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by

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The role of relocation mobility in tax and subsidy competition

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Abstract

In this paper, we analyse the role of relocation mobility in tax and subsidy competition. Our primary result is that increasing mobility of firms leads to increasing 'net' tax revenues under plausible assumptions. While enhanced relocation mobility intensifies tax competition, it weakens subsidy competition. The resulting fall in government subsidy payments can overcompensate the decline in tax revenues, leading to a rise in net tax revenues. Interestingly, the opportunity costs of subsidy competition can rise along with net tax revenues. We derive these conclusions in a model in which two governments are first engaged in subsidy competition and thereafter in tax competition, and firms locate and potentially relocate in response to successive policy choices on taxes and subsidies.

JEL classifications: F21; H25; H71; H87; R38; R51
Keywords: Tax competition; subsidy competition; capital and firm mobility; foreign direct investment

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1 Motivation

Standard tax competition models suggest that the more mobile the tax base, the fiercer tax competition and, thus, the lower tax revenues. In this paper, we ask whether this negative effect of mobility on public revenues continues to hold when we take into account that international investors need to be attracted through subsidies first before their then established firms can be taxed. To this end, we analyse the interaction between tax and subsidy competition for firms. We ask how increasing relocation mobility affects the interplay between taxes and subsidies, the opportunity costs of subsidy competition and, particularly, net tax revenues. In this context, relocation mobility refers to the costs that arise when an already established firm moves to another jurisdiction. Importantly, and in contrast to models of tax competition only, relocation mobility has an impact not only on taxes but also on subsidies and thus on each country's net tax revenues, defined as the difference between a government's tax revenues and its subsidy payments.

Our primary result is that increasing relocation mobility leads to *increasing* net tax revenues under plausible assumptions. We derive this conclusion in a four-stage model in which two symmetric jurisdictions compete for firms with subsidies and taxes, each aiming at maximising its net revenues. In the first stage, the non-cooperative governments simultaneously set subsidies for attracting international investors. In the second stage, the investors decide where they set up their firms and receive subsidies. After subsidies have been phased out, governments simultaneously choose corporate taxes in the third stage. In the fourth stage, firms decide whether to stay or to relocate, and pay taxes accordingly.

Firms differ in their country-specific set up costs in the second stage. More importantly, they face relocation costs in the fourth stage, reflecting their imperfect relocation mobility. That is, reversing the investor's location choice from the second stage is possible but costly. This implies that firms are, in general, locked in once they are set up in a country. The lock-in effect, in turn, allows governments to levy higher taxes on firms than is otherwise possible, and it provides incentives to pay subsidies to attract new firms in the first place. It thus sets the stage for the interaction between tax and subsidy competition.

As a result of this interaction, a decline in relocation costs leads to a rise in net tax revenues in the two countries under reasonable conditions, although it weakens the lock-in effect and intensifies tax competition. This outcome occurs because the induced fall in taxes softens the preceding subsidy competition and is more than offset by the resulting decline in subsidy payments. To identify more clearly the conditions under which this outcome prevails, we establish the *What-You-Give-Is-What-You-Get* (WYGIWYG) *principle* and differentiate between the positive distribution effect and the ambiguous density effect on net tax revenues.

The WYGIWYG principle states that the opportunity costs of subsidy competition completely offset the additional tax revenues generated by attracting international investors through subsidies. The distribution effect and the density effect isolate the impact of an increase in relocation mobility on the resulting net tax revenues caused by a rise in the overall number of relocating firms and in the number of relocating firms at the margin, respectively. Our analysis suggests that net tax revenues are particularly prone to rise as mobility increases if the initial mobility is rather low, whereas they are more likely to decline if mobility is already fairly high to begin with. Interestingly, the opportunity costs of subsidy competition can go up along with net tax revenues.

A key feature of our analysis is increasing relocation mobility. For sure, relocation can still cause substantial opportunity costs. Firms often forge strong links with local business networks and suppliers and acquire location-specific knowledge once they have become established in a region. Local links and knowledge are worthless in the case of relocation. Also, relocation requires not only the transfer of financial capital, but also the movement of real capital goods and human capital, which is particularly costly.

Nevertheless, we argue that relocation costs have declined over time. For instance, the reduction of international information asymmetries, the development of modern communication and transportation technologies, the growth of modern logistics and the internationalisation of former national economies have diminished the role of the established local networks. Also, international legal and economic harmonisation, market liberalisation and regional integration (such as the European single market or NAFTA) have simplified the movement of financial, real and often even human capital across borders. Progress in production technologies can further reduce relocation costs.[1] All these measures and developments have made relocation of firms less costly, not only for big multinationals, whose relocation choices receive media attention, but also for small and medium-sized enterprises, whose decisions usually take place below the public radar.

A prime example of a firm that was attracted by subsidies and relocated its production facilities within a short period of time at the expense of public finances is Nokia (Haufler and Mittermaier, 2011). From 1995 onwards, the Finnish company had received approximately €90 million in subsidies for setting up and securing mobile phone production in Bochum, Germany. Despite this substantial financial

[1] In the semiconductor industry, the pace of the technological progress has, in some sense, substantially reduced relocation costs. Since product life cycles became extremely short in this industry (Henisz and Macher, 2004) and new production lines are set up very frequently, it is only a small step from replacing production facilities to relocating an entire factory.

support, Nokia closed down its Bochum plant and relocated production to Cluj, Romania, in 2008 (Financial Times, 2008).[2] At the time, politicians were taken by surprise, as their expectation of having 'locked-in' Nokia in Bochum as a long-term future taxpayer proved to be an illusion, with dire financial implications.

Berlin, Germany's capital, also learned that subsidies can fail to keep firms for long, albeit on a much larger scale. Like most OECD countries, Germany has long supported firms which set up new businesses or relocate old ones. During the East-West division of Europe, and until the late 1980s, firms were particularly attracted to settle in West Berlin through the very generous *Berlin subsidy* ('Berlinförderung'), a place-based public support scheme offered only there. Among others, several major cigarette producers, such as British-American Tobacco (BAT), Rothmans and Philip Morris, came to Berlin after being granted annual subsidies of up to 100,000 Deutschmarks (approximately €50,000) per job created and a generous preferential sales tax treatment for goods produced in Berlin and shipped to West Germany. While this financial support might have had the desired political effect of buttressing the 'front city of the Cold War', it was unsuccessful in creating a sound and long-lasting tax base (Ahrens, 2015; Koglin, 2015). Once the subsidy programme was abandoned after German reunification in 1990 and regular taxation set in, all cigarette producers but one left Berlin, and so did firms of other industries (Tagesspiegel, 1999, 2008). Indeed, firm relocation is a world-wide phenomenon. For instance, analysing the US manufacturing sector, Lee (2008) shows that on average 12% of plant openings were relocations from one US state to another.

Having noticed that the lock-in effects of initial location choices are often much weaker than expected, politicians have become more and more critical of subsidies to attract firms. Anticipating this problem, the semiconductor memory producer Qimonda already mentioned in its 2006 IPO prospectus that "[a]s a general rule, we believe that government subsidies are becoming less available in each of the countries in which we have received funding in the past" (Qimonda, 2006, pp. 26–27). This reasoning is in line with our model, which shows that an increase in relocation mobility leads to lower subsidies and can be a blessing in disguise in terms of net tax revenues.

Our paper is related to the 'tax holiday' literature. In this literature, governments initially grant tax holidays, or upfront subsidies, to attract foreign direct investments (FDIs) and to compensate firms for high time-consistent taxes in the future (e.g., Bond and Samuelson, 1986; Doyle and van Wijnbergen, 1994; Janeba, 2002; Kishore

[2]A similar case occurred with Motorola, another mobile phone maker. It received about €26 million for its plant in Flensburg, Germany, before shifting production to Asia and closing down in Flensburg in 2007, only nine years after opening new facilities in this location (Detje et al., 2008).

and Roy, 2014; Thomas and Worrall, 1994).[3] This policy outcome is similar to our subsidy and tax structure. But, unlike the papers above, we analyse the impact of changes in relocation mobility on net tax revenues. We also examine how the mobility of firms affects the strategic interaction between the governments in the subsidy and tax stages. By contrast, the articles referred to cannot explore this issue, as they either consider the unilateral policies of a single host country or assume a large number of potential host countries, thus excluding strategic interaction from the outset.[4]

Closer to our approach is Lee's (1997) model. He analyses a two-period model in which capital is perfectly mobile in the first period and imperfectly mobile in the second period. Governments non-cooperatively levy a tax on capital and use each period's revenues to provide a public good in the very same period. Lee (1997) focuses on the question whether the public good is oversupplied or undersupplied in the second period, ignoring the first-period outcome. By contrast, we focus on the overall impact of changes in relocation mobility on net tax revenues in the two periods together, and on how these changes affect the interaction between tax and subsidy competition.

More recently, Langenmayr and Simmler (2017) analyse the market entry and relocation choices of firms. In their model, local governments only compete in taxes (and not in subsidies and taxes at different stages, as in our model), and strategic interaction between jurisdictions plays no role. While Langenmayr and Simmler (2017) can thus not tackle our questions in their theoretical model, their empirical analysis shows that taxes indeed rise as more immobile firms are set up in a jurisdiction. This result confirms the importance of relocation costs.

In a different vein, Ferrett et al. (2019) also explicitly consider relocation decisions. In their model, two governments compete for a single firm in each of two periods. They focus on changing characteristics of the competing regions as the main driver of relocation and show that fiscal competition can make relocation not only more likely but location choices also more efficient. By contrast, we analyse the impact of mobility on public budgets and argue that declining relocation costs of firms can increase net tax revenues.

Like our paper, the literature on tax competition in models of the 'new economic

[3] In an alternative and complementary approach to the tax holiday literature, Chisik and Davies (2004) explain how a succession of bilateral treaties on the taxation of FDIs can actually lead to a gradual reduction in tax rates over time.

[4] Haufler and Wooton (2006) analyse regional tax and subsidy coordination within an economic union when the two members of this union compete with a third country. However, governments have only one policy instrument at their disposal, either a subsidy or a corporate tax. This paper thus differs considerably from the tax holiday literature and from our contribution.

geography' raises some doubts about whether increasing economic integration necessarily erodes government revenues (e.g., Baldwin and Krugman, 2004; Borck and Pflüger, 2006; Kato, 2015; Kind et al., 2000). In contrast to our paper, the arguments in this literature hinge on the presence of significant agglomeration economies and the emergence of a core-periphery pattern, which allow the core to tax agglomeration rents. Finally, Konrad and Kovenock (2009) is related to both the tax holiday and the new economic geography literature. They analyse tax competition for 'overlapping FDIs' in a dynamic model with agglomeration advantages. In their model, the vintage property of FDIs prevents a ruinous race to the bottom as long as governments only have non-discriminatory taxes at their disposal.[5]

Our paper proceeds as follows. In section 2, the model is presented. Section 3 investigates the outcome of the subsidy and tax competition stages. We analyse and discuss the impact of increasing relocation mobility in section 4. Section 5 concludes with a brief discussion of some policy implications.

2 Governments and firms

We start by presenting our two-period, four-stage model of tax and subsidy competition for firms. In the first period (consisting of the first and second stages; see below), the governments of two jurisdictions non-cooperatively grant subsidies to attract international investors. Given these subsidies, investors then decide in which country they set up their firms. In the second period (consisting of the third and fourth stages), the two governments non-cooperatively levy corporate taxes. Since the firms are now established in a country, they are locked-in, but only imperfectly, as we will explain in more detail below. Firms can still relocate in response to the tax policies of the jurisdictions. So there is competition for mobile firms in both periods, albeit to a different degree.

Firms Consider two symmetric countries, A and B, and a continuum of international investors of size 2. In the first period, each of the investors sets up a single firm in either country A or B. A firm's set-up costs are $c + z_i$ if it is located in country i, $i = A, B$. While all firms face an identical basic cost component c, they differ with respect to their country-specific cost components (z_A, z_B). (For notational convenience, firm indices are omitted.) These country-specific elements capture how 'familiar' international investors are with the two countries. This 'familiarity' affects the easiness, and thus costs, with which investors can set up firms in specific

[5] Following a different line of reasoning, Wilson (2005), among others, argues that tax competition can be welfare-enhancing because it can force countries to reduce 'government waste'.

countries.[6]

We denote an investor's cost differential between setting up a firm in country i and j by $\Delta z_{ij} = z_i - z_j$. This cost disadvantage of investing in country i is distributed according to the distribution function $G(\Delta z_{ij})$. The function $G(\Delta z_{ij})$ is twice continuously differentiable and strictly increasing over the interval $\left[-\overline{\Delta z_{ij}}, \overline{\Delta z_{ij}}\right]$, with $\overline{\Delta z_{ij}} > 0$, $G(-\overline{\Delta z_{ij}}) = 0$ and $G(\overline{\Delta z_{ij}}) = 1$. Its corresponding density function $G'(\Delta z_{ij})$ is assumed to be symmetric around $\Delta z_{ij} = 0$. More precisely, distribution $G(\Delta z_{ij})$ satisfies $G'(\Delta z_{ij}) = G'(-\Delta z_{ij})$ and thus $G(0) = 0.5$. That is, each investor with a cost differential of Δz_{ij} is counterbalanced by one with a cost differential of $-\Delta z_{ij}$. Finally, we impose a standard restriction on the slope and curvature of the distribution function $G(\Delta z_{ij})$ over the interval $\left(-\overline{\Delta z_{ij}}, \overline{\Delta z_{ij}}\right)$.

Assumption 1:
(i) $\partial(G/G')/\partial\Delta z_{ij} > 0$ or, equivalently, $G''(\Delta z_{ij}) < G'(\Delta z_{ij})^2/G(\Delta z_{ij})$,
(ii) $\partial[(1-G)/G']/\partial\Delta z_{ij} < 0$ or, equivalently, $G''(\Delta z_{ij}) > -G'(\Delta z_{ij})^2/[1-G(\Delta z_{ij})]$.

Property (i) states that the inverse hazard rate $G(\Delta z_{ij})/G'(\Delta z_{ij})$ be increasing in Δz_{ij}. Property (ii) is the counterpart to property (i), saying that the ratio $[1 - G(\Delta z_{ij})]/G'(\Delta z_{ij})$ be decreasing in Δz_{ij}.[7] These two restrictions are common in the literature (e.g., Bergemann and Strack, 2015; Konrad and Thum, 2014). They provide sufficient conditions for well-behaved objective functions by excluding too steep a slope in the density functions.

In the second period, a firm realises the gross return π if it continues to stay in the country where it was established in the first period. Its return drops to $\pi - m$ if it relocates in the second period, since it then faces relocation costs m. While π is the same for all firms, relocation costs m differ across firms. As these relocation costs need not be prohibitive, firms are only imperfectly locked in.

