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AN INVESTIGATION INTO THE IMPACT OF
VISUAL-SPATIAL DIFFICULTIES ON LEARNING GEOMETRY

by

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Abstract

Sridhar Nagubandi

AN INVESTIGATION INTO THE IMPACT OF VISUAL-SPATIAL DIFFICULTIES ON LEARNING GEOMETRY

This thesis reports the findings of my study of students with visual-spatial deficits in my specialist school in the city of New York. It is comprised of a pilot study, mathematical interviews, and interventions with students and teachers. This study is qualitative and primarily uses case studies to explain the interventions with both the students and the teachers. The study is made up of interventions with two students, and interventions with several teachers who work in my specialist K-12 school which includes both primary and secondary school teachers.

Since very little research has been conducted in this field to this point, the findings presented in this thesis aim to give teachers, especially secondary school mathematics teachers, an understanding of the challenges that secondary school students with visual-spatial deficits face when they are learning mathematics. In addition, this research also discusses intervention sessions that I conducted with teachers that gives some insights into educating secondary school mathematics teachers about mathematics learning disabilities and their impact on the students that they teach.

The main findings of this research are that there are effective interventions for both students and teachers that help students with visual-spatial deficits learn mathematics. A successful theme that has emerged is centring which helps students to start questions that they find challenging, and also focus their attention on obtaining a solution. It can sometimes lead to a greater understanding of mathematics as well.
AUTHOR’S DECLARATION

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award without prior agreement of the Graduate Sub-Committee.

Work submitted for this research degree at the Plymouth University has not formed part of any other degree either at Plymouth University or at another establishment.

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Conferences Attended:

4th National Dyscalculia & MLD Conference
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Chapter 1: An Introduction to and the Motivation for My Research

1.1 An Introduction to My Research

In my work as a secondary mathematics teacher in a New York City K-12 school that is exclusively for students with learning disabilities, I have taught students with a variety of learning disabilities. My students mainly have language-based, non-verbal and mathematical disabilities, in addition to some who have visual-spatial deficits. Whilst there has been extensive training provided for teachers who work with students who have language-based disabilities such as dyslexia, and even some training for teaching students with non-verbal learning disabilities, there was virtually no professional development that addressed teaching mathematics to students with mathematical disabilities and visual-spatial deficits. The minimal training that was provided was mainly geared to help primary school teachers and only addressed mathematical learning disabilities superficially. Furthermore, there was never any substantial training or discussion on how to teach students with visual-spatial deficits.

The visual-spatial deficits that some of the students I have taught, and currently teach are quite substantial and have a significant impact on how they learn mathematics. In fact, for some students, the visual-spatial deficits may be much more impactful than other learning disabilities. An initial examination may reveal that these issues would primarily be encountered in geometry courses, but the integrated nature of algebra, trigonometry and other secondary mathematics courses ensures that students will have to rely on their visual-spatial skills in all of their secondary mathematics courses. In the United States and specifically in New York State, whilst the main mathematics courses in high school – Algebra I,
Geometry, and Algebra II and Trigonometry – tend to be standalone, they are very much integrated, meaning that each of the courses will have topics from all of the main high school courses. For example, a typical Algebra I course will have several chapters that cover analytic geometry, a Geometry course will have a substantial amount of algebra throughout the entire course, and an Algebra II & Trigonometry course will have several chapters that require a solid knowledge of analytic and plane geometry. This makes it essential for students with visual-spatial deficits to learn strategies to ensure they can learn mathematics in all of their courses more effectively.

In addition to the mathematics courses, a variety of Regents examination courses in other subjects ranging from Global History and Geography, and U.S. History and Government to Living Environment (biology) and Physical Setting/Earth Science (geology) have a substantial amount of work that require students to rely extensively on their visual-spatial skills. All of the aforementioned courses require students to interpret maps, charts, tables, and a variety of graphs including pie charts, line graphs, and bar graphs. In addition, the history courses require students to interpret and create their own timelines. This would also be true for students who take non-Regents courses in these subjects.

There is a diversity in the nature and intensity of these visual-spatial deficits. Some students have difficulty recognising the important features in a diagram that represents one and two-dimensional figures, whilst others may only have difficulty interpreting three-dimensional figures. Some students have difficulty interpreting figures with any number of dimensions. The intensity of
the deficits varies significantly too. Several students that I have taught were able to recognise some of the important features of a two-dimensional figure, but could not recognise their most important features. Whilst some can recognise that there are two angles on a page, they cannot determine which angle has a larger measure by observation even if the angles have substantially different measures. It appeared that students who did not have visual-spatial difficulties were able to perform these tasks with no relative difficulty.

1.2 Motivation for My Research

The motivation for my research was largely spurred by my work with one student. She could not distinguish between the sizes of the areas that The Russian Federation and Australia occupied on a map even when the pieces of the map that these countries occupied were cut out and the smaller piece (Australia) was placed on top of the larger piece (The Russian Federation), as shown in Figure 1. My work with this student especially sparked my interest in developing a system for helping her and students with similar difficulties that she faced to learn more effectively despite having significant deficits.
1.2.1 Dearth of Research about Mathematics Learning Disabilities (MLDs)

This thesis includes my work with students with a variety of MLDs. I was very surprised by the results of existing research in this field. Jordan (2007, p.63) stated that approximately 5-8% of school aged children in the United States and other countries have an MLD. Whilst this figure has been verified by other researchers such as Murphy (2007, p.458), Shalev and Gross-Tsur (2005, p.121), and Barbaresi (2005, p.481), state that this proportion can range from 5.9% to 13.8% for 19 year olds depending on the standards that are used to determine if a student has a MLD. Jordan’s estimate for the proportion of children who have an MLD is similar to or greater than the proportion of children who have dyslexia. Despite this, there has been remarkably little research dedicated to MLDs. However, there has been an abundance of research conducted on language-based learning disabilities such as dyslexia. According to Murphy (2007, p.458) a PsycInfo search of peer reviewed articles published in English from 1985 to 2006 resulted in only 231 papers that included one or more of the terms: mathematical
disability, dyscalculia, or a combination of these terms in any location of the article. In contrast, during the same time period there were more than 1000 articles that included just the term dyslexia in the title alone. Of the articles that discussed MLDs, very few had implications for secondary school students. Fewer still specifically addressed student difficulties in geometry. My own Primo search conducted in July 2016 for the terms learning, disability and geometry in the title yielded only one result. A different Primo search for the former two terms in the title and geometry in any location of the paper resulted in only fourteen different papers. Even when the research has addressed geometry, it has done so in a very cursory manner, and is usually geared to aid primary school teachers. Because of this dearth of research, secondary school mathematics teachers are at a tremendous disadvantage when compared to their colleagues who teach liberal arts and social sciences, subjects that benefit from the extensive research on reading and writing for students with learning disabilities.

As a teacher in my specialist school, I have encountered this problem first hand. Whilst there is a plethora of research and professional development courses for teaching reading comprehension and writing, in the sixteen years that I have taught at my school there has never been a professional development course that addressed MLDs in an in-depth manner. Furthermore, often well-intentioned head teachers and curriculum coordinators who tend to be well versed in language-based learning disabilities tend to know little about MLDs, and try to apply the strategies that they know for teaching reading and writing, to teaching mathematics. Their efforts were usually not only futile, but were confusing for teachers, especially those new to teaching students with learning disabilities that
impact on mathematics. It was also problematic for students since these strategies, whilst successful in teaching language were, with few exceptions, often ineffective or sometimes even harmful and hindered a student's ability to learn mathematics.

1.2.2 Work with Teachers and Other Education Professionals

As part of my research, I will give presentations and facilitate workshops that will help both mathematics and non-mathematics teachers who teach different age groups in my school and other schools to help them not only learn about different MLDs, but perhaps more importantly show them how they will appear in their classes with specific examples. I will also do this with parents who have children with learning disabilities, as well as mental health professionals who work with these children.

The goal of this research is to create a powerful resource for teachers, head teachers, curriculum coordinators, evaluators and parents of students with MLDs. In particular, I will conduct research to explore how teachers can identify and classify visual-spatial deficits through formal and informal assessments, and determine what impact these deficits have on how their students learn geometry specifically, and mathematics in general.
Chapter 2: The Literature Review

2.1 An Overview of Learning Disabilities

This chapter of my thesis will summarise both mathematical and non-mathematical learning disabilities. This will include not just a description of MLDs, but also an analysis of the most recent and relevant research in this field. There will also be a discussion about the difficulties of identifying MLDs and the reasons that will help readers to understand why they do not have the same status as language-based learning disabilities.

2.2 Overview of Learning Disabilities (LDs)

There are a variety of learning disabilities that have been classified by different criteria. Researchers have classified learning disabilities into several categories. I will classify learning disabilities into three categories: language-based learning disabilities, non-verbal learning disabilities and other learning disabilities that do not fit into either of these categories. Through my research I have become aware that there is quite a bit more research on language-based learning disabilities than any other type of learning disability. In my discussions with teachers in my specialist school, even the teachers who do not teach English or history have familiarity with language-based LDs much more than they have knowledge of MLDs and non-verbal learning disabilities. Research on non-verbal learning disabilities is relatively new. Whilst more research is being conducted in this area, there is still far less research literature available than for language-based disabilities.
2.2.1 Language-Based Learning Disabilities

The American Speech-Language-Hearing Association (ASHA) described language-based learning disabilities as “…problems with age-appropriate reading, spelling, and/or writing” for children with average to superior intelligence (ASHA, 2012). The most commonly diagnosed language-based learning disabilities include: reading disorder (according to the terminology used in DSM-IV), dysgraphia and information-processing disorders. The International Dyslexia Association (IDA) reported that approximately 15-20% of children have a language-based learning disability; of this group “70-80% have deficits in reading” (Interdys.org, n.d.).

2.2.2 Dyslexia

Dyslexia is perhaps the most commonly diagnosed learning disability, and as a result the most easily recognised disability by evaluators, educators and school administrators such as school principals. Lawrence (2009, p.9) wrote that dyslexia is a term that is used for children who have literacy difficulties. This term was not commonly used until the late twentieth century. However, the origins of the diagnosis of the disability and the term dyslexia itself go back to the nineteenth century. Sir Francis Galton was the first educator to describe literacy difficulties in 1869, and introduced research in this field. In 1877, the German neurologist Adolph Kussmaul started to study adult patients who had difficulty with reading. He noticed that some people who had neurological impairments had difficulty reading properly and frequently used words in the incorrect order. He termed these difficulties as “word blindness” (Shaywitz, 2005, p.14; Lawrence, 2009, p.10). During Kussmaul’s time these difficulties
were considered to be medical in nature, and as such the terms to describe these deficiencies originally came from the medical field. Children with literacy difficulties were considered to have medical issues, poor motivation or underlying mental limitations. In 1887, the ophthalmologist Rudolf Berlin was the first to use the term *dyslexia*. In 1891, Dejerne described a case of a person who had reading difficulties because of a brain injury caused by a strike to his head with a crowbar in *The Lancet*. Following this report, other reports were published that reinforced the medical view of dyslexia and further supported Kussmaul’s research (Lawrence, 2009, p.9-11).

The medical view of this disability persisted into the first half of the 1900s with the contributions of the Scottish eye surgeon James Hinshelwood and more importantly the American neurologist Orton who “*was probably the first to recognise that children with reading difficulties often reversed letters. He called this phenomenon strephosymbolia. He also introduced the term developmental alexia to describe these children with reading difficulties. There were now three different terms in existence, all used to describe this learning difficulty. The medical view of these literacy difficulties does not inform teachers as to how to educate these students properly*” (Lawrence, 2009, p.12).

In the latter half of the twentieth century a shift started to occur from a medical view of reading difficulties to more of an educational one. Lawrence (2009, p.12-13) wrote that even when the cause was thought to be medical, “*it was agreed that the primary management of the problem was best conducted within an educational environment.*” In 1978, *The Warnock Report* (Educationengland.org.uk, 1978) which considered learning disabilities was
released. It helped to change the role of the medical officials in schools in the United Kingdom by placing more importance on the role of teachers and educational specialists and diminishing the role of medical professionals (Lawrence, 2009, p.13; Educationengland.org.uk, 1978). Since the 1970s there have been other major research programs in dyslexia. Of particular note, as suggested by Shaywitz (2005, p.25-26), was a research initiative started as a consequence of *A Report to the U.S. Congress* submitted by the Interagency Committee on Learning Disabilities (1987) submitted to the United States Congress. The report began an “*intensive, all-out drive to understand learning disabilities.*” As a result, The National Institutes of Health (NIH) established the Centers for the Study of Learning and Attention. A national effort helped to develop multiple studies including the *Connecticut Longitudinal Study* (Shaywitz, 2005, p.25-26) that addressed the number of students who have dyslexia, and what happens to children with dyslexia over time.

Dyslexia is a very important factor to consider when educating students with MLDs. Gillum wrote that “*there is a higher than expected co-morbidity of diagnosis of dyscalculia with dyslexia and ADHD*” (2012, p.4). This was confirmed by Söbel (Butterworth, 2005, p.17) who reported there is a strong connection between the two disabilities. Estimates of the comorbidity (Butterworth, 2005, p.17) of the disabilities range from 40% to 67% as reported by Lewis, Hitch, and Walker (1994, p.284) who stated this as the proportion of students with both dyscalculia and reading difficulties. The big differences in the estimates stem from the variety of criteria that assessors use to determine if a student has a MLD.
2.2.3 Dysgraphia

Put simply, dysgraphia is a disability that affects a student’s ability to write. The National Institute of Neurological Disorders and Stroke (NINDS) reported “Dysgraphia is a neurological disorder characterized by writing disabilities. Specifically, the disorder causes a person's writing to be distorted or incorrect” (NINDS, 2011). Dysgraphia affects a student’s ability to learn mathematics in general, and arithmetic in particular because his or her handwriting may make it so difficult to line up numbers properly that they cannot carry out calculations accurately. Whilst there is not much research about the comorbidity of dysgraphia and MLD, Desoete (2008, p.16) wrote the “co-morbidity of mathematical and writing disabilities is about 50%.” This figure was also confirmed by the work of Shalev and Gross-Tsur, (2005, p.121-125).

2.2.4 Other Language-Based Learning Disabilities

Other language-based disabilities are not diagnosed as frequently. This could be because evaluators are not familiar with these disabilities or because they naturally occur less frequently. Sometimes the disability does not occur frequently in children because it is caused by factors such as traumatic brain injury or stroke, the latter of which does not occur frequently in children.

2.2.4.1 Aphasia

Aphasia is a disorder that affects speaking, listening, reading and writing (ASHA(2), n.d.). Not surprisingly it can have a significant impact on a student’s ability to learn mathematics. Aphasia is caused by an injury to one or more of the language areas of the brain. Sometimes, this disability is caused by traumatic
brain injury, dementia, illness, progressive neurological disorders or a stroke, the latter of which occurs rarely in children (ASHA(3), n.d.). McCormick (2017, p.1345) wrote that aphasia “can occur along with significant impairments in memory, attention, executive functioning, which further complicates the decision-making processes.” The more difficult decision making processes may have an impact on learning mathematics, especially when students have to select a method to solve a problem. However, I am not aware of any research that shows its impact on learning mathematics.

According to Damico (2012, p.318), “The exact incidence and prevalence of aphasia is unknown, in part because of the variety of conditions that have been or can be labelled aphasia.” Damico (2012, p.318) further stated that the number of people who have aphasia will be based on how it is defined. Since not many children are affected by aphasia, it is not frequently diagnosed in the school context.

2.2.4.2 Childhood Apraxia of Speech (CAS)

ASHA defines CAS as:

“a neurological childhood (paediatric) speech sound disorder in which the precision and consistency of movements underlying speech are impaired in the absence of neuromuscular deficits (e.g., abnormal reflexes, abnormal tone). CAS may occur as a result of known neurological impairment, in association with complex neurobehavioral disorders of known or unknown origin, or as an idiopathic neurogenic speech sound disorder. The core impairment in planning and/or programming spatiotemporal parameters of movement sequences results in errors in speech sound production and prosody” (ASHA(5), n.d.).

Shahin (2015, p.49) who described CAS as a neuromotor disorder wrote that the treatment for a child who is diagnosed with CAS “involves extended one-
on-one therapy with a speech language pathologist.” The origin of apraxia is not known. Damico (2012, p.393) wrote that the etiology of apraxia (not just CAS) is unknown. It appears that not much is known about how apraxia affects the extent to which a child learns mathematics.

2.3 Non-Verbal Learning Disabilities (NLDs or NVLDs)

There are a variety of NLDs that have a significant impact on a student’s ability to learn mathematics. The research in this field started with the work of Doris J. Johnson and Helmer R. Myklebust in 1967. Wasserstein (2008, p.484) reported that they discovered clinical evidence to support a new subtype of a “psychoneurological learning disability” in which students demonstrated problems with social perception and adjustment. Byron Rourke, M.D., a noted neuropsychologist and another trailblazer in this field wrote about what he called the NLD Syndrome (1995, p.1). First, Rourke (1995, p.13) described students with NLDs as having poor bilateral tactile perception. This pertains to students having a poor sense of touch on both the left and right sides of their bodies. However, Rourke stated that the poor tactile perception is more prominent on the left side than on the right side of the body. Whilst these students tend to have very poor tactile perception when they are younger, as they get older, this deficiency becomes less prominent.

NLD students also have difficulty learning new material and tend to deal with learning new material in a very inappropriate manner. As these students get older, they often learn to adjust and make proper accommodations to enable more
effective learning. In addition, they have a tendency to over-assimilate novel information so they can attempt to learn this new information.

Of particular importance for my research are the visual perception abilities of NLD students. Often, these students tend to have poor visual perception skills. Unlike tactile perception skills which improve over time, visual perception deficits usually increase with age. These students have a lot of difficulty recognising the relationships between the entities that they see, and sometimes even the details of what they see. These deficits are very apparent in many of the NLD students that I have taught.

Similarly, complex psychomotor skills are deficient in NLD students. As with visual perception, these severity of the deficits increases over time. However, Rourke (1995, p.14) describes one exception to this, which is handwriting. NLD students often practise handwriting so much, that they may even grow to develop neat handwriting. Perhaps, some of their other psychomotor skills could also be improved with extensive practice over an extended period of time.

Attention for NLD students is bifurcated. Attention is quite good for simple, repetitive, verbal material – especially when it is delivered through an auditory mode. However, attention to tactile or visual input is quite poor. These deficiencies increase over time unless information is overlearned and presented in a repeating programmatic fashion. It does not come as a surprise that attention to non-verbal information is low when delivered in visual or haptic modes (i.e. through touch or feeling).
Dougherty and Johnston (1996, p.289) defined overlearning as “*training that leads to improved retention.*” These researchers argued that overlearning is procedurally achieved by practise beyond “*successful performance, determined by some predetermined criterion.*” However, the authors also cited Ivarie (1986, p.25-30) who proposed an alternative definition to overlearning. Ivarie stated that overlearning is learning a skill to the point that it is learned beyond immediate recall. Dougherty and Johnston (1996, p.289) stated that the literature defines overlearning as a “*procedure for learning and also as an intensity or degree of learning.*”

Since NLD students learn most effectively when information is presented in verbal and auditory modes, exploratory behaviour is limited. They tend to be very sedentary, and do not take part in much physical exploration. NLD students tend not to explore anything in a visual or tactile manner, even when the object is nearby and within easy reach. This tendency not to explore physically increases as the student becomes older. There appears to be no evidence or research to suggest that this tendency has an impact on a student’s ability to explore mentally since this type of exploration does not necessarily rely on his or her tactile abilities.

NLD students have marked memory deficiencies. Not surprisingly they have poor tactile and visual memories. This type of memory deficiency tends to become worse as the students get older. However, tactile and visual information can be remembered if the information is overlearned.

Memory for nonverbal material tends to be poor in NLD students. When nonverbal information is presented in auditory, visual or tactile modes these
students have difficulty remembering the material. Unless the information is readily coded in a verbal fashion, it becomes very difficult for these students to retain the information. This is especially true for complex, meaningful, or novel information – even if the novel information is nonverbal. These memory impairments generally increase with age.

Verbal deficits are also significant in children who are diagnosed with an NLD. They tend to have a mildly deficient oral-motor praxis. Praxis is defined as an ability to successfully interact with the physical environment by planning, organising, visualising and carrying out a sequence of unfamiliar actions. In addition, these students do have limited speech prosody which means they do not pick up on the rhythm, stress or intonation of speech. As a consequence, NLD students frequently do not pick up on sarcasm, irony, emphasis, contrast, focus, or any other element that is not coded in a familiar fashion by grammar or choice of vocabulary. With the exception of oral-motor praxis, all of these deficiencies get worse with age.

In terms of specific academic deficits, students with NLD tend to have poor graphomotor skills in their early school years; however, with extensive practice, these students can become quite skilled at handwriting.

Reading comprehension is limited for these students. Even though they tend to have very good single word decoding skills, comprehension becomes more challenging with novel information in advancing years (Rourke, 1995, p.3-7).

In mathematics, the deficits are marked and generally advance with years. These students have relatively significant difficulties with mechanical arithmetic
when compared with word recognition and spelling. As they progress in their education, the gap between reading and spelling, and mechanical arithmetic performance widens. Rourke (1995, p.3-4) wrote that mechanical arithmetic performance rarely exceeds the fifth or sixth (equivalent to Years 5 and 6, respectively) grade level. Not surprisingly, mathematical reasoning remains low for these students. The deficits that NLD students have in visual-spatial perception, psychomotor skills, concept formation, appreciation of novel data, strategy generation and hypothesis testing are the basic neuropsychological deficits that influence how these students learn mathematics. In addition, a student’s limitation in judgment and reasoning leads to a failure in recognising the appropriateness of answers to arithmetic problems which also will have an impact on problem solving in higher level mathematics. (Rourke, 1995, p.7; p.15).

Science also presents a challenge since it contains a lot of complex and novel information, and sometimes includes mathematics that requires higher-order reasoning. It combines two areas of potentially significant deficits.

NLD students also have considerable socioemotional and adaptational deficits. Since they have difficulty with countenancing, organising, analysing and synthesising novel and complex situations, adapting to new situations is a great challenge for these students. Since these students rely almost exclusively on prosaic and rote language, they are sometimes unprepared to react to an unfamiliar situation. This is complicated by the fact that NLD students have significant deficits in social perception, judgment, and interaction skills. Not
surprisingly, they tend to withdraw socially with advancing age (Rourke, 1995, p.6-7).

2.4 Attention Deficit Hyperactivity Disorder (ADHD)

*The Diagnostic & Statistical Manual of Mental Disorders – Fourth Edition* (DSM-IV-TR) (1994) published by the American Psychological Association (APA) defined three subtypes of ADHD: *predominantly inattentive type*, *predominantly hyperactive-impulsive type*, and *combined type*. The diagnostic criteria listed in DSM-IV (LDAWE, n.d.) for ADHD are shown in Appendix A of this thesis. The definitions for ADHD between DSM-IV-TR and DSM-V are similar, but are a bit more general in DSM-V. This flexibility seemed to be part of the philosophy that has guided the authors of the DSM-V.

ADHD can have a significant impact on a student’s ability to learn mathematics. Zentall and Ferkis (1993, p.6) reported that students with learning disabilities and ADHD have lower achievement in mathematics than others their age. Furthermore “*slow computation affects problem solving by increasing attention load*” (1993, p.6). More about ADHD and its effect on learning mathematics will be discussed in further detail later in this thesis.

2.5 Introduction to Mathematics Learning Disabilities (MLDs)

Mazzocco (2005, p.143) stated that there is an “*absence of a consensus definition of MLD.*” This term includes disabilities such as developmental dyscalculia (DD) (Murphy, 2007, p.457), Gerstmann’s Syndrome (McLean, 2014, p.1-2, p.10), acalculia and other disabilities that are not necessarily mathematical in nature, but affect a student’s ability to learn mathematics. These
terms are described in greater detail in section 2.7 of this thesis. Students with visual-spatial deficits, (Feifer, 2005, p.44; Marjoram and Nelson, 1985) for example, may have greater difficulty learning mathematics — especially geometry. Students with attentional deficits may have greater difficulty in completing multi-step arithmetic problems that require them to have an increased attention to detail when they perform their computations and show their written work. The research has not yet pointed to one specific definition for this term, and its definition will likely be debated for many years to come.

Jordan (2007, p.63) stated that approximately 5-8% of school aged children in the United States and other countries have a mathematics learning disability. This figure has been confirmed by other researchers such as Murphy (2007, p.458), and Shalev and Gross-Tsur (2005, p.121). However Barbaresi (2005, p.481), stated that this proportion can range from 5.9% to 13.8% for 19 year olds depending on the standards that are used to determine if a student has an MLD. Jordan (2007, p.63) wrote that though 5-7% of children have dyslexia, which is similar to or less than the percentage of students who have MLDs. Remarkably, there is very little research on MLDs whilst there is an abundance of research that has been conducted on language-based LDs such as dyslexia.

According to Murphy (2007, p.458) a PsycInfo search of peer reviewed articles that were published in English from 1985 to 2006 resulted in only 231 papers that included one or more of the terms: mathematical disability, dyscalculia, or a combination of these terms in any location of the article. In contrast, during the same time period there were more than 1000 articles that
included just the term *dyslexia* in the title alone. Of the articles on MLDs that I have read in the course of this research, very few had implications for secondary school students with these disabilities. Fewer still specifically addressed student difficulties in geometry. My own Primo search conducted in July 2016 for the terms *learning, disability* and *geometry* in the title yielded only one result. A different Primo search that I conducted for the former two terms in the title and *geometry* in any location of the paper resulted in only fourteen different papers. In the papers that I have read in the course of this research, when the literature has addressed geometry, it has done so in a very cursory manner, and is usually geared to aid primary school teachers. Because of this dearth of research, secondary school mathematics teachers are at a significant disadvantage when compared to their colleagues who teach liberal arts and social sciences, subjects that benefit from the extensive research on reading and writing for students with learning disabilities.

As a secondary school mathematics teacher in a K-12 school that is exclusively for students with a variety of mainly language-based learning disabilities, I have encountered this problem first hand. Whilst there is a plethora of research and professional development courses for teaching reading comprehension and writing, in the twelve years that I have taught at my school there has never been a professional development course that addressed MLDs in an in-depth manner. Furthermore, often well-intentioned school principals and curriculum coordinators who tend to be well versed in language-based learning disabilities know little about MLDs, and try to apply the strategies that they know for teaching reading and writing to teaching mathematics. Their efforts are
usually not only futile, but are confusing for teachers, especially those who are new to teaching students with learning disabilities that impact on mathematics. It is also problematic for students since these strategies whilst successful in teaching language are, with a few exceptions, often ineffective or sometimes even harmful and hinder a student's ability to learn mathematics.

The goal of this research is to create a powerful resource for teachers, principals, curriculum coordinators, evaluators and parents of students with MLDs. In particular, I will conduct research to explore how teachers can identify and classify visual-spatial deficits through formal and informal assessments, and determine what impact these deficits have on how their students learn geometry specifically, and mathematics in general. After the deficits have been clearly identified, I will develop intervention strategies that will benefit students with an MLD and also students who have visual-spatial difficulties that are not extensive enough to warrant a diagnosis of or do not fit any of the many criteria that are used to determine if a student has an MLD. In addition, I will also develop interventions for teachers which will help them to better teach students who have difficulties in learning mathematics.

2.6 The Diagnosis of MLDs

The diagnosis of MLDs in school children is a relatively new field that is becoming increasingly important. As more educators, principals, curriculum coordinators and parents recognise the importance of identifying and helping students with MLDs learn effectively, this issue of how to properly meet the unique needs of these students is being discussed more frequently. Whilst there is a wealth of resources available for teachers of dyslexic students and students
with other language–based learning disabilities, there is very little that teachers can refer to inform how to teach their students with MLDs. According to Jordan (2007, p.63) and Murphy (2007, p.458) approximately 5 to 8% of students have some type of MLD, including: dyscalculia, Gerstmann's Syndrome, and visual-spatial deficits. The resources that are currently available are mainly geared towards primary school students, and almost no research is available for students at the secondary or postsecondary levels. Birsh (2005, p.8) stated that even though a similar number of school children, 5% to 10%, have dyslexia, there is disproportionately more research on dyslexia.

