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Trubridge, Nicola Jane

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AN INVESTIGATION INTO HOW TEACHERS DEVELOP CONNECTED APPROACHES TO SCHOOL MATHEMATICS

by

NICOLA JANE TRUBRIDGE

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AUTHOR'S DECLARATION

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award without prior agreement of the Doctoral College Quality Sub-Committee. Work submitted for this research degree at the University of Plymouth has not formed part of any other degree either at the University of Plymouth or at another establishment. This study was self-financed by the author with sponsorship from the school where the author was employed. Relevant scientific seminars and conferences were regularly attended at which work was often presented and several papers prepared for publication.

Conference Papers

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An investigation into how teachers develop connected approaches to school mathematics.

ABSTRACT

This thesis details the longitudinal case study of a mathematics department as they implemented aspects of the Collaborative Connected Classroom (CCC) Model, a literature informed framework exemplifying teaching for understanding. The CCC Model describes the nature of mathematical activity as: activities build on learners' prior knowledge; tasks connect different areas of mathematics or connect different ideas using multiple representations; links are made between procedures and concepts; tasks involve making comparisons and application tasks are presented as challenges.

A CPD programme was designed to demonstrate the CCC Model to the department through a series of engaging, challenging and inspiring activities. Then, over a sustained period the department actively experimented with different aspects. Data was collected using four in-depth semi-structured interviews, triangulated with learning walks, book scrutinies and presentations given by department members.

A Teacher Development Model (TDM) emerged to measure which aspects of the CCC Model teachers were implementing as they progressed through the phases of: awareness; guided exploration; independent exploration; independent development and transformation of their practice. The derivation of the Professional Mathematical Growth Model which is situated entirely within a change environment that is both social and professional was theorised to explain the mechanisms that supported the change process.

The study concludes that: research informed, inspiring, sustained CPD; trust and collaboration; responses from learners; the strategy of 'what's the same and what's different?' combined with an increased professionalism and school focus on action research were all positive stimuli to support 'teaching for understanding'.

These findings will be of interest to individuals who design and implement CPD programmes particularly those within mathematics education. There are applications of the emerged TDM and the Professional Mathematical Growth Model to others interested in looking at the process of teacher change or for teachers themselves as a reflective tool.

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LIST OF ABBREVIATIONS

ACME	Advisory Committee on Mathematics Education
AO	Assessment Objective
AST	Advanced Skills Teacher
B.Ed.	Bachelor of Education
CCC	Collaborative Connected Classroom
CPD	Continuing Professional Development
DCSF	Department for Children, Schools and Families
DfE	Department for Education
DfEE	Department for Education and Employment
FMSP	Further Mathematics Support Programme
FTM	Framework for Teaching Mathematics
GCSE	General Certificate of Secondary Education
HEI	Higher Education Institution
HLTA	Higher Level Teaching Assistant
ITE	Initial Teacher Education
KS3	Key Stage 3
KS4	Key Stage 4
MKT	Mathematics Knowledge for Teaching
NC	National Curriculum
NCETM	National Centre for Excellence in Teaching Mathematics
NQT	Newly Qualified Teacher
Ofqual	Office of Qualifications and Examinations Regulations
PCK	Pedagogic Content Knowledge
PGCE	Post Graduate Certificate of Education
PISA	Programme for International Student Assessment
QCA	Qualifications and Curriculum Alliance
RECME	Researching Effective CPD in Mathematics Education
SKE	Subject Knowledge Enhancement
TAM	Teaching Advanced Mathematics
TDM	Teacher Development Model

CHAPTER 1: OVERVIEW OF THE RESEARCH

1.0 Introduction

This chapter introduces this research study. It considers the contextual information that has led to the study of this area and then provides an overview of what will appear in each chapter of this thesis.

1.1 Contextual information

There is a vast range of theories and opinions about what is meant by the term understanding and in particular by the term mathematical understanding. There was one startling moment in my career when this came to the forefront of my thinking, and the question about what mathematical understanding is and how teachers can be supported to develop this with their learners has remained in my mind ever since.

The changing moment for me was in my role as a mathematics consultant. Whilst observing a year 7 lesson on writing algebraic expressions the teacher used a text book for the main part of the lesson and students 'happily' worked through four exercises. Near the end of the lesson the teacher read out the solutions and students marked their work and both the teacher and students were 'content' as most students got all the questions correct.

I was familiar with these exercises as I had used the same textbook myself when I was a new teacher, but in my consultant role I had the opportunity to look at them with a fresh view. I noticed that to be 'successful' in the first exercise all you had to do was insert a plus sign into all the gaps that were given (e.g. $x \square 3$), the second exercise all involved subtraction, the third multiplication and the fourth division. At no point did students have to think which operation would arise out of the real-life situation given. Students did not even have to decide what variables or numbers were needed as they were already printed on the page.

During the feedback session, the teacher felt that all students had 'understood' the objective and was ready to move on to the next topic. I was left convinced that the students did not actually understand how to write an algebraic expression but had simply learnt how to successfully follow the pattern within the exercise.

From that point on I noticed many lessons where similar events happened and I began to think about how the term understanding is used within the mathematics classroom and across the mathematics education community. This led me to read Skemp (1976) and later on Swan (2005). These experiences led me to be interested in connected knowledge.

On a personal level, I know that my style of teaching has changed over time and this has been informed by the experiences that I feel privileged to have had. I worked as a mathematics teacher for five years before being appointed to the local authority as a mathematics consultant for seven years which meant I had the luxury of time to think about how mathematics teaching could be developed and the opportunity to engage with pedagogic discussions with a wide range of people.

Over my career, I have become involved with working with teachers at various points from those entering the profession to those leading on mathematics learning. Throughout this time, it became obvious to me that there is a range of Continuing Professional Development (CPD) opportunities with teachers engaging in them at varying levels. I became interested in what makes effective CPD and how this can be implemented within the constraints of school and teaching life. These experiences led me to want to explore:

- How can a programme of professional development engage and support a mathematics department to teach for understanding?

1.2 Theoretical framework

The first part of the literature review (Chapter 2) looks at what is meant by knowledge and understanding within the mathematics classroom. This enables the following sub-research question to be answered.

- What is meant by ‘teaching for understanding’ and what would this look like in a mathematics classroom?

The interplay between procedural and conceptual knowledge (instrumental and relational knowledge) are explored in depth and the importance of making connections is established. This review provides an overview of other research studies that have been carried out where connections are explored within the mathematics classroom. The synthesis of the range of literature led to the development of the theoretical framework the Collaborative Connected Classroom (CCC) Model. A consideration of curriculum and assessment models within England was considered to see whether these constraints support or are a barrier to teachers working within the theoretical framework.

1.3 Continuing Professional Development (CPD)

Throughout the ten years of my career, prior to starting this research, I have been involved in delivering, designing and supporting professional development activities. This has ranged from delivering the ‘top down’ day courses as prescribed by the National Strategies, designing courses that address a specific local need, supporting colleagues through coaching and mentoring and later working at University on Initial Teacher Education (ITE) courses such as the Post-Graduate Certificate in Education (PGCE) and Bachelor of Education (B.Ed.) in Secondary Mathematics. I also led the Mathematics Subject Knowledge Enhancement (SKE) course for potential teachers who did not have a degree in mathematics but wanted to teach the subject.

Having had the opportunity to work across the range of activities described above I have had chance to informally reflect and review the effectiveness of these professional development activities. These personal experiences led me to want to explore the sub research questions:

- What is meant by the term professional development?
- What factors will contribute to an effective professional development programme?

The second part of the literature review (Chapter 3) considers CPD. First looking at what it means to be a professional in the field of education and then looking at the nature of teacher development and change. The literature review describes what makes effective CPD in general and then focusses on the specific needs for mathematics CPD.

1.4 Gap in current research

Whilst Chapter 2 details the vast range of research already available looking at connected knowledge, Askew *et al.* (1997, p. 101) recommend that future research is needed on ‘exploring the nature of 'connected' knowledge in more detail’.

It is acknowledged in Chapter 3 that there is a wide range of research about CPD and more specifically mathematics CPD, however the literature suggests various areas where future research is needed.

Askew *et al.* (1997, p. 101) recommends ‘exploring changes in teachers' beliefs over time, including the role of different elements in the change process’. The National Centre for Excellence in Teaching Mathematics (NCETM) recommended, in 2009, that further research is needed to investigate the barriers to engagement with CPD and to investigate aspirations of teachers who do not currently participate in CPD. The NCETM (2009) also recommend that further research is needed into the kind of research that is used in CPD, the way it is used and the effect this has on the professional development of the teachers.

The original element of this study is to combine these two key areas of 'teaching for understanding' and mathematics teacher CPD together. There is evidence to suggest that research is needed in this area. Askew *et al.* (2010, p. 47) noted that 'investigation is needed into how teachers can develop classroom tasks that encourage understanding through deeper thinking about mathematical concepts and inter-relationships as well as procedural fluency'. This agrees with the NCETM (2009, p. 8) which recommends that research is needed to 'investigate different approaches to engage teachers with students' conceptual development in mathematics'.

Given the current gaps in the research this thesis addresses the research question below.

- Q What would an effective CPD programme look like that supports the implementation of the CCC Model?

Chapter 4 then describes the development of the CPD programme which was used within the study and the range of professional development activities that were used to encourage implementation of the CCC Model.

1.5 Research methodology and research methods

As the research questions aim to explore ‘how’ teachers develop and ‘how’ the CPD informs change, a case study was chosen as the methodology. To gather data in-depth semi-structured interviews were used as well as additional data to support triangulation. The rationale for the methodology and methods used are presented in Chapter 5.

1.6 The pilot study

Chapter 6 details the pilot study which included working with several different cohorts of trainee teachers from different institutions and Higher-Level Teaching Assistants (HLTAs) from within the school where the study was based. The refinements, after the pilot study, to the research methods and the CPD tasks are also presented here.

1.7 The main study

Chapter 7 starts with a summary profile of each of the teachers involved in the main study, which is drawn from an analysis of their initial interviews. This chapter then describes the CPD sessions carried out with colleagues and provides a descriptive account of their responses to interviews throughout the study.

1.8 Analysis of the main study

Chapters 8 and 9 consider the themes generated from the pilot study and main study and draws together evidence from a range of sources to answer the sub research questions:

The CPD programme

- Q Does the professional development result in teacher change?
- Q What is the impact of exposing teachers to academic literature within a programme of CPD?
- Q What are the barriers to engagement with the CPD programme?

The CCC Model

- Q Which approaches will engage teachers with students' development of connections?
- Q Which approaches will teachers explore with students?
- Q How can teachers develop tasks that encourage connections to be made?
- Q What are the barriers to engagement with the CCC Model?

The case and its sub cases

- Q Are there any differences in how teachers develop across the mathematics department?
- Q What has influenced these differences?

Chapter 8 presents the results and findings and Chapter 9 provides an analysis of the data from the main study and compares this to what other researchers have found. The literature reviewed in Chapters 2 and 3 is returned to when presenting the interpretation of the results.

1.9 Conclusions

The final chapter returns to the overarching main research question and provides a summary of findings from the study. It summarises the contribution that this thesis makes to new knowledge and where it fits into the bigger educational landscape. The chapter provides possible applications of emerging theory and recommendations where additional work could build on these findings and conclusions.

CHAPTER 2: COLLABORATIVE CONNECTED CLASSROOM MODEL

2.0 Introduction

This chapter sets out to answer the first sub research question:

- Q What is meant by 'teaching for understanding' and what would this look like in a mathematics classroom?

The first section collates a range of theories and opinions from the mathematics education community extending back to the 1970s and up to the present day as to what is meant by individual mathematical knowledge and understanding. It considers the various dichotomies between types of student understanding for example relational versus instrumental and procedural versus conceptual and teases out some of the key similarities and differences in the definitions put forward over the years.

The second section looks at how social learning theory has developed over the years both in the UK and internationally.

The third section of this chapter moves from the theories arising from Sections 2.1 and 2.2 and considers what this might look like in the secondary mathematics classroom i.e. how do we see it in practice? This draws on both research literature from primary and secondary and guidance materials from organisations such as the Office of Qualifications and Examinations Regulations (Ofqual) and the Advisory Committee on Mathematics Education (ACME).

The fourth section draws together: the theory from section one and two; with what this might look like in the classroom from section three; and leads into a theoretical framework for the study which was developed and named the ‘Collaborative Connected Classroom Model’.

The final section considers the constraints that teachers work within in England such as the curriculum and assessment that might support or be a barrier to working in a Collaborative Connected way. It then compares with international priorities.

2.1 Defining mathematical knowledge and understanding

There are many different views of mathematical understanding and the debate has been on-going for decades. Simon (2017) suggests that for more than 25 years, mathematics educators have been stressing the goal of mathematical understanding. Often researchers and educators refer to a distinction between two types of understanding or knowledge that an individual might have. This section outlines examples of these and looks at similarities between them.

2.1.1 Instrumental and relational understanding

One of the first major theories put forward that categorised understanding into two types was back in the 1970s by Richard Skemp. Skemp (1986) was a teacher of mathematics who became interested in the challenges of teaching and learning. One of his main areas of interest and study was trying to figure out what it means to understand mathematics. In his well-known article, ‘Relational Understanding and Instrumental Understanding’ (1976), he acknowledges Stieg Mellin-Olsen for drawing his attention to two meanings of understanding using the terms, relational and instrumental. Skemp goes on to define relational understanding as ‘knowing both what to do and why’ (1976, p. 2) and instrumental understanding as ‘rules without reasons’ (1976, p. 2).

Skemp (1976) initially positioned himself with the notion that the only real understanding was that of the relational type but questioned his own thinking by exploring what the advantages were of both types of understanding. He proposed three advantages that instrumental understanding might provide. First that ‘within its own context instrumental mathematics is usually easier to understand; sometimes much easier’ (Skemp,1976, p. 8). For example, dividing by a fractional number is a difficult concept to get but being able to just multiply by the reciprocal is a much easier task. Or similarly, if you want to multiply a pair of negative numbers it is easy to learn the rule a minus times a minus equals a positive. The second advantage is that ‘rewards are more immediate and more apparent’ (Skemp, 1976, p. 8). This has been observed, by the researcher, in lessons that students like the success of seeing a page of correctly marked work giving them the confidence that they can do mathematics. The third advantage put forward is that ‘because less knowledge is involved, one can often get the right answer more quickly and reliably by instrumental thinking than relational’ (Skemp, 1976, p. 8).

Skemp proposed four advantages that relational understanding might provide. The first being ‘it is more adaptable to new tasks’ (1976, p. 8). The second ‘it is easier to remember’ (Skemp, 1976, p. 9). The example that is used to explain this is finding area of simple shapes. For example, if you wanted to find the area of a triangle you could just learn the formulae and have success in an instrumental way, but then if you wanted to extend your learning to finding areas of parallelograms and trapeziums etc. there would be different rules to learn and apply for each. If, however there is a relational understanding that links together how these are derived from rectangle areas then connections have been made and it should indeed be easier to remember. The third advantage is that ‘relational knowledge can be effective as a goal in itself’ and the fourth ‘relational schemas are organic in quality’ (Skemp, 1976, p. 10). Linking the third and fourth points, Skemp claims that ‘if people get satisfaction from relational understanding, they may not only try to understand relationally new material which is put before them, but also actively seek out new material and explore new areas’ (1976, p. 10).

Tall (1994) states that whilst procedural learning may work at one level in routine examples it produces an escalating degree of difficulty at successive stages because it is more difficult to co-ordinate processes than manipulate concepts. He comments that ‘the failing student fails because he or she is doing a different kind of mathematics which is harder than the flexible thinking of the successful mathematician' (Tall, 1994, p. 9).

Another interesting point that Skemp (1976) referred to is the potential for a mis-match between the students' goals and the teachers' ideas. For example, there could be the case when the teacher wants a class to understand relationally but the students own goal is to understand instrumentally. This is certainly something the researcher has experienced both as a classroom teacher within secondary school when students have asked 'just tell us the rule I do not want to know why it works' and when working with trainee teachers. Alternatively, there is the opposite case when the teacher is teaching in an instrumental way, but students question why and perhaps are told 'you don't need to know why for the exam just be able to get the correct answers'.

One of the main arguments against instrumental understanding that Skemp puts forward is that it involves knowing lots of rules rather than considering a smaller number of principles and being able to apply them to different situations.

> 'Learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans from getting to any starting point within his schema to any finishing point.' (Skemp, 1976, p. 14)

The work of Skemp has since influenced many academics, researchers and teachers. Byers and Herscovics (1977) reviewed the article referred to above and in discussion with teachers and drawing on their own experience put forward an agreement in principle of both relational and instrumental but also put suggestions that there were some types of understanding that did not fall into either of the two categories. They concluded that there are in fact four different kinds of understanding:

- **Instrumental understanding** is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.
- **Relational understanding** is the ability to deduce specific rules or procedures from more general mathematical relationships.
- **Intuitive understanding** is the ability to solve a problem without prior analysis of the problem.
- **Formal understanding** is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning. (Byers and Herscovics, 1977, p. 26)

For example, consider the case of a student solving an equation of the form $x + 4 = 9$, an instrumental approach might be 'change the side change the sign', a relational approach might be to add the inverse of $+4$ to both sides. An intuitive approach might not have a method attached but a feeling that the answer 5 is about the right size. Rather than illustrate this example with what a student that has achieved formal understanding would do, Byers and Herscovics (1977) consider what it might look like if they do not. In this case for example this would be of the form:

$$\begin{aligned} x + 4 &= 9 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Buxton (1978) proposed four different levels of understanding, rather than suggesting four different types of understanding. The first level is **rote** which is purely instrumental. For example if students were given the question "7×9?"

they answer 63 without going through any thinking process. The second level is **observational** which is 'slightly deeper than purely instrumental but not fully relational' (Buxton, 1978, p. 36), for example they might notice that in the nine times table the digits sum to nine. The third level is **insightful** which is said to be relational so in the case of the nine times tables students might make links to addition on a number line to acknowledge that you go forward a ten and then back one. At this level, students feel that they understand why a specific concept works rather than just how. The fourth level refers to the definition from Byers and Herscovics and is **formal** which ‘is only appropriate after insightful or relational understanding is achieved and at a stage in the student's development where some idea of the need for and the nature of proof is accepted’ (Buxton, 1978, p. 36). In 1987, Skemp adapted his model of understanding and included revisions by considering Byers and Herscovics (1977) work. He then reclassified it as follows:

- **Instrumental understanding** is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.
- **Relational understanding** is the ability to deduce specific rules or procedures from more general mathematical relationships.
- **Formal** [= Logical in my table] **understanding** is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning. (Skemp, 1987, p. 166)

Relational Learning	Instrumental Learning
1. Learning consists of building up a conceptual structure	1. There is no awareness of overall relationship
2. Goal is 'to enlarge or consolidate my mental map'	2. Goal is to get 'required finishing points (answers)'
3. 'Mistakes' involve learning.'.... if he does take a wrong term he will .... be able to correct his mistake	3. Mistakes result in being lost unless you can retrace your steps ' and get on the right path'
4. As schemas grow 'our awareness of possibilities enlarges'	4. Learning is merely the 'learning of an increasing number of fixed plans'
5. Works against memory limitations; much less memory work is involved	5. Relies on 'memorising ... a different method for every new class of problems'
6. Generates confidence in 'finding new ways of getting there without outside help'	6. 'Learner dependent on outside guidance for learning each new way to get there'
7. An intrinsically satisfying goal in itself	7. Extrinsic rewards are necessary
8. Leads to enjoyment of mathematics	8. Leads to ultimate failure
9. 'It is easier to remember .... it is certainly harder to learn'	9. 'Within its own context ... easier to understand'

Figure 1. Classification of relational and instrumental learning (Reason, 2003, p. 6)

As well as academics drawing on and extending on Skemp's theoretical work there has been evidence of classroom practitioners considering and reflecting on the terms relational and instrumental understanding. One example of this is in the journal 'Mathematics Teaching' where Reason (2003) summarised the fundamental differences between the two in Figure 1.

2.1.2 Procedural and conceptual understanding

As well as the distinction into instrumental versus relational understanding, the distinction of understanding into the categories of procedural and conceptual has been a focus of many research articles and is perhaps the most common.

> 'Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information … a unit of conceptual knowledge cannot

> be an isolated piece of information; by definition it is part of conceptual knowledge only if the holder recognises its relationship to other pieces of information.' (Hiebert and Lefevre, 1986, p. 3-4)

Using the above definition from Hiebert and Lefevre (1986) it follows that constructing relationships between the pieces leads to the development of conceptual knowledge. These relationships are so important that Hiebert and Lefevre distinguish two levels at which these can occur, the **primary** and the **reflective**. At the **primary** level 'the relationship connecting the information is constructed at the same level of abstractness ... than that at which the information is represented' (1986, p. 4) whereas at the **reflective** level 'the relationships transcend the level at which the knowledge currently is represented' (1986, p. 5).

Consider the case of addition of decimal numbers, at the **primary** level, a student may recognise that when lining up the decimal points what you are doing is adding tenths to tenths and so on. Whereas if the student also makes the connection that this is just a special case of the general idea of adding like things and applies this to fractions with a common denominator then this involves the 'process of stepping back and reflecting on the information being connected' (Hiebert and Lefevre, 1986, p. 5).

Hiebert and Lefevre divide procedural knowledge into two distinct parts 'one part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks' (1986, p. 6). Despite the distinction into two types of procedural knowledge, they are similar in that all procedural knowledge relies on a 'sequential nature' (1986, p. 6). Perhaps it is this sequential nature of relationships that makes it different from conceptual where the relationships can be of many different types.

The notion of ‘connections’ occurs regularly in the literature. Hiebert and Carpenter (1992) define conceptual knowledge so that it is identified with knowledge that is understood 'conceptual knowledge is equated with connected networks' and procedural knowledge is defined as a sequence of actions (1992, p. 78). Miller and Hudson (2007) also acknowledge that conceptual knowledge involves understanding but adds the importance of relating it to the meaning of mathematics.

The notion of procedural and conceptual understanding has informed and been built on by many other researchers including: Sfard (1991); Gray and Tall (1994); Kadijevich and Haapasalo (2001); Peled and Segalis (2005); Long (2005); Rittle-Johnson and Star (2007) and Kazemi and Stipek (2008). Their work will be considered in Section 2.1.3.

2.1.3 The interplay between types of understanding

Not only has there has been considerable debate about defining the two types of understanding but there has also been much debate as to which is the more important or which one should be taught first. Skemp (1976) acknowledges that even mathematicians who would classify themselves as relational still use instrumental thinking.

Byers and Herscovics (1977) assert that a good teacher can help a student to progress from **intuitive** understanding to **formal** understanding and similarly can support the move from instrumental to relational but that ‘the effective learning of mathematics cannot be based on one type of understanding. Nor, in the long run, can the different kinds of understanding be arranged in a linear order’ (1977, p. 27). They conclude that for optimal learning to happen the best approach is a spiral one so that ‘different types of understanding are used consecutively and repeatedly at even greater depth’ (Byers and Herscovics, 1977, p. 27).

Hiebert and Lefevre (1986) acknowledge that the debate regarding two different types of knowledge has been ongoing for many years but recognise that the discussion has evolved over time and has moved from purely defining the types to looking at the relationships between them.

> ‘Although it is possible to consider procedures without concepts, it is not so easy to imagine conceptual knowledge that is not linked with some procedures. This is due, in part, to the fact that procedures translate conceptual knowledge into something observable. Without procedures to access and act on the knowledge we would not know it was there.’ (Hiebert and Lefevre, 1986, p. 9)

Hiebert and Lefevre (1986, p. 16) claim that examining the relationships between conceptual and procedural knowledge is worthwhile since ‘being competent in mathematics involves knowing concepts, knowing symbols and procedures and how they are related’. Figure 2 summarises their key ideas.

Benefits for Procedural Knowledge	Benefits for Conceptual Knowledge
Developing meaning for symbols Building relationships between conceptual knowledge and the formal symbol system of mathematics is the process that gives meaning to symbols	**Symbols enhance concepts** Viewed as cognitive aids symbols help to organize and operate on conceptual knowledge (e.g. place value notation)
Recalling procedures Connecting procedures with their conceptual underpinnings is the key in producing procedures that are stored and retrieved more successfully	**Procedures apply concepts to solve problems** Procedures can facilitate application of conceptual knowledge whereby highly routine procedures can be used thus reducing the mental effort required. This frees up space for other processes including planning how to tackle problems
Effective use of procedures If conceptual knowledge is linked to procedures it can a) enhance problem representations and simplify procedural demands b) monitor procedure selection and execution c) promote transfer and reduce the number of procedures required	**Procedures promote concepts** The introduction of new procedures can sometimes trigger the development of concepts for example where children use their counting procedures to develop the ordinal concept of number

Figure 2. Benefits of linking conceptual and procedural knowledge (Hiebert and Lefevre, 1986

Whilst Hiebert and Lefevre (1986) state the importance of linking the relationships together they also acknowledge that even if these relationships are made explicit that students often fail to recognise or construct the relationships for themselves and therefore are not always internalised. Three reasons are given to expand on this: the first being a deficiency in knowledge base; the second that children tend to 'overlook or fail to encode relationships that may be obvious to adults' (p. 17) and the third that there is a tendency for knowledge to become compartmentalised whereby things learned in a particular context are initially linked to the characteristics of that context. This prevents learners from noticing the similarities between the newly and previously acquired knowledge.

This agrees with Tall (1988, p. 37) 'empirical research has emphasised that individuals build up their mental imagery of a concept in a way that may not always be coherent and consistent'. Tall (1988) refers to the importance of giving students rich experiences with varieties of examples and non-examples to enable them to form a more coherent concept developing both the concept definition and associated concept image.

A slightly different stance on the procedural / conceptual divide is put forward by Sfard (1991). In her research, she considered and referred to the range of understandings as mentioned in the previous sections but also looked at the dual nature of mathematical conceptions. She concluded that 'abstract notions such as number or function, can be conceived in two fundamentally different ways: structurally as objects, and operationally as processes' (Sfard, 1991, p. 1). One of the main differences stressed is that the model of operational and structural is in fact a 'duality rather than dichotomy' (Sfard, 1991, p. 9). To make this clearer consider the case of rational number which has a structural conception as pairs of integers but equally has an operational conception as division of integers.

Gray and Tall (1994) also suggest there is a duality but this time between process and concepts. They consider the situation where the same symbol is used to represent both; for example, in the case $5 + 7$ the addition sign represents the process of addition and the concept of a sum. In the case of $\frac{3}{4}$ the symbol stands for both the process of division and the concept of a fraction. The term ‘procept’ is used to represent this duality. They suggest that more able students can use ‘proceptual thinking ... the ability to compress stages in symbol manipulation to the point where symbols are viewed as objects which can be decomposed and recomposed in flexible ways’ (Gray and Tall, 1994, p. 18).

Hiebert and Carpenter (1992) also acknowledge that both kinds of knowledge are required for mathematical expertise. They claim that uncovering relationships between conceptual and procedural knowledge is more useful than trying to establish which one is more important (this continues from the work of Hiebert and Lefevre, 1986).

Rittle-Johnson and Alibali's (1999) study on mathematical equivalence (sample size of 89) investigated how conceptual instruction influenced children's problem-solving procedures and how procedural instruction influenced conceptual understanding. The study suggested that there is a causal relationship between conceptual and procedural knowledge. They proposed that the relationship between the two types of knowledge is not unidirectional but that 'conceptual and procedural knowledge appear to develop iteratively, with gains in one type of knowledge leading to gains in another' (p. 188). Rittle-Johnson and Alibali (1999) proposed that, although the two types of knowledge can support each other; overall conceptual knowledge may have a greater influence on procedural knowledge rather than the other way around.

Kadijevich and Haapasalo (2001) recognised the importance of making links between procedural and conceptual knowledge. They drew on two approaches from their previous work (2000). They defined the **developmental approach** where procedural knowledge is used and then the outcome is reflected on which leads to a greater conceptual knowledge and the **educational approach** where meaning is built for procedural knowledge before mastering it. They recognise that the different approaches may be suitable at different points and for different topic areas. For example, if there is a need to introduce the concept of a limit that promotes its dynamic definition, then the **developmental approach** may be needed whereas when teaching fractions and decimals then the **educational approach** may be better.

> ‘For most topics, the educational approach may be more relevant than the developmental one. However, the utilisation of an interplay of these approaches may, for some topics, be a better strategy than the application of one of them.’ (Kadijevich and Haapasalo, 2001, p. 157)

Long (2005, p. 61), claimed that ‘conceptual knowledge is intricately linked with procedures and algorithms. In fact, knowledge of procedures is nested in conceptual knowledge’.

In a later study in Israel, of 58 children, Peled and Segalis (2005) took a different approach that considered whether students could abstract mathematical principles by making connections between the procedures that they had learnt already (in this case they focused on subtraction). This study was built on the fact that ‘in most schools in Israel traditional (standard) procedures are still taught explicitly’ (2005, p. 207), the researchers wanted to build on the current situation. Their findings showed that using their abstraction and mapping instruction was positive and about three quarters of the children in the study group were then able to make connections which led them to create a generalised subtraction schema. Peled and Segalis argue that ‘the effort to construct general principles pays off by improving performance and understanding in specific domains and by facilitating transfer to new situations’ (2005, p. 208).

In England, Askew *et al.* (2010, p. 34) state that 'procedural fluency and conceptual understanding are largely seen as mutually exclusive aims'. They acknowledge that this polarisation is not helpful. They make comparisons between England and Pacific Rim countries.

> 'Pacific Rim teaching is largely dominated by procedures and hence supportive of procedural fluency, but the procedures used tend to be explicitly grounded in mathematical principles and consequently more mathematically coherent and meaningful than those most commonly used in the United Kingdom. In the Pacific Rim, mathematically informed procedural teaching is introduced and promoted through carefully constructed textbooks.' (Askew *et al.*, 2010, p. 34)

In a Mathematical Needs Report (ACME, 2011), the importance of procedures was still recognised (in terms of recall, accuracy and fluency) but it was also acknowledged that learning mathematics is different from other subjects and the following was put forward:

> 'Mathematics is a highly interconnected subject that involves understanding and reasoning about concepts, and the relationships between them. It is learned not just in successive layers, but through revisiting and extending ideas. As such, the mathematical needs of learners are distinctive from their more general educational needs. For mathematical proficiency, learners need to develop procedural, conceptual and utilitarian aspects of mathematics together.' (ACME, 2011, p. 1)

2.1.4 Conclusions

From the 1970s onwards, debate has continued about the nature of individual mathematical understanding. There are similarities between the relational and conceptual models in that both acknowledge the importance of making connections. Skemp's (1987) relational definition, given earlier, emphasises the learner's ability to deduce new rules or relationships for themselves, this I feel corresponds to constructing relationships at Hiebert and Lefevre's (1986) reflective level.

The formal language of mathematics appears as an important feature in both models considered so far. However, within the work from Byers and Herscovics (1977) and the later work from Skemp (1986) formal understanding is seen as a different type of understanding in its own right whereas Hiebert and Lefevre (1986) classify this as being part of procedural knowledge.

A common theme throughout the literature is that neither procedural / instrumental nor conceptual / relational knowledge is more important, but evidence suggests that the different types should be taught together. The researcher believes, drawing from the literature and experience, that the most important aspect in teaching for understanding is the importance of making connections.

From this point on the term **connected** will be used to encompass the notion of both relational / conceptual understanding and including making connections between procedures and their corresponding concepts and vice versa. For example, the use of Peled and Segalis' (2005) generalisation subtraction schema would be included within the connected model and also making connections between the formal language of mathematics and the concepts that are being developed.

2.2 Development of learning theory

Section 2.1 considered the most prominent theme of mathematics education research in the 1970s and 1980s, the construction of individual understanding, with discourses principally rooted in psychology (Davis and Simmt, 2006). Over the past few decades the situation has changed and there are a wide range of learning theories from psychological, behavioural, cognitive and social. This section looks at how learning theories have developed over the years both within the UK and internationally and explores in more depth those which are deemed to be the most relevant within the research of mathematics education.

2.2.1 Constructivism

Constructivism is a theoretical perspective concerning how people ‘come to know’ (Jaworski, 2002). Over time the constructivist viewpoint has been refined and amended, and there are some fundamental differences in interpretation, but it is constructivism that is widely accepted as best describing the process by which pupils learn mathematics. Chambers and Timlin (2013, p. 102) suggest that ‘it is the constructivist viewpoint that underpins many of the changes in mathematics teaching that have taken place over the past 20 years’. Constructivism is founded on Piaget’s (Swiss psychologist) belief that learning is an active process. Piaget was interested in child development and believed that development precedes learning, ‘development was seen to be a natural process through interactions with physical and social worlds, and learning was seen to be derivative of the developmental process' (Jaworski, 2002, p. 70). Piaget identifies four stages of cognitive development as sensori-motor, pre-operational, concrete operational, formal operational with children moving from one stage to another when they are ready (Chambers and Timlin, 2013).

We learn, according to Piagetian theory, by forming mental structures or schemata to represent our perceptions of what we experience in the world around us. New experiences result in either new schemata, or the reinforcement or modification of existing schemata (Jaworski, 2002). Piaget (1950) suggested two processes of mental (re) structuring: *assimilation*, the process of fitting new knowledge into existing schemata and *accommodation*, the process of adapting schemata to fit new perceptions that challenge existing structures. Watson and Dawes (2017, p. 41) define these processes; *assimilation* 'is where the individual interprets reality in order that it becomes consistent with the person's worldview' and *accommodation* 'describes thinking being adapted to reflect reality'.

Watson and Dawes (2017) suggest that a constructivist-orientated teacher aims to encourage accommodation. This not only aids the students' procedural fluency, but also contributes to their conceptual understanding. Chambers and Timlin (2013, p. 102) suggest that 'learners progress when they notice a discrepancy between what they currently believe or what is commonly believed, and what appears to be true'.

2.2.2 Social constructivism

Piaget's ideas of cognitive development have been adapted by many modern writers to consider social interaction, known as the social constructivist viewpoint. Social constructivism is often linked to the work of Vygotsky and his colleagues in Russia and to Ernest (1991) in the UK. Vygotsky (1987) suggested that effective learning can only take place in a social context. Vygotsky viewed learning as fundamentally a social process. 'Human learning presupposes a special social nature and a process by which children grow into the intellectual life of those around them' (Vygotsky, 1978, p. 88). Vygotsky has been influential in mathematics education, prompting interest in student collaborative work in which students can discuss mathematical ideas and 'construct' their understanding. Vygotsky has also contributed to interest in dialogue in the classroom, where teachers encourage students to articulate their misconceptions rather than correcting their errors, prompting interest in the social and cultural aspects of learning mathematics.

> 'Social constructivism suggests that learning takes place on two planes: the social plane though interaction with other people, and then the internal, psychological plane. Discussion, therefore, becomes a central part of learning, much more than the teacher transmitting knowledge.' (Chambers and Timlin, 2013, p. 103).

From the social constructivist viewpoint 'mathematics is not just knowledge and facts to be learnt, but it is rather a set of established cultural practices that students become acculturated into as they engage in mathematical activity' (Watson and Dawes, 2017, p. 43).

Within the classroom and outside, mathematical knowledge forms part of a social structure and access to that structure comes through communication. The view of mathematics from a social constructivist is of a practice in which ‘understanding is negotiated before acceptance by a wider community of mathematicians’ (Chambers and Timlin, 2013, p. 103). We learn from being part of and interacting within a social environment and individual construction of knowledge is a derivative of that social construction (Jaworski, 2002).

There are common features between the work of Piaget and Vygotsky with both considering the way in which the cognitive processes are connected to the physical environment or the social world. The common feature being the construction of individual knowledge. ‘For Piaget, it was about making sense of experience and thought to develop and adapt processes. For Vygotsky, language and communication were an important aspect of developing cognitive and psychological processes’ (Watson and Dawes, 2017, p. 44).

2.2.3 Enactivism

A more recent emerging elaboration of constructivist epistemologies is that of enactivism, a theory of cognition that has its roots in biological and evolutionary understandings. Reid (2016, p. 138) suggests that ‘enactivism was introduced into mathematics education at a time when the main theoretical debate concerned how to describe the social interactions between individuals’. Proulx defines enactivism as:

> ‘An encompassing term given to a theory of cognition that views human knowledge and meaning-making as processes understood and theorized from a biological and evolutionary standpoint. By adopting a biological point of view on knowing, enactivism considers the organism as interacting with/in an environment.’ (Proulx, 2013, p. 313)

Francis, Khan and Davis’ (2016), work which is grounded in the perspective of Varela (Chilean biologist), Thompson (Canadian philosopher) and Rosch (American psychologist), have described enactivism as a theory of ‘engagement that is simultaneously attentive to the coupling of organisms and their environments, to action as cognition and to sensori-motor co-ordinations’ they comment that the environment plays a significant role in ‘understanding the dynamic unfolding of cognitive processes: that is to say, the environment is always a (potential) learning environment in providing resources for thinking, for doing/knowing and for being’ (Khan and Davis, 2016, p. 3).

It therefore doesn’t make sense to study the emergence of an individual's understandings without considering the social and political contexts in which those understandings arise. Knowledge is embedded in a series of increasingly complex systems which could be groups, schools, communities and cultures (Davis, 1995, Begg, 1999). Reid (2016, p. 164) suggests that ‘enactivism offers a ‘grand theory’ that can be brought to bear on most of the phenomena of interest to mathematics educators’. Reid pronounces the strength of the theory in describing interactions between cognitive systems which can include human beings, their conversations and larger social systems.

> ‘The school, as an agent of society, does not merely transmit the knowledge of one generation to the next; it participates in the transformation of that knowledge in focusing on this idea and not that one, it is assigning a value to both; in teaching it this way and not that way, it is privileging particular ways of acting over others.’ (Davis, 1995, p. 8)

The subject of mathematics and alongside it mathematical understanding are collective phenomena, not individual ones (Davis, 1995). To learn mathematics involves becoming socialized into the ways of knowing used in the community of mathematicians and mathematics teachers (Nunes, 1999). It is suggested that ‘learning mathematics affects who we are, what we do, how we stand in relationship to others, and how we situate ourselves in our world’ (Davis, 1995, p. 8). The enactivist view suggests that individual conceptions and collective knowledge take shape simultaneously. Davis (1995) suggests that mathematical knowledge only exists in conversing and puts forward the question as to whether mathematics is a process, or a product is replaced with the assertion that the two are inseparable.

Within the UK, Brown and Coles (2012, p. 221) say that learning is equivalent to action ‘perception is not the passive receipt of information, but an active process of categorization made possible by our history of interaction’. Enactivism, focuses on this learning in action – as opposed to learning from action (Francis, Khan and Davis, 2016). Davis, Sumara and Kieren (1996, p. 153) see learning and action as one and the same ‘learning should not be understood in terms of a sequence of actions, but in terms of an ongoing structural dance – a complex choreography – of events which, even in retrospect, cannot be fully disentangled and understood, let alone reproduced’.

Proulx (2013, p. 314) suggest that 'students change, reactions or strategy development are not seen as causal events determined by external stimuli' but that they arise from the participants interacting with and in their environment. The environment is not static, but evolves through this interaction, which triggers back additional reactions. Enactivist thought helps to emphasize the critical role of the teacher as a trigger, however it is in the interaction between the learner and environment that learning happens, not 'because of' the learning environment (Towers, Martin and Heater, 2013). In a classroom, the most significant features of the environment for any individual are the other individuals in the room. So, this interaction between teacher and student or between student and student must, in the process, alter the structure of both (Coles and Brown, 2016). When working within an enactivist framework, instruction is impossible as the teacher cannot know the connections that a learner might make (Brown and Coles, 2012). The teacher's role is to use their previous experience and awareness in response to the connections that the learner makes, in response to actions from their peers and should ask why, to enable learners to justify their actions. Enactivism shifts the role of the teacher from instructor with knowledge to one of co-learner and facilitator (Begg, 2002).

2.2.4 Neuroscience and the neurology of learning

Within the last decade we have seen advances in technologies that have given researchers the opportunity to study the workings of the mind and brain. Neuroscience is concerned with the study of the central nervous system, which is the brain, spinal cord and the system that controls voluntary actions.

Neuroscience has been of interest in the UK with Maguire's (University College London) study of London Taxi drivers and British author Boaler's work and in America with Dweck's (Stanford University) research. The key contribution of brain research has been the idea of brain plasticity (Boaler, 2015) and through experience we can develop new connections in our long-term memory. This suggesting that the brain itself does not present limits to learning mathematics, limitations are more likely to come from self-theories and self-beliefs (Watson and Dawes, 2017). Studies by Dweck (2006) looked at whether students had fixed or growth mindsets which impacted on whether they gave up when problems were challenging.

2.2.5 Conclusions on learning theory

Learning theory has evolved over time, across international arenas, with a diverse range of influences from behaviourism, constructivism, cognitive psychology, enactivism and finally, neuroscience. Learning theory is a complex phenomenon and Watson and Dawes (2017, p. 50) state that 'no single theoretical perspective is sufficient to analyse and explain the learning processes that might occur in a secondary mathematics classroom'. What is clear however is that learning in secondary mathematics classrooms involves students working with teachers and their peers in the social environment of the classroom.

2.3 Features of a classroom where there is connected teaching

So far, theory and definitions about what is meant by individual mathematical understanding have been considered and the importance of **connected** teaching has been established in Section 2.1. How learning develops was considered in Section 2.2 and the importance of developing within social contexts and the importance of the teacher facilitating rather than being the transmitter of knowledge was stated. This next section considers what **connected** teaching might look like within the secondary classroom. Studies from different countries are considered and drawn together to provide a theoretical framework for use within the research study. Although a model is proposed that is **connected** in nature rather than merely relational or conceptual, where journal papers have been drawn on language is used as in the papers then they are synthesised to be consistent with the researchers developed theoretical framework that of the Collaborative Connected Classroom Model.

2.3.1 Classrooms that promotes understanding

Building on the literature already referred to, Hiebert *et al.* (1997) wrote a practical book to support American teachers in making sense of the agenda of teaching and learning for understanding. Within the introduction to their book they acknowledge that by understanding they are meaning ‘we understand something if we see how it is related or connected to other things we know’ (p. 4). This definition describes the connected model, described above. It is appropriate and a useful starting point in considering what a connected classroom might look like.

Hiebert *et al.* (1997) considered a range of dimensions to teaching and the core features that were present where understanding was promoted within the mathematics classroom. Figure 3, from Hiebert *et al.* (1997, p. 12) is a useful summary of their thinking.

Dimensions	Core Features
Nature of classroom tasks	Make mathematics problematic Connect with where the students are Leave behind something of mathematical value
Role of the teacher	Select tasks with goals in mind Share essential information Establish classroom culture
Social culture of the classroom	Ideas and methods are valued Students choose and share methods Mistakes are learning sites for everyone Correctness resides in mathematical argument
Mathematical tools as learning supports	Meaning for tools must be constructed by each user Used with purpose -- to solve problems Used for recording, communicating and thinking
Equity and accessibility	Tasks are accessible to all students Every student is heard Every student contributes

Figure 3. Summary of dimensions and core features of classrooms that promote understanding (Hiebert et al., 1997, p. 12)

2.3.2 Connectionist, transmission and discovery teachers

Askew *et al.* (1997) studied 90 primary teachers and 2000 students looking at what made effective teachers of numeracy. In the study numeracy was defined to be ‘the ability to process, communicate and interpret numerical information in a variety of contexts’ (Askew *et al.*, 1997, p. 6). The identification of effective teachers was not based on presumptions of 'good practice' but on evidence of increases in pupil attainment.

From observing teachers' behaviour and through questionnaires, they were categorised into having a predominantly **connectionist**, **transmission** or **discovery** orientation. Figure 4, adapted from Askew *et al.* (1997, p. 35-36), shows the key features of the connectionist teachers.

	Connectionist
Beliefs about what it is to be a numerate pupil	The use of methods of calculation which are both efficient and effective. Confidence and ability in mental methods. Selecting a method of calculation on the basis of both the operation and the numbers involved. Awareness of the links between different aspects of the mathematics curriculum. Reasoning, justifying and, eventually, proving, results about number.
Beliefs about pupils and how they learn to become numerate	Pupils become numerate through purposeful interpersonal activity based on interactions with others. Pupils learn through being challenged and struggling to overcome difficulties. Most pupils are able to become numerate. Pupils have strategies for calculating but the teacher has responsibility for helping them refine their methods. Pupil misunderstanding need to be recognised, made explicit and worked on.
Beliefs about how best to teach pupils to become numerate	Teaching and learning are seen as complementary. Numeracy teaching is based on *dialogue* between teacher and pupils to explore understandings. Learning about mathematical concepts and the ability to apply these concepts are learned alongside each other. The connections between mathematical ideas need to be acknowledged in teaching. Application is best approached through challenges that need to be reasoned about.

Figure 4. Connectionist teachers' orientations towards teaching numeracy (Askew et al.,1997)

The study findings showed that teachers with a strongly **connectionist** orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strong **discovery** or **transmission** orientations.

When talking to the highly effective teachers it was concluded by Askew *et al.* (1997) that they believed that being numerate required having a rich network of connections between different mathematical ideas and being able to select and use strategies, which are both efficient and effective.

Although this study was based in the primary setting and the focus was on numeracy there are useful elements within the characterisations that could be explored within the secondary context and to expand to mathematics. The work of Swan (2005) as part of the Improving Learning in Mathematics publication, draws on similar characteristics that of **transmission** and **connected**.

	A 'Transmission' View	'Connected', 'challenging' view
Mathematics is	A given body of knowledge and standard procedures that has to be 'covered'.	An interconnected body of ideas and reasoning processes.
Learning is	An individual activity based on watching, listening and imitating until fluency is attained.	A collaborative activity in which learners are challenged and arrive at understanding through discussion.
Teaching is	Structuring a linear curriculum for learners.	Exploring meaning and connections through non-linear dialogue between teacher and learners.
	Giving explanations before problems. Checking that these have been understood through practice exercises.	Presenting problems before offering explanations.
	Correcting misunderstandings.	Making misunderstandings explicit and learning from them.

Figure 5. Transmission and connected view of mathematics, learning and teaching (Swan, 2005)

Figure 5 summarises how he defines beliefs about mathematics teaching and learning. The materials that were developed as part of Swan's research project were designed to promote what is called the **connected challenging** model. The noticeable difference between Swan (2005) and Askew *et al.* (1997) is the apparent loss of the discovery model. This is addressed within the Swan publication where it was put forward the view that in discovery teaching 'the teacher simply presents tasks and expects learners to explore and discover the ideas for themselves' (p. 5). Swan's (2005) model is different because he sees the teacher having a more proactive role in lessons. What is not clear is whether this difference is because the Askew report focuses on primary education and the Swan guidance is aimed at teachers of secondary and adult education. Both these key research documents draw on the importance of making connections. Swan included eight effective principles for teaching mathematics (2005, p. 7-10) which are:

i. Build on the knowledge learners bring to sessions
ii. Expose and discuss common misconceptions
iii. Develop effective questioning
iv. Use cooperative small group work
v. Emphasise methods rather than answers
vi. Use rich collaborative tasks
vii. Create connections between mathematical topics
viii. Use technology in appropriate ways

In Haylock and Cockburn's 5th edition (2017) of their book for primary teachers they aim to help the reader understand what constitutes understanding in mathematics. Their main theme is that 'understanding involves establishing connections' (2017, p. 8). Instead of creating connections between different topics they look at the connections between language, pictures, symbols and concrete situations. Figure 6 from Haylock (1982) shows the links between the categories.

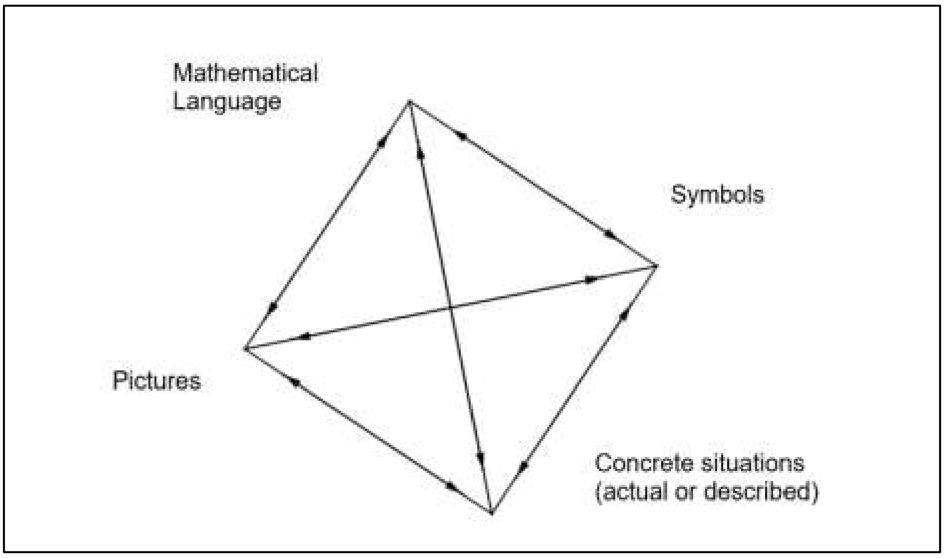


Figure 6. 'Connections between language, symbols, pictures and concrete situations' (Haylock, 1982, p. 55)

So, for example in the case of place value this might look like Figure 7.

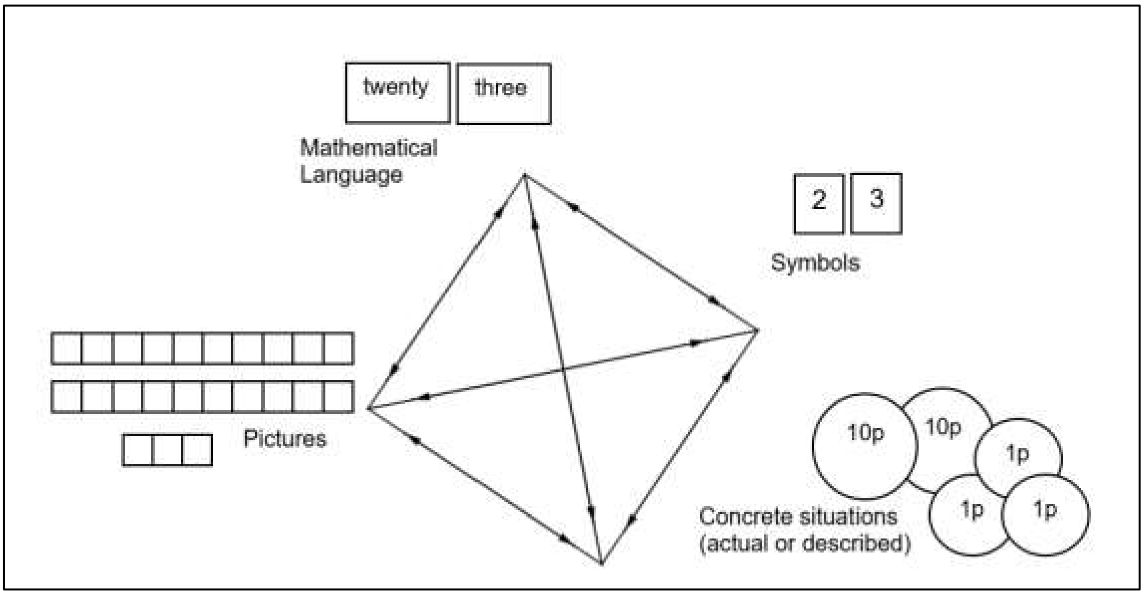


Figure 7. Connections between language, symbols, pictures and concrete situations for place value

This image was refined in Haylock and Cockburn (2008, 2017) so that the links remain the important part and the image of the square above is no longer present. Although Haylock and Cockburn (2017) is written for foundation stage and primary teachers, the key ideas that; mathematical activity involves the manipulation of concrete materials, symbols, language and pictures and that the connections between these four types of experience constitute important components of mathematical understanding is also relevant to secondary educators. The authors also acknowledge some of the same ideas as Gray and Tall (1994) about how the equals sign, can relate to a wide variety of different situations, language and pictures.

This importance of drawing on connections is also prevalent in guidance documentation. The ACME (2011) report describes the features of mathematics that distinguish what is different between mathematical needs and general needs. They say that mathematics requires understanding and reasoning about real and imagined objects, and is defined by a range of different kinds of knowledge, including the following:

- facts, methods, conventions and theorems
- mathematical concepts and structures
- connections between concepts
- notations, models and representations of situations within and outside mathematics
- symbols that are defined by formal rules of combination
- numerical, spatial, algebraic and logical reasoning within and outside mathematics
- mathematical ideas and contextual problems and applications
- deductions from axioms, hypotheses, generalizations and proofs
- generalizations from mathematical results and abstract higher-order concepts (ACME, 2011, p. 5)

Making connections between concepts is a valued component of knowledge. However, there is no explicit mention of other connections that might be important.

2.3.3 Socio-mathematical norms

In an American study, Kazemi and Stipek (2008) looked at a range of classroom practices and considered socio norms that were evident in lessons. From this they developed a model of socio-mathematical norms which they felt were evident in classrooms where there was a push for conceptual thinking. Their model of socio-mathematical norms included a set of expectations about what constitutes mathematical thinking and can be seen in Figure 8.

	Socio Norm	Socio-mathematical Norm
Topic	Students	Teachers
Dialogue	Students describe their thinking	Ask students to justify their strategies mathematically – not simply a procedural description
Strategies	Students find multiple ways to solve problems, and they describe their strategies	Ask students to examine the mathematical similarities and differences among multiple strategies
Errors	Students can make mistakes as part of the learning process	Believe mistakes are opportunities to reconceptualise a problem, explore contradictions and try out alternative strategies
Engagement in discussion	Students collaborate to find solutions	Hold each student accountable for thinking through the mathematics in a problem
		Promote the idea that consensus should be reached through mathematical argumentation

Figure 8. Features of socio-mathematical norms evident in high press conceptual classrooms (Kazemi and Stipek, 2008).

They also raised the point that additional research is needed to understand how the norms are created and sustained within the classroom and how they influence students' mathematical understanding.

2.4 Developing a theoretical framework

Having looked at a range of different models and studies for classrooms that are conceptual / relational / connectionist in nature, this section considers the similarities between the different models. A theoretical framework is put forward to work with which states the position that will be followed within the research. Appendix 1.1 shows the collation of different ideas and definitions from a range of literature enabling comparisons to be made.

Considering the range of areas that are included in Appendix Tables 1-6, the researcher developed a theoretical framework that shows a model of what teaching for understanding would look like within a mathematics classroom. It has been categorised under the following four headings.

1. Teachers beliefs about mathematics and learning
2. Nature of mathematical activity
3. Social culture of the classroom
4. Characteristics of learners

The latter three areas could be measurable through looking at classroom practice, but the first area is highly important as it is potentially the beliefs of the teacher that will inform the nature of the classroom tasks and the social culture that is developed. These in turn will influence the learners and the characteristics that they display.

2.4.1 Teachers beliefs about mathematics and learning

This section details the beliefs that teachers would hold if they were teaching for understanding. The important overriding theme, that is consistent throughout the literature (Skemp, 1976; Askew *et al.*, 1997; Swan, 2005; ACME, 2011), is that mathematics is a subject that contains a wide range of connections. These connections can be between different areas of mathematics (for example the use of proportional reasoning within the topics of similar triangles and conversions) and between different representations (for example seeing an arithmetic sequence represented in its numerical, graphical and mapping forms).

The literature suggests that, within mathematics, mistakes are an important part of the learning process (Hiebert *et al.*, 1997; Reason, 2003) and that they should be made explicit within lessons and developed as part of the lesson (Askew *et al.*, 1997; Swan, 2005). These mistakes provide essential opportunities to reconceptualise a problem (Kazemi and Stipek, 2008).

Learning consists of building a conceptual structure (Skemp, 1976) and is a collaborative activity in which learners are challenged arriving at understanding through discussion with peers and teachers (Swan, 2005). The nature of this collaborative activity will be expanded within the social culture of the classroom section.

2.4.2 Nature of mathematical activity

If teachers believe that mathematics is ‘connected’ then teaching is about making learners engage with these connections. Mathematical activity will therefore involve connecting different areas of mathematics or connecting different ideas in the same area of mathematics by creating opportunities for a variety of words, symbols, diagrams and concrete situations (Askew *et al.*, 1997; Haylock and Cockburn, 2017; Hiebert *et al.*, 1997).

Whilst the notion of mathematical connections is not a new idea, the CCC Model aims to make more explicit the nature of the interplay of the connections between procedures and concepts. With this in mind, mathematical tasks may make specific links between procedural and conceptual knowledge (Kadijevich and Haapasalo, 2001).

Tasks may take the **educational approach** where meaning is built for procedural knowledge before mastering it (Kadijevich and Haapasalo, 2001), for example learners are encouraged to invent their own strategies before learning traditional algorithms (Franke *et al.*, 1998). Or they may take the **developmental approach** where procedural knowledge is used and then reflected on (Kadijevich and Haapasalo, 2001), for example comparison tasks are used where teachers encourage connections between the procedures being used to make generalisations resulting in conceptual understanding (Peled and Segalis, 2005).

Whichever approach (**developmental / educational**) is taken; mathematical tasks need to be accessible (Hiebert *et al.*, 1997) and build on the knowledge that learners have (Swan, 2005) by connecting ideas to their current conceptual schema (Skemp, 1976). Misunderstandings should be made explicit, so students can learn from them (Askew *et al.*, 1997; Hiebert *et al.*, 1997; Swan, 2005). Misconceptions are productive (Barmby, 2009) they provide the opportunity for teachers to build deeper understanding (Watson and Dawes, 2017).

It is important that mathematical tasks are problematic, and that application should be approached by challenges that need to be reasoned about (Hiebert *et al.*, 1997; Askew *et al.*, 1997). One method is that the teacher presents problems before explanations are offered (Swan, 2005) and they share essential information after selecting tasks with a goal in mind (Hiebert *et al.*, 1997).

2.4.3 The social culture of the classroom

There are many features apparent in a classroom where there is a focus on developing a social culture of more connected teaching. Research shows there will be a high degree of focussed discussion between teacher and whole class, teacher and groups of pupils, teachers and individual pupils and pupils themselves (Askew *et al.*, 1997).

It is acknowledged that learners should emphasise methods rather than answers (Swan, 2005). However, in the CCC Model the importance is on enabling learners to examine the mathematical similarities and differences between multiple strategies (Askew *et al.*, 1997; Kazemi and Stipek, 2008). Teachers will work actively with the pupils' explanations, refining them and drawing pupils' attention to differences between methods (Askew *et al.*, 1997; Kazemi and Stipek, 2008).

The important feature is that all learners will be encouraged to contribute and share their methods where they justify their strategies mathematically – not simply a procedural description (Kazemi and Stipek, 2008). There will be a strong emphasis on developing methods, reasoning and justification (Askew *et al.*, 1997). Learners will be each held accountable and consensus should be reached through mathematical argumentation (Hiebert *et al.*, 1997; Kazemi and Stipek, 2008).

Section 2.4.2 emphasised the importance of multiple representations. Barmby suggests that we need to distinguish between 'internal' and 'external' representations. External representations being words, graphs, diagrams, etc., whereas internal representations are the mental representations that we possess at a personal level. For a given mathematical concept the internal and external forms may be similar, but not necessarily the same 'the internal representations are always personally derived' (Barmby, 2009, p. 6). When teaching mathematics, we are often concerned with external representations, which may help pupils to develop flexible ways of working with concepts. 'Reasoning is the process by which the learner articulates and demonstrates connections between representations' (Barmby, 2009, p. 6) and drawing out student reasoning is therefore integral to developing their understanding.

Collaborative activity remains a key topic in mathematics education and there exists a ‘documented need for a better understanding of how mathematical learning evolves in social settings’ (Francisco, 2013, p. 417).

Canadian academics Martin, Towers, and Pirie, (2006), reviewed a broad body of literature that led them to offer a perspective on collective mathematical understanding as an improvisational and emergent process. They define collective mathematical understanding as ‘acts of mathematical understanding that cannot simply be located in the minds or actions of any one individual but instead emerge from the interplay of ideas of individuals as these become woven together in shared action’ (Martin and Towers, 2015, p. 5). From this implies that the growth of collective mathematical understanding is ‘a dynamical ever-changing interactive process, where shared understandings exist and emerge in the discourse of a group working together’ (Martin and Towers, 2015, p. 6).

2.4.4 Characteristics of learners

In a classroom where there is a focus on developing a more connected understanding of mathematics, learners will use strategies, which are both efficient and effective (Askew *et al.*, 1997). They will know what to do and why they are doing it (Skemp, 1976) as they will be fluent with connections in mathematics (ACME, 2011). Thus, they will be more confident in looking at new problems and attempting them without outside help (Skemp, 1976).

Learners who have made connections between procedures and their underpinning concepts will know a range of concepts, symbols and procedures and how they are related (Hiebert and Lefevre, 1986).

2.4.5 Summary of the Collaborative Connected Classroom Model

Table 1 summarises the theoretical framework with features that would be evident in a classroom that is teaching for mathematical understanding.

Teachers Beliefs about Mathematics and Learning	• Mathematics is a highly-interconnected body of ideas that involves understanding and reasoning about concepts and the relationships between them • Mistakes should be recognised and made explicit. They are opportunities to reconceptualise a problem explore strategies and try out alternative strategies • Learning consists of building a conceptual structure whereby ideas are revisited and extended • Learning is a collaborative activity where learners are challenged to arrive at understanding through discussion

Nature of Mathematical Activity	• Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
	• Tasks either connect different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
	• Links are made between procedures and concepts
	  o meaning is built for procedural knowledge before mastering it ('educational approach')
	  o procedures are evaluated to promote conceptual understanding ('developmental approach')
	• Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
	• Application tasks are presented as challenges that may be problematic and need to be reasoned about

Social Culture of the Classroom	• Ideas and methods are valued, and each student is held accountable for thinking through the mathematics in a problem until a consensus is reached. • There is an emphasis on reasoning and justification and not simply giving a procedural description • High degree of focussed non-linear discussion between teacher and groups of pupils, teachers and individual learners and between learners themselves • Discussion involves examining mathematical similarities/differences/connections among multiple strategies and refining learners' explanations
Characteristics of Learners	• Know what to do and why they are doing it • Know a range of concepts, symbols and procedures and how they are related. • Use strategies which are both efficient and effective • Are aware of connections within mathematics • Are confident in tackling unfamiliar problems

Table 1. Collaborative Connected Classroom Model

This is also included in Appendix 2.1 for ease of later referencing.

2.5 Encouragement for and barriers to teaching in a connected way

Within the current setting, of the English education system, there are several constraints including curriculum and assessment models that teachers work within. There was also support for teachers provided by the late National Strategies. This section explores how these systems might support or detract from the implementation of the CCC Model. Curriculum changes occurred during the time of this study, so both are detailed as teachers involved in the study were teaching from the 2008 curriculum and moved into the 2014 curriculum as the study progressed.

2.5.1 The 2008 National Curriculum

At the beginning of this study, teachers were working with the 2008 version of the National Curriculum (NC). The NC programmes of study, for Key Stage 3 (KS3) and Key Stage 4 (KS4), (DCSF and QCA, 2007); emphasise the importance of mathematics as a subject and the important links to society and the workplace. There is a section on key concepts (competence, creativity, applications and implications of mathematics and critical understanding) that underpin mathematical study and the statement that ‘pupils need to understand these concepts to deepen and broaden their knowledge, skills and understanding’ (DCSF and QCA, 2007, p. 140) but no mention as to what is meant by understanding or how learning might take place.

The next section of the programmes of study refers to the key processes (representing, analysing, interpreting and evaluation and communicating and reflecting) and how these skills are essential for learners to make progress. The fourth section details the range and content that needs to be covered. The final section of the document outlines opportunities that the curriculum should provide including the statement that there should be opportunities for pupils to ‘work on tasks that bring together different aspects of concepts, processes and mathematical content’ (DCSF and QCA, 2007, p. 147).

Whilst the NC documents provided the statutory guidance for schools there was surprisingly little focus on teasing out the importance of mathematical understanding. In fact, the term understanding was used twelve times in the KS3 programmes of study but nowhere was it defined so it is not a surprise that teachers have been left to form their own definition of the term.

Interestingly the 1999 mathematics NC (DfEE and QCA, 1999) makes explicit reference to making connections between topic areas ‘Teaching should ensure that appropriate connections are made between the sections on number and algebra, shape, space and measure, and handling data’ (DfEE and QCA, 1999, p. 29). However, there was no guidance as to what appropriate connections might be.

From experience gained working with trainee teachers the researcher observed that many were not aware themselves of the range of connections between topics so were not able to share this with their learners. ACME proposed in 2011 that ‘the curriculum must show the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion’ (ACME, 2011, p. 19) and that the cross-curriculum ideas should be represented explicitly.

Chambers (2008, p. 29) highlights ‘one criticism of the National Curriculum has been it encourages an atomized approach to mathematics, where pupils learn a series of mini-skills that they find hard to bring together’. However, the guidance for the 2008 National Curriculum makes it clear that teachers should avoid the atomized approach and that links should be drawn together and they should also provide pupils with the opportunity to work on extended problems.

2.5.2 The National Strategies

In addition to working with the 2008 NC, many teachers had been supported with published guidance, or through attending training events from the National Strategies. The work of the disbanded National Strategies made attempts to make the connections explicit in the Framework for Teaching Mathematics (FTM). The guidance section reports that the linking will result in better standards of mathematics and the planning section emphasises the importance of making connections.

> ‘Good planning ensures that mathematical ideas are presented in an interrelated way, not in isolation from each other. Awareness of the connections helps pupils to make sense of the subject, avoid misconceptions and retain what they have learnt. So, when you plan: .... present each topic as a whole, rather than as a fragmented progression of small steps.... bring together related ideas across strands.’ (DfEE, 2001, p. 46)

Not only did the National Strategies recognise the importance of these connections, but there was also an attempt to provide supporting ideas and materials to help teachers. Within the supplement of examples, expected outcomes were detailed for years 7 - 9 and next to these there were links to other curriculum areas. For example, for the objective 'generate points and plot graphs of functions' it suggested linking to properties of linear sequences, proportionality, enlargement and trigonometry (DfEE, 2001).

What is interesting though, is that the work of the National Strategies led many teachers into an atomized approach to teaching where they felt that they had to use one objective each lesson from the framework.

Some parts of the FTM gave explicit pointers about what might be involved in understanding certain topics e.g. in the case of algebra 'understanding that algebra is a way of generalising from arithmetic' (DfEE, 2001, p. 15) but this is not present for all topic areas.

When the 2008 curriculum was launched the National Strategies produced a set of guidance papers (DCSF, 2008) to help schools implement the curriculum changes. Thinking had not changed and it was reassuring to see the same statements regarding connections as were in the original FTM.

2.5.3 The 2014 National Curriculum

As the study progressed teachers moved to teach the 2014 curriculum. The purpose of study of the 2014 NC sets out with the initial statement that 'mathematics is a creative and highly inter-connected discipline' (DfE, 2013, p. 2 and DfE, 2014, p. 3). The importance of the interconnected relationships is at the forefront of what mathematics is defined to be about. There are three main aims of the mathematics NC:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and nonroutine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. (DfE, 2013, p. 2 and DfE, 2014, p. 3)

The first aim importantly mentions both fluency and conceptual understanding. Within the following guidance on what becoming fluent might mean there is the specific reference to different representations in both the programmes of study. 'Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas' (DfE, 2013, p. 2 and DfE, 2014, p. 3).

There is the acknowledgement that although the curriculum is organised into distinct domains; at KS3 pupils should build on KS2 and build connections across mathematical ideas to develop their fluency (DfE, 2013) and at KS4 they should develop and consolidate connections across mathematical ideas (DfE, 2014).

Examples are given at KS3 where pupils should 'move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]' (DfE, 2013, p. 4) and they should 'extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations' (DfE, 2013, p. 4).

At KS4, they move freely between different numerical, algebraic, graphical and diagrammatic representations is extended to include linear, quadratic, reciprocal, exponential and trigonometric functions (DfE, 2014). Pupils are expected to 'make and use connections between different parts of mathematics to solve problems' (DfE, 2014, p. 6).

The programmes of study reflect the importance of spoken language in pupils' development across the whole curriculum and the use of discussion to probe and remedy their misconceptions (DfE, 2013, and 2014).

Whilst there are statements about the interconnected nature of mathematics and the importance of making connections this new curriculum hasn't been perceived by all educational professionals in the same way. Many see the curriculum as the implementation of a personal vision of Michael Gove (then Secretary of State for

Education). The fluency versus conceptual debate has been interpreted in different ways.

> 'One frequent presumption of Gove's new curriculum is teaching through explicit rules. The explicit assumption is that teachers should announce a rule of grammar, spelling, calculation or nature prior to the learner engaging in any activity.' (Wrigley, 2014, p. 36)

In March 2013, a letter was written to the Independent newspaper, signed by over 100 academic professionals to warn of the dangers posed by 'Michael Gove's new National Curriculum' suggesting that it could severely erode educational standards.

> 'The proposed curriculum consists of endless lists of spellings, facts and rules. This mountain of data will not develop children's ability to think, including problem-solving, critical understanding and creativity. Much of it demands too much too young. This will put pressure on teachers to rely on rote learning without understanding.' (Bassey, Wrigley and Maguire, 2013)

Although these points are about the whole curriculum, they are commenting more generally that the curriculum values memorisation and recall over understanding and inquiry, this also impacts on mathematics. Bassey, Wrigley and Maguire (2013) state that Mr Gove has misunderstood England's decline in the Programme for International Student Assessment (PISA) tests and that the schools in high-achieving Finland, Massachusetts and Alberta emphasise cognitive development, critical understanding and creativity, not the rote learning that he puts forward. This is echoed by Wrigley (2014, p. 35) 'the ultimate irony of Gove's PISA envy is that PISA tests require intellectual process: problem-solving and application of knowledge rather than the regurgitation of a series of facts' he then goes on to say, 'it is counterproductive to design education around competition for PISA; paradoxically, a high ranking is more likely to result from in-depth learning and co-operation than testing and competition' (Wrigley 2014, p. 39).

2.5.4 Ofsted and accountability

As well as international competition with the PISA tables, schools compete within leagues tables and are held accountable by Ofsted. One of the key findings from Ofsted (2008) was that the best teaching in both primary and secondary was by teachers who were enthusiastic, knowledgeable and focused clearly on developing pupils' understanding of important concepts. Their findings fed into a later report (Ofsted, 2009) where the essentials of good mathematics teaching were considered. There was a high emphasis placed on understanding concepts throughout the reports and Appendix 1.2 shows the difference between good and satisfactory features.

Many of the features identified as good teaching support the CCC Model put forward in Section 2.4.5. However, this wasn't observed across the board.

> 'The fundamental issue for teachers is how better to develop pupils' mathematical understanding. Too often, pupils are expected to remember methods, rules and facts without grasping the underpinning concepts, making connections with earlier learning and other topics, and making sense of the mathematics so that they can use it independently. The nature of teaching and assessment, as well as the interpretation of the

> mathematics curriculum, often combine to leave pupils ill equipped to use and apply mathematics.' (Ofsted, 2008, p. 5)

Wrigley (2014, p. 39) acknowledges that current notions of accountability were designed to promote competition among schools and individuals, however highlights 'they lead to superficial learning for short-term assessment and grading, rather than intellectual engagement and enduring cognitive development'.

2.5.5 Assessment

The General Certificate of Secondary Education (GCSE) mathematics subject criteria sets out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in mathematics. At the beginning of this study, Ofqual (2009), in a similar way to the National Curriculum, mentioned the workplace and society and the importance of learners having a positive experience of the subject. The aims of GCSE study are condensed into five key areas that GCSE specifications must address.

> GCSE specifications in mathematics must enable learners to:
>
> - develop knowledge, skills and understanding of mathematical methods and concepts
> - acquire and use problem-solving strategies
> - select and apply mathematical techniques and methods in mathematical, every day and real-world situations
> - reason mathematically, make deductions and inferences and draw conclusions
> - interpret and communicate mathematical information in a variety of forms appropriate to the information and context. (Ofqual, 2009, p. 3-4)

So again, in a key supporting document understanding is referred to but not actually defined.

The later GCSE specifications in mathematics (DfE, 2013b, p. 3) say teachers ‘should encourage students to develop confidence in, and a positive attitude towards mathematics and to recognise the importance of mathematics in their own lives and to society. They should also provide a strong mathematical foundation for students who go on to study mathematics at a higher level post-16’. The aims have been revised to the following:

> GCSE specifications in mathematics should enable students to:
> - develop fluent knowledge, skills and understanding of mathematical methods and concepts
> - acquire, select and apply mathematical techniques to solve problems
> - reason mathematically, make deductions and inferences and draw conclusions
> - comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context. (DfE, 2013b, p. 3)

The researchers experience as a consultant found when talking to teachers about tasks that they felt would aid students in developing their understanding, often was faced with a response ‘we haven’t got time to do that we have all this content to cover before the exams’. This finding was echoed by Ofsted ‘too much teaching concentrates on the acquisition of sets of disparate skills needed to pass examinations' (2008, p. 6).

Ofsted (2008, p. 7) recommend that the Qualifications and Curriculum Authority (QCA) should 'ensure current and future developments in external assessment place increased emphasis on pupils' understanding of mathematics and readiness for the next stage in their education and avoid forms of assessment that fragment the mathematics curriculum'.

However, it is acknowledged that there are real challenges assessing mathematical understanding. Skemp (1976) acknowledges that it is difficult to assess whether a person understands relationally or instrumentally. He states, 'from the marks he makes on paper, it is very hard to make valid inference about the mental processes by which a pupil has been led to make them' (Skemp, 1976, p. 12).

When writing examination papers, the examination boards must adhere to weighting of marks against assessment objectives. In some respects, the examinations are beginning to be less focussed on content and more on whether students can choose the appropriate part of mathematics from their toolkit as can be seen from the 2009 Assessment Objectives (AO) in Figure 9.

Assessment objectives		Weighting (%)
AO1	Recall and use their knowledge of the prescribed content	45–55
AO2	Select and apply mathematical methods in a range of contexts	25–35
AO3	Interpret and analyse problems and generate strategies to solve them.	15–25

Figure 9. Weighting of assessment objectives (Ofqual, 2009, p. 9)

Note that the weighting of AO1 was previously 80% before the 2009 change.

This notion of less assessment of content continues with the 2013 GCSE subject criteria, shown below in Figure 10.

Assessment objectives		Weighting (%)	
		Higher	Foundation
AO1	Use and apply standard techniques	40	50
AO2	Reason, interpret and communicate mathematically	30	25
AO3	Solve problems within mathematics and in other contexts	30	25

Figure 10. Weighting of assessment objectives (DfE, 2013b)

However, explicitly mentioned in the detail for AO3 is that students should be able to 'make and use different connections between different parts of mathematics' (DfE, 2013b, p. 13).

There was a move in assessment procedures (September 2014 for GCSE and September 2017 for A Level) from modular to linear examinations, supporting the belief that mathematics is an interconnected subject rather than breaking it into discrete topics and examinations. If students have a more connected understanding of mathematics they will be better prepared to deal with questions that are assessing their ability to reason, interpret and problem-solve.

2.5.6 International comparisons

The English curriculum and assessment models have been discussed, however it is worth commenting on priorities that are prevalent in other countries. We teach in a period where mathematics is a high stakes subject 'it is a period where international comparative test results and their ranking within the world are seen by some politicians as the only measure of success' (Mooney *et al*. 2018, p. xiiv).

The idea of a mathematical interconnection between other disciplines and real life was a central point for mathematics education when the National Council of Mathematics Teachers incorporated it into the curriculum for the United States (NCTM, 1991). Today, García-García and Dolores-Flores (2018) note that mathematical connections are included in the curriculum of countries such as South Africa, Mexico, and Australia, to name just a few. This led them to develop their research with Mexican high-school students with mathematical connections and Calculus as the object of their study.

There is well-established and growing literature on group learning internationally within the mathematics education field. 'Researchers recommend that teachers employ group processes in the classroom for various reasons, including to enhance students' communication and reasoning skills in mathematics and to foster equity in the classroom' (Towers, Martin and Heater, 2013, p. 425).

2.6 Chapter conclusions

This chapter explored a range of literature about mathematical understanding. The interplay between procedural and conceptual knowledge was considered and the evidence suggests that the connections between the two types of knowledge are important and worthy of further consideration. The development of learning theory over time was considered and the importance of learning within a social culture was established.

The theoretical framework of the CCC Model was developed from the synthesis of research literature and for this study answers the sub research question:

- Q What is meant by 'teaching for understanding' and what would this look like in a mathematics classroom?

The English NC and assessment procedures were discussed to see if they would support the proposed CCC Model. In conclusion both the curriculum guidance and Ofsted support the importance of developing mathematical understanding. The importance of making connections is highlighted in key curriculum guidance such as; QCA (2007), DfEE (2001), DCSF (2008), DfE (2013) and DfE (2014). However, reports such as Ofsted (2008) observe that pupils had too few opportunities to make connections across different areas of the mathematics. Whilst the importance of connections is evident within guidance documents how these connections might be made is often left to the readers' interpretation.

Ofsted (2008, p. 8) recommended that schools should 'encourage teachers to focus more on developing pupils' understanding and on checking it throughout lessons'. They also acknowledged that many teachers would benefit from professional development on planning and teaching for understanding with the fundamental area for improvement being the subject knowledge of non-specialist teachers and the pedagogical skills of secondary teachers. The next chapter considers professional development of teachers and identifies what makes effective CPD for mathematics teachers.

CHAPTER 3: CONTINUING PROFESSIONAL DEVELOPMENT

3.0 Introduction

The main research question is to investigate whether a programme of professional development can engage and support a mathematics department to teach for understanding. Whilst Chapter 2 set out what 'teaching for understanding' might look like, this chapter sets out to explore professional development in general and then to consider what factors might support the design of an effective programme. The sub research questions that are addressed within this chapter are:

- What is meant by the term professional development?
- What factors will contribute to an effective professional development programme?

This chapter begins by considering what it means to be a professional within the education setting and then considers the nature of development and teacher change. Different professional development models are considered and then issues specific to mathematics are drawn out. Barriers and tensions are considered before deciding on the design of the CPD that was used within this research study.

3.1 Being a professional

The term 'professional development' suggests a process whereby teachers become more professional. The term 'professional' implies several things. The most commonly accepted definitions of a profession ‘are of an occupation which requires a long training, involves theory as a background to practice, has its own code of behaviour and has a high degree of autonomy’ (Dean, 1991, p. 5). Eraut (1995) identifies that teachers themselves use the term professional in different ways and different contexts. The term may be followed by any of the following: conduct, status, quality, judgement or responsibility and has a slightly different meaning in each case.

The notion of being an educational professional is critical to the idea that training is not complete when you become qualified to do the job. There are new skills, techniques and technological advancements that can be developed over time. Eraut (1995, p. 230) believes that ‘it is the moral and professional accountability of teachers which should provide the main motivation for their continuing professional development’.

He suggests further that being a professional practitioner implies:

1. A moral commitment to serve the interests of students by reflecting on their well-being and their progress and deciding how best it can be fostered and promoted.
2. A professional obligation to review periodically the nature and effectiveness of one’s practice in order to improve the quality of one’s management, pedagogy and decision making.

3. A professional obligation to continue to develop one's practical knowledge both by personal reflection and through interaction with others (Eraut, 1995, p. 232)

3.1.1 Professional knowledge

There are a range of ideas on what knowledge is needed to work as an educational professional. Eraut (1995) distinguishes among three areas of knowledge. The first of which '**subject-matter knowledge**' is that which is found in the school curriculum and formally taught to pupils. The second type is '**education knowledge**' which combines both theoretical and practical knowledge which supports the teaching process. The third area he considers is that of '**societal knowledge**' which incorporates both experiential and common-sense knowledge acquired by living in a society. He puts forward that this more organised and focused knowledge of society is important for good citizenship.

Joubert and Sutherland (2008) build on Eraut's social ideas and suggest that a profession is better understood as an applied field rather than as a discipline. In the case of mathematics education, which not only draws on the areas of mathematics the subject but also on psychology and sociology, this seems a sensible distinction. As the educational professional knowledge base is continually growing it is interesting to consider how the theories and knowledge are generated. Joubert and Sutherland (2008) suggest this generation occurs through a combination of empirical research (which has focused on mainly formal professional development), the elaboration of practitioner maxims and practical principles; and the preferred view or ideology of the profession.

The ideology of the teaching profession in England is in part determined by the government and the guidance of documents such as the National Curriculum (DfES and QCA, 2004) and the National Strategies FTM (DfEE, 2001). The view of Forde *et al.* (2006, p. 20) is that ‘government forces have become ever more intrusive into the teacher's world, to the point where teacher professionalism has become seriously affected’. Forde *et al.* acknowledge a view that currently carries weight that the 'good professional' is someone who delivers government educational strategies, without having a great deal of influence in the formation of those strategies. Within the English education system there has been distancing of the role of the teacher in terms of writing and preparation of all aspects of the curriculum content. Teachers do not have ownership of what is to be taught. This role has shifted from teachers to government with the statutory NC.

3.1.2 Teachers standards

Whatever the research community believe and put forward, the fact is that all teachers within England should comply with nationally set standards that inform the profession. September 2012 saw the introduction from the Department for Education (DfE, 2012) of a set of common standards for all teachers, which replaced different standards for trainees and qualified teachers. These standards focus on two areas ‘teaching’ and ‘personal and professional conduct’. The standards define the minimum level of practice that can be expected from the point of being awarded Qualified Teacher Status and are used in appraisals.

3.2 Professional development

The previous section outlined the characteristics of professional knowledge that are required within education, this section considers how these professional attributes can grow and develop and discusses the notion of CPD.

The teaching standards (DfE, 2012) refer to fulfilling wider professional responsibilities. Within this there is expectation that teachers will take responsibility for improving teaching through engaging with appropriate professional development. At this point it seems sensible to look at what is meant by the term professional development.

3.2.1 What is professional development?

Day's (1999) definition acknowledges the process of professional development that sits alongside the moral obligation already referred to in Section 3.1.

> 'Professional development consists of all natural learning experiences and those conscious and planned activities which are intended to be of direct or indirect benefit to the individual, group or school and which contribute through these, to the quality of education in the classroom. It is the process by which, alone and with others, teachers review, renew and extend their commitment as change agents to the moral purposes of teaching; and by which they acquire and develop critically the knowledge, skills and emotional intelligence essential to good professional thinking, planning and practice with children, young people and colleagues through each phase of their teaching lives.' (Day, 1999, p. 4)

A more recent definition by Avalos (2011) acknowledges the complexities of the learning within policy and school cultures.

> 'Professional development is about teachers learning, learning how to learn, and transforming their knowledge into practice for the benefit of their students' growth. Teacher professional learning is a complex process, which requires cognitive and emotional involvement of teachers individually and collectively, the capacity and willingness to examine where each one stands in terms of convictions and beliefs and the perusal and enactment of appropriate alternatives for improvement or change. All this occurs in particular educational policy environments or school cultures, some of which are more appropriate and conducive to learning than others.' (Avalos, 2011, p. 10)

Avalos also mentions the emotional involvement of the teachers. This is focussed on and considered in more depth later within this chapter. From a mathematics point of view the NCETM (2009, p. 14) defined that 'CPD for mathematics teachers should stimulate teachers to re-think, to experiment, to make fresh distinctions and to probe those distinctions to explore how they are informative in enabling choices related to teaching and learning'.

What is clear is that professional development should be a process where teachers are encouraged to think and consider themselves as learners while trying to improve practice so that the quality of the educational experience is improved in some way. Professional development cannot be optional, not only is there the moral obligation highlighted by Eraut (1995) and Day (1999), but within the current teaching standards in England there is an expectation that all teachers will engage.

> ‘Appropriate self-evaluation, reflection and professional development activity is critical to improving teachers’ practice at all career stages. The standards set out clearly the key areas in which a teacher should be able to assess his or her own practice and receive feedback from colleagues. As their careers progress, teachers will be expected to extend the depth and breadth of knowledge, skill and understanding that they demonstrate in meeting the standards, as is judged to be appropriate to the role they are fulfilling and the context in which they are working.’ (DfE, 2012, p. 4)

This on-going professional development is not a new idea, Cockcroft (1982) identified that even if initial teacher training is of a high standard all those who teach mathematics need continued support throughout their careers to be able to develop their professional skills and to enable them to maintain and enhance the quality of their work.

3.2.2 Formal and informal professional development

It is useful at this stage to consider the different types of professional development and the vast range of opportunities available for teachers. These include: formal organised sessions like departmental meetings; in school development sessions; award bearing courses run by Higher Education Institutions (HEIs); courses run by examination boards; conferences run by professional subject associations to name a few and more informal opportunities like reflections of own classroom practice and discussions within the staff room.

Fullan (1995, p. 265) describes professional development as ‘the sum total of formal and informal learning pursued and experienced by the teacher in a compelling learning environment under conditions of complexity and dynamic change'.

Due to the nature of informal development, it is often unplanned and therefore unrecorded so there is less research on the subject. However, Joubert and Sutherland in their comprehensive literature review (2008) conclude that while informal learning is clearly important it is frequently not fully optimised to address the improvement of development of aspects of teacher knowledge and understanding.

3.2.3 Classifications of formal professional development

Eraut (1995) classified in-service training into three different models; first the model of personal learning which is linked to the concept of the reflective practitioner, secondly the range of institutional development models centred on the notion of school improvement and lastly the curriculum implementation models which are associated with centrally planned change.

This more 'traditional' in-service training has moved towards new forms of professional developments including being part of learning communities. Joubert and Sutherland (2008) classify the more formal planned learning into three different types: firstly, with teachers working together within schools or wider areas; secondly, courses often run by external experts and thirdly, with teachers working as researchers in their own classrooms. The NCETM (2009) Researching Effective CPD in Mathematics Education study (RECME) found that the initiatives they examined in their study could be categorised as courses, within school initiatives and networks. Across the literature that has been reviewed it seems that these classifications are defined either by the setting where the learning activity takes place or by the people involved and the level of outside help.

One thing that seems to have become more prevalent in recent years is the culture of learning. As Fiszer (2004, p. 5) puts it ‘to sustain teacher learning that directly affects classroom practice, we must provide a culture that requires and supports on-going professional development’. There is also still the continued debate as to which aspects of professional knowledge are relevant in the current climate. ‘Professional development needs to be based on a more holistic understanding of what constitutes relevant professional knowledge’ (Forde *et al.*, 2006, p. 71).

3.3 Teacher development and change

Having identified that the educational knowledge base is continually growing and developing, the notion is that teachers should develop and change to keep relevant and successful with their learners.

Guskey (2002, p. 381) describes professional development programmes as ‘systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students.’

This section looks at the generic process of change and what this means within the educational setting. It then considers what makes people want to change from looking at personal motivation factors, attitudes and beliefs to whole school or nationally imposed change.

3.3.1 Process of change

Clarke and Hollingsworth (1994) suggest that the notion of teacher change is open to multiple interpretations, and that each could be associated with a perspective on teacher professional development. In a later publication, they describe six different perspectives on teacher change:

- Change as training—change is something that is done to teachers; that is, teachers are ‘‘changed’’.
- Change as adaptation—teachers ‘‘change’’ in response to something; they adapt their practices to changed conditions.
- Change as personal development—teachers ‘‘seek to change’’ in an attempt to improve their performance or develop additional skills or strategies.
- Change as local reform—teachers ‘‘change something’’ for reasons of personal growth.
- Change as systemic restructuring—teachers enact the ‘‘change policies’’ of the system.
- Change as growth or learning—teachers ‘‘change inevitably through professional activity’’; teachers are themselves learners who work in a learning community. (Clarke and Hollingsworth, 2002, p. 948)

It is also acknowledged by many researchers that the process of change is complex and time consuming. Fullan (1995, p. 257) writes that to be in the change business ‘means that you are in the business of having to learn autonomously and collaboratively because so much is happening, much of it unpredictable’ and

Eraut (1995, p. 249) that 'change in the classroom is a complex and lengthy process, requiring not only orientation and preparation but appropriate support during the implementation process itself'.

3.3.2 Teacher motivation, attitudes and beliefs

It seems sensible that for change to be successful at an individual level teachers need to be motivated to want to change. Dean (1991) suggests a range of factors that motivate teachers including:

- Children and young people developing learning
- Enthusiasm for subject matter
- Recognition, interest, praise and encouragement
- A chance to contribute and to shine
- A chance to take responsibility
- A challenge to professional skill
- The inspiration of others
- Career prospects.

Hargreaves (1995) draws on the importance of schools being places of learning for both students and teachers.

> 'What we want for our children we should also want for our teachers – that schools be places of learning for both of them and that such learning be suffused with excitement, engagement, passion, challenge, creativity and joy. Meeting such challenges is not only a challenge for teacher development but also fundamentally a challenge to our beliefs about and our commitments to the kinds of schools and education we want in the postmodern world.' (Hargreaves, 1995, p. 28)

Joubert and Sutherland (2008) acknowledge that there are some researchers who believe that a change in knowledge and beliefs comes before any changes that may occur in classroom practice where there are other researchers that argue that the noticeable change in student learning will help change teachers' knowledge and beliefs. 'It is not the professional development itself that provokes change, but the experience of successful implementation of change that will lead to changes in teachers' knowledge and beliefs' (Joubert and Sutherland, 2008, p. 13).

This is in line with the model proposed by Guskey (2002) where he proposes that any significant change that occurs in teachers' attitudes and beliefs will occur after they see evidence of improvements in the learning of their students. These improvements may result from a change that has been made within the classroom.

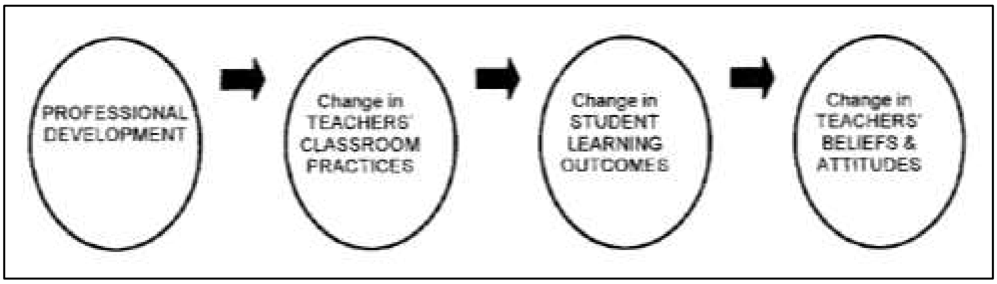


***Figure 11. 'A model of teacher change'** (Guskey, 2002, p. 383)*

The important point from Guskey's model is that 'it is not the professional development per se, but the experience of successful implementation that changes teachers' attitudes and beliefs. They believe it works because they have seen it work, and that experience shapes their attitudes and beliefs (2002, p. 384).

On the other hand, Franke *et al.* (2001) put forward that changes in beliefs and practices occur in a mutually interactive process. Teachers' thoughts influence their classroom practices. Their reflections on these activities and the outcomes of changed practice influence the teachers' beliefs about mathematics learning and teaching. Changes in attitudes and behaviours are iterative.

These ideas are echoed by Eraut (2001) where it is emphasised that it is important to consider the relationships between knowledge, beliefs and practice and recognising that all three aspects have to change to achieve sustained change in student learning. Eraut (2001) also emphasises that there is often a great deal more learning to occur after any CPD event when trying to use it in practice.

3.3.3 The interconnected model of professional growth

Whilst Guskey's model is linear in nature a more interconnected model (Figure 12) is put forward by Clarke and Hollingsworth (2002).

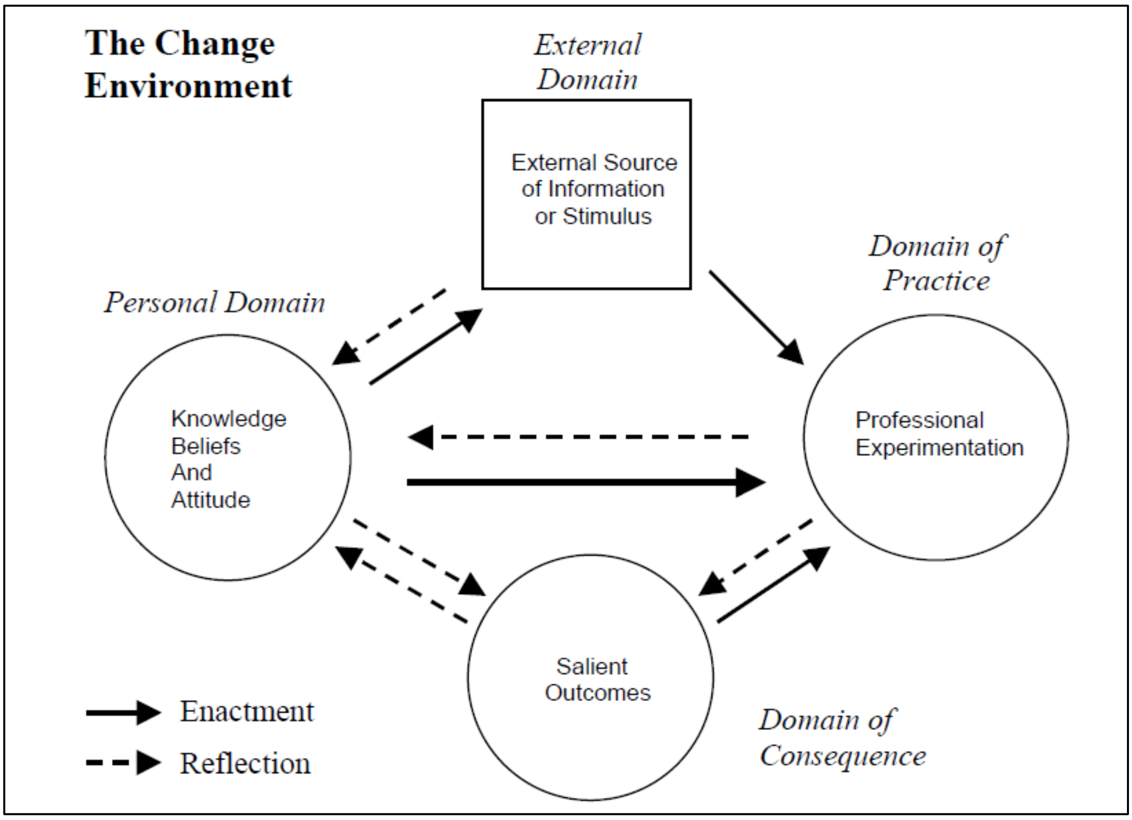


Figure 12. 'The interconnected model of professional growth' (Clarke and Hollingsworth, 2002, p. 951)

In this model of interconnected professional teacher growth, change in one domain is translated into change in another, through the processes of reflection and enaction. The term enaction is used to 'distinguish the translation of a belief or a pedagogical model into action from simply ''acting'', on the grounds that acting occurs in the domain of practice, and each action represents the enactment of something a teacher knows, believes or has experienced' (Clarke and Hollingsworth, 2002, p. 951).

Clarke and Hollingsworth (2002) put forward that the interconnected model compliments a range of theories of teacher learning and that it can be interpreted as 'consistent with either the cognitive or the situative perspective, dependent upon whether we take teacher growth as being the development of knowledge or of practice' (p. 955).

This model of professional growth of teachers has informed many recent research studies. Witterholt *et al.* (2012) used the model to assess the development of a mathematics teacher that was involved in a series of network meetings. Voogt *et al.* (2011) report the use of the model as an analytical tool to identify processes of teacher learning during the collaborative re-design of curriculum.

The important features to be drawn from the research literature are that change is a complex process which is not only affected by professional development opportunities but by teachers' motivation to engage with such tasks and their own attitudes and the beliefs that they hold.

3.4 Mathematics specific issues

Since this research study is investigating the effect of a CPD programme that supports teachers teach for mathematical understanding it is important to address the context of mathematics. Throughout the years there has been on-going discussion and debate about the specific needs of the teachers of mathematics. This section outlines some of the issues, and the nature of the CPD that is needed to address these issues.

3.4.1 Shortage of mathematics specialists

One of the specific issues that arises in England is that there are teachers of mathematics who are not specialists. In 1982, Cockcroft highlighted the fact that there were many teaching mathematics in primary, middle and secondary schools whose qualifications for this task were weak or non-existent. Smith (2004) acknowledges that this under-supply of appropriately qualified mathematics teachers remains an issue.

> 'The shortage of specialist mathematics teachers teaching mathematics is the most serious problem we face in ensuring the future supply of sufficient young people with appropriate mathematical skills. We think it likely that there is a current shortfall of around 3,400 specialist mathematics teachers in maintained secondary schools in England. We also note a recent survey finding that over 30 per cent of those currently teaching mathematics do not have a post A-level qualification in mathematics.' (Smith, 2004, p. 4)

In agreement with Cockcroft, ACME (2002) contends that one of the most effective ways to raise the quality of mathematical provision in schools would be to expand CPD for teachers of mathematics. They argue, that due to the technical nature of mathematics and the subtle links within it, there is a special requirement for mathematics teachers.

These ideas are echoed in Smith (2004) where the 'Making Mathematics Count' inquiry recommended to the government that an urgent priority was to create a large-scale programme of subject specific CPD for teachers of mathematics in England, Northern Ireland and Wales. It was felt that, in addition to retaining and attracting greater numbers of mathematics teachers, a successful CPD programme would lead to a more motivated and enthusiastic teaching force in mathematics, with improved subject matter and subject related pedagogical expertise. The report is clear that there is a need for CPD for teachers of mathematics at all stages of their careers, whatever their knowledge and experience. This report led later to the setup of the NCETM.

In addition to the funding allocated to the NCETM to support mathematics CPD there have been a range of government initiatives implemented to help recruit mathematics teachers. There have been high bursaries for those with mathematics degrees and funded subject knowledge courses to ‘boost’ knowledge of non-specialists, however despite these attempts the number of non-specialists teaching mathematics remains high. However Askew *et al.* (1997) comment that it is not the level of formal qualification those mathematics teachers have but the nature of the knowledge about the subject. Two initiatives that address these issues are the SKE courses for teachers to boost their mathematical knowledge prior to starting an initial teacher training course (DfE, 2013c) and the SKE+ course for teachers that are already qualified (DfE, 2013d). Despite these initiatives, Carmichael (2017) still shows there is a shortage of mathematics teachers.

3.4.2 Pedagogical content knowledge

It is not just the technical nature of the subject content that is important but how this content is presented to students to ensure learning occurs and misconceptions are addressed. Shulman (1986) considers how subject content knowledge grows in the mind of teachers. He distinguishes into three categories of content knowledge, firstly the subject matter content knowledge, then the pedagogical content knowledge, and finally the curricular knowledge. Within the category of pedagogical content knowledge, he includes the ways of representing and formulating the subject that make it comprehensible to others. He argues that teachers must have at hand a range of alternative forms of representation; these may come from research whereas others come from experience.

> 'Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates.' (Shulman, 1986, p. 10)

This idea of pedagogic content knowledge (PCK) has been used by many other researchers and teacher educators. Smith (2004, p. 107) writes that a teacher's overall competence involves three separate elements: subject matter knowledge and confidence, general pedagogical skills and subject specific pedagogical skills.

French (2003) developed a subject knowledge workbook for his trainee teachers which was specifically designed to enhance trainees' subject knowledge and highlight common errors and misconceptions with questions such as suggesting various ways to help pupils understand $\frac{2}{3}$ divided by $\frac{3}{4}$.

What is clear, however the types of knowledge are defined, is that moving from a separation of mathematics content knowledge (mathematics) and management of teaching into the blended PCK has had an important influence on professional development programmes (Joubert and Sutherland, 2008).

3.4.3 Mathematics knowledge for teaching

Within the field of mathematics teacher education, researchers have been expanding the notion of PCK and developing more fine-grained conceptualizations of this knowledge for teaching mathematics. One such conceptualization is 'mathematical knowledge for teaching' (MKT), mathematical knowledge that is specifically useful in teaching mathematics (Silverman and Thompson, 2008).

It is acknowledged that the mathematics that teachers need to know is substantially different from that mathematics that other adults need to know. To address these issues, Ball and Bass (2000, 2003) began their Study of Instructional Improvement. The study focused on producing a survey instrument that could measure the mathematical knowledge used in teaching elementary school mathematics. With the phrase "used in teaching," the developers meant to capture not only the mathematical content that teachers teach but also the specialized knowledge of mathematics needed for the work of teaching. Hill, Rowan and Ball (2005, p. 377) define specialized content knowledge, as the mathematical knowledge, not pedagogy 'it includes knowing how to represent quantities such as $1/4$ or $.65$ using diagrams, how to provide a mathematically careful explanation of divisibility rules, or how to appraise the mathematical validity of alternative solution methods for a problem such as $35 - 25$'.

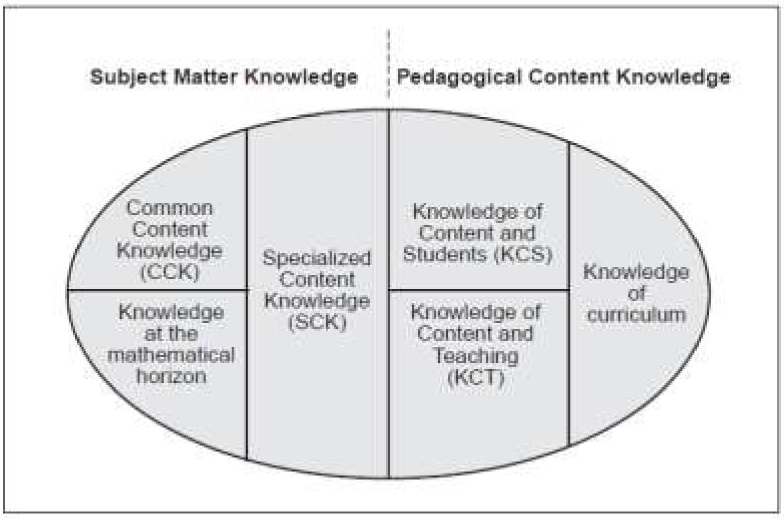


Figure 13. Domain map for mathematical knowledge for teaching (Hill, Ball and Schilling, 2008, p. 377)

Ball, Hill and Bass (2005) suggest that MKT demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably. Figure 13 shows a domain map for MKT. Hill, Ball and Schilling when describing MKT state the following:

> 'We mean not only the mathematical knowledge common to individuals working in diverse professions, but also the subject matter knowledge that supports that teaching, for example, why and how specific mathematical procedures work, how best to define a mathematical term for a particular grade level, and the types of errors students are likely to make with particular content'. (Hill, Ball and Schilling, 2008, p. 241)

3.4.4 Mathematics specific CPD

In 2008 the NCETM published their Mathematics Matters final report which collated the experiences and opinions of over 150 mathematics educators.

> ‘Participants felt that many teachers lack confidence in the subject and an awareness and understanding of appropriate resources (including ICT). They also lack time for continuing professional development. Participants commented that many teachers ‘lack confidence’ in mathematics, ‘can’t see the bigger, interconnected picture’ and ‘can only see one learning pathway through the curriculum’. When unexpected insights, answers or misconceptions arise, for example, many teachers don’t have sufficient subject knowledge to depart from their predetermined plan. This is also evidenced by the overdependence on pre-packaged schemes and textbooks.’ (NCETM, 2008, p. 21)

ACME (2011) identified that learners need teachers who have sound mathematical, pedagogical and subject- specific pedagogical knowledge. Despite this importance of PCK or MKT, the teaching standards (DfE, 2012, p. 7) don’t mention the associated pedagogy but simply say ‘demonstrate good subject and curriculum knowledge.’ Whereas, the previous standards (TDA, 2008) specifically mention, not only secure knowledge and understanding of the subject but also knowledge of the associated pedagogy to enable it to be taught effectively.

