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Plymouth Undergraduate Research in Mathematics

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Abstract
We briefly discuss some peculiarities of undergraduate research in mathematics and present two examples from our educational practice.

Introduction

Mathematics is a mature subject with a recorded history spanning about 4000 years. As a consequence it is not entirely straightforward to produce new results that warrant publication in, say, a peer reviewed journal. A typical result in pure mathematics would be the formulation of a proposition or a theorem that will then have to be proved using the machinery of the relevant sub-fields — which often cover a vast range of previous findings. The nature of the problems will vary considerably as well. Some 'old' problems remain open and will continue to do so for the time being. Even if the statements seem rather simple on the surface, their actual proof may be very hard (sometimes having defied a solution for centuries). A recent example is the proof of Fermat's Last Theorem, a story told brilliantly in Simon Singh's popular account [1]. Other problems are difficult to grasp without a thorough knowledge of specialised mathematical topics that go well beyond the skills to be acquired by mathematics undergraduates.

A large part of the mathematics education in Plymouth deals with applied mathematics. Here, one may produce new results by investigating problems that have been solved previously, say by putting them into another context, or by drawing analogies, or by modifying and generalising the assumptions. This may be simpler than presenting proofs, but typically will meet with substantial challenges of a more technical nature.

These caveats notwithstanding, we regard it as an important enhancement of the maths student's experience to be exposed to these fundamental aspects of mathematics (research). As with any lecturer, though, we are confronted with a range of student backgrounds, capabilities and skills, and, in principle, we want to accommodate all of them. We will hence discuss two somewhat complementary examples from our educational practice which are supposed to resolve at least part of the difficulties met in having undergraduates undertake mathematical research.

Example 1: Scientific Article

The first example is essentially a (four page) mini-review of a specific maths problem to be done through group work by (effectively) all final year students. For several years now, this has been part of the assessment in the project module "Mathematics in Context" and has resulted in five publications in The Plymouth Student Scientist (TPSS) [2–6]. In these projects, which have to be feasible for *everyone* in the cohort, students do not actually produce *new* results, but try to understand and explain a reasonably well defined (hence concrete) mathematical problem. Topics have covered a wide range from applied themes such as sustainable energy, astronomy and geodesy, biology and engineering to pure mathematics such as spectral theory, large numbers or the Riemann hypothesis 'explaining' the distributions of the primes.

The practical task for the students is to write a scientific article on a randomly allocated topic using an original journal template produced with the mathematical typesetting software LaTeX. They are thus required to present mathematical ideas from a novel context in a logically convincing way, obeying all the formal requirements of a scientific publication. Apart from the actual mathematical content, this also includes the structure (such as title, abstract, main text, references) as well as typesetting, layout and design (formulae, figures and tables with proper enumeration). The students hence learn all the skills required to produce a written report of their findings, assisted by an up-to-date software tool (LaTeX). The work is to be done by independent research of whichever relevant resources they can find. This typically will be web sites (starting from Wikipedia), but they have to (properly) cite at least one book.

Feedback has generally been positive, with the students enjoying the opportunity for independent study. Some find the task somewhat daunting but tend to cope despite their feeling of being somewhat 'stretched'. The opportunity to publish in TPSS is announced as an incentive from the very beginning. Some students rise to the challenge, while others choose a 'minimal effort' approach. Needless to say that the outcomes vary in quality. Nevertheless, the very few failures were due to issues of process, such as not meeting deadlines, rather than content. The best reports, on the other hand, are proof of the students' engagement and enthusiasm and hence are a pleasure to read.

Example 2: A summer research project

The London Mathematical Society (LMS) funds a number of research bursaries for undergraduates to undertake research in mathematics. The funding is awarded as a grant to a university staff member based on the academic merits of the student to perform the task and on the project to be addressed. In recognition of the difficulty for an undergraduate research project to result in a peer reviewed publication even these prestigious grants do not specify that the research should lead to a publication. They rather state that their goal "*is to give experience of research to undergraduates with research potential and to encourage them to consider a career in scientific research*" [7].

