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28 Abstract

29 Integrating wave energy converters into coastal structures such as breakwaters, seawalls or jetties not only offers benefits in terms of construction costs but also improves wave energy extraction. 30 In this paper a novel theoretical model based on linear potential flow theory is developed to study 31 the performance of an oscillating water column (OWC) integrated into a vertical structure in water 32 33 of finite water depth. The model has three fundamental advantages relative to previous works: no 34 thin-wall restriction (the thickness of the OWC chamber wall is considered), no singularities, and 35 far fewer truncating terms in the eigen-function expansions. The OWC chamber is a vertical 36 cylinder semi-embedded in the structure with a submerged, open bottom. As water waves impinge 37 on the structure, the water column in the chamber oscillates and drives an air turbine installed at 38 the chamber top to extract wave power. Using linear wave theory, the velocity potential in the 39 water domain is decomposed into scattering and radiation potentials whose unknown coefficients 40 are determined by the eigen-function matching method. Upon successful validation, the model is used to investigate the influence of the thickness of the chamber wall and the radius and 41 42 submergence of the chamber on wave power absorption.

43

Keywords: Oscillating Water Column; Breakwater-integrated OWC; Wave energy; Potential flow;
Excitation volume flow; Hydrodynamic coefficients

46

47 1. Introduction

48 A large number of wave energy conversion concepts have been proposed since 1970s, which 49 can be roughly classified as: oscillating water column (OWC) (e.g., [1, 2]), point absorber (e.g., [3]), attenuator (e.g., [4]), oscillating wave surge converter (e.g., [5]), overtopping (e.g., [6]), and 50 others. Despite the large number of concepts proposed and investigated so far, only a few wave 51 52 energy converters (WECs) have been tested at a large scale, and even fewer have achieved the 53 fully commercial stage [7, 8]. The challenges in bringing WECs to the market include: high cost 54 of construction, installation and maintenance; negative environmental impact; poor reliability; and 55 low power extraction efficiency [9, 10]. It is not easy to solve all these problems concurrently since some of them might be in conflict - and therein lie the challenges. For example, the 56 57 improvement of the reliability of WECs generally results in an increased cost of construction; to 58 enhance the power capture efficiency more sophisticated systems are typically necessary, but this 59 very sophistication is generally detrimental to the overall cost and survivability of the system.

60 The integration of a WEC into a marine structure, e.g., a breakwater, as opposed to its stand-alone 61 deployment in the open sea is an effective means to overcome a number of these challenges and 62 significantly increase the attractiveness of wave power exploitation [9, 11]. The integration not only offers benefits in terms of shared costs of construction, but also improves the robustness of 63 the WEC and minimizes its environmental impact. Additionally, thanks to the wave power 64 65 absorbed by the WEC, wave reflection at the structure is diminished, which is often advantageous 66 from the points of view of coastal protection and non-interference with shipping. The synergies 67 between wave energy and marine structures have been investigated in a number of works, e.g., 68 integration of an array of WECs with a breakwater [12], integration of an OWC into an offshore 69 wind turbine [13] or breakwater [14].

Among wave energy conversion technologies, OWC systems are especially simple, for the only

moving mechanical part is an air turbine/generator located above the water; therefore, 71 72 OWC-breakwater integration has received considerable attention [9]. Evans and Porter [15] 73 developed a theoretical model to simulate a two-dimensional (2D) OWC device composed of a 74 thin vertical surface-piercing barrier in front of a vertical wall. An integral equation for the 75 horizontal velocity across the gap under the thin barrier was adopted to deal with the singular 76 behaviour in the velocity field. Theoretical results showed that increasing the distance between the 77 barrier and the wall decreased the frequency at which resonance occurred. Later, Morris-Thomas, 78 Irvin [16] examined effect of the front barrier geometry on the performance of the OWC experimentally. The hydrodynamic efficiency in short waves was found to decrease with the 79 80 increase of the barrier's submergence or thickness. More recently, the impact of the underwater 81 lips of an offshore OWC device in terms of both thickness and submergence was investigated by 82 Elhanafi, Fleming [17] with a two-dimensional computational fluid dynamics (CFD) model. By 83 selecting the optimal combination of the submergence and thickness of the lips, a peak efficiency 84 exceeding 0.79 was achieved, much larger than the 0.3 for a device with simpler, typical geometry. 85 Other aspects such as the role of the turbine Power Take-Off (PTO) system and the environmental conditions in the power extraction of an onshore or bottom-fixed, breakwater-integrated OWC 86 87 have also been investigated. Sheng, Alcorn [18] proposed a numerical method based on potential 88 flow theory to assess the primary energy conversion of two generic OWC WECs (one bottom 89 fixed and another floating). The hydrodynamics and thermodynamics with consideration of the air 90 compressibility for different types of the air turbine PTOs (i.e., Wells turbine, impulse turbines and bi-radial turbines) were coupled in the time-domain, and the numerical results appeared accurate 91 enough for the OWC power extraction assessment, especially for the bottom-fixed OWC. López 92 93 and Iglesias [19] developed a virtual laboratory based on artificial neural networks that can be 94 employed to obtain the pneumatic efficiency of a given OWC under specific wave condition, tidal level and turbine damping. Physical model tests of these parameters were carried out as well [20]. 95 96 In order to achieve an optimal energy transfer, Pereiras, López [21] described a methodology for 97 matching a nonlinear turbine to the OWC chamber. Elhanafi, Fleming [22] adopted a CFD model 98 to learn the impacts of both the PTO damping and incoming wave height on the performance of an 99 onshore OWC. The reflection coefficient and the energy absorption coefficient generally increase 100 and decrease with wave height. Research has also been directed towards other types of onshore or 101 bottom-fixed, breakwater-integrated OWCs, e.g., the U-type OWC [23, 24] and the multi-chamber 102 OWC [25]. Additionally, the integration of OWCs with floating breakwaters was considered by 103 He, Huang [26], He, Leng [27].

104 The above studies are focused on 2D problems of OWC-breakwater integration. In contrast, 105 there are few studies on its 3D aspects. For experimental work these require a wave basin rather 106 than a flume, with a scale model of the breakwater as well as the OWC itself [28]. For numerical work, if the boundary element method is employed, the surfaces of the OWC device and the 107 surrounding breakwater or coastline must be divided into elements [29]; if the finite element 108 109 method is adopted, a numerical wave basin shall be established with the entire water volume 110 discretized [30]. Due to the considerable experimental and computational cost, 3D studies of 111 OWC-structure integration are not common. If the shapes of the structure and OWC are regular, 112 theoretical models may be used to solve the 3D hydrodynamic problem. Martins-rivas and Mei 113 [31] proposed a theoretical model based on the 3D wave radiation/diffraction theory and the usual method of eigen-function expansions to study wave power extraction from an OWC at the tip of a 114

long and thin breakwater. The thin-walled OWC was represented by a hollow cylinder in their 115 model, in which the method for solving the integral equation of Evans and Porter [15] was used to 116 treat the singular behaviours in the velocity field beneath the thin wall of the OWC chamber. The 117 linearized air compressibility in the chamber was taken into account as part of the PTO system. 118 The effects of the radius and submergence of the OWC chamber, the air compressibility and the 119 120 incident wave direction were investigated. It was found that the free surface outside was strongly dependent on the incident wave direction, whereas the power extracted was roughly insensitive to 121 122 the incident direction. Subsequently, Martins-rivas and Mei [32] applied the same theoretical approach to a thin-walled OWC installed on a straight cliff-like coast. The performance of the 123 124 OWC was found to strongly depend on the incident wave direction. Wave reflection at the coast 125 could lead to up to a doubling in the power absorbed by the OWC. The role of either the radius or submergence of the OWC was not considered. This theoretical model was later applied by Lovas, 126 127 Mei [33] to a vertical OWC at the tip of a general wedge-shaped coast.

