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Flexural buckling of sandwich beams with thermal-induced non-uniform sectional properties

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Abstract – An analytical study is presented on the flexural buckling of sandwich beams considering thermal-induced non-uniform cross-sectional properties. The formula for determining the buckling loads of sandwich beams under a linearly varied non-uniform temperature distribution scenario, both with and without considering the influence of transverse shear deformation, is derived. The effects of local reductions in Young's modulus and shear modulus on the flexural buckling of sandwich beams are discussed. The obtained results demonstrate the importance of considering non-uniform cross-sectional properties and associated shear effects in the buckling analysis of sandwich beams when they are exposed to a thermal environment.

Keywords: Sandwich beam; non-uniform temperature; flexural buckling; shear effect; thermal effect.

1. Introduction

The flexural buckling of beams/columns under an axially compressive load is normally analysed using the Bernoulli–Euler theory, in which the transverse shear deformation effect is ignored. The approach is only applicable to homogeneous beams that are not very deep. For deep, composite or sandwich beams the transverse shear deformation of the beams could have an important influence on the bending, vibration and buckling behaviour of the beams and thus should be considered in the analysis. The bending theory of beams with shear effect was developed first by Timoshenko in 1921 [1]. However, the theory involves a shear correction factor that needs appropriately to represent the associated shear strain energy. Recently, Elishakoff et al. [2] presented an excellent literature review on the historical development of Timoshenko's beam theory and the elucidation on how to determine the shear correction factor.

There is increasing use of laminated composite and sandwich beams in building and construction industries due to their attractive properties such as light weight, high strength and high stiffness [3,4,5,6]. Numerous studies have been published in literature on the buckling analysis of laminated and sandwich beams. For instance, Gutierrez and Webber [7] derived a differential equation for analysing the wrinkling of honeycomb sandwich beams. The equation was solved by using sine functions for the displacement of the buckled face. The obtained critical buckling loads were presented for typical beams according to the face-plate thickness and core thickness. Farkas and Jármai [8] performed the static bending, shear stresses and deflections analyses of sandwich beams made by two rectangular tubes glued by a damping layer and presented an optimal design procedure for sandwich beams with constant cross-section. Tripathy and Rao [9] presented a study on the stability of curved laminated beams made of repeated sub-laminate construction using finite element method and discussed the lay-up optimization for buckling by ranking individual composite curved beams. Potzta and Kollár [10] developed the replacement sandwich beams of building structures and derived corresponding stiffnesses of the replacement beams. The effectiveness of the replacement beams was demonstrated through the examples of the in-plane and flexural–torsional buckling and vibration analyses of high-rise buildings. Chen and Li [11] investigated the axially compressed elastic buckling of battened columns in which the shear effect on the buckling of columns was considered. Yuan et al. [12] investigated the buckling of castellated steel columns subjected to axial compression and examined the effect of web openings on the buckling of the columns. Cheng et al. [13] presented a thermal buckling analysis of thin-walled open-section columns with non-uniform sectional properties, subjected to axially compressive loads. Vo et al. [14] presented the free vibration and buckling analyses of functionally graded sandwich beams using a finite element analysis model, which considers shear deformation and thickness stretching effects. Costa et al. [15] proposed a new formulation for evaluating the flexural and torsional stiffness and the lateral-torsional buckling of multilayer laminated glass beams, which can be used to characterize the behaviour of simply supported laminated glass columns and beams up to five layers, subjected to various different types of loads. Do and Grognec [16] presented a finite element model to analyse the buckling problem of sandwich structures, whose core layer is made of a homogeneous foam periodically strengthened by orthogonal reinforcements. Rasheed et al. [17] developed a generalized analytical approach for analysing the lateral-torsional buckling of simply supported anisotropic hybrid, thin-walled, rectangular cross-section beams subjected to pure bending. An excellent literature review was recently provided by Sayyad and Ghugal [18] on the bending, buckling and free vibration analyses of shear deformable isotropic, laminated and sandwich beams using various different theoretic and numerical analysis methods.

The use of laminated composite and sandwich beams in buildings needs to consider the effect of thermal environment on their performance [19,20,21,22]. Recently, Li et al. [21] presented a spectral element model for analysing thermal effect on free vibration and buckling behaviours of laminated composite beams. More recently, Nguyen et al. [22] proposed a new hybrid shape functions for buckling and vibration analysis of laminated composite beams under thermal and mechanical loads, in which the displacement field was assumed based on a higher order shear deformation beam theory. However, neither in Li et al. [21] model nor in Nguyen et al. [22] model the non-uniform temperature distribution has been taken into account. The above survey of literature shows that, despite the considerable amount of work published in literature on the buckling analysis of laminated composite and sandwich beams, there is very little research focussing on the effect of the non-uniform mechanical properties induced by the non-uniform temperature distribution on the buckling behaviour of laminated composite and sandwich beams. It is anticipated that if the mechanical property is not uniform in the cross section of a beam, the bending centre of the beam will not be at the geometric centroid of the section. In this case either the bending or the shear of the beam will behave differently. In this paper an analytical study on the flexural buckling of sandwich beams with non-uniform sectional properties is presented. The non-uniform mechanical properties are assumed to be induced by the non-uniform temperature distribution in the beam cross-section. The formula for calculating the critical buckling load of sandwich beams under a linearly varied non-uniform temperature distribution scenario, both with and without considering the effect of transverse shear deformation, is derived.

