Flexural behaviour of BFRP rebar reinforced concrete beams

Rudzinskis, E.

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1. Appendices

10.01 Basalt Rebar Tensile Test

Photo 1 – BFRP Rebar Sample Ends Sheared Off During Tensile Strength Test.

Photo 2 – BFRP Rebar Pulled Out from Aluminium Encasement.
Photo 3 - Extensometer BFRP Rebar Inside The Testing Machine

Photo 4 – BFRP Spiral Sanded Off at Each End. Test Failed by one end crushed.
Damaged “teeth” on the left-hand side.

*Photo 5 – Damaged Upper Tensile Strength Testing Machine Clamp.*
Concrete mix design was based on BS 8500-1-2005+A1 and “Design of Normal Concrete Mixes” design manual (refer to list of references).
Concrete mix design was based on BS 8500-1-2005+A1 and “Design of Normal Concrete Mixes” design manual (refer to list of references).

### Table 1 Concrete mix design form

<table>
<thead>
<tr>
<th>Stage</th>
<th>Item</th>
<th>Reference or calculation</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Characteristic strength</td>
<td>Specified</td>
<td>Cube: 35 N/mm² at 28 days</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Proportion defective</td>
</tr>
<tr>
<td>1.2</td>
<td>Standard deviation</td>
<td>Specified</td>
<td>N/mm² or no data</td>
</tr>
<tr>
<td>1.3</td>
<td>Margin</td>
<td>(k = ...) Specified</td>
<td>N/mm²</td>
</tr>
<tr>
<td>1.4</td>
<td>Target mean strength</td>
<td>C2</td>
<td>N/mm²</td>
</tr>
<tr>
<td>1.5</td>
<td>Cement strength class</td>
<td>Specified</td>
<td>42.5 kg/m³</td>
</tr>
<tr>
<td>1.6</td>
<td>Aggregate type: coarse</td>
<td>Crushed</td>
<td></td>
</tr>
<tr>
<td>1.7</td>
<td>Free-water/cement ratio</td>
<td>Specified</td>
<td>Use the lower value</td>
</tr>
<tr>
<td>1.8</td>
<td>Maximum free-water/cement ratio</td>
<td>Specified</td>
<td>0.68</td>
</tr>
</tbody>
</table>

| 2     | Slump or Vee time                 | Specified                | 52.8 mm or Vee time |
| 2.2   | Maximum aggregate size            | Specified                | 20 mm           |
| 2.3   | Free-water content                | Specified                | 225 kg/m³       |

| 3     | Cement content                    | C3                       | 225 kg/m³       |
| 3.2   | Maximum cement content            | Specified                | 330 kg/m³       |
| 3.3   | Minimum cement content            | Specified                | 330 kg/m³       |

| 3.4   | Modified free-water/cement ratio  | Specified                | 330 kg/m³       |

| 4     | Relative density of aggregate (SOI)|                          | 2.7 known assumed |
| 4.2   | Concrete density                  | Fig 5                    | 2375 kg/m³       |
| 4.3   | Total aggregate content           | C4                       | 2375 kg/m³       |

| 5     | Grading of fine aggregate         | Percentage passing 600 μm sieve | 53 % |
| 5.2   | Proportion of fine aggregate      | Fig 6                     | 42 % |
| 5.3   | Fine aggregate content            | C5                       | 764 kg/m³       |
| 6.4   | Coarse aggregate content          |                           | 1056 kg/m³      |

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Cement (kg)</th>
<th>Water (kg or litres)</th>
<th>Fine aggregate (kg)</th>
<th>Coarse aggregate (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>per m³ (to nearest 5 kg)</td>
<td>330</td>
<td>225</td>
<td>765</td>
<td>350</td>
</tr>
<tr>
<td>per trial mix</td>
<td>0.065 m³</td>
<td>21.45</td>
<td>14.625</td>
<td>49.725</td>
</tr>
</tbody>
</table>

**Per 4 beams = 0.18 m³**

\[(4 \times (0.2 \times 0.15 \times 1.5)) = 4 \times (Depth \times Breadth \times Length)\]
Photo 6–Aggregate Quantities for Concrete Mix.