Denote the 'number' or, more correctly, mass of firms which locate in jurisdiction i in period 1 by N_i. Then, the characteristic m is distributed across these N_i firms according to the distribution function $F(m)$. This function is twice continuously differentiable and strictly increasing over the interval $[0, \overline{m}]$, with $F(0) = 0$ and $F(\overline{m}) = 1$.

In line with assumption 1, the distribution $F(m)$ fulfils the following properties, already explained above, over the intervals $(0, \overline{m})$.

[6] The fact that investors are biased towards specific countries is an empirically well and long established result. Leblang (2010) mentions cultural affinity, economic familiarity, information asymmetries and diaspora networks as reasons why investors tend to invest in a certain country.

[7] Instead of explicitly deriving the one-dimensional distribution of the cost differentials Δz_{ij} from a two-dimensional distribution of the investors' country-specific set-up costs (z_A, z_B), we directly characterise the distribution function $G(\Delta z_{ij})$. This 'reduced-form' distribution function turns out to be sufficient for the further analysis, since it contains all the information needed.

Assumption 2:
(i) $\partial(F/F')/\partial m > 0$ or, equivalently, $F''(m) < F'(m)^2/F(m)$,
(ii) $\partial[(1-F)/F']/\partial m < 0$ or, equivalently, $F''(m) > -F'(m)^2/[1-F(m)]$.

The functions G and F are common knowledge to firms and their investors and to governments. Each investor learns about the realisation of her firm's country-specific cost differential Δz_{ij} before she makes the initial location decision in the first period. Similarly, each firm learns about the realisation of its relocation costs m before it makes its relocation decision in the second period. For simplicity, we assume that a firm's first-period cost differential and second-period relocation costs are not correlated. This assumption enables us to put forward our arguments as simply as possible.[8]

Governments When competing for mobile firms, the non-cooperative governments have subsidies and corporate taxes at their disposal. In the first period, each investor who sets up a firm in country i receives subsidy s_i. In the second period, each government can differentiate between 'domestic' firms, i.e., firms that were already set up in its jurisdiction, and 'foreign' firms, i.e., firms that newly relocate to its jurisdiction, and can implement preferential tax regimes accordingly. That is, domestic firms pay tax t_i^n, while foreign firms face tax t_i^m in country i.

We interpret these variables as effective taxes that include the range of tax breaks particularly available to internationally mobile firms. Also, we allow for preferential regimes, since the national tax codes and instruments often result in differential 'effective' tax treatment of foreign and domestic firms even if nominal taxes are not preferential in nature. In this context, OECD (2017), for instance, continues to review scores of financial instruments and assess their harmfulness. In fact, Genschel and Rixen (2014) criticise that the OECD's reports on preferential tax regimes tend to systematically underestimate the extent of these practices and even to 'whitewash' them. In section 4, we discuss how our assumption of preferential tax regimes affects our conclusions.

Objectives and timing Each country maximises its 'net' revenues NR_i, i.e., the difference between tax revenues R_i and subsidy payments P_i, given the decisions of its opponent. As usual, investors maximise the (expected) net profits of their firms, taking into account gross return, set-up costs, relocation costs, subsidies and taxes.

[8] In fact, it is far from clear whether set-up cost differentials and relocation costs are correlated. An investor who is very 'familiar' with both countries can face a very minor cost differential. But if this investor establishes a steel factory, the relocation costs will be substantial—if not prohibitive. Low set-up cost differentials do not imply low relocation costs, and vice versa.

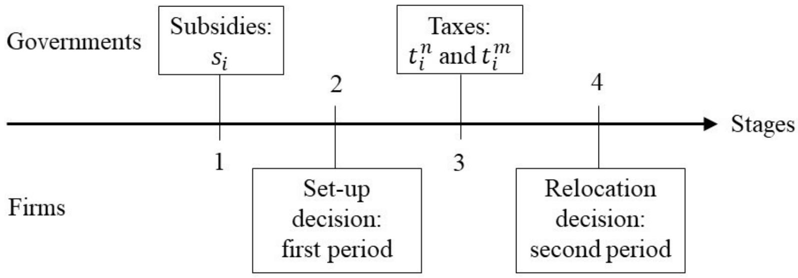

Figure 1: Timing and decisions

The precise timing of the subsidy and tax competition game between the two governments is illustrated in figure 1. In the first stage, the non-cooperative governments simultaneously set subsidies s_A and s_B. Given these subsidies, investors decide in the second stage whether their firms locate and receive subsidies in either country A or B. In the third stage, the governments simultaneously set their taxes t_A^n, t_A^m, t_B^n and t_B^m, again non-cooperatively. In the fourth stage, firms decide whether they stay or relocate, and pay their taxes accordingly.

In terms of time periods, the first two stages can be interpreted as constituting period 1, the third and fourth stages as constituting period 2. As mentioned above, each investor learns about her country-specific set-up costs prior to her location decision at the beginning of the second stage. Similarly, each firm learns about its relocation costs prior to its relocation decision at the beginning of the fourth stage.

3 Subsidy and tax competition

Solving our model for the subgame-perfect Nash equilibrium by backward induction, we start with the tax competition stages and then go on to the subsidy competition stages.

3.1 Tax competition

The firms' decisions in the fourth stage are straightforward. A firm that was set up in region i in the first period can stay in this region and receive the net return $\pi - t_i^n$ (first period costs and subsidies are sunk at this stage). Alternatively it can move to region j and gain the net return $\pi - m - t_j^m$. A profit maximising firm thus stays in region i (relocates to region j) if, and only if,

$$m \geq t_i^n - t_j^m \qquad (m < t_i^n - t_j^m), \tag{1}$$

i.e., if, and only if, the tax differential between the countries is smaller (strictly greater) than the firm-specific relocation costs.[9] Consequently, the share of firms relocating from region i to j is

$$\gamma_{ij} = \begin{cases} 0 & \text{if} \quad t_i^n < t_j^m \\ F(t_i^n - t_j^m) & \text{if} \quad t_i^n \in \left[t_j^m, t_j^m + \overline{m}\right] \\ 1 & \text{if} \quad t_i^n > t_j^m + \overline{m} \end{cases} . \tag{2}$$

Then, the tax revenues of government i are

$$R_i\left(t_i^n, t_i^m\right) = t_i^n\left(1 - \gamma_{ij}\right)N_i + t_i^m \gamma_{ji} N_j, \tag{3}$$

where N_i and N_j result from the investors' decisions in the second stage. The term $t_i^n\left(1 - \gamma_{ij}\right)N_i$ captures the tax revenues from all firms that were already located in country i in the first period and stay there in the second period. By contrast, the term $t_i^m \gamma_{ji} N_j$ refers to the revenues from those firms that were initially located in country j and only enter country i in the second period.

In the third stage, government i chooses taxes t_i^n and t_i^m that maximise revenues R_i, given the choices of its competitor (recall that previous subsidy payments are sunk at this stage). As usual, higher taxes increase the revenues from the firms ultimately located in country i, but reduce the number of those firms. The optimal taxes balance these opposing effects. Also, governments never choose non-positive taxes in equilibrium, so we can focus on positive taxes (see proof of lemma 1). Then, country i's reaction functions are implicitly given by

$$t_i^n = \begin{cases} \frac{1 - F(t_i^n - t_j^m)}{F'(t_i^n - t_j^m)} \in \left(t_j^m, t_j^m + \overline{m}\right) & \text{if} \quad t_j^m F'(0) < 1 \\ t_j^m & \text{if} \quad t_j^m F'(0) \geq 1 \end{cases} , \tag{4}$$

$$t_i^m = \begin{cases} \frac{F(t_j^n - t_i^m)}{F'(t_j^n - t_i^m)} \in \left(t_j^n - \overline{m}, t_j^n\right) & \text{if} \quad \left(t_j^n - \overline{m}\right) F'(\overline{m}) < 1 \\ t_j^n - \overline{m} & \text{if} \quad \left(t_j^n - \overline{m}\right) F'(\overline{m}) \geq 1 \end{cases} . \tag{5}$$

As condition (4) shows, country i's optimal tax t_i^n on domestic firms will strictly exceed (equal) the tax t_j^m these firms face in country j if tax t_j^m is low (high). Any tax t_i^n that is below t_j^m cannot be optimal, since country i could raise tax t_i^n without losing any domestic firms, thereby increasing tax revenues. Also, condition (5) means that country i's optimal tax t_i^m strictly undercuts any positive tax t_j^n,

[9] In principle, subsidies could be contingent on performance. In reality, incomplete contracts and other problems will make it difficult for governments to reclaim subsidies even if firms fail to comply with performance requirements and relocate their production facilities. At most, a firm will be forced to pay back a part of its subsidy in the case of plant closure and relocation. This would obviously increase the relocation costs of the firm and modify condition (1), but it would not change our conclusions qualitatively.

since this is the only way to attract firms initially located in country j and extract some tax revenues from these firms.

Together, the reaction functions of the two governments implicitly determine the equilibrium taxes

$$t_i^n = \frac{1 - F(t_i^n - t_j^m)}{F'(t_i^n - t_j^m)} \quad \text{and} \quad t_i^m = \frac{F(t_j^n - t_i^m)}{F'(t_j^n - t_i^m)} \tag{6}$$

and the positive tax differential[10]

$$\Delta t_{ij} := t_i^n - t_j^m = \frac{1 - 2F(t_i^n - t_j^m)}{F'(t_i^n - t_j^m)} > 0. \tag{7}$$

Since equilibrium taxes are symmetric, i.e., $t_i^n = t_j^n$ and $t_i^m = t_j^m$, so are equilibrium tax differentials, i.e., $\Delta t_{ij} = \Delta t_{ji} = \Delta t_{jj} = \Delta t_{ii} := t_i^n - t_i^m > 0$.

These solutions contain three important conclusions. First, government i's tax on firms already established in country i in the first period exceeds the tax on firms that move to region i in the second period, i.e., $t_i^n > t_i^m$. This tax differential arises because firms are locked in, at least imperfectly, once they have settled in a country. Since firms respond less elastically to an increase in the 'domestic' tax t_i^n compared to that in the 'foreign' tax t_j^m, they end up with higher tax payments if they stick to their initial location choice, leading to positive tax differentials $\Delta t_{ij} = \Delta t_{ii}$. Nevertheless, the majority of firms stay in the country in which they were set up, i.e., $F(t_i^n - t_j^m) < 0.5$ in equilibrium.[11] This outcome simply reflects the lock-in effect.

Second, the lock-in effect and the induced positive tax differential $\Delta t_{ii} := t_i^n - t_i^m$ translate into a positive revenue differential $\Delta\Omega_{ii} := \Omega_i^n - \Omega_i^m$, where $\Omega_i^n := t_i^n \left[1 - F(t_i^n - t_j^m)\right]$ and $\Omega_i^m := t_i^m F(t_j^n - t_i^m)$. The terms Ω_i^n and Ω_i^m stand for the tax revenues that country i can expect in the second period from a firm initially set up in country i and j, respectively. The revenue component Ω_i^n exceeds Ω_i^m, since, due to the lock-in effect, the share $\left[1 - F(t_i^n - t_j^m)\right]$ is greater than $F(t_j^n - t_i^m)$ and the tax t_i^n is higher than t_i^m. More precisely, using conditions (6) and (7), we get

$$\Delta\Omega_{ii} := \Omega_i^n - \Omega_i^m = \Omega_i^n - \Omega_j^m = t_i^n - t_j^m = t_i^n - t_i^m =: \Delta t_{ii} > 0 \tag{8}$$

in equilibrium.[12] That is, the revenue differential is exactly equal to the tax differential. As the revenue differential is positive, attracting investors in the first

[10] The tax differential can only be positive, since a non-positive one, i.e., $t_i^n - t_j^m \leq 0$, would require $F(t_i^n - t_j^m) \geq 1/2$ according to solution (7). This implies $t_i^n - t_j^m > 0$, which obviously contradicts $t_i^n - t_j^m \leq 0$ (see also proof of lemma 1).

[11] Formally, a positive tax differential $\Delta t_{ij} := \left[1 - 2F(t_i^n - t_j^m)\right]/F'(t_i^n - t_j^m) > 0$ directly implies $F(t_i^n - t_j^m) < 0.5$.

[12] The equality signs follow from $t_i^n = t_j^n$, $t_i^m = t_j^m$ and $\Omega_i^n - \Omega_j^m = \left[1 - F(t_i^n - t_j^m)\right]^2/F'(t_i^n - t_j^m) - \left[F(t_i^n - t_j^m)\right]^2/F'(t_i^n - t_j^m) = \left[1 - 2F(t_i^n - t_j^m)\right]/F'(t_i^n - t_j^m) = t_i^n - t_j^m$.

period increases tax revenues in the second period. This sets the stage for subsidy competition in the first period.

Third, taxes are independent of the number of firms N_i and thus independent of subsidies. By contrast, the optimal subsidies in the first stage are shaped by the future taxes, as will soon become evident. In this sense, there is a one-way link between tax and subsidy competition.

The equilibrium values (6) and (7) are analogous to the results in Haupt and Peters (2005). We derive these results in a more micro-founded setting, which is, in some sense, more general with respect to mobility. More importantly, Haupt and Peters (2005) only consider tax competition and completely ignore subsidy competition, while we are interested precisely in the relationship between tax and subsidy competition, and we analyse the resulting net tax revenues. Let us therefore turn next to the subsidy competition between the governments.

3.2 Subsidy competition

Since the tax t_A^n (t_A^m) is equal to t_B^n (t_B^m), and since the distributions of relocation costs m are the same in the two countries, a firm's expected net return in the second period, i.e., gross return net of expected tax payments and expected relocation costs, is independent of its location in the first period. The location choice in the second stage, however, affects a firm's overall net profit through its country-specific set-up costs and received subsidy. A firm has net costs of $c + z_i - s_i$ ($c + z_j - s_j$) in the first period if it is set up in country i (country j). This firm is thus located in country i (country j) in the second stage if, and only if,

$$\Delta z_{ij} \leq s_i - s_j \qquad (\Delta z_{ij} > s_i - s_j), \tag{9}$$

i.e., if, and only if, the subsidy differential between the countries is greater (strictly smaller) than the potential cost disadvantage of investing in country i. The resultant shares of investors who locate their firms in country i and j are, respectively, $G(s_i - s_j)$ and $1 - G(s_i - s_j)$. As the total number of investors is normalised to 2, the numbers of firms established in country i and j amount to[13]

$$N_i = 2G(s_i - s_j) \qquad \text{and} \qquad N_j = 2[1 - G(s_i - s_j)]. \tag{10}$$

In the first stage, government i chooses its subsidy s_i, taking the subsidy of its opponent as given. It maximises its net tax revenues

$$NR_i = 2\{\Omega_i^n G(s_i - s_j) + \Omega_i^m [1 - G(s_i - s_j)] - s_i G(s_i - s_j)\}, \tag{11}$$

[13] For notational convenience, we now focus on 'interior' solutions (i.e., $|s_i - s_j| < \overline{\Delta z_{ij}}$ and thus $G(s_i - s_j) \in (0, 1)$), which indeed prevail in equilibrium. See proof of lemma 1.

where $\Omega_i^n := t_i^n \left[1 - F(t_i^n - t_j^m)\right]$ and $\Omega_i^m := t_i^m F(t_j^n - t_i^m)$ are the expected second-period tax payments of a firm initially set up in country i and j, respectively, as already discussed in section 3.1. The first two terms in the braces thus capture future tax revenues, while the third term gives the subsidy payments to investors.