In this context we propose a novel theoretical model based on linear potential flow theory to evaluate the hydrodynamic performance and power extraction of the OWC. This novel approach has three fundamental advantages relative to previous works. First, the thin-wall restriction is removed, i.e., the thickness of the OWC chamber wall is taken into consideration. Second, there is no singularity. Finally, far fewer truncating terms of the eigen-function expansions are required to obtain accurate results. The effects of wall thickness, radius and submergence of the OWC on power extraction can thus be investigated with the present model – as shown below.

The rest of the paper is organized as follows. Section 2 presents the relation between the PTO 135 system and the hydrodynamic problem. The basic governing equation, the boundary conditions for 136 137 wave scattering and radiation problems, and the expressions of the scattering and radiated velocity 138 potentials in different regions of the water domain are developed in Section 3, alongside the method for solving the unknown coefficients. The expressions of excitation volume flow and 139 hydrodynamic coefficients are derived in Section 4. The model validation can be found in Section 140 141 5. The influence of the radius, wall thickness and submergence of the OWC chamber on wave power absorption are investigated in Section 6. Finally, conclusions are drawn in Section 7. 142

143 2. Mathematical model

144 Consider an oscillating water column (OWC) installed on a vertical wall in a water domain of 145 uniform depth *h* (see Fig.1). The OWC chamber is composed of a vertical circular cylinder with a 146 ring shape cross section and it is half embedded in the wall. The outer and inner radii of the OWC 147 chamber are denoted as *R* and *R*_i, respectively. On the seaside, the chamber is open from a finite 148 submergence, denoted as *d*, to the seabed. As water waves propagate in the direction of β , the 149 water column enclosed by the OWC chamber oscillates and drives a Wells turbine (not plotted in 150 Fig. 1) installed at the chamber top to extract wave power.



152 153

Fig. 1. Oscillating water column integrated into a coast/breakwater: (a) bird view; (b) top view.

As shown in Fig. 1, a general Cartesian coordinate system Oxyz is adopted with the Oz axis at the location of the symmetrical vertical axis of the OWC pointing upward and the Ox axis along the waterline at the coast/breakwater. A polar coordinate ($Or\theta_z$) is defined as given in Fig. 1b.

Subjected to regular waves of angular frequency ω with small amplitude, the flow problems may be treated in the linear potential theory regime in the frequency domain based on the assumption that the fluid is isotropic, incompressible inviscid, and the time-harmonic flow is irrotational. The fluid motion can be described by the velocity potential Re[$\Phi(r,\theta,z)e^{-i\omega t}$], where Φ is a complex spatial velocity potential independent of time, *t*, and satisfies the Laplace equation, i represents imaginary unit. In a similar way, the air pressure inside the OWC chamber can be written as Re($pe^{-i\omega t}$), where *p* is the complex air pressure amplitude inside the OWC chamber.

164 The spatial velocity potential Φ can be decomposed into the wave spatial potential, $\Phi_{\rm I}$, which 165 represents the wave field when the vertical wall without OWC (i.e., a flat wall) is subjected to 166 monochromatic incident waves, the diffracted wave spatial potential $\Phi_{\rm D}$ induced by the existence 167 of the OWC, and the radiated wave spatial potential as follows

168
$$\Phi = \Phi_{\rm I} + \Phi_{\rm D} + p\Phi_{\rm R} , \qquad (1)$$

169 where $\Phi_{\rm R}$ is the spatial velocity potential due to unit air pressure oscillation inside the OWC 170 chamber. $\Phi_{\rm I}$, $\Phi_{\rm D}$, and $\Phi_{\rm R}$ all satisfy the Laplace equation, the boundary condition at the side wall 171 of the coast/breakwater and the seabed boundary condition. Moreover, $\Phi_{\rm D}$ and $\Phi_{\rm R}$ must satisfy a 172 radiation condition at infinite distance. Hereinafter, the sum of the incident and diffracted velocity 173 potentials, which is the so-called scattering velocity potential (i.e., $\Phi_{\rm S}=\Phi_{\rm I}+\Phi_{\rm D}$), is adopted for the 174 sake of simplicity.

Considering the air turbine employed in the OWC is an idealized lossless linear Wells turbine and assuming the mass flux through the Wells turbines is proportional to the chamber air pressure, following Sarmento and Falcão [34] and Martins-rivas and Mei [32], the complex air pressure amplitude, *p*, is related to the scattering and radiated velocity potentials by:

179 $\left[-i\left(a_{\rm PTO}+a\right)+\left(c_{\rm PTO}+c\right)\right]p=Q_{\rm e},$ (2)

where a_{PTO} is used to take into account the effect of air compressibility, and can be expressed as $a_{\text{PTO}} = \omega V_0 / (v^2 \rho_0)$, in which V_0 is the air chamber volume, v denotes the sound velocity in air and ρ_0 represents the static air density; c_{PTO} is the damping of the PTO system depending on the rotational speed of turbine blades, the scales of turbine rotor, the design of turbines and ρ_0 as well; Q_e , the so-called excitation volume flow, is the rate of upward displacement of the water surface inside the column contributed by the scattering potential:

186
$$Q_{\rm e} = \int_0^{2\pi} \int_0^{R_{\rm i}} \frac{\partial \Phi_{\rm s}}{\partial z} \bigg|_{z=0} r dr d\theta , \qquad (3)$$

187 c and a are hydrodynamic coefficients that can be derived from the volume flow inside the column 188 induced by the radiated potential (i.e., $Q_{\rm R}$),

189
$$-(c-ia) = Q_{R} = \int_{0}^{2\pi} \int_{0}^{R_{i}} \frac{\partial \Phi_{R}}{\partial z} \Big|_{z=0} r dr d\theta.$$
(4)

190 The time-averaged power extraction by the PTO system (i.e., the Wells turbine), P, can be 191 calculated by:

192
$$P = \frac{c_{\rm PTO}}{2} \left| p \right|^2 = \frac{c_{\rm PTO}}{2} \frac{\left| Q_{\rm e} \right|^2}{\left(c + c_{\rm PTO} \right)^2 + \left(a + a_{\rm PTO} \right)^2}.$$
 (5)

193 The efficiency of wave power extraction is generally expressed by the relative wave capture194 width

195
$$\eta = \frac{kP}{P_{\rm in}} = \frac{2kP}{\rho g A^2 c_{\rm g}},\tag{6}$$

where P_{in} is the incident wave energy per unit width of the wave front; c_g is the group velocity of the incident wave.