In a report looking at over 200 pieces of research Coe *et al.* (2014) identify the elements of teaching with the strongest evidence of improving attainment. Coe *et al.* (2014) found that the two factors that they found had the strongest evidence of improving pupil attainment were teacher’s PCK and quality of instruction (including strategies like effective questioning and the use of assessment). Coe *et al.* (2014) define great teaching as that which leads to improved student progress so in their study of effective teaching they looked for strong evidence of impact on student outcomes. They found that:

> ‘The most effective teachers have deep knowledge of the subjects they teach, and when teachers’ knowledge falls below a certain level it is a significant impediment to students’ learning. As well as a strong understanding of the material being taught, teachers must also understand the ways students think about the content, be able to evaluate the thinking behind students’ own methods and identify students’ common misconceptions.’ (Coe *et al.*, 2014, p. 2)

There is an agreement about the shortage of mathematics teachers and the importance of developing mathematics subject specific CPD. The Advisory Committee of Mathematics Education (ACME) concluded that ‘all teachers of mathematics should be entitled to subject- specific CPD. Incentives and funding should be found for non-specialists to undertake subject courses at an appropriate level. Such courses should focus on key mathematical ideas, the latest research on teaching and learning, and the nature of mathematics’ (ACME, 2011, p. 14). However, Joubert and Sutherland (2008, p. 12) note that there is ‘no agreement in the literature about the most effective way of structuring professional development so that teachers learn about the interrelated aspects of mathematical knowledge for teaching’.

Haylock and Cockburn (2017, p. 9) recognise that to be able to teach for understanding ‘the teachers must themselves understand clearly the mathematical concepts, principles and processed they are teaching’. The implication of this is that researching how CPD can be structured to support teaching for understanding is an area worthy of studying.

3.5 Tensions and barriers

It has been identified above that teachers' mathematical subject knowledge is an important basis for effective teaching of mathematics. However Askew *et al.* state that:

> 'The relationship between teacher subject knowledge and pedagogy is not simple and pedagogy depends upon tacit values and expectations as well as knowledge. What a teacher emphasises within a lesson may depend on cultural factors such as beliefs about learners as much as on the teacher's subject knowledge.' (Askew *et al.*, 2010, p. 46)

If it is considered that effective CPD involves a change in teachers' learning, then there are many challenges and barriers that prevent CPD from being effective.

> 'Teacher learning and development is a complex process that brings together a host of different elements and is marked by an equally important set of factors. But also, that at the centre of the process, teachers continue to be both the subjects and objects of learning and development.' (Avalos, 2011, p. 17)

Section 3.3.2 highlighted that there are many reasons why teachers may be motivated to partake in professional development activities and Section 3.1.2 showed the professional requirement. There are also a wide range of factors that might mean teachers either choose to opt out or attempt to opt in unsuccessfully.

3.5.1 Anxiety and rejection of new ideas

It is acknowledged (Fiszer, 2004) that the introduction of new concepts can cause a period of cognitive conflict and disequilibrium. During this challenging period, adult learners are likely to feel anxiety and frustration.

> 'If anxiety and frustration are overwhelming, the learner will not be able to resolve the conflict but will retreat to the stability of old assumptions and patterns of thinking. Rather than learning, a teacher in this situation might become rigidly entrenched in comfortable old ways of thinking.' (Fiszer, 2004, p. 14)

In addition to reverting to old ways when exposed to or trained in new knowledge and skills teachers often resist or reject them or simply select only the bits that suit them (Hargreaves, 1995). This new 'knowledge' is more likely to be rejected when it is imposed or encountered in the context of multiple and what might be contradictory and overwhelming innovations.

Little (1993) suggests that this rejection can occur if the new knowledge and skills are packaged in off-site courses or one-shot workshops that are alien to the purposes and contexts of teachers work.

The researcher, from experience and discussions with colleagues, has noticed that there is the feeling of an overwhelming number of innovations which is often the result of government and political agendas with England.

3.5.2 Political agenda

'Education in the UK in the twenty-first century is largely policy driven: school improvement and pupil attainment are regarded politically as being affected by policy directives' (Forde *et al.*, 2006, p. 5). Forde *et al.* (2006) identifies the policy shift towards performativity, which has resulted in a move away from professional agency towards surveillance of the professional's role. This surveillance and distrust of the profession may be another reason why teachers choose to opt out of professional development as they do not feel they have any control.

> 'Issues of change and improvement are at the core of government policy and public priority within education …if we are to bring about change and improvement that is meaningful and which positively affects children's learning, then it must be done in a way that places teacher professionalism, and the professional community, at the centre.' (Forde *et al.*, 2006, p. 13)

Ofsted (2008) emphasised the importance to shift from a narrow emphasis on disparate skills towards a focus on pupils' mathematical understanding but school accountability policies can be seen to encourage 'teaching to the test', and strategies such as early entry, or a focus on particular groups, which in turn leads to a procedural approach to mathematics (ACME, 2011). These school policies can seem contradictory to the belief that mathematics should be taught in a connected way.

In the ACME (2011, p. 1) Mathematical Needs report, it was identified that learners need 'institutions and systems that take into account the needs of the different subjects in the criteria for qualifications, in methods of assessment and in accountability measures' and a 'school and college management who do not prioritize superficial learning for test results'.

Avalos (2011) also comments that it is the policy environments and reforms, as well as teacher working conditions and historical factors that determine what is deemed to be acceptable forms of professional development.

3.5.3 Curriculum reform

Another challenge identified by Adler, Ball, Krainer, Lin and Novotna (2005), is that many teachers have not learned some of the content they are now required to teach, or they have not learned it in ways that enable them to teach what is now required.

> 'Curriculum reform processes in mathematics across different countries resulted in many teachers now having to teach a curriculum that is quite different from the one for which they were educated, and from one with which they had become experienced – and often also successful.' (Adler *et al.*, 2005, p. 361).

3.6 Characteristics of effective CPD

So far it has been considered what is meant by CPD and some of the different classifications of both the type of activities and people involved. Section 3.5 has highlighted reasons that might prevent teachers from opting in to the development process. It has also been shown why mathematics CPD plays a pivotal role in education. This section critiques strengths and weaknesses of different types of CPD and then draws on a range of evidence to look at characteristics of effective CPD. It concludes with putting forward the characteristics of effective mathematics CPD.

3.6.1 Criticisms of a one-day course

Craft (2000) notes criticisms of the course model; explaining that one-day workshops are widely used but often ineffective as they are rarely directed to the needs of the department or school, they have limited impact on practice. This is echoed by Smith (2004, p. 119) where he identifies that 'cascade training, in particular, is widely identified as a weak link in CPD programmes'. The criticisms concerned lack of time and opportunity to cascade the training back in schools and the fact that the effectiveness of the process and message diminishes as you move down the chain. There is also a high cost associated with either providing a venue to deliver courses or time to release teachers to attend work-shops. Cordingley *et al.* (2005, p. 11) put forward the case that 'most of the effective CPD in the research included learning which took place in the teachers' own schools and classrooms'.

3.6.2 Meeting the needs of teachers

Another important issue to consider, when developing CPD, is the needs of the teachers who are taking part in it.

> ‘Even when new techniques have demonstrable merit, training in them may be ineffective when it does not address the real conditions of teachers' work, the multiple and contradictory demands to which teachers must respond, the cultures of teachers' workplaces and teachers' emotional relationships to their teaching, to their children, and to change in general.’ (Hargreaves, 1995, p. 26)

Fiszer (2004) suggests that if you want to motivate teachers to participate then there needs to be a strong connection between the current teacher needs and the focus of the professional development effort.

> ‘Not only must teachers be involved in determining the theories or strategies presented but professional development activities must also model skills, provide practice opportunities in simulated and actual settings, and allow structured and open-ended feedback about performance. Professional development is most influential when ongoing support of the sessions involves collaboration, testing of selective ideas, reflective practice, and peer observation.’ (Fiszer, 2004, p. 7)

Focused specifically on mathematics, De Geest (2011) carried out a study listening to the views of teachers needs for their own professional development. Where this professional development (PD) was deemed to be successful teachers felt that the following categories were evident:

1. The teachers felt enabled to respond, and at times find solutions, to issues they had identified as problematic, or not knowing how to address, in their classroom practice.
2. The PD made them think at a high level, had challenged them which had made it interesting.
3. The PD made them look afresh at things and had inspired them with new ideas.
4. They had been able to follow/satisfy their own interests within the PD.
5. They felt strengthened in their views, their opinions, their thinking because the input into the PD had been theirs, based on their needs. Some teachers reported this had made them feel stronger, enabled them to argue their views better.

6. They knew how to put the theory of their professional development into their classrooms by having concrete examples and practical skills, and had experienced positive responses from the students. (De Geest, 2011, p3)

3.6.3 Effective CPD

Cordingley *et al.* (2005) published a systematic review on the impact of collaborative CPD on classroom teaching and learning. Their findings show evidence of positive benefits of CPD that:

- made use of peer support;
- made explicit use of specialist expertise;
- made explicit mention of involving the teachers in applying and refining new knowledge and skills and experimenting with ways of integrating them in their day-to-day practice:
- involved consultation with the teachers, about their own starting points, the focus of the CPD, the pace of the CPD or the scope of the CPD;
- involved teachers observing one another as an integral part of the CPD; and
- involved specialists in observation and reflection. (Cordingley *et al.*, 2005, p. 7)

As well as looking at the impact and characteristics of collaborative CPD, they then began to explore the nature of collaboration in more detail and put forward some tentative hypotheses about the characteristics of effective collaboration. These are as follows:

- Classroom-based activities may be a helpful factor in increasing the effectiveness of the CPD.
- Collaboration between teachers, which is coupled with active experimentation, may be more effective in changing practice than reflection and discussion about practice alone.
- Collaboration may be an effective vehicle for securing teacher commitment and ownership of CPD in cases where it is not possible for the teachers to select a CPD focus of their choice.

- Paired or small group collaboration may have a greater impact on CPD outcomes than larger groups. (Cordingley *et al.*, 2005, p. 7)

Some of the features that seem important here are the facts that active experimentation needs to occur in teachers own classroom, so change is more likely to occur in this context and the importance of collaboration in helping to develop ownership.

3.6.4 Effective mathematics CPD

Within mathematics specific research, many of the findings from Cordingley *et al.* (2005) are echoed. Smith (2004) recommended that there should be a move away from the cascade model towards school-based developments in which members of a mathematics department work together within the school context which is led or facilitated by an expert mathematics teacher.

Joubert and Sutherland (2008), in their extensive review of literature, suggest that CPD works best when programmes of professional development:

- include commitment to the enterprise by both institutions and teachers
- encourage purposeful networking amongst teachers
- are grounded in classroom practice
- are based on sound educational practice
- build on what teachers already know, taking into account the voice of the teacher
- avoid adopting a 'deficit model' of teacher knowledge and practice
- focus on ‘mathematics for teaching’
- centre around activities that reveal aspects of 1) teachers’ awareness, beliefs, and knowledge 2) teachers’ practice and 3) students’ learning
- support reflection and inquiry by teachers on both their own learning and their own classroom practice

- are explicit about ‘what change counts as improvement’
- support the development and evaluation of classroom-based activities. (Joubert and Sutherland, 2008, p. 28)

These suggestions not only emphasise the importance of classroom practice but also on the focus of mathematics knowledge for teaching.

In discussion with teachers, the NCETM (2009) highlighted effective features of organisation and structure of the CPD. It was felt important to teachers that the leaders of their CPD are knowledgeable about mathematics education and have recent and relevant experience of classroom practice. The teachers valued practical advice that was directly applicable to the classroom and they appreciated CPD grounded in classroom practice. They valued CPD that was stimulating, enjoyable and challenging and appreciated the time that their involvement in the CPD gave them to focus on their professional practice. They also enjoyed the opportunity to network with colleagues from the same or different schools. Whilst these are some useful insights from teachers about what they found important and valuable it is not clear whether these are enough to stimulate change.

Joubert and Sutherland’s (2008) review informed the NCETM (2009) document so it is not surprising to see many things in common in their list of recommendations. One distinct difference in these suggestions is that the NCETM suggests the need to be explicit about research that has informed the design.

Returning to the Askew *et al.* (1997) study mentioned in Chapter 2, it was found that four of the five highly effective **connectionist** orientated teachers identified an emphasis on the importance of working with pupils' meanings and understandings as significant elements of the CPD that they had engaged in. Also, they acknowledged that 'they had not been aware of the importance of developing pupils' meanings and mental strategies until the CPD made them focus upon this' (p. 80). The CPD involved a variety of practical activities where the teachers had chance to develop their own methods and mental strategies.

3.6.5 The NCETM CPD standard

With the end of the National Strategies leading to the demise of centrally planned cascade training, many organisations and individuals are leading their own professional development training. The NCETM developed a CPD standard that is awarded to organisations / individuals that can demonstrate they are providing effective CPD. Drawing on the work of ACME (2002, 2006) the NCETM identified three 'strands' of effective CPD:

- broadening and deepening mathematics content knowledge;
- developing mathematics-specific pedagogy, which includes appreciating how learners engage with mathematics and likely obstacles to progression;
- embedding effective mathematics pedagogy in practice.
(NCETM, 2012)

The NCETM seeks to promote CPD opportunities for teachers that impinge on all three strands in ways that are cumulative and sustained over the career of a teacher. These are a useful reminder of the need for mathematics specific CPD and of the importance of addressing MKT.

3.6.6 Studies of professional learning in mathematics

This section details recent studies looking at mathematics teachers' professional development and change. Ball, Ben-Peretz and Cohen (2014), acknowledge that in the past decade interest has increased in creating opportunities for teachers to work together on improving their practice. Examples include teacher research groups, lesson study and professional learning communities.

Brown and Coles (2011), when looking at how enactivism re-frames mathematics teacher development, found that encouraging the making of distinctions through the use of similarities and differences has shown the importance of narrowing the gap between action and communication in the continuing professional development of teachers.

In the Canadian study, of learners' interaction with their teachers, Towers, Martin and Heaters (2013, p. 430) found that 'actions serve to strengthen the collective, which provides the sustenance for the individual to flourish'. They acknowledge the mutual determination between the organism (learner) and environment each enhances and adapts to the other.

In England, Cajkler, Wood, Norton and Pedder (2014) report the outcome of a lesson study project explaining that the process offered opportunities for participants to develop individual expertise through collaboration in a community of teachers which led to greater confidence to make changes and willingness to take risks.

Tarling and Ng'ambi (2016) studied teachers in South Africa, looking at how to describe teachers’ existing uses of emerging technologies and the underlying pedagogical orientations using their developed Teaching Change Frame. They found that they could map teacher change in IT using their framework (which was developed from Blooms taxonomy). They comment that within South African schools ‘classroom practices are part of much greater historic, economic and socio-cultural factors that influence and are in turn influenced by the activities in the classroom’ (Tarling and Ng’ambi, 2016, p. 559).

Brown (2017), studied mathematics teachers from Australian secondary schools’ perceptions of change in their own practice in digital technology use during the time of their participation in a three-year project. The study found that participation in the program could lead to (perceived) change in technology use. It was evident from the study that several features supported change these were:

- willingness to, and school leadership support for, participation in the project
- previous teaching experience (including with technology)
- congruent expectations of curriculum documents
- access to a range of digital technologies
- on-going opportunities to collaborate with teachers and researchers

However, Brown (2017) concluded that whilst these features were supportive of teacher change, to teaching practices of integrating technology use to increase cognitive demand and higher order thinking, they were not sufficient for substantive change.

> ‘Whilst participation in a research project can promote teacher change, transformative change requires a focus on the interdependent strands of knowledge (MCK and PCK), beliefs, and practice.’ (Brown, 2017, p. 63)

3.7 Further research needed

Whilst there is already a vast range of research on CPD models the literature suggests that further research is needed in a variety of specific areas.

It was acknowledged in Section 3.3 that beliefs are an important part of the process however, ‘the literature is divided as to how to organise professional development in order to achieve changes in teacher knowledge and belief’ (Joubert and Sutherland, 2008, p. 13).

Askew *et al.* (1997, p. 101) recommend research on ‘exploring changes in teachers' beliefs over time, including the role of different elements in the change process’. The NCETM (2009) recommended that further research is needed to investigate the barriers to engagement with CPD and to investigate aspirations of teachers who do not currently participate in CPD.

Chapter 2 detailed the importance of connected knowledge. Askew *et al.* (1997, p. 101) recommend future research on 'exploring the nature of 'connected' knowledge in more detail'. This agrees with the NCETM (2009 p. 8) who also recommend that research is needed to 'investigate different approaches to engage teachers with students' conceptual development in mathematics'. Askew *et al.* (2010, p. 47) also noted that 'investigation is needed into how teachers can develop classroom tasks that encourage understanding through deeper thinking about mathematical concepts and inter-relationships as well as procedural fluency'.

The NCETM (2009) recommend that further research is needed into the kind of research that is used in CPD, the way it is used and the effects this has on the professional development of the teachers.

While category-based perspectives based on content knowledge and pedagogical content knowledge have received the most visibility in studies of the mathematics teacher, Chapman (2015) suggests that they provide a limited or biased representation of this knowledge. They suggest that ongoing research of this knowledge is needed to reveal details of it and issues associated with it from different perspectives and contexts.

Ball, Ben-Peretz and Cohen (2014, p. 318) acknowledge that 'the teaching profession lacks adequate structure, in the main, to support the development of shared knowledge on a widespread basis'.

3.8 Analytical framework

This section draws together literature and theory already referred to and presents a framework (Figure 14) for the study. This framework draws and builds on other theories and blends them with a mathematical perspective. In Section 3.3.1 it was shown that Clarke and Hollingsworth (1994) proposed six perspectives on teacher change. For this study the last perspective is the one that is used. When teachers work together as learners as part of a community the 'change' is their growth in knowledge and their implementation of this changed knowledge into their practice. The aim of running PD programmes is to support teachers to change in some way, this may involve a change in their knowledge or in how it is enacted. For PD to be 'effective' the change needs to continue over a sustained period.

Within Figure 14, the pink hexagon shows that teacher knowledge might grow because of the input from the external domain. This knowledge would be a specific aspect of MKT in this study, as the aim is an emphasis on teaching for mathematical understanding using the ideas underpinned in the CCC Model.

Guskey's model (Figure 11) has been incorporated with the belief that teachers seeing a change in student outcomes might lead them to continue to experiment and that over time this might change their beliefs and attitudes. Clarke and Hollingsworth's model (Figure 12) has been drawn on to incorporate the fact that the change environment is affected by the personal, external and practice domains. Within this refined model the personal domain is shown in the middle as an individual's motivation to engage is central to whether they engage with the PD input and whether they 'choose' to enact.

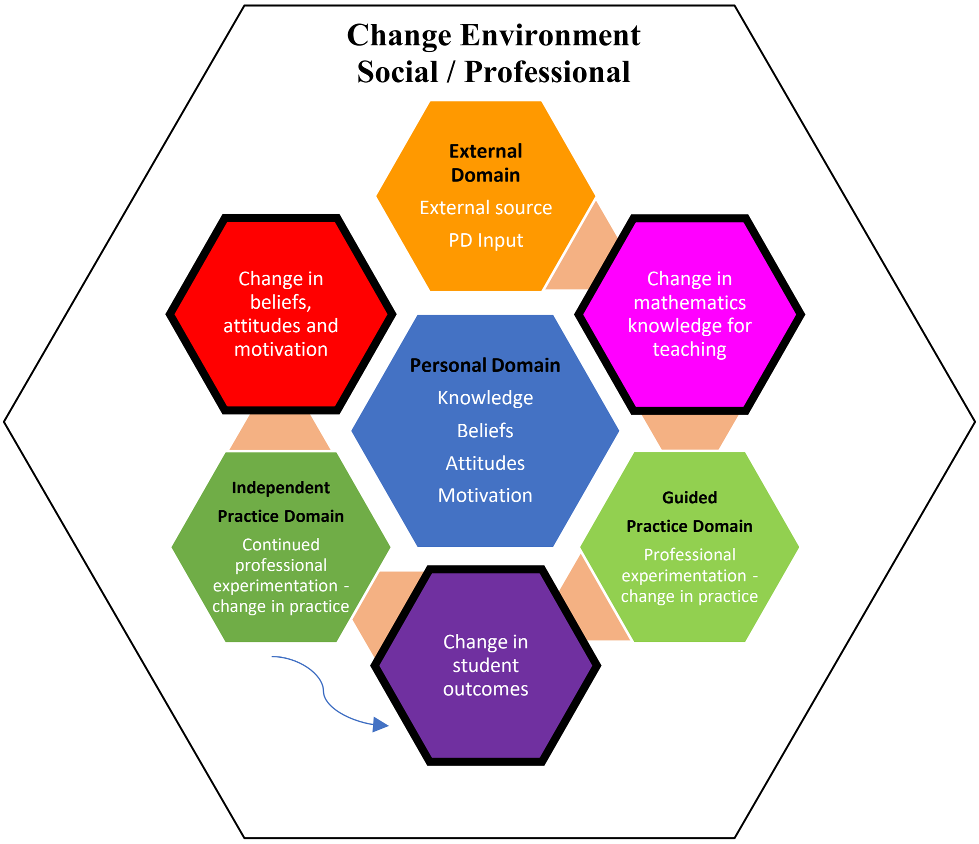


Figure 14. Professional Mathematical Growth Model

The two practice domains have been included within the model to demonstrate that it wouldn't be effective if teachers experimented once and then reverted to previous ways of teaching. Putting into practice the new mathematical knowledge for teaching must be sustained with teachers continuing to experiment. It is suggested that first there is a guided practice domain where teachers can implement new ideas arising from the PD sessions but that to be effective this needs to be embedded in a sustained way with teachers then continuing to change practice without support from the external domain.

For change to happen, it needs to be recognised that teachers are themselves learners and therefore the development of learning theory identified in Section 2.2 has also been incorporated.

> ‘Because of its dynamic and nested character, mathematics-for-teaching cannot be considered a domain of knowledge to be mastered by individuals. It always occurs in contexts that involve others – and, hence, an awareness of how others might be engaged in productive collectivity is an important aspect.’ (Davis and Simmt, 2006, p 309)

This theoretical model of Professional Mathematical Growth has therefore been placed in its entirety within the social and professional change environment. The PD that is being run does not sit in isolation but occurs within the social and professional context of the school and this model reflects that. Effective PD would incorporate teachers supporting each other to enact new ideas and pedagogy so the social dimension to the change environment is imperative.

Coles and Brown (2016, p. 154) reinforce ‘we are quite literally changed through interaction with others, or, more precisely, we change ourselves through interaction with others who likewise change themselves’. They suggest that change may be minimal, but we cannot not change as a result of interacting with others. Shown within the model in Figure 14 the interaction could be between the teacher and the external domain, between the teacher and the learners or in the wider context of the mathematics faculty or the whole school setting.

3.9 Chapter conclusions

This chapter has answered in Section 3.1 what it means to be a professional within the education setting. Section 3.2 defined the different types of professional development and Section 3.3 looked at the process of teacher change.

As this study focusses on mathematics, Sections 3.4 and 3.5 highlight issues specific to the subject. Teachers PCK and more recently MKT has received significant attention in mathematics education research. ‘It has been considered from different perspectives, with different constructs to describe it, resulting in a complex landscape of what it is about and what it entails’ (Chapman, 2015 p. 101).

Combined with Section 3.6 this chapter answers the sub research questions:

- What is meant by the term professional development?
- What factors will contribute to an effective professional development programme?

Section 3.7 adds additional weight to the argument that further research is needed into how CPD might be used to explore connections. Section 3.8 detailed the analytical framework used to support later analysis. The next chapter considers the design of the professional development programme that would be used to support the implementation of the CCC Model to enable teaching for understanding.

CHAPTER 4: DESIGN OF CPD PROGRAMME AND RESEARCH QUESTIONS

4.0 Introduction

This chapter provides a rationale for this research study and outlines the research questions. It draws on the research evidence from Chapter 3 about CPD and looks at themes arising through the literature to inform the design of the CPD programme that was used in this study. It then uses the theoretical framework of the CCC Model from Chapter 2 (Appendix 2.1) and the process of writing and choosing the professional development activities are explained. These tasks were developed and refined during various stages of the pilot study (further details of their refinement can be found in Chapter 6). This chapter sets out to answer the sub research question:

- Q What would an effective CPD programme look like that supports the implementation of the CCC Model?

4.1 Rationale for the research

Chapter 2 showed the literature that led to the CCC Model and Chapter 3 showed that there is a need for further research into several key areas:

1. The nature of CPD and teacher change; including how teacher's beliefs may change over time or what the barriers might be to that change.
2. The nature of CPD and connected knowledge; including investigating different approaches to engage teachers with students' conceptual development and how teachers can develop tasks for use in the classroom to develop deeper thinking around concepts and relationships.
3. The use of research within CPD programmes.

4.1.1 Development of research questions

Within this study the research aim is to explore the notion of teaching for mathematical understanding and how this might be developed through a CPD programme.

Cohen, Manion and Morrison (2011, p. 126) refer to the process of operationalization as an important aspect of effective research question design, 'operationalization means specifying a set of operations or behaviours that can be measured addressed or manipulated'. This process involves translating a very general research aim or purpose into specific concrete questions to be answered. Day Ashley (2012b) recommends an overarching research question which encompasses the set of research questions. The main research question for this study is:

- Q How can a programme of professional development engage and support a mathematics department to teach for understanding?

From this overarching theme, sub questions follow to enable important themes to be addressed. Punch (2009) states that good research questions are:

- clear - they can be easily understood, and are unambiguous;
- specific - their concepts are at a specific enough level to connect to the data connectors;
- answerable - we can see what data are required to answer them, and how the data will be obtained;
- interconnected - they are related to each other in some meaningful way rather than being unconnected
- substantively relevant - they are interesting and worthwhile questions for the investment of research effort (Punch, 2009, p. 76)

Bearing in mind Cohen's process of operationalization and Punch's definitions of research questions, the process of deriving specific concrete sub questions that are clear, interconnected and worthwhile are explained throughout this chapter. These are summarised at the end of the chapter.

4.2 Themes considered when developing a CPD programme

Several themes arise from Chapter 3 that were considered when developing a CPD programme in general. There were also specific areas that needed to be considered when developing a more '**connected**' approach through the implementation of the CCC Model.

4.2.1 The use of a subject specialist

Cordingley *et al.* (2005) acknowledge the importance of using a specialist expert and NCETM (2009) fed back the importance that leaders of CPD are knowledgeable about mathematics education and have recent and relevant experience of classroom practice. In this study, the researcher designed, led and evaluated the professional development sessions as the ‘subject expert’. It was felt that their career background had enabled them to be knowledgeable about mathematics and current work as a teacher meant there was relevant classroom experience. However, there were obvious challenges with being both the researcher and the leader of the professional development programme. This is discussed in Chapter 5.

4.2.2 The use of research

Alongside sharing knowledge from an ‘expert’ is the under researched topic of whether to be explicit about the research underpinning the design of the CPD. ‘Professional development should become characterised by reflective teachers researching their own practice and in engaging with the research of others’ (NCETM, 2008, p. 24).

NCETM (2009) propose that research underpinning CPD should be explained as part of the programme. This is an area they say needs to be investigated further. Within this CPD programme research will underpin the design and will be shared with the department.

4.2.3 Challenge and inspiration

De Geest (2011, p. 3) showed that where professional development was successful, teachers commented that 'it made them think at a high level, it had challenged them which had made it interesting' it had also made them look afresh at things and had inspired them with new ideas. This agrees with NCETM (2009) who said professional development should include stimulating and challenging mathematical activities which get teachers to re-think. The CPD programme incorporated a range of activities that were designed to be both challenging and inspirational.

4.2.4 Moving from CPD to the classroom

Professional development is successful when teachers know how to put the theory into their classrooms, by having concrete examples and practical skills, and had experienced positive responses from the students (De Geest, 2011). It is therefore important that professional development activities model skills and provide practice opportunities in actual settings (Fiszer, 2004; Cordingley *et al.*, 2005). In this study, the professional development took place within the school which enabled teachers to then experiment in their own classrooms. It is important to acknowledge that there is often a great deal more learning to occur after the professional development event when trying to use it in practical situations (Eraut, 2001).

4.2.5 Sustained activity

Forde *et al.* (2006) identified part of the reason for dissatisfaction with CPD provision and delivery was its 'one-off' nature providing few opportunities to develop in a sustained way over time. Not only does there need to be a chance to practise the ideas in the teacher’s own classroom but these experiments need to be sustained over time if it is to directly affect classroom practice (Fiszer, 2004). It is important to develop a culture that supports this on-going professional development. The support not only needs to be provided at the orientation and preparation stage but also throughout the whole implementation process (Eraut, 1995). An adequate amount of time needs to be built into the programme for teachers to try out new ideas and reflect on their learning (NCETM, 2009).

ACME (2002) also acknowledge that there should be opportunities to generalise so that teachers can experiment with their theories and begin to see how they might influence their own practice in the classroom or school. This interchange of new ideas with practice will help the development to be sustained over time and has been incorporated into the CPD programme design.

4.2.6 Collaboration

Section 3.6.3 showed the importance of collaboration within CPD programmes. There is evidence that collaboration between teachers may be more effective in changing practice than reflection and discussion about practice alone (Cordingley *et al.*, 2005; Fiszer, 2004). Cordingley *et al.*’s (2005, p. 11) research, suggests that ‘collaborative CPD is linked with positive outcomes regarding teachers’ attitudes to working and reflecting collaboratively with colleagues on a sustained basis’.

Raywid (1993) also acknowledged that teachers who have been very successful in their careers have the habit of finding time to collaborate. This collaboration may include designing materials together or informing and critiquing one another.

As well as formal collaboration, within this study, there was opportunity to informally get together over break times within the normal school day. Joubert and Sutherland (2008) stated that informal learning is frequently not fully optimised to address the improvement of development of aspects of teacher knowledge and understanding. As the researcher worked alongside colleagues four days a week there was opportunity to support the informal learning that occurred at unplanned sessions.

4.2.7 Personalisation to teachers’ needs

To encourage teachers to be motivated to take part in professional development there needs to be a connection to their own personal needs (Fiszer, 2004). ACME (2002) also believe that CPD programmes should be personalised, to address teachers’ needs and should be designed to support them in developing their own versions of theories or understanding of mathematics and mathematics teaching. De Geest (2011) also found that professional development was successful when teachers could follow their own interests. Whilst the CPD programme was designed to focus on teaching for understanding there were different areas that teachers could choose to focus on when experimenting within their own classrooms.

4.3 Principles informing the design of the CPD

The literature referred to in Sections 3.1 – 3.6 and the themes highlighted in Section 4.2 have guided the principles that informed the design of the CPD programme and the professional development sessions within it. This research study is based on the notion that professional development activities must provide opportunities:

1. to develop teachers' mathematics knowledge for teaching through collaborative working
2. for teachers to be challenged and inspired by new ideas or ways of working
3. for sustained active classroom experimentation
4. to be personalised to the teachers' and department's needs
5. for participants to be supported by a subject expert to engage with research documentation

Building on these principles, a programme was designed that would encompass these ideas.

4.4 The CPD programme for this study

Research informed the initial professional development activity. The use of academic literature and how it informed the development of the CCC Model was shared with the department. There were opportunities to demonstrate what the CCC Model might look like in the classroom. This was followed by different negotiated activities where colleagues were involved for example in choosing which topic area they would like to develop, with the aim of building ownership and a buy-in to the concepts. There was a mix of formal and informal development due to the nature of the researcher's role within the school.

Figure 15 (shown in full in Appendix 2.2) shows an overview of the CPD programme that was implemented within this study.

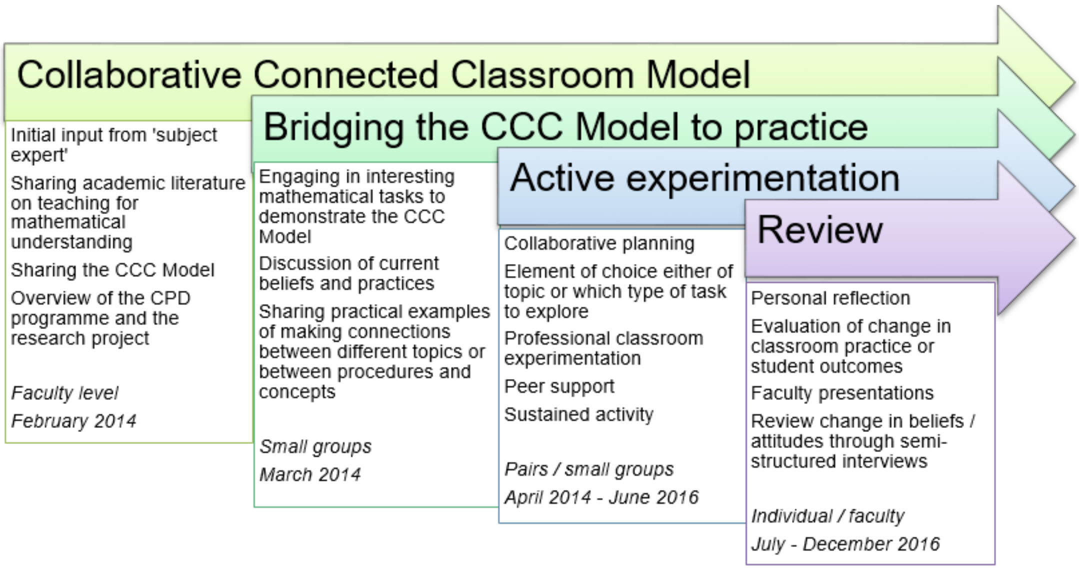


Figure 15. CPD programme for the main research study

4.5 Sharing the CCC Model phase

Section 3.7 acknowledged that the NCETM (2009) recommended further investigation into the way research can be used and the effect that this has on professional development of teachers.

The nature of the role of research within CPD appears to be an under researched topic. Hargreaves (1996, p. 1) reported in an annual lecture for the Teacher Training Agency that 'teaching is not at present a research-based profession'. However maybe times are changing as De Geest (2010), as part of the report for the RECME study, found that three quarters of the CPD initiatives which they studied entailed some form of research in their set-up and running, offering a different picture to the one Hargreaves portrayed in the 1990s.

The NCETM (2009) noted that research is used in many ways whether it is the involvement of a HEI encouraging participants to read literature or whether a CPD initiative is based on resources that have been developed from research.

De Geest (2010) concluded that using research can be an effective means for teachers to become aware of different perspectives about teaching and learning and to become engaged in deep thinking. It was also noted that using research can give status and credibility to the CPD initiative itself and to the teacher.

As this study aimed to involve teachers in working together to develop a deeper understanding of mathematics and connections within it, it was felt appropriate to start the professional development with a session sharing academic literature.

A summary of the academic literature about ‘teaching for understanding’ was put together from the material in Chapter 2 which outlined the historical developments in thinking. This provided the basis of the research session which concluded with sharing the theoretical framework of the CCC Model. This presentation session can be found in Appendix 2.3. The inclusion of this professional development activity enabled the focus on the following research question.

- Q What is the impact of exposing teachers to academic literature within a programme of CPD?

4.6 Bridging the CCC Model to practice phase

Section 3.7 highlighted the need to ‘investigate different approaches to engage teachers with students’ conceptual development in mathematics’ (NCETM, 2009, p. 8). Askew *et al.* (1997, p. 101) also recommended ‘exploring the nature of connected knowledge in more detail’ and Askew (2010) put forward that further ‘investigation is needed into how teachers can develop classroom tasks that encourage understanding through deeper thinking about mathematical concepts and inter-relationships as well as procedural fluency’.

Alongside this, Section 3.4.2 noted that there is the challenge that many teachers lack confidence in mathematics and cannot themselves see the bigger interconnected picture (NCETM, 2008). Therefore, it is important that the professional development provides opportunities to develop teachers' own connected knowledge as well as consider how this can be implemented within classrooms.

> 'Part of any CPD programme should be structured so as to allow opportunities to relate theory to practice in the classroom, and to provide time for informed and collaborative reflection with peers and with those with appropriate expertise.' (ACME, 2002, p. 4)

These ideas have led to the need to research:

- Which approaches will engage teachers with students' development of connections?

The CCC Model (Appendix 2.1) highlights what the nature of mathematical activity might look like within a classroom that is 'teaching for understanding'. Therefore, it is important that these ideas are modelled within the professional development sessions. Activities were chosen or developed to show what the CCC Model might look like in the classroom for each of the bulleted points from the table. Whilst many activities link to various aspects of the CCC Model, it was decided to categorise the activities under the headings to make bridging the CCC Model to practice more explicit for colleagues. Around thirty activities were chosen or developed that could be used with teachers to clarify and make sense of the research documentation of the CCC Model, a rational for a subset are detailed within this section.

Part way through the pilot study another activity was included that immediately followed the research presentation. The aim of this was to use a card sorting task (Appendix 2.4) to give teachers a chance to discuss and clarify for themselves what the 'new terminology' meant. Further through the pilot study an additional task looking at three different scenarios for teaching how to find the area of a parallelogram (Appendix 2.5) was added to bridge the gap between the academic literature and what it might look like in the classroom.

4.6.1 Activities that build on knowledge that learners have by connecting ideas to their current conceptual schema

In early stages of the pilot, an algebraic connection mapping activity was carried out to encourage teachers to collaborate to extend their own conceptual schema. This is detailed in the pilot study chapter. Part way through the pilot study it was decided that it was not realistically possible to design a specific activity that would model the bigger idea of connecting ideas to current conceptual schemas because the teachers within the project all have very different backgrounds, and routes into teaching, so are not coming with the same conceptual schemas. When activities were shown for each aspect of the CCC Model there was chance to reflect on what prior knowledge learners would need to access the tasks.

4.6.2 Activities that connect different areas of mathematics or connect different ideas in the same area using different representations

A key component of CPD is the broadening and deepening of mathematical knowledge and understanding (ACME, 2002). This is to enable teachers to become increasingly aware of key ideas, new ways to promote mathematical reasoning, different representations and links within mathematics, as well as links to other subjects where mathematics plays a role. De Geest (2011) identified that teachers who found professional development successful engaged in thinking at a high level, which had challenged them and therefore made it interesting. They also had the opportunity to look at things afresh and were inspired with new ideas.

With these ideas in mind the researcher chose to explore the concept of straight line graphs using several different avenues. Straight line graphs are a topic that spans all secondary key stages, so all teachers had some understanding of it; however, the task enabled teachers to consider arriving at equations in different ways. The first activity, ‘linear equations’ (Figure 16), was chosen due to its open nature to see how teachers attempted to find the equation for the given line. This enabled discussion around methods chosen and whether they were instrumental or relational in their nature.

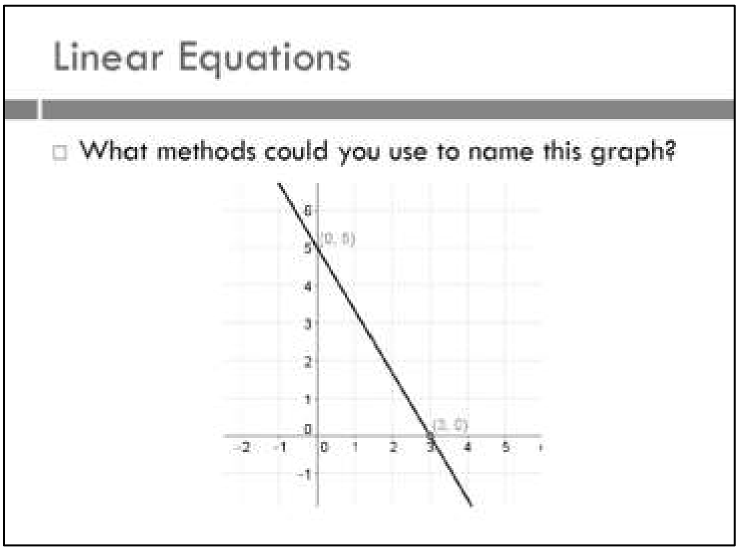


Figure 16. Linear equations

The second activity, ‘algebraic and graphical representations’ (Figure 17), then considered the same activity from a different perspective.

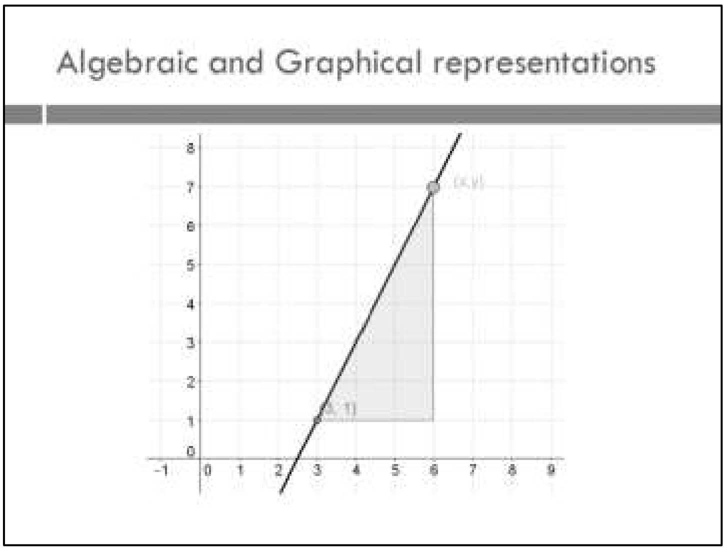


Figure 17. Algebraic and graphical representations

The point (x, y) slides along a line of fixed gradient making the connection with the bigger picture of rates of change and calculus.

The third activity, ‘algebraic and geometric representations’ (Figure 18), considered looking at the problem from a geometric point of view and deriving an expression for the right-angled triangle contained within the first quadrant in two different ways. This was then extended to consider what happens if the variable point is not contained within the first quadrant.

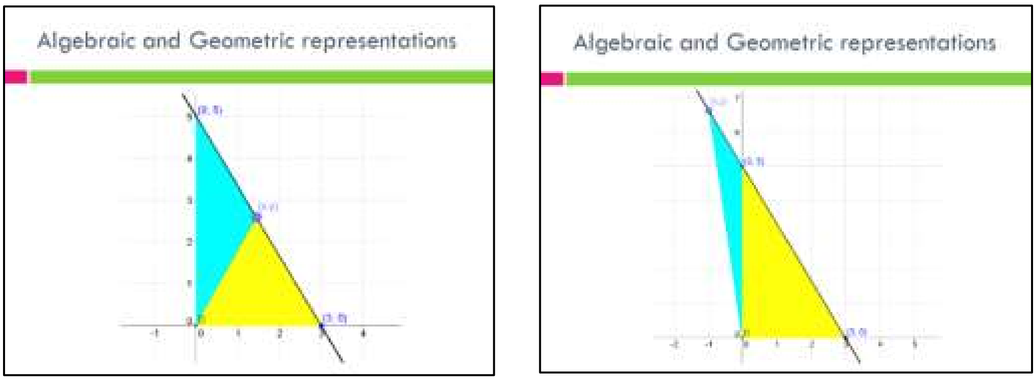


Figure 18. Algebraic and geometric representations

The activity from Swan (2005), ‘interpreting algebraic expressions’ (Figure 19), was chosen as an example of connecting different ideas in the same area of mathematics. The activity provided an opportunity to translate between different representations to deepen understanding of the concept of algebraic expressions.

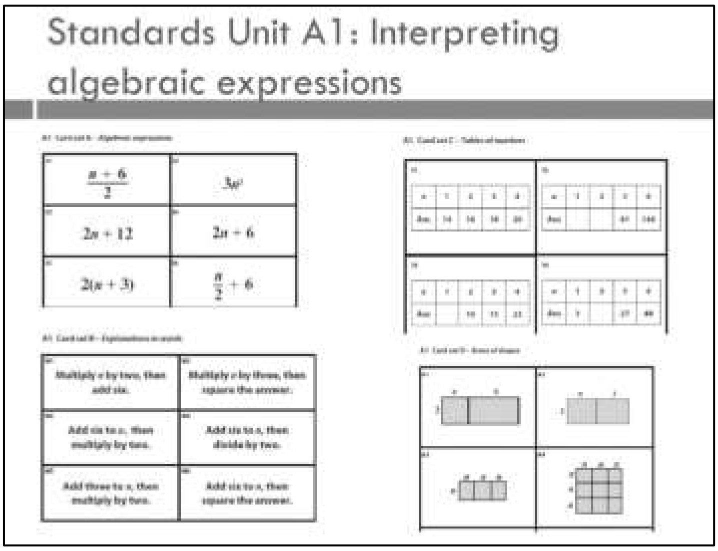


Figure 19. Interpreting algebraic expressions

The ‘images of fractions’ poster (Figure 20) was chosen to show how the CCC Model could be applied to a contrasting topic.

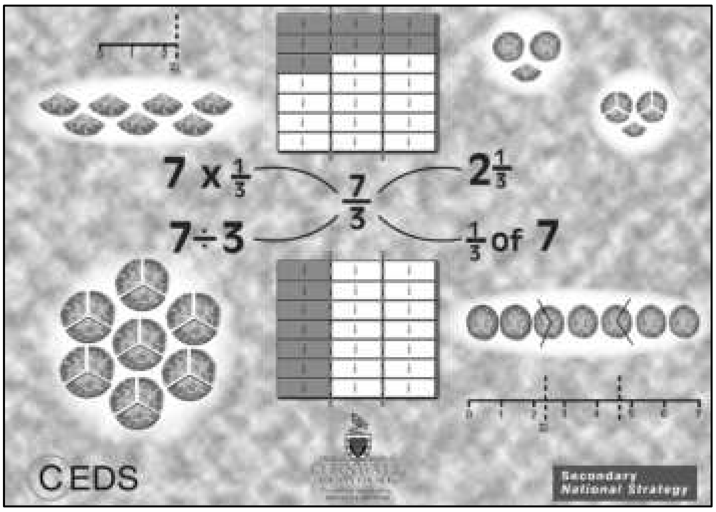


Figure 20. Images of fractions

This poster was developed by the researcher as part of their work for the local authority when working in conjunction with the National Strategies.

4.6.3 Activities that make links between procedures and concepts

Section 2.1.3 considered the interplay between procedures and concepts and whilst the general conclusion was there will be times when Kadijevich and Haapasalo's (2001) **educational approach** (concepts considered before arising at procedures) is better, there will be other topics when the **developmental approach** (procedural knowledge is used then reflected on) is more effective. The CPD programme planned to show activities that demonstrated the possible links and then encourage discussion and evaluation with colleagues as to which approach might be beneficial for different topics. For example, the activity 'sum of natural numbers' (Figure 21) was an opportunity to think about the concept of summation alongside different visual representations before arising at the procedure of the rule $\sum_1^n n = \frac{n(n+1)}{2}$.

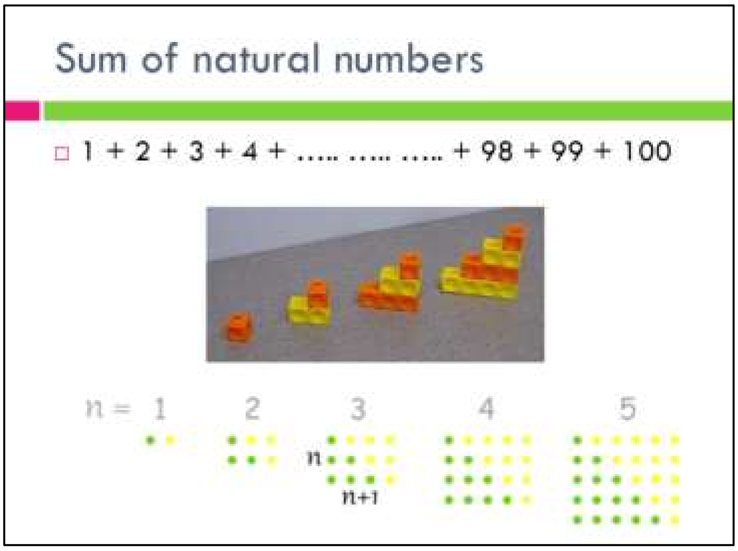


Figure 21. Sum of natural numbers

To show the concept of evaluating procedures the topic of percentages was chosen. This is often taught in a procedural way i.e. to find 23% divide by 100 to find 1% and then multiply by 23. This task (DfES, 2005) gave rise to the opportunity to reflect on the bigger picture of how percentages are just one aspect of direct proportion and can be linked to scale factors and the visual images of number lines and straight-line graphs through the origin (Figure 22).

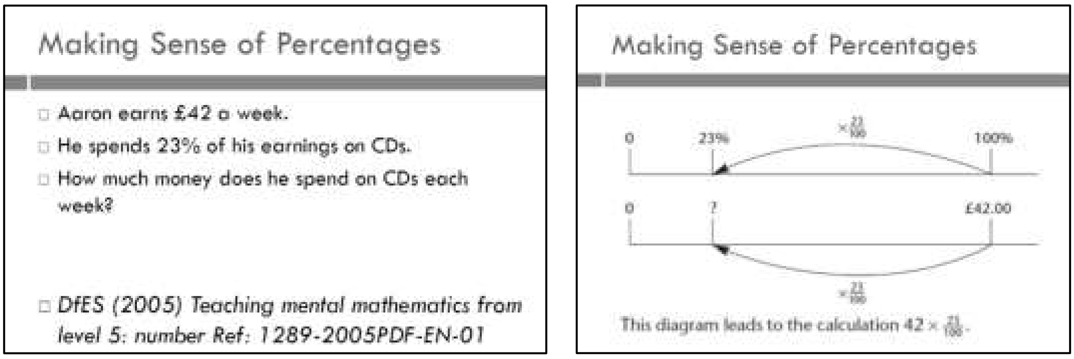


Figure 22. Making sense of percentages (DfES, 2005)

4.6.4 Activities that make comparisons

To develop conceptual understanding 'the teacher must encourage learners to notice 'sameness' and 'difference', examine alternative representations and share and discuss interpretations' (NCETM, 2008, p. 18).

The following quick starter activities were designed to engage discussion and a deeper understanding of quadratic equations. The activity 'quadratic graphs' (Figure 23) enables links to be drawn out with the nature of the roots alongside the symmetry and orientation properties of the quadratic. The activity 'quadratic equations' (Figure 24) was designed to make links between factorised and expanded forms and how these can be used to solve equations. There is a non-unit coefficient of x^2 to enable students to discuss different strategies that would be needed to solve this and a difference of two squares example was added to further the discussion.

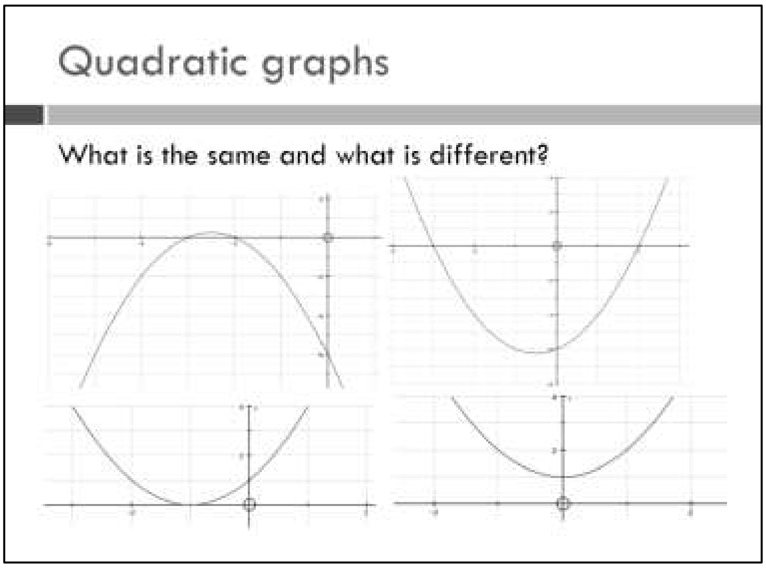


Figure 23. Quadratic graphs

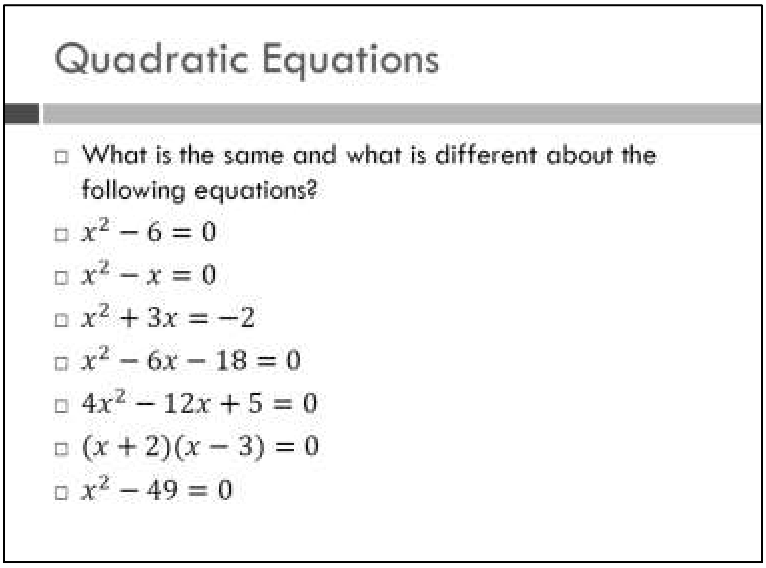


Figure 24. Quadratic equations

The other activity that was chosen was from Teaching Mental Mathematics from Level 5 (DfES, 2006). The ‘find as many ways as you can’ activity (Figure 25) is about finding area in several different ways. This rich task provides opportunities for learners to consider which would be the most efficient way to find the answer.

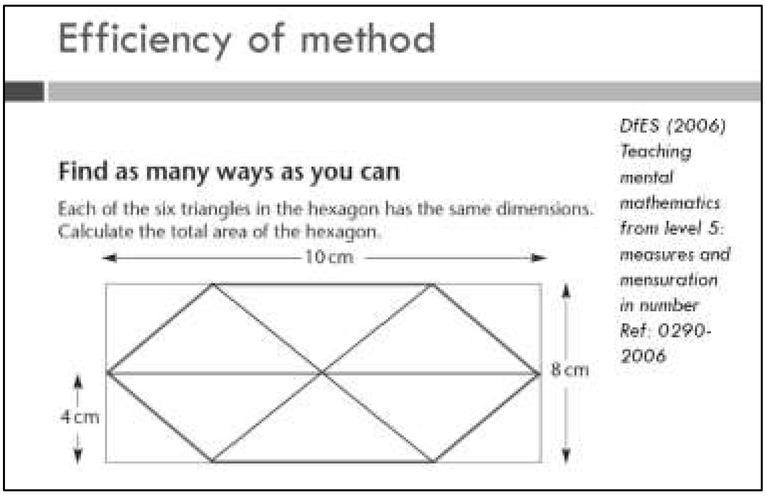


Figure 25. Find as many ways as you can

4.6.5 Activities that are application tasks, presented as challenges that may be problematic and need to be reasoned about

This NCETM task (NCETM, 2012b) 'factorisation' (Figure 26) was chosen as a problem that is potentially challenging for the teachers within the professional development session.

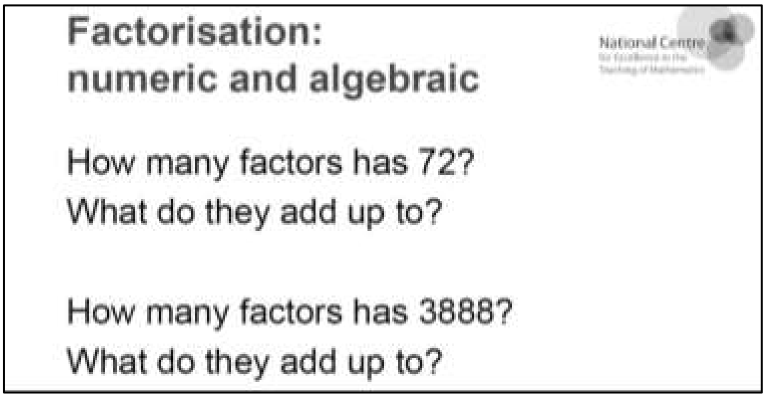


Figure 26. Factorisation numeric and algebraic (NCETM, 2012b)

There is no obvious procedure to tackle it; however, the solution relies on a more relational understanding of products of factors and connections within the structure of arithmetic.

Alternatively, involving only an understanding of areas of triangles, the activity ‘cutting hexagons’ (Figure 27) was chosen. Again, it was not obvious initially what the answer might be so reasoning was needed to challenge correct and incorrect ideas that arose. It demonstrates that a relational approach of understanding of perpendicular heights is important.

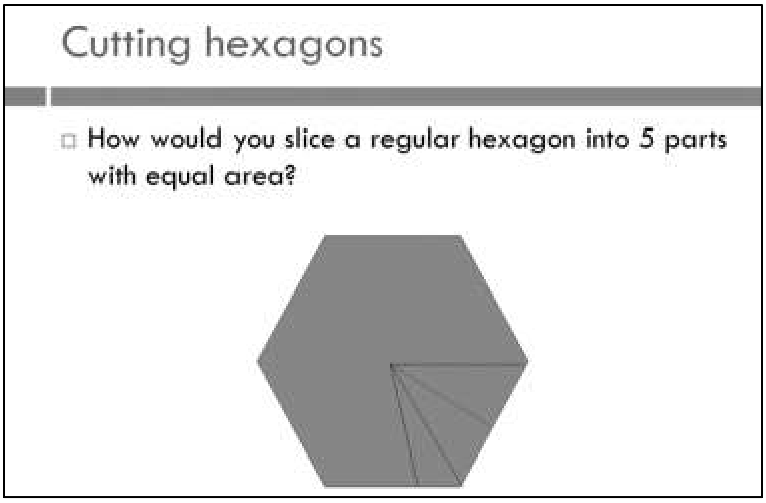


Figure 27. Cutting hexagons

A summary of the full presentations that were used during the bridging the CCC Model to practice phase are shown in Appendices 2.6 and 2.7.

4.7 Active experimentation phase

Fiszer (2004, p. 2) recommends that 'effective professional development should ensure follow-up to the ideas discussed where collaboration, testing of selective ideas, and reflective practice are involved'. Unfortunately these needs of teachers are not evident in typical professional development programmes (Fiszer, 2004). The design of this CPD programme aimed to give teachers a chance, during the active experimentation phase, to explore how the ideas from the research and exploratory sessions could be put into practice.

To encourage as much engagement as possible from across the department there were chances for teachers to reflect and explore professionally within their own classroom, aspects that they found interesting or would like to take further. Examples of these were: developing tasks that explore similarities and differences; exploring topics that could be taught in a more conceptual way; making links between different areas of mathematics. Appendices 2.8, 2.9 and 2.10 show the additional inputs that were used with the department during the active experimentation phase of the CPD programme. It was planned for the researcher to work with small groups of teachers as they collaborated on their chosen theme to provide support and guidance as they developed ideas, with a focus on the research questions:

- Which approaches will teachers explore with students?
- How can teachers develop tasks that encourage connections to be made?

4.8 Review phase

Section 3.7 highlighted the need for further research into different elements in the change process (Askew *et al.*, 1997) and to investigate barriers to engagement with CPD (NCETM, 2009). It was thought that, during the active experimentation phase, there may be teachers that chose to opt out or might only explore a particular area, so time was built in to reflect and review the process. This enabled gathering of data to answer the research questions:

- Does the professional development result in teacher change?
- If so which elements support the process of change?
- What are the barriers to engagement with the CPD programme?
- What are the barriers to engagement with the CCC Model?

Whilst the research was looking at implementation of the CPD programme with a whole department, it was anticipated that teachers would change in different ways and therefore this would need to be explored in more depth to answer the following sub research questions:

- Are there any differences in how teachers develop across the mathematics department?
- What has influenced these differences?

4.9 Conclusions and summary of the research questions

This chapter has drawn on the research literature from Chapters 2 and 3 and provided a rationale for the need of this study with the overarching research question being:

- How can a programme of professional development engage and support a mathematics department to teach for understanding?

Using a range of academic literature, the professional development programme was derived (Appendix 2.2) which was then used to support the design of individual presentations and tasks (Appendices 2.3 – 2.11) which answer the research question:

- What would an effective CPD programme look like that supports the implementation of the CCC Model?

The design of the CPD programme and the tasks within the professional development sessions enabled the need for exploration of further interconnected sub questions:

- Does the professional development result in teacher change? If so which elements support the process of change?
- What is the impact of exposing teachers to academic literature within a programme of CPD?

- What are the barriers to engagement with the CPD programme?
- Which approaches will engage teachers with students' development of connections?
- Which approaches will teachers explore with students?
- How can teachers develop tasks that encourage connections to be made?
- What are the barriers to engagement with the CCC Model?
- Are there any differences in how teachers develop across the mathematics department? If so what has influenced these differences?

The research questions from the study are summarised in Appendix 3.1.

CHAPTER 5: RESEARCH METHODOLOGIES AND METHODS.

5.0 Introduction

Hedges (2012, p. 23) states 'research design is the organisation of data collection so that the data collected will support unambiguous conclusions about the problem being studied'. This chapter discusses the research methodology and the tools used within this study. It provides details on the design of the research study with the crucial objective being to ensure transparency of the process (Hedges, 2012).

The first section considers the different types of research study and the process that was undertaken when developing research questions stated in Chapter 4. The second section outlines the rational for using a case study approach and considers the advantages and disadvantages of this methodology for this study. Within this section the important factors of reliability, validity and trustworthiness are considered.

The third section describes the research instruments used and the methods of analysis that were employed. Within this study the research instruments were interviews, learning walks, book scrutinies, analysis of audio recording of professional development sessions, presentations from teachers and notes from informal discussions.

The fourth section defines key terms such as codes, coding and themes and sets out how the data was analysed as the study progressed. The fifth section gives a rationale for how the data is presented. It explains the need for summary profiles of individuals, then for organisation by theme and finally the need for presenting data to answer the research questions.

5.1 Research methodology

Cohen, Manion and Morrison (2011) suggest that research design is governed by the notion of fitness for purpose. This section provides a theoretical and philosophical justification of why the research study was designed in the way it was and why certain methods were chosen.

5.1.1 The research paradigm

Creswell (2013, p. 15) states that ‘whether we are aware of it or not, we always bring certain beliefs and philosophical assumptions to our research’. These beliefs could arise from our own educational training or from discussions with other scholarly communities. Creswell (2013) identifies that the difficulty arises in first becoming aware of our own assumptions and beliefs and then in whether we actively incorporate them into our qualitative studies. When the study started, the researcher was not aware of her own philosophical beliefs or how they might impact on the study.

Punch (2009) identifies two main ways of planning a research project. The first is to start with a paradigm, to articulate it and then to develop research questions and methods from it. The second is the pragmatic approach where you begin with the research questions that need answers and from these choose methods to answer them.

For this study, the literature review and personal experience led the researcher to believe that 'teaching for understanding' is important and to want to explore how this can be developed with colleagues and other teachers. Therefore, this study followed the pragmatic approach where appropriate methods were chosen to answer the research questions (Appendix 3.1).

5.1.2 Qualitative versus quantitative research

Qualitative research is a general term. Lichtman (2013, p. 7) suggests it is a way of knowing in which 'a researcher gathers, organizes, and interprets information obtained from humans using his or her eyes and ears as filters'. Qualitative research can be contrasted with quantitative research, which 'relies heavily on hypothesis testing, cause and effect, and statistical analyses'. The main purpose of qualitative research is 'to provide an in-depth description and understanding of the human experience' (p. 17).

Denzin and Lincoln (1998) provide a generic definition of qualitative research as one that involves an interpretive, naturalistic approach to its subject matter, where the researcher studies things in their natural settings and attempts to make sense of and interpret the meanings people bring.

Since the situation was to be studied in its entirety, in the natural school setting, rather than just looking at specific pre-determined variables mainly qualitative methods were used. This permitted ‘inquiry into selected issues in great depth with careful attention to detail, context, and nuance; so that data collection need not be constrained by predetermined analytical categories’ (Patton, 2002, p. 227).

In Section 5.1.1, it was stated that the researcher initially was unaware of their own philosophical beliefs. Since working on the pilot study and considering ontological and epistemological beliefs Creswell’s (2013) philosophical assumptions that underpin the nature of qualitative research are a useful summary that aligns with how the study was approached. The characteristics and implications given in Figure 28 are demonstrated within the main study for this thesis.

Assumption	Questions	Characteristics	Implications for Practice (Examples)
Ontological	What is the nature of reality?	Reality is multiple as seen through many views	Researcher reports different perspectives as themes develop in the findings
Epistemological	What counts as knowledge? How are the knowledge claims justified? What is the relationship between the researcher and that being researched?	Subjective evidence from participants; researcher attempts to lessen distance between himself or herself and that being researched	Researcher relies on quotes as evidence from the participant; collaborates, spends time in the field with participants, and becomes an 'insider'
Axiological	What is the role of values?	Researcher acknowledges that research is value-laden and that biases are present	Researcher openly discusses values that shape the narrative and includes his or her interpretations of participants
Methodological	What is the process of research? What is the language of research?	Researcher uses inductive logic, studies the topic within its context, and uses an emerging design	Researcher works with particulars (details) before generalizations, describes in detail the context of the study, and continually revises questions from experiences in the field.

Figure 28. Philosophical assumptions with implications for practice (Creswell, 2013, p. 215)

5.1.3 The research questions

The research design is the logic that links the data to be collected (and the conclusions to be drawn) to the questions of the research study stated in Appendix 3.1. The research questions are 'the one component that directly links to all the other components of the design' (Maxwell, 1995, p. 65).

However, different types of study suggest the use of research questions in different ways. Maxwell (1995) points out that often during qualitative studies researchers must go into the study with an open mind and see what needs to be investigated. It is important that the research begins with goals and a base of experience and theoretical knowledge which will highlight issues and generate questions about these. The goals of wanting to explore the effect of CPD on teaching for understanding were set at the outset but research questions were not set at the start of the study.

Maxwell (1995, p. 65) acknowledges that ‘qualitative researchers often don’t develop their eventual research questions until they have done a significant amount of data collection and analysis’ and that ‘often well-constructed, focussed questions are generally the result of an interactive design process, rather than being the starting point for developing a design’ (p. 66). Within this study there were provisional questions that guided the study and the way data was collected however these were refined as the study progressed to the final version in Appendix 3.1.

5.2 Case study methodology

The overarching theme of this study is to find out how a group of teachers, through professional development, can develop their teaching for mathematical understanding. In general Yin (2014) suggests that case studies are the preferred method when “how” or “why” questions are being posed. This section outlines the nature of using a case study approach and considers the advantages and disadvantages of this for this research study.

5.2.1 The rationale for the use of a case study methodology

Yin (2014) defines a case study with a two-fold definition; the first part begins with the scope of a case study and the second with a technical definition.

1. A case study is an empirical inquiry that
    - investigates a contemporary phenomenon (the "case") in depth and within its real-life context, especially when
    - the boundaries between phenomenon and context are not clearly evident.
2. The case study inquiry
    - copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result
    - relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result
    - benefits from the prior development of theoretical propositions to guide data collection and analysis (Yin, 2014, p. 16-17)

Creswell (2013, p. 97) defines case study research as 'a qualitative approach in which the investigator explores a real-life contemporary bounded system (a case) or multiple bounded systems (cases) over time, through detailed in-depth data collection involving multiple sources of information'.

In this situation, the case that is being researched is the mathematics department where the researcher is employed, and the contemporary phenomenon is the study of their response to the implementation of the CPD programme. The case is bounded in two ways firstly by context and the people that are involved and secondly by the time frame under which the CPD programme was run.

The investigation is mainly qualitative with the main source of data coming from audio recordings of professional development sessions and transcriptions of interviews. The case study methodology lends itself most appropriately to this type of investigation. Yin (2009) identifies four different applications of using a case study approach.

> 'The most important is to explain the presumed causal links in real-life intervention that are too complex for the survey or experimental strategies. A second application is to describe an intervention and the real-life context in which it occurred. Third case studies can illustrate certain topics within an evaluation, again in a descriptive mode. Fourth the case study strategy may be used to enlighten those situations in which the intervention being evaluated has no clear, single set of outcomes.' (Yin, 2009, p. 19-20)

The appropriate application in this situation is the second one referred to above. The intervention is the CPD programme and the real-life context is the study of the mathematics department where the CPD programme is being implemented. The case study method allows the holistic and meaningful characteristics of real-life events to be retained (Yin, 2009).

5.2.2 Advantages and disadvantages of a case study methodology

There are several advantages to using a case study. Day Ashley (2012, p. 102) outlines that 'the strength of case study research lies in its ability to enable the researcher to intensively investigate the case in-depth, to probe, drill down and to get at its complexity, often through long term immersion in, or repeated visits to/encounters with the case'.

In this study, it was important to have repeated visits with the case as the CPD programme continued for a couple of years, and that the uniqueness of real individuals and situations through accessible accounts were investigated and interpreted to ‘catch the complexity of situatedness and behaviour’ (Cohen, Manion and Morrison, 2011, p. 129).

Cohen, Manion and Morrison (2011) identify that it is important in case studies for events and situations to be allowed to speak for themselves, rather than to be largely interpreted, evaluated or judged by the reader. Nisbet and Watt (1984) identify weaknesses of case study research to be that the results may not be generalisable and that they are not easily open to cross checking so may be selective, biased or subjective. These areas are addressed in Sections 5.2.4 - 5.2.6.

5.2.3 Defining the case

Case study research begins with the identification of a specific case which may be a concrete entity such as an individual, a group or an organisation or at a less concrete level maybe a community, a relationship, a decision process or a specific project (Creswell, 2013). When selecting a case, Lichtman (2013) suggests that you consider the typical, the exemplary or model, or the unusual or unique.

In this study, the case chosen is deemed to be typical. The case is defined to be the implementation of a CPD programme with the mathematics department at the secondary school where the researcher works. The department is ‘typical’ for several reasons. The profile of the department includes a range of ages, experiences and qualifications that would be like other large secondary comprehensive school.

The researcher joined the school as an Advanced Skills Teacher (AST) at the same time as beginning the research study. On arriving at the school there was the acknowledgement that, although there were no major issues or concerns in terms of performance or attainment of the department, improvements could be made to encourage colleagues to work collaboratively to further develop attainment of students. Therefore, the role as an AST was to support the head of department (new in post) with leading on the development of teaching and learning within the mathematics department.

In relation to this specific case looking at ‘teaching for understanding’ the department would be described as typical. There were some teachers that were already using approaches, such as multiple representations, from documents like the Swan (2005) report and other teachers that taught in a more procedural way.

Although the whole department (the single case) were involved in the CPD programme there would also be the need for incorporated subunits of analysis. These subunits can add opportunities for ‘extensive analysis, enhancing the insights into the single case’ (Yin, 2009, p. 52). In this case these subunits refer to individual teachers within the team that might be exemplary, unusual or special in some way.

Lichtman (2013) identifies that with qualitative research there is often not sufficient breadth to make generalisations so ‘it is not important to get a case that represents all other cases. Your goal is to get detailed and rich descriptions of the case you select’ (p. 92).

5.2.4 The role of the researcher

In a qualitative study the role of the researcher needs to be considered and the dilemma of trying to be unbiased and objective. The researcher plays an important role in the process. Data is collected, information is gathered, and realities are constructed through their eyes and ears (Lichtman, 2013). There is potential for 'observer bias' since the method involves some sort of subjectivity (Angrosina, 2012).

All researchers will have a research position and whether an insider or an outsider there will be different strengths and weaknesses. In this study, the researcher was a teacher in their own school. This has obvious advantages and disadvantages. Planning of research needed to consider this 'position' as an insider.

Angrosina (2012, p. 166) uses the term participant observation where researchers are active members of the group they are observing, acknowledging that is it important to 'become enough of a member of the group to gain an insider's perspective on what is going on, without losing the credibility of the objective scientist'.

Punch (2009) identifies advantages as: convenience; access to the research situation and consent is often easier; the relevance of connecting research to your own professional issues and the fact that teacher-researchers studying their own school can bring an insider's understanding to the social, cultural and political aspects. Within this study these advantages identified by Punch (2009) were apparent and supported analysis.

Punch (2009) also identifies possible disadvantages to be: the insider knowledge has potential to bring bias and subjectivity; the teacher-researcher may have a vested interest in the results 'this is especially possible when a new or different method of teaching - perhaps teacher-developed - is the focus of the research' (p. 44); the challenges in generalizability, for example positive results about a new method may be down to the commitment of the teacher as much as the method itself and the challenge between what is research data and normal professional data. Within this study these had to be considered as the researcher had designed the CPD programme and there was therefore a vested interested in the successful implementation. This point is discussed further in the analysis within Chapter 9.

Coe (2012, p. 46) acknowledges that it is important that the interpretation is not influenced by other spurious or inappropriate features of the research process. 'Different perspectives or beliefs may lead to different interpretations; in order to make sense of a particular interpretation we may need to understand the perspective and beliefs that led to it and how they might have been influential'.

Coe (2012) identifies bracketing as an approach to do this. Bracketing is a process where:

> ‘The researcher attempts to identify, state, suspend or disassociate from the research process aspects such as their own ontological and epistemological positions and theoretical frameworks, suppositions based on the researcher's personal knowledge, history, supposition culture, assumptions, beliefs, experiences, values and viewpoints, supposition base on the academic and scientific theoretical orientation and theories, and pre-existing assumptions about the phenomenon being investigated.’ (Coe, 2012, p. 47)

Whilst at the beginning of the pilot stage the researcher was unaware of their own position and the theoretical framework that was worked within, the process of engaging with the pilot informed understanding. The researchers’ ontological and epistemological position is stated within Sections 5.1.1 and 5.1.2 as part of the bracketing approach identified by Coe (2012).

5.2.5 Reliability, validity and trustworthiness in case studies

Yin (2014) suggests that when designing a case study, the quality can be maximised through four critical conditions (a) construct validity, (b) internal validity, (c) external validity, and (d) reliability.

Validity is used to judge whether the research accurately describes the phenomenon that it is intended to describe (Bush, 2007). Internal validity relates to the extent that research findings accurately describe the phenomenon being investigated whereas external validity relates to the extent that findings can be generalised to a wider setting.

Yin suggests that you can increase reliability by maintaining a chain of evidence so that an external reader can 'follow the derivation of any evidence from initial research questions to ultimate case study conclusions' (2014, p. 127).

Instead of reliability and validity, Bassey (2007, p. 144) uses the term trustworthiness as he says that 'reliability is an impractical concept for case study since by its nature a case study is a one-off event and therefore not open to exact replication'. One way he suggests is providing evidence so that others can examine the evidence and check for trustworthiness, by what he calls an audit trail where a flow chart of the data, the analysis and the interpretation is kept. Using NVIVO software enabled a trail of data interpretation to be kept.

Bassey (2007) suggests several tests for trustworthiness in case study research. These include ensuring prolonged engagement with the data with persistent observation of emerging issues. He recommends that there is sufficient triangulation of data that leads to any analytical statements. Any hypothesis, evaluation, or emerging story needs to be systematically tested and it is advantageous to use a critical friend to challenge findings.

These principles to increase reliability / trustworthiness were employed throughout the study. An independent researcher that didn't know the department was involved in reading interviews and checking conclusions were reliable. The teachers involved were also involved in their own self-reflection against the Teacher Development Model (TDM) that was generated within the study.

5.2.6 Generalisability of the case study approach

If a researcher conducts a study with a particular group of participants in a particular context on a particular occasion, then the claims that can be made about the interpretation of what was observed must be validated (Coe, 2012).

While single case-studies are rarely generalisable, Dressman, Journell and Mann (2012 p. 186-7) identify ‘a need for researchers to make greater efforts to improve the validity of their interpretations through repeated studies of similar cases or by taking a longitudinal approach that studies the same case over an extended period of time’.

Within this study the longitudinal approach was taken. However, case studies are generalisable to theoretical propositions and not to populations or universes.

> ‘In this sense, the case study, like the experiment, does not represent a “sample,” and in doing a case study, your goal will be to expand and generalize theories (analytic generalization) and not to extrapolate probabilities (statistical generalization).’ (Yin, 2014, p. 21)

Coe (2012) suggests that if we claim that the phenomenon, interpretation or inference has applicability or meaning beyond this context that have been directly studied then we are making what is called a transfer claim.

5.2.7 Ethics and informed consent

It was ethically important that informed consent was gained from participants in both the pilot and the main study. Cohen, Manion and Morrison (2011) say this requires an explanation of the purposes of the research, the right to withdrawal, the rights and obligations to confidentiality and opportunities for participants to ask questions about any parts of the research.

One difficulty that needed to be considered in this case was brought up by Bassey (2007, p. 144) ‘the closer that one comes to the people being studied the more important it is to ensure that they are willing to be studied and that what they say or do is reported in such a way that is not prejudicial to their best interests’. This is echoed by Lichtman:

> ‘Individuals participating in a research study have a reasonable expectation that they will be informed of the nature of the study and may choose whether or not to participate. They also have a reasonable expectation that they will not be coerced into participation. On the face of it, this might seem to be relatively easy to follow. But if a study is to be done in an organization, individuals within that group (e.g. students, workers) might feel that they cannot refuse when asked. There might be pressure placed on them by peers or superiors.’ (Lichtman, 2013, p. 53)

This was a challenge in this situation as the CPD programme was planned for all teachers in the department as part of the plans to develop collaborative work involving improving teaching and learning.

With this in mind a research information sheet was designed and given to all participants in the study (Appendix 3.2). It was made clear to the department that whilst the whole department would be involved in the professional development activities that participation in the research study (i.e. for comments and interviews to be included in the analysis) was voluntary and that teachers could withdraw from that part of the process at any time.

Throughout this research study guidelines of the ethical principles of the University of Plymouth have been followed and the study was approved by the Ethics Committee of the Faculty of Science and Engineering. All participants were asked to complete a consent form as shown in Appendix 3.3.

5.3 Research methods

This section outlines a description in practical terms of how data was collected. It explains why participants were chosen and why the methods of data collection were chosen.

5.3.1 The use of interviews as a research method

Atkins and Wallace (2012, p. 86) describe interviews as a 'flexible research tool which can be used to gather a range of different types of information, including factual data, views and opinions, personal narratives and histories' which makes them useful for answering a wide range of research questions.

Forsey (2012, p. 364) acknowledges that the research interview provides an opportunity for ‘creating and capturing insights of a depth and level of focus rarely achieved through surveys, observational studies, or the majority of casual conversations help with fellow human beings’.

It was important within this case study that data that was gathered was both factual and involved teachers’ opinions with depth that wouldn’t be achieved by other research methods. Interviewing for research is different from common conversation and requires design, preparation, purposeful conduct and attentive listening.

> ‘In depth interviews are purposeful interactions in which an investigator attempts to learn what another person knows about a topic, to discover and record what that person has experienced, what he or she thinks and feels about it, and what significance or meaning it might have.’ (Mears, 2012, p. 170)

5.3.2 Advantages and disadvantages of using interviews

One main advantage of using an interview is the opportunity for dialogue means that ‘the interviewer can press not only for complete answers but for responses about complex and deep issues’ (Cohen, Manion and Morrison, 2011, p. 409).

This direct interaction can be both an advantage and a disadvantage. The advantage is it allows for in-depth discussion as mentioned in the previous section, however it can be prone to subjectivity and bias on the part of the researcher. For the main study, it was decided that the researcher would not carry out the interviews to avoid potential bias, the rationale for this is explained in the Chapter 6 pilot study.

One disadvantage of interviewing is the process of transcribing and analysing, which Atkins and Wallace (2012) acknowledge can be time consuming.

5.3.3 The semi-structured interview

There are a wide range of defined interview types. Patton (2015) classifies three approaches by informal conversational interview, the interview guide, the standardised open-ended questions interviews. Oppenheim (1992) splits them into essentially two different kinds: exploratory and standardised. Newby (2009) classifies individual interviews into structured, semi-structured, evolving in depth and evolving cognitive whereas Cohen, Manion and Morrison (2011) focuses on the classifications of structured, unstructured, non-directive and focused. What is clear is that these different types of interviews have different purposes and need to be carried out in different ways.

Kvale (1996) argues that interviews differ in their openness of their purpose, their degree of structure, the extent to which they are exploratory or hypothesis-testing, whether they seek description or interpretation, or whether they are largely cognitive focused, or emotion focused.

This study used a semi-structured interview that was exploratory in its nature. The advantages of using a semi-structured approach according to Newby (2009) are that it reflects the research question, it can be used to clarify any misunderstanding. It also allows questioning to explore the issues and should reveal rich data. The disadvantages of this approach are that it is more time consuming and if a large sample were to be used interviewers would need to be trained which would result in a high cost.

Cohen, Manion and Morrison (2011) suggest that the schedule for a semi-structured interview should contain information on the topic to be discussed, specific possible questions to be included in each topic, the issues within each topic, together with some possible questions about each issue and a series of prompts and probes to use throughout for each topic, issue and question.

> ‘Probes are used to deepen the response to a question, increase the richness and depth of responses, and give cues to the interviewee about the level of response that is desired.’ (Patton, 2015, p. 407)

Kvale (1996) suggests the interviewer’s questions should be straightforward and brief even if the responses may not be. During the ideal interview, the interviewer will attempt to verify any interpretations that have occurred. Patton (2015) suggests that these probes should be natural and conversational and used to follow up initial responses.

Patton (2015) recommends that information is provided prior to interview and then again at the beginning of the interview. These informed consent protocols typically cover the following:

- What is the purpose of collecting the information?
- Who is the information for? How will it be used?
- What will be asked in the interview?
- How will responses be handled, including confidentiality?
- What risks and or / benefits are involved for person being interviewed? (Patton, 2015, p. 497)

Considering the points from Sections 5.3.1- 5.3.3 a pre-interview script was designed which can be seen in Appendix 3.4. Participants were informed of the nature and purpose of the interview, it was deemed important to be honest without risking bias to strive to put the participant at ease (Cohen, Manion and Morrison, 2011).

As the study progressed the research questions were refined through the iterative process and that the interview questions were revised to support emerging theories.

5.3.4 Recording and transcribing the interviews

There are many ways to record an interview including note-taking, audio recording and video recording. Oppenheim (1992) suggests that it is essential for exploratory interviews to be recorded so that they can be analysed in depth afterwards. If recorded, this also gives the opportunity for the interview to be examined by more than one person so to avoid bias where possible. This agrees with Hammersley (2012) who notes that since 'the data' are preserved and can be reproduced it means that they are open to repeated analysis, and furthermore can be made available to readers of research reports so that analysis can be checked and replicated by others.

For this study, it was decided to use the method of audio recording. The advantages of this as detailed by Atkins and Wallace (2012) are that it enables the entire interview to be captured which then allows for careful review of data from a complete transcription. However, one disadvantage is there is a possibility that technology could fail resulting in no record at all. Fortunately, in this study technology only failed once in the final couple of minutes of one interview.

Atkins and Wallace (2012) also highlight that the use of such equipment may make the interviewee self-conscious which may therefore inhibit their responses. Audio recording is not able to capture body language. However, whilst video recording would record the additional features that an audio recording cannot it is not possible to preserve the anonymity of the interviewee, therefore this method was dismissed.

One challenge was how to transcribe interviews. Hammersley (2012, p. 439) writes that ‘transcription is a process of 'construction' rather than simply a matter of writing down what was said’. There are a variety of decisions involved in transcription and for that reason Hammersley (2012) notes that neither transcripts nor electronic recordings should be treated as data that are simply given, in an unmediated fashion. Decisions had to be made which included how much of them to transcribe, whether to try and capture things like pace and pitch, whether to include expressive elements such as laughter and silence.

For this study, it was decided to record in full all interviews so a clear record of what happened in each case could be accessed. A decision was made to transcribe interviews in full as this would support others to triangulate interpretations. The researcher transcribed the interviews, as Maxwell (1995) identifies that the process of transcribing is also an opportunity for analysis, where relevant this enabled expressive elements to be added to the transcripts.

5.3.5 The interview sample

At the beginning of the main study it was decided to interview all colleagues within the department. Forsey (2012) recommends interviewing as many people as necessary to find out what one needs to know. It was not possible to know, before starting, different people's beliefs and opinions and as there were only a small number of colleagues this seemed a pragmatic and feasible approach.

5.3.6 Generalisation

As with case studies, interview studies do not aspire to generalisability, however their findings can have implications for other settings.

> 'Semi-structured or open ended interviews invite participants to share their experience and understanding, thereby revealing the possibilities and limits of what people may do in similar circumstances, even when we cannot predict what they will do.' (Mears, 2012, p. 174)

Mears (2012) suggests that as findings are generated you can point out potential significance for other settings and situations.

5.3.7 Triangulation

Although the in depth semi-structured interviews formed a major part of the data collected for this study, triangulation was needed to strengthen the study. Bush (2007) describes triangulation as the process of comparing many sources of evidence to determine the accuracy of information. This could involve methodological triangulation where several methods are used to explore the same issue. It could also involve respondent triangulation where the same questions are asked to many different participants.

> 'Triangulation strengthens a study by combining methods. This can mean using several kinds of method or data, including both quantitative and qualitative approaches.' (Patton, 2015, p. 316)

It is essential to use multiple sources of evidence, with data needing to converge in a triangulating fashion (Yin, 2014). The use of triangulation as a strategy 'reduces the risk that conclusions will reflect only the systematic biases or limitations of a specific source or method and allows you to gain a broader and more secure understanding of the issues you are investigating' (Maxwell, 1995, p. 69). With this issue critical to ensuring confidence in any conclusions that were drawn, data was also gathered from recording the professional development sessions and from general conversations that occurred at other points. This data was supported by carrying out learning walks, book scrutinies and by transcribing presentations that teachers delivered.

Professional development sessions

Professional development sessions often went on for over an hour and the process of transcribing these would be timely and costly. It was decided that these would be listened to by the researcher and sections that supported or disputed what was being said in the interviews would be transcribed. Photographs of outcomes of planning or notes written on white boards were taken as another source to triangulate with.

Learning walks and book scrutinies

Learning walks were carried out on two occasions within the study. These were carried out by two people to help reach a shared understanding of what was being observed. All teachers were observed, and notes were made from these walks alongside photographs of pupil work and displays. A similar process was carried out when looking at a sample of students' books.

Teacher presentations

At the end of the CPD programme, teachers presented their own action research projects to the rest of the department. These were video recorded and the presentations and questions (asked by peers) were transcribed for use as triangulated data.

Informal conversations

Due to the nature of the researcher's role within school, as well as providing the formal professional development sessions, the researcher also worked alongside these colleagues daily. There were times when colleagues would chat, and additional relevant information would be available outside of the 'recorded' sessions. Therefore, sometimes additional notes were made, and teachers were made aware when these comments were written.

5.4 Analysis of data

This section defines data, code, coding and themes and then considers different methods of coding. It explains the process of coding adopted in this study and how themes were developed throughout.

5.4.1 Definitions

Within this study the term data is used as defined by Guest, Macqueen and Namey (2012, p. 50) to mean 'the textual representation of a conversation, observation or interaction'. So, this refers to parts of transcribed interviews or descriptions of photographs from learning walks and student books, and to transcribed presentations that were delivered by teachers within the department.

Miles, Huberman and Saladana (2014, p. 71) say that 'codes are labels that assign symbolic meaning to the descriptive or inferential information compiled during a study' and Saldana defines it as 'most often a word or short phrase that symbolically assigns a summative, salient, essence-capturing, and/or evocative attribute for a portion of language-based or visual data' (Saladana, 2013, p. 3). In this study as Cohen, Manion and Morrison (2011) suggest the codes have been derived from the researcher's own creation, or they have been derived from the words used spoken by one of the participants in the transcribed data.

Cohen, Manion and Morrison (2011) describe coding as the ascription of a category label to a piece of data, that is either decided in advance or in response to the data that has been collected. It is the process by which a qualitative analyst links specific codes to specific data segments (Guest, Macqueen and Namey, 2012). Coding is not a precise science but an interpretive act, 'an exploratory problem-solving technique without specific formulas or algorithms to follow' (Saldana, 2013, p. 8).

Throughout the coding process for this study the researcher needed to reflect and interpret meaning from the data (Miles, Huberman and Saladana, 2014). The outcomes of the coding and analytic reflection are the themes. Where a theme is defined to be 'a unit of meaning that is observed (noticed) in the data by the reader of the text' (Guest, Macqueen and Namey, 2012, p. 50).

5.4.2 The analysis process

The essence of data analysis is to highlight and clarify patterns that have been observed (Angrosina, 2012). This may not be done in a linear way, often in qualitative studies the researcher moves back and forth between the data gathering and the data analysis stage (Lichtman, 2013). In qualitative research, the exploration and linking of theoretical and other organising concepts is creative and not merely mechanical (Denzin and Lincoln, 1998).

Lichtman (2013) suggests that analysis is an ongoing process throughout the life of the project, with the ideal model of researchers following a circular model of gathering and analysing data. There is general agreement (Lichtman, 2013; Cohen, Manion and Morrison, 2011) that the goal of analysing data collected is to arrive at common themes. However, drawing conclusions and constructing theories can come down to a matter of interpretation so there needs to be a sound argument presented for any claims that we make based upon the results of the data analysis (Atkins and Wallace, 2012).

This section identifies the process of coding involved in analysing interviews and other data within this study. The process of analysis is one of organising and categorising. Lichtman (2013) suggests that you begin coding at the first interview and then move to the next, either using previous codes or adding more codes as necessary. This iterative process continues until all the data has been analysed. At this point it may be necessary to rename and re-categorise codes as well as disregard ones that may be redundant.

The other way to analyse qualitative material is to tell the story in the form of a narrative. Lichtman (2013) suggests that you either conduct an analysis in which you identify themes or provide an interpretation in the form of a narrative, whichever is chosen depends on the end goal. This study uses a mixture of both types. The process of coding, as shown in Figure 29, was used to generate conclusions from the study however the narrative approach was helpful to provide a picture of the professional development sessions that happened and gave extra evidence to justify the interpretations of data.

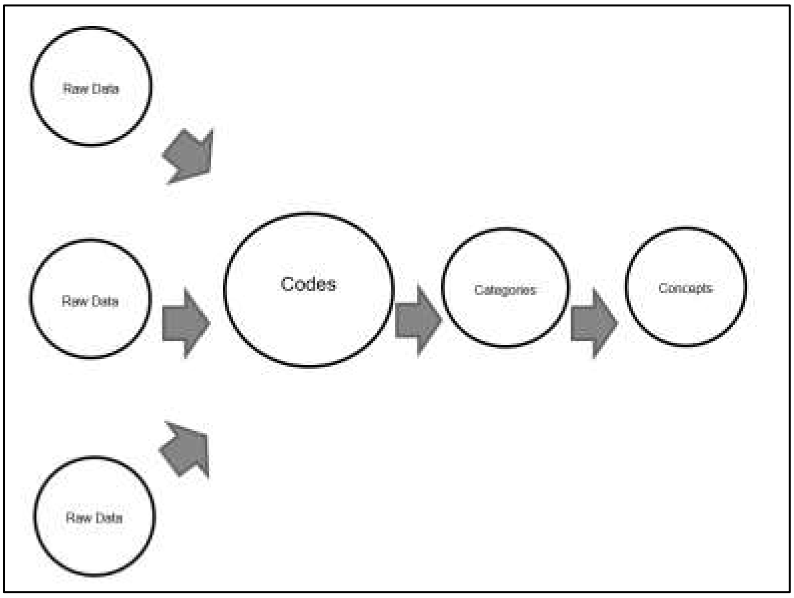


Figure 29. 'Three C's of data analysis: codes, categories and concepts' (Lichtman, 2013, p. 252)

Trying to make sense and meaning from qualitative data is a process that moves between the questions, the data and the meaning.

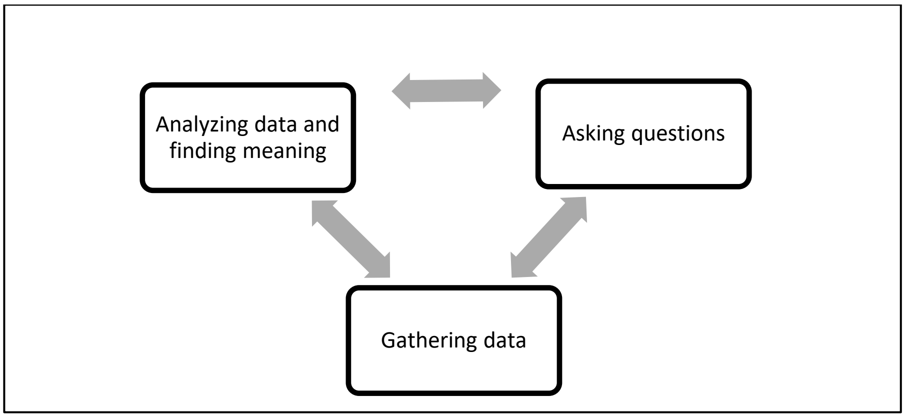


Figure 30. 'Relationship between questions, data and meaning' (Lichtman, 2013, p. 255)

The iterative cyclical model, provided by Lichtman (2013) and shown in Figure 30, shows the process followed throughout the data coding process of this study.

5.4.3 Apriori and empirical codes

Various distinctions are made between different types of codes and coding; for example, Neuman (2011) distinguishes between open, axial and selective coding and Saldana (2013) defines a range of coding including elemental, affective, exploratory, procedural and grammatical. However, for this study, perhaps the most helpful distinction is the one that Gibson and Brown (2010) make between **apriori** and **empirical** codes.

Apriori codes are created to reflect categories that are of interest before the research has begun whereas **empirical** codes are derived by reading through the data, as points of importance and commonality are identified. For this study, it was appropriate to use a mixture of both. **Apriori** codes can be used as part of the deductive approach since these were derived from the researcher's previous reading and **empirical** codes can be used in an inductive way for issues and things emerged that were not anticipated from the researcher's prior reading in this subject area (Harding, 2013). So, for example there were some pre-determined avenues to explore prior to the semi-structured interviews were carried out. These were set up in NVIVO in advance of data analysis. Figure 31 shows an example of this.

Main Study Initial Interview Question Themes	0	0	28/03/2014 17:36
1. What makes Effective CPD	10	59	24/10/2013 11:32
2. What makes Less Effective CPD	6	14	24/10/2013 11:32
3. How is Research used	10	32	24/10/2013 11:49
4. What is understanding	7	19	24/10/2013 11:32
5. Interview Image	9	11	24/10/2013 11:51
6. Linking Curriculum	9	24	24/10/2013 13:39
Main Study Nov 14 Interview Themes	0	0	11/02/2015 09:46
Q1a. Instrumental vs Relational	9	20	11/02/2015 08:33
Q1b.Reflections on Research Presentation	9	18	11/02/2015 08:27
Q2 Reflections on CPD activities	10	37	11/02/2015 08:28
Q3a Reflections on active experimentation stage impact	10	63	11/02/2015 08:28
Q3b Activities that are more relational	0	0	11/02/2015 09:49
Q4 Moving forwards as a department	9	25	11/02/2015 08:30

Figure 31. Examples of apriori codes

Then as the iterative cycle of coding was carried out new empirical codes arose and were added and amended in an inductive way. Figure 32 shows **empirical** codes that arose when looking at constraints.

Constraints	0	0	24/10/2013 14:17
Being taken out of comfort zone	2	2	23/01/2013 13:12
Challenge of implementing new approaches	4	8	03/07/2014 12:31
Curriculum Changes	2	2	18/03/2015 13:15
Exams	9	14	24/10/2013 11:50
Feeling of frustration	1	2	03/07/2014 12:26
General lack of effort	1	2	18/03/2015 14:48
Lack of PD opportunities	3	3	24/10/2013 12:00
Not got the classes to try it with	2	2	18/06/2014 09:07
Not supported enough with external courses qualifications	2	2	13/09/2014 15:48
Not sure how to (SK wise)	2	5	18/03/2015 14:46
Not wanting to change routine	1	2	10/09/2014 20:58
Ofsted	1	1	18/03/2015 13:14
Students not wanting to think	2	2	18/03/2015 14:29
Time Constraints	4	7	24/10/2013 11:48
Transferability of new approaches	2	2	03/07/2014 12:33

Figure 32. Examples of empirical codes

The process used in this study was the one suggested by Harding (2013). The process of using empirical code is broken down into four steps which are:

1. Identifying initial categories based on the reading of the transcript;
2. Writing codes alongside the transcripts;
3. Reviewing the list of codes, revising the list of categories and deciding which code should appear in which category;
4. Looking for themes and findings in each category. (Harding, 2013, p. 83).

In this study, as Neuman (2011) suggests, coding began with a very thorough reading of the full transcript. It was important for the researcher to engage with every line of the transcripts and to underline key phrases or make a note of what was of interest to assist in the process of thinking holistically about the data. Initial coding took a variety of forms including, using code words, phrases or segments of the text.

> ‘Coding is not a "one-off" exercise; it requires reading and rereading, assigning and reassigning codes, placing and replacing codes, refining codes and coded data; the process is iterative and requires the researcher to go back and forth through the data on maybe several occasions to ensure consistency and coverage of the codes and data.’ (Cohen, Manion and Morrison, 2011, p. 560).

As the process continued and more data was coded it was necessary to revisit and change the initial codes. Some additional codes were needed, and other codes needed to be rephrased. Using specialist NVIVO software enabled a more efficient approach and facilitated the researcher to think more clearly about the data (Harding, 2013). The researcher used her judgement to identify broad subject areas under which the data could be categorised. Several practical measures were taken with an initial list of codes and categories to make better sense of the data. These include as suggested by Harding (2013);

- Identifying codes which should belong in the initial categories but were not placed there on the coding first place.
- Creating subcategories within the initial categories.
- Identify new categories which can bring together a number of codes. (Harding, 2013, p. 93)

Interpreting phenomena in context is a key feature of qualitative research. Harding (2013) identifies that to correctly interpret the words of respondents, the qualitative researcher must be empathetic. In this study, in the case of comments where the meaning may not be obvious the researcher needed to consider the context of what had been said and apply the code that reflects the most likely meaning of the speaker. This was triangulated by an external analyst who also applied codes to interview transcripts.

5.4.4 Generating conceptual themes

Conception of conceptual themes seems to vary substantially, however Harding (2013) recognises that they are likely to have five different characteristics. First, they are likely to be drawn from different sections of the interview transcripts and usually code is taken from the analysis of different issues. Second, the conceptual theme may not be referred to directly. The third characteristic is that the conceptual theme may not be spotted on the first reading of the transcripts. The identification and analysis of such themes illustrates particularly well the need for the qualitative researcher to return to and re-analyse the data.

As Barbour (2014) notes:

> ‘Sometimes issues don't ‘jump out’ at you until someone says something particularly vehemently or articulately. However, this does not mean that it isn't present in earlier transcripts. Once sensitized, you may be surprised to find how many other instances you can find.’ (Barbour, 2014, p. 266).

Fourth, the use of conceptual themes is likely to achieve the most difficult aspect of finding a thematic analysis. Fifth, identifying conceptual themes enables the researcher to move beyond identifying findings to building theory. Throughout this study, the characteristics identified by Harding (2013) were evident.

Harding (2013) notes that the process of identifying conceptual themes is onein which it is difficult to suggest an approach that can be generalised however it may involve the following four steps which were employed throughout this study:

1. Identifying the conceptual theme and creating a category;
2. Bringing together codes from different illustrative issues into the category;
3. Creating subcategories to reflect different elements of the conceptual theme;
4. Using the conceptual theme to explain relationships between different parts of the data and to build theory. (Harding, 2013, p. 112)

8.3 Barriers to CPD		
Name	References	Sources
8.3.1 Departmental constraints	1	1
1. Continuity of development	17	9
2. Challenges in collaboration	5	3
8.3.2 College constraints	0	0
1. Other daily priorities	16	9
2. Conflict between college and departmena	9	6
8.3.3. External factors within education	0	0
1. Change of curriculum	5	4
2. Exams	15	10
8.3.4 Implementation issues	0	0
1. Need to observe it happening	3	2
2. Need to repeat it a number of times	4	3
3. Extra time needed to make lessons inspira	7	6
4. Mentor dependence	19	11
5. Lack of embedding in SOW	4	3
8.3.5 Attitudinal barriers	0	0
1. Being taken out of comfort zone	3	3
2. Easier to revert to what you know already	9	7
8.3.6. Personal constraints	0	0
1. Family Commitments	5	4
2. End of career	3	2

Figure 33. Using NVIVO to generate conceptual themes

Figure 33 shows an example of using NVIVO to develop conceptual themes.

5.4.5 Objectivity, sensitivity, validity, reflexivity and confidentiality

Data collection and analysis have traditionally called for 'objectivity'. However, Corbin and Strauss (2008) acknowledge that objectivity in qualitative research is a myth. Guba and Lincoln (1998) highlight that researchers bring to the situation their own paradigms, including perspectives, training, knowledge, and biases.

These aspects then become woven into all aspects of the research process. Corbin and Strauss (2008) suggest aiming for sensitivity, which stands in contrast to objectivity. It requires that the researcher put themselves into the research. ‘Sensitivity means having insight, being tuned in to, being able to pick up relevant issues, events and happenings in the data. It means being able to present the views of participants and taking the role of the other through immersion in the data…. mostly it is a trait that develops over time through close association and work with both people and data’ (p. 32).

They suggest that professional experience can enhance sensitivity. Although at times experience can prevent analysts from reading data correctly, experience can also enable researchers to understand the significance of something more quickly. Within this study the researcher has stated their own paradigm and any biases have been made explicit throughout. Due to the nature of close working with colleagues throughout the study a sensitive approach has also been employed.

There are some simple techniques that have been used to enhance the validity of this study; Schmidt (2004) suggests reading thoroughly interview transcripts before beginning analysis, this was employed in this study. Another particularly important concept associated with enhancing the validity of qualitative research is reflexivity. The following definition is offered by Heaton:

> ‘Reflexivity in primary qualitative research generally involves the self-examination of how research findings were produced and, particularly the role of the researcher(s) in their construction.’ (Heaton, 2010, p. 94)

Within this study the researcher has played a critical role in the examination of research findings, this has been stated throughout and any bias has been declared at appropriate points within the findings. The involvement of the supervisor carrying out interviews and external researchers supporting the analysis process have been critical to support the validity of the analysis process.

Lincoln and Guba (1985) use the method referred to as ‘member checking’ which entails participants themselves, reviewing the summarized data to see if it accurately reflects their intent and meanings. They acknowledge that this can be done after an individual data collection event has been summarized. During the analysis phase, when the TDM was generated, the teachers themselves were involved in stating where they were on the model this was used as a way of member checking and validating the data.

Confidentiality is a key ethical requirement of any research project. In addition to comments in Section 5.2.7, during this study the researcher has sought to avoid making teachers identifiable and so in addition to referring to everyone by code/pseudonym, some of the more specific details that would have made identification possible have been taken out of the transcripts, as suggested by Harding (2013).

5.5 Presentation of data

Cohen, Manion and Morrison (2011) highlight seven different ways of organizing and presenting data analysis. These are: by group; by individual; by theme; by research question; by instrument; by case study; and by logical narrative. Several of these were chosen at different stages for the organisation and presentation of the data. This section describes the rationale for each.

5.5.1 Organisation by individual

One way to organise the data is by individual where the total responses of a single participant are presented, and then the analysis moves on to the next individual. Summaries of the initial interviews are presented in this way in Section 7.2 with the advantage that it 'preserves the coherence and the integrity of the individual's response and enables a whole picture to be presented' (Cohen, Manion and Morrison 2011, p. 551). It was felt that this was an important feature to set the scene for the study as it enables the reader to gain an insight into the background of the individuals involved within the case study.

Schmidt (2004) highlights the importance of thoroughly reading and re-reading transcripts before beginning analysis. He states that the simple technique will enhance validity and make it more likely that the findings of the study accurately reflect the original data. Schmidt acknowledges that this is time consuming, but it should ensure that the researcher does not neglect any ideas or sections of the transcripts when conducting their analysis. Once the reading is complete, the process of summarising can begin. It is often helpful to summarise one section of the transcript at a time. Harding (2013) suggests that the process of summarising usually involves the following steps:

1. Identify the research objective(s) that the section of the transcript is most relevant to.
2. Decide which pieces of information or opinion are most relevant to this objective / these objectives and which are detailed that do not need to be included in the summary.
3. Decide where (if at all) there is repetition that needs to be eliminated.
4. On the basis of these decisions, write brief notes. (Harding, 2013, p. 57).

Miles and Huberman (1994) argue, the full transcript should be reduced to a summary that fits onto one sheet of paper and so is easy to the researcher to compare with other summaries.

5.5.2 Organisation by theme

However, leaving data just in individual form wouldn't help answer the research questions, so Cohen, Manion and Morrison (2011) highlight the need for a second level of analysis looking then for issues arising across the individuals to look for themes, patterns arising within the data. The researcher decided to use these themes as a second layer as to use them in the first instance, risked the coherence and integrity of everyone's responses being lost during the data reduction process.

