web-
265 flange corner. Moreover, it is interesting to observe that the half-wave lengths were different
266 along the beam length. This is because when the beam is subject to uniformly distributed
267 transverse load, the longitudinal stresses vary parabolically along the beam length. The highest
268 compressive stress appears in the central region, which leads to the shortest half-wave length;
269 whereas the half-wave lengths near the two beam ends are the largest because the stresses are
270 the lowest there. The varied compressive stresses produce the distortional buckling modes with
271 several buckling waves, each has different half-wave lengths.

272 To validate the proposed analytical approach, Fig.9 compares the critical stress of PCFS beams
273 with different hole sizes obtained from Eq. (23) and those obtained from FEA, where σ_{cr} is the
274 critical stress and σ_y is the yield stress ($\sigma_y=450$ MPa, obtained from Albion sections). It can be

275 observed from the comparison that the tendency of the critical stress curves was similar. As the
276 size of holes increased, the relative critical stress of the beam decreased. It is mainly because
277 the restraint of the compressed flange and lip from the web is weaker for the PCFS beam with
278 larger holes.

279 The results given in Fig.9 shows that when the beam length is longer than 3500 mm, the present
280 analytical solution matches very well with the data obtained from FEA. However, for the short
281 beams, the results calculated from Eq. (23) are much higher than that obtained from FEA, as
282 detailed in the red box in Fig.9. This is because when the beams are subject to the uniformly
283 distributed transverse load, the shear stress near the support in the short beam is much severer
284 than corresponding bending stress. Consequently, short PCFS beams may buckle due to the
285 shear stress rather than the bending stress. In addition, stress concentration exists near the web
286 opening which may induce a local buckling. All of these buckling modes are included in the
287 FEA but not in the present analytical solution. This is why the present critical stress is higher
288 than that FEA predicted ones for short beams. Nevertheless, with the increase of the beam
289 length, the present solution becomes very close to the FEA predicted results.

290 It should be mentioned that, the stress gradient affects not only the distortional buckling
291 behavior but also the behavior of other buckling types of PCFS beams, such as the shear
292 buckling and the lateral-torsional buckling. This is generally owing to the variation of pre-
293 buckling stresses along the longitudinal direction. However, the effect mechanisms of the stress
294 gradient on different types of buckling may be also different. In the present paper, only
295 distortional buckling of PCFS beams has been investigated. The other buckling types of PCFS
296 will be investigated as our future work.

297 **4. Conclusions**

298 This paper has presented an analytical and numerical study on the distortional buckling
299 behaviour of the PCFS beams subject to uniformly distributed transverse loads. The equations
300 for predicting the distortional buckling stress of PCFS beams have been derived using the
301 energy method. The model has been compared against the results obtained from the FEA. From
302 the analytical and numerical results obtained in this research, the conclusions can be made as
303 follows:

- 304 • The distortional buckling stress of PCFS beams decreases with the increase of the
305 diameter of the perforation. The influence of circular holes can be evaluated by
306 reducing the vertical spring stiffness of the compressed flange-lip component adopted
307 in the buckling model.
- 308 • The critical stress of distortional buckling of a PCFS beam subject to uniformly
309 distributed transverse load is larger than the same beam subject to pure bending.
310 However, the difference between their two critical stresses decreases with the increase
311 of their beam lengths.
- 312 • The effect of stress gradient on the distortional buckling is highly depending on the
313 length of the beam, it reduces with the length increases.
- 314 • The half-wave lengths of the distortional buckling mode of the PCFS beams are
315 different along the beam length when it is subject to uniformly distributed transverse
316 loads and the largest deflection occurred at the mid-span.