Much of this scarcity can be traced to the lack of consistent standards for assessing MLDs. Murphy (2007, p.458) reported that there is no universally accepted definition for MLD as there is for dyslexia, or other more commonly diagnosed learning disabilities. Researchers can select from at least twenty-four different evaluation criteria to determine if a child has a MLD. Some researchers such as Mazzocco used the Test for Early Mathematics Ability, Second Edition (TEMA – 2) and others used a variety of assessment tools ranging from the more commonly used Wide Range Assessment Test – Revised (WRAT – R) and Broad Mathematics Composite portion of the Woodcock Johnson – Revised (WJ – R). In addition, the cut-off scores to determine if a student has an MLD range from scoring below the 10th percentile on the TEMA – 2, as defined by Mazzocco, to the scoring below the 46th percentile on the WJ – R as defined by Geary, Brown & Samaranayake (Murphy, 2007, p.460). Furthermore, the selection process for which students are recommended for assessment of a MLD varies dramatically. Passolunghi and Siegel (2001) used only teacher–nominated participants,
whereas Geary, Brown & Samaranayake (Murphy, 2007, p.460) only assessed students who were already receiving remedial education. These varying assessments, cut-off scores, and selection processes for students who actually are assessed, make it difficult to determine if a student has a MLD. Also, because of these varying standards, it is difficult to provide continuous intervention to students with MLDs (Murphy, 2007, p.461-462). Murphy (2007, p.461), cited the work of Silver (1999) who reported the persistence rate, the rate at which a student initially diagnosed with an MLD is still considered to have a mathematical disability after 19 months, was only 47% to 63% for children who are 9 to 13 years old. Because of these issues, it is very difficult for a student who has been diagnosed with an MLD to continue to receive the proper intervention services and additional assistance that is needed for optimal learning of any subject, but especially mathematics. Table 1 displays the numerous assessments and standards that are used to determine if a student has an MLD (2007, p.460).

<table>
<thead>
<tr>
<th>Study</th>
<th>Evaluation Criteria</th>
<th>N</th>
<th>MLD n</th>
<th>Age at Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alarcón, DeFries, Light, &amp; Pennington (1997)b,c</td>
<td>Composite of WRAT-R Arithmetic and PIAT Mathematics subtests 1.5 SD² mean for control sample Verbal or Performance IQ³ 90. Single assessment</td>
<td>647</td>
<td>95</td>
<td>8 – 20 years</td>
</tr>
<tr>
<td>Light &amp; DeFries (1995)b,c</td>
<td>Composite of WISC-R Arithmetic and PIAT Mathematics subtests 1.5 SD² mean for control sample Verbal or Performance IQ³ 90. Single assessment</td>
<td>487</td>
<td>260d</td>
<td>8 – 20 years</td>
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<tr>
<td>Study</td>
<td>Test Description</td>
<td>Percentile</td>
<td>Grade</td>
<td>Age Range</td>
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<td></td>
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<td>percentile</td>
<td></td>
<td>Grade 3 (Age 9)</td>
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<tr>
<td>Geary &amp; Brown (1991)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Receiving Chapter 1 remedial education services in math, which required use of a</td>
<td>41</td>
<td>15</td>
<td>Grades 3, 4</td>
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<tr>
<td></td>
<td>standard math achievement test</td>
<td></td>
<td></td>
<td>(Ages 10 – 11)</td>
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<td></td>
<td>National percentile rank &lt; 46</td>
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<tr>
<td>Geary, Bow-Thomas, &amp; Yao (1992)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>WJ-R Math Composite Index</td>
<td>37</td>
<td>13</td>
<td>Grade 1 (Age 7)</td>
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<tr>
<td></td>
<td>National percentile rank &lt; 46</td>
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<tr>
<td>Geary, Brown, &amp; Samaranayake (1991)</td>
<td>Receiving Chapter 1 remedial education, which required use of a standard</td>
<td>38</td>
<td>12</td>
<td>Grades 1,2</td>
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<td></td>
<td>mathematics achievement test</td>
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<td>(Ages 7 – 8)</td>
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<td>National percentile rank &lt; 46</td>
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<td>Two assessments</td>
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<td>Geary, Hoard, &amp; Hamson (1999)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>WIAT Math Reasoning</td>
<td>90</td>
<td>40</td>
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<td></td>
<td>&lt; 30th percentile</td>
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<td>IQ &gt; 80</td>
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<td>Jiménez Gonzalez &amp; Garcia Espínel (1999)</td>
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<td>148</td>
<td>104</td>
<td>7 – 9 years</td>
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<td>&lt; 25th percentile</td>
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<td>Hanich, Jordan, Kaplan, &amp; Dick (2001)</td>
<td>WJ-R Math Composite score</td>
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<td>105</td>
<td>Grade 2 (Age 8)</td>
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<td>≤ 35th percentile</td>
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<td>Jordan, Hanich, &amp; Kaplan (2003)</td>
<td>WJ-R Math Composite score</td>
<td>180</td>
<td>88</td>
<td>7 – 9 years</td>
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<tr>
<td></td>
<td>≤ 35th percentile</td>
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<td>Grade 2</td>
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<tr>
<td>Landerl, Bevan, &amp; Butterworth (2004)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Composite score 3 SD above the control mean on response time or number of errors</td>
<td>49</td>
<td>21</td>
<td>8 – 9 years</td>
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<td></td>
<td>on a timed arithmetic test</td>
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<td></td>
<td>All children monolingual</td>
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<td>Study</td>
<td>Test/Diagnosis</td>
<td>Percentile / Score</td>
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<td>Notes</td>
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<tr>
<td>Mazzocco (2001)</td>
<td>TEMA-2 &lt; 10th percentile Single assessment</td>
<td>34e 13</td>
<td>5 – 7 years Grades K – 2</td>
<td></td>
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<tr>
<td>Ostad (1997)c</td>
<td>Standard mathematics achievement test &lt; 14th percentile Two assessments</td>
<td>927 101</td>
<td>7.5 – 11.67 years</td>
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<tr>
<td>Passolunghi &amp; Siegel (2001)c</td>
<td>Standardised mathematics test &lt; 30th percentile Teacher-nominated participants Single assessment</td>
<td>49 23</td>
<td>9.33 years</td>
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<tr>
<td>Räsänen &amp; Ahonen (1995)c</td>
<td>WRAT-R or arithmetic test score 1–1.5 SD &lt; mean of sample IQ &gt; 85 Single assessment</td>
<td>160 80</td>
<td>9 – 12 years</td>
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</tr>
<tr>
<td>Gross-Tsur, Manor, &amp; Shalev (1996)g</td>
<td>Score on arithmetic battery equal to or below the mean score for typical children two grades younger FSIQ &gt; 80 Two assessments</td>
<td>140 140</td>
<td>11 – 12 years</td>
<td></td>
</tr>
<tr>
<td>Shalev et al. (1998)g</td>
<td>Standardised arithmetic battery performance ≤ 5th percentile FSIQ &gt; 80 Three assessments</td>
<td>123 58</td>
<td>13 – 14 years</td>
<td></td>
</tr>
<tr>
<td>Shalev, Manor, &amp; Gross- Tsur (2005)g</td>
<td>Standardised arithmetic battery performance ≤ 5th percentile FSIQ &gt; 80 Four assessments</td>
<td>104 42</td>
<td>16 – 17 years</td>
<td></td>
</tr>
<tr>
<td>Silver et al. (1999)b,c</td>
<td>WRAT-Arithmetic, WJ-R Calculations, or WJ-R Applied Problems standard score &lt; 90 WISC-R FSIQ &gt; 90 Two assessments</td>
<td>80 80</td>
<td>9 – 15 years</td>
<td></td>
</tr>
<tr>
<td>Study</td>
<td>Assessment</td>
<td>Mathematics subtest (New Zealand)</td>
<td>Percentile</td>
<td>Grade</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>----------------------------------</td>
<td>------------</td>
<td>-------</td>
</tr>
<tr>
<td>White, Moffitt, &amp; Silva (1992)(^b)</td>
<td>Progressive Achievement Test</td>
<td>≤ 30th percentile</td>
<td>Single assessment</td>
<td>157</td>
</tr>
</tbody>
</table>

Note. Studies are organized by research group. Cut-off criteria listed in bold type. Grade reported for U.S. samples only. MLD = mathematics learning disability; PIAT = Peabody Individual Achievement Test (Dunn & Markwardt, 1970); WISC-R = Wechsler Intelligence Scale for Children–Revised (Wechsler, 1974); WJ-R = Woodcock Johnson Psychoeducational Battery–Revised (Woodcock & Johnson, 1989); WIAT = Wechsler Individual Achievement Test (The Psychological Corporation, 1992); TEMA-2 = Test of Early Math Ability, 2nd ed. (Ginsburg & Baroody, 1990); FSIO = Full Scale IQ; WRAT-R = Wide Range Achievement Test–Revised (Jastak & Wilkinson, 1984).

\(^{a}\)These criteria apply only to the definition of MLD, not to any specific MLD subgroups (e.g., MLD and reading disabilities). ^{b}\)In addition to MLD criteria, participants were excluded based on school or clinic history, concurrent diagnosis, or teacher report. ^{c}\)In addition to MLD criteria, no known medical or psychological etiology was excluded. ^{d}\)Twin pairs. ^{e}\)Sample size does not include matched control groups because these groups were not mutually exclusive. ^{f}\)Mean age range. ^{g}\)In addition to MLD criteria, participants were selected from those performing in the lowest 20% of an original cohort of 3,029 school children (n = 600).

Table 1: Murphy (2007) Table of Criteria for Assessment of MLDs

Murphy (2007, p.461-462) also reported that there are no major intervention programs provided for students beyond the primary school level. As such, students with a MLD do not receive the proper intervention they require just as they are entering a period in their mathematics education where they require it the most. The courses that students generally take in American secondary schools: introductory algebra, geometry (which contains a substantial amount of algebra), and advanced algebra and trigonometry require not only higher order cognition and greater visual-spatial reasoning than their previous
mathematics courses, but also demand that the students take on board a lot of new information at a faster pace and without as much repetition. Figures 32, 62, 67, and 68 give examples of the type of questions that students must answer on state exams which require these skills. The questions that are shown in figures 32 and 62, and to a lesser extent the question shown in Figure 67 also place demands on each student’s ability to process language. This further diminishes a student's ability to succeed in his or her mathematics classes.

In the United States, the necessary repetition that students need is usually not incorporated in their mathematics lessons. Many teachers that I have spoken to believe they cannot have more practice because they are pressured to complete the many topics in a given curriculum. In my experience teaching students with LDs, this lack of repetition makes it very difficult for students to store information in their long–term memory. Memory plays an essential role in learning mathematics. Maehler and Schuchardt (2008, p.9) stated that “working memory deficits might be so dominant in causing learning disorders that intelligence does no longer make a difference when working memory functioning falls below a certain threshold.” This makes intervention strategies where students receive assistance with repetition even more important. The extra practice may allow students with an MLD a better opportunity to develop their long–term memory. Noël’s (2012) research that is based on the work of Sarnecka and Carey (2008), indirectly supported the importance of repetition. Her research suggested that children, especially those who have been diagnosed with an MLD, work slowly to build new representations in number.
These factors when combined with the general anxiety that many students and also teachers, especially primary school teachers who do not have a substantial mathematical background, have about mathematics, has created a situation where the language disabilities have been extensively studied, but the mathematical issues have been largely ignored. Carey (2017) wrote that students who have “*high levels of mathematics anxiety are more likely to have other forms of anxiety, such as general anxiety and test anxiety and tend to have some math performance decrement compared to those with low math anxiety.*” She additionally stated that it is unclear to what extent other forms of anxiety affect mathematics anxiety and performance. Beilock (2010, p.1860-1862) discussed the role that a teacher’s mathematics anxiety, affects his or her students. In particular, she noted that female teachers with a lot of mathematics anxiety have female students who have lower mathematics performance. However, these same teachers did not seem to have much of an impact on the performance of their male students. In addition, she wrote (2010, p.1861) that gender ability beliefs may play a greater role than teacher anxiety in mathematics performance. The work by both of these researchers suggested that more research needs to be done on how mathematics anxiety affects a student’s mathematics performance.

It is not surprising that very few intervention plans are in place, and are not common in most schools. Ann Dowker (2005, p.326-327) stated that there are very few intervention programs for young children. Most notably the *Mathematics Recovery Program* and the *Numeracy Recovery Program* both seek to improve a child's numeracy skills. The former focuses on “*broad developmental stages*” such as counting and number representation, whereas the
latter focuses on independent skills such as estimation. Whilst these are relatively well known, most primary school teachers (including the teachers in my specialist school) do not know that these programs exist.

When it comes to research on the effects of visual–spatial disorders on how mathematics is learned at the secondary level, there is almost no research available. Murphy (2007, p.464), cited the work of Hammill (1994) who stated that students with MLDs do not do as well as non–MLD students on the Developmental Test for Visual Perception, 2nd edition (DVTP-2). Whilst these researchers stated that visual–spatial ability does not determine mathematical ability by itself, it does affect how students with an MLD learn mathematics. Unal (2009, p.1009) reported on the level of improvement of the geometric knowledge and understanding of university students who were taking a pre-service geometry class for middle and high school teachers. He reported that the pre-service teachers who had the lowest visual–spatial ability at the beginning of the course showed little or no improvement in a variety of geometric skills, as determined by the Van Hiele levels. Students with the highest visual-spatial ability showed greater improvement than the students with the lowest visual–spatial ability. Students who were initially between the students with the lowest and highest visual–spatial ability showed more improvement than the students with the lowest visual–spatial ability, but were not as successful as the students with the highest visual–spatial ability. If this is applied to secondary school students, it necessitates the need to have a strong intervention program in place to help the students with low visual-spatial ability enhance their geometric knowledge and understanding.
Smith and Olkun (2005, p.98-99) described how technology has proved to be somewhat successful in helping students with low visual–spatial ability. In particular, they explain that Logo, HyperGami, and Newton's World have interactive features that engage students and allow them to improve their mental rotation skills which are very important in geometry. They indicated that the computers provide students with an opportunity to focus on improving these skills without outside distractions (2005, p.109).

2.7 Types of Mathematics Learning Disabilities

There are a variety of MLDs, many of which educators, administrators (head teachers and other professionals in leadership positions in schools), parents, and sometimes perhaps evaluators are not familiar with. The most prominent MLD is dyscalculia which is also called developmental dyscalculia (DD or DC). The papers that I have read in the course of this research have discussed DD more than any other MLD. Other MLDs include acalculia, Gerstmann’s Syndrome and Mathematics Disorder.

2.7.1 Dyscalculia or Developmental Dyscalculia

Piazza (2010, p.33), stated that “developmental dyscalculia is a learning disability that affects the acquisition of knowledge about numbers and arithmetic.” Whilst there are no universally accepted criteria for the assessment of dyscalculia, the definition offered by Piazza and her colleagues is accepted by most researchers in this field. The etiology of dyscalculia is not known; Price and Ansari (2013, p.2) reported that “progress in understanding the root causes of
DD and how best to treat it have been impeded by lack of widespread research and variation in characterizations of the disorder across studies.”

Whilst the exact proportion of students who have DD may not be known, the prevalence of DD is difficult to determine because of the numerous criteria and various terminologies (such as MLD) that are used by different researchers and other professionals. Price and Ansari (2013, p.2) reported that about “3-6% of individuals have DD.” The research from Jordan (2007), Murphy (2007), Shalev and Gross-Tsur (2007), and Barbaresi (2005), may provide upper bounds for the occurrence of this MLD.

2.7.2 Possible Root of Dyscalculia

Cohen Kadosh (2007) and his research team discovered that the roots of dyscalculia may be based in the right parietal lobe of the brain. Cohen Kadosh said that:

“... Most people process numbers very easily – almost automatically – but people with dyscalculia do not. We wanted to find out what would happen when the areas relevant to maths learning in the right parietal lobes were effectively knocked out for several hundred milliseconds. We found that stimulation to this brain region during a maths test radically impacted on the subjects’ reaction time.”

Cohen Kadosh and his colleagues were able to cause dyscalculia for a few hundred milliseconds by using neuronavigated transcranial magnetic stimulation (TMS) to stimulate the brain. They then had the subjects compare the values of two numbers that were presented to them and measured the reaction times of each of the subjects. The results showed that the subjects who did not have dyscalculia had similar reaction times to those who were diagnosed with dyscalculia, thus suggesting that deficiencies in the right parietal lobe may
play a part in the determining the origins of dyscalculia.

2.7.3 Subtypes of Dyscalculia & Dehaene and Cohen’s Triple Code Model

Feifer (2005, p.39), used Von Aster’s categorisation of dyscalculia into three subtypes: verbal, procedural and semantic. These three subtypes are based on Dehaene’s triple code model of numeric representation. Von Aster (2000, p.41) used this model as a theoretical framework for these subtypes that he developed.

The triple code model developed by Dehaene and Cohen (1995, p.85) offers “three categories of mental representations in which numbers can be manipulated in the human brain.” Figure 2 gives a concise summary of this model.

![Figure 2: Dehaene and Cohen’s Triple Code Model](image)

The first code is the visual Arabic numeral form. In this form the numbers are written to show an “ordered list.” For example, 52 represents the ordered list: <5> <2>. In this code, numbers are “represented on an internal visuo-spatial
“scratchpad” where “they are manipulated in the human brain” and interpreted as 52.

The second code is a verbal word frame. In this form, 52 is represented as \{5\} tens and \{2\} ones. Dehaene and Cohen (1995, p.85-86) wrote “in this notation, symbols such as “Ones” and \{2\} denote abstract addresses that together constitute a word lemma linked to the phonological and graphemic forms of the word.”

The third form is the analogue, magnitude, representation code. In this code, the human brain can retrieve the “quantity or magnitude associated with a given number.” This allows the brain to make comparisons between quantities, and also helps to make estimations. For example, the brain can determine that 52 is somewhere between 0 and 100 (Dehaene and Cohen, 1995, p.85-86).

Von Aster built on this framework as a basis for his model for the three subtypes of dyscalculia. Each of these subtypes corresponds to a nonstandard function in a given part or parts of the brain.

### 2.7.3.1 Verbal Subtype of Dyscalculia

Students with the verbal subtype tend to have difficulty with “counting and rapid number identification skills” and even recalling overlearned mathematical facts. Students who are diagnosed with this subtype of dyscalculia have a disorder that makes it difficult to comprehend the “verbal representations of numbers” which is marked by the “inability to use language-based procedures to assist in arithmetic fact retrieval skills” (Feifer, 2005, p.39). Feifer, referenced Von Aster (2000, p.49) who stated that “reading and spelling difficulties as well as developmental disorders of speech and language are also relatively common
in this group of children.” Von Aster also stated that in the sample of students that he investigated, 6 of the 11 students were also diagnosed with ADHD and this may have affected their inability to count accurately (2000, p.49). He (2009, p.49) further stated that the verbal subtype of dyscalculia is also associated with “lesions along the left-hemispheric perisylvian areas [of the brain],” which are the areas of the brain are responsible for reading and written language. However, since these regions do not affect magnitude comparison, students with this subtype generally do not have difficulty comparing two numbers (Feifer, 2005, p.39)

2.7.3.2 Procedural Subtype of Dyscalculia

The procedural subtype of dyscalculia is a “disorder in the ability to transcode numeric systems into a meaningful language system” (Feifer, 2005, p.39). Von Aster reported that students with this subtype have difficulty reading and writing Arabic numerals that are read aloud to them (2000, p.46). Unlike the verbal subtype, this subtype does not affect the retrieval of learned facts. Instead, students will have difficulty in developing and using efficient algorithms when they are solving problems. Dehaene and Cohen (1997, p.220-221) explained that the procedural error coding of numbers is mainly processed in the left and right inferior ventral occipital-temporal regions of the brain. Kadosh and Walsh (2009, p.315) wrote that these regions of the brain where the Arabic code resides is responsible for “multi-digit calculations.” Often students with this subtype have difficulty learning mathematics but do not have any other type of learning difficulties (Dehaene and Cohen, 1997, p.220-221; Feifer, 2005, p.39-40).
2.7.3.3 Semantic Subtype of Dyscalculia

Students with the semantic subtype of dyscalculia are unable to distinguish between the magnitudes of numbers. For example, Figure 3, which shows a graphical comparison of 2, 20, and 200, may be difficult for a student with semantic dyscalculia. Whilst a student with semantic dyscalculia may be able to accurately determine the relationship between the three values and recognise that $2 < 20 < 200$, and perhaps even be able to accurately calculate that 20 is ten times larger than 2, and 200 is one hundred times as large as 2, the student will not be likely to have an intuitive feeling for the sizes of these values. At a first glance this may seem only to affect the comprehension of elementary mathematical concepts. However, it has a significant impact on a student’s ability to immediately determine the plausibility of an answer. Feifer, (2005, p.40-41) claimed that the semantic subtype directly affects the transcoding of expressions which is essential for success in higher mathematics. Dehaene and Cohen (1995) claimed that the bilateral inferior parietal areas are important because these areas control how semantic information is processed and are responsible for how different magnitudes are compared.
Ardila and Rosselli (2002, p.180) wrote that Henschen proposed the term *acalculia* in 1925 and defined it as “the impairment in computational skills resulting from brain injury.” However, Henschen was not the first to describe individuals with what is now called acalculia. Ardila and Rosselli referred to research by Lewandowsky and Stadelmann in 1908.

These researchers (2002, p.179), using the definition from the *INS Dictionary of Neuropsychology* (Loring, 1999), defined acalculia as:

“The loss of the ability to perform calculation tasks resulting from a cerebral pathology is known as acalculia or acquired dyscalculia. Acalculia has been defined as an acquired disturbance in computational ability.”

**2.75 Gerstmann’s Syndrome**

NINDS (n.d.), defined Gerstmann’s Syndrome as a “cognitive impairment that results from damage to a specific area of the brain -- the left parietal lobe in
the region of the angular gyrus. It may occur after a stroke or in association with damage to the parietal lobe.” Often, this damage is caused by a stroke.

Gerstmann’s Syndrome (NINDS, n.d.) has four main symptoms:

- Dysgraphia or agraphia — a writing disability
- Acalculia or dyscalculia
- An inability to distinguish between left and right
- Finger agnosia — an inability to distinguish the different fingers that are on the hands

Typically children who are diagnosed with Gerstmann’s Syndrome (NINDS, n.d.) will be likely to learn to adjust to their deficits, but will probably not overcome them. However, adults who are diagnosed with Gerstmann’s Syndrome, will have symptoms that fade over time.

This syndrome has become a point of interest amongst my colleagues at my specialist school because it appears in three students — one female fourth grade student (equivalent to Year 5), and two male students, one seventh grade and one eighth grade student (equivalent to Years 8 and 9, respectively). The seventh and eighth grade students had strokes in utero, and the eighth grade student also had a stroke when he was a few months old. It is unclear if the fourth grade student had a stroke when she was younger. Her doctor had recorded what the primary school principal described as a “SIDS [Sudden Infant Death Syndrome] type episode” when she was younger. Unfortunately, more information was not available about this incident. This incident may very well have been a stroke, but the student’s doctor had never diagnosed her with one.
2.7.6 Mathematics Disorder

Mathematics Disorder is now considered an obsolete term. The *DSM-IV-TR* described Mathematics Disorder by the following criterion:

A. *Mathematical ability, as measured by individually administered standardized tests, is substantially below that expected given the person's chronological age, measured intelligence, and age-appropriate education.*

B. *The disturbance in Criterion A significantly interferes with academic achievement or activities of daily living that require mathematical ability.*

C. *If a sensory deficit is present, the difficulties in mathematical ability are in excess of those usually associated with it.*

In the United States and other countries, this was usually the name that was given to a mathematics disability.

2.7.7 Specific Learning Disorders (SLDs)

There were many controversial changes to the *DSM-V*. Most notably, this edition of the manual eliminated Dyslexia, Mathematics Disorder, Disorder of Written Expression, and Asperger’s Syndrome as disorders. These terms could, however, be used as descriptive language when diagnosing a student as having an SLD.

The diagnostic criteria for SLD are:

A. *Difficulties learning and using academic skills, as indicated by the presence of at least one of the following symptoms that have persisted for at least 6 months, despite the provision of interventions that target those difficulties:*

1. *Inaccurate or slow and effortful word reading (e.g. reads single words aloud incorrectly or slowly and hesitantly, frequently guesses words, has difficulty sounding out words).*
2. Difficulty understanding the meaning of what is read (e.g., may have read text accurately but not understand the sequence, relationships, inferences, or deeper meanings of what is read).

3. Difficulties with spelling (e.g., may add, omit, or substitute vowels or consonants).

4. Difficulties with written expression (e.g., makes multiple grammatical or punctuation errors with sentences; employs poor paragraph organization; written expression of ideas lacks clarity).

5. Difficulties mastering number sense, number facts, or calculation (e.g., has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the math facts as peers do; gets lost in the midst of arithmetic computation and may switch procedures).

6. Difficulties with mathematical reasoning (e.g., has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems).

2.7.8 Williams Syndrome (WS)

O’Hearn and Luna (2009, p.11) described WS as “a developmental disorder characterized by relatively spared verbal skills and severe visuospatial deficits.” In addition, they also stated that those who have been diagnosed with WS tend to have serious impairments in mathematics.

O’Hearn and Landau (2007, p.238) cited the work of Ansari and Karmiloff-Smith (2002) and Paterson (2006), and stated that people who have WS “have particular problems with mathematics in addition to, and possibly related to, their visuospatial deficits and parietal lobe abnormalities.”

Whilst more research is being conducted in this area, not much is known about the impact of WS on children who are learning mathematics. Van
Herwegen (2015, p.144) wrote that children with WS “have revealed that arithmetic skills are severely impaired even in adulthood.” She went on to write that even though children with WS are adept at counting sequences, they have difficulty attaining an understanding of the underlying mathematical concepts. Van Herwegen (2015, p.144-145) further wrote that “typically developing” children are able to compare magnitudes accurately and efficiently, but this ability is impaired for those who have WS.

Whilst children with WS have diminished ability in some areas, interestingly their visuospatial skills tend to fall in the normal range (Van Herwegen, 2015, p.145). More research has to be conducted to determine why this is the case.

2.7.9 Turner’s Syndrome (TS)

Attout, (2015, p.1) described TS as a “genetic condition characterized by a cognitive profile with a relative weakness in visuo-spatial abilities and preserved verbal abilities.” He further cited the work of Simon, (2008, p.86) who stated that people with TS had significant impairments in mathematics because of a “possible core deficit of numerosity processing.”

Not surprisingly, the weaknesses in visuo-spatial abilities will make learning geometry more difficult for students with TS. Attout (2015, p.1) wrote that “numerical magnitude comparison of discrete non-symbolic quantities is not influenced by the visual cognitive load in TS.”
2.8 Fragile X Syndrome (FXS)

The NIH (2016) defined FXS as “a genetic condition that causes a range of developmental problems including learning disabilities and cognitive impairment” that affects about 1 in 4000 boys, and 1 in 8000 girls. Boys are affected much more severely than girls are. Most males and about one-third of females who have FXS are “intellectually disabled.” Children with FXS tend to have higher anxiety and attentional issues which could make it difficult to learn mathematics especially in primary school.

2.9 The Causes of MLDs

The causes of various MLDs are still unknown and widely debated. Even for dyscalculia or DD, the most commonly known of the MLDs, not only is the cause unknown, but the methods used to diagnose it are varied and have not been standardised. Landerl (2004, p.100) stated that there is no standard definition for “maths achievement” and it is “hard for researchers to pinpoint the key deficits in dyscalculia, or to be sure how to define dyscalculics for study”. As such, it is difficult to determine the cause of this learning disability. However, Gross (2006, p.665) indicated that the intraparietal sulcus plays an important role in learning mathematics. She stated that sophisticated symbolic number processing in adults occurs in this region of the brain. Noël’s research supported this claim; in particular, she claimed that the right intraparietal sulcus reacts to numerical changes in children just as in adults (Noël, 2012). This is to say that when a person is shown a diagram with an increasing or decreasing number of shapes on it, the right intraparietal sulcus reacts to the change in the number of figures. This
research is also supported up by Wedderburn (2012), who stated that dyscalculia is associated with “inefficient parietal lobes”.