2. Buckling analysis model of sandwich beams

Consider the flexural buckling of a symmetric rectangular sandwich beam with thermal-induced non-uniform sectional properties as shown in Fig.1a. Let t be the thickness of the outer layers of the beam, h be the half depth of the middle part of the beam, and b be the width of the beam. When the underneath surface of the beam is exposed to an elevated temperature, the heat will transfer from the bottom surface to the top surface. Since for most sandwich beams the heat transfer is much quicker in the two outer layers than in the middle part the temperature in each of the two outer layers can be assumed to be uniformly distributed, whereas the temperature in the middle part can be assumed to be linearly distributed along its depth direction, as shown in Fig.1b. Because of the non-uniform distribution of temperature in the cross-section, the mechanical properties of the beam are expected to be different at different places on the cross-section (see Fig.1c).

For some sandwich beams, although the material of the middle part is weaker than that of the outer layers, the bending rigidity of the middle part may still not be negligible. Thus, not only the shear deformation but also the bending deformation of the middle part need to be considered when establishing the equilibrium equations. By doing so, herein, the two outer layers of the beam are assumed to deform according to the Bernoulli's hypothesis and the middle part of the beam is assumed to deform according to Timoshenko's beam theory. For the convenience of presentation, the origin of the cross-sectional coordinate system is established at the geometric centroid of the cross-section of the beam (see Fig.1a). The axial displacements of the lower and upper layers at their own centroid are denoted as $u_1(x)$ and $u_3(x)$ as indicated in Fig.2. The transverse displacements of the two outer layers are assumed to be the same, denoted as $v(x)$. Thus, the axial strain in the lower and upper layers can be expressed as follows,

At lower layer: $-(h+t) \le y \le -h$

$$\varepsilon_1(y) = \frac{du_1}{dx} - \left(y + h + \frac{t}{2} \right) \frac{d^2 v}{dx^2} \tag{1}$$

At upper layer: $h \le y \le (h+t)$

$$\varepsilon_3(y) = \frac{du_3}{dx} - \left(y - h - \frac{t}{2} \right) \frac{d^2 v}{dx^2} \tag{2}$$

The axial strain in the middle part can be expressed as follows,

$$\varepsilon_2(y) = \frac{1}{2}\left(\frac{du_1}{dx} + \frac{du_3}{dx} \right) - \frac{y}{2h}\left(\frac{du_1}{dx} - \frac{du_3}{dx} \right) + \frac{yt}{2h}\frac{d^2 v}{dx^2} \tag{3}$$

The resultant force of the axial stress on the whole cross-section of the beam can be calculated as follows,

$$N_x = E_1 b \int_{-(h+t)}^{-h} (\varepsilon_1 - \varepsilon_{th1}) dy + E_3 b \int_{h}^{h+t} (\varepsilon_3 - \varepsilon_{th3}) dy + b \int_{-h}^{h} E_2 (\varepsilon_2 - \varepsilon_{th2}) dy \tag{4}$$

where E_1, E_2, and E_3 is the Young's modulus of the materials in lower layer, middle part and upper layer, respectively, ε_{th1}, ε_{th2}, and ε_{th3} is the thermal strain of the materials in lower layer, middle part and upper layer, respectively. Similarly, the resultant bending moment of the axial stress about z-axis can be calculated as follows,

$$M_z = -E_1 b \int_{-(h+t)}^{-h} y(\varepsilon_1 - \varepsilon_{th1}) dy - E_3 b \int_{h}^{h+t} y(\varepsilon_3 - \varepsilon_{th3}) dy - b \int_{-h}^{h} E_2 y(\varepsilon_2 - \varepsilon_{th2}) dy \tag{5}$$