198 Note that $Q_{\rm e}$, *c* and *a* are fundamental for evaluating the performance of the OWC. In order to 199 obtain these parameters, wave scattering and radiation problems, i.e., $\Phi_{\rm S}$ and $\Phi_{\rm R}$, should be solved 200 first.

201 3 Solution to scattering/radiated potentials

203

202 The governing equation and the boundary conditions for Φ_{χ} (χ =S, R) can be written as follows:

$$\frac{\partial \Phi_{\chi}}{\partial z} = 0, \qquad z = -h, \tag{7}$$

204
$$\frac{\partial \Phi_{\chi}}{\partial z} = 0, \qquad R_{i} \le r \le R, \quad z = -d, \quad 0 < \theta < \pi, \tag{8}$$

205
$$\frac{\partial \Phi_{\chi}}{\partial z} - \frac{\omega^2}{g} \Phi_{\chi} = 0, \quad r \ge R, \quad z = 0, \quad 0 < \theta < \pi, \tag{9}$$

206
$$\frac{\partial \Phi_{\chi}}{\partial z} - \frac{\omega^2}{g} \Phi_{\chi} = \begin{cases} 0, & \chi = S \\ \frac{i\omega}{\rho g}, & \chi = R \end{cases}, \quad r \le R_i, \quad z = 0, \quad 0 \le \theta \le 2\pi, \quad (10)$$

207
$$\frac{\partial \Phi_{\chi}}{\partial \theta} = 0, \quad r > R, \quad -h \le z \le 0, \quad \theta = 0, \pi, \quad (11)$$

208
$$\frac{\partial \Phi_{\chi}}{\partial \theta} = 0, \quad R_{i} < r < R, \quad -h \le z \le -d, \quad \theta = 0, \pi, \quad (12)$$

209
$$\frac{\partial \Phi_{\chi}}{\partial r} = 0, \quad r = R, \quad -d \le z \le 0, \quad 0 \le \theta \le \pi, \quad (13)$$

210
$$\frac{\partial \Phi_{\chi}}{\partial r} = 0, \quad r = R_{i}, \quad -d \le z \le 0, \quad 0 \le \theta \le 2\pi, \quad (14)$$

211
$$\frac{\partial \Phi_{\chi}}{\partial r} = 0, \quad r = R_{i}, \quad -h \le z \le -d \quad , \quad \pi \le \theta \le 2\pi \,, \tag{15}$$

212 where ρ is the water density and g represents the gravity acceleration.

213 The entire fluid domain can be divided into three regions: I, inner region enclosed by the OWC, 214 i.e., $r \le R_i$, $0 \le \theta \le 2\pi$, $-h \le z \le 0$; II, ring region beneath the OWC chamber, i.e., $R_i \le r \le R$, $0 \le \theta \le \pi$, $-h \le z \le -d$; III, outside region, i.e., $r \ge R$, $0 \le \theta \le \pi$, $-h \le z \le 0$. Φ_{χ} ($\chi = S$, R) in these three regions are denoted as 215 $arPsi_{\chi}^{
m in}$, $arPsi_{\chi}^{
m ring}$ and $arPsi_{\chi}^{
m out}$, respectively. 216

3.1 Expressions of scattering/radiated potentials in different regions 217

218 In different regions, with the application of the method of separation of variables, Φ_{χ} (χ =S, R) 219 can be expressed by orthogonal series as follows [35, 36]: 220

I, inner region

221
$$\varPhi_{\chi}^{\text{in}}(r,\theta,z) = \sum_{m=-\infty}^{\infty} \sum_{l=0}^{\infty} \frac{\tilde{I}_m(k_l r)}{k_l \tilde{I}'_m(k_l R_i)} A_{m,l}^{\chi} Z_l(z) e^{im\theta} + \varPhi_{p,\chi}^{\text{in}},$$
(16)

222 where $A_{m,l}^{\chi}$ is the unknown coefficients to be determined; $\Phi_{p,\chi}^{in}$ is a particular solution,

223
$$\boldsymbol{\Phi}_{\mathbf{p},\chi}^{\mathrm{in}} = \begin{cases} 0, & \chi = \mathbf{S} \\ -\frac{\mathbf{i}}{\rho\omega}, & \chi = \mathbf{R} \end{cases}$$
(17)

224
$$\tilde{I}_{m}(k_{l}r) = \begin{cases} J_{m}(k_{l}r), & l = 0\\ I_{m}(k_{l}r), & l \neq 0 \end{cases}$$
(18)

225 in which J_m and I_m denote the Bessel function and the modified Bessel function of the first kind,

226 respectively. k_0 is the wave number and k_l (l>0) is the eigenvalue given by [37, 38],

227
$$\omega^2 = -k_l g \tan(k_l h), \qquad l=1,2,3,...$$
(19)

228
$$Z_0(z) = N_0^{-0.5} \cosh\left[k_0(z+h)\right]; \ Z_l(z) = N_l^{-0.5} \cos\left[k_l(z+h)\right];$$
(20)

229 where

230
$$N_0 = \frac{1}{2} \left[1 + \frac{\sinh(2k_0h)}{2k_0h} \right], \quad N_l = \frac{1}{2} \left[1 + \frac{\sin(2k_lh)}{2k_lh} \right].$$
(21)

231 II, ring region

232
$$\mathcal{P}_{\chi}^{\mathrm{ring}}\left(r,\theta,z\right) = \sum_{m=0}^{\infty} \left[F_{m,0}^{\chi}\left(r\right) + \sum_{l=1}^{\infty} \left(C_{m,l}^{\chi} \frac{I_{m}\left(\beta_{l}r\right)}{I_{m}\left(\beta_{l}R\right)} + D_{m,l}^{\chi} \frac{K_{m}\left(\beta_{l}r\right)}{K_{m}\left(\beta_{l}R\right)} \right) \cos\left[\beta_{l}\left(z+h\right)\right] \right] \cos m\theta ,$$
233 (22)

234 where

235
$$F_{m,0}^{\chi}(r) = \begin{cases} C_{m,0}^{\chi} + D_{m,0}^{\chi} \left[1 + \ln\left(\frac{r}{R}\right) \right], & m = 0\\ C_{m,0}^{\chi} \left(\frac{r}{R}\right)^{|m|} + D_{m,0}^{\chi} \left(\frac{r}{R}\right)^{-|m|}, & m \neq 0 \end{cases}$$
(23)

236 in which $C_{m,l}^{\chi}$ and $D_{m,l}^{\chi}$ are the coefficients to be solved; K_m is the modified Bessel function of 227 the second kind: β is the *l* th signaryalus which is given by