The optimal subsidy is implicitly given by the first-order condition

$$\frac{dNR_i}{ds_i} = -2G\left(s_i - s_j\right) + \left[\Omega_i^n - \Omega_i^m - s_i\right] 2G'\left(s_i - s_j\right) = 0. \tag{12}$$

A marginal rise in subsidy s_i increases government spending by the number of recipients $2G\left(s_i - s_j\right)$. This negative effect of today's subsidy on net tax revenues is captured by the first term on the right-hand side of the derivative. By contrast, the second term shows the positive impact of today's subsidy on net revenues. As government i's expected future tax revenue from a firm is Ω_i^n if the firm is set up in country i, but only Ω_i^m if the firm is set up in country j, attracting an additional investor in the first period increases future tax revenues by the differential $\Delta\Omega_{ii} := \Omega_i^n - \Omega_i^m$. Taking into account the subsidy payments, the net benefit of gaining an additional international investor is $\Delta\Omega_{ii} - s_i$. Finally, the term $2G'(s_i - s_j)$ tells us how the number of firms established in country i changes in response to a marginal rise in subsidy s_i.

Analogously to government i, government j sets its subsidy s_j such that it maximises its net tax revenues

$$NR_j = 2\left\{\Omega_j^n \left[1 - G\left(s_i - s_j\right)\right] + \Omega_j^m G\left(s_i - s_j\right) - s_j \left[1 - G\left(s_i - s_j\right)\right]\right\}, \tag{13}$$

yielding the first-order condition

$$\frac{dNR_j}{ds_j} = -2\left[1 - G\left(s_i - s_j\right)\right] + \left[\Omega_j^n - \Omega_j^m - s_j\right] 2G'\left(s_i - s_j\right) = 0. \tag{14}$$

The interpretation of this condition is completely in line with that of condition (12). We only need to replace $G\left(s_i - s_j\right)$ with $\left[1 - G\left(s_i - s_j\right)\right]$, and vice versa.

The two countries' first-order conditions (12) and (14), which implicitly determine their reaction functions, and the symmetry of the distribution function G imply that the equilibrium subsidies are symmetric and, since $G(0) = 0.5$ holds, given by

$$s_i = \Delta\Omega_{ii} - \tau_i = \Delta\Omega_{jj} - \tau_j = s_j, \quad \text{where} \quad \tau_i = \frac{1}{2G'\left(0\right)} = \tau_j. \tag{15}$$

To understand the equilibrium policy, and to facilitate our economic interpretation of the model, briefly consider a standard tax competition game in which investors only have one location choice. In this variation of our model, the two governments non-cooperatively set taxes τ_i and τ_j in the first stage, aiming at maximising their tax revenues Υ_i and Υ_j, and investors decide on their firms' location in the second stage, facing country-specific cost differentials as captured by the distribution function $G(\Delta z_{ij})$. Firms cannot relocate, as the game ends after the second stage. All other assumptions remain valid. Then, the tax revenues of country i and j are, respectively, $\Upsilon_i(\tau_i;\tau_j) = \tau_i 2G(\tau_j - \tau_i)$ and $\Upsilon_j(\tau_j;\tau_i) = \tau_j 2[1 - G(\tau_j - \tau_i)]$. The corresponding first-order conditions $d\Upsilon_i/d\tau_i = 2G(\tau_j - \tau_i) - \tau_i 2G'(\tau_j - \tau_i) = 0$ and $d\Upsilon_j/d\tau_j = 2[1 - G(\tau_j - \tau_i)] - \tau_j 2G'(\tau_j - \tau_i) = 0$ are analogous to those of the extended model, (12) and (14). Clearly, the equilibrium is again symmetric. As $G(0) = 0.5$, we get

$$\Upsilon_i = \tau_i = \frac{1}{2G'(0)} = \tau_j = \Upsilon_j. \tag{16}$$

We coin the terms *hypothetical taxes* for τ_i and τ_j and *hypothetical tax revenues* for Υ_i and Υ_j, as they refer to the outcome in the 'hypothetical' case in which the game ends after the second stage.

With this brief exploration in mind, the equilibrium subsidies (15) have a straightforward interpretation. If there were no revenue differentials $\Delta\Omega_{ii}$ and $\Delta\Omega_{jj}$, there would be no incentive to attract investors with subsidies, and firms would have to pay the hypothetical taxes τ_i and τ_j in the first period (that is, the same taxes that would result if there were no second period). These taxes are 'cut' by the expected revenue differential (8). In this sense, governments give up current revenues for the benefit of having future ones. But only if the future gains $\Delta\Omega_{ii}$ and $\Delta\Omega_{jj}$ strictly exceed the hypothetical taxes τ_i and τ_j, will the subsidies s_i and s_j indeed be positive (see equilibrium solution (15)). This outcome, in turn, requires a sufficiently strong lock-in effect.

We have so far side-stepped the more technical topics of existence and uniqueness of the equilibrium. These issues are taken up in lemma 1.

Lemma 1 *Tax and subsidy competition.*
A subgame-perfect equilibrium exists and is unique. Equilibrium taxes and subsidies satisfy conditions (6), (7) and (15). Moreover, $N_i = N_j = 1$ results for the numbers, or masses, of firms.

Proof: *See appendix A.* □

3.3 Net tax revenues

In a one-period tax competition game that encompasses the first two stages of our current model, governments would levy the hypothetical tax τ_i, as argued above. In our extended model, governments aim at attracting investors in the first period for the sake of higher tax revenues in the second period. To this end, they are willing to forego the hypothetical tax τ_i in the first period and to additionally pay a subsidy s_i if the lock-in effect is sufficiently strong. With this in mind, let us define country i's opportunity costs C_i of attracting an international investor in the first period as the sum of the subsidy s_i paid to this investor and the foregone hypothetical tax τ_i of this investor, i.e., the tax this investor who set up a firm in country i would have paid if there were no second period. These opportunity costs of subsidy competition, as we also call them, then amount to

$$C_i = s_i + \tau_i = \Delta\Omega_{ii} - \tau_i + \tau_i = \Delta\Omega_{ii}, \tag{17}$$

where we used equilibrium condition (15). That is, the opportunity costs equal the second-period revenue differential $\Delta\Omega_{ii}$.

Recall that the revenue differential $\Delta\Omega_{ii}$ is exactly the difference between the expected second-period tax revenues from a firm initially set up in country i (i.e., $\Omega_i^n = \Delta\Omega_{ii} + \Omega_i^m$) and those from a firm initially set up in country j (i.e., Ω_i^m). Hence, the expected second-period gain from attracting an investor in the first period is exactly offset by the associated opportunity costs. We refer to this outcome as the *What-You-Give-Is-What-You-Get* (WYGIWYG) *principle.* This principle carries over from the individual to the aggregate level and thus reappears in each country's net tax revenues:

$$NR_i = \overbrace{\underbrace{\Delta\Omega_{ii}}_{\text{rev diff}} + \underbrace{2\Omega_i^m}_{\text{basic rev}}}^{\text{revenues } R_i} - \overbrace{[\underbrace{\Delta\Omega_{ii}}_{\text{opp costs } C_i} - \underbrace{\Upsilon_i}_{\text{hypo tax rev}}]}^{\text{subsidy payments } P_i}. \tag{18}$$

where we used the equilibrium outcome (15), $\tau_i = \Upsilon_i$ and $N_i = N_j = 1$.

Rearranging revenues $R_i = \Omega_i^n + \Omega_i^m = \Delta\Omega_{ii} + 2\Omega_i^m$, we can split up country i's aggregate revenues into the two components $\Delta\Omega_{ii}$ and $2\Omega_i^m$. The first component (*rev diff*) captures the additional tax revenues that arise because firms of mass 1 are initially set up in the country i. Due to the lock-in effect, these firms contribute $\Omega_i^n = t_i^n [1 - F(\Delta t_{ij})]$ to country i's second-period revenues, while they would have paid only $\Omega_i^m = t_i^m F(\Delta t_{ji})$ in taxes in country i if they had initially been set up in country j, with only $F(\Delta t_{ji})$ of them relocating to country i and paying a tax of only t_i^m. As each country attracts investors of mass 1 in the first period, the aggregate revenue differential $\Delta\Omega_{ii} N_i$ equals the expected individual revenue differential $\Delta\Omega_{ii}$.

The second component captures the remaining tax revenues and is coined basic revenues (*basic rev*). By construction, they are equal to the tax revenues that would accrue to country i if no firm were located there in the first period. In this case, all firms would be set up in country j, but the share $F(\Delta t_{ji})$ of these firms of mass 2 would relocate in the second period, generating revenues of $t_i^m 2F(\Delta t_{ji}) = 2\Omega_i^m$.

The subsidy payments P_i can also be decomposed into two elements, the aggregate opportunity costs C_i of attracting investors (*opp costs*) and the aggregate hypothetical tax revenues Υ_i (*hypo tax rev*), which the governments forego. As investors of mass 1 set up their firms in each country, aggregate opportunity costs $\Delta\Omega_{ii}N_i$ coincide with individual ones $C_i = \Delta\Omega_{ii}$. For the same reason, the aggregate hypothetical tax revenues Υ_i are identical to the individual hypothetical tax τ_i (see eq. (16)). These are exactly the revenues that would arise if there were no second period, as discussed in section 3.2. They are hypothetical since, despite being part of equation (18), they do not materialise, precisely because governments give them up in the pursuit of international investors. By definition, the opportunity costs of attracting investors in the first period consist of subsidy payments and foregone tax revenues, and thus the aggregate subsidy payments P_i are the difference between the aggregate opportunity costs C_i and the hypothetical tax revenues Υ_i.

As at the individual level, the first-period opportunity costs (*opp costs*) of and the additional second-period revenues (*rev diff)* resulting from attracting investors cancel each other out at the aggregate level. That is, the WYGIWYG principle holds at both the individual level and the aggregate level.

Intuitively, this outcome is fairly straightforward. The maximum opportunity costs a regional government is willing to accept to attract an investor are exactly equal to the difference in expected second-period tax payments between a firm set up domestically and a firm set up abroad, given by $\Delta\Omega_{ii}$. As this difference is the same for both regions (i.e., $\Delta\Omega_{AA} = \Delta\Omega_{BB}$), the maximum opportunity costs both governments are ready to tolerate are the same as well. Thus, a Bertrand-type upward competition induces them to offer a subsidy that erodes any potential net gains from attracting investors. As the opportunity costs exactly offset the revenue differential, all that remains of the second-period tax revenues, once they are consolidated for the opportunity costs of attracting investors initially, are the basic revenues. Taking the WYGIWYG principle into account, net tax revenues are

$$NR_i = 2t_i^m F(t_j^n - t_i^m) + \tau_i. \tag{19}$$

They are ultimately determined by only two terms, the basic revenues and the hypothetical tax revenues. The latter revenues turn out to be key because of the WYGIWYG principle, although they do not literally materialise.

The WYGIWYG principle suggests an alternative interpretation of the basic revenues $2\Omega_i^m$. The expected revenues $\Omega_i^m = t_i^m F(\Delta t_{ji})$, which are generated from firms initially set up abroad, count twice: First, they increase aggregate revenues in the second period by $t_i^m F(\Delta t_{ji})$. Second, they reduce subsidy payments in the first period by $t_i^m F(\Delta t_{ji})$ because they cut the revenue differential $\Delta\Omega_{ii}$, thus reducing the incentives to attract investors. This 'double-counting' interpretation is illustrated in figure 2, where the shaded area shows the second-period tax revenues net of the opportunity costs of attracting investors in the first period.

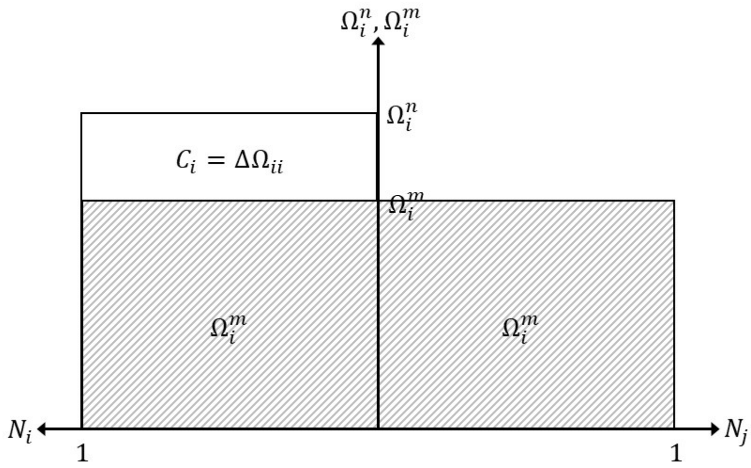

Figure 2: Decomposition of second-period tax revenues

4 The impact of relocation mobility

We now turn to our key issue, the impact of an increase in relocation mobility on net tax revenues and the opportunity costs of subsidy competition. As argued above, even firms that are well established in a country are for various reasons becoming more and more mobile. In our model, this increase in mobility comes as a reduction in the firms' relocation costs. More specifically, we interpret the rise in mobility as an increase (decrease) in the share of firms with low (high) relocation costs. This is expressed as changes in the values of the distribution and density functions $F(m;\alpha)$ and $F'(m;\alpha)$ caused by an increase in a mobility parameter α such that $\partial F'(m;\alpha)/\partial\alpha \gtreqless 0 \Leftrightarrow m \lesseqgtr \widetilde{m} \in (0,\overline{m})$ and thus $\partial F(m;\alpha)/\partial\alpha > 0$ for $m \in (0,\overline{m})$. We stick, for convenience, to our notation $F' = \partial F/\partial m$ and $F'' = \partial^2 F/\partial m^2$. The partial derivatives with respect to the parameter α are explicitly expressed as $\partial F/\partial\alpha$ and $\partial F'/\partial\alpha$.

