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Efficient Frontiers in Portfolio Optimisation with Minimum Proportion Constraints

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ABSTRACT

This work chronicles research into the solution of portfolio problems with metaheuristic solvers. In particular, a genetic algorithm for solving the cardinality constrained portfolio optimisation problem with minimum asset proportions is presented and tested on the datasets of [1]. These datasets form benchmark instances used to test portfolio optimisers and are based upon indices ranging from 31 to 225 assets. The results of the GA are indicatively compared to solutions of [2] for a variety of minimum proportions, suggesting that solutions exhibit certain clustering characteristics for higher proportions. Further work is also discussed. This research is based upon the first part of the ongoing PhD thesis of the first author.

CCS CONCEPTS

• **Theory of computation → Bio-inspired optimization**; *Non-convex optimization*; • **Mathematics of computing →** *Evolutionary algorithms*;

KEYWORDS

Genetic algorithm, efficient frontier, cardinality constraint, minimum proportion, copula.

1 INTRODUCTION

Portfolio optimisation aims to efficiently choose optimal proportions of portfolio assets subject to given constraints. The first approach, Modern Portfolio Theory (MPT), that solved modern "portfolio problems" was introduced by Markowitz [8]. This so-called mean-variance model of a financial portfolio supposes that the historical returns of the assets influence their future returns. In the real world, however, various constraints are commonly used in order to make the problem reflect reality more closely and also seem more intriguing. Introducing these constraints often renders the resulting portfolio problems NP-hard [7]. Computing solutions to these problems thus often requires approximation algorithms such as meta-heuristics [3] (or hybrids; e.g., the work of [6]). Meta-heuristics are capable of finding optimal (or indeed, near-optimal) solutions efficiently. The work of [2] introduced a modified integer programming method to solve a Limited Asset Markowitz model. Most notably, the authors give optimal solutions for each of the datasets considered here; the solutions are highly nonlinear.

2 THE CARDINALITY CONSTRAINED PROBLEM WITH MINIMUM PROPORTIONS

The cardinality constrained portfolio optimisation problem with minimum asset proportions is as follows [3]:

$$\text{Compute } \min \; r = \sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j$$

subject to

$$\overline{R} = (\mathbf{x}, \mu) = \sum_{i=1}^{n} \mu_i x_i$$

$$\sum_{i=1}^{n} x_i = 1$$

$$\sum_{i=1}^{n} z_i \le K \tag{1}$$

$$l_i z_i \le x_i \le u_i z_i \quad \text{for } i = 1, 2, \ldots, n \tag{2}$$

$$z_i \in \{0, 1\} \text{ for } i = 1, \ldots, n,$$

where x_i is the weight of asset i, μ_i is the expected return of asset number i, $\overline{R}$ is the expected return of the portfolio and r is the variance (risk). Note that σ_{ij} is the covariance between the returns for assets i and j. Inequality (1) limits the number of assets to a maximum of K and (2) imposes the lower l_i and upper u_i bounds on assets. The lower bound thus gives the minimum proportion constraint (in practice, $u_i = 1$) and is a common practical restriction.

The optimal solution to the above continuous problem is known as the cardinality-constrained efficient frontier (CCEF).

3 COMPUTATIONAL RESULTS

We present preliminary results of a parallel GA to find the CCEFs of the datasets in [1]. The GA is coded in R, using the 'GA', 'foreach' and 'Rmpi' packages, and attempts to optimise each CCEF point (i.e., (risk, return) pair) separately. The advantage of this approach is that it is easily parallelisable and may run on as many cores as desired. All results presented in this work were run on 64 cores of an HPC cluster at the home institution. Note the CCEFs of [2], provided for an illustrative comparison, has a 1% minimum proportion.

The GA operators used were linear ranking selection, random position mutation and Laplace crossover, with crossover probability 0.5, mutation probability 0.4 and maximum number of iterations $2000n$ (n being the number of assets). The population size was 50, and a given number of points were optimised to approximate the true CCEF that gives the best possible trade off between risk and return. The following figures give typical output - details of full sets of runs are omitted due to space considerations.