Working memory deficits are associated with DD. Landerl (2005) cited the work of Geary (2005, p.102) who suggested “that poor working memory resources not only lead to difficulty in executing calculation procedures, but may also affect learning of arithmetic facts.” The causes of poor working memory are largely unknown but researchers such as Gathercole and Alloway (2007, p.11) suggested that “genes play an important role in the frontal areas of the brain that support working memory.”

Hale and Fiorello (2004, p.214) referenced Rourke’s (1995) work about the visual–spatial subtype of MLD. These authors summarised his research which claimed that the visual–spatial subtype is due to a “white matter syndrome affecting the right hemisphere” of the brain. Hale and Fiorello (2004, p.214) also stated the “right hemisphere has more association cortex, as well as more white than gray matter to allow for intersensory integration.”

Molko (2003, p.847) referred to Dehaene and Cohen’s (1997) work that also suggested that a link exists between the intraparietal sulcus and understanding of numerical quantities. More importantly to this research, there is evidence that showed certain genetic disorders such as Williams, Turner, and Fragile X syndromes, suggested “an association of developmental dyscalculia and visuo-spatial-processing deficits in TS may reflect the intimate relationship between number and visuo-spatial representations.”
2.10 Nonverbal Learning Disabilities, Visual-spatial Deficits, and a Basis for Research

Often students who have NLDs have significant difficulty with mathematics. Dianne Matthaei (2008, p.12) referenced the work of Rourke (1995), a neuropsychologist and researcher who has written extensively about these types of disabilities. Matthaei (2008, p.12) stated that Rourke and many other prominent researchers in this field even made poor mathematical performance a requirement for a diagnosis for an NLD. More recently however, researchers do not necessarily make this a requisite for a diagnosis for an NLD (Matthaei, 2008, p.12).

Since visual-spatial disorders have typically been classified as NLDs, this is especially important for my research. As noted before, not much research has been conducted in this area. Dianne Matthaei's M.A. thesis included an intervention plan for two secondary school geometry students who were identified with visual-spatial disorders. I intend to build on her work — the only work of its kind that I have found — by creating a system to identify and classify visual-spatial disorders in a manner that will be more useful for mathematics teachers. In particular, my goal is to create a system where teachers could use the SmartBoard, or other interactive technologies including tablets such as the Apple Ipad, to give an assessment and classify the type of visual-spatial disability a student has. After giving the assessments to some high school students in my school, I will create an intervention plan for the students who have visual-spatial deficiencies since these deficits may significantly affect their ability to learn mathematics, and in particular geometry.
2.11 Current Research on Visual-Spatial Deficits

The current research on teaching geometry to students with visual spatial deficits is quite limited. Any substantial research in this field seems to have been developed in just the past ten years. Of note is Dianne Matthaei’s 2008 M.A. thesis from Pacific Lutheran University in the United States. As of December 2016, this is the only research that I am aware of that specifically addresses teaching geometry to secondary school students who have visual-spatial deficits.

Matthaei’s research is made up of two clinical case studies as defined by Miles and Huberman (Matthaei, 2008, p.37). These researchers (1994, p.404) defined a clinical case study as an approach that:

“... is aimed at understanding a particular type of individual, such as a child with a specific learning disability. Such case studies usually employ clinical interviews and observations but may also involve testing and other forms of data collection. The usual goals are to better understand the individual and the disability and identify possible treatments.”

Matthaei used this as a model for her research. She worked with two secondary school students with visual-spatial deficits and developed intervention plans to help them learn geometry more effectively. By using this method, she wanted to gain a deeper understanding of how these deficits affected each student’s ability to comprehend mathematics. I will be employing a clinical interview model that Dr. Michael Telch (n.d.), a psychologist at the University of Texas at Austin, defined as a clinical interview as a “situation of primary vocal communication” in which the subject shares his or her experience in order to gain benefit. Since my goal is to develop an intervention plan for some of my interviewees, the latter part of this definition is especially appropriate and relevant to my research.
The students who Matthaei studied were selected based on their diagnosis of a spatial learning disorder (2008, p.38-39). She wrote that the diagnosis is based on several factors including scores on standardised tests of spatial processing and interviews with the parents of the students, and the students themselves. In addition, performance on non-standardised measures were considered, which Matthaei claimed is consistent with the diagnosis for a spatial learning disorder.

The standardised tests (Matthaei, 2008, p.90-92) include the Test of Nonverbal Intelligence, 3rd Edition (TONI-3), and four subtests of Woodcock-Johnson Test of Cognitive Abilities: Visual Matching, Spatial Relations, Concept Formation, and Retrieval Fluency. In addition, the Beery-Buktenica Developmental Test of Visual-Motor Integration, 5th Edition was also used. The non-standardised tests were developed by the Ark Institute of Learning, an organization that seeks to “diminish the impact of learning disorders in the lives of individuals” (Ark Institute, n.d.) in 1992. They include the: Perceptual Forms, Parquetry, Hidden Objects and Word Pairs tests. Both of these students scored very low on almost all of these tests. The exception is the Word Pairs test, but only when they were instructed to use imagery (Matthaei, 2008, p.41).

In addition to these tests and interviews, Matthaei also looked at reports from other professionals – including teachers and psychologists that the students work with – in order to confirm the validity of the finding of a spatial learning disorder (Matthaei, 2008, p.39).

Matthaei (2008, p.42-43) collected qualitative and quantitative data from a variety of sources. The data included field notes that were taken at the end of
each session that Matthaei had with each participant. In addition, she used a quarterly summary of instruction reports, which gave a synopsis of Matthaei’s work with each student, scores from teacher-constructed class chapter tests and finals for the semester (term).

Matthaei worked with each student from the start to the end of each course. Ryan, the male student, was officially excused from his geometry class at his high school and worked with Matthaei in a one-to-one situation on a daily basis. He was taught by Matthaei who employed a variety of intervention strategies that are described in the next few pages. Ryan was subject to the all of the requirements set by his geometry teacher, including those for testing. The only exceptions to these rules were the accommodations that were set in Ryan’s Individualised Educational Plan (IEP). Since the IEP is a legally binding document that dictates what services and accommodations a school must provide to a student with a learning disability, one of the accommodations to Ryan’s testing was allowing extended time for his examinations. Though not in his IEP, Ryan was allowed to use written scripts that are described in the next section of this thesis. In the second semester, Ryan’s new teacher insisted that the scripts should not be allowed to be used on class examinations, and Ryan agreed to this over Matthaei’s strong objection. She reported (2008, p.51) that Ryan said “I needed the scripts at first, but I don’t think I need them anymore.”

Hailey, the female student that Matthaei (2008, p.47) remained in with her geometry class in school. She worked with Matthaei individually after school for two to three hours each week. She was not permitted to use any scripts during her testing. Matthaei did not report on the accommodations that she was allowed to
have during her class such as extended time for her teacher-constructed class examinations.

During her work with these students, Matthaei (2008) developed three effective interventions: picture notes, written scripts and using physical three-dimensional models to supplement the drawings of these models on a printed page.

Matthaei (2008, p.29) wrote that picture notes were a “non-linguistic representation which Marzano listed as one of the nine categories of instructional strategies that yielded the greatest effect size in terms of student achievement.” Matthaei found that this intervention was very useful in helping students learn mathematical terminology. This is one of the few interventions typically used for teaching language that are successful in teaching mathematics. It is not surprising that this intervention was successful considering that this method was originally used to develop imagery when teaching reading and is useful for teaching novel and complex terminology. The picture notes method has a strong connection to language acquisition and can be applied to teaching the language of mathematics. When the teachers, parents and the students from Matthaei’s study were asked to rate the effectiveness of this strategy on a Likert scale from 1 to 5 (where 5 corresponds to “very effective”) the median rating was 5; in fact, all of the respondents rated this method a 5, except for one teacher who rated it a 2. The teacher who gave the lower rating for this method stated this technique was not especially helpful in enabling Hailey to learn vocabulary. However, Hailey’s mother remarked that this method helped her daughter learn vocabulary in her Spanish class (2008, p.53-54); perhaps this method could be
helpful in learning new mathematical terminology which for some students may be similar to learning a foreign language. This further supported the claim that this strategy is an effective technique to teach reading. Figure 4 shows two examples of these picture notes.

![Figure 4: Matthaei’s Picture Notes Strategy Cards](image)

Figure 4 shows the picture notes strategy that was adapted from strategies that have been used by teachers who have taught reading to learning disabled students, and especially dyslexic students.

Matthaei (2008, p.56) reported that students with spatial problems “frequently describe feeling lost in a problem.” To help students overcome this difficulty, Matthaei introduced writing scripts to them. These scripts are comprised of the sequence of steps necessary to solve a problem or perform a calculation (such as finding the volume of a rectangular prism) on a 4 inch by 6 inch note card. These scripts were also an effective intervention. Figure 5 shows an example of a writing script.
Figure 5 shows an example of a writing script that Ryan used to calculate the volumes of a rectangular prism and a right circular cylinder.

Ryan remarked “Those script cards we made with examples on them were really great. When I see an example, it helps me relate how one problem is like another and then I can do it.” This strategy was discussed in Polya’s (1945) research on problem solving and outlined in his classic text How to Solve It. The four respondents rated this strategy higher than the picture notes strategy. All of the respondents rated this method a 5, except for one teacher who rated it a 4; the median score for this strategy was 5. The difficulty with these scripts is that some teachers allowed their use for class exams that they created, and other teachers did not. One teacher who did not allow the scripts to be used for the exams that he administered stated that if the goal of the exam is to assess acquired knowledge, then the scripts should not be permitted. The purpose of the
assessment should dictate whether these scripts should be used. Another aspect to consider is whether these scripts may be used on standardised exams; on most exams, including the New York State Regents Examinations, students will not be permitted to do so (Matthaei, 2008, p.57).

The third strategy that Matthaei used is to have physical, three-dimensional models to supplement the two-dimensional figures on a printed page. Matthaei (2008, p.58-59) reported “While not used often, it seemed to be a strategy that resulted in a conceptual breakthrough when employed.” She stated that since difficulty with mental rotation is a hallmark of visual-spatial disorders, the students in the study who manipulated the physical objects were able to make a breakthrough in their conceptual understanding. Figure 6 shows an example of how this strategy was used. The question about the cone, that is shown on the left, would be accompanied with a wooden cone, that is shown to the right of the question. The student could manipulate it as he or she needs to more clearly understand the problem.

Figure 6: Matthaei’s Physical Three-Dimensional Model Approach
The respondents scoring supported Matthaei’s claim. This strategy had a median score of 5; with one rating of three, one of four, and five ratings of five along with two abstentions. The two teachers who abstained from responding did not comment on this strategy because they did not use this strategy when they were teaching their students (2008, p.59).

Another aspect of learning that Matthaei placed importance on is memorisation. Both of her subjects for her clinical case studies reported without prompting that memorisation was an important aspect of how they learn effectively. Ryan said that he needed a longer time to memorise information, but once he had learned it, he generally did not forget it. More importantly, Ryan’s teacher noted that memorisation allowed for improvisation, a skill that is essential for success in open-ended questions and especially proofs which is an important part of most secondary school geometry courses in the United States (2008, p.59-60). Unfortunately, the teacher did not justify his reason. However, in my experience I have also found this to be true. This is not only true in manipulating the equations for basic formulas, such as the two equations for the circumference of a circle, but perhaps more importantly in writing two-column proofs in geometry.

2.12 Mathematics and the Brain

All learning disabilities, whether they are mathematical in nature or otherwise, have their origins in the brain. The study of differences in the brain of students with and without learning disabilities and their effects on how students learn mathematics or language is relatively new. The advent of functional magnetic resonance imaging (fMRI) has significantly enhanced how the brain is
studied, and has consequently enhanced the quality of the research. As Clay (2007, p.2) of the American Psychological Association (APA) has stated “psychologists and other researchers aren’t using fMRI just to see what lights up in people’s brains as they perform different mental tasks” they are using it to “help answer classic questions within psychology” such as the nature of how people make decisions and the best ways to treat people with learning disabilities. Even with language-based learning disabilities such as dyslexia, where there is a larger volume and more in-depth research than mathematical learning disabilities, fMRI is a relatively new tool whose uses and benefits have not been fully established.

2.13 The Localisation View of Brain Function

Feifer (2005, p.25) stated that historically many researchers and medical professionals assumed that the localisation or cerebral localisation view of brain function was true, meaning that specific parts of the brain were responsible for certain functions. He (2005, p.25) additionally stated that the brain was viewed as a dichotomous organ with language abilities being attributed to its left hemisphere and mathematical skills to its right hemisphere. According to Feifer (2005, p.25), this view still persists among some educators today. He wrote that “most practitioners not schooled in a cognitive science domain” tend to consider the brain to be a dichotomous organ with the left and right hemispheres having discrete and non-overlapping functions. However, this leads to some rather substantial logical inconsistencies. For example, there are students with visual-spatial difficulties (a right hemisphere skill) that have a good number
sense. If specific functions were localised to different parts of the brain, localisation model of brain function would be contradicted. The research by Goldberg (1990) and Von Aster (2000) suggested that the cerebral localisation view is implausible and can lead to faulty diagnoses and possibly poor interventions as a consequence.

2.14 The Lurian View of Brain Function

Alexander Luria developed his own view of how the brain functions about 40 years ago. Luria (Feifer, 2005, p.32) proposed “the brain consists of three basic functional units or processing blocks, that control arousal-activation, perceptual integration, and higher level programming and executive functioning.” In his model, Luria (Feifer, 2005, p.32) rejected the idea that a specific task is attributed to a specific hemisphere of the brain. Instead, he developed a “vertical model in which lower brain systems such as the reticular activating system orient our attention towards a stimulus, and posterior brain regions involving the occipital, temporal, and parietal lobes, that process and integrate the information.” In addition, he proposed the frontal lobes help with functional task outputs by directing our attention towards a goal-based activity. Luria’s view (Feifer, 2005, p.33) has influenced the modern neuropsychological views “in that higher cortical functioning stems from an amalgamation of functional units and subprocesses.”

Current technology, mainly fMRI, seems to give validity to Luria’s model. Feifer (2005, p.35-36) referred to the research ofstonescu-Cosson (2000) who showed with fMRI that parietal lobes were activated during both numerical
processing tasks and (Feifer, 2005, p.35) “visually guided hand and eye movements.”

2.15 The Analogous Nature of Brain Function for Reading Disabilities and MLDs

Research (Feifer, 2005, p.36) has shown that there is a basic neural circuitry system that is analogous to that of other learning disabilities. In particular, there is evidence to show that the brain processes numeric information in a manner that is analogous to the way it processes verbal information. Whether the brain is processing verbal or numeric information, there are many neural pathways located throughout the brain, and additional “parallel networks” that all work simultaneously to efficiently deal with the great volume of data that our brain are presented with in everyday interactions.

Feifer (2005, p.36) referred to the work of Paulesu (1996) who conducted a study on how the brain processes verbal information. According to the latter group of researchers, novice and proficient readers use different neural pathways. Feifer (2005, p.36) referred to Shaywitz’s (2003) research and wrote dyslexic students activate “more frontal regions” because they are unable to use the neural pathways responsible for automatic word recognition in print.

In a similar manner, (Feifer, 2005, p.37) referred to the research by Stanescu-Cossom (2000), who noted there is an increase in activity in the inferior frontal gyrus when “students attempt to process larger numbers within the verbal fact retrieval system.” Feifer (2005, p.37) referred to Shaywitz (2003) and stated that for dyslexics, this same part of the brain becomes overactivated as they try to break down large words that they cannot instantly recognise. Whereas the
inferior frontal gyri are mainly responsible for breaking larger units into smaller and meaningful units that can be processed more easily. This is analogous to how the brain processes large verbal information into smaller units that can be processed more easily.

The angular gyrus is the location in the brain where the second analogous system between mathematics and reading can be established (Feifer, 2005, p.37). The angular gyrus is where the occipital lobes (which is associated with vision) and parietal lobes (which is associated with spatial awareness) meet in the brain. Feifer, (2005, p.37) again referred to Stanescu-Cosson (2000) who reported that the left hemisphere of this location of the brain are especially active in “exact calculation tasks such as the verbal retrieval of rote additional facts.” In addition, Feifer (2005, p.37) also make note of Goldberg’s research (1989) who stated that the angular gyrus is where symbolic representation is seated in the human brain. This is especially important since students who have difficulty with reading fluency have difficulty with memorising mathematical facts due to a “shared inability to recall and retrieve information stored in a language dependent code quickly.”

The left hemisphere of the occipital-temporal region of the brain is where the third analogous system between reading and mathematics can be established (Feifer, 2005, p.38). Feifer (2005, p.38) referred to Shaywitz (2003) who stated this rapid neural pathway is used by proficient readers to automatically recognise printed words. Feifer (2005, p.38) reported that “the left fusiform gyrus, an extremely long convolution extending lengthwise across the occipital and
temporal lobes, may participate in the cerebral network underlying multiplication and number identification.” Feifer (2005, p.38) cited Dehaene (1996) who stated that PET scan analyses showed the occipital and temporal lobes may be part of the cerebral network that is used in “automatic number recognition tasks, particularly with multiple digits.”

2.16 The Visual-Spatial Connection

Feifer (2005, p. 44) referenced Marjoram & Nelson (1985) who stated that many cognitive neuroscientists recognised the connection between visual-spatial and mathematical abilities. In addition, Feifer (2005, p.44-45) referenced Benbow and Lubinski (1993) who reported on the vast advantage that male students have over female students in scoring 700 out of 800 (roughly the top 5% of scores) on the mathematics section of the Scholastic Aptitude Test (SAT), a standardised examination that many students take for admission to universities in the United States. Benbow and Lubinski (1993) wrote that the ratio of male to female students who score in this range is roughly 16:1. However, this difference does not exist between male and female students for the verbal ability section of the SAT.

There are several theories that attempt to explain why men have stronger visual-spatial skills than women. An anthropological explanation (Feifer, 2005, p.44) stems back to pre-historical times when men were responsible for hunting. These men, the theory suggests, had to take long trips to unfamiliar areas and then return home with the prey that they had killed. Since they did not have any of the current technological tools to assist them, such as a global positioning system or even a compass, they had to learn to “assess their relative positions in
the world by sharpening their visual orientation skills with respect to fixed positions to their immediate environment.” However, the theory continues, women were not usually permitted outside of their settlements, and as such did not develop their visual-spatial skills as much as men did.

Feifer (2005, p.44-45) also offer a sociological explanation that attempts to explain the differences in men and women’s visual-spatial abilities. Feifer (2005, p.44-45) stated that some societies, including the United States, have not always encouraged women to enter traditionally male dominated science, engineering, and mathematics based occupations. In fact, they may have been discouraged. Toy companies bombard the parents of young children with specific messages that reinforce stereotypical behaviours for both boys and girls. Boys were typically shown with action figures and girls were typically shown with dolls and cooking sets. The former set of toys help boys develop visual-spatial skills in a manner that the toys marketed to girls probably do not.

Feifer (2005, p.46) cited the work of Levine (1999) who conducted a study of boys and girls 4 and 5 years of age. These children were given the WPPSI-R Mazes subtest, stated that whilst a difference does exist in the visual-spatial abilities between the genders, it is not always easy to assess. Feifer (2005, p.46) stated:

“…the magnitude of gender differences in spatial skill development is often difficult to assess, as effect for young children may be attributed to varying task demands, the extent to which the task itself actual taps spatial skill prowess and previous practice and exposure to these measures.”

Despite these difficulties, there was strong evidence to support gender differences in visual-spatial skills (Feifer, 2005, p.46). In particular, according to
Weiss (2003) and Voyer (1995), mental rotation tasks produced the greatest gender differences among all of the neuropsychological test constructs. Feifer (2005, p.46-47) referenced Weiss (2003) who wanted to determine the neural network involved in mental rotation tasks. He (2005, p.46-47) reported that in a study of 20 right-handed university students who were given mental rotation tasks, fMRI analysis showed that there was “increased signal intensities bilaterally in both the parietal lobes and the frontal lobes.” This information about the frontal lobes in especially valuable because it suggests that executive functioning plays a part in visual-spatial abilities. Feifer (2005, p.47) again referenced Weiss (2003) in stating this gives an insight into how men and women use different cognitive strategies to complete the mental rotation tasks.

2.17 Memory, Anxiety, and Brain Function

There are many researchers who have studied the link between anxiety and memory. Feifer (2005, p.55) stated that there are a lot of empirical studies that suggest that students who have anxiety (whether it is mathematically or non-mathematically based) tend to have poorer memory capabilities and lower academic performance. In fact, Feifer (2005, p.55) referenced the research of Ashcraft and Faust (1994) who reported that their study showed that students who specifically had increased maths anxiety had “lowered academic performance on more complex mathematical procedures.” This research was later confirmed by Maehler and Schuchardt (2008, p.9) who stated that “working memory deficits might be so dominant in causing learning disorders that intelligence does no longer make a difference when working memory functioning falls below a certain threshold.” Feifer (2005, p.58) referenced Kellogg (1999)
who reported that students with higher mathematics anxiety did not perform as well as students with lower mathematics anxiety even on not just problem solving questions, but also basic arithmetic questions.

Feifer (2005, p.55-56) stated that mathematics anxiety has been attributed to gender. In addition, he claimed that anxiety has been associated with women more than men because of “sociological stereotypes that math and science require no emotional intuition and thus seem better suited for males.” Feifer (2005, p.55-56) continued that some researchers view men as having an inherent advantage over women because mathematics is very visual-spatial in nature, an area in which men seem to have an advantage over women. However, whilst acknowledging that gender differences do exist in mathematics, he said that they usually do not occur until high school (college) or university levels, as shown in the research by Hyde (1990). Feifer (2005, p.56) further suggested that sometimes, sociological biases may be involved in determining success in mathematics (or any other field), it is “perceived success or failure that ultimately influences and reinforces internalized beliefs about potential success for math” as suggested by Eccles (1983).

Most neuropsychologists (Feifer, 2005, p.57) have noted the “counterproductive relationship between heightened anxiety levels and limitations with working memory.” Feifer (2005, p.56) reported that the “net result of this deadly concoction is often a severe limitation of cognitive flexibility when problem solving.” Foley’s (2017, p.52) research seemed to confirm this. She wrote “Data from the Program for International Student Assessment (PISA), which tests 15-year-olds’ academic achievement worldwide, shows that math
Anxiety may also severely reduced a student’s ability to shift sets, that is the ability to change gears when dealing with complex tasks with probably large amounts of data. Passolunghi’s (2016, p.2) research also suggested that mathematics anxiety has a negative impact on shifting sets. These skills are of course essential for success in mathematics at any level. Even with all of this data, Feifer (2005, p.58) warned readers that mathematics anxiety alone “was not directly responsible for the difference in performance between boys and girls.”

In terms of this research and the influence of anxiety on visual-spatial abilities, Feifer (2005, p.57) referenced Casey (1997) who showed that there was not only a correlation between gender and anxiety, with female students being more anxious, there was also a correlation between gender and self-confidence with male students being more confident in their own abilities. Most importantly though, Casey showed there is a correlation between gender and performance on mental rotation tasks, with male students having higher performance than female students. Ferguson’s (2015, p.1) research, which according to Schillinger (2018, p.109-110) is the only study thus far on how mathematics anxiety affects visual-spatial abilities, suggests that mental rotation skills are negatively impacted by this type of anxiety.
Chapter 3: Research Methodology and Research Methods

3.1 Introduction and Rationale for this Research

As I have mentioned before in this research, there is very little information available about visual-spatial deficits and their impact on how secondary school students learn mathematics, and in particular geometry. The only research of significance that I have found is an M.A. thesis by Dianne Matthaei. In her thesis, Matthaei uses two, single case studies, for each of the students for whom she created intervention plans. Matthaei conducted qualitative research and considered each student’s history in mathematics as well as their psychoeducational evaluations.

As stated before, my research will aim to answer three questions:

- Firstly, I will determine what types of visual-spatial deficits students have, and how they may be classified by extending Karagiannakis’ model. I will build on this model by creating a more detailed view of the Visual-Spatial category of this model by separating this category into two subcategories of deficits: two-dimensional and three-dimensional figures and relationships.

- In the second stage of my research, I will determine what components will form an effective intervention plan for students with visual-spatial deficits. This will include a discussion as to how teachers could select the best components for each of their students based on their individual set of deficits.
Finally, I will determine what type of interventions will work with teachers who are teaching students with diagnosed learning disabilities — both mathematical, and non-mathematical. In particular I will determine what interventions are best for helping teachers in my school teach mathematics to students who have visual-spatial deficits.

My research will use student interviews, interventions with students, and training sessions for teachers that will aim to serve as a different type of intervention for the teachers. I firmly believe that these approaches will help me to answer my research questions. Matthaei, who has conducted similar research has used some of these strategies. She found this to be very effective. Ultimately this approach will help me to answer my research questions:

- What are the types of visual-spatial deficits that the students I have interviewed have? Also, how can these deficits be classified by extending Karagiannakis’ model?
- What components will form an effective intervention plan for students with visual-spatial deficits?
- What components will form an effective intervention plan for teachers in my school who work with students that have LDs, and specifically those who have visual-spatial deficits?

My research will primarily be qualitative. I have chosen to use a case study approach for a variety of reasons that will be outlined in the next section of
this chapter. Furthermore, this method seems to be the preferred method since my aim is to answer the “how” and perhaps “why” questions as described by Yin (2014, p.9-10), about the interviewees’ disabilities, how they may learn mathematics, and perhaps why they learn best in a certain manner.

3.2 Qualitative Research

Corbin and Strauss (2008, p.12-13) reported that there are many benefits of qualitative research. As they stated (2008, p.12), the most frequent and accurate reason for doing qualitative research is determined by the research questions. My research questions have sought to gain an insight into my students’ interpretations of two and three-dimensional diagrams, and reveal information into the thought processes of the students who appear to have visual-spatial deficits. A qualitative approach would best answer these types of questions. A quantitative approach may indicate how often a particular misconception occurs, but due to the small scale of my research, it would not necessarily yield reliable information. Also it would not reveal possible reasons why students interpret diagrams in a particular manner or suggest how to support these students.

3.3 Qualitative Analysis

Corbin and Strauss (2008, p.1) defined qualitative analysis as “a process of examining and interpreting data in order to elicit meaning, gain understanding, and develop empirical knowledge.” Since the nature of my research is based on only some of the students at my unique special needs school,
I felt this was the best approach to use due to the small sample size. It is highly unlikely that the knowledge garnered from my research can be generalised to the mainstream student population, to special needs students in a mainstream school, or even to another special needs school. In fact, I am not certain that these results can even be generalised to all of the students in my school.

### 3.4 Rationale and Benefits of a Case Study Approach

Since my research is primarily qualitative I have chosen to use a case study approach. Yin (2014, p.15) starts with Schramm’s (1971) definition of case study. Schramm (1971) wrote:

> “The essence of a case study, the central tendency among all types of case study, is that it tries to illuminate a decision or set of decisions: why they were taken, how they were implemented, and with what result.”

In addition, Yin added a twofold definition of case study. Firstly, he (2014, p.16) wrote that a case study must investigate a “contemporary phenomenon ... in depth and within its real-word context” especially when “the boundaries between phenomenon and context may not be clearly evident.”

The second part of Yin’s (2014, p.17) definition of a case study inquiry is that it:

- Copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result
- Relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result
- Benefits from the prior development of theoretical propositions to guide data collection and analysis
The first two points in this part of Yin’s definition of a case study are especially relevant to my research. The results of this research will be especially important for the students and teachers who are in my school. Several teachers have stated that they have had students who have had difficulty interpreting graphs, not just in mathematics, but also in their science and social studies classes. In the latter subject, these students tend to also have difficulty reading maps. Whilst the teachers have been able to develop some strategies to help these students, there is no resource that they can refer to for assistance. Furthermore, some of these teachers struggle to understand the difficulties that their students face because they have never had experience teaching students with these difficulties, and have never been learned about these types of learning disabilities.