Photo 7 – Plasticiser “MasterPolyheed”
## Control of Substances Hazardous to Health (COSHH) Assessment

### Faculty/Department: Science and Engineering  School/Section: School of Engineering / Civil and Coastal Engineering

**Assessment No.**  **Assessor:** Tony Tapp

**Description of procedure or experiment (include how long and how often this is carried out and the quantity of substance used):** BASF MasterPolyheed 410, a mid range plasticizer admixture based on polycarboxylate ether, used in concrete mixes to increase workability or to lower the water/cement ratio while maintaining workability. Added to a mix at a rate of 0.5 to 2% by weight of cement by incorporating into the final 40% of the mixing water. Product is supplied in 1 litre plastic screw-top container.

**Identify the persons at risk:**
- [ ] Staff
- [ ] Contractors
- [x] Public/students

**Name the substance involved in the process, the supplier (if known) and the information source:** BASF MasterPolyheed 410, supplied as a light brown coloured alkali liquid which is soluble in water. Supplier: BASF plc, PO Box 4, Earl Road, Cheadle, Cheshire, SK8 6QG, 0161 485 6222, product-safety-north@basf.com; Emergency: 0049 180 2273 112

### Classification (state the category of danger, tick all that apply)

- [x] Irritant
- [ ] Very Toxic
- [ ] Toxic
- [ ] Corrosive
- [ ] Harmful
- [ ] Carcinogenic
- [ ] Gas under pressure
- [ ] Sensitiser
- [ ] Explosive
- [ ] Highly flammable
- [ ] Oxidising
- [ ] Flammable
- [ ] Environment

### Hazard Type (tick all that apply)

- [x] Gas
- [ ] Vapour
- [x] Mist
- [ ] Fume
- [ ] Dust
- [x] Liquid
- [ ] Solid
- [ ] Other *(Details)*

### Route of Exposure (tick all that apply)

- [x] Inhalation
- [x] Skin
- [x] Eyes
- [x] Ingestion
- [ ] Other *(Details)*

**Workplace Exposure Limits (WELs) please indicate n/a where not applicable**

**Long-term exposure level (8hr TWA):** No limits given.

**Short-term exposure level (15 mins):** No limits given.

**State the Risks to Health from Identified Hazards (include Risk Phrases):**

R36/38 - Irritating to eyes and skin; S33/39 - Wear suitable gloves and eyeface protection.
Control Measures: (for example extraction, ventilation, training, supervision). Include special measures for vulnerable groups, such as disabled people and pregnant workers. Take account of those substances that are produced from activities undertaken by another employer’s employees. (include Safety Phrases)

Wear PPE. Avoid aerosol generation.

Can a less hazardous substance be used?  
Yes ☐  No ☑

If YES why is it not being used?

Is health surveillance or monitoring required?  
Yes ☐  No ☑

Personal Protective Equipment (state type and standard)

- Dust mask  
- Visor  
- To avoid splashes.
- Respirator  
- Goggles  
- EN166 class 2FT or better.
- Impermeable / chemical resistant; EN374  
- Overalls  
- Other

First Aid Measures

In all cases seek medical attention if symptoms persist. Inhalation: Remove casualty to fresh air. Skin contact: Wipe off immediately; wash with soap and plenty of warm water. Seek medical attention if irritation develops. Eye contact: Flush with water for at least 15 minutes. Seek immediate medical attention. Ingestion: Rinse mouth, then drink plenty of water; seek medical attention.

Spillage/Uncontrolled Release Procedures

Do not breathe vapours. Wear PPE. Avoid discharge into watercourses or drains. Contain spill and absorb spill with an inert absorbent or cloth, place collected product into suitable container for disposal.