An increase in mobility corresponds to an upward shift of the distribution curve, as illustrated in figure 3. The slope of the distribution curve becomes initially steeper and then flatter. Such changes certainly increase the share of relocating firms at the old equilibrium level Δt_{ji} (i.e., $\partial F(\Delta t_{ji};\alpha)/\partial\alpha > 0$), while they might raise or lower the number of firms which alter their relocation choice in response to a marginal increase in the tax differential (i.e., $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha \gtreqless 0$), depending on whether the equilibrium tax differential Δt_{ji} is smaller or greater than the critical value $\widetilde{m}$.

So changes in relocation mobility manifest themselves in changes in the distribution function and the density function. Also, the distribution and density functions completely determine the equilibrium policy choices and, more generally,

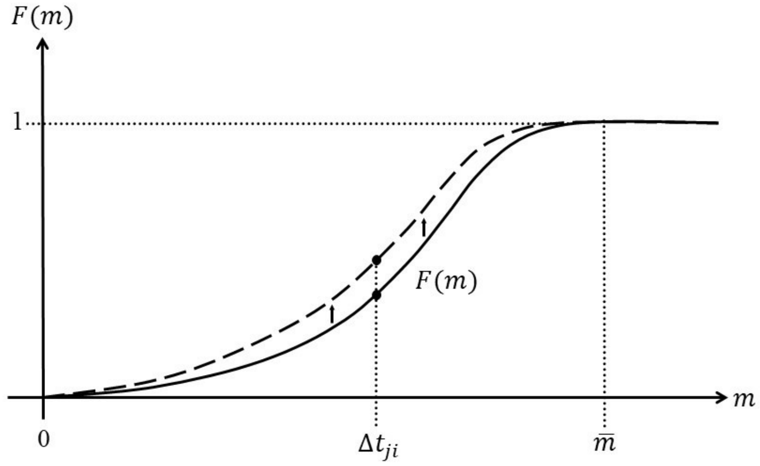

Figure 3: Increasing relocation mobility and distribution function

the equilibrium outcome. In particular, inserting policy choices (6) and (16) into net tax revenues (19) yields $NR_i = 2\left[F(\Delta t_{ji};\alpha)\right]^2 / F'(\Delta t_{ji};\alpha) + 1/\left[2G'(0)\right]$, where $\Delta t_{ji} = \left[1 - 2F(\Delta t_{ji};\alpha)\right] / F'(\Delta t_{ji};\alpha)$ (see eq. (7)).

Together, the clear equilibrium results and the straightforward specification of increasing mobility enable us to explore the role of mobility in a very tractable manner. Higher relocation mobility affects the equilibrium outcome in two distinct ways. First, there is the distribution effect, which captures the implication of a shift of the distribution function (i.e., $\partial F(\Delta t_{ji};\alpha)/\partial\alpha > 0$). Second, there is the density effect, which refers to the impact of a change in the density function (i.e., $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha \gtreqless 0$). We study these two effects, which can reinforce or counteract each other, separately before turning to the overall effect.

4.1 Distribution effect

Let us start by analysing the distribution effect in isolation. More specifically, we now analyse $dNR_i/d\alpha$ and $dC_i/d\alpha$ for

$$\partial F(\Delta t_{ji};\alpha)/\partial\alpha > 0 \text{ and } \partial F'(\Delta t_{ji};\alpha)/\partial\alpha = 0 \text{ (distribution effect).}$$

It turns out that such an increase in relocation mobility actually raises net tax revenues, as stated in our first proposition and explained afterwards.

Proposition 1 *Distribution effect of increasing relocation mobility.*
The net tax revenues NR_i increase with the firms' mobility parameter α. That is, $dNR_i/d\alpha > 0$.
Proof: See appendix A. □

As discussed above, net tax revenues can be reduced to the sum of basic and hypothetical tax revenues due to the WYGIWYG principle (see eq. (19)). Moreover, relocation mobility leaves hypothetical tax revenues $\Upsilon_i = \tau_i = 1/\left[2G'(0)\right]$ unaffected. All that ultimately matters in the context of proposition 1 is thus the impact of an increase in mobility on basic revenues, as captured by the derivative

$$\frac{dNR_i}{d\alpha} = 2t_i^m \frac{\partial F(\Delta t_{ji}; \alpha)}{\partial \alpha} + 2t_i^m F'(\Delta t_{ji}; \alpha) \frac{dt_j^n}{d\alpha}, \tag{20}$$

where the envelope theorem, i.e., $\partial NR_i/\partial t_i^m = 2\partial(t_i^m F)/\partial t_i^m = 2(\partial R_i/\partial t_i^m) = 0$, is used and $dt_j^n/d\alpha$ is calculated for $\partial F/\partial\alpha > 0$ and $\partial F'/\partial\alpha = 0$.

The first term on the right-hand side captures the direct distribution effect of increasing mobility. For given taxes t_j^n and t_i^m, the number of firms relocating from country j to country i rises, since the lock-in effect is weakened. This positive effect raises country i's 'basic' tax base by $2[\partial F(\Delta t_{ji}; \alpha)/\partial\alpha]$ and, for given taxes, net revenues by $2t_i^m[\partial F(\Delta t_{ji}; \alpha)/\partial\alpha]$.

The second term shows the indirect distribution effect of increasing mobility through the tax changes in equilibrium. In fact, the tax t_j^n can go up or down as firms become more mobile.[14] If the tax t_j^n increases with parameter α, country i's tax base will grow, reinforcing the positive direct effect on net revenues. Country i's 'basic' tax base would only erode if the tax t_j^n decreased with parameter α, thus counteracting the positive direct effect. But even in this case, the overall impact on net revenues remains positive, since the drop in the tax t_j^n in response to the rise in the number of relocating firms is only of secondary importance to the direct effect. The secondary fall in the number of relocations resulting from a decline in the tax t_j^n could not outweigh the initial increase in the number of relocating firms that caused the tax adjustment in the first place. All in all, the 'basic' tax base increases, i.e., $\partial F(\Delta t_{ji}; \alpha)/\partial\alpha + F'(\Delta t_{ji}; \alpha)(dt_j^n/d\alpha) > 0$, and so do net tax revenues.

As the focus of this paper is on the interaction between tax and subsidy competition, another natural question of interest is how increasing relocation mobility affects the opportunity costs of subsidy competition. These opportunity costs tell us how much net revenues governments lose when engaged in an 'integrated' tax and subsidy competition game in the first and second period (which generates net tax revenues NR_i), as opposed to two 'separated' tax competition games (which would generate tax revenues $R_i + \Upsilon_i$). Ignoring subsidy competition understates the loss from government competition, but this loss declines with mobility, as stated in proposition 2.

[14]This ambiguity reflects two counteracting effects. On the one hand, an increase in relocation mobility puts downward pressure on the tax t_j^n, since more domestic firms consider relocation. On the other hand, the potential influx of foreign firms tends to increase the tax t_i^m. This in turn pushes t_j^n up because the taxes t_j^n and t_i^m are strategic complements (i.e., $dt_j^n/dt_i^m > 0$).

Proposition 2 *Distribution effect of increasing relocation mobility continued.*
The opportunity costs C_i of subsidy competition decline with the firms' mobility parameter α. That is, $dC_i/d\alpha < 0$.
Proof: See appendix A. $\square$

The explanation for proposition 2 is straightforward. An increase in relocation mobility weakens the lock-in effect and thus the governments' ability to levy a tax t_i^n on domestic firms that exceeds the tax t_j^m these same firms face abroad. Thus, the tax 'top up' that governments impose on domestic firms compared to newly relocated foreign ones, i.e., the tax differential Δt_{ii}, declines in equilibrium.[15] The revenue differential $\Delta\Omega_{ii}$ and the opportunity costs C_i of subsidy competition decrease along with the tax differential Δt_{ii}, since the two differentials and the opportunity costs are all equal (see solutions (8) and (17)). Intuitively, countries simply gain less tax revenues from attracting investors in the first period, as firms become more likely to relocate. Hence, they are less willing to pay high subsidies in the first period, and subsidies and the opportunity costs of attracting international investors fall.

It seems obvious that a rise in net tax revenues occurs in conjunction with a decline in the opportunity costs of subsidy competition, as propositions 1 and 2 imply. But as we show next, this conjunction does not hold when it comes to the density effect.

4.2 Density effect

Having analysed the distribution effect, our attention turns to the density effect. That is, we now take account of the fact that marginal changes in relocation mobility can also affect the slope of the distribution function. The additional effects that arise if $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha \neq 0$ holds at the 'old' equilibrium level Δt_{ji} are stated in proposition 3.

Proposition 3 *Density effect of increasing relocation mobility.*
The positive distribution effect of a marginal rise in relocation mobility on net tax revenues is reinforced, while the decline in the opportunity costs of subsidy competition is counteracted, if $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha < 0$ holds. By contrast, the positive distribution effect on net tax revenues is counteracted, while the decline in the opportunity costs of subsidy competition is reinforced, if $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha > 0$ holds.

Proof: See appendix A. $\square$

[15] As mentioned above, the tax t_i^n can increase with the firms' mobility parameter α. Even in this case, the tax differential Δt_{ii} would decline, simply because the rise in the tax t_i^n would be less pronounced than the rise in the tax t_j^m.

The ambiguities of the density effect can be explained as follows. If density F' decreases (increases) with mobility parameter α, the firms' responses to tax increases become less (more) elastic. This causes a rise (decline) in tax t_j^n. Such a tax change increases (erodes) the 'basic' tax base $2F(\Delta t_{ji};\alpha)$ and thus basic revenues of country i. Formally, $dNR_i/d\alpha = 2t_i^m F'(\Delta t_{ji};\alpha)\left(dt_j^n/d\alpha\right) \gtreqless 0 \Leftrightarrow \partial F'\left(\Delta t_{ji};\alpha\right)/\partial\alpha \lesseqgtr 0$ results, where $dt_j^n/d\alpha$ is calculated for $\partial F\left(\Delta t_{ji};\alpha\right)/\partial\alpha = 0$ and $\partial F'\left(\Delta t_{ji};\alpha\right)/\partial\alpha \neq 0$ (in contrast to derivative (20), where $dt_j^n/d\alpha$ is calculated for $\partial F\left(\Delta t_{ji};\alpha\right)/\partial\alpha > 0$ and $\partial F'\left(\Delta t_{ji};\alpha\right)/\partial\alpha = 0$; see proofs for details).

In the case of the density effect, the same forces that raise (lower) net tax revenues also raise (lower) the opportunity costs of attracting international investors in the first place. That is, a positive (negative) density effect on net tax revenues is perfectly consistent with more (less) wasteful subsidy competition. The reason for this result is as follows. As discussed above, a smaller density F' implies less elastic tax bases and higher taxes, which in turn increase net tax revenues. Moreover, less elastic tax bases allow governments to charge domestic firms a higher tax 'top up'. That is, the tax differential Δt_{ii} increases, as does the revenue differential $\Delta\Omega_{ii}$, which in turn reinforces subsidy competition and drives up the opportunity costs of attracting international investors. Reversing this line of reasoning, a larger density F' reduces taxes and narrows the tax and revenue differentials. While the decline in taxes depresses revenues, the fall in the revenue differential reduces the losses through subsidy competition.

4.3 The overall effect

While the distribution effect is clear-cut, the density effect is ambiguous. This ambiguity simply mirrors that the value of the density function at the old equilibrium can go up or down as mobility increases. After all, an increase in the value of the distribution function $F(\Delta t_{ji};\alpha)$ for each tax differential $\Delta t_{ji} \in (0,\overline{m})$, which is a natural specification of what an increase in relocation mobility means, does not sufficiently restrict the possible changes in the density function. Despite the ambiguity, we can identify plausible patterns of how relocation mobility overall affects the equilibrium outcome.

We do so by considering the specific distribution function $F(m;\alpha) = (1+\alpha)m - \alpha m^2$ and the corresponding density function $F'(m;\alpha) = 1+\alpha-2\alpha m$, where $m \in [0,1]$ and $\alpha \in [-1,1]$. This quadratic distribution satisfies assumption 2 and provides a tractable example, which is also used by, e.g., Mongrain and Wilson (2018). It also satisfies the required properties regarding relocation mobility; that is, $\partial F(m;\alpha)/\partial\alpha = m(1-m) > 0$ for all $m \in (0,1)$ and $\partial F'(m;\alpha)/\partial\alpha = 1-2m \gtreqless 0 \Leftrightarrow m \lesseqgtr 0.5 =: \widetilde{m}$. Using this specification, we can show

Proposition 4 *Overall effect of increasing relocation mobility.*
(i) In general, the overall effect of a marginal increase in relocation mobility on the equilibrium outcome is ambiguous.
(ii) For the specification above, the overall effect of a marginal increase in relocation mobility on net tax revenues is positive (negative) if, and only if, the initial relocation mobility is below (above) the threshold level $\widehat{\alpha} = -0.089$. That is, $dNR_i/d\alpha \gtreqless 0 \Leftrightarrow \alpha \lesseqgtr \widehat{\alpha}$. Moreover, the opportunity costs of subsidy competition decline as relocation mobility increases. That is, $dC_i/d\alpha < 0$.

Proof: See appendix A. □

Inserting our specific distribution and density functions into equilibrium condition (7) and rearranging the resulting terms, we obtain $1-3(1+\alpha)\Delta t_{ji}+4\alpha\Delta t_{ji}^2 = 0$, which implicitly gives the equilibrium tax differential. Using this condition, it can be easily shown that the tax differential Δt_{ji} monotonically decreases, with $\Delta t_{ji} = 0.5 = \widetilde{m}$ and $\Delta t_{ji} = 0.191$ at the interval boundaries $\alpha = -1$ and $\alpha = 1$, respectively. Consequently, the derivative $\partial F(\Delta t_{ji};\alpha)/\partial\alpha = \Delta t_{ji}(1 - \Delta t_{ji}) > 0$ declines from 0.25 to 0.155 as mobility α increases and thus tax differential Δt_{ji} decreases, while $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha = 1 - 2\Delta t_{ji}$ rises from 0 to 0.618.

These changes in the derivatives $\partial F/\partial\alpha$ and $\partial F'/\partial\alpha$, in turn, reflect important shifts in the magnitudes of the distribution and density effects. First, increasing relocation mobility tends to weaken the positive distribution effect on net tax revenues, as indicated by the fall in $\partial F/\partial\alpha$. Second, the density effect, which vanishes for $\alpha = -1$ and thus $\partial F'(0.5;-1)/\partial\alpha = 0$, not only counteracts the revenue-increasing distribution effect for $\alpha > -1$ and thus $\partial F'/\partial\alpha > 0$ (see proposition 3), but tends to gain in strength as mobility increases. The overall effect on net tax revenues is positive (negative) if, and only if, mobility falls short of (exceeds) the threshold level $\widehat{\alpha}$. In contrast to this ambiguous outcome, the distribution effect and density effect both reduce the opportunity costs of subsidy competition.