Substituting Eqs.(1-3) into (4) and (5) and using $\varepsilon_{thk} = \alpha_k \Delta T_k$ (k=1,2,3) where α_k is the thermal expansion coefficient, T_k is the temperature in lower layer (k=1), middle part (k=2) and upper layer (k=3), and $\Delta T_k = T_k$ -20 where 20 °C is the assumed ambient temperature, it yields,

$$\begin{aligned} N_x &= E_1 bt\left(\frac{du_1}{dx} - \alpha_1 \Delta T_1\right) + E_3 bt\left(\frac{du_3}{dx} - \alpha_3 \Delta T_3\right) + \frac{bh}{2}(E_{21}+E_{23})\left(\frac{du_1}{dx} + \frac{du_3}{dx} - \alpha_2(\Delta T_1 + \Delta T_3)\right) \\ &+ \frac{bh}{2}(E_{23}-E_{21})\left(\frac{du_3}{dx} - \frac{du_1}{dx} - \alpha_2(\Delta T_3 - \Delta T_1) + t\frac{d^2v}{dx^2}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} M_z &= E_1 bt\left(h+\frac{t}{2}\right)\left[\frac{du_1}{dx} - \left(h+\frac{t}{2}\right)\frac{d^2v}{dx^2} - \alpha_1\Delta T_1\right] - E_3 bt\left(h+\frac{t}{2}\right)\left[\frac{du_3}{dx} + \left(h+\frac{t}{2}\right)\frac{d^2v}{dx^2} - \alpha_3 \Delta T_3\right] \\ &+ bt\left(h^2 + ht + \frac{t^2}{3}\right)(E_1+E_3)\frac{d^2v}{dx^2} - \frac{bh^2}{3}\left(E_{23}\frac{du_3}{dx} - E_{21}\frac{du_1}{dx} + t(E_{21}+E_{23})\frac{d^2v}{dx^2}\right) \\ &+ \frac{\alpha_2 bh^2}{3}\left(E_{23}\Delta T_3 - E_{21}\Delta T_1\right) \end{aligned} \tag{7}$$

Note that for given axial force, N_x, and bending moment, M_z, Eqs.(6) and (7) are not sufficient enough to determine the three displacement variables (u_1, u_3, v). An additional equation that can be developed is to link the average shear strain and average shear stress in the middle part of the sandwich beam. The average shear stress in the middle part can be obtained by examining the shear stress in the two outer layers, which can be expressed as follows,

$$\tau_{xy} = \frac{\tau_{xy}^3 + \tau_{xy}^1}{2} = \frac{1}{2b}\left(\frac{dN_3}{dx} - \frac{dN_1}{dx}\right) = \frac{t}{2}\left(E_3\frac{d^2u_3}{dx^2} - E_1\frac{d^2u_1}{dx^2}\right) \tag{8}$$

where τ_{xy} is the average shear stress in the middle part, τ_{xy} with superscript 1 or 3 represents the corresponding shear stress in lower or upper layer, N_1 and N_3 is the axial force in lower and upper layer, respectively. The average shear strain in the middle part can be calculated based on the axial and transverse displacements of the middle part (see Fig.2) as follows,

$$\gamma_{xz} = \frac{1}{2h}\left[\left(u_3 + \frac{t}{2}\frac{dv}{dx}\right) - \left(u_1 - \frac{t}{2}\frac{dv}{dx}\right)\right] + \frac{dv}{dx} = \frac{u_3 - u_1}{2h} + \left(\frac{t}{2h}+1\right)\frac{dv}{dx} \tag{9}$$

By applying the shear stress-strain constitutive relation for the middle part material, τ_{xy}= $(G_{21}+G_{23})\gamma_{xy}/2$, where G_{21} and G_{23} is the shear modulus of the middle part material at temperatures T_1 and T_3, respectively, the following equation can be obtained,

$$\frac{t}{2}\left(E_3\frac{d^2u_3}{dx^2} - E_1\frac{d^2u_1}{dx^2}\right) = \frac{G_{21}+G_{23}}{2}\left[\frac{u_3-u_1}{2h} + \left(\frac{t}{2h}+1\right)\frac{dv}{dx}\right] \tag{10}$$

It is obvious from Eq.(10) that if the shear deformation effect of the middle part is ignored, that is $(G_{21}+G_{23})\rightarrow\infty$, then the following equation holds,

$$\frac{u_3-u_1}{2h} + \left(\frac{t}{2h}+1\right)\frac{dv}{dx} = 0 \tag{11}$$

which indicates that the section normal of the middle part of the beam is identical to the section normal of the two outer layers of the beam. In this case only two variables are independent among the three displacement variables (u_1, u_3, v) because of the displacement constraint condition given by Eq.(11).