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323

324

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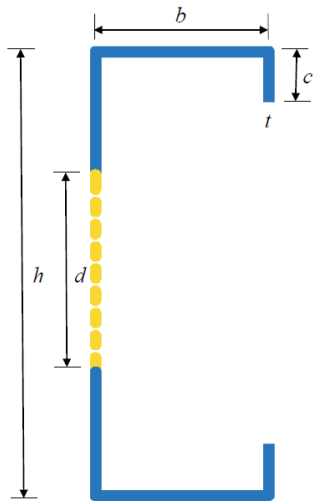
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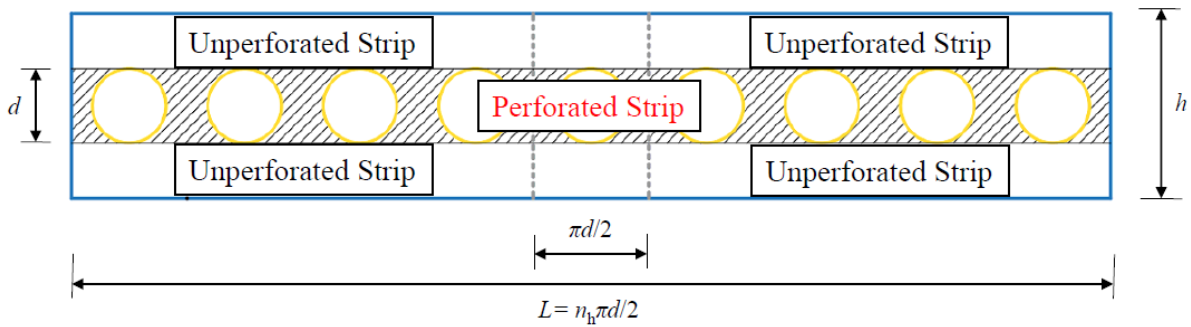
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(a) Cross section



(b) Longitudinal direction

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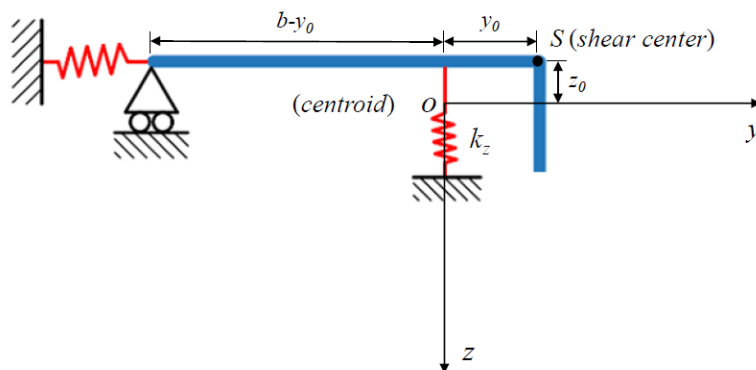
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397 Fig.1 Notation and geometry used for PCFS beams (dimensions are defined based on the
398 middle line of the section).

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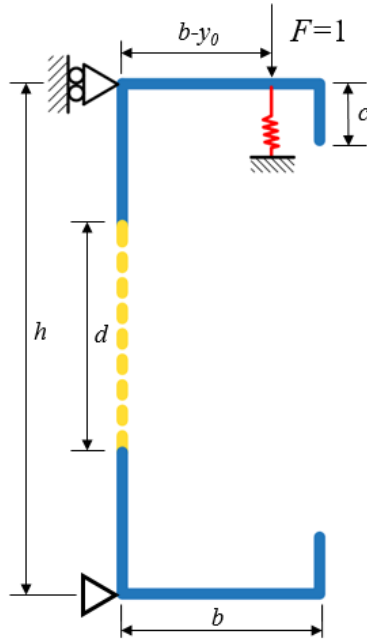


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402 Fig.2 Analytical model proposed by Li and Chen [5].

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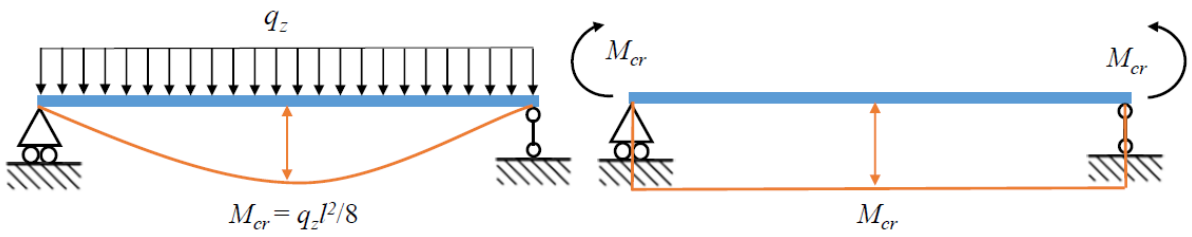


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406 Fig.3 Model used for determining the stiffness of the vertical spring of the PCFS beam.