Honey (2008, p.99) cited Cohen (2001, p.181) who wrote “Case studies establish cause and effect, indeed one of their strengths is that they observe effects in real contexts, recognising that context is a powerful determinant of cause and effects.” The last part of this statement is especially important for my research. The special needs school where I teach and conduct research is indeed a very unique place that significantly impacts the context of my research, and also how any reader should read this very subject specific thesis. Based on the different methods presented by Yin (2014, p.9-10): experiment, survey, archival analysis, history, and case study, I strongly believe that the case study approach — case studies of individual students supported by a history would be best suited for my research. This approach will have several benefits, with the most important being that it gives the readers of this research a better picture of the
students who will participate in the research. The benefits of the case study are discussed in detail in a later section of this chapter. The survey will allow the participants’ mathematics teachers to comment on their knowledge of the students, and possibly provide insights about the students that did not emerge from the mathematical interviews. Finally, the history will give more detailed background information about each student’s learning disability as determined by his or her psychologist or psychiatrist. The psychoeducational evaluation that these mental health professionals write also gives information from the students’ parents or guardians, and former teachers, tutors, and others who have known the students well.

Yin (2014, p.147) noted numerous benefits of using the case study method including the ability to build explanations. Carey (2012, p.238) delineated four advantages very succinctly. Firstly, he wrote that case studies can provide “rich raw material for advancing theoretical ideas.” This may not be the case for my research, but it is nevertheless important. Secondly Carey reported that case studies help to “provide insight” in all stages of the “theory-building process.” The final two ideas that Carey offered will be especially important for my research. Carey further stated that “new information that holds across many cases can stimulate new theoretical thinking,” and that this approach can be used as an effective research tool. From the student interviews that I have conducted so far, and to a lesser extent the interventions, I have found both of these points to be applicable to my research. In the training sessions and interventions that I have conducted with the teachers, I have found that whilst there is no new
theoretical thinking, there appears to be a new set of techniques that they are developing to create practical interventions for the students.

3.5 A Description of the Mathematical Interviews

One research question related to the identification and classification of misconceptions. The idea of a mathematical interview that would expose these misconceptions was considered to be a research tool.

Each mathematical interview was comprised of a set of questions that students would be asked to work through. The idea being that misconceptions would be exposed, not only so that they could be seen, but also the context in which they arise. The context and content of the initial set of questions were influenced by the previous experience of teachers in my school. These fell into the broad areas listed below:

• Issues with the identification of vertices
• Issues with parallel and perpendicular lines
• Comparison of area and volume
• Relative positions and perspective

The mathematical interviews that I have conducted have sought to determine three main aspects:

• What types of visual-spatial deficits that the students in my school have
• The students’ thought processes they have when they answer questions correctly with hesitation, or incorrectly with or without hesitation
• Identify the students with visual-spatial deficits who would perhaps need an intervention program in order to improve these difficulties

In the initial pilot study, students were given a short set of questions on the SmartBoard and asked to write their answers on it. The Recorder feature of the SmartBoard permitted the answers to be recorded. Also, it allowed me to ask questions which the students answered orally, and sometimes with further writing and drawings on the SmartBoard. The students’ oral and written responses were also recorded on the SmartBoard. This allowed me to start to identify and categorise the misconceptions and note the frequency of each type of misconception.

The next phase of the mathematical interviews added a few more questions. These questions helped to further refine the categories of misconceptions. Furthermore, the additional questions allowed me to determine if a student’s misconception was limited to one question (in particular The School of Athens question) or if it emerged in more than one question of the same type. Also, two questions with physical objects were introduced. In the first question, students were given the nets of two three-dimensional figures. They had to determine which net had the greater surface area. In the second question, the students had to determine which figure had the greater volume. In the second question, the three-dimensional figures did not have the same nets as the figures in the first question. This was done to minimise the chance that students were able to directly utilise information in the surface area and volume comparison questions. The responses for these two questions were recorded on a tablet.
computer. In this phase of the interviews, the total interview was usually split up into two different sessions so the students would not be overwhelmed by the number of questions, and would not be fatigued so the visual-spatial issues were not affected by these non-academic issues.

3.5.1 Strengths of the Mathematical Interviews

The mathematical interviews were a good starting point for a lot of my research. As mentioned before, the interviews were meant to gain meaningful insights into the students’ thought processes and to begin to understand why they interpret diagrams in a given way — whether they interpreted them correctly or incorrectly. The structured nature of the interview, but informal style, afforded students an opportunity to ask questions when they were uncertain of a question or part of a question. The informality was in the language that the students used in answering the questions. For example, a student may have said corner when referring to the vertex of a polygon. It further permitted them to express their difficulties with minimal fear of criticism or judgement. This, of course, is also a unique aspect of my specialist school which is discussed in greater detail in section 4.3 of this thesis. Even though some students reported feeling anxious when answering a question, on the whole students did not feel anxious about the entire interview process. This was especially true for some students who appeared to have visual-spatial deficits because this issue was finally being addressed. One of the students, Barbara, that wanted to have an intervention plan, said that she and her parents had known that she had these difficulties, but never had a plan to help her to address them.
3.5.2 Drawbacks of the Mathematical Interviews

Given the unique nature of my school, I have known several students that I interviewed very well even if I did not currently or previously teach them. They have felt comfortable to seek help with me, and in a few cases have sought help with me more than they have with their own mathematics teachers at the time. In some cases, the relationship that I developed with the students extended to their families. Occasionally, the students’ siblings, and sometimes even parents, would see me for tutoring in their mathematics courses or research. Given all of this, it was sometimes difficult for me as the interviewer to be perfectly objective. In the initial interviews that I had conducted, I would often not ask probing questions because I thought that I already had known why a student would answer a question in a certain manner; I thought the main reason for this was because they had done so previously in their work with me. When I asked more probing questions in subsequent interviews, sometimes I was surprised by the responses students gave. One student, answered the question in Figure 34 in a manner which surprised me even though I felt that I had known him well. He circled the people in the “back” as the small group of people on the leftmost part of the painting, the small group of people on the rightmost part of the painting, and the people farthest away from the centre of the painting, because he interpreted the painting as a movie and felt that the “ends” were the “back.”

Another difficulty with the interviews was with scheduling. It was very difficult to set an appointment at a time that was convenient for the student and myself. This caused many difficulties for both parties. In addition, as I split the
interview into two sessions, it was even more difficult to arrange a time to interview the students for both parts of the interview.

3.5.3 The Purpose of the Mathematical Interviews

The purpose of the mathematical interviews will be to provide an insight into the nature of each interviewee’s visual-spatial deficits that a questionnaire cannot. Since these interviews will allow students to answer questions that are presented on the SmartBoard, I will be able to record not only their answers, but also gain some understanding of the thought process that led them to their answers by asking probing questions. The questions will potentially help to determine if there are patterns of misconceptions that lead to what is interpreted as visual-spatial deficits. Also, the interviews will allow students an opportunity to declare any objections or uncertainty about a question that is asked. This is especially important because many students in my specialist school have been diagnosed with language-based learning disabilities which will make a potentially difficult task even more challenging.

Finally, some of the students may not have been exposed to, or may not remember some of the terminology (such as the vertex of an angle) in a particular question. The interviews will provide an opportunity for me to give an explanation and to more accurately determine if there is a visual-spatial deficit, and minimise the chance that the questions were not answered accurately because of a language deficit, and not a visual-spatial one. This will also provide an opportunity for me to reword questions so they will become less ambiguous for
future interviewees, and make it easier to establish if they have visual-spatial deficits.

3.5.4 Mathematical Interviews vs. Questionnaires

As mentioned before in this research, since many of the students that I will interview have been diagnosed with learning disabilities, mostly language-based learning disabilities, having a page that is filled with text, even if they have accompanying diagrams for each of the questions, would not yield the most useful results. Certainly the interpretation of the language will be a difficulty, and it will be challenging to determine to what extent language played a role when students answered the questions. In addition to what I have already written about the benefits of the mathematical interviews, this approach has an additional advantage. I can observe each interviewee’s body language, and speech tone to gauge their anxiety. This would not be possible even if I was watching the students complete a questionnaire on a one-to-one basis. In fact, if I were to do this, just the act of me watching the students who complete the questionnaires may cause them to have anxiety. Furthermore, since many of the students have also been diagnosed with ADHD, it may be difficult for students to concentrate on answering the questions if I were watching them.

If I were to select students for further interventions based on the results of a questionnaire, it may lead to a selection of students who appear to have a visual-spatial deficit when in fact they do not. The students who are ultimately selected for the interviews may have a language deficit, and may interpret the
questions incorrectly. These students will not necessarily have a visual-spatial deficit.

Finally, since I have a small group of students that I can select in my school, the questionnaires are not going to be the best tool to use. Mathers (2014, p.727) argued that questionnaires are best suited for collecting large amounts of data from a large group of people. This certainly will not apply to my research. Furthermore, he noted that there are several disadvantages to using questionnaires. Most importantly for my research (2009, p.6), he also stated that there is no way of determining how much thought and effort a person has placed into answering a question. In addition, it also difficult to determine a respondent’s emotions and behaviour when answering a question. This is such an important part of my research because the interviews will help me to give a better picture of each student that I am working with, than if I were to ask students to complete a questionnaire. For these reasons, I feel that a questionnaire will not be the best way to gather information for my research.

3.6. **Explanation Building and Drawbacks of a Case Study Approach**

An advantage of using case studies is that it helps to build explanations. Yin (2014, p.147) wrote that explaining a phenomenon “is to stipulate a presumed set of causal links about it, or ‘how’ or ‘why’ something happened.” He further wrote that that these causal links may be complex and very difficult to measure in a precise manner.

The explanations that most case studies have given are in narrative form. The origins for the explanations will come from evidence gathered from the
interviews and intervention plans that are a part of my research. As I focus more on obtaining rich descriptions from my interviews and interventions, the explanations may have a stronger base.

Yin (2014, p.150) noted the drawbacks to using case studies. He referenced the work of Diane Vaughn (1992) who reported that two major problems may arise. First, the iterative process of explanation building may lead the researcher to stray from his or her original research questions. What Yin and Vaughn are referring to is the iterative process of this aspect of case studies. Yin (2014, p.149) wrote that since this process can lead to rival explanations, it may cause the researcher to stray from his or her original research questions.

The second drawback that Yin (2014, p.150) reported is that an “unwanted selective bias may creep into the process, leading to an explanation that glosses over some key data.” This is something that I must be especially careful to avoid since my school is such a unique setting, and because I know the students that I am interviewing so well. I already have preconceived notions of the interviewees since I have taught so many of them and am already familiar with some of their difficulties. However, to ensure that I minimise any type of bias, I have asked the students more probing questions that established why they have interpreted a diagram or physical object in a given matter. This questioning has led to some interesting and very unexpected responses from the students when they have explained their reasoning.
3.6.1 Other Drawbacks of Case Studies

Carey (2012, p.7) noted some other challenges of using case studies. Of particular importance to my research, he stated five drawbacks, all of which are applicable to my research. Carey (2012, p.7) reported that the five disadvantages are:

- Data that are collected are unique to the process. I suspect that this is especially true because of the reasons stated in the last section
- It is difficult to establish validity or reliability
- There is case selection bias. This may be somewhat unavoidable considering the small group of students who are in my school and how well I know them academically and personally
- The conclusions are highly subjective
- The results are not generally predictive.

The last two disadvantages that Carey gave may be especially applicable to my research. Whilst there is evidence to support my conclusions, I fully realise that since I know my interviewees well and have created interventions for them in their mathematics classes (in a context outside of my research), I may have a tendency to view conclusions with my experiences with them in mind.

Finally, Carey’s statement about the results not being predictive are especially true for my research. In fact, the results of my interviews have shown that the results are not predictive. Though Cohen and Mannion (1994, p.123) referred to Adelman (1980) and stated that: “Case studies allow generalisations either about an instance or from an instance to a class,” I do not believe my research will allow for this. The students who have significant visual-spatial deficits tend to be exceptions rather than the rule in my experience. Whilst there
are some similarities in the deficits that these students have, the way they appear in class is usually not the same. However, Cohen and Mannion (1994, p.123) also noted the unique strength of case studies is “in their attention to the subtlety and complexity of the case in its own right.”

3.6.2 Case Studies and their Importance to my Research

Yin (2014, p.9) listed different research methods that are often used in social science research:

- Experiment
- Survey
- Archival analysis
- History
- Case study

Since my research aims to answer “how” and “why” questions, Yin reported that the appropriate methods for this type of research are experiments, histories, and case studies. Since I cannot control behavioural events, experiments are excluded. Since a history does not focus on contemporary events — namely determining the visual-spatial deficits that students have, and how to develop interventions for them — and would not be very useful since they have not focused on the topics that I am researching, a case study is the most appropriate method for my research.

In addition, the case study approach will allow me to try to gain insights into the thought process that each student uses. This could help provide insight
into how each student thinks about a problem. The other methods will not provide this opportunity.

3.7 Grounded Theory Approach

Corbin and Strauss (2008, p.1) reported that Glaser and Strauss developed the grounded theory approach in 1967 to build theory from data. Glaser and Strauss (1967, p.2) defined the grounded theory approach as “the discovery of theory from data systematically obtained from social research.” Charmaz (2008, p.81) further added that “Grounded theory methods offer you a flexible set of inductive strategies for collecting and analysing qualitative data.” She (2008, p.82) further added that this is a:

“... comparative and interactive method. You begin to construct your analysis by comparing bits of data — ideas an incidents — with each other. Your comparisons involve you in the analysis. Grounded theory keeps you interacting with the data and your emerging ideas about it.”

Since almost all of the data was from my social research which was conducted at my school, this seemed to be the best approach to use. This approach was not selected by default, but rather because of its usefulness, and appropriateness.

Trochim (n.d.) wrote that there are several key analytic strategies involved in the grounded theory approach. Carey (2012, p.7) wrote that these include: coding, memoing, and integrative diagrams and sessions. Trochim (n.d.) defines these terms as:

Coding is a process for both categorizing qualitative data and for describing the implications and details of these categories. Initially one does open coding, considering the data in minute detail while developing
some initial categories. Later, one moves to more selective coding where one systematically codes with respect to a core concept.

Memoing is a process for recording the thoughts and ideas of the researcher as they evolve throughout the study. You might think of memoing as extensive marginal notes and comments. Again, early in the process these memos tend to be very open while later on they tend to increasingly focus in on the core concept.

Integrative diagrams and sessions are used to pull all of the detail together, to help make sense of the data with respect to the emerging theory. The diagrams can be any form of graphic that is useful at that point in theory development. They might be concept maps or directed graphs or even simple cartoons that can act as summarizing devices. This integrative work is best done in group sessions where different members of the research team are able to interact and share ideas to increase insight.

My data has included the first two of these three strategies. I have coded the visual-spatial misconceptions according to their type, such as two-dimensional, three-dimensional, and physical objects, and subtypes such as two-dimensional angle, or three-dimensional ordering. This has helped to categorise the visual-spatial deficits in an effective manner.

The memoing that I have done was both in the manner that Trochim has outlined, and also in a different manner. Other than taking my own notes, I have taken notes that have been recorded on the SmartBoard videos. These notes have been in the form of the written and oral responses that students have provided, in addition to my own comments.

3.8 An Explanation of Insider Research, and the Benefits and the Drawbacks

Unluer (2012, p.1) wrote that there are advantages and disadvantages for being an insider who is doing research. This section will discuss both of these
aspects of insider research in greater detail. Unluer (2012, p.2) referenced Breen (2007, p.1) who stated that “insider-researchers are those who chose to study a group to which they belong, while outsider-researchers do not belong to the group under study.” Whilst I am not a student with learning disabilities, I do consider myself an insider because I have been part of the school’s community for almost 17 years. Unluer’s work is especially relevant to my work because she too has worked in a school for students with disabilities. Whilst she worked in a Turkish higher education institute, the School for the Handicapped, where she taught students with hearing disabilities, I feel that there are enough similarities between our institutions that it will be easier to make connections between her research and mine instead of other researchers’ work.

Unluer (2012, p.1) cited the work of Bonner and Tolhurst (2002) who stated that there were three key stages of being an insider. Firstly, these researchers stated that the inside researcher should have a greater understanding of the group that is being studied than someone who is not an insider. Given the very unique nature of my specialist school, which is discussed in greater detail in the next chapter of this thesis, I believe that I have gained an insight to working with students in my school that most outsiders, and even teachers who are new to the school do not have. This includes an understanding of the nature of the students’ disabilities, but also the hierarchy of the school, and the politics associated with the school. As Unluer (2012, p.1) wrote, the insider has a special understanding of how the school “works.”

Unluer (2012, p.1) went on to state that Bonner and Tolhurst (2002) wrote that the second stage of being an insider is “not altering the flow of social
interaction unnaturally.” Throughout my work with the students in my school, I have done my best to allow for a natural social interaction. Before students participated in the research, they were given an explanation of the research, my goals, and why it is important. Students were also given several opportunities to ask questions about the research. This process allowed the students to ask any questions that they have, and express their doubts. It also allowed them a chance for students to speak about their anxiety, and hopefully reduce it. This also allowed our interactions to flow without any major roadblocks. In addition, the loosely structured, and informal but respectful nature of the interventions also allows for an open exchange that allows the students to benefit from the meetings where they get the specific help that they need. Referring to the first stage of Bonner and Tolhurst’s (2002) research, having an insider’s knowledge of how to approach the students about my research, and how to teach them in a more effective manner was especially helpful in achieving this.

The third and last stage of being an insider (Unluer, 2002, p.1) as stated by Bonner and Tolhurst (2002) is “having an established intimacy which promote both the telling and the judging of truth.” Because of the relationship that I was able to develop, which probably would not be possible in larger or even some similar-sized schools, I believe that the students will feel comfortable telling the truth during the interviews that I have conducted, and the interventions that I will conduct in future stages of this research.
3.8.1 Advantages of Insider-Researchers Conducting Research

There are several advantages that insider-researchers have. Unleur (2012, p.5) built on the work of Coughlan (2003), Herrmann (1989), and Rouney (2005) and stated that insider-researchers know:

- The local values of an institution
- Particular knowledge and taboos associated with the group that is being studied
- The formal and informal power structure of an institution
- How to obtain permission to conduct research
- How to access records and documents to easily facilitate the research process
- How to ask clarifying questions more easily

In addition, she reported that administrators (managers) who are part of the institution that is being studied give importance to the research, which has been very true at my school. Also, since I have been teaching at the school for more than almost 17 years, my colleagues have also afforded my work with a lot of respect.

3.8.2 Disadvantages of Insider-Researchers Conducting Research

Unleur (2012, p.1) refers to Delyser (2001, p.441-442) who noted that insider researchers make wrong assumptions about the research that they do because of the prior knowledge that they have. This can lead to a lack of awareness of the potential issues that appear during the research. This will especially be true as I conduct my interviews. I will work to avoid making assumptions about the responses that the students will give,
and come to an inaccurate conclusion. Also, from the students’ responses I may also assume that they are processing the information a certain way because so many students that I have previously worked with have processed that type of question in the same manner. In doing this, the objectivity that an insider-researcher has may be lost.

Another problem that insider-researchers face is that they may not account for important information (Unleur, 2012, p.2, p.6). This is easy to overlook when certain behaviours and actions that the people who are being studied demonstrate are not described by the researcher because they are so used to what outsiders would deem to be important and notable. This type of difficulty may be an issue as I conduct my interviews and interventions. It is important to be especially aware of this particular issue as I conduct my research. Unleur (2012, p.6) built on the work of Hermann (1989), Rooney (2000), Sikes and Potts (2008), and Smyth and Holian (2008) and said the insider-researcher’s closeness to the situation could hinder the researcher from “seeing all of the dimensions of the bigger picture.”

Unleur (2012, p.6) noted that participants in the research may assume that the insider-researcher already know what they are speaking about. However, this is not always true, and can lead to missing information and an incomplete picture, which is especially difficult when conducting qualitative research. This may especially be an issue when I conduct teacher interviews to determine the effectiveness of the teacher training sessions.
The insider-researcher role duality can be a major disadvantage (Unleur, 2012, p.6). She referenced DeLyser (2001) who referenced Bogdan and Biklen (1998, p.52). DeLyser (2001, p.442) wrote “Those with a preexisting role can find that role in contradiction with the separate status of researcher; the transition between roles may cause personal difficulties; and ethical issues may arise when studying coworkers, particularly if a researcher is in a position of power over them.” This is something that I especially have to consider when working with the mathematics teachers since I am one of their supervisors, and they may feel compelled to agree with my point of view.

Unleur (2012, p.2) noted that an insider-researcher must be especially careful to do credible research. She refers to the work of Smyth and Holian (2008, p.407-408) who wrote that an insider must especially have an awareness of bias. Unleur (2012, p.2) wrote:

“To conduct credible insider research, insider-researchers must constitute an explicit awareness of the possible effects of perceived bias on data collection and analysis, respect the ethical issues related to the anonymity of the organization and individual participants and consider and address the issues about influencing researcher’s insider role on coercion, compliance, and access to privileged information, at each and every stage of the research.”

3.9 Analysing SmartBoard Interviews

The SmartBoard interviews that I will conduct will help me to determine if the students that I am interviewing have visual-spatial deficits. The SmartBoard will allow me to capture the most important information from the interviews whilst simultaneously maintaining each student’s anonymity. The recordings made on the SmartBoard will allow me to easily provide evidence for any visual-spatial deficits that the students who I interview have, and will
additionally allow me to categorise the visual-spatial deficits that emerge in the interviews. Ultimately, the SmartBoard recordings will allow me to develop meaning from the interviews which is an essential aspect of the case study approach that I will ultimately use in my research.

3.10 Research Context

The context for the research that I am conducting is unique. Since the school that I teach and conduct research in is atypical, it should be a constant reference point by any reader of this research.

There are several issues that should be considered in the research context. Firstly, my specialist school is a very unique environment where the teachers tend to be well versed in teaching students with learning disabilities, and the students are generally eager to learn about their learning disability. In addition, the teachers in the school often have a lot of specialised training through CPD courses, and on the job training, usually starting as an assistant teacher. This is so important that the next chapter of this thesis will specifically address this very issue.

Secondly, since there is not much research in this field that is dedicated to MLDs, the conclusions that will be drawn from the research should be especially considered. There is not much research that will verify or contradict my conclusions.

Finally, the research that I am aware of has not provided a neurological or scientific underpinning for my conclusions. As mentioned before, Birsh (2005,
p.8) wrote about the disproportional amount of research on dyslexia, and my own
Primo search also suggested that this is still the case.

3.11 Ethical Issues

Working with students who have diagnosed LDs brings forth several important ethical issues. Firstly, since the students that will participate in my research can sometimes become easily upset when they are given feedback that may reaffirm that they have a disability or suggest that they have difficulties that they were not previously aware of, I will take care to ensure that they will have an effective way for them to process their emotions. In addition to the positive relationship that I have developed with them, which will be discussed in greater detail in Chapter 4 of this thesis, I will take several steps to ensure that either the high school psychologist, social worker, or principal (head teacher) will be available to speak to the students if they become upset and I could not help them to process their emotions. Usually, these professionals will be on the same floor or one floor away, and will be available to meet with the students almost immediately.

In addition, the students’ parents will be required to give consent for their children to participate in my research. They too will likely be available to speak to their children if needed. The heightened level of trust that the parents have in the teachers of the school — again, something that will be discussed in Chapter 4 of my thesis is a unique aspect of my school that will also help to engender trust in the students that will participate in my research.
Chapter 4: My Specialist School

4.1 The Unique Nature of My Specialist School

The school that I teach and conduct research in is very unique. In 1973 it started as a primary school exclusively for students with learning disabilities in the basement of a church. As the school grew, a middle school was added and the school’s mission was amended to meet the needs of more students. In 2000, a year before I started working at the school, a high school was added, and the mission statement was further amended to meet the needs and challenges of the both the high school students who were previously in the middle school as well as new students from other schools. Since the school has a unique mission of educating students with learning disabilities, there is a specific subset of all students who attend my school. These students receive a very specialised education that has many significant aspects that are different from the general education schools that their peers attend.

4.1.1 The School’s Mission Statement

The school’s mission statement which was most recently revised in 2015 reflects the school’s goals as determined by a committee that was comprised of teachers, school principals, mental health professionals, members of the Board of Trustees (some who have children who attend the school, and some who have children that have graduated from the school), and teachers. The mission statement (Reference withheld to maintain the anonymity of the school) is:

“Our Mission [school’s name withheld], a K-12 coeducational college [university] preparatory day school, is dedicated to working collaboratively with students, educators, and families to help children with language-based learning
disabilities realize their full potential. By building upon their strengths, we provide a rigorous program that teaches children perseverance, resilience and the importance of self-awareness and self-advocacy. We prepare our children to become courageous, confident, productive and caring people who will embrace the challenges and opportunities of the 21st century. Tolerance, respect, and active engagement are hallmarks of our intentionally diverse community. [School’s name withheld] recognizes its responsibility to the wider educational community and is committed to remaining a leader in its field.”

This statement is included to give the reader an understanding of the principles that guide all of the employees, students, and families in the school. In particular, it shows the specific skills that the students should aim to develop despite their learning disabilities.

4.2 Retention of Students

Most of the students who attend my school start in the elementary school, and tend to stay in the school until they graduate from the high school. A few students leave the school at the end of fifth grade, and a few more usually leave at the end of eighth grade and choose to go to a general education high school, some with extensive, and others with minimal learning supports. Typically 3 to 5 out of each cohort of about 36 students leave the school early; these students tend to have milder learning disabilities than their peers at the school, and have often developed better compensatory skills that enable them to utilise strategies in order to overcome their learning disabilities. In the last seven high school graduating classes, the average time that a student attended the school was between nine and ten years. In fact, at least four or five of the students from each of these graduating classes were in the school for at least ten years. There are numerous educational, financial, social, and emotional reasons that the students
Because the students have been at the school for such a long period of time, they tend to feel very comfortable with their peers. Since there is a minimal amount of bullying, and what appears to be an excess of understanding of each other’s academic and emotional challenges, the students tend to feel that the school is a very safe place. Often, students have come from schools where their classmates, or even their teachers have ridiculed them for their learning difficulties, and all of the social and academic difficulties that they bring. Many of the students in the school have pragmatic language difficulties or what are perceived to be poor social skills (especially for NLD students), and they tend to have a significant impact on their social lives — especially for people who do not understand or have the patience to work with students with these difficulties. In addition, since the teachers in the school are trained to be especially sensitive to these issues, the students and their parents have expressed that this was an especially important feature of the school. This has been particularly true with parents and students who have come to the high school from a general education setting where the other students and teachers had difficulty comprehending their learning disabilities.

This statement is taken very seriously by the faculty, school administrators, parents, and students. In fact, representatives from each of these groups were responsible for constructing the mission statement two years ago. The mathematics teachers, even before this Mission Statement was released,
made an effort to strive towards these goals because they were considered especially important.

4.4 Low Student to Teacher Ratio

The student to lead teacher to assistant teacher (or paraprofessional) ratio can never exceed 12:1:1 in any special education class as mandated by U.S. federal, and New York State laws. Because the academic and most elective classes, whether academic or co-curricular, are so small, and students are able to receive daily individualised instruction, the parents do not want their children to leave what they find to be a successful environment for their children to learn. In addition, since over 90% of students are funded entirely by the New York City Department of Education, or the local educational authority that is determined by the student’s residence, if they live outside of New York City, the parents want to take advantage of a free independent school education that costs over $50,000 U.S. per annum. Whilst the public (government) schools in the U.S. are tuition free, the ratio of students to teachers can be as high as 36:1:1 in New York City.

4.5 Parental Participation

The parental participation in their children’s education is quite high. Because it is so difficult for the students to gain admission to the school (less than 3% of the applicants are admitted) the parents had to be extensively involved in getting their children into the school. In fact, the admissions process is quite lengthy and requires that each student’s parent or legal guardian obtain a psychoeducational evaluation, a long and expensive process which usually costs
$2000 - $3000 with insurance, and sometimes even more, if completed by a licensed psychologist or psychiatrist in New York City.

In addition to the initial application and submission of the psychoeducational evaluation, the parents or legal guardians of the applicants must additionally go through the other parts of the application including interviews with school principals, social workers, and psychologists.

Furthermore, the parents or legal guardians are interviewed themselves. Finally, each applicant must also submit letters from teachers that describe the nature of the applicant’s learning difficulties, and an explanation of the child’s willingness to work and cooperate with teachers and other students.