There are two distinctions to be made at this point. Some of the areas of interest and issues were 'decided pre-ordinately' (Cohen, Manion and Morrison, 2011, p. 551) based on the initial research questions (labelled with apriori codes) and secondly some new factors emerged responsively from the data (labelled with empirical codes). The researcher had to as Cohen, Manion and Morrison describes (2011, p. 551) 'trawl through the residual data to see if there were other important issues that have emerged that have not been caught in the pre-ordinate selection of categories and issues for attention’.

5.5.3 Presentation by research question

Another method of organising and presenting the analysis of the data is by research question. Cohen, Manion and Morrison (2011, p. 552) acknowledge that this a useful method of organisation as it draws together all the relevant data for each of the research questions at the beginning of the enquiry and therefore 'closing the loop' for the reader.

With these aspects in mind, initially the findings are presented by individual to preserve the whole picture of the person and then these are followed with a restatement of findings under the themes being explored, this included any new issues that arose. Finally, the data was reorganised to answer the original research questions. Due to the electronic qualitative software NVIVO being used, the reorganisation into these three different presentations was straightforward.

5.6 Chapter conclusions

This chapter has detailed the methodologies and methods employed during this study. The rationale for using a case study was presented and validity and trustworthiness were considered. Research instruments were presented, and the process of data analysis was described.

CHAPTER 6: THE PILOT STUDY

6.0 Introduction

Day Ashley (2012b) and Yin (2014) acknowledge that a pilot study can be useful to help refine the data collection plans that are to be used in the main study. The refinements can be made to both the data content, the tools and procedures that are followed.

The pilot study was important for several reasons. It enabled the researcher to learn how to use technology such as audio recording software and analysis packages such as NVIVO. The pilot also enabled the trial of different CPD activities so that these ideas could be refined before they were used with the mathematics department in the main study.

This chapter presents details of the pilot study and considers themes that emerged from the pilot and how methodologies and research instruments were adapted before the main case study.

6.1 Design of the pilot study

In general, when selecting pilot cases Yin (2014) suggests that the main criteria can be convenience, access, and geographic proximity. The initial pilot study was designed to make use of the opportunities that were readily available to the researcher.

Initially the plan was to work with two groups of professionals, a whole cohort of B.Ed. Secondary Mathematics students (Group 1) and two HLTAs within the school (Group 2). The B.Ed. students were from a partner HEI and had a scheduled session each fortnight with the researcher to help them develop as part of their school-based training.

Challenges arose with these initial pilot groups, (detailed in Section 6.3.1 and 6.3.2) so part way through the pilot year work also began with a cohort of trainees on a PGCE course (Group 3) at a different partner HEI.

> ‘If more than a single pilot case is planned, the report from one pilot case also can indicate the modifications to be attempted in the next pilot case. In other words, the report can contain the agenda for the ensuing pilot case. If enough pilot cases are done in this manner, the final agenda may actually become a good prototype for the final case study protocol.’ (Yin, 2009, p. 94).

In addition to data gathered from the whole PGCE cohort, a more in-depth follow up was carried out with an individual case (Maggie, Group 3, Trainee 3.11) to see how the ideas generated from the sessions could be used within school. Maggie was chosen as a pilot case for several reasons. Throughout the CPD sessions she was keen to take part and it appeared obvious that she was interested in the idea of teaching in a more conceptual way, however wasn’t sure how to. She welcomed the CPD and additional time that would be spent with her and her school was willing to let the researcher work with her. Also, due to the locality of her training school the researcher could get there easily not incurring additional costs to time or diary.

In this case the pilot cases were formative and assisted the researcher 'to develop relevant lines of questions — possibly even providing some conceptual clarification for the research design as well' (Yin, 2014, p. 96). Working with the first three groups led to the refinement of some tasks and activities before piloting a second phase (Phase B) of the pilot with the next cohort of PGCE trainees (Group 4).

The pilot ran for just over twelve months. Findings from each group are detailed and then themes were pulled together when considering implications for the main study.

6.2 Details of the pilot study

The pilot study activities are shown in a chronological timeline in Appendix 4.1 and an overview is given in Figure 34. The first session for all participants was a session where literature review findings from Chapter 2 were explained and the CCC Model was shared. The session presentation is given in full in Appendix 2.3.

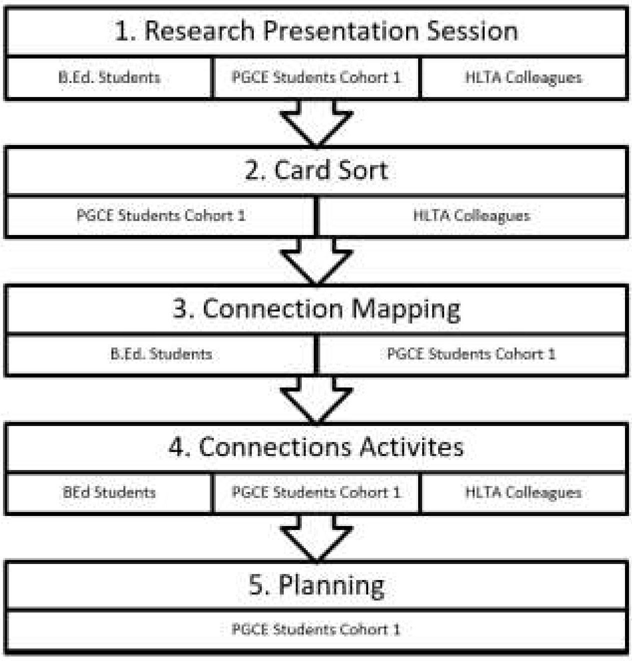


Figure 34. Phase A, outline of pilot study

The B.Ed. group worked on a connection mapping activity after the research presentation, the outcome of this is in Appendix 4.3. Whereas with the PGCE cohort, Appendix 2.4 shows an instrumental and relational card sorting task that was trialled (NCETM, 2010) to clarify the meaning of instrumental and relational, before going on to create a connection map. The outcomes from their card sort can be seen in Appendix 4.4 and their connection mapping in Appendix 4.5. PGCE cohort 1 then had an opportunity to discuss their thinking and to begin to plan for their own classes. In addition to the activities shown in Figure 34, Maggie experienced two more collaborative planning sessions (Appendix 4.7) and an opportunity to teach and reflect on these before being interviewed.

Throughout the pilot stage, the researcher continued to develop and source a variety of tasks that promoted teaching for understanding. Therefore, the trainee teachers (Group 4) involved in the second phase of the pilot experienced additional activities including the classroom scenarios task (Appendix 2.5) as shown in Figure 35.

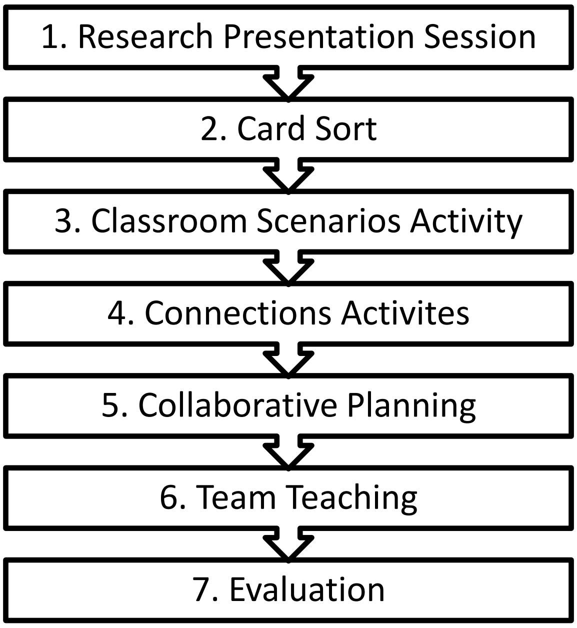


Figure 35. Phase B outline of pilot study

6.3 Description of the pilot study

This section describes the professional development sessions carried out and adds quotes that may be useful as part of later analysis when looking for themes to explore further in the main study.

6.3.1 Group 1 B.Ed. trainees

The initial pilot plan was to work with a cohort of five B.Ed. trainees (Trainees 1.1 – 1.5) for a year and for this to form the main part of the pilot study. However, in the early stages due to different members of the cohort being absent for various sessions it was difficult to get continuity. Four were present for the research presentation session (Appendix 2.3) and then two weeks later a different four were present for the multiple representation tasks (Appendix 4.2) and connection mapping (Appendix 4.3).

The outcome of the connection mapping task didn't turn out as had been anticipated. Trainees were asked to think about their chosen topic of algebra and to consider: different topics, how they linked to each other; linked to other areas of the curriculum; to think about different ways of representing ideas and to consider the notion of procedures versus concepts. Much of the conversation revolved about why students didn't like algebra and a discussion around history. Seven times throughout the task they were questioned about what the big ideas were and asked if there were links between topics. Twenty minutes into the activity a sample of books and resources were handed out and they were asked if there were other ideas that they had missed. Additional topics like quadratic equations were added. The researcher kept probing how the topics might link and there generally was a quiet response. Thirty minutes into the task the group were still not forthcoming with any links, so it was decided to end the task.

The group moved on to looking at multiple representations (Appendix 4.2) and various activities were explored. After considering slide 16 about completing the square, Trainee 1.4 commented 'I wish I was taught that way, that makes so much sense I have never understood completing the square before' he commented that people had tried to teach him algebraically and that he had to relearn the rule before each exam as he never understood it.

Due to ethical reasons, later sessions about connected teaching with this cohort were cancelled. There were several trainees that were not on target to successfully complete their studies and although they enjoyed the ideas that were presented to them, it was felt that the researcher's role needed to revert to professional mentor and involving them in the research study would be detrimental to their own professional studies.

Reflecting on the sessions it was clear that even though the presentation session was delivered on conceptual and procedural understanding, when faced with the connection mapping task the trainees were unable to put these ideas into practice. There may be several reasons for this; perhaps they did not have the subject knowledge themselves to make the connections or perhaps the task wasn't clear. The reason cannot be concluded from the evidence gathered. Before working with other groups, it was decided to add the additional card sort task (Appendix 2.4) with the aim that it would provide an opportunity to discuss what the differences were between topics being presented in a relational versus instrumental way.

6.3.2 Group 2 HLTAs within school

At the same time as working with the B.Ed. students the research presentation was also delivered to two HLTAs within the department, Annette and Brian. These HLTAs work with small groups of students and intervention groups. There was the opportunity to trial the card sort (Appendix 2.4) with these colleagues.

As soon as the task was given the first comment was a clarification between the two of them of the terminology and one of then stated ‘so instrumental is learning by rote …..’. They carried on rearranging the cards saying, ‘that is a procedure although maybe done in an understanding way’. Listening to the conversation it was felt that the task had provided the outcome that was required and was a useful addition to the CPD programme. There was discussion about what some of the cards meant and they looked to the researcher to see if their answers were ‘correct’.

A few weeks later the pilot multiple representations session (Appendix 4.2) was shared with these colleagues. During the session, discussion around teaching concepts versus procedures resulted in some useful reflections. Brian commented ‘as a reflection of what I am doing, I seem to be doing a lot of procedural, because I am doing things in a rush in preparation for the exam and there is not much conceptualising going on at all’.

Later in the session it became clear that they wanted to work in different ways. The initial response from Brian was to try each task with numbers and then to generalise with algebraic notation whereas Annette was more comfortable exploring the visual images that extended from the grid method of multiplication. A discussion between them followed ‘I don’t think geometrically, do you?’ (Brian), Annette replied, ‘yes I do’ and commented that she liked using a range of kinaesthetic resources such as Cuisenaire rods with students.

When the session was concluding, it was explained that in later sessions they would explore the ideas further and reflect on the procedures and concepts. Without any prompting, this led the HLTAs to reflect on their own experiences as to how they had learnt mathematics. Brian said, ‘I had procedures beaten into me’ the other agreed and commented ‘but I don’t think my depth of understanding of mathematics is very deep, but I can do the procedures like long division’ (Annette).

Whilst it had been planned to continue work with this cohort of HLTAs, one of them needed to take a long period off work and the other’s role changed which meant they would become part of the main study. It was therefore decided not to continue working with these colleagues as part of the pilot stage.

6.3.3 Group 3 PGCE cohort 1

This section summarises the outcomes from the pilot work with PGCE cohort 1 (Trainees 3.1 – 3.11). The research presentation and the card sort activity were shared. Appendix 4.4 shows photos of the outcome.

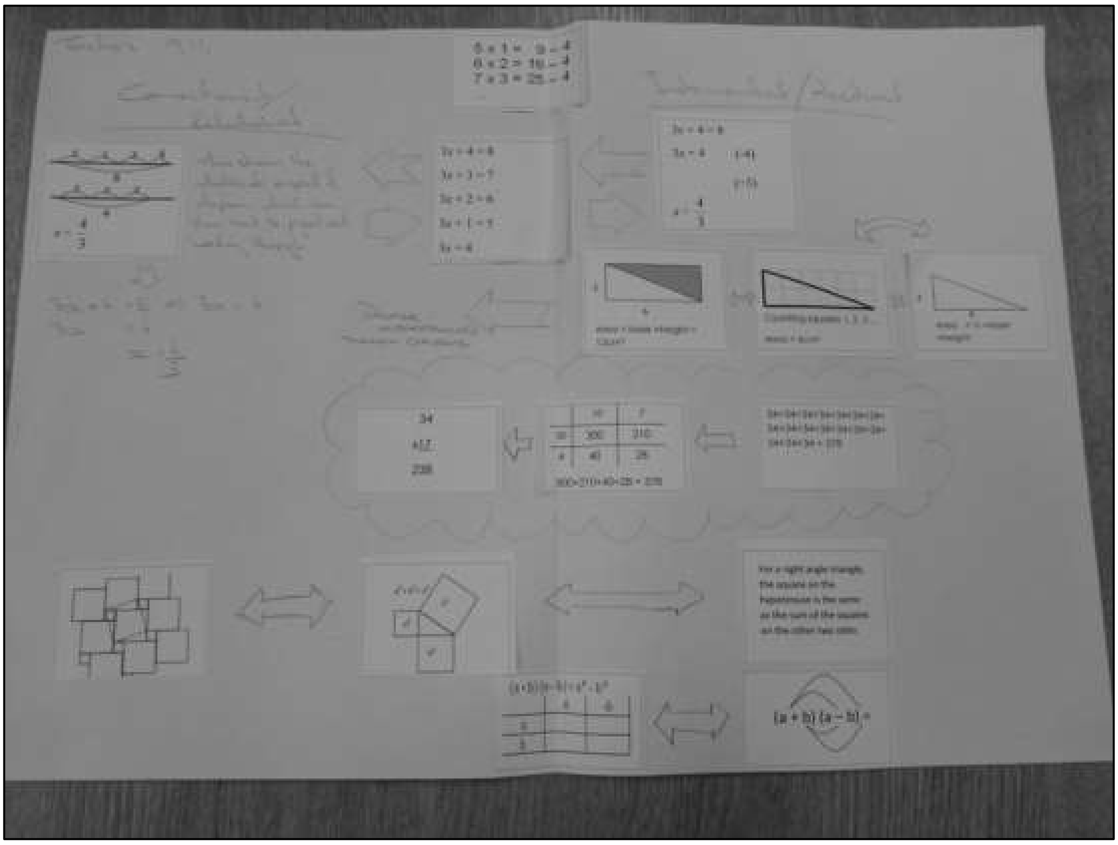


Figure 36. PGCE cohort 1, Trainees 3.9 and 3.11 card sort outcome

All groups engaged in productive discussion; in addition to grouping the cards Trainees 3.9 and 3.11 also annotated their poster (Figure 36) emphasising deeper understanding could be generated through dialogue. Their poster acknowledges the importance of the links between the cards.

The group split into two subgroups to follow up this work. They were given the task of making connections between different aspects of algebra.

The first group (Trainees 3.1- 3.6) began by individually adding lots of separate post it notes, four minutes in they started to move and group cards. There was a lot of supportive laughing showing that they were comfortable working with each other, however there were a few comments that caused a professional debate.

Trainee 3.2 said ‘I really want to put fruit’ this prompted a discussion about other experienced teachers they had seen introducing algebra using fruit in a fruit bowl to introduce the idea of collecting like terms ‘he uses a fruit bowl when they are adding but he uses a blender when they are multiplying’. This idea seemed to divide opinion; it could be sensed that Trainee 3.4 was concerned about this practice and questioned ‘but they don’t multiply in a blender they just add’ despite the challenge the initial trainee was adamant it was a successful approach ‘it did work quite well’.

Over the next few minutes the discussion focused on the fact that some of their ideas were about what to do whereas others were about how to do it. They began to identify where the links were and how the ideas linked together. Their dialogue referred to curriculum levels and the order in which you might teach things, mentioning Piaget’s learning theory moving from concrete objects to representations and then to the use of letters.

Ten minutes into the discussion one trainee put ‘a letter representing an unknown number’ in the middle saying ‘this is central to it all as this is what algebra is’. They then went on to reflect making a key point about the different types of algebra ‘the only thing is you have got the two types of algebra, you have got the find x, so x actually is a number … and then you have got x is an unknown and you don’t need to find it’ this promoted reflection on the difference between an expression and an equation; at this point a card for identity was added to the poster. Thirteen minutes into the task it was decided to take most post it notes off so they could be reorganised.

During this reorganisation, their dialogue continued about the idea of doing the same thing to both sides of an equation. This prompted a comment from Trainee 3.4 (the same trainee that challenged the fruit bowl idea) ‘you have also got the thing that stupid teachers say, like cross the train tracks and change the sign and one of them I heard, what was that stupid thing I saw at school X ….if it has got two things together like $4g$ because it is a letter and number they can’t be lovers so you have to separate them’. The researcher questioned whether these were procedures or concepts the trainee replied, ‘they are just nonsense, aren’t they?’.

Sixteen minutes into the task, quadratics and polynomial post it notes were added, the dialogue continued about how algebra was linked to the real world and whether this was the experience for students in school or whether students just did $4x + 6 = 15$. The pressure of just teaching for exams was mentioned as a constraint.

Twenty-three minutes into the task one trainee commented, ‘if we are thinking about understanding, function machines are a really strong procedure’ they then went on to say that the image of jumps on the number line was more conceptual. At this point the group started to colour code whether their post it notes were more conceptual or procedural.

The two quadratic cards were marked as procedural, so the researcher asked could they add a card that might be more conceptual. They thought for a bit and came up with the area model. Doing the same thing to both sides was initially colour coded as a procedural method then a discussion about what equality meant resulted in the idea that if you really understood the concept of equality then doing the same thing to both sides would be conceptual. Arrows continued to be added between post it notes. One trainee commented ‘I reckon everything could have an arrow connecting them’ another said, ‘not between Pythagoras and simultaneous equations’ another said, ‘I am sure you could link it somehow!’.

Thirty minutes into the task the trainee that led the conversation about doing the same thing to both sides pointed to the gradient card and commented ‘with things like this you could put it in both blue and red, depending on how you are teaching it’. A few more trainees nodded at this point and black lines began to be added to the poster. The post it with proof on caused debate some of the trainees felt there were procedures to be learnt whereas others were adamant it was a conceptual idea. At thirty-seven minutes, more black labels were added as the consensus was it depends on how you teach it. One trainee wasn’t in agreement with finding a root ‘there is a clear procedure to follow to find the root’ at this point four of the six trainees were now saying it depends on how you teach it. During this period five trainees were in debate and one was quiet listening. He then said reflectively ‘this is the conclusion that I am coming to, most of it you can teach either conceptual or procedural it is just the methods that you use’. The debate continued then the trainee that was earlier saying finding a root was procedural now said ‘all of it is both’. The final reflection was about whether procedural and conceptual were the correct words to use on their poster and whether relational and instrumental would have been better. Figure 37 shows the outcome at the end of the session.

Figure 37. PGCE cohort 1, Trainees 3.1 - 3.6 connection mapping outcome

The second group (Trainees 3.7 - 3.11) were given the same task, they started by adding their individual ideas. Six minutes in one trainee started to move the post it notes around and began to classify their groups into symbols, language and why we can use it. A couple of minutes later, ‘why’, ‘what’, ‘how’ and ‘when’ were decided as headings and added. Twelve minutes into the task one trainee suggested ‘I was thinking how it is more about the thinking about why it works’ …’you mean like relational or procedural’ commented another trainee. The trainees then decided they could categorise them like they did in the earlier activity. There was some confusion exactly as to which terminology to use but agreed on relational/conceptual being green and procedural being red. They decided that the whole section on why should be green ‘that is the point of the why – the why is supposed to be relational because that is what relational means'. There was a short discussion to say that problem solving was more conceptual because there was no set procedure for it. Sixteen minutes in one trainee decided that solving for an unknown could also be procedural.

One trainee said 'Do we think the how's are green as well? As number lines are conceptual', the others agreed and referred to sessions they had attended promoting use of a number line. At this point, there were only a couple of trainees putting forward ideas and there was some uncertainty about the historical development of mathematics one trainee at this point said, 'with my own understanding at this point of what these are (pointing to relational/procedural) I don't know'. At twenty-five minutes, there was a comment that we need to add some red (procedural) to the diagram as there isn't any there yet. However, they couldn't agree on where to add red and what the procedures would be.

The defining moment of clarity for Maggie occurred towards the end of the session when a discussion occurred about using speed, distance, time triangles 'that is a super procedure' she said acknowledging that you could get the answer without understanding. I questioned how you might do the same thing in a more conceptual way. After quite a long silence someone said, 'how about using units?' this led to a discussion about dimensional analysis. At this point, Maggie acknowledged she had got the idea and that example clarified the differences for her. Figure 38 shows outcome the for the group.

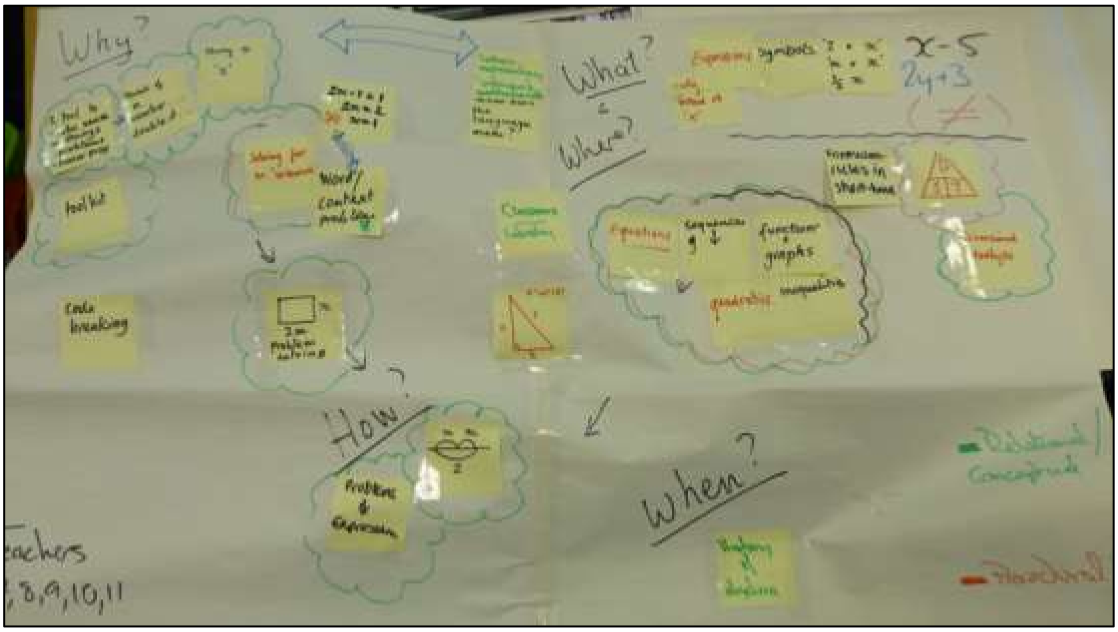


Figure 38. PGCE cohort 1, Trainees 3.7 - 3.11 connection mapping outcome

After these sessions, there was the opportunity to reflect with their course tutor (also the research supervisor). It was concluded that despite being given the same task groups had engaged at very different levels and the outcomes were very different. The task provided both groups the opportunity to discuss and clarify for themselves the ‘new’ terminology that had been shared in the research presentation.

PGCE cohort 1 (Trainees 3.1 - 3.6) came together a week later and looked at their connection map intending to plan something from the ideas. They worked together for fifty minutes with dialogue continuing around ideas from the connection map. One focus of the conversation was how do you introduce algebra, one trainee was certain that function machines were the only way to do this and others were not convinced, they talked about different representations including mappings, functions and graphs one trainee commented, ‘I think that linking graphs with algebra is a really important thing that doesn’t get done enough’. This led onto other trainees saying they would start with physical objects to help get across the concept of balancing, the difference between procedures and concepts was clarified again within the group.

During a discussion about linking different concepts the trainees identified that they felt that they had to conform to departmental schemes of learning and how disjoint the objectives were. An example was given of three consecutive units on construction, Pythagoras and proportionality and how the researcher had linked them by constructing a Pythagorean fractal tree using the student’s construction skills from the first unit, Pythagoras was then used to calculate unknown lengths in the tree and then the proportional relationship between triangles in the fractal was considered in the proportionality unit. This example provided, led to mixed opinions from the PGCE trainees. Some thought it was great and they requested moving their children to the researcher’s class others suggested that you wouldn’t be allowed to do that in all schools saying, ‘I think this is me being quite naive and young to the teaching system but I would see a scheme of work and think I have got to do that’ other trainees commented that they would like to do that but wouldn’t think of it ‘maybe it just comes with experience where the links are’.

A discussion was led about thinking about the big picture when planning, one response was ‘I can’t wait until I have got the time to do that at the moment I am just trying to get through the next week’ others echoed the concerns and pressures of planning on the course.

Referring back to the notes (Appendix 4.5) that had been made in this collaborative session one trainee questioned ‘would everybody teach the conceptual and then the procedural or would you do the procedure and then the concept?’ one reply was ‘I think it is easier to teach conceptual then procedural it think it is harder for them to go from a procedure to a concept’ another trainee agreed ‘I think you are right because once they have got the procedure they don’t care, they know how to do it’. The consensus was to start with the concept to get them to understand it first. The point was then raised as to whether this was applicable with all attainment levels and that lower sets wouldn’t cope with the concepts.

Different ways of solving equations were discussed, and for each method there was a divided opinion among the trainees. They agreed on the importance of variety of methods with their learners. A further ten-minute discussion continued around the use of function machines and their limitations.

This dialogue continued to the theme of how algebra is introduced, some talked about shopping lists and other reinforced the importance of variable and that a could not represent apple but needed to be cost or weight of apples. The researcher talked about the concept of introducing algebra as the notion of generalised arithmetic and the comments were ‘can you be my mentor’ from one trainee and ‘can you be my maths teacher’ from another with laughter that acknowledged they hadn’t considered doing this before.

The outcome of the connection activity with PGCE cohort 1 (Trainees 3.7 – 3.11) showed an uncertainty in the process of deciding on tangible examples of what the procedural and conceptual approach might look like. Based on this outcome, rather than going straight to a planning session, it was decided to model some mathematical activities that would help exemplify the ideas. The 'exploring expressions for areas task' was looked at (Figure 39) writing expressions for the blue area in different ways.

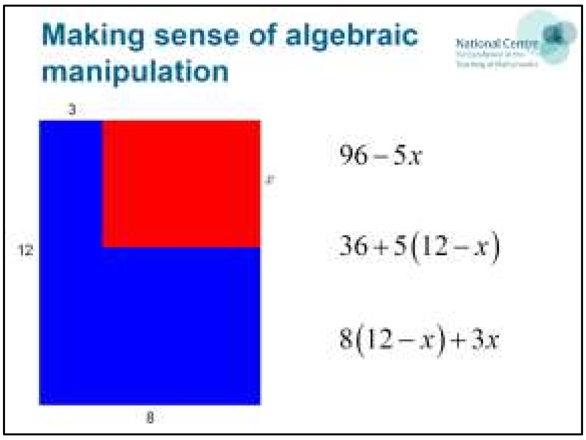


Figure 39. Exploring expressions for areas (NCETM, 2012b)

The ideas were then consolidated by considering the task of generalising different ways of finding the perimeter of a rectangle (Appendix Figure 10). The idea was to develop a more conceptual understanding of the distributive rule. This promoted a difference in opinion when Maggie said laughing 'there is that face thing to multiply out double brackets' and Trainee 3.7 said 'I like that, we got taught to do that'. It was clarified that obviously the 'face rule' worked but we were looking to show why. Maggie pointed to the notes on the board (Appendix Figure 8) and said, 'they would follow that though and know what you mean'. Trainee 3.9 who had been quietly reflecting then spoke up 'that bit that you have boxed there; I think that is a fantastic way of expanding an expression over a bracket'. The trainees continued to work with several problems and were encouraged to use visual representations to see if they could show their results in a different way.

Whilst exploring the task about 'squaring numbers ending in a five' (Figure 40) a few interesting points arose from Maggie 'do you find this a bit of a prejudice thing about drawing stuff, drawing boxes and pictures?' she recounted examples of trying to encourage students to draw things on their white boards and was faced with the response that students didn't want to because it was for little kids.

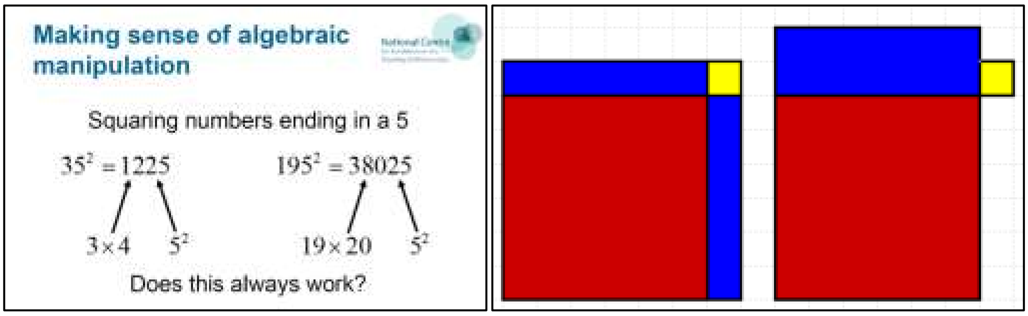


Figure 40. Squaring numbers ending in a five (NCETM, 2012b)

Trainee 3.8 commented that they had felt the same whilst engaging in mathematical problems on the PGCE course; they had wanted to draw sketches to help with the problem and were made to feel that they were not proper mathematicians. Maggie agreed then referring to the squaring task said, 'but this is proving that this can be interesting and technical and not just for the thick people'. As the conversation continued Maggie said:

> 'But there is the sort of, I just want to do the procedure because that is what clever people do and I don't want to do that because that is what thick people do. Even some of the teachers think like that.' (Maggie)

This promoted some concern within the group with Trainee 3.7 expressing 'we don't make you feel like that do we because we just do the algebra'. The response quickly moved back to the students feeling embarrassed rather than the focus of the trainees themselves as different learners.

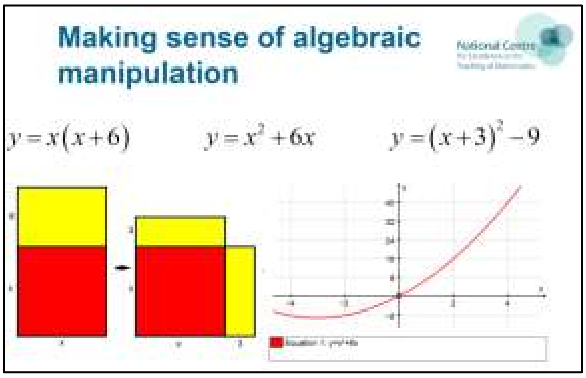


Figure 41. Completing the square (NCETM, 2012b)

The session continued with the researcher modelling activities where visual representations could be used to prove different things. The session was concluded by showing the completing the square slide (Figure 41) the response from Maggie was smiling 'that is good isn't it' and Trainee 3.7 said 'oh yeah, …… I have never really understood completing the square, but that makes complete sense now…. hurray I love it'.

The debate continued as to which way things could be taught. Maggie questioned whether since going back into school the researcher had been able to add conceptual understanding to students that may have only been taught procedures or whether 'it has not ruined it for ever that they have only learnt procedures?'. Trainee 3.7 said that they had loved the smiley face method for multiplying out brackets and had only learnt the rules for completing the square commented 'it hasn't ruined it for us'. Maggie added 'it is a shame they didn't do it the right way around'. Following on from these discussions the group were asked what this all might look like in the classroom and whether there was a better way to do things. 'I just instinctively say it is better to learn the procedure first, but that is probably because that was how I was taught, and it worked for me' (Trainee 3.7).

Maggie referred to being shown how to differentiate from first principles on the SKE course and whether you should start with that or not (two other trainees commented that they were not shown that until they went to university) or whether you should start with the rules about powers. Trainee 3.9 commented 'I was taught to the test; this is called differentiation, and this is how you do it' then Trainee 3.7 commented 'looking back maybe it would be better if I had been shown from first principles' which had contradicted their previously made comments about wanted to know the rules first. There was a general agreement at this point and then Maggie commented 'but at the end of the day they don't test that', she commented that you were only tested on the rules and not whether you understood it. Most of the trainees commented at this point if they were asked to differentiate something today they could use the rule to do it but wouldn't be able to do it from first principles.

Maggie again echoed that it wasn't the rules or the concepts that were important, but both were needed. Trainee 3.7 said 'I would probably just teach the rules', Trainee 3.9 then said, 'I think it is about having the confidence to teach the concepts', Maggie agreed and commented 'there is still a lot of stuff that I don't know why'. Trainee 3.9 added 'I think you have to be really knowledgeable and know your subject inside out before you can start teaching conceptually'. The session concluded with showing how carrying out a standard subtraction calculation could be made more conceptual by referring to the place value (Appendix Figure 11).

6.3.4 Individual case Maggie

A couple of months after the session with the PGCE cohort 1, the researcher met with Maggie at her placement school. This involved two planning sessions, the opportunity for Maggie to teach her planned lessons and an evaluative interview.

Maggie's interview transcript was transcribed in full and shared with two experienced researchers to gather their opinions to help support analysis and conclusions. This was done for two reasons: firstly, to use the skills of other colleagues that had more experience in analysing qualitative data and secondly to avoid any bias due to the researcher knowing Maggie well.

The first interesting point from Maggie's interview is that she referred to herself as a learner on the SKE course where she felt that being taught in a more relational way 'made things easier for me'. She commented this was the 'first time

I had been taught in a certain way'. Before starting this study the researcher was Maggie's course tutor and teacher on this SKE course she is referring to. Maggie mentioned that as a pupil at school she was given a couple of examples and then lots of questions to practise. However, she wanted to know why things worked. She commented that seeing the conceptual ideas 'made me feel better about my own maths'.

When asked about which aspects of the collaborative planning were useful she referred to the session that the researcher ran with the PGCE cohort and commented that she had benefitted from thinking about 'what can I do with my classes that's not completely way out there'. This triangulated with discussions that we had about how we might teach subtraction (Appendix Figure 11). When looking at these ideas with her peers Maggie referred to just a small 'tweak' to what you would say that would make it much more conceptual. This notion of 'tweaking' things is referred to again in her interview

> 'I think... the understanding was fine but thinking of little tweaks and tools to make it obvious what the connections are for students learning some of the instrumental stuff at the same time was good, and I wouldn't have got there on my own. Even though I liked the conceptual stuff I didn't have the ideas, I didn't have the creative ideas, how could you show these things in different ways.' (Maggie)

When Maggie reflected on her taught lessons she mentioned confidence on several occasions 'it made me more confident to get them to look at completely different methods' and 'it gave me confidence to draw out things'. This confidence seems to have come from a personal increase in her own subject knowledge for teaching that arose during the sessions. When asked about which parts were useful, Maggie replied:

> ‘One was actual proper knowledge which was, ... I am not that familiar with the grid method and I'd sort of taught myself how to help them with it but then you showed me, the kind of more area-based grid method which I just thought was superb.’ (Maggie)

Although Maggie thought this area representation was superb and made sense to her she expressed her frustration that the students didn’t think the same ‘they were not interested at all because they had already moved on from that’. Although the students didn’t need the area representation at this point in their learning Maggie reassured herself that she would need to use it with another class. Alongside the new representations for teaching multiplication Maggie commented ‘I needed the knowledge from you and I needed the tips’.

When asked for additional feedback on the process, Maggie commented on the time pressures that makes teachers just want to reuse something they have done already. She felt that if there was a sense that collaborative planning would save time then others would buy in. She also commented ‘I think, when we jointly planned lessons, it has helped me, look at things slightly differently on everything that I do, so it is like a nice message for everything’ however there was a sense that she still needed the mentoring process to continue ‘you kind of don’t want it to stop’.

6.3.5 Group 4 PGCE cohort 2

The PGCE trainees from cohort 2 (Trainees 4.1 – 4.14) were given an evaluation form (Appendix 4.9) to complete during their final session. The data can be viewed in Appendix 4.10. The trainees were asked to comment on the research presentation. All evaluations agreed that it was informative with 62% strongly agreeing. All evaluations agreed that the presentation was a useful aspect of the professional development session with 54% strongly agreeing. The statement 'I believe all professional development should be underpinned by research' promoted difference in opinion; 15% disagreed, 69% agreed and 15% strongly agreed.

One of the main aims of the pilot study was to gain feedback on the tasks that had been chosen / designed to be part of the professional development sessions. The NCETM (Appendix 2.4) card sort was included because engaging in discussion with the cards would help to develop an understanding of the associated terminology. 92% of the trainees agreed that is was helpful with 54% strongly agreeing. One response (Trainee 4.14) disagreed with the statement and commented specifically on this in the additional comments 'was unclear about the card sort task, it seemed a grey area or maybe I missed the point of it'.

The next task, an activity looking at different scenarios, hoped to give teachers a practical example of what a relational lesson might look like. All evaluations agreed that it did, with 77% strongly agreeing. A range of other activities were used to make sense of the research theory 92% either agreed or strongly agreed that they did. Several trainees commented they enjoyed the completing the square task with Trainee 4.13 writing 'I really enjoyed the completing the square activity it made me see maths in a way never seen before'. All the trainees agreed or strongly agreed that the activities helped to develop aspects of their own subject knowledge.

The last aspect where feedback was gathered was the process of collaborative planning and team teaching. All responses agreed or strongly agreed that the process was useful, that it enabled them to put ideas into practice. A high proportion (86% of responses) strongly agreed that it was useful to reflect on the team teaching with peers with the other 14% agreeing.

In addition to the quantitative data gathered the trainees were invited to give additional feedback. There were two trainees (4.6 and 4.7) that seemed to particularly gain from the experience.

> 'Thoroughly enjoyable and productive two days. The relational/instrumental research was very interesting and very beneficial. Team teaching and being able to discuss ideas was very thought provoking, as well as the reflection period.' (Trainee 4.6)
>
> 'I found it really useful to implement/try out tasks that meant pupils had to have a more relational understanding. It was interesting to see how they took on the task. The scenarios highlighted and influenced the activities that we chose for the lesson.' (Trainee 4.7)

These comments have been triangulated with the lesson plans and observations made during the lessons that were taught. These two trainees spent time in their lesson considering multiple representations and developed an activity to support the teaching of the objective they were given (to be able to find the original amount given a percentage increase / decrease and the new amount). Their lesson plan can be seen in Appendix 4.11. The part that specifically addresses the ideas explored was a card sort (Appendix 4.12) making links between the different ways to calculate reverse percentages.

The pair of trainees wrote this activity and identified pupil success criteria in the lesson plan that all students should be able to identify the procedure behind reverse percentages and that some students should be able to explain and justify why the procedure works. A set of the cards is shown in Figure 42.

Figure 42. Reverse percentage multiple representations

Whilst the evaluations were very positive about the process there is no clear evidence in the lesson plans, except in the case above, the key ideas behind the CCC Model were implemented. The researcher was only able to watch part of each lesson however in discussion afterwards some trainees commented the part that had made the most difference to them was challenging students to explain their thinking and this was the area they focussed on. Many of the reflections in the debrief afterwards were focussed on nerves and how the practicalities had gone rather than on the specific focus of the CPD which they had engaged with, however since this was the first lesson that the PGCE students had ever taught this was not unexpected.

6.4 Overall themes emerging from the pilot study

The data from the pilot study led to several themes developing that were explored further in the main study.

6.4.1 The way individuals were taught

Throughout the pilot study, people referred how they were taught both at school and as adult learners on their journey to their teaching career. This manifested in two different ways, firstly with them liking the methods that they were taught and the acknowledgement that the rules worked for them, and secondly with them reflecting on the CPD activities and wishing they had been taught in that way instead.

When showing the more procedural method for expanding double brackets Trainee 3.7 commented ‘I like that, we got taught to do that’ and when discussing whether procedures or concepts should be taught first ‘I just instinctively say it is better to learn the procedure first, but that is probably because that was how I was taught and it worked for me’ (Trainee 3.7).

The HLTAs commented that they had the procedures drilled into them, with Brian saying, ‘I had procedures beaten into me’ but that because of that they felt they didn’t have a deep understanding.

After considering the completing the square slide, Trainee 1.4 commented ‘I wish I was taught that way, that makes so much sense I have never understood completing the square before’ he reflected on his own experience and explained that people had tried to teach him algebraically and that he had to relearn the rule before each exam as he never understood it. This response was echoed by Maggie ‘that is good isn’t it’ and Trainee 3.7 said ‘oh yeah … I have never really understood completing the square, but that makes complete sense now…. hurray I love it’.

Maggie referred to herself as a learner on the SKE course where she felt that being taught in a more relational way ‘made things easier’. She commented that this was the ‘first time I had been taught in a certain way’, comparing to her school experience of being given a couple of examples and then lots of questions to practise. She commented that she wanted to know why things worked and that seeing the conceptual ideas made her ‘feel better about my own maths.’

6.4.2 Beliefs about mathematics and learners

At various points throughout the CPD sessions, HLTAs and trainees explicitly referred to their beliefs about mathematics or their beliefs about learning. Some refer to beliefs systems that they hold and others throughout the CPD process suggest that there is a change in their thinking on some areas.

One point worthy of further consideration arose when discussing with Group 3 whether to start with the procedure or the concept first. The consensus was to start with the concept to get them to 'understand' it first before moving to the procedure. Maggie questioned whether conceptual understanding could be added to students who may have only been taught procedures in the past 'it has not ruined it for ever that they have only learnt procedures?'. Trainee 3.7, who had loved the smiley face method for multiplying out brackets and had only learnt the rules for completing the square commented 'it hasn't ruined it for us'. Maggie added 'it is a shame they didn't do it the right way around'.

One other theme that arose, when Group 3 decided in general that you should start with the concepts and then move to the procedures, was whether this was applicable with all attainment levels and that perhaps lower sets wouldn't cope with the concepts.

Interestingly this contradicts some of the other comments from the other groups where there was the feeling that the 'clever' children would be using only the procedures and perhaps the more visual representations (that might have potential to lead to a greater conceptual understanding) would be used with lower attaining or younger students. Maggie questioned 'do you find this a bit of a prejudice thing about drawing stuff, drawing boxes and pictures?' she recounted examples of trying to encourage students to draw things on their white boards and was faced with the response that students didn't want to because it was for little kids.

This led to teachers referring to their own experiences as learners on their PGCE course. Whilst engaging in mathematical problems on the PGCE course; Trainee 3.8 wanted to draw sketches to help with their problems and said they were made to feel that they were not proper mathematicians. Maggie echoed these ideas suggesting that the feeling was that students 'needed' to do the procedure because that is what clever people do and they didn't want to engage with other representations because 'that is what thick people do'. There was the acknowledgement from some in Group 3 that not only did students have these beliefs but some teachers did too.

After engaging in a variety of tasks as part of the multiple representations part of the CPD Maggie commented 'but this is proving that this can be interesting and technical and not just for the thick people'.

One change in belief arose when discussing calculus, Trainee 3.7 commented 'looking back maybe it would be better if I had been shown from first principles' which had contradicted previously made comments about which was a better way of thinking.

6.4.3 The use of an external source of information or stimulus

One theme that arose was trainees needing ideas from an external source. Maggie on several occasions referred to the CPD and the new ideas and information that she had gained. She commented ‘I needed the knowledge from you and I needed the tips’. There was also the sense that she still needed the mentoring process to continue ‘you kind of don’t want it to stop’ as she was worried that she wouldn’t then gather these new ideas.

With the other trainees from Group 3, when the researcher talked about the concept of introducing algebra as the notion of generalised arithmetic the comments were ‘can you be my mentor’ from one teacher and ‘can you be my maths teacher’ from another there was the acknowledgement that they wouldn’t have come up with these ideas by themselves. Maggie also commented on the impact of the planning time ‘I think, when we jointly planned lessons, it has helped me, look at things slightly differently on everything that I do, so it is like a nice message for everything’.

6.4.4 Developing confidence with subject and pedagogic knowledge

There were several times throughout sessions where it became apparent that teachers either didn’t have the subject knowledge themselves or the confidence to experiment with new pedagogic ideas.

The algebraic discussion with Group 3 where Trainee 3.2 had liked the idea of using fruit in a fruit bowl to introduce the idea of collecting like terms demonstrated either a lack of subject knowledge of what algebra is or a narrow approach to what success is. The teacher commented ‘it did work quite well’ and maybe in that lesson students had been able to collect like terms however this approach would not lead to an understanding of the concepts underpinning algebraic manipulation.

When doing the connection mapping task Group 1 struggled to make connections, this was emphasised with Group 3 ‘maybe it just comes with experience where the links are’.

The issue of teacher confidence to be able to teach in a more conceptual way arose on several occasions. Trainee 3.9 said, ‘I think it is about having the confidence to teach the concepts’, Maggie agreed and commented ‘there is still a lot of stuff that I don’t know why’. Trainee 3.9 added ‘I think you have to be really knowledgeable and know your subject inside out before you can start teaching conceptually’.

Throughout the collaborative planning sessions, it was apparent that Maggie gained confidence to try new ideas. When asked to reflect on the lessons she taught she mentioned confidence on several occasions firstly ‘it made me more confident to get them to look at completely different methods’ and ‘it gave me confidence to draw out things’.

> ‘I think... the understanding was fine but thinking of little tweaks and tools to make it obvious what the connections are for students learning some of the instrumental stuff at the same time was good, and I wouldn't have got there on my own. Even though I liked the conceptual stuff I didn't have the ideas, I didn't have the creative ideas, how could you show these things in different ways.’ (Maggie)

Part of this confidence seems to have come from an increase in subject knowledge for teaching that arose during the sessions. When asked about which parts were useful, Maggie replied ‘one was actual proper knowledge which was, ... I am not that familiar with the grid method and I'd sort of taught myself how to help them with it but then you showed me, the kind of more area-based grid method which I just thought was superb’.

6.4.5 Constraints from implementing ideas

When new ideas, from the CCC Model, and approaches to teaching and learning were proposed there were several reasons suggested as to why trainees felt it would be difficult to implement them. There was a perception, from some, that they wouldn’t be ‘allowed’ to teach that way because you must follow the scheme of work.

There was also the pressure of time. The focus was often on what to teach tomorrow and not being able to stand back and consider the big picture ‘I can’t wait until I have got the time to do that….. I am just trying to get through the next week’. These ideas were echoed with the rest of Group 3. Maggie also commented on the time pressures that makes teachers just want to reuse something they have done already rather than try something new.

There was reference to not having time to explore concepts deeply because of the deadlines of exams. Brian commented ‘as a reflection of what I am doing, I seem to be doing a lot of procedural, because I am doing things in a rush in preparation for the exam and there is not much conceptualising going on at all’. Others referred to themselves as learners being taught to the test ‘I was taught to the test; this is called differentiation, and this is how you do it’ (Trainee 3.9).

Other trainees in Group 3 commented they would like to explore and work with the ideas in a more connected way but wouldn’t think them up ‘maybe it just comes with experience where the links are’. They were having difficulty identifying the connections for themselves.

6.5 Implications from the pilot for the main case study

Yin (2014, p. 98), states that reports from a pilot should ‘be explicit about the lessons learned for both the research design and the field procedures’. This section details changes planned for the main study that arose from reflection and analysis of the pilot studies.

6.5.1 Refinement of the CPD programme

The CPD programme continually developed throughout the study. The card sorting task (used with Groups 2, 3 and 4) was chosen to engage discussion around the key terminology from the literature review and it did the job intended. As soon as the task was given to Group 2 the first comment was a clarification between the two of them of the terminology 'so instrumental is learning by rote …..'. They carried on rearranging the cards 'that is a procedure although maybe done in an understanding way'. Group 3 engaged in productive discussion and 92% of Group 4 agreed that the task was useful. This task was used within the main study for the same purpose.

Group 4 were the only pilot group to experience the classroom scenarios task regarding area of parallelograms and as all evaluations agreed (with 77% strongly agreeing) that it gave them a practical example of what a relational lesson might look like, therefore this was used within the main study.

As part of the multiple representation activity, Group 1 and 3 were shown a slide that showed different representations of completing the square, a trainee from Group 1 expressed that he had never understood it before and that it was useful. This was echoed when Group 3 saw it. These comments from across these two groups led the researcher to develop a full activity which was then used with Group 4. Although the evaluations from Group 4's sessions didn't have a specific question about the completing the square activity, several teachers commented that they enjoyed the completing the square activity with Trainee 4.13 writing 'I really enjoyed the completing the square activity it made me see maths in a way never seen before'. This activity was therefore kept as a main task for the main study with the aim of developing teachers own pedagogic knowledge.

The connection mapping task was more difficult to evaluate. Whilst the outcome was not always what was anticipated and in the case of Group 1, no real connections were made, the process of undertaking the activity led to useful discussions 'this is the conclusion that I am coming to, most of it you can teach either conceptual or procedural it is just the methods that you use'. It was decided that the connection mapping activity would not form part of the CPD programme, however it would be used as a tool during the collaborative planning sessions when there was a narrower focus.

6.5.2 Refinement of research methods

As part of the pilot, interviews with Maggie were carried out, which led to the need to address the questions of trustworthiness and reliability. Atkins and Wallace (2012, p. 86) question 'to what extent we can know what the interviewee is telling us is 'true' and how certain we can be that a different interviewer asking the same questions of the same interviewee would receive the same answers as we did'. As the researcher had known Maggie for several years, it was not possible to tell for sure whether she was being truthful or was saying maybe what she thought would want to be heard.

Another issue was ‘careful transcription of the words spoken does not, in itself, tell us what someone was meaning to say or what they were doing’ (Hammersley, 2012, p. 442). There were occasions throughout these interviews where the interviewees referred to situations that the researcher had been involved in and therefore knew about, and at the time the researcher felt like they had probed deep enough during the interview. On reflection, with the words from the transcription there was not always the written evidence that was felt during the interview process itself.

Bush (2007) acknowledges that it is difficult to ensure reliability using semi-structured interviews because of the deliberate strategy of treating each participant as a potentially unique respondent. In this case the interviewer contributed to shaping the conversation. With these issues arising during the pilot it was felt that it was more beneficial to increase validity and trustworthiness of the study if someone else were to carry out interviews.

Therefore, the final research design has been informed both by prevailing theories and by a fresh set of empirical observations (Yin, 2009).

6.5.3 Conclusions

The pilot study served the aims that were intended, the CPD programme was refined and an open mind led to themes arising as detailed in Section 6.4 that were explored further in the main study.

CHAPTER 7: THE MAIN STUDY

7.0 Introduction

This chapter outlines the activities carried out and the data that were collected as part of the main case study. The case was defined in Section 5.2.3 as the mathematics department within the secondary school where the researcher is employed. At the start of the study the mathematics department consisted of seven full time teachers (Charlotte, Daniel, Elliot, Frazer, Georgie, Heidi, Ian) and the researcher.

In addition to these teachers there was a HLTA employed to work entirely within mathematics (Annette) and an additional colleague who initially was employed to work with intervention groups (Brian). As the study progressed, Brian's role changed, and he followed the assessment only route to become qualified within the school, with the researcher as his mentor. He then was appointed to a teaching post within the department. Both Annette and Brian were involved in the pilot study and are already referred to in Chapter 6. Jenny was a PGCE trainee from a partner PGCE programme and was with the school on placement (January – June 2014).

Mid-study, Georgie had two terms off school (January – July 2015) on maternity leave and was covered by Kate. In September 2015, they both became part time and shared classes. From September 2015 Louise was appointed as a School

Direct trainee. She trained within the school and then was employed to work at the school from September 2016.

7.1 An overview of the main study

Section 6.5 identified the changes and refinements made to the design of the CPD programme after the implementation of the pilot study. Figure 43 shows the chronological order of the activities within the main research study.

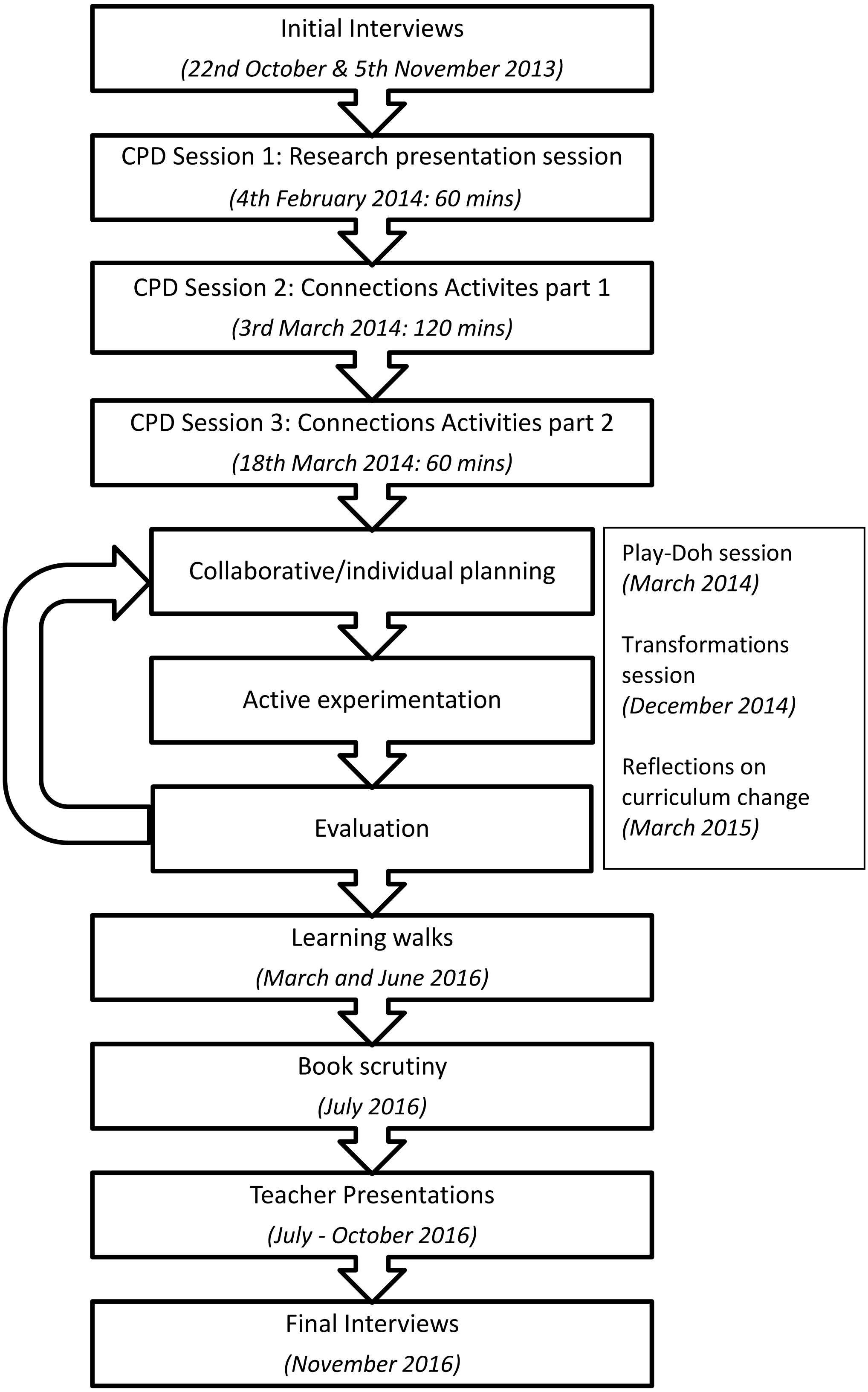


Figure 43. Outline of main study

7.2 Initial interviews and summary profiles

Interviews were carried out prior to beginning the main study in October / November 2013. The semi-structured interview schedule can be seen in Appendix 5.1. All nine colleagues were interviewed, and their interviews were transcribed and stored within NVIVO. These transcripts were explored during Graduate Student Week at a session with the Centre for Teaching Mathematics. The supervisory team and other colleagues from the University of Plymouth read the transcripts, highlighted key features and suggested possible codes. This helped avoid bias that having an insider researcher might bring to the analysis of the data. Each transcript was coded, example below in Figure 44, before codes were revisited and amended.

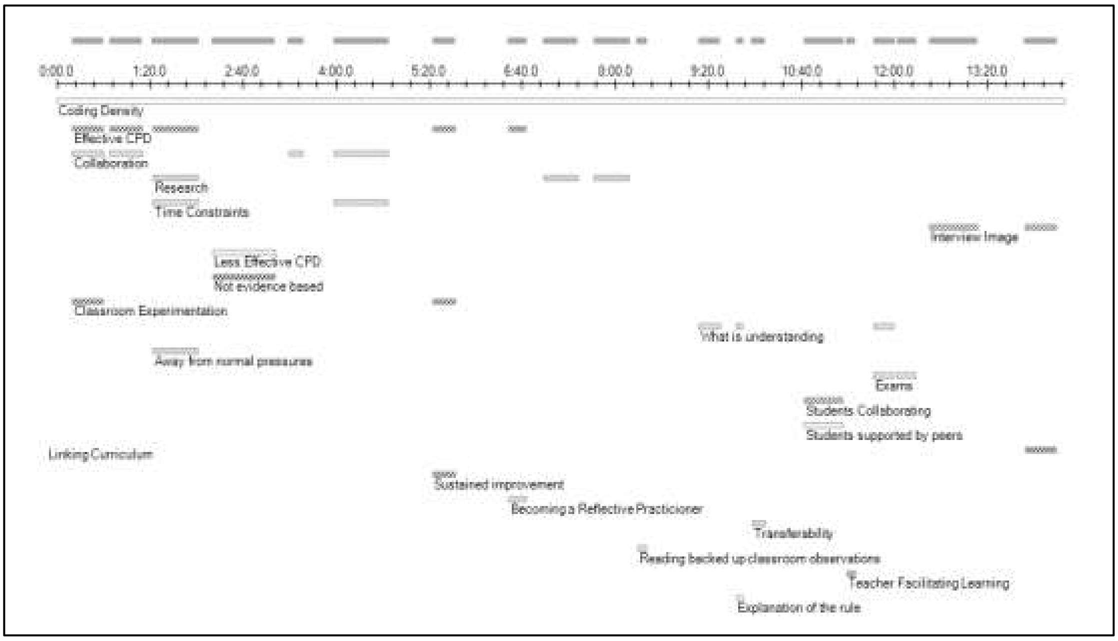


Figure 44. Summary of initial coding from Charlotte's initial interview

A summary of the teacher profiles and their general responses to the initial interview questions are detailed next. In the first instance, these were written and then an independent researcher, who didn't know the department, looked at the data and produced their own thoughts which were triangulated.

7.2.1 Summary profile of Annette

Annette is qualified as a primary school teacher and works as a HLTA within the school. She had recently engaged in an online professional development course, Mathematical Thinking in Schools run by the Open University. The positive aspects of such CPD included being challenged to think and gaining a variety of practical ideas that could be implemented in the classroom 'there are strategies that I have learnt, things like using NRICH activities - I will go to those first now rather than looking at a text book first when I am lesson planning'. The less positive aspects were identified to be the lack of collaboration due to the distance learning aspect and the lack of feedback on how to get better from marked course assignments.

One key comment that had been taken from a locally run day course was 'never tell what you can ask'. Annette had found this idea important and tries to use it within her teaching and when she is supporting students with intervention work. However, when learning herself; there were aspects that she wanted to be told, 'I needed to improve my own skills in algebra and I found it frustrating because I didn't actually get that help because the whole thing was you have to find it out for yourself'. She felt she was happy to start exploring new things but would like a facilitator to nudge in the correct direction whereas the nature of online training didn't allow for that.

When questioned about research, Annette identified searching for a variety of resources to help make teaching more practical. She had begun to read key texts identified by the online course ‘it is right, but it is finding the time and motivation to actually do it’.

When questioned about what is means for students to understand mathematics she felt they ‘can transfer it to another area’ and it is ‘taking it deeper rather than just being able to answer the question there and then to actually be able to go back and do it in weeks and months’. Her teaching beliefs include the importance of ‘trying to boost their confidence’ and ‘trying to help them make connections with real life’. She gave examples of bringing in patchwork quilts to consider angle properties and acknowledged that she tries to be practical with her teaching.

In addition to the quotes and facts above the external analyst felt when reading the interview transcripts that Annette came across as very empathic and that she cares about her students succeeding.

7.2.2 Summary profile of Brian

Brian is training within the school and he highlighted the main form of CPD has been from the researcher as his school-based mentor. He acknowledged the process of writing lesson plans and getting feedback to improve. He articulated the areas that have informed his development so far, including behaviour management and becoming less didactic. ‘What we are doing now is tailoring me to get more from the children than I am putting in and it is quite interesting because I am starting to emulate Nic in the way that I am doing activities rather than me at the front directing the lesson'. When probed further about what is meant in practice to emulate his mentor he comments that 'Nic likes lessons where the children are doing lots and then because they are doing things and discussing things they can engage, and their performance levels rise. I am starting to get that now'. He mentioned observing his mentor and the process of osmosis from that. He says that he can see that it is effective and wants to teach like that.

When asked about research, he referred to books given to support his early stages of development as a new teacher referring to the planning process. He describes students understanding mathematics as 'to be comfortable with the principles of what they are doing and be able to employ those skills into a wider problem'. He acknowledged that you can drill pupils to do things but that is not the way to do it as you don't get understanding 'you have to give them a chance to play with the mathematics themselves; you have to lay it out and give them scope to try and help themselves to acquire the knowledge'. He stated he personally enjoys working on challenging mathematics problems with other colleagues in the department.

The external analyst felt that during the interview Brian wanted to impress with his practical knowledge and made a lot of references to his previous navy career.

7.2.3 Summary profile of Charlotte

Charlotte has the role of mathematics department lead at the school and has been in post for a couple of years. She described one of the effective features of CPD as being able to talk to other colleagues. At external courses, she has appreciated being shown new ideas and then having opportunity to discuss them with colleagues in her school before putting into practice. Recently she has been on a middle management course where an advantage of being away from school meant she ‘could concentrate on what you were trying to achieve’ without interruption. The course involved lots of additional reading and the challenge for her was finding time amongst all the demands of her head of department role. She highlighted less effective CPD as being told that something was the case without any evidence or reasons to back this up giving the example of senior leaders showing a video example of ‘outstanding teaching’. She recognises the importance of collaboration and is trying to move the department forward by providing opportunity to let them plan collaboratively.

Charlotte reflects on effective CPD during her first five years ‘I had some good CPD about how to be a more interactive classroom, to work together, and I think that worked well for me and I continued that into the rest of my lessons. From then on, I have sort of tried different things and tweaked it myself, so it was using that grounding that I had quite a few years ago’. She felt it was significant because it happened early on in her career when she had an open mind and could evaluate herself in a reflective way.

When asked about research she highlighted materials that have been looked at again from ITE and a range of policy documents that informed action research within her own classroom. She felt that the research was backing up what she found in her own classroom.

When questioned about what it means for students to understand mathematics, she believes that if students understand something they are not following rules and patterns but know why they are doing it. She refers to students building their own methods and provides examples of encouraging students to share their ideas within lessons. ‘I would say if you can teach them to understand it then by understanding it they become a stronger mathematician and will become that grade C student, rather than teaching them rules to just get through the exam’.

The external analyst felt that Charlotte was keen to have more subject specific CPD and wanted to promote opportunities for staff but gets caught up in the practicalities of the timetable.

7.2.4 Summary profile of Daniel

Daniel has recently been actively engaged in a long-term subject knowledge enhancement course. He has enjoyed being away from the school environment to have the opportunity to ‘do maths’. The course has given him a ‘fresh approach to teaching different topics’ and he has valued sharing these ideas in department meetings and with colleagues over coffee.

He is trying to develop an approach to teaching for a deeper understanding, wanting students to know why things work rather than rote learning. He has enjoyed seeing a more algebraic approach on the course. This has helped him to develop his own subject knowledge and gain the confidence to trial new things.

He acknowledges a frustration of wanting to be able to implement these ideas with students, however timetable constraints have meant that he has not been able to try these algebraic approaches (such as completing the square) with classes that he has had due to attainment of students within his classes. The challenge, for him, has been to adapt the approach to work with students at a lower level. This is an area that he feels yet to solve. There is a sense of sometimes being directed to a more rote approach to prepare for exams which he is not finding fulfilling.

He would like to be able to say that he uses research within his subject teaching however honestly says he doesn't have the time. However, he acknowledges the benefit of talking to colleagues and other professionals and having time for reflecting on pedagogy.

He believes that if students are encouraged to understand mathematics more fully that they will be more willing to ask and create questions themselves rather than simply do questions. He also highlighted the challenge of getting students to remember things from one year to the next and on reflection considers that perhaps he doesn't make the links between topics as explicit as maybe he could.

The external analyst added that Daniel comes across as bored with working at low levels. It is obvious that he wants to pass on his ‘improved’ subject knowledge but makes no mention of the students developing this for themselves.

7.2.5 Summary profile of Elliot

Elliot has attended several day courses during his career and was also part of the Teaching Advanced Mathematics (TAM) course at the time of this initial interview. He described on several occasions liking to get resources and materials ‘that I can take away and use in school’. He has also benefitted from visiting topics on TAM that he has never taught so has found it useful ‘to be able to see different ways of being able to teach it and different ways to approach questions’.

He acknowledged that he has a routine that he is used to with his teaching and when trying to implement new ideas ‘it is quite hard to break that, so it is a real effort for me to make that change but when it does it becomes quite natural and it works quite well’. He is happy to try to accommodate things ‘to suit the way the school do things but for me that is a lot of effort’.

When questioned about research he referred to a masters’ module where he had the freedom to choose his topic to explore, he looked at how learning objectives were used in lessons and whether this restricted creativity. The research reading was guided by his university tutor.

When questioned about what it means to understand mathematics he thinks ‘it is finding the links within the subject’. He thinks that if a pupil understands you should be able to present the student with a new problem and they will ‘piece it together and are able to work parts of it out’. He refers to the greatest mathematicians of history and acknowledges that ‘the new rules were not there, they discovered them from the knowledge they had’. He acknowledges that it is difficult to observe mathematical understanding but through listening to discussion-based group work you can observe the language that they use and see what strategies they apply.

When asked about linking topic areas together within teaching ‘I would often link parallel lines and angles all together with trigonometry… because you quite often see GCSE questions similar but often ‘pupils come up with the links rather than me pointing them out’.

The impression from the transcript, gained by the external analyst, was that Elliot comes across as very co-operative, but a ‘reluctant changer’ and that possibly he is a bit lazy.

7.2.6 Summary profile of Frazer

Frazer acknowledged that ‘going on a recognised course’ has been more effective for him than in house CPD. The external day courses included one on whole class teaching and another on behaviour management. He attended these courses as an NQT and ‘it very much shaped how I perform in the classroom’.

He felt that in house CPD was less effective, for example implementing information technology or literacy policies was felt to be just about the school ticking boxes rather than efficient training that would have an impact on the experience of his students.

When questioned about mathematics CPD, he refers to sharing good practice within department meetings ‘even 5-10 minutes of having a fresh idea is useful’ and to mathematics sessions within a whole school training day such as on GeoGebra which is great, but he would like more time to develop skills with new technologies and more follow up afterwards.

Frazer hasn’t been involved in any subject knowledge training sessions other than sharing ideas at departmental meetings. He hasn’t taught A Level and feels that if he had to, he would need to study it himself again.

When questioned about research Frazer said, ‘I read articles, I keep an eye on mathematics websites more than journals these days, really interested in the BBC news and the sort of results they put there as a result of research’. However, he acknowledges that these don’t usually have an impact on his teaching but ‘teaching is very much a life experience, whatever you get gives you an insight into how you approach a lesson or a child in difficulty, your teaching day is always developing’. One of his personal beliefs is to continue to adapt his teaching to ensure that his students are happy in the classroom.

When questioned about what it means to have mathematical understanding he stated that ‘the deeper understanding is an insight into the concepts of maths, there is a huge difference between someone being able to process things,… it is very easy to give the ingredients of an answer doing a sharing ratio question but to actually have the knowledge of the underlying fractions and the proportional relationships which lie beneath is I think where the understanding starts’.

In addition to this the external analyst felt that Frazer was very negative about whole school CPD and it came across that he doesn’t think there was sufficient opportunity to engage with subject specific CPD.

7.2.7 Summary profile of Georgie

Georgie has taken part in local network meetings for middle leaders and found the opportunity to collaborate with groups of mathematics teachers an effective form of CPD. She identifies the importance of informal professional development opportunities ‘here at break times and lunch times are when we get the best collaborative planning, now that is not official collaborative professional development but just a simple conversation and that I am feeling like I am developing’.

She acknowledges a change within the department and previous tensions between colleagues meant this informal development didn’t happen before. Due to a change in personnel within the department and a more positive atmosphere ‘I love to talk about maths, and that is what has happened a lot more recently and the more we talk the more confident you feel to want to push yourself and to want to work with other people'. 'It is the openness and the way we discuss the collaboration and I think that has done more for me here than any professional courses that I have been on'. She identifies the importance of colleagues being open and having faith in each other and trusting that resources can be shared for everyone's benefit. Georgie commented on her increased confidence to take part in department meetings when doing mathematical problems.

Georgie identifies some of the less effective CPD to be whole school training and puts this down to the assumption that all teachers are at the same starting point, for example implementing literacy policies, sessions are not differentiated so that individual teachers can make progress with their own professional development.

Georgie has started a post-graduate certificate in education and has actively been finding her own reading around the subject and trying to implement and share ideas from this.

When questioned about what it means to understand mathematics she said, 'I am not sure I honestly understand mathematics, I think I have the confidence to be able to use it…., I certainly haven't got a degree'. She identified that if a student could explain to their partner then that would demonstrate understanding. She also acknowledges that understanding requires retention of knowledge and being able to use it in the long term.

When questioned about making links within different mathematical curriculum areas Georgie mentions an example of linking congruence, similarity and proportion and explains that the new scheme of learning has influenced these links. She commented that she doesn't make links explicit because she doesn't feel that is her job but that students come up with links themselves.

The external analyst picked up that Georgie seems enthusiastic and committed to making learning fun. From analysing the transcripts, it was felt that she possibly lacks confidence in her own ability and is slightly timid. She needs nurturing and a supportive environment to flourish, however comes across as keen to improve herself and her students learning.

7.2.8 Summary profile of Heidi

Heidi joined the school as a NQT and prior to that had completed the mathematics Subject Knowledge Enhancement (SKE) Course (at the time the researcher was running the programme) and then completed her PGCE at a local university.

As well as the recent professional development to become a teacher she has also engaged with the TAM course and the Princes Teaching Institute (PTI) new teacher subject knowledge days. Both have involved a considerable commitment in terms of time and reflection on practice and she has identified them as effective types of CPD. She would like more time at department meetings to talk about teaching and learning mathematics 'we are doing it already, but sometimes we don't really have enough time to think about the problems'. The main impact from attending CPD sessions has been an increase in confidence in both subject knowledge and subject pedagogy. 'I am just feeling generally more confident with my teaching, I am also more confident to make different connections between different areas of maths'. The CPD has given her chance to 'see the overall picture rather than topics as standing alone'. She feels that because of her training and CPD her explanations are clearer, and she is more confident in allowing students to work independently instead of being so didactic. She acknowledges trying to get students to understand why things are happening 'compared to just learning the method and the formula'. She acknowledged that this change in thinking happened when she took part in the SKE course where teachers and tutors modelled this pedagogy. This was a 'big revelation' compared to how she learnt mathematics in Russia 'where there was rote learning a lot, you literally have the formula, you apply it and don't think about how it works and you get a good mark in the exam'.

Within her PGCE she has researched PCK such as the teaching of fractions and this has informed her teaching with a greater understanding of misconceptions. Since working in school, she has searched for academic literature to enhance the progress of boys within mathematics.

When asked about what it means to understand mathematics Heidi comments that understanding means different things for different types of students 'one group of students, which for me I would be happy that they understood if they could apply it outside of school, the other kind of group of students I would expect to see them getting the right answers each time but not being afraid to tackle problems'.

The external analyst felt that Heidi wants to learn more about teaching and her subject. It was obvious that she is keen to make lessons fun, but afraid students sometimes get caught up enjoying and not necessarily learning. Heidi is concerned about the progress of boys and wants to improve this so is researching how. The transcript analysis showed that Heidi appears very conscientious and student oriented.

7.2.9 Summary profile of Ian

Ian has attended several one-day courses in his career but hasn't had opportunity to attend anything recently. He has taken part in courses on stretching higher attaining students and on making lessons outstanding which he felt were 'actually quite useful'. One of the important aspects that he found effective was the opportunity to collaborate with 'other teachers not from this department, nothing against the department, just fresh ideas'.

He identifies some of the in-school training as less effective due to either 'being talked at' or 'being taught how to suck eggs'.

Ian acknowledges that he doesn't use research articles or journals however he takes on other people's ideas and resources from places like the Times Educational Supplement website.

He describes mathematical understanding as ‘to understand the mechanics of the mathematical manipulation and calculation’. When questioned, what might be observed if a pupil understood mathematics, an example was given where a pupil that had been absent for a lesson had ‘solved the problem by a completely different method’. He also said that a pupil can understand ‘if they can explain it clearly, what it is they are doing and why they are doing it logically’.

Ian identified lots of different aspects of mathematics within the interview image including Pythagoras, parallel lines, proportion, trigonometry etc. when asked if these topics would be linked within normal teaching he felt ‘not on a regular basis’.

The external analyst commented that Ian ‘seems very fed up, and a bit disinterested – like he doesn’t want to be there’. It was felt that he doesn’t seem to want to put his own time in to developing resources or linking topics.

7.2.10 Summary profiles of Jenny, Kate and Louise

Jenny, Kate and Louise were not at school when the initial interviews were taken so there is not an initial summary of their comments, but they are included here as were involved in later interviews and quotes from their involvement are included where appropriate during this chapter. Jenny was on PGCE placement at the school with the researcher as her mentor. Kate joined the department to cover for Georgie’s maternity leave, she had previously been a head of department in a different school and had chosen to relinquish this role to have children herself. Louise joined the school as a School Direct trainee and later was appointed as a NQT.

7.3 Summary of initial interviews

Whilst Section 7.2 gave a summary profile for each person involved within the study this next section summarises the responses from the department by question topic.

7.3.1 Initial responses to what is effective CPD

All teachers cited examples of effective CPD. The common theme being able to get out of school and having the opportunity to discuss with other teaching staff from around the country. This allowed a clearer focus and opened fresh ideas. Often, they provided examples that you could bring back to school and implement. It was noted that having external professionals brought added value to the delivery and ideas.

Much of the CPD referred to was pedagogy based, typically around behaviour management, developing an interactive classroom, becoming a reflective teacher and stretching higher attaining students. Only Daniel, Elliot, and Heidi mentioned subject knowledge CPD. However, neither Daniel nor Elliot have used this in their classrooms though.

7.3.2 Initial responses to what is less effective CPD

Charlotte, Elliot, Frazer, Georgie, Heidi and Ian all cited examples of less effective CPD. The common theme was school-based CPD. Although they gave different reasons, it was seen to be too basic, just a tick box exercise and the themes did not always fit with mathematics. Heidi came across as frustrated that there was a lack of time within department meetings which made developing teaching and learning less effective because it could not be fully explored or discussed.

7.3.3 Initial responses to using research to inform teaching

Only Daniel and Ian have not actively used any research. They have searched for resources but not researched in its academic sense. Annette and Elliot researched because it was part of a course they were doing. Everyone who researched looked at pedagogy with only Heidi looking at subject knowledge as well. Heidi was clearly very interested in using research to inform all aspects of her work whilst Annette and Frazer cited lack of time, lack of motivation and lack of relevance as reasons for making little use of research.

7.3.4 Initial responses to what is mathematical understanding

Although the words used were different, the ethos was the same as colleagues described what it means to understand mathematics. The whole group agreed in being able to explain, to apply in different circumstances, to make connections, to have deeper understanding beyond applying rules and processes. To be able to explain clearly the what, the how and the why, by way of logical reasoning.

7.3.5 Initial responses to the connected curriculum

Annette, Frazer and Heidi say they always make links between topic areas and practical applications. Brian, Daniel and Ian say they sometimes make the links. Brian finds it easier for older students but feels that year seven have nothing to link with. Daniel doesn't appear to use this as much as he would like. Whereas, Charlotte, Elliot and Georgie say that they get the students to make the links for themselves.

7.3.6 Reflection on initial interviews

In analysing the initial interviews keywords sprung to mind that were used later as part of the coding process. These were; self-confidence, attitude, enjoyment, subject knowledge, pedagogy, changes in practice, barriers, collaboration, professional CPD and empathy.

Overall the initial interviews show the wide range of experiences and expertise present within the teachers in the main study. It is worth noting that although these were called 'initial interviews' and were carried out at the beginning of this study, the researcher had already been working with colleagues at the school for 18 months at this point.

During this time alongside the head of department, the researcher had already been involved in making curriculum changes, including implementing a new scheme of learning and although hadn't shared the CPD sessions at this point colleagues were aware of the researcher's background and that there was a focus on developing deeper understanding and connections.

As referred to in the summary profiles the researcher had led sessions on the SKE course for Heidi and was in the process of training Brian so these teachers were aware of the beliefs of the researcher as a mathematics educator and had opportunity to observe these in practice. So, it would be incorrect to conclude that these initial summary profiles were before any exposure to the thinking behind the CCC Model.

7.4 CPD session sharing research

The initial sharing research session was carried out on the 4th February 2014. Unfortunately, due to personal circumstances and other after school teaching commitments not all the department were present. The following teachers attended; Brian, Charlotte, Elliot, Frazer, Georgie and Ian. During this session, the PowerPoint slides shown in Appendix 2.3 were presented. The Trubridge and Graham (2013) paper summarising the CCC Model (Appendix 7.1) was emailed to absent teachers as prior reading before the next CPD session.

7.5 CPD bridging the CCC Model to practice

The bridging the CCC Model to practice sessions, were followed up on the 3rd and the 18th March 2014. A wide variety of tasks were shared with the department, these can be seen in Appendix 2.7. The research supervisor was also present for one of these sessions, so he could engage with the tasks and observe how the department responded.

7.6 Additional inputs at department meetings

In addition to the two researcher-led collaborative sessions, where colleagues were engaged in mathematical tasks, time was also dedicated in several departmental meetings to share other ideas and activities.

The researcher within their own practice had been experimenting with how to teach volume of prisms in a more conceptual way. Exploration with Play-Doh and stencil cutters was used to demonstrate the importance of the cross section when finding the prism volume. This practical task (Appendix 7.2) was shared with the department during a departmental meeting and was later written up and published in Mathematics in School (Trubridge, 2015).

In addition to sharing time at department meetings, display boards were created with the help of Jenny. Three boards were situated outside of the department office with the following headings: similarities and differences; multiple representations and developing concepts. Colleagues were encouraged to showcase ideas that they had trialled in their classes in the form of a poster on these boards.

7.7 Mid-study hypothesis

In September 2014 during a regular supervisory meeting, progress of the project and CPD was discussed and reviewed. It was felt that the department had enjoyed the departmental training sessions and were generally on board with the ideas, however it wasn't clear at this stage how much impact there had been with teachers transferring ideas to their classrooms. As evidence was needed to confirm these thoughts, a set of mid-study interviews was planned and constructed to review progress and to identify the next steps forward.

7.8 Mid-study interviews part 1

Interviews were carried out in November 2014 (Appendices 5.2 and 5.3) to gather people's thoughts and opinions on the CPD programme so far and test the mid-study hypothesis. These interviews were carried out with all mathematics teachers (Charlotte, Daniel, Elliot, Frazer, Georgie, Heidi and Ian) and with Brian who now was training within the school. A week later the interview was carried out with Jenny whilst in her NQT year at a different school. She had been part of the departmental sessions, so it seemed appropriate to see impact on her thinking even though she no longer worked within the department.

There were four focus areas considered within the interviews. The aim was to give colleagues a chance to reflect and feedback on the research presentations and what effect if any it had on their thinking. The second area was to gather feedback on the CPD activities and to ascertain which activities had been useful and why. The third aspect was to gather information about what had been trialled at an individual level and the final feedback required was what colleagues thought should happen next at departmental level.

This first section gives impressions from each teacher that was interviewed, and then later sections show responses grouped by interview question. As before, to remove potential bias, the interviews were carried out by the project supervisor. These were transcribed in full and analysed using NVIVO software and findings were discussed at a meeting.

7.8.1 Mid-study interview part 1 summary by person.

Teacher	Summary comments
Brian	CPD opened my eyes to different ways of introducing topics. Have used several of the topics in class. Am getting class to reflect and discuss more. Becoming less traditional and trying to be more relational. Ultimately this approach will make me a better teacher.
Charlotte	CPD made me more aware of relational teaching, which I always try to use, but I struggle with some students who do not care about understanding. Same/difference is an easy, quick way to start probing. Need a department approach to key topics to avoid tricks and tips.
Daniel	Haven't really engaged with the CPD due to lack of maths teaching, but I could see the logic of the methods, and I liked the visual forms for algebra which were new to me. Would be keen to be involved in developing more topics in this way with the department. The session itself was useful because it opened my eyes to different ways of introducing things to children, so it was good.
Elliott	I liked the different approaches, but I didn't understand some of them. I find multiple representations very difficult, and I have difficulty remembering all of the different ways. It would be useful if they were added into the scheme of work, so we could use them more readily.
Frazer	I have used other relational tasks and they have been very effective in connecting topics, such as geometric and algebraic proofs. I like to have practical approaches.
Georgie	Seeing the approach work has had more impact than the actual CPD, the knowing it will work with our students. Although the session helped me to make more connections. It was both exciting and inspiring. Unfortunately, some of the scheme of work units do not include a lot of relational learning, and it needs to be there to build on. We all need to share more resources. The new approach gives me more confidence. Learning more about maths personally and becoming more interested in the details. I am becoming excited, and I find it inspiring.
Heidi	The students need to be able to explore topics to make their learning relational, but time constraints are an issue. I would like to use more investigations.
Ian	Using the CPD activities gets the students to make more connections. I like visual and kinaesthetic activities. We need to bring forward our ideas and share them.
Jenny	The CPD activity changed the way I would teach certain topics. I have taken similarities and differences to my new school and always include this. The approach empowers learners. They find lots of connections. Students need to know why they are doing stuff and how it all links together. I need to emphasise the students' development of methods to solve things rather than prescribing.

Table 2. Summary of mid-study interviews part 1 by person

Table 2 shows a summary of comments from each person interviewed in November 2014.

7.8.2 Reflections on research presentation.

Initially colleagues were asked to describe in their own words the differences between instrumental and relational understanding. Responses varied from the presentation of very clear articulated answers with use of appropriate terminology to some showing a general understanding of the ideas but less confidence in the words to use. There was a consensus from most that instrumental understanding involved rules and instructions that were provided by the teacher. Quotes are shown in Table 3.

Teacher	Quotes about instrumental understanding
Charlotte	Instrumental teaching would be give them the formula.
Elliott	Instrumental is kind of a set of rules to be able to solve a problem.
Frazer	Instrumental is just having a set of instructions and following it.
Heidi	Instrumental understanding is what has been practiced for a long time in classrooms, you give the rules, children know the rules and then just follow it.
Jenny	Instrumental understanding is more about applying methods.

Table 3. Quotes about instrumental understanding

Other colleagues were less confident in giving a specific definition however when prompted further could talk about important ideas. Georgie felt that it regarded the tips and tricks that were often talked about and Ian felt that it was about rote learning.

When talking about relational understanding, slightly different elements were revealed by different people. The majority referred to relational as being about making links and that students would understand the processes underpinning the ideas. This is shown by their quotes in Table 4.

Teacher	Quotes about relational understanding
Charlotte	Relational understanding is the students having an understanding of where something comes from.
Elliott	Relational is trying to link ideas and topics to what they already know and then to extend it a little bit further.
Frazer	Relational is when experiences are used to deepen the understanding and to apply those learnt bits of knowledge in a more applied way rather than just a rote way.
Georgie	Relational understanding was being able to see the relationships between the different topics.
Ian	Understanding the processes.
Jenny	Relational is more about making links between different subjects for example knowing that maybe quadratic expressions can be related to a graph which can be related to something else. So just relating all areas of maths together.

Table 4. Quotes about relational understanding

Whilst not presenting answers as definitions Heidi talked about adjusting schemes of learning to make more connections and to develop students' intuition in transferring their knowledge and the importance of 'starting with exploration and then coming up with the rule'. Similarly, Daniel without presenting definitions said, 'it was all very clear for me at the time, but if you don't engage with it you lose the thread' he was very clear that there are different levels of understanding and that he wanted to work with students to help them understand relationships about mathematical concepts.

Although not asked specifically, Brian instead of giving differences between the two types of understanding, responded in a way that reflected further on what he thought the possible outcomes might be.

> 'If we are relying on purely teaching them by rote then all we will be able to expect them to do is repeat facts and the ability for them to repeat those facts is based on the extent of their memory and to an extent their intellect as well…….If we can make connections between different areas of maths we can model things in different ways so we can actually access the different kinds of learner, then we can help them understand what is happening because if they understand it there is a greater chance they will be able to repeat it and to use it in a practical context.' (Brian)

In addition to describing the differences between relational and instrumental some teachers gave practical examples in their responses. Charlotte referred to using areas of rectangles to help students derive areas of triangles and Frazer referred to the topic of proportion which was being taught at the time ‘I would expect it as a wider understanding of knowing proportionality compared to a whole as well as using the relationships, with some understanding of using them rather than this is the unit method and this is the result you get from it’.

Some additional, perhaps unexpected responses arose from two teachers during this question about whether all students could think in a relational way or whether different attainment students needed to work in different ways. This theme will be detailed further during the analysis in Chapter 9.

A follow up question looked at whether the exposure to the research presentation had any personal impact on what it means to understand mathematics. There was no consensus to this question. For Charlotte and Elliot, the presentation had refocused them. For Jenny, it had challenged the way she had been taught and whether she personally had a relational understanding herself. A summary of quotes is provided in Table 5.

Teacher	Quotes about impact of research presentation on what it means to understand mathematics
Charlotte	Yes, it has shifted me a bit, it was nothing that I wasn't already thinking of but it has made me more aware.
Elliott	It has a little bit, yes … it did make me think again to sort of refresh that, and to hear it from a different point of view and specifically about maths and how it can be applied and how the benefits can be seen from it so yes.
Georgie	Yes, although I was always aware it was important.
Heidi	Yes, definitely.
Ian	I think it did for quite a while but as we have seen I have forgotten about it. It did influence me in the recent past, it certainly did at the time.
Jenny	I think personally it made me question whether or not I actually understood maths … in that I think I went through my A Levels and degree just knowing how to do things rather than what I was doing. So, it certainly made me question whether I understand maths or not.

Table 5. Quotes about impact of research presentation on what it means to understand mathematics

One interesting point to note here was although in the first part of the question Daniel hadn't been confident to define the differences between instrumental and relational at this point he could bring the ideas together in his own mind:

> 'Yes because, there are generic issues across all subjects, teaching for understanding rather than just teaching to pass an exam and when we were going through things you can see why a change of emphasis in your approach would be more beneficial in the long run. I was able to reflect on what I was taught as a teacher, some people looked at methods for methods sake and seeing that was just a way of remembering tricks to pass exams rather than understanding. So, when Nic did the presentation I could absolutely see where she was coming from and why it was a better approach to aspire to teach like it.' (Daniel)

Although the question was aimed at finding out about teachers own thinking about understanding, many commented in their responses about their practice of teaching and their learners. This can be seen in Table 6.

Teacher	Quotes reflecting on what it means to understand mathematics
Charlotte	I think it has made me definitely make sure that everything that I try to teach, I try to teach in a more relational way and get the students to understand where it has come from. I think sometimes you do creep into here it is get on with it and it has made me think more about what I am teaching and why.
Frazer	To some extent, I suppose, the workshops that we have done have been useful in that regard, thinking about not just teaching it but trying to emphasise the understanding.
Heidi	We did a meeting about teaching volume of prisms etc. and before I was doing mostly instrumental understanding sort of this is the rule this is what you can do, and you can do these questions. But I found there was lots of confusions between the volumes and the areas and surface areas and what we were looking for.
Ian	I am aware all of the time of trying to get them to understand rather than follow basic rules.
Jenny	I think it had an impact on, when I was trying to teach pupils to understand mathematics, making sure that they are aware of concepts and why they are doing things rather than just being able to apply methods to a problem.

Table 6. Quotes reflecting on what it means to understand mathematics

Three additional themes arose from this question: whether the multiple representations were useful; whether the students wanted to be taught that way, and the importance of the CPD being carried out within the context of the school people were working in. These ideas will be teased out in the analysis within Chapter 9.

In addition to the quotes stated in the tables within this section, the external analyst also provided some insights as an outsider. It was felt from reading the transcripts that both Georgie and Heidi seem to be fully on board and that Brian, Charlotte, Daniel, Elliot and Frazer can see the changes and where it will lead. They felt that Daniel can see that research is useful, but it doesn’t look like he will take it further at this stage. Elliot, Frazer, and Ian all come across as if they can’t be bothered with research as it means they should change what they do.

7.8.3 Reflections on CPD activities

Colleagues were asked to comment on the effectiveness of the CPD activities in terms of making sense in a practical way of the research presentations. They were asked if there were any activities that were useful to help clarify terminology or to develop meaning with regards to developing a more relational understanding. The interviewer was provided with a hand out (see Appendix 5.3) which contained prompt images for colleagues to refer to specific activities.

When asked to comment on the effectiveness of the CPD in terms of making sense of the research presentation. Jenny offered a useful insight:

> ‘It gave us the opportunity to see what was meant by those terms. I think we all interpreted instrumental and relational understanding ourselves but when we actually applied it to the tasks and we were able to see the connections, it helped to clarify that.’ (Jenny)

The consensus as shown in Table 7 was that the CPD was effective with different reasons being: they liked the different methods; visual images helped to give a better understanding, and that it made them think as teachers. There were also positive responses to the use of similarities and differences and how to teach the volume of prisms. Some comments indicated a motivation for themselves to attempt to change their practice.

All agreed effective	B, E, F, G –gave them different methods C, G – made me think C, G, J – gave a better understanding D, I – visual pictures were useful J – made connections clearer
Positive responses	E – thought it was a good way in showing what Nic was trying to do G – springboard to get going, less lazy at looking for challenging resources J – changed the way I would teach, click moment when connections made F – more effective than I had planned, eureka moments evident from pupils, student collaboration is important H – prisms were fantastic C – similarities and differences easy way to start, not hard to plan, not time consuming

Table 7. Summary of effectiveness of the CPD session in terms of making sense of the research presentation

Not surprisingly five of the teachers referred to having seen and used the visual area method for expanding brackets before. One of the colleagues who hadn't seen visual representations in this way before commented more generally on that approach.

> 'The visual aspect was something that I hadn't considered or experienced before. To see it presented in that way, I could see how it would work for a cohort of students. That was the one thing that I really picked up.' (Daniel)

When asked whether there were any tasks that were useful to clarify the terminology or to develop meaning there were two responses prevalent. Colleagues that had been familiar with the area representations of expanding brackets specifically mentioned the completing the square task as a new way for them personally to see the idea and suggested that it was an idea that they wouldn't have come up with themselves. Quotes are shown in Table 8.

Teacher	Quotes about the completing the square task
Charlotte	From a personal perspective, the one with the completing the square, that made me think and made me think about how I teach that.
Elliott	I would have though the quadratic one (area representation) was the best to show that we can use this with our grid method and we can use it algebra and we can use it with proof and we can use it with completing the square so I thought that was the most useful showing that if they have a good understanding of using the grid then you can use that in a wide range of other topics.
Georgie	The one that really strikes me was the completing the square activity and seeing all the different connections with quadratic equations. I never would have thought about coming up with an activity like that.
Ian	This one - the missing square…. It was useful having the visual picture, with my year 11 set three last year. That was useful.
Jenny	The one on completing the square, I will always remember that because again I knew how to complete the square, but I didn't know what I was doing so on a personal level rather than a teaching level. I then understood how to complete the square and how it all linked together and that we are actually trying to make a square.

Table 8. Quotes about the completing the square task

The other task that was specifically referred to by the two colleagues newer to teaching was the area of the parallelogram task. For Brian because it exemplified how a relational task might look and for Heidi because she recognised that students struggle to learn and apply a lot of different procedures. Their quotes are shown in Table 9.

Teacher	Quotes about the parallelograms task
Brian	For me and my development the exploring the three different ways of teaching area was useful with the range of different scenarios, the initial scenario was much more procedural whereas scenario three was much more complex and probably spread over three lessons is a very good way of accessing the different ways of thinking putting across as a conceptual piece.
Heidi	Yes, I found the parallelogram task really interesting because it is a topic that pupils struggle with and are always forgetting, only because I think they learn the formula and then they forget which side they need to use.

Table 9. Quotes about the parallelogram task

There were some general other positive areas that arose during this phase of the interviews. Georgie and Heidi commented that the sessions were interesting whereas Brian commented that the sessions were grabbing his attention. Georgie said, 'it was probably the most exciting CPD session that I have had' the reasons were two-fold firstly the content and presentation of new ideas but secondly due to the climate that had been created for the department to explore the ideas together. When answering questions about the CPD tasks many teachers referred to examples they had already tried with their classes since the sessions. These responses are included in the next section as they are about the active experimentation stage.

7.8.4 Reflections on active experimentation phase

The third phase of the interview asked teachers to reflect on any impact since the CPD sessions for either themselves or their learners. General keywords that appeared included enthusiastic, engaged, fun, empowering and visual impact. However, it was acknowledged by Elliot that multiple representations are 'difficult if you don't know them yourself first' and that the 'momentum can drop over time' (Charlotte). Table 10 shows a summary of responses.

Reflections on personal impact	C, D, F - enthused/inspired D - liked different approaches F - fully applied and visual G - liked similarities and differences, easy way in, scheme of work needs to incorporate more relational activities but two of department not on board. E - struggle with multiple representations – I need to know them before I can use them. C - initial push but drops off.
Reflections on impact of learners	• Enthusiastic, empowering, deeper understanding, connections, engagement, fun, exploring, experimental, visual impact I - Venn diagrams bring out more connections E - similarities and differences is good

Table 10. Summary of reflections on impact for teachers and learners

Teachers explained which tasks they had used from the CPD sessions. The examples from the training sessions that had been trialled with classes and were specifically mentioned in the interview responses included the ideas behind the parallelogram scenarios task (Brian) and the completing the square activity (Georgie and Ian). When referring to using the completing the square task Georgie commented ‘seeing the way that the students made the connections after I had made the connections in a similar way was inspiring. It was brilliant’. Whereas Ian commented that ‘it was useful having the visual picture’ when using it with his year eleven class.

The activity that generated the most enthusiasm was one that wasn’t planned as part of the initial CPD session, but one that was developed by the researcher for use with their own classes and later shared at a departmental meeting (Appendix 7.2). The activity was introducing the concept of volumes of prisms through layering up cross sections of Play-Doh pieces. Five teachers said that they had used the activity with their own classes.

Some teachers used the activity because it was different, inspiring or exciting. Charlotte referred to the departmental response ‘when Nic brought in the Play-Doh everyone got enthused’ and in response of her trialling it with pupils ‘I can see it in the lessons the students getting more engagement out of it and getting more fun out of it’. Daniel commented that he had thought it was a good idea and had used it immediately ‘it was a different approach and, in some ways, an inspiring approach rather than looking at cubes or maths objects’. Others referred to using the ideas because of the more visual approach ‘I have used this one because some children do have problems visualising what a prism is and using the plasticine to be able to cut it up to reveal the cross section was really useful especially with lower year groups' (Brian).

Heidi highlighted several key points in terms of noticing that when she had previously taught a year ten class 'they were not sure really of the volume of the prism' she commented her students could not identify what a prism was and they were getting confused between spheres, pyramids and prisms. She also commented some of these confusions had arisen from using prisms in Science lessons on light. So, after the departmental meeting and being shown the Play-Doh idea she trialled the task and commented 'I found that my year elevens actually got the concept that what we were looking for was a constant cross section'. When probed more deeply during the interview process a useful summary of her personal reflection is given below.

> 'Um, ..., I think, I think because they are using their intuition, I remember asking them about if you cut this you have one heart and then another and how you can build it up to create a volume and they started to think about the levels of cross section so it was more intuitive so the whole, the volume must be something to do with what we can see from the bird's eye so the cross section and then how high it is. I remember some of them saying if I only have three hearts stuck together their volume must be a lot less than if I had twenty, so they did work it out from there. I think it is a combination of things, the first that students are given complete freedom just to have a go. They created different prisms and had the chance to think about what a prism is and seeing that if you combined shapes such as bone and heart it wouldn't work. So, they experimented in different ways. The first step was they had the freedom to explore, I think the second step was them discussing on the tables. This activity promoted loads of discussion, I was afraid that they might be distracted by the Play-Doh, but they actually stayed on task fully for the whole lesson, and they used a medium which we don't normally use in maths well and they were talking amongst themselves so working out things. There was a minimum input from me in that lesson, I remember at the end of the lesson when we started putting it in a general way then I had to point them a little bit. I think they made connections for example that the prism in Science is not the same as in

> maths so how it compared to cuboids, so they made a connection with other shapes. I think for me, to give them open ended task first and give them time to see what they can come up with themselves.' (Heidi)

This quote from her transcript shows that not only has she tried the task shown in the department meeting but has reflected very clearly on why she felt it was successful, both in terms of the students being able to explore and discuss with minimum input from the teacher and in terms of them having a greater understanding of the importance of the cross section of a prism.

In addition to teachers trialling activities that they had been shown, some had made up their own or used activities from other sources. Frazer referred to a visual approach to developing a proof of Pythagoras Theorem.

> 'It was a very simple chopped up square and moving a couple of triangles around and their understanding of proving Pythagoras was superb and we went on to prove algebraically from that and there was no difficulty in knowing why the geometric proof had worked and that it could be transformed into an algebraic proof.' (Frazer)

Whereas Charlotte referred to trialling an activity (based on an article that had been read) that explored integration by considering areas rather than introducing it as the opposite of differentiation. This led to the students deriving the new knowledge themselves.

Daniel took the similarities and differences idea and looked at cuboids with different dimensions. He reflected that students noticed the number of edges, vertices and faces would stay the same whereas the volume and surface area would change. Heidi also explored similarities and differences, but this time with a range of different sequences teasing out the importance of a common difference for a linear sequence. She also mentioned when teaching ratio specifically connecting it to graphs.

Whilst some teachers gave specific examples of activities that they had tried up to the interview point in November, others began to reflect more generally on changes in their overall practice. Charlotte commented on some subtle changes 'I think being more aware of saying what is the same and what is different a little bit more. Probably just standing back and allowing the students to just spot things and perhaps even when I have got differentiated questions up I would say these ones are easier because and these ones are harder because and now they tell me'. Since several people had commented on using the similarities and differences idea the interviewer asked Charlotte (as head of department) why she thought this idea was used so readily. She responded, 'I think because it is the easiest thing to do' she went on to expand 'I can just make the teaching a little bit better very quickly'. She then commented that some of the other ideas take more time to organise and to consider how you are going to teach using them.

Daniel also commented on making small tweaks that would be changed when trying to develop understanding 'when I am planning and reflecting on things that are going well or not, when I am trying to develop the understanding of different contexts... it is almost.... like you tinker and change, and you pull things out because it didn't work and put something else in'.

Brian spoke more generally about how he thinks about delivering lessons. He reflects that at the start of his training he was more traditional and then comments on his reflections from watching the researchers lessons ‘when I was in with Nic I saw the way she was doing things and her carousels and ways of not giving solutions or black or white answers just answering a question with a question, giving them open ended things to think about, going away to reflect in their books and ask each other’ he then comments about his own journey as a teacher ‘I think I am on the road towards this but wouldn’t be arrogant enough to say that I am there’.

During the interview process some constraints to implementing the activities/ideas arose from the transcripts. Daniel talked about the challenges that he has motivating a difficult year eleven class and how his focus has been on ‘trying to build motivational levels rather than on trying to develop understanding’. One additional issue that arose was that it was easy to use ideas that had been introduced and discussed in departmental meetings but when trying to come up with new ideas from scratch it was more difficult, a comment arose regarding presenting students with different images ‘I need to know what the multiple representations are before I can give them to the kids is the main thing’ (Elliot).

During this phase of the interview, the teachers were also asked what they do when trying to make their teaching more relational. Heidi commented that it was the opportunity for students to explore that was paramount and giving students the chance to talk to each other during that process. Elliot also said that he would find tasks where ‘you could just play with the maths’ and that ‘if they can explore in some way then that makes it more relational'. Jenny referred to the need to going back to basics and checking students had a deeper understanding before moving onto new content. Table 11 shows responses when teachers were asked to describe activities that were relational in nature.

Examples of activities	• Exploring, making own connections, what's the same and what's different frequently used, reflection, interpreting, student discussions, questioning, group work, practical tasks, multiple representations, developing concepts, visual and kinaesthetic, card sorts, connections All - similarities and differences C, E, I, J, H - similarities and differences is easy to use, no extra work involved, easiest to implement, simple open questions, can explore, have tasks to play with, helps remembering, brings out misconceptions and challenges G - development of new tasks. Check internet, other tasks and activities, good balance and variety

Table 11. Summary of examples of activities that are more relational

Having had the opportunity to engage with all the interview transcripts up to this point the external analyst picked up from reading that they felt teachers had a positive attitude to the ideas that had been suggested within the CCC Model but that most teachers were working with what they had been given with only really Georgie actively finding her own resources. The impression from the external analyst was that teachers expected to be spoon fed the activities at the time they needed them but then they were happy and positive to use them.

7.8.5 Moving forwards as a department

The final question asked colleagues what they felt should be done, at either an individual level or departmental level, to further develop a connected approach to mathematics teaching and learning within the school.

Departmental level	C, D, F - pick out key topics, teach same way cross department E - put representations into scheme of work so all can use F, G - more workshops/regular sessions to develop activities I, G - everyone to bring an idea, share, build up activities B – action plan H - extra unit based on John Mason's 'Thinking Mathematically' for problem solving tasks
Individual level	B - regular, more frequent, development sessions F - more awareness of possibilities, more sharing and talk of approaches, more time to reflect B - embrace for NQT, make me better teacher D - appeals to me/want to push on further I – problems – time, with everything else that has to be done

Table 12. Summary of next steps

Table 12 shows a summary of comments. The following themes arose from their interview transcripts.

Enjoyment of opportunity to collaborate

There were several teachers that commented that they had enjoyed working collaboratively. Daniel commented 'I really enjoyed the collaborative element of sharing the examples' and Georgie requested wanting 'more of the same really' and commented that during the CPD session in March 'I came away really excited'. Ian commented that he wanted to continue 'like we have been doing, pooling ideas, ideas and success and what doesn't work.'

More frequent development time

Whilst it was acknowledged that 'there is time in every meeting spent on developing teaching and learning which never used to be the case' (Daniel) there was also the request to 'spend more time collaborating in departmental meetings' (Daniel). This was echoed by other colleagues suggesting that the amount of CPD session time as a department should be greater 'more frequent sessions on developing this, it would be useful, but there is a lot of pressure on other things' (Brian) and 'I don't think my understanding is quite there and I need to be pulled along. I think we should have much more regular sessions on how to build up these activities' (Georgie), alongside 'more sharing and talk of approaches…. and more time to think about it' (Frazer).

Focus on developing certain topics

Another theme arising was to highlight certain topics to work on with the aim of developing a more consistent approach on how we teach the chosen topic area. The topics mentioned were either curriculum topics like fractions or negative numbers (Charlotte), ratio and proportion (Daniel), or more generic themes like problem solving and the development of the key ideas of specialising and generalising (Heidi). The department lead had a very clear steer on the best way to move forward and the rationale behind it.