For a given temperature distribution (that is T_1 and T_3) Eqs.(6), (7) and (10) can be used to determine the three displacement variables (u_1, u_3, v). For the flexural buckling problem of the sandwich beam subjected to an axial compressive load P, the resultant force and resultant moment are balanced by the force, P, and moment, Pv, and thus the following expressions hold,

$$N_x + P = 0 \tag{12}$$

$$M_z + Pv = 0 \tag{13}$$

Substituting Eqs.(6) and (7) into (12) and (13), it yields,

$$\left(E_1bt + E_{21}bh\right)\left(\frac{du_1}{dx}\right) + \left(E_3bt + E_{23}bh\right)\left(\frac{du_3}{dx}\right) + \frac{bht}{2}(E_{23} - E_{21})\left(\frac{d^2v}{dx^2}\right) = \tag{14}$$
$$E_1bt\alpha_1\Delta T_1 + E_3bt\alpha_3\Delta T_3 + bh\alpha_2(E_{21}\Delta T_1 + E_{23}\Delta T_3) - P$$

$$\left[E_1bt\left(h + \frac{t}{2}\right) + \frac{E_{21}bh^2}{3}\right]\left(\frac{du_1}{dx}\right) - \left[E_3bt\left(h + \frac{t}{2}\right) + \frac{E_{23}bh^2}{3}\right]\left(\frac{du_3}{dx}\right) +$$
$$\left[\frac{\left(E_1 + E_3\right)bt^3}{12} - \frac{(E_{21} + E_{23})bh^2t}{3}\right]\left(\frac{d^2v}{dx^2}\right) + Pv = \tag{15}$$
$$bt\left(h + \frac{t}{2}\right)\left(E_1\alpha_1\Delta T_1 - E_3\alpha_3\Delta T_3\right) - \frac{\alpha_2bh^2}{3}\left(E_{23}\Delta T_3 - E_{21}\Delta T_1\right)$$

Eqs.(10), (14) and (15) describe the nonlinear load-displacement curve of the sandwich beam, subjected to axially compressive loads, under the influence of nonuniform temperature distribution. For a give temperature distribution one can use these three equations to determine the load-deflection curve of the beam. Assume that when the flexural buckling occurs the displacement function of the beam can be approximated as follows,

$$u_1(x) = C_1 \cos\frac{\pi x}{L} \tag{16}$$

$$v(x) = C_2 \sin\frac{\pi x}{L} \tag{17}$$

$$u_3(x) = C_3 \cos\frac{\pi x}{L} \tag{18}$$

where C_1, C_2 and C_3 are the constants and L is the beam length. Note that, for the flexural buckling problem of the beam, it is only the homogeneous form of Eqs.(10), (14) and (15) needs to be examined. Substituting Eqs.(16), (17) and (18) into (10), (14) and (15), the following 3x3 determinant, representing the coefficient matrix of the homogeneous form of Eqs.(10), (14) and (15), is obtained:

$$\|A\| = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & P + a_{32} & a_{33} \end{Vmatrix} \tag{19}$$

in which,

$$a_{11} = \frac{E_1t}{2}\left(\frac{\pi}{L}\right)^2 + \frac{G_{21} + G_{23}}{4h},\ a_{12} = -\frac{G_{21} + G_{23}}{2h}\left(h + \frac{t}{2}\right)\left(\frac{\pi}{L}\right),\ a_{13} = -\frac{E_3t}{2}\left(\frac{\pi}{L}\right)^2 - \frac{G_{21} + G_{23}}{4h},$$

$$a_{21} = -b\left(E_1t + E_{21}h\right)\left(\frac{\pi}{L}\right),\ a_{22} = -\frac{bht}{2}\left(E_{23} - E_{21}\right)\left(\frac{\pi}{L}\right)^2,\ a_{23} = -b\left(E_3t + E_{23}h\right)\left(\frac{\pi}{L}\right),$$

$a_{31} = -b\left[E_1 t\left(h + \frac{t}{2}\right) + \frac{E_{21}h^2}{3}\right]\left(\frac{\pi}{L}\right)$, $a_{32} = -bt\left[\frac{(E_1 + E_3)t^2}{12} - \frac{(E_{21} + E_{23})h^2}{3}\right]\left(\frac{\pi}{L}\right)^2$,

$a_{33} = b\left[E_3 t\left(h + \frac{t}{2}\right) + \frac{E_{23}h^2}{3}\right]\left(\frac{\pi}{L}\right)$.

The condition that the buckling of the beam occurs is when the determinant given by Eq.(19) becomes zero, from which the critical buckling load, $P_{cr} = P$, can be obtained as follows,

$$P_{cr} = \frac{a_{22}(a_{11}a_{33} - a_{13}a_{31}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) - a_{32}(a_{11}a_{23} - a_{13}a_{21})}{a_{11}a_{23} - a_{13}a_{21}} \tag{20}$$

It is interesting to notice from Eqs.(14) and (15) that, all terms associated to the thermal expansion are located in the right-hand-side of the equations. They thus have no effect on the coefficient matrix of the homogeneous form of Eqs.(10), (14) and (15), which indicates that the thermal expansions do not affect the critical buckling load, although they may have influence on the load-deflection curve of the beam. Note that the critical load given by Eq.(20) is only for the flexural buckling of the beam about its major axis (z-axis). Thus, the buckling discussed herein is only for beams that are laterally restrained, in which case the flexural buckling about y-axis and torsional buckling of the beams are prevented by the lateral restraints.