In terms of observable variables, the following picture emerges (see proof of proposition 4 for details). Subsidy payments P_i decline as relocation mobility α rises, since attracting increasingly mobile firms becomes less beneficial. Interestingly, a marginal increase in relocation mobility also raises tax revenues R_i if the mobility level is sufficiently low. In this case, the revenue gains from foreign firms that relocate to country i more than compensate for the revenue losses from domestic firms that move to country j. Intuitively, although increasing mobility reduces the governments' ability to impose a tax 'top up' on domestic firms (i.e., the tax differential declines), governments can initially (i.e., for low mobility levels) raise the overall tax levels, with a larger share of relocating firms driving up taxes t_i^m and

t_j^m. Initially, countries thus benefit from an increase in relocation mobility through both lower subsidy payments and higher tax revenues.

For higher mobility levels, however, a rise in mobility intensifies tax competition and lowers tax revenues. Then, a growing number of relocating firms and a decline in taxes bite deeper and deeper into revenues as firms become even more mobile. However, any decrease in tax revenues falls short of the fall in subsidy payments as long as mobility is below the threshold level $\widehat{\alpha}$, and net tax revenues increase with relocation mobility. Only if mobility levels exceed $\widehat{\alpha}$ will the decline in tax revenues caused by a further increase in mobility become strong enough to dominate the fall in subsidy payments, thereby yielding lower net tax revenues. Overall, an inverse U-shaped relationship between net tax revenues and relocation mobility emerges. While countries initially benefit from increased mobility, they ultimately suffer.

We consider our example as particularly insightful, since it shows a range of outcomes and a clear pattern of the relationship between net tax revenues and mobility.[16] Key to the emergence of this pattern are preferential tax regimes and the resulting tax differentials. To see this point intuitively, consider the case without tax differentials (i.e., $t_i^n = t_i^m = t_i$). In this case, no relocation takes place in a symmetric equilibrium (i.e., $t_i = t_j$), irrespective of the degree of relocation mobility (i.e., $F(0;\alpha) = 0$ for all α). Thus, there is nothing like a distribution effect. However, there is still a density effect, which is now unambiguous because $\partial F'(m;\alpha)/\partial\alpha \gtreqless 0 \Leftrightarrow m \lesseqgtr \widetilde{m} \in (0,\overline{m})$ implies $\partial F'(0;\alpha)/\partial\alpha > 0$. In fact, in our example increasing mobility reduces the tax differential, thereby ensuring that at some point the distribution effect fades and the density effect comes to dominate.

The issue of the existence of the distribution effect reemerges in appendix B. There we analyse the effects of a decline in inter-country cost differentials Δz_{ij}, which can be interpreted as an increase in the initial location mobility. We show that such a decline intensifies subsidy competition and thus leads to lower net tax revenues, but leaves the opportunity costs of subsidy competition unaffected. In particular, we argue that the absence of a distribution effect, which is linked to the absence of subsidy differentials, drives the fall in net tax revenues (see appendix B).

[16] There are alternative specifications that produce a narrower result, with either the increasing or decreasing part of the inverse U-shaped relationship between net tax revenues and mobility missing. Consider, e.g., the distribution function $F(m;\alpha) = F(\alpha m)$, $m \in [0, \overline{m}(\alpha)]$ and $\alpha > 0$. In this example, the equilibrium tax differential is implicitly given by $\alpha\Delta t_{ij} = [1 - F(\alpha\Delta t_{ji})]/F'(\alpha\Delta t_{ji})$, implying that $\alpha\Delta t_{ij}$ does not vary with α. Thus, net tax revenues $NR_i = 2[F(\alpha\Delta t_{ji})]^2/[\alpha F'(\alpha\Delta t_{ji})] + [1/G(0)]$ decline with mobility α for all α, as the fall in tax revenues always dominates the decrease in subsidy payments. We are grateful to one of the referees for bringing up this example. In a previous version of this paper, we discuss an example in which net tax revenues always increase with mobility. Details will be provided on request.

4.4 Tax regimes and the effects of mobility

As preferential tax regimes and tax differentials turn out to be important for our analysis, it is worthwhile to discuss some similarities and differences between our results and those in papers on preferential tax regimes (e.g., Bucovetsky and Haufler, 2007, 2008; Haupt and Peters, 2005; Janeba and Smart, 2003; Keen, 2001; Mongrain and Wilson, 2018). Ignoring subsidy competition, these contributions consider tax competition when countries differentiate their taxes levied on domestic and foreign firms operating in their jurisdictions. They analyse whether limiting the differentials between the taxes paid by domestic and foreign firms can raise the countries' tax revenues compared to the outcome without any cap. According to Haupt and Peters (2005), the paper closest to our contribution, a moderate restriction on preferential tax regimes always increases revenues.

In our model, the distribution effect of an increase in relocation mobility raises net tax revenues. Since it also weakens the lock-in effect, the initial gap Δt_{ii} cannot be maintained any more and the tax differential declines in equilibrium. As long as the distribution effect is dominant, an increase in relocation mobility in our model leads to similar outcomes as an externally imposed cap on preferential tax regimes in Haupt and Peters (2005). Despite these similarities, key mechanisms are different.

In Haupt and Peters (2005), limiting preferential tax regimes curbs reciprocal dumping. That means—translated into our model—that the tax t_i^m on foreign firms goes up for sure, leading to a drop in the share of relocating firms and a rise in tax revenues from domestic firms (i.e., firms which were initially set up in the country considered). This increase in revenues more than compensates for the potential loss of tax revenues from foreign firms, thus raising overall tax revenues. In our model, by contrast, additional tax payments of domestic firms in the second period are not relevant, since they are completely offset by higher subsidies in the first period. Hence, the interaction between tax and subsidy competition, on which we focus, and the resulting WYGIWYG principle eliminate an effect central to the conclusions in Haupt and Peters (2005). Key to our results is the positive direct distribution effect of an increase in relocation mobility (i.e., the induced rise in net revenues for given taxes). This effect, in turn, has no counterpart in Haupt and Peters (2005), who do not consider changes in mobility.

As this brief discussion illustrates, some effects are paramount in our model, but irrelevant in Haupt and Peters (2005), and vice versa. Nevertheless, the papers on restricting preferential tax regimes and our paper can mutually enrich each other. For instance, analysing tax and subsidy competition together might weaken the argument in favour of restricting preferential tax regimes, as any increase in tax revenues from domestic firms might be offset by higher subsidy payments. Also, restricting preferential tax regimes might change the implications of increasing relocation mobility on net tax revenues. As our discussion in section 4.3 implies, the positive distribution effect might fade if the tax differential is capped, and the overall effect might turn negative.

4.5 Repeated relocation choices

Up to this point, we have assumed that firms can relocate only once after the initial set-up period. Obviously, this assumption is a crude simplification, since firms can repeatedly reconsider their location choice in their usual life spans.[17] To get an idea of whether our results remain valid once we allow for repeated relocation over time, let us thus extend the number of periods in our model from two to K, with $K > 2$. In addition to the initial set-up period (period 1) and the final period (now period K), we now have intermediate periods (periods 2 to $K-1$), and firms can relocate in each of the periods 2 to K.

The distribution function $G(\Delta z_{ij})$ still describes country-specific set-up costs in the first period, while the distribution function $F_k(m;\alpha_k)$ characterises the relocation costs in period k, $k = 2, ..., K$. The subscript k is added, since the distribution of relocation costs can vary over the firms' life span. Other than extending the number of periods to K, we leave the assumptions of our model unchanged. In particular, assumption 2 still applies to each function $F_k(m;\alpha_k)$. Also, government i still grants subsidy s_i in the initial set-up period and levies taxes $t^n_{i,K}$ and $t^m_{i,K}$ in the final period. In the intermediate periods, firms pay taxes $v^n_{i,k}$ or $v^m_{i,k}$, depending on whether a firm was already located in country i in period $k-1$ or whether it relocates to country i in period k. Governments aim at maximising net tax revenues as before.

Following the line of reasoning in section 3, we arrive at the equilibrium taxes and subsidies of the extended system:

$$t^n_{i,K} = \frac{1 - F_K(t^n_{i,K} - t^m_{j,K})}{F'_K(t^n_{i,K} - t^m_{j,K})} \quad \text{and} \quad t^m_{i,K} = \frac{F_K(t^n_{j,K} - t^m_{i,K})}{F'_K(t^n_{j,K} - t^m_{i,K})}, \tag{21}$$

$$v^n_{i,k} = t^n_{i,k} - \Delta\Omega_{ii,k+1}, \quad \text{where} \quad t^n_{i,k} = \frac{1 - F_k(t^n_{i,k} - t^m_{j,k})}{F'_k(t^n_{i,k} - t^m_{j,k})}, \tag{22}$$

$$v^m_{i,k} = t^m_{i,k} - \Delta\Omega_{ii,k+1}, \quad \text{where} \quad t^m_{i,k} = \frac{F_k(t^n_{j,k} - t^m_{i,k})}{F'_k(t^n_{j,k} - t^m_{i,k})}, \tag{23}$$

$$s_i = \Delta\Omega_{ii,2} - \tau_i, \quad \text{where} \quad \tau_i = \frac{1}{2G'(0)}. \tag{24}$$

[17] For instance, consider the example of Nokia, which moved its production from Bochum, Germany, to Cluj, Romania, in 2008, as discussed in section 1. The Finnish firm closed the Cluj factory, which was replaced by Asian plants, only three years later (Financial Times, 2011).

Consistent with our previous notation, the definition $\Delta\Omega_{ii,k+1} := \Omega^n_{i,k+1} - \Omega^m_{i,k+1} = t^n_{i,k+1}\left[1 - F_{k+1}(t^n_{i,k+1} - t^m_{j,k+1})\right] - t^m_{i,k+1}F_{k+1}(t^n_{j,k+1} - t^m_{i,k+1})$ is used. As the equilibrium solution is again symmetric, i.e., $t^n_{i,k} = t^n_{j,k}$ and $t^m_{i,k} = t^m_{j,k}$, the revenue differential exactly equals the tax differential in each period, i.e., $\Delta\Omega_{ii,k} = t^n_{i,k} - t^m_{i,k} =: \Delta t_{ii,k} = v^n_{i,k} - v^m_{i,k} > 0$.

The new equilibrium conditions for the taxes in the last period and the subsidy in the first period, (21) and (24), are identical to those in section 3, (6) and (15), but for the time subscript, and so are the explanations for these results. Taxes $v^n_{i,k}$ and $v^m_{i,k}$ consist of two components, as does subsidy s_i. First, there are the taxes $t^n_{i,k}$ and $t^m_{i,k}$. These taxes are hypothetical taxes in the periods 2 to $K-1$, since they would only be levied in these periods if there were no ensuing period, or if a one-period tax competition game were played in each of these periods. Second, there is the revenue differential $\Delta\Omega_{ii,k+1}$. It again captures the revenue gain that arises in the ensuing period through attracting firms in the current period. The actual taxes $v^n_{i,k}$ and $v^m_{i,k}$ result from 'correcting' the hypothetical taxes by the amount of the revenue differential in very much the same way that subsidy s_i follows from 'cutting' the hypothetical tax τ_i by the revenue differential $\Delta\Omega_{ii,2}$. Only, in periods 2 to K, the governments can apply preferential regimes, since they can differentiate between domestic and foreign firms based on the location of firms in the previous period. As discussed in section 3, subsidy s_i will 'truly' be a subsidy only if the initial location mobility is sufficiently high. Otherwise, it would be a tax (i.e., s_i would be negative). Similarly, taxes $v^n_{i,k}$ and $v^m_{i,k}$ will 'truly' be taxes only if the mobility is sufficiently low. Otherwise, they would be subsidies (i.e., $v^n_{i,k}$ and $v^m_{i,k}$ would be negative).

Using equilibrium taxes and subsidies (21) to (24), we can calculate the net tax revenues:

$$NR_i = \underbrace{2t^m_{i,K}F(t^n_{j,K} - t^m_{i,K})}_{\text{basic rev period } K} + \underbrace{2\sum_{k=2}^{K-1}\left[t^m_{i,k}F(t^n_{j,k} - t^m_{i,k})\right]}_{\text{hypo basic rev period 2 to } K-1} + \underbrace{\tau_i}_{\text{hypo tax rev period 1}} \quad . \tag{25}$$

The similarity between net tax revenues (19) and (25) is striking. The first term and the third term on the right-hand side of expression (25) are identical to the terms on the right-hand side of expression (19) and need no further explanation. The second term on the right-hand side of (25) is the new component in the case of K periods and captures the hypothetical basic revenues in the intermediate periods 2 to $K-1$. These revenues are 'hypothetical' and 'basic' in the same sense as the hypothetical tax revenues in the first period are hypothetical (i.e., they do not literally materialise) and the basic revenues in the final period are basic (i.e., they do not include the revenue differential).

Fundamentally, introducing intermediate periods does not alter the WYGIWYG principle. Government i continues to pay a subsidy s_i in the first period. In every intermediate period k, government i levies taxes $v_{i,k}^n$ and $v_{i,k}^m$ that fall short by $\Delta\Omega_{ii,k+1}$ of the taxes $t_{i,k}^n$ and $t_{i,k}^m$ it would charge if this period were the last one. The resulting opportunity costs of keeping firms at home or attracting new firms in period 1 (2, 3, ... $K-1$) exactly offset the generated revenue differentials in the immediately ensuing period 2 (3, 4, ... K). Each country is left with the hypothetical tax revenues in the initial period and the hypothetical or real basic revenues in the following periods. In line with our previous analysis, all that matters when it comes to increasing relocation mobility in any period 2 to K is the impact on basic revenues in this period. Given the similarity between the two-period case and the K-period case, it is not surprising that extending the model to K periods leaves our key conclusions unaltered, as stated in proposition 5.

Proposition 5 *Increasing mobility in the K-period case.*
Consider the K-period extension of our model as specified above. Then, an increase in the relocation mobility parameter α_k in any period k has qualitatively the very same effects on net tax revenues NR_i and opportunity costs C_i of subsidy competition as an increase in the mobility parameter α in the two-period model.

Proof: See appendix A. □

5 Concluding remarks

Governments compete for firms in both subsidies and taxes. We have analysed the resulting interplay between subsidy and tax competition, leading to the WYGIWYG principle. That is, the additional revenues generated by attracting firms through subsidies are exactly offset by the opportunity costs of subsidy competition. This result has helped us to shed some light on the impact of an increase in relocation mobility on net tax revenues, thereby distinguishing between the distribution and density effects.