If the applicant is admitted, the parents then usually seek funding from the local educational authority. Since these authorities are not always willing to fund the student, the parents must generally hire a solicitor, make a case in a court and convince a judge that the local schools cannot provide an appropriate education for their children. This process, which is quite expensive, is also very lengthy and requires the persistence and dedication of the parents.

4.6 **Teacher Training**

The teachers at my specialist school generally receive inservice training for a longer period of time in teaching students with learning disabilities than teachers in general education schools that offer push-in services, (where a learning support teacher assists the general education teacher in educating a learning disabled student in a general education setting) or even self-contained classes to these type of students. Since almost all of the lead teachers in the
elementary, middle, and high schools started as assistant teachers, they have a longer inservice training period of at least one year and usually two years or longer, which is considerably longer than teachers in most general education settings. Since teacher training for new secondary teacher trainees in New York involves six weeks of training in each a middle school, and a high school for only part of a school day, the training that assistants in our school receive is for a full academic year, and for a complete day where they learn about all of the facets of teaching a full set of classes including lesson planning, grading homeworks and class assignments, and constructing and marking examinations. Typically, most of the lead teachers in the school have been assistant teachers for two or more years, and some as long as seven years before they become lead teachers. This crucial development time for new teachers is especially important because not only do teachers gain a much deeper understanding of how to teach learning disabled students, but they are also immersed in the unique culture of our school.

In the high school mathematics department, the assistant teacher training has had a significant beneficial impact for the students in a variety of ways. The most striking aspect of the teaching is the consistency in the approaches to teaching between the teachers in all of the different courses in the high school ranging from Algebra I to Calculus. This consistency has appeared in multiple ways and appears to be very helpful for the students.

The first similarity in the mathematics classes is the structure of each teachers’ lessons. Generally, if there was homework assigned in the previous class, the homework is reviewed first, and students are given a chance to ask questions about the assignment. Usually, the students will have a chance to ask
about any questions that they have. If the teachers feel that they need to move on to the lesson, the students will be offered a time outside of the lesson to seek extra assistance.

After the homework, the lesson is started. From the start of the lesson, the topics are clearly defined. Often, only one type of problem is covered for the day in order to ensure that there is ample time for the students to have guided practice with the teachers and their peers, and then finally independent practice. If the students are especially having difficulty with a given topic, the teachers usually do not assign homework for the topic until they feel that the students have some comfort with the work, and they have enough problems that were completed in class to use as a reference.

Another consistent aspect of the lessons is that the students are usually given handouts. These handouts generally have the question and accompanying diagrams, as would be given in textbooks or examination papers, organise the questions in a manner that is easy for the students to follow, and gives an adequate space for students to show their workings. Since so many of the students have executive functioning issues which impact the organisation of their work, the handouts reduce these difficulties and allow them to focus on learning mathematics.

In addition to these similarities, the mathematics teachers in the high school tend to utilise the same methods to teach a specific topic. Some of this is by design, and some of this is because the teachers in the department (made up of only four lead and four assistant teachers) tend to have the same view of teaching. One of the other lead teachers in the department worked with me as an
assistant teacher for her first two years at the school. As a result, she tends to use many of the same techniques and similar language that I have developed. When she became a lead teacher, she trained one of the other lead teachers, who in turn uses many of the same techniques and similar language. The third lead teacher, who had several years of teaching experience in general education settings, also uses the same techniques that the other teachers use. When all of the lead teachers train new or returning assistant teachers, we make a conscious choice to train them in the same manner. This provides a lot of consistency across the classes. This also helps to lower the students’ mathematical anxiety because they have a reasonable idea of what will occur during the class and there is a set routine.

The methods that are used tend to simplify the language as much as possible. This is especially important given the language-based learning disabilities that many of the students in the school have. Also, when technical terms specific to mathematics, such as hypotenuse, or function are used, they are repeatedly used and redefined in the class by both the teachers and students multiple times in a lesson. This also helps the students to remember these terms, and use them properly.

In addition to the language, the other techniques that he mathematics teachers use are very consistent. For example, when the teachers teach the students how to solve linear equations in one variable, the students are usually first taught to draw lines around the equal sign so they can more easily distinguish between the left and right sides of the equation, which can be difficult for some of the students who have difficulties with directionality. In fact, some
students are even taught to write “left” over the left side of the equation, and “right” over the right side of the equation. Then the students are always taught to move the variables to the left side of the equation, and the constants to the right side of the equation (if this is not already the case). Lastly, students are taught to solve for the variable in the given equation by using the techniques that they have learned in the class. A similar strategy is used to teach students how to solve equations with multiple variables. Figure 7 shows an example of this strategy that was used in one of my ninth grade Algebra I classes.

Figure 7: Example of Vertical Lines Technique for Solving Linear Equations

When students are taught to solve word problems, they are taught a routine that they could apply to most types of word problems. The method that they are taught is to write a list of the given information, followed by what they must solve. In addition, students are then taught to write any equations they might use to solve the problems. Since the procedure is relatively simple, it is
This last aspect of the teacher training is especially important because the school’s culture is so unique in many different ways. Whilst there is no simple way to describe what is unique, the most important differentiating features include an understanding of how the learning disabilities of the students appear in the classroom, and having an abundance of patience for the challenges that the students face in learning new information.

4.7 Lead Teacher and Assistant Teacher Working Relationship and Training

The lead teacher and assistant teacher training varies by department. The mathematics teachers have numerous CPD training sessions which are usually held during department meetings. Recently, I facilitated a full day workshop about MLDs for all of the new teachers in the entire school which all of the mathematics teachers of course, participated in. All of the new mathematics teachers, and most of the teachers who attended the workshop (even those who had a degree in special education) reported in a school constructed survey that this was the first time that they heard of MLDs including dyscalculia.

In addition to this full day workshop, all of the high school mathematics teachers regularly attend department meetings where there are different themes that guide our meetings. The professional development that takes place in these meetings will be discussed further in Chapter 8 of this thesis.
Chapter 5: Pilot Study and Mathematical Interviews Results

5.1 Initial Pilot Study

The initial pilot study will be used to determine the types of visual-spatial deficits that the students that I interview have. I will use a modified version of Karaganakkis’ model for classifying the MLDs, which is described in section 5.2 of this thesis.

5.2 A Theoretical Framework for this Research

Karaganakkis (2012, p.3) proposed a theoretical model for classifying MLDs into four subtypes: non-verbal, verbal, logical-procedural, and visual-spatial. These four subtypes correspond to the parts of the brain that regulate each of their functions as shown in Figure 8.

![Figure 8: Karaganakkis’ MLD Model](image-url)
I intend to build on Karagiannakis’ model and further separate the visual-spatial section into two parts, where two-dimensional and three-dimensional deficits are studied in greater detail. My revised version of this model is shown in Figure 9.

![Figure 9: My Revised Version of Karaginakkis’ Model](image)

This revised model is based on my teaching experience and the results of my pilot questionnaire. Some students with significant visual-spatial deficits had difficulty with questions that had both two and three-dimensional figures, whilst others had difficulty with only three-dimensional figures. Classifying the differences in two and three-dimensional figure visual-spatial deficits and their effects on learning mathematics will be a very important part of my research.
5.3 A Rationale for Mathematical Interviews and a Specific Classification of Visual-Spatial Deficits

As mentioned before in this research, there are several standardised and non-standardised tests available to screen students for visual-spatial deficits. These tests whilst helpful, do not give mathematics teachers specific information about the type of visual-spatial deficits that would appear in a mathematics class and especially in a geometry class. The Beery-Buktenica Development Test for Visual-Motor Integration, 6th edition (Beery VMI), for example, is a standardised tests that “offers a convenient and economical way to screen for visual-motor deficits that can lead to learning, neuropsychological, and behaviour problems” (McCrimmon, 2012, p.1). Whilst this certainly is important, it does not help a mathematics teacher determine if the student has difficulty with two-dimensional or three-dimensional representations, or to what extent the students have these specific deficits. This test does not show how the visual-spatial deficits would manifest themselves in their classes. Finally, teachers are typically not permitted to see the questions that are given on these tests, so it does not provide a reference for understanding their students’ visual-spatial deficits.

Because of the significant drawbacks of this and other existing assessments, I believe that the structure of my interviews will prove to be a more practical resource for teachers. The interviews that I have constructed will help teachers classify the deficits as they could possibly appear in their mathematics classes. In particular, since the interviews that I have constructed also include state exam and textbook questions, which many teachers use in their instruction, it will be especially useful for teachers who will potentially encounter students with these visual-spatial deficits in this scenario. Furthermore, my interviews will
require students to demonstrate their functional knowledge. For example, a student may memorise the relationships between the angles formed by two parallel lines that are cut by a transversal, or the basic properties of a rectangular prism. Teachers may interpret this memorisation and recitation of these facts as understanding. However, if students cannot draw the figures that represent these scenarios, this shows a different dimension of their visual-spatial deficits. In addition, this gives teachers perhaps the most useful important information, namely that students who have difficulty drawing diagrams will have a greater challenge.

5.3.1. Specific Categories of Misconceptions

I believe that having specific categories of misconceptions will prove to be powerful. When teachers, especially those new to the profession, are given specific classifications of the visual-spatial deficits that their students have, then they will be better able to help their students learn mathematics. This enhanced knowledge will help them to develop better instructional techniques, and if needed, longer-term intervention plans. Furthermore, these intervention plans could include a variety of strategies including those that were used by Matthaei (2008). Of particular importance, is having physical three-dimensional models to support a student’s understanding of three-dimensional diagrams on a page.

5.4 Initial Pilot Study Results

In my fieldwork, the selection of students was an important factor. At first, all of the ninth grade students (ages 14-15) and then the tenth grade students (ages 15-16) were invited to participate in the mathematical interviews.
However, since even the students who expressed interest in participating in the interviews did not always follow through by submitting their consent forms, or could not find a mutually convenient time to be interviewed, I had to also invite both the eleventh and twelfth grade students (ages 16-17 and 17-18, respectively) to participate. This led to the interviews being conducted with a wide range of students in both age, mathematical knowledge, and capabilities. For example, some students were enrolled in an Algebra I course, whilst others were enrolled in Calculus and had completed all of its prerequisite courses, namely Algebra I, Geometry, and Algebra II & Trigonometry.

Since all of the students that participated in this study are learning disabled and almost all of them have Individualised Education Plans (IEPs) – a government document that is recognised at the federal, state and local levels – that identifies a student’s classification of learning disabilities and entitles him or her to a variety of supplemental services and accommodations, all of the participants are vulnerable. As such, extra care was taken to ensure their privacy. In addition, the interview process was explained in detail to both the students and their parents. Both were informed that the students could withdraw from the study at any time and for any reason up to the point of publication of this research.

After the interview was completed, if the results suggested the possibility that there was a visual-spatial deficit, these children were given this information and informed that they could further participate in the study and perhaps have an intervention planned for them in the future, if appropriate. In case a student had a negative reaction to the results (which did not happen), both the psychologist and
social worker in the high school were nearby (within a two minute walk) and available to process the results of the interview and the student’s feelings with him or her. This was the case in all of the stages of my research.

In the initial pilot study, the results of the sixteen interviews that I conducted seemed to verify what I have seen in my teaching experience at my school. The main difficulties that my students have are of three different types: identifying the relationships between figures when there is a lot of information on a page (such as identifying pairs of parallel and perpendicular lines in a diagram with many lines); identifying the properties of two-dimensional figures that do not have the same orientation (such as determining which angle has a larger measure or which shape has a larger area); identifying the properties of three-dimensional figures represented in two dimensions on a SmartBoard screen (such as determining which three dimensional figure has a larger volume). This was especially true if the figures did not have the same orientation.

5.5 Visual-Spatial Deficits with Angle Properties and Measurements

The interviewees answered questions about angle properties and the relationships between two angles. In the interviews, I wanted to determine if students could correctly determine two aspects of angles: correctly identifying the vertex of an angle and determining which angle has a larger measure when given a pair of angles.

5.6 Determining the Vertex of an Angle

My interest in this field of research was first piqued when I taught a student who had such profound visual-spatial deficits that she could not identify
the vertex of an angle. Even when she was able to verbally explain what the vertex of an angle was and seemed to genuinely have an understanding of its meaning, she still had difficulty identifying it. Whilst I have never taught any other student who had demonstrated this difficulty, I wanted to determine if any interviewees could not identify the vertex. All of the 16 students who I interviewed identified the vertex of the given angles correctly. At the start of the interviews I expected most students to answer this question correctly. However, I thought at least one may answer this incorrectly based on my past experience. The orientation or measure of the angle did not matter. Whilst some students who had not previously studied geometry formally or recently did not know or remember the definition of vertex, they were able to answer the question correctly once its definition was stated for them.

All of the students that I have interviewed were able to correctly place a movable red dot on the vertex of an angle. Figure 10 shows an example of one of these questions that the students had to answer.

![Figure 10: Angle Vertex Identification Question 3](image)

(All 23 interviewees answered this question correctly)
There were three questions of this type and all 16 interviewees answered each of these questions correctly. In additional interviews with seven students who were not part of the initial pilot study, all of the students answered this question correctly, and it appeared that they were able to do so without difficulty.

### 5.6.1 Comparing Angle Measures

The next set of questions regarding angles required students to identify the angle with the larger measure by circling it. The students were informed that they were able to place the diagrams of one of the angles over the other for easier comparison. There were three questions of this type. All of the 16 interviewees answered each of these questions correctly. One of the questions, shown in Figure 11, shows angles with the same orientation. However, the angle with the larger measure is drawn with smaller rays and the angle with the smaller measure is drawn with larger rays. Based on my experience, I thought that a few students would answer this question incorrectly because they would attribute the angle with the larger measure to the angle with larger rays. However, none of the students were persuaded to circle the angle with the smaller measure despite this. Perhaps because they were able to move the angles over each other on the SmartBoard it was easier for them to accurately make comparisons.

This will be something that I will use when I develop an intervention plan in future stages of my research. The intervention plan will be discussed later in this thesis.
In additional interviews with seven students who were not part of the initial pilot study, all of the students answered this question correctly. Even though these students did answer this question correctly, three students seemed to hesitate when they were answering the question. When I questioned them about their hesitation, they said that they were thinking about the length of the rays that were shown in the diagram for the smaller angle that is on the lower right side of the diagram. Student 22 said that “I had to think ... and realised that the length probably didn’t matter.”
5.6.2 Parallel Lines

The questions that required the interviewees to identify parallel lines or indicate if a set of lines are not parallel proved to be more problematic. Whilst the questions with less visual information were answered quickly and correctly, questions with more visual information were answered with greater hesitation and less accuracy. These results once again verified what I have experienced in teaching at my school.

The first type of question presented a pair of parallel lines and asked students to circle the lines that were parallel as shown in Figure 12. To my surprise, all 16 interviewees answered this question correctly. Since the students could not move one line on top of another, I thought some students would not be able to notice that they are parallel.

![Figure 12: Parallel Lines Identification Question 1](image)

In a further 7 interviews that I conducted, which were not part of the initial pilot study, all of the students answered this question correctly. There did
not seem to be any misconceptions about this question, and some students said that this question was easy to answer.

The second question regarding parallel lines had slightly different results. Figure 13 shows the question with two non-parallel lines. The students were told if the lines were not parallel, they could simply write “none” or give a similar answer. Whilst most of the students answered this question correctly, Student 3 did not. He indicated that both lines were parallel. When I asked him if he could define parallel lines, he said that they are lines “that go on forever without ever touching.” Despite knowing the properties of parallel lines, it appears the student may have some visual-spatial deficit that prevented him from noticing that the lines would eventually intersect. Because this student had been diagnosed with mild cerebral palsy by his doctor, this may perhaps have been the cause of this difficulty. Rosenbaum (2006, p.13) wrote that visual impairment is an aspect of cerebral palsy. His difficulty would be consistent with his condition.

In interviews with seven students who were not part of the initial pilot study, all seven students answered the question correctly. Just as with the preceding question, these seven students did not indicate that they had any significant difficulties.
The next three parallel lines identification questions were more difficult. Figures 13, 14, and 16 show these questions which required students to identify parallel lines when they are presented with three lines. In each diagram, at least two of the lines are parallel. Whilst most of the students answered these questions correctly, the incorrect answers and even some of the correct answers did reveal some of the misconceptions that students had.

The question shown in Figure 13 did have some interesting results. Of the 16 interviewees, 2 answered this question incorrectly. Student 4 who answered this question incorrectly said that the two diagonal lines may look parallel. She continued “I guess if you move them closer together they may be parallel.” This response is quite revealing. Firstly, what she stated about the lines being closer together, or having a diagram with information presented in a small area is something that students seem to prefer and understand more in the classes that I have taught. Secondly, the student who correctly stated the definition of parallel, seemed to have some misconception of parallel; it appeared that she believed that
the size of the diagram can change the relationship between the lines and alter if they are parallel or not.

Student 5, whose response is shown in Figure 15, even extended the middle line so it would intersect the other two lines. At first, I thought she was trying to determine if the alternate interior angles or corresponding angles that the lines formed were equal in measure to determine if the lines were parallel. Instead, at the end she stated none of the lines were parallel. She extended the middle line until it intersected the other two lines because it was “visually easier to see.”

**Figure 14: Parallel Lines Identification Question 3**
(2 of 16 interviewees answered this question incorrectly)
Student 6 had a very different interpretation of the diagram that was presented in this question. Whilst he answered the question correctly, he asked if it was possible for none of the lines were parallel. When I stated that at least two lines were parallel, he kept his answer. However, he said that he was “leaning towards none of the lines being parallel” if it were an option.

The responses to the question shown in Figure 16 also revealed some interesting information. Student 4 circled only the top two lines – which are closer together, to indicate that they were the only parallel lines shown in the diagram. Not surprisingly, she said that since the top two lines were closer together it was easier to see that those lines were parallel. The extra distance between the top two lines and the bottom line made it more difficult to see that the bottom line was also parallel to the others. Again, this is consistent with what I have noticed with the students that I have taught. When visual information is presented close together, students tend to understand the information more easily.

Student 18, however, circled only the bottom two lines to indicate that they were parallel. She said that the top line seemed to “curve in” to the middle
line and would eventually intersect. This was an unexpected result as I expected students would circle all three lines or only the top two lines that are closer together.

![Figure 16: Parallel Lines Identification Question 4](image)

(4 of 23 interviewees answered this question incorrectly)

In further interviews with seven students who were not part of the initial pilot study, two additional students answered this question incorrectly. Student 19’s answer echoed Student 4’s comments; he only circled the top two lines were parallel because it was easier for him to see. He said “I couldn’t see that the third line was parallel [to the other two lines]” because the bottom line “looked different.” Student 23 stated that she thought that none of the lines were parallel because they looked like they would intersect “if I drew them all the way.” When I asked her what she meant by this, she said that she would extend the lines all of the way across the SmartBoard. She was somewhat convinced that these lines were parallel when I extended the lines on the SmartBoard and she was able to see that they would not intersect. Even with this, she was not fully convinced of this. She said “I am still not sure” referring to the nature of the parallel lines.
The next parallel line identification question of this type had the non-parallel line intersecting the two parallel lines. The diagram in this question is similar to the diagrams that students have seen in questions from their mathematics courses where they have to find the measures of the angles in the diagrams. In fact, Student 5 who answered the question correctly noted that she was able to do so without hesitation because she recognised this type of diagram from the previous questions of this type that she completed in class. Student 10, who answered this question incorrectly made a mistake in determining if the lines were parallel. He put the SmartBoard marker so it would coincide with one of the lines that were parallel. He moved the marker to the other line to see if it had the same slope. It appeared that he inadvertently moved the marker and changed the manner in which he was holding the marker before comparing it to the other line. He incorrectly determined the other line was not parallel because he incorrectly placed the marker and thought the second parallel line did not have the same slope as the first.

![Diagram of parallel lines](image)

**Figure 17: Parallel Lines Identification Question 5**  
(3 of 23 interviewees answered this question incorrectly)
Two other students, Student 20 and Student 23, both recognised this type of question from their coursework but still did not answer the question correctly. After I showed Student 20 which two lines were parallel, she said that the “middle line,” referring to the transversal, changed how the parallel lines looked. When I deleted the transversal from the diagram, she was then able to see that the remaining lines were parallel.

Student 23, however, said that she just could not see that the lines were parallel even when I deleted the transversal to show her. I even moved the top line to coincide with the bottom line and she still could not see that the lines were parallel.

Finally, a question with four pairs of parallel lines was given. Students had to identify each pair of parallel lines by highlighting or circling them in the same colour. Figure 18, which shows this question, caused more difficulty for students. In the initial pilot study, even though only one student answered this question incorrectly, many students showed greater hesitation in answering this question. In addition to the increased amount of visual information, students took a longer time to answer this question because they had to develop a strategy to keep track of which lines were parallel.
Student 4, whose incorrect response is shown in Figure 19, did not notice the parallel lines that have the greatest positive slope were parallel at all. In fact, perhaps the main reasons she circled the lines with negative slopes as parallel is because I asked about those. More importantly, she stated that each pair of lines that were close to each other were parallel even though they had unequal slopes. A point for further investigation is to determine whether or not this student or other students believe parallel lines have to be directly next to each other. This misconception is something that I have encountered with other students.
In further interviews with seven students that were not part of the initial pilot study, the same difficulties re-emerged. Though only one additional student, Student 23, answered the question incorrectly, the other students also took a longer time to answer the question. All seven students struggled to answer the question, and it seemed had to spend time to determine a strategy to answer it correctly. Student 21, who answered every question in the interview correctly, spent more time on this question, and said it was the most difficult one up to this point in the interview. He said that he had to “think longer” and “see which ones [lines] are parallel.”

5.7 Perpendicular Lines and Planes

Just as with parallel lines, the students had more difficulty with correctly answering perpendicular line questions that had more visual information. However, with some of the simpler questions, students had more difficulty identifying if a pair of lines are perpendicular or not. Figure 20, shows a pair of perpendicular lines, where one line is vertical and the other horizontal. All 16

Figure 19: Student 4’s Incorrect Response to Parallel Lines Identification Question 6

In the diagram below, each line is parallel to exactly one other line. Highlight the pairs of lines that are parallel in different colors.
interviewees in the initial pilot study answered this question correctly. The additional seven students that I interviewed later also answered this question correctly. Several students said that they were able to determine that the lines were perpendicular because it was very easy to see. This confirms what I have seen with the students that I have taught.

![Figure 20: Perpendicular Lines Identification Question 1](image)

(All 23 interviewees answered this question correctly)

Whilst students were able to determine if lines were perpendicular when the lines were horizontal and vertical, if the lines were not displayed in this manner, then several students were not able to properly determine if the lines were perpendicular. Student 5, whose response to the question shown in Figure 21, seemed to have a genuine misconception of perpendicularity. When asked to define perpendicular lines, she stated these lines “intersect, but parallel lines don’t.”

Student 23 who also answered this question incorrectly stated that the lines “look like they form an L when they meet,” referring to the way that she knows that lines are perpendicular if they intersect and form an “L shape.”
Figure 21: Student 5’s Incorrect Response to Perpendicular Identification Question 2
(2 of 23 interviewees answered this question incorrectly)

Student 4 whose response to a different but similar question is shown in
Figure 22. She stated that the lines are not perpendicular because the lines do not
bisect each other, which she believed was a requirement for perpendicularity.

Figure 22: Student 4’s Incorrect Response to Perpendicular Identification Question 3
(5 of 23 interviewees answered this question incorrectly)
Of the additional seven students that I interviewed after the initial pilot study, all of the students answered the question correctly. Even Student 23, who had difficulty with many of the questions, said that the lines were perpendicular because they made an “L shape in the upper right” part of the point of intersection of the lines. However, she said that she could not see the other L shapes in the other three sections, even when I drew them for her on the SmartBoard.

This misconception was consistent throughout her interview. In Figure 23, shown below, she answered the question incorrectly. Again, she gave the same reason for her answer.

![Figure 23: Student 4’s Incorrect Response to Perpendicular Identification Question 4](image)

(6 of 23 interviewees answered this question incorrectly)

The difficulties that students had with more visual information were not just limited to parallel lines. The difficulties persisted with questions that had diagrams that displayed perpendicular lines. Figures 24 and 25 show some of the
different mistakes that the students made. These errors demonstrated the scope of their visual-spatial deficits. Student 5’s work is shown in Figure 24 identified almost all pairs of intersecting lines as being perpendicular. Interestingly, this student also made a circle in the upper-left part of the diagram where the segments did not intersect. This student thought that segments did intersect there even though there was no intersection there. Student 4’s response to the same question, shown in Figure 25, incorrectly identifies some pairs of lines as being perpendicular.

![Image of a diagram with marked lines and circles indicating perpendicular lines.]

**Figure 24: Incorrect Response Provided by Student 5 to Perpendicular Identification Question 5**

(6 of 23 interviewees answered this question incorrectly)

Interestingly, Student 23 answered this question correctly. She said that she was able to tell that the segments in the lower-left section of the diagram were perpendicular immediately because they formed the L shape. Secondly, she noticed that the segments that intersected in the upper-right section of the diagram were also perpendicular. She then guessed about the other perpendicular
segments. Student 23 said “I sort of ... circled where I thought the segments were perpendicular.”

![Diagram of segments and colored lines](image)

**Figure 25: Incorrect Response Provided by Student 4 to Perpendicular Identification Question 5**

Even when students know the definitions of parallel and perpendicular lines, and can apply their knowledge to solve questions with simpler visual representations, they have difficulty with three-dimensional figures shown on a page. This is a point for further investigation in my future research. Figure 26 displays a question that appeared on the June 2003 New York State *Integrated Geometry* Regents examination, an exam that students in government and some non-government secondary schools take at the end of their geometry course. The error made on this question by Student 4 is not unique and is the type of question that students answer incorrectly, even if they do not readily demonstrate visual-spatial deficits.
Interestingly, Student 22 (who did not participate in my initial pilot study) answered this question correctly and noted that planes $P$ and $R$ had to be parallel because the diagram “looked like a pamphlet that was unfolded.” Since the pages that would be made up of the aforementioned planes do not touch, and “look the same but in different places,” they had to be parallel to each other.

Perhaps the difficulty that students have with this question is that they have to combine the language presented in this problem with their knowledge of mathematics which is especially difficult for students with language-based learning disabilities. This would especially make sense considering that Student 16 had been diagnosed with dyslexia in primary school. In this question, she correctly highlighted the parallel planes that were being discussed, but maybe did not read the question carefully enough to notice that she selected the planes were perpendicular and not parallel. In my work with other students in my school,
even if they do not have visual-spatial deficits, they tend to become more anxious with questions that have three dimensional representations on a two dimensional surface. In this particular question, there was the aforementioned difficulty in addition to the challenges of reading the question that may have been caused by her dyslexia.

The anxiety that the students have in answering these questions is not limited to any exam. The language is really the more important issue. Their language-based disabilities, especially dyslexia, which many students were diagnosed with, add a level of difficulty that is a lot to overcome. The question shown in Figure 26 has quite a bit of difficult language. It required students to know the definitions of parallel and perpendicular, terms that students with dyslexia may incorrectly interchange. In addition, students must know the symbol for a line that is presented in the problem. Finally, the visual information must be processed quickly, and the words have to be matched to the diagram. Not surprisingly, students have had difficulty completing these complex tasks accurately.

5.8 Two–Dimensional Area Questions

In the initial pilot study, the interviewees mainly answered questions where they had to determine which two-dimensional figure has the larger area. The students who were interviewed were able to move one figure over the other on the SmartBoard to make the comparison easier.

Student 3, who as reported earlier in this research, has mild cerebral palsy. He answered 3 of the 6 area comparison questions incorrectly. Again, this may
be attributed to his condition which may have detrimentally affected his visual capabilities.

Two questions however, did cause a bit of difficulty for other students. Figures 27 and 28 show the third and fourth area comparison questions respectively which some students answered incorrectly. It is not surprising that the question shown in Figure 27 was more difficult to answer because the figures could not be moved. Even though the figures could be moved in the diagram shown in Figure 28, the students who answered this question incorrectly (except Student 3) tried to determine the area by just looking at the figures as they did for other questions. Since the areas of these figures were more similar than the figures in the other questions, this task proved to be more difficult and led to erroneous selections; 3 of the 16 interviewees answered this question incorrectly.

