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Wind Energy

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Abstract
This paper will discuss the principles of harnessing wind power through windmills and wind turbines and its limitations (Betz's law). It will also provide an estimate of the available wind energy per day and person in the UK and clearly distinguish between energy and power.

1 Introduction

With the world increasingly looking for ways to “go green”, wind energy is the fastest growing renewable energy source [1]. People have been harnessing wind power from as early as 5000 BC for powering boats along the Nile, and windmills have been used from as early as 2000 BC for pumping water and grinding grain [2]. The use of windmills to generate electrical energy started with low power battery charging during the twentieth century. Today’s wind turbines can be considered to be highly advanced versions of a windmill. There are numerous types of wind turbines, for this paper we will concentrate on a three blade horizontal axis turbine. Wind turbines use wind to generate electricity. They work by converting wind kinetic energy first into rotational kinetic energy in the turbine and then into electrical energy [3]. Power is defined as the rate at which energy is generated and will be measured in watts (W), which is defined as joules per second. Energy is the ability of a system to perform work and will be measured in joules (J) [4]. Mathematically it is defined as follows,

$$\text{Power}\,(\mathsf{W}) = \frac{\text{Energy}\,(\mathsf{J})}{\text{Time}\,(\mathsf{s})}.$$

Wind power then refers to the rate at which the wind energy is converted into electrical energy.

2 Harnessing Wind Power

Many countries are now embracing the benefits of wind power. While some countries are just starting up wind farms, other countries have fully formed plans to utilise wind energy. Much of Europe has begun to build and use wind farms, i.e. groupings of wind turbines which harness the energy of the wind and then convert it into electricity. As the wind passes over the blades of a turbine, the ensuing rotation turns the low-speed shaft. This shaft is connected to a gearbox which multiplies the rotation speed, which in turn is connected to the electric generator by a high-speed shaft (see Fig. 1). The rotational energy is converted into electrical energy inside the generator and outputs the correct voltage for distribution, usually 33,000 V [1].

Mathematics is relevant in many ways for the harnessing of wind power and the production of useful energy. Large scale weather models are used to find the most suitable places to position the wind farms and where to position each wind turbine. The energy gain is optimised by means of fluid dynamics to design the best shape of the blades, structurally and aerodynamically. There are, however, limitations to the amount of energy that can be obtained due to losses through sound and friction (see next section). Mathematics also answers the question why three blades? The underlying reason is (cost) efficiency. The energy gain grows rather slowly with the number of blades. Having more than three blades leads to an increase in energy output of just about three percent. Therefore, the extra cost of a wind turbine with more than three blades cannot be justified.

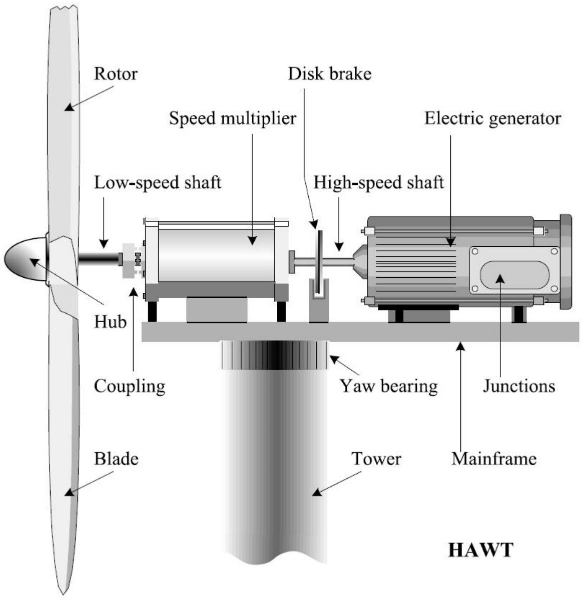

Figure 1: Diagram showing the internal components of a HAWT (horizontal axis wind turbine) [5].

3 Limitations

3.1 Betz's Law of Limitation and Mathematical Model of a Wind Turbine

Betz's law states that the maximum fraction of kinetic energy which can be extracted from a wind current is 59.3%, known as Betz's limit [6]. This is shown by comparing the available kinetic energy to the extracted energy at the turbine. To model the wind flow, a closed system must be considered to employ the conservation laws of mass, momentum and energy.

The mass flow rate $\dot{m}$ of a Newtonian fluid is given by

$$\dot{m} = \rho A v, \tag{1}$$

where ρ is the mass density, taken to be that of air, A is the fluid area and v the flow velocity. By conservation of mass the mass flow rate is constant and must have the same value upstream, at the turbine and downstream. The associated fluid areas and velocities are A_1, A and A_2 and v_1,v and v_2, respectively (see Fig. 2). Thus, the mass flow rate can be written in three equivalent ways,

$$\dot{m} = \rho A_1 v_1 = \rho A v = \rho A_2 v_2. \tag{2}$$

By Newton's second law, the force F is the derivative of momentum p with respect to time,

$$F = \dot{p}, \tag{3}$$

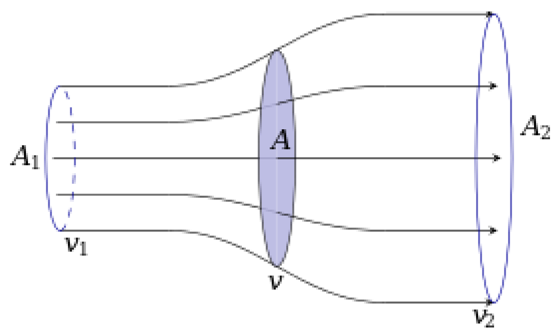

Figure 2: Diagram showing the flow of air through a wind turbine, where A is the cross sectional area, and v is the velocity [7].