Storage

Store at room temperature in tightly closed original container.

Disposal of Substances & Contaminated Items

- Hazardous Waste
- Skip
- Run to Drain
- Return to Supplier
- Other

Name of Assessor: Tony Tapp  
Signed:  
Date: 22/12/2016

Name of Safety Manager/ HOS/ HOD.
Signed:  
Date:

Review Date:
Signed:  
Date:
## Bar Bending Schedule

### Bending schedule

**Job no.:**

**Date prepared:** 29/11/2016  
**Date revised:**

**Prepared by:** E.R.  
**Checked by:** D.E

<table>
<thead>
<tr>
<th>Member</th>
<th>Bar mark</th>
<th>Type &amp; size</th>
<th>No. of mbrs.</th>
<th>No. bars in each</th>
<th>Total no.</th>
<th>Length of each bar (L) mm</th>
<th>Shape code</th>
<th>A° mm</th>
<th>B° mm</th>
<th>C° mm</th>
<th>D° mm</th>
<th>E° mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top/Bottom</td>
<td>1</td>
<td>H8</td>
<td></td>
<td></td>
<td>10</td>
<td>1450</td>
<td>00</td>
<td>1450</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear Links</td>
<td>2</td>
<td>H8</td>
<td>4</td>
<td>6</td>
<td>24</td>
<td>660</td>
<td>51</td>
<td>100</td>
<td>150</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-Bars</td>
<td>3</td>
<td>H8</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>399</td>
<td>11</td>
<td>115</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Material Safety Data Sheet

<table>
<thead>
<tr>
<th>Basalt Continuous Filament Fibres</th>
<th>Continuous Basalt Fibre Distributor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raesbergenstraat 47</td>
<td></td>
</tr>
<tr>
<td>2804TJ Gouda</td>
<td></td>
</tr>
<tr>
<td>The Netherlands.</td>
<td></td>
</tr>
<tr>
<td>Tel: +31 182 535520</td>
<td></td>
</tr>
<tr>
<td>Mob: +31 6 54 907 214</td>
<td></td>
</tr>
<tr>
<td>E-mail: <a href="mailto:j.dewit@vulkan-europe.com">j.dewit@vulkan-europe.com</a></td>
<td></td>
</tr>
<tr>
<td>Web site: <a href="http://www.vulkan-europe.com">www.vulkan-europe.com</a></td>
<td></td>
</tr>
</tbody>
</table>

### 4. Emergency and First Aid Procedures.

<table>
<thead>
<tr>
<th>Inhalation:</th>
<th>No specific treatment is necessary as this material is not likely to be inhaled, unless the material is in dust form: see infra 8.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin Contact:</td>
<td>Wash with soap and running water.</td>
</tr>
<tr>
<td>Eye contact:</td>
<td>Immediately flush with plenty of water. Get medical attention if problem develops.</td>
</tr>
<tr>
<td>Ingestion:</td>
<td>Ingestion of this material is unlikely. If it does occur, get medical attention.</td>
</tr>
</tbody>
</table>
6. Accidental Material Release or Spillage.

Personal protective equipment:
- See infra 8.

Environmental protection:
- Prevent spread of basalt fibre dust and avoid dust-generating conditions.

Cleaning procedures:
- Vacuum clean dust. If sweeping is necessary, use a dust suppressant.

---

8. Exposure Limits and Personal Protective Equipment.

<table>
<thead>
<tr>
<th></th>
<th>OSHA PEL (8 h TWA)</th>
<th>ACGIV TLV (8 h TWA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibrous basalt (Fibrous basalt dust), CAS No 65997-17-3</td>
<td>5 mg/m³ (respirable dust)</td>
<td>5 mg/m³</td>
</tr>
<tr>
<td>Basalt fibre continuous filament</td>
<td>15 mg/m³ (total dust)</td>
<td>1 fibre/cm³ (suggested)</td>
</tr>
</tbody>
</table>

Size (µm)

Personal protective clothing and equipment.