In my further interviews with seven additional students, three more and two more students respectively answered the questions that were presented in these figures incorrectly. Student 22 answered both of these questions incorrectly. She said that the question in Figure 27 was “easy” because the triangle was larger since “it opened up more at the top,” referring to the measure of the top angle of the triangle. When I asked her about the top angle of the hexagon, she said that it did not have as big of an opening, and cited the lengths of the segments as her justification for her answer. For the second question, which is shown in Figure 28, she did not move the figures at all and stated that the circle was bigger because “the round figure” had to be bigger.
5.9 Three–Dimensional Visual Spatial Deficits

Several students in both my initial pilot study and further interviews had difficulty interpreting the relationships between three-dimensional objects when
they were represented in two dimensions on a page or on a board. This sometimes made some of the questions that I posed to the students challenging for them in ways that I did not expect. However, I gained a wealth of information from one of the student’s responses to these questions.

5.9.1 Comparing the Volume of Three-Dimensional Figures

The question in Figure 29 required students to determine if the cylinder or the sphere had the larger volume.

![Figure 29: Volume Comparison Question 1](image)

(3 of 23 interviewees answered this question incorrectly)

The response that Student 6 gave to this question was quite surprising. He stated that he could not determine which figure had the greater volume because there was not enough information. He recognised that both the cylinder and sphere were “3-d shapes that looked 2-d” so a lack of understanding between two-dimensional and three-dimensional figures did not appear to be an issue. The first difficulty that the student had was that he did not know how to interpret the representation of the cylinder. He circled the cylinder on the SmartBoard and stated “for all I know I could be looking at the top of the cylinder” which showed
that he did not have a very strong understanding of how I expected him to interpret the diagram.

![Diagram of a sphere and a cylinder with notes](image)

**Figure 30: Student 6’s Incorrect Response to Volume Comparison Question 1**

Figure 30 shows Student 6’s final response to the question. I have realised from his response that the questions that I present have to be represented much more clearly.

Student 22 who also answered the question incorrectly said that the sphere was larger than the cylinder. She placed the sphere over top of the cylinder and said that “it [the sphere] cannot fit into the can,” referring to the cylinder, because she “cannot see how it can be done.” She further remarked that she cannot see “how it [the sphere] can go inside the can because I can't see it” even if she had physical models to represent the two figures.

Student 25 who also answered the question incorrectly also said that he thought that the “ball could hold more water than the cylinder.” He did not move
the sphere or cylinder even when he was told that he was allowed to. This student, who answered all but two of the other questions correctly, appeared to be tired, and may have said this to move on to the other questions. I did split the interview into two sessions in order to avoid fatigue, but it still emerged as an issue since the interview was conducted in the afternoon on the final day of the school week. I suspect that if the second part of this interview was conducted at a different time, the student’s answer to this question would have been different.

The importance of representation became very clear in the next question of the interview. I asked Student 6 to determine which pyramid had the larger volume. Since the student interpreted the diagram much more easily, he was able to answer the question much more easily and quickly. Figure 31 shows Student 6’s response to this question. Even though the pyramids have a different orientation and the same colour, both of which may make it harder to answer the question according to several students that I interviewed, Student 6 circled the pyramid with the larger volume without hesitation, and used the features of the SmartBoard effectively to verify his answer by sliding the smaller pyramid over the larger pyramid.
Two students that I interviewed who did not take part in my pilot study also answered this question incorrectly. They both stated quickly that the pyramid that was shown to the right was larger. They did not spend much time on these questions. Student 25, said that the figure to the right was larger because “it just looked bigger” and the sides of the base were larger because “they went deeper.” When I asked him what he meant, he said that he thought that the sides of the base of the pyramid that was shown on the right were larger than the sides of the base of the pyramid on the left.

5.9.2 Visualisation Questions

The interviews had three questions where the students were given questions that did not have an accompanying diagram. The first question asked students to determine if a building or a car had a larger volume, and the second question asked them to determine if a baseball or basketball had a larger volume, a context that most American students would be familiar with. Some students asked what greater volume meant, and I defined it within the questions as which
object takes up more space. All 23 of the interviewees answered both of these questions correctly and quickly.

In my further interviews, I asked students to answer a Geometry Regents examination question where they had to visualise the information that was given in the question. The question that is shown in Figure 32 was asked to 7 students who did not participate in my pilot study, and 9 students who did participate in the pilot study.

![Figure 32: Visualisation Question from the June 2012 New York State Geometry Regents Examination (6 out of 16 students answered this question incorrectly)](image)

All of the students, whether they answered it correctly or incorrectly, noted that the language in the question was confusing. This was not at all surprising given that the vast majority of the students in my school have a language-based learning disability, most notably, dyslexia. Of the 16 students who answered this question, 7 drew a diagram. All of these students answered this question correctly. Student 5, who answered this question incorrectly, said that she did not know what type of diagram that she could have drawn. She was able to accurately define the terms plane, perpendicular, point and line. This
student said that “I didn’t know what kind of picture I could’ve drawn here.” This was a common remark from most students, and even the students who drew a diagram were uncertain if they were drawing it correctly. One student who did answer the question correctly, said that he simply guessed the answer. He said that he “just can’t read through it again” even though the question was read to him. Not surprisingly, this student was diagnosed with dyslexia by her psychologist.

5.10 Analysing Paintings and Photographs

In the pilot study interviews, there were two questions that provided a lot of insight into how students interpreted paintings or photographs. The responses that students gave may provide some information about how I should develop my intervention plan in future stages of my research.

In one question, shown in Figure 33, I displayed Raphael’s The School of Athens. The students were asked to circle the people at the back of the scene in the painting. What was surprising was that the students had very different ideas of what the front and back of the painting were. This painting was purposely selected because many students in the high school had learned about this painting in at least one of their art courses.
The most important insight that I gained from the responses to this question is that I should not assume that students see even basic aspects of diagrams in the same manner that I see them. Figure 34, shows Student 8’s response to this question. He selected a small group of people to the left and right of the painting because he said that he views the painting “like a movie” and believed that the left and the right are the “ends of the painting” and as such are the back. I never assumed any student would interpret this painting as a movie.
Student 6, whose response is shown in Figure 35, also had a very different interpretation of who is in the back of this painting. He said that anyone who is not circled is in the front, and all of the people who are circled are in the back. He chose the circled people for the back because it was very hard for him to determine the order. I made the assumption that the steps shown in the painting suggested an order, but it is clear that some students did not interpret the painting in the same manner.

The most puzzling answer though came from Student 5, whose response is shown in Figure 36. She stated that no one is in the back and instead all of the people appeared to be “on the same level.”
In the subsequent interviews that I conducted, two additional students answered this question incorrectly. Student 22 who answered the question incorrectly also said that everyone seemed to “be on the same line” after
observing the painting for over a minute. She chose not to circle any people and asked if this was a “trick question.” I told her that none of the questions were trick questions, and she should not expect any deception in any of the interview questions. At the end, she decided to keep her original answer.

Perhaps the difficulty that students had in answering *The School of Athens* question stemmed from the perception that there was a lack of order. In Figure 37, a photograph of pyramids with a clearer order was shown. Students were asked to determine how many pyramids were in the front row. All of the students who were interviewed answered this question correctly.

![Pyramids Question](image)

**Figure 37: Pyramids Question**  
(All 23 of the interviewees answered this question correctly)

5.11 Questions with Physical Figures

After I conducted the pilot study, I conducted further interviews with 7 additional students who did not participate in the pilot study, and 6 students who did participate in the pilot study. The two questions that I asked the students
involved comparing the surface areas of the nets of a pyramid and a rectangular prism which are shown on the top and bottom of Figure 38 respectively.

![Figure 38: Comparison of the Nets of a Rectangular Prism and Pyramid](image)

(2 out of 13 students answered this question incorrectly)

The students were asked to determine which net had the larger area. Whilst most of the students did answer this question correctly, two students, Student 5, and Student 22 both answered that they thought that the area of the pyramid’s net was larger than the area of the rectangular prism’s net. Student 5 said that the “height of the triangle,” referring to the height of one of the triangles of the pyramid’s net is greater than the height of the of one of the rectangles (that are not the small squares) that make up the net of the rectangular prism. Whilst this is true, this misconception was shared by both students. Though this question was difficult because the areas of the nets of these solids are close, there was
enough evidence to suggest that that rectangular prism’s net had the larger area. In fact, Student 23, who initially answered the question incorrectly changed her answer. She had initially said that the pyramid’s net had the larger area because of the difference in the heights of the sections. However, when I asked her to explain her reasoning, she said “wait, I think I got it wrong.”

![Figure 39: A Still Shot of Student 23’s Manipulation of the Nets of the Pyramid and Rectangular Prism](image)

She then moved the net of the pyramid on to the top of the net of the rectangular prism, stopped for a moment, and then stopped for about 15 seconds and changed her answer. In particular, she said “the extras of the bottom one,” referring to the parts of the net of the rectangular prism that were not covered by the net of the pyramid, “made the area of the box bigger.”
5.11.1 Comparison of Volumes of a Pentagonal and Hexagonal Prism

The final question with the physical models asked the students to determine if the volume of the pentagonal prism or the volume of the hexagonal prism was greater. Both prisms had bases with similar areas, but the height of the hexagonal prism was noticeably different.

![Figure 40: A Photo of the Pentagonal and Hexagonal Prisms](image)

(All 13 students answered this question correctly)

All of the students answered this question correctly. Twelve of the thirteen students answered the question very quickly, and one student, Student 23, answered the question correctly after putting one prism on top of the other, as shown in Figure 41. When I asked the other students to explain how they determined their answer, they provided an explanation that was similar to Student 23’s explanation. She said “these pieces,” referring to the bottom face of the hexagonal prism and the top face of the pentagonal prism “are almost the same [area], but this [the hexagonal prism] is taller.”
The interviews that I have conducted during the pilot study seemed to have revealed some very interesting information about the difficulties that the students had answering questions that required them to use their visual-spatial skills. The results of the pilot study may also suggest the difficulties that will emerge in future interviews that I will conduct with other students. In addition, the varied strategies that students have used gave me some insight into their ability to solve problems, and also their ability to use the tools that were available to them. In particular, the ways that students utilised the interactive features of the SmartBoard may have revealed an ability to think in a non-routine manner which could potentially help them to solve problems despite their visual-spatial deficits.
Chapter 6: Description of the Interventions with Students

In my research, I conducted interventions with two students, Carol and Diana. I carried out interventions with Carol from when she was in the tenth grade to the twelfth grade, and for Diana during each year that she was in high school. This chapter will set the stage for the reader to understand the nature and rationale for the interventions.

6.1 Structure of the Interventions

The interventions that I conducted with each student were sometimes different according to the topic that was covered, but very similar in how they were structured. There were several features of both which I believe are especially noteworthy and important to this research.

Firstly, the interventions almost always focused on only one topic. These topics were directly related to what the students were studying in their mathematics classes, or were designed to help them develop the skills that they needed to successfully learn a specific set of lessons in their mathematics courses.

Secondly, the interventions were almost always short. Most of the sessions were between 10 to 20 minutes in length, and some were just five minutes long. These short sessions were important for both students, but especially for Diana because she was diagnosed with ADHD by her psychologist, and was prescribed medication for this by her psychiatrist.

Thirdly, the interventions usually sought to provide simple solutions to the challenges that the students encountered. This was done in order to make it easier for the students to recall the strategies that were being taught, and also because it
made it easier for the students to utilise the strategies. Some of the interventions were not as simple because the topics themselves were challenging and required more explanation. For example, both Carol and Diana had difficulty with using the graphics calculator and interpreting the slopes of two lines that were graphed on the calculator. Whilst this approach was ultimately simple, the process for approaching these problems was difficult for the students because they had to recall a lot of steps for entering the equations that needed to be graphed into the calculator.

6.2 Selection of Topics for the Interventions

The topics for the interventions were usually selected by Carol and Diana. For some of the sessions, I recommended the topics from the difficulties that they described to me, or from my previous work with them. Whilst I never taught Carol in a class, except for a few days when I covered for her mathematics teacher who was absent from school, Carol did seek my help on numerous occasions before I formally started work with her for this research. I learned more about her through these informal help sessions, from her mathematics teachers, and also her psychoeducational evaluation which was conducted by her psychologist who was independent of the school.

I taught Diana when she was in ninth grade for the entire year. She too sought my help on numerous occasions, and my work with her was informed by the regular class lessons and these additional help sessions that were not necessarily part of my research.
### 6.2.1 Carol’s Topics for the Intervention Sessions

In total, I conducted 13 intervention sessions with Carol over about three academic years. The topics for the interventions are shown in Table 2.

<table>
<thead>
<tr>
<th>Session</th>
<th>Title of the Intervention Session</th>
<th>Brief Explanation of Carol’s Difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Determining if the graph of a relation is a function.</td>
<td>Carol had difficulty with these questions because she would think that the x-axis and y-axis were part of the graph of the relation.</td>
</tr>
<tr>
<td>2</td>
<td>Determining which line or line segment has a greater slope.</td>
<td>Carol had difficulty answering questions that required her to interpret graphs in both mathematics and science.</td>
</tr>
<tr>
<td>3</td>
<td>Identifying the sides of a right triangle for the Pythagorean Theorem.</td>
<td>Carol had difficulty identifying the legs and the hypotenuse of a right triangle and was answering many questions incorrectly despite knowing how to solve the problems algebraically.</td>
</tr>
<tr>
<td>4</td>
<td>Identifying the sides of a right triangle for trigonometry questions.</td>
<td>Carol had difficulty identifying the hypotenuse, opposite side, and adjacent side of a right triangle when given an acute angle of the right triangle.</td>
</tr>
<tr>
<td>5</td>
<td>Using a graphics calculator to determine which line has a greater slope by visual inspection.</td>
<td>Carol had difficulty determining the line that had a greater slope when it was graphed on the graphics calculator.</td>
</tr>
<tr>
<td>6</td>
<td>Determining the lengths of the segments of a right triangle that had an altitude drawn from its right angle to its hypotenuse.</td>
<td>Carol could not find the lengths of the segments of these types of triangles because she seemed to be overwhelmed by the amount of visual information presented in these problems.</td>
</tr>
<tr>
<td>7</td>
<td>Determining the measures of the opposite and consecutive angles of a parallelogram.</td>
<td>Carol had difficulty identifying the opposite and consecutive angles of a parallelogram.</td>
</tr>
<tr>
<td>8</td>
<td>Determining the measures of the angles that are formed by parallel lines that are cut by a transversal.</td>
<td>Carol could not determine the measures of the angles. It also seemed that she was overwhelmed by the amount of visual information presented in the question.</td>
</tr>
<tr>
<td>9</td>
<td>Determining the measures of the angles that are formed by two or more intersecting lines.</td>
<td>Carol had difficulty identifying vertical as opposed to consecutive angles.</td>
</tr>
<tr>
<td>10</td>
<td>Graphing trigonometric functions.</td>
<td>Carol had difficulty graphing the sinusoidal and cosinusoidal functions.</td>
</tr>
<tr>
<td>11</td>
<td>Determining the lengths of the segments of chords, and tangents to a circle.</td>
<td>Carol had difficulty identifying and finding the measures of these segments in the circle.</td>
</tr>
<tr>
<td>12</td>
<td>Determining the angles formed by chords, secants, and tangents to a circle.</td>
<td>Carol had difficulty identifying and finding the measures of these angles in and outside the circle.</td>
</tr>
<tr>
<td>13</td>
<td>Visual difficulties in solving algebraic problems.</td>
<td>Carol had difficulty with solving algebraic problems because some of the difficulty that stemmed from noticing the sides of an equation.</td>
</tr>
</tbody>
</table>

**Table 2: A List and Brief Explanation of Carol’s Interventions Sessions**

**6.2.2 Diana’s Topics for the Intervention Sessions**

Diana’s sessions were mostly different. However, a few were the same. The approach to working with Diana was not always the same as working with Carol. I conducted 14 intervention sessions with Diana which are shown in Table 3. Since Diana needed more help and time with some of the topics, the
interventions were held more than once. With Diana in particular, some of the sessions would help her with upcoming work in her mathematics classes. I coordinated with her teachers to ensure that I would cover topics that would make it easier for her to learn.

<table>
<thead>
<tr>
<th>Session</th>
<th>Title of the Intervention Session</th>
<th>Brief Explanation of Diana’s Difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Determining if the graph of a relation is a function.</td>
<td>Diana, much like Carol had difficulty with these questions because she would think that the x-axis and y-axis were part of the graph of the relation.</td>
</tr>
<tr>
<td>2</td>
<td>Determining which side of a triangle is the largest, middle length, and smallest when given the measures of the angles of the triangle.</td>
<td>Diana had difficulty understanding that the largest side of a triangle is opposite the angle with the largest measure, and that the smallest side of a triangle is opposite the side with the smallest measure.</td>
</tr>
<tr>
<td>3</td>
<td>Identifying the sides of a right triangle (the opposite, adjacent, and hypotenuse) for trigonometric functions.</td>
<td>Diana was exposed to these terms which helped her when she started to learn the three basic trigonometric functions: sinusoidal, cosinusoidal, and tangent.</td>
</tr>
<tr>
<td>4</td>
<td>Addressing difficulties with directionality.</td>
<td>Diana has significant difficulty with directionality, and has difficulty with determining directions.</td>
</tr>
<tr>
<td>5</td>
<td>Solving linear equations: a visual-spatial viewpoint.</td>
<td>Diana had difficulty solving linear equations. Her difficulty stemmed from understanding where she had to write her workings and solution. In addition, she had difficulty starting to solve the question, which often started with her not knowing where to look.</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>Solving and graphing the solution to a linear inequality on a number line.</td>
<td>Diana who was able to solve linear inequalities relatively well, but had a lot of difficulty graphing the solution on a number line.</td>
</tr>
<tr>
<td>7</td>
<td>Comparing and contrasting chords, secant lines, and tangent lines to a circle.</td>
<td>Diana had difficulty identifying each type of segment or line, and understanding the nature of tangent lines.</td>
</tr>
<tr>
<td>8</td>
<td>Recognising and understanding the nature of similar triangles when a larger triangle has a segment drawn with endpoints on two different sides, and the drawn segment is parallel to the third side.</td>
<td>Diana had considerable difficulty with this because there was a lot of information for her to consider when viewing a diagram.</td>
</tr>
<tr>
<td>9</td>
<td>Recognising interior and exterior angles in a polygon, and establishing basic relationships between these angles.</td>
<td>Diana had difficulty recognising the difference between the interior and exterior angles of a polygon when they formed a linear pair.</td>
</tr>
<tr>
<td>10</td>
<td>Determining the measures of the angles formed by parallel lines that are cut by a transversal.</td>
<td>Diana, much like Carol had difficulty finding the measures of these angles. However, her difficulties were manifested in slightly different, but important ways.</td>
</tr>
<tr>
<td>11</td>
<td>Addressing difficulties with translations.</td>
<td>Diana had difficulty with translations which likely stemmed from her difficulty with directionality.</td>
</tr>
<tr>
<td>12</td>
<td>Addressing difficulties with rotations.</td>
<td>Diana had difficulty with rotations which may have stemmed from her issues with directionality.</td>
</tr>
<tr>
<td>13</td>
<td>Addressing difficulties with dilations (enlargements).</td>
<td>Diana had difficulty plotting the points and drawing the images of the dilations of a preimage.</td>
</tr>
</tbody>
</table>
144

| 14 | Addressing issues with compound transformations. | This session with Diana helped her to incorporate what she and I worked on in the three previous sessions. |

**Table 3: A List and Brief Explanation of Diana’s Interventions Sessions**

6.3 **The Implementation of Intervention Sessions and Revision Sessions for Examinations**

The purpose of the intervention sessions was to help the students develop strategies to overcome their visual-spatial deficits and learn their mathematics coursework more effectively. It was *not* meant to help students to revise for examinations. To ensure that this goal was met as much as possible, the intervention sessions were not held before exams. However, the relationship that I developed with these students led to individualised revision sessions that I did not expect, and that are not directly a part of this research. Most of the revision sessions did however, use the techniques that were discussed and used during some of the intervention sessions.

6.3.1 **Scheduling of the Intervention Sessions**

The intervention sessions were usually held during towards the end of the day. This was the best time for the students and I to meet. Ideally the sessions would have been held at a different time when the students and I were not as fatigued, but this was usually not possible because of our conflicting schedules. In addition, we did not always plan an intervention session during a regular time. The intervention sessions were held when the students had a difficulty come up in their coursework. I feel this was a good time to have these sessions because it
responded to the students’ needs and made them feel that the sessions were valuable for their own development.

6.4 Exclusion of Other Possible Intervention Sessions

There were some revision sessions that I held with both Carol and Diana. Whilst an outside observer may consider some of these sessions as intervention sessions because of the strategies and problem solving techniques that were discussed in these meetings, I am not including these sessions as intervention sessions because the purpose of these sessions was primarily exam preparation.

6.5 Gathering Information During the Intervention Sessions

The information that I gathered during the sessions was recorded on a tablet, or a laptop computer. For a few sessions, I took notes during the sessions as well. This allowed me to provide a rich description of the intervention sessions.

6.6 Selection of Intervention Sessions for Deeper Discussion

It is not possible or beneficial to provide a detailed discussion of all of the intervention sessions for Carol and Diana. I will provide a rich and detailed description of some of the intervention sessions for each student in order to give a better description of each student’s difficulties as well as provide insight of both successful and unsuccessful strategies for teaching these students.
This chapter will discuss the intervention sessions that I have conducted with Carol and Diana in greater detail. Whilst it will not be possible or effective to discuss all of the sessions in detail, I will provide a detailed discussion of five of the intervention sessions that I conducted with Carol and an additional five intervention sessions that I have conducted with Diana. I have selected the sessions to give a good description of each student’s visual-spatial challenges, how these difficulties were addressed, and if the interventions were successful.

7.1 A Description of Carol and her Challenges

I started to formally work with Carol at the end of her ninth grade year in high school. I was first introduced to her earlier that year when the principal of the high school informed me about her general difficulties in mathematics. Later that year, I learned that Carol and her mother believed that she had significant visual-spatial deficits. What was frustrating for the both of them, and for Carol’s teachers was that Carol was never diagnosed with these deficits, and her teachers did not always know that she had these difficulties, and were not able to teach her effectively even if they were aware of these difficulties. As such, Carol and her mother said that she had a lot of difficulty in her mathematics classes when any work that required her to rely on her visual-spatial skills was taught. At one point in her ninth grade coursework, Carol was exempt from being tested on any questions that required her to interpret graphs or charts in her geology and world history courses. However, this was not the case in her mathematics courses where she still had to contend with the challenges of charts and graphs.
7.2 Direct Instruction for the Intervention Sessions

Throughout the interventions for both Carol and Diana, direct instruction was the main mode of instruction. There is ample evidence (Gurganus, 2017, p.201), that direct instruction tends to be the most effective form of instruction for students with learning disabilities. Also, since the intervention sessions tended to be short, the direct instruction allowed us to use the available time in an optimally. In particular, the direct instruction approach probably allowed us to examine more questions than if another method of instruction was used. Finally, the direct instruction approach also reduced anxiety for students for several reasons. Firstly, since the interventions sometimes previewed new topics, the students were much more likely to feel at ease if they were given more information from the start. This is also something that I have noticed in teaching new topics to many students at my specialist school. If the intervention reviewed a topic that they did not understand, the direct instruction usually cleared up any misconceptions and difficulties that they may have had much more so than another approach. Finally, the direct instruction also models the language that the students need to learn to use, and it also introduced or reminded them of the language that their teachers will use, or textbook or worksheet problems will present. This is especially important since so many of the students have language-based learning disabilities.

7.3 Routines and Structure

The literature (Strain and Sainato, 1987, p.26-28; Elbaum, 2001) strongly suggested that students with learning disabilities, especially those diagnosed with
NLDs, do better with routines and structure. Since some teachers, including myself, believe that mathematics inherently has a deep structure, the mathematics classes are built on this structure. As such, the interventions also had an easily defined structure. Since the difficulty was explained by the student, or I decided ahead of the intervention session, the questions to be covered during the intervention will generally be written ahead of time, and the session time will be used to the maximum benefit for the student.

7.4 The Intervention Sessions with Carol

I will describe the following five sessions that I have conducted with Carol in detail:

- Determining if the graph of a relation is a function
- Determining which line or line segment has a greater slope
- Determining the lengths of the segments of a right triangle that had an altitude drawn from its right angle to its hypotenuse
- Determining the measures of the angles that are formed by parallel lines that are cut by a transversal
- Determining the lengths of the chords in a circle, and the tangents to a circle.

7.4.1 Determining if the Graph of a Relation is a Function

Carol had difficulty understanding how to determine if the graph of a relation is also the graph of a function such as the graph of the relation shown in Figure 42. The first difficulty that Carol had was understanding the definitions of the terms relation and function. Once I defined these terms for her, using what Carol said was a simple language, I started to show her the graphs of relations where she had to determine if relations were also functions.
I showed her how to use the Vertical Line Test to determine if the graph of the relation is also the graph of a function. After giving her five graphs of relations, three of which were functions, Carol insisted that none of the graphs displayed functions. She said that the “[vertical] lines [that were drawn on the graph] touched in more than one place” for every graph. When I asked her to show me where the vertical lines that were drawn touched the graph of a straight line with a positive slope in more than one place, she pointed to two different places, which I circled. The first point that I circled was where the vertical line touched the given line, and the second place was where the vertical line touched the x-axis on the graph. This misconception that Carol held persisted whether or not the graph of the relation was or was not a function. I had Carol use a highlighter to highlight the x-axis and y-axis, and then told her that unless the graph of the relation was directly “on the x-axis,” meaning that the graph coincided with the x-axis, or unless the graph touched the x-axis, the x-axis should not be considered to be a point of intersection with the vertical line. Once I explained this to her, she was able to understand this. I then gave her 5 additional questions
with graphs of relations where three of the graphs were those of functions, and two were not. Carol answered all of these questions correctly, and left the session feeling more confident. She said at the end of the session “I feel much better about this now.”

On a test that Carol took the following week which was created by her mathematics teacher, Carol showed me that she was able to correctly answer the two questions on this topic that her teacher put on the test. A few months later in the school year, Carol answered the question about identifying if the graph of a relation was also the graph of a function correctly on the Common Core (Algebra) Regents examination. She said that she thought back to this session when answering the question. “It all came back to me” she said about the techniques that came up in the intervention session.

7.4.2 Determining which Line or Line Segment has a Greater Slope

Another difficulty that Carol had was determining which line or line segment that is on a graph had a greater slope. This difficulty came up with not only two lines that had slopes with different signs, but most commonly with lines that have slopes with the same sign. Carol further had difficulty understanding why two lines with no slope had equal slope.

This session, much like the previously described session, focused on teaching Carol simpler techniques that allowed her to focus on using the technique even when she worked independently, and in a high-pressure situation such as an exam.

The session started with two lines with positive slopes that were shown on the graph, as shown in Figure 43. Carol’s initial tendency was to say that the line
that was above the other line had a greater slope because of its location, when she
saw problems of this type. This type of problem was especially important
because Carol encountered this in not just her mathematics classes, but also her
science and history classes, in addition to the American College Test (ACT),
which whilst not an admissions test, is a factor for admission to many
universities in the United States.

Figure 43: An Example of the Type of Slope Question Shown at the
Start of the Intervention Session

Carol was able to tell if a line had a positive slope or negative slope. This
made it easier for her to determine which line had a greater slope when she was
given two lines that had slopes with opposite signs. However, even for these
questions, Carol had difficulty keeping the answers in her memory. I showed her
a technique with a vertical number line labelling system that is shown in Figure
44. Even when Carol was able to correctly determine the slope of the line, she
had difficulty stating which number for the value of the slope was greater.
Writing the numbers on a vertical number line helped her to correctly identify the line with the higher slope.

![Vertical Number Line](image)

**Figure 44: An Example of the Vertical Number Line that was Shown to Carol**

The main focus of this intervention session was however, to help Carol determine which line on a graph had a greater slope. Since Carol was able to tell if the graph of a line had a positive or negative slope, we focused on lines that slopes with the same sign. I told Carol to draw a horizontal line segment that was an inch long to the right of a point that was on each given line. Then Carol drew a vertical segment from the end of the horizontal segment to the to the corresponding point on the line. The length of the vertical segment was measured with the ruler, as shown in Figure 45. Carol was then asked to put the two lengths of the vertical segments, including the signs, on a vertical number line as described before. This enabled her to take a topic that was open-ended and make it something that was more concrete, and perhaps easier to understand. Carol stated that this was “something I could do in history and science,” referring to
using this approach to interpreting graphs in these courses. Whilst we did not cover how to interpret graphs in different scenarios and subjects, Carol did later report that this session was helpful for her other courses.