> 'I think we need to spend some time together going through the scheme of learning, picking out keys topics that are perhaps taught in a manner that doesn't help understanding like negative numbers, which some of the department do, we hear it all the time like two negatives make a positive getting rid of those comments would be nice. and just pulling apart some of those big topics and deciding as a department how we are going to teach them so when a bell goes from class to class they are taught in a similar way, so it is not confusing for the child. That's a big ask and a big thing for me.' (Charlotte)

These ideas were echoed in a similar way by Brian.

> 'What we are moving towards is developing our ideas about the way we want to deliver certain topics and we will agree that as a department and then we will refine the ways in which we want to deliver those, we will move

> more and more towards a connected approach because that is by far preferential to teaching by rote. That’s the way I think we will go.’ (Brian)

This underlying theme was echoed by a request for more workshops to ‘come up with some particular lessons like the prism lesson that we could all decide that is how we are going to teach that topic throughout KS3 and KS4 with some continuation’ (Frazer). Also, to spend ‘more time informally at break times considering the topics that are coming up’ (Daniel).

Building into schemes of learning

There were also comments suggesting the ideas that were shared in the CPD sessions should be written as prompts into the scheme of learning ‘so we can remind ourselves, I think that would be the easiest thing as a classroom teacher to implement in the future’ (Elliot) because they felt that if the resources weren’t used immediately there is a danger that teachers would forget them when needed.

Moving towards others sharing ideas

Whilst there was a mention in the section above about needing the ideas that had been provided in training when discussing moving forwards there was a feeling from some that others should and could be responsible for sharing their own ideas. The comments began to reflect that all colleagues could at later meetings ‘come with at least one idea of what we have done, because you can bring them and lay them out to see’ (Ian) and ‘taking it in turns, perhaps every departmental meeting someone has to come up with something inspirational’ (Georgie).

Barriers

When questioned about how the department could move forward, several people referred to constraints that could stop this developmental work happening. The notion of constraints to implementing successful CPD is explored in much more depth in Chapter 9. However, some of the comments that arose at this point in the interview process were ‘it is it is just getting the time to do it around everything else that has to be done’ (Ian) and time was mentioned again with specific mention of curriculum changes ‘we need time to actually do it which is always a precious commodity especially when you have got curriculum changes and we are moving to a system without levels’ (Brian).

Desire to move forwards

Despite some of the barriers highlighted above there were several comments that show the positivity to moving forwards along this development agenda. ‘I think we have got the desire to do it and buy in from the department …. and if I can embrace this as an approach it will make me a better teacher’ (Brian). ‘I would certainly want to engage in developing the understanding of students in what we have been discussing. It is something that appeals to me and something that I would want to push on further’ (Daniel). Whilst there were no negative comments with regards to the engagement in the CPD programme from individuals themselves, when interviewed Georgie highlighted ‘I would say there are a couple of members of the department that are slightly more cynical than others, but I would say that actually the enthusiasm from the rest of the department is enough to pull them along’.

Additional reflections from the external analyst felt that at this point Ian was the only person to come out with negative comments and citing time as a main barrier. Daniel was felt to now be much more on board and wanting to work as part of the department team despite not being in mathematics all the time.

7.9 Developing conceptual understanding of transformations

Following on from the outcomes of the mid-study interviews it was decided to run an additional CPD session with the department. After liaising with the head of department it was decided to focus on transformations, as this unit would be taught by all colleagues after the Christmas break. It was decided there would be an opportunity to look at progression in the subject and to develop some of the ideas of the CCC Model to highlight key ideas.

The aim of the session was twofold. Firstly, to demonstrate a range of resources that could be used to support the teaching of transformations through the CCC principle of ‘what stays the same and what is different’ and secondly to show how the same principle could be applied to develop teachers own subject knowledge of matrix transformations.

The CPD session (Appendix 2.8) was delivered on 16th December 2014, with teachers Daniel and Georgie absent. It began with the activity shown in Figure 45. Colleagues were asked to consider the similarities and differences between the two images. This task was one that had been used as part of the researcher’s teaching with a further mathematics A Level group.

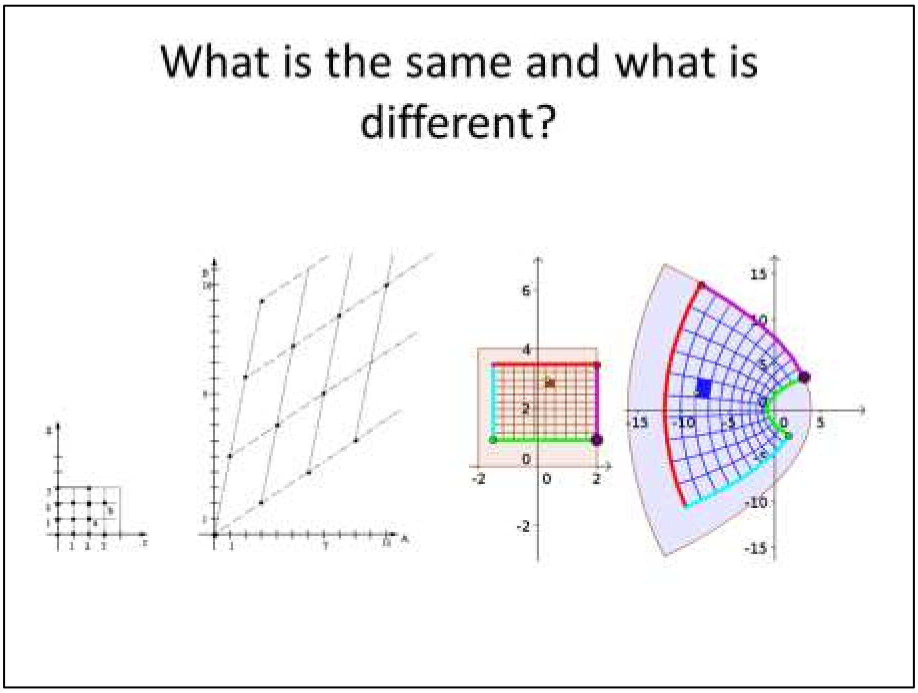


Figure 45. Transformations

A general discussion followed about what were the big ideas and concepts that underpinned transformations and their study within secondary school. Colleagues were given tablet computers to explore a variety of activities created in GeoGebra (Figure 46) and to consider the following questions throughout the session.

- What are the key conceptual ideas underpinning the study of transformations?
- What are the key connections to make with other topics?
- How does the principle of what stays the same and what changes help to develop conceptual understanding?

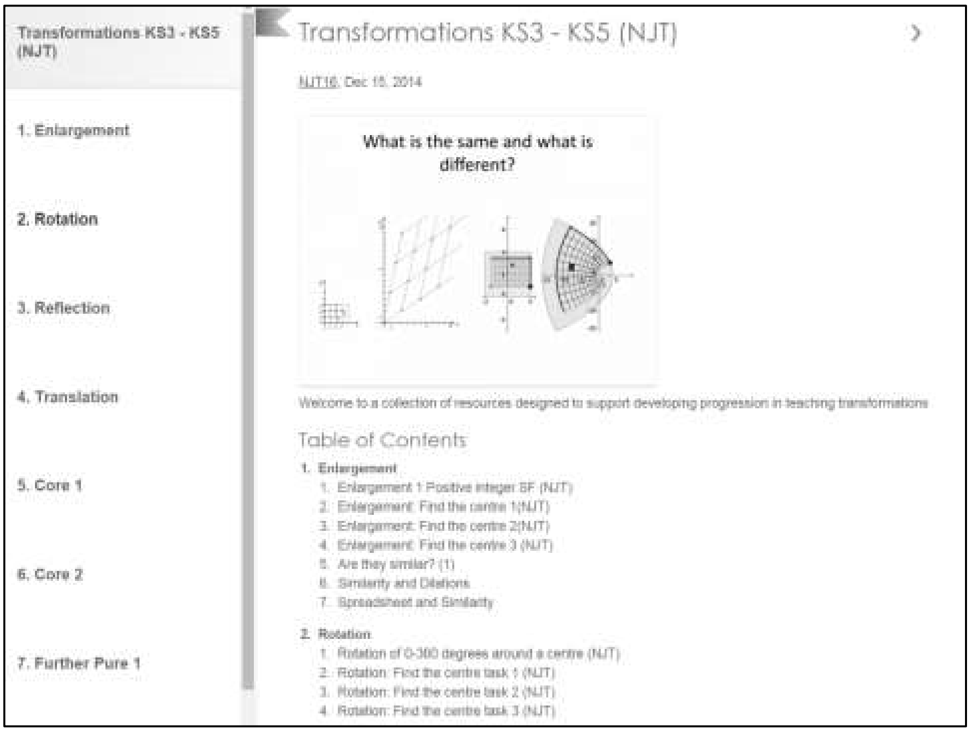


Figure 46. GeoGebra book with files to explore during the CPD session

Samples of the activities are shown in Figures 47 – 50.

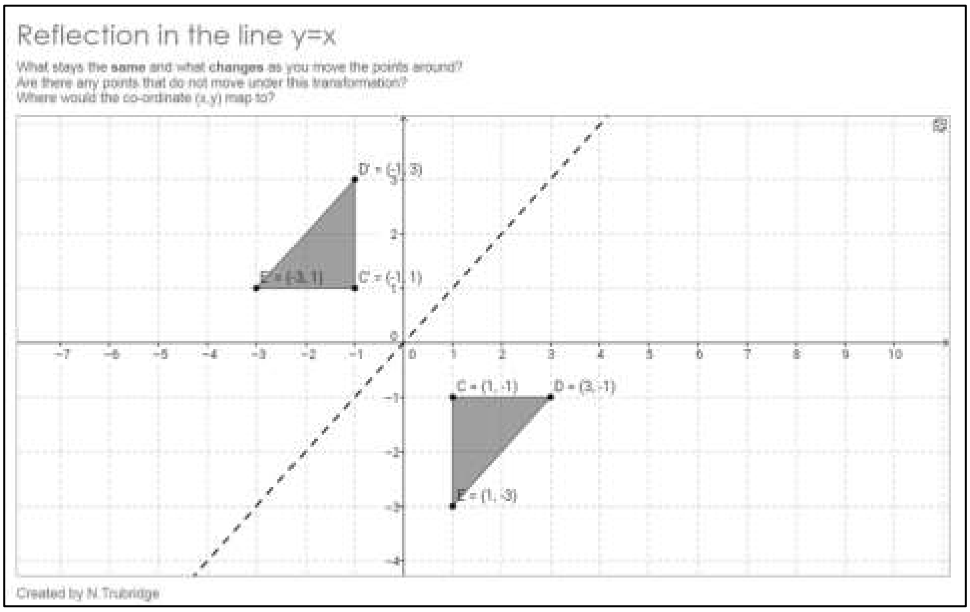


Figure 47. Reflection in the line y = x

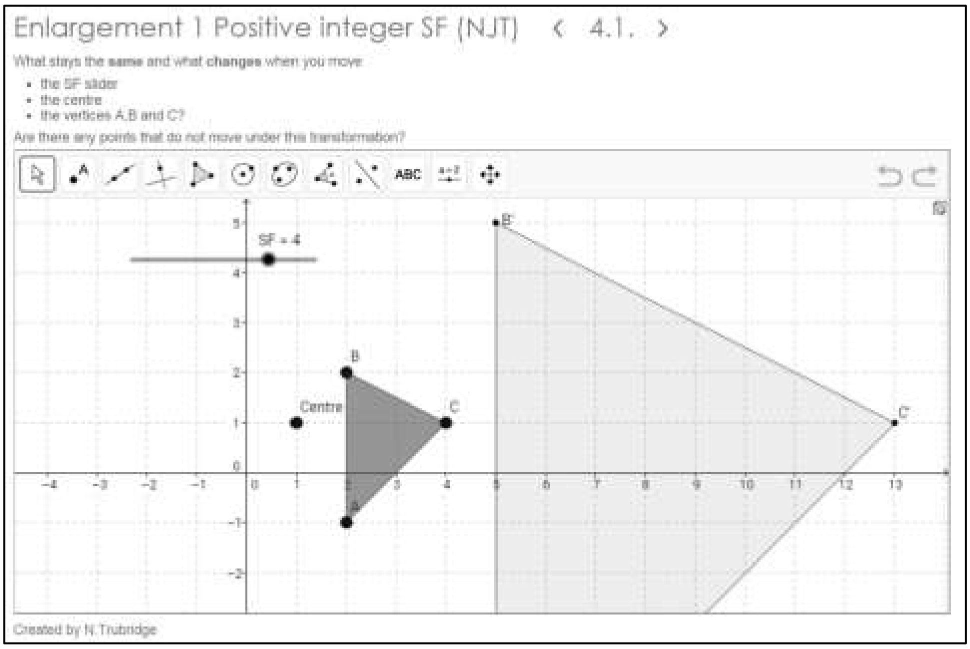


Figure 48. Enlargement

After exploring a range of resources at KS3 and KS4 the department considered how the same principles of exploring similarities and differences could be applied to transformations using matrices from the A Level further mathematics syllabus.

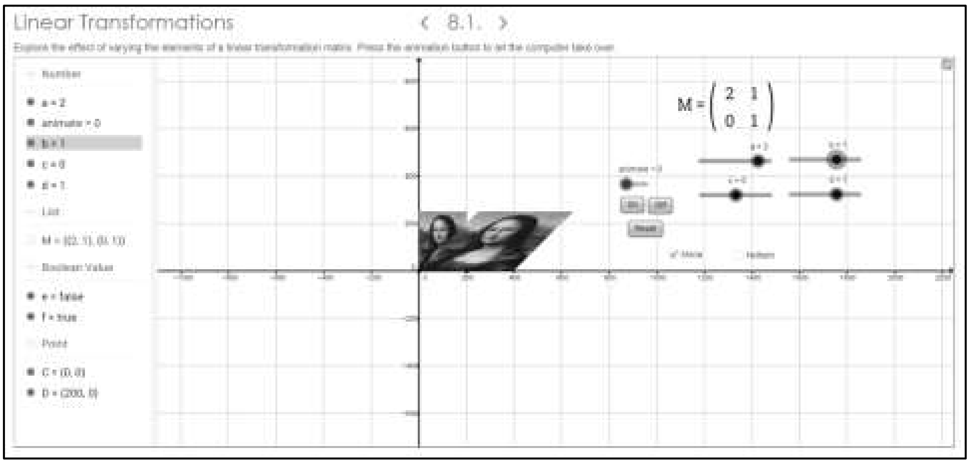


Figure 49. Matrix transformations

After having chance to explore the learning activities a discussion revolved around the key concept of invariance, the terminology of invariant points, invariant lines and lines of invariant points was introduced. This terminology was new to all colleagues. However, is a requirement of further mathematics A Level. The consensus was that although matrix transformation was a new concept the idea of exploring similarities and differences meant everyone could connect to their prior learning and begin to access the ideas. Figure 49 shows another idea from the further mathematics A Level topic of transformations using matrices. This was used as a plenary slide and colleagues were quick to engage with the activity and to develop their own thinking of the topic.

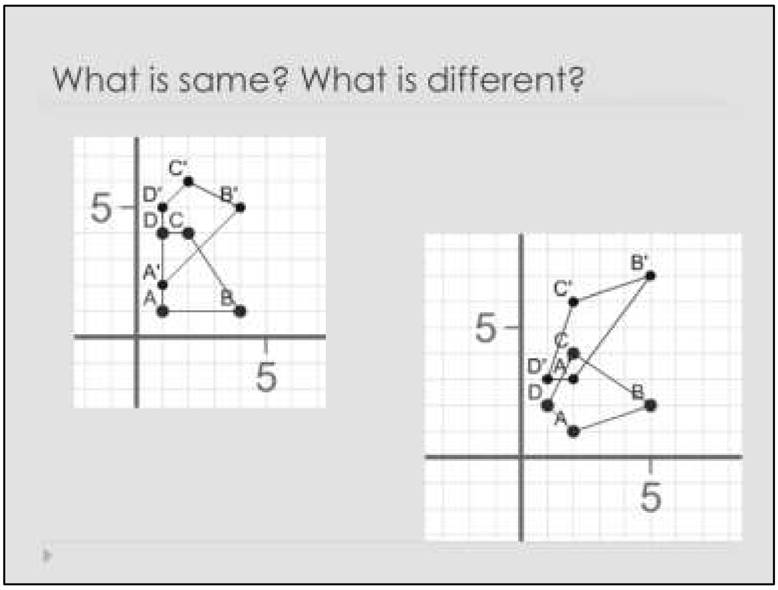


Figure 50. Matrix transformations 'what's the same, what's different?'

A concept map was started during the session and it was agreed that it would be completed as a reminder of the new terminology explored within the CPD session. The concept map following on from the session was shared with all colleagues (Appendix 2.11).

At the beginning of the next departmental training feedback was gathered with regards to the trial of the transformations activities. At this point Elliot, Frazer and Heidi had used the GeoGebra reflections and enlargements activities with their classes. They commented on students being able to visualise and predict what would happen as the various sliders were moved to explore scale factors. The consensus was that exploring where generalised co-ordinates (x, y) would map to was something new but Charlotte thought this would make progression to A Level easier. Quotes about the tasks are given in Table 13.

Teacher	Quotes about the GeoGebra transformations activity
Charlotte	If they notice the coordinates earlier in younger years it is going to make the A Level easier.
Elliot	It was good because kids could visualise where the shape would be, scale factor of 2 would be there, 1 would be there, where would zero be then, then they could work out what would happen. Every student gets something out of it and everyone notices something new including myself. Like somethings I didn't realise, like with the diagonal line that the coordinates just swapped over for the x and y. Um I think the only issue with every kid noticing something is that it is really hard to get it all back off them. There is one kid there with an absolute nugget of information, but it is trying to get it out to the rest of the class.
Frazer	I think identifying what type of transformation has taken place was probably the easiest one to do, they were picking those up quickly.
Heidi	I think enlargements worked really well finding the centre, all my year 9 class really enjoyed this, they really liked it. When they started to draw their own lines from the centre of enlargement, I said next lesson we will look at negatives, and fractional - they said there must be something that goes on the other side. When I was doing it with top said what was good was they came to the conclusions themselves, when they were reflecting that lines connecting their coordinates should be crossing at 90 degrees, they just came to this conclusion themselves, I did not explain it to them but quite a handful of them picked this up and yeah, that's why it has to be in that place.

Table 13. Quotes about the GeoGebra transformations activity

7.10 Reviewing the CCC in the context of curriculum change

As part of ongoing departmental planning meetings, a session was led with the department looking at the 2014 curriculum documents. The session was held with the department on 5th March 2015. The following teachers were present, Charlotte, Elliot, Frazer, Heidi and Ian. The aims of this session were:

- To review curriculum changes and implications for future developments in mathematics
- To review where the model of the CCC fits into the latest educational landscape
- To consider roles within the whole school development planning process
- To establish collaborative planning groups and to identify ways forward

The session began with a mathematical task as shown in Figure 51.

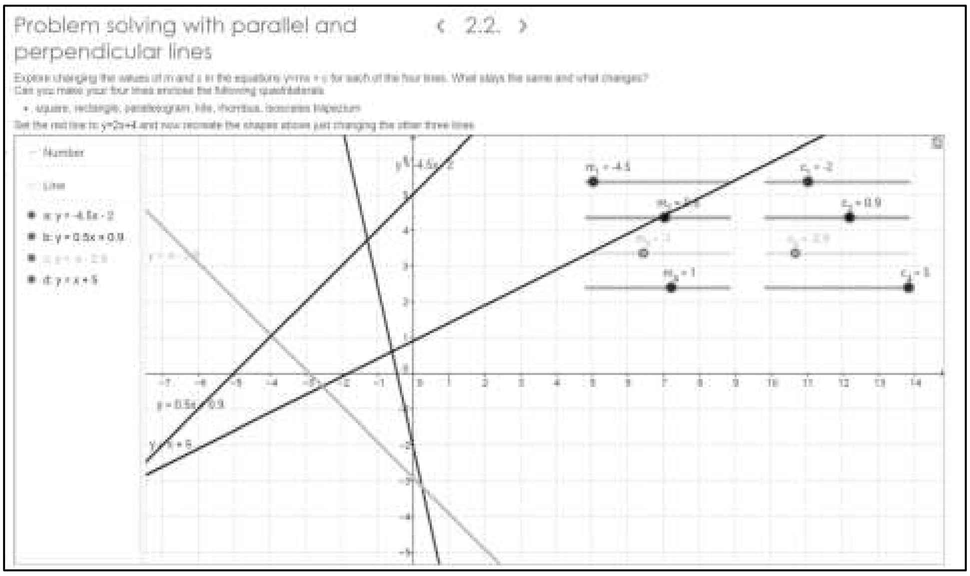


Figure 51. Problem solving with parallel and perpendicular lines.

The session then highlighted the change in emphasis between the 2008 and the 2014 curriculum documentation as detailed in Sections 2.4.1 and 2.4.3. Colleagues were given the opportunity to explore the DfE (2013) and DfE (2014) documents. A discussion was led highlighting the relevant links to the CCC Model (Appendix 2.9).

Colleagues were made aware of strands that would underpin the next academic year's whole school development plan and how the work of the department could dovetail with this. Figure 52 shows the links. At this point it is worth noting that there is was a change in emphasis within the whole school development planning cycle to incorporate more action research projects across the school. There was an expectation that all colleagues would engage at some level.

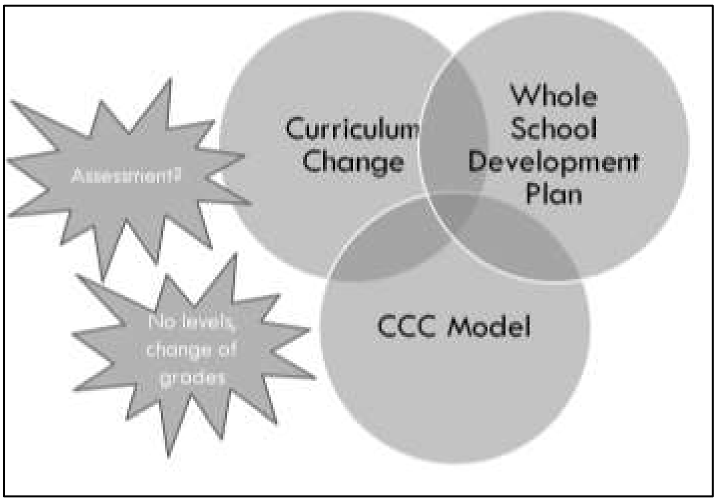


Figure 52. Links between department and whole school development

7.11 Moving forward with curriculum change and the CCC Model

The topics addressed in Section 7.10 were revisited as a department on 23rd April 2015 during a two-hour training session (Appendix 2.10). The same five teachers (Charlotte, Elliot, Frazer, Heidi and Ian) were present for this and Brian joined the meeting half way though.

Trying to incorporate the overlap in the Venn diagram from Figure 52 a draft planning template ‘Exploring the features of the CCC Model within the context of the 2014 curriculum - Mathematics Collaborative Action Research’ was developed (Appendix 2.12). This was shown as an optional document; however, the aims of embedding curriculum change alongside active experimentation of the CCC Model were prevalent whilst meeting proposed whole school aims of all colleagues engaging with action research. The themes needed to meet whole school project requirements are detailed in the left-hand column as elements of study.

After the planning document was shared, colleagues were grouped into teams to consider the new curriculum documents and to identify within the strands of number and ratio and proportion which topic areas would need to be added or developed further in the current scheme of learning. At the end of the session all groups shared their findings.

At KS3 it was agreed that additional work would need to be developed to ensure learners had a deeper understanding of fractions and the manipulation between mixed and improper fractions. At the top end of the foundation tier it was agreed that an additional unit of work would need to be developed to address inverse proportion. At the top end of higher tier, it was agreed there would need to be development of a unit of work looking at rates of chance to address new aspects of the GCSE subject criteria.

The training session concluded these areas would be crucial to ensure students could access the required content for the new GCSE specifications. It was also noted at this point that using multiple representations and generating ideas to develop these concepts would be an appropriate way to tackle these areas and therefore that the ideas underpinning the CCC Model were helpful for this developmental work.

7.12 Planning for curriculum change

Time was allocated for teachers to get together and to discuss how they were going to address the topics identified in Section 7.11. Some groups quickly divided up their topic areas for individual work and others chose to spend more time collaborating. The researcher provided a sample of research papers on the topics identified previously.

7.13 Mid-study interviews part 2

Additional interviews were carried out in September 2015 to gather thoughts and opinions on the curriculum changes that were happening and the change in focus for CPD at whole school level. These interviews were carried out with Annette, Brian, Charlotte, Daniel, Elliot, Heidi and Ian. Frazer was absent from school when these interviews were scheduled, and Georgie had only just returned to work part time and was not available that day. The interview schedule and prompt images can be viewed in Appendices 5.4 and 5.5.

7.13.1 Mid-study interview part 2 summaries by person

As before to remove potential bias the interviews were carried out by the project supervisor. These were later transcribed in full and analysed using NVIVO software. Findings were discussed at a meeting and a summary of the outcome is given in Table 14.

Teacher	Comments
Annette	Don't go much to training but do a lot of reading. Problem solving from the new curriculum makes it practical, so I like it. Action research is an opportunity to develop my pedagogic content knowledge in my own way. I particularly like using different representations and diagrams. I have been able to implement changes and I can see it is really much more beneficial to the students, but time is an issue. CPD has to become part of your normal practice and going on day courses doesn't always make it happen whereas long term if it is expected it will happen. Fear in maths is one of the worse things because your brain seizes up and you can't do it. The atmosphere of being accepting of making mistakes has helped student motivation.
Brian	I've taken Nic's model on board. It works very well with the curriculum change. I think multiple representations are good because it targets different kinds of learner but also get them to stand back and look at similarities and differences. Fluency in fundamental skills as it unlocks so many different areas. Multiple representations will be very useful to me. Action research will provide opportunity to develop pedagogic content knowledge as well as dovetailing into my outstanding teacher programme. Nic's model is helping implement change but there are pressures of time, thinking and planning and creating. Bench marking is missing from Nic's model but otherwise it is the same as the school model. The whole school approach makes everyone comply, but I want to develop professionally anyway. Students can conceptualise at different levels. Motivation comes from approaches taken – stimulating interest by multiple representations, different approaches, outside interests.
Charlotte	Nic's model goes nicely with the curriculum change about mastery and understanding. We are starting to change the teachers' beliefs. Fluent in fundamentals supported by similarities and differences. I am also linking in with growth mind set. I think implementing the change is very important, particularly to improve the girls understanding. Students can be a barrier. The department has become much more open and supportive. I would be researching and working on the project anyway. Nic's model is more collaborative than the school approach. I believe in growth mind set but students are not motivated to understand at the moment. Many don't want to take the risk of failing so won't try.
Daniel	I haven't got a firm grasp of the curriculum changes in maths but am trying elements of what Nic is doing. I am trying to use similarities and differences in all my subject areas. I find creating the scenario difficult so I steal others' ideas, but I want to produce my own. Directed time has been beneficial in moving forwards. Ongoing CPD is preferable to one off courses as you put it into practice. Conceptual thinking requires time so some groups, such as GCSE retakes, have more limited opportunity to develop this. Motivation is improving due to higher expectations.

Table 14. Summary of mid-study interviews part 2 by person

Teacher	Comments
Elliot	Nic's model is a natural way of going through the transitional phase. I'm looking at problem solving skills supported by building connections and ideas. Refreshing to think about different ways of doing things. Given directed time is good for me as it gives me a better chance of working on the projects and research. Nic's model is transitional rather than tick boxes and the CPD is more purposeful now. The younger you start the more conceptual thinking you will develop whilst there is less of a time constraint. Motivation lessens as the students get older.
Heidi	Nic's model matches with the changes in the new curriculum in challenging the students more. Fluent in fundamentals supported by problem solving. Provides an opportunity to develop my own pedagogic content knowledge. My explanations need to be clearer, but the students don't all find it easy to think so openly about the problems. I would be researching and engaging with the action research anyway so directed time is irrelevant to me. Being able to pursue your own interests is more beneficial than the usual CPD courses you get sent on. Conceptual thinking needs to be developed over time. Motivation still needs to be developed by making topics relevant and giving sufficient thinking time.
Ian	Model looks good but needs to be put into practice. Fits with government direction, pushes less routine tasks. Solving problems by similarities and differences. Definitely makes you think more about how you teach. Similarities and differences, helps you connect ideas. Works in most areas. Easy to implement. Action research is helping me stay motivated. Using directed time is helpful. Better to be able to work on own interests as opposed to imposed CPD, but easy to fall behind with everything else there is to do. Conceptual thinking is available at different levels, but student motivation is more to do with success rather than understanding.

Table 14 continued. Summary of mid-study interviews part 2 by person

7.13.2 Reflections on curriculum change

Whilst the study progressed there was a change of curriculum. Colleagues were asked to think about the CCC Model and to articulate whether the ideas and ways of working were more or less important than before considering the curriculum changes. Table 15 shows a summary of responses.

A, C, E, I, H	Felt the CCC Model worked better and fitted in well
E	Felt it was a natural way to go through the transitional phase
D	Didn't know how it fitted
B, H	Felt changes needed to be included such as areas for development, B felt the areas could be found through a gap analysis

Table 15. Summary of how the CCC Model fits considering curriculum change

Both Annette and Daniel acknowledged that due to after school teaching commitments or being in other department meetings respectively that they had missed some of the departmental sessions so felt distant and out of the loop from discussions. However, Annette commented that there was a lot more problem solving in the new curriculum and the ideas of students working more collaboratively with their reasoning would support this. Daniel commented that 'I'd take elements of it (the CCC Model) and try and develop them across my teaching and that's how I have been doing it' he was less confident in how this linked to the new curriculum.

Brian's initial response reflected on the organisational logistics as to how we had planned to divide up and share the new topics that we would be developing 'I think we are doing it very pragmatically we have gone through and looked at what we already had, we have done a gap analysis to see what is missing, based on that gap analysis we have then gone through the scheme of learning and put in areas which need further development'.

This was echoed by Elliot who felt that the CCC Model was a good 'transitional phase' which was supporting the department to move to the ideas in the new curriculum and to get to the endpoint of 'a different way of teaching maths for the department'. Ian was less confident with his response but when questioned whether the direction of travel being taken by the department was in line with government suggestions he agreed.

Both Charlotte and Heidi referred to specifics from the new curriculum and how they linked to the CCC Model. Charlotte's response referred to mastery in the new curriculum and that it would be unlikely that students could master a topic if they had learnt in a procedural way. Heidi talked more generally about the connections that could be made and the valuing of different methods.

> 'I know what we are doing and what we have done. I think we are getting more emphasis on understanding or giving the understanding to our students that mathematics is a highly-interconnected body of ideas but trying to link different concepts and different ideas in lessons from the activities. Also, we are trying to emphasise that for the question you could come up with different ideas and different methods and that all of them will be valued within the new curriculum, we are also looking at problem solving and reasoning behind it so whenever they look at the problem they say if I know this then I also know this etc. and they can use this kind of reasoning. Obviously, we have identified any changes in the new curriculum and we are including them.' (Heidi)

Heidi felt the CCC Model was now 'more important because the emphasis is obviously a lot of their reasoning, conjecturing, justifying findings' she referred to the change in style of questions and that the level of challenge was higher.

7.13.3 Reflections on mathematics action research

At this stage, small teams within the mathematics department had been put together to develop a unit of work / resources to support aspects of the new curriculum. This section of the interview gave colleagues opportunity to explain which aspect of the new curriculum they felt needed developing and which feature of the CCC Model they felt might support this (Appendix 5.6). All colleagues were aware of the topics that they were developing.

Fluency	B - because it connects so much together C, H
Reasoning	
Problem solving	E - because it is a big part of the GCSE H, I

Table 16. Summary of which curriculum aspect teachers thought needed developing

Table 16 shows that three colleagues felt that developing fluency in their topic area was important. Brian felt that developing fluency in ratio and proportion was important because ‘if you can master that skill it can unlock so many different areas’. Charlotte felt that becoming fluent in the fundamentals of mathematics in the context of Calculus was important and ‘being able to start with a basic graph and slowing making it bit by bit a more complex problem’ would help students’ reason. Heidi felt that fractions are part of being fluent in the fundamentals of mathematics. Developing problem solving was referred to by Elliot for the introduction of Calculus, and this was deemed an ‘obvious’ area to develop for fractions by Heidi and Ian.

Colleagues were then asked to consider the features of the CCC Model (Appendix 5.6) and to say which feature they felt they would be using within their own planning. Responses to this were quick and to the point. Most people stated using different representations (Annette, Heidi, Brian), using similarities and differences (Charlotte, Brian) or application tasks (Heidi, Ian). Elliot referred to building connections and ideas. The feature about making links between procedures and concepts was not referred to by anyone (Table 17).

Builds on knowledge	E
Making connections	A - because it helps students H, B
Linking procedures and concepts	
Similarities and differences	B - because it develops mathematical thinking C
Application tasks	H, I

Table 17. Summary of which feature of the CCC Model teachers thought might support their development

When questioned what they had planned to do Brian commented that he had already bought books and downloaded several articles that he wants to read. Charlotte, Elliot and Heidi had some early formed ideas of what they might explore as shown in Table 18.

Multiple representations	A B – research multiple representations C – picture and graphs
Connections	E – illustrate connections but get students to identify them for themselves
Collaboration	H – bank of ideas for everyone to use

Table 18. Summary of ideas and plans as to how development work will be carried out

When colleagues were asked whether they felt the collaborative planning was an opportunity to develop their own pedagogic content knowledge, all responses were yes, as shown in Table 19.

Yes	A, B, C, D, E, F H, I – all agreed this was an opportunity to develop their own pedagogic content knowledge A – in my own way B – gave me ways to dissect lessons C – changing student attitudes to develop conceptual understanding E – refreshing to look at things in a new way H – better way of teaching I – focusing on it may get different ideas

Table 19. Summary whether the CPD was an opportunity to develop PCK

Most responses were brief, but some extended reasons were given by Heidi and Elliot as shown in Table 20.

Teacher	Quotes about opportunities to develop PCK
Elliott	It is good because you get bored of teaching the same things the same way year in year out so actually it is really refreshing to think about how else can I do this and not doing it the same old way that I have been doing it. So, I have really embraced that.
Heidi	Well we have done teaching fractions, but it is probably the first one exploring the pedagogy behind it, better ways of teaching it, because it is not going to be new concept to any of us, it is finding the ways of incorporating it into the new curriculum the way it should be done.

Table 20. Quotes about opportunities to develop PCK

Ian also commented how easy it was to get stuck in a rut and teach things in the same way, but this should enable ideas to be shared. Brian commented how he wanted to continue to develop as a teacher and that the collaborative work would also support his research on the Outstanding Teacher Programme (a school led programme delivered by AST's).

7.13.4 Reflections on implementing change

At the time of the second mid-study interview colleagues were in the active experimentation phase where the expectation was that ideas would be explored with classes. This section of the interview asked, ‘Do you feel that you recognise what Nic is trying to encourage you to do?’ and ‘Do you feel that you have been able to implement the sort of changes that have been encouraged by the department?’.

Yes	A, B, C, D, H, I all said yes. B - developing multiple representations C - encouraging her as HOF I - particularly the similarities and differences A - using a lot of ideas but not sure which is Nic’s or Open University module stuff
No	

Table 21. Summary of whether teachers recognised what the project has set out to achieve

When questioned whether colleagues recognised what they were being encouraged to do there were a mixture of responses. Table 21 shows that all said yes, but with varying degrees of certainty and clarity.

Annette said, ‘a little’ but referred again to this being due to her not being at all sessions. Brian commented ‘yes I think so’ and then referred to the tasks that he would undertake during the active experimentation stage and Ian ‘yes, I think so particularly with similarities and differences’.

Daniel, Charlotte and Heidi were more confident in their replies. Daniel referred to principles that he had explored in maths and how he had extended them to other subjects. Heidi’s response was ‘yes of course’ and commented that all resources shared would be tried with classes. Charlotte felt that she not only had a secure understanding of what was to be implemented but also ‘obviously, I do, it is being part of the head of department, I wouldn’t be encouraging her if I didn’t know what it was, and I didn’t believe in what she was doing’.

Collaboration	A – really interesting way of doing things B – regular updates in curriculum meetings C
Similarities and differences	D, H, I
Seeing effect size from INSET	D
Stealing ideas	D

Table 22. Summary of things that have helped implementation

Table 22 shows the areas that were supporting teachers make progress. To a certain extent everyone had implemented aspects of the CCC Model with the experimentation of exploring similarities and differences as being the main avenue explored. Things that helped were ‘regular updates at curriculum meetings’ (Brian), and getting ideas from others - Daniel refers to ‘stealing ideas’ and Annette specifically mentioned getting new ideas from chatting with Nic, Charlotte and Heidi. Daniel commented that although he had been using the similarities and differences strategy for a while and believed himself it was working when effect sizes were mentioned at whole staff training ‘I was always going to try and extend what I had started last year but I thought if that is going to have that effect size then that is something I really do want to discover a bit more about’. So, seeing the research evidence that backed up the strategy spurred him to explore further.

Time constraints	A, B, D
Planning/creating resources	B, D
Student attitudes	C – maths for a qualification rather than enjoyment
Student ability	H – the way the students think and the language they use I

Table 23. Summary of implementation barriers

Table 23 shows barriers evident at this stage of the active experimentation phase. When discussing barriers to implement the changes, time pressures were raised by Annette, Brian and Daniel. Charlotte raised the barrier of the students themselves wanting to be spoon fed answers and the barrier was coming up with ways to challenge them to think for themselves. An interesting barrier raised by Ian was when discussing with students ‘you can say similarities and differences and you can compare them, but you have still got to break down the barrier of what the actual difference is’. This echoes some of the ideas shared by Heidi with the confusion of language used in classifying tasks and how different meanings are sometimes used across the curriculum.

7.13.5 Reflections on whole school move towards action research

The fourth phase of the interview acknowledged the drive at whole school level for everyone to engage with action research and asked colleagues how they felt about this. A summary is shown in Table 24.

Positive that directed time is allocated	A – good to work as a team doing stuff B – used to working a lot in own time C – would be doing it anyway so this is a bonus D – good idea as it wouldn't happen otherwise E – good that it is directed as important for personal CPD H – would do it anyway but important for some who wouldn't otherwise I – better and makes more sense
Another thing to do	D – initially thought that but I can see the rationale behind it

Table 24. Summary of how teachers feel about engaging with action research

The feeling was generally positive, with Heidi 'quite enthusiastic about it' and hoping she will learn a lot, with the rationale that teachers should be looking to develop, and action research is one way this can happen. Brian considered it to be a big demand on time but felt that it was something he 'should be doing' to develop as a teacher.

There was acknowledgement that directed time had been dedicated to working on action research and for Heidi and Charlotte they felt they would be working on action research anyway, so this was a bonus to support them. However, Ian commented that being towards the end of his career it was more difficult to keep motivated to try new things. There was acknowledgement of a change of culture within the school from Daniel 'I think the professionalism over the last three years has increased within the staff. It's not the expectation that you can get your masters, but it is certainly at the forefront of a lot more people's minds now than it has been'. However, when teachers were asked whether they had planned to carry out their own research. Daniel commented that he had no plans to carry out his own research. Whereas Brian, Charlotte and Elliot had ideas as to what they would do as shown in Table 25.

Yes	B – have got lots to read already C – looking into mind sets of students and how this holds them back E – already looking at cross curricular links and leadership in general H
No	D – not at the moment I – probably not

Table 25. Summary of whether teachers plan to carry out their own research

The interviewer asked for comments on the whole school model of CPD versus the one from within the mathematics study. Both Brian and Daniel articulated the similarities between mathematics CPD programme and the whole school one. Brian highlighted 'external input doesn't seem to figure in Nic's model, but I am sure it is there embedded somewhere I have just not picked it out' when explored further with the interviewer it was agreed that the research input could be classified as external input.

Some interesting points were raised by some colleagues in their responses. Elliot acknowledged that after doing a PGCE and entering the profession the use of research usually stops so the models gave the opportunity to be 'continually reading / developing and learning because teaching is continually developing'. When reflecting on the images given by the two models:

> 'If you look at them just quickly, you can see that Nic's model is going forward and you can see that the development is leading you along a path, you can also see what parts affect you at different stages. The school model is a little bit…. cold shall we say in that it looks like it is just tick boxes and it looks like it is just boxes that need to be filled in and that it doesn't look like it is taking you anywhere, just from that.' (Elliot)

The notion of going on a forward journey was also brought up by Heidi 'well for me as a mathematician and a maths teacher what we are doing in maths seems to be more, it has got more validity … I think it is, for me personally I don't know how to explain it but what we are doing in maths is a little bit more of a personal journey, the whole school model is more like you have to do things'.

Charlotte raised the main difference between the two models as the CCC Model is 'about having the time to collaboratively work and to collaborate discuss things' whereas at school level 'everyone is still going off and working on their own little projects'. When probed further by the interviewer as to which was preferable 'I think there is value in both, because although I am working with Nic in doing this and I am still doing my own research in the background and although it connects to this it is still its own project in its own right'. She recognised the need to go away and independently explore things herself but that 'to actually get it to properly work we have still come back to it as a department'. A summary of comments is shown in Table 26.

CCC CPD programme	A - long term become part of practice B - well established C - about collaborative working E - development is leading you down a path. You can see what parts affect you at different stages H - more applicable to maths
School Action Research Model	C - individualised projects E - just looks like tick boxes to be filled in, doesn't appear to lead anywhere
Comparisons	B - same other than external input D - lots of similarities between both H - same thing just different language

Table 26. Summary comments on CPD programme and school action research model

The interviewer asked were these models preferable to previous CPD. Some flippant comments and laughs referred to a lack of previous CPD. Table 27 shows a summary of comments.

Yes	C - yes, when you go away there is no time to feedback so it doesn't go anywhere, however some conflicts between levels of control D - effective as ongoing, stats showing effectiveness are good, professionalism has increased E - previously felt like ticking boxes, now doing it for a purpose, similarities and differences has had a big effect in development and progression of student understanding H - more engaging, interest of the person I - because you have chosen it to improve on, have to be careful to make sure you put the time into it B - needed to move forwards
No	
Indifferent	A - I enjoy all of it

Table 27. Summary comments on whether these new CPD programmes were preferable to previous ways of working

Annette recognised the change in emphasis evident from previous CPD programmes.

> 'It's also long term what Nic is doing isn't it? Whereas previously it tends to be going out and doing a day course and then actually putting it into practice when you come back doesn't always happen whereas if she is here with us doing it on a long term you are more likely to actually, eventually get into doing some of the things. It has to become part of your normal practice, doesn't it?' (Annette)

This was echoed by Daniel 'CPD was only ever about going on a course for a day or a weekend or whatever and you always used to go away and get enthused about what you were doing and then come back and never have the time to put it into practice'. He also commented that he had initially been sceptical about having more in house CPD and reflecting on a long-term management course he had been on which was ongoing over a long period of time felt that overall things were 'going quite well'.

Elliot felt that previous CPD was just going on a course to tick a box during performance management whereas now 'it feels like we are actually doing something for a purpose'. There was also the acknowledgement that the direction of whole school travel was in some ways now following what had been implemented in maths already.

> 'I was really, surprised,…not surprised but happy to see the similarities and differences when the whole school CPD lead did his presentation and seen that it was a 1.6 effect size,….. it was good to see that using similarities and differences which is something that we all have embraced as a department show that is has such a big effect in the development and progression in understanding of students, so that was really good to see. Hopefully the rest of the school would kind of embrace that as well.' (Elliot)

7.13.6 Reflections on themes that have arisen

Several developing themes arose in previous interviews with the department and the final section of the interview explored these further. Colleagues were asked whether they believed that all attainment levels of students are capable of conceptual thinking and whether students within the school were motivated to understand. Table 28 summarises comments to the first question.

Yes	A - particularly because of my reading B, I - at different levels C - otherwise wouldn't believe in growth mind set E - if we can start at a young enough age. Can get stuck in their ways, particularly lower ability I H - not all capable at first, but can develop ability
No	D - GCSE retakes aren't. Limited by previous ways of working perhaps together with time limits

Table 28. Summary of whether teachers believe all students are capable of conceptual thinking

The question about whether all attainment levels were capable of conceptual thinking was responded with a resounding yes from Annette and Charlotte who both referred to their own reading and beliefs on developing growth mind-sets.

There were some comments that reflect that potentially everyone can but we need to start them thinking in that way early on with Elliot commenting ‘Yes, I think if we start from a young enough age and they get used to this way of thinking’ and Heidi commenting that ‘I think up to now I have seen that there is an assumption that not everyone is capable but what I can see is that they could develop this ability’. Both Brian and Ian feel that to a degree everyone can but at their own level.

> ‘I think everyone has limited intelligence which is an obvious factor and therefore children with a lesser ability probably have less ability to conceptualise than those with more ability…… so if you come in at a lower level they can still conceptualise but not at the same degree as someone who has the brain size of a planet.’ (Brian)

Daniel reflected on the type of GCSE resit classes that he teaches

> ‘It is too easy to say no, sorry it is too easy to say that some students that just can’t and when you see some students towards the end of their schooling career and have perhaps been taught over the years in the way that hasn’t allowed them to explore that type of thinking, then you try and introduce it – you just no nowhere fast. But the type of mathematics I have to teach, or am asked to teach, is that for example I have picked up year 10 for half of their lessons this year having not taught them before and you have to get up to speed on where are they in terms with their ability to think in the conceptual way and you are trying to ascertain their levels before you start. In an ideal world, it would be nice to collect your Group in year 7 and to go through with them and develop that thinking all the way through, then I would have a much better answer for you in terms of whether everybody can or not.’ (Daniel)

He continued to reflect ‘I want to say that everybody can, but I haven’t had that experience or evidence to say that they can’.

When questioned about whether learners were motivated within the school there was a mixed response as shown in Table 29. Many teachers appear in both categories citing reasons as to why they might be motivated but also why some are not. There was the feeling the climate was changing 'I think we are getting there, I feel there is definitely more of an atmosphere of being accepting of making mistakes and improving through that, yes' (Annette) but an acknowledgment that not all students wanted to understand.

Yes	A - getting there. Practical questions that are real life B - very much so because of the approach taken not didactic model C - not all at the moment once they achieve, feel making progress D - much more so than ever before expectations are higher, behaviour for learning are correct, breeds success E - some are, younger students are keen, eager to please, youth and innocence H - mixture they want to learn and do well I - 20-30% motivated to understand, rest only motivated to succeed. Their upbringing, mind set
No	C - not all at the moment for some it is risk of failing and others no point to it so "what's it for" E - some are, some need to be sold the bigger picture one understood concept is easier than 20 rules H - mixture don't see where they will use it outside of school rushed too quickly through topics I - large percentage happy to succeed by providing answers but not interested in understanding.

Table 29. Summary of whether teachers believe students are motivated to understand

7.14 Triangulation of data

In addition to the vast amount of interview data it was decided additional data should be collected that could be used to validate and triangulate findings. The findings of these are not presented in this section merely an explanation of what was collected and when. The data will be used to support or dispute any conclusions that arise from the interviews with teachers involved in the project.

Specific extracts from this data will be used in the results and findings chapter as conclusions are made.

7.14.1 Learning walks

During March 2016, a learning walk was carried out by the researcher and the second supervisor. All teachers were visited and the majority with two different classes. During the visits to lessons, notes were made on the climate, what was happening in the lesson and the learner's responses. Books and displays (including the ones created for the study) were looked at and photographs were taken where it was felt there was evidence that teachers were using elements of the CCC Model or indeed whether it gave evidence that they had not engaged with the principles. This learning walk was carried out as part of the ongoing whole school review of teaching and learning. A learning walk was repeated in June 2016 this time with the researcher and the project supervisor.

7.14.2 Book scrutiny

As the learning walks were constrained by the timetable, and what lessons happened to be on when the researcher and the supervisor was available, it was felt that there may be other practice that needed to be considered from other classes. So, in July 2016 all teachers were asked to submit a sample of books that would be reviewed. This gave a valuable opportunity to see the work from the academic year. Photos were taken, and these were used to triangulate comments arising during the interviews.

7.14.3 Presentations

As part of the whole school increased engagement with action research colleagues had to disseminate their findings to a wider audience. This was a necessary requirement as part of the performance management cycle. Within mathematics, colleagues presented to each other at department training evenings and department meetings as shown in Table 30. These presentations were filmed and then transcribed so they could be used to triangulate data from the study. This triangulated data is drawn on to back up conclusions that are made within Chapter 8 and 9.

Colleague	Presentation	Date
Brian	Ratio and proportion	19th July 2016
Charlotte	Creating a growth mind-set	19th July 2016
Elliot	Iteration	19th July 2016
Frazer	Solving quadratic inequalities	19th July 2016
Kate	Collaborative Connected Classroom – ideas and examples	19th July 2016
Heidi	Using multiple representations for division of fractions and solving problems	5th September 2016
Ian	Maths CPD investigation	20th September 2016
Georgie	Using visual images for fractions	18th October 2016
Louise	Place value	18th October 2016

Table 30. Action research presentations

7.15 Stages of progression in teacher development

Throughout the study, it became obvious that teachers were on a journey and some were engaging with some features of the CCC Model and others were engaging with using academic research to inform practice. A Teacher Development Model (TDM) emerged that modelled the phases that teachers were moving through on their own journeys with the project. This can be viewed in full in Appendix 6.1.

7.15.1 Pilot of the CCC TDM

At the end of July 2016, Elliot left the school as he was moving to take up a job in another school starting in September 2016. Before he left he was asked to annotate the draft version of the model of teacher development when implementing the CCC Model. He was then interviewed about the model and his rationale for his own annotations on it. The feedback from his interview informed further development of the TDM before it was used with other colleagues.

7.15.2 Self-evaluation against TDM

Colleagues were asked, at the end of October 2016, to consider the model of teacher development when implementing the CCC Model and were asked to highlight where they felt there were on each theme. They were instructed to highlight in green areas that they were currently doing and highlight in pink aspects that they felt they hadn't met.

7.16 Final interviews

Final interviews were carried out on 2nd November and 9th December 2016. The interviewer used the responses from everyone on the TDM as a prompt for these discussions. The aim of these interviews was to tease out for each theme what had supported colleagues in moving forwards and what were the barriers to engaging with or making progress in each area. Findings are presented in Chapter 8. Interview schedules are presented in Appendix 5.7.

7.17 Chapter conclusions

Chapter 7 has chronologically set out the main study activities that were undertaken and a summary of findings that were drawn from the initial and mid-study interviews. This data along with the final interviews and triangulated data is drawn together in Chapters 8 and 9.

CHAPTER 8: RESULTS AND FINDINGS

8.0 Introduction

Whilst Chapter 7 provided a descriptive overview of the main research study with data presented in a chronological way. This chapter details the main themes that have arisen and draws on evidence and findings from across the whole study to answer the original research questions. The overarching question for this study is:

- How can a programme of professional development engage and support a mathematics department to teach for understanding?

The initial sub research questions that were answered using literature in Chapters 2, 3 and 4 are:

- What is meant by 'teaching for understanding' and what would this look like in a mathematics classroom?
- What is meant by the term professional development?
- What factors will contribute to an effective professional development programme?
- What would an effective CPD programme look like that supports the implementation of the CCC Model?

This chapter addresses the research questions that needed empirical research. The sub questions have been divided into three categories. Firstly, those that relate to the design of the CPD programme and the use of academic literature within the programme.

- Q What is the impact of exposing teachers to academic literature within a programme of CPD?

The second section looks at the implementation of the CCC Model and how teachers responded to different elements of it.

- Q Which approaches will engage teachers with students' development of connections?
- Q Which approaches will teachers explore with students?
- Q How can teachers develop tasks that encourage connections to be made?

Thirdly, the questions that acknowledge the department case study is made up of several teachers that responded differently.

- Q Are there any differences in how teachers develop across the mathematics department?
- Q What has influenced these differences?

The overarching theme of teacher change and barriers to change draws together many elements so is covered within the analysis in Chapter 9.

8.1 What is the impact of exposing teachers to research within a programme of CPD?

This section brings together the data gathered throughout the study considering the use of research both as it was presented in the CPD sessions and then teachers using it themselves through the study.

8.1.1 Research used prior to the study

At the start of the study, in the initial interviews, six colleagues referred to independent exploration of research they had already done. This varied from Annette and Frazer carrying out informal research from reading teaching magazines and using internet sites to more formal research from Charlotte, Elliot, and Georgie as part of requirements for masters courses they had begun. Heidi identified independent exploration of research that she had already done for PGCE and TAM courses and then in addition; 'recently I was reading about, it is not completely mathematics, but I wasn't very happy about the progress of the boys in my lessons and I have been reading more about white boys and why they are achieving or not achieving in class.' (Heidi, 2013, Initial interview) showing that she was beginning to independently follow an enquiry raised from her classroom.

8.1.2 The use of research throughout the study

The research presentation of the CCC Model (Appendix 2.3) was shared on 4th February 2014 and following that, activities were trialled that aimed to make sense of the terminology presented in the research. The first mid-study interview in November 2014 gave colleagues a chance to reflect on the terminology of instrumental and relational understanding. These findings were presented in Section 7.8.2.

Between the two mid-study interviews the school moved towards a more action research focused model of CPD so the later interview gave an opportunity for teachers to voice their opinions on this. The expectation from the school was that as part of the performance management review cycle everyone would engage with a piece of action research, although there was no expectation that this would involve academic reading.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Initial	1	3	6	0	0
Mid-study 1	1	1	2	1	0
Mid-study 2	1	0	3	3	0
Final	1	2	5	7	2

Table 31. Evidence from interviews for each stage in the model (cell shows number of cases)

Table 31 shows a count of the evidence arising from the interviews for each stage of the development model. As time progressed, there is more evidence from interview comments that teachers are at the independent development and transformation phase of the TDM.

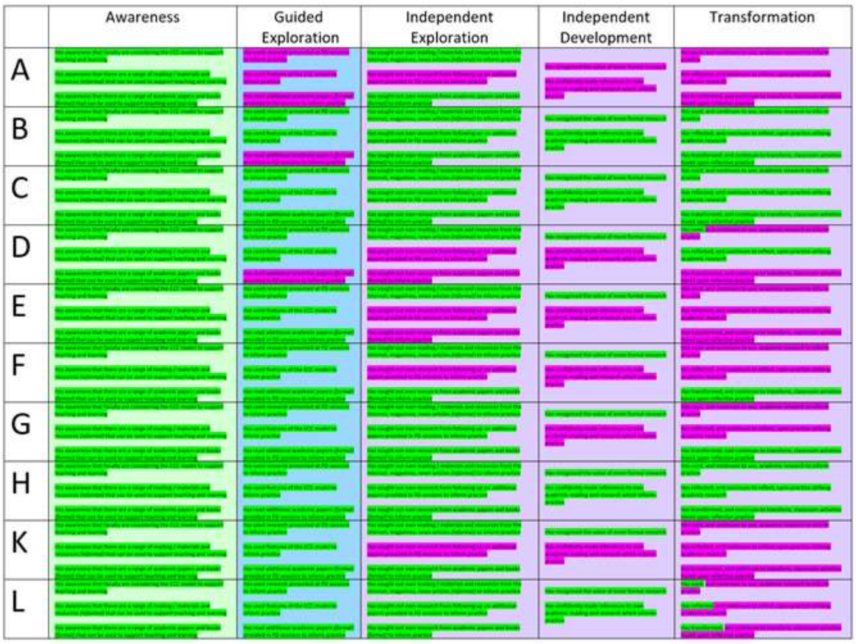


Figure 53. Teachers self-reflection on research

Figure 53, shows the teachers self-reflection on the TDM prior to the final interviews. Teachers highlighted parts green when they felt they were at that stage and pink areas they didn't think they had fully met (a full version can be found in Appendix 6.2).

Whilst the data from interviews only shows two people talking about how research has transformed their practice, when combined with the extra triangulated data (Table 32) the data observed is more in line with teacher's self-reflection. In this case it was not possible to gather conclusions from learning walks, book scrutinies and displays whether research had been used to transform practice. The additional evidence was drawn from both the presentations that teachers did for the department and in the transcriptions of their comments made during the presentations.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of coding references	4	6	37	24	6
Number of coding references (interviews)	4	6	26	16	2
Number of coding references (triangulated data)	0	0	7	8	4
Number of cases coded	3	4	11	7	3
Cases coded	B, G, I	A, B, E, K	A, B, C, D, E, F, G, H, I, K, L	B, C, D, F, H, K, L	B, C, H

Table 32. Evidence from interviews and triangulated data for each stage in the model.

8.1.3 Findings at each phase from data analysis

This section describes the phases from the TDM and gives examples of evidence coded during the process of data analysis.

Awareness phase

Whilst all colleagues highlighted the awareness phase green, the data at a superficial level doesn't look like there is evidence to support this claim. However, when coding elements of text or data the higher stage would have been coded where appropriate. For example, if someone talked about a paper that they had read from a CPD session this would have been coded as guided exploration when it could have been 'double coded' as awareness as by reading a paper they would have been aware that papers exist.

Prior to starting the study, all teachers were aware that there are formal and informal research resources/papers however the codes referred to them talking about the CCC Model and how this has been discussed at department meetings. At this stage codes where given for example to someone referring to similarities and differences being part of the CCC Model but not actually having used it at that point in the study.

Guided Exploration Phase

Evidence presented in the guided exploration phase shows four colleagues (Brian, Daniel, Georgie, Kate) referring specifically to reading papers that had been presented at the CPD sessions. With Elliot and Annette also referring to additional papers from external courses.

Independent Exploration Phase

All colleagues in the self-reflection highlighted aspects of the independent exploration phase green and there is some evidence from the data to agree with this. Five colleagues (A, C, G, H, E) refer specifically to formal reading that they are doing for their other courses (TAM, Open University, masters) and seven (B, C, E, G, H, K, L) refer to formal reading for their school led action research projects. Annette gives examples of informal resources that have been gathered and both Elliot and Frazer refer to informal research involving news articles and lesson resources.

Independent Development Phase

Except for Annette all colleagues have highlighted they recognised the value of more formal research. Brian, Charlotte, Heidi and Louise in their self-reflection highlighted the whole of the independent phase green. There is evidence from the interviews to support this: Brian making confident references to how his reading on Blooms taxonomy has informed his classroom practice; Charlotte referring on numerous occasions to her work on growth mind-set; Heidi referring to reading that has informed her action research project on developing multiple representations of fractions alongside other areas such as using John Mason's work from Thinking Mathematically (Mason, Burton and Stacey, 2010), and Louise talking about her multiple representations action research project and also about Jo Boaler and number sense (Boaler, 2015). Triangulated data from presentations backs up these comments from interview and it was pleasing to see the level of reference to formal academic research presented back to the rest of the department.

Daniel, Kate and Frazer highlighted the phase 'has recognised the value of more formal research'. Frazer acknowledges that moving with the times there is a need to change and things he was reading was backing up conclusions that he had come to over his career. Daniel refers to Marzano's (1998) publication on effect size and how this impacted on his thinking. Kate gives the impression that since joining the department she is happy to spend more time thinking about teaching pedagogy and is moving in the direction of independent development in the research strand.

> 'Since moving here, I am just focusing on my teaching, so I have got more time within that to actually think. So, I am certainly looking at more research within my teaching.' (Kate, 2016, Final interview)

Georgie and Elliot have also both highlighted they recognise the value of more formal research, this however is not necessarily evidenced and will be returned to in the barriers section.

Transformation Phase

In the self-reflection activity Brian, Charlotte, and Heidi highlighted all aspects green whereas Daniel, Frazer and Louise coloured aspects green. There is evidence in both interview data and triangulated data that to a certain extent using research has transformed each of their practices within the classroom. However, this transformation is at different levels and perhaps on one specific area of mathematics rather than curriculum wide.

Both Charlotte and Heidi's presentations were based on formal academic research which they evaluated after they trialled aspects with their learners. Informal statistics of evidence were referred to in both cases as they reflected on the input of their action research.

8.1.4 Exploring anomalies in the data

A gap in phases

In Figure 53 it appears that Annette has a ‘gap’ where the guided exploration phase is highlighted pink. On further exploration in interview this was highlighted as not met due to Annette not being present in the training sessions so missing out on some of the resources that were shared. Progress had been made in the independent development phase though because other discussions and resources had led to the same ideas being considered.

Does transformation mean transformation?

Looking back at the self-reflection grid of the teacher development (Figure 53) and the evidence collection (Table 32) shows Brian, Charlotte and Heidi at the transformation phase. Over the study Heidi refers to research on several different areas, attainment of boys, development of fractions, problem solving and visual proofs showing that using academic research is widely embedded into practice. In contrast Brian’s comments and evidence all comes from one piece of action research on Blooms taxonomy and whilst this is confidently referred to in interview and presentation, not all data from triangulation support that the ideas discussed are embedded.

Recognising the value of formal research

All teachers except for Annette by the end of the study highlighted that they recognised the formal value of research. At the beginning of the study she commented 'I like things that I am going to take back and use and I did find some of the Open University stuff a bit hypothetical and a lot about different theories' (Annette, 2013, Initial interview). However, Annette didn't attend many of the mathematics afterschool CPD sessions or the whole school training on action research and wasn't required to complete a project for performance management. She didn't have the experiences over the study as other colleagues, which for the rest supported them to recognise the value of formal research.

8.1.5 Barriers to using research

There was opportunity during the final interview to explore barriers to using research, time was the major factor quoted (A, B, C, F, G, H, K, L). Specific elements can be seen in Table 33 below.

Time	A - Easier to revert to what you already know works; it takes more time to do it differently B - Pressures of time with other commitments C - Time is a barrier, but must make time to do it D - Free time is spent on new specifications F - Takes more time to consider a decent lesson that is well constructed with lots of theory being put into practice F - Getting time to sit and read G - Due to family commitments at home H - To dedicate to reading, to process and to understand K - No free time at school and family commitments at home L - Finding time to sit and read with increased pressure
Academic jargon	H - You need to be able to understand the language that the papers are using and bring it down to the classroom A - Prefers practical applications, some of the research is a bit hypothetical and a lot about different theories
Access	F - Knowing where to look and find research when you need it

Table 33. Barriers to using research

Although time is quoted as the main reason for not engaging in academic research this splits into two different areas. Firstly, time is limited due to pressures of family commitments or other initiatives and secondly, the acknowledgement that is takes more time to change practice and to do things differently as time is needed to reflect and consider a lesson activity that is based on research theories.

Little (1993, p. 142-143) states that teachers ‘have less routine access to sources of research, less time to read and evaluate it, and less familiarity with its arcane language’. Although access to research was mentioned it wasn’t as prevalent an issue as was expected. Due to the nature of Universities having subscriptions to journals those on masters courses had access to academic reading however other teachers would only be able to get open access materials available via the internet or more ‘mainstream’ published books. This wasn’t raised as an issue by teachers within the study, however coming from an academic perspective it could be argued that as schools don’t have access to peer reviewed journals, teachers are perhaps left to read less reliable sources.

8.1.6 The use of research over the study

In the final interview teachers were asked whether they felt they were researching more now than they were at the beginning of the study. Their responses are summarised in Table 34.

Same	B - Always been researching since training and continued
No	D - Has greater awareness but hasn't had time to research L - Wants to but increased work load as NQT has meant less time E - Not time for formal research
Yes	K - Now focused on teaching rather than leadership so more time to C - Whole school drive is supporting too C - Increased momentum to do so F - Way the department has been encouraged to go A - Other influences too other than just CCC G - Lots of informal stuff from t-drive H - Building on research from ITE training

Table 34. Responses to whether use of research has increased over study

Whilst most people say they have increased their use of research over the time of the study it cannot be claimed that this is as a direct result of the CPD programme. The increased emphasis at whole school level on action research and the link with performance management would have notably affected the data in this section even though there was no specific requirement to include academic research. Over the study, Charlotte returned to her masters and continued to engage with research 'I think also it is having someone around me that is doing it as well, yeah, some of the things you are talking about are quite interesting and I have got to get back into this' (Charlotte, 2016, Final interviews).

Two of the three teachers that haven't increased the amount of research acknowledge they have a greater awareness and want to do more but time is now the limiting factor.

8.2 Which approaches will engage teachers with students' development of connections?

This section shows the data that has been collected across the main study broken down into the different aspects of the CCC Model. First data is looked at from an overall perspective and then a section is dedicated to each aspect of the model. Data throughout the study was coded against the strand of the CCC Model and the phase of the TDM it was relevant to. Table 35 shows a count of evidence in the form of a two-way table.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation	Total
Conceptual structure	6	16	17	5	0	44
Connect areas of mathematics	16	5	24	8	0	53
Multiple representations	19	22	64	30	4	139
Procedures and concepts	17	29	23	20	2	91
Comparisons	5	11	51	19	3	89
Application tasks	4	3	21	5	0	33
Total	67	86	200	87	9	449

Table 35. Strands of CCC Model mapped against phases of teacher development (number in cell is count of references coded)

In total 449 pieces of data were coded from segments of interview text, photographs of students' work and comments from learning walks and teacher presentations. Coded data was also tagged to the teacher that it related to 'the sub case' so department reports referring to the whole team and not specific individuals are not included. Table 35 shows most data evidenced was in the independent exploration phase and then approximately equal amounts in the guided exploration and independent development phase. There is much less evidence in the transformation phase of the TDM. It also shows that most data was found for the multiple representations aspect.

Using the data from Table 35, NVIVO was used to look at how many teachers (sub cases) had evidence in each cell. This is shown below in Table 36. Where there is eleven in a cell, it shows that there was evidence from all teachers from the department (including HLTA).

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Conceptual structure	3	9	8	3	0
Connect areas of mathematics	11	2	7	4	0
Multiple representations	10	10	11	6	2
Procedures and concepts	10	10	8	3	1
Comparisons	5	8	11	6	3
Application tasks	4	3	10	2	0

Table 36. Strands of CCC Model mapped against phases of teacher development (number in cell is count of cases coded)

Table 36 shows that all teachers independently explored both the multiple representations and making comparisons aspects of the CCC Model. Whilst looking at a count of how many teachers gives a useful picture Table 37 shows which cases were referred to in each cell.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Conceptual structure	E, G, H	A, B, C, D, E, F, I, K, L	B, C, D, E, F, G, H, L	E, G, H	
Connect areas of mathematics	A, B, C, D, E, F, G, H, I, K, L	E, K,	B, C, E, G, H, K, L	B, E, H, L	
Multiple representations	A, B, C, D, E, F, G, H, K, L	B, C, D, E, F, G, H, I, K, L	A, B, C, D, E, F, G, H, I, K, L	C, E, F, H, K, L	F, H
Procedures and concepts	B, C, D, E, F, G, H, I, K, L	B, C, D, E, F, G, H, I, K, L	B, C, D, E G, H, K, L	C, G, H	H
Comparisons	B, C, E, F, H	A, B, E, F, H, I, K, L	A, B, C, D, E, F, G, H, I, K, L	C, D, E, H, K, L	D, E, H
Application tasks	B, C, H, K	E, G, I	A, B, C, D, E, F, G, H, I, L	E, H,	

Table 37. Strands of CCC Model mapped against phases of teacher development

Table 37 shows that of the teachers in the transformation phase Heidi is in all three areas whereas Frazer is in the multiple representation aspect and Daniel and Elliot in the making comparisons aspect. Data was collected over time so for example in mid-study interview one many teachers were talking about using resources that they had been given (guided exploration stage) later in the process they referred to resources they had found themselves (independent exploration).

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Conceptual structure		A*, I, K,	B, C, D, F, L	E, G, H*	
Connect areas of mathematics	A*, D*, F*, I*		C, G, K,	B*, E, H, L	
Multiple representations			A*, B, D*, G, I,	C, E, K, L	F*, H
Procedures and concepts		F, I	B, D, E, K, L	C, G	H
Comparisons			A, B, F, G, I,	C*, K, L	D*, E*, H*
Application tasks	K		A, B, C, D, F, G, I, L	E*, H,	

Table 38. Teachers positioned in cell with highest evidence (* no triangulated evidence)

Table 38 shows the highest phase there was evidence collected for. Tables 37 and 38 show the department has gone further along the TDM for the multiple representations and the making comparisons strand. Possible explanations for this are explored with Sections 8.2.3 and 8.2.5. The strand with the least progress against the model is connecting different areas of mathematics this will be considered in Section 8.2.2.

Anomalies with triangulation

NVIVO was used to filter and see which evidence sources were interview data (so the teachers were saying they had worked on things) and to see where there was additional evidence from triangulated sources to confirm these comments. Where there is a * in the tables that follow it shows, the data was taken from an interview source that is not backed up by additional triangulated data. Tables 43 and 44 show a marked discrepancy in the transformation phase of the comparisons strand which will be explored in Section 8.2.5.

8.2.1 Making links to students' conceptual structure

This section explores one aspect of the CCC Model, the building of students' prior knowledge by making links to their conceptual structure. There was 44 pieces of data coded with 17 of those from interviews and 27 from triangulated data. Table 39 shows a breakdown of where the data is from and which teachers were coded in each phase. Most teachers were at the independent exploration stage with Annette, Ian and Kate at the guided phase and Elliot, Georgie and Heidi at the independent development phase. There is no evidence of any teachers being at the transformation phase for this aspect of the CCC Model.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of references coded	6	16	17	5	0
Number of references coded (interviews)	5	4	5	3	0
Number of references coded (triangulated data)	1	12	12	2	0
Number of cases coded	3	9	8	3	0
Cases coded	E, G*, H*	A*, B, C, D*, E, F, I, K, L*	B, C, D, E, F, G, H, L	E, G, H*	
Highest phase coded		A*, I, K,	B, C, D, F, L	E, G, H*	

Table 39. Making links to students' conceptual structure versus each phase

Figure 54. Teachers self-reflection on making links with conceptual structure

In the self-reflection task (Figure 54, in full in Appendix 6.2) Brian, Frazer and Heidi all positioned themselves in the transformation phase highlighting the phrase 'has transformed and continues to transform practice to enable curriculum connections to be developed from students' prior knowledge' however data from other sources doesn't explicitly show that. Appendix 6.5 shows examples of tasks used with learners in the exploration phases.

8.2.2 Connecting areas of mathematics

This section explores the data for one aspect of the CCC Model, connecting different areas of mathematics. There was 53 pieces of data coded with 27 of those from interviews and 26 from other triangulated data. Table 40 shows a breakdown of where the data is from and which teachers were coded in each phase.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of references coded	16	5	24	8	0
Number of references coded (interviews)	14	0	11	2	0
Number of references coded (triangulated data)	2	5	13	6	0
Number of cases coded	11	2	7	4	0
Cases coded	A*, B*, C*, D*, E*, F*, G*, H*, I*, K, L	E, K,	B, C, E, G, H, K, L	B*, E, H, L	
Highest phase coded	A*, D*, F*, I*		C, G, K,	B*, E, H, L	

Table 40. Connecting different areas of mathematics versus each phase

Table 40 shows a mixed picture. There are 7 teachers that have explored and some then gone on to develop in this area. Appendix 6.6 shows examples of tasks that have been used with learners at the exploration phases. However, four teachers seem to remain only at the awareness phase.

Figure 55 shows only Brian and Heidi highlighting green aspects of the transformation phase in their self-reflection and even they haven't highlighted the independent phase all green. Figure 55 (full version in Appendix 6.2) shows there is much more pink highlighting at the later phases of development. Barriers to development in this aspect will be considered in Chapter 9.

Figure 55. Teachers self-reflection on connecting different areas of mathematics

8.2.3 The use of multiple representations

This section explores data for the use of multiple representations. There was 139 pieces of data coded with 55 of those from interviews and 84 from triangulated data. Table 41 shows a breakdown of where the data is from and which teachers were coded in each phase.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of references coded	19	22	64	30	4
Number of references coded (interviews)	15	10	19	8	3
Number of references coded (triangulated data)	4	12	45	22	1
Number of cases coded	10	10	11	6	2
Cases coded	A*, B*, C*, D*, E, F*, G*, H*, K, L	B*, C, D*, E, F, G, H, I, K, L	A*, B, C, D*, E, F, G, H, I, K, L	C, E, F, H, K, L	F*, H
Highest phase coded			A*, B, D*, G, I,	C, E, K, L	F*, H

Table 41. Multiple representations versus each phase

Appendix 6.7 shows examples of tasks that have been shared with learners at the exploration phases of the model. Teachers in general were much more to the right of the TDM for this aspect of the CCC Model with triangulated evidence showing six teachers at independent development or higher.

The self-reflection (Figure 56, full version in Appendix 6.2) also provides the perception from most (except A, D and G) that they are along the journey to transformation in this phase. There was certainly evidence in books and learning walks that six of the seven teachers (C, E, F, H, K, L) that say they were at the independent development phase were. The only anomaly was Brian who in the reflection activity positioned himself in the transformation phase however the highest triangulated evidence was at the independent exploration phase where given tasks making links between area and expanding brackets had been trialled with students.

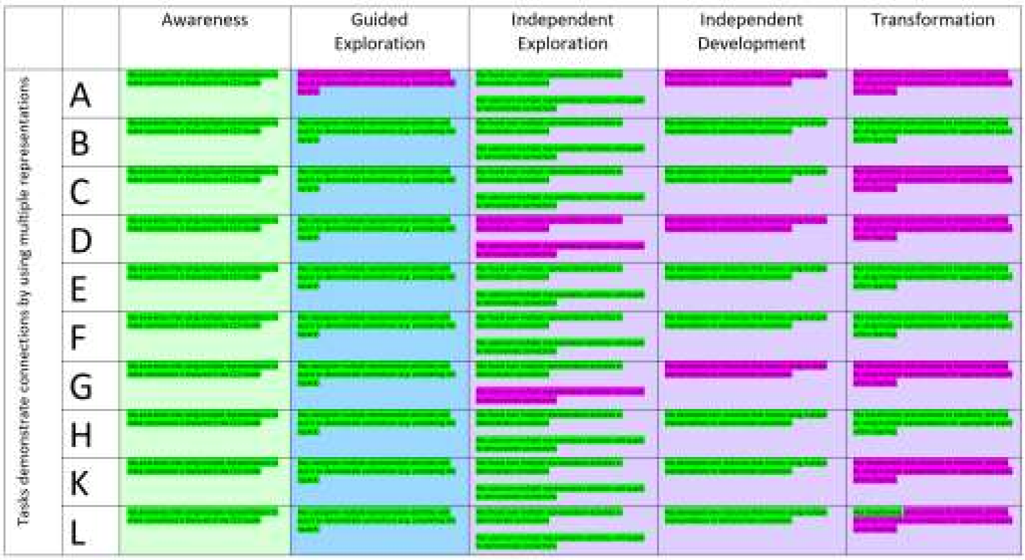


Figure 56. Teachers self-reflection on using multiple representations

The transformation phase bullet point, has transformed, and continues to transform, practice by using multiple representations for appropriate topics within teaching, was difficult to gather triangulated data (Appendix 6.1). Both Heidi and Frazer could confidently refer to lots of different examples of using multiple representations that had been gathered from sources other than the researcher, however it is difficult to measure whether this practice would extend to other topics that would follow.

8.2.4 Making links between procedures and concepts

This section explores the data for one aspect of the CCC Model, making links between procedures and concepts. There are 91 pieces of data coded with 41 of those from interviews and 50 from triangulated data. Table 42 shows a breakdown of where the data is from and which teachers were coded in each phase.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of references coded	17	29	23	20	2
Number of references coded (interviews)	16	13	8	4	0
Number of references coded (triangulated data)	1	16	15	16	2
Number of cases coded	10	10	8	3	1
Cases coded	B, C, D, E, F, G, H, I, K, L	B, C, D, E, F, G*, H, I, K, L*	B, C, D, E, G, H, K, L	C, G, H	H
Highest phase coded		F, I	B, D, E, K, L	C, G	H

Table 42. Making links between procedures and concepts versus each phase

Within the making links between procedures and concepts aspect there was triangulated evidence for all teachers at their highest stage of development.

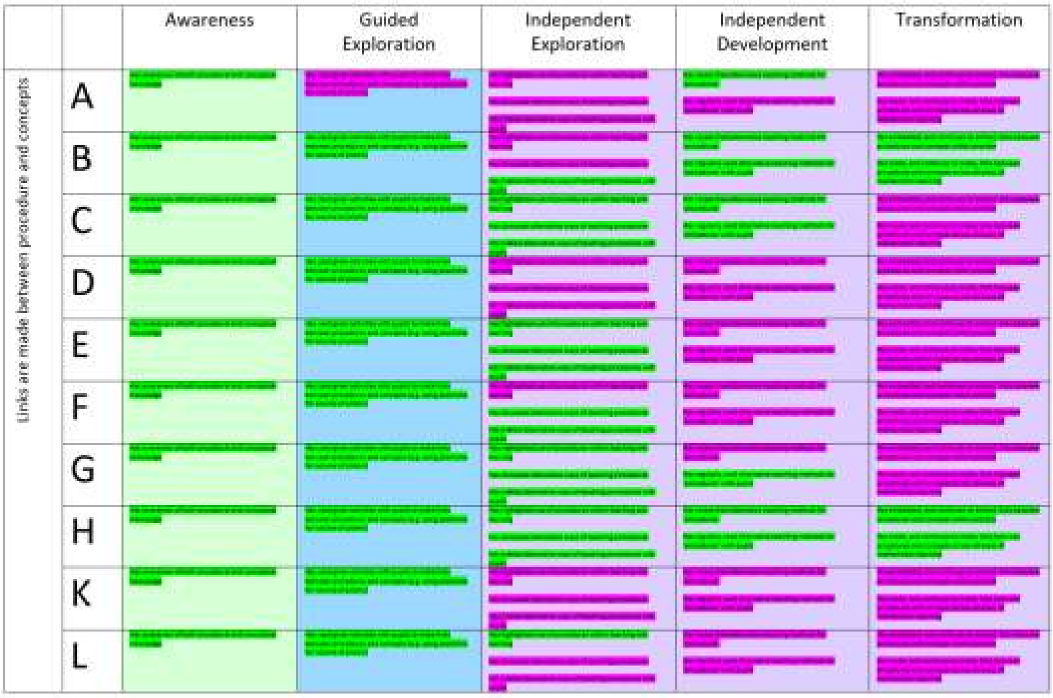


Figure 57. Teachers self-reflection on making links between procedures and concepts

However, comparing to their self-reflection in Figure 57 (full version in Appendix 6.2) Brian has highlighted the transformation phase green yet not referred to this in interview or other triangulated data. The self-reflection grid perhaps shows a lack of confidence in this area as Kate, Daniel and Louise haven't highlighted green the following phrases

- Has highlighted use of procedures within teaching and learning
- Has discussed alternative ways of teaching procedures
- Has trialed alternative ways of teaching procedures with pupils (TDM, Appendix 6.1)

However, the examples shown in Appendix 6.8 do show Kate and Daniel have explored alternative ways of teaching procedures.

8.2.5 Making comparisons

This section explores the data for one aspect of the CCC Model, making comparisons of either similarities and differences in concepts or comparing efficiency in different methods. There are 89 pieces of data coded with 43 of those from interviews and 46 from triangulated data. Table 43 shows a breakdown of where the data is from and which teachers were coded in each phase.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of references coded	5	11	51	19	3
Number of references coded (interviews)	4	5	20	11	3
Number of references coded (triangulated data)	1	6	31	8	0
Number of cases coded	5	8	11	6	3
Cases coded	B, C, E, F, H	A*, B*, E, F, H, I*, K, L	A, B, C, D, E, F, G, H, I, K, L	C*, D*, E, H*, K, L	D, E, H
Highest phase coded			A, B, F, G, I,	C*, K, L	D*, E*, H*

Table 43. Making comparisons versus each phase

Examples of tasks at the exploration phases are shown in Appendix 6.9. Table 43 shows that for this aspect of the CCC Model all teachers are at least at the independent exploration phase of the TDM. This agrees with the self-reflection task shown in Figure 58 (full version in Appendix 6.2). Noticeable here is that the first bullet point in the independent development phase (has used examples of comparisons, similarities and differences, with pupils in new topic areas) is highlighted green by everyone, whereas the second bullet point (comparisons in the form of looking at efficiency of method) isn't. The coding carried out on NVIVO shows there is data in this cell but doesn't split it down to the respective part though.

Figure 58. Teachers self-reflection on making comparisons

8.2.6 Application tasks

This section explores the data for one aspect of the CCC Model, using application tasks where they are presented as challenges. There are 33 pieces of data coded with 9 of those from interviews and 24 from other triangulated data. Table 44 shows a breakdown of where the data is from and which teachers were coded in each phase. Examples of tasks at the exploration phases are shown in Appendix 6.10. Table 44 shows most teachers at the independent development phase when considering the use of application tasks. This strand has triangulated evidence for most to support what has been said in interviews.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Number of references coded	4	3	21	5	0
Number of references coded (interviews)	3	1	3	2	0
Number of references coded (triangulated data)	1	2	18	3	0
Number of cases coded	4	3	10	2	0
Cases coded	B*, C*, H*, K	E, G, I*	A, B, C, D, E, F, G, H, I, L	E*, H,	
Highest phase coded	K		A, B, C, D, F, G, I, L	E*, H,	

Table 44. Use of application tasks versus each phase

However, when looking at Figure 59 (full version in Appendix 6.2) which shows the self-reflection more teachers have highlighted the independent and transformation phase than was seen in evidence collection. Considering the two bullet points in the independent development phase in the model:

- Has trialled a range of application tasks presented as challenges
- Has given students time to explore problems themselves before offering support (TDM, Appendix 6.1)

Seven teachers highlighted the second point green whereas only two highlighted the first green. Both Brian and Heidi highlighted all the transformation phase green however, this was not observable during the book scrutiny. Partly this could be due to the statements referring to management of learners and encouraging them to tackle tasks which are actions that the teacher would be doing and not easy to observe with the triangulated data collected.

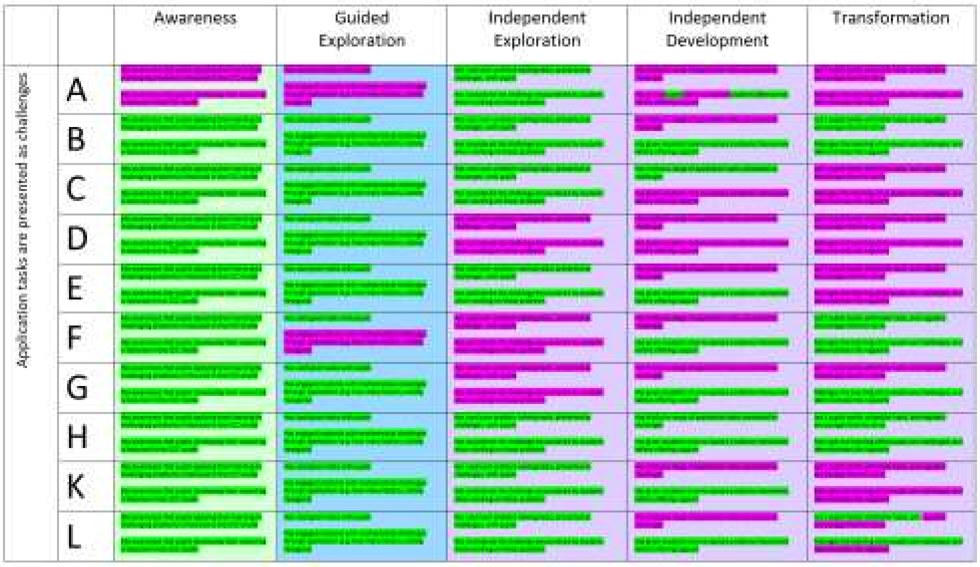


Figure 59. Teachers self-reflection use of application tasks

8.2.7 Summary reflections on the CCC Model

Figure 60 (full version in Appendix 6.2) shows the overall summary page of the CCC Model that teachers highlighted as part of the final evaluation process. Overall Annette, Daniel and Frazer highlighted more areas as not met than other members of the department when considering the whole CCC Model. However other sections have shown they have made progress with areas they have chosen to work on.

Figure 60. Teachers self-reflection summary of CCC Model

8.2.8. Conclusions on which approaches were used to develop connections with learners

During the bridging theory to practice sessions the department were exposed to a range of different activities that modelled the aspects of the CCC Model. Figure 62 shows that everyone; tried a multiple representations activity and explored tasks that made comparisons. Examples of these are shown in Appendix 6.7 and 6.9.

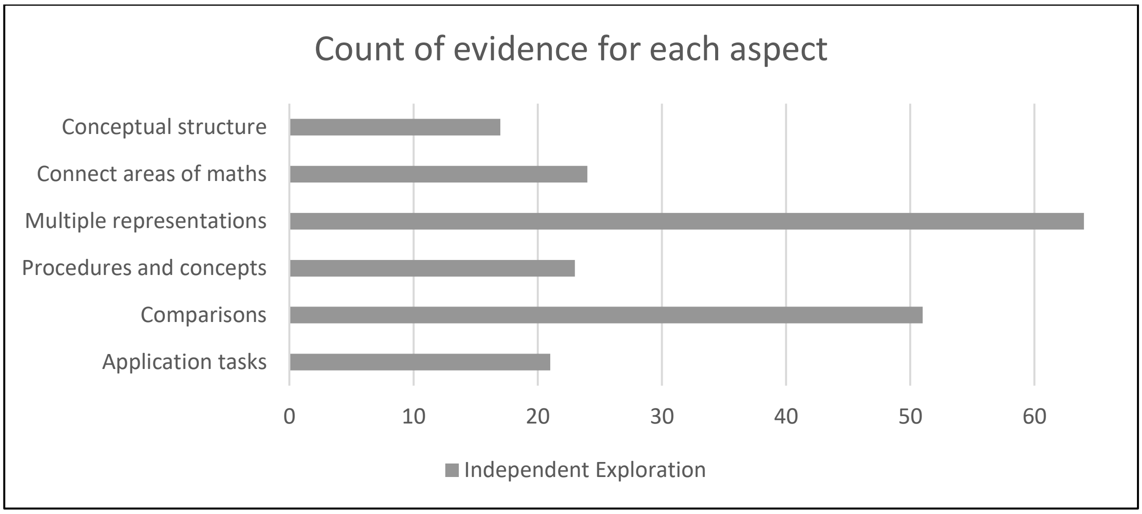


Figure 61. Evidence at the independent exploration phase

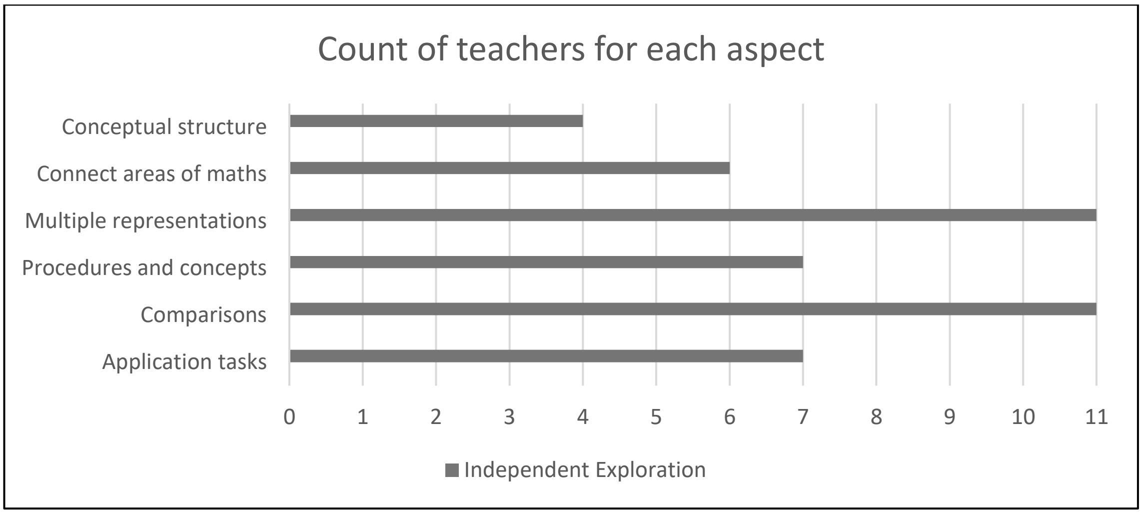


Figure 62. Number of teachers at the independent exploration phase

8.3 How do teachers develop tasks that encourage connections to be made?

Whilst early phases of the study focussed on the presentation of research and exploring activities to model the key ideas of the CCC Model, the active experimentation phase from April 2014 to June 2016 encouraged teachers to collaborate and to develop resources and their pedagogy. This section details evidence gathered at the last two phases of the TDM (Appendix 6.1) as the evidence in these phases moved beyond activities and tasks that had been provided by the researcher. This section shows where teachers had moved on with their journey what types of things they were doing with their learners.

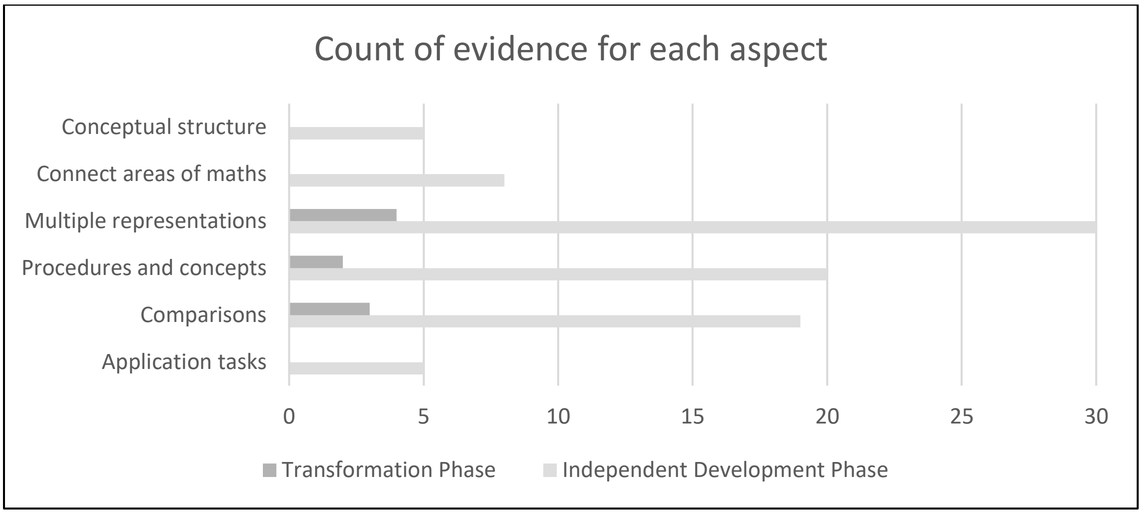


Figure 63. Evidence at the independent development and transformation phase

Figure 63 shows a count of evidence observed for each aspect of the CCC Model and for the use of research.

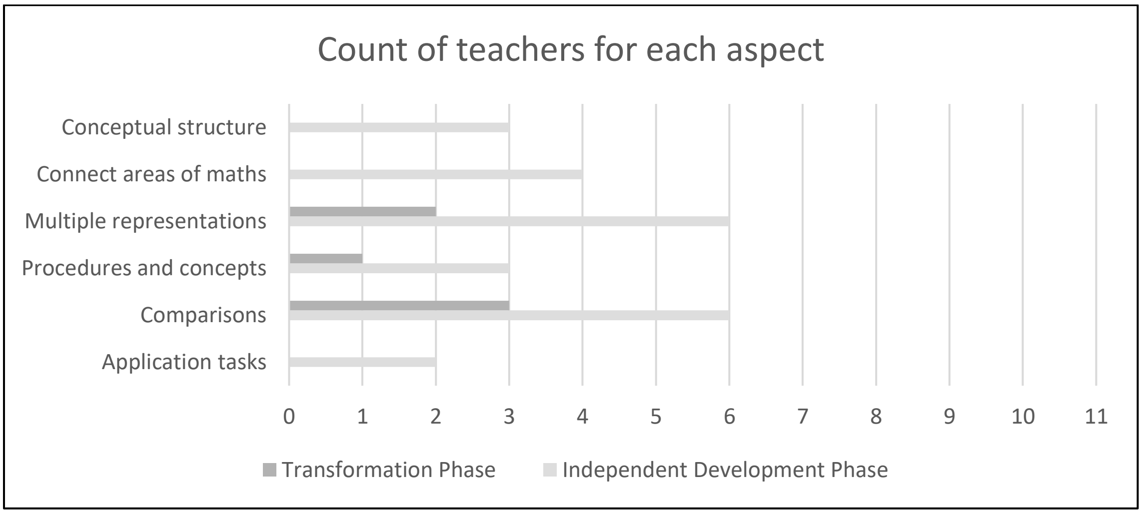


Figure 64. Number of teachers at the independent development and transformation phase

Figure 64 shows the same data but looking at the number of teachers (where the maximum would be eleven).

8.3.1 Teachers making links to students' conceptual structure

At the independent development phase, there was the least amount of evidence for the strand making links to students' conceptual structure with only five pieces of evidence in total from three teachers (E, G and H). Elliot discussed in his final interview a couple of examples where he had tried to build on students' knowledge. The first:

> 'I feel that within myself I am trying to actively build on pupils' knowledge and to try to develop the different ways of teaching it and things like that, my example would be the interview lesson with the percentages, …. it was like what would be a better token to have in the super market £20 or 20% off.' (Elliot, 2016, Final interview).

And secondly, he discussed a task about different lengths of sticks which led to averages without learners realising that they were deriving this from their knowledge. He also said:

> 'With the estimating roots, I have done, it was like the mid rule and then there was like an estimating rule and there was loads of different methods and the way that I explained that this method worked was by making the links between things they already know.' (Elliot, 2016, Final interview)

Georgie (2016, Final interview) felt she was working at the independent development phase by using arrows on her display to link new knowledge to previous knowledge.

8.3.2 Teachers connecting areas of mathematics

During the learning walks and book scrutiny there were a few examples of teachers making connections within topic areas or across areas of mathematics. Louise had trialled a range of resources that she had adapted making links between place value charts, fractions and multiplying by powers of 10. Whereas Elliot had organised lessons that made explicit connections between, Pythagoras, cosine rule, areas of triangles and trigonometry. Brian also in interview acknowledged:

> ‘Well as part of my ratio and proportion thing I have connected straight line graphs with ratio and proportion with two column tables, with gradients, with sequences, so yes I have done lots of connections there in my project.’ (Brian, 2016, Final interview).

In these cases, the ideas had been discussed at department level, but resources hadn’t been provided so the teachers had presented these links themselves to their learners.

Charlotte had numerous examples (Figure 65) where links had been made between ratio, proportion and straight-line graphs.

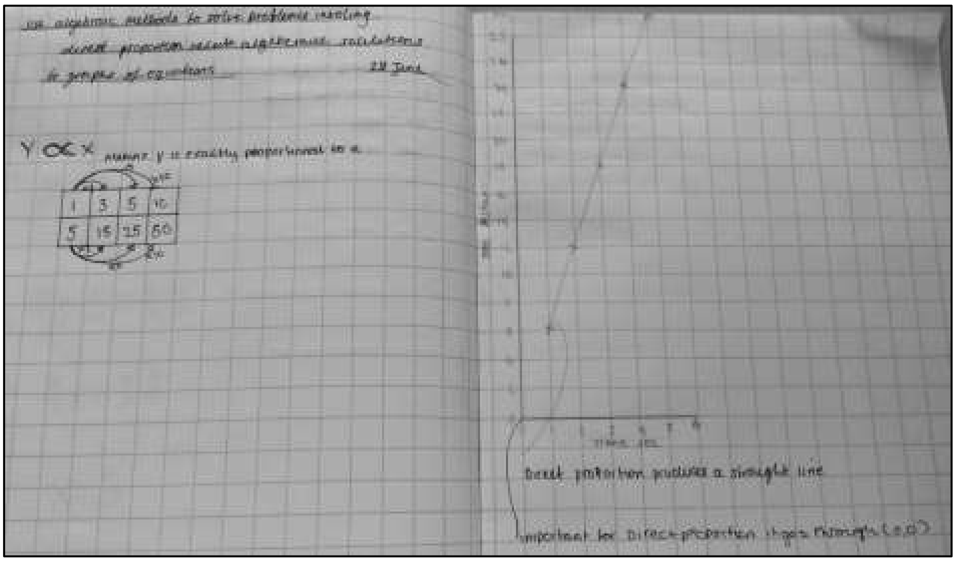


Figure 65. Proportion and linear graphs (Charlotte, 2016, Book scrutiny)

8.3.3 Teachers using multiple representations

There were six teachers that had independently explored the multiple representations strand within their lessons with classes with thirty pieces of evidence in total.

Frazer's books revealed examples that introduced trigonometry by making links with the unit circle and explicitly mentioning similar triangles and links to the graphical representation of the sine curve. These can be seen in Figures 66 and 67.

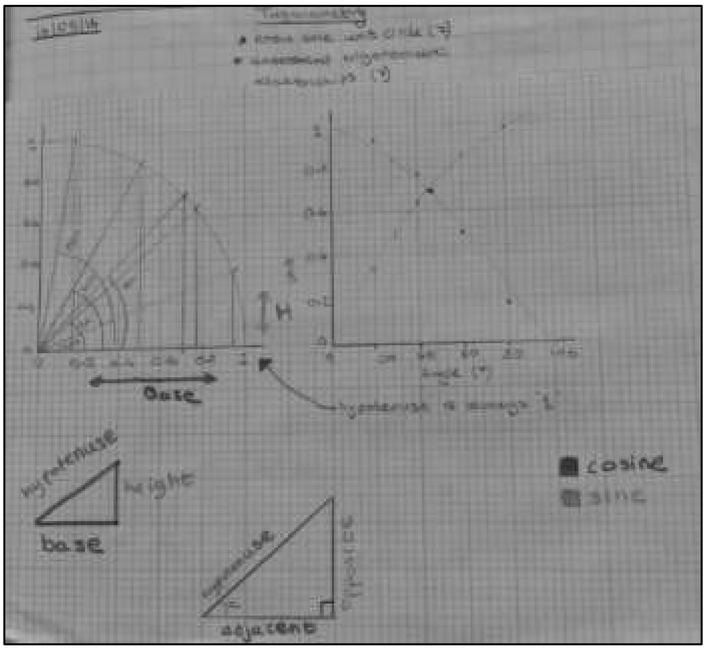


Figure 66. Making links within trigonometry 1 (Frazer, 2016, Book scrutiny)

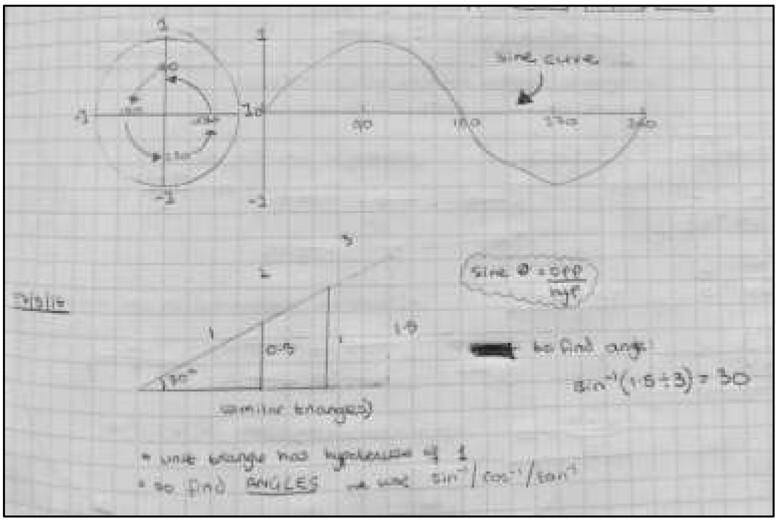


Figure 67. Making links within trigonometry 2 (Frazer, 2016, Book scrutiny)

Frazer focussed his action research on the use of multiple representations and there were many examples that had been used from the Further Mathematics Support Programme (FMSP) alongside tasks he had developed himself, for example Figure 68 encouraging students to make links between the algebraic and graphical form of quadratic inequalities.

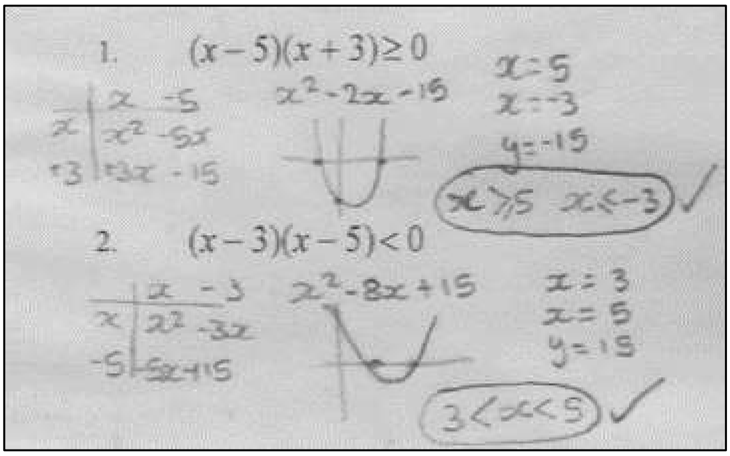


Figure 68. Quadratic inequalities (Frazer, 2016, Book scrutiny)

Charlotte designed a task making links between function machines, tables and graphs. Whereas Kate found and adapted resources that matched improper fractions, mixed numbers and visual images of fractions. Elliot used visual images to explore percentages. Heidi used multiple representations in great depth, these are shared in the next section as she was using them with the purpose of deriving fraction calculation procedures. Louise also explored in depth multiple representations and chose to write her PGCE subject specific action research project on the question ‘Can presenting a mathematical concept in different representations help pupils make links and develop a conceptual understanding?’.

> ‘I reviewed the topics I would be teaching and selected appropriate times to apply teaching with multiple representations. I used multiple representations in teaching linear functions and graphs, and fractions. The planning for these lessons involved developing my own resources and selecting resources from others (for example the Standards Unit, Swan, 2005) which would allow pupils to experience a variety of representations.’ (Louise, PGCE Assignment)

8.3.4 Teachers making links between procedures and concepts

When looking at making links between procedures and concepts there were 20 pieces of coded data at the independent development phase, coming from three teachers Charlotte, Georgie and Heidi.

However, except for Charlotte's example (Figure 69) where she was encouraging students to derive the formula of area of a circle from the principle of limits, the remaining evidence was linked to the topic of fractions which was explored in depth by Georgie and Heidi as part of their action research.

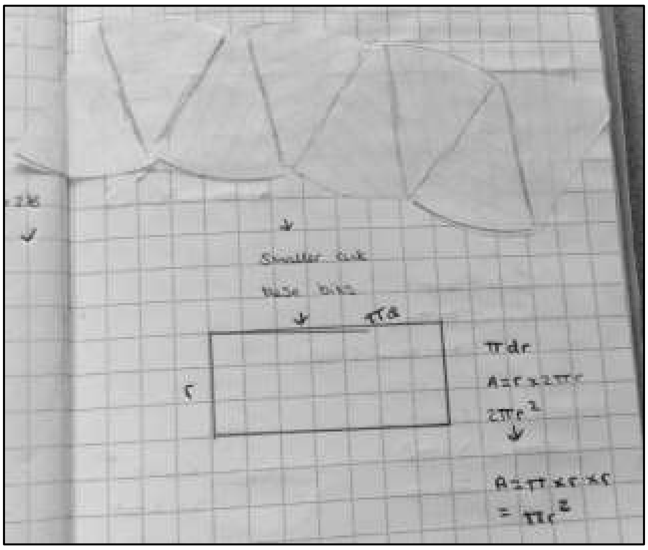


Figure 69. Area of a circle (Charlotte, 2016, Book scrutiny)

A variety of methods were explored that linked using multiple representations to help derive the procedures from exploring fraction concepts in a visual way. Georgie (Figure 70) explored with her learners how to multiply fractions by an integer.