3. Numerical examples

As a numerical example, a sandwich beam with beam depth-to-width ratio of $(2h+2t)/b=8/3$ and skin-to-core thickness ratio of $t/h=1/3$, at different beam length-to-beam depth ratios ranging from $L/(2h+2t)=2$ to $L/(2h+2t)=8$ is analysed. To demonstrate the effect of temperature variation on the critical buckling load of sandwich beams, four different temperature scenarios given in Table 1 are considered herein. In all of the four scenarios the temperature of the upper layer of the beam is assumed to be in an ambient state, whereas the temperature of the lower layer of the beam has a varying temperature from ambient 20 ^{o}C in case 1 to 600 ^{o}C in case 4. The assumed Young's modulus and shear modulus of the outer layer and/or middle part material of the beam at different temperatures are also given in Table 1, which is based on the reduction of Young's modulus of most composite materials [23]. Considering the core material is generally weaker than the skin material in most sandwich beams, the Young's modulus of the middle part material of the sandwich beam is assumed to be a half of that of the outer layer material of the beam for the same temperature.

Fig.3 shows the calculated critical buckling load curves of the sandwich beam obtained from Eq.(20) for the ambient temperature case 1. The three buckling curves shown in the figure represent the results obtained from different theories. The first one, denoted as "present solution", is directly calculated from Eq.(20) using the mechanical properties given in Table 1, which represents the model presented in this paper. The second one, denoted as "$E_{21}=E_{23}=0$", is calculated from Eq.(20) using the mechanical properties given in Table 1 except for E_{21} and E_{23} that are assumed to be zero (i.e. assuming the middle part has no bending rigidity), which is normally used for sandwich beams. The third one, denoted as "$G_{21}=G_{23}=\infty$", is also calculated from Eq.(20) using the mechanical properties given in Table 1 except for G_{21} and G_{23} that are assumed to be infinity (i.e., assuming the middle part has no shear deformation), which treats the sandwich beam as a Bernoulli-Euler beam. The critical buckling loads in all of the three buckling curves are normalised by a reference load $P_o = 2Eb(h+t)$.

It can be seen from Fig.3 that, the critical buckling load calculated by ignoring the shear deformation is much higher than that calculated from the present model, particularly for beams with short length, indicating that the shear deformation of the middle part of the sandwich beam has an important effect on the buckling behaviour of the beam. In contrast, the critical buckling load calculated by ignoring the bending rigidity of the middle part is consistently lower than that calculated from the present model, indicating that the bending rigidity of the middle part of the sandwich beam may have some positive effect on the critical buckling load. However, the quantity of the effect is dependent on the relative dimensions and relative mechanical properties of the middle part to that of the two outer layers. Also, it can be seen from the figure that all three critical buckling loads decrease with the increase of the beam length; the differences between the three critical buckling loads are also found to decrease with the increased beam length. This indicates that the effect of both the shear deformation and bending rigidity of the middle part of the sandwich beam on the critical buckling load reduces with the increased beam length.

To examine the effect of the temperature-induced non-uniform cross-sectional properties on the buckling behaviour of the sandwich beam, Figs.4-6 show the variation of the three critical buckling loads of the sandwich beam for three different temperature scenarios (cases 2-4), calculated using three different theories as explained in Fig.3. For the convenience of comparisons, all critical buckling loads shown in the figures are normalised by using the critical buckling load at ambient temperature obtained from Bernoulli-Euler beam theory (i.e. the critical buckling load with "$G_{21}=G_{23}=\infty$" shown in Fig.3). It can be seen from these figures that, the critical buckling loads calculated from all three different theories are found to decrease in different extent with the increase of the temperature or the reduction of the Young's modulus and shear modulus. The gaps between the critical buckling loads obtained from different theories are also found to reduce with the increase of the temperature or the reduction of the Young's modulus and shear modulus. It is interesting to notice that the gap between the critical buckling load obtained from the "present solution" and that obtained from "$G_{21}=G_{23}=\infty$" decreases with the increase of beam length; whereas the gap between the critical buckling load obtained from the "present solution" and that obtained from "$E_{21}=E_{23}=0$" increases with increased beam length. Also, because of the non-symmetric mechanical properties caused by the non-uniform temperature distribution, the normalized critical buckling load obtained from "$G_{21}=G_{23}=\infty$" decreases slightly with the increased beam length. This indicates that, for the sandwich beam with non-uniform cross-sectional properties even when the shear deformation effect is neglected the critical buckling load is not exactly linearly proportional to the factor $1/L^2$.