Figure 45: A Diagram of Carol’s Method for Calculating the Slopes of Lines

7.4.3 Determining the Lengths of the Segments of a Right Triangle that had an Altitude Drawn from its Right Angle to its Hypotenuse

This topic is quite challenging even for students who do not have visual-spatial deficits. For Carol, it was especially challenging, and caused a lot of anxiety for her. The multiple steps that can often be involved in solving these problems, the algebraic challenges, along with the amount of visual information that these problems presented made it very challenging for Carol. At the start of the session she said “I just freeze up” when I see these questions. She said that
these problems were difficult for her not only for the aforementioned reasons, but also because of the time given in class to do these questions.

The technique that I showed Carol in this session was again a simple one. Figure 46 shows the three diagrams that I drew for her along with the markings. The three diagrams that are shown in this figure pertain to the segment lengths and how they are related to each other. The loops, both round and rectangular, for each segment show how many times they have to be multiplied to each other. Then the product of the lengths of the segments with the rounded curves and the product of lengths of the segments with the rounded curves are set equal to each other.
Carol said that this was “something that I can make by myself,” referring to the fact that she could recreate this diagram on her own. Once we had practised six questions with these three different scenarios, Carol seemed to have a better understanding of this work. When I spoke to her the day after this session, she said “I was able to do the questions! I’m so excited.” When I checked her homework questions, she answered 4 out of the 6 questions correctly. The two questions that she did not answer correctly had minor errors. One question had a small arithmetic error, and the other question, the most difficult of the homework questions, had an algebraic error.

This intervention session helped Carol to learn how to solve these types of problems, but did not teach her the underlying reasons why the problems can be solved in this way. With students who face visual-spatial challenges, and mathematical difficulties in general, it is sometimes better to have students learn how to solve problems even if they do not gain a deeper understanding of what they are learning. Whilst I aimed to help Carol and other students understand the underlying concepts for each lesson or intervention session, this is not always possible, and is a tradeoff that unfortunately must sometimes be made.

7.4.4 Determining the Measures of the Angles that are Formed by Parallel Lines that are Cut by a Transversal

This topic often causes a lot of difficulties for students with visual-spatial deficits. Carol said that the diagrams were challenging for her because these problems are “super confusing” and “[it is] hard to tell where to start looking.” Carol further said of the parallel lines, it is hard to tell “where they’re going.” When I asked her what she had meant by this, it seemed that she had difficulty
understanding how the angles were formed and also found it hard to keep track of all of the angles in the diagram.

The strategy that I taught her for this was again something that she could recreate without major difficulty. Figures 47 and 48 show the two diagrams that I drew for Carol that helped her to keep track of the angles and remember the relationships between the angles. I first told Carol to draw a circle around each set of four angles that were formed by the transversal that intersects with each of the parallel lines. I then had Carol label each set of four angles as \( UL, UR, LL, \) and \( LR \) for the upper left, upper right, lower left, and lower right angles respectively. When I asked Carol if this was a helpful first step, she said “it is good to know where to start each time.” This routine seemed to help Carol start the problem, and perhaps diminish some of her anxiety.

![Figure 47: First Parallel Lines Cut by a Transversal Marking System](image-url)
Once Carol had marked each of the angles in each set, I instructed her to set the measures of the two angles that were labelled $UL$, $UR$, $LL$, or $LR$ equal to each other. Carol already recognised that the vertical angles, the angles marked $UL$ and $LR$, and the angles marked $UR$ and $LL$, also had equal measures. I also mentioned to her that these two sets of vertical angles can be easily recognised because they have opposite labels. Finally, I told Carol the other pairings of angles that are next to each other, but not vertical angles, are supplementary. She said that she had already known this too.

### 7.4.5 Determining the Lengths of the Segments of Chords, and Tangents to a Circle

Carol had a lot of difficulty determining the lengths of the segments formed by the chords of a circle, and also the lengths of the tangents to a circle that start from the same point that is external to a circle. She asked that we cover these two topics because she found it to be exceptionally difficult. Carol said “I
don’t know what I don’t get about it [the circles].” Since there were two topics to be covered, this intervention session was about thirty minutes in length.

Figure 49 shows two chords in a circle that intersect to form an “X” shape. I purposely drew the diagram to form an “X” to remind Carol that the product of the lengths of the segments of one chord will equal the product of the lengths of the segments of the other chord. Carol said that she thought that the chords “had to go through the centre [of the circle]” and that was “confusing.” Once I assured her that chords did not have to go through the centre of the circle, I was able to help her with the other aspects of the difficulties that she had. Since Carol could not see the different segments that made up each chord, I had her start by putting a large dot on the point of intersection of the two chords that were in the circle as shown in Figure 50. Once this was done, Carol was able to see the different segments of each chord more easily. She stated that “putting the dot down” was very helpful in helping her see the different parts of each segment. In addition, I also had her draw rounded loops for the two segments that made up one chord, and rectangular loops that made up the two segments of the other chord. Once this was done, Carol was able to see what terms she had to multiply together. Carol practised several problems of this type and answered all of the questions correctly. Since there was not too much complexity in the algebraic work as she was only required to solve linear equations, Carol said that she did not find this to be too difficult.
The second topic that Carol and I discussed in this intervention session was tangent lines to a circle that shared an endpoint that was external to the
circle. This portion of the intervention session was shorter as Carol’s misconceptions about the tangents seemed to be cleared up more quickly.

Carol seemed to have difficulty understanding the definition of the terms **tangent line**, **point of tangency**, and **external point**. Once these terms were defined with an accompanying diagram, as shown in Figure 51, Carol was able to overcome her initial difficulties. It seemed that language, and not just the visual-spatial deficits played a role in her misconceptions. Carol’s visual-spatial difficulties stemmed from her not knowing how to process the information in the diagram that was given to her. As she said before, she did not know where to look in the diagram. Once I told her to look for a “**triangular, pointy corner outside the circle**” to find where the tangents have a shared endpoint, she was able to more easily see the tangent lines. I then had her highlight the tangents and told her that these tangents to the circle have equal lengths. Carol then did several problems with tangents and was able to answer all of the questions incorrectly.

![Figure 51: Circle with Tangents from a Common External Point with Highlighting](image.png)

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This session did not necessarily focus on establishing the underlying reasons the properties of the chords and tangents that were discussed. Rather, it was mainly to help Carol to overcome her visual-spatial deficits, and to some extent her language difficulties.

7.5 A Description of Diana and her Challenges

I started to work with Diana when she was in the ninth grade, when I was her mathematics teacher. From my earliest work with her, it was clear that Diana appeared to have some significant visual-spatial deficits. This was apparent even in the Algebra I course that I taught her in the ninth grade. One of the first difficulties that I noticed about Diana is that she had difficulty solving linear equations. A large part of her difficulties stemmed from her difficulties with where to start answering questions. She sometimes said, much like Carol, that she did not know where to look when starting the questions. As the Algebra I course progressed, Diana had difficulty with linear inequalities where she had to graph the solution to the inequalities on number lines, and had significant difficulty constructing the graphs of linear equations and inequalities on a coordinate plane.

Much like Carol, Diana’s psychoeducational evaluation which was conducted by an independent licensed psychologist, did not discuss these difficulties in any significant extent. Unfortunately, this did not help Diana obtain the help that she needed. I also suspected that her teachers were very confused by her difficulties since she is very artistically talented. In fact, Diana was admitted to one of the best universities in the United States for art. Figure 52
shows a painting that she did during her eleventh grade art class. The painting would suggest that Diana has a solid understanding of proportions, directionality, and symmetry, all visual-spatial aspects that she struggles with immensely.

Figure 52: Diana’s Painting, *Mermaid*

### 7.5.1 Intervention Sessions with Diana

I will describe the following five sessions that I have conducted with Diana in detail:

- Determining which side of a triangle is the largest, middle length, and smallest when given the measures of the angles in the triangle

- Solving linear equations: a visual-spatial viewpoint

- Solving and graphing the solution to a linear inequality on a number line

- Recognising interior and exterior angles in a polygon, and establishing basic relationships between these angles
Addressing difficulties with translations

7.5.2 Determining which Side of a Triangle is the Largest, Middle Length, and Smallest when Given the Measures of the Angles in the Triangle

Diana had a lot of difficulty determining the order of the measures of the sides of the triangle from largest to smallest when given the measures of the angles in a triangle. This included questions where she is given the measures of all three angles in the triangle, and the measures of two of the angles in the triangle where Diana would have to find the measure of the third angle, and where she was given the measure of a base or vertex angle in an isosceles triangle.

Diana did need reminders on how to determine the measures of the angles in the triangles in the latter two cases. However, the primary focus of this intervention session was to address the angle to side relationship. As such, the first set of triangles that I showed her were drawn to scale to make this task easier.

In this session, unlike some of the other sessions with both her and Carol, I did seek to establish a fundamental understanding as to why the largest side is opposite the largest angle, and the smallest side is opposite the smallest angle of a triangle. I initially started with a scalene triangle, as shown in Figure 53, that did not have the measures of the angles. I did this to see if Diana could determine which angle had the largest measure, and which one had the smallest measure.
Whilst Diana was able to determine which angle was the largest, she did have difficulty identifying the smallest angle. She almost immediately said that the “largest angle was the widest” meaning it had the greatest opening, but could not easily tell the smallest angle. When I asked her to repeat this task for a different triangle that is shown in Figure 54, she was able to identify the smallest angle almost immediately, and hesitated, but then identified the largest angle correctly. When I asked her why she had difficulty with identifying the largest angle, she said that since it was on the left side of the diagram, and not at the top, as it was in the previous diagram, it made it more difficult for her to determine. This was somewhat surprising to me, and was not something that I thought about before.
To help Diana overcome this difficulty, I taught Diana a very simple technique. I had her start at the top angle of the triangle, and had her move her index and middle fingers along the two sides that made up the top angle. When both of her fingers were spread out and reached the other two vertices of the triangle, I had her measure this distance between her fingers with a ruler. She then repeated this process for the other two sides. This seemed to help Diana, and she was able to correctly order the sides of the triangle correctly in seven questions that I gave her. These questions did not require Diana to determine the measures of the angles as the main focus of this session was to address Diana’s visual-spatial deficits, and not her algebraic difficulties.

### 7.5.3 Solving Linear Equations: A Visual-Spatial Viewpoint

When I worked with Diana on addressing the visual-spatial challenges that she had with solving linear equations, a few weeks before she was taking her *Algebra (Common Core)* Regents examination. Diana said that the difficulty that she had with solving linear equations persisted since I taught her in ninth grade.
the previous year. Diana said that she had difficulty identifying the two sides of
the equation. She was confused by a common phrase that other mathematics
teachers and I have used “do on to one side what you do on the other,” referring
to the two sides of an equation. I addressed the first difficulty that she had by
drawing vertical lines around the equal sign between the two sides of the
equation as shown in Figure 55. Then I showed Diana to write “LEFT” over the
left side of the equation and “RIGHT” over the right side of the equation.

![Image](image.png)

**Figure 55: Vertical Lines Strategy for Solving Linear Equations**

I had shown this strategy to Diana when I taught her when she was in
ninth grade, but she said that she had forgotten about it and had not used this
strategy in a long time. She said that this “centred the problem” for her, and it
was easier for her to see what follows. I then instructed Diana to circle the
variable and its corresponding coefficient, and then follow the steps to isolate the
variable that she was more familiar with.

Diana said that centring the equation with the two vertical lines helped her
because the “numbers and letters floated all over [the page].” This description is
consistent with those who have been diagnosed with dyslexia as Diana had been.
In addition, since Diana was diagnosed with ADHD by her psychologist, and takes medication that was prescribed by her psychiatrist, this difficulty in focusing may also have been a product of her attentional issues.

7.5.4 Solving and Graphing the Solution to a Linear Inequality on a Number Line

The day after the session described in section 7.5.3, Diana and I worked on her difficulty with solving linear equalities and graphing their solutions on the number line. The previous day’s session helped Diana to complete the first part, which required her to solve for the given variable. The only reminder that I gave to her was that she should remember to switch the inequality when she divides both sides of the inequality by a negative value.

The second part, graphing the solution to the inequality on the number line, was especially difficult for Diana. She had difficulty determining if the circle that the variable was bounded by should be filled in or kept unfilled. In addition, she did not know which way to shade on the number line. Diana associated the greater than, and greater than or equal to inequalities with going up, and the less than, and less than or equal to inequalities with going down. This confused her and gave her pause when she had to shade the number line.

Diana and I started the session by having her solve the inequalities that are shown in Figure 56. I did not have her graph the solutions on the number line since we would do that part together. After she solved for the variable for the inequalities correctly, we looked at each solution. We looked at the first two inequalities where the solutions had the greater than, and greater than or equal to symbols. I asked Diana to make these symbols into arrows, which she did. I then
told her that the arrow shows the way in which the line should be shaded, which she did correctly for the first question. She also shaded correctly for the second question, but then I told her to fill in the circle because the line under the greater than symbol “is a like a pen that you can use to fill in the circle.” Since the first inequality did not have the line underneath the inequality, I told her that she did not “have a pen to fill in the circle.” This concrete approach seemed to work, and Diana was able to independently graph the solution the other two questions on number lines. I then gave her five additional inequalities that were already solved where she had to graph the solution on the number lines. She graphed all of these solutions on the number line correctly too.

Figure 56: Linear Inequalities with Solutions Graphed on the Number Line
The concrete step-by-step approach seemed to make it easier for Diana to solve these questions more easily. Diana said that “it took the pressure off to go in pieces,” referring to the step-by-step approach.

7.5.5 Recognising Interior and Exterior Angles in a Polygon, and Establishing Basic Relationships Between these Angles

Diana found it challenging to also determine if an angle of a polygon is an interior or exterior angle. I was able to address this quickly with Diana by highlighting its perimeter as shown in Figure 57. She said “I see now ... the interior angles have to be between the highlighted part,” referring to the highlighted sides of the polygon. I had Diana put a star where the interior angles were located, and circles where the exterior angles were located. Diana was able to do this very quickly, and said that she was satisfied with this explanation.

Figure 57: Hexagon with Interior and Exterior Angles Marked with Stars and Circles Respectively
When I asked Diana why she had so much difficulty with this, she again said that “things just float around.” I asked her if highlighting the perimeter of the polygon was helpful in the same way that drawing lines around the equal sign of a linear equation was, she simply said “yeah.” It seemed the highlighting had a centring effect that helped Diana to focus on the problem.

Diana’s second difficulty was understanding the relationship between the interior angle of a polygon and its corresponding exterior angle. Diana had difficulty understanding why the sum of their measures was 180°. I asked if she knew the measure of the angle created by the straight line that I drew for her, and she did state that it was 180°. After that I drew a pentagon with the exterior angles that is shown in Figure 58. I then had Diana put a ruler on one of the sides of the pentagon so it coincided with one of the interior angles and its corresponding exterior angle. Once Diana saw that the two angles formed a straight line, she was able to understand that the sum of these pairs of angles was 180°. This time the ruler seemed to have a centring effect. Diana said that using the ruler helped to focus her attention in a way that she could not without it.
7.5.6 Addressing Difficulties with Translations

Diana had difficulties learning about transformations in general, and translations in particular. Whilst she did not have difficulty determining the points for the image after a translation, she did have difficulty plotting points, and also found it challenging to interpret the graph with a pre-image and image as shown in Figure 59.
The plotting of points, an issue that I remember when I taught her in ninth grade, re-emerged with the translations. Diana said that she was overwhelmed with the existing pre-image that is already on the graph when she has to plot the points for the image. She said “it [the pre-image] throws me off, and ... I kind of get confused when I see it.”

To help Diana to overcome her difficulties with plotting points, I had her write $x$ and $y$ next to the $x$-value and $y$-value of each coordinate, respectively. I had her do this because Diana said that she sometimes confuses these values whilst plotting points on a graph. After my prior experience working with her in these intervention sessions, I had Diana highlight the $x$ and $y$ axes, to try to centre her focus on the graph. Once this was done, Diana did not have much
difficulty plotting the points even with a pre-image already on the graph. She seemed to like the concreteness that the centring provided.

The other difficulty that Diana had was determining the rule for a translation that led from a pre-image to an image. I showed her the diagram shown in Figure 60 that had a pre-image and image. I had Diana identify the pre-image and the image, which she appeared to do easily. I then instructed her to write *first* next to the pre-image and *second* next to the image, which she did quickly. Once this was done, I had Diana circle corresponding, or as I told her “matching” points on the pre-image and image which she was also able to do without hesitation. I then drew two arrows, one horizontal and one vertical, to indicate the direction from the corresponding points of the pre-image to the image, and had Diana put a signed number next to each arrow to indicate the direction of the movement and the distance of the movement. Diana hesitated with the signs at first, but was able to do so after a brief pause. I then had her put \( x \) next to the horizontal, and \( y \) next to the vertical arrows respectively. Diana needed to practise this a few times, but then seemed to have understood what she needed to do. She said that she left the session “feeling better.” The next day, she reported that she was able to her homework accurately and without “feeling stressed.” Whilst I did not see her answers to the questions, I believed her because she had always been very honest with me when she has not understood her homework.
Figure 60: Figure of Pre-Image and Image with Directional Arrows

7.6 Developing Understanding and Centring

During all of the interventions with both Carol and Diana, the goal of the intervention sessions was not to always develop an underlying understanding of the work that was covered. Instead, the goals of the sessions were to help them use strategies that would help them mitigate the effects of their visual-spatial deficits, so they could more successfully learn mathematics. Whilst it is true that they did not always attain a level of understanding that mathematics teachers hope that all of their students do, it is sometimes not possible to achieve this. In particular, the students did not always gain a deeper understanding where they could generalise what they have learned and apply their knowledge to solve different types of problems that they were not familiar with. However, that does not mean the students did not gain a greater competency with mathematics.
Finally, another theme that emerged with Carol and especially Diana was centring. Once there was a focus point developed, where the students could start to look, both Carol and Diana had an easier time understanding and interpreting diagrams. This gave them greater confidence, and enabled them to reduce their anxiety.
Chapter 8: Analysis of the Student Interventions

This chapter will focus on analysing the intervention sessions that I conducted with Carol and Diana. The primary goals of this chapter are to determine if each intervention worked, and why they may or may not have worked. Whilst a variety of methods may be used to determine if the interventions were successful, I have chosen to look at the results of each student’s independent work, including homework assignments, teacher constructed exams, and standardised exams. I believe that this will more effectively allow me to see if the students were able to use what they learned in the interventions without my direct assistance, and also to see if they have retained what I taught them during these sessions.

8.1 An Analysis of Carol’s Intervention Sessions

As I mentioned in Chapter 6 of this thesis, the thirteen intervention sessions that I conducted with Carol are shown in Table 2. This section will explore the efficacy of each of the intervention sessions. Whilst it is not possible to provide a proof in the mathematical sense that these interventions have worked, I will provide supporting evidence to show that they may have worked. In addition, if the interventions have not worked, I will aim to show why they may not have worked.

8.1.1 Determining if the Graph of a Relation is also the Graph of a Function

The first intervention that I conducted with Carol was aimed to help her determine if the graph of a relation is also the graph of a function. The
intervention seemed to have helped. Carol was able to answer the questions on her homework assignment that she did the night of the intervention without difficulty. She particularly noted that she used the strategy that we discussed. Carol said that she found it very helpful to highlight the axes, and once she did this, she “didn’t really have any issues.”

On her teacher constructed exam the following week, she had two questions that required her to determine if the graph of the relation was also the graph of a function, as shown in Figure 61. She answered both of these questions correctly as well. In these questions, Carol did not highlight the axes. She said “I don’t need [to do] this anymore.” I am not sure if Carol was able to transfer these types of skills to answering other questions, but it does seem that she was able to gain confidence in her ability by having developed these skills.

Figure 61: Figure of Two Functions Identification Question
8.1.2 Determining which Line or Line Segment has the Greater Slope

This intervention session too seemed to have been successful. Carol said that she was able to “finally see what [line] was going up or down” much more easily. What I found interesting was that months later Carol was able to apply these skills to interpreting graphs in her American history class. I was pleasantly surprised when Carol said that she was able to interpret a line graph about unemployment during the Great Depression in the United States.

Carol did say that she was not able to apply these skills as easily in her science classes. In particular, she did not understand how to interpret some of the line graphs that she encountered in her biology class. When I asked her why she thought it was more difficult in her science class, Carol said that “I couldn’t understand what the graph was about.” When I probed a bit further, it seemed that she did not understand the context of the graph. This seemed to sow confusion in her mind.

On the June 2014 Algebra I (Common Core) Regents examination, Carol did answer question 18 correctly, shown in Figure 62, which required her to use this strategy. Carol said that she did use this strategy, but did not draw the vertical line with a ruler. In addition, she said “I guessed an answer first … and when I drew the [vertical] lines [without a ruler], I got it right.”
8.1.3 Identifying the Sides of a Right Triangle for the Pythagorean Theorem

Carol found this session to be very useful. The centring theme re-emerged in this intervention session. Once she learned to identify the right angle and noted that the side opposite to it was the hypotenuse which should be labelled with \( c \), Carol was able to solve Pythagorean Theorem problems more easily. She did however, still make mistakes. These mistakes tended to be algebraic, and sometimes arithmetic in nature. The main purpose of the session was not to address these difficulties, but rather to help Carol develop strategies to overcome her visual-spatial deficits.
8.1.4 Identifying the Sides of a Right Triangle for Trigonometry

Much like the session that I described in section 8.1.3, centring the diagram of a right triangle was important. Once Carol identified the right triangle, and noted that the side opposite to this angle was the hypotenuse, she then identified the acute angle that was already marked in the problem. She was then able to proceed to label the side opposite this angle with an \( O \) to represent the opposite side, and the remaining unlabelled side as \( A \) to represent the adjacent side.

Whilst Carol was able to label the sides of the right triangle correctly, she did have difficulty answering questions where she had to use one of three trigonometric functions: sinusoidal, cosinusoidal, or tangent, to find the length of a side of a right triangle when given the measure of an acute angle. The errors that Carol made were either algebraic in nature, or more commonly she set up the functions incorrectly. For example, she would sometimes write that the sine of an angle was equivalent to the ratio of the hypotenuse to the opposite side. After she practised this skill by completing several question, she was able to solve these types of problems more accurately.

Carol did however, identify the three sides of the right triangle correctly almost every time. This session’s purpose was to directly address these difficulties and not necessarily focus on solving these questions.

8.1.5 Using a Graphics Calculator to Determine which Line has a Greater Slope by Visual Inspection

This topic was especially challenging for Carol. Part of the difficulty was getting Carol to remember the steps to accessing the graph features on the Texas
Instruments TI-84 Plus Silver Edition graphics calculator. When she was able to do this, we started graphing lines with the different slopes but the same sign. Carol recognised how to determine which line had the greater slope fairly quickly by using a modified form of the strategy that was described in section 8.1.2 of this chapter. She used the y-intercept of each line as a starting point, and “saw how far the line went up [or down] for every tick mark,” referring to one horizontal tick mark on the x-axis of the graph. I was again surprised by Carol’s ability to develop a similar, albeit different strategy of her own.

A difficulty that she did have was determining which line had a greater slope when she had two lines with negative slopes. Carol said that the line with the greater negative slope had the higher slope. This was not necessarily due to her visual-spatial deficits, but maybe attributed to difficulties that she had with basic numeracy skills. This is not surprising since her psychologist diagnosed her with Mathematics Disorder.

8.1.6 Determining the Lengths of the Segments of a Right Triangle that had an Altitude Drawn from its Right Angle to its Hypotenuse

Carol was able to use the strategy that I presented to her in this intervention session effectively when she had my assistance during the session. She reported that she was able to use the strategy in her classwork and her homework when she referred to our notes. However, when Carol took her examination, she said that she “froze and couldn't draw the pictures” referring to the diagrams that are shown in Figure 46. When she took her examination, Carol could not accurately recreate two of the diagrams. She could only remember the diagram that showed the length of the altitude of the right triangle that was drawn
from the right angle to its hypotenuse was the geometric mean of the lengths of the two segments that form the hypotenuse.

This is perhaps one of the biggest drawbacks with some of the strategies that I have used with Carol and other students that I have taught in my school. Developing a foundation built on memorisation instead of understanding makes it difficult, and perhaps in some cases impossible, for the student to re-create previously learned information. Whilst this is not the most ideal approach, when working with students who have significant mathematical deficits, this may be the best of the options that are available.

8.1.7 Determining the Measures of the Opposite and Consecutive angles of a Parallelogram

Carol had difficulty identifying the opposite and consecutive angles of a parallelogram. The CON-OPP-CON method where she would start at the interior angle of her choice and write “CON,” “OPP,” and “CON” for each angle other that the first angle that Carol chose, in a clockwise direction.

Carol said that “this was easy” to do and she was able to do it in all of her work. After a few problems that we completed together, she started to write C for consecutive angles, and O for opposite angles. Even when I worked with her for an exam review, she was able to do this independently and quickly.

It seemed again that selecting an angle in the beginning and labelling the other angles served as a centring strategy. When I first started to work with Carol during this session, she said that she could not “figure out where to look and start,” a theme that has re-emerged.
8.1.8 Determining the Measures of the Angles that are Formed by Parallel Lines that are Cut by a Transversal

This intervention session seemed to have been successful. Carol said that “labelling the angles,” referring to each set of four angles that is formed by one of the parallel lines and the transversal, made it much easier to establish the relationships between the angles. She said “I could see what my teachers were talking about.” Carol answered most of these questions correctly when she did her homework assignments and class exam independently. The mistakes that she made were algebraic and arithmetic in nature. For the homework questions and exam questions that I saw, Sarah showed that she had known the correct relationships between the angles that were formed by the parallel lines and the transversal that intersects them.

8.1.9 Determining the Measures of the Angles that are Formed by Two or More Intersecting Lines

This intervention session with Carol also seemed to have led to successful results. The strategy that I taught Carol was similar to the method described in section 8.1.8 of this chapter. I instructed her to label the angles that are congruent to each other by using one star, two stars, or three stars, and finding the other congruent angles by extending the sides of the angles.

Carol said that she was able to use this strategy independently on homeworks, and also on her teacher constructed examinations. Whilst this topic appears in the curriculum before the topic in section 8.1.8, it may have been easier to conduct the intervention session after, because Carol said “it helped a lot to do the parallel lines” session before.
8.1.10 Graphing Trigonometric Functions

This session focused on only graphing sinusoidal and cosinusoidal functions. Though Carol’s visual-spatial deficits have often made graphing challenging, she found this topic to be especially difficult. There were three aspects of graphing that were difficult for Carol: determining the points on the graph of the function, plotting the points, and drawing the graph of the function accurately.

Carol’s executive functioning difficulties played a role in making these three tasks difficult for her. Once I reminded her to make a table for specific $x$-values, namely 0, $\pi/2$, $\pi$, $3\pi/2$, and $2\pi$, she was able to determine the corresponding $y$-value for each $x$-value.

Plotting points was initially difficult for Carol. Part of this difficulty stemmed from her not knowing how to create a proper scale on the axes for the graph. After I instructed her how to create a simple scale, the remaining difficulty was knowing how to plot the points accurately. She often interchanged the $x$-coordinate with the $y$-coordinate when plotting each points. When I noticed that she was doing this, I had her write $x$ and $y$, next to the $x$-value and $y$-value of each coordinate, respectively. This seemed to have helped as Carol make fewer mistakes.

Another graphing error that Carol made was incorrectly counting the number of boxes when graphing. She would often count the origin as 1 instead of 0, and this led to errors when she was plotting the $y$-value of each coordinate. When I told her to count the origin as 0, this problem seemed to have been substantially diminished.
The final problem that Carol had was drawing the graph of the functions. She would draw the maximums and minimums of the function with a point instead of a smooth curve. When I mentioned to her that she should “think of the [graphs of] the functions like a snake,” she remembered to draw smoother curves. She said “the snake thing really helped” her to remember how to draw the graphs of the functions.