This example focusses on one such LMS funded research project[1]. The second author obtained such an LMS grant for one of our students to undertake a research project between their second and third years (last summer). From a supervisor's perspective, the desired outcomes were for the student to:

(a) gain experience with mathematical research by focussing on examples which provide intuition to tackle a general problem;

(b) be introduced to topics that extend material covered in their degree programme;

(c) become familiar with various topics in pure mathematics that would be of benefit for further studies (e.g. a PhD) in other areas of pure mathematics (not just continuations of the project); and

(d) gain experience in independent study of mathematics that is focussed towards a particular research goal.

Overall the LMS undergraduate research project undertaken over summer 2015 was successful, both in terms of the goals (a) to (d) above and in terms of the research progress made. Although the problem has turned out to be more complicated than the supervisor had expected, the complications were interesting in their own right. Moreover the student was able to reach several of the planned milestones and made significant progress in resolving the research problem in question. The student is currently in the process of preparing a short submission to TPSS.

Part of the supervisor's report to the LMS is to provide a summary of the project for the lay reader. To give some flavour of the project contents we directly quote from the report:

> "*Consider edges (lines) between vertices (points) drawn on a sphere so that all the faces are triangles (have three edges) and each face is coloured with one of two colours so any two faces that share an edge have different colours. Such an embedding in which the faces are each coloured with one of two colours and no faces that share an edge have the same colour is referred to as a properly face* 2*-coloured triangulation of the sphere. A finite abelian group (a finite set of objects along with a rule that tells you how to add them together to get to other objects in the set), called the canonical group of the triangulation, can be derived from such a triangulation.*
> *An open question, due to Cavenagh and Wanless and motivated from the theory of latin trades, is, given a finite abelian group, Γ, does there exist a face 2-coloured triangulation of the sphere, whose canonical group is isomorphic to Γ?*
>
> *In this project [the student] began by covering background material on the problem; namely: quotients of free abelian groups, partial latin squares and planar embeddings.*

[1] London Mathematical Society Undergraduate Research Bursary, grant number URB 15–49.

> *[They] next used the computer program Maple to form a catalogue of the canonical groups of 'small' properly face* 2- *coloured spherical triangulations. [The student] used these computational results to draw insight into what the answer to the open question and proved several results on the existence of families of abelian groups that occur as canonical groups. [The student] also made informed conjectures as to families of groups that may not be obtained in this manner.*"

There are several challenges that face a supervisor when establishing such an undergraduate research project (whether or not it is supported by a grant). As mentioned in the introduction, due to the hierarchical nature of mathematics as a discipline, even by the end of their undergraduate degree students may not have seen the mathematical structures under discussion or indeed even have been introduced to their underlying concepts.

As such a project needs to be carefully selected so that covering background material does not consume the entire duration of the project. This means it is likely that the supervisor has already drafted an approach and formulated a summary of the additional material required by the student. Then the supervisor can take a hands-on approach in guiding the student through the necessary material. In other words, the supervisor needs to 'trail-blaze a path' towards the research goal that the student can then follow.

An additional challenge is that until a project is well under way it may not be clear how difficult the project will be to accomplish or even if the chosen approach will (or even can) produce any results. In other areas a researcher may have a hypothesis as to the outcome of an experiment and regardless of whether or not the hypothesis is true the results of the experiment may be discussed. However, in pure mathematics if an attempted approach to prove a result fails (and does not yield a counter example) all the researcher is left with is the knowledge that the approach did not work. The best case scenario may be that the researcher is left with a slightly better understanding of the problem. This makes providing a timetable for a research project exceedingly hard – working on a problem for two years before making significant (or in fact any) progress is quite common[2].

An undergraduate research project typically runs for a time between six weeks and several months. So if a 'dead end' is reached, the student may not have time to recover, and the project may potentially reduce to a literature survey (and a frustrated and demoralised student). Such difficulties are mitigated during a postgraduate research degree as the additional time means that a student can afford to be stuck on a problem for considerable time or to work on multiple problems and hopefully make progress towards one or more of them. Hence, it may be advisable that the supervisor of an undergraduate project has a very clear idea of how the project could be approached and to select one that has a series of smaller milestones that can be reached.

[2] Recall that it is plausible for an innocuous question to take hundreds of years to solve (for example again see [1]).

In summary, we conclude that undergraduate research in mathematics is entirely feasible if carefully tailored with regard to student needs and capabilities. Both our examples discussed here met this requirement so that we clearly view them as success stories that we are keen to continue. So please keep watching this space for further high quality achievements from Plymouth mathematics undergraduates!

References

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