where momentum p is

$$p = mv. \tag{4}$$

By momentum conservation the momentum at the turbine is the momentum lost between the upstream and downstream flow of wind,

$$p = p_1 - p_2 = m(v_1 - v_2). \tag{5}$$

The force is then found by substituting this into (3),

$$F = \frac{dm}{dt}(v_1 - v_2) = (v_1 - v_2)\dot{m} = \rho A v(v_1 - v_2). \tag{6}$$

Power P is the rate of change of energy, E, or work done [8],

$$P = \frac{dE}{dt} \tag{7}$$

where work done is the usual line integral

$$E = \int_C \mathbf{F} \cdot \mathbf{v}\, dt. \tag{8}$$

Combining (7) and (8) yields the power in the form

$$P = F \cdot v = \rho A v^2 (v_1 - v_2)\ , \tag{9}$$

where we have assumed that the force $\mathbf{F}$ acts along the velocity $\mathbf{v}$ to evaluate the scalar product. Using conservation of energy (for a closed system), the kinetic energy E at the rotor is the difference between the kinetic energy before and after it passes the rotor,

$$E = \frac{1}{2} m(v_1^2 - v_2^2)\ . \tag{10}$$

This result can be be used to find the power from (7)

$$P = \frac{1}{2}\dot{m}(v_1^2 - v_2^2). \tag{11}$$

Recalling the mass flow rate $\dot{m}$ from (2) gives another equation for the power at the turbine,

$$P = \frac{1}{2}\rho A v(v_1^2 - v_2^2). \tag{12}$$

Comparing the two power equations at the turbines (9) and (12) results in a formula for the velocity,

$$v = \frac{1}{2}(v_1 + v_2)\ , \tag{13}$$

which turns out to be the arithmetic mean of upstream and downstream velocities, v_1 and v_2, respectively. By substituting the velocity v back into either of the power equations gives the power in terms of only v_1 and v_2,

$$P \;=\; \frac{1}{4}\rho A(v_1 + v_2)(v_1^2 - v_2^2) = \frac{1}{4}\rho A v_1^3\left(1 + \frac{v_2}{v_1}\right)\left(1 - \frac{v_2^2}{v_1^2}\right)\ . \tag{14}$$

The ratio

$$b \equiv \frac{v_2}{v_1} \tag{15}$$

between downstream velocity and upstream velocity is known as the interference parameter. In terms of the latter, the power output becomes

$$P = \frac{1}{4}\rho A v_1^3(1 + b)(1 - b^2)\ . \tag{16}$$

The efficiency of the turbine can be measured by the *performance coefficient* C_p, which is the ratio between extractable power (16) at the turbine to the power of the upstream wind at the cross sectional area, A,

$$C_p = \frac{\frac{1}{4}\rho A v_1^3(1 + b)(1 - b^2)}{\frac{1}{2}m\rho A v_1^3} = \frac{1}{2}(1 + b)(1 - b^2)\ . \tag{17}$$

To find the maximum of the performance coefficient we differentiate with respect to b, which yields a quadratic equation,

$$\frac{dC_p}{db} = \frac{1}{2}(1 + b)(1 - 3b) = 0\ . \tag{18}$$

This may be solved for a maximum interference parameter b_{max},

$$b_{max} = \frac{1}{3}, \tag{19}$$

which, via (15), can be translated back to an optimal velocity relation,

$$v_2 = v_1/3\ . \tag{20}$$

Substituting $b_{max} = 1/3$ into the performance coefficient (17) yields Betz's limit [9],

$$C_p(b_{max}) = \frac{16}{27} \simeq 59.3\,\%\ . \tag{21}$$

As a consequence, the maximum power P_{max} that can be extracted from the upstream wind is

$$P(b_{max}) = \frac{1}{2}\rho A v_1^3 C_p(b_{max}) = \frac{16}{54}\,\rho A v_1^3 \ . \tag{22}$$

This is the power output if the final velocity of the wind is a third of the initial velocity, cf. (20). In this case, the turbine has converted the optimal amount of kinetic energy from the wind to spin the turbine.

3.2 Other Limitations

As useful as wind energy is, it does not come without its limitations. These range from trivial limitations such as displeasing appearance and noise disturbance to more significant constraints including efficiency, cost, construction limitations, no energy being produced in low winds and, as we have discussed, Betz's law.