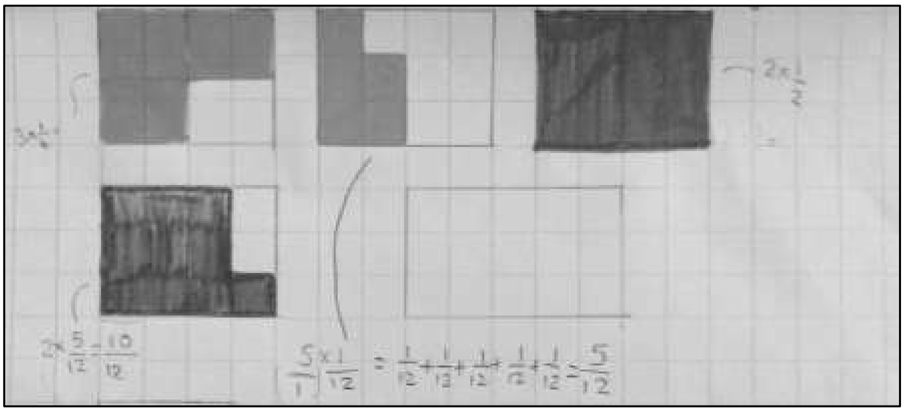


Figure 70. Multiplying fractions by integers (Georgie, 2016, Book scrutiny)

Heidi (2016, Final interview) acknowledged she had developed her subject pedagogy ‘well for me personally I have really enhanced my understanding of fractions…. I noticed that the way I was teaching was not sufficient for students…. for me personally it was me developing the ideas and finding the different ways of explaining to students’. There was a range of evidence in different forms to triangulate Heidi’s comments and to validate she had changed her practice. Figures 71 and 72 show her student work on multiplying and dividing fractions respectively.

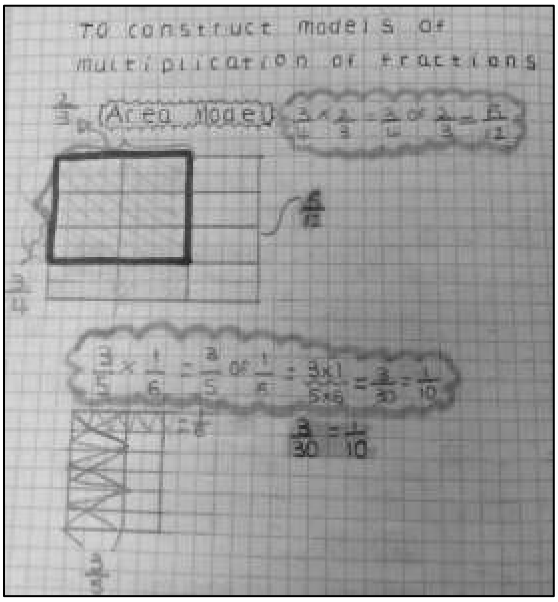


Figure 71. Models of multiplication of fractions (Heidi, 2016, Book scrutiny)

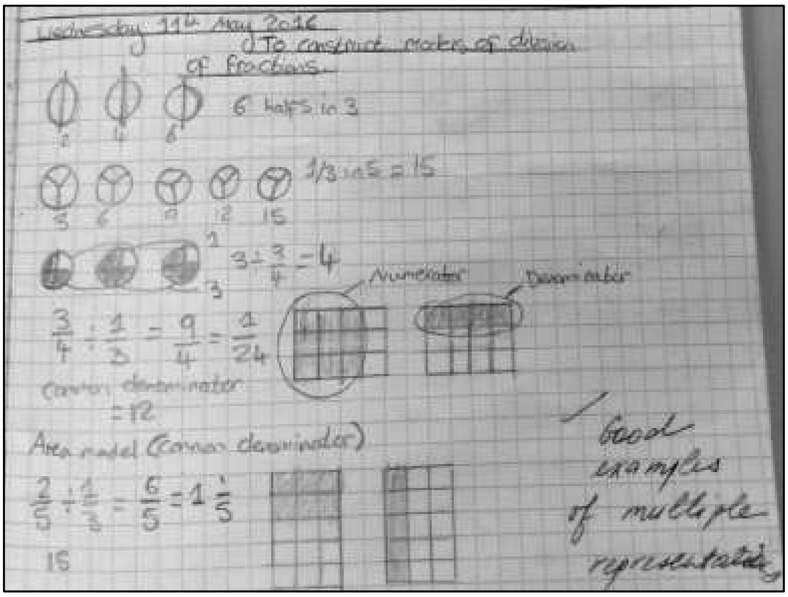


Figure 72. Models of division of fractions (Heidi, 2016, Book scrutiny)

Figures 73 and 74 show the approach Heidi had researched and trialled using practical examples of worded questions and the use of recipes to help students gain a deeper understanding of division of fractions.

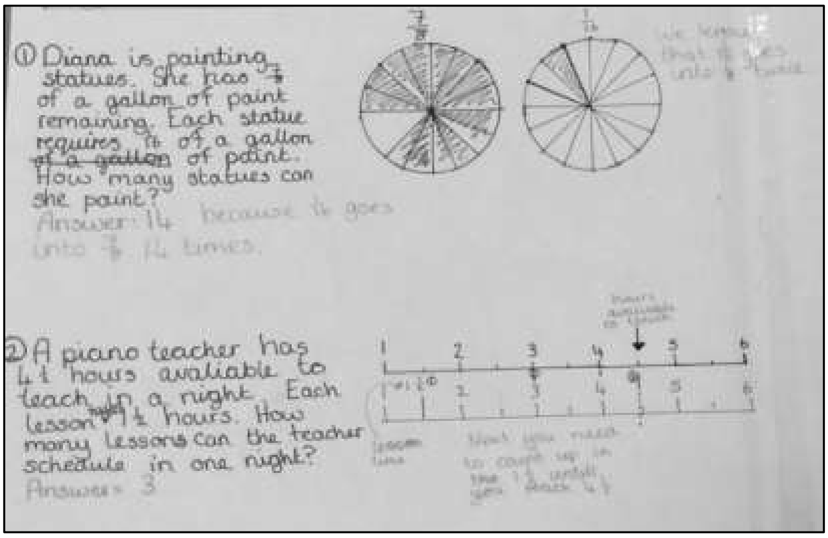


Figure 73. Division of fractions (Heidi, 2016, Presentation)

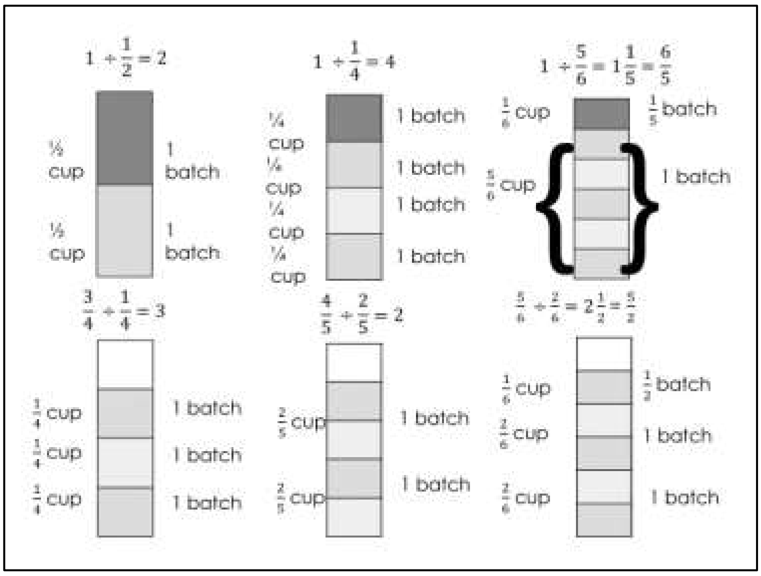


Figure 74. Division of fractions using recipes (Heidi, 2016, Presentation)

Heidi discussed these ideas with colleagues as she was trialling them and when they were shared at the final presentation Charlotte and Georgie had also used them. During Heidi's presentation to the department she reflected on how she had approached the teaching of fractions using visual representations and concluded that:

> ‘As we know we need to find a good balance between their conceptual and procedural understanding just to guarantee the advance of mathematical knowledge.’ (Heidi, 2016, Presentation)

There was also evidence gathered in the book scrutiny that showed students had been encouraged to reflect on the use of these new visual methods to help them understand the concept of multiplicative inverses. One example is given in Figure 75.

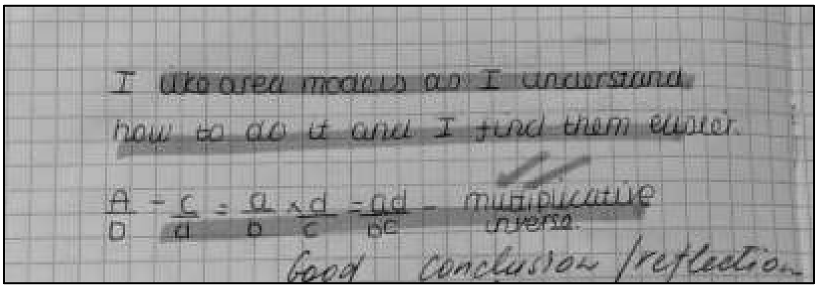


Figure 75. Area models when dividing factions (Heidi, Book scrutiny)

These examples support the triangulation that Heidi has moved to the transformation phase when making links between procedures and concepts when teaching fractions.

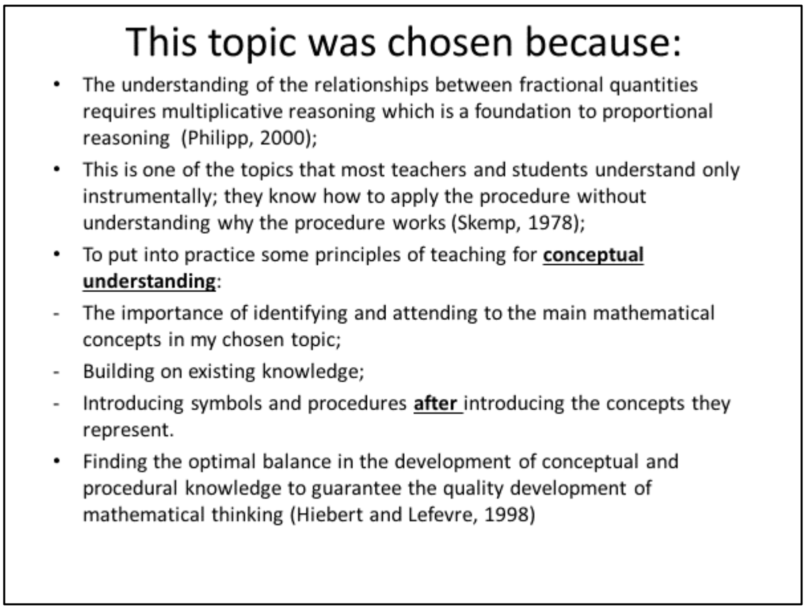


Figure 76. Rationale for fractions development (Heidi, 2016, Presentation)

Heidi not only provided academic references that were new to the ones the researcher provided as can be seen in her starting presentation slide (Figure 76) but she also articulated her own opinions as to why this was important for her to study further:

> ‘I chose this topic because understanding of the relationships between fractional quantities requires multiplicative reasoning which is a foundation to proportional reasoning and we know it is going to be big in the new curriculum. This is one of the topics that most teachers and students understand only instrumentally; they know how to apply the procedure without understanding why the procedure works. I also wanted to put into practice some principles of teaching for conceptual understanding as well so by identifying and attending to the main mathematical concepts in my chosen topic; building on their existing knowledge; and introducing symbols and procedures after they understand it.’ (Heidi, 2016, Presentation)

8.3.5 Teachers making comparisons

Whilst the making comparisons strand of the CCC Model had two elements to it (exploring similarities and differences and efficiency of method). There was a marked discrepancy as to which was trialled and developed. Looking at similarities and differences was embraced by everyone, at least at the independent exploration phase, with six teachers (C, D, E, H, K, L) taking things further to come up with their own examples.

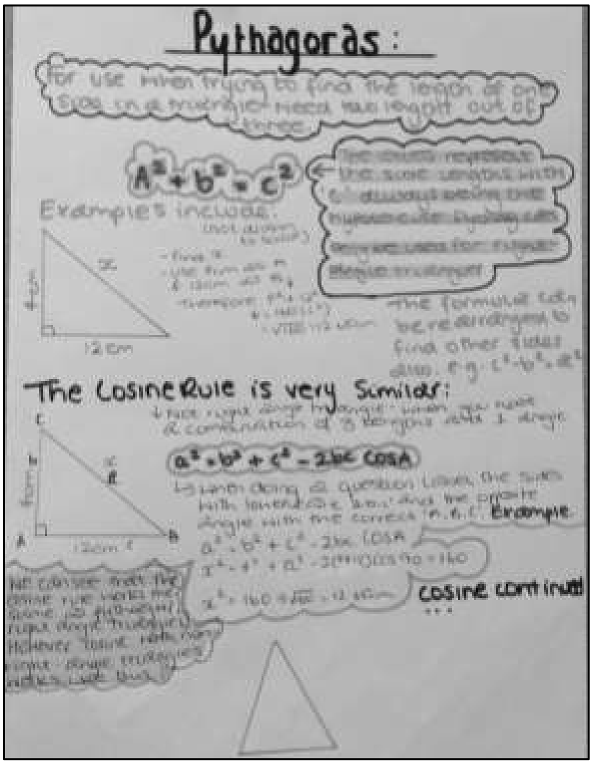


Figure 77. Pythagoras and cosine rule (Elliot, 2016, Learning walk)

Whilst in Elliot's room, during a learning walk, there was a slide on the screen asking learners to complete a poster showing the similarities and differences in questions about triangles. Figure 77 shows one response from a learner.

On the same learning walk, a visit to Heidi, showed students engaged in looking at the similarities and differences of the structure of a problem-solving exam question (Figure 78).

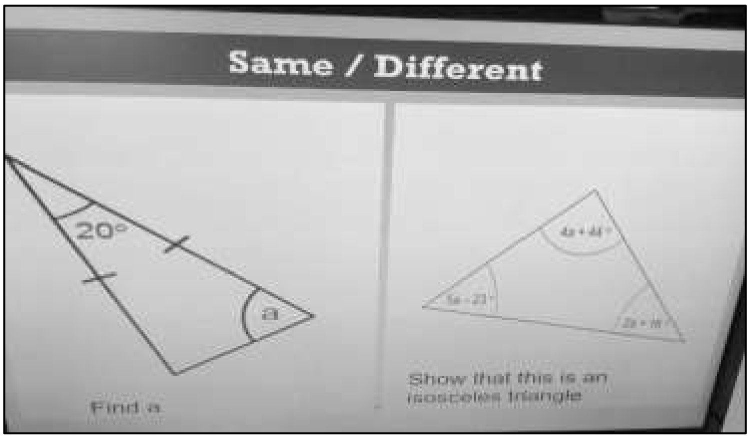


Figure 78. Problem solving with triangles (Heidi, 2016, Learning walk)

When questioned if this was something that happened often the students said it did most weeks and showed the researcher another example in their books that referred to congruent and similar shapes which can be seen in Figure 79.

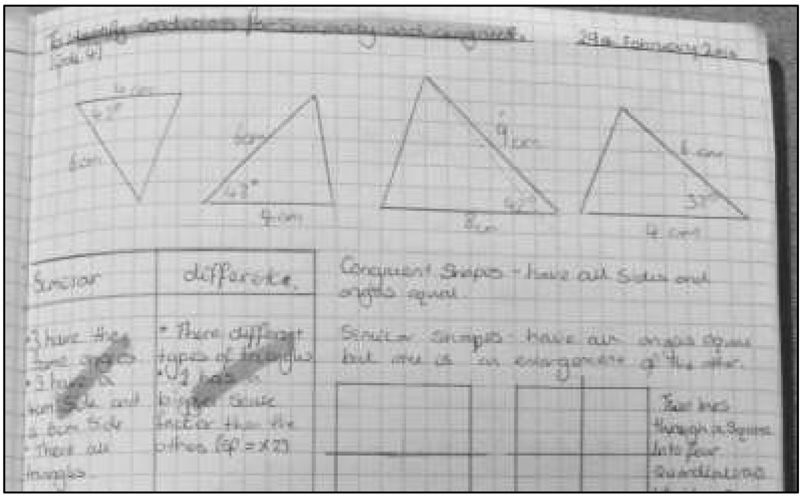


Figure 79. Similarity and congruence (Heidi, 2016, Learning walk)

Interviews showed teachers confidently talking about how they had used similarities and differences as a starter type activity with their classes. Heidi (2015, Mid-study interview) used sequences to tease out arithmetic and geometric properties and Daniel (2015, Mid-study interview) used the athletes Usain Bolt and Mo Farrah in a physical education lesson to explore ‘different muscle fiber types and the different energy systems in a completely contextual situation, without having to teach anything’.

There were examples in Kate’s and Louise’s presentations that they had used with their learners. Kate explored similarities and differences of properties of numbers such as factors, multiple etc. and Louise used it to develop a deeper understanding of multiplying by a tenth as shown in Figure 80.

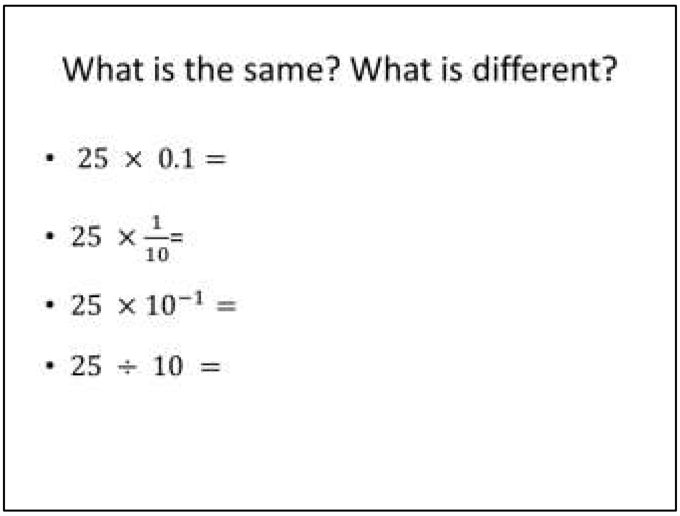


Figure 80. Multiplying by a tenth (Louise, 2016, Presentation)

Charlotte gave examples of using the strategy with A Level students, to look at vector equations and their relationships. She also reflected on how the department developed in this area ‘I think we are all very much happy about the similarities and differences and I think that one is coming through again and again, so everyone seems to be quite happy with that' (2015, Mid-study interview).

When looking at efficiency of method there was only a couple of examples in Heidi's books that were evident. Students had been asked to group together cards (Figure 81, left hand side) and then had discussed the different methods for finding percentage change.

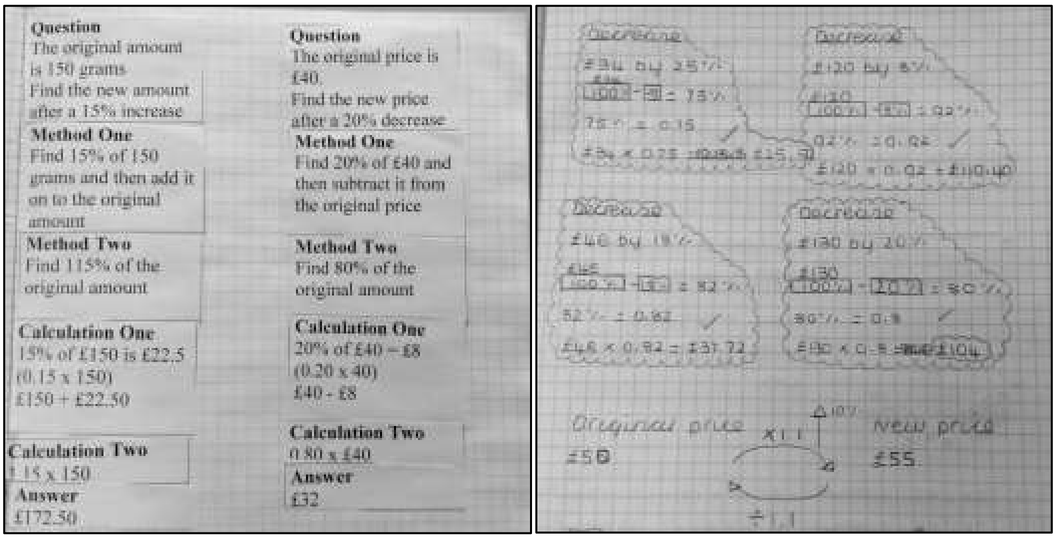


Figure 81. Percentage change (Heidi, 2016, Book scrutiny)

They had later extended these ideas to using multipliers as can be seen in a pupil book in Figure 81 (right hand side).

There is evidence from interviews that three teachers (D, E, H) believe they have transformed their practice using the aspect of making comparisons in the form of looking at similarities and differences. When Elliot was questioned how often he used the strategy his response was 'once a day at least and at other times it would be all the lessons that you would say those words' (2016, Final interview). Heidi provided a similar response 'what I do a lot of in my lessons is similarities and differences, I have started to introduce it basically at the beginning of each topic to encourage students, to give them the overall picture and to see how the topic will look' (2016, Final interview). The similarities and differences aspect was also explored in depth by Daniel who comments 'I embedded that in a lot of my other subjects and in my maths .... now that is becoming just off the cuff, what's the same and what's different is becoming more everyday speak for me in my math's lessons' (2016, Final interview).

For these teachers, due to the frequency of them using the strategy it is embedded into their practice. Since a lot of the similarity and differences work involved teachers showing a slide which prompted discussion in lesson there wasn't written evidence that this was happening in exercise books. A deeper analysis of teachers' planning/ resources would need to have been looked at to gather triangulated evidence in this aspect at the higher phases of development.

8.3.6 Teachers using application tasks

There was only Elliot and Heidi that referred to using application tasks. Elliot described his confidence in the classroom:

> 'I am confident in letting pupils tackle unfamiliar tasks, I am quite happy to just let them get on with it and just try their different approaches. And I will sit there for twenty minutes at the front of the lesson whilst they struggle to start it and give those clues.' (Elliot, 2016, Final interview)

He also gives exampleshe had used with students, including getting a year ten class to work out the net of a cone that involved Pythagoras and arc lengths. Heidi’s presentation (Figure 82) showed examples of tasksshe had used to encourage learners to apply their knowledge in the topic of fractions.

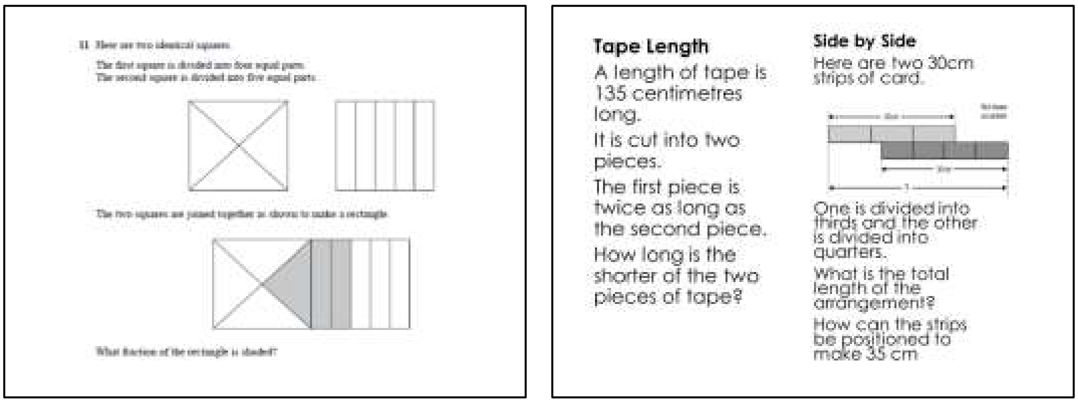


Figure 82. Application tasks used with learners (Heidi, 2016, Presentation)

8.4 The department versus individuals

Whilst Sections 8.2 and 8.3 consider which areas of the CCC Model were implemented and reasons for this, this section considers individual cases from within the department that appear interesting and different. This section presents an answer to the questions:

- Are there any differences in how teachers develop across the department?
- What has influenced these differences?

Whilst analysing data it became obvious that if teachers were changing their practice there may be very different reasons as to why they had engaged. This section looks at the specific cases of Brian, Daniel and Georgie as they are deemed to be exemplary, unusual or special in some way (Yin, 2009).

8.4.1 The case of Brian

The case of Brian is considered as his perception of his implementation of the CCC Model is somewhat different to that observed. Appendix 6.2 shows he positioned himself mainly at transformed practice in most aspects of the CCC Model. However, when triangulating with what was seen in practice there were some contradictions.

During a task in the pilot study he commented ‘I don’t think geometrically, do you?’ and when referring to his own experiences as a mathematics learner he commented, ‘I had procedures beaten into me’ (Brian, 2013, Pilot study). One thought from the researcher was, as Brian was taught procedurally himself and that had been successful as a result, he saw no reason to do anything different with his learners. Brian came to the teaching profession late in his career and from conversations with him he has a very algebraic dominated view of mathematics.

Throughout the whole study there was only one quote that questioned whether the CCC Model was the right thing to be embedding into practice and that was Brian voicing concerns as to whether we needed multiple representations at all:

> ‘In some ways I can understand the analogies in the diagrams that are being used such as the expanding brackets and I can see that, Nic is going to hate me for saying this but I am going to say it, these might help some people to understand but for me it is an analogy for analogies sake it is comparing the expansion of brackets with areas and it is a model that fits but for the students that I have shown it too it doesn't really help them understand, in fact in many ways it confuses. I think we have to be careful whether the model is appropriate or not and perhaps for other people it works, for me it overly complicates the picture (apologies to Nic!).’ (Brian, 2014, Mid-study interview)

There was also evidence in his presentation of these beliefs, when talking about ratio he was adamant that $a{:}\, b$ would only be expressed as $\frac{a}{b}$ stating ‘it is what I was taught a long time, but I know that some of us have a problem with it. It is a recognised way of expressing a ratio. …. waiting to be hacked down!’ (Brian, 2016, Presentation). He was referring to the discussions held between the researcher and the department about different fractions that arise from ratio pairs depending on whether you are considering part: part or part: whole relationships.

Several pieces of evidence that show that even though Brian states on the CCC Model that practice was transformed he still teaches in a procedural way. During the visit to year 9 lessons in March 2012 the external visitor felt the lesson observed of Brian was ‘very procedural’ (2012, Learning walk). The book scrutiny also shows an example of using completing the square to solve quadratic equations (Figure 83) which was traditional in its format despite the enthusiasm from others to experiment with the resources provided by the researcher.

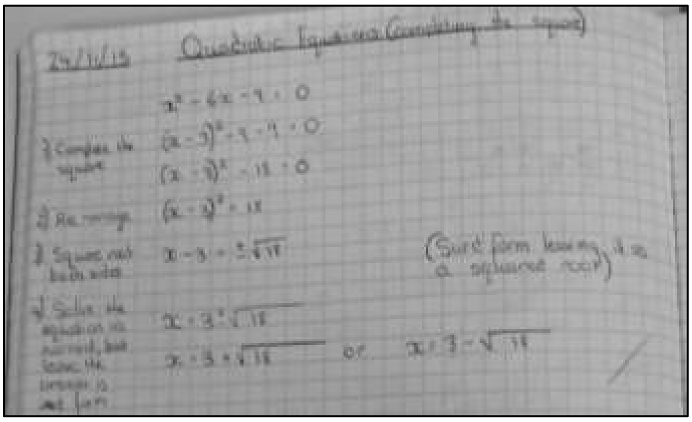


Figure 83. Solving quadratic equations (Brian, 2016, Book scrutiny)

Figure 84 shows an example of how to convert fractions to percentages in a procedural way.

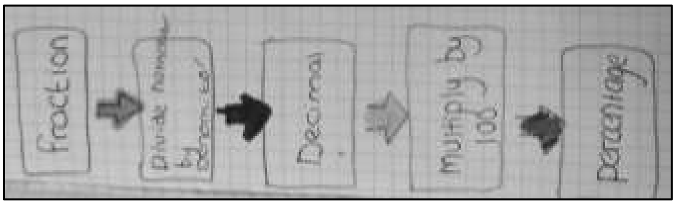


Figure 84. Converting fractions to percentages (Brian, 2016, Book scrutiny)

Throughout the process there were also quotes from other teachers about work put into the shared area being very procedural. When Elliot was reflecting on how a trainee teachers lesson had gone he said, ‘she has used a lot of Brian’s stuff which is okay, but it is very procedural and is just telling them how it is done’ (2015, Conversation with researcher).

It was felt by the external analyst that Brian wanted to say the right things during interviews to please the interviewer and researcher. During a review meeting the interviewer commented ‘Brian seemed to think that he was teaching conceptually but I don’t think that he really understands what this entailed’ (2016, Conversation with researcher). Brian has a strong mathematical background and secure subject knowledge; however, although he was happy to acknowledge his weaknesses in not knowing about new resources such as Cuisenaire rods (2016, Final interview) it was felt that he was unconsciously not aware of gaps in his own MKT and he perceived that the procedural way that he was taught would suit all his learners.

8.4.2 The case of Daniel

Daniel was an interesting sub case for many reasons. When he started the study, he had been in the process of engaging with a SKE+ course which had already opened his eyes to different ways of doing things. This alongside the CCC Model led him to consider how mathematics should be taught.

> 'For a man that has been teaching for over 20 years, the amount of reflection that I have been doing over the last couple of years is the most that I have ever done in my career without a shadow of a doubt and that has been really focused on it has to go away from just putting down facts and they have got to have an understanding of it.' (Daniel, 2016, Final interview)

In this case, Daniel is an example of a teacher that was on board with the ideas of teaching for understanding and the aspects of the CCC Model, but other than exploring similarities and differences, he wasn't always able to take the principles and develop his own resources for other areas of the curriculum. For example, he referred to being shown the proof of the quadratic formula on the SKE course 'I would love to explore it more by exploring it with kids, but I haven't had the kids that I have been able to go on and do that with' (Daniel, 2013, Initial interview).

He acknowledged on many occasions the challenges of adapting the ideas to his classes.

> ‘My challenge at the moment is to try and get that approach to the maths at the lower level and I am finding that quite a challenge to be fair. I have some kids with a really basic understanding and to get them to go beyond putting things in a box and to get to the process is quite difficult and a challenge I haven’t had for a while….. Once I get a handle on that I will be able to move the techniques and understanding that I have down to their level, but I am finding that I come up with an idea and I try it in the lesson and it doesn’t work and I have to try something else again.’ (Daniel, 2013, Initial interview)

His main feeling in interviews was of frustration, because he had so many new ideas but felt he didn’t have the classes to explore with. This was partly due to a limited mathematics timetable and because he had experienced activities at higher tier topics that wouldn’t fit with his foundation tier scheme of learning.

> ‘The amount of maths teaching is now very small, and it is only to bottom set year 11 which is a narrow focus, so any of the collaborative stuff we have been looking at with the department, when I have been involved with it, has been a challenge to get it in those lessons has been a challenge.’ (Daniel, 2014, Mid-study interview)

However, he experimented and later embedded the aspect of similarities and differences not only in mathematics but also in physical education and psychology. There were a few reasons that could be attributed to this, he could see how the strategy could be used in different settings and for students at different levels. He could quickly come up with questions that prompted deep thinking and discussions which he could see the benefit in. Also, later in the process he saw the reported success from Marzano’s research (1998) and this confirmed what he believed to be the case in a positive way.

> ‘It was good to see that using similarities and differences, which is something that we all have embraced as a department, has such a big effect in the development and progression in understanding of students, so that was really good to see.’ (Daniel, 2015, Mid-study interview)

He referred to needing to try new ideas with lots of different classes ‘I think you would try it with year 8 and then again with year 9’ and different occasions ‘I don’t teach enough of it to keep it fresh and keep coming back to it all of the time’ to be able to embed them into his teaching repertoire (Daniel, 2016, Final interview). He had success with the strategy of exploring similarities and differences because he had been able to use with all classes and across his other teaching subjects too.

Daniel as a sub case exemplifies two points. First, it shows even if teachers are on board with the ideas, they are not always able to take the messages underpinning a resource or activity that has been shared and to apply it to a different topic due to lack of subject knowledge or lack of pedagogic transferability. Secondly, that several different attempts at something new is needed before it becomes part of a teacher’s repertoire.

8.4.3 The case of Georgie

Georgie is an interesting sub-case with an emotional personal story. On several occasions, she opened up to the interviewer about her feelings and how her mind-set had changed over time. Appendices 6.11 – 6.14 show extracts from her interview transcript so as Yin suggests (2014), from Section 5.2.5, the reader can follow the derivation of evidence to the conclusions being drawn.

In the initial interview (extracts shown in Appendix 6.11), she comments that she is now developing and ‘that is only something that has happened fairly recently here’. Reasons for this show previous tensions under the management of the previous head of department. She used to be guarded about people watching her but recently under leadership from the new head of department and the researcher she is now open to discussing mathematics and feels the department are ‘leading’ in professional development. She said, ‘we have got faith in each other now’ and this results in teachers sharing resources and ideas informally and formally. She acknowledges the different methods and ways that teachers attempt problems, some algebraically and some with pictures. When questioned what it means for a student to understand maths she said ‘I am not sure I understand mathematics … I certainly haven’t got a degree’.

As soon as the CPD sessions were carried out Georgie was interviewed again (extracts shown in Appendix 6.12) because the researcher could see a change in her behaviours and wanted to explore this further. During this interview Georgie opened up to the interviewer about her fears and lack of confidence ‘I have always been quite honest about the fact that I don't believe my mathematics skills are up there with everyone else in the department. I don't think that I am a mathematician in the same way that everyone else is’. She referred to the CPD sessions and the support that had been provided to help tease solutions from her. She was excited to engage in a task that previously she felt would have been inaccessible to her. Previously she wouldn’t have engaged for fear of being ridiculed, now the culture was such that challenges could be overcome together.

She now felt supported and able to make mistakes, giving reasons such as the approach that the researcher took and the types of tasks that were chosen.

> 'I suppose during the last 5 years, I felt I was capped at an average mathematician who has got strength in mental arithmetic and tricks and things like this, now I am being made to feel like with support I could be a mathematician, nothing here makes me feel like Nic is better than I am, we are equal and that is such a nice environment to be a part of. It really does make a difference.' (Georgie, 2014, Mid-study interview)

What is not clear from the transcripts is whether Georgie just values the support and maybe it wouldn't have mattered what the CPD topic was, she had bought into the change in culture and for her that was all that counted. She admitted to coming out of her comfort zone 'I wouldn't do it for anybody, I probably wouldn't do it for anybody else if I am honest, but because Nic has got this love she makes me want to be a better mathematician and a better maths teacher' (2014, Mid-study interview).

The November 2014 interviews (extract shown in Appendix 6.13) show Georgie repeating comments about how she lacked mathematical knowledge, 'I have always been a little bit embarrassed really when we have these department meetings about my understanding about maths' and 'when people are saying prove it, I often take a back seat because I am not convinced that I know what I am talking about really'. She commented again on feeling comfortable to work with peers and now had more confidence to inspire students with the tasks too. This confidence in being able to do tasks had motivated her to be 'less lazy' because 'I will work harder to find things that possibly will work because I stand a better chance of finding out how they work'.

Georgie was then off school for maternity leave but re-joined the department later in the study and was interviewed again for the final interviews (extract shown in Appendix 6.14). At this stage Georgie had been working on her fractions research project and was proud of what she had done ‘the outcome of what I had achieved I was pleased with and I thought it was important that I did share because I thought it would positively inform my peers teaching’.

She again referred to how it used to be, ‘we were made to feel like our teaching wasn’t good enough and we weren’t good enough but now it is like an aspirational department and we are made to feel like you want to achieve what everyone else is achieving’.

It can be concluded that, Georgie started the study in an unhappy place afraid to take part in department meetings, feeling her thoughts would be ridiculed. She was not willing to do mathematics in front on colleagues in case she made a mistake. Her journey engaging with the external domain of the CPD programme, changed her knowledge, beliefs and attitudes as she moved to a settled place trusting the researcher and the climate that had been built. She became willing to enact on these and professionally experiment. By the end of the study she was also willing to share her work believing in herself and that it would be of value to others.

CHAPTER 9: ANALYSIS OF FINDINGS

9.0 Introduction

This study set out to design and implement a programme of professional development that would support teachers change their practice to enhance teaching for understanding. All teachers experimented and changed their practice in some way. Analysis of interviews, supported by triangulated data, revealed there were several elements of the programme that supported teachers with this change. There were aspects of the initial design, including the sustained focus on the CCC Model and the chance to collaborate with peers. There were also themes that arose that were important to the change process, including trust between colleagues and an accountability to the profession and to whole school action research. The analysis of these areas is presented in relevant sections below.

Chapter 3 provided the analytical framework of the Professional Mathematical Growth Model (Figure 14) which was constructed from reviewing literature. This chapter presents the discussion that identifies the mechanisms by which teachers have developed their practice over the study within relevant sections of the framework. The overall change process is explored at the end.

9.1 External domain

Figure 14 suggested that input by the external domain can be an initial trigger for teachers to develop their own mathematics knowledge for teaching. Within this study, one element of the external domain was to share research and to encourage teachers to use research within their practice. The other element was for the designed CPD to be inspiring so that teachers would want to engage.

9.1.1 Academic literature

Over the course of the study it has been shown that teachers within the department have increased the amount of formal and informal research. However, conclusions cannot be drawn about whether this was due to the implementation of the CPD programme as the professional change environment also altered, with the unplanned factors of the whole school focus on action research supporting the movement of teachers in this direction. This is reflected in a comment from the head of department; ‘we were definitely trying to start it here first, in mathematics, but I think that Nic has probably been helped a little bit by the school saying that you have got to do a project’ (Charlotte, 2016, Final interview).

Increased discussions about literature

The level of discussion within the department about research increased across the study and teachers presented to each other with confidence and many included specific references to academic reading. Within the department there was momentum to move forwards with academic research 'I think it gave me that, I had been trying to do my masters for a long time and kept coming up against it as I couldn't find the time, and I think that it gave me that momentum again, yes there is a need to do this, I did need to carry this on, I do need to do it' (Charlotte, 2016, Final interview).

Whilst the trend was for teachers to be moving and continue moving towards the transformation phase of the TDM it is important to note that confidence on this journey was restricted to the personal areas that had been studied and transformation using research is not necessarily embedded in other mathematical topics or pedagogic areas.

Validity

There was the feeling when comparing the CPD programme from this study with others, that teachers had experienced previously at college, it had more validity. 'For me as a mathematician and a maths teacher what we are doing in maths seems to be more, it has got more validity' (Heidi, 2015, Mid-study interview). This supports the findings from De Geest (2010) that research can give status and credibility to the CPD initiative.

9.1.2 PD input

Chapter 3 answered the question ‘what factors will contribute to an effective professional development programme’ then Section 4.2 drew on these factors and the CPD was designed for this study. Section 4.4 described the phases that would be incorporated into the CPD programme. The mid-study interviews gave teachers a chance to reflect on the research presentation (covered in Section 8.1) and on the activities shared as part of the bridging the CCC Model to practice phase. Reflecting on De Geest’s (2011) study of successful professional development of mathematics teachers (discussed in Section 3.6.2) there were several categories from her research evident when analysing the interviews from teachers within this study.

Challenging, interesting and inspired them with new ideas

Table 2 showed that teachers ‘liked the different approaches’ (Elliot), it opened eyes to different ways of introducing topics (Brian, Daniel) and ‘changed the way I teach certain topics’ (Jenny). There was a general positive atmosphere ‘it was both exciting and inspiring’ (Georgie). Elliot commented ‘it did make me think again’ and Charlotte referred to an activity looking at similar triangles and Pythagoras theorem ‘for me that was one of the first times I have done proportion in that kind of way’ (2014, Mid-study interviews).

They knew how to put the theory in classrooms with concrete examples

It was shown in Table 7 the CPD was effective in supporting all teachers to make sense of the academic literature presentation, for reasons such as it gave them different methods and the visual images were useful. Daniel commented ‘to see it presented in that way, I could see how it would work for a cohort of students’ (2014, Mid-study interview). Section 7.8.3 highlighted the effectiveness of the completing the square task and the parallelograms task and later in the study the Play-Doh activity and the GeoGebra transformation tasks were a favourite for many. ‘I remember the task about squaring and I had never seen it before and I found it interesting, well as I said before I found this was fantastic (referring to Play-Doh task on handout)’ (Heidi, 2014, Mid-study interview).

The study provides additional evidence supporting the NCETM (2009), Section 3.6.4, that teachers valued practical advice that was directly applicable to the classroom and valued stimulating, enjoyable and challenging CPD. Teachers enjoyed the opportunity to work together ‘everyone gets enthused’ (Charlotte, 2014, Mid-study interview). They liked exploring mathematics and developing their practice ‘it was a different approach and, in some ways, an inspiring approach’ (Daniel, 2014, Mid-study interview). ‘It is good because you get bored of teaching the same things the same way year in year out so actually it is really refreshing to think about how else can I do this and not doing it the same old way that I have been doing it’ (Elliot, 2015, Mid-study interview).

Sustained nature of the PD input

The regular updates and the sustained nature of the CPD were seen to be positive in supporting change.

> 'Previously it tended to be going out and doing a day course and then actually putting it into practice when you come back doesn't always happen whereas if she is here with us doing it on a long term, you are more likely to eventually get into doing some of the things. It has to become part of your normal practice, doesn't it?' (Annette, 2015, Mid-study interview).

Charlotte commented on having 'the time to introduce it in the first place, the time to think about it, the time for them to go and try things' and that 'the more we do it the more natural it will become' (2016, Final interview).

9.1.3 Reflections on external domain

The external domain can be seen to trigger thoughts and discussions across colleagues, some of these were prompted by literature that had been provided or by activities and discussions with the researcher in their role as PD provider. Throughout the CPD process teachers had repeatedly gone back to the external domain in some cases for clarification or for 'reassurance' they were on the 'right track'. The ongoing availability of the external domain, as the researcher was situated in school, was found to be a significant feature in this study.

The NCETM (2009) RECME study, based on a wide range of well-established literature, recommended further research was needed into the kind of research that is used in CPD, the way it is used and the effects this has on the professional development of the teachers.

This case study has shown teachers are willing to and can engage with research but collaborative working is important. Teachers were able to engage rather than shy away from academic literature with the researcher facilitating understanding where teachers struggled to do this independently. There is potential for continued impact on classroom practice as teachers now acknowledge and recognise the importance of the use of academic literature. In this study the external domain was critical and provided ‘new ideas’ that teachers would not have thought of themselves. The use of academic literature was also seen to raise the credibility and professionalism of the CPD.

9.2 Mathematics knowledge for teaching

Section 4.2 highlighted one of the principles of the CPD was to develop teachers’ mathematics knowledge for teaching through collaborative working. The area of MKT being developed was getting teachers to make connections. This can be thought of in two ways; firstly, which approaches engaged teachers themselves to make connections within their own MKT and secondly which ones were then used with students to develop connections within the classroom i.e. the teachers first had to collectively understand in action for them to be able to carry out action. The original research question asked which approaches engaged teachers with students’ development of connections. During the bridging theory to practice phase of the CPD programme (Appendix 2.2) there were several tasks that engaged teachers with making connections themselves.

9.2.1 Completing the square with multiple representations

Early interviews, detailed in Section 7.8.3, showed teachers were ‘engaged’ with the completing the square task. Most were already comfortable with using area representations for basic algebra and this was a natural extension for them. Elliot saying ‘I thought that was the most useful showing that if they have a good understanding of the grid then you can use that in a wide range of other topics’ (2014, Mid-study interview). It also appeared the completing the square task was new MKT for them ‘I would never have thought about coming up with an activity like that’ (Georgie, 2014, Mid-study interview) and ‘I then understood how to complete the square and how it all linked together’ (Jenny, 2014, Mid-study interview). Charlotte (2014, Mid-study interview) commented ‘that made me think about how I teach that’. This task was an example of teachers understanding in action, where the activity of them taking part socially supported the cognitive development of collective knowledge.

9.2.2 Developing conceptual understanding of volumes of prisms

The next task that had the greatest enthusiasm from the department was the Play-Doh task as described in the Mathematics in School journal article (Trubridge, 2015). Section 7.8.4 showed five teachers at that early stage using the activity because it was ‘different, inspiring or exciting’. There was also collective realisation that spending time teaching the topic well would mean less time revisiting it as later stages in the course. This resulted in the teachers questioning the teaching practices they had been acculturated into as they engaged themselves with mathematical activity.

9.2.3 Invariant lines and points

Teachers within the social environment of faculty meetings, generated new knowledge when considering the topic of transformations (detailed in Section 7.9). The strategy of 'what's the same and what's different' combined with GeoGebra as a dynamic tool enabled teachers as learners themselves to build on their own knowledge of transformations and to extend to a more conceptual understanding of invariance as they explored at a level beyond where they were teaching themselves.

9.2.4 Reflections on developing mathematics knowledge for teaching

The strategy of 'what's the same and what's different' had been modelled to teachers by the researcher in different situations. When experiencing the strategy as learners themselves, exploring transformations, they had experienced first-hand how it could generate productive social learning and a growth of new knowledge. In a similar way, when the completing the square task was used collaboratively with the faculty their own understanding deepened through social interaction.

In Section 3.7 it was noted, that most studies of mathematics teachers focused on category-based perspectives based on content knowledge and pedagogical content knowledge (Chapman, 2015). This study has added new findings from a different perspective, within the context of a faculty working together over several years to develop their shared understanding of what teaching for understanding might entail.

There are many elements of MKT that teachers developed over the CPD programme. Referring to Figure 13 from Chapter 3, some teachers developed their own subject matter knowledge as they explored invariance leading to transformation of matrices which was an unfamiliar topic. For most the concept of completing the square was not new but the PCK of how it could be taught using multiple representations was seen to be a revelation. Findings showed a greater depth in teachers thinking about how the curriculum could be organised to enable teaching of volumes of prisms to have more meaning with their learners. When teachers themselves learnt new subject content knowledge they then experienced a frustration if they couldn't 'impart' it quickly with their groups of learners. This frustration was also felt when pressure of examinations loomed, and teachers reverted to more procedural methods.

Whilst part of this study hoped teachers would develop cognitive knowledge themselves the important element was for teachers to change their practice in the context of their teaching, so most of the analysis is referred to within the domains of practice section.

9.3 Domains of practice – enabling factors

Figure 14 proposed that within the Professional Mathematical Growth Model there are two distinct practice domains. The first where teachers were able to experiment by either implementing their changed MKT using resources and ideas shared from the external domain. The second domain of practice focusses of teachers moving away from the reliance of the external domain and becoming more independent as they chose which aspects to take further and change over a sustained period. This section presents the analysis of data that moved teachers along the journey from guided to independent experimentation.

Chapter 8 showed that, of the ideas shared within the CCC Model, the use of multiple representations, similarities and differences and making links between procedures and concepts had the biggest impact.

9.3.1 Why was multiple representations embedded?

Early interviews show some teachers were already familiar with using multiple representations particularly the use of the area method to help explain multiplying algebraic expressions. As they understood this already, extending these ideas to activities such as completing the square or using visual methods for proof could be seen to be natural growth. This is a topic area with readily available resources for people to use, Frazer drew on resources from the FMSP and Kate from research papers, these were adapted for use in the classroom. Kate acknowledged she was already familiar with using multiple representations ‘so they came a bit more naturally for me to stretch myself to prepare more things’ (2016, Final interview).

9.3.2 Why was similarities and differences embedded?

There were perhaps a couple of reasons why this aspect was so successful with the department. It was seen to be an ‘easy’ thing to implement that didn’t take much time to plan and the ‘value added’ relative to the time to plan was great.

> ‘It took five minutes to set up with the equations where you had to think a little bit carefully but then it is a good hour’s lesson of good discussion going on between the students to find out what is actually going on.’ (Charlotte, Mid-study interview)

Daniel also commented on the importance of the strategy enabling learners to generate ideas, so they derived knowledge themselves giving them a deeper understanding of the concepts being covered and on the need of ‘getting those really good leading questions that you can bring into maths to start churning out those really good discussions’ (2015, Mid-study interview).

This notion of learners generating new knowledge themselves was echoed by Charlotte (2016, Final interview) ‘when I do it and when they look at the differences and similarities I sort of point out look I have taught you nothing and you have worked it all out. I suppose if I do that more often they will be more inclined to take that on themselves’.

There was another unplanned aspect to the research study that also moved teachers forward in the aspect of similarities and differences. The slide in Appendix Figure 18 was shown in a whole staff training evening (September 2015) which gave additional evidence from another source that similarities and differences would have a greater effect on gains in learning than areas such as repetition and practice or cooperative learning. This was after the department had moved to the active experimentation phase of the CPD programme and reassured Daniel and Elliot they were doing the 'right thing'.

> 'I was really, surprised, well not surprised, but happy to see the similarities and differences when the assistant principal did his presentation and saw that it was a 1.6 effect size, …….it was good to see that using similarities and differences, which is something that we all have embraced as a department, has such a big effect in the development and progression in understanding of students, so that was really good to see.' (Elliot, 2015, Mid-study interview)

Daniel (2015, Mid-study interview) also referred to the same slide being shown and commented 'I think I was always going to try and extend what I had started last year but I thought if that is going to have that effect size then that is something I really do want to discover a bit more about'. He then went on to explore this further for his action research CPD project 'it was all kicked off by Marzano's research on effect sizes knowing that had a huge effect size if you get it right. That's why I wanted to investigate it really' (2016, Final interview).

9.3.3 Why was progress made linking procedures and concepts?

Heidi transformed her practice making links between procedures and concepts when considering operations with fractions. This was due to her initially recognising there were gaps in student understanding and trying to think how she could address them. Heidi's progress in this area was largely due to extensive research looking at literature and other people's ideas to help formulate ways to use visual representations to support the development of fractions procedures from concepts.

9.3.4 Reflections on the domains of practice

Whilst Sections 9.3.1 - 9.3.3 refer to specific aspects of the CCC Model there are some generic enabling factors that supported teachers whilst working within the two practice domains.

Resources to use in the classroom

In the early days, teachers valued practical resources they could just pick and up use. Some of these were provided by the researcher: 'the GeoGebra ones, they are good I have used those with higher groups' (Brian, 2014, Mid-study interview); 'there are a lot of algebraic examples which hopefully are going to be useful' (Frazer, 2014, Mid-study interview); and 'being able to have the resources in front of me' (Georgie, 2014, Mid-study interview) was an incentive to try them.

As the study progressed some teachers continued to use resources that were developed by the researcher 'because Nic set up the GeoGebra programmes, I have used those…. it was there and easily accessible and I used it' (Annette, 2016, Final interview). Whereas others found resources they recognised followed the principles of the CCC Model. Frazer used a number of multiple representation tasks from the FMSP 'I loved the resources from the FMSP they were super. Brilliant, I have used a lot of that stuff, actually. I think they have put together some really good thinking type tasks' (2016, Final interview) and Louise had 'used different resources like the standard unit ones with the cards sorts and stuff in there because most of the activities generally link in with this' (2016, Final interview).

Towards the end of the study teachers were also using each other's resources that had been shared at department presentations. Daniel commented on using Louise's place value ideas and Charlotte using Heidi's fractions resources.

CPD in own context

Another factor that supported change was the importance of the CPD happening within their own school context. Brian commented 'I saw the way Nic was doing things and her carousels and ways of not giving solutions or black or white answers just answering a question with a question' (2014, Mid-study interview). Louise also commented on being able to observe 'I think, observing other teachers like Nic or Charlotte the way that they do things, or the way they present ideas, I think I like that, or I really like that and how can I use that' (2016, Final interview). These two teachers had been training throughout this case study, so observation and reflection was part of this which they valued.

However, in addition, although Georgie didn't observe these lessons first hand she was still able to reflect that these ways of working could work within the school context with students that she knew.

> 'Being able to sit down and have informal conversations at break times and lunch times about this student did this, and this student did that and because you know who they are you think oh my God when I taught them they would never have been able to do that, being able to see that is really quite nice. Um, and so, like I say, being told this will work is one thing, seeing it for yourself and doing it for yourself and seeing it for yourself through the activities was brilliant and having the confidence to be able to say yes, I can do this, it is not rocket science it makes sense.' (Georgie, 2014, Mid-study interview)

She also commented 'seeing it work with the results that they get that really does cement that this is what we should be doing' (2014, Mid-study interview). These findings echo what was stated in Section 3.6.1 where Cordingley *et al.* (2005, p. 11) put forward 'most of the effective CPD in the research included learning which took place in the teachers' own schools and classrooms'.

Role of subject mentor

Alongside the resources provided and seeing things working in the school context, many teachers came to the researcher, the external domain, throughout the programme for either guidance or reassurance. Annette acknowledged that she had gone to the researcher to bounce ideas and for support with her Open University course 'Nic will know about that anyway as she helped me with some of the work that I had to do' (2016, Final interview). Louise commented 'Nic is probably the one person that I go to' (2016, Final interview) and Georgie 'having Nic in the department to sort of say that this is the way I do it' has helped to share ideas and provide support on how to teach topics' (2014, Mid-study interview).

Pedagogic transferability

Where teachers had moved to the independent practice domain they were able to transfer the associated pedagogy to new topics or new classes. Whether this was the use of an image to support making connections or the use of 'what's the same, what's different' as a tool to construct knowledge through social interaction.

9.4 Student outcomes

Whilst this study focussed on teacher's practice and didn't analyse student feedback, there were many instances when teachers referred to what they had observed in their lessons when they had implemented new ideas/strategies.

9.4.1 Motivation to change

Several teachers trialled resources and became motivated by the responses they were seeing from their learners. Georgie and Heidi refer to the 'lightbulb' moment, Ian the 'click' and Frazer the 'eureka' moment demonstrating that they could see a change in students understanding when engaging with tasks that followed the principles of the CCC Model. Table 45 shows quotes to evidence this.

Teacher	Quotes about student responses	Evidence source
Charlotte	I can see it in the lessons the students getting more engagement out of it and getting more fun out of it.	2014, Mid-study interview
Daniel	I used the Play-Doh as I thought wow that’s a good idea. I used it immediately with a year 7 group, in terms of them getting some understanding from it, it was quite good.	2014, Mid-study interview
Frazer	I did a Pythagorean proof with my top set year ten, it wasn't one of these but same ideas, but it was surprisingly useful in how it worked and did grab from the lowest ability. The Eureka moments were evident and lovely. It was a very simple chopped up square and moving a couple of triangles around and their understanding of proving Pythagoras was superb and we went on to prove algebraically from that and there was no difficulty in knowing why the geometric proof had worked and that it could be transformed into an algebraic proof.	2014, Mid-study interview
Georgie	Seeing the way that the students made the connections after I had made the connections in a similar way was inspiring. It was brilliant.	2014, Mid-study interview
Heidi	I did this activity and (refers to Play-Doh task on handout) I found that my year 11s got the concept that what we were looking for was a constant cross section. I was afraid that they might be distracted by the Play-Doh, but they stayed on task fully for the whole lesson and they were talking amongst themselves so working out things.	2014, Mid-study interview
Ian	And then we did it in area of circles and getting them to see they were doing it again and there was a click.	2014, Mid-study interview
Heidi	Initially we worked out the sequence and then showed the general idea for linear sequences and they quite quickly came up with, I don’t know I was thinking about it later on what helped, I was thinking after the lessons it was really the moment from literally they were writing all the sequences and looking at the differences thinking they had 15 different categories and then then literally there was a light bulb moment where they recognised that this goes down by the same amount and there was one with decimals but the difference is going down by 0.3 so I can still put them in the same category. So, I really don’t know, but there was literally like this (clicks fingers) I didn’t tell them they literally said they are the same but square and cubed numbers they realised they were different but then said they could be put together because they were using powers.	2015, Mid-study interview
Ian	Particularly in comparing differences and similarities it does make students think quite a lot and see the connections hopefully as well.	2015, Mid-study interview
Georgie	Seeing the students and how they react to different ways of teaching. Suddenly a student becomes excited about what they are doing, and they want to move forwards and I think that inspires you as well. That inspires you to want to improve and change the way you are teaching. Also, the lightbulb moment when you see it working. When you see them excited to come to lessons.	2016, Final interview

Table 45. Quotes about student responses

Teacher	Quotes about student responses	Evidence source
Kate	I have seen a change in me and in the students and I want to keep doing it. I have found it particularly useful I must say with lower ability; I have found their engagement is so much better because they understand it. I had a low ability group of year 11 last year and I put up a what's the same and what's different for prisms just putting pictures up and they ended up having a 20-minute discussion with me about, corners, and we talked about proper terminology, but for them it was amazing they were talking about the number of faces, number of edges and then they looked at the fact that some of them were regular and some of them weren't so we talked about that. They talked about parallel lines and perpendicular lines and the amount of stuff they came out with was incredible and because they just bounced off each other, it was probably one of the best lessons I have had with them.	2016, Final interview

Table 45 continued. Quotes about student responses

There were also comments about students being 'engaged' (Charlotte, Kate), 'staying on task' (Heidi) and becoming 'excited' (Georgie). With Kate saying, 'I have found their engagement is so much better because they understand it' (2016, Final interview) citing an example of how a what's the same and what's different task led to a 20-minute discussion. This observed change in their learners has led to teachers to want to continue to change their practice 'I have seen a change in me and in the students and I want to keep doing it' (Kate, 2016, Final interview) 'it inspires you to want to improve and change the way you are teaching' (Georgie, 2016, Final interview).

9.4.2 Resistance to change

On the opposite side, there are several teachers that acknowledged that students were reluctant at times to engage with this perceived different way of thinking and working. In the final interviews, Heidi said that students have a 'perception of what mathematics is and what they have to do in a mathematics lesson …. for them mathematics is working with the numbers and doing calculations so for example, I did geometrical proof today and one of them said it is not English why are we writing so much, so they have preconceived ideas on what mathematics looks like and can be reluctant to engage' (2016, Final interview).

This lack of engagement by students was also highlighted as frustration by Charlotte 'some students want to be shown how to do it and they have got that argument against what you are doing all of the time because they just want to do it and get on with it and actually not understand what they are doing' which was making her question whether her effort was useful 'I think some students make you question whether it is the right thing, there are times when students test you and you stand back and think I am not going to do it for you, you have got to come up with it yourself' (2014, Mid-study interview).

A similar response from Daniel who also voiced concerns about his students' attitude to understanding mathematics 'their level of understanding comes from almost a need to please me rather than a desire to learn intrinsically themselves which is very temporal. I would love to work with students that were much more proactive in what they want to learn and therefore understand relationships about mathematical concepts and why they work and what it can lead to' (2014, Mid-study interview). Later in the same interview this was raised again 'it is frustrating that I can't have that depth of understanding with the students for one of two reasons they either don't want to or are not cognitively able to' (Daniel, 2014, Mid-study interview). Heidi commented, 'for some students it is not a natural way of thinking and you have got to get them thinking in that way so that is a barrier' (2015, Mid-study interview).

9.4.3 Reflections on student outcomes

The analysis of data from this study also agrees with the findings from Joubert and Sutherland (2008, p. 13) that were put forward in Section 3.3.2 that 'it is not the professional development itself that provokes change, but the experience of successful implementation of change that will lead to changes in teachers' knowledge and beliefs'. The findings from this study provide extra evidence to support Guskey's (2002) model of teacher change (shown in Figure 11) showing that the observed improvements in the learning of their students led to teachers developing positive attitudes to the implementation of the CCC Model. However, it can also be seen that the change is not linear, the process is more iterative as shown in Figure 14. The change in student outcomes was a catalyst for further independent practice which when sustained over time changed teachers' beliefs, attitudes and motivation.

9.5 Personal domain

Central within the model in Figure 14 is the knowledge, beliefs, attitudes and motivation of the teachers involved within the study. This section details the motivating factors and the beliefs and attitudinal responses that were observed within the study.

9.5.1 Motivation factors

In Section 3.3.2 possible factors that would motivate teachers were stated from Dean (1991). Some of these were evident within this study.

Teacher belief - professionalism to keep learning and doing the best

For a few teachers within the study it was clear they were motivated to develop their knowledge to support learners and this would have happened whether this case study and CPD programme had occurred or not. They had rooted within them a professionalism to keep learning. Annette was engaged with an Open University course and as such engaged in academic reading 'I am in the middle of reading The Elephant in the Classroom by Jo Boaler' (2015b, Mid-study interview). Similarly, 'personally, I would be doing it anyway' (Charlotte, 2015, Mid-study interview) and 'it doesn't matter whether it is directed time or not because I would be doing it in my own time anyway' (Heidi, 2015, Mid-study interview). Brian also acknowledged that he didn't want to stagnate as a teacher 'I want to develop and grow, I am a professional person' (2015, Mid-study interview). The rationale about why people enter the profession was commented on by Kate:

> 'Well that's the root of why we go into teaching isn't it. I do want to give the students the best experience that I can, and I think the bottom line is, I have always tried to do that and that hasn't changed for me no matter how frustrating your day is. This is why I am in the job. I want to teach them, and I want to do a good job.' (Kate, 2016, Final interview)

Frazer also commented that ‘I always want to feel that I can walk out of a lesson feeling like it was a good lesson and that children have made progress’ (2016, Final interview), keeping learners at the heart of what happens each day.

For these teachers, the moral commitment and professional obligation as quoted in Section 3.1 from Eraut (1995) was evident within the analysis of the study.

Teacher beliefs – principles of mathematics teaching

Some teachers were quite passionate in their own beliefs about what skills they hoped to develop in their classrooms. Annette for example referred to ‘reasoning is really important’ and ‘if I can find practical problems then I do’ (2015, Mid-study interview). She commented specifically on the importance of her developing confidence in her intervention sessions:

> ‘My underlying principle, certainly with the after-school work and catch up stuff has been to increase the students’ confidence with their mathematics. The levels and grades they get at the end I am not that bothered about, I have to be, because we have to, but for me it is more important that they are actually more comfortable in mathematics lessons.’ (Annette, 2016, Final interview)

When asked what prompts her to change practice she replied ‘to make more enjoyable lessons’ (Annette, 2016, Final interview), which is a belief also held firmly by Frazer ‘students need to enjoy their time learning and it doesn’t matter what subject they are doing, what task they are doing they need to be happy. Happy children learn, if they are not happy they are not going to be interested’

(2016, Final interview). So, if activities developed confidence or made lessons more enjoyable, resulting in happy children Frazer and Annette bought into them.

Teachers beliefs - align with CCC Model

The beliefs of the head of department aligned with the CCC Model so she was supportive and motivated to engage the department throughout the process. There was a legacy of previous ways of working that she was keen to change.

> ‘Departmentally I think we need to spend some time together going through the scheme of learning, picking out keys topics that are perhaps taught in a manner that doesn't help understanding like negative numbers, which some of the department do, we hear it all the time like 2 negatives make a positive getting rid of those comments would be nice.’ (Charlotte, 2014, Mid-study interview)

Throughout the study she expressed on numerous occasions to the interviewer ‘I wouldn’t be encouraging her if I didn’t believe in what she was doing. I think it is important’ (Charlotte, 2015, Mid-study interview) she acknowledged results were important but ‘it is not just about results it is about making the students be prepared for their future and being good all-around thinkers’ (Charlotte, 2016, Final interview). Having the support of the head of department was a vital area in being able to support the department on their development journey.

Addressing gaps in understanding

Heidi reflected that previously her students could not identify prisms and ‘they were jumbling everything from spheres to square based pyramids still thinking they were all prisms’ (2014, Mid-study interview) and ‘we are teaching fractions, but students did not understand why the multiplication and division of fractions work the way that they do' (2016, Final interview). These are examples where Heidi was motivated to try something new because she recognised what had been done before wasn't effective. Ian was also keen to work on fractions because 'it's amazing how many of the kids I teach, you say you want a quarter, how do you find a quarter? And they don't really understand the concept of a quarter…. there is still a bit of spark missing somewhere' (2015, Mid-study interview).

Circumstances changed so could focus on teaching

In the case of Kate and Daniel it could be seen that a change of circumstances to lifestyle and work had enabled them to have more time to think about their practice. Daniel commented on a change in lifestyle since giving up training for sports each week so now 'having more time to reflect on what I am doing' (2016, Final interview). Whereas Kate when moving schools had changed from being the head of department to being a part time teacher 'since moving here, I am just focusing on my teaching so I have got more time within that to actually think ….. I have found it refreshing to be able to concentrate on my teaching and actually seeing the change in what the children can do when you actually take this on board is quite something' (2016, Final interview).

Aspirational

One final motivating factor was identified only by Georgie, (2016, Final interview), but perhaps significant to her as it was mentioned in some way six times. She said, ‘I am really proud to be part of this department’ and ‘when we were in a whole staff meeting and we were being held up as the department that had done particularly well with our presentations that felt nice but it didn’t feel fake, it felt like we had done it because we wanted to do it and we want to work together as a team to improve but like I say it is an aspirational department’. Her motivation to keep working was she didn’t want to be left behind ‘I don’t want to feel like I am standing still because everybody is making progress, and everybody is doing this that and the other, oh I have tried this, and it makes you want to be a part of it. It makes you want to succeed and to learn from everyday experiences in the classroom and how to improve in the classroom’. She has been inspired by the department and ‘you aspire to be like the people that are doing it right in my mind’.

9.5.2 Personal constraints

There were some factors outside of work and education that could be seen to be personal barriers for teachers not being able to or not seeing it as a highest priority to engage with CPD.

Family commitments

There were several comments relating to family coming first and the difficulties of juggling work and home commitments. Three teachers referred to their young families as to reasons for finding it difficult to dedicate time to research and development. Georgie said, ‘I started my masters before my child was born’ and that she hadn’t time to return to it (2016, Final interviews). In a similar vein from

Elliot, ‘my issue with that is if I go home I have the kids to look after’ (2015, Mid-study interview) and ‘at the moment when I am driving into school I am thinking about the kids and what they have done and about what I need to do or remember from the shops and that used to be what am I am going to teach and how am I going to teach and things like that’ (2016, Final interview). Kate also felt that school work was difficult to do at home ‘at home I have got two small children so spending time research just wouldn’t happen’ (2016, Final interview).

End of career

Another barrier to dedicating time to CPD is the incentive to keep developing at the end of a career ‘it’s a bit tricky as I am nearly at the end of the road …as I have only got another couple of years it would be easy just to turn off and keep on going as you were’ (Ian, 2015, Mid-study interview). Also, from Annette ‘I don’t actually know how much longer I am going to be here which would be another barrier. The incentive to try new things suddenly stops, that’s only the past few months that I have been thinking that’ (2016, Final interviews).

9.5.3 Attitudinal barriers

Zimmerman (2006) suggested that one attitudinal barrier is the fear of change and acknowledge that it can be stressful for someone to step out of their comfort zone and make changes to what they are accustomed. There were three teachers within the study that exhibited signs of an attitudinal barrier at some point. During the pilot study Annette commented during the card sorting task ‘none of us like being put on the spot, being out of comfort zone' (Annette, Pilot study CPD session). Elliot referred to this in general in the initial interviews when talking about CPD that he had previously been involved with:

> 'It does change the way that I do things in the classroom but not dramatically. I think for me I get into my routine and it is quite hard to break that, so it is a real effort for me to make that change but when it does it becomes quite natural and it works quite well. I am quite happy to sort of change the way that I do things to suit the way that the school do things but for me that is a lot of effort.' (Elliot, Initial interview)

He was then questioned further by the interviewer about whether he was reluctant to change, and his response was:

> 'I wouldn't say reluctant, I am willing to change but I get into my habit so my lessons go a certain way and I quite like the way that they go and all the rest of it and if I need to remember to point out the PLTS skills or independent learners or something like that I forget because that is not part of the routine of the classroom, it is not that I am being reluctant just forget.' (Elliot, Initial interview)

These comments about working within comfort zone and reverting to normal practice were also referred to by Annette (2016, Final interview) 'It is easier to revert to what you already know works and Daniel (2016, Final interview) 'perhaps I go back to, slip back to my comfort zone of what I have done before rather than taking a risk and moving on'.

9.5.4 Teacher belief barriers

There were some beliefs that teachers portrayed that were barriers to change in practice when implementing the CCC Model.

Teacher beliefs about learners

A barrier to engaging with the CCC Model could be attributed to teachers underlying beliefs about their learners. Throughout the process Brian referred to lower attaining students that needed to think in a procedural way. In his initial interview, he said ‘there are people who have the intellect to do these things and they are in the higher sets and there are others that will struggle. Some you may never unlock that door’. These types of comments were repeated in a similar way in later interviews.

> ‘Your lower level thinkers are more capable of the procedural end and less able to think conceptually, however the reverse view is that we would like them to think conceptually because that allows them to access the higher levels of thinking.’ (Brian, 2014, Mid-study interview)

When asked about whether the CCC Model was useful the response was ‘particularly for the more able groups - those that are more capable of conceptual thinking’ (Brian, 2014, Mid-study interviews). On the other hand, Jenny recognised the importance of lower attaining students needing to conceptualise basic skills ‘I think with the bottom set year 7, there are certain things that they need to have conceptualised like the rounding on a number line, I think that is quite important just for their basic understanding of decimals and just general number and the same with teaching them fraction' (2014, Mid-study interview).

During the study, the researcher and the head of department had many conversations about how to encourage more students to think in a conceptual way. The head of department acknowledged their interest in developing a growth mindset with students and felt that one barrier to this was teachers themselves within the department having a fixed mindset of what students could achieve. It was hypothesised that recent learning developments of neurological science referred to in Section 2.2.4 were evident with teachers providing the limiting pupil-theories and pupil-beliefs.

No desire to improve in this area

One potential reason for not engaging with the CCC Model would be teachers didn't want to or didn't agree with the principles of it. However due to the accountable nature of the action research and the wide number of aspects that could be explored this didn't come across. There were two things that did arise, one being whether aspects of the CCC Model were useful (Brian not believing in multiple representations, Section 8.4.1) and the second being a personal desire to improve in other areas of practice.

Wanting to explore other research projects

There were a couple of teachers that engaged with the CCC Model but also explored other areas they were passionate about themselves. Charlotte successfully completed her own masters qualification and Heidi also continued to explore problem-solving strategies and proof. Both teachers were ‘on board’ with the CCC Model however, it could be seen that they divided their time between the different areas.

9.5.5 Reflections on the personal domain

Whilst PD can be designed with the aim of developing mathematics knowledge for teaching at a personal level, the intrinsic motivation from within the personal domain is likely to be informed by prior experiences. Although creating a culture that supports each individual there will be constraints and barriers outside of school that will be outside of the control of the PD provider.

This case study has shown that teachers develop at different rates. Often there is a culture where teacher practice is assessed by school leadership teams, within this longitudinal study teachers have become more aware of what they are doing and more reflective on their practice. Through the exposure of academic literature and external ideas they developed a realisation of the need to change for themselves rather than having strategies imposed on them.

9.6 Change environment – enabling enactment

Clark and Hollingsworth position their ‘Interconnected model of professional growth’ within ‘The Change Environment’ (Figure 12) whereas within this study the Professional Mathematical Growth Model (Figure 14) is situated in its entirety within a change environment that is both social and professional. This section presents analysis from the study relevant to the environment that surrounded the professional growth of the mathematics faculty.

9.6.1 Departmental enabling factors

There were several elements at departmental level that supported the process of change when engaging with CPD. Teachers enjoyed and benefited from collaborative exploration. This was achieved by changing the culture to one of professional learning built on mutual respect and trust. This resulted in a momentum to work together on a journey that would continue after the case study had finished.

Collaborative exploration

There were many quotes throughout the study that recognised how teachers valued the opportunity to collaborate. Table 46 shows some of these.

Teacher	Quotes about collaboration	Evidence
Daniel	I really enjoyed the collaborative element of sharing the examples.	2014, Mid-study interview
Georgie	And I have never been an advocate of tips and tricks but being able to discuss it with other people and having these ideas about where things come from and how you can introduce it to students is really helpful.	2014, Mid-study interview
Annette	I think it's great because it's good to work as a team doing stuff anyway and so that we are all moving in the same sort of direction.	2015, Mid-study interview
Elliot	It is the planning together that is really important, just to sort of get an idea as to how other teachers are doing it and perhaps sort of sharing ideas and things like that.	2015, Mid-study interview
Charlotte	I think having the opportunity to bounce ideas off other people, sometimes someone will give you an idea and then you will roll with it and make it better as well.	2016, Final interview
Daniel	I think the conversations, what this has done for me is, the learning conversations that I have with staff are very much more focused.	2016, Final interview
Elliot	Just having those conversations either formally or informally with you and other people in the department has been really powerful to look at an approach has been really useful. If you were just trying to get me to do it I think it would be really difficult as I would only have one person to bounce off and I think you need lots of different minds to bounce off.	2016, Final interview
Louise	I like that idea of being able to share what I think and what I feel about teaching. I think it is important to share ideas, even if I am new, there are still might be that I have used a different approach and they have not.	2016, Final interview

Table 46. Quotes about opportunities to collaborate

In addition, during the final interview, teachers were asked where they went when they wanted to develop their practice. Charlotte and Frazer mention using websites and resources but everyone else referred to going to each other. Whilst some single out that they would talk to the researcher in the first instance there are others that reflect that they would go to any of the department as shown in Table 47.

Teacher	Quotes about where you go to develop practice
Annette	I have a host of colleagues that I would go to depending on what it was particularly, I would ask any of them.
Brian	Colleagues as a starting point. I talk to Nic and Charlotte a lot about different ways and different things.
Daniel	It is all too straightforward to say if I wanted to do something mathematically, to develop my understanding of mathematics I would go to Nic, that is all too easy to say, but I think the culture that is developing in the department is that I feel comfortable going to anyone in the mathematics department and say how do you do this. I think I need to change the way I do this have you seen anything, or I would feel comfortable discussing it with any member of staff.
Elliot	Well Nic obviously, Kate has been quite good, um, Heidi and Charlotte have been quite good.
Georgie	Colleagues, absolutely colleagues.
Heidi	Well first obviously other members of the department looking at the resources that we have already, but I am also using a lot of my own stuff.
Kate	Probably I would talk to people, spending time research just wouldn't happen so yes, I would always talk to people and I think seeing it in action is always good. Well I think they are all brilliant, I talk to Charlotte quite a lot, Heidi and Nic too. I have seen Brian teach and Louise teach so that is always helpful.
Louise	Nic is probably the one person that I go to.

Table 47. Quotes about where you go to develop practice

Charlotte felt that 'because we have had a common theme that we keep coming back and discussing and talking about has enabled us to move forward together rather than one person on their own trying to change one little thing. It is harder to change on your own than working together as a team. I think that has probably helped a lot of people' (2016, Final interview).

Professional relationships and trust

Perhaps the reason for colleagues now looking to each other for support in developing their practice is due to the professional relationships and trust that has built up over the period of the study. King and Newmann (2000) suggest that when colleagues collaborate, professional relationships are strengthened because of shared experiences, achieving successes and working through challenges as a group. This was certainly the case within this study.

> ‘I think the department have become much more open to say when they have seen something and much more happy to say actually how do I do it the other way, can you help me do it the other way and I think it is that changing the feeling within the department that we are here to support each other and we are not here to judge each other, and I think sometimes they have felt a bit judged and didn’t feel like saying how do I do this was okay. There is more sharing I believe going on within the department and I think encouragement for that to keep happening will carry on.’ (Charlotte, 2015, Mid-study interview)

Jao and McDougall state ‘in order to engage in collaborative work, teachers must trust their colleagues and be willing to experiment with a collaborative approach’ (2016, p. 560). This trust was crucial for the change in Georgie ‘to be able to sit with a group of people that I feel comfortable with and develop my thinking and not be told, you don’t understand this and you are not getting this right, to be encouraged to explore my thinking I was able to then think okay and think I can do this and I can inspire others to do this as well’ (2014, Mid-study interview).

Momentum to go on a journey which will continue

There was evidence that the department now had momentum to work together. Annette commented there was ‘huge encouragement from the rest of the department’ (2015, Mid-study interview) and from Charlotte ‘as soon as someone sees something like this everyone goes and tries it, they do go and try it. They are a good department and want to try things and see how it works’ (2014, Mid-study interview). Later in the study she commented ‘I think hopefully we are starting fully to change the beliefs of the teachers’ (2015, Mid-study interview).

When reflecting on how CPD had changed, Elliot felt there was now a purpose to what he was doing:

> ‘It felt before like you just did CPD because that was in your performance management meetings and you would have to do something then and you might go on a course and that ticks that box. Whereas it feels like we are actually doing something for a purpose.’ (Elliot, 2015, Mid-study interview)

Similarly, Heidi commented, ‘for me personally what we are doing in maths is a little bit more of a personal journey’ (2015, Mid-study interview). Charlotte returned to study and complete her masters qualification ‘I think that it gave me that momentum again, yes there is a need to do this, I did need to carry this on’ (2016, Final interview). There were several comments about the culture ‘that has changed over the last few years a lot, it is much better than it ever used to be’ (Charlotte, 2016, Final interview).

> ‘I think Nic has been really successful with the support of the whole department in creating a cultural change and so I think ethos is fairly well embedded just because I stop having interviews with you won’t mean what is moving forwards will stop.’ (Daniel, 2016, Final interview)

This climate and ways of working were cited by Louise as reasons for wanting to focus her PGCE assignment on the use of representations in mathematics ‘I could have done it on anything …. but I chose to do a more maths focused one because of what was going on here, just because I had that support to help me and help me with the ideas’ (2016, Final interview). She commented on the positive attitude from most of the department who wanted to improve their practice and reflected on her own motivations for wanting to join the department.

> ‘I know for me that is the reason I really wanted a job here because I knew that all of this was going on in the background, so it is definitely going to help me become a better teacher. That is even what made me want to train here, I came and was like Nic is doing her PhD project and that’s really interesting, I read around it a bit and was like oh yeah that’s really

> important because I didn't want to get stuck somewhere and not making progress and to me it is about the maths.' (Louise, 2016, Final interview)

Kate, who joined the school mid-study, reflected that at her last school they did start to look at different ways of teaching 'but it wasn't quite the same about how we can improve a child's understanding, that is the big change for me it has completely shifted the way I teach' (2016, Final interview).

9.6.2 School enabling factors

Whilst this case study focuses on a mathematics department, and their engagement with a CPD programme, there were several whole school factors that supported the process of change within the mathematics department.

Culture of professional learning

It was acknowledged by Heidi, Daniel and Elliot that the school had been developing in a way that supported professional learning and the culture had changed over the years. Daniel comments 'I think the professionalism over the last 3 years has increased within the staff as well' and he acknowledged the increase in people considering doing masters qualifications (2015, Mid-study interview). Heidi was quite excited about engaging in professional learning 'I felt enthusiastic about it, I don't see it as something extra, I don't see it as, obviously it will take time, but I don't see it as a burden in other words' (2015, Mid-study interview).

Elliot acknowledged the possibility that after learning on a PGCE course ‘you go into teaching and it suddenly stops’ he commented ‘I think we should be continually reading / developing and learning because teaching is continually developing’. He felt the culture was changing so that everyone could see how they could ‘contribute to becoming an outstanding school’ (Elliot, 2015, Mid-study interview). This temptation to stand still was echoed by Daniel.

> ‘I think it’s good that you are always looking to increase your knowledge base and that no one should stand still, which is the temptation’ whilst he felt that there was already a lot to do and then ‘the action research came in on top and I thought it’s just another thing that I have got to get my head around, but then when you stop moaning about these things and you look at it and you think actually that’s not arduous its spread over the year, that’s just going to push me in a direction I wanted to go anyway but it’s just now being given that final shove to get going’ (Daniel, 2015, Mid-study interview).

At the end of the study Daniel reflected:

> ‘I think that having this new approach to CPD that we have had in the school over the last few years has been the biggest thing that has opened my eyes because you can get set in your ways and I have just found myself enjoying talking about learning a bit more, when you have courses like the OTP one that Nic was heading here just having those conversations about learning and being immersed in that a bit more has just brought it out of me.’ (Daniel, 2016, Final interview)

In this quote Daniel is referring to the Outstanding Teacher Programme (OTP) that was run by the researcher and the AST team. This programme enabled him to engage in professional learning through dialogue with other colleagues across the school. His presentation that he did, as part of this, was on the use of similarities and differences that he had taken from the CCC Model and then used in the teaching of all subjects and shared more widely as part of the OTP programme.

Balance between central control and choice

Supporting the creation of a culture of professional learning was the acknowledgement in interviews from Elliot, Heidi and Ian that the leadership team had relinquished some control which enabled teachers to choose their own areas to work on for action research projects which was valued. There were comments that this change in balance would support the transfer to becoming an outstanding school. Elliot commented ‘the principal is definitely trying to release some control of that leadership ……so we are all starting to see how we can all contribute to becoming an outstanding school’ (2015, Mid-study interview). In a similar vein ‘there is a good balance between central control and letting people do things they are interested in, which hopefully will bring outstanding performance or results for us’ (Heidi, 2015, Mid-study interview).

This element of choice was valued ‘they are allowing you to explore your own interests within the subject’ (Heidi, 2015, Mid-study interview) and ‘because it is more motivating and is something you are more interested and wanted to do, because it is something you have chosen’ (Ian, 2015, Mid-study interview).

Dedicated time for action research

The requirement for teachers to take part in their own action research wasn’t part of the original planned programme of CPD. However, during the study this became a feature at whole school level. Section 7.13.5 showed that everyone valued the fact time was dedicated to work on action research for example ‘I think it is good that directed time is allocated… I think we should be given the time to do this because it is important for our CPD’ (Elliot, 2015, Mid-study interview) and ‘if you’ve got directed time the expectation is that you have been given the opportunity to research something you are really interested in there is no excuse’ (Daniel, 2015, Mid-study interview). Charlotte also felt it was positive to allow people time and refers to teachers who ‘are gradually coming in to why they are actually doing it’ (2015, Mid-study interview).

Accountability

Since the action research at whole school level was tied to performance management there was an element of accountability. Teachers knew they had to present their action research to an audience and members of the leadership team would be observing. With this brought a level of professionalism and accountability to the teams people had been working with.

Several teachers had worked together on developing a conceptual understanding of fractions. They had divided aspects of the resource development between them which had encouraged motivation to participate as they needed each other’s contribution to succeed. This was also found by Little (1990). The response echoed the findings from Johnson *et al.* (1990) that were cited in Jao and McDougall (2016) that when team members feel accountable towards the team and invested in meeting the team’s goals, members were more apt to participate in the process. There was also an increased motivation evident to not let the team down. An example of this was Ian, after seeing the high standard of earlier presentations he requested an extension, from the head of department, so he could improve his own presentation. This is additional evidence supporting the claim from Jao and McDougall (2016):

> 'If teams set shared goals and all team members feel like their contributions are valued, it is only natural that members will feel a sense of accountability in completing any tasks assigned, reaching the goal and will be motivated to do so.' (Jao and McDougall 2016, p. 561)

Charlotte as head of department acknowledged that although the initial ideas and active experimentation were considered in mathematics 'I think that Nic has probably been helped a little bit by the school saying that you have got to do a project which has made some of them do it' (2016, Final interview).

There was an acknowledgement from some that they initially felt pushed to do things but when reflecting later they were glad they had explored mathematics topics.

> 'I was proud of what I had discovered and I mean if we hadn't been pushed to do these subject developing things within our projects then I probably wouldn't have focused on something that was subject based so I was really thankful because fractions are something that everybody struggles to teach and now I have this method that actually seems to work, I have tested it out and tried it.' (Georgie, 2016, Final interview)

Georgie later commented that if she had to do another project again she would choose another subject focused one and choose a topic that she has struggled to teach.

9.6.3 External enabling factors

In addition to whole school influences there were also external factors prevalent that supported the need for teachers to engage with the CPD programme.

Curriculum and examinations

The curriculum change leading to the change in the mathematics examinations were supporting drivers and supported the engagement. Section 7.13.2 showed most teachers felt that the CCC Model supported work that was needed on the new curriculum with Charlotte, Annette, Elliot, Brian and Heidi acknowledging it was needed to meet the new aims of mastery, reasoning and problem solving.

> 'Because there is a lot more problem-solving expected in the new curriculum that is going to be much better dealt with using collaborative work and reasoning strategies by the students.' (Annette, 2015, Mid-study interview)

There was a need to change with the times and the CCC Model supported this 'I think the model very nicely goes with the curriculum change about mastery and understanding because I mean they can't do understanding and they can't possibly do mastery really until they properly understand their mathematics' (Charlotte, 2015, Mid-study interview). Charlotte clearly articulated the changes to the new GCSEs and how in particular the use of similarities and differences can support this process.

> 'The new GCSE is much more complex it is much more challenging, it needs the students to think for themselves, it needs the students to be challenged and I think our students here and even myself sometimes we were far too happy just to tell them how to do things and they can't do that anymore, the mathematics in the classroom has to change for the new GCSE and we have to make students that are happy to think things through, to be able to compare to be able to contrast things and to be able to just look at a question and be able to pull out the information themselves and being able to link, because the questions are not just one topic, there is lots of different topics in it and being able to link those topics together and be able to pull out the mathematics that is in a question has become much more important. I think the students have to be able to stand back and do this more themselves and I think by doing the similarities and differences we are getting them to look at subtleties and differences between them and giving them strategies to just help think things through.' (Charlotte, 2016, Final interview)

Elliot also commented that the increased emphasis on problem solving and making connections would 'be a big part of the GCSE from what I have seen from the textbooks that we have got a lot of the harder questions that are taking lots of elements from different parts of maths' and that these skills would need to be built because 'I know it is going to help with their further education as well because I know the A Levels are going in the same direction' (2015, Mid-study interview).

Professional development work was needed anyway to incorporate new areas of the curriculum. Heidi acknowledged 'we have identified any changes in the new curriculum and we are including them' (2015, Mid-study interview). Elliot commented he would be thinking about how to 'introduce differentiation' and developing a unit on the new topic of iteration 'for me personally I am sort of looking at the problem-solving skills that we are looking at that level as well' (2015, Mid-study interview). Charlotte saw this as an opportunity to think afresh about how she was going to teach these new concepts 'they are probably going to be reasoning why as the straight-line changes what happens to that tangent' so they could gain a deeper understanding of rates of change (2015, Mid-study interview).

There was also highlighted by Heidi the need to change how we teach some current topics to suit the new curriculum ‘we have done teaching fractions but it is probably the first one exploring the pedagogy behind it, better ways of teaching it, because it is not going to be new concept to any of us it is finding the ways of incorporating it into the new curriculum the way it should be done’ (2015, Mid-study interview). This led to her developing her action research on using multiple representations to teach fractions in a conceptual way.

Prior experiences

Heidi also commented on one of the main external drivers for her wanting to engage on the journey ‘I was taught by Nic on the SKE course and I think that initially I had a good start. … I experienced really good teaching and experienced a variety of strategies of teaching, different sort of approaches to teaching and I think that because it was not instructional, and we did a lot of exploration and investigations as well and I think it started me in this way’ (2016, Final interview). Daniel’s interest in working in a collaborative way exploring subject content had already been heightened at his SKE course ‘my own professional development in maths currently is currently concerned with improving my subject knowledge and in terms of how effective that has been, it has been really actually very good actually…it has sparked that thinking’ (2013, Initial interview). This was also the case for Maggie where she referred to how being taught in a relational way on the SKE course ‘made things easier for me’ (Pilot study).

These prior experiences meant that they were ready to hear the messages and willing to take on board the ideas ‘I certainly want to engage in developing the understanding of students in what we have been discussing. It is something that appeals to me and something that I would want to push on further’ (Daniel, 2014, Mid-study interview).

9.6.4 Reflections on the change environment

In Section 2.4.3 it was stated that there is a ‘documented need for a better understanding of how mathematical learning evolves in social settings’ (Francisco, 2013, p. 417). This study provides findings that extend what is already known.

The change environment, surrounding the PD programme, was essential to the department making changes and growing their own mathematical knowledge for teaching. Trust and collaboration were needed, and teachers needed to feel safe to experiment with new ideas. They also valued time provided for collaboration.

In 2014, Cajkler, Wood, Norton and Pedder reported that their lesson study project provided opportunities for participants to develop individual expertise through collaboration in a community of teachers and this led to greater confidence to make changes and willingness to take risks. This study suggests that collaboration in a different form (teachers working together on mathematical tasks) can lead to the same effect building confidence to try new things in the classroom.

Referred to in Section 3.6.6, Towers, Martin and Heaters (2013, p. 430) found that ‘actions serve to strengthen the collective, which provides the sustenance for the individual to flourish’. They acknowledge the mutual determination between the organism (learner) and environment each enhances and adapts to the other. This was seen to be the case for Georgie within this study. Her being part of the collective of the faculty and the professional aspirations that came from that supported and encouraged her to flourish.

Referred to in Section 2.4.3 was Martin and Towers (2015) notion of collective mathematical understanding within the classroom and how understanding arises from shared action. This study suggests that the same idea needs to be extended to teachers working together. Teacher’s working together in shared action grew their own collective mathematical understanding of what it meant to teach for understanding. These shared understandings not only emerged because of teachers discussing together but also due to the going forwards and back again to the external domain.

Referred to in Section 3.6.6, Brown’s longitudinal study of Australian secondary school mathematics teachers (2017) found several features that supported change (with the use of digital technology). Generic features that concurred with both research studies are:

- willingness to, and school leadership support for, participation in the project
- congruent expectations of curriculum documents
- on-going opportunities to collaborate with teachers and researchers

Whilst Brown (2017) felt that previous teaching experience was a feature this wasn't deemed to be within this study. Teacher beliefs and the way they had experienced learning new cognitive knowledge themselves could be argued to be more important than how long they had been teaching themselves.

Brown (2017) concluded that whilst these features were supportive of teacher change they were not sufficient for substantive change. This was also the case within this study. Brown (2017, p. 63) stated that 'whilst participation in a research project can promote teacher change, transformative change requires a focus on the interdependent strands of knowledge (MCK and PCK), beliefs, and practice.' This study would agree with Brown's point but extend it further to suggest that transformative change is more likely to happen if the social and professional change environment is such that teachers are opened up to the idea of change first, then they are more likely to embrace the ideas provided by ongoing opportunities to collaborate with the external domain of both researchers and academic literature.

9.7 Barriers to change

Whist Sections 9.1 – 9.6 identified within the Professional Mathematical Growth Model (Figure 14) factors and mechanisms that supported the process of change. The NCETM (2009), based on their extensive RECME study, recommended further research was needed to investigate the barriers to engagement with CPD. This study found there were several barriers experienced by teachers or by the researcher when implementing the CPD programme.

9.7.1 Barriers to engagement with the CPD programme

This section details some of the barriers highlighted by colleagues through the research study for not engaging fully with the CPD process. The first two phases of the CPD programme (Appendix 2.2) were sharing the research of the CCC Model and then bridging the theory to practice through practical CPD tasks for the department to engage with. Barriers to engaging with research in general and throughout this study have been detailed in Section 8.1.5. Barriers to engaging with the bridging theory to practice phase are limited in number as it has been shown this was a positive experience for colleagues who valued having time to work together solving mathematical problems and sharing ideas and beliefs. The challenges arose when moving to the active experimentation phase of the CPD programme. The barriers that arose were categorised into several themes.

Implementation issues

During the active experimentation phase of the CPD programme (Appendix 2.2), the expectation was for teachers to collaboratively plan the agreed topics and then to experiment within their own classrooms. There were several barriers when implementing new ideas. If these are deemed to be generic they are recorded in this section whereas if it is specific to implementing aspects of the CCC Model they are stated in Section 9.8.2.

Need to observe in classrooms

Whilst Louise and Brian referred to the strength of seeing the principles in action not everyone had that opportunity. This was a barrier for Kate ‘that is something I would prefer to see others do because maybe I am doing it and not realising I am doing it’ she looked for reassurance ‘I would prefer to see other people doing it’ so she could then say if she was doing the right thing (2016, Final interview). Elliot also felt that more people would come on board if they had the opportunity to see it in lessons.

> ‘I think there needs to be more observing each other's types of lessons that highlight it. I mean I have really jumped on board with what Nic has been doing and I think my teaching has been transformed as a result. I don't know if other people have jumped on-board so much and I don't know if other people have been as emerged in it enough. So, if those teachers saw those lessons that I really like that kind of model this then I think they would get a better idea of what we are trying to do.’ (Elliot, 2016, Final interview)

Need to repeat it to embed it

It was acknowledged that to embed something in practice it would need to be repeated with different classes or in different contexts. So, there was a challenge with specific tasks like some of the proof examples as it may have been taught to one class and then potentially not done again until the next year. 'I think because I am working on proofs now I am starting to look at efficiency of method there as well but is not every class or every topic or every year group I am teaching' (Heidi, 2016, Final interview). With more generic ideas Daniel found it a challenge to repeat the ideas due to his lack of mathematics classes.

> 'I find that despite the fact that I reflect on a lot of these points quite regularly it is not as embedded in my teaching as I would like it to be yet. When you are trying to put something new into your teaching when you have only got one group and you put it in and take it out for a bit if you are a maths teacher 100% I think you would try it with year 8 and then again with year 9.' (Daniel, 2016, Final interview)

Extra time needed to embed to embrace change

Whilst lack of time was already highlighted as a barrier to being able to collaborate there was also the acknowledgement 'it takes a lot more time to find out and do things differently' (Annette, 2015, Final interview) because 'you are moving out of your comfort zone' (Daniel, 2015, Mid-study interview). Time is needed to read academic research to 'process and understand and find relevant parts to teaching' (Heidi, 2016, Final interview), then to develop inspirational lessons with 'theory being put into practice' takes a huge amount of time (Frazer, 2016, Final interview). Quotes are shown in Table 48.

Teacher	Quotes about time needed to embrace change	Evidence source
Annette	It takes a lot more time to find out and do things differently doesn't it.	2016, Final interview
Daniel	Because you are moving out of your comfort zone and you are teaching in a different way than you have done before its coming up with those approaches and being able to set aside that time and thinking this is the way I want to do it, and I want to do it with this particular question and this particular focus. It's sitting down and coming up with those really good questions that allow the kids to sort of want to…… intrigue - generate the intrigue into the answer. It's creating that scenario that I find I need to find the time to do that and everything else takes over and you think I am not there yet with that one and it kind of gets pushed back and pushed back and that's my barrier and I just want to create those resources for myself. Once I get those resources I think I will become much more, much better, at organising the way I question within the classroom to get the outcomes I want as a result of taking on this style.	2015, Mid-study interview
Frazer	To then sit down and try to consider a decent lesson that is well constructed with lots of theory being put into practice, if you can do one a week you are doing well. So, time is a huge factor there really.	2016, Final interview
Georgie	As we moved through, with the independent development it is finding time to do these additional things. It is difficult enough with the time that I have got let alone planning these inspirational things that are going to work in every lesson and we are not given the time to do it.	2016, Final interview
Heidi	One of the smaller issues was time limits because as I found with academic research you have to actually dedicate time to read and process and understand and find relevant parts to teaching.	2016, Final interview
Louise	It is more difficult to find the time to sit down and think I am going to read about proportion. It is not that I am not reading anything, but I am not doing it consistently.	2016, Final interview

Table 48. Quotes about time needed to embrace change

Evans (2014), quote is a useful insight into what was also seen in this study.

> 'New ideas or ways of thinking that have been planted within teachers' consciousness may take time to blossom and to become gradually assimilated into their practice – and in the interim such ideas or perspectives may have been augmented (or diluted) through interaction with a myriad of other (often unrecognisable or unidentifiable) influences on practice. To assume that any generative impact of professional learning or development will be (immediately) evident represents over-simplistic reasoning that fails to incorporate consideration of the complexity and, I argue, the multidimensionality, of professional learning and development.' (Evans, 2014, p. 188)

Mentor dependence

Whilst the support of the researcher was shown to be positive, it could also be a barrier when teachers became dependent on the mentor for the answers. Annette referred to needing someone when working on her Open University course ‘you haven’t got that person to ask at the end of it,…..sometimes you just need a clue you need something to sort of draw you on and sometimes just to give you an example’ (2013, Initial interview). Throughout this study many colleagues relied on going to the mentor for advice ‘I am going back to start preliminary discussions with Nic, if you like, over what research should I be going to look at?’ (Daniel, 2015, Mid-study interview). Georgie and Kate kept showing the researcher their presentations and asking whether the content was okay. Elliot in his final interview talked about his plans for his new department that he was moving to and how he wanted to take the ideas from the CCC Model to them ‘I want to think about how I would do the same thing with my department in the future and the only thing that I can think of is buying Nic in to do it!’ (2016, Final interview). This mentor dependence extends back to the pilot study too with Maggie saying ‘I needed the knowledge from you and I needed the tips’.

Lack of embedding in scheme of learning

One challenge was seeing new ideas that were liked but not having them to hand when teaching the topic ‘because if you do it once it is easy to forget and if you don’t use it yourself you will forget them’ (Elliot, 2014, Mid-study interview). Elliot felt that ‘I don't feel that there is enough yet of work going into schemes of work of development of what we are trying to do’ (2016, Final interview), and he needed a reminder of when to use things. Frazer also referred to this as an issue when he could recall ‘seeing a really good lesson on that’ and then knowing where to look for it when needed (2016, Final interview).

9.7.2 Barriers to engagement with the CCC Model

Whilst Section 9.7.1 identified the barriers to engaging with a CPD programme in general this section considers the barriers that teachers faced when trying to implement the CCC Model.

Teachers at times referred either explicitly or implicitly to their own lack of mathematical knowledge. This fell into two different areas; those that referred to lack of subject knowledge, or to a lack of pedagogical confidence in the ability to make connections or to come up with new ideas independently.

Not taught at a higher level

The first area showed a couple of teachers lacking in confidence due to not having taught A Level or only having foundation classes for GCSE. Georgie for example said ‘I think it was the fact that it was a topic that I had struggled to teach previously as a higher-level topic’ (Georgie, 2014, Mid-study interview) this was also echoed by Frazer ‘I have not taught A Level, I taught in 11-16 in London and then went into an office for 5 years before I came here so I haven’t touched A Level for years. And the thought of having to pick up A Level maths, is almost I would have to do the whole thing myself again (2013, Initial interview).

Heidi identified a barrier within a topic 'subject knowledge for me, when it comes to geometrical proof I have actually started to read around papers and books to gather some other ideas to see how other people are introducing the topic, what resources they are using and also the barriers, that is from my perspective, my own personal perspective is I need to enhance my subject knowledge' (2016, Final interview).

Can't make the multiple representations themselves

Another area where there was a barrier was teachers agreeing in principle with multiple representations but not being able to come up with their own. Elliot identified he was struggling to do this in lessons 'because I need to know what the multiple representations are before I can give them to the kids' (2014, Mid-study interview). There was a similar response from Frazer where he felt it was challenging to do this without support 'as I say the multiple representations especially, I can see the worth of it but it has to be with resources and with planning and structure that is well designed to support the learning and to get that right is a difficult thing, it is not something that is easy to do unsupported' (2016, Final interview).

Not having the ideas in the first place

Not specifically just to do with multiple representations but more generally teachers also found a barrier to be 'sometimes having the ideas in the first place, sometimes you want things that are different and sometimes it is nice to have that discussion time with people as well' (Charlotte, 2016, Final interview) this was similar to Georgie's comment 'I think that was I was struggling because I wasn't necessarily coming up with the activities before, although I thought they were getting a deeper understanding, it wasn't quite hitting the point' (2014, Mid-study interview). This was like the response from Maggie in the pilot study 'even though I liked the conceptual stuff I didn't have the ideas, I didn't have the creative ideas, how could you show these things in different ways' (Pilot study).

This was echoed with teachers enjoying the completing the square and the prisms because they were creative but that the teachers wouldn't have considered doing it that way until they were shown the resources.

It was also recognised that some of these approaches that support the CCC Model might involve resources and these were unfamiliar to Brian coming from a different career into teaching. 'I don't instinctively know what they are talking about like using Cuisenaire rods or other bits and pieces that is probably a weak area of mine' (2016, Final interview).

Need to teach it to then reflect on how to teach it

Similar to not having the ideas in the first place, Louise as an NQT identified the challenges of teaching things for the first time and how it is easy to be too teacher led and then reflections can be made on how to teach it more effectively the second time around 'for example, at the moment I am doing circle theorems and I think next year how can I do that in a different approach, how can I make it more student led, I feel like I have probably led them a bit too much this year' (2016, Final interview).

Difficult to transfer ideas to classroom

There were a couple of barriers transferring ideas to the classroom, Georgie said 'the first time we were introduced to this I was thinking this is all well and good but I am not sure how I can translate this into my teaching' (2014, Mid-study interview) and Daniel at times trying things unsuccessfully at first 'once I get a handle on that I will be able to move the techniques and understanding that I have down to their level but I am finding that I come up with an idea and I try it in the lesson and it doesn't work and I have to try something else again' (2013, Initial interview).

Findings from this area would echo those from Borko (2004) that helping teachers develop their subject and pedagogical knowledge may seem simple but it isn't and improving their actual classroom practices has proven to be even more complicated.

9.7.3 Departmental inhibiting factors

There were several logistical factors raised as barriers to engaging with aspects of the CPD programme.

Continuity of development

It was felt the department were developing as a group however for a variety of reasons not everyone could be at every session or meeting (Table 49). Part time teachers were not always in school the days meetings were scheduled and the school timetable-imposed barriers with teachers not being able to attend meetings as they had lessons to teach at the same time as department meetings. This resulted in some teachers feeling like they had missed out on the development work through no fault of their own ‘unfortunately, I have missed a lot of the work that Nic has done with the group because I am always teaching after school so that hasn't helped me understanding what some of these things are talking about’ (Annette, 2016, Final interview).

The team also changed in dynamic throughout the study with Georgie being off in the middle of the study on maternity leave and Kate joining late as she had come from another school. This was observed as a lack of confidence as parts of the CPD had been missed by Kate (2016, Final interview) ‘I didn’t feel perhaps as confident as the others do with the model because they have been to more sessions’. There was also absence from sessions and interviews due to illness and Ian was off for a long period at the end of the study so wasn’t there for the final interviews. Daniel raised continuity issues as he taught across three departments so had to spread his attendance at meetings across them all.

Teacher	Quotes about continuity of development	Evidence source
Daniel	It was all very clear for me at the time, but if you don't engage with it you lose the thread. I remember at the time (sorry Nic) leading on from the presentation and going into the CPD it was a logical progression and everything was making sense and I could see where the pieces were fitting together, however since then my engagement with it has been negligible due to the reduced amount of mathematics that I have been teaching this year.	2014, Mid-study interview
Annette	I sort of feel slightly out of what is happening as I don't get to go to much of the training as I am teaching afterschool maths. I think I missed the last interview for that reason, but as I say I have missed out on a lot of the actual training since then and I am not quite sure exactly where we are at on that.	2015, Mid-study interview
Daniel	With myself being split over three faculties, one of the conversations I have with Charlotte over time is that I get aware of change, but I am not necessarily in on all the departments discussions, so I find it, you know, just keep me in the loop, just keep me in the loop. That is one of the things that I find it difficult being spread so thin across different areas of the school that I can't seem to be able to focus on things such as Nic is doing, I just thought I'd take elements of it and try and develop them across my teaching.	2015, Mid-study interview
Annette	Unfortunately, I have missed a lot of the work that Nic has done with the group because I am always teaching after school so that hasn't helped me understanding what some of these things are talking about.	2016, Final interview
Georgie	It was just the timetable and that I ended up teaching at the time but…. I have missed out on seeing how they have progressed with their presentations which is a shame.	2016, Final interview
Kate	By the time I had joined the school, I had missed the training session and because I am part time I also don't attend Thursday meetings, so I have been to one session, so I don't feel perhaps as confident as the others do with the model because they have been to more sessions. So, I have used some of it but haven't had the same opportunities as the others so I said I did look for some research I did look through Nic's things and I did read the things that she gave me but I haven't had the same sessions so a lot of it I said I wasn't confident to say yes, I have transformed everything in my practice. It was hard to join a project half way through, it was quite scary because when I came in everybody was talking about this CCC Model and I thought, what on earth are they talking about and I don't know, I found like I was on the back foot I was trying to put these things in but not entirely sure I was putting them in right and I guess that was my lack of confidence from not being in the sessions.	2016, Final interview

Table 49. Quotes about continuity development

Challenges in collaboration

Whilst there were many positives of collaborative work previously highlighted, the process of collaboration was also at times a barrier to moving forwards in a small number of examples. Georgie commented early on about using the schemes of work that were shared:

> ‘First step is to see who has written the scheme for our medium term scheme of learning because I know if it is certain members of staff that have written it I can take it and run with it, if other members of staff have written it, you appreciate it has not been done in a similar method it is the difference between this type of relational understanding and here is a worksheet and here is the text book and here is MyMaths to get you there. So, it really does depend on who has written the scheme.’ (Georgie, 2014, Mid-study interview).