Respiratory protection:

Protective gloves:

Eye protection:

Body protection:

- When, through mechanical processing, basalt dust is produced and when the dust levels exceed the recommended levels, use an approved respirator and local exhaust for processing machines.
- Impervious gloves are recommended.
- Impervious glasses are recommended.
- Protect skin as much as possible by clothing.

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11. Toxicology.

The basalt continuous filament has a diameter > 5 µm and does not split longitudinally. Only when, through a mechanical process, the filaments are broken into dust particles, is the OSHA standard for nuisance dust of...
<table>
<thead>
<tr>
<th>Application</th>
<th>(if the respirable fraction of 5 mg/m³ is reached)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Ecology.</td>
<td>N/A. Basalt has its origin in nature.</td>
</tr>
<tr>
<td>16. Other Information.</td>
<td>All data in the MSDS are based on the current state of knowledge, however they do not certify product properties and they do not justify legal liability. Vulkan-Europe bv. as the distributor cannot control the use of the end-product, the consumer has to determine under which circumstances the product can be used safely. <strong>This Material Safety Data Sheet is exclusively intended for professional use.</strong></td>
</tr>
</tbody>
</table>

Raesbergenstraat 47  
2804TJ Gouda  
The Netherlands.  
Tel: +31 182 535520  
Mob: +31 6 54 907 214  
E-mail: j.dew@vulkan-europe.com  
Web site: www.vulkan-europe.com
Reinforcing Cage Photos

**Photo 8** – BFRP and Steel Reinforcement Setup in Beams B2 and B4.

**Photo 9** - BFRP and Steel Reinforcement Setup in Beams B1, B2 and B4
10.07 Concrete Cube Curing

*Photo 10* – Concrete (C28/35) Concrete Cube Curing Inside the Water Chamber.

10.08 Location of external displacement gauge.

*Photo 11* – External Vertical Displacement Gauge Location 1.
Photo 12 - External Vertical Displacement Gauge Location 2.

10.09 Crack Propagation

10.09.1 Beam B1 (Control Beam)
10.09.3  Beam B3
Steel Ultimate Load Calculations

\[ d = h - c_{nom} - \phi_{link} - \frac{\phi_{bar}}{2} \]

\[ \Phi = 200 - 25 - 8 - \frac{8}{2} = 163 \text{ mm} \]

\[ T = \text{Area of Steel Reinforcement} \times \text{Allowable Stress} = 50.5 \text{ (kN)} \]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T</strong></td>
<td>Area of Steel Reinforcement x Allowable Stress</td>
<td>50.5 (kN)</td>
<td></td>
</tr>
<tr>
<td><strong>Area of</strong></td>
<td>Reinforcement</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td><strong>Allowable</strong></td>
<td>Stress</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td><strong>Materials Factor (Concrete)</strong></td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Materials Factor (Steel)</strong></td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FOR EQUILIBRIUM COMPRESSION MUST BE EQUAL TENSION

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C = 0.8 \times X \times (\text{material factor} \times F_{ck} / \text{material factor})</strong></td>
<td>15.02976 (mm)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ X = \frac{C \times 10^3}{F_{ck} \times b} \]

L = 1.5 (m)

h = 200 (mm)

\[ C_{nom} = 25 \text{ (mm)} \]

\[ \Phi = 8 \text{ (mm)} \]
BM Capacity = $T \cdot (d - 0.8 \cdot X/2)$

BM Capacity = 7.93 (kNm)

\[ P_{\text{Total}} \text{ (kN)} = 31.71 \text{ (kN)} \]
\[ P \text{ in (kN)} = 15.86 \text{ (kN)} \]
\[ P \text{ in (kN)} = 22.65 \text{ (kN)} \]

\[ \delta \text{ Uncracked(Max)} = 0.25 \text{ (mm)} \]

\[ a(m) = 0.35 \]
\[ E = 210000000 \]
\[ I = 0.00005625 \]
BFRP Ultimate Load Calculations

\[ d = h - c_{\text{nom}} - \phi_{\text{link}} - \phi_{\text{bar}} / 2 \]