A major limitation for wind turbines is the cost factor. Many people are concerned that they are not cost effective. At the cheaper end of the scale there are micro turbines (2.5 kW – 6 kW) which can range in price from £10,000 – £30,000 [10]. These turbines can be erected in your own back garden. However, they are not very popular due to the fact that many will not pay back within the lifetime of the turbine making them neither cost effective nor a viable option for many home owners. At the other end of the scale there are large turbines (1 MW – 2.5 MW) ranging in price from £2 million – £3.3 million [10]. While they come with considerable start-up cost they also produce a lot more energy resulting in a payback period of 1 – 5 years [10]. This makes them a more viable option for governments and energy companies. Hence, until wind turbines become more affordable at the lower end of the scale, it will be difficult for home owners to switch to small scale renewable wind energy.

Another of the main limitations of wind energy is the *capacity factor*. The capacity factor is the ratio of the actual power output of a power plant, in our case a wind farm, over a period of time compared to the power plant's output if operating at full capacity over the same period of time. In most cases wind farms have a capacity factor of between 25 – 30 percent, although some can be as high as 40 percent [11].

Fig. 3 shows an example of how the power output for a wind turbine is affected by the wind speed. As you can see, the turbines have a maximum power output, at which point any increase in wind speed has minimal effect on the output. This, coupled with the protection shut down at high wind speeds, shows how important stable wind conditions are for optimal power output and explains why the capacity factor is only 25 – 30 percent.

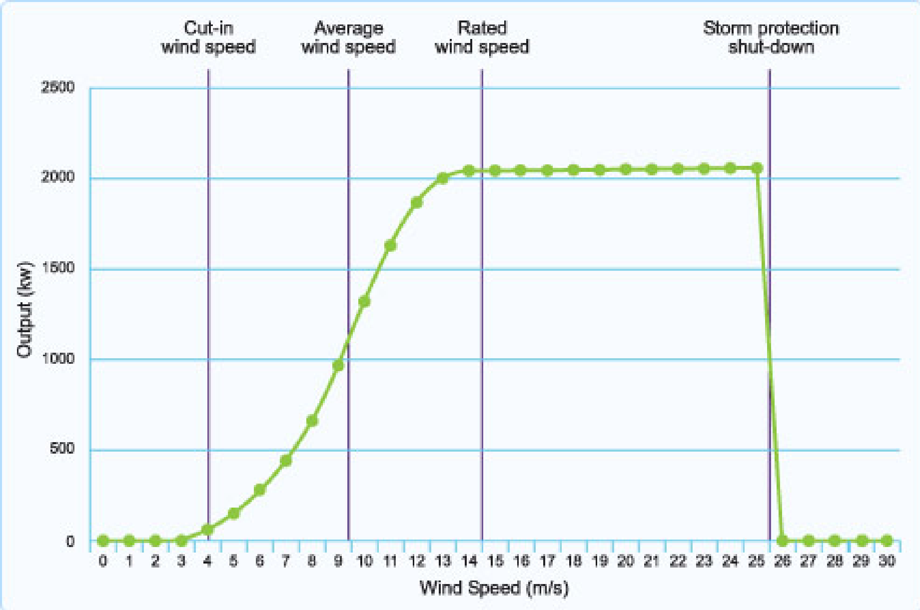

Figure 3: Wind Speeds in relation to power output of a wind turbine [11].

4 Brief Estimates

By using data from the home office for the generation of electricity from renewable sources for 2014, it is possible to estimate the available wind energy per day and person in the UK. The total power available from wind energy sources is 32×10^9 kWh [12] and, given that we know 1 kWh equates to 3.6×10^6 J, the total energy available per year is 1.15×10^{17} J. Therefore, the energy available per day is 3.15×10^{14} J. The most recent estimate of the UK population is 64.6 million people [13]. Therefore, we can estimate the available wind energy per person per day to be approximately 4.88×10^6 J.

5 Conclusion

The demand for renewable sources of energy has resulted in an increase in the number of wind farms across the UK. However, many people remain unconvinced by their efficiency and reliability to provide a stable stream of power to the country. There is also the problem of creating a way to store electrical energy and, in particular, any temporary surplus of energy. There is no doubt that turbine efficiency has increased significantly over the last 10 years [14], but there will always be a limit as outlined by Betz's law. These are not the only obstacles standing in the way of wind power; their appearance and noise are huge drawbacks for many people, which accordingly would not want any wind farms in their area. Another issue with wind farms is the massive cost involved in planning and building them, with repay of cost easily taking five years and more. This is an even bigger issue for home based turbines, which in many cases will not pay off their cost during their lifetime. Overall, though, the increasing efficiency combined with the unlimited quantity of wind across the UK ensures wind energy will be a crucial part of the countries renewable energy plan for the future. The limit imposed by Betz's law is the major drawback for commercial wind energy, at least in principle. However, this limit has yet to be reached in practice,so that there is still significant potential for wind energy to become an increasingly efficient and reliable source of energy. Until this potential is fully realised, wind energy will continue to be supplementary to other, more reliable forms of energy production.

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