4. Conclusions

In this paper, an analytical study has been presented on the flexural buckling of sandwich beams considering thermal-induced non-uniform cross-sectional properties. The formula has been derived for determining the buckling loads of sandwich beams under a linearly varied non-uniform temperature distribution scenario, both with and without considering the influence of transverse shear deformation. The effects of local reductions in Young's modulus and shear modulus on the flexural buckling of sandwich beams have been also discussed. From the results obtained the following conclusions can be drawn:

- The thermal expansion induced by the temperature variation in sandwich beams has no influence on the buckling loads of sandwich beams, although it may affect the axial and bending deformations of the beams.

- The bending stiffness of the middle part of sandwich beams may have some positive influence on the buckling loads of the beams, depending on its dimensions and material properties. Neglecting the bending stiffness of the middle part of a sandwich beam will provide an underestimated critical buckling load of the beam.
- Non-uniform distribution of temperature can lead to non-uniform distribution of mechanical properties. The latter can significantly affect the buckling behaviour of sandwich beams.
- The reduction in the Young's modulus of bottom layer or in the shear modulus of middle part near the bottom layer can considerably reduce the buckling loads of sandwich beams. The reduction of the buckling load increases with the reduction of these two moduli.
- The transverse shear deformation in the middle part of sandwich beams has a large effect on the critical buckling load of the sandwich beams. However, its effect decreases with increased beam length.

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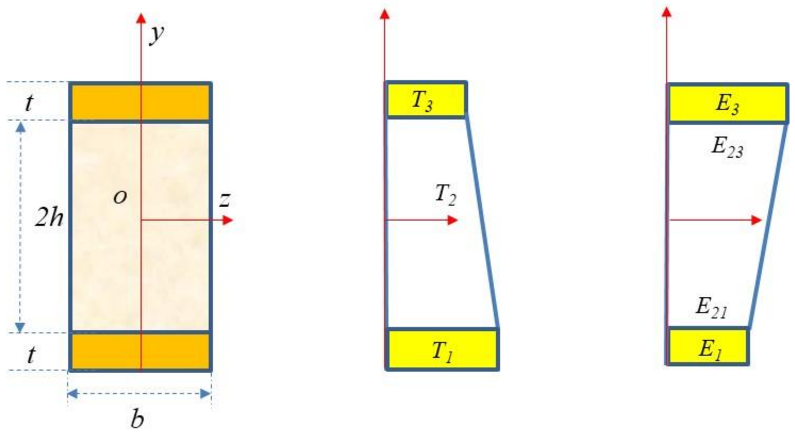
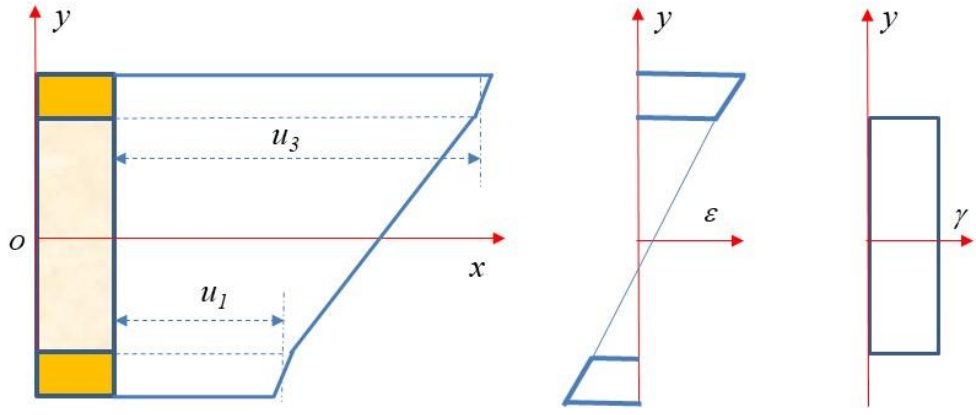
Table 1. Mechanical properties used in examples

Description	Parameter	Case 1	Case 2	Case 3	Case 4
Temperature in lower layer, °C	T_1	20	200	400	600
Young's modulus in lower layer	E_1	E	4E/5	3E/5	2E/5
Young's modulus in middle part	E_{21}	E/2	2E/5	3E/10	E/5
Shear modulus in middle part	G_{21}	E/6	2E/15	E/10	E/15
Temperature in upper layer, °C	T_3	20	20	20	20
Young's modulus in upper layer	E_3	E	E	E	E
Young's modulus in middle part	E_{23}	E/2	E/2	E/2	E/2
Shear modulus in middle part	G_{23}	E/6	E/6	E/6	E/6
Note: *E* is a reference Young's modulus					



(a) (b) (c)

Fig.1 (a) Cross-section of a sandwich beam. (b) Temperature (*T*) distribution on cross-section. (c) Young's modulus (*E*) distribution on cross-section.