237 the second kind;
$$\beta_l$$
 is the *l*-th eigenvalue which is given by

238
$$\beta_l = \frac{l\pi}{h-d}, l=0, 1, 2, 3, \dots$$
(24)

239 III, outside region

240
$$\boldsymbol{\varPhi}_{\boldsymbol{\chi}}^{\text{out}}\left(r,\theta,z\right) = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} E_{m,l}^{\boldsymbol{\chi}} \frac{\tilde{K}_{m}\left(k_{l}r\right)}{\tilde{K}_{m}\left(k_{l}R\right)} \cos\left(m\theta\right) Z_{l}\left(z\right) + \boldsymbol{\varPhi}_{\mathbf{p},\boldsymbol{\chi}}^{\text{out}}, \qquad (25)$$

in which $E_{m,l}^{\chi}$ is the unknown coefficients to be determined; and $\Phi_{p,\chi}^{out}$ is a particular solution,

242
$$\boldsymbol{\Phi}_{\mathrm{p},\chi}^{\mathrm{out}} = \begin{cases} \boldsymbol{\Phi}_{\mathrm{I}}, & \chi = \mathrm{S} \\ 0, & \chi = \mathrm{R} \end{cases}$$
(26)

243 where, following Zheng and Zhang [37],

244
$$\Phi_{\rm I}(r,\theta,z) = -\frac{2{\rm i}gA}{\omega} \frac{Z_0(z)}{Z_0(0)} \sum_{m=0}^{\infty} \varepsilon_m \left(-{\rm i}\right)^m J_m(k_0 r) \cos m\beta \cos m\theta , \qquad (27)$$

245 in which $\varepsilon_m = 1$ for m = 0, whereas $\varepsilon_m = 2$ for $m \neq 0$.

246
$$\widetilde{K}_{m}(k_{l}r) = \begin{cases} H_{m}(k_{l}r), & l=0\\ K_{m}(k_{l}r), & l\neq 0 \end{cases}$$
(28)

247 where H_m denotes the Hankel function of the first kind.

248 3.2 Method of computation for unknown coefficients

249 The scattering and radiated spatial potentials as expressed in Sections 3.1 satisfy all the

boundary conditions shown in Eqs. (7) ~ (12) already. Additionally, the boundary conditions at r=R and $r=R_i$, i.e., Eqs. (13)~(15), together with the pressure and velocity continuity conditions on the interfaces of two adjacent regions should be satisfied as well, which can be used to determine

- 253 the unknown coefficients in Φ_{χ} . These continuity conditions for Φ_{χ} are given as follows:
- 254 1) Continuity of normal velocity at the boundary $r=R_i$:

255
$$\frac{\partial \Phi_{\chi}^{\text{in}}}{\partial r} = \begin{cases} 0, \ -d < z < 0, r = R_{\text{i}}, 0 \le \theta \le \pi; \text{ and } -h < z < 0, r = R_{\text{i}}, \pi \le \theta \le 2\pi \\ \frac{\partial \Phi_{\chi}^{\text{ring}}}{\partial r}, \quad -h < z < -d, r = R_{\text{i}}, 0 \le \theta \le \pi \end{cases}$$
(29)

256 2) Continuity of normal velocity at the boundary r=R:

257
$$\frac{\partial \Phi_{\chi}^{\text{out}}}{\partial r} = \begin{cases} 0, & -d < z < 0, r = R, 0 \le \theta \le \pi \\ \frac{\partial \Phi_{\chi}^{\text{ring}}}{\partial r}, & -h < z < -d, r = R, 0 \le \theta \le \pi \end{cases}$$
(30)

258 3) Continuity of pressure at the boundary $r=R_i$:

259
$$\Phi_{\chi}^{\text{ring}} = \Phi_{\chi}^{\text{in}}, \quad -h < z < -d, r = R_{\text{i}}, 0 \le \theta \le \pi$$
(31)

260 4) Continuity of pressure at the boundary r=R:

$$\Phi_{\chi}^{\text{out}} = \Phi_{\chi}^{\text{ring}}, \quad -h < z < -d, r = R, 0 \le \theta \le \pi$$
(32)

After inserting the expressions of Φ_{χ} as given in Section 3.1 into these continuity conditions, i.e., Eqs.(29)~(32), and making use of orthogonality of trigonometric functions and eigen-functions, the unknown coefficients in Φ_{χ} can be determined. For convenience, details of the derivations are given in Appendix A.

266 4 Excitation volume flow and hydrodynamic coefficients

267 4.1 Excitation volume flow

261

268 Once the unknown coefficients are determined, the excitation volume flow Q_e as given in Eq. (3) 269 can be easily calculated by:

270
$$Q_{\rm e} = \frac{2\pi\omega^2 R_{\rm i}}{g} \left(-\frac{A_{0,0}^{\rm D}}{k_0^2} Z_0(z) + \sum_{l=1}^{\infty} \frac{A_{0,l}^{\rm D}}{k_l^2} Z_l(0) \right).$$
(33)

271 4.2 Hydrodynamic coefficients

272 In a similar way, the hydrodynamic coefficients as given in Eq. (4) can be rewritten in terms of 273 $A_{m,l}^{R}$ as:

274
$$-(c-ia) = Q_{R} = \frac{2\pi\omega^{2}R_{i}}{g} \left(-\frac{A_{0,0}^{R}}{k_{0}^{2}}Z_{0}(z) + \sum_{l=1}^{\infty}\frac{A_{0,l}^{R}}{k_{l}^{2}}Z_{l}(0)\right).$$
(34)

The method, as shown in Eq. (4) or Eq. (34), which is derived from the radiated volume flow
inside the column is a straightforward way for calculating the hydrodynamic coefficient, *c*.
Actually there are two indirect methods as well for evaluating *c*, the one expressed by far-field
coefficients as

279
$$c = 2\rho\omega h \sum_{m=0}^{\infty} \frac{1}{\varepsilon_m} \frac{\left|E_{m,0}^{R}\right|^2}{\left|H_m(k_0 R)\right|^2},$$
 (35)

which can be derived from Green's identity [36, 39]; and the other one derived from the excitation volume flow Q_e based on Haskind Relation [32, 39]:

282
$$c = \frac{\omega Z_0(0)^2}{8\pi\rho g^2 h} \int_0^{\pi} \left| Q_{\rm e}(\beta) \right|^2 \mathrm{d}\beta \,. \tag{36}$$

The comparison of the results of c by using these two indirect methods as given in Eqs. (35) and (36) with that of the direct method, i.e., Eq. (4) or Eq. (34), can be adopted as an approach to validate the theoretical model.

286 5 Model validation

Martins-rivas and Mei [32] solved the hydrodynamic problems from a thin-walled (i.e., $R_i=R$) OWC along a straight coast for R/h=0.5, d/h=0.2 subjected to regular waves propagating at different angles β with different values of kh. The present theoretical model without the thin-wall restriction is adopted to re-simulate the same case, in which the inner radius is chosen as $R_i/h=0.49$, i.e., $(R-R_i)/R=0.02$, to represent the thin chamber wall.