She reflected that ‘there are a couple of members of the department that are slightly more cynical than others’ but the enthusiasm from the rest of the department was enough to pull them along (2014, Mid-study interview). Elliot also commented on trying to collaborate with Brian who he perceived to be teaching in a procedural way, he felt that discussions didn’t help him progress but ‘also talking to people who are against it and don't do it like that has made me think that mine is better too!’ (2016, Final interview).

9.7.4 School inhibiting factors

When questioned about what was stopping progress the standard answer was time. It was felt that too much time was needed just dealing with normal day to day school priorities of planning, marking, assessing, reporting etc. Table 50 shows some of the comments regarding time being a barrier to engage with CPD in the active experimentation phase of the programme. These are examples of the contradictory demands to which the teachers must respond (Hargreaves, 1995) as discussed in Section 3.6.2.

Teacher	Quotes about time constraints	Evidence source
Charlotte	Admin jobs interfere and become more important and they shouldn't be, it should be about the learning and understanding of mathematics.	2014, Mid-study interview
Brian	Time pressure, NQT year, creating stuff on top of everything, increased work load as well as developing a research project, that's a bit of a barrier.	2015, Mid-study interview
Annette	Time pressure as I have said.	2016, Final interview
Brian	Only pressures of time.	2016, Final interview
Charlotte	Time to do it is a potential barrier. I think you have to do that or the everyday things just stop you doing it.	2016, Final interview
Daniel	Sometimes it is not at the forefront of my mind because something in A Level Psychology or PE has taken over for that morning of whatever.	2016, Final interview
Frazer	First one is time, hugely so If you think about the normal day to day work you have to do.	2016, Final interview
Georgie	It is time, time constraints, trying to do my job. I have got the desire to do it, but time constraint is the only thing really stopping me. If you could say to me, you have some hours every week and you must sit down and do this then that would be wonderful I would love that.	2016, Final interview
Louise	Time as an NQT going from 10 hours to 18 hours teaching, that extra 8 hours of planning and preparing.	2016, Final interview

Table 50. Quotes about time doing other daily tasks being a constraint

Conflict between school and department priorities

At times throughout the study there was perceived to be a conflict between department and school priorities for CPD. The initial interview responses to what makes less effective CPD shown in Section 7.3.1 cited examples of not wanting to engage with school-wide based CPD as it often didn't fit with mathematics and 'we haven't had time to do subject specific CPD' (Charlotte, 2013, Initial interview).

Initially, when action research projects were suggested at school level there was the expectation that groups would work on school identified theories of action or school run leadership and outstanding teacher programmes (Appendix Figure 17). The work the English department were doing with the Royal Shakespeare Company was featured but the mathematics development already in progress wasn't. Charlotte acknowledged the frustration in this lack of recognition.

> 'Nic has fought really hard to get the CCC listed in there and I don't know why the principal has not physically put it there, he knows that we are carrying it on to do that, I don't think Nic is winning with that at the moment but I believe we are still allowed to do this and present our own but it is not labelled there exactly.' (Charlotte, 2015, Mid-study interview)

She commented on the difficulty of getting time together as a department 'Nic's project is about having the time to collaboratively work and to collaboratively discuss things' (2015, Mid-study interview) whereas the English department were given whole days off timetable together to work with the Royal Shakespeare Company. This discrepancy in allocation of time was an additional barrier alongside a couple of mathematics collaborative planning sessions being cancelled due to issues with cover. Jao and McDougall (2016, p. 565) found 'repeatedly, teachers reported that time was one of the biggest barriers to collaboration' these findings would be echoed in this study.

There were also concerns raised by the head of department about the pressure coming from the leadership team to focus on feedback rather than teaching:

> 'It feels like that is coming and taking over and so for me it is having the balance with what is going on in the classroom and also having the time

> to mark their work and at the moment I am having too much pressure to mark their books.' (Charlotte, 2016, Final interview)

Conflict on teaching approaches

Whilst everyone wanted the best outcomes for learners at the school there was conflict in how that might be achieved. Daniel commented that teaching his C/D borderline class had 'become quite unfulfilling' as he had been directed by the school leadership team to teach in a rote way.

> 'We were asked to do this was via loads of practice papers and it was a lot of rote learning and exam practice to get the kids over that final line and it was demoralising in a way you knew that a lot of the students weren't actually understanding the processes that were involved behind the maths they just knew that if they did 2+2 they would get four. If you changed the context they would struggle and it was really quite hard to just keep going down that road of exam practice rather than trying to teach for understanding and trying to teach for them to be able to change things and to be able to move things to another context and still to be able to come out with some problem-solving skills and they were obviously lacking because time was against us.' (Daniel, 2013, Initial interview)

Numerous conversations occurred between colleagues at department meetings as to whether they should teach to the examination or ensure learners can understand and develop skills for later study or life and sometimes 'the pressure of achievement and attainment overtakes that' (Charlotte, 2016, Final interview). These concerns that ACME (2011) raised in the Mathematical Needs report, stated in Section 3.5.2, can be seen to be still prevalent in the current education climate with a school management who prioritize superficial learning for test results, which in turn leads to a procedural approach to mathematics. As Ofsted (2008) recognised, school accountability policies can be seen to encourage teaching to the test.

> 'I have always taught the C/D borderline group so I am probably more prone to using tips and tricks than a lot of the other teachers in the department because I have had to, the school wants the results so what do I do, do I get the school the results or do I give the students in 6 months lifelong understanding of maths and now I feel like I have got the rest of the department, well Nic and Charlotte and people that matter saying if we are going to start this from year 7 you can do it like that and there doesn't have to be the tips and tricks, I think it is going to help my teaching and the rest of the department as well the way we are able to start from scratch almost to rip it all up and it is daunting the thought of having to do everything but it will be so worth it if we only have to teach the things once and they have got a proper understanding of it, so in 10 years' time I could say area of a parallelogram and they remember it because they learnt it properly.' (Georgie, 2013, Initial interview).

9.7.5 External inhibiting factors

Section 9.6.3 showed that a change in curriculum and examination structure was seen by many as supportive of the CCC Model due to the alignment of principles and aims. However, the time needed to dedicate to these was also a constraint. 'Some of the significant barriers are the change of syllabus for example with the new GCSE we are aiming at a point and we don't know what that point is, we are not sure on the grading structure or the marks they have got to achieve' (Brian, 2016, Final interview). This resulted in time being spent debating what a grade 5 might look like rather than spending time discussing pedagogy. Similarly, for Daniel 'both A Level Psychology and PE are new specifications which are taking up a lot of time in terms of getting them right' (2016, Final interview).

The pressure of the examination structure was also seen as a constraint. Whilst Daniel commented 'teaching for understanding rather than just teaching to pass an exam, when we were going through things you can see why a change of emphasis in your approach would be more beneficial in the long run' (2014, Mid-study interview) it was noted that this philosophy would change as exams approached. This was echoed by Brian (Pilot, Study) ‘I seem to be doing a lot of procedural, because I am doing things in a rush in preparation for the exam and there is not much conceptualising going on at all’ and Georgie (2014, Mid-study interview), ‘for the first 18 months of a GCSE course you teach for deeper understanding and the week before the exam when they are panicking you teach the tips and tricks’.

9.7.6 Conclusions on barriers

There were several barriers that stopped the CPD programme being as successful as it could have been: logistical ones at departmental level resulted in a lack of continuity in development which was a challenge for some; and time was reported as the biggest barrier with an overload of jobs to do within school life. The external constraints such as new curriculums and examinations led to an increased pressure on colleagues with school accountability policies leading to potential conflict between teaching for understanding and the perceived need for procedural work to prepare for examinations. These barriers follow those from Section 2.5.4 suggested by Wrigley (2014, p. 39) that current notions of accountability were designed to promote competition among schools however, can ‘lead to superficial learning for short-term assessment and grading, rather than intellectual engagement and enduring cognitive development’.

Whilst these pressures from the leadership team at times promoted a procedural approach, teachers themselves were keen to change their practice leading to implementation issues: needing to observe the CCC Model in classrooms; needing to repeat aspects to embed them; extra time needed to embed to embrace change; and finding resources when they were needed.

At times teachers within the study experienced attitudinal barriers such as frustration and anxiety to change. A few teachers also cited personal reasons for not embracing ideas fully due to family commitments or coming to the end of their career.

The first main barrier to engagement with the CCC Model has been identified as a lack of subject or pedagogical content knowledge which incorporates a range of subthemes from the teachers: not having taught at that level; not having the ideas or can't make the connections themselves which presents challenges transferring new knowledge into classroom practice. It perhaps is summed up nicely by Maggie's quote 'I think you have to be really knowledgeable and know your subject inside out before you can start teaching it conceptually' (2013, Pilot study). Other barriers have come from the students themselves, with a preconceived idea of what mathematics education is and how they should be taught and a lack of desire to truly understand but a requirement to pass exams. From some teachers, there is maybe the belief embedded that not all students can learn in a more conceptual way.

9.8 The PD change process

The process of change can be seen to be a complicated and lengthy process. The complexities of learning within policy and school cultures was raised by Avalos (2011) in Section 3.2.1. Whilst all teachers changed in some way, the amount of change was varied and would be impossible to measure in a statistical sense. However, this section has drawn together evidence to show which elements supported the process of teacher change.

Referred to in Section 3.7, Ball, Ben-Peretz and Cohen (2014) acknowledged that the teaching profession lacked the structure to support the development of shared knowledge on a widespread basis. This study provides a possible structure (Appendix Figure 1) that could be applied to other areas to help support the development of shared knowledge at a departmental level.

There were a variety of environmental change factors. At school level: an increased professional culture; a change in balance between central led to chosen action research projects; accountability aligned to performance management and allocation of time were all aids to the moving forwards with CPD. External factors due to changes of examinations and curriculum requirements also supported as the CCC Model was seen to align with new curriculum aims.

However, the factors that had the most supporting evidence were the importance of departmental collaboration; due to professional relationships and trust that had been established. This alongside teachers being inspired by what they had seen working with students in their own school. Many teachers were motivated by seeing the responses from their learners which provides additional evidence to support the inclusion of this element within Figure 14 (building on Guskey's model, 2002). This study also adds additional evidence to the argument from Avalos (2011) that developing school cultures can be conducive to professional development and teachers learning. It also adds additional evidence to support a recent study by Attard, Tonna and Shanks (2017, p. 97-98) who found that 'the teachers expressed their need for professional learning in relation to the challenges they feel they need to address in their classes'.

Within this study teacher change was viewed from the perspective of growth in learning (Clarke and Hollingsworth, 2002) with the teachers changing through engagement in professional activity, the teachers themselves were learners within the community (department, situated within the context of the whole school). The understanding developed was related to the purpose of teaching secondary mathematics in a more connected way. Towers, Martin and Heater (2013) emphasized the critical role of the teacher as a trigger, this study has shown with PD the external domain can be a critical trigger and theorises it is in the interaction between the external domain and the environment that teacher learning happened.

CPD was deemed to be 'effective' when teachers continued to explore and develop their practice independently. It was found within this study that long-term development needs to be gradual, teachers need to go on a journey. The development needs to be evolutionary as opposed to revolutionary. Rather than teachers moving from external input to the guided domain and then to the independent domain this case study found that numerous cycles transferring between the external domain and the guided practice domain (Figure 85) were needed before teachers moved to the independent domain. This longer-term model was found to break down the resistance to change and supported teachers understand what teaching for understanding meant in order to carry out action and experiment with their practice in the context of the school environment.

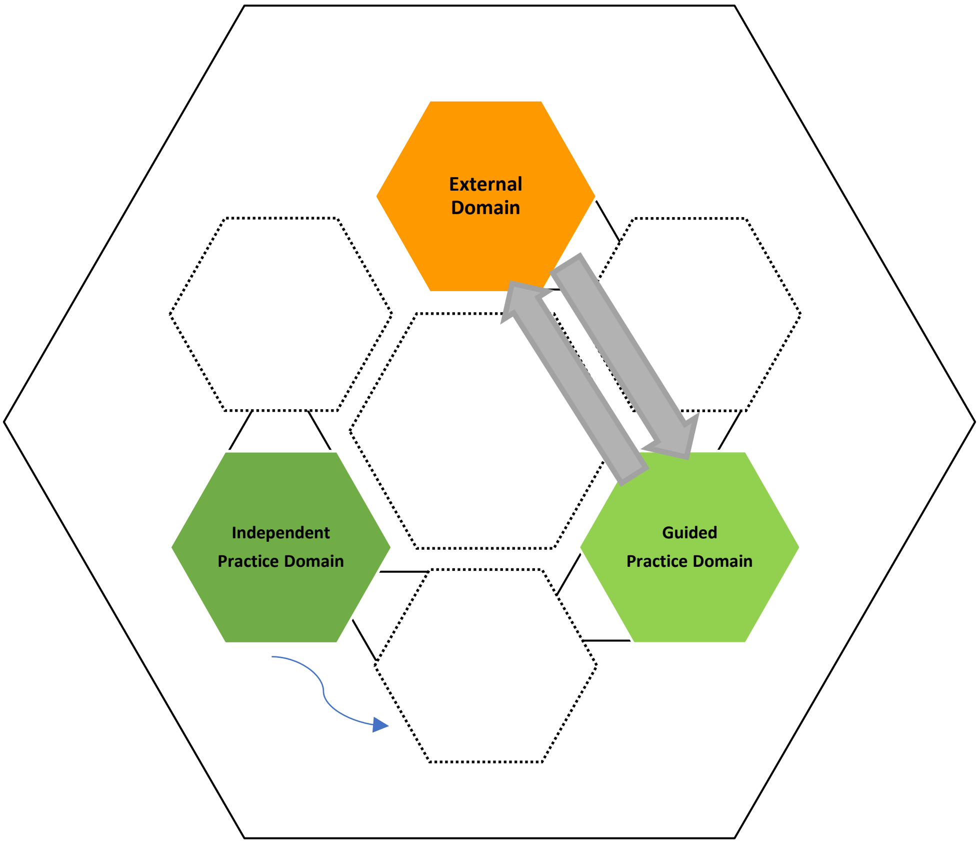


Figure 85. Process of change

Whilst providing a range of resources for colleagues was useful in the short term, if the end goal of CPD is independent change then this study has shown that not dealing with specific mathematical topics but mathematical principles that help develop understanding will support longer term development and growth. Providing principles for teachers within professional development is not new (for example Swan, 2005) however just sharing the principles is not in itself enough for teachers to change.

Brown and Coles (2011) found in their study (referred to in Section 3.6.6) when working with teachers encouraging the use of similarities and differences narrowed the gap between action and communication in the continuing professional development of teachers. This study would agree with their finding and also provide new evidence suggesting that the strategy of what's the same and what's different should first be modelled with an 'unknown topic' (in this case transformations of matrices and invariant lines) so teachers learning in the cognitive sense is developed, this is then more likely to lead to teacher growth in the situative perspective as the strategy is employed with their learners. This complements the work of Clarke and Hollingsworth (2002) (referred to in Section 3.3.3) where teacher growth combines the development of knowledge and practice.

In this study teachers learned in different domains, some learned new mathematics knowledge for teaching from the activities that were provided within the PD sessions 'bridging theory to practice phase' and some extended their understanding by using academic literature to support using new representations with learners. The PD was most 'effective' when teachers were able to move from the guided practice domain to the independent practice domain and there were several mechanisms that supported this transition. The most significant was when teachers were able to tweak their practice using small steps, see the impact on learners and then evolve further the principles underpinning ‘what’s the same and what’s different’ to enable them to readily develop their own ideas and prompts for learner discussion.

Section 2.2.2 emphasised the importance of learning within social settings and this can be interpreted at two levels, the theory applies to pupil learners but within this study was equally applicable to teachers on a learning journey when engaging with new knowledge. The incorporation of both social and professional elements into the change environment (Figure 14) was informed by literature and the study findings confirm the explicit inclusion of these is important.

Within Section 5.2.6 it was stated that although single case studies themselves are rarely generalisable the findings can be generalisable to theoretical propositions. This case study provides original contributions to knowledge recognising that the domain of practice has two parts the guided domain and the independent domain. This subtle distinction will support other CPD designers as they think about what participants might need initially to support their guided practice and what mechanisms can be employed over a sustained period to ensure participants are able / willing to move successfully to the independent domain of practice. Without mechanisms to support transition to the independent domain the PD would be less effective in achieving long term change. This study theorises that ‘techniques and strategies’ that can be applied to numerous topics and classes can support teachers evolve their independent practice and are more likely to lead to transformation of practice. The techniques and strategies should be first modelled at a ‘higher level’ than teachers currently teach so they can experience the power of learning new knowledge themselves in that way.

CHAPTER 10: REFLECTIONS

This study set out to answer the following research question:

- Q How can a programme of professional development engage and support a mathematics department to teach for understanding?

This chapter reiterates the key findings that have arisen throughout the study in its entirety and emerging theories are restated with implications for future research.

10.1 Key enabling features

When considering what was meant by professional development (Section 3.2.1) Avalos (2011, p. 10) commented that 'teacher professional learning is a complex process, which requires cognitive and emotional involvement of teachers individually and collectively'. This was certainly found to be the case within this study. Teachers had to engage with the CPD that was being delivered and be motivated to take it into the classroom. The motivation to take part was often down to the culture that was created within the department over the period of the study and the responses from the students to activities. Whilst there was an initial engagement and buzz of ideas during the delivered CPD sessions this itself would not have had the required impact within the classroom. This study provides additional evidence to support the quote from Fiszer (2004, p. 5), cited in Section

3.2.3, that ‘to sustain teacher learning that directly affects classroom practice, we must provide a culture that requires and supports on-going professional development’.

Chapter 9 provided detail on various aspects that supported development. However, to summarise, the key enabling features that supported professional mathematical growth are summarised in Figure 86.

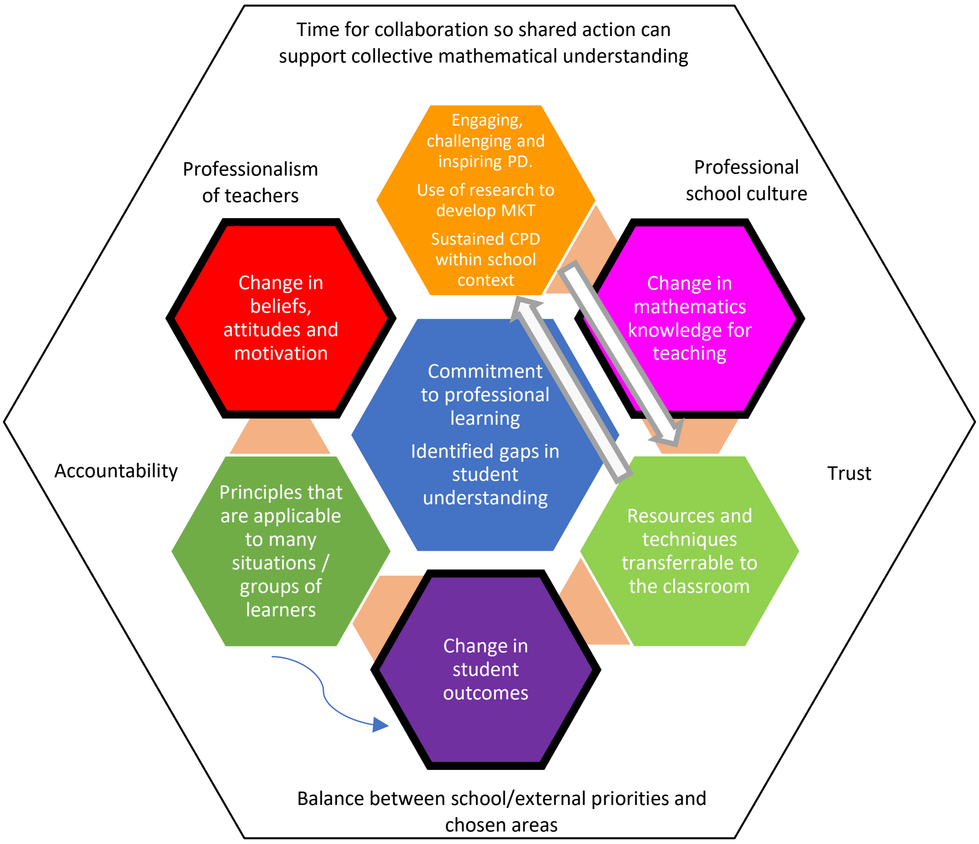


Figure 86. Professional Mathematical Growth Model: key enabling features

10.2 Emerging theory with personal reflections

It was quoted in Section 3.4.3 from Joubert and Sutherland (2008, p. 12) that there is ‘no agreement in the literature about the most effective way of structuring professional development so that teachers learn about the interrelated aspects of mathematical knowledge for teaching’. This study suggests that the CPD programme (Appendix 2.2) is one way this could be done. The sharing of research and the bridging theory to practice sessions were valued and the researcher recommends these features to others looking to provide CPD to mathematics teachers. As Eraut (2001) emphasised, there is often a great deal more learning to occur after any CPD event when trying to use it in practical situations, so the incorporation of the active experimentation stage was an important feature.

A future recommendation would be to add observations to the active experimentation stage, both to observe the ‘expert’ model ideas and then to support the implementation process with others. However, it is felt that a positive climate for teacher learning would need to be established to enable observations to be productive.

The TDM (Appendix 6.1) that emerged to help track the progress of teachers on their journey was a valuable tool. This was of interest to those that attended the researcher’s presentations at the British Society of Research into Learning Mathematics Conference (Trubridge and Graham, 2017, Appendix 7.3) and the University of Plymouth Postgraduate Research Conference (Trubridge, 2017,

Appendix 7.4). The TDM could be adapted to support evaluation of the implementation of other CPD programmes. The researcher recommends that to improve the model, clarification is needed with regards to how often or in how many different situations something would need to happen before teachers progressed to the next phase. There are some other improvements that could be made to specific wording in each strand as for example, when using multiple representations, it could be possible to transform practice without teachers developing their own resources as there are many high-quality resources readily available.

Section 3.6.6 detailed recent and relevant academic research in the context of mathematics CPD and this study extends well established literature. This study has revealed details and issues from the perspective of a group of teachers within the context of their own school as they develop their MKT with the researcher supporting their journey.

This thesis has stated that when planning for sustained PD it is important to recognise the different domains of practice, that of guided and independent. Within the mathematics specific context, the use of 'what's the same and what's different' was a strategy that enabled teachers to change their practice independently. It is theorised that when planning mathematics PD, programme designers should focus on what techniques or strategies model the principles they are trying to embed and spend time emphasising and modelling those. This study contributes new evidence that teachers need to learn themselves through shared action (working together on unfamiliar mathematical tasks) as seeing principles in action supports the motivation to experiment and their willingness to enact ideas with their classes. This will help aid transition from guided to independent practice. Within this study the availability of the external domain was critical throughout the process as teachers returned for clarification, reassurance and extra ideas. The external domain was seen to trigger teacher learning through CPD that was encompassed in its entirety within the social and professional change environment.

10.3 Implications for future research

The CCC Model that was derived within this study (Appendix 2.1) had four different areas. This study focussed on the design of CPD tasks that modelled the 'nature of mathematical activity' and encouraged teachers to develop these types of activities. An area to research in the future that was beyond the limitations of this study would be to explore within the classroom how the social culture can be developed. This would help to answer the question, Section 2.3.3, from Kazemi and Stipek (2008) about how to create and sustain socio-mathematical norms that press for conceptual thinking.

This study has researched which approaches engaged teachers with students' conceptual development in mathematics and found that the strategy of 'what's the same and what's different?' was embraced by teachers, partly because of the ease of teachers planning for it and partly due to seeing positive response from learners. A natural area to research in the future would be to look at the impact of this strategy on learners within the mathematics classroom. Another phase of research would be to work with teachers to build on Tall's (1988) distinction between concept image and concept definition and to see how the strategy of 'what's the same and what's different?' could be used to tackle the various learner conceptions of the same concept definition, helping to further develop MKT of teachers.

One challenge that arose when implementing the CCC Model was not all pupils wanted to 'understand' mathematics or were motivated to think more deeply about their learning. Although statistical data was not collected in this study from the pupils, it was felt by the faculty that those students going on to study or currently doing A Level were readier to take on the principles of the CCC Model. This raises the unanswered questions of 'what motivates learners to want to understand mathematics?' and 'can teachers, and the systems they work within, change the motivation of leaners to think conceptually?'.

Within this study it was seen that teachers developed an understanding of what teaching for understanding looked like, but some remained within the guided practice domain and it would need to be explored further 'how do you move someone who is trapped in a cycle of dependence?'.

The Professional Mathematical Growth Model (Figure 14) was provided as a framework to describe the factors that would influence teachers when they are engaged in PD that is designed to support the development of specific areas of MKT and the implementation of these ideas within classrooms. Whilst this study focussed on using the CCC Model to support teachers make connections, it is possible that this framework could be generalised to implementing other aspects of MKT so would need to be tested within other PD programmes. It was also theorised in Figure 85 that additional arrows need to be included, as teachers continued to move forwards and backwards with knowledge from the external domain to the guided practice domain. This emerging theory should be tested in more research studies before generalisation is possible.

APPENDICES

APPENDIX 1: LITERATURE ANALYSIS

Appendix 1.1 Similarities between models and theories

Dimension	Core Features	Researcher	Model, Report or Theory
Mathematics	Is an interconnected body of ideas and reasoning processes	Swan (2005)	Connected Challenging View
	Is a highly-interconnected subject that involves understanding and reasoning about concepts, and the relationships between them. It is learned not just in successive layers, but through revisiting and extending ideas Requires understanding and reasoning about real and imagined objects, and is defined by a range of different kinds of knowledge, including connections between concepts	ACME (2011)	Mathematical Needs Report
	Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information … a unit of conceptual knowledge cannot be an isolated piece of information; it is part of conceptual knowledge only if the holder recognises its relationship to other pieces of information	Hiebert and Lefevre (1986)	Conceptual knowledge
	Conceptual and procedural knowledge develop iteratively, with gains in one type of knowledge leading to gains in another Conceptual knowledge may have a greater influence on procedural knowledge than the other way around	Rittle-Johnson and Alibali (1999)	Linking procedures and concepts
	Conceptual knowledge is intricately linked with procedures and algorithms. Knowledge of procedures is nested in conceptual knowledge	Long (2005)	Linking procedures and concepts
	Being competent in mathematics involves knowing, concepts, knowing symbols and procedures and how they are related	Hiebert and Lefevre (1986)	Linking procedures and concepts

Appendix Table 1. Similarities between models and theories for mathematics

Dimension	Core Features	Researcher	Model, Report or Theory
Learning	Consists of building up a conceptual structure	Skemp (1976)	Relational Understanding
	Is a collaborative activity in which learners are challenged and arrive at understanding through discussion	Swan (2005)	Connected Challenging View
	Teaching and learning are complementary	Askew et al. (1997)	Connectionist Model
Learners	Know what to do and why Can adapt their knowledge to new tasks	Skemp (1976)	Relational Understanding
	Need to become aware of, familiar with, and fluent in connections in mathematics	ACME (2011)	Mathematical Needs Report
	Being numerate involves the use of methods of calculation which are both efficient and effective confidence and ability in mental methods selecting a method of calculation based on both the operation and the numbers involved;	Askew et al. (1997)	Connectionist Model
	Are encouraged to invent their own strategies before learning traditional algorithms not only exhibit better conceptual knowledge but also have fewer algorithmic bugs than children who have only learned the standard algorithms	Carpenter, Franke, Jacobs, Fennema, and Empson (1998)	Linking procedures and concepts

Appendix Table 2. Similarities between models and theories for learning and learners

Dimension	Core Features	Researcher	Model, Report or Theory
Teaching is	Connecting different ideas in the same area of mathematics using a variety of words symbols and diagrams Connecting different areas of the mathematics curriculum Pupils have strategies for calculating but the teacher has responsibility for helping them refine their methods Assisting pupils to develop efficient conceptually based strategies, and in doing so uses discussion and challenge to introduce links between different meanings and representations	Askew *et al.* (1997)	Connectionist Model
	Exploring meaning and connections through non-linear dialogue between teacher and learners Presenting problems before offering explanations.	Swan (2005)	Connected Challenging View
	Creating connections between mathematical topics	Swan (2005)	Principles for effective teaching
	Making links between procedural and conceptual knowledge. 'Developmental approach' is where procedural knowledge is used and then the outcome is reflected on which leads to a greater conceptual knowledge 'Educational approach' is where meaning is built for procedural knowledge before mastering it For most topics, the educational approach may be more relevant than the developmental one. However, the utilisation of an interplay of these approaches may, for some topics, be a better strategy than the application of one of them	Kadijevich and Haapasalo (2001)	Linking procedures and concepts
Role of the teacher	With the choice of carefully selected comparison tasks, after bringing students to some minimal knowledge level, teachers can encourage connections and generalizations by scaffolding	Peled and Segalis (2005)	Linking procedures and concepts
	The connections between mathematical ideas need to be acknowledged in teaching	Askew *et al.* (1997)	Connectionist Model
	Select tasks with goals in mind Share essential information Establish classroom culture	Hiebert *et al.* (1997)	Classrooms that promote understanding
	Develop effective questioning	Swan (2005)	Principles for effective teaching

Appendix Table 3. Similarities between models and theories for teaching and the role of the teacher

Dimensions	Core Features	Researcher	Model, Report or Theory
Nature of Classroom Tasks	Make mathematics problematic Leave behind something of mathematical value	Hiebert *et al.* (1997)	Classrooms that promote understanding
	Opportunities to make connections between language, pictures, symbols and concrete situations	Haylock and Cockburn (2017)	Understanding Mathematics
	Building relationships between conceptual knowledge and the formal symbol system of mathematics is the process that gives meaning to symbols Procedures can be used to promote concepts. (For example, where children use their counting procedures to develop the ordinal concept of number) If conceptual knowledge is linked to procedures, it can a) enhance problem representations and simplify procedural demands b) monitor procedure selection and execution c) promote transfer and reduce the number of procedures required	Hiebert and Lefevre (1986)	Linking procedures and concepts
Application	Is best approached through challenges that need to be reasoned about Pupils learn through being challenged and struggling to overcome difficulties Learning about mathematical concepts and the ability to apply these concepts are learned alongside each other	Askew *et al.* (1997)	Connectionist Model
	Students have confidence in finding new ways of getting there without outside help	Skemp (1976)	Relational Understanding
	Connecting procedures with their conceptual underpinnings is the key in producing procedures that are stored and retrieved more successfully Procedures can facilitate application of conceptual knowledge whereby highly routine procedures can be used thus reducing the mental effort required. This frees up space for other processes including planning how to tackle problems	Hiebert and Lefevre (1986)	Linking procedures and concepts

Appendix Table 4. Similarities between models and theories for the nature of classroom tasks and application

Dimension	Core Features	Researcher	Model, Report or Theory
Errors and Misconceptions	'Mistakes' involve learning.'.... if he does take a wrong turn he will .... be able to correct his mistake	Reason (2003)	Relational understanding
	Teachers believe mistakes are opportunities to reconceptualise a problem, explore contradictions and try out alternative strategies	Kazemi and Stipek (2008)	Socio-mathematical norm in conceptual classrooms
	Pupil misunderstanding needs to be recognised, made explicit and worked on.	Askew *et al.* (1997)	Connectionist Model
	Teaching is making misunderstandings explicit and learning from them	Swan (2005)	Connected Challenging View
	Mistakes are learning sites for everyone	Hiebert *et al.* (1997)	Classrooms that promote understanding
Mathematical tools as learning supports	Meaning for tools must be constructed by each user. They should be Used with purpose, to solve problems Used for recording, communicating and thinking	Hiebert *et al.* (1997)	Classrooms that promote understanding
	Viewed as cognitive aids symbols help to organize and operate on conceptual knowledge (e.g. place value notation)	Hiebert and Lefevre (1986)	Linking procedures and concepts
	Use technology in appropriate ways	Swan (2005)	Principles for effective teaching
Equity and accessibility	Tasks are accessible to all students Connect with where the students are	Hiebert *et al.* (1997)	Classrooms that promote understanding
	Build on the knowledge learners bring to sessions	Swan (2005)	Principles for effective teaching
	Most pupils can become numerate	Askew *et al.* (1997)	Connectionist Model

Appendix Table 5. Similarities between models and theories for the errors and misconceptions, mathematical tools and learning supports and equity and accessibility

Dimensions	Core Features	Researcher	Model, Report or Theory
Social culture of the classroom	Teachers ask students to justify their strategies mathematically – not simply a procedural description Teachers ask students to examine the mathematical similarities and differences among multiple strategies Teachers hold each student accountable for thinking through the mathematics in a problem Teachers promote the idea that consensus should be reached through mathematical argumentation	Kazemi and Stipek (2008)	Socio-mathematical norm in conceptual classrooms
	Emphasise methods rather than answers Use cooperative small group work	Swan (2005)	Principles for effective teaching
	Being numerate involves reasoning, justifying and, eventually, proving, results about number Pupils become numerate through purposeful interpersonal activity based on interactions with others. Numeracy teaching is based on dialogue between teacher and pupils to explore understandings. Teachers place a strong emphasis on developing reasoning and justification leading to the proof aspects of UAM High degree of focussed discussion between teacher and whole class, teacher and groups of pupils, teachers and individual pupils and pupil themselves Teachers work more actively with the pupils' explanations refining them and drawing pupils' attention to differences between methods, raising questions of efficiency.	Askew *et al.* (1997)	Connectionist Model
	Correctness resides in mathematical argument Every student is heard Every student contributes Ideas and methods are valued Students choose and share their methods	Hiebert *et al.* (1997)	Classrooms that promote understanding

Appendix Table 6. Similarities between models and theories for the social culture of the classroom

Appendix 1.2 Features of good and satisfactory mathematics teaching

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Lesson objectives involve understanding.	Lesson objectives are procedural, such as descriptions of work to be completed, or are general, such as broad topic areas.
Lesson activities are structured around key concepts and misconceptions, so that carrying out the activities enhances understanding; for example, involving pupils in developing suitable methods to solve problems, selecting questions carefully from exercises. Pupils can explain why a method works and solve again a problem they solved a few weeks earlier.	There is a successful focus on developing skills and obtaining correct answers rather than enhancing understanding; such as providing examples which do not illustrate why the method works, or doing questions identical to worked examples, too many of which are similar and are not carefully selected. These skills may be short-lived, so pupils cannot answer questions which they have completed correctly a few weeks earlier.
Work requires thinking and reasoning and enables pupils to compare approaches.	Methods are clearly conveyed by teachers and used accurately by pupils; pupils rely on referring to examples, formulae or rules rather than understanding or remembering them.
Practical, discussion and ICT work enhance understanding, for example, using demonstration and mental visualisation of shapes being rotated, with pairs deciding which method gives the correct answer and why.	Practical, discussion and ICT work is motivating and enables pupils to reach correct answers but is superficial and not structured well enough to enhance their understanding, such as unfocused pair work on a book exercise, group tasks where the highest attainer does all the work or free choice of hands-on ICT.
Pupils give explanations of their reasoning as well as their methods.	Questioning is clear and accurate but does not require explanation or reasoning; pupils describe the steps in their method accurately but do not explain why it works; for example, discussion activities enable pupils to share approaches but do not ensure they explain their reasoning.

Appendix Table 7. Understanding concepts and explaining reasoning: Features of good and satisfactory mathematics teaching (Ofsted, 2009, p. 4-6)

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Pupils spend enough time working to develop their understanding.	Teachers give effective exposition that enables pupils to complete work correctly but restricts the time they have to develop their understanding through their own work; for example, teachers talk for too long, pupils spend too long copying examples, notes or questions, or drawing diagrams.
Good use of subject knowledge capitalises on opportunities to extend understanding, such as through links to other subjects, more complex situations or more advanced mathematics.	Any small slips or vagueness in use of subject knowledge do not prevent pupils from making progress.
Teachers introduce new terms and symbols meaningfully; they expect and encourage correct use; pupils and teachers use mathematical vocabulary and notation fluently.	Teachers introduce new terms and symbols accurately and demonstrate correct spelling.
Lesson forms clear part of a developmental sequence and pupils recognise links with earlier work, different parts of mathematics or contexts for its use.	Lesson stands alone adequately but links are superficial, for example, pupils know it is lesson two of five on a topic but not how it builds on lesson one. Contexts or applications are mentioned without indicating how the mathematics may be used in a way the pupils can understand.
Non-routine problems, open-ended tasks and investigations are used often by all pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.	Typical lessons consist of routine exercises that develop skills and techniques adequately, but pupils have few opportunities to develop reasoning, problem solving and investigatory skills, or only the higher attainers are given such opportunities.

Appendix Table 7 continued. Understanding concepts and explaining reasoning: Features of good and satisfactory mathematics teaching (Ofsted, 2009, p. 4-6)

OFSTED. 2009. Mathematics: understanding the score - Improving practice in mathematics teaching at secondary level. London: Ofsted.

APPENDIX 2: CPD

Appendix 2.1 Collaborative Connected Classroom Model

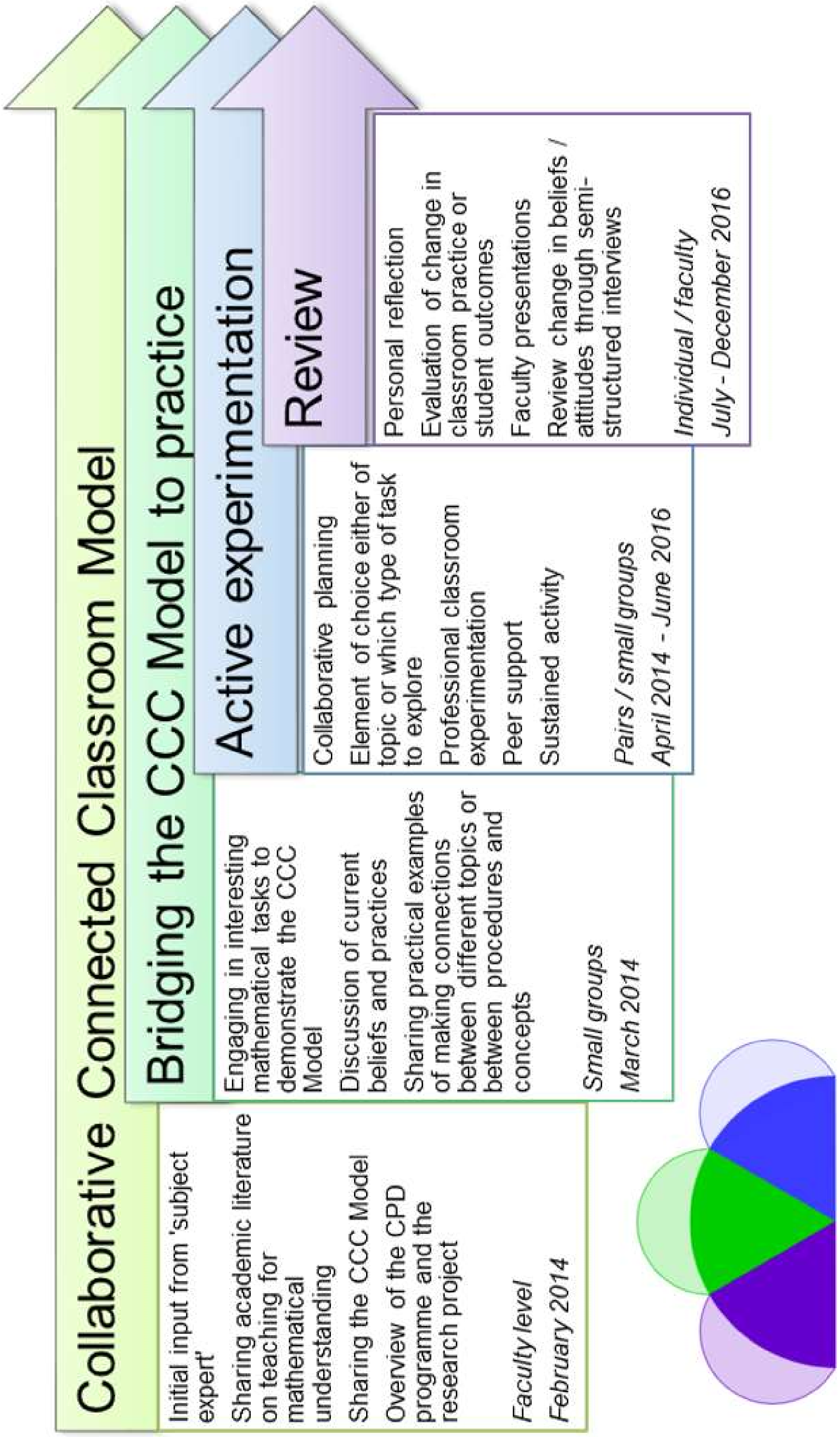
Teachers Beliefs about Mathematics and Learning	• Mathematics is a highly interconnected body of ideas that involves understanding and reasoning about concepts and the relationships between them • Mistakes should be recognised and made explicit. They are opportunities to reconceptualise a problem explore strategies and try out alternative strategies • Learning consists of building a conceptual structure whereby ideas are revisited and extended • Learning is a collaborative activity where learners are challenged to arrive at understanding through discussion

Nature of Mathematical Activity	• Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
	• Tasks either connect different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
	• Links are made between procedures and concepts
	    o meaning is built for procedural knowledge before mastering it ('educational approach')
	    o procedures are evaluated to promote conceptual understanding ('developmental approach')
	• Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
	• Application tasks are presented as challenges that may be problematic and need to be reasoned about

Social Culture of the Classroom	• Ideas and methods are valued and each student is held accountable for thinking through the mathematics in a problem until a consensus is reached. • There is an emphasis on reasoning and justification and not simply giving a procedural description • High degree of focussed non-linear discussion between teacher and groups of pupils, teachers and individual learners and between learners themselves • Discussion involves examining mathematical similarities/differences/connections among multiple strategies and refining learners' explanations
Characteristics of Learners	• Know what to do and why they are doing it • Know a range of concepts, symbols and procedures and how they are related. • Use strategies which are both efficient and effective • Are aware of connections within mathematics • Are confident in tackling unfamiliar problems

Appendix Table 8. Collaborative Connected Classroom Model

Appendix 2.2 CPD programme



Appendix Figure 1. CPD programme

Appendix 2.3 Research presentation

Slide 1

SHARING RESEARCH

Nicola Trubridge

Slide 2

Overview of Session

1. Personal journey and reflections
2. Exploring models of understanding
   - Relational vs. Instrumental
   - Conceptual vs. Procedural
   - Conceptual / Relational / Connectionist
3. Features of a classroom where there emphasis on Conceptual / Relational / Connectionist understanding
4. The Collaborative Connected Classroom Model

Slide 3

Initial Observation

- Do all teachers mean the same as me when they talk about mathematical understanding?

Slide 4

Findings from Literature Review

- Relational vs. Instrumental understanding
- Conceptual vs. Procedural understanding
- Conceptual / Relational / Connectionist
- Features of a classroom where there emphasis on Conceptual / Relational / Connectionist understanding

Slide 5

Relational vs. Instrumental

Relational and Instrumental understanding

- Skemp (1976)
- Byers and Herscovics (1977)
- Buxton (1978)
- Reason (2003)

Slide 6

Skemp (1976, p2)

- Defines relational understanding as ‘knowing both what to do and why’ and instrumental understanding as ‘rules without reasons’.

Slide 7

Skemp (1976, p8)

- Advantages that instrumental understanding might provide:
    1. Easier to understand
    2. Rewards are more immediate and more apparent
    3. Because less knowledge is involved, one can often get the right answer more quickly and reliably by instrumental thinking than relational

Slide 8

Skemp (1976, p8-10)

- Advantages that relational understanding might provide:
    1. It is more adaptable to new tasks
    2. It is easier to remember
    3. Relational knowledge can be effective as a goal in itself
    4. Relational schemas are organic in quality

Slide 9

Skemp (1976)

- Potential mismatch between the pupils' goals and the teachers' ideas

Slide 10

Skemp (1976, p14)

- 'Learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans from getting to any starting point within his schema to any finishing point'

Slide 11

Byers and Herscovics (1977, p26)

- **Instrumental understanding** is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.
- **Relational understanding** is the ability to deduce specific rules or procedures from more general mathematical relationships.
- **Intuitive understanding** is the ability to solve a problem without prior analysis of the problem.
- **Formal understanding** is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning.

Slide 12

Buxton (1978, p36)

- Four different levels of understanding
  1. Rote
  2. Observational
  3. Insightful
  4. Formal

Slide 13

Skemp (1987, p166)

- **Instrumental understanding** is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.
- **Relational understanding** is the ability to deduce specific rules or procedures from more general mathematical relationships.
- **Formal** [= Logical in my table] **understanding** is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning.

Slide 14

Reason (2003, p6)

RELATIONAL LEARNING	INSTRUMENTAL LEARNING
1. Learning consists of building up a conceptual structure	1. There is no awareness of overall relationship
2. Goal is 'to enlarge or consolidate my mental map'	2. Goal is to get 'required finishing points (answers)
3. 'Mistakes' involve learning.'.... if he does take a wrong term he will .... be able to correct his mistake	3. Mistakes result in being lost unless you can retrace your steps ' and get on the right path'
4. As schemas grow 'our awareness of possibilities enlarges'	4. Learning is merely the 'learning of an increasing number of fixed plans'
5.Works against memory limitations; much less memory work is involved	5. Relies on 'memorising ... a different method for every new class of problems'
6. Generates confidence in 'finding new ways of getting there without outside help	6. 'Learner dependent on outside guidance for learning each new way to get there'
7. An intrinsically satisfying goal in itself	7. Extrinsic rewards are necessary
8. Leads to enjoyment of mathematics	8. Leads to ultimate failure
9. 'It is easier to remember .... it is certainly harder to learn'	9. 'Within its own context ... easier to understand'

Slide 15

Conceptual vs. Procedural

Conceptual and Procedural understanding

- Hiebert and Lefevre (1986)
- Hiebert and Carpenter (1992)
- Miller and Hudson (2007)

Slide 16

Hiebert and Lefevre (1986, p3-4)

- Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information … a unit of conceptual knowledge cannot be an isolated piece of information; by definition it is part of conceptual knowledge only if the holder recognises its relationship to other pieces of information.

Slide 17

Hiebert and Lefevre (1986 p4,5)

- Development of conceptual knowledge
  - Primary level
  - Reflective level
- Procedural knowledge
  - Formal language
  - Algorithms
  - Sequential nature

Slide 18

Hiebert and Carpenter (1992, p78)

- Define conceptual knowledge so that identifies it with knowledge that is understood: 'Conceptual knowledge is equated with connected networks' and procedural knowledge is defined as a sequence of actions

Slide 19

Miller and Hudson (2007, p49)

- Acknowledge that conceptual knowledge involves understanding but adds the importance of relating it to the meaning of mathematics.

Slide 20

Byers and Herscovics (1977, p27)

- A good teacher can help a student to progress from intuitive understanding to formal understanding and similarly can support the move from instrumental to relational
- 'the effective learning of mathematics cannot be based on one type of understanding. Nor ... can the different kinds of understanding be arranged in a linear order'
- For optimal learning to happen the best approach is a spiral one so that 'different types of understanding are used consecutively and repeatedly at even greater depth'

Slide 21

Hiebert and Lefevre (1986, p10-16)

- Benefits of linking conceptual and procedural knowledge

Benefits for Procedural Knowledge	Benefits for Conceptual Knowledge
Developing meaning for symbols	Symbols enhance concepts
Recalling procedures	Procedures apply concepts to solve problems
Effective use of procedures	Procedures promote concepts

Slide 22

Conceptual / Relational / Connectionist

- Sfard (1991)
- Gray and Tall (1994)
- Kadijevich, D, & Haapasalo, L (2001)
- Peled and Segalis (2005)
- Rittle-Johnson and Alibali, (1999)
- Askew et al (1997)
- Swan (2005)
- ACME (2011)

Slide 23

Rittle-Johnsons and Alabali (1999)

- Investigated how conceptual instruction influenced children's problem solving procedures and how procedural instruction influenced conceptual understanding
- *'conceptual and procedural knowledge appear to develop iteratively, with gains in one type of knowledge leading to gains in another'* (p 188).

Slide 24

Kadijevich, D, & Haapasalo, L (2001)

1. Developmental approach
2. Educational approach

- *'For most topics, the educational approach may be more relevant than the developmental one. However the utilisation of an interplay of these approaches may, for some topics, be a better strategy than the application of one of them'* (p157)

Slide 25

Peled and Segalis (2005)

- Considered whether students could abstract mathematical principles by making connections between the procedures that they had learnt already
- *the effort to construct general principles pays off by improving performance and understanding in specific domains and by facilitating transfer to new situations'* (p208).

Slide 26

Askew et al (1997)

Types of teachers

- Connectionist
- Transmission
- Discovery
- Teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations.

Slide 27

Askew et al (1997, 35)

Teachers with a connectionist orientation towards numeracy teaching

Beliefs about what it is to be a numerate pupil	Being numerate involves: • the use of methods of calculation which are both efficient and effective; • confidence and ability in mental methods; • selecting a method of calculation on the basis of both the operation and the numbers involved; • awareness of the links between different aspects of the mathematics curriculum; • reasoning, justifying and, eventually, proving, results about number.

Slide 28

Askew et al (1997, 35)

Teachers with a connectionist orientation towards numeracy teaching

Beliefs about pupils and how they learn to become numerate	• Pupils become numerate through purposeful interpersonal activity based on interactions with others. • Pupils learn through being challenged and struggling to overcome difficulties. • Most pupils are able to become numerate. • Pupils have strategies for calculating but the teacher has responsibility for helping them refine their methods. • Pupil misunderstanding need to be recognised, made explicit and worked on.

Slide 29

Askew et al (1997, 36)

Teachers with a connectionist orientation towards numeracy teaching

Beliefs about how best to teach pupils to become numerate	• Teaching and learning are seen as complementary. • Numeracy teaching is based on *dialogue* between teacher and pupils to explore understandings. • Learning about mathematical concepts and the ability to apply these concepts are learned alongside each other. • The connections between mathematical ideas need to be acknowledged in teaching. • Application is best approached through challenges that need to be reasoned about.

Slide 30

Swan (2005, p5)

	A 'Transmission' View	'Connected', 'challenging' view
Mathematics is	A given body of knowledge and standard procedures that has to be 'covered'.	An interconnected body of ideas and reasoning processes.
Learning is	An individual activity based on watching, listening and imitating until fluency is attained.	A collaborative activity in which learners are challenged and arrive at understanding through discussion.
Teaching is	Structuring a linear curriculum for learners.	Exploring meaning and connections through non-linear dialogue between teacher and learners.
	Giving explanations before problems. Checking that these have been understood through practice exercises.	Presenting problems before offering explanations.
	Correcting misunderstandings.	Making misunderstandings explicit and learning from them.

Slide 31

ACME (2011, p1)

- *'Mathematics is a highly interconnected subject that involves understanding and reasoning about concepts, and the relationships between them. It is learned not just in successive layers, but through revisiting and extending ideas. As such, the mathematical needs of learners are distinctive from their more general educational needs. For mathematical proficiency, learners need to develop procedural, conceptual and utilitarian aspects of mathematics together.'*

Slide 32

Features of a classroom where there emphasis on Conceptual / Relational / Connectionist understanding

- Hiebert et al (1997)
- Haylock (1982)
- Kazemi, E, and Stipek, D (2008).

Slide 33

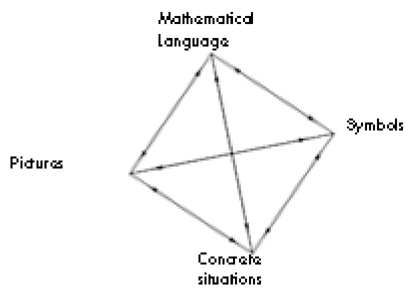
Hiebert et al (1997, p12)

Dimensions	Core Features
Nature of classroom tasks	Make mathematics problematic Connect with where the students are Leave behind something of mathematical value
Role of the teacher	Select tasks with goals in mind Share essential information Establish classroom culture
Social culture of the classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument
Mathematical tools as learning supports	Meaning for tools must be constructed by each user Used with purpose -- to solve problems Used for recording, communicating and thinking
Equity and accessibility	Tasks are accessible to all students Every student is heard Every student contributes

Slide 34

Haylock (1982, p55)

- Connections between language, pictures, symbols and concrete situations.



Slide 35

Kazemi, E, and Stipek, D (2008).

	Socio Norm	Sociomathematical Norm
Topic	Students	Teachers
Dialogue	Students describe their thinking	Ask students to justify their strategies mathematically – not simply a procedural description
Strategies	Students find multiple ways to solve problems, and they describe their strategies	Ask students to examine the mathematical similarities and differences among multiple strategies
Errors	Students can make mistakes as part of the learning process	Believe mistakes are opportunities to reconceptualise a problem, explore contradictions and try out alternative strategies
Engagement in discussion	Students collaborate to find solutions	Hold each student accountable for thinking through the mathematics in a problem Promote the idea that consensus should be reached through mathematical argumentation

Slide 36

Comparing Models

- Features of a Conceptual / Relational / Connectionist Classroom

Slide 37

Collaborative Connected Classroom

- Teachers Beliefs about Mathematics and Learning
- Nature of Mathematical Activity
- Social Culture of the Classroom
- Characteristics of learners

Slide 38

Teachers Beliefs about Mathematics and Learning

- Mathematics is a highly interconnected body of ideas and that involves understanding and reasoning about concepts and the relationships between them
- Mistakes should be recognised and made explicit. They are opportunities to reconceptualise a problem explore strategies and try out alternative strategies
- Learning consists of building a conceptual structure whereby ideas are revisited and extended
- Learning is a collaborative activity where learners are challenged to arrive at understanding through discussion

Slide 39

Nature of Mathematical Activity 1

- Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema
- Tasks either connect together different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
- Links are made between procedures and concepts
  - meaning is built for procedural knowledge before mastering it ('educational approach')
  - procedures are evaluated to promote conceptual understanding ('developmental approach')

Slide 40

Nature of Mathematical Activity 2

- Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
- Application tasks are presented as challenges that may be problematic and need to be reasoned about

Slide 41

Social Culture of the Classroom

- Ideas and methods are valued and each student is held accountable for thinking through the mathematics in a problem until a consensus is reached.
- There is an emphasis on reasoning and justification and not simply giving a procedural description
- High degree of focussed non-linear discussion between teacher and groups of pupils, teachers and individual learners and between learners themselves
- Discussion involves examining mathematical similarities/differences/connections among multiple strategies and refining learners explanations

Slide 42

Characteristics of Learners

- Know what to do and why they are doing it
- Know a range of concepts, symbols and procedures and how they are related.
- Use strategies which are both efficient and effective
- Are aware of connections within mathematics
- Are confident in tackling unfamiliar problems

Slide 43

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Slide 44

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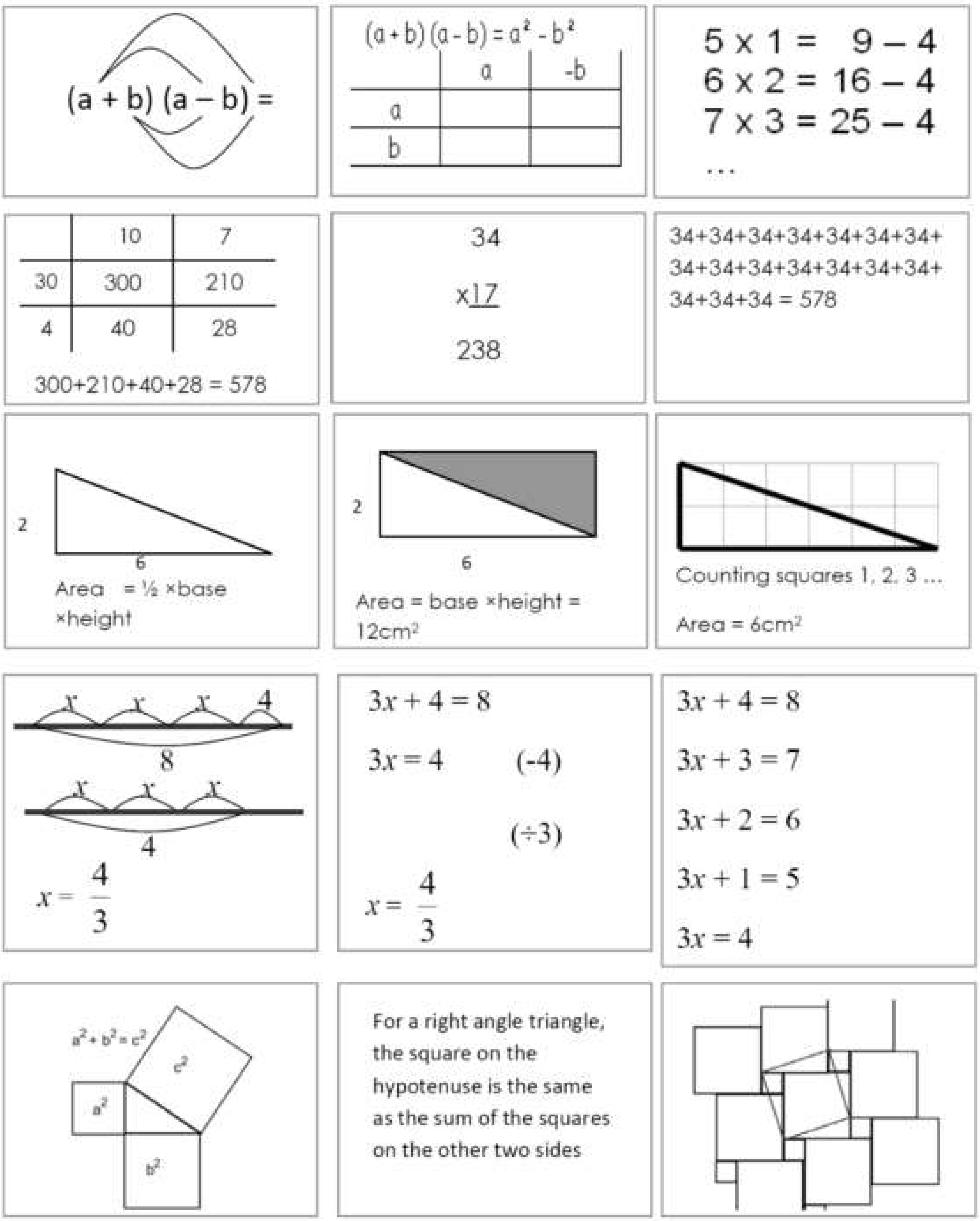
Appendix 2.4 Instrumental and relational card sort



NCETM. 2010. *Mathematics Departmental Workshops: Effective day to day provision for able, gifted and talented students Instrumental and relational card sort* [Online]. NCETM. Available: https://www.ncetm.org.uk/public/files/705724/Resource+sheet+3.pdf. *Permission to reproduce this has been granted by the NCETM*

Appendix 2.5 Area of parallelogram scenarios CPD task

Learning Objective: We are learning to find the area of a parallelogram

---

Scenario 1:
Outcomes: Students will:

- evaluate simple algebraic expressions by substituting natural numbers for the variables

Lesson:

1. Teacher writes on the board the formula for the area of a parallelogram.
2. Teacher does several examples on how to use the formula.
3. Students complete similar questions on their own, using the examples as models.

Scenario 2:
Outcomes: Students will:

- evaluate simple algebraic expressions by substituting natural numbers for the variables
- develop the formula for finding the area of a parallelogram

Lesson:

1. Teacher explains to students that the formula for the area of a parallelogram is related to the formula for the area of a rectangle, since the parallelogram can be cut and reassembled into a rectangle.
2. Using the developed formula, teacher then provides examples of how to use the formula to find the areas of parallelograms.
3. Students complete several questions using the examples as models.

Scenario 3:
Outcomes: Students will:

- develop the formula for finding the area of a parallelogram
- describe measurement concepts using appropriate measurement vocabulary
- solve problems involving the congruence of shapes
- identify two-dimensional shapes that meet certain criteria
- recognise patterns and use them to make predictions
- interpret a variable as a symbol that may be replaced by a given set of numbers
- translate simple statements into algebraic expressions or equations
- evaluate simple algebraic expressions by substituting natural numbers for the variables

Concepts: geometric vocabulary, properties of 2-D shapes, area, base, height, congruence

Part 1:

Outcomes: Students develop and practice geometry vocabulary, and form a concept of base and height of a parallelogram.

1. Distribute a parallelogram diagram to each group of 2 students.
2. Without showing the diagram to student B, student A will give instructions to student B that will enable student B to draw a parallelogram congruent to student A's diagram.
3. Using a different diagram, the students switch roles.
4. After the activity, students will share their drawing instructions in the large group.
5. Vocabulary can be recorded and clarified with diagrams.
6. Teacher should make sure that base and height are clearly established.

Facilitating questions:

Q. What are some other ways of saying that?

Q. What does that mean?

Part 2:

Purpose: Students calculate the areas of parallelograms by counting, refine their counting method using calculations, and arrive as a class at the formula for area of a parallelogram.

1. Ask students to find the areas of several different parallelograms by measuring them with a grid and counting the squares.
2. Ask them if there is a way to make the counting faster.
3. Some groups will discover the congruent triangles on either end of the parallelogram, put together, make up full squares.
4. Other groups will notice that the base and height are related to the area of the parallelogram.
5. Students must be able to defend their calculation method by explaining how it works.
6. In the large group, students will share their observations from the activity, build on each other's answers, and establish the formula for the area of a parallelogram.
7. Students can test the formula on a few parallelograms by calculating the area by multiplying base and height, by calculating the area by counting squares, and then comparing.

Part 3:

Purpose: Consolidating understanding.

1. Ask students to draw on grid paper a parallelogram that has an area of 15 square units.
2. Compare the diagrams.
3. There are an infinite number of these; students will probably produce parallelograms with a base and a height using 3 and 5 units, but notice that the sides have different slants.
4. Ask students to draw on grid paper a parallelogram that has an area of 24 square units. Compare the diagrams.

Extension: Ask students to draw a triangle that has an area of 12 square units. Have them explain how they know their answer is correct.

Notes from scenarios

Some things to note about scenario 1 are these:

- The students do not know how or why the formula works.
- The formula needs to be memorized to be used again.
- Students can complete the exercise and get correct answers without having an understanding of what area is.
- The lesson only addresses one expectation.

Some things to note about scenario two are these:

- The teacher does the math (i.e. achieves the outcome), and explains it to the students.
- There is only one explanation for the development of the formula; there is no opportunity for students to develop alternative explanations.
- Students can complete the exercise and get correct answers without having understood the explanation, and without having an understanding of what area is.

Some things to note about scenario three are these:

- All students can participate in this activity.
- The tasks asked of students can be handled in more than one way.
- The tasks allow students to practice using language.
- The tasks help students to develop concepts for themselves, and relate procedures to concepts.
- The extension question sets the stage for developing the formula for the area of a triangle
- The lesson takes longer than one day.
- The lesson allows students to meet or revisit several expectations.

Scenarios adapted from
http://www.wcdsb.ca/programs/curriculum/math/pdf/math-developing-doc.pdf

Waterloo Catholic District School Board Math Action Team (2003) *'There's More to Math. A Framework for Learning and Instruction'*

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Appendix 2.6 CPD presentation 7th March 2013

Slide 1

SETTING THE SCENE

07/03/13 Nicola Trubridge

Slide 2

Work so far

- Chapter 1. Rationale for my research
- Chapter 2. Literature review leading to theoretical framework 'The collaborative connected classroom'
- Chapter 3. Literature review leading to proposed model of CPD
- Chapter 4. Exploring research methods and methodologies (i.e. pilot study!)

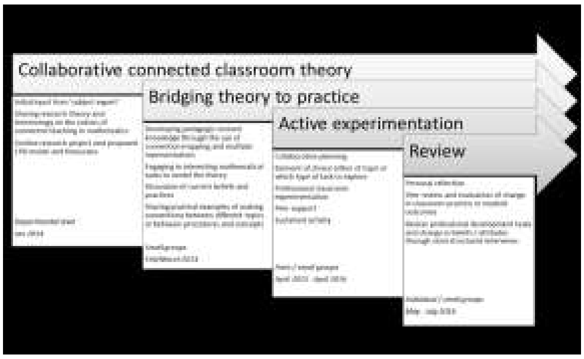
Slide 3

Ethics and Consent

- Permission from headteacher
- Ethical clearance from Plymouth University Ethics Committee
- Consent forms
- Research Information form

Slide 4

CPD Model for Promoting Connected Teaching



Slide 5

School Improvement Agenda

- Securing increased numbers of students attaining a grade C
- Securing increased numbers of pupils making at least expected levels of progress.

- Wave 3 vs Wave 1 intervention?

Slide 6

Mathematics Improvement Agenda

- Positive attitude to the department and the subject
- Increased intake to A Level Mathematics and Further Mathematics
- Awareness of transferability of skills to the workplace or other courses.

Slide 7

Professional Development

- 'Appropriate self-evaluation, reflection and professional development activity is critical to improving teachers' practice at all career stages. The standards set out clearly the key areas in which a teacher should be able to assess his or her own practice, and receive feedback from colleagues. As their careers progress, teachers will be expected to extend the depth and breadth of knowledge, skill and understanding that they demonstrate in meeting the standards, as is judged to be appropriate to the role they are fulfilling and the context in which they are working.' (DfE, 2012, p4)

Slide 8

Professional Development Planning

- Which areas of the mathematics curriculum do you feel would be beneficial in devoting professional development time to?
- Consider in your response
  - Which areas of mathematics do our students struggle to 'understand' (retain/recall and apply)?
  - Are there any areas where you would personally like to explore new/different ways of teaching?

Slide 9

Response 1 March 2013

- Algebra – factorising
- Circles – volumes of cylinders
- Angles in polygons
- Irrational numbers
- Practise of computation

Slide 10

Response 2 March 2013

- Solving equations
- Simplifying
- Indices
- Straight line graphs
- Proportion – fractions of quantities

Slide 11

Response 3 March 2013

- Fractions/Decimals/Percentages
- Geometrical Reasoning and Proofs
- Place value
- Ratio and Proportion
- Contsructions and loci (esp loci)
- Transformations

Slide 12

Response 4 March 2013

- Direct and Inverse proportion
- Reasons to use circle theorems
- Units – conversions
- Inequalities
- Multiplying out brackets
- Fractions/decimals/percentages

Slide 13

Response 5 March 2013

- Units of measure – Imperial/Metric
- Multiplying out brackets
- y=mx + c gradients and intercepts
- Concept of negative numbers

Slide 14

Response 6 March 2013

- Further Pure 4
- Vectors and Matrices

Slide 15

Response 7 March 2013

- Introduction to solving equations
- Graph sketching and how to sketch key points algebraic techniques – link to solving etc
- Multiplication of negatives
- Fractions and how they work

Appendix 2.7 CPD presentation 3rd and 18th March 2014

Slide 1

BRIDGING THEORY TO PRACTICE

03/03/14

Nicola Trubridge

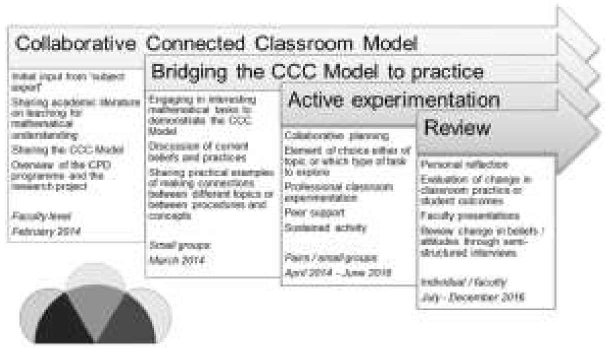
Slide 2

Aims of CPD programme to develop connected approaches to mathematics

- to develop teacher's mathematical subject and pedagogical content knowledge through collaborative working
- for teachers to be challenged and inspired by new ideas or ways of working
- for sustained active classroom experimentation
- to be personalised to the teacher's and department's needs
- for participants to be supported by a subject expert to engage with research documentation

Slide 3

CPD Model for Promoting Connected Teaching



Slide 4

Collaborative connected classroom theory

- Teachers Beliefs about Mathematics and Learning
- Nature of Mathematical Activity
- Social Culture of the Classroom
- Characteristics of learners

Slide 5

Bridging theory to practice

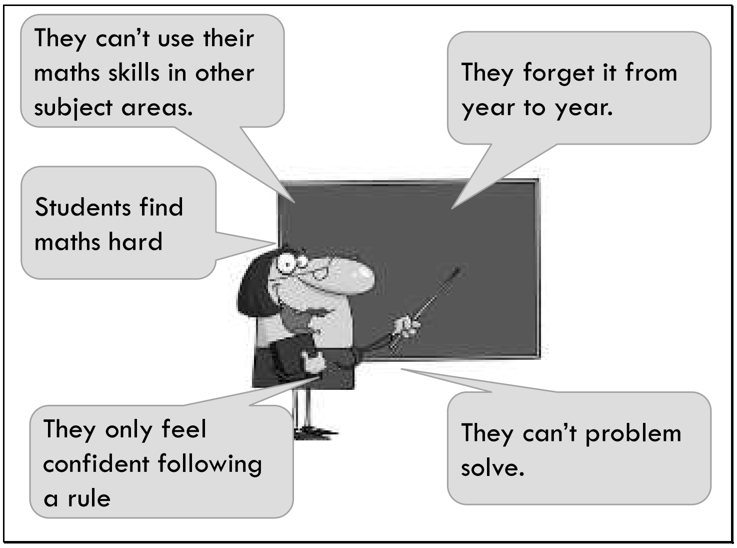
- Developing pedagogic content knowledge through the use of connection mapping and multiple representations
- Engaging in interesting mathematical tasks to model the theory
- Discussion of current beliefs and practices
- Sharing practical examples of making connections between different topics or between procedures and concepts

Slide 6

Programme for 3rd March 2014

The big picture	(5 mins)
Summary of research presentation	(5 mins)
Activity to consolidate terminology	(15 mins)
Compare and contrast scenarios	(15 mins)
Maths activities	(70 mins)
Where next	(10 mins)

Slide 7



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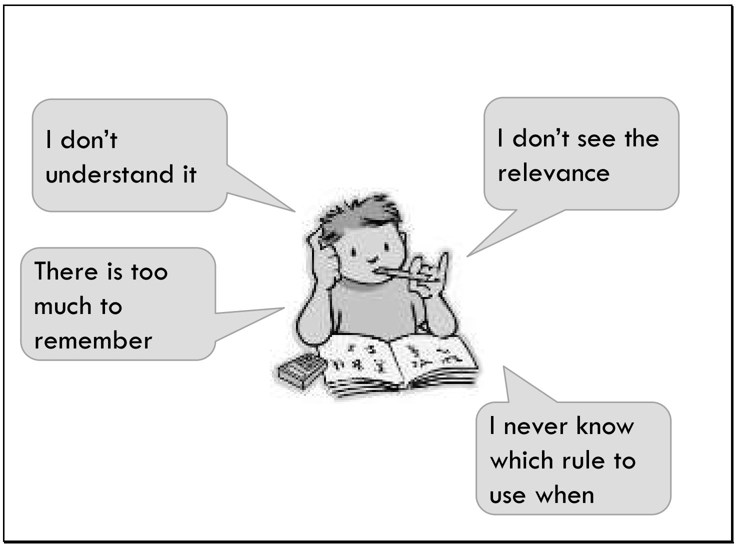
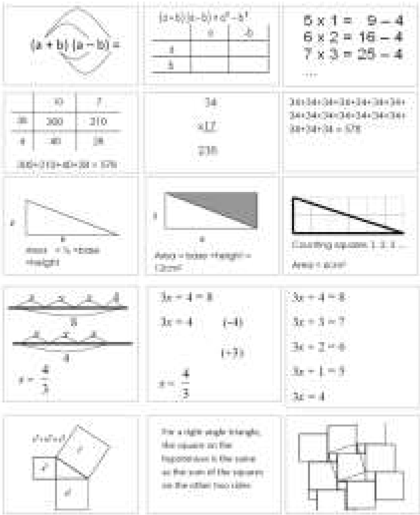


Slide 9

Activity: Instrumental vs. Relational

- Consider the cards
- Would you describe them as instrumental or relational approaches?
- Classify your cards and annotate your thoughts

Slide 10



Slide 11

Area of a parallelogram

- Imagine you were to teach area of parallelograms
- Compare the three different teaching scenarios on your **handout**
- What do you notice from a teaching and learning perspective?

Scenarios adapted from There's More to Math. A Framework for Learning and Instruction
http://www.wcdsb.ca/programs/curriculum/math/pdf/math-developing-doc.pdf

Slide 12

Scenario 1

- The students do not know how or why the formula works.
- The formula needs to be memorized to be used again.
- Students can complete the exercise and get correct answers without having an understanding of what area is.
- The lesson only addresses one outcome

Slide 13

Scenario 2

- The teacher does the math (i.e. achieves the outcome), and explains it to the students.
- There is only one explanation for the development of the formula; there is no opportunity for students to develop alternative explanations.
- Students can complete the exercise and get correct answers without having understood the explanation, and without having an understanding of what area is.

Slide 14

Scenario 3

- All students can participate in this activity.
- The tasks asked of students can be handled in more than one way.
- The tasks allow students to practice using language.
- The tasks help students to develop concepts for themselves, and relate procedures to concepts.
- The extension question sets the stage for developing the formula for the area of a triangle
- The lesson takes longer than one day.
- The lesson allows students to meet or revisit several outcomes

Slide 15

Aims of CPD programme to develop connected approaches to mathematics

- to develop teacher's mathematical subject and pedagogical content knowledge through collaborative working
- for teachers to be challenged and inspired by new ideas or ways of working
- for sustained active classroom experimentation
- to be personalised to the teacher's and department's needs
- for participants to be supported by a subject expert to engage with research documentation

Slide 16

You were asked March 2013

- Which areas of the mathematics curriculum do you feel would be beneficial in devoting professional development time to?
- Consider in your response
  - Which areas of mathematics do our students struggle to 'understand' (retain/recall and apply)?
  - Are there any areas where you would personally like to explore new/different ways of teaching?

Slide 17

Departmental Responses

- Circles – volumes of cylinders
- Angles in polygons
- Irrational numbers
- Practise of computation
- Indices
- Geometrical Reasoning and Proofs
- Constructions and loci (esp loci)
- Transformations
- Reasons to use circle theorems
- Inequalities
- Graph sketching and how to sketch key points algebraic techniques – link to solving etc

- Algebra
  - Factorising, Simplifying, Expanding (4)
  - Solving equations (2)
  - Straight line graphs (2)
- Number
  - Ratio and Proportion (3)
  - Fractions
  - Units – conversions (2)
  - Fractions/decimals/percentag es (2)
  - Place value
  - Negative Numbers (2)

Slide 18

Theory underpinning the session

Trubridge & Graham (2013) Proceedings of the British Society for Research into Learning Mathematics 33 (2)

http://www.bsrlm.org.uk/IPs/ip33-2/BSRLM-IP-33-2-09.pdf

Resources:

- Secondary National Strategy
- Cornwall Learning
- NCETM
- Standards Unit
- MEI
- N.Trubridge

Slide 19

CCC: Nature of Mathematical Activity 1

- Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema
- Tasks either connect together different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
- Links are made between procedures and concepts
  - meaning is built for procedural knowledge before mastering it (‘educational approach’)
  - procedures are evaluated to promote conceptual understanding (‘developmental approach’)

Slide 20

CCC: Nature of Mathematical Activity 2

- Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
- Application tasks are presented as challenges that may be problematic and need to be reasoned about

Slide 21

Pupils' perceptions of algebra

I just don't get algebra!

What do your pupils struggle with most?

Slide 22

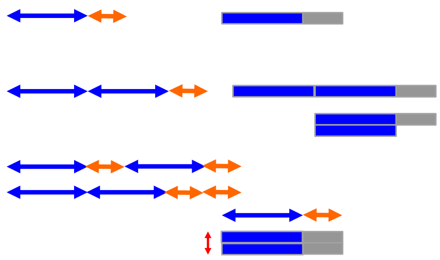
Discussion

Why do you think we should teach algebra?

Slide 23

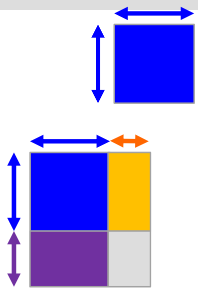
Show me a 'picture' for the following

- n + 3
- 2n + 3
- 2(n + 3)



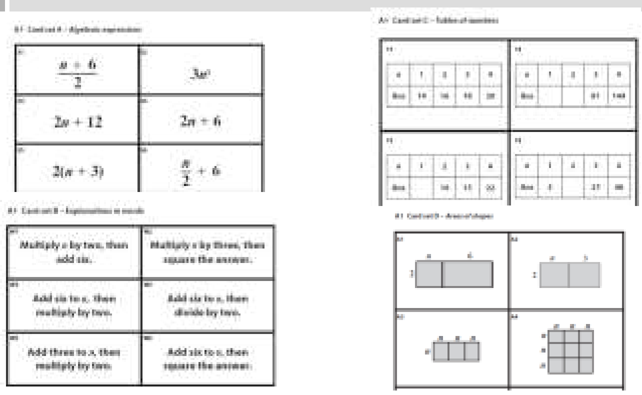
Slide 24

What could these represent?



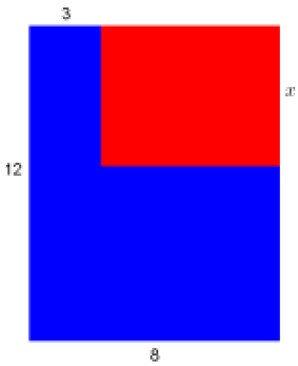
Slide 25

Standards Unit A1: Interpreting algebraic expressions



Slide 26

Write an expression for the blue area



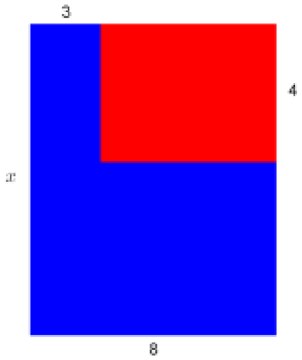
$96-5x$

$36+5(12-x)$

$8(12-x)+3x$

Slide 27

Write an expression for the blue area



Slide 28

Always, Sometimes or Never True

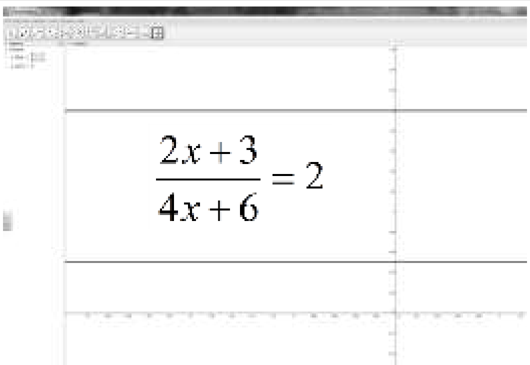
For your given cards divide them into 3 different groups

1. Always true
2. Sometimes true
3. Never true

Be prepared to justify your reasons

Slide 29

Never True



Slide 30

Always True

$x + x = 2x$

$4(x + 3) = 4x + 12$

$(x + 3)(x + 2) = x^2 + 5x + 6$

$(x + 6)^2 = x^2 + 12x + 36$

$x^2 + 8x + 12 = (x + 4)^2 - 4$

$(n + 1)(n - 1) = n^2 - 1$

$a^2 - b^2 = (a + b)(a - b)$

$cos^2\theta + sin^2\theta = 1$

Show me visually WHY these are always true

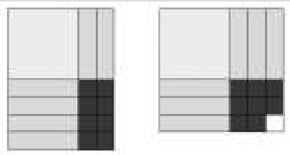
Identities

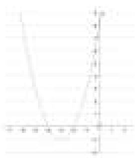
$\equiv$

Slide 31

The missing square

- Match together equivalent cards
- How does the use of multiple representations help to understand the concept of completing the square?





$(x + 4)(x + 2)$

$x^2 + 6x + 8$

$(x + 3)^2 - 1$

Slide 32

Pairs of factors that differ by 2

We are learning to use algebra to express generality

Outcome

- By the end of the activity you will be able to find a pair of factors of 359999 that differ by 2

Slide 33

Expressing generality

$0 \times 2 =$
$1 \times 3 =$
$2 \times 4 =$
$3 \times 5 =$
$4 \times 6 =$

Slide 34

Expressing generality

$0 \times 2 = 1^2 - 1$
$1 \times 3 = 2^2 - 1$
$2 \times 4 = 3^2 - 1$
$3 \times 5 = 4^2 - 1$
$4 \times 6 = 5^2 - 1$

$(n-1)(n+1) = n^2 - 1$
$n(n+2) = (n+1)^2 - 1$

Slide 35

Extend backwards to include negative numbers?

$(-3) \times (-1) = (-2)^2 - 1$
$(-2) \times 0 = (-1)^2 - 1$
$(-1) \times 1 = 0^2 - 1$
$0 \times 2 = 1^2 - 1$
$1 \times 3 = 2^2 - 1$

Slide 36

Work out a pair of factors that differ by 2

899

3599

4899

359999

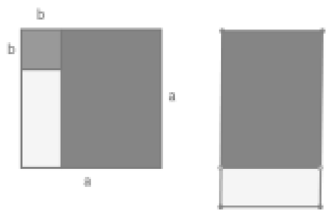
$(n-1)(n+1) = n^2 - 1$

$n(n+2) = (n+1)^2 - 1$

Slide 37

Visual Identity

$$a^2 - b^2 = (a-b)(a+b)$$



Slide 38

As many ways as you can

Using the commutative law and inverses, write the equation as many ways as you can until you find an equation you can solve in your head or with jottings

E.g. $2x + 5 = 13$

$5 + 2x = 13$

$2x - 8 = 0$

$5 - 13 = -2x$

$x + x + 5 = 13$

$2x = 13 - 5$

$5 - 5 + 2x = 13 - 5$

$5 + 2x = 8 + 5$

$2x = 8$

Slide 39

As many ways as you can

'as many ways as you can' requires an understanding of:

- The order of operations
- Commutative law
- Inverses

This is very different from the application of a given rule such as 'change side , change the sign'.

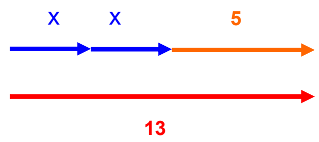
This is the key to seeing algebra as generalised arithmetic.

Slide 40

Matching, using a number line

The matching method involves transforming one or both sides and matching terms, so they can be cancelled. This is supported with the use of a number line

E.g. 2x + 5 = 13



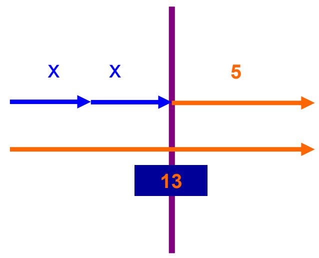
Slide 41

Matching, using a number line

By separating the 13 into 8 + 5 you can match the 5

E.g. 2x + 5 = 13

2x + 5 = 8 + 5



Slide 42

Matching, using a number line

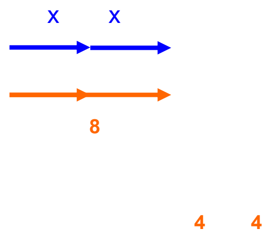
The 5 will then cancel and you are left with a simple equation to solve

E.g. $2x + 5 = 13$

$2x + 5 = 8 + 5$

$2x = 8$

$x = 4$



Slide 43

Matching, using a number line

Solve the following equations using a number line:

1. $3x + 10 = 22$
2. $4m + 1 = 3m + 6$
3. $4(n + 1) = 40$
4. $5x - 3 = 2x + 9$
5. $4x+3=6x+7$

Do the number line diagrams help to support this technique?

Slide 44

Matching, using a number line

Matching requires an understanding of:

- The order of operations
- Use of conventions including brackets
- Ability to construct number line diagram

Most suitable for equations with positive integer coefficients and positive solutions

Slide 45

Balancing

With balancing you do the same operation to both sides of the equation. Eventually you simplify one side to the unknown

E.g.	5x - 2	=	22 + x	
(-x)	5x – 2 –x	=	22 + x –x	(-x)
	4x – 2	=	22	
(+2)	4x – 2 +2	=	22 +2	(+2)
	4x	=	24	
(÷4)	4x/4	=	24/4	(÷ 4)
	x	=	6	

Slide 46

Balancing

Balancing requires an understanding of:

- The order of operations and use of conventions
- Inverses operations to support cancelling

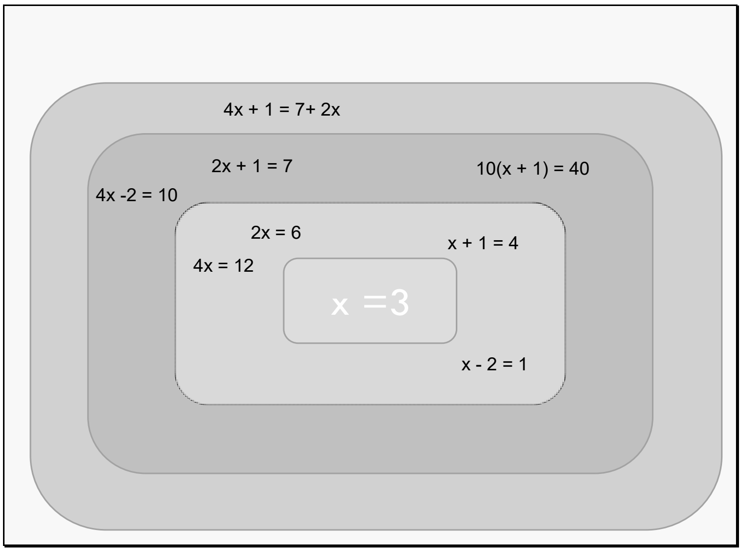
Suitable for all equations, but where do students make their mistakes ?

Slide 47

Which would your students find easy/hard?

2 -2x = 11 - 5x	12x + 8 = 44
0.3x + 0.1 = 1	3x = 9
0.03x + 0.01 = 0.1	3x + 2 = 11
0.06x + 0.04 = 0.22	3x + 1 = 10
$x + {}^{1}/_{3} = 3{}^{1}/_{3}$	6x + 4 = 22
1.5 x + 0.5 = 5	9x + 6 = 33
4x + 2 = 11 + x	6x + 2 = 11 + 3x
4 = 22 - 6x	2 = 11 - 3x

Slide 48



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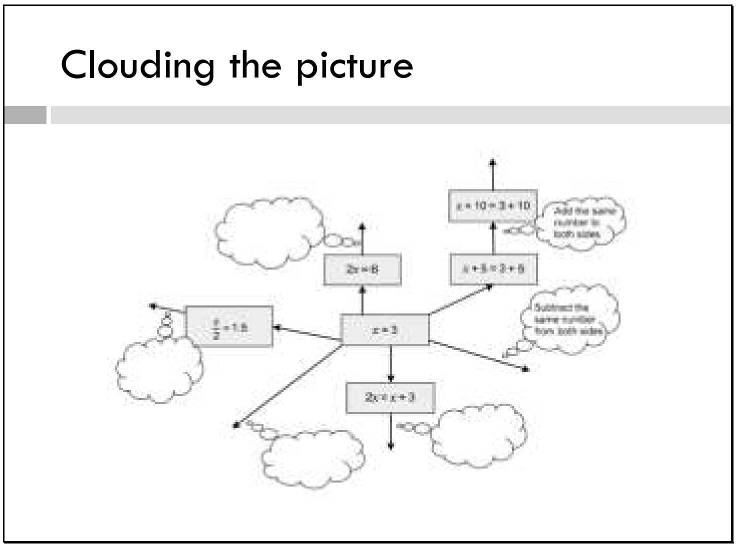

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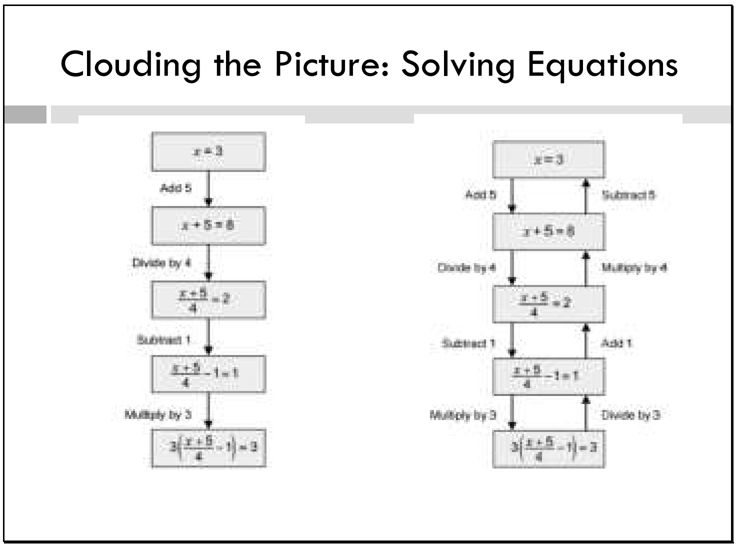
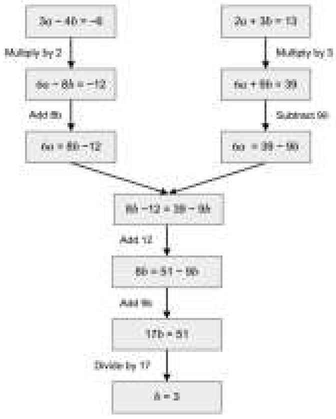
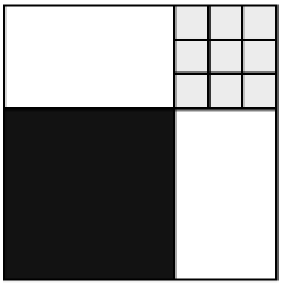


Slide 51

Clouding the picture: Simultaneous Equations



Slide 52



$$(x+3)^2 \neq x^2 + 9$$

Slide 53

Exploring Tasks Using the Ideas Developed

1. Squaring numbers ending in a 5
2. Sum of the squares of two consecutive integers is one greater than twice the product of the integers
3. Adding two square numbers and doubling the answer
4. Four digit numbers
5. Linear Equations by considering graphs and geometry

Slide 54

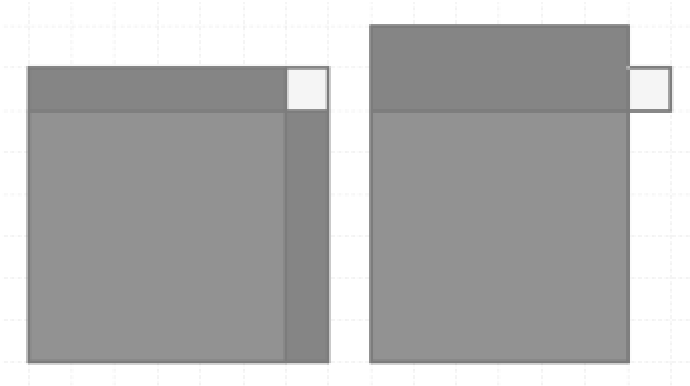
Activity 1

Squaring numbers ending in a 5

$$35^2 = \underbrace{12}_{3\times 4}\underbrace{25}_{5^2} \qquad 195^2 = \underbrace{380}_{19\times 20}\underbrace{25}_{5^2}$$

Slide 55

Activity 1



Slide 56

Activity 2

"Reasoning and proof were not well developed in most of the secondary schools visited…A recent GCSE examination question asked pupils to prove that 'the sum of the squares of two consecutive integers is one greater than twice the product of the integers' and provided an illustrative example, $9^2 + 10^2 = 181$ and $2 \times 9 \times 10 = 180$. The principal examiner's report stated that 'the concepts of algebraic proof were rarely demonstrated well'." (Ofsted: Made to Measure)

Slide 57

Activity 2 Solution

Slide 58

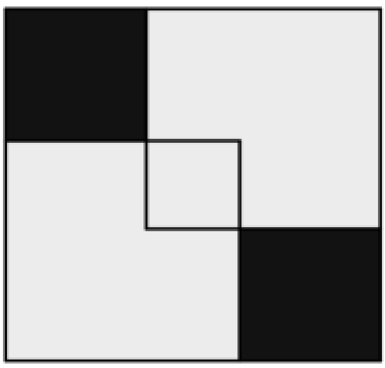
Activity 3

Add two square numbers and double the answer.

Prove that the result is also the sum of two squares.

Slide 59

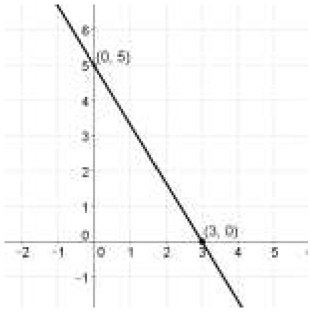
Activity 3 Solution



Slide 60

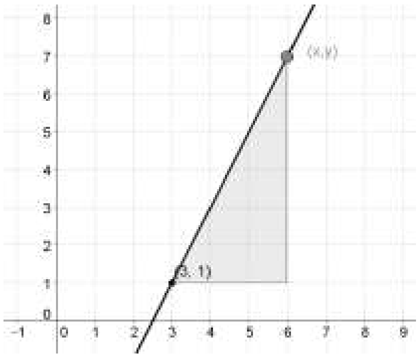
Activity 4: Linear Equations

- What methods could you use to name this graph?



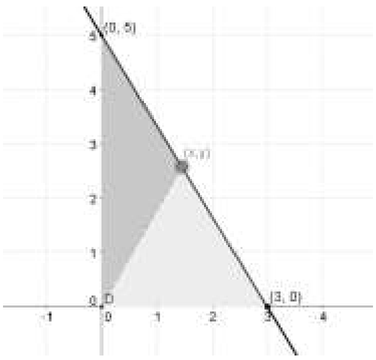
Slide 61

Activity 4



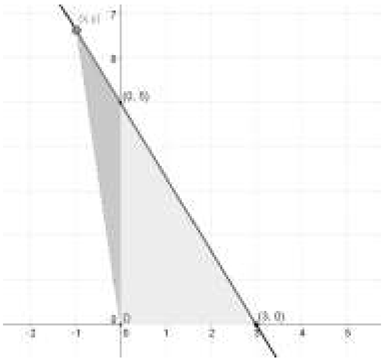
Slide 62

Activity 4 (GeoGebra link)



Slide 63

Activity 4



Slide 64

BRIDGING THEORY TO PRACTICE PART 2

18/03/14

Nicola Trubridge

Slide 65

Aims of CPD programme to develop connected approaches to mathematics

- to develop teacher's mathematical subject and pedagogical content knowledge through collaborative working
- for teachers to be challenged and inspired by new ideas or ways of working
- for sustained active classroom experimentation
- to be personalised to the teacher's and department's needs
- for participants to be supported by a subject expert to engage with research documentation

Slide 66

Bridging theory to practice

- Developing pedagogic content knowledge through the use of connection mapping and multiple representations
- Engaging in interesting mathematical tasks to model the theory
- Discussion of current beliefs and practices
- Sharing practical examples of making connections between different topics or between procedures and concepts

Slide 67

Programme for 18th March

- Reflections on last session (5 mins)
- Sharing good practice (10 mins)
- Additional activities to bridge theory to practice
- Collaborative working

Slide 68

Active Experimentation

- Collaborative planning
- Element of choice either of topic or which type of task to explore
- Professional classroom experimentation
- Peer support
- Sustained activity

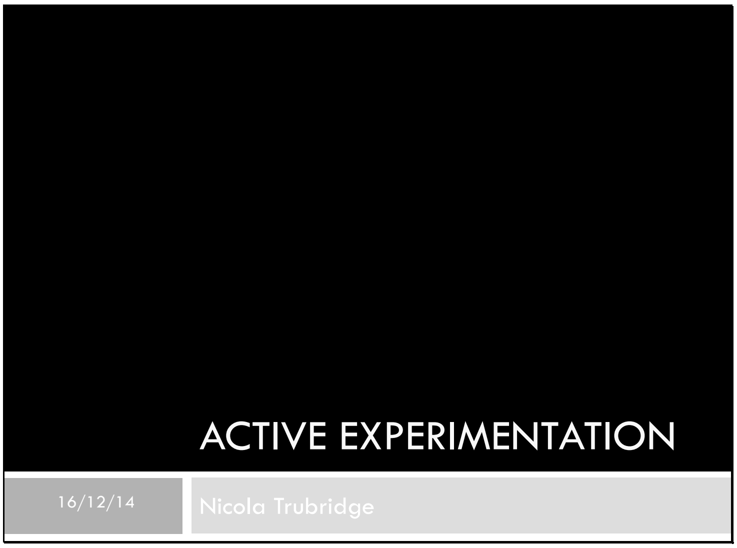
Slide 69

Things to consider

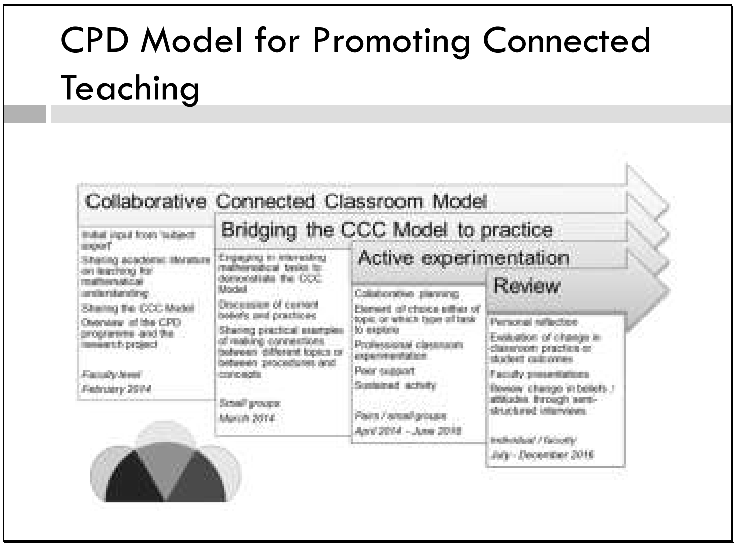
- Are their any procedures you would like to have a deeper conceptual understanding of?
- Are there any specific strategies i.e. multiple representations or similarities and differences that you might like to explore?
- Are there connections that could be made between mathematical topics that you would like to explore?

Appendix 2.8 CPD presentation 16th December 2014

Slide 1



Slide 2



Slide 3

Bridging theory to practice

- Developing pedagogic content knowledge through the use of connection mapping and multiple representations
- Engaging in interesting mathematical tasks to model the theory
- Discussion of current beliefs and practices
- Sharing practical examples of making connections between different topics or between procedures and concepts

Slide 4

Active Experimentation

- Collaborative planning
- Element of choice either of topic or which type of task to explore
- Professional classroom experimentation
- Peer support
- Sustained activity

Slide 5

Initial feedback: Presentations etc.

- Activities made sense of aspects of the research
- Liked different tasks e.g. similarities and differences, completing the square, use of play dough
- Time to work with colleagues

Slide 6

Active Experimentation

- Used activities modelled at CPD sessions e.g. play dough, multiple representations for multiplying out brackets etc
- Developed additional similarities and differences tasks for pupils to use

Slide 7

Barriers/Constraints to moving on

- Time constraints
- Class constraints
- Difficult to come up with multiple representations
- Not sure how to develop the concepts/subject knowledge

Slide 8

Suggested ways forward

- More time to refresh ideas
- Build into scheme of learning
- Look at topics we are approaching together

Slide 9

Transformations and Coordinates

- Exploring the big picture/big ideas?
- Developing our own pedagogic content knowledge
- How can we incorporate this ‘new’ knowledge into our teaching
  - Similarities and differences
  - Multiple representations
  - Developing concepts

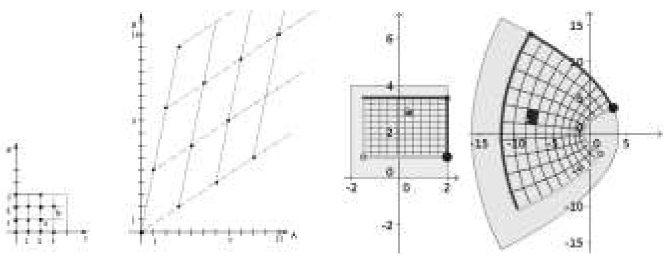
Slide 10

Transformations and Coordinates

- What does this topic mean to you
- KS3
- KS4
- KS5
- Beyond

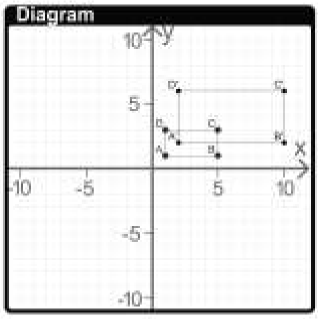
Slide 11

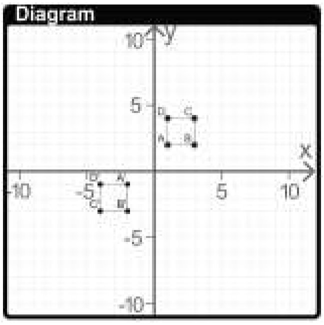
What is the same and what is different?



Slide 12

What transformations?





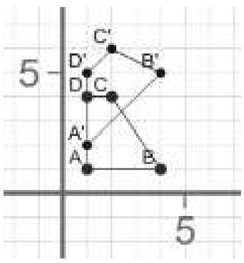
Slide 13

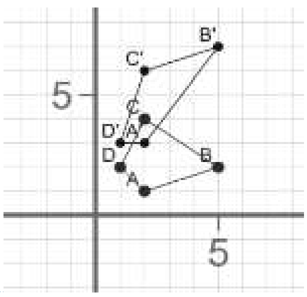
Explore the activities on the learnpad

- What are the key conceptual ideas underpinning the study of transformations?
- What are the key connections to make with other topics?
- How does the principle of what stays the same and what changes help to develop conceptual understanding?

Slide 14

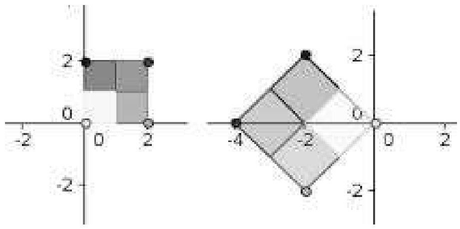
What is same? What is different?



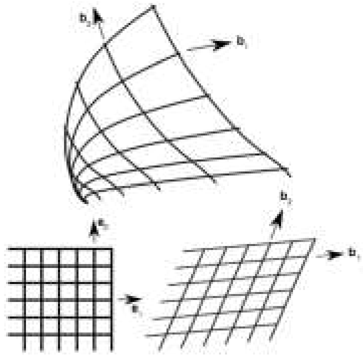


Slide 15

What transformations?



Slide 16



Appendix 2.9 CPD presentation 5th March 2015

Slide 1

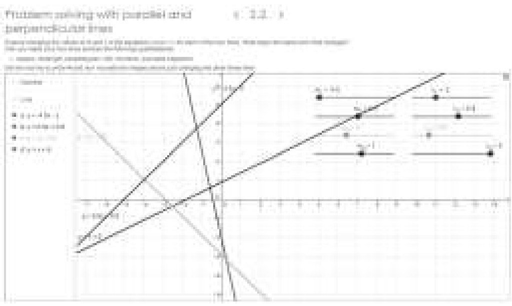
COLLABORATIVE PLANNING PART 1

05/03/15 Nicola Trubridge

Slide 2

Quick maths starter

- http://tube.geogebra.org/student/b465029#material/465009



Slide 3

Objectives

- To review curriculum changes and implications for future developments in mathematics
- To review where the model of CCC fits into the latest educational landscape
- To consider our roles within the whole school development planning process
- To establish collaborative planning groups and to identify ways forward

Slide 4

National Curriculum 1999

- The 1999 mathematics national curriculum (DfEE and QCA, 1999) makes explicit reference to making connections between topic areas 'Teaching should ensure that appropriate connections are made between the sections on number and algebra, shape, space and measure, and handling data' (DfEE and QCA, 1999, p. 29). However there is no guidance as to what might be appropriate connections.

Slide 5

National Strategies 2001

- Good planning ensures that mathematical ideas are presented in an interrelated way, not in isolation from each other. Awareness of the connections helps pupils to make sense of the subject, avoid misconceptions and retain what they have learnt. So when you plan: .... present each topic as a whole, rather than as a fragmented progression of small steps.... bring together related ideas across strands.