(a) (b) (c)

Fig.2 (a) Displacement distribution on cross-section of a sandwich beam. (b) Bending strain distribution on two outer layers. (c) Shear strain distribution in middle part.

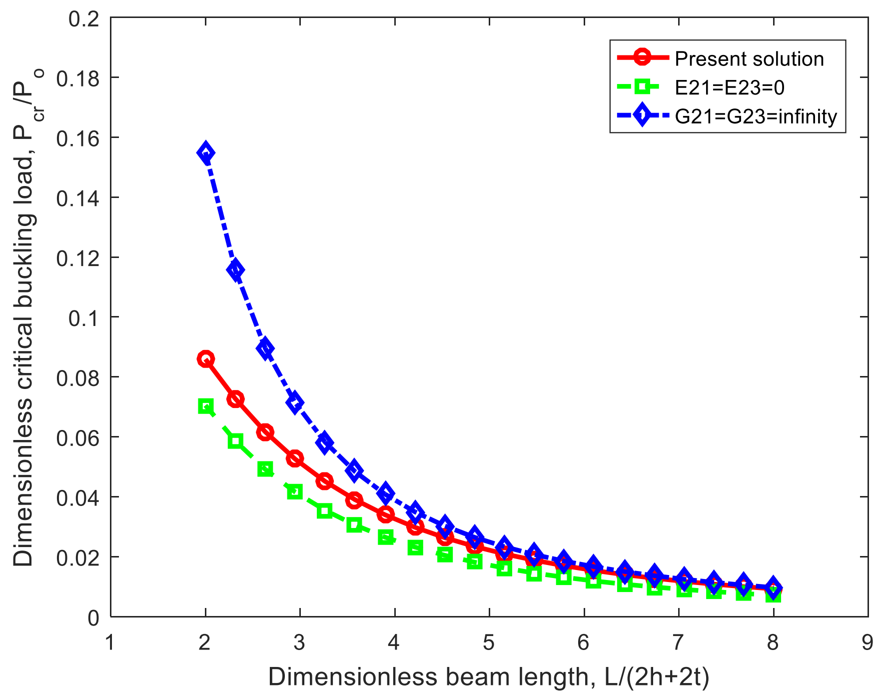


Fig.3 Influence of bending and shear stiffnesses of middle part on critical buckling load of sandwich beam (h/t = 3, h/b = 1, $P_o = 2Eb(h+t)$) for temperature case 1.

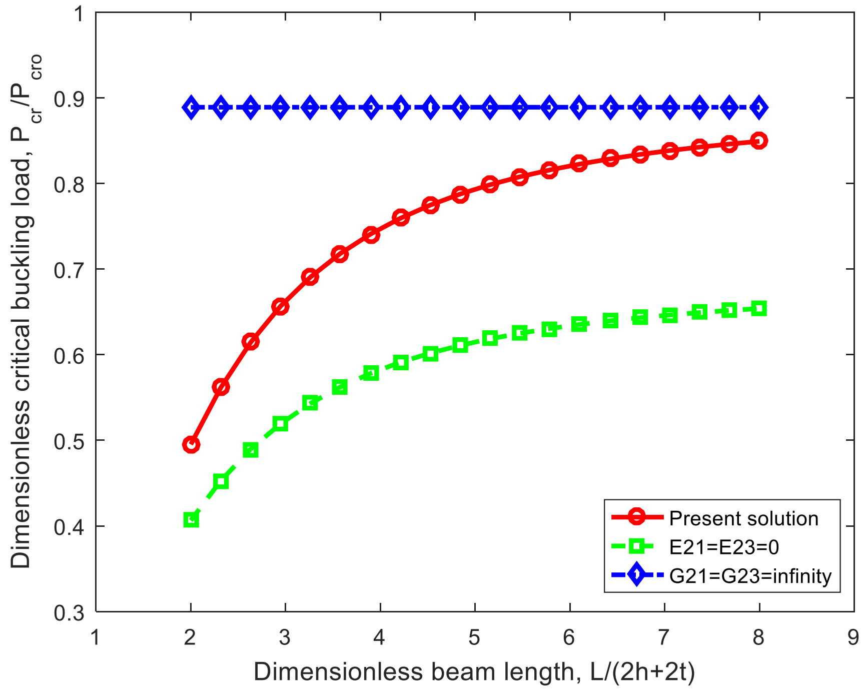


Fig.4 Influence of bending and shear stiffnesses of middle part on critical buckling load of sandwich beam (h/t = 3, h/b = 1, P_{cro} is the critical buckling load shown in Fig.3 with "G_{21}=G_{23}=∞") for temperature case 2.