To make a comparison with the published results, the method as adopted in Martins-rivas and Mei [32] for nondimensionalizing Q_e and hydrodynamic coefficients, *c* and *a*, i.e.,

294
$$Q_{\rm e} = \omega Q_{\rm e} / (ARg), \ (\overline{c}, \overline{a}) = (c, a) \omega \rho / R$$
, is re-employed in the present section.

5.1 Wave scattering problem

The comparison between the present results of the free surface elevation pattern inside and outside the OWC for kh=1.64 and those of Martins-rivas and Mei [32] is given in Fig. 2.





299 Fig. 2. Free surface elevation inside and outside the OWC for R/h=0.5, d/h=0.2, kh=1.64, $t=\pi/2\omega$. (left) results of Martins-rivas and Mei [32] for thin-walled OWC, i.e., R_i=R; (right) present results 300 with $R_i/h=0.49$. The incidence angles $\beta=0, 0.25\pi, 0.5\pi$. 301

303 In addition, comparison of the excitation volume flow of the OWC as a function of incidence β for R/h=0.5, d/h=0.2, kh=3.170 and 1.802 by using the present model with that of Martins-rivas 304 305 and Mei [32] is illustrated in Fig. 3.



306

308

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Fig. 3. Excitation volume flow of the OWC as a function of incidence β for *R/h*=0.5, *d/h*=0.2. 307 symbols: results from Martins-rivas and Mei [32] for thin-walled OWC, i.e., $R_i=R$; lines: present results with $R_i/h=0.49$.

The excellent agreement of the present results with those of Martins-rivas and Mei [32], as 310 311 shown in Figs. 2 and 3, proves that the present theoretical model works pretty well in solving the 312 scattering problem.

5.2 Wave radiation problem 313

Figure 4 illustrates the frequency response of the hydrodynamic coefficients of the OWC with R/h=0.5, d/h=0.2. It can be learnt that the present results of *c* and *a* with $R_i/h=0.49$, i.e., $(R-R_i)/R=0.02$, are in rather good agreement with those under thin-wall restriction [32]. What is more, results of *c* for the case with $R_i/h=0.4$ by using direct method (denoted as DM) and by adopting the other two indirect methods (denoted as FFC and HR, respectively) agree with each other pretty well, meaning the correctness of the present model in solving wave radiation problem.



320

321Fig. 4. Frequency response of radiation damping and added mass of the OWC with R/h=0.5,322d/h=0.2: (a) radiation damping; (b) added mass. Circles: results from Martins-rivas and Mei [32]323for thin-walled OWC, i.e., $R_i=R$; lines and crosses: present results.

324

Note there is an obvious difference of the frequency response of radiation damping by using $R_i/h=0.4$ and $R_i/h=0.49$, reflecting the significant effect of the OWC chamber's thickness on the hydrodynamic characteristics of the OWC along a vertical wall. Influence of the thickness on the performance of the OWC deserves more attention and such effect will be discussed in the next section.

330 6 Results and discussion

Hereinafter, following Lovas, Mei [33], the dimensionless coefficients of Q_e and hydrodynamic coefficients, *c* and *a*, are redefined as follows:

333
$$\overline{Q}_{e} = \frac{\sqrt{g/h}}{Ahg} Q_{e}; \ \left(\overline{c}, \overline{a}, \overline{c}_{PTO}, \overline{a}_{PTO}\right) = \frac{\rho \sqrt{g/h}}{h} \left(c, a, c_{PTO}, a_{PTO}\right), \tag{37}$$

with which, Eq. (6) can be rewritten as

335
$$\eta = \frac{khg}{c_{\rm g}\sqrt{g/h}} \frac{\overline{c}_{\rm PTO} \left|\overline{Q}_{\rm e}\right|^2}{\left[\left(\overline{c} + \overline{c}_{\rm PTO}\right)^2 + \left(\overline{a} + \overline{a}_{\rm PTO}\right)^2\right]}.$$
 (38)

Following Martins-rivas and Mei [32] and Lovas, Mei [33], a_{PTO} is calculated based on $\rho/\rho_0=1000$, $\nu=340$ m/s, h=10 m, and $V_0=\pi R^2 h$. The corresponding optimal PTO damping for a fixed OWC chamber (fixed V_0) is obtained by requiring $\partial P/\partial c_{\text{PTO}}=0$,

339
$$c_{\rm PTO} = \sqrt{c^2 + (a + a_{\rm PTO})^2}$$
 (39)

6.1 Comparison with an isolated offshore OWC and effect of incident wave direction

- Figure 5 presents the frequency responses of wave excitation volume flux, hydrodynamic coefficients, turbine parameter and wave power capture factor when the coast/breakwater integrated OWC suffers from different incident directions. For reasons of symmetry, only the results for $\beta = \pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$ are presented. For comparison, the results for the same OWC in the open sea are also displayed [36]. Only one blue dash curve as plotted in each figure of Figs.
- 346 5b~5d is used to represent the results of \overline{c} , \overline{a} and \overline{c}_{PTO} for the coast/breakwater integrated
- 347 OWC, respectively, since they are all independent of β .
- For wave scattering problem of the isolated offshore OWC, in the computed range of *kh*, there
- is only one peak of $|\bar{Q}_e|$ -kh curve at kh \approx 2.44 (see Fig. 5a). Whereas for the coast/breakwater
- integrated OWC, apart from the main peak of $|\overline{Q}_e|$ -kh curve at kh \approx 1.73, a second sharp peak is
- also observed at a higher frequency, i.e., $kh\approx 4.82$. Such a feature can be identified from the view 351 of natural modes in a closed cylinder in the radiation problem. For the isolated offshore OWC, the 352 only peak of the \overline{c} -kh curve occurs at kh \approx 2.44 (Fig. 5b), which corresponds to a piston-like 353 354 motion, i.e., the so-called Helmholtz mode of oscillation. In a coast/breakwater integrated OWC, the Helmholtz mode cannot exist alone because of the asymmetry of the opening; another mode, 355 i.e., the sloshing mode, is excited [32], and dominates the water motion inside the OWC chamber 356 357 at $kh \approx 4.82$. As shown in Fig. 5c, the \overline{a} -kh performs like a N letter shaped and a two-N letter shaped curves for an isolated offshore OWC and the integrated case, respectively, and the sign of 358 \overline{a} changes rapidly at the kh where the peak of the \overline{c} -kh curve occurs. These values of kh can be 359 360 called the natural frequencies of the OWC in the absence of the PTO. The spiky behaviour of \bar{a} around these natural frequencies is connected to the peak of the \overline{c} -kh curve. Note that the 361 chamber coefficient ($-\overline{a}_{PTO}$) is also plotted in Fig. 5d as a gray solid curve, which intersects the 362 363 \overline{a} -kh curve at kh \approx (2.48, 4.18) and (1.86, 2.90, 4.90) for different cases. The kh-values where these intersections occur correspond the resonant frequencies of the OWC with the PTO system. 364 For wave conditions corresponding to these resonant frequencies, \bar{a} and $\bar{a}_{_{\rm PTO}}$ cancel each 365 other, and it can be readily known from Eq. (2) that the air pressure inside the OWC chamber is in 366 phase with the excitation volume flux. The frequency response of the optimal turbine parameter 367 \overline{c}_{PTO} is illustrated in Fig. 5d, in which $\overline{c}_{PTO} = \overline{c}$ is satisfied at the resonant frequencies. 368
- 369 Compared with the single offshore OWC, the variation of \overline{c}_{PTO} for the coast/breakwater 370 integrated OWC is less marked and less abrupt (except for $kh\approx4.9$), which means that it may be 371 easier to achieve in practice.
- Finally, η as a function of *kh* for different values of the incident wave angle β is given in Fig. 5e. It is apparent that η for the isolated offshore OWC reaches the theoretical maximum value (i.e., 1.0) at these two resonant frequencies. Peaks of the η -*kh* curve for the integrated case are also observed at the corresponding resonant frequencies. Thanks to the wave reflection from the coast/breakwater, the value of η for the coast/breakwater integrated OWC at 1.6<*kh*<3.1 can be around twice as large as the theoretical maximum for the offshore case. Note that there is a