- \( L = 1.5 \text{ (m)} \)
- \( h = 200 \text{ (mm)} \)
- \( c_{\text{nom}} = 25 \text{ (mm)} \)
- \( \phi = 8 \text{ (mm)} \)

\[ \text{T} = \text{Area of Steel Reinforcement} \times \text{Allowable Stress} = 101 \text{ (kN)} \]

- Area of Reinforcement = 101
- Allowable Stress = 1000
- Materials Factor = 1

FOR EQUILIBRIUM COMPRESSION MUST BE EQUAL TENSION

\[ C = 0.8 \times X \times \text{(materials factor} \times \text{fck/materials factor)} \rightarrow X = \left( C \times 10^3 \right) / (\text{fck} \times b) \]

\[ X = 30.05952 \text{ (mm)} \]

- Statistical Factor = 1
- \( \text{fck} = 28 \)
- Section Width (b) = 150

\[ \lambda = 0.8 \]

- 0.8

BM Capacity = $T \cdot (d - 0.8 \cdot X/2)$

BM Capacity = 15.25 (kNm)

$P_{\text{in}}$ (kN) = 43.57

$P_{\text{Total}}$ in (kN) = 87.13

$a(m) = 0.35$

$E = 50000000$

$I = 0.00005625$

$\delta_{\text{max}} = 2.8$ (mm)
10.12  Bending Moment and Shear Force Calculation (Beam B4)

**CALCULATIONS**

- BFRP REBAR REINFORCED BEAM B4

**OUTPUT**

\[
\text{BM} = 40.275 \times 0.251 = 10.109 \text{ kNm}^2
\]

\[
\text{SFD:} \quad V_{Ed} = 40.275 \text{ kN}
\]

Approx. location of BFRP Rebar Failure
CALCULATIONS

BENDING CHECK

\[ K = \frac{M}{6d^2 f_{ck}} \]

\[ = \frac{10 \times 10^9 \times 10^6}{150 \times 163^2 \times 28} = 0.0906 \]

\[ z = \frac{d}{2} \left[ 1 + \sqrt{1 - 3.53 K} \right] \leq 0.95d \]

\[ z = 163 \left[ 1 + \sqrt{1 - 3.53(0.0906)} \right] \leq 0.95(163) \]

\[ = 148.72 \leq 154.85 \quad \checkmark \]

\[ N_s = \frac{N}{f_{yd} z} \]

\[ f_{yd} = f_{ck} \times y_m = 1000 \]

\[ y_m = 1.0 - nice \ material \ factor \ not \ established. \]
**CALCULATIONS**

- $f_{yk} = 1000$

- $f_{yd} = 1000/1$

\[ \frac{1000}{1} = 1000 \text{N/mm}^2 \]

- $A = \frac{10 \times 10^9 \times 10^6}{1000 \times 141.92}$

\[ = 68 \text{ mm}^2 < A_{prov \ (Barab)} = 101 \text{ mm}^2 \]

Calculation above signifies that at point of failure BFRP had more than enough capacity to withstand the tension.