(DfEE, 2001, p. 46)

Slide 6

Framework for Teaching Maths (2001)

- Not only does the National Strategy recognise the importance of these connections, but there is also an attempt to provide supporting ideas and materials to help teachers. Within the supplement of examples expected outcomes detailed for years 7 - 9 next to these contain links to other curriculum areas. For example for the objective ‘Generate points and plot graphs of functions’ it suggests linking to properties of linear sequences, proportionality, enlargement and trigonometry.

Slide 7

National Curriculum 2008

- The 2008 National Curriculum, KS3 and KS4 programmes of study (DCSF and QCA, 2007); start with emphasising the importance of mathematics as a subject and the important links to society and the workplace.
- There is a section on key concepts (competence, creativity, applications and implications of mathematics and critical understanding) that underpin mathematical study and the statement that ‘Pupils need to understand these concepts in order to deepen and broaden their knowledge, skills and understanding’ (DCSF and QCA, 2007, p. 140) but no mention here as to what is meant by understanding or how learning might take place.

Slide 8

National Curriculum 2008

- The guidance for the 2008 National Curriculum makes it clear that you should avoid the atomised approach; you should draw links wherever possible, and should also provide pupils with the opportunity to work on extended problems. You should give all pupils experience of problems that draw learning together from different parts of the mathematics programme of study. Chambers (2008)

Slide 9

National Curriculum 2008

- The next section of the programmes of study refers to the key processes (representing, analysing, interpreting and evaluation and communicating and reflecting) and how these skills are essential for learners to make progress.
- The fourth section details the range and content that needs to be covered. The final section of the document outlines opportunities that the curriculum should provide including the statement that there should be opportunities for pupils to 'work on tasks that bring together different aspects of concepts, processes and mathematical content' (DCSF and QCA, 2007, p. 147).

Slide 10

Ofsted (2008 and 2009)

- The fundamental issue for teachers is how better to develop pupils' mathematical understanding. Too often, pupils are expected to remember methods, rules and facts without grasping the underpinning concepts, making connections with earlier learning and other topics, and making sense of the mathematics so that they can use it independently. The nature of teaching and assessment, as well as the interpretation of the mathematics curriculum, often combine to leave pupils ill equipped to use and apply mathematics.

(Ofsted, 2008, p. 5)

Ofsted Hand-out

Slide 11

ACME 2011

- ACME proposed in 2011 that 'The curriculum must show the sophisticated connections and relationships between key mathematical ideas in a non-linear fashion' (ACME, 2011, p. 19) and that the cross curriculum ideas should be represented explicitly .

Slide 12

National Curriculum 2014

- Mathematics is a creative and highly inter-connected discipline

(DfE, 2013, p. 2 and DfE, 2014, p. 3).

- Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas.

(DfE, 2013, p. 2 and DfE, 2014, p. 3)

Slide 13

Aims of 2014 Curriculum

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

(DfE, 2013, p. 2 and DfE, 2014, p. 3)

Slide 14

- There is the acknowledgement that although the curriculum is organised into distinct domains; at key stage 3 pupils should build on key stage 2 and build connections across mathematical ideas to develop their fluency (DfE, 2013) and at key stage 4 they should develop and consolidate connections across mathematical ideas (DfE 2014).

Slide 15

Examples in KS3 PoS

- Pupils should 'move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]' (DfE 2013, p4)
- They should 'extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations (DfE 2013, p4).

Slide 16

Examples in KS4 PoS

- They move freely between different numerical, algebraic, graphical and diagrammatic representations is extended to include linear, quadratic, reciprocal, exponential and trigonometric functions (DfE 2014) and pupils are expected to 'make and use connections between different parts of mathematics to solve problems' (DfE 2014, p6).

Slide 17

Challenge

- One frequent presumption of Gove's new curriculum is teaching through explicit rules. The explicit assumption is that teachers should announce a rule of grammar, spelling, calculation or nature prior to the learner engaging in any activity.

(Wrigley, 2014, p. 36)

Slide 18

Letter to Gove

- The proposed curriculum consists of endless lists of spellings, facts and rules. This mountain of data will not develop children's ability to think, including problem-solving, critical understanding and creativity. Much of it demands too much too young. This will put pressure on teachers to rely on rote learning without understanding.

(Bassey et al, 2013)

Slide 19

The PISA debate

- Bassey et al (2013) state that Mr Gove has misunderstood England's decline in PISA international tests and that the schools in high-achieving Finland, Massachusetts and Alberta emphasise cognitive development, critical understanding and creativity, not the rote learning that he puts forward. This is echoed by Wrigley (2014, p. 35) 'the ultimate irony of Gove's PISA envy is that PISA tests require intellectual process: problem-solving and application of knowledge rather than the regurgitation of a series of facts'...... 'It is counterproductive to design education around competition for PISA; paradoxically, a high ranking is more likely to result from in-depth learning and co-operation than testing and competition (Wrigley 2014, p. 39).

Slide 20

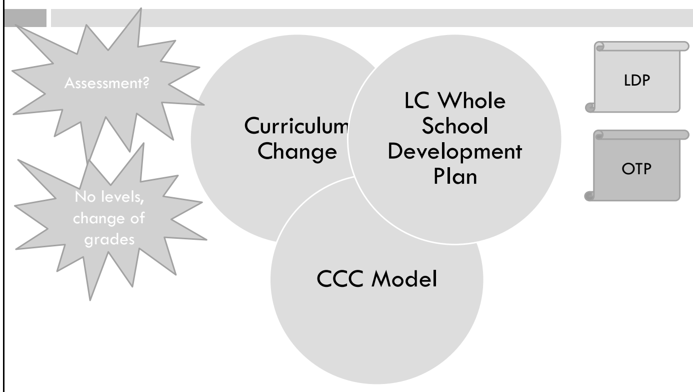
The CCC Model

- How are the theories underpinning the CCC model still relevant to the new curriculum?
- Which aspects are most prevalent in the new PoS?

CCC Handout

Slide 21

Working smart



Slide 22

GCSE Subject Criteria

The expectation is that:

- All students will develop confidence and competence with the content identified by standard type
- All students will be assessed on the content identified by the standard and the underlined type; more highly attaining students will develop confidence and competence with all of this content
- Only the more highly attaining students will be assessed on the content identified by **bold type.** The highest attaining students will develop confidence and competence with the **bold** content.

Slide 23

Our stages versus grades!

SoL stage	Current Level/Grade	New numbering system!	Needed from GCSE Criteria
1	3/4	1,2	Standard
2	4/5	2,3	Standard
3	5/6	3,4	Standard
4	5/6/7/E/D/C	3,4,5	Underlined
5	7/8/C/B	4,5,6	Underlined
6	B/A	5,6,7	**Bold**
7	A/A*	7,8,9	**Bold**

Slide 24

Planning groups

Key Stage	SoL Stage		Things to consider
	0	A	KS2 and KS3 PoS
3	1,2,3	H, I	KS3 PoS
4	4,5	B, K	KS4 PoS GCSE Subject Criteria foundation
4	6,7	C, F, E	KS4 PoS GCSE Subject Criteria higher Adding in FMSP Problem solving
5		E	Core maths and KS5 curriculum changes

KS3 and KS4 PoS

GCSE Subject Criteria

Slide 25

Ongoing Collaborative tasks

- Check that the relevant parts of GCSE criteria are embedded within the appropriate stage of our SoL
- Are we incorporating
  - Fluency vs developing concepts
  - Reasoning opportunities
  - Problem solving tasks

Appendix 2.10 CPD presentation 23rd April 2015

Slide 1

COLLABORATIVE PLANNING PART 2

23/04/15

Training Evening

Slide 2

The aims of this project are to work collaboratively to

Prepare our learners for the new 2014 curriculum changes by:

- Ensuring progression is appropriate for different stages of learner
- Ensure new ‘content’ is embedded into schemes of learning
- Experiment with an aspect of the Collaborative Connected Classroom model to encourage development of a more relational understanding

Slide 3

The outcome is that learners:

- Know what to do and why they are doing it
- Know a range of concepts, symbols and procedures and how they are related.
- Use strategies which are both efficient and effective
- Are aware of connections within and outside of mathematics
- Are confident in tackling unfamiliar problems

Slide 4

The overarching aims of the 2014 curriculum are for learners to:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Slide 5

The feature of the CCC model that this piece of work sets out to explore is:

- Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
- Tasks either connect together different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
- Links are made between procedures and concepts
  - meaning is built for procedural knowledge before mastering it ('educational approach')
  - procedures are evaluated to promote conceptual understanding ('developmental approach')
- Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
- Application tasks are presented as challenges that may be problematic and need to be reasoned about

Slide 6

Collaborative tasks

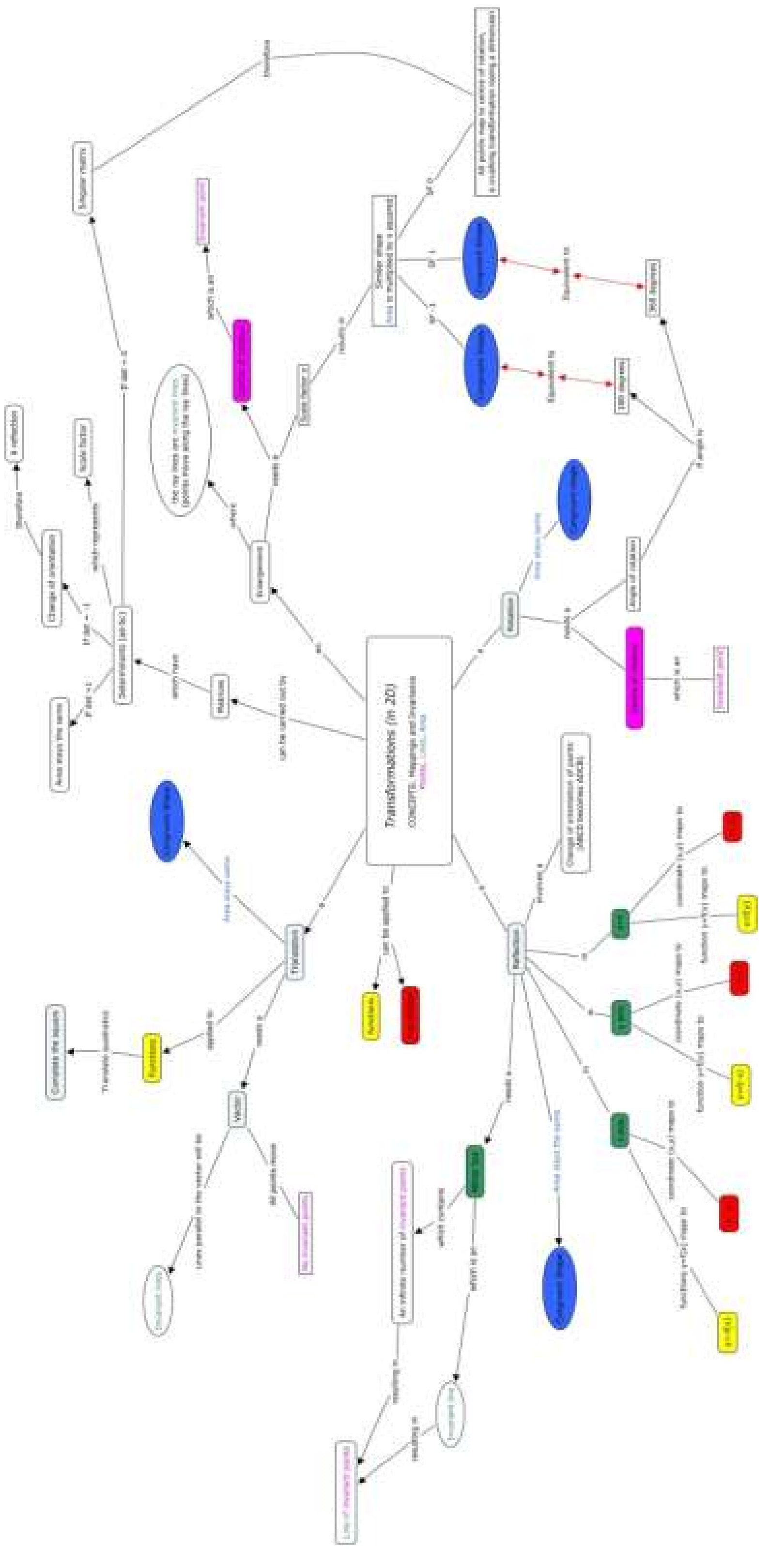
- Compare the number strand of the new national curriculum with the number strand from appropriate stages from our current SoL
- Highlight objectives that are already included (annotate with stage) consider where additional objectives need to be placed.

Slide 7

Collaborative tasks

- Choose the focus area from new curriculum that you feel needs developing
- Choose the focus area from CCC model that shows how you are addressing your chosen area
- Start to develop the unit of work to incorporate these aspects.

Appendix 2.11 Transformations concept map



Appendix Figure 2. Transformations concept map

Appendix 2.12 Action research template

Exploring the features of the ‘Collaborative Connected Classroom’ within the context of the 2014 curriculum

Teacher Name____________________________

<table>
<tr><th>Element of Study</th><th>Activity</th></tr>
<tr><td>Identifying need for improvement
(delete and add as appropriate!)</td><td>New content in GCSE 2014 curriculum that is not currently taught or in schemes of learning
The departmental review identified the need to expand pedagogical repertoire across the department
___ group of students are not performing as well as other groups
_____ has been highlighted as a need through lesson observation feedback
Analysis of mock exams has led to the topic of _____ being chosen as an area to develop</td></tr>
<tr><td>External Input
(delete and add as appropriate!)</td><td>Departmental CPD sessions led by N.Trubridge (Feb 14, Mar 14, Dec 14, Mar 15)
Reading of key academic research papers on chosen subject area</td></tr>
<tr><td>Experiment by changing practice</td><td>Collaborative planning with ____ to develop stage ____ scheme of learning
Adding / amending objectives to ensure progression and pitch is appropriate for new curriculum
Development of key activities that incorporate CCC Model in the context of the new curriculum
Trial of activities with learners</td></tr>
<tr><td>Review impact of practice change</td><td>Reflection on activities as an individual
Discussion with pupils to gather evidence
Reflection on activities with collaborative planning partner/trio</td></tr>
<tr><td>Disseminate findings</td><td>Dissemination at department meeting
Completion of summary case study
Interviews with Ted/Nicola</td></tr>
<tr><td colspan="2">The aims of this project are to work collaboratively to
o Prepare our learners for the new 2014 curriculum changes by:
o Ensuring progression is appropriate for different stages of learner
o Ensure new ‘content’ is embedded into schemes of learning
o Experiment with an aspect of the Collaborative Connected Classroom model to encourage development of a more relational understanding
The outcome is that learners:
o Know what to do and why they are doing it
o Know a range of concepts, symbols and procedures and how they are related.
o Use strategies which are both efficient and effective
o Are aware of connections within and outside of mathematics
o Are confident in tackling unfamiliar problems</td></tr>
</table>

The aspect of the new curriculum aims that this piece of action research sets out to develop is highlighted.	The overarching aims of the 2014 curriculum are for learners to: o become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately. o reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language o can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The feature of the CCC Model that this piece of work sets out to explore is highlighted	o Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
	o Tasks either connect different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
	o Links are made between procedures and concepts
	    o meaning is built for procedural knowledge before mastering it ('educational approach')
	    o procedures are evaluated to promote conceptual understanding ('developmental approach')
	o Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
	o Application tasks are presented as challenges that may be problematic and need to be reasoned about

Bibliography (delete and add as appropriate)	o DfE. (2013). Mathematics programmes of study: key stage 3 National curriculum in England. (DFE-00179-2013). o DfE. (2013b). Mathematics GCSE subject content and assessment objectives. (DFE-00233-2013). o DfE. (2014). Mathematics programmes of study: key stage 4 National curriculum in England. (DFE-00496-2014). o Skemp, R. R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teaching* (77), 20-26. o Trubridge, N., & Graham, T. (2013). *Exploring the features of a collaborative connected classroom.* Paper presented at the British Society for Research into Learning Mathematics, Sheffield.

APPENDIX 3: RESEARCH QUESTIONS AND ETHICS

Appendix 3.1 Summary of research questions

Q How can a programme of professional development engage and support a mathematics department to teach for understanding?

Sub research questions	Why	Addressed
1) What is meant by 'teaching for understanding' and what would this look like in a mathematics classroom?	There are many definitions of this	Literature review in Chapter 2 resulting in the CCC Model
2a) What is meant by the term professional development?	There are many definitions of this	Literature review in Chapter 3
2b) What factors will contribute to an effective professional development programme?	Want teachers to change their practice so need to plan for this to happen	Literature review in Chapter 3
2c) What would an effective CPD programme look like that supports the implementation of the CCC Model?	Because the CPD programme has to address the specific nature of the implementation of the CCC Model	Literature review and resulting design of CPD programme in Chapter 4
3a) Does the professional development result in teacher change? If so what elements support the change process?	Need to know if teachers change	Empirical research presented in Chapter 9
3b) What is the impact of exposing teachers to academic literature within a programme of CPD?	Area highlighted as needing further research from literature	Empirical research presented in Section 8.1
3c) What are the barriers to engagement with the CPD programme?	Area highlighted as needing further research from literature	Empirical research presented in Chapter 9
4a) Which approaches will engage teachers with students' development of connections?	Area highlighted as needing further research from literature	Empirical research presented in Section 8.2
4b) Which approaches will teachers explore with students? 4c) How can teachers develop tasks that encourage connections to be made?	Want to know how teachers use the new knowledge from CPD	Empirical research presented in Section 8.3
4d) What are the barriers to engagement with the CCC Model?	Want to know if teachers don't engage, why	Empirical research presented in Chapter 9
5a) Are there any differences in how teachers develop across the mathematics department? 5b) What has influenced these differences?	There are lots of different sub-units that impact on the 'case'	Empirical research presented in Section 8.4

Appendix 3.2 Research information sheet

Name of principal investigator: Mrs Nicola Trubridge

Title of Research
An investigation into how teachers develop connected approaches to school mathematics

Aim of research

I am conducting a piece of research as part of my thesis whilst working towards a PhD in Mathematics Education. The aim is to investigate through a range of professional development activities how a mathematics department can develop a more connected approach to mathematics teaching and learning.

Description of procedure

The research study will involve four distinct aspects and methods of data collection. These are detailed below.

1. **Departmental Professional Development**

   The whole department (including any trainee teachers / HLTAs) will be invited to take part in a collaborative professional development activity. This activity will be videoed; the film will focus on the outcome of the activity itself and not on the teachers taking part. Extracts from the video files will be kept to use at conferences and professional development events for teachers. These extracts will be shown to you and your permission will be sought to use the videos for this purpose. Any part of the video for which this permission is not obtained will be destroyed upon completion of the project.

2. **Collaborative Planning**

   After the departmental activity, subgroups will be formed to take part in collaborative planning sessions where there will be opportunities to reflect, review and refine the process. Subgroups will choose an aspect that they feel is beneficial to them in developing a more connected approach to school mathematics. It is anticipated that this will involve different people working on the following areas

- Adaptations to schemes of learning
- Developing units of learning
- Writing lesson plans
- Writing lesson resources
- Discussion of peer observations

Collaborative planning sessions will be audio recorded; these recordings will be destroyed when they have been transcribed. Outcomes of planning sessions (unit plans/ resources) will also be used to inform the work.

3. Interviews

A representative sample of the participants involved will be invited to be interviewed to ascertain their thoughts and opinions. The interviews will be between 30-60 minutes and will occur once a term. The interviews will be audio recorded; these recordings will be destroyed when the project has been completed. Participants will be interviewed a maximum of three times.

4. Journal

I will be keeping a journal of other comments that are relevant to the study that occur within the department at other times e.g. in departmental meetings or over coffee. If I note in the journal something that you have said I will show you the journal entry and ask you to initial it to say it is correct and that you give permission for it to be used. If you do not agree the journal entry will be destroyed.

Right to withdraw

Participation in the project is voluntary and you may withdraw from it at any time without penalty up to the stage of the publication of the final report. You have a right not to answer specific questions and to ask for recording or note-taking to cease at any point.

Confidentiality

Every effort will be made to ensure that you are not identifiable in the report of the research unless you choose to be so.

If you are dissatisfied with the way the research is conducted, please contact me in the first instance. If you feel the problem has not been resolved, please contact the secretary to the Faculty of Science and Engineering Human Ethics Committee: Mrs Paula Simson 01752 584503.

If you wish to discuss any aspects of the study, then please do not hesitate to contact me. May I thank you in advance for your valuable contribution.

Nicola Trubridge

Contact details: University of Plymouth
Faculty of Science and Engineering
School of Computing, Electronics and Mathematics
nicola.trubridge@plymouth.ac.uk

Appendix 3.3 Ethics consent form

Name of principal investigator: Mrs Nicola Trubridge

Title of Research
An investigation into how teachers develop connected approaches to school mathematics

Brief statement of purpose of work

The aim is to investigate through a range of professional development activities how a mathematics department can develop a more connected approach to mathematics teaching and learning.

- The objectives of this research have been explained to me and I have been given a research information sheet.
- I understand that I am free to withdraw from the research at any stage, and ask for my data to be destroyed if I wish.
- I understand that my anonymity is guaranteed, unless I expressly state otherwise.
- Under these circumstances, I agree to participate in the following aspects of the research.

Aspect of Data Collection	Please tick to confirm
Departmental Professional Development	
Collaborative Planning	
Interviews	
Journal Entries	

Name: ………………………………………

Signature: ....……….... Date: ....…

Appendix 3.4 Pre-interview script read to participants

Name of principal investigator: Mrs Nicola Trubridge

Additional researchers: Dr Ted Graham

Title of Research
An investigation into how teachers develop connected approaches to school mathematics

Aim of research

The aim is to investigate through a range of professional development activities how a mathematics department can develop a more connected approach to mathematics teaching and learning.

This interview will be audio recorded. Audio recordings will be destroyed when the project has been completed.

Participation in the project is voluntary and you may withdraw from it at any time without penalty up to the stage of the publication of the final report. You have a right not to answer specific questions and to ask for recording or note-taking to cease at any point.

Every effort will be made to ensure that you are not identifiable in the report of the research unless you choose to be so.

The interview schedule is flexible so you can give your own ideas and opinions on the departmental development work that we have carried out. If I am unsure what you mean I will ask for clarification. I anticipate that the discussion will take between 30-60 minutes and your responses will be used to inform next steps in this project.

APPENDIX 4: PILOT STUDY DATA

Appendix 4.1 Pilot study timeline of activities

Date	Group	Cohort	Present	Activity
23/11/2012	1	B.Ed. Trainees	T1,3,4,5	Sharing Research Session
07/12/2012	1	B.Ed. Trainees	T1,2,4,5	Connection activity
07/12/2012	1	B.Ed. Trainees	T1,2,4,6	Multiple Representations Tasks
13/12/2012	2	School HLTAs	A, B	Sharing Research Session
13/12/2012	2	School HLTAs	A, B	Instrumental vs Relational Card Sort
04/01/2013	3	PGCE Cohort 1	T1-11	Sharing Research Session
04/01/2013	3	PGCE Cohort 1	T1-11	Instrumental vs Relational Card Sort
04/01/2013	3	PGCE Cohort 1	T1-6	Connection activity
04/01/2013	3	PGCE Cohort 1	T7-11	Connection activity
09/01/2013	3	PGCE Cohort 1	T1-5	Planning activity (no scaffold)
16/01/2013	3	PGCE Cohort 1	T7-11	Planning activity
17/01/2013	2	School HLTAs	A, B	Multiple Representations in Algebra 1
24/04/2013	3	PGCE Cohort 1 Case Study	Maggie T11	Review research, collaborative planning
01/05/2013	3	PGCE Cohort 1 Case Study	Maggie T11	Collaborative Planning Session
08/05/2013	3	PGCE Cohort 1 Case Study	Maggie T11	Interview
22/10/2013	4	PGCE Cohort 2	T1-14, no T2	Initial task for pilot study (Appendix 4.8)
22/10/2013	4	PGCE Cohort 2	T1-14, no T2	Sharing Research Session
22/10/2013	4	PGCE Cohort 2	T1-14, no T2	Instrumental vs Relational Card Sort
22/10/2013	4	PGCE Cohort 2	T1-14, no T2	Range of Multiple representation tasks
29/10/2013	4	PGCE Cohort 2	T1-14	Collaborative Planning
05/12/2013	4	PGCE Cohort 2	T1-14	Team Teaching
06/12/2013	4	PGCE Cohort 2	T1-14	Meeting to reflect on process
05/12/2013	4	PGCE Cohort 2	T1-14	Evaluation forms

Appendix Table 9. Pilot study timeline of activities

Appendix 4.2 Multiple representations CPD session

Slide 1

Multiple Representations in Algebra

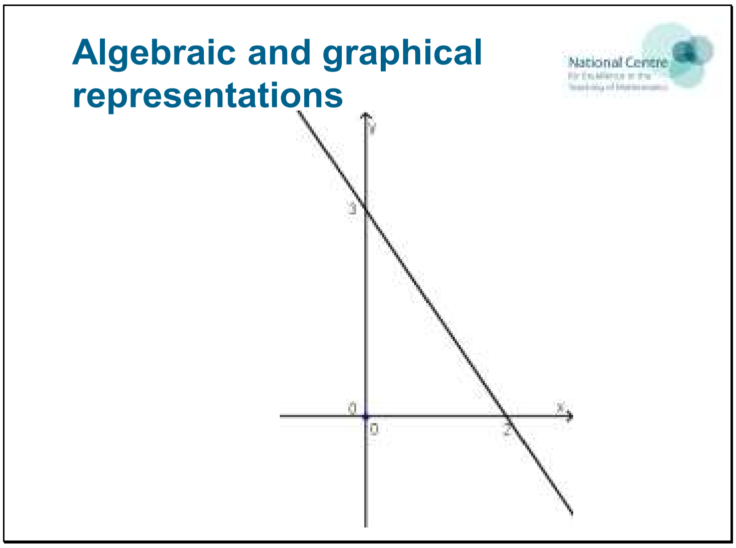
- The following slides are from the NCETM Professional Development Lead programme (2012 cohort)

Slide 2

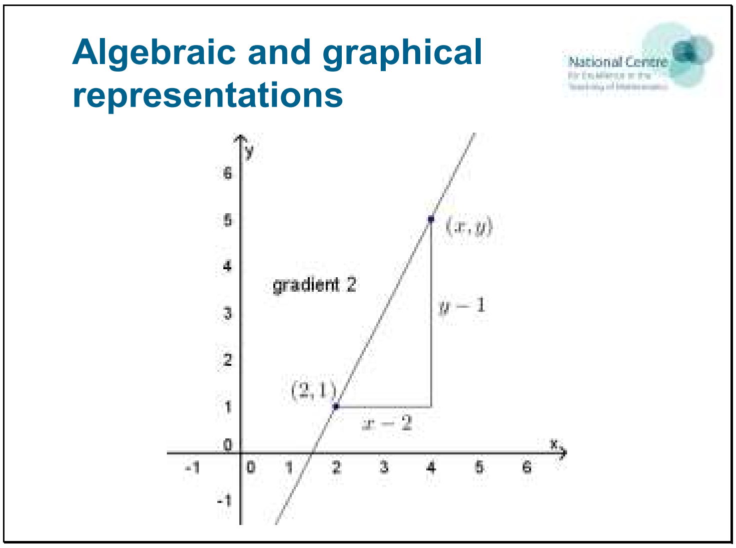
Rethinking algebra

- Algebraic and graphical representations
- Making sense of algebraic manipulation
- Factorisation: numeric and algebraic

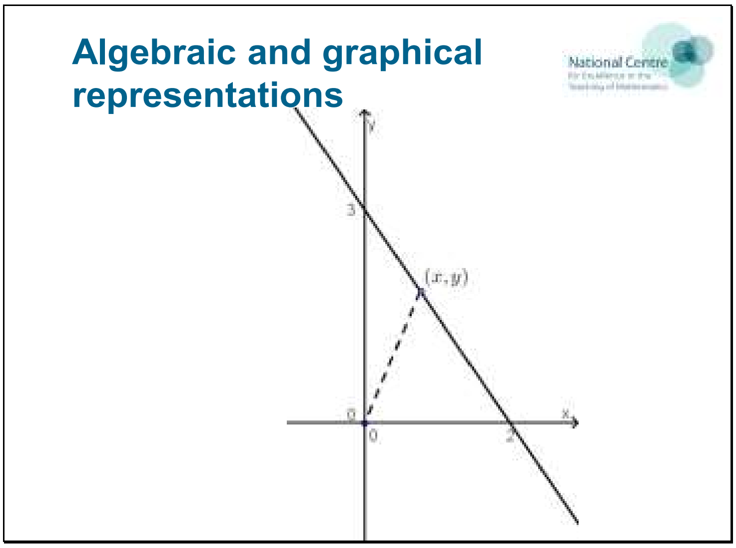
Slide 3



Slide 4

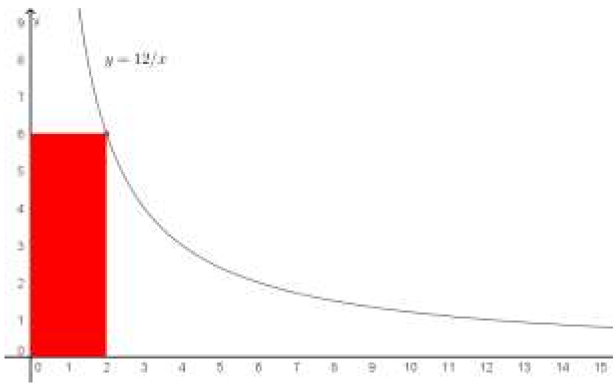


Slide 5



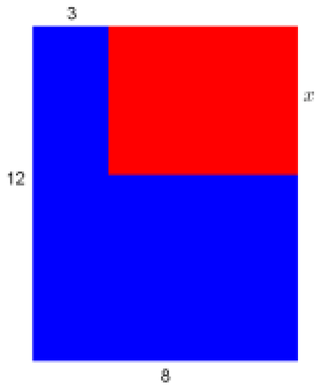
Slide 6

Algebraic and graphical representations



Slide 7

Making sense of algebraic manipulation



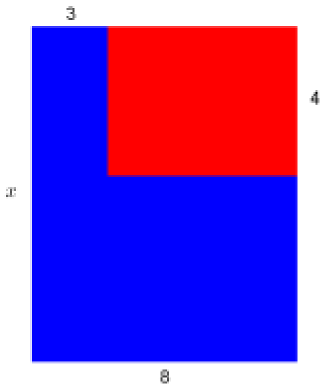
$96 - 5x$

$36 + 5(12 - x)$

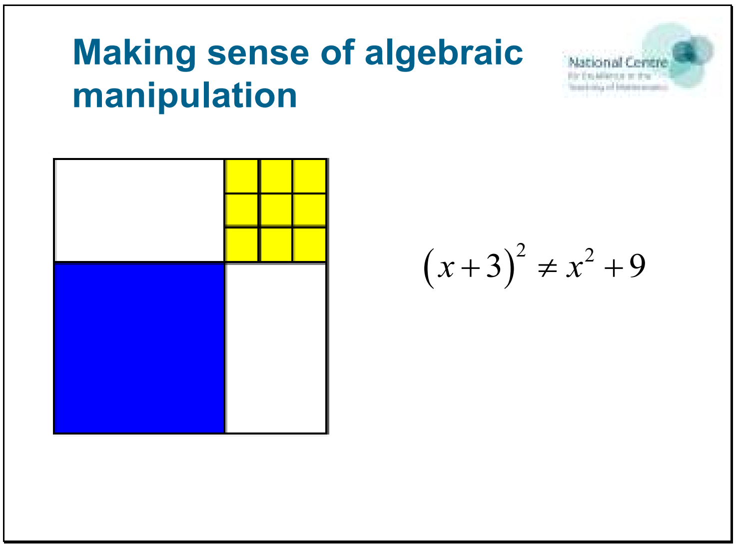
$8(12 - x) + 3x$

Slide 8

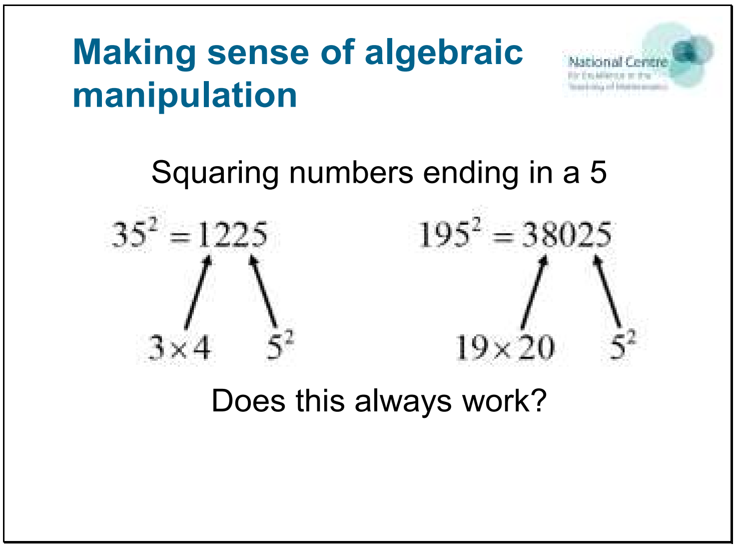
Making sense of algebraic manipulation



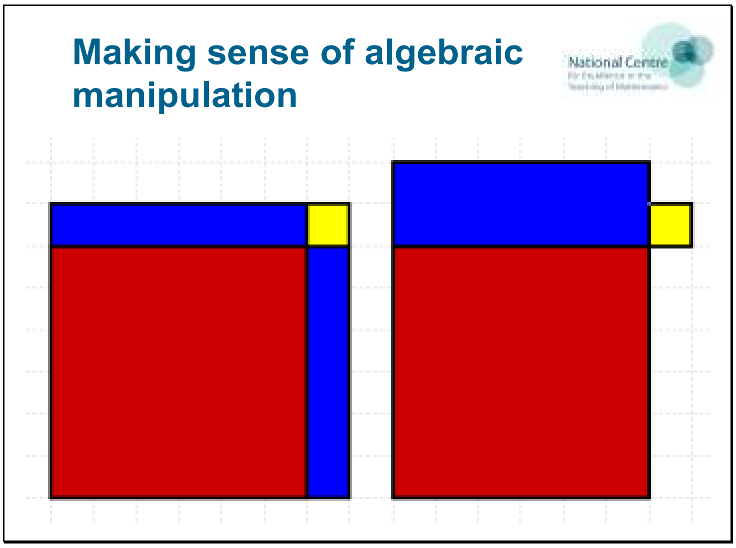
Slide 9



Slide 10



Slide 11

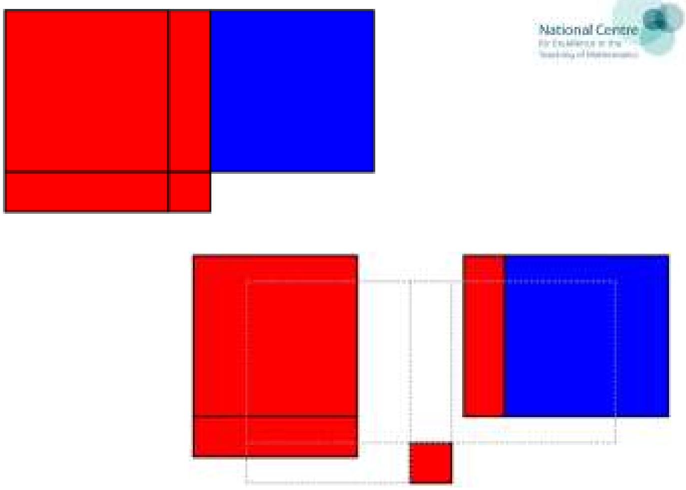


Slide 12

Making sense of algebraic manipulation

"Reasoning and proof were not well developed in most of the secondary schools visited…A recent GCSE examination question asked pupils to prove that 'the sum of the squares of two consecutive integers is one greater than twice the product of the integers' and provided an illustrative example, $9^2 + 10^2 = 181$ and $2 \times 9 \times 10 = 180$. The principal examiner's report stated that 'the concepts of algebraic proof were rarely demonstrated well'." (Ofsted: Made to Measure)

Slide 13



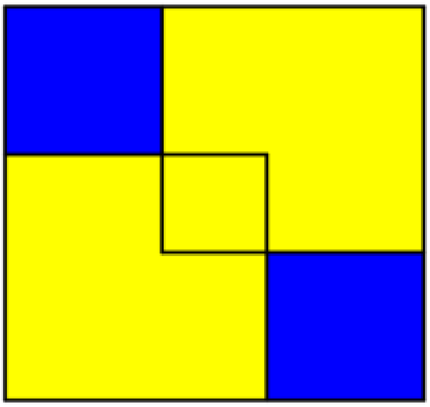
Slide 14

Making sense of algebraic manipulation

Add two square numbers and double the answer.

Prove that the result is also the sum of two squares.

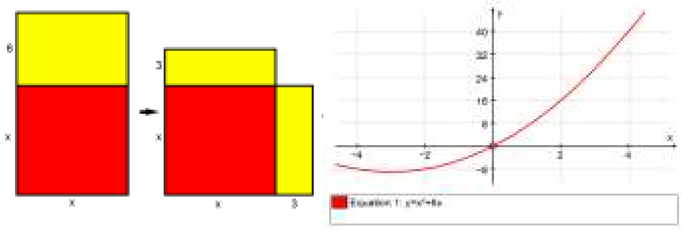
Slide 15



Slide 16

Making sense of algebraic manipulation

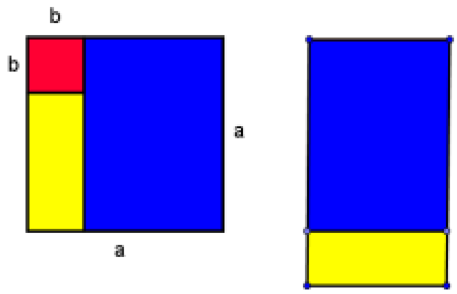
$$y = x(x+6) \qquad y = x^2 + 6x \qquad y = (x+3)^2 - 9$$



Slide 17

Factorisation: numeric and algebraic

$$a^2 - b^2 = (a-b)(a+b)$$



Slide 18

Factorisation: numeric and algebraic

How many factors has 72?

What do they add up to?

Slide 19

Factorisation: numeric and algebraic

	1	a	a^2	a^3
1	1	a	a^2	a^3
b	b	ab	a^2b	a^3b
b^2	b^2	ab^2	a^2b^2	a^3b^2

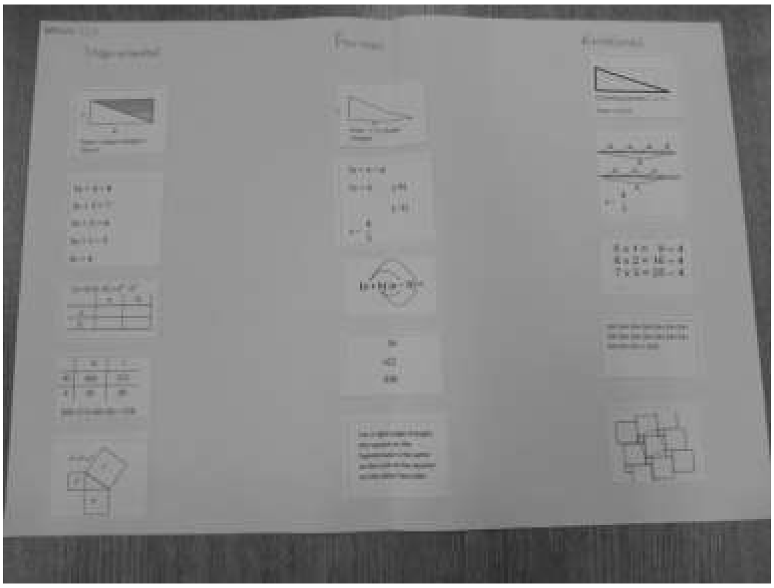
	1	2	2^2	2^3
1	1	2	2^2	2^3
3	3	2×3	$2^2\times3$	$2^3\times3$
3^2	3^2	2×3^2	$2^2\times3^2$	$2^3\times3^2$

NCETM 2012b. Multiple representations in algebra. Presentation shared as part of training for 2012 Professional Development Leads: NCETM.

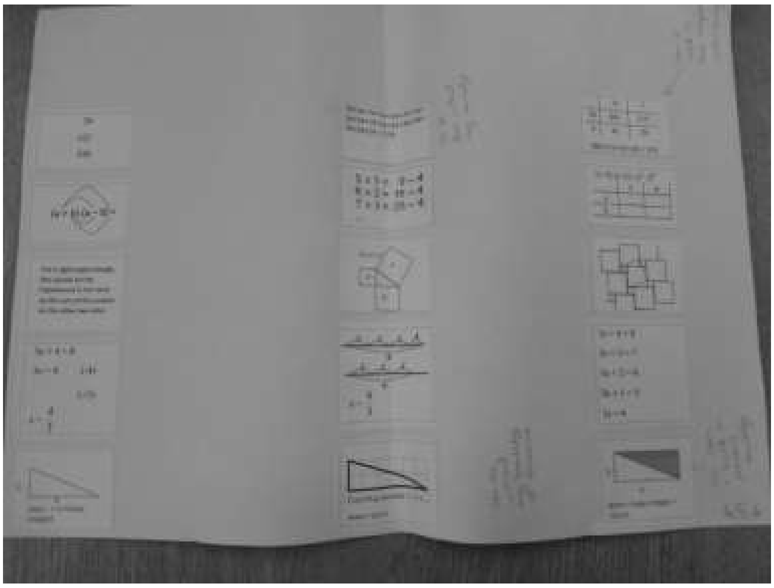
Appendix 4.3 Group 1 connection mapping outcome

Appendix Figure 3. Outcome of connection mapping (Group 1)

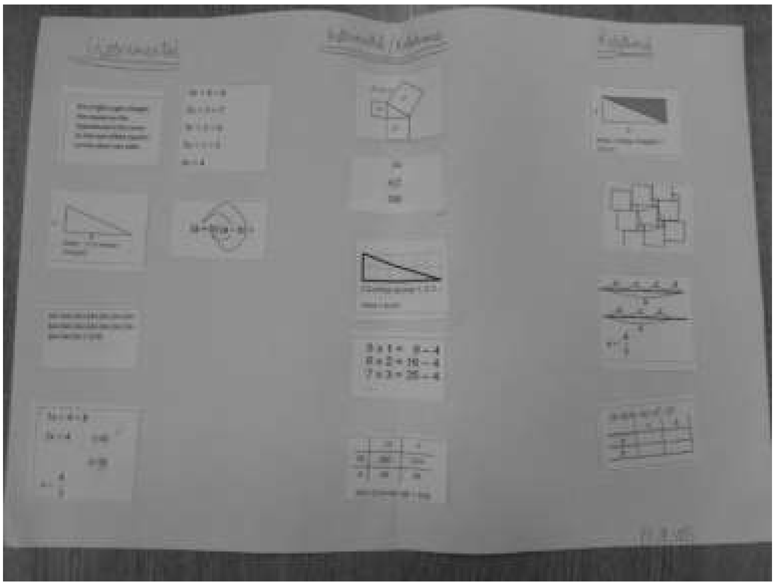
Appendix 4.4 Group 3 Card sort outcomes



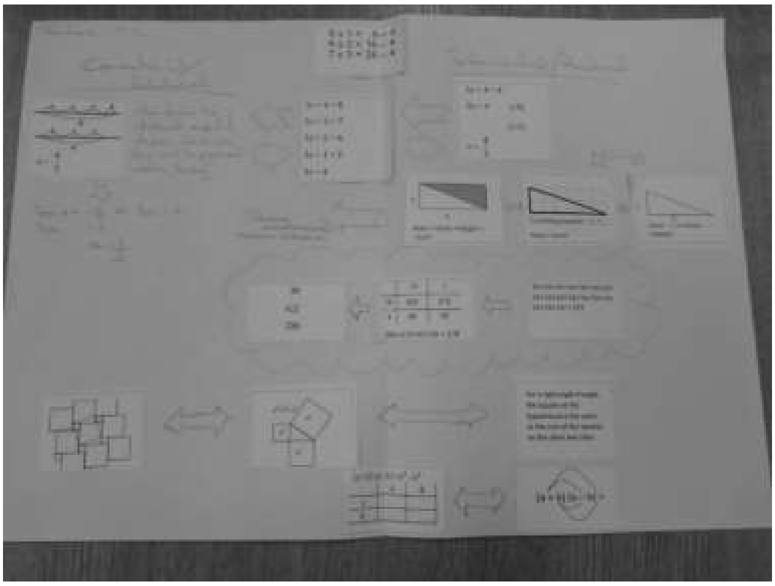
Appendix Figure 4. Outcome of card sort (Trainees 3.1,3.2,3.3)



Appendix Figure 5. Outcome of card sort (Trainees 3.4,3.5,3.6)

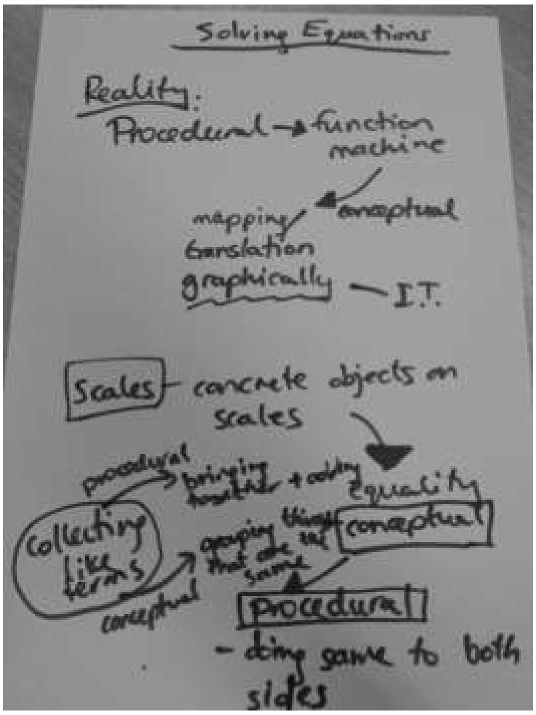


Appendix Figure 6. **Outcome of card sort (**Trainees 3.7,3.8,3.10)

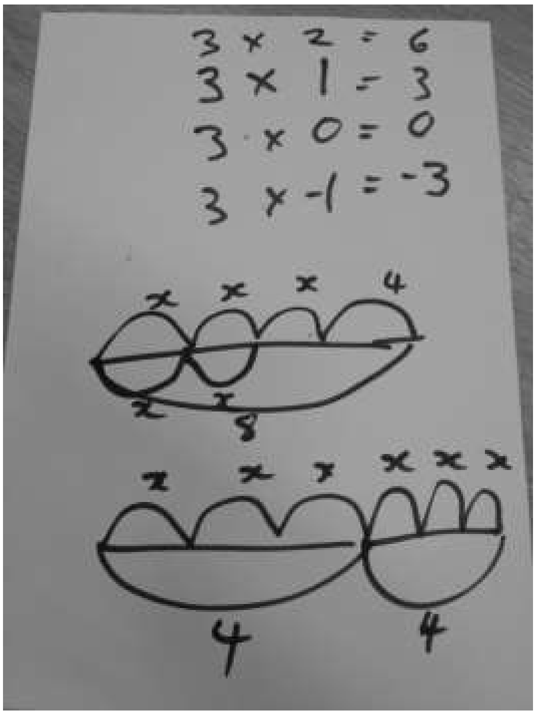


Appendix Figure 7. **Outcome of card sort (**Trainees 3.9,3.11)

Appendix 4.5 Group 3a collaborative planning

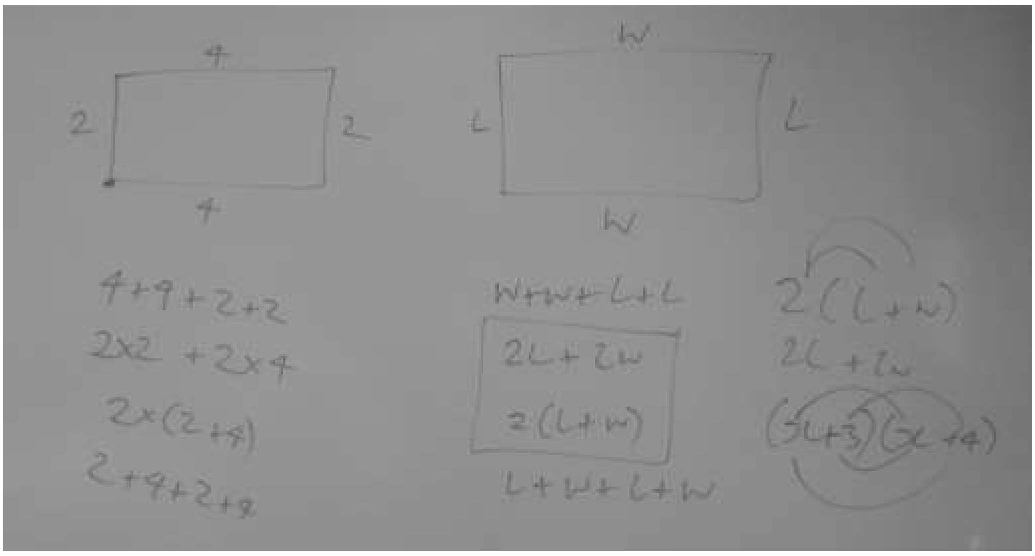


Appendix Figure 8. Notes from collaborative planning solving equations 1

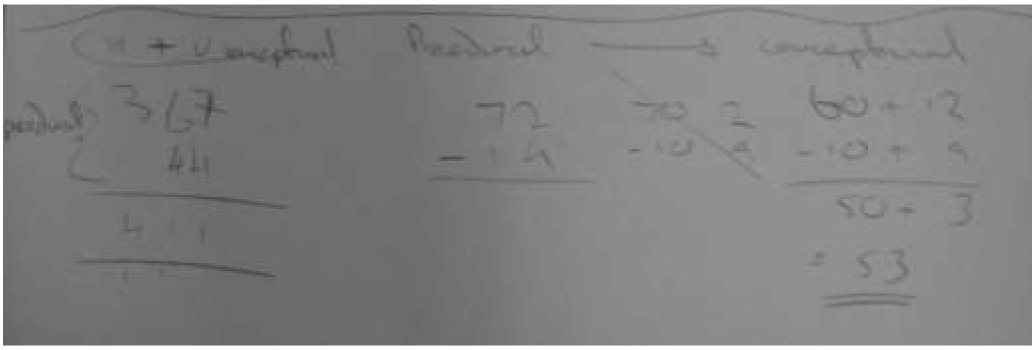


Appendix Figure 9. Notes from collaborative planning solving equations 2

Appendix 4.6 Group 3b multiple representations session

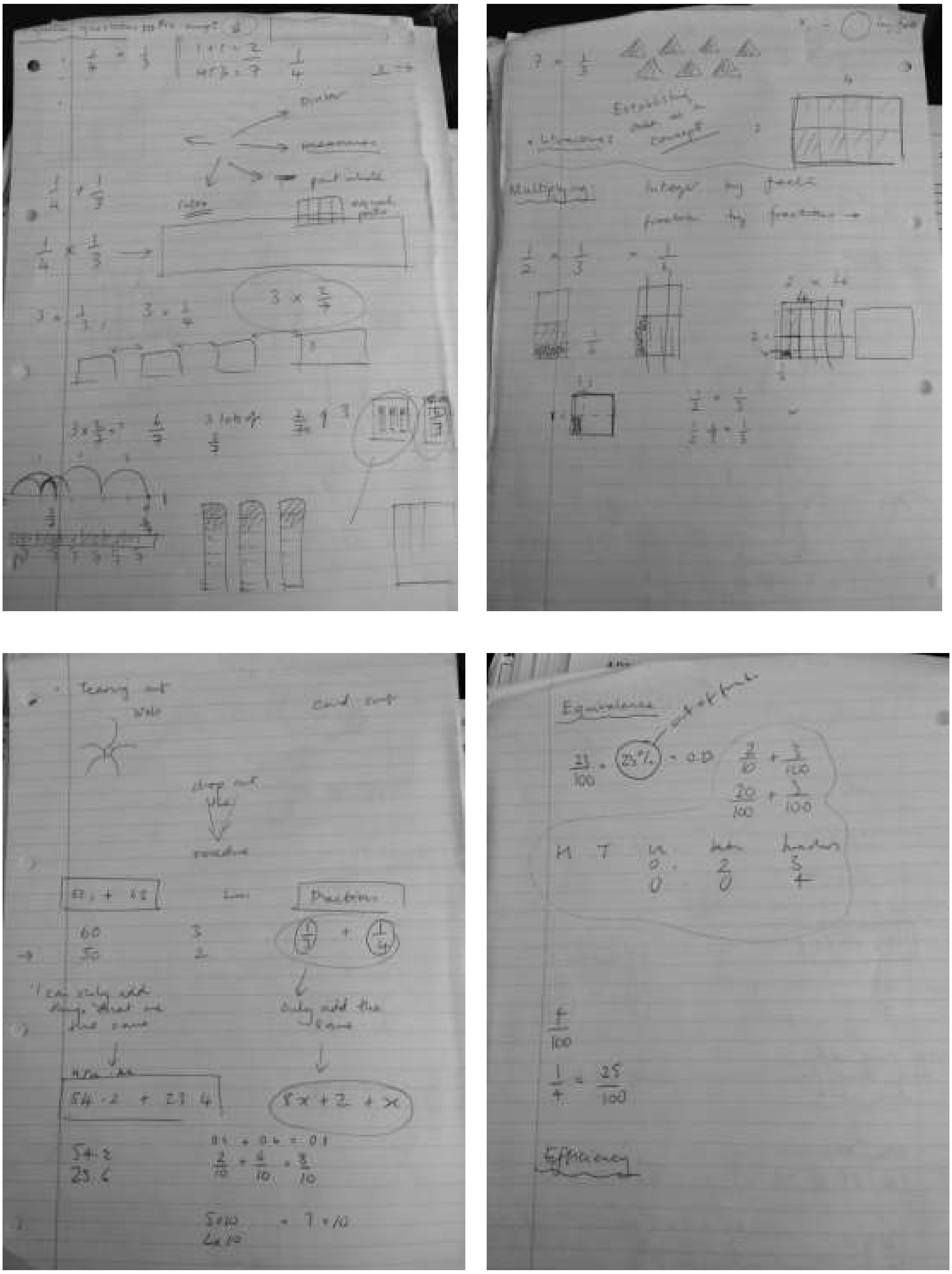


Appendix Figure 10. Expanding brackets

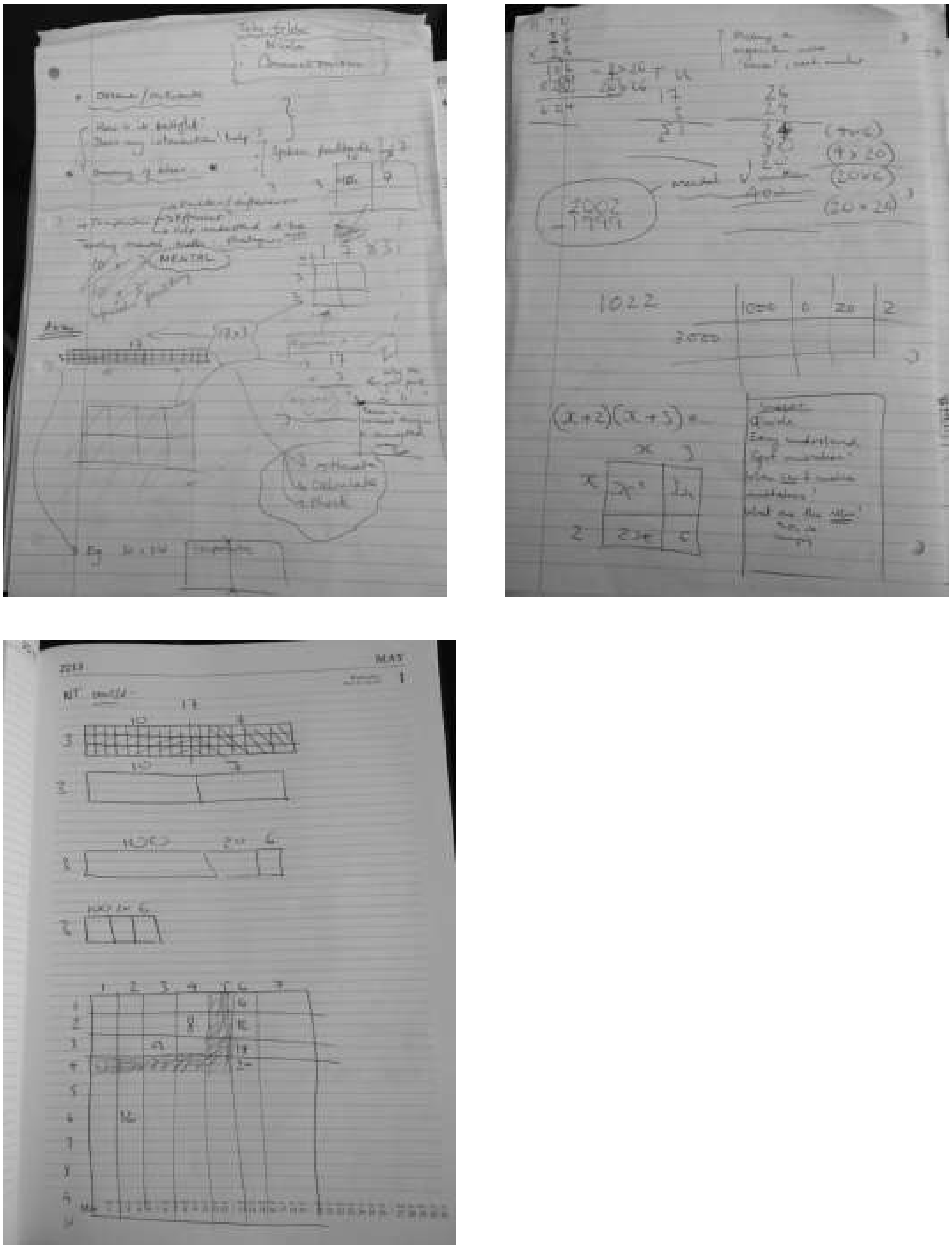


Appendix Figure 11. Subtraction

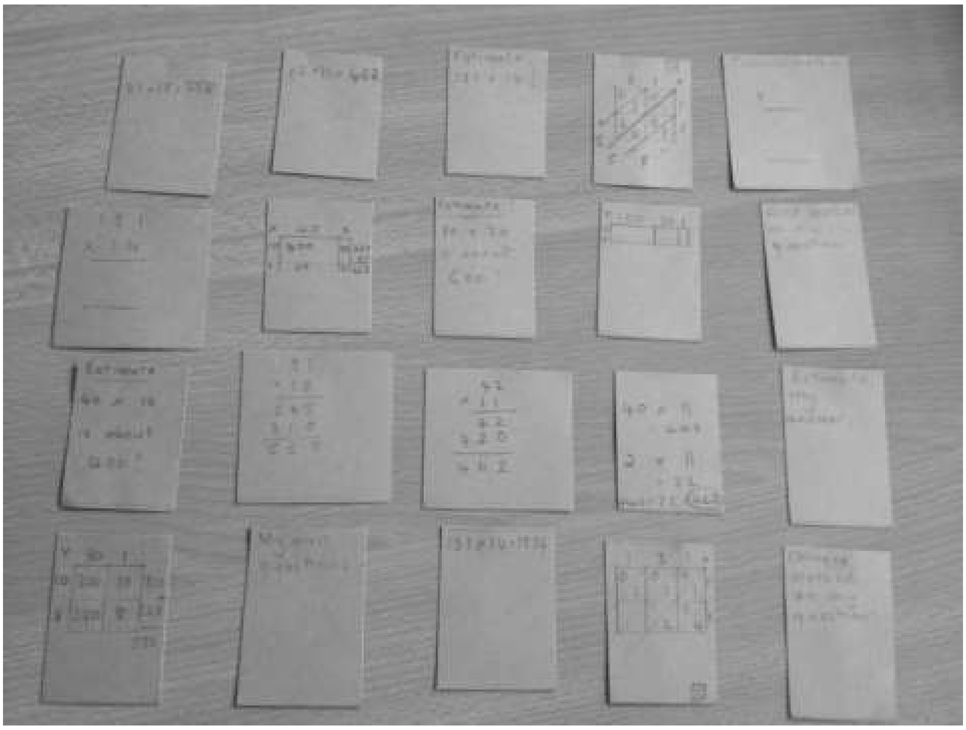
Appendix 4.7 Collaborative planning with Maggie



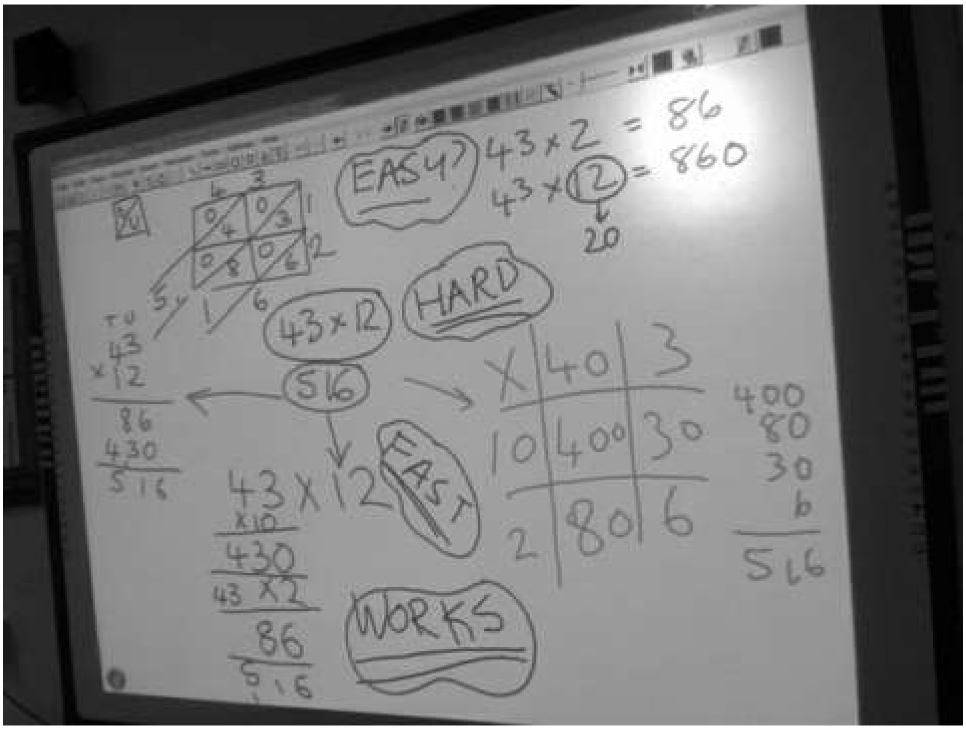
Appendix Figure 12. Notes from collaborative planning session 1



Appendix Figure 13. Notes from collaborative planning session 2



Appendix Figure 14. Resources used in Maggie's lesson



Appendix Figure 15. Interactive white board after Maggie's lesson

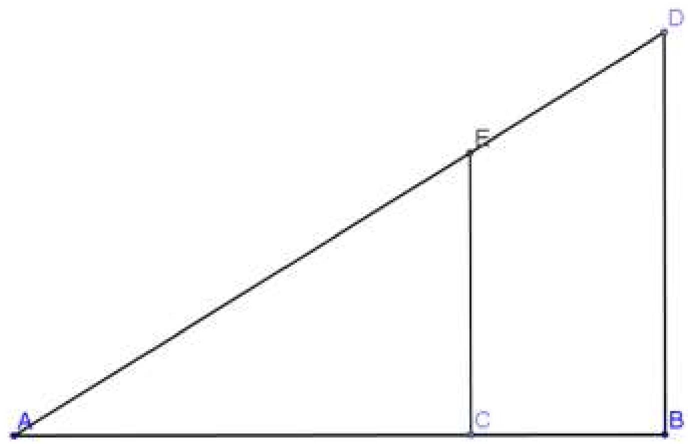
Appendix 4.8 Initial tasks for pilot study

Task One

- *Q* In your opinion, can you describe what it means for students to understand mathematics?

Task Two

- *Q* What aspects of mathematics does this image bring to mind?



Appendix 4.9 Initial evaluation of pilot study

Thank you for taking part in the professional development session and for engaging in activities leading to collaborative planning and team teaching with a focus on developing connected teaching and learning. This pilot stage will inform future work with other colleagues. Please tick the corresponding box to show whether you agree or disagree with the statements below.

Aspect of Professional Development	Statement	Strongly Disagree	Disagree	Agree	Strongly Agree
Research Presentation	The presentation was informative				
	The presentation was a useful aspect of the professional development session				
	I believe all professional development should be underpinned by research				
Activities to bridge theory and practice	The Relational vs Instrumental card sort task helped to develop an understanding of the associated terminology				
	The activity looking at the 3 classroom scenarios for teaching area of parallelogram demonstrated what a more relational lesson might look like				
	The activities (completing the square, how many factors?) helped to make sense of the research theory				
	The activities helped to develop aspects of my own subject knowledge				
Collaborative planning and team teaching	The process of collaborative planning was useful				
	Collaborative planning and team teaching enabled us to put some of the ideas into practice				
	It was useful to reflect on the team teaching with peers				
Please give any comments below that you feel would be useful to help shape the future development of the project.					

Appendix 4.10 Initial evaluation of pilot study responses

Aspect of Professional Development	Statement	Strongly Disagree	Disagree	Agree	Strongly Agree
Research Presentation	The presentation was informative			5	8
	The presentation was a useful aspect of the professional development session			6	7
	I believe all professional development should be underpinned by research		2	9	2
Activities to bridge theory and practice	The Relational vs Instrumental card sort task helped to develop an understanding of the associated terminology		1	5	7
	The activity looking at the 3 classroom scenarios for teaching area of parallelogram demonstrated what a more relational lesson might look like			3	10
	The activities (completing the square, how many factors?) helped to make sense of the research theory		1	6	6
	The activities helped to develop aspects of my own subject knowledge			7	6
Collaborative planning and team teaching	The process of collaborative planning was useful			8	6
	Collaborative planning and team teaching enabled us to put some of the ideas into practice			6	8
	It was useful to reflect on the team teaching with peers			2	12

One trainee was absent from the initial research session so their evaluation form was incomplete and therefore there are no results for them for the first two sections. Percentages are calculated from the 13 responses to the first 2 sections and from 14 responses from the last section.

	Group A	Group B	Group C
Year 10	Problem solving with similar shapes 4.01, 4.02, 4.03	Reverse Percentages 4.06, 4.07	Listing systematically leading to sample space diagrams 4.10, 4.11
A Level	Tangents & Normals 4.04, 4.05	Tangents & Normals 4.08, 4.09	Harmonic Form 4.12, 4.13, 4.14

Additional Comments

Teacher	Comment
4.01	Would be useful nearer the start of the course
4.02*	I certainly learnt from it in terms of what was good and what could be changed *Absent from research session so no evaluation for first 2 sections
4.03	As this was my first lesson it was substantial to my understanding of pace. The research and presentation reinforced certain aspects of planning and learning which worked well within the lesson e.g. getting students to explain and demonstrate understanding by explaining answers. The whole experience was so beneficial for me. Thank you
4.04	Gaining more information and knowledge about different ways of teaching and the advantages of doing so helped with my development
4.05	Gaining knowledge of the relational teaching theory helped me to reflect on my own teaching styles and how I implement it. I do feel that there needs to be an instrumental / relational balance but I am going to try and tip the scales in the relational favour.
4.06	Thoroughly enjoyable and productive two days. The relational/instrumental research was very interesting and very beneficial. Team teaching and being able to discuss ideas was very thought provoking, as well as the reflection period.
4.07	I found it really useful to implement/try out tasks that meant pupils had to have a more relational understanding. It was interesting to see how they took on the task. The scenarios highlighted and influenced the activities that we chose for the lesson
4.08	The session really made me look at how I approach teaching and learning and then adapt our plan to try and incorporate this into our teaching. Thankyou, to improve more exploring of the different in relational etc and perhaps more comparative with other teaching styles.
4.09	I think this is a fantastic idea and opportunity. I feel like I have learned a lot and this approach is close to my instinctively preferred method. The discussion and attempted classroom teaching were particularly useful but having various methods helped cement my understanding and felt in keeping with the theory being taught.
4.10	Enjoyed all sessions Presentation very engaging and informative Particularly enjoyed different approach to completing the square card sort
4.11	I found it very thought provoking and inspiring. It gave me ideas and theories that I'd like to use in my lessons in the future. I like the way we can challenge the way maths is normally taught.
4.12	I think this whole experience brought a lot of knowledge towards my development. Working as a team was beneficial. Having the opportunity to deliver the lesson was even more. I could learn from my mistakes and reflect on it.
4.13	Really enjoyed all of the sessions and activities. The feedback on our lesson was really useful. I really enjoyed the completing the square activity it made me see maths in a way never seen before.
4.14	Really enjoyed the activities (completing the square, graphs etc). Was unclear about the card sort task, it seemed a grey area or maybe I missed the point of it. Collaboration and team teaching was very useful.

Appendix 4.11 Group 4 collaboratively planned lesson

PGCE Secondary Lesson Plan

Trainee Name……4.06 and 4.07

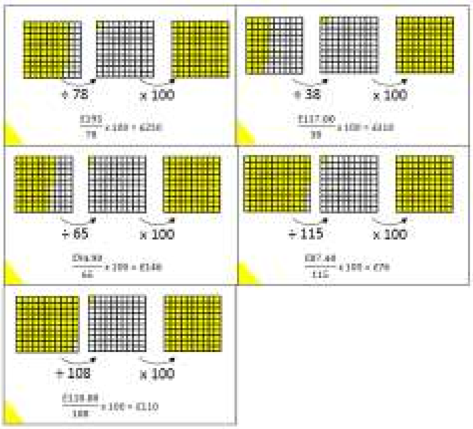
Date 05/12/13 Duration 60 mins			Class No. of Pupils	Ability
Big Picture Percentages			Assumed Prior Learning for this lesson Calculating percentage increases, decreases and percentages of amounts.	Personal provision, incl. collaboration with support
Lesson Title & Learning objectives Reverse Percentages Objective: To be able to find the original amount given a percentage increase/decrease and the new amount.			Assessment Questioning Justification of answers Comparison of scenarios Observation	Risks/ Health & Wellbeing Ensure all bags are under desks with no obstacles as pupils walk to the front. Ensure no leads are in the way.
Time	**Activity**	**Teacher activity**	**Pupil activity + Success criteria**	**Resources + Planned differentiation**
11:30	Starter	Mixed Percentages Bingo • Percentage of amount • Percentage increases • Percentage decreases Display choice of 16 numbers on the board and ask pupils to pick 8 different numbers. Allow 1 minute 30 for pupils to choose their numbers. Teacher 1 walks around the classroom monitoring pupil's on the question. Indicate to Teacher 2 when majority of pupils have completed question. Teacher 2 changes slides on PowerPoint. When a pupil has crossed off all of their numbers, declare them the winner.	Pupils should draw a 4 x 2 grid in the back of their books and fill in the numbers. Pupils are permitted to use calculators to solve the percentage questions. Success Criteria: All pupils should be able to recall basic percentage calculations	Bingo PowerPoint Exercise Books Whiteboard

11:40	Transition	Ask pupils to discuss with the person next to them how they solved the problems. (30 seconds) Discuss with class the methods that they used when solving the percentage questions and whether there were any differences. Ask pupils to raise hands to offer explanations. Write methods on board. Ask how they would work out 50% of 250? If no-one offers 'multiplier' method, ask: • Does anyone know of a quicker method? • How else can a percentage be represented? (Hopefully leads on to decimal representation) Offer class two scenarios. • £105 in a 30% sale. • £135 in a 25% sale. Ask them which of the two scenarios they think saves the most money. Ask them to raise their hands for the scenario that they think offers the best saving, record numbers on board. Introduce the concept of reverse percentages and how they should be able to give an informed decision at the end of the lesson	During this time, pupils are discussing the methods they used with their partners. Pupils are raising their hands to offer suggestions and listening to each other. Pupils are raising their hands to indicate which they think is the best option.	PowerPoint slides Whiteboard

11:50	Main Activity 1- Formulation	Hand out pre-prepared card sort activity. Explain to class that they need to group together the cards so that there are 5 sets of cards in each group. Display example grouping on board and ask class what they think the reverse percentage diagram means. (See example set of cards in Appendices) 1) Original Price & Price in Sale (to be filled in) (GREEN) 2) Diagram from original price to sale price with multiplier (RED) 3) Sale % (BLUE) 4) Diagram from sale price to original price (to be filled in) (PURPLE) 5) Unitary method with diagram (YELLOW) Allow 15 minutes for pupils to group together cards. Each group will be missing a card and pupils are required to create the missing card.	Pupils are to work in pairs to group cards together. The green and purple cards have information missing which the pupils need to fill in. They then need to stick one group of cards down in their exercise books.	18 x Set of 25 cards Glue PowerPoint Whiteboard
12:05		Discuss the two different methods with class – dividing by multiplier and unitary method. • Which two cards will help us find the original amount? • How do they work? • What is happening in the yellow cards? • Which method do you think is easier? Straw poll. • Identify that they both work, but using the multiplier is probably quicker.	Pupils should be listening and offering contributions to the class discussion. Success criteria. All: Should be able to identify the procedure behind reverse percentages. Some: Should be able to explain and justify why the procedure works.	

12:10	Main Activity 2	Display an exam style question on the board and read it through. Ask one pupil to identify the new amount and one to identify the percentage change. Ask one pupil to come up to the board and draw a diagram going from the original amount to the new amount. • Do we all agree? Ask another pupil to come up and draw the diagram in reverse – from new amount to original amount. • Do we all agree? Ask class to calculate answer and raise hands to offer suggestion. Hand out sheet (one between two) containing four exam style questions. Ask pupils to answer questions in book, with diagrams. (The sheet can be found in the appendices) Read out answers so that pupils can self-assess their work.	Pupils should be listening and thinking about the exam sheet question. Pupils should be offering to come to the board and one pupil should be chosen to draw diagram on the board. Pupils should be offering to come to the board and one pupil should be chosen to draw diagram on the board. Pupils should be writing their solutions, with diagrams, in their exercise books. Success Criteria: Most pupils should be able to use reverse percentages in a context when answering the exam style questions with a diagram. Depending on time; some pupils may get onto the extension task, the Tarsia puzzle where they would be able to identify the five reverse percentage mistakes and annotate the appropriate calculation.	PowerPoint Presentation Whiteboard Pen Exam Style Questions sheet Extension task: Completed Tarsia puzzle. Pupils need to find 5 mistakes within the completed Tarsia and correct them. (The sheet can be found in the appendices)
12:25	Plenary	Reintroduce the two scenarios that were shown earlier in the lesson. Ask class if they now feel comfortable that they could give us an informed answer. Allow class 2 minutes to calculate their answers and then ask if anybody would now like to change their minds. Ask a pupil who feels confident to come to the board and justify their answer on the whiteboard in front of the rest of the class.	Look at the question again and think about if they can now answer the question confidently. Work out the original price for each of the scenarios and then state whether they would like to change their original answer. A pupil will come to the board and show the method that they used in order to find which scenario had the best savings.	PowerPoint slides and whiteboard

Appendix 4.12 Group 4 card sort from lesson



APPENDIX 5: INTERVIEWS

Appendix 5.1 Initial interview schedule (October/November 2013)

Semi Structured Interview Schedule: Topics, Sample Questions and Prompts/Probes

Continuous Professional Development

- Q Reflecting on your own professional development, can you describe aspects that have been effective / less effective?
    - Enjoyment of CPD vs change in practice
    - Development of subject knowledge vs pedagogy
    - Effect on teaching and learning
    - If not taken part in any recent CPD why?
    - Barriers to implementation

Research

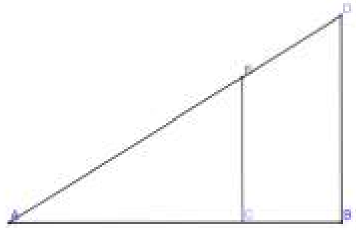
- Q Have you ever used research to inform teaching and learning?
    - How was it used?
    - How did you come across the research?
    - Was is beneficial?

Mathematical Understanding

- Q In your opinion, can you describe what it means for students to understand mathematics?
    - Constraints
    - How is understanding observed?

Connected Curriculum

- Q What aspects of mathematics does this image bring to mind?



- Different areas of maths vs. different representations
- Have you linked these areas in lessons?
- Give other examples of other curriculum areas where you make links explicit for students

Appendix 5.2 Mid-study interview 1 schedule (November 2014)

<u>Semi Structured Interview Schedule: Topics, Sample Questions and Prompts/Probes</u>

CPD Session 1: Research presentation (Feb 2014)

First can we reflect on the research presentation that was shared in February 2014.

- Q In your own words, can you describe the differences between instrumental and relational understanding?
- Q Did this 'exposure' to research have any impact on your thinking about what it means to understand mathematics?

CPD Sessions 2 and 3: Activities to explore making connections (March 2014)

Now can we think back to the sessions in March when the department got together and Nicola shared examples of activities that could promote making connections (*show hand out with prompt images*).

- Q Could you comment on the effectiveness of this CPD in terms of making sense of the research presentation?
- Q Were there any tasks/activities that were useful to clarify the terminology/develop meaning?
    - o Did activities challenge / engage
    - o Subject knowledge vs Pedagogic content knowledge
    - o Collaboration with peers

Active Experimentation Stage

Since March I am aware that further ideas have been shared at departmental meetings (e.g. use of play dough to develop the concept of prism volume) and colleagues are beginning to showcase their ideas on the display boards in the corridor.

- Q Can you reflect on any impact for you personally or your learners?

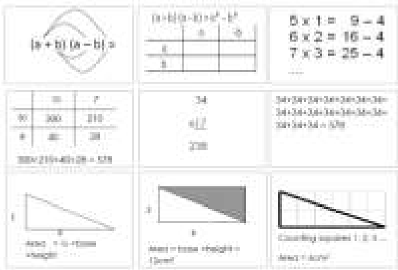
- Can you give me an example of an activity that is more relational?
- What is it about the activity that makes it relational?
- What do you need to do with a topic to try and develop a more relational approach?
    - Using tasks modelled vs development of new ones
    - Which approach? Similarities and differences, multiple representations / developing concepts
    - Barriers to engaging with the ideas

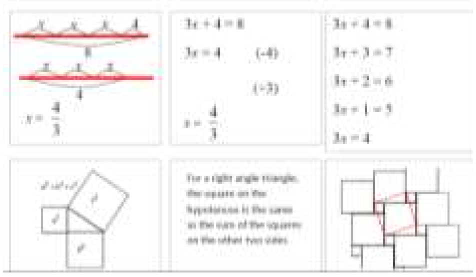
Moving forwards

- To further develop a connected approach to mathematics teaching and learning. What do you feel should be done next at a departmental / individual level?

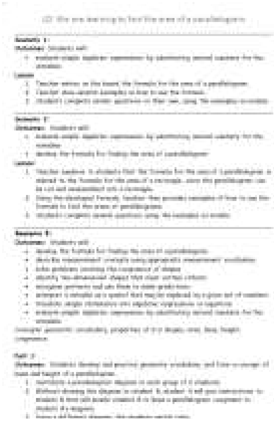
Appendix 5.3 Prompt images for mid-study interview 1

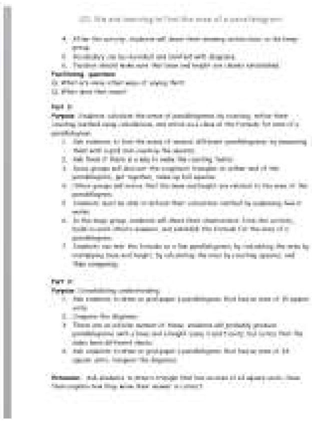
Card sort activity looking at the type of understanding shown in the images



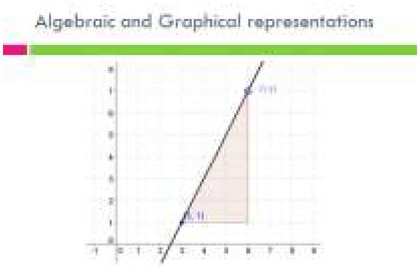


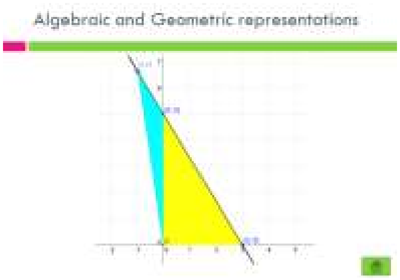
Exploring 3 different scenarios for teaching area of a parallelogram



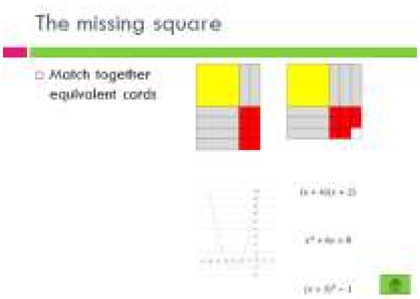


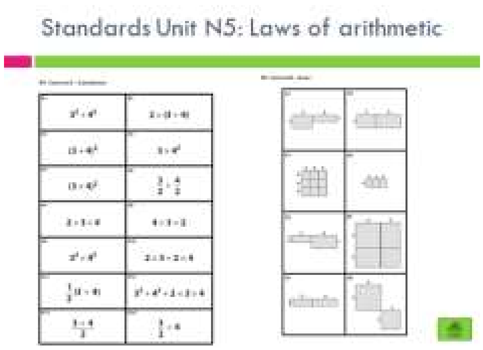
Different ways of writing linear equations



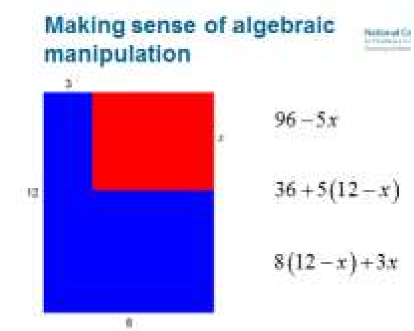


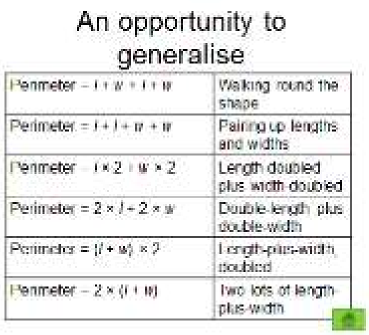
Developing multiple representations



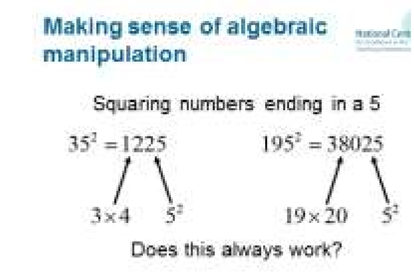


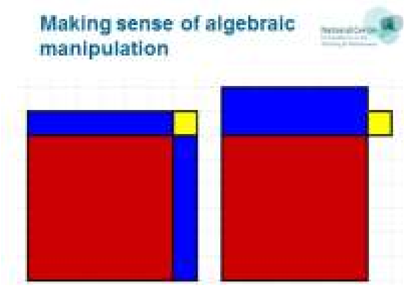
Building meaning for procedures



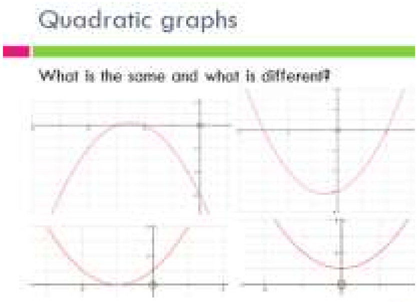


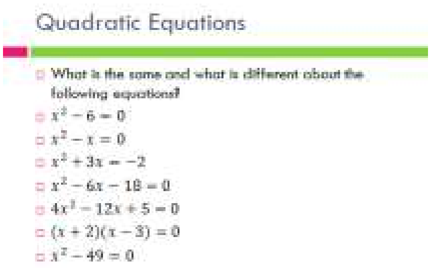
Linking algebraic proof with geometric proof



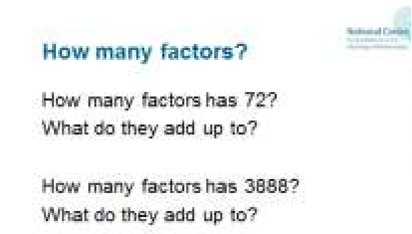


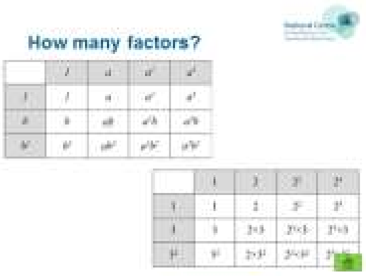
Exploring similarities and differences

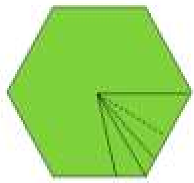




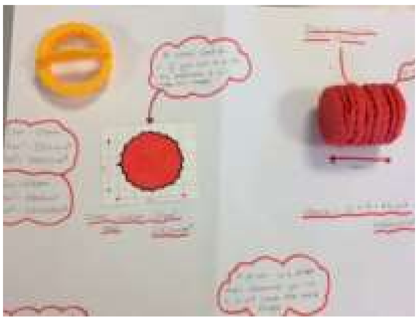
Problem solving that requires a relational understanding







Developing concepts of the volume of prism



Appendix 5.4 Mid-study interview 2 schedule (September 2015)

Semi Structured Interview Schedule: Topics, Sample Questions and Prompts/Probes

Curriculum Change

Since the department have been working together on elements of the Collaborative Connected Classroom Model there has been a curriculum change.

- What do you think about the model that Nicola has proposed in light of this curriculum change?
- Are the ideas and ways of working more/less important than before?

Mathematics Action Research (refer to action research planning template attached)

I understand that small teams within the maths department have been put together to develop a unit of work / resources to support aspects of the new curriculum.

- Which aspect of the new curriculum do you feel needs developing for your area? Why?
- Which feature of the CCC Model do you feel might support this?
- Have you any ideas yet what you might do / how you plan to go about this?
- Do you feel this is an opportunity to develop your own pedagogic content knowledge (i.e. explore new ways to teach the maths topics that you might not have done before)?

Implementing Change

After the initial phase of CPD input from Nicola, I understand that there is an expectation that you will explore ideas within your own classes.

- Do you feel that you recognise what Nicola is trying to encourage you to do?
- Do you feel that you have been able to implement the sort of changes that have been encouraged by the department?
  - What barriers have you experienced?
  - What has helped you?

Whole school move towards action research

I understand that this academic year there is a drive at whole school level for everyone to engage with action research?

- How do you feel about this?
  - Is it positive that directed time is being allocated or seen as another thing to do?
  - Do you plan to carry out your own research too?
- Can you comment on the CPD models (Nicola's CCC CPD programme and the one at whole school INSET*)* that have been shared? *(show handout Appendix Figure 16 and 17 for prompt images).*
  - Are these models preferable to previous methods of CPD?
- Are there any other comments that you would like to make about this direction of travel within the school *(show handout Appendix Figure 18 and 19)*

As part of this, I understand that the head of department has made available funding from the mathematics budget to support you with any reading you might want to do.

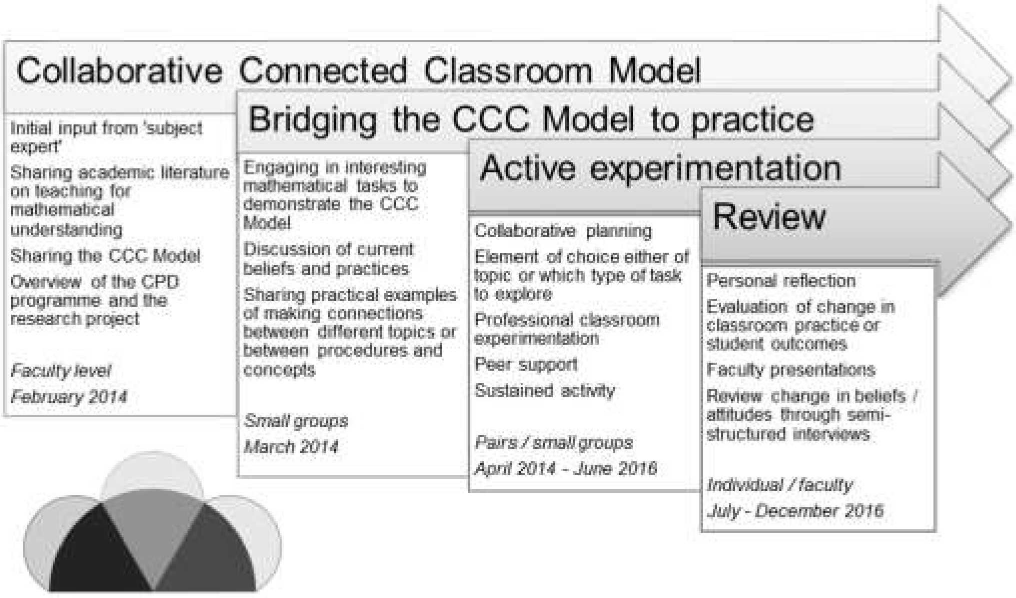
- Did you choose a book when offered?
  - If yes, what did you choose and why?
  - If not, why not?
- Extra question for head of department: Why did you choose to allocate funding to books for colleagues?

Themes that have arisen from previous interviews

There are several interesting themes that have arisen in previous interviews with the department and I would like to explore your thoughts on a couple of them further.

- Do you believe that ALL attainment levels of students are capable of (should be taught) conceptual thinking?
  - If not, which groups do you feel are / aren't?
- Are students within this school motivated to understand?
  - If yes, what makes them motivated?
  - If no, how can we make them motivated?

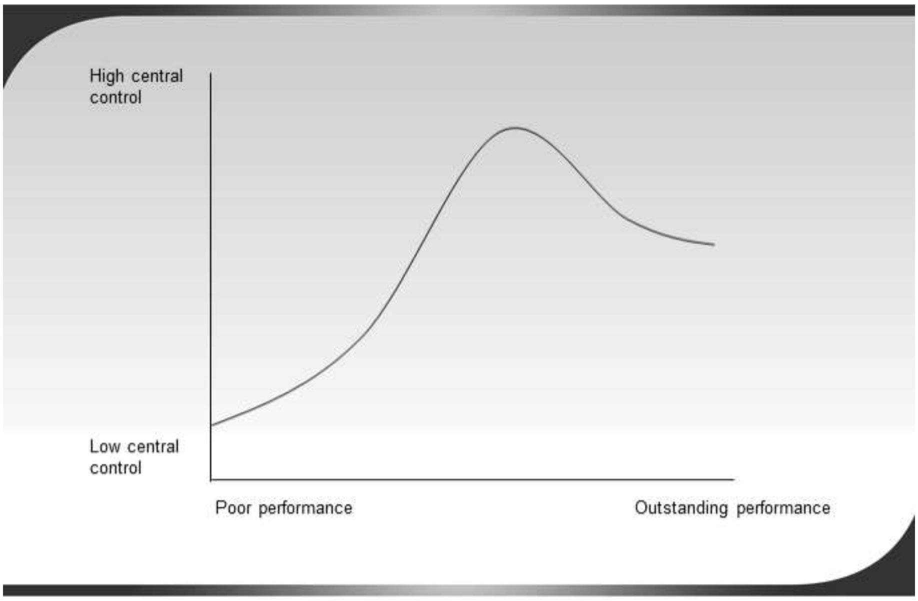
Appendix 5.5 Prompt images for mid-study interview 2



Appendix Figure 16. CPD programme for implementing the CCC Model

	Identify need	External input	Action Research	Evaluate impact	Disseminate outcomes
Personal CPD project					
Royal Shakespeare Company					
Theories of Action					
Prince's Teaching Institute					
Outstanding Teacher Programme					
Leadership Development Programme					
Masters/Doctorate Degree					

Appendix Figure 17. Whole school CPD model (2015, School presentation)



Appendix Figure 18. Performance graph (2015, School presentation)

	Effect Size	Top Effective Classroom Methods
1	1.60	Identifying similarities and differences, using analogies
2	1.00	Summarising and effective note-taking
3	0.80	Developing a 'Growth Mindset'
4	0.77	Repetition and Practice
5	0.75	Non-linguistic representations
6	0.73	Cooperative Learning
7	0.68	Effective Goal setting and feedback

Marzano, 2000

Appendix Figure 19. Effect size (2015, School presentation)

Appendix 5.6 Action research template

<table>
<tr><td colspan="2">The aims of this project are to work collaboratively to

Prepare our learners for the new 2014 curriculum changes by:

Ensuring progression is appropriate for different stages of learner
Ensure new ‘content’ is embedded into schemes of learning

Experiment with an aspect of the Collaborative Connected Classroom model to encourage development of a more relational understanding

The outcome is that learners:

Know what to do and why they are doing it
Know a range of concepts, symbols and procedures and how they are related.
Use strategies which are both efficient and effective
Are aware of connections within and outside of mathematics
Are confident in tackling unfamiliar problems
</td></tr>
<tr><td>The <u>aspect</u> of the new curriculum aims that this piece of action research sets out to develop is highlighted.</td><td>The overarching aims of the 2014 curriculum are for learners to:

become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.
</td></tr>
<tr><td>The <u>feature</u> of the CCC Model that this piece of work sets out to explore is highlighted</td><td>
Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
Tasks either connect together different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
Links are made between procedures and concepts

meaning is built for procedural knowledge before mastering it (‘educational approach’)
procedures are evaluated to promote conceptual understanding (‘developmental approach’)

Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
Application tasks are presented as challenges that may be problematic and need to be reasoned about
</td></tr>
</table>

Appendix 5.7 Final interview schedule (November/December 2016)

Semi Structured Interview Schedule: Topics, Sample Questions

Use of research to inform practice

- Q Why have you highlighted the sections in that way?
- Q What are the barriers to moving forwards?
- Q Do you feel you are using research more now than at the beginning of the study?

Strands of the CCC Model

- Q Are there some aspects of the CCC Model that you feel you have made more progress with? Why?
- Q If you have changed your practice – why did you?
- Q What are the barriers to moving forwards overall / in each aspect?
- Q Do you think you will continue moving forwards on this journey towards the transformation stage?

Your own presentation to department

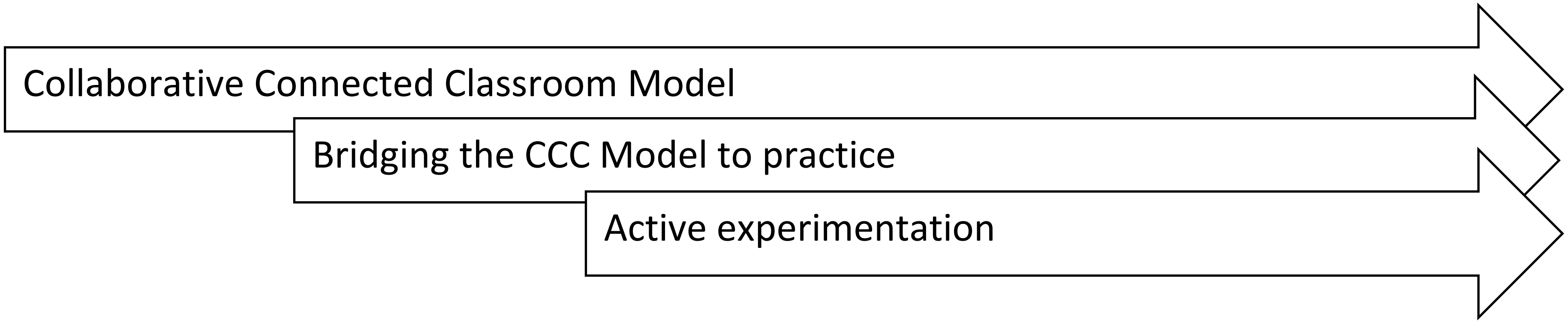
- Q How did you feel your presentation went to the department?
- Q Were you confident in presenting to others? Why / Why not?

Additional themes

- Q What prompts you to want to change your practice?
- Q Where/who do you 'go to' when wanting to develop your practice?