378 frequency between the second and the third resonant frequencies where no power can be 379 extracted.



387 The effect of the incident wave direction β on $|Q_e|$ is not obvious (Fig. 5a). As β increases 388 from $\pi/6$ to $\pi/2$, this dependence is slightly visible at the natural frequencies. As a comparison, a significant influence of β on η can be observed for 1.6<*kh*<3.1 and 4.9<*kh*<5.0 (Fig. 5e), where the more perpendicular the incident wave direction relative to the coast/breakwater, the more wave power that can be captured. Hereinafter, the effects of the other parameters will all be examined with $\beta = \pi/2$.

393 6.2 Radius of the OWC chamber

The coast/breakwater integrated OWCs with $R/h=0.3\sim0.7$ are selected as five cases to investigate the effect of the radius of the chamber on the performance of the OWC (Fig. 6). As R/hincreases, the highest peak of the $|\overline{Q_e}|$ -kh curve (Fig. 6a) shifts toward a lower frequency and gains intensity. Similar changes affect \overline{c} , \overline{a} and $\overline{c_{PTO}}$ (Figs. 6b~6d). As R/h increases, more natural frequencies can be observed in the computed range of kh. For R/h=0.3, there is only one natural frequency in the range of kh plotted, whereas for R/h=0.4, 0.5 and 0.6, there are two. For R/h=0.7, three natural frequencies are readily observable.



403



Fig. 6. Comparison for different radii of the OWC chamber, R/h. (a) wave excitation volume flux $|\overline{Q}_{e}|$; (b) radiation damping \overline{c} ; (c) added mass \overline{a} and chamber coefficient - \overline{a}_{PTO} (thin solid lines, each of which corresponds to the line of \overline{a} plotted in the same color); (d) turbine parameter \overline{c}_{PTO} ; (e) wave power capture factor η . In every case, $(R-R_{i})/h=0.1$, d/h=0.2, $\beta=\pi/2$.

409 Given that the chamber volume V_0 ($V_0 = \pi R^2 h$) is dependent on R, there are also five ($-\overline{a}_{PTO}$)-kh curves plotted in Fig. 6c corresponding to different values of R/h. For R/h=0.3, there are only two 410 resonant frequencies in the computed range of kh. For the other cases, e.g., R/h = 0.6, there could 411 412 be four resonant frequencies in the same range of kh. As plotted in Fig. 6d, the larger the R/h, the higher and more abrupt the variation of \overline{c}_{PTO} . The plot of η (Fig. 6e) shows that when kh is 413 between the first two resonant frequencies, as R/h increases, the η -kh curve turns higher and flatter, 414 and shifts toward lower frequencies. Here, Δkh is adopted to denote the difference between the 415 416 first two resonant frequencies, and η_1 and η_2 are employed to represent the η -values corresponding to the first two resonant frequencies, respectively. Figure 7 presents η_1 , η_2 and Δkh as three 417 functions of R/h. It is clear that, as R/h increases from 0.3 to 0.7, both η_1 and η_2 increase in a linear 418 419 way approximately, whereas Δkh decreases dramatically from 1.19 to 0.56. Ideally the R/h ratio 420 should be selected to achieve the balance between the peak value of η and its bandwidth, so that

421 the OWC can capture the most power for a specified range of wave conditions.





Fig. 7. Variation of η_1 , η_2 and Δkh with R/h for $(R-R_i)/h=0.1$, d/h=0.2, $\beta=\pi/2$

424 6.3 Thickness of the OWC chamber

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Figure 8 presents the results of the OWC chamber with $(R-R_i)/h=0.05, 0.1, 0.15$ and 0.2, and the 425 426 other parameters fixed at R/h=0.2, $\beta = \pi/2$. For comparison, some results of the thin-wall 427 case, i.e., $(R-R_i)/h = 0$, which were previously displayed by Martins-rivas and Mei [47] and Lovas 428 et al. [48], are replotted in Fig. 8 as well. As $(R-R_i)/h$ increases, the inner radius of the chamber decreases, and the highest peak of $|\bar{Q}_e|$ -kh curve loses intensity and moves toward higher 429 frequency as expected (see Fig. 8a). Meanwhile, the main peak of \overline{c} shifts toward higher 430 frequency and turns higher and narrower. Similar changes are also found for \overline{a} and \overline{c}_{PTO} as 431 given in Figs. 8c and 8d. It should be noted from Fig. 8c that with the increase of $(R-R_i)/h$, the first 432 two intersection points of \bar{a} and $-\bar{a}_{PTO}$ get closer and closer to each other horizontally, hence 433 the frequency band of η -kh as plotted in Fig. 8e turns narrower and narrower, whereas the 434 frequency position of the middle of the band remains almost the same. In addition, the peaks of η 435 436 corresponding to the first two resonant frequencies are lower as $(R-R_i)/h$ increases. Therefore, generally, the thickness of the OWC chamber should be as small as possible, so that larger and

broader main peaks can be achieved and more wave power absorbed. Needless to say, the

minimum thickness will be dictated in practice by structural considerations.

437 438

439





444 Fig. 8. Comparison for different thicknesses of the OWC chamber wall, $(R-R_i)/h$. (a) wave 445 excitation volume flux $|\overline{Q}_e|$; (b) radiation damping \overline{c} ; (c) added mass \overline{a} and chamber

446 coefficient - \overline{a}_{PTO} (gray solid line); (d) turbine parameter \overline{c}_{PTO} ; (e) wave power capture factor η .

In every case, R/h=0.5, d/h=0.2, $\beta=\pi/2$.