Thus this suggest that beam failure did not occur in bending. Stress check is required...
**CALCULATIONS**

**SHEAR CHECK**

\[ V_{ed} = \frac{V_{ed}}{b_w \cdot t} \]

\[ = \frac{110.275 \times 10^3}{150 \times 148.72} \]

\[ = 1.19 \text{ N/mm}^2 \]

\[ H_{ew} = \frac{2V_{ed} \cdot b_w \cdot S}{\sqrt{f_{yd} \cdot \cot \theta}} \]

\[ = \frac{1.19 \times 120 \times 150}{433 \times 2.5} \]

\[ = 30 \text{ mm}^2 \]

*3 H/8 links provide: 151 mm² > 30 mm²*

The supplier stated at the point of failure shear links had more than enough capacity to withstand shear force applied.
### Sheet 01

<table>
<thead>
<tr>
<th>Experimental Load Applied (kN)</th>
<th>Actual Deflection Recorded by External Gage (mm)</th>
<th>Predicted Deflection (mm)</th>
<th>Difference (mm)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.11</td>
<td>0.10</td>
<td>0.01</td>
<td>4.6</td>
</tr>
<tr>
<td>8</td>
<td>0.10</td>
<td>0.18</td>
<td>0.08</td>
<td>4.9</td>
</tr>
<tr>
<td>12</td>
<td>0.25</td>
<td>0.20</td>
<td>0.05</td>
<td>5.3</td>
</tr>
<tr>
<td>16</td>
<td>0.36</td>
<td>0.34</td>
<td>0.02</td>
<td>4.9</td>
</tr>
<tr>
<td>20</td>
<td>0.46</td>
<td>0.42</td>
<td>0.06</td>
<td>13.6</td>
</tr>
</tbody>
</table>

### Sheet 02

<table>
<thead>
<tr>
<th>Experimental Load Applied (kN)</th>
<th>Actual Deflection Recorded by External Gage (mm)</th>
<th>Predicted Deflection (mm)</th>
<th>Difference (mm)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.20</td>
<td>0.00</td>
<td>4.9</td>
</tr>
<tr>
<td>8</td>
<td>0.10</td>
<td>0.18</td>
<td>0.00</td>
<td>2.2</td>
</tr>
<tr>
<td>12</td>
<td>0.20</td>
<td>0.26</td>
<td>0.06</td>
<td>25.0</td>
</tr>
<tr>
<td>16</td>
<td>0.34</td>
<td>0.34</td>
<td>0.07</td>
<td>20.5</td>
</tr>
<tr>
<td>20</td>
<td>1.20</td>
<td>0.42</td>
<td>0.86</td>
<td>47.1</td>
</tr>
</tbody>
</table>

### Sheet 03

<table>
<thead>
<tr>
<th>Experimental Load Applied (kN)</th>
<th>Actual Deflection Recorded by External Gage (mm)</th>
<th>Predicted Deflection (mm)</th>
<th>Difference (mm)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>4.9</td>
</tr>
<tr>
<td>8</td>
<td>0.10</td>
<td>0.18</td>
<td>0.00</td>
<td>2.2</td>
</tr>
<tr>
<td>12</td>
<td>0.20</td>
<td>0.26</td>
<td>0.06</td>
<td>25.0</td>
</tr>
<tr>
<td>16</td>
<td>0.34</td>
<td>0.34</td>
<td>0.07</td>
<td>20.5</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
<td>0.42</td>
<td>0.08</td>
<td>15.7</td>
</tr>
</tbody>
</table>

### Sheet 04

<table>
<thead>
<tr>
<th>Experimental Load Applied (kN)</th>
<th>Actual Deflection Recorded by External Gage (mm)</th>
<th>Predicted Deflection (mm)</th>
<th>Difference (mm)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>4.9</td>
</tr>
<tr>
<td>8</td>
<td>0.17</td>
<td>0.18</td>
<td>0.01</td>
<td>5.2</td>
</tr>
<tr>
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<td>0.20</td>
<td>0.26</td>
<td>0.06</td>
<td>3.2</td>
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<tr>
<td>16</td>
<td>0.34</td>
<td>0.34</td>
<td>0.02</td>
<td>4.9</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
<td>0.42</td>
<td>0.08</td>
<td>15.7</td>
</tr>
</tbody>
</table>
10.14 Cracked Section Deflection Calculations

neutral axis position

\[ x = 30.1 \]

ii) second moment of area of the cracked section

\[ I_C = \frac{b x^3}{3} + a S \left( d - a \right)^2 \]

\[ b = 150 \text{ mm} \]
\[ a e = 5.81 \text{ mm} \]
\[ E = 50 \text{ kN/mm}^2 \]
\[ S = 101 \text{ mm}^2 \]
\[ d = 163 \text{ mm} \]
\[ I_C = 11723848.4 \text{ mm}^4 \]