The key conclusion is that a rise in relocation mobility increases net tax revenues under plausible conditions. Higher relocation mobility reinforces tax competition, but weakens subsidy competition. Overall, the fall in subsidy payments can overcompensate for the decline in tax revenues, yielding higher net tax revenues. This outcome seems to be particularly likely if the initial relocation mobility is low and tax differentials are substantial. By contrast, if the initial mobility is already high and tax differentials are small, an increase in relocation mobility seems more likely to reduce net tax revenues. In any case, an increase in net tax revenues needs not go hand in hand with a decline in the opportunity costs of subsidy competition, but it tends to do so.

Our conclusions qualify the common belief that increasing mobility erodes national revenues—a belief that is backed by 'pure' tax competition models (see for a review, e.g., Genschel and Schwartz, 2011, and Keen and Konrad, 2013). Notably, our contrasting results are derived in a 'conventional' tax competition framework, but in one that is supplemented by subsidy competition stages. The model's prediction that subsidy payments fall over time as a result of decreasing relocation costs is in line with the (anecdotal) evidence of recent years. A decline in subsidy payments also reflects the expectation of the industry, as explored in the introduction.

Our findings have important policy implications. They indicate that fiercer tax competition (here, due to an increase in relocation mobility) might be advantageous to governments because of its feedback effect on subsidy competition. In the public debate, however, the focus is on measures that weaken tax competition. In our model, such measures intensify subsidy competition, with potentially adverse effects on net tax revenues. So an exclusive concentration on tax cooperation might be misleading and thus detrimental to future revenues, for reasons different from those previously discussed in the literature (see, e.g., Becker and Fuest, 2010, and Han et al., 2017, who analyse partial cooperation when countries decide on taxes and infrastructure). Instead, more attention should be paid to subsidy competition, since reducing it might indeed be a more successful avenue for larger tax revenues than restricting tax competition.

As discussed in section 4, exploring how limitations on preferential tax regimes affect our conclusions could be an interesting extension of our analysis.[18] Going one step further, we could endogenise relocation mobility. As briefly indicated in section 1, relocation costs are at least partly driven down by political decisions, such as the creation of the European single market. Also, firms themselves influence their relocation mobility.

Appendix A

Proof of lemma 1 We start by analysing the tax competition equilibrium (third and fourth stage). As argued above, this equilibrium is independent of the governments' subsidies (first stage) and the investors' initial set-up choices (second stage). In step 1, we determine the relevant interval in which the equilibrium taxes lie.

[18] See the papers on preferential tax regimes referred to in section 4 and also Krieger and Lange (2010) for a discussion of the implications of preferential and non-preferential regimes in the context of student and graduate mobility.

Uniqueness and existence of the tax competition equilibrium are proved in step 2. In step 3, we show that our lines of reasoning can easily be repeated to prove existence and uniqueness of the subsidy competition equilibrium, and thus of the subgame-perfect equilibrium.

Step 1 (Relevant equilibrium tax interval) With governments maximising tax revenues $R_i = \Omega_i^n(t_i^n; t_j^m)N_i + \Omega_i^m(t_i^m; t_j^n)N_j$ in the third stage, non-positive taxes can never occur in equilibrium. Formally, $t_i^n < 0$ and $t_j^m < 0$ imply $\Omega_i^n(t_i^n; t_j^m) < 0$ or $\Omega_j^m(t_j^m; t_i^n) < 0$, $t_i^n < 0$ and $t_j^m \geq 0$ imply $\Omega_i^n(t_i^n; t_j^m) < 0$, and $t_i^n \geq 0$ and $t_j^m < 0$ imply $\Omega_j^m(t_j^m; t_i^n) < 0$. The choice $t_i^n < 0$ ($t_j^m < 0$) cannot occur in equilibrium, since governments can always set $t_i^n = 0$ ($t_j^m = 0$), yielding $\Omega_i^n(t_i^n; t_j^m) = 0$ ($\Omega_j^m(t_j^m; t_i^n) = 0$). Then, $t_i^n = 0$ cannot be an equilibrium either, as governments can always choose some $t_i^n \in \left(0, t_j^m + \overline{m}\right)$ for $t_j^m \geq 0$ and achieve $\Omega_i^n(t_i^n; t_j^m) > 0$. In this case, $t_j^m = 0$ cannot be an equilibrium either, since governments can always levy some $t_j^m \in (0, t_i^n)$ for $t_i^n > 0$ and get $\Omega_j^m(t_j^m; t_i^n) > 0$. Hence, we can focus on positive taxes, i.e., $t_A^n, t_B^n, t_A^m, t_B^m > 0$. Also, we consider the relevant case of $N_i > 0$ and $N_j > 0$ and prove in step 3 that these inequalities are indeed satisfied in equilibrium.

To analyse the relevant tax interval further, we differentiate tax revenues (3), which yields

$$\frac{\partial R_i}{\partial t_i^n} = \begin{cases} N_i & \text{if} \quad t_i^n < t_j^m \\ \left[1 - F(t_i^n - t_j^m) - t_i^n F'(t_i^n - t_j^m)\right] N_i & \text{if} \quad t_i^n \in \left[t_j^m, t_j^m + \overline{m}\right] \\ 0 & \text{if} \quad t_i^n > t_j^m + \overline{m} \end{cases}, \tag{26}$$

$$\frac{\partial R_i}{\partial t_i^m} = \begin{cases} N_j & \text{if} \quad t_i^m < t_j^n - \overline{m} \\ \left[F(t_j^n - t_i^m) - t_i^m F'(t_j^n - t_i^m)\right] N_j & \text{if} \quad t_i^m \in \left[t_j^n - \overline{m}, t_j^n\right] \\ 0 & \text{if} \quad t_i^m > t_j^n \end{cases}. \tag{27}$$

From derivative (26) follows that $\partial R_i/\partial t_i^n|_{t_i^n < t_j^m} = N_i > 0$ and thus, as function R_i is continuous, $R_i^n|_{t_i^n = t_j^m > 0} > R_i^n|_{t_i^n \in (0, t_j^m)} > \Omega_i^m N_j = R_i^n|_{t_i^n = 0}$, and that $\partial R_i/\partial t_i^n|_{t_i^n = t_j^m} \gtreqless 0 \Leftrightarrow t_j^m F'(0) \lesseqgtr 1$. Also, $R_i^n|_{t_i^n \in (0, t_j^m + \overline{m})} > \Omega_i^m N_j = R_i^n|_{t_i^n \geq t_j^m + \overline{m}} = R_i^n|_{t_i^n = 0}$ holds. Then, the optimal tax t_i^n lies in the interval $\left(t_j^m, t_j^m + \overline{m}\right)$ for $t_j^m F'(0) < 1$ and equals t_j^m for $t_j^m F'(0) \geq 1$. For $t_j^m F'(0) < 1$, the first-order condition

$$\frac{\partial R_i}{\partial t_i^n} = \left\{\left[1 - F(t_i^n - t_j^m)\right] - t_i^n F'(t_i^n - t_j^m)\right\} N_i = 0 \tag{28}$$

implicitly defines the governments' continuous reaction functions, since the second-order condition

$$\frac{\partial^2 R_i}{\partial (t_i^n)^2} = -\left[2F'(t_i^n - t_j^m) + \frac{\left[1 - F(t_i^n - t_j^m)\right] F''(t_i^n - t_j^m)}{F'(t_i^n - t_j^m)}\right] N_i < 0 \tag{29}$$

is satisfied according to assumption 2 (ii), and since F is a twice continuously differentiable function. Thus, the best responses are implicitly given by condition (4).

Analogously, from eq. (27) follows that $\partial R_i/\partial t_i^m|_{t_i^m<t_j^n-\overline{m}} = N_j > 0$ and thus, as function R_i is continuous, $R_i^n|_{t_i^m=t_j^n-\overline{m}>0} > R_i^n|_{t_i^m\in(0,t_j^n-\overline{m})} > \Omega_i^n N_i = R_i^n|_{t_i^m=0}$, and that $\partial R_i/\partial t_i^m|_{t_i^m=t_j^n-\overline{m}} \gtreqless 0 \Leftrightarrow \left(t_j^n-\overline{m}\right) F'(\overline{m}) \lesseqgtr 1$. Moreover, $R_i^n|_{t_i^m\in(0,t_j^n)} > \Omega_i^n N_i = R_i^n|_{t_i^m\geq t_j^n} = R_i^n|_{t_i^m=0}$ holds. Thus, the optimal tax t_i^m lies in the interval $\left(0,t_j^n\right)$ for $\left(t_j^n-\overline{m}\right) F'(\overline{m}) < 1$ and equals $t_j^n-\overline{m}$ for $\left(t_j^n-\overline{m}\right) F'(\overline{m}) \geq 1$. For $\left(t_j^n-\overline{m}\right) F'(\overline{m}) < 1$, the first-order condition

$$\frac{\partial R_i}{\partial t_i^m} = \left[F(t_j^n-t_i^m) - t_i^m F'(t_j^n-t_i^m)\right] N_j = 0 \tag{30}$$

implicitly defines the governments' continuous reaction functions, since the second-order condition

$$\frac{\partial^2 R_j}{\partial (t_j^m)^2} = \left[-2F'(t_j^n-t_i^m) + \frac{F(t_j^n-t_i^m)F''(t_j^n-t_i^m)}{F'(t_j^n-t_i^m)}\right] N_i < 0, \tag{31}$$

is fulfilled according to assumption 2 (i), and since F is a twice continuously differentiable function. Thus, the best responses are implicitly given by condition (5).

From conditions (4) and (5), we can conclude that $t_i^n \in \left[t_j^m, t_j^m+\overline{m}\right)$ and $t_j^m \in [t_i^n-\overline{m}, t_i^n)$. Hence, any equilibrium satisfies $t_i^n \in \left(t_j^m, t_j^m+\overline{m}\right)$ (or, equivalently, $t_j^m \in (t_i^n-\overline{m}, t_i^n)$) and is thus characterised by conditions (6) and (7). (We implicitly assume that the firms' gross returns π are sufficiently large so that they do not constrain government taxation, and that the density function is finite for $m=0$.)

Step 2 (Existence and uniqueness) Having shown that $t_i^n \in \left(t_j^m, t_j^m+\overline{m}\right)$ holds in equilibrium, we next prove that a solution to condition (7), or equivalently to condition $\Delta t_{ij} - \left[1-2F(\Delta t_{ij})\right]/F'(\Delta t_{ij}) = 0$, and thus to condition (6) exists and is unique. To this end, we differentiate the term $\left[1-2F(\Delta t_{ij})\right]/F'(\Delta t_{ij}) =: \Phi(\Delta t_{ij})$ with respect to Δt_{ij}, leading to

$$\frac{\partial \Phi(\Delta t_{ij})}{\partial \Delta t_{ij}} < 0 \Leftrightarrow F''(\Delta t_{ij}) > -2\frac{\left[F'(\Delta t_{ij})\right]^2}{1-2F(\Delta t_{ij})} \tag{32}$$

for $F(\Delta t_{ij}) \in [0, 0.5] \Leftrightarrow \Delta t_{ij} \in [0, m^{crit}]$, where m^{crit} is defined as $m^{crit} : F(m^{crit}) = 0.5$ with $m^{crit} > 0$. Furthermore, inequality $F''(\Delta t_{ij}) > -2\left[F'(\Delta t_{ij})\right]^2/\left[1-F(\Delta t_{ij})\right]$ is satisfied (see assumption 2 (ii)), and inequality $-2\left[F'(\Delta t_{ij})\right]^2/\left[1-F(\Delta t_{ij})\right] \geq -2\left[F'(\Delta t_{ij})\right]^2/\left[1-2F(\Delta t_{ij})\right]$ is also fulfilled for $\Delta t_{ij} \in [0, m^{crit}]$. Thus, $F''(\Delta t_{ij}) > -2\left[F'(\Delta t_{ij})\right]^2/\left[1-2F(\Delta t_{ij})\right]$ indeed results for $\Delta t_{ij} \in [0, m^{crit}]$, and $\Phi(\Delta t_{ij})$ continuously declines with Δt_{ij} in the interval $[0, m^{crit}]$.

Also, we know that $\Phi(0) = 1/F'(0) > 0$, $\Phi(m^{crit}) = 0$, and, for $\Delta t_{ij} \in (m^{crit}, \overline{m}]$, $\Phi(\Delta t_{ij}) < 0$ hold. As a result, the term $\Delta t_{ij} - \Phi(\Delta t_{ij})$ continuously increases with Δt_{ij} in the interval $[0, m^{crit}]$, with $[\Delta t_{ij} - \Phi(\Delta t_{ij})]|_{\Delta t_{ij}=0} < 0$ and $[\Delta t_{ij} - \Phi(\Delta t_{ij})]|_{\Delta t_{ij} \geq m^{crit}} > 0$. Given these properties, the intermediate value theorem implies that a solution Δt_{ij} to condition $\Delta t_{ij} - \Phi(\Delta t_{ij}) = 0$ (or, equivalently, to condition (7)) exists and is unique, with $\Delta t_{ij} \in (0, m^{crit})$. Then, equilibrium taxes $t_A^n = t_B^n$ and $t_A^m = t_B^m$ exist and are uniquely determined by (6).

Step 3 (Subsidy competition and subgame-perfect equilibrium) The first-order conditions (12) and (14) implicitly characterise the governments' reaction functions for $s_i \in \left(s_j - \overline{\Delta z_{ij}}, s_j + \overline{\Delta z_{ij}}\right)$, since the second-order conditions

$$\begin{aligned}
\frac{\partial^2 NR_i}{\partial s_i^2} &= -4G'(s_i - s_j) + 2\frac{G(s_i - s_j)}{G'(s_i - s_j)}G''(s_i - s_j) < 0, & (33)\\
\frac{\partial^2 NR_i}{\partial s_i^2} &= -4G'(s_i - s_j) - 2\frac{1 - G(s_i - s_j)}{G'(s_i - s_j)}G''(s_i - s_j) < 0 & (34)
\end{aligned}$$

are satisfied under assumption 1. We can indeed show that equilibrium subsidies can only lie in the interval $\left(s_j - \overline{\Delta z_{ij}}, s_j + \overline{\Delta z_{ij}}\right)$. As the proof is in the same vein as our proof in step 1, we omit the proof this time, but provide details upon request.

The first-order conditions (12) and (14) imply $\Delta s_{ij} - [1 - 2G(\Delta s_{ij})] / G'(\Delta s_{ij}) = 0$ in equilibrium, where $\Delta s_{ij} = s_i - s_j$. Obviously, as $G(0) = 0.5$, the equality $\Delta s_{ij} - [1 - 2G(\Delta s_{ij})] / G'(\Delta s_{ij}) = 0$ is satisfied for $\Delta s_{ij} = 0$, yielding equilibrium subsidies (15). We are left to show that the solution $\Delta s_{ij} = 0$ is unique. Also, $\Psi := [1 - 2G(\Delta s_{ij})] / G'(\Delta s_{ij}) \gtreqless 0 \Leftrightarrow \Delta s_{ij} \lesseqgtr 0$, since $G(\Delta s_{ij}) \lesseqgtr 0.5 \Leftrightarrow \Delta s_{ij} \lesseqgtr 0$. Thus, $\Delta s_{ij} - [1 - 2G(\Delta s_{ij})] / G'(\Delta s_{ij}) \lesseqgtr 0 \Leftrightarrow \Delta s_{ij} \lesseqgtr 0$, which confirms that the solution $\Delta s_{ij} = 0$ and the corresponding equilibrium subsidies are unique. The symmetric solution $s_i = s_j$ implies $N_i = N_j = 1$.