447

448 6.4 Submergence of the OWC chamber

Figure 9 compares the results for the coast/breakwater integrated OWC with different 449 submergence of the chamber, i.e., d/h = 0.1, 0.15, 0.2, 0.25 and 0.3, with $R/h=0.5, (R-R_i)/h=0.1$, 450 $\beta = \pi/2$. The results of $|\overline{Q}_e|$ (Fig. 9a) show that with the increase of d/h, the highest peak of $|\overline{Q}_e|$ 451 452 turns higher and narrower, and shifts toward lower frequencies. Similar changes apply to \overline{c} and \overline{a} (Figs. 9b and 9c). This is reasonable, for the radiation loss becomes weaker. All the resonant 453 frequencies in the computed range of kh reduce as d/h increases. The larger the d/h ratio, the 454 higher and more abrupt the variation of the corresponding \overline{c}_{PTO} with kh (Fig. 9d), which may be 455 456 more difficult to achieve in practice. The plot of η in Fig. 9e shows that, due to the change in the 457 resonant frequencies, the peaks of η are shifted toward lower frequencies as well with the increase in d/h. Meanwhile, both η_1 and η_2 are found to decrease slightly, and the main bandwidth in terms 458 of Δkh also decreases. The η corresponding to the kh between the first two resonant frequencies 459 460 decreased more dramatically than η_1 and η_2 . It might be concluded that a better result can be obtained by using a smaller value of d/h. 461



465

466 Fig. 9. Comparison for different submergence of the OWC chamber, d/h. (a) wave excitation 467 volume flux $|\overline{Q}_{e}|$; (b) radiation damping \overline{c} ; (c) added mass \overline{a} and chamber coefficient - \overline{a}_{PTO}

468 (gray solid line); (d) turbine parameter \overline{c}_{PTO} ; (e) wave power capture factor η . In every case,

469 $R/h=0.5, (R-R_i)/h=0.1, \beta=\pi/2.$

470 Note that d/h cannot be too small in practice, otherwise the opening might not be continuously 471 submerged, especially when the OWC is subjected to incident waves with a large amplitude, to a 472 large tidal range, or both.

473 7 Conclusions

474 In this paper a theoretical model based on linear potential flow theory is proposed to study the 475 performance of an OWC along a vertical coast/breakwater without the thin-wall restriction of 476 previous works. The water domain is divided into three regions, i.e., the interior region enclosed 477 by the OWC chamber, the half-ring shaped region beneath the OWC chamber and the exterior 478 region in front of the coast/breakwater extending towards infinite distance horizontally. Subjected 479 to small amplitude incident regular waves, wave-structure interaction is decomposed into wave scattering and wave radiation problems. In order to determine the unknown coefficients of the 480 481 scattering and radiated potentials in these three regions, the eigen-function matching method is 482 employed. The wave power extraction of the OWC with linear PTO system is then evaluated in 483 the frequency domain.

The influence of the vertical coast/breakwater is briefly discussed by comparing the performance of the integrated OWC with that of a similar isolated OWC deployed in the open sea. Finally, the effects of the radius, thickness and submergence of the chamber on the performance of the OWC along a coast/breakwater are investigated by means of the theoretical model. The following conclusions may be drawn.

489 The value of η for the coast/breakwater integrated OWC at specified ranges of *kh* can be around 490 twice as large as the theoretical maximum of η for the offshore case due to the wave reflection 491 from the coast/breakwater. The more perpendicular the incident wave direction relative to the 492 coast/breakwater, the more wave power that can be captured by the OWC.

493 As the R/h ratio increases, more natural and resonant frequencies can be observed in the 494 computed range of kh. The main peaks of η shift toward lower frequencies and the peak values 495 increase nearly linearly with R/h, whereas the bandwidth reduces drastically.

The smaller the $(R-R_i)/h$ ratio, the larger and broader the main peaks of η , i.e., more wave power absorbed, and the frequency position of the middle of the band remains almost the same. Needless to say, an appropriate thickness, rather than zero thickness of the OWC chamber, will be dictated in practice by overall considerations, including not only wave power extraction but also structural survivability.

501 With the increase of d/h, the peaks of η are shifted toward lower frequencies. Meanwhile, both 502 η_1 and η_2 are found to decrease slightly, and the main bandwidth in terms of Δkh is reduced.

To capture wave power on a large scale, it is expected that multiple OWCs along a
coast/breakwater will be required. It is possible to extend the present theoretical model to multiple
OWCs, as will be reported elsewhere.

506

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- 511 Appendix A. Integral equations of the scattering and radiation problems
- 512 After inserting Eqs. (16) and (22) into Eq.(29), multiplying both sides by $Z_{\zeta}(z)e^{-i\tau\theta}$ and 513 integrating for $z \in [-h, 0]$ and $\theta \in [0, 2\pi]$, for any pair of integer (τ, ζ) , it can be obtained that

514
$$2\pi h A_{\tau,\zeta}^{\chi} = \sum_{l=0}^{\infty} \left[\frac{\pi}{\varepsilon_{|\tau|}} \left(X_{|\tau|,l}^{(1)} C_{|\tau|,l}^{\chi} + Y_{|\tau|,l}^{(1)} D_{|\tau|,l}^{\chi} \right) + i \sum_{\substack{m=0\\m\neq|\tau|}}^{\infty} \frac{\tau \left[\left(-1 \right)^{\tau-m} - 1 \right]}{\tau^2 - m^2} \left(X_{m,l}^{(1)} C_{m,l}^{\chi} + Y_{m,l}^{(1)} D_{m,l}^{\chi} \right) \right] L_{l,\zeta} ,$$
515 (A.1)

where

517
$$X_{\tau,\zeta}^{(1)} = \begin{cases} \frac{\tau}{R} \left(\frac{R_{i}}{R}\right)^{\tau-1}, & \zeta = 0\\ \frac{\beta_{\zeta} I_{\tau}'\left(\beta_{\zeta} R_{i}\right)}{I_{\tau}\left(\beta_{\zeta} R\right)}, & \zeta \neq 0 \end{cases}; \quad Y_{\tau,\zeta}^{(1)} = \begin{cases} \frac{1}{R_{i}}, & \zeta = 0, \tau = 0\\ -\frac{\tau}{R} \left(\frac{R}{R_{i}}\right)^{\tau+1}, & \zeta = 0, \tau \neq 0. \end{cases}$$
(A.2)

$$L_{l,\zeta} = \int_{-h}^{-d} \cos\left[\beta_{l}(z+h)\right] Z_{\zeta}(z) dz$$

$$= \begin{cases} \frac{\left(-1\right)^{l} \left(h-d\right)^{2} k_{0} Z_{0}(0) \sinh\left[k_{0}(h-d)\right]\right]}{\left[\left(h-d\right)^{2} k_{0}^{2}+l^{2} \pi^{2}\right] \cosh\left(k_{0}h\right)}, & \zeta = 0 \\ \frac{\left(-1\right)^{l} \left(h-d\right)^{2} k_{\zeta} Z_{\zeta}(0) \sin\left[k_{\zeta}(h-d)\right]}{\left[\left(h-d\right)^{2} k_{\zeta}^{2}-l^{2} \pi^{2}\right] \cos\left(k_{\zeta}h\right)}, & \zeta \neq 0 \end{cases}$$
(A.3)