Curvature Cracked Section

\[ (1/r)_C = \frac{M}{E I_C} \]

\[ M = 15.25 \text{ kNm} \]

\[ E I_C = 8.61 \text{ kN/mm}^2 \]

\[ (1/r)_C = 1.51 \times 10^{-4} \text{ /mm} \]

c) The Average of cracked and uncracked curvature

\[ M_{cr} = f_{cr} \left( 0.5 \frac{b h^3}{12} \right) \]

\[ f_{cr} = 2.6 \text{ N/mm}^2 \text{ or Mpa} \]
\[ b w = 150 \text{ mm} \]
\[ h^2 = 40000 \text{ mm}^2 \]

\[ M_{cr} = 2.8 \text{ kNm} \]

\[ \varepsilon = 1 - \beta \frac{M_{cr}}{M} \]

\[ \beta = 1 \text{ (assumed for a single short-term load). Also for sustained loads or cyclic loading is 0.5} \]

\[ \varepsilon = 0.966 \]

\[ (1/r)_C = (1/r)_C + (1/c) (1/r)_uc \]

\[ (1/r)_C = \frac{M}{E I_C} \text{ or } \frac{1}{150} \times 100000000 \text{ mm}^4 \]
\[ (1/r)_uc = 1.77108 \times 10^{-5} \text{ /mm} \]

\[ 1/r = 1.47 \times 10^{-4} \]
The deflection of the beam can be calculated from the total curvature using elastic bending theory which for small deflections is based on the expression:

\[ M = \frac{EId^2y}{dx^2} \]

\[ \frac{EIdy}{dx} = Mx + A \]

\( A \) – constant of integration

\[ A + x = \frac{L \ dy}{2 \ dx} = 0 \]

\[ \therefore A = \frac{-ML}{2} \]

\[ -Ely = \frac{Mx^2}{2} + Ax + B \]

\( B \) – constant of integration

\[ -Ely = \frac{Mx^2}{2} - \frac{MLx}{2} + B \]

when \( x = 0 \) and \( y = 0 \)

\( B = 0 \)

\[ y = \frac{M}{EI} \left( \frac{x^2}{2} - \frac{Lx}{2} \right) \]

*Maximum Deflection Occurs at Midspan When \( x = \frac{L}{2} \)*

\[ y = -\frac{M}{EI} \left( \frac{L^2}{8} \right) \]

substitute \( -\frac{M}{EI} \) with \( \frac{1}{r} \) (refer to ciii)

\[ y = \frac{1}{r} \left( \frac{L^2}{8} \right) \]

\[ y = 1.47 \times 10^{-4} \left( \frac{1300^2}{8} \right) \]

\[ y = 31.0 \text{ mm} \]
Transverse Steel Beam Deflection

Analysis undertaken using Two-Frame Structural Analysis software.
(Deflection Units in mm; External Load in kN)
**Base Rebar Modulus of Elasticity:**

\[ E = \frac{\text{Stress}}{\text{Strain}} = \frac{400}{0.0081} = 49383 \, \text{MPa} = 49.4 \, \text{kN/mm}^2 \]

**Steel Rebar Steel Modulus of Elasticity:**

\[ E = \frac{\text{Stress}}{\text{Strain}} = \frac{290}{0.0014} = 207143 \, \text{MPa} = 207.1 \, \text{kN/mm}^2 \]