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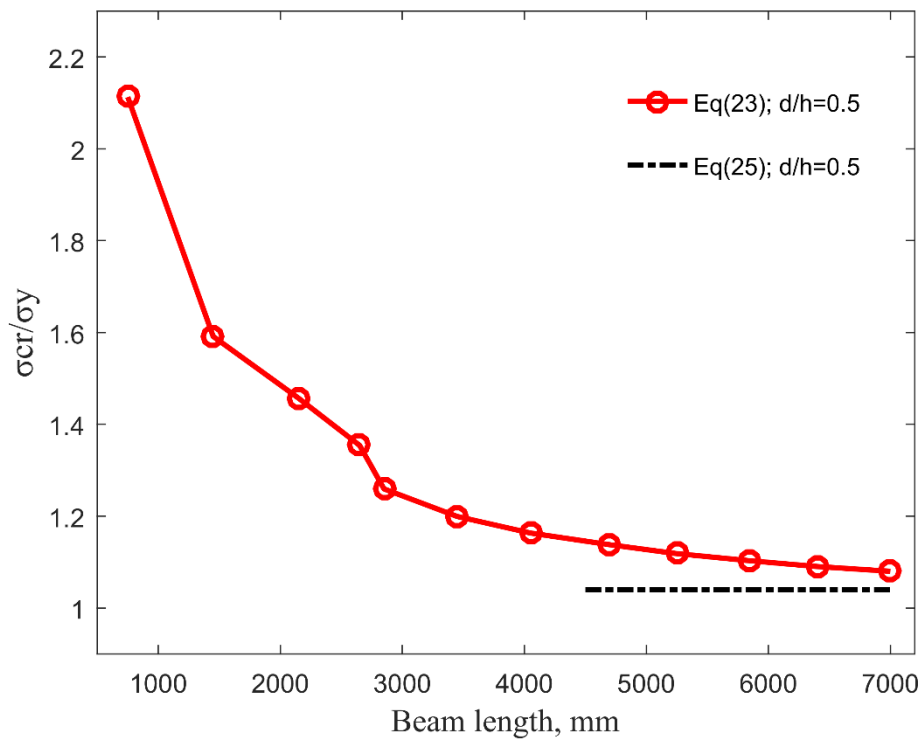
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410 Fig.4 Bending moment diagram for uniformly distributed transverse loads (left) and pure
411 bending (right).

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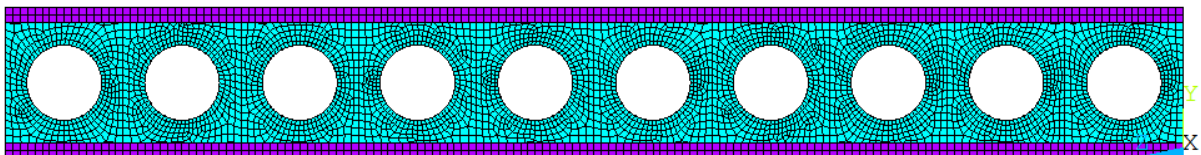
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414 Fig.5 Comparison of the critical stress of PCFS beams between uniformly distributed
 415 transverse loads (continuous line) and pure bending (dashed line) ($h=200$ mm, $b=65$ mm,
 416 $c=20$ mm, $t=2$ mm, $d=100$ mm, $\sigma_y=450$ MPa).