After inserting Eqs. (22) and (25) into Eq.(30), multiplying both sides by $Z_{\zeta}(z)\cos(\tau\theta)$ and integrating for $z \in [-h,0]$ and $\theta \in [0,\pi]$, for any pair of integer (τ, ζ) , we have

521
$$\sum_{l=0}^{\infty} \left(X_{\tau,l}^{(2)} C_{\tau,l}^{\chi} + Y_{\tau,l}^{(2)} D_{\tau,l}^{\chi} \right) L_{l,\zeta} - h Z_{\tau,\zeta}^{(2)} E_{\tau,\zeta}^{\chi} = f_2^{\chi}, \qquad (A.4)$$

in which

523
$$f_{2}^{\chi} = \begin{cases} -\frac{2\delta_{\zeta,0}igAk_{0}h}{\omega Z_{0}(0)}\varepsilon_{\tau}\left(-i\right)^{\tau}J_{\tau}'\left(k_{0}R\right)\cos\left(\tau\beta\right), \ \chi = S\\ 0, \qquad \chi = R \end{cases}$$
(A.5)

$$X_{\tau,\zeta}^{(2)} = \begin{cases} \frac{\tau}{R}, & \zeta = 0\\ \frac{\beta_{\zeta}I_{\tau}'(\beta_{\zeta}R)}{I_{\tau}(\beta_{\zeta}R)}, & \zeta \neq 0 \end{cases}; \quad Y_{\tau,\zeta}^{(2)} = \begin{cases} \frac{1}{R}, & \zeta = 0, \tau = 0\\ -\frac{\tau}{R}, & \zeta = 0, \tau \neq 0, \quad (A.6)\\ \frac{\beta_{\zeta}K_{\tau}'(\beta_{\zeta}R)}{K_{\tau}(\beta_{\zeta}R)}, & \zeta \neq 0 \end{cases}$$

525
$$Z_{\tau,\zeta}^{(2)} = \begin{cases} \frac{k_0 H_{\tau}'(k_0 R)}{H_{\tau}(k_0 R)}, & \zeta = 0\\ \frac{k_{\zeta} K_{\tau}'(k_{\zeta} R)}{K_{\tau}(k_{\zeta} R)}, & \zeta = 1, 2, 3 \cdots \end{cases}$$
(A.7)

526 After inserting Eqs. (16) and (22) into Eq.(31), multiplying both sides by $\cos[\beta_{\zeta}(z+h)]\cos(\tau\theta)$ 527 and integrating for $z \in [-h, -d]$ and $\theta \in [0, \pi]$, for any pair of integer (τ, ζ) , it can be obtained that

$$\sum_{l=0}^{\infty} \left[\frac{\pi}{2} \left(\frac{\tilde{I}_{\tau}\left(k_{l}R_{i}\right)}{k_{l}\tilde{I}_{\tau}'\left(k_{l}R_{i}\right)} A_{\tau,l}^{\chi} + \frac{\tilde{I}_{-\tau}\left(k_{l}R_{i}\right)}{k_{l}\tilde{I}_{-\tau}'\left(k_{l}R_{i}\right)} A_{-\tau,l}^{\chi} \right) - i \sum_{\substack{m=-\infty\\m\neq\pm\tau}}^{\infty} \frac{m\left\lfloor \left(-1\right)^{m-\tau}-1\right\rfloor}{m^{2}-\tau^{2}} \frac{\tilde{I}_{m}\left(k_{l}R_{i}\right)}{k_{l}\tilde{I}_{m}'\left(k_{l}R_{i}\right)} A_{m,l}^{\chi} \right) L_{\zeta,l}$$

$$= \frac{\pi\left(h-d\right)}{\varepsilon_{\tau}\varepsilon_{\zeta}} \left(X_{\tau,\zeta}^{(3)}C_{\tau,\zeta}^{\chi} + Y_{\tau,\zeta}^{(3)}D_{\tau,\zeta}^{\chi} \right) + f_{3}^{\chi}$$
529 (A.8)

529

530 where

531
$$f_{3}^{\chi} = \begin{cases} 0, \qquad \chi = S \\ \frac{\delta_{\tau,0}\delta_{\zeta,0}i\pi(h-d)}{\rho\omega}, \chi = R \end{cases}$$
(A.9)

532

$$X_{m,l}^{(3)} = \begin{cases} \left(\frac{R_{i}}{R}\right)^{m}, & l = 0 \\ \frac{I_{m}\left(\beta_{l}R_{i}\right)}{I_{m}\left(\beta_{l}R\right)}, & l \neq 0 \end{cases}; \quad Y_{m,l}^{(3)} = \begin{cases} 1 + \ln\left(\frac{R_{i}}{R}\right), & l = 0, m = 0 \\ \left(\frac{R}{R_{i}}\right)^{m}, & l = 0, m \neq 0 \end{cases} .$$
(A.10)

$$\frac{K_{m}\left(\beta_{l}R_{i}\right)}{K_{m}\left(\beta_{l}R\right)}, & l \neq 0 \end{cases}$$

533 After inserting Eqs. (22) and (25) into Eq.(32), multiplying both sides by $\cos[\beta_{\zeta}(z+h)]\cos(\tau\theta)$ 534 and integrating for $z \in [-h, -d]$ and $\theta \in [0, \pi]$, for any pair of integer (τ, ζ) , we have

535
$$\frac{h-d}{\varepsilon_{\zeta}} \left(C_{\tau,\zeta}^{\chi} + D_{\tau,\zeta}^{\chi} \right) - \sum_{l=0}^{\infty} E_{\tau,l}^{\chi} L_{\zeta,l} = f_4^{\chi}, \qquad (A.11)$$

536 where

537
$$f_{4}^{\chi} = \begin{cases} -\frac{2\varepsilon_{\tau} igAL_{\zeta,0}}{\omega Z_{0}(0)} (-i)^{\tau} J_{\tau}(k_{0}R) \cos(\tau\beta), \ \chi = S\\ 0, \qquad \chi = R \end{cases}$$
(A.12)

538 Eqs.(A.1), (A.4), (A.8) and (A.11) form a linear algebraic system, which can be used to solve $A_{m,l}^{\chi}$, $C_{m,l}^{\chi}$, $D_{m,l}^{\chi}$ and $E_{m,l}^{\chi}$ numerically after truncation. In the present model, the infinite 539 terms of $e^{-im\theta}/\cos(m\theta)$, and $Z_l(z)/\cos[\beta_l(z+h)]$ are truncated at m=M and l=L, respectively. Accurate 540 results can be obtained by choosing M=12, L=20. 541 542

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