Consequently, we can conclude that (i) a subgame-perfect equilibrium exists and is unique, (ii) equilibrium taxes and subsidies are characterised by (6), (7) and (15), and (iii) $N_i = N_j = 1$ results.

Proof of propositions 1, 2 and 3

Preliminary results Inserting the optimal taxes (6) and the hypothetical taxes (as stated in equilibrium solution (15)) into the net tax revenues (19) and rearranging to resulting terms lead to

$$NR_i = 2\frac{F^2(\Delta t_{ji}; \alpha)}{F'(\Delta t_{ji}; \alpha)} + \frac{1}{2G'(0)}. \tag{35}$$

Differentiating net tax revenues (35) with respect to mobility parameter α yields

$$\frac{dNR_i}{d\alpha} = \frac{\partial NR_i}{\partial \alpha} + \frac{\partial NR_i}{\partial \Delta t_{ji}}\frac{d\Delta t_{ji}}{d\alpha}. \tag{36}$$

The components of this derivative are given by

$$\frac{\partial NR_i}{\partial \alpha} = 2\frac{2F'(\Delta t_{ji};\alpha)F(\Delta t_{ji};\alpha)\frac{\partial F(\Delta t_{ji};\alpha)}{\partial \alpha} - [F(\Delta t_{ji};\alpha)]^2 \frac{\partial F'(\Delta t_{ji})}{\partial \alpha}}{[F'(\Delta t_{ji};\alpha)]^2}, \tag{37}$$

$$\frac{\partial NR_i}{\partial \Delta t_{ji}} = 2\frac{2[F'(\Delta t_{ji};\alpha)]^2 F(\Delta t_{ji};\alpha) - [F(\Delta t_{ji};\alpha)]^2 F''(\Delta t_{ji};\alpha)}{[F'(\Delta t_{ji};\alpha)]^2}, \tag{38}$$

$$\frac{d\Delta t_{ji}}{d\alpha} = -\frac{2F'(\Delta t_{ji};\alpha)\frac{\partial F(\Delta t_{ji};\alpha)}{\partial \alpha} + [1-2F(\Delta t_{ji};\alpha)]\frac{\partial F'(\Delta t_{ji};\alpha)}{\partial \alpha}}{[F'(\Delta t_{ji};\alpha)]^2 [3+\rho]}, \tag{39}$$

where

$$\rho = \frac{\Delta t_{ji} F''(\Delta t_{ji};\alpha)}{F'(\Delta t_{ji};\alpha)} = \frac{[1-2F(\Delta t_{ji};\alpha)] F''(\Delta t_{ji};\alpha)}{[F'(\Delta t_{ji};\alpha)]^2} \tag{40}$$

is the elasticity of the density function $F'(\Delta t_{ji};\alpha)$ with respect to changes in the tax differential Δt_{ji}, evaluated at the equilibrium. Note that derivative (39) follows from tax differential (7) and the associated comparative statics: $d\Delta t_{ji}/d\alpha = -(\partial\kappa/\partial\alpha)/(\partial\kappa/\partial\Delta t_{ji})$, where $\kappa(\Delta t_{ji};\alpha) := \Delta t_{ji} - [1-2F(\Delta t_{ji};\alpha)]/F'(\Delta t_{ji};\alpha)$ and $\partial\kappa/\partial\Delta t_{ji} = 3+\rho$.

We can prove propositions 1, 2 and 3 in a more convenient and shorter manner by making use of derivatives (36)-(39) instead of derivative (20) and the comparative statics that leads to $dt^n/d\alpha$ (which can be provided upon request).

Proposition 1 We analyse $dNR_i/d\alpha$ for $\partial F(\Delta t_{ji};\alpha)/\partial\alpha > 0$ and $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha = 0$ at the equilibrium value $\Delta t_{ji} > 0$. With $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha = 0$, derivatives (37) and (39) simplify considerably. To prove proposition 1, we insert derivatives (37), (38) and (39) into derivative (36) and rearrange the resulting terms (using eq. (40)):

$$\frac{dNR_i}{d\alpha} = 4\frac{F(\Delta t_{ji};\alpha)\frac{\partial F(\Delta t_{ji};\alpha)}{\partial\alpha}}{F'(\Delta t_{ji};\alpha)(3+\rho)}\left[1 + \frac{[1-F(\Delta t_{ji};\alpha)] F''(\Delta t_{ji};\alpha)}{[F'(\Delta t_{ji};\alpha)]^2}\right] > 0 \tag{41}$$

The terms $F(\Delta t_{ji};\alpha)$, $F'(\Delta t_{ji};\alpha)$ and $\partial F(\Delta t_{ji};\alpha)/\partial\alpha$ are all positive for $\Delta t_{ji} > 0$. Also, the inequality $F''(\Delta t_{ji};\alpha) > -[F'(\Delta t_{ji};\alpha)]^2/[1-F(\Delta t_{ji};\alpha)]$ (see assumption 2 (ii)) implies that the sign of the terms in the square brackets is also positive, and that the inequality $3+\rho > 3-[1-2F(\Delta t_{ji};\alpha)]/[1-F(\Delta t_{ji};\alpha)] > 2$ is fulfilled, where eq. (40) is used. Hence, the derivative (41) is positive, too.

Proposition 2 From eqs. (8) and (17), $C_i = \Delta\Omega_{ii} = \Delta t_{ii} = \Delta t_{ji}$ follows in equilibrium, and thus $dC_i/d\alpha = d\Delta t_{ji}/d\alpha$. Using derivative (39), we then get

$$\frac{d\Delta t_{ji}}{d\alpha} = -\frac{2F'(\Delta t_{ji};\alpha)\frac{\partial F(\Delta t_{ji};\alpha)}{\partial\alpha}}{[F'(\Delta t_{ji};\alpha)]^2(3+\rho)} = \frac{dC_i}{d\alpha} < 0 \tag{42}$$

for $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha = 0$. The inequality $dC_i/d\alpha < 0$ holds because $F'(\Delta t_{ji};\alpha)$, $\partial F(\Delta t_{ji};\alpha)/\partial\alpha$ and $3+\rho$ are all positive, with $3+\rho > 2$ as shown in the proof of proposition 1.

Proposition 3 To calculate the additional impact of an increase in mobility parameter α on net revenues NR_i that arises if $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha \neq 0$, we evaluate derivatives (37) and (39) for $\partial F(\Delta t_{ji};\alpha)/\partial\alpha = 0$ and $\partial F'(\Delta t_{ji};\alpha)/\partial\alpha \neq 0$ at the equilibrium value $\Delta t_{ji} > 0$. Inserting again derivatives (37)-(39) into derivative (36) yields, after some rearrangements,

$$\frac{dNR_i}{d\alpha} = -2\frac{\frac{\partial F'}{\partial\alpha}}{(F')^2}\left[F^2 + \left(\frac{2(F')^2 F - F^2F''}{(F')^2}\right)\left(\frac{1-2F}{3+\rho}\right)\right] \gtreqless 0, \tag{43}$$

where we suppress the functions' argument Δt_{ji} and parameter α. Assumption 2 (i), i.e., $F'' < (F')^2/F$, implies that $2(F')^2 F - F^2F'' > 0$ holds. Also, assumption 2 (ii), i.e., $F'' > -(F')^2/(1-F)$, implies that the inequality $3+\rho > 3-[(1-2F)/(1-F)] > 2$ is satisfied, with eq. (40) being used. Finally, $F < 0.5$ and thus $1-2F > 0$ hold in equilibrium (see tax differential (7) and the explanations in footnotes 10 and 11). Thus, all terms in the square brackets are positive, resulting in

$$\frac{dNR_i}{d\alpha} \gtreqless 0 \Leftrightarrow \frac{\partial F'(\Delta t_{ji};\alpha)}{\partial\alpha} \lesseqgtr 0. \tag{44}$$

Also, using derivative (39) and following the line of the proof of proposition 2, we get

$$\frac{d\Delta t_{ji}}{d\alpha} = -\frac{[1-2F(\Delta t_{ji};\alpha)]\frac{\partial F'(\Delta t_{ji};\alpha)}{\partial\alpha}}{[F'(\Delta t_{ji};\alpha)]^2[3+\rho]} = \frac{dC_i}{d\alpha} \gtreqless 0 \Leftrightarrow \frac{\partial F'(\Delta t_{ji};\alpha)}{\partial\alpha} \lesseqgtr 0 \tag{45}$$

for $\partial F(\Delta t_{ji};\alpha)/\partial\alpha = 0$. The inequality signs result from $1-2F > 0$ and $3+\rho > 0$ (see above). Together, inequalities (44) and (45) prove proposition 3.

Proof of Proposition 4 Part (i): Derivatives (36)-(39) imply that the total effect of an increase in relocation mobility on the equilibrium outcome can be decomposed into a distribution effect and a density effect. The ambiguity of the overall effect stems from the ambiguity of the distribution effect (see proposition 3). As we show below, an increase in relocation mobility can indeed raise or lower net tax revenues.

Part (ii): Inserting our specific distribution and density functions $F(m;\alpha) = (1+\alpha)m - \alpha m^2$ and $F'(m;\alpha) = 1+\alpha-2\alpha m$ with $m = \Delta t_{ji}$ into eqs. (35), (37) and (38), we obtain the specific net tax revenues

$$NR_i = 2\frac{\left[(1+\alpha)\Delta t_{ji} - \alpha\Delta t_{ji}^2\right]^2}{1+\alpha-2\alpha\Delta t_{ji}} + \frac{1}{2G'(0)} \tag{46}$$

and, after some rearrangements, the corresponding partial derivatives

$$\frac{\partial NR_i}{\partial \alpha} = 2\Delta t_{ji}^2 \frac{(1+\alpha-\alpha\Delta t_{ji})\left(1+\alpha-3\alpha\Delta t_{ji}+2\alpha\Delta t_{ji}^2\right)}{(1+\alpha-2\alpha\Delta t_{ji})^2}, \tag{47}$$

$$\frac{\partial NR_i}{\partial \Delta t_{ji}} = 4\Delta t_{ji} \frac{(1+\alpha-\alpha\Delta t_{ji})\left[(1+\alpha)^2-3\alpha(1+\alpha)\Delta t_{ji}+3\alpha^2 t_{ji}^2)\right]}{(1+\alpha-2\alpha\Delta t_{ji})^2}. \tag{48}$$

The corresponding equilibrium tax differential is implicitly defined by

$$\Xi(\Delta t_{ji}) := 1-3(1+\alpha)\Delta t_{ji}+4\alpha\Delta t_{ji}^2 = 0 \quad \Leftrightarrow \quad \alpha = \frac{1-3\Delta t_{ji}}{3\Delta t_{ji}-4\Delta t_{ji}^2}, \tag{49}$$

which follows from inserting our distribution and density functions into solution (7) and rearranging the resultant condition $\Delta t_{ji} = \left\{1-2\left[(1+\alpha)\Delta t_{ji}-\alpha\Delta t_{ji}^2\right]\right\} / (1+\alpha-2\alpha\Delta t_{ji})$. As the specific distribution satisfies assumption 2 (details will be provided upon request), the equilibrium condition (49) has a unique solution Δt_{ji} in the interval $(0,\overline{m})$, with $\overline{m}=1$ in our example (see proof of lemma 1). Alternatively, note that the continuous function $\Xi(\Delta t_{ji})$ is either linear in Δt_{ji} (for $\alpha=0$) or quadratic (for $\alpha \neq 0$). Thus, as $\Xi(0)=1>0$ and $\Xi(1)=\alpha-2<0$, a unique tax differential Δt_{ji} satisfies condition $\Xi(\Delta t_{ji})=0$ in the relevant interval $(0,1)$.

Using condition (49) and conducting comparative statics analysis, we get

$$\frac{d\Delta t_{ji}}{d\alpha} = -\frac{3\Delta t_{ji}-4\Delta t_{ji}^2}{3(1+\alpha)-8\alpha\Delta t_{ji}} \tag{50}$$

$$= -\frac{\left(3\Delta t_{ji}-4\Delta t_{ji}^2\right)^2}{3-8\Delta t_{ji}+12\Delta t_{ji}^2} < 0 \tag{51}$$

in equilibrium (see eq. (39)). Consider the fraction in eq. (51). Obviously, the numerator is non-negative for all Δt_{ji} and positive for all $\Delta t_{ji} \neq 0$ and $\Delta t_{ji} \neq 3/4$. Moreover, the denominator $3-8\Delta t_{ji}+12\Delta t_{ji}^2$ reaches its minimum of 5/3 at $\Delta t_{ji}=1/3$, as can easily be checked, and is thus positive for all Δt_{ji}. Hence, $d\Delta t_{ji}/d\alpha<0$ results, with $\Delta t_{ji}=0.5$ and $\Delta t_{ji}=0.191$ at the boundaries $\alpha=-1$ and $\alpha=1$ (see eq. (49)). That is, $\Delta t_{ji} \in [0.191, 0.5]$ is satisfied in equilibrium. Also, as $d\Delta t_{ji}/d\alpha = dC_i/d\alpha$ (see proofs of propositions 2 and 3), $dC_i/d\alpha<0$ holds, which proves the second part of part (ii).