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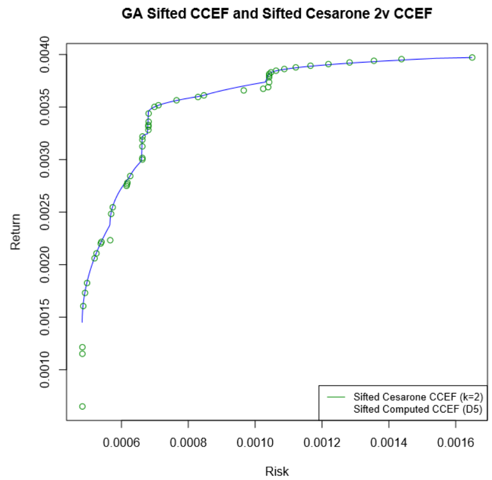

Figure 1: A comparison between the sieved CCEF of [2] and the GA solution for dataset (D5) [n = 225], K = 2 and l_i = 0.01.

Figure 1 is an example of GA output on instance (D5), $K = 2$ and minimum proportion 0.01. The GA originally used 250 points - these have been processed by a sifting algorithm [4], resulting in the points (green dots) with lowest risk for a given level of return being kept and all other points discarded. In this figure, the blue curve is the *sifted* CCEF of [2] for (D5) and $K = 2$. The minimum proportions for both EFs on this figure are identical, and hence the results are comparable, with the full spread of the blue curve covered by the green dots. The green dots below the return range of the comparison CCEF are due to the (blue) comparison CCEF kinking to the right and so being removed by sifting.

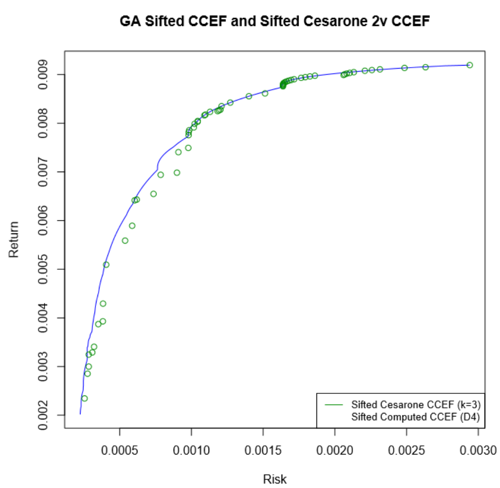

Figure 2: A comparison between the sieved CCEF of [2] and the GA solution for dataset (D4) [n = 98], K = 3 and l_i = 0.3.

Figure 2 gives an example of sieved GA output with instance (D4), $K = 3$, and minimum proportion 0.3 (originally 500 points before sifting). It can be seen that there is definite clustering behaviour of the sifted GA-computed solution (green dots) at three locations in the CCEF of [2], suggesting that the spread of green dots along the CCEF is restricted by virtue of the relatively high minimum proportion constraint. Unsurprisingly, the green dots are further away from the CCEF as given risk/return values are hard to achieve using a minimum proportion of 0.3.

4 CONCLUSIONS AND FURTHER RESEARCH

This work briefly presented a GA for solving cardinality constrained portfolio optimisation problems with minimum proportion constraints, and results were given for two datasets. Further research indicates that increasing the minimum proportion constraint causes a considerable increase in computational, thus increasing problem difficulty. To the best knowledge of the authors, there have been no results previously presented using high minimum proportions and a paucity of results for metaheuristics approximating solutions for small K. These are the two main contributions of this work.

There have been solvers for varying types of portfolio optimisation problems of assets with varying constraints (e.g., [3, 6]). However, MPT approaches assume the underlying joint distribution of assets is normal, or ignore the fact that investors may prefer other asset distribution characteristics than mean and standard deviation. In fact, asset distributions of real-world financial data are rarely normal and exhibit so-called “heavy tails” behaviour. Copulae model each asset marginal and, in addition, interdependencies between assets (e.g., [5]). Current extensions of this work include development of novel risk measures based upon copulas and metaheuristic computation using these measures.

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