APPENDIX 6: PRESENTATION OF DATA

Appendix 6.1 Teacher Development Model



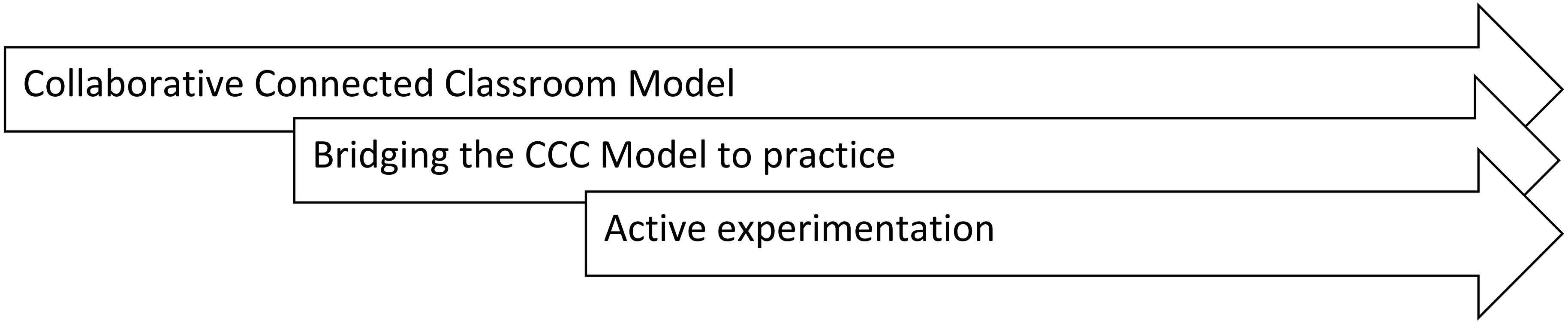
	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Use of research to inform practice	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research

	resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has read additional academic papers *(formal)* provided in PD sessions to inform practice	provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice		Has transformed, and continues to transform, classroom activities based upon reflective practice
Activities build on pupils' knowledge by connecting ideas to conceptual structure	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
Tasks connect different areas of mathematics	Has awareness that connecting different areas of mathematics is featured in the CCC Model	Has used given links with pupils to connect different areas of mathematics (e.g. making links between ratio and proportion) Has changed teaching methods with given links connecting different areas of mathematics	Has identified new links between topics which have not previously been used Has shared with pupils' new links that have not been previously used	Has developed own resources that make links between different areas of mathematics Has developed own strategies to make links between different areas of mathematics	Has transformed, and continues to transform, practice to enable connections between different areas of mathematics to be embedded within teaching

Tasks demonstrate connections by using multiple representations	Has awareness that using multiple representations to make connections is featured in the CCC Model	Has used given multiple representation activities with pupils to demonstrate connections (e.g. completing the square)	Has found own multiple representation activities to demonstrate connections Has used own multiple representation activities with pupils to demonstrate connections	Has developed own resources that involve using multiple representations to demonstrate connections	Has transformed, and continues to transform, practice by using multiple representations for appropriate topics within teaching
Links are made between procedure and concepts	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for ‘procedures’ Has regularly used alternative teaching methods for ‘procedures’ with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
Tasks involve comparisons (similarities and differences or efficiency of method)	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons

Application tasks are presented as challenges	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
Overall summary of use of CCC Model	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are 'connected' in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching

Appendix 6.2 Teacher self-reflection of development model



		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Use of research to inform practice	A	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	B	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	C	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice

	D	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	E	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	F	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	G	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	H	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	K	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice
	L	Has awareness that department are considering the CCC Model to support teaching and learning Has awareness that there are a range of reading / materials and resources *(informal)* that can be used to support teaching and learning Has awareness that there are a range of academic papers and books *(formal)* that can be used to support teaching and learning	Has used research presented at PD sessions to inform practice Has used features of the CCC Model to inform practice Has read additional academic papers *(formal)* provided in PD sessions to inform practice	Has sought out own reading / materials and resources from the internet, magazines, news articles *(informal)* to inform practice Has sought out own research from following up on additional papers provided in PD sessions to inform practice Has sought out own research from academic papers and books *(formal)* to inform practice	Has recognised the value of more formal research Has confidently made references to own academic reading and research which informs practice	Has used, and continues to use, academic research to inform practice Has reflected, and continues to reflect, upon practice utilising academic research Has transformed, and continues to transform, classroom activities based upon reflective practice

		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Activities build on pupils' knowledge by connecting ideas to conceptual structure	A	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	B	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	C	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	D	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	E	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	F	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	G	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
	H	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge
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	L	Has awareness that building on pupils' conceptual structure is featured in the CCC Model	Has used given activities that build on prior knowledge with pupils (e.g. use of grid method to support multiplying out algebraic brackets)	Has used own activities with pupils on a number of occasions to build on pupils' conceptual structure Has used own activities with a number of different classes to build on pupils' conceptual structure	Has implemented new ways of introducing topics in the classroom building on pupils' prior knowledge Has developed more effective strategies for assessment of pupils' current stage of concept development	Has transformed, and continues to transform, practice to enable curriculum connections to be developed from pupils' prior knowledge

		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Tasks connect different areas of mathematics	A	Has awareness that connecting different areas of mathematics is featured in the CCC Model	Has used given links with pupils to connect different areas of mathematics (e.g. making links between ratio and proportion) Has changed teaching methods with given links connecting different areas of mathematics	Has identified new links between topics which have not previously been used Has shared with pupils' new links that have not been previously used	Has developed own resources that make links between different areas of mathematics Has developed own strategies to make links between different areas of mathematics	Has transformed, and continues to transform, practice to enable connections between different areas of mathematics to be embedded within teaching
	B	Has awareness that connecting different areas of mathematics is featured in the CCC Model	Has used given links with pupils to connect different areas of mathematics (e.g. making links between ratio and proportion) Has changed teaching methods with given links connecting different areas of mathematics	Has identified new links between topics which have not previously been used Has shared with pupils' new links that have not been previously used	Has developed own resources that make links between different areas of mathematics Has developed own strategies to make links between different areas of mathematics	Has transformed, and continues to transform, practice to enable connections between different areas of mathematics to be embedded within teaching
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		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Tasks demonstrate connections by using multiple representations	A	Has awareness that using multiple representations to make connections is featured in the CCC Model	Has used given multiple representation activities with pupils to demonstrate connections (e.g. completing the square)	Has found own multiple representation activities to demonstrate connections Has used own multiple representation activities with pupils to demonstrate connections	Has developed own resources that involve using multiple representations to demonstrate connections	Has transformed, and continues to transform, practice by using multiple representations for appropriate topics within teaching
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		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Links are made between procedure and concepts	A	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	B	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	C	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	D	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	E	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	F	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	G	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	H	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	K	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching
	L	Has awareness of both procedural and conceptual knowledge	Has used given activities with pupils to make links between procedures and concepts (e.g. using plasticine for volume of prisms)	Has highlighted use of procedures within teaching and learning Has discussed alternative ways of teaching procedures Has trialled alternative ways of teaching procedures with pupils	Has researched alternative teaching methods for 'procedures' Has regularly used alternative teaching methods for 'procedures' with pupils	Has embedded, and continues to embed, links between procedures and concepts within practice Has made, and continues to make, links between procedures and concepts across all areas of mathematics teaching

		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Tasks involve comparisons (similarities and differences or efficiency of method)	A	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	B	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	C	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	D	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	E	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	F	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	G	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	H	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	K	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons
	L	Has awareness that exploring similarities and differences is featured in the CCC Model Has awareness that looking at efficiency of method is featured in the CCC Model	Has used given examples of comparisons (similarities and differences) with pupils in some classes (e.g. transformations on GeoGebra) Has used given examples of comparisons (efficiency of method) with pupils in some classes	Has used examples of comparisons (similarities and differences) with pupils in new topic areas Has used examples of comparisons (efficiency of method) with pupils in new topic areas	Has used comparisons regularly within mathematics teaching Has regularly reflected upon impact of using comparisons with pupils Has improved teaching methods for using comparisons based upon reflective practice	Has embedded, and continues to embed, the principle of making comparisons into everyday practice Has made, and continues to make, pupils value the importance of working with comparisons

		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Application tasks are presented as challenges	A	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	B	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	C	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	D	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	E	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	F	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	G	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	H	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	K	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly
	L	Has awareness that pupils applying their learning to challenging problems is featured in the CCC Model Has awareness that pupils developing their reasoning is featured in the CCC Model	Has used given tasks with pupils Has engaged students with mathematical challenges through application (e.g. how many factors, cutting hexagons)	Has used own problem-solving tasks, presented as challenges, with pupils Has considered the challenge encountered by students when working on these problems	Has trialled a range of application tasks presented as challenges Has given students time to explore problems themselves before offering support	Let's pupils tackle unfamiliar tasks, and regularly encourages them to do so Manages the learning when pupils are challenged, and demonstrates this regularly

		Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Overall summary of use of CCC Model	A	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	B	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	C	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	D	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	E	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	F	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	G	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	H	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	K	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching
	L	Has awareness of the differences between procedural and conceptual knowledge Has awareness of all of the features of the CCC Model	Has engaged with features of the CCC Model during PD sessions Has trialled features of the CCC Model with a class Recognises activities that are ‘connected’ in nature	Has used alternative resources that exemplify features of the CCC Model Has experimented with a number of classes Has experimented with a number of topics Has refined given activities and ideas to produce alternative resources	Has taken on board the generic principles and features of the CCC Model Has developed own teaching resources from academic research to exemplify the features of the CCC Model Has developed own strategies from academic research to exemplify the features of the CCC Model	Has embedded, and continues to embed, the principles underpinning the CCC Model in everyday practice Has refined, and continues to refine, both practice and activities as additional academic research transforms teaching

Appendix 6.3 Main study data by source and count of evidence

INITIAL INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	1	1	1	0	0	3
Connect areas of maths	0	0	0	0	0	0
Multiple representations	1	0	0	0	0	1
Procedures and concepts	1	0	0	0	0	1
Comparisons	0	0	2	0	0	2
Application tasks	0	0	0	0	0	0
	3	1	3	0	0	7

MID STUDY 1 INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	4	2	2	0	0	8
Connect areas of maths	9	0	4	0	0	13
Multiple representations	9	7	5	1	0	22
Procedures and concepts	8	8	4	1	0	21
Comparisons	2	1	8	1	0	12
Application tasks	1	0	0	0	0	1
	33	18	23	3	0	77

MID STUDY 2 INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	0	0	1	0	0	1
Connect areas of maths	2	0	3	0	0	5
Multiple representations	3	1	2	0	0	6
Procedures and concepts	1	0	0	0	0	1
Comparisons	1	2	4	8	0	15
Application tasks	1	1	0	0	0	2
	8	4	10	8	0	30

FINAL INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	0	1	1	3	0	5
Connect areas of maths	3	0	4	2	0	9
Multiple representations	2	2	12	7	3	26
Procedures and concepts	6	5	4	3	0	18
Comparisons	1	2	6	2	3	14
Application tasks	1	0	3	2	0	6
	13	10	30	19	6	78

ALL INTERVIEWS	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	5	4	5	3	0	17
Connect areas of maths	14	0	11	2	0	27
Multiple representations	15	10	19	8	3	55
Procedures and concepts	16	13	8	4	0	41
Comparisons	4	5	20	11	3	43
Application tasks	3	1	3	2	0	9
	57	33	66	30	6	192

TRIANGULATED DATA	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	1	12	12	2	0	27
Connect areas of maths	2	5	13	6	0	26
Multiple representations	4	12	45	22	1	84
Procedures and concepts	1	16	15	16	2	50
Comparisons	1	6	31	8	0	46
Application tasks	1	2	18	3	0	24
	10	53	134	57	3	257

ALL DATA	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation	
Conceptual structure	6	16	17	5	0	44
Connect areas of maths	16	5	24	8	0	53
Multiple representations	19	22	64	30	4	139
Procedures and concepts	17	29	23	20	2	91
Comparisons	5	11	51	19	3	89
Application tasks	4	3	21	5	0	33
	67	86	200	87	9	449

Appendix 6.4 Main study data by source and count of teachers

INITIAL INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	1	1	1	0	0
Connect areas of maths	0	0	0	0	0
Multiple representations	1	0	0	0	0
Procedures and concepts	1	0	0	0	0
Comparisons	0	0	1	0	0
Application tasks	0	0	0	0	0

MID STUDY 1 INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	3	2	2	0	0
Connect areas of maths	7	0	3	0	0
Multiple representations	7	5	4	1	0
Procedures and concepts	6	6	3	1	0
Comparisons	2	1	5	1	0
Application tasks	1	0	0	0	0

MID STUDY 2 INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	0	0	1	0	0
Connect areas of maths	2	0	2	0	0
Multiple representations	2	1	2	0	0
Procedures and concepts	1	0	0	0	0
Comparisons	1	2	3	4	0
Application tasks	1	1	0	0	0

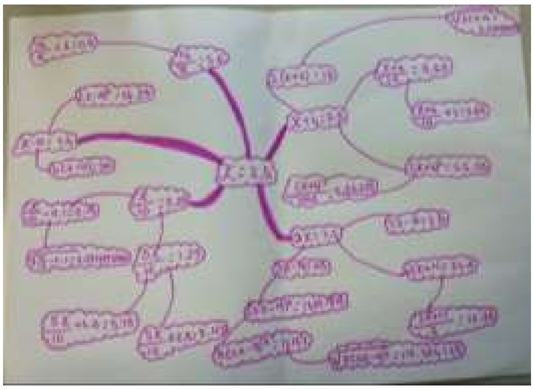
FINAL INTERVIEW	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	0	1	1	3	0
Connect areas of maths	3	0	3	2	0
Multiple representations	2	2	7	4	2
Procedures and concepts	4	4	2	1	0
Comparisons	1	1	6	2	3
Application tasks	1	0	3	1	0

ALL INTERVIEWS	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	4	4	3	3	0
Connect areas of maths	10	0	4	2	0
Multiple representations	9	7	10	4	2
Procedures and concepts	10	8	5	2	0
Comparisons	4	3	11	6	3
Application tasks	3	1	3	1	0

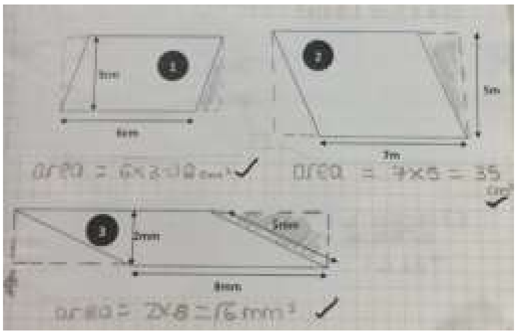
TRIANGULATED DATA	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	1	1	1	1	0
Connect areas of maths	2	1	5	2	0
Multiple representations	3	4	8	5	1
Procedures and concepts	1	3	4	2	1
Comparisons	1	4	9	3	0
Application tasks	1	0	4	1	0

ALL DATA	1. Awareness	2. Guided Exploration	3. Independent Exploration	4. Independent Development	5. Transformation
Conceptual structure	4	4	4	3	0
Connect areas of maths	11	1	6	3	0
Multiple representations	11	10	11	5	2
Procedures and concepts	11	9	7	3	1
Comparisons	5	7	11	7	3
Application tasks	4	1	7	2	0

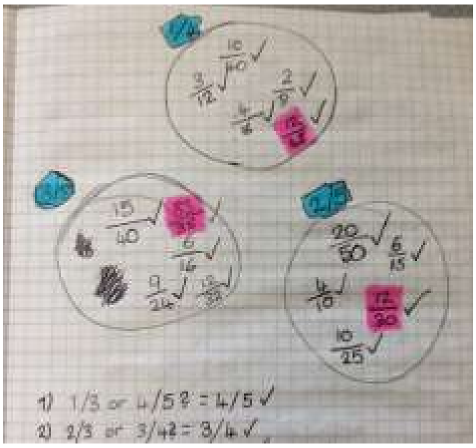
Appendix 6.5 Making links to students' conceptual structure



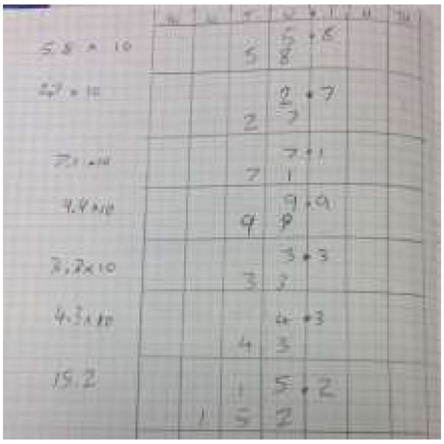
Charlotte, 2016, Book scrutiny



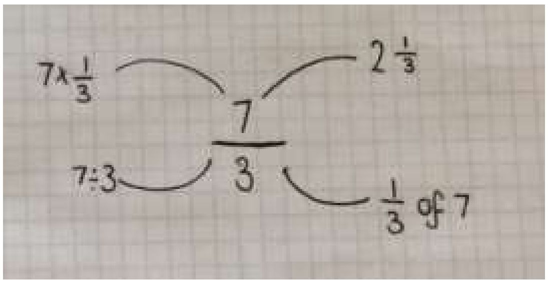
Ian, 2016, Lesson observation



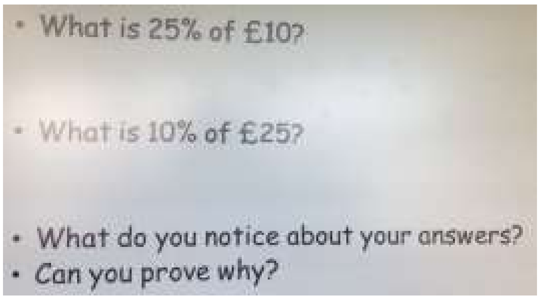
Elliot, 2016, Book scrutiny



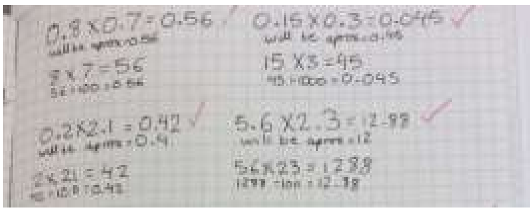
Frazer, 2016, Learning walk



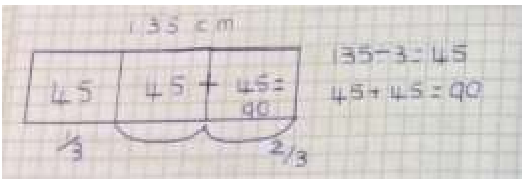
Louise, 2016, Book scrutiny



Elliot, 2016, Learning walk

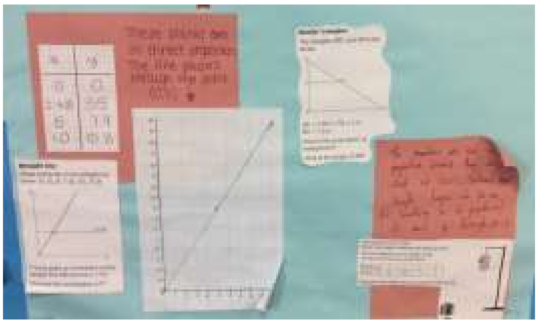


Kate, 2016, Book scrutiny

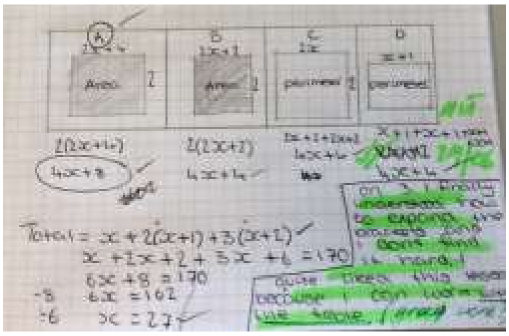


Heidi, 2016, Book scrutiny

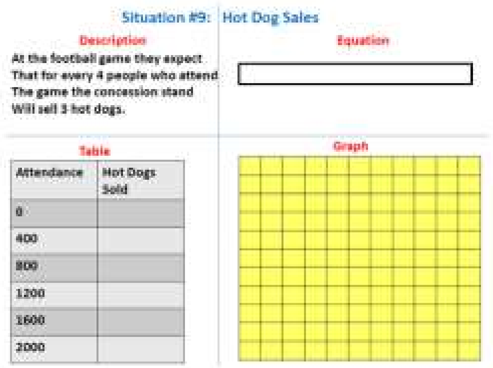
Appendix 6.6 Connecting areas of mathematics



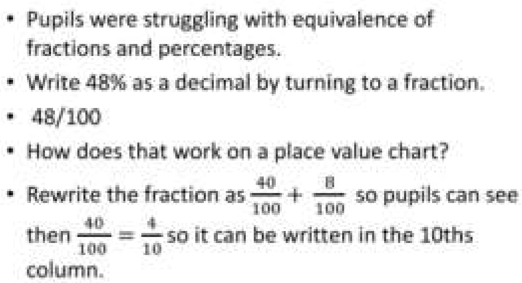
Charlotte, 2016, Learning walk



Heidi, 2016, Book scrutiny



Kate, 2016, Presentation



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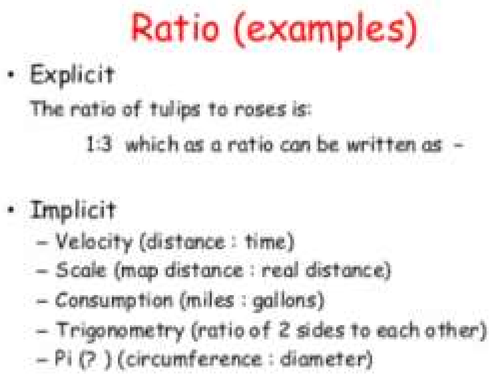
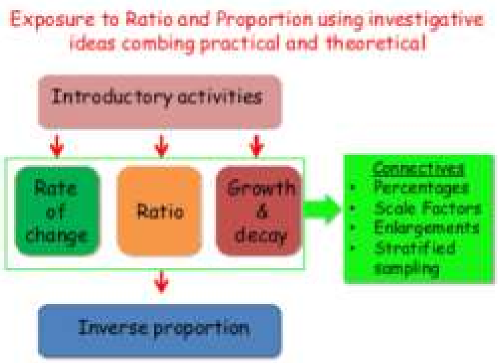
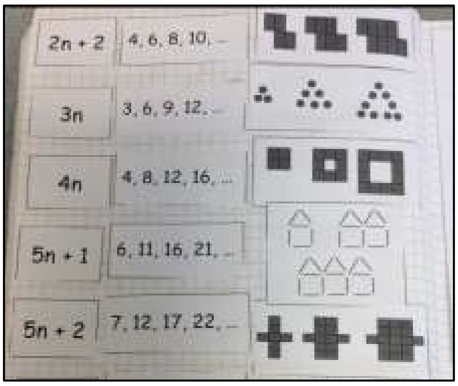


Brian, 2016, Presentation

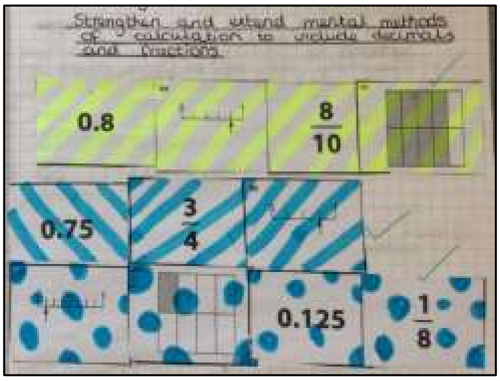


Brian, 2016, Presentation

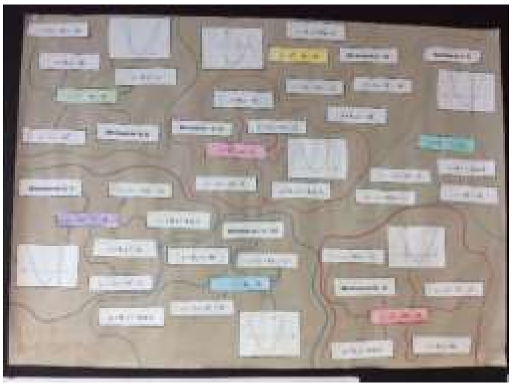
Appendix 6.7 Multiple representations

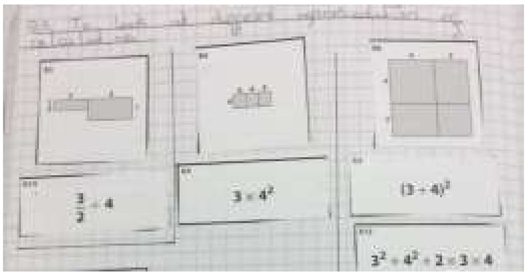


Ian, 2016, Book scrutiny

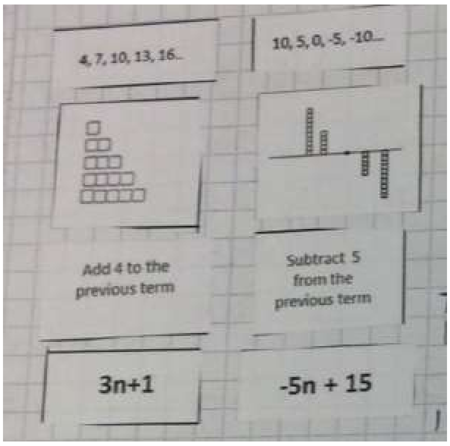


Louise, 2016, Book scrutiny

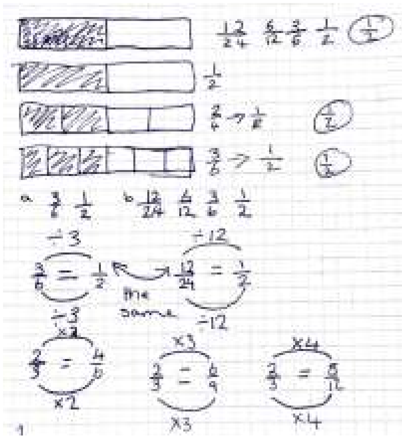

Charlotte, 2016, Learning walk

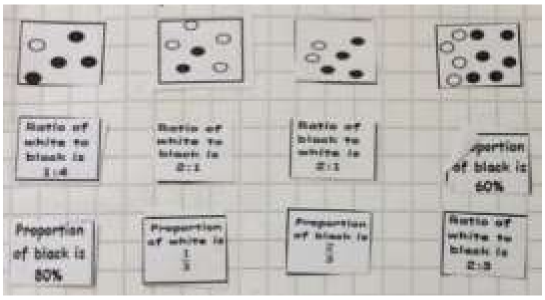


Elliot, 2016, Learning walk

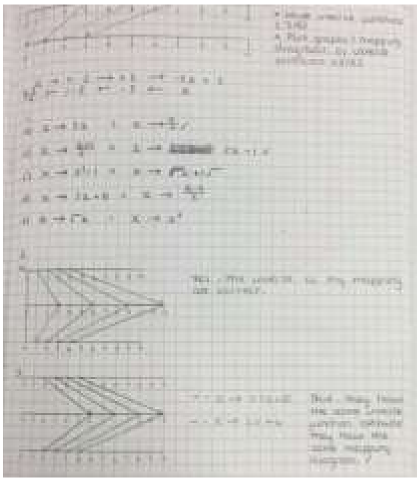


Elliot, 2016, Learning walk

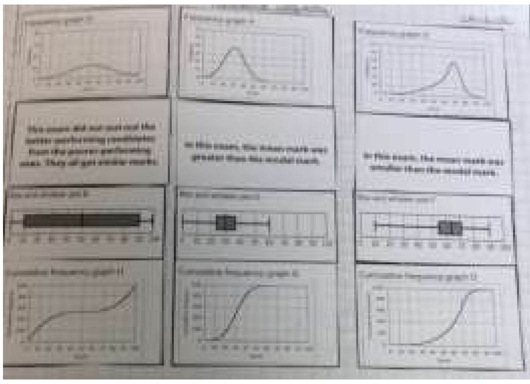

Elliot, 2016, Book scrutiny



Elliot, 2016, Book scrutiny



Frazer, 2016, Learning Walk



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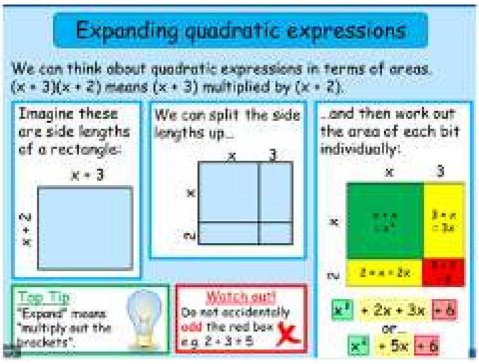
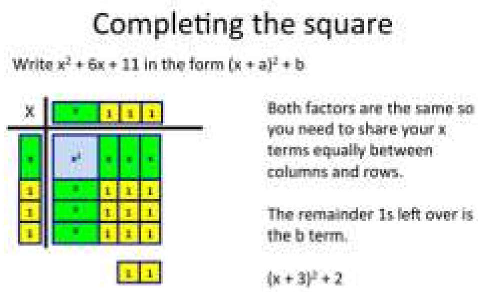
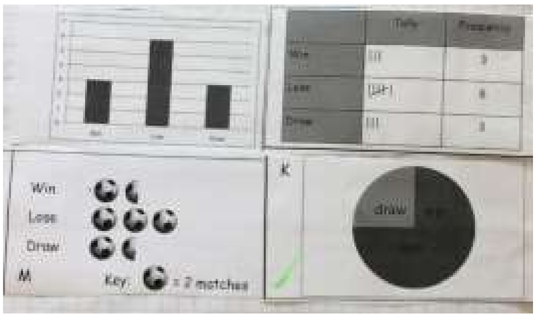


Kate, 2016, Presentation

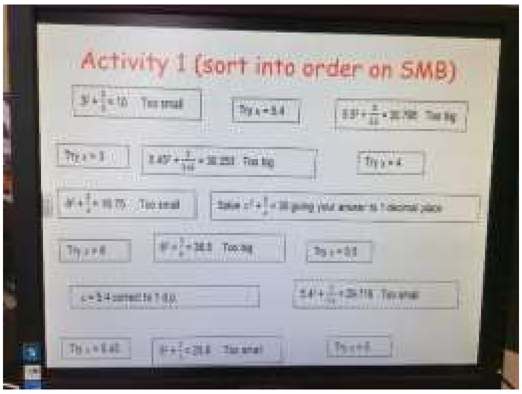


Kate, 2016, Presentation

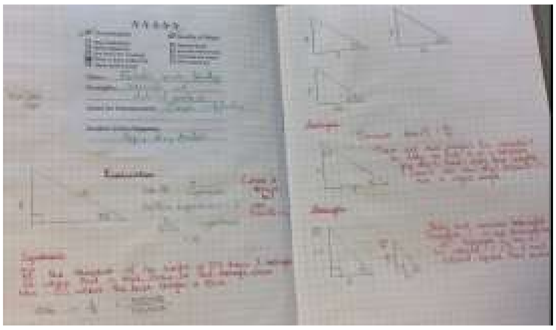


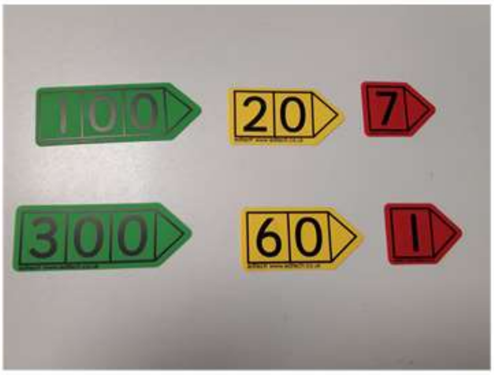
Louise, 2016, Book scrutiny

Appendix 6.8 Making links between procedures and concepts



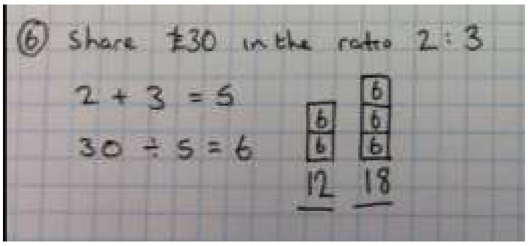
Brian, 2016, Learning walk


Brian, 2016, Book scrutiny

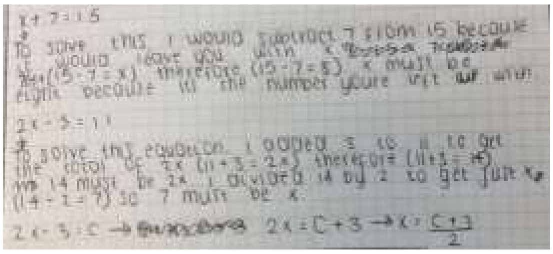


Kate, 2016, Presentation


Charlotte, 2016, Learning walk

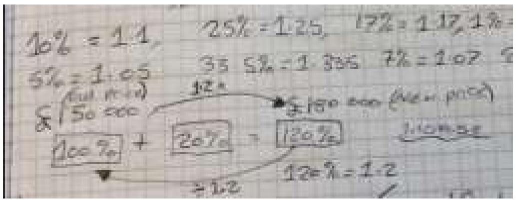


Daniel, 2016, Learning walk

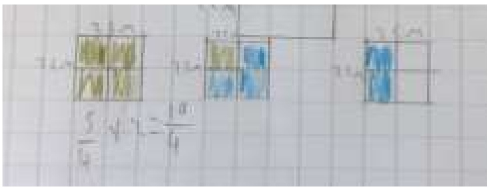

Daniel, 2016, Book scrutiny


Frazer, 2016, Learning walk

Ian, 2016, Learning walk

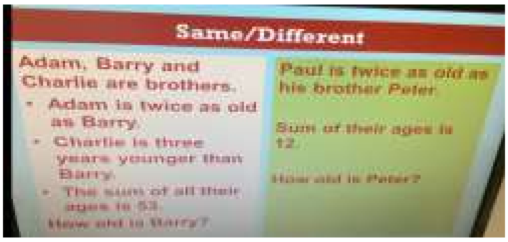


Heidi, 2016, Book scrutiny

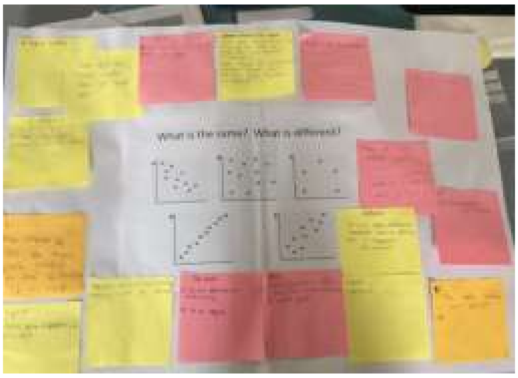

Georgie, 2016, Learning walk

Appendix 6.9 Making comparisons

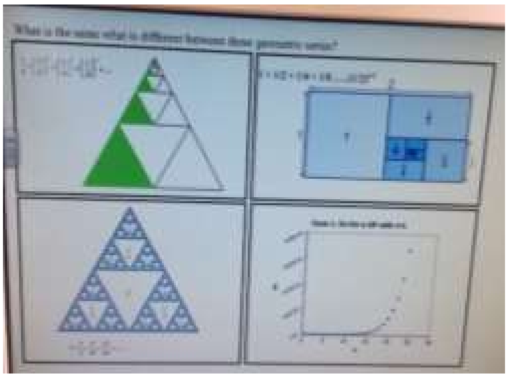
Elliot, 2016, Book scrutiny



Heidi, 2016, Learning walk

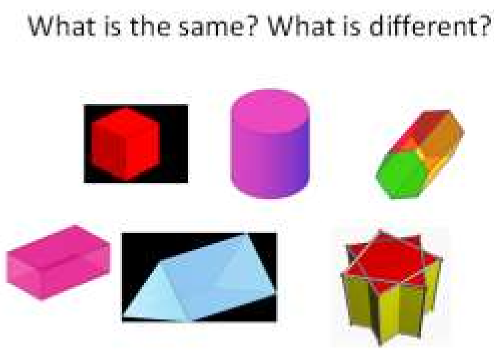


Kate, 2016, Presentation

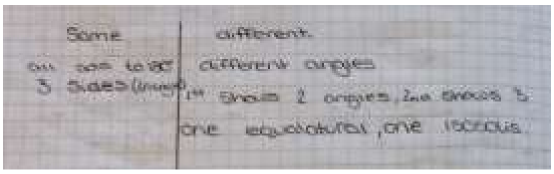


Charlotte, 2016, Learning walk

Elliot, 2015, Display

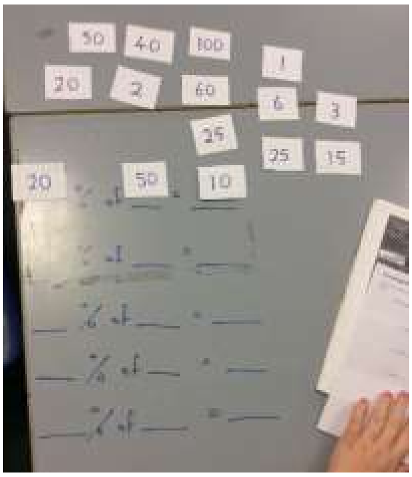


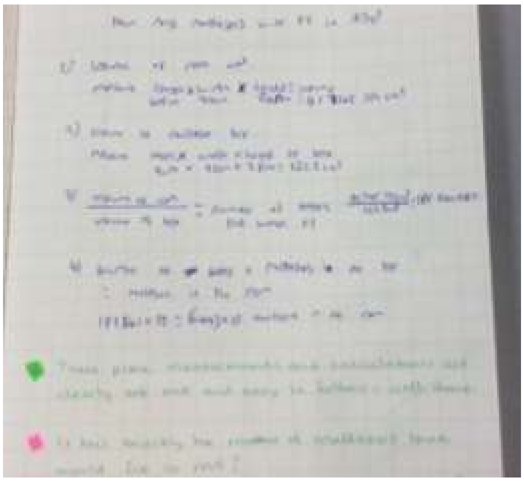
Kate, 2016, Presentation

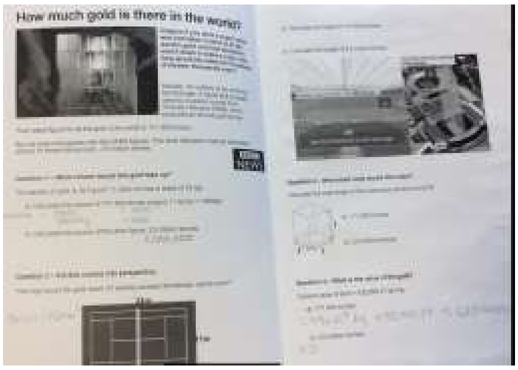


Heidi, 2016, Book scrutiny

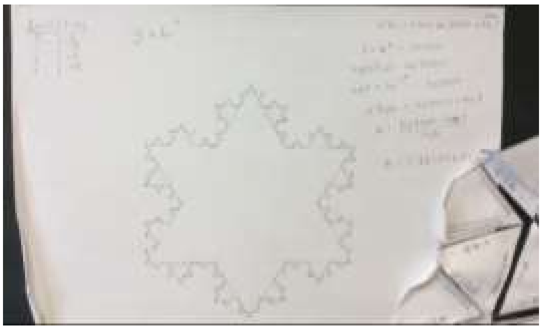
Appendix 6.10 Application tasks


Annette, 2016, Learning Walk

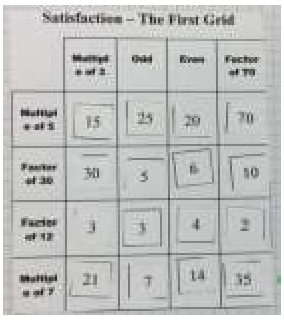

Annette, 2016, Learning Walk



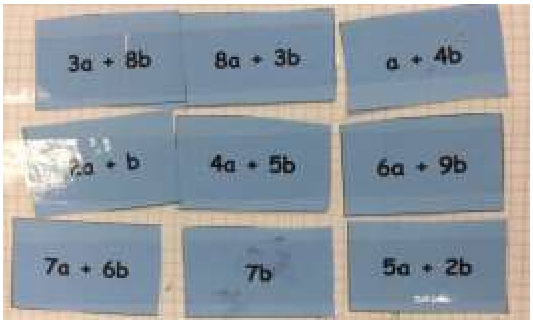
Brian, 2016, Book scrutiny


Charlotte, 2016, Learning Walk

Elliot, 2016, Lesson observation

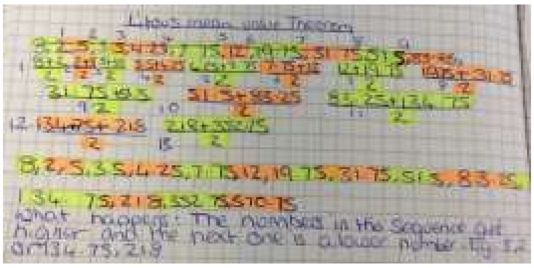


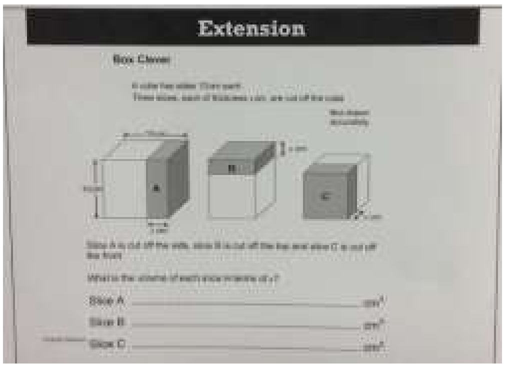
Elliot, 2016, Learning walk



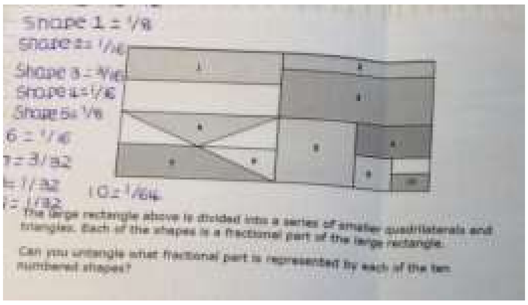
Elliot, 2016, Learning walk

Elliot, 2016, Book scrutiny


Georgie, 2016, Book scrutiny



Heidi, 2016, Learning walk



Heidi, 2016, Book scrutiny

Appendix 6.11 Interview extract Georgie (November 2013)

	Timespan	Content	Person
1	0:00.0 - 0:23.3	Reflecting on your own CPD can you describe aspects that have been either effective or less effective	INT
2	0:23.3 - 1:07.5	Yes, things that I feel that have been particularly effective both at school and outside are collaboration with groups of maths teachers. I took part in the local leader's network meetings last half term and it was really useful to go along and just discuss ideas informally with them and the same thing here break times and lunch times are when we get the best collaborative planning now that is not official professional development but just a simple conversation and that I am feeling like I am developing like a teacher and that is only something that has happened fairly recently here.	Georgie
3	1:07.5 - 1:11.7	Why has it only happened recently here?	INT
4	1:11.7 - 1:29.6	Tensions within the department meant that there wasn't ever a time when we would gather informally to have a discussion over a sandwich and people would tend to go and do their own things in different places since Nic has been here, she has encouraged us to do this and when we are sitting around there will be a maths problem flying around the table or something like that. That's the way, you know I am a maths geek I love to talk about maths, and that is what has happened a lot more recently and the more we talk about it the more confident you feel to want to push yourself and to want to work with other people. I feel like now I am at a stage where I would be happy to open my door to anybody to come in and watch me and I would be happy to go and watch them whereas before I would have been a bit guarded about being worried about people coming to watch me	Georgie
5	2:01.4 - 2:05.4	So, something has really changed within the department then?	INT
6	2:05.4 - 2:52.9	Yes, and now it is the openness and the way we discuss maths the collaboration and I think that has done more for me here than any professional courses that I have been on. What I have found when I go outside the school is that we are implementing a lot of things that are being recommended nudge boards, red and green pens and so on, part of that is influenced by Nic and part by the head of department whose has been an outstanding head of department and a lot of the time we are doing things that are exciting and new and I feel like we are leading in professional development and this year we are not going to middle managers meetings because it was us doing it.	Georgie
7	2:52.9 - 3:02.9	That is really good. So, give me one example of how this informal development has influenced something that you have done in the classroom	INT
8	3:02.9 - 3:14.0	Well schemes of learning particularly, before we were encouraged to collaborate with schemes of learning but now we are told you are going to use your professional development time to write these schemes of learning so we sat around at lunch time and someone might say I am not sure what I am going to teach tomorrow for this particular thing and someone might say I have got a brilliant resources and before you know it there will be 200 sheets being copied so everyone in the department can use it. That never used to happen before you would have 8 teachers teaching the same thing	Georgie

		8 different ways and now, a particular example would have been with loci I started doing part of my masters on learning outside of the classroom and so I was taking my students out and trying things with it and people were saying I like the idea of that, I am going to try that, by the end there were quite a few members of staff that were willing to take it out and try it which wouldn't have happened before	
9	4:03.6 - 4:16.0	You mentioned a couple of interesting things there, everybody doing the same thing. Is that a valuable thing? How does that benefit the department and the students?	INT
10	4:15.9 - 4:24.6	Well it doesn't necessarily but if everyone is singing from the same hymn sheet it can work against or for it depends what the activity is	Georgie
11	4:24.6 - 4:25.7	Assume it was a good one	INT
12	4:25.6 - 5:15.0	If you think it is a good one, what it means we have got faith in each other now, that if someone says I have got this brilliant activity instead of going well I am going to have to go home and go through it and through it and check we trust each other that yes this a good resource for everybody. When I talk about everybody using it and whether that is a good thing or not if it is a good resource yes it is a good idea because, what would be ideal is if a student could go from one class one year to another class another year and not feel like they have got to learn a new way of learning maths because everyone is singing from the same hymn sheet and doing activities that were similar, we haven't got one person using a text book and one uses worksheets and one person that only uses learning outside of the classroom it is a good it a good melange of things	Georgie
13	5:15.0 - 5:19.8	So, you are kind of in a funny sort of way you are taking the strengths from everybody	INT
14	5:19.8 - 5:21.2	Yes	Georgie
15	5:21.2 - 5:22.6	and trying to	INT
16	5:22.5 - 5:23.5	disseminate them across the department	Georgie
17	5:23.5 - 5:24.8	and trying to get a more sort of homogenous department	INT
18	5:24.7 - 5:33.3	yes, collaborative which is what has always been in the development plan but I honestly think feel that until Nic turned up and the new head of department started running the department we didn't have that	Georgie
19	5:33.2 - 5:48.4	Ok that is really interesting. Um, I guess most of that is about how you teach or how teach a particular topic	INT
20	5:48.4 - 5:49.7	I would of thought so yes	Georgie
21	5:49.6 - 5:55.3	Have you got any examples of less effective professional development	INT
22	5:55.3 - 6:49.2	Um, I find some of the professional development within school, so whole school training, more difficult and less effective, because they have come up with a problem that they perceive us to have and then try and solve it by making all of us do the same thing at the same time, a recent example of this is the literacy policy, now I consider myself to be literate and I can spell and I can use grammar but we spent 2 hours of a training evening doing literacy a couple of weeks ago and there are so many more things that I could be doing within that 2 hours that I felt that within that 2 hours I didn't make any progress from where I was, and surely the point of	Georgie

		professional development is that you start from a base point and move forward	
23	6:49.2 - 6:54.1	Just going back to the kind of stuff that you like	Georgie
24	6:54.1 - 7:08.4	A lot of what you have described to me sounds about pedagogy and how you teach do you do anything as a department about subject knowledge or subject content is that something you have experienced recently	INT
25	7:08.4 - 8:03.3	Yes, every department meeting we do have a discussion about, using Heidi that brings it in because she has been off with the PTI, something to look through together and we will discuss different methods because we have got some very mathematical people and people that like to draw pictures and we have got people that like to use algebra and some of the harder questions I really struggle with so often I will be sitting back just watching people. Now firstly because it a more collaborative department I now feel more confident to just start it and give it a go but it is really nice to see everyone's different methods of doing it and having the confidence when someone like Heidi explains something to you she is so calm and patient it makes you think yes, I can do this let's have a go at it so yes I think department meetings is probably the time that we do it most	Georgie
52	11:38.2 - 11:46.8	Okay we had better move on to mathematical understanding. Can you describe in your opinion what is means for a student to understand mathematics	INT
53	11:46.8 - 12:27.1	Um, for a student to understand mathematics (repeats the question to self), ...I am not sure I understand mathematics, um I think I have the confidence to be able to use it and to do it so I suppose it stems from being able to have the confidence to try something to go out and to have a go at it because understanding maths ranges from, in my mind, the very basic level up to degree level and I certainly haven't got a degree.	Georgie

Appendix 6.12 Interview extract Georgie (March 2014)

	Timespan	Content	Person
1	0:01.0 - 0:13.8	First question is how did you feel about doing the mathematical tasks in the session that you did 2 weeks ago and in the session this morning	INT
2	0:13.8 - 1:40.3	Right, I have always been quite honest about the fact that I don't believe that my mathematics skills are up there with everyone else in the department. I don't think that I am the mathematician in the same that everyone else is. I believe that my skill lies in being able to see it from the kid's point of view. I didn't do a maths degree. So honestly when Nic presented the tasks a couple of weeks ago I first panicked a little bit because I was embarrassed about how little I felt that I would be able to do, um, I then actually realised with the department being so different now it didn't make a difference anymore. Whereas before Christmas I was made to feel like it mattered that I couldn't do things. The first session that we did two weeks ago I genuinely felt like there was support there for me and so if I wasn't quite sure where I was going there was scaffold questions and things like that to help tease it out of me. So, when we started today I was almost excited to see if I could remember the sort of things I had learnt and think about where I could move forward from there and I have never felt that before. So, I think for me the two parts of it have brought out an excitement in me in something that I didn't think I had. I thought that was something was other people did, it was way over my head, far too much for me.	Georgie
3	1:40.3 - 1:43.0	So, what is different before and after Christmas?	INT
4	1:42.9 - 1:46.2	Person X (names the previous head of department).	Georgie
5	1:46.2 - 1:53.9	So, go on expand,	INT
6	1:53.9 - 2:41.5	When person X was in charge and that goes back a long way but even up to the day he left he would make you feel like if you didn't quite understand something that wasn't acceptable, um, if I had a slight gap in my knowledge or whatever - for example I had never heard the word composite used before if I am honest (referring the session just run by Nic where the word had been used and exemplified) and I have no qualms with going and telling Nic that whereas with person X if I said I am not actually sure how to do that he would be-little you and make you feel like that wasn't acceptable and there was a flaw in you and you would do everything you could to get over that but there wouldn't be that support and you would be made to feel like this was your problem not this is something that we can overcome together which is how I feel now.	Georgie
7	2:41.5 - 2:49.8	I think that..	INT

8	2:49.8 - 3:07.6	Before I would say that differences were not a good thing everyone had to be the same and everyone had to be degree at first level mathematician and everyone had to like the same things whereas now people in the department appreciate that we all have different strengths and weaknesses and appreciate this and that okay sometimes you are not going to know something and it is okay to go and ask other people. That's the difference I think	Georgie
9	3:07.5 - 3:14.9	So is it just because person X isn't here or is it because of the sorts of things that you have been doing with Nic or is it something else as well	INT
10	3:14.9 - 3:17.0	It is partly to do with the confidence that Nic has instilled in with the activities that we have been doing so the fact that we do activities like this all of the time, and again we used to do them with person X but it was the case of here is the correct answer that's the end if, it wasn't a case of so what did you find easy what did you find hard, what can we do to develop you etc. So partly it is Nic but partly it is the fact that none of us now feel we have this cloud hanging over us and all of us feel a lot more content to chat with each other about strengths and weaknesses.	Georgie
11	3:49.2 - 3:54.1	Has it been helpful that you have done tasks that people don't know the answer to	INT
12	3:54.0 - 3:55.1	Yes	Georgie
13	3:55.1 - 4:04.0	I guess one of the things with person X's era is that person X knew the answer to everything	INT
14	4:03.2 - 4:05.4	There was a right and wrong answer	Georgie
15	4:05.3 - 4:17.0	Whereas just even looking at what we have done this morning, um, one of the things that Nic has emphasised with the similarities and differences is that there are no wrong answers	INT
16	4:16.9 - 4:21.2	Exactly yes	Georgie
17	4:21.1 - 4:26.3	And in that question we did about the factors, none of use knew the answers, so none of us knew and we kind of got on the case	INT
18	4:26.2 - 5:18.4	Exactly but we felt free to be able to do that whereas before it would have been I am not writing anything on my show me board in case he tells me of for writing that. That was the difference I think. But also Nic knows from what I have said that what we were doing this morning that I don't necessarily enjoy or find easy so her support of me in that, coming over and asking the questions about what do you think is going on here and how do you feel we should move on from here it is just so totally different. I felt supported, if I had said I don't have a clue, please, then there would still have been that support rather than 'for goodness sake'. I feel able to make mistakes in these sorts of sessions which I never have done before	Georgie
19	5:18.4 - 5:25.1	It is not just because Nic is different from person X,	INT
20	5:25.1 - 5:47.0	No, it is because of the discussion technique; it is the way that tasks are discussed and the way that Nic doesn't come across as I know the answer to the task and this is the right way and the wrong way.	Georgie

		The way that she is so free with 'okay the first time I looked at this I didn't know what to do and I didn't know how to approach it' and that makes you feel more relaxed. So, it is Nic partly and it is the approach that she takes	
21	5:46.9 - 5:51.8	The approach	INT
22	5:51.8 - 5:52.8	Yes, definitely without a doubt	Georgie
23	5:52.8 - 5:59.8	Did you do this question I think she did last time the one about squaring numbers that end in a 5 and there is that funny little rule that you get	INT
24	5:59.7 - 6:00.7	Yes	Georgie
25	6:00.7 - 6:03.9	How did you find that one?	INT
26	6:03.9 - 6:47.2	I love stuff like that and actually that's where my interest in maths comes from, um and I think that's where my skill in the classroom lies because I know lots of little tips and tricks that will get a class hooked. Look I can times by 11 incredibly quickly and thinks like that and I don't necessarily know the algebra behind it but I can use the tips and tricks so for Nic to then say here is the algebra behind it - oh brilliant because now I do the tips and tricks and now I know where they are coming from more and I am more willing to explore that I can honesty see just from that little session that my interest in maths is going to grow almost exponentially now. There is so much more I can explore and I feel like I should go back to uni and do a maths degree	Georgie
27	6:47.2 - 6:48.5	It has given you a kind of way in	INT
28	6:48.5 - 7:31.1	Yes it has opened the door to,.. I suppose the last 5 years I felt was capped at an average mathematician who has got a strength in mental arithmetic and tricks and things like this, now I am being made to feel like with the support you could be a mathematician, nothing here makes me feel like Nic is better than I am we are equal and that is such a nice environment to be a part of. It really does make a difference. These tasks, the fact that we are able to explore and try things is brilliant the five squared task was fascinating	Georgie
29	7:31.0 - 7:32.0	So it is stimulating your mathematical interest?	INT
30	7:32.0 - 7:34.4	Yes, because I love maths	Georgie
66	18:03.6 - 18:10.9	Good, is there anything else while we are on this topic that you would like to add that we haven't talked about	INT
67	18:10.8 - 18:16.8	No, just that I am really excited about this	Georgie

68	18:16.8 - 18:20.4	That is interesting in itself	INT
69	18:20.4 - 18:22.0	I think it is really important	Georgie
70	18:22.0 - 18:23.2	It is good in your job if you are excited about it isn't it?	INT
71	18:23.2 - 18:52.8	Yes, I suppose so, but that is what I have said before that is what Nic inspires in you, she wants you to ..., you want to be part of her team and honestly, I am not necessarily a team player and I think that Charlotte and Nic would agree with that and so for me to get on board with what she is doing	Georgie
72	18:52.8 - 18:53.8	so, you are coming out of your comfort zone	INT
73	18:53.8 - 19:09.0	yes, because that is what she inspires in us, I wouldn't do it for anybody, I probably wouldn't do it for anybody else if I am honest but because she has got this love and she makes me want to be a better mathematician and a better maths teacher and I think that is what it is for me	Georgie

Appendix 6.13 Interview extract Georgie (November 2014)

	Timespan	Content	Person
29	9:10.8 - 9:44.7	So, what we will do now is to move on to the session that you did in March when Nicola did all these activities with you which you printed out and I have got some here. And, what she did really was try to show you some activities that would promote connections. So, the first thing that I want to ask you is could you comment on the effectiveness of the CPD in terms of making sense of the research presentation that she did in February	INT
30	9:44.7 - 12:18.9	It sort of brings it all home really. I have always been a little bit embarrassed really when we have these department meetings about my understanding about maths, because I understand maths and I understand what everyone is talking about but when people are saying prove it, I often take a back seat, I am quite a confident member of staff and I think the department would say that I am probably one of the more confident members of staff but in these CPD meetings that we have had previously I generally take a back seat and wait for everyone else to start talking because I am not convinced that I know what I am talking about really. And probably the first time we were introduced to this I was thinking this is all well and good but I am not sure how I can translate this into my teaching so being able to have the resources in front of me and to be able to sit with a group of people that I feel comfortable with and develop my thinking and not be told, you don't understand this and you are not getting this right, to be encouraged to explore my thinking I was able to then think okay and think I can do this and I can inspire others to do this as well. So, I think the activity really did back it up. Because it is all very good being told this is the way it can work, being able to do it myself from I situation where I couldn't do it before. I came away from the CPD session thinking my goodness me I am much better at maths than I thought I was and that was really nice and really good for me. And now being able to use the activities, I have got a high ability set in year 11, and being able to use the activities with them. You sort of introduce an activity to them and they sort of say I can't see the connection, the one that really strikes me was the completing the square activity and seeing all the different connections with quadratic equations. I never would have thought about coming up with an activity like that, but seeing the way that the students made the connections after I had made the connections in a similar way was inspiring. It was brilliant. Um, and so, like I say, being told this will work is one thing, seeing it for yourself and doing it for yourself and seeing it for yourself through the activities was brilliant and having the confidence to be able to say, yes I can do this, it is not rocket science it makes sense	Georgie
31	12:18.9 - 12:20.0	So, there was something very useful in that CPD session for you personally in terms of your own understanding	INT
32	12:20.0 - 12:29.4	yes absolutely	Georgie

33	12:29.4 - 12:43.6	Maybe, tell me if I am wrong, that was quite critical in kind of your decision to take it to your class because you could see it, if it works for me it will work for them	INT
34	12:43.5 - 12:44.4	absolutely	Georgie
35	12:44.4 - 12:53.3	and you seeing the impact in the class because you have had things to take forward	INT
36	12:53.3 - 13:17.6	yes, completing the square was a topic that I was struggling to teach, I taught it at A Level as revision which was fine but to actually go in from the very beginning and to try to explain completing the square, to have an activity like this that backs it up was brilliant and now it inspires me to think how can I teach other topics in a similar way to let them to see the connections themselves I think	Georgie
37	13:17.6 - 13:24.6	So, this stuff that Nic did with you back in March gave you some sort of spring board to get going	INT
38	13:24.4 - 13:25.7	Yes, it was probably the most exciting CPD session that I have had because I felt like I was able to use the stuff that we had done and not be embarrassed about it anymore.	Georgie
39	13:25.7 - 13:44.8	How has it been taking that forward and thinking about how can I do this in other places because obviously this doesn't cover the whole GCSE course	INT
40	13:44.7 - 14:28.9	No, it gives me the, instead of when you are searching for resources and things like that, I mean generally we have the schemes of learning mapped out, but when searching for additional activities, there are some outstanding websites out there that I probably would have avoided in the past because I didn't necessarily understand what the activity was demanding of you and I would have just looked at it and thought at first glance I don't understand so I am not going to bother with it. But now I think okay, I have got the understanding but I need to sit and think about it a bit more, so it has probably made me less lazy in a way in that I will work harder to find things that possibly will work because I stand a better chance of finding out how they work. So, I think that is the main thing that I have taken from it.	Georgie
63	25:18.6 - 25:29.0	Final question about moving forwards, to further develop a connected approach to mathematics teaching and learning what do you think should be done next at departmental/individual level?	INT
64	25:29.0 - 27:04.0	At a departmental level in order to build my confidence and the confidence of the more cynical members, and I am not a cynical member I am completely on board with everything that we are doing but I put myself in the group with the cynical people because I don't think my understanding is quite there and I need to be pulled along. I think if we could have much more regular sessions on how to build up these activities, and I am not saying that Nicola needs to be the one running them but taking it in turns perhaps every departmental meeting someone has to come up with something inspirational because I know that a lot of people have been to the Princes Teaching Institute and have come up with activities and brought them back so I think working as a team to do that. Individually I probably need to stop being quite so lazy and relying on glad that we have got schemes of work written and thinking about how I could get my teaching to move on. Because although it is there not everything suits every class. So just more of the same really and the second	

		CPD session in March I came away really excited thinking I can do this and not only can I do this but I could write a lesson based on this so just practising really.	
65	27:04.0 - 27:13.8	I think you kind of mentioned getting other people does that convey the message that we can all do it	Georgie
66	27:13.7 - 28:03.0	yes, but we have all had such different experiences, life experiences and teaching experiences, and educational training experiences that I could come up with some perfectly good activities but to have eight or ten people coming up with activities from all of their different experiences would be hugely beneficial. I have been to PTI but people come back inspired, because I haven’t got that opportunity they are seeing things that I wouldn’t see so it is a case of sharing because everybody has got different experiences I would say.	INT
67	28:03.0 - 28:06.8	Is there anything you want to add?	INT
68	28:06.8 - 28:10.6	I am going to go away and reread the stuff on instrumental and relational stuff!	Georgie

Appendix 6.14 Interview extract Georgie (December 2016)

Use of research to inform practice

Q The first bit is about the use of research, you have got quite a bit of green but it is turning pink towards the end, so tell me a bit about that?

For the project that we undertook, the fractions work, we did a lot of work together as a group. The awareness bit that is just knowledge of and we spent quite a bit of time in department meetings talking about where we could access things then when we broke off into smaller groups, because I was part of a smaller group, we talked about where we could find other things. So that is where the awareness comes from the CCC Model, we had done a lot of work in our department meetings on that and how we could use it. The guided exploration was using research presented at CPD sessions that was used in the project that we had done in the presentation to show everyone else what we had done. As we moved through, with the independent development it is finding time to do these additional things. So, for the academic reading and research, I had done quite a lot based around what was here and there was always the intention of this is really peachy and interesting or I am going to go off and look at that in a bit more detail but it is finding the time to go and do that on top of everything else. So, what I was finding was moving from being aware of it to completely changing the way I think and teach is quite a big jump so as I moved down it became more difficult to implement.

Q Getting hold of academic articles is not so easy and then finding time to actually sit and think about them is what you are saying

Yes, there is quite a lot, there is quite a lot on the t-drive that we can use and Heidi and I had looked together at different articles to sort out our own reading materials and read lots about different things and in fact part of the presentation that I had done come from something that we had looked at together on the internet so that is why, to a point it is fairly easy to get the information then. I mean independent development, like I said, it is finding the time so I started a masters before my child was born but I don't know when I am going to have the chance to finish it really, um and I would love to be able to because I really feel that it would be useful because I really enjoyed doing it and have been able to use it in my teaching, I think that has really helped. So, from there it is just moving forwards really.

Strands of the CCC Model

Q So, let's look at these strands here which are more related to teaching really, again you have got a lot of green and then it goes pink as you move across. So, tell me about that. Perhaps pick out strands that you have made more progress than others and say why?

And so again, it is being able to use things that I have found. So, other people have come with ideas about how you teach these sorts of things and sort of implementing that in the classroom is one thing but sitting down to come up with a completely new way of thinking I find, firstly quite daunting and for something like fractions it is quite difficult because a lot of the stuff has been done. I really enjoyed this part of the project and I really enjoyed integrating that into the classroom and part of the independent exploration finding our own things but then transforming and developing my own methods, I suppose in a way I was taking what I had read and using that but in my own way, but not enough to be able to call it my own if you see what I mean.

Q So, you are kind of adapting and using things that other people have created

But I wouldn't call it creating

Q Maybe extending them

Yes, extending them a little bit and then adapting them for my class because every class is different and thinking about different ways of use. But it is not something that I could call my own, not by any means.

Q In a way isn't that quite a good place to be?

Yes, absolutely and I was proud of what I had discovered and I mean if we hadn't been pushed to do these subject developing things within our projects then I probably wouldn't have focussed on something that was subject based so I was really thankful because fractions are something that everybody struggles to teach and now I have this method that actually seems to work, I have tested it out and tried it. I wouldn't have come up with that by myself I don't think so the fact that we have been pushed that way and then I found this it inspired me in the future so this time when they said to us as part of the whole school you have to do another project straight away I said I want to do proportional reasoning because I know it is a topic that I have struggled, well not struggled to teach but there must be better ways to teach it and then the department said we are going to do proportional reasoning and I thought great we can all do things together and run along together with this, whereas probably previously I would have said I want to think about my own teaching something that I could do in the classroom like behaviour or homework or something like that whereas now I am much more inclined to go on a subject based route

Q And to go with the whole department rather than going off on your own subgroup or

No, no because I am happy to go off on my own or subgroup and in a way, it is better for me because I know what I need to be doing in my teaching. For example, in the department meeting we had last week we were discussing things and a lot of what was being discussed was something that I already use anyway because this project has made me think about lots of other things in a different way. One of the examples, and this is all about the collaborative connectedness of the classroom, I used arrows (referring to the proportional reasoning INSET the week before) a lot in my teaching because if I use it on one topic and can use it in another topic students are sort of looking and going but you have used these before, and now you are using them for this and I can see that connection. That fits with this a bit

Q This connecting ideas to their conceptual structure

Yes, absolutely. So, introducing new ways of building on pupils' prior knowledge so what I mean, with something like the arrows, I have used arrows to show connections with other topics and what I bring in when I start a new topic is how can I use that again. Because I have used it before the students see it and go, actually I can do it like this which is really useful. I am not at the stage where I am transforming and enabling curriculum connections to be developed from pupil's prior knowledge because I am not pushing onto that yet, but next year I think that box would go green because we are all going to be looking at it and how we can implement it across the curriculum.

Q At the back of mind, I am thinking, Georgie can take ideas adapt, extend and use, but what you are kind of stop developing is when you have to be more original?

Yes, that is exactly it

Q So, what you have just seems to imply to me that in actual fact working with the department you as an individual and with the ability to spark off each other you could start to be much more original

Yes, and I think quite a lot of us feel like that, another member of the department has taken something that she has always struggled with in the past but because the department meetings have given her the opportunity to practice certain things and actually see the best way to move forwards it to take something that you are not 100% confident with and that's how you move forwards. Don't practice things that you are good at but things that you are not so good at. And I think with me it has been nice to say as a department we need to work on this, go ahead and do it and like I said I wouldn't have had the inspiration to go and do it on my own.

Q Let's look at this last page, you have gone a lot of green with just a couple of bits' pink, Yes, and again this links back to the projects and the fact that we have developed teaching resources so although I have been taking lots of information from the internet and things that I have read. This particular project and the presentation that I used was brilliant but then it meant taking that and using my own resources and developing those for my own individual classes and making sure there was differentiation and things like that but, from that being able to use, because we were looking at pictorial representations as well, making sure that we were using pictorial representations in lots of things and the students are feeling more confident to be able to do that. We have been using show me boards a lot more in my lessons rather than saying everything that we do has to go in our exercise books why don't we look at drawing pictures and really getting to grips with how your brain works rather than me telling you what you should be doing.

Q So, thinking about you developing, you seem to suggest that you have changed your practice and I think you talked about the project being a big part of that, are there any other things that have prompted you to change your practice?

I think I mentioned this before, the inspiration in the department, you feel like, not just during department meetings, but during break times and lunch times, you aspire to be like the people that are doing it right in my mind. So, you listen to what people are saying and think that sounds like it is going to work and I am going to try that, but we didn't have that before, we didn't have that for a long time. We were made to feel like our teaching wasn't good enough and we weren't good enough but now it is like an aspirational department and we are made to feel like you want to achieve what everyone else is achieving.

Q So that is something that is really important for the rest of the team

Yes, definitely and that is something that, I am really proud to be part of this department

Q That is good, isn't it?

Yes, it's nice but when we were in a whole staff meeting and we were being held up as the department that done particularly well with our presentations because we had done presentations and that felt nice but it didn't feel fake, it felt like we had done it because we wanted to do it and we want to work together as a team to improve but like I say it is an aspirational department.

It is a changed culture, isn't it?

Yes, absolutely. I want to be as good as the best teachers in the department um and we have got, everyone has got their different strengths so it is not just one or two people.

Q They don't want to remain on a pedestal
No, they want to

Q They want to take you with them

Yes, exactly, aspirational everybody, well not everybody but most people have got these desires to move forwards and I suppose we are quite lucky that we have got a couple of mature teachers but people that have not been long qualified and so they are looking to make progress as well so it is not just the people that are doing masters and things like that it is everybody which is lovely.

Q What are the barriers to moving forwards in these different strands or are there just general barriers?

It is time, time constraints, trying to do my job, it is difficult enough with the time that I have got let alone planning these inspirational things that are going to work in every lesson and we are not given the time to do it. This term has been particularly tough; I know by the time we get to the summer term we have a lot more time but by then we are mentally and physically exhausted from the rest of the year. It is time really, it is time. I have got the desire to do it but time constraint is the only think really stopping me. If you could say to me you have some hours every week and you must sit down and do this then that would be wonderful I would love that.

Q So, you have got lots of pink over here on the right hand side and you have talked a bit already about moving onwards with this new proportional reasoning theme. So, it sounds to me like your development is continuing and you are heading to the far side of Nicola's grid. Is that fair comment?

Yes, I think it is, I am definitely interested in developing as far as I can but I wouldn't be this far without the support and without the project going ahead in the first place.

Action research project presentation to the department

Q How did you feel your presentation went to the department?

Not great, it was, I didn't do it at the same time as everybody else as we ran out of time and by the time I eventually got to do it I came into the meeting late and it felt a bit rushed. It was the last 10 minutes of a meeting that was an hour and a half that everyone else had been in. I also, because I hadn't seen anyone else's presentations as I had been teaching the first time we had done them so I felt a little bit, in a way it was good as I don't feel the pressure of so and so did extremely well but I also felt like it's probably not good enough so I felt a bit of pressure like that.

Q Do you feel like you missed out from a developmental perspective by not seeing what everyone else did?

Absolutely and finding the time now to sit down and go through their presentations is not something I have done so I have missed out on seeing how they have progressed too which is a shame. In a way it has made me feel like, because mine was left, that it wasn't really as important as everyone else's.

Q I am sure that was not the intentional

No it was just the timetable and that I ended up teaching at the time but….

Q But that presentation bit doesn't take away from what you got out of doing the project?

No and if I am honest the presentation was, I would have preferred to have spent the time doing more work on it rather than having to prepare a presentation in the time, because the presentation took me so long to prepare when I could have spent more time on the project.

Q Aside from you having the duff slot so to speak were you confident in presenting to others?

I was confident in the outcome as in I was confident in what I had done was successful but I am not a presenter really, even though as a teacher my job is as a presenter really, it is difficult when you are doing it to your peers. Certainly, the outcome of what I had achieved I was pleased with and I thought it was important that I did share because I thought it would positively inform my peers teaching.

Q So, you were happy because you had something of value to offer?

Yes, absolutely that is exactly it.

Changing practice

Q What prompts you to want to change your practice?

My aspirational department, the fact that I don't want to feel like I am standing still because everybody is making progress and everybody is doing this that and the other, oh I have tried this and it makes you want to be a part of it. It makes you want to succeed and to learn from everyday experiences in the classroom and how to improve in the classroom. I think that is probably the biggest thing and also seeing the students and how they react to different ways of teaching.

Q That is interesting say a little bit more about that.

So, in my first five years of teaching we were all under a lot of stress and pressure to teach in a certain way from our head of department at the time and I didn't get the sort of excitement from classes I had during training and things like that. There was so much pressure on us that there wasn't time to plan these different ways of teaching things

Q So that was here?

Yes, and then it all changed and it was made clear to us that actually there are better ways to teach, and since I have been doing that you get that, you see that advert on the tele with the whole lightbulb moment, and suddenly a student becomes excited about what they are doing and they want to move forwards with themselves and I think that inspires you as well. That inspires you to want to improve and change the way you are teaching.

Q You have a good base here?

Yes, it really helps, and then we had the first five years here and the current head of department came here at the same time as me as she also went through the same horrendous stress and difficulty with me and then everything became so much easier when she became department head and we have been this inspirational, aspirational department. It is partly my colleagues and also the lightbulb moment when you see it working. When you see them excited to come to lessons, especially as a maths teacher, you get parents saying they have always hated maths but they love it now and I think I have done that it is amazing and the best job in the work for that reason.

Q I think you have probably answered this already but where do you go when wanting to change your practice?

Colleagues, absolutely colleagues,

APPENDIX 7: PUBLICATIONS

Appendix 7.1 Proceedings of the BSRLM (June 2013)

Exploring the features of a collaborative connected classroom

Nicola Trubridge and Ted Graham

Plymouth University

This article considers the various dichotomies between types of mathematical understanding. It concludes that whilst the different categorisation is useful, it is the interplay and connections between these types of understanding that is more beneficial to student learning. Theories are drawn from a wide literature base to consider what this might look like in the secondary mathematics classroom and the Collaborative Connected Classroom Model is proposed.

Keywords: making connections, collaboration, understanding.

Dichotomy of types of understanding

There are many different views of mathematical understanding and the debate has been on-going for decades. Often researchers and educators refer to a distinction between two types of understanding or knowledge. One of the first references to this is by Skemp where relational understanding is defined as "knowing both what to do and why" and instrumental understanding as "rules without reasons" (1976, p.2).

Byers and Herscovics (1977, p.26) were in agreement in principle of both relational and instrumental understanding but also put forward suggestions that there were some types of understanding that did not fall into either of the two categories. These were intuitive understanding, "the ability to solve a problem without prior analysis of the problem" and formal understanding, "the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning".

Buxton (1978, p.36) rather than suggesting different types of understanding proposed four different levels of understanding. The first is rote which is the purely instrumental. The second is observational which is "slightly deeper than purely instrumental but not fully relational". The third level is insightful which is said to be relational. The fourth level, formal, refers to the definition from Byers and Herscovics which "is only appropriate after insightful or relational understanding is achieved and at a stage in the student's development where some idea of the need for and the nature of proof is accepted".

As well as the distinction into relational vs. instrumental understanding, the distinction of understanding into the categories of procedural and conceptual has been a focus of many research articles and is perhaps the most common.

> Conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information … a unit of conceptual knowledge cannot be an isolated piece of information; by definition it is part of conceptual knowledge only if the holder recognises its relationship to other pieces of information. (Hiebert & Lefevre 1986, pp.3-4)

It follows that constructing relationships between the pieces of information leads to the development of conceptual knowledge. These relationships are so important that Hiebert and Lefevre distinguish two levels at which these can occur, the primary and the reflective. At the primary level "the relationship connecting the information is constructed at the same level of abstractness ... than that at which the information is represented" (1986, p.4) whereas at the reflective level "the relationships transcend the level at which the knowledge currently is represented" (1986, p.5).

Hiebert and Lefevre divide procedural knowledge into two distinct parts "one part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks" (1986, p.6). Despite the distinction into two types of procedural knowledge, they are similar in that all procedural knowledge relies on a "sequential nature" (1986, p.6). It appears to us that it is perhaps this sequential nature of relationships that makes it different from conceptual where the relationships can be of many different types.

The notion of 'connections' occurs regularly in the literature. Hiebert and Carpenter (1992) define conceptual knowledge so that it is identified with knowledge that is understood: "Conceptual knowledge is equated with connected networks" and procedural knowledge is defined as a sequence of actions (1992, p.78).

Interplay between types of understanding

Not only has there has been considerable debate about defining the two types of understanding, there has also been much debate as to which is the more important and which one should be taught first. Skemp (1976) acknowledges that even mathematicians who would classify themselves as relational still use instrumental thinking.

Byers and Herscovics (1977) assert that a good teacher can help a student to progress from intuitive understanding to formal understanding and similarly can support the move from instrumental to relational but that "the effective learning of mathematics cannot be based on one type of understanding. Nor ... can the different kinds of understanding be arranged in a linear order" (1977, p.27). They conclude that for optimal learning to happen the best approach is a spiral one so that "different types of understanding are used consecutively and repeatedly at even greater depth" (1977, p.27).

Hiebert and Lefevre (1986) acknowledge that the debate regarding two different types of knowledge has been ongoing for many years but recognise that the discussion has evolved over time and has moved from purely defining the types to looking at the relationships between them.

> Although it is possible to consider procedures without concepts, it is not so easy to imagine conceptual knowledge that is not linked with some procedures. This is due, in part, to the fact that procedures translate conceptual knowledge into something observable. Without procedures to access and act on the knowledge we would not know it was there. (Hiebert & Lefevre 1986, p.9)

Hiebert and Carpenter (1992) also acknowledge that both kinds of knowledge are required for mathematical expertise. They claim that uncovering relationships between conceptual and procedural knowledge is more useful than trying to establish which one is more important. Long (2005, p.61) claims that conceptual knowledge is intricately linked with procedures and algorithms. In fact, "knowledge of procedures is nested in conceptual knowledge".

Rittle-Johnson and Alabali (1999, p.188) propose that "conceptual and procedural knowledge appear to develop iteratively, with gains in one type of knowledge leading to gains in another". However Askew, Hodgen, Hossain and Bretscher (2010, p.34) state that "procedural fluency and conceptual understanding are largely seen as mutually exclusive aims". ACME (2011, p.1), recognise the importance of procedures but they also acknowledge "for mathematical proficiency, learners need to develop procedural, conceptual and utilitarian aspects of mathematics together".

The Collaborative Connected Classroom Model

The concluding theme is that neither procedural/instrumental nor conceptual/relational knowledge is more important. They should be taught together with the importance on making connections. With this in mind literature, where there is a focus on developing classrooms that are connected in nature, has been reviewed. These ideas have been synthesised and classified into four domains detailed below.

Teachers Beliefs about Mathematics and Learning

The important overriding theme, that is consistent throughout the literature (Skemp, 1976; Askew et al., 1997; Swan, 2005; ACME, 2011), is that mathematics (is a subject that) contains a wide range of connections. These connections can be between different areas of mathematics (for example the use of proportional reasoning within the topics of similar triangles and conversions) and also between different representations (for example seeing an arithmetic sequence represented in its numerical, graphical and mapping forms).

Learning consists of building a conceptual structure (Skemp, 1976) and is a collaborative activity in which learners are challenged and arrive at understanding through discussion (Swan, 2005). The nature of this collaborative activity will be expanded within the social culture of the classroom section.

The theory suggests that, within mathematics, mistakes are an important part of the learning process (Hiebert et al., 1997) and that they should be made explicit within lessons and developed as part of the lesson (Askew et al., 1997; Swan, 2005). These mistakes can provide essential opportunities to reconceptualise a problem (Kazemi & Stipek, 2008).

Nature of Mathematical Activity

If teachers believe that mathematics is 'connected' then teaching is about making learners engage with these connections. Mathematical activity will involve connecting different areas of mathematics or connecting different ideas in the same area of mathematics by making opportunities for a variety of words, symbols, diagrams and concrete situations (Askew et al., 1997; Haylock, 1982; Hiebert et al., 1997).

Whilst the notion of mathematical connections is not a new idea, the Collaborative Connected Classroom model aims to make more explicit the nature of the interplay of the connections between procedures and concepts. With this in mind mathematical tasks may make specific links between procedural and conceptual knowledge (Kadijevich & Haapasalo, 2001).

Tasks may take the 'educational approach' where meaning is built for procedural knowledge before mastering it (Kadijevich & Haapasalo, 2001), for example learners are encouraged to invent their own strategies before learning traditional algorithms. Or they may take the 'developmental approach' where procedural knowledge is used and then reflected on (Kadijevich & Haapasalo, 2001), for example comparison tasks are used where teachers encourage connections between the procedures being used to make generalisations resulting in conceptual understanding (Peled & Segalis, 2005).

Whichever approach (developmental/educational) is taken; mathematical tasks need to be accessible (Hiebert et al., 1997) and build on the knowledge that learners have (Swan, 2005) by connecting ideas to their current conceptual schema (Skemp, 1976). Misunderstandings should be made explicit so students can learn from them (Askew et al., 1997; Hiebert et al., 1997; Swan, 2005).

It is important that mathematical tasks are problematic and that application should be approached by challenges that need to be reasoned about (Hiebert et al., 1997; Askew et al., 1997). One method is that the teacher presents problems before explanations are offered (Swan, 2005) and they share essential information after selecting tasks with a goal in mind (Hiebert et al., 1997).

The Social Culture of the Classroom

There are many features apparent in a classroom where there is a focus on developing a social culture of more connected teaching. Research shows there will be a high degree of focussed discussion between teacher and whole class, teacher and groups of pupils, teachers and individual pupils and pupil themselves (Askew et al., 1997).

It is acknowledged that learners should emphasis methods rather than answers (Swan, 2005). However in the Collaborative Connected Classroom model the importance is on enabling learners to examine the mathematical similarities and differences between multiple strategies (Askew et al., 1997; Kazemi & Stipek, 2008). Teachers will work actively with the pupils' explanations, refining them and drawing pupils' attention to differences between methods (Askew et al., 1997; Kazemi & Stipek, 2008)

The important feature is that all learners will be encouraged to contribute and share their methods where they justify their strategies mathematically – not simply a procedural description (Kazemi & Stipek, 2008).

There will be a strong emphasis on developing methods, reasoning and justification (Askew et al., 1997). Learners will be each held accountable and consensus should be reached through mathematical argumentation (Hiebert et al., 1997; Kazemi & Stipek, 2008).

Characteristics of Learners

In a classroom where there is a focus on developing a more connected understanding of mathematics, learners will use strategies, which are both efficient and effective (Askew et al., 1997). They will know what to do and why they are doing it (Skemp, 1976) as they will be fluent with connections in mathematics (ACME, 2011). As a result they will be more confident in looking at new problems and attempting them without outside help (Skemp, 1976).

Learners who have made connections between procedures and their underpinning concepts will know a range of concepts, symbols and procedures and how they are related (Hiebert & Lefevre, 1986).

The table below summarises the model of the Collaborative Connected Classroom.

Table 1: Collaborative Connected Classroom Model

Teachers Beliefs about Mathematics and Learning	• Mathematics is a highly interconnected body of ideas that involves understanding and reasoning about concepts and the relationships between them • Mistakes should be recognised and made explicit. They are opportunities to reconceptualise a problem explore strategies and try out alternative strategies • Learning consists of building a conceptual structure whereby ideas are revisited and extended • Learning is a collaborative activity where learners are challenged to arrive at understanding through discussion

Nature of Mathematical Activity	• Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
	• Tasks either connect together different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams)
	• Links are made between procedures and concepts
	  ○ meaning is built for procedural knowledge before mastering it ('educational approach')
	  ○ procedures are evaluated to promote conceptual understanding ('developmental approach')
	• Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method
	• Application tasks are presented as challenges that may be problematic and need to be reasoned about

Social Culture of the Classroom	• Ideas and methods are valued and each student is held accountable for thinking through the mathematics in a problem until a consensus is reached. • There is an emphasis on reasoning and justification and not simply giving a procedural description • High degree of focussed non-linear discussion between teacher and groups of pupils, teachers and individual learners and between learners themselves • Discussion involves examining mathematical similarities/differences/connections among multiple strategies and refining learners explanations
Characteristics of Learners	• Know what to do and why they are doing it • Know a range of concepts, symbols and procedures and how they are related. • Use strategies which are both efficient and effective • Are aware of connections within mathematics • Are confident in tackling unfamiliar problems

Concluding comments

In conclusion I have drawn together a range of research literature and acknowledge that whilst it is useful to explore the nature of procedural vs. conceptual or instrumental vs. relational knowledge, it is the interplay between these that might lead to gains in student learning. I have proposed a model that acknowledges the importance of making connections and have considered what this might like look like within the mathematics classroom. Further research will be needed to explore how this model could be implemented.

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Appendix 7.2 Mathematics in School (November 2015)

Using Play Doh to develop conceptual understanding of volume of prisms

by **Nicola Trubridge**

Introduction

I am currently a mathematics AST at an 11–19 rural comprehensive school in Cornwall. I am also a part time research student at the University of Plymouth. One area that I have been researching within my teaching career is how to make mathematics more connected so that pupils gain a relational (Skemp, 1976) or conceptual (Hiebert and Lefevre, 1986) understanding. This article briefly describes the aims of my PhD study and then demonstrates in a practical context what the ideas might look like in the mathematics classroom for the topic of volumes of prisms.

The Collaborative Connected Classroom

My study looks at what makes effective CPD for teachers of mathematics and how research literature can be used to inform professional development activities to enable the development of what Trubridge and Graham (2013) refer to as the 'Collaborative Connected Classroom'.

The model of the Collaborative Connected Classroom has been developed through the synthesis of a large range of literature regarding theoretical and practical ideas underpinning the notion that mathematics is an interconnected web of ideas, and the belief that this should be addressed when teaching the subject. There is also an emphasis on the importance of collaboration and learner discussion. The model suggests that where teaching is connected, the nature of mathematical activity might incorporate some of the following principles:

- Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema.
- Tasks either connect together different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams).
- Links are made between procedures and concepts.
- Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method.
- Application tasks are presented as challenges that may be problematic and need to be reasoned about.

Background Context to the Lesson

This particular lesson was planned for my Year 10 foundation class. The majority of students have aspirational targets of grade C for the end of their GCSE course. When introducing a topic on shapes I used anagrams of keywords to see which shape names pupils were familiar with (and also to develop their literacy skills). Pupils could correctly rearrange to get the word 'prism' but no one within the class actually knew what one was or what properties would need to be present for a solid to be categorized as a prism.

Looking at questions these pupils had attempted previously, it became obvious that when faced with any volume question they would simply multiply all lengths together (perhaps they were trying to apply the rule for volume of a cuboid incorrectly to other situations).

This lesson was therefore designed to develop a deeper, more conceptual, understanding of what a prism is and why the cross-section area is a necessary feature in calculating the volume. It was also designed to engage students by using a kinaesthetic approach.

Lesson Episode 1: Exploring the Concept of Volume vs Surface Area

At the beginning of the lesson, students were each given a pot of *Play Doh*, and asked to use all of the dough in the pot to make a 3D shape. They were then questioned about what were the similarities and differences of the produced solids and to reflect on what assumptions had been made. Although they were not asked to perform any mathematical calculations, after some discussions they came to the conclusion that the volumes of all of the shapes were the same. They based this discussion on the assumption that the new *Play Doh* pots contained the same amount of dough in each. However, since they had created very different types of solid they noticed that the surface areas would be different.

Lesson Episode 2: Calculation of Cross-section Area

Using a range of different stencil cutters pupils were asked to create as many of their chosen shape as they could from their pot. The remaining *Play Doh* was put back in the tubs, which resulted in students now having different volumes of dough.

Fig. 1 Creating shapes from stencils

They were then asked the question "How could you calculate or estimate the area on the top of your shape?"

Depending on the shape of the cutter, pupils used different strategies, some estimated by considering the area of the rectangle it fell within (Fig. 2), others copied their stencil shape onto squared paper (Fig. 3) and some realized they could calculate using the area of a circle formula (Fig. 4).

Discussion continued to consider which methods would give the most accurate answers for each type of shape.

Lesson Episode 3: Developing a Rule for Calculating Volume of Prisms

Pupils were then asked to stack their shapes onto each other and to think about how they might calculate the volume of their prism. I did not define a prism; however, I showed them Figure 5 and said "If this is a gingerbread man prism what might your shape be called?"

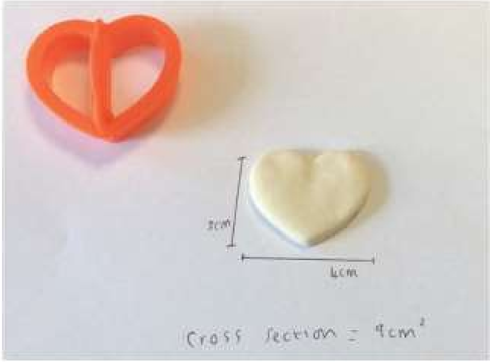

Fig. 2 Estimating area from a rectangle

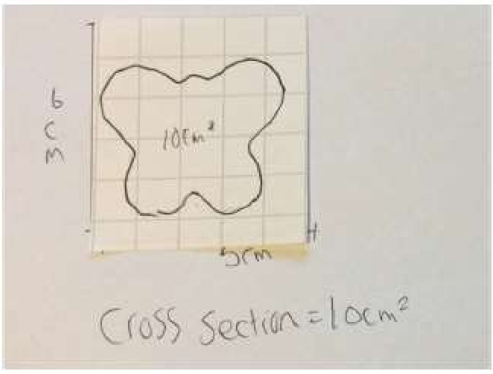

Fig. 3 Estimating area from counting squares

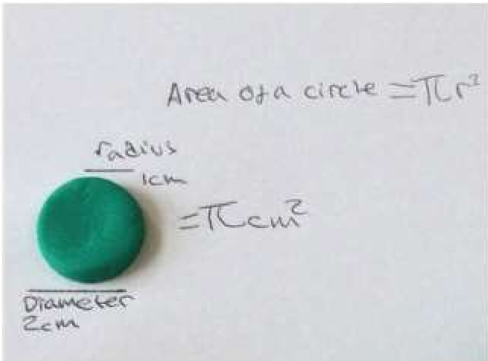

Fig. 4 Calculating area using formulae

Initially there was a lot of discussion around the classroom about the number of shapes in the stack being important to the calculation for the volume of prism.

Ideas and methods were valued and each student was held accountable for thinking through the mathematics; then pupils began to realize that some had thinner shapes whereas others had thicker ones. A consensus was reached that it must be the length of the prism that was important to the volume calculation.

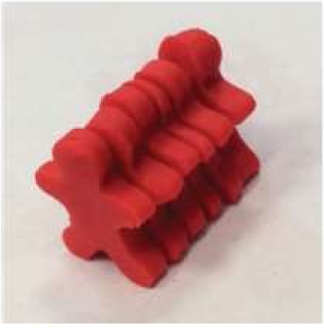

Fig. 5 A gingerbread man prism

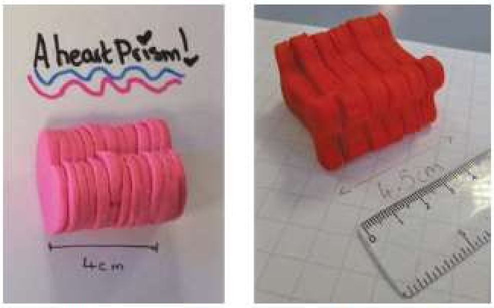

Fig. 6 The significance of the length of a prism

Lesson Episode 4: Developing a Procedural Rule and Reviewing Terminology

During the final phase of the lesson pupils were encouraged to write a generalized 'rule' (Fig. 7) showing how to find the volume of any prism.

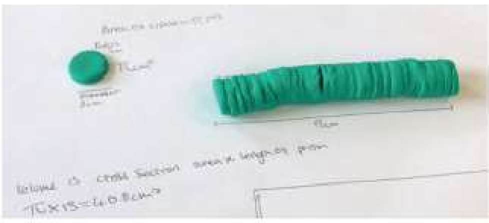

Fig. 7 Developing a rule

They were also asked to write in their own words what they now thought a prism was (Fig. 8).

Reflections and Next Steps

Pupils developing their own understanding of the concept of volume and building meaning of how to calculate it (before deriving the rule that volume of prism = cross-section area × length) is what Kadijevich and Haapsalo (2001) describe as the 'educational approach'. This approach is where meaning is built for

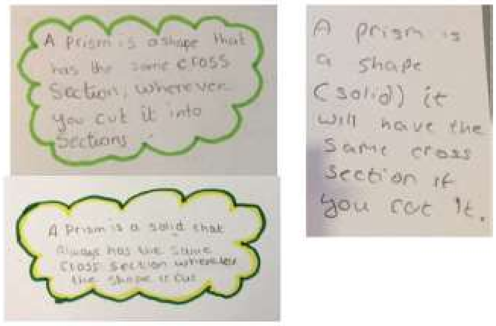

Fig. 8 Prism definitions

procedural knowledge before mastering it. Franke *et al.* (1998) comment that learners that are encouraged to invent their own strategies before learning traditional algorithms not only exhibit better conceptual knowledge but also have fewer algorithmic bugs than learners that have only considered the standard algorithms by rote. In following lessons, pupils demonstrated a greater confidence in identifying prisms; deciding on the cross-sectional face; and then calculating the volume. Six months later in their mock examination more pupils were able to calculate the volume of the prism shown in Figure 9 below.

This lesson idea was shared at a departmental meeting and other colleagues have also taught it this way to their learners. It was concluded that since a cuboid and cylinder are just special cases of prism that there was no need to learn the separate 'rules' for these and that students could apply their derivation of volume of prisms to all of these solids. Therefore the volume of cuboid (base × height × width) formula that was

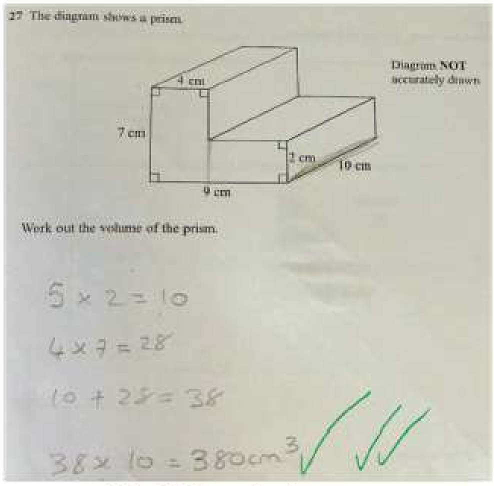

Fig. 9 Mock examination response

previously taught as a precursor to prisms was in fact no longer needed. We may even remove finding volumes of cuboids from the scheme of learning to free up more time to explore the concepts of volume and area in more depth.

Further Information

The mathematics department at Launceston College are continuing to work together to develop and reflect on other topics and how they may be taught in a more conceptual way.

We plan to delay some instrumental learning of specific formulae in order to promote the development of greater and more generalizable conceptual understanding. More detail on the theories underpinning the design of this lesson and further ideas about what a Collaborative Connected Classroom might look like can be found in the Trubridge and Graham (2013) article detailed below in the references.

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Keywords: Conceptual understanding; Engagement; Volumes of prisms.

Appendix 7.3 Proceedings of the BSRLM (June 2017)

A case study to explore approaches that help teachers engage with students' development of mathematical connections

Nicola Trubridge and Ted Graham

Plymouth University

This research study considers the Collaborative Connected Classroom (CCC) model and how it might be implemented within a school via a programme of sustained CPD that incorporates: research sharing; engagement with activities that bridge theory and practice; active collaboration and exploration of ideas. This paper reports the findings when looking at which aspects of the CCC model engaged teachers themselves and then which of these tasks were used with their learners to develop mathematical connections.

Keywords: CPD; connections; conceptual understanding; secondary teachers

Introduction

It is accepted that pupils' understanding of mathematics can be developed by exploring connections between concepts and different representations (ACME, 2011; Swan, 2005). However, there is a shortage of mathematics specialists and many reports (ACME, 2002; Cockcroft, 1982; Smith, 2004) contend that one of the most effective ways to raise the quality of mathematical provision is to expand continuing professional development (CPD) for teachers of mathematics. This study considers the Collaborative Connected Classroom model (Trubridge & Graham, 2013) which was implemented via a programme of CPD that was designed to support a group of teachers; the aim being to develop a more connected approach to school mathematics. This paper evaluates which approaches engaged teachers as they took part in the CPD programme.

Project outline

Trubridge and Graham (2013) identified the nature of mathematical activity within a Collaborative Connected Classroom to be as shown in Figure 1 below.

Nature of Mathematical Activity	1. Builds on the knowledge that learners bring by connecting ideas to their current conceptual schema. 2. Tasks either connect different areas of mathematics or connect different ideas in the same area using different representations (symbols, words, diagrams) 3. Links are made between procedures and concepts a. meaning is built for procedural knowledge before mastering it b. procedures are evaluated to promote conceptual understanding 4. Tasks involve comparisons; this may be looking for similarities or differences between ideas or looking at efficiency of method 5. Application tasks are presented as challenges that may be problematic and need to be reasoned about

Figure 1. Nature of mathematical activity in the CCC model

There were many aims of the CPD, with the focus being to develop teacher's pedagogical content knowledge through collaborative working. The 'subject expert' led an initial session sharing research papers and findings that informed the development of the CCC model. This was followed by sharing activities that were 'challenging and inspirational' to model each of the aspects of Figure 1; showing what they might look like in the mathematics classroom, hence bridging theory and practice. The next aim was for teachers to explore and develop these new ideas through sustained active classroom experimentation and collaboration with action research.

Methods

The case is a 'typical' mathematics faculty within an 11-18 college. Semi-structured interviews were carried out with all eleven members (A, B, C, D, E, F, G, H, I, K, L) of the faculty on four different occasions throughout the longitudinal four-year study. Additional data was collected to enable triangulation in the form of: book scrutinies, learning walks and presentations by the teachers of their action research projects.

Teacher development model

As the study advanced, it became evident that teachers progressed along a continuum (although not necessarily in a linear way). The first phase was the 'awareness' phase where teachers learned about the elements of the CCC model. They then experimented and used resources that were provided by the 'subject expert' which is named the 'guided exploration' phase. The biggest change in practice was as teachers moved to the 'independent exploration' and then 'independent development' phases where their use and development of ideas went beyond those provided in CPD sessions. Some teachers moved to the 'transformation' phase whereby the new way of working became the norm in their practice. Data was coded in the form of a two-way table where each cell had a description assigned to enable consistency in coding the data to the most appropriate stage.

Findings

Figure 2 shows most data at the 'independent exploration' phase with approximately equal amounts in 'guided exploration' and 'independent development' phases.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation	Total
Conceptual structure	6	16	17	5	0	**44**
Connect areas of mathematics	16	5	24	8	0	**53**
Multiple representations	19	22	64	30	4	**139**
Procedures and concepts	17	29	23	20	2	**91**
Comparisons	5	11	51	19	3	**89**
Application tasks	4	3	21	5	0	**33**
Total	**67**	**86**	**200**	**87**	**9**	**449**

Figure 2. Strands of CCC model mapped against phases of teacher development (number in cell is count of references coded)

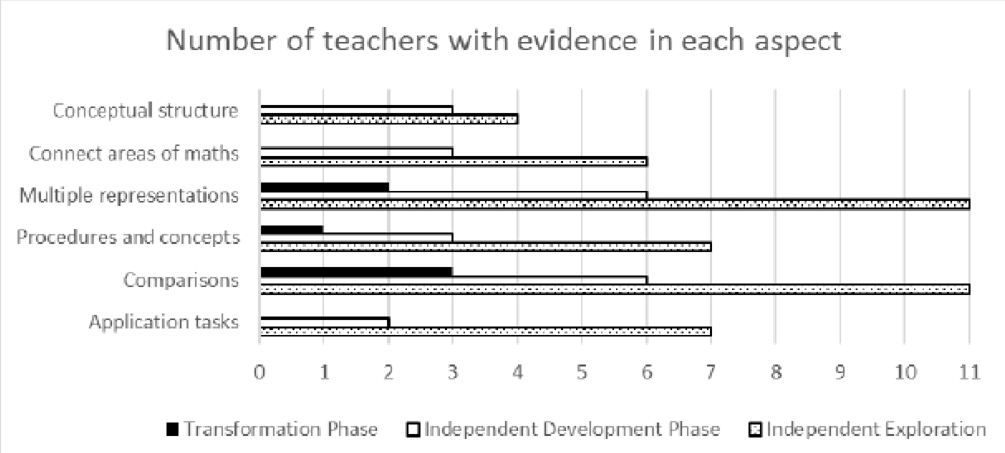

Figure 3. Number of teachers at the independent exploration/development and transformation phase for each aspect

Figure 3 shows that all teachers had independently explored using multiple representations with six of these teachers that had moved to the independent development phase and beyond. Fraser's books revealed examples that introduced trigonometry by making links with the unit circle and explicitly mentioned similar triangles and links to the graphical representation of the sine curve. Charlotte had designed a task making links between function machines, tables and graphs, whereas Kate had found and adapted resources that matched improper fractions, mixed numbers and visual images of fractions. Elliot used visual images to help explore percentages. Heidi used multiple representations in great depth to support the derivation of fraction calculation procedures. Louise also explored in depth and wrote her PGCE action research project on the question 'Can presenting a mathematical concept in different representations help pupils make links and develop a conceptual understanding?'.

	Awareness	Guided Exploration	Independent Exploration	Independent Development	Transformation
Conceptual structure		A*, I, K,	B, C, D, F, L	E, G, H*	
Connect areas of mathematics	A*, D*, F*, I*		C, G, K,	B*, E, H, L	
Multiple representations			A*, B, D*, G, I,	C, E, K, L	F*, H
Procedures and concepts		F, I	B, D, E, K, L	C, G	H
Comparisons			A, B, F, G, I,	C*, K, L	D*, E*, H*
Application tasks	K		A, B, C, D, F, G, I, L	E*, H,	

Figure 4. Teachers positioned in cell with highest evidence (* no triangulated evidence)

Whilst the 'making comparisons' strand of the CCC model had two elements to it (exploring similarities and differences and efficiency of method), there was a marked discrepancy as to which was trialled and developed. The strategy of exploring similarities and differences was embraced by everyone at least at the independent exploration phase with six teachers (C, D, E, H, K, L) taking things further to come up with their own examples. Figure 4 shows evidence from interviews that three teachers (D, E, H) believe they have transformed their practice using the aspect of making comparisons in the form of looking at similarities and differences although triangulated evidence was difficult to gather due to the nature of the tasks, being discussion-based with less written evidence. When Elliot was questioned how often he used the strategy his response was "once a day at least and at other times it would be all the lessons that you would say those words" (2016, Final interview). Heidi provided a similar response:

> What I do a lot of in my lessons is similarities and differences. I have started to introduce it at the beginning of each topic to encourage students, to give them the overall picture and to see how the topic will look. (2016, Final interview)

The similarities and differences aspect was also explored in depth by Daniel who comments "I embedded that in a lot of my other subjects and in my maths .... now that is becoming just off the cuff, what's the same and what's different is becoming more everyday speak for me in my maths lessons" (2016, Final interview).

The final aspect where there was evidence of teachers making changes to practice was making links between procedures and concepts. This progress however was restricted to two curriculum areas; fractions and volumes of prisms. The teachers themselves engaged with the Play Doh task (Trubridge, 2015) that was shared at a CPD session as it was engaging and inspiring "I love it and think it is exactly what volume needs really, to have the resources" (Fraser, Mid-study interview) and five teachers used this with their learners. Developing a greater understanding of fraction calculations was chosen as an action research project by Heidi, Ian and Georgie.

Why was progress made in multiple representations?

Early interviews show that most teachers were already familiar with using multiple representations and the use of the area method to help explain multiplying algebraic expressions. Building and extending these ideas to activities such as completing the square or using visual methods for proof could be seen to be a natural extension for both learners; "we started linking back in the work they had done on quadratics, they had already seen the various representations of quadratics" (Fraser, Presentation) and for teachers "so they came a bit more naturally for me to stretch myself to prepare more things" (Kate, 2016, Final interview). When teachers themselves were exposed to several proofs that would require sophisticated algebraic skills but found that they could be explained to a much lower age of audience using visual representations there was an incentive to engage with the use of multiple representations across the curriculum. "I did a Pythagorean proof …it was surprisingly useful how it worked and did grab attention from the lowest ability. The Eureka moments were evident and lovely" (Fraser, Mid-study interview).

One of the tasks that was modelled for the faculty, completing the square, was seen to be new pedagogic knowledge for teachers with Georgie saying "I would never have thought about coming up with an activity like that and seeing the way that the students made the connections after I had made the connections in a similar way was inspiring" (Georgie, Mid-study interview).

Another reason for progress in the aspect of multiple representations was that it is an approach where there are readily available resources. Fraser drew on lots of resources from the Further Mathematics Support Programme, Kate from research papers, Louise from Swan (2005) and these were then adapted for use in the classroom.

Why was progress made in similarities and differences?

There were perhaps a couple of reasons why this aspect was so successful with the faculty. The strategy was seen to be relevant to all topics so could frequently be used "probably most lessons I say what is the same and what is different" (Georgie, Mid-study interview). It was portrayed as an 'easy' thing to implement that didn't take much time to plan however 'value added' relative to the time to plan was great.

> It took five minutes to set up with the equations where you had to think a little bit carefully but then it is a good hour's lesson of good discussion going on between the students to find out what is actually going on. (Charlotte, Mid-study interview)

Elliot (Mid-study interview) commented "It is probably the easiest one to implement as a teacher because you are just asking a simple open question and I think it can have a big effect".

Teachers were motivated by the responses from their learners. Daniel commented on the importance of the strategy enabling learners to generate ideas so they derived knowledge themselves that gave them a deeper understanding of the concepts being covered and on the need for "getting those really good leading questions that you can bring into maths to start churning out those really good discussions" (Mid-study interview). This was reiterated by Charlotte "when they look at the differences and similarities I sort of point out: look, I have taught you nothing and you have worked it all out" (2016, Final interview). This was echoed by Elliot "it gets them thinking for themselves and they are not relying on you telling them" (Mid-study interview).

There was another unplanned aspect to the research study that also moved teachers forward in the aspect of similarities and differences. A slide referring to effect sizes of different pedagogic approaches was shared at whole staff training which gave additional evidence from another source that similarities and differences would have a greater effect on gains in learning than areas such as repetition and practice or cooperative learning. This was after the faculty had moved to the active experimentation phase of the CPD model and reassured Daniel and Elliot that they were doing the 'right thing'.

> I was really surprised; well not surprised, but happy to see the similarities and differences when the assistant principal did his presentation and saw that it was a 1.6 effect size, …….it was good to see that using similarities and differences, which is something that we all have embraced as a department, has such a big effect in the development and progression in understanding of students, so that was really good to see. (Elliot, Mid-study interview)

Daniel (Mid-study interview) also referred to the same slide being shown and commented "I was always going to try and extend what I had started last year but I thought if that is going to have that effect size then that is something I really do want to discover a bit more about". He then went on to explore this further for his action research CPD project "it was all kicked off by Marzano's research on effect sizes knowing that had a huge effect size if you get it right. That's why I wanted to investigate it really" (2016, Final interview).

Why was progress made linking procedures and concepts?

Heidi's progress in this area, and Georgie's to a lesser extent, was largely due to extensive research looking at readily available literature and other people's ideas to help formulate ways to use visual representations to support the development of fraction procedures from concepts. Heidi provided a range of academic references within her presentation to the faculty. She also articulated her own opinions as to why this was important for her to study further "this is one of the topics that most teachers and students understand only instrumentally" (Heidi, Presentation). Heidi had a personal desire and commitment to improve the teaching of fractions and challenged herself with the support of her academic reading to explore and transform her practice in this area.

Georgie referred to the legacy within the faculty of using the trick of 'times and twiddle' (for dividing by fractions) and how she wanted to move on from this collaboratively;

> I am probably more prone to using tips and tricks because I have had to and now I feel like I have got the rest of the department saying if we are going to start this in year 7 then there doesn't have to be tips and tricks. (Mid-study interview)

Progress in developing a more conceptual understanding of volumes of prisms was due to teachers themselves being inspired and enthused by tasks that were modelled in the CPD sessions and recognising that if time was dedicated to teaching these concepts more effectively then they wouldn't need to consider every case of prism within schemes of learning.

Conclusion and further questions

This study has shown that the aspects of the CCC model that engaged teachers were: the use of similarities and differences, multiple representations and linking fraction procedures and concepts. Whilst in this study teachers were quick to use the strategy of 'what is the same and what is different' with their learners on a regular basis, reporting an increase in class discussion and deeper thinking, further study should be carried out to research the impact on learners within the mathematics classroom. Another area to explore is the challenge of moving teachers from guided to independent exploration and it raises the question how do you enable teachers to be more independent thinkers rather than relying on what someone else has thought about?

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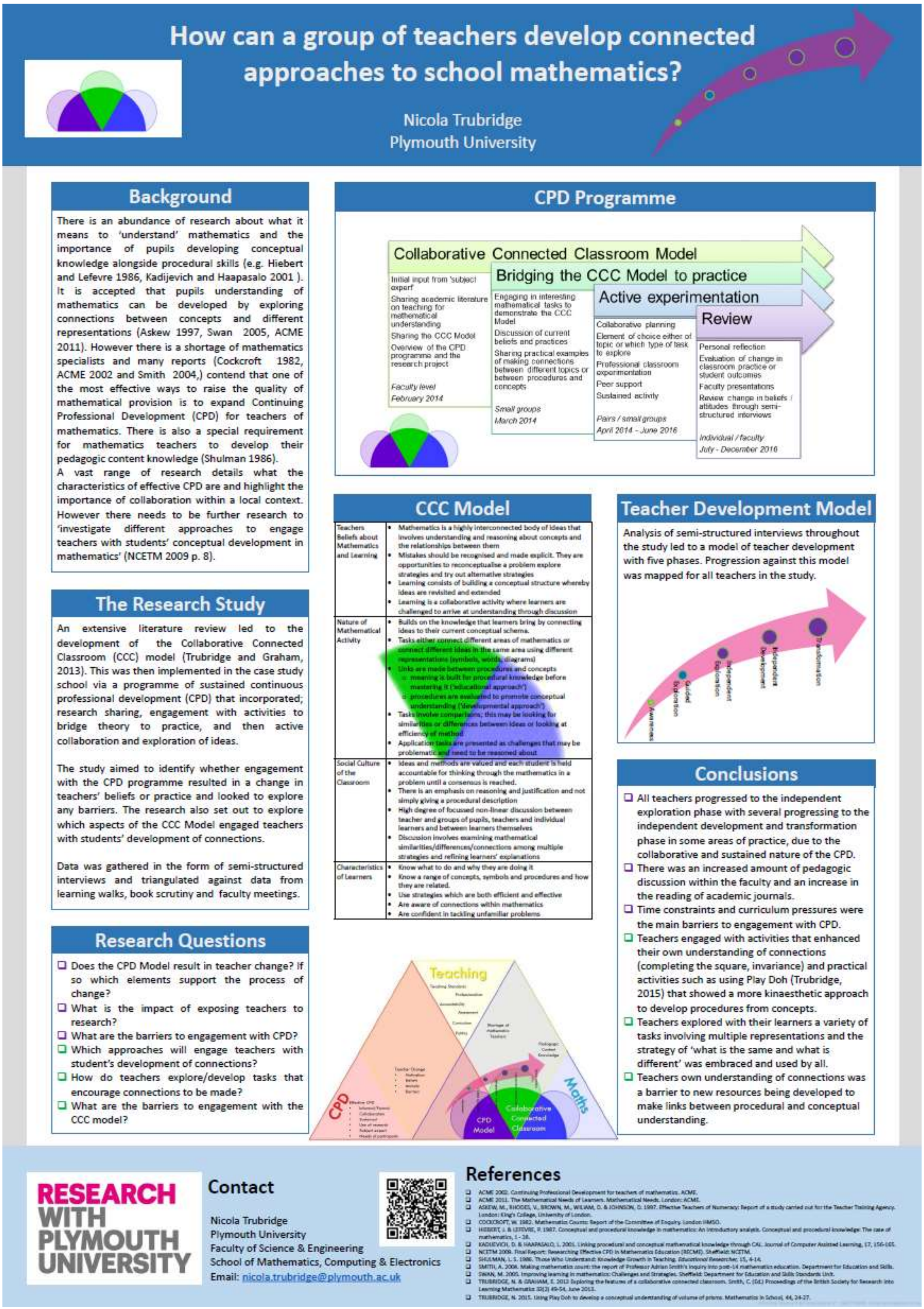
Appendix 7.4 Poster shared at Plymouth University (June 2017)



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