Inserting the derivatives (47), (48) and (50) into (36) yields, after some rearrangements,

$$\frac{dNR_i}{d\alpha} = \frac{2(1+\alpha-\alpha\Delta t_{ji})\Delta t_{ji}^2}{(1+\alpha-2\alpha\Delta t_{ji})^2\left[3(1+\alpha)-8\alpha\Delta t_{ji}\right]}\Theta \gtreqless 0 \Leftrightarrow \Theta \gtreqless 0, \text{ where} \tag{52}$$

$$\Theta := (1+\alpha)^2(8\Delta t_{ji}-3)+\alpha(1+\alpha)(1-18\Delta t_{ji})\Delta t_{ji}+\alpha^2(6+8\Delta t_{ji})\Delta t_{ji}^2.$$

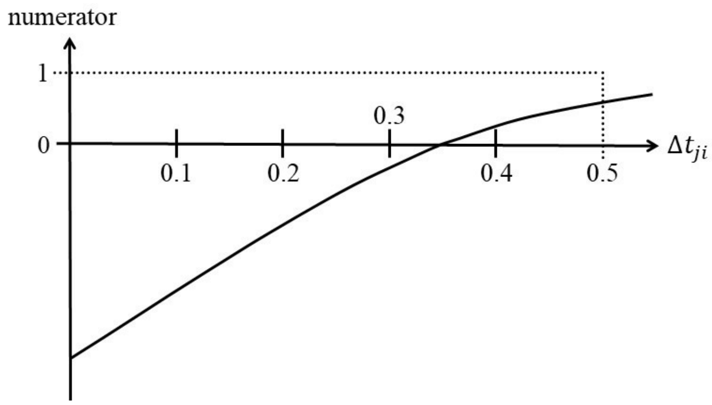

Figure 4: Net tax revenues and relocation mobility

The fraction of derivative (52) is positive for $\alpha \in [-1, 1]$ and the corresponding equilibrium tax differentials $\Delta t_{ji} \in [0.191, 0.5]$. Hence, the sign of $dNR_i/d\alpha$ is equal to the sign of $\Theta(\Delta t_{ji}; \alpha)$. Using condition (49) to replace α, we obtain

$$\Theta = \frac{-3 + 9\Delta t_{ji} + 9\Delta t_{ji}^2 - 42\Delta t_{ji}^3 + 42\Delta t_{ji}^4 - 16\Delta t_{ji}^5}{(3\Delta t_{ji} - 4\Delta t_{ji}^2)^2}. \tag{53}$$

The denominator is positive for $\Delta t_{ji} \in [0.191, 0.5]$. By contrast, the numerator can be positive or negative. In fact, as Δt_{ji} increases, the numerator switches its sign from negative to positive exactly once in the relevant interval $[0.191, 0.5]$, namely for $\widehat{\Delta t}_{ji} = 0.350$ (see figure 4 for illustration). Condition (49) gives us the corresponding threshold level $\widehat{\alpha} = -0.089$. As $d\Delta t_{ji}/d\alpha < 0$, $dNR_i/d\alpha \gtreqless 0 \Leftrightarrow \Delta t_{ji} \gtreqless \widehat{\Delta t}_{ji} \Leftrightarrow \alpha \lesseqgtr \widehat{\alpha} = -0.089$, which proves the first part of part (ii).

In section 4.3, we claim that not only net tax revenues but also tax revenues themselves initially increase and then decrease as relocation mobility increases. We prove this result along the lines of our reasoning above. Note that $R_i = NR_i + P_i = NR_i + \Delta\Omega_{ii} - \Upsilon_i = NR_i + \Delta t_{ji} - \tau_i$ follows from eqs. (8), (16) and (18), and thus $dR_i/d\alpha = dNR_i/d\alpha + d\Delta t_{ji}/d\alpha$ results. Then, using the derivatives (47), (48) and (50) again, we obtain, after some tedious rearrangements,

$$\frac{dR_i}{d\alpha} \gtreqless 0 \quad \Leftrightarrow \quad \widetilde{\Theta} \gtreqless 0, \quad \text{where} \tag{54}$$

$$\widetilde{\Theta} := \frac{-15 + 84\Delta t_{ji} - 178\Delta t_{ji}^2 + 208\Delta t_{ji}^3 - 206\Delta t_{ji}^4 + 192\Delta t_{ji}^5 - 116\Delta t_{ji}^6 + 32\Delta t_{ji}^7}{(3\Delta t_{ji} - 4\Delta t_{ji}^2)^2}.$$

Again, the denominator is positive, while the numerator switches its sign from negative to positive exactly once as the tax differential increases from 0.191 to 0.5, namely for $\widetilde{\Delta t}_{ji} = 0.463$. Using again condition (49) gives us the corresponding threshold level $\widetilde{\alpha} = -0.732$ and thus, as $d\Delta t_{ji}/d\alpha < 0$, $dR_i/d\alpha \gtreqless 0 \Leftrightarrow \alpha \lesseqgtr -0.732$.

Finally, as $P_i = \Delta\Omega_{ii} - \Upsilon_i = \Delta t_{ji} - \tau_i$, $dP_i/d\alpha = d\Delta t_{ji}/d\alpha < 0$ results (see eq. (50)). That is, subsidy payments decline as relocation mobility increases.

Proof of Propositions 5 Applying backward induction to solve the K-period model, we begin by analysing period K. The two-stage game in the last period of the K-period model is completely identical to the two-stage game in the last period of the two-period model, and so are the equilibrium taxes, given by eqs. (6) and (21), respectively. Only, we now need to add time subscripts.

Next, we turn to period $K-1$. Analogously to the two-period model, the location choices in two succeeding periods are independent of each other. A firm is domestic in country i in period $K-1$ if it was already located there in period $K-2$. Such a domestic firm stays in (relocates to) country i (country j) in period $K-1$ if, and only if,

$$m_{K-1} \geq v^n_{i,K-1} - v^m_{j,K-1} \qquad (m_{K-1} < v^n_{i,K-1} - v^m_{j,K-1}), \tag{55}$$

i.e., if, and only if, the mobility costs are greater (strictly smaller) than the tax differential between the countries. The resulting share of firms which relocate from country i to country j is $F_{K-1}(v^n_{i,K-1} - v^m_{j,K-1})$. Consequently, the number of firms that are in country i after relocation choices have been made is

$$N_{i,K-1} = \underbrace{\left[1 - F_{K-1}(v^n_{i,K-1} - v^m_{j,K-1})\right] N_{i,K-2}}_{=H_{i,K-1}} + \underbrace{F_{K-1}(v^n_{j,K-1} - v^m_{i,K-1}) N_{j,K-2}}_{=N_{j,K-2}-H_{j,K-1}}, \tag{56}$$

where $H_{i,K-1}$ is the number of i's domestic firms that stay put in period $K-1$ and $(N_{j,K-2} - H_{j,K-1})$ is the number of j's domestic firms that move to country i in period $K-1$.

Each government non-cooperatively chooses taxes $v^n_{i,K-1}$ and $v^m_{i,K-1}$ that maximise its net tax revenues, which are given by

$$\begin{aligned} NR_{i,K-1} &= \Omega^n_{i,K}\left[H_{i,K-1} + (N_{j,K-2} - H_{j,K-1})\right] + \Omega^m_{i,K}\left[(N_{i,K-2} - H_{i,K-1}) + H_{j,K-1}\right] \\ &\quad + v^n_{i,K-1} H_{i,K-1} + v^m_{i,K-1}(N_{j,K-2} - H_{j,K-1}) \end{aligned} \tag{57}$$

for $v^n_{i,K-1} > v^m_{j,K-1}$ and $v^m_{i,K-1} < v^n_{j,K-1}$. These inequalities indeed hold in equilibrium, since it is always optimal to attract some foreign firms through undercutting the opponent's tax on its domestic firms, in line with the reasoning in section 3 and the proof of lemma 1.

The optimal taxes are implicitly characterised by the first-order conditions

$$\begin{aligned} \frac{dNR_{i,K-1}}{dv^n_{i,K-1}} &= \left[1 - F_{K-1}(v^n_{i,K-1} - v^m_{j,K-1})\right] N_{i,K-2} \\ &\quad + \left[v^n_{i,K-1} - \Delta\Omega_{ii,K}\right] F'_{K-1}(v^n_{i,K-1} - v^m_{j,K-1}) N_{i,K-2} = 0, \end{aligned} \tag{58}$$

$$\frac{dNR_{i,K-1}}{dv^m_{i,K-1}} = F_{K-1}(v^n_{j,K-1} - v^m_{i,K-1})N_{j,K-2}$$
$$+ \left[v^m_{i,K-1} - \Delta\Omega_{ii,K}\right] F'_{K-1}(v^n_{j,K-1} - v^m_{i,K-1})N_{j,K-2} = 0. \quad (59)$$

Then, the equilibrium taxes are given by (22) and (23) for $k = K-1$. Inserting these equilibrium taxes into objective function (57) yields, after some rearrangements, net tax revenues

$$NR_{i,K-1} = 2\Omega^m_{i,K} + \Omega^n_{i,K-1}N_{i,K-2} + \Omega^m_{i,K-1}N_{j,K-2}. \quad (60)$$

Repeating this analysis for periods $2, ..., K-2$ gives net tax revenues

$$NR_{i,2} = 2\sum_{k=3}^{K}\Omega^m_{i,k} + \left\{\Omega^n_{i,2}N_{i,1} + \Omega^m_{i,2}N_{j,1}\right\}. \quad (61)$$

It remains to investigate subsidy competition in the first period, which is completely analogous to the one in the two-period model. The investors' location choice problem is exactly the same as in the two-period model, and so is the number of firms set up in country i and j, respectively, but for the time subscript: $N_{i,1} = 2G(s_i - s_j)$ and $N_{j,1} = 2[1 - G(s_i - s_j)]$ (see solution (10)).

Inserting the number of firms into net tax revenues (61), we get

$$NR_{i,1} = 2\sum_{k=3}^{K}\Omega^m_{i,k} + 2\left\{\Omega^n_{i,2}G(s_i - s_j) + \Omega^m_{i,2}[1 - G(s_i - s_j)] - s_iG(s_i - s_j)\right\}, \quad (62)$$

$$NR_{j,1} = 2\sum_{k=3}^{K}\Omega^m_{j,k} + 2\left\{\Omega^n_{j,2}[1 - G(s_i - s_j)] + \Omega^m_{j,2}G(s_i - s_j) - s_j[1 - G(s_i - s_j)]\right\} \quad (63)$$

for the two countries. Besides subscripts, the only differences between these two expressions and their counterparts (11) and (13) are the new first terms on the right-hand side of net tax revenues (62) and (63). However, these differences do not affect the equilibrium subsidies, which are given by solution (24) and are identical to solution (15) but for a subscript.

Plugging the optimal subsidies back into net tax revenues (62) and (63) gives, for both countries, $NR_{i,1} = 2\sum_{k=2}^{K}\Omega^m_{i,k}(\alpha_k) + \tau_i$ (or, alternatively, eq. (25)). In equilibrium, we also get (aggregate) opportunity costs $C_i = \sum_{k=2}^{K}\Delta\Omega_{ii,k}(\alpha_k)$, with $\Delta\Omega_{ii,k}(\alpha_k) = \Delta t_{ii,k}(\alpha_k) = \Delta t_{ji,k}(\alpha_k)$. The comparative statics results $dNR_{i,1}/d\alpha_k = 2d\Omega^m_{i,k}(\alpha_k)/d\alpha_k$ and $dC_i/d\alpha_k = d\Delta t_{ji,k}(\alpha_k)/d\alpha_k$ are completely analogous to those in the two-period model.

Appendix B

In this appendix, we analyse the impact of a decline in the inter-country cost differentials Δz_{ij} on net tax revenues and the opportunity costs of subsidy competition. Hence, we investigate the case in which international investors become less biased towards one of the countries when they decide where their firms are set up in the first period. We interpret such a change as an increase in location mobility and compare the implications with those of a rise in relocation mobility. More specifically, we capture the increase in location mobility as a rise (fall) in the share of firms with a low (high) absolute value of set-up cost differential Δz_{ij}. This is expressed by changes in the value of the density function $G'(\Delta z_{ij};\beta)$ caused by a mobility parameter β such that $\partial G'(\Delta z_{ij};\beta)/\partial\beta \gtreqless 0 \Leftrightarrow |\Delta z_{ij}| \lesseqgtr \widetilde{\Delta z_{ij}} \in \left(0, \overline{\Delta z_{ij}}\right)$ and $\partial G'(\Delta z_{ij};\beta)/\partial\beta = \partial G'(-\Delta z_{ij};\beta)/\partial\beta$. Importantly, such changes leave the distribution function unaffected at $\Delta z_{ij} = 0$, i.e., $G(0;\beta) = 1/2$ for all β.

As the density function $G'(\Delta z_{ij};\beta)$ is still symmetric around $\Delta z_{ij} = 0$, the equilibrium subsidies and hypothetical taxes also remain symmetric, i.e., $s_i = s_j$ and $\tau_i = \tau_j$, and are still given by solution (15). Thus, investors are still evenly split up between the two countries, with $2G(0) = 1$ firms being set up in each country. However, since increasing location mobility shifts investors from higher to lower set-up cost differentials (i.e., $\partial G'(\Delta z_{ij};\beta)/\partial\beta \gtreqless 0 \Leftrightarrow |\Delta z_{ij}| \lesseqgtr \widetilde{\Delta z_{ij}}$), and since the cost differential remains zero in equilibrium, the equilibrium value of the density function unambiguously increases. That is, investors respond more elastically to changes in subsidies or, equivalently, hypothetical taxes, and subsidy competition becomes more intense. Consequently, subsidies surge and net tax revenues decrease. The density effect on net tax revenues, which was ambiguous in the case of increasing relocation mobility, is now unambiguously negative. Additionally, the positive distribution effect analysed in section 4 now has no counterpart, since the equilibrium value of the distribution function G is independent of the location mobility in the case of a perfectly symmetric density function G' around the set-up cost differential of zero. All that remains is the negative density effect.

Using equilibrium outcome (15) and (19), we get

$$\frac{dNR_i}{d\beta} = -\frac{\partial G'\left(0;\beta\right)/\partial\beta}{2\left[G'\left(0;\beta\right)\right]^2} = \frac{dNR_j}{d\beta} < 0, \tag{64}$$

where the inequality sign follows from $\partial G'(0;\beta)/\partial\beta > 0$ (which is in turn implied by $\partial G'(\Delta z_{ij};\beta)/\partial\beta \gtreqless 0 \Leftrightarrow |\Delta z_{ij}| \lesseqgtr \widetilde{\Delta z_{ij}} \in \left(0, \overline{\Delta z_{ij}}\right)$).

In contrast to net tax revenues, the opportunity costs of attracting international investors depend only on relocation mobility (see equilibrium outcome (8) in conjunction with eqs. (7) and (17)) and are left unaltered by changes in location mobility, i.e., $dC_i/d\beta = 0$. After all, the opportunity costs C_i are simply the sum of subsidy s_i and hypothetical tax τ_i, and any rise in subsidies in response to increasing location mobility is completely offset by a decline in hypothetical taxes of the same amount because these two effects are just two sides of the same coin. In this sense, subsidy competition does not become more wasteful as location mobility increases, although net tax revenues decline. The negative impact on the public budget in the two-period model is exactly the same as in the hypothetical case in which the game ends after the first period (and thus consists of tax competition only). Hence, subsidy competition itself neither reinforces nor weakens the decline in revenues.

Proposition 6 *Increasing location mobility.*
The net tax revenues NR_i decline, while the opportunity costs C_i of subsidy competition remain unaffected, as the mobility parameter β increases. □

Our discussion about repeated relocation choices in section 4 directly implies that proposition 6 also holds in the K-period extension of our model.

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