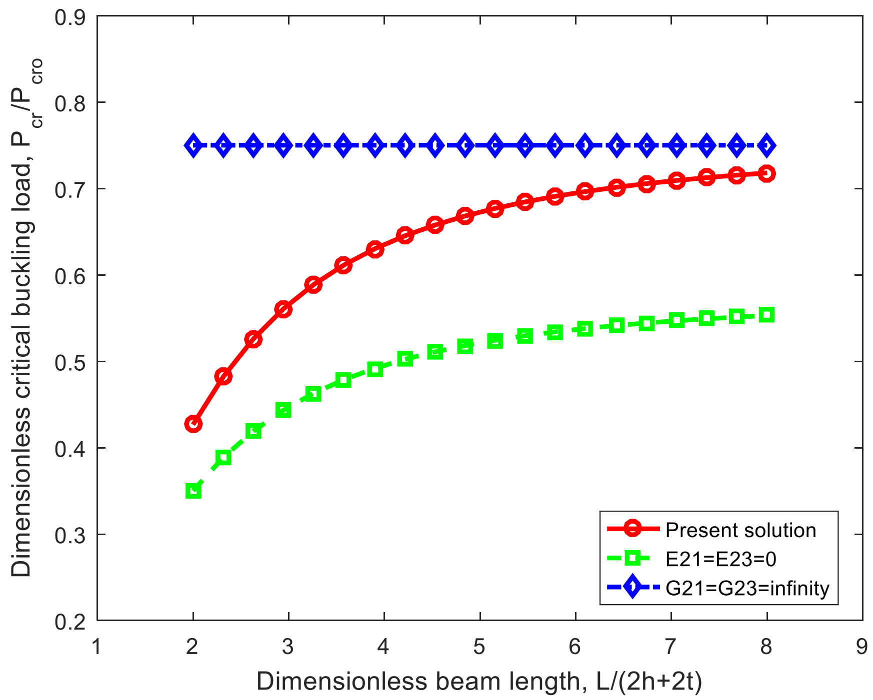


Fig.5 Influence of bending and shear stiffnesses of middle part on critical buckling load of sandwich beam (h/t = 3, h/b = 1, P_{cro} is the critical buckling load shown in Fig.3 with "G_{21}=G_{23}=∞") for temperature case 3.

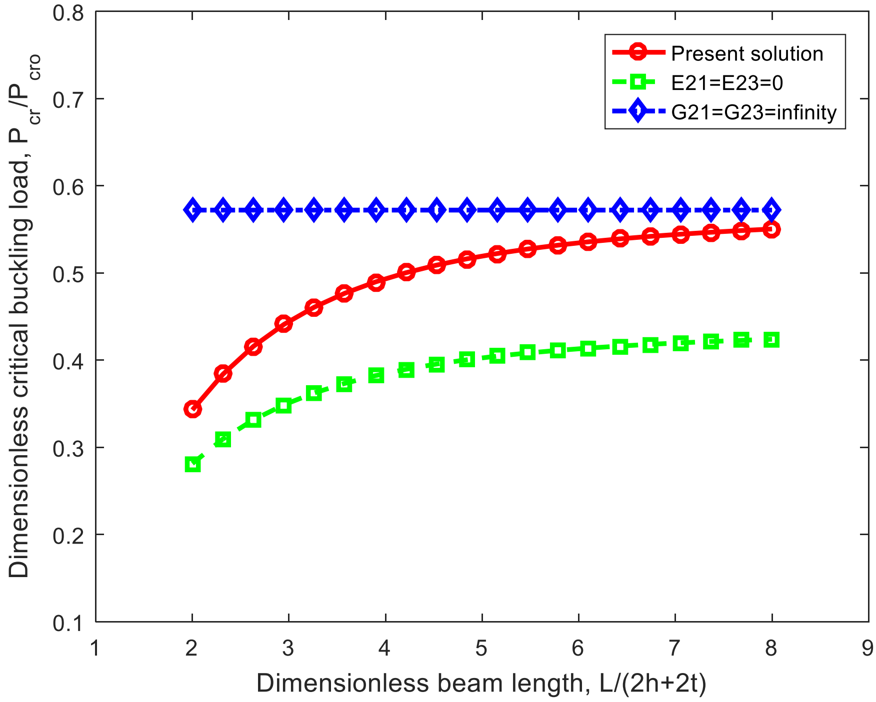


Fig.6 Influence of bending and shear stiffnesses of middle part on critical buckling load of sandwich beam (h/t = 3, h/b = 1, P_{cro} is the critical buckling load shown in Fig.3 with "G_{21}=G_{23}=∞") for temperature case 4.