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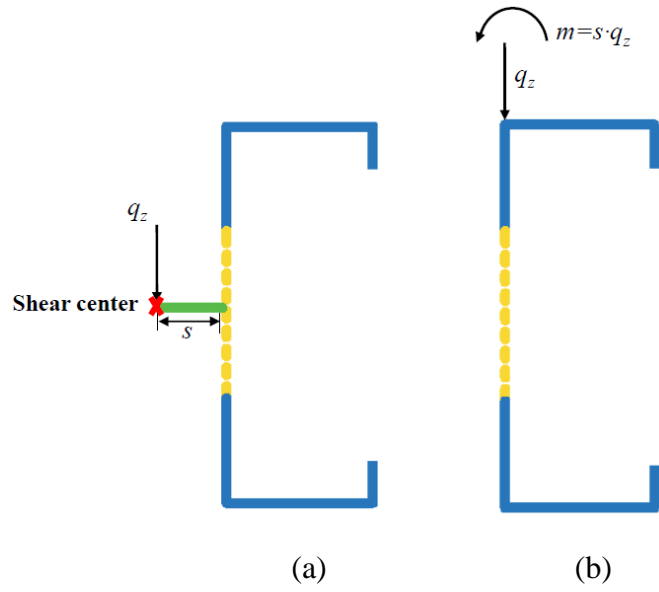
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421 Fig.6 Typical finite element mesh of the PCFS beam ($h=200$ mm, $b=65$ mm, $c=20$ mm, $t=2.5$
 422 mm, $d=100$ mm).

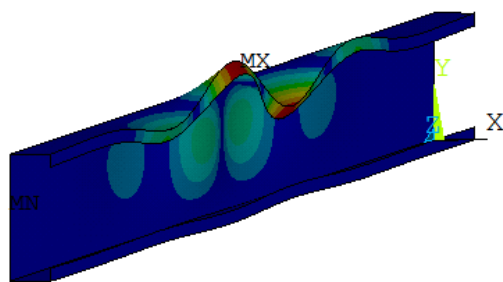


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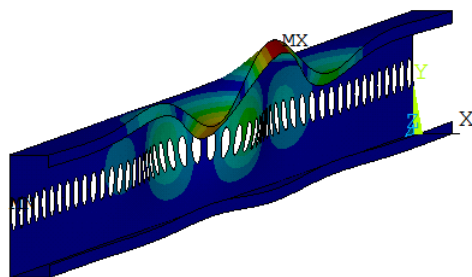
425 Fig.7 Loading conditions of PCFS beams.

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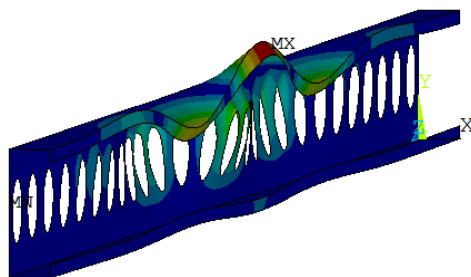
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428 (a) $d/h=0$; $l=3900$ mm



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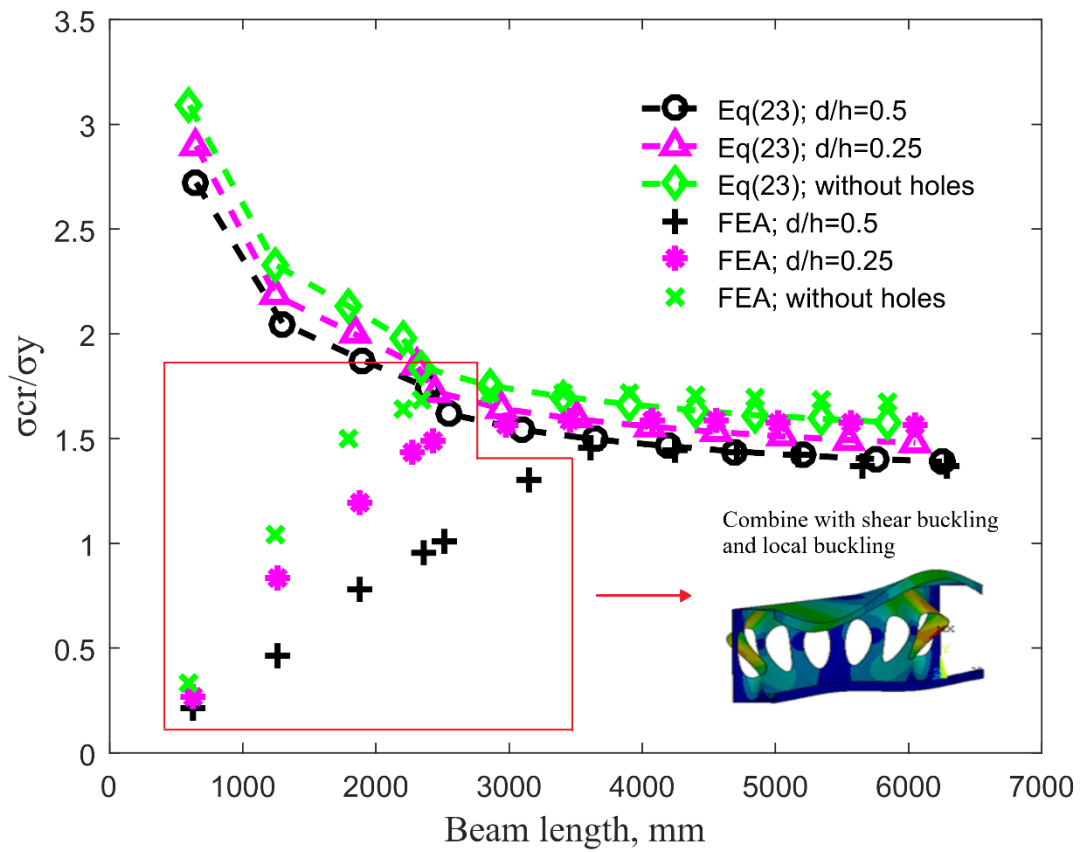
430 (b) $d/h=0.25$; $l=4028$ mm



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432 (c) $d/h=0.5$; $l=4239$ mm

433 Fig.8 Typical distortional buckling mode shapes of PCFS beams.



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435 Fig.9 Comparison between analytical model and FEA of PCFS beams subjected to uniformly
 436 distributed transverse loads.

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451 Table 1 Distortional half-wave lengths of CFS channel-sections from Albion Section

| Section | Web depth (mm) | Flange width (mm) | Lip length (mm) | Thickness (mm) | Half-wave length (mm) |
|---------|-------------------|----------------------|--------------------|-------------------|--------------------------|
| C12515 | 120 | 50 | 15 | 1.5 | 450 |
| C12516 | 120 | 50 | 15 | 1.6 | 450 |
| C14614 | 145 | 62.5 | 20 | 1.4 | 700 |
| C14515 | 145 | 62.5 | 20 | 1.5 | 650 |
| C14616 | 145 | 62.5 | 20 | 1.6 | 650 |
| C14618 | 145 | 62.5 | 20 | 1.8 | 600 |
| C14620 | 145 | 62.5 | 20 | 2 | 550 |
| C17616 | 175 | 62.5 | 20 | 1.6 | 650 |
| C17618 | 175 | 62.5 | 20 | 1.8 | 600 |
| C17620 | 175 | 62.5 | 20 | 2 | 600 |
| C17623 | 175 | 62.5 | 20 | 2.3 | 550 |
| C17625 | 175 | 62.5 | 20 | 2.5 | 500 |
| C20168 | 200 | 65 | 20 | 1.8 | 650 |
| C20620 | 200 | 65 | 20 | 2 | 600 |
| C20623 | 200 | 65 | 20 | 2.3 | 550 |
| C20625 | 200 | 65 | 20 | 2.5 | 550 |
| C22620 | 225 | 65 | 20 | 2 | 600 |
| C22623 | 225 | 65 | 20 | 2.3 | 600 |
| C22625 | 225 | 65 | 20 | 2.5 | 550 |
| C24623 | 240 | 65 | 20 | 2.3 | 600 |
| C24625 | 240 | 65 | 20 | 2.5 | 550 |
| C24630 | 240 | 65 | 20 | 3 | 500 |
| C26625 | 265 | 65 | 20 | 2.5 | 550 |
| C26630 | 265 | 65 | 20 | 3 | 500 |
| C30725 | 300 | 75 | 20 | 2.5 | 650 |
| C30730 | 300 | 75 | 20 | 3 | 600 |

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