Despite this process having many steps, Carol seemed to remember how to graph these functions correctly on her independent homework assignments and teacher constructed class examinations. On her class examination a few weeks after the intervention session, Carol answered the two questions on this topic mostly correctly. On both questions, she made an error when she was plotting each point. She made an error when counting on the y-axis. She said that the “small graph paper [on the examination paper] made it difficult for her to count [correctly].”

8.1.11 Determining the Lengths of the Segments of Chords, and Tangents to a Circle

Carol seemed to have success with the strategies that were presented in this intervention session. Again, as in other intervention sessions, centring was a theme that re-emerged. Once Carol was able to know where to start by putting a point where the chords of the circle intersected, she was able to apply the other strategies that were discussed in this intervention session.

The errors that Carol did make in her homework and examination questions were often arithmetic and algebraic in nature. She was able to correctly use the techniques that I taught her most of the time. However, she did make a
mistake on an examination question where she thought that a secant line was a tangent line. Carol said she made this error because “the [secant] line was barely in [the circle],” and that she could not see it clearly.

8.1.12 Visual Difficulties in Solving Algebraic Problems

Carol had a lot of difficulty in solving algebraic problems, most notably linear equations and inequalities. Whilst it may seem odd that I would classify solving linear equations and inequalities, and algebraic problems as visual-spatial in nature, the difficulties that Carol, just like Diana had, were in part about centring. Carol said “drawing the lines [around the equal sign or inequality symbol] was really good” because she “didn’t know where to start if the problem was long” referring to a problem with many terms.

Whilst Carol still had difficulty completing the algebraic aspects of the problems, she was able to better solve these types of questions. Even for quadratic equations, Carol said that it was very useful, and helped to remind her what steps she needed to take to solve a problem. It seemed that the lines may have acted as a visual cue that prompted her to activate her prior knowledge.

The lines also helped Carol, much like it helped Diana, with directionality as well. When she wrote “LEFT” and “RIGHT” over the respective sides of the equation or inequality, it made it easier for her to think about the sides she was moving the variables and the constants to which she found to be very helpful.
8.2 An Analysis of Diana’s Intervention Sessions

The fourteen intervention sessions that I conducted with Diana are shown in Table 3. This section will explore the efficacy of each of Diana’s intervention sessions. This analysis will be similar to the analysis of Carol’s interventions.

8.2.1 Determining if the Graph of a Relation is the Graph of a Function

The approach that I used to teach Diana was very similar to the approach that I used to teach Carol. In fact, they had many of the same difficulties. Just like Carol, Diana initially thought that none of the graphs of the relations were graphs of functions because the vertical lines that she drew intersected with the graph and the $x$-axis. Once I told Diana that she should not consider the $x$-axis to be a point of intersection with the vertical line, unless the graph intersected the $x$-axis, her misconception seemed to be cleared up. I even taught Diana that she should highlight the axes, which she did in the beginning, and later said “I don’t need [to do] this anymore.”

Diana was able to answer her homework questions independently and correctly. She was further able to answer all of the questions on her teacher-constructed class examination and midterm examination, the latter of which about two months later, correctly. She said “I found it easy to remember … the colouring [highlighting] really helped.”

8.2.2 Determining which Side of a Triangle is the Largest, Middle Length, and Smallest when Given the Measures of the Angles in the Triangle

Diana had a lot of difficulty determining which side of a triangle had the greatest and least measures when she was given the measures of the interior angles of the triangle. Whilst her coursework included questions where she had
to determine the measures of at least one angle in the triangle, this intervention
session was meant to focus on addressing the visual-spatial difficulties that she
had. Even though Diana sometimes had difficulty finding the measures of angles
in the triangle, this session was not meant to address this difficulty.

The method that was used to address this difficulty was very similar to
other sessions in that the approach taken was a simple one. I simply had Diana
write “L” next to the largest angle, “S” next to the smallest angle, and “M” for
the remaining middle angle. I then had her draw an arrow from each of these
angles to the opposite side, and write S, M, or L for each side respectively.

I also helped her to understand why the largest side was opposite the
largest angle, and the smallest side opposite the smallest angle. I had her start by
placing her index and middle fingers on the largest angle, and then spreading
them out until they coincided with the endpoints of the largest side of the
triangle. I had her repeat this process for the smallest side. This seemed to help
her understand the relationship between the angle measures and the side lengths.
At the end of the session, she said “I didn’t know it was that simple.”

8.2.3 Identifying the Sides of a Right Triangle for Trigonometry

This session focused on a skill that Diana would need to know for a lesson
that her teacher would present to her class later in the week. Based on my
previous experience teaching Diana, I thought that she would have difficulty
identifying the hypotenuse, and adjacent and opposite sides of an acute angle in
the right triangle. Whilst some of her challenges stemmed from an unfamiliarity
and complexity with the terminology and their use, it was apparent in the session
that Diana’s main difficulty was visual-spatial in nature.
I started by drawing a right triangle ABC, where C was the right angle, as shown in Figure 63. Since she remembered that the hypotenuse was opposite the right angle, she was able to identify that side very quickly. When I marked angle A with a loop, I had her draw an arrow to the side opposite this angle. In then instructed her to label this as the opposite side. Finally, I told her to label the remaining side as the adjacent side.

Figure 63: Figure with Determining the Sides of a Right Triangle
After this first question, I had Diana complete two questions with assistance, shown in Figure 64. I then proceeded to give her 4 additional questions where the orientations of the triangles were varied, as shown in Figure 65. When the orientations were different, Diana hesitated at first to answer these questions, but ultimately did answer all of these questions correctly. She said that “I needed to think about it for a second.”

Figure 64: Two Side Identification Questions with Right Triangles
I spoke to Diana after her first trigonometry lesson where she would have to know how to determine the different sides of the right triangle. Diana said “it was good that we went over this … because it was confusing learning the trig functions.” She continued, “It would have been really confusing if I didn’t,” referring to her learning this information ahead of time. Diana said that she did not have any difficulty labelling the sides of the right triangle even though some of her classmates did.

8.2.4 Addressing Difficulties with Directionality

Diana had significant difficulty with directionality, in her mathematics coursework and outside of her academic work. This was also noted by her psychologist who conducted her psychoeducational evaluation. He did not use a
standardised test such as the Benton Right-Left Orientation Test, and did not formally diagnose Diana with deficits in directionality. Rather, he mentioned how it affected some of her work on different subtests of the WISC-IV, including the Block Design Test.

I tried to help Diana with this by using physical aids. Diana usually wore a ring on her right hand, and a watch on her left hand. When her teachers gave her directions to look at the left or the right, Diana often made errors and looked in the incorrect direction.

To try to help her to overcome these difficulties, Diana and I spent the time during this intervention session to look at her ring that was on her right hand when she had to look to the right, and look at her watch when she was told to look to her left.

When Diana tried this method in class over the next week of lessons, it did not seem to be very successful. She said that she sometimes forgot about this method, but even when she remembered it, she did not do it correctly. In the end, this method was not simple for her, even though it seemed to be simple to me. I should have considered her word retrieval issues, especially considering that she was diagnosed with dyslexia. These reasons may explain why this method was ultimately unsuccessful.

8.2.5 Solving Linear Equations: A Visual-Spatial Viewpoint

After the unsuccessful intervention that was described in section 8.2.4 of this chapter, I worked with Diana to help her to develop strategies that would address an ongoing difficulty that she had with directionality in solving linear equations. The issue with this was not just directionality, but much like Carol,
Diana said that “I didn’t know where to start.” When I asked her what she meant, she said that she did not always know where to look when she started the problem.

I used the same strategy with Diana that I did with Carol. I had her draw lines around the equal sign, and then had her write “LEFT” and “RIGHT,” above the left and right sides of the equation respectively.

Diana said that this really helped because “it was easy to memorise,” and it made “starting the problem easier.” When I further questioned Diana, she said that it was better to write “LEFT” and “RIGHT” above the respective sides of the equation because “once I write it … I didn’t have to think [about the sides].”

It seemed that eliminating the need to have Diana actively think about the sides was the key to making this method successful. In addition, the lines around the equal sign seemed to have activated her prior knowledge about solving linear equations.

### 8.2.6 Solving and Graphing the Solution to a Linear Inequality on a Number Line

Diana had difficulty correctly graphing the solution to a linear inequality on a number line. In particular, she would not always know which way to shade the solution on the number line, and also determine if she should fill in the circle that corresponded to the solution of the inequality on the number line.

In this session, I showed Diana a strategy that was similar to the strategy that was described in section 8.2.5 of this chapter. We did not spend time working on strategies to solve the inequalities, but rather dedicated the session to graph solutions such as $x < 5$, or $x \geq 6$. Whilst Diana did have some difficulty
actually solving the inequalities, the purpose of the session was to address the visual-spatial difficulties that she had.

I had Diana draw an arrow for both the less than and greater than symbols, as shown in Figure 66. Once the arrows were drawn, Diana said “I see how they go now,” referring to the direction that she should shade the solution on the number line.

![Figure 66: Inequality Shading on the Number Line](image)

To help her determine if the circle on the number line should be filled or left unfilled, I had her think about the line underneath the less than or equal to, and the greater than or equal to symbols as pens that she could use to fill in the circle. She said “that is easy to remember.”
I had her shade the solutions to ten questions on the number line. Diana was able to complete all of these questions correctly. She did not see this topic again until her revision sessions for her June 2015 Algebra I (Common Core) Regents examination. Diana was able to answer question 35, shown in Figure 67 partially correctly. Whilst this question did not ask her to graph the solutions to the inequalities on a number line, and instead asked her to shade the solutions on a coordinate plane, she was able to remember how to shade correctly for each inequality. For question 9, which is shown in Figure 68 which again did not ask Diana to shade a solution on the number line, Diana correctly identified the
domain of the function. It required her to know which direction the graph would be drawn when given an inequality.

![Graph of a function](image)

**Figure 68: Question 9 from the June 2015 Algebra I (Common Core) Regents Examination**

(Diana’s examination paper could not be copied because of the restrictions set by the New York State Board of Regents)

When I asked Diana about these two questions, she said that “in this one,” referring to question 9, I remembered which way the “thing [inequality] should go because I drew the arrows.” For question 35 however, she said “I don't remember what I did … I was just too tired.” Her fatigue, towards the end of the 37 question examination, may have played a role in her answering the question partially incorrectly.

### 8.2.7 Comparing and Contrasting Chords, Secant Lines, and Tangent Lines to a Circle

The purpose of this session was to help Diana distinguish between a chord, a secant line, and tangent to a circle. Whilst language played a role in
Diana’s difficulties, there were also some very specific visual-spatial issues that also made it challenging for Diana.

The session started with discussing tangent lines to the circle. I told Diana that the word *tangent* is derived from the Latin word *tangens* which means “barely touching, so it can only touch the circle barely, or just once.” Diana said that this helped her to remember the term. She said that “*I’ll probably forget your story,*” but that she will still remember that the tangent line will be outside the circle except for the one point that touches he circle.

For secant lines, I told Diana that “*since the word ‘secant’ starts with ‘sec,’ it has to touch the circle a second time.*” She seemed to understand that the secant line would extend outside the circle on her own. She said “*the sec thing will help.*” Without any prompting from me, Diana wrote “SEC” over the secant line in the circle.

Finally, with chords, I told Diana that chords have to be entirely in the circle because the chord “*starts with a c*” so it has to be entirely in the circle. Diana said that she understood this, but I sensed some hesitation from her facial expression and body language.

When Diana was taught this lesson by her teacher, she said that she was able to remember the different segments. She said that learning the terms with the diagrams ahead of time were helpful and that it was “*a lot less confusing [when] I learned it before.*” Diana additionally stated that she did not always know whether a given segment or line was a chord, secant, or tangent, but when her teacher said each term, she remembered what we had discussed in the intervention session.
8.2.8 Recognising and Understanding the Nature of Similar Triangles when a Larger Triangle has a Segment Drawn with Endpoints on Two Different Sides, and the Drawn Segment is Parallel to the Third Side

Diana had difficulty distinguishing between the two triangles that were presented in the diagram that is shown in Figure 69. In particular, in these types of problems, Diana did not recognise that there were two separate triangles which were similar to each other. In addition, whilst she said that she noticed the midsegment of the triangle, the segment whose endpoints are the midpoints of two different sides and is parallel to the third side of the triangle, she did not seem to know “what to do with it [the midsegment]” even when she was given a formula that stated the relationship between the midsegment and the parallel side.

![Figure 69: Triangle with a Midsegment](image)

The focus of this session was to help Diana identify the midsegment and the side that it was parallel to. In addition, another goal, though not the primary
goal, was to show her that the corresponding angles of the two triangles, including the shared angle, are congruent to each other.

I had Diana reorient the triangle if needed by turning the page around, so that the midsegment was shown horizontally. This allowed Diana to see the triangle the way that she first saw it in class and in our sessions. This served as a centring activity and it allowed Diana to focus her attention. Once this was done, I had Diana write “TOP” for the angle at the upper part of the diagram, “MID” for the segment that represented the midsegment, and “BOTTOM” for the base of the triangle, as shown in Figure 70. This seemed to help Diana recognise the midsegment and the base of the triangle that was parallel to it. She was able to do identify these segments in the six triangles that I showed her.

![Figure 70: Triangle with a Midsegment and Notations](image)

I also taught Diana about the relationship between the lengths of the midsegment and base of the triangle since she had trouble remembering it. Since
the measure of the midsegment is half the measure of the base, and the word “MID,” which was used to label the midsegment had half as many words as “BOTTOM” which marked the base of the triangle, I told her that the length of the midsegment was equivalent to the half the length of the base.

Diana said that both strategies did help her, and she was able to answer four questions on her exam a few weeks after the session was held correctly or partially correctly. Diana was able to identify the base and midsegment correctly in all of the problems, but made algebraic errors on two of the questions. It seemed that she did find success with the strategy in identifying the important segments.

As the time that we had for the session ended, we were not able to discuss the relationship between the angles in the two similar triangles. Whilst we planned to cover this in a later session, we were not able to do so.

8.2.9 Recognising Interior and Exterior Angles in a Polygon, and Establishing Basic Relationships Between these Angles

Diana had significant difficulty determining if an angle in a polygon was an interior or exterior angle. Whilst she was able to do this easily for quadrilaterals, she had difficulty doing this for triangles, and polygons with more than four sides.

The strategy that I used to teach Diana was very basic. I had her highlight the sides of the polygon in one colour. On the inside of this boundary, I had her write “IN.” On the outside of this boundary, I had her write “OUT.” She had some hesitation about the inside. I described the inside of the polygon to her as the “area between the sides [of the polygon],” and everything else that was not
between the sides to be the outside. This seemed to have helped her, as she was able to correctly identify the inside and outside fairly quickly in the problems that we did in this session.

The other part of the session focused on establishing that the sum of the measures of an interior angle of a polygon and its corresponding exterior angle have a sum of $180^\circ$. Again, I sought to give Diana a basic explanation. I had her use a ruler to see that any interior angle and its corresponding exterior angle form a straight line, which she already knew had a measure of $180^\circ$. Once I showed her the relationship between these angles by using a ruler, Diana seemed to understand this. She reported that she did not have difficulties with either aspect of this intervention session.

8.2.10 Determining the Measures of the Angles Formed by Parallel Lines that are Cut by a Transversal

The strategy that I used in this intervention session was identical to the labelling strategy that I used to teach Carol which was described in section 8.1.8 of this chapter. Just as with Carol, Diana found success with this strategy. She used it whilst completing her homework questions independently, and also when she completed questions in her teacher constructed examinations. Diana did struggle with directionality, and hesitated before she labelled each angle with a left or right, and up or down. Nevertheless, she did it accurately.

8.2.11 Addressing Difficulties with Translations

Diana had difficulty understanding how translations moved figures on a graph. She did not have too much difficulty with the arithmetic and algebraic
aspects of the work, which often is the reason that students have difficulty with translations.

However, Diana had challenges with what she described as “seeing how they move,” referring to how figures moved even when a preimage and image after a translation are shown on a graph. To help her overcome this difficulty, I again sought to use a simple strategy that she could recreate independently.

I had Diana highlight the preimage in one colour, and the image in a different colour. I then had Diana select a point on the preimage, and its corresponding figure on the image. To my surprise, it seemed that she as able to do this without much difficulty in all of the figures that I showed to her in the problems that we completed. Diana then counted the number of units to the left or right, and up or down that the corresponding points from the preimage and image were. Since I instructed Diana to select a prominent point in each figure, she was able to count the number of units between the two points more easily. However, the main problem that she had was to count the number of units correctly. She often counted the point that she selected as one instead of zero. Despite this, Diana seemed to gain an understanding of how the translation rules dictated how the figures would be moved. She said at the end of the session “I get it now.”

As mentioned before, Diana for the most part was able to graph the image, but did not have an understanding of what she was doing and was just applying the given rules. Her statement after this session suggested that she seemed to gain an understanding of translations.
8.2.12 Addressing Difficulties with Rotations

The difficulty that Diana faced with rotations were similar to her difficulties with translations. I asked her to explain what her difficulties were at first. She said that “I can’t tell where to look [for the change from the preimage] … and everything looks jumbled up.”

In order to help her to overcome these difficulties, I had Diana highlight the preimage and image in different colours. I then asked Diana “What do you think a rotation is?” She replied “it's a spin.” I then had her mark two points, one on the preimage and one on the image that corresponded to each other. I had her use her finger to move from the corresponding points from the preimage to the image in a circular manner, since it is a spin, as Diana described it. I had to remind Diana that all of the movements had to be anticlockwise since rotations in the Geometry Regents course are almost always in this direction, and always centred at the origin unless otherwise indicated. This course sometimes covers rotations that are clockwise, and never has rotations that are not centred at the origin. I also explained to her that the more she moved her finger from one point to another, the rotation has a greater degree measure.

After I had Diana explain how the rotations occurred, she seemed to understand the direction and magnitude of the rotation. At the end of the session she said “I think I get it now.” As with the session on translations, Diana’s statements indicated that it seemed she understood how rotations created the image from the preimage.
8.2.13 Addressing Difficulties with Dilations (Enlargements)

In the United States, enlargements are called dilations. The strategy that I used to teach her about dilations in this session was very similar to the sessions on translations and rotations.

Again, I asked Diana to highlight the preimage and image in different colours. Once she did this, she said “Oh, I think I see it now.” The highlighting almost served as a mechanism for centring where she could know what to focus her attention on in the diagram. I realised this when I asked her to explain what she was able to see that was new. She replied “I think I know where to start now,” referring to her ability to focus her attention on the important aspects of the graph. Diana further said “once I highlighted these … I know where everything is going.” When I asked her what she meant, she said that she had known what to look at first, and what to look for after highlighting the preimage.

Based on her statements, and her ability to explain how the preimage becomes the image by the end of this session, it seemed again that Diana had a better understanding of dilations.

8.2.14 Addressing Issues with Compound Transformations

The final intervention session with Diana focused on addressing difficulties with compound transformations where she would have to combine two or more of the transformations that we discussed. Whilst at first Diana may have had difficulties with the visual-spatial deficits that these types of problems presented, it seemed that after the three intervention sessions that I conducted with Diana immediately before this session helped her to overcome her difficulties.
The problems that Diana seemed to have stemmed from her executive functioning challenges. Diana’s psychologist wrote about this specific deficiency in her psychoeducational evaluation. This may make organising the steps in the procedure for these compound transformations, such as a rotation followed by a translation, difficult for her to carry out.

It was especially difficult to develop a strategy to help Diana with this topic. I started with instructing Diana with the highlighting strategy as before, but she seemed to be confused when she had to highlight three different figures. This caused a lot of confusion for her. When I asked Diana to note the corresponding points on the three figures, she seemed to shut down. She said “I can't do it.” It seemed that she was very tired, and the timing of the session was not ideal as it was held on a Friday afternoon, at the end of a full school week. Diana then asked to stop the session, and I agreed that we should end the session.

Whilst she was able to make some progress in understanding how single transformations were formed, she did not have much success with this topic. Diana opted not to follow through with another session on this topic, so we did not meet again to discuss this.

8.3 Successful Intervention Themes from the Sessions

It seemed that the main themes that emerged from the sessions which were successful were simplicity, ability to independently reproduce certain strategies in high stress situations, and centring, where students were given a cue to focus on the problem in a helpful manner. If one or more of these criteria were met, it seemed that the intervention worked. This is not surprising to me. In my
work with Carol, Diana, and the other students that I have taught in my school, these criteria often determine if a strategy is successful.

One of the most important themes that emerged from the intervention sessions was centring. Whilst I had known this before I started the research, the interventions with both Carol and Diana have helped me to formalise and utilise it much more directly, and seemingly with greater efficacy. I feel that it is also a theme that I am more comfortable sharing with my colleagues at my specialist school. I think in the future that this may perhaps become the most important of the themes that have emerged from the interventions that I have conducted with Carol and Diana. In fact, the techniques that are based on centring incorporate the other themes: simplicity and reproducibility during stressful situations, that have also emerged during the interventions.

8.4 Intervention Themes that were Not Successful

Whilst the intervention sessions were generally successful, there were several cases where it seemed that the interventions were not helpful in helping Carol and Diana learn mathematics. One of the themes that emerged was that physical, mental, and emotional fatigue were sometimes factors in a less than successful intervention. Though teachers may find it easier to notice physical fatigue, it is probably not as easy to notice mental and emotional fatigue. I believe that building a relationship with a student will make it easier to notice the latter two types of fatigue. Firstly, the students are likely to simply admit that they are tired or distracted as Carol and Diana have on several occasions. I also believe that they are less fearful or worried about disappointing their teachers if they feel more comfortable working with them. In fact, Diana has said this about
the teachers that she has found the greatest success with. She said that the positive relationships that she developed with her teachers helped to reduce her anxiety. This is a sentiment that has been echoed by many of the students that I have taught at my specialist school.
This chapter will discuss the work that I did with the mathematics teachers in my school, and a little about the work that I did with teachers outside of my school. I will primarily describe the work with the teachers in the high school of my specialist school and aim to show how it was a form of an intervention.

9.1 An Unexpected Outgrowth of My Research

I did not anticipate having intervention sessions with teachers who are in my department when I started this research. However, it became an integral part of my research when I started to give presentations about MLDs to the elementary, middle, and high schools, teachers in my school, and then later to teachers outside of my school. When I spoke to my Director of Studies about these presentations, especially with teachers in my department, he suggested that this could be an important addition to my research.

9.2 The Structure of the Intervention Sessions with Teachers in my Department

The structure of the intervention sessions that I conducted with the teachers in my department for the most part reflected the intervention sessions that I conducted with Carol and Diana. The sessions typically focused on one topic, and were usually ten to twenty minutes in length. My goal was to ensure that the teachers would be able to ask questions, and also make sure that there was a chance for an open discussion.
The other intervention sessions that I had with the teachers were individual meetings where a lead teacher, assistant teacher, or a teaching team that was made up of one lead and one assistant teacher, and I would meet. We would discuss certain issues that they had with a student, or with the content that they were teaching.

9.3 The Topics for the Intervention Sessions with the Teachers in My Department

The topics for the intervention sessions were either requested directly by the teachers, or they had their origin from my lesson observations and listening to the concerns and frustrations that the teachers in my department shared.

The five intervention sessions that I conducted with the teachers in the department as a whole are shown in Table 4.

<table>
<thead>
<tr>
<th>Topic for the Intervention Session</th>
<th>Brief Description of Intervention Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction to MLDs</td>
<td>This session was a one hour session that introduced teachers to a variety of MLDs, as well as visual-spatial deficits.</td>
</tr>
<tr>
<td>Difficulties with Writing</td>
<td>This session focused on helping teachers, especially new teachers understand the difficulties that students in my specialist school have.</td>
</tr>
<tr>
<td>Helping Teachers Develop Strategies to Teach Students with Visual-Spatial Deficits</td>
<td>This session focused on finding a strategy to teach the Midsegment Theorem.</td>
</tr>
<tr>
<td>Understanding Difficulties with Mental Mathematics for Students with Learning Disabilities</td>
<td>This session focused on helping teachers understand why mental mathematics can be difficult for the students in my specialist school.</td>
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<td>---</td>
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</tr>
<tr>
<td>Psychoeducational Evaluations for Mathematics Teachers</td>
<td>These sessions focused on what information that mathematics teachers should look for in a student psychoeducational evaluation in order to gain meaningful information that will aid in instruction, and discussion with parents and the students themselves.</td>
</tr>
</tbody>
</table>

Table 4: Table of the Intervention Sessions Held with the Mathematics Department in my Specialist School

9.3.1 Introduction to MLDs

This session focused on introducing MLDs to the teachers in my department and other departments in the three divisions of my school. The session, which was attended by about twenty five teachers included 5 of the 7 high school mathematics teachers (not including myself). Most of the teachers who attended this session taught in the middle school or high school. In addition, one high school mathematics teacher who was from a different specialist school that is very similar to my school also attended the session.

This session introduced MLDs to almost all of the teachers for the first time. The principal of the middle school, who has a master’s degree in special education, and who attended this session said that “*this was the first time I heard about these [mathematics] disabilities other than dyscalculia.*” She continued to say that she only had a very basic understanding of dyscalculia, and that in all of her undergraduate and graduate coursework she had a minimal exposure to
MLDs. In fact, she said “we only spent a few minutes [in all of my graduate coursework] on it [dyscalculia].” This sentiment was also echoed by the elementary school principal, and one of the co-principals of the high school. The other co-principal of the high school did say that she learned about the different subtypes of dyscalculia from her studies in neuropsychology.

This success of this session led to another session for the elementary school teachers which was held a few weeks later. Again, these teachers and the elementary school principal (who had also earned a master’s degree in special education) that attended this session, also said that she “never really learned about math disabilities in all of my [undergraduate and graduate] courses.”

Over the years since the first presentation that I made, I have also started to present a regular one day workshop for new lead and assistant teachers who in the summer before the school year started in 2016. My school’s new elementary and middle school mathematics specialist, who had taught mathematics for over ten years, and also had earned a master’s degree in special education, again echoed the sentiments of the principals and said “I’ve heard [about] dyscalculia, but not much other than a two minute discussion [in a graduate course] ... I never learned about these other disabilities before.” Almost all of the teachers who attended this presentation also did not previously learn about MLDs, and even if they did, only did so in a superficial manner.

As I was invited to other schools to make presentations about MLDs, I would often have teachers who taught other subjects that would say that they thought that they had a certain subtype of dyscalculia, or believed that they had a visual-spatial deficit. I felt that these comments suggested that there needs to be
more education on MLDs even for teachers who are familiar with learning disabilities.

9.3.2 A Session on the Difficulties with Writing

One of the difficulties that all of the high school mathematics teachers, both experienced and new, have said is that they have difficulty helping students with writing and keep track of their work. The teachers stated that they had difficulty understanding why their students found it challenging to write numbers in algebraic equations accurately from the previous lines. The difficulties, which may be associated with dyslexia, perhaps visual-spatial deficits, dysgraphia, or attentional issues made accurate copying challenging for the students.

Whilst I am not certain why the students have difficulty copying information correctly, it is certainly something that I have seen many times before. My approach to help my colleagues to understand this difficulty was to increase the teachers’ understanding of the difficulties that their students had.

This session was attended by three lead teachers, and three assistant teachers. All of the lead teachers started as assistant teachers, and they had been teaching at the school for varying amounts of time ranging from three years to ten years. Two of the assistant teachers were new to the school, and in fact new to teaching. The other assistant teacher was starting her second year at the school.

Figure 71 shows the first part of the activity that was conducted during a short session that was about 20 minutes in length. This first part was about 10 minutes in length. The teachers, all of whom were very competent in high school level geometry, were required to answer question 36 from the August 2015 NYS
Geometry (Common Core) Regents examination. However, they were required to hold the page so “Page 1” appeared right side up on the page. The teachers were required to read the question upside down, and write all of their workings and solutions on the page. However, they did not have to write upside down. Whilst I did not tell them to write their answers in specific locations, since the Regents guidelines do not require that the workings and solutions to be written in certain sections on the page, they reported that they felt their answers should be written in the given spaces. This assumption that they made limited their ability to make the work easier for themselves, and is something that I have seen many of the students in my school do. Even though the entire page was available, just as it was on the actual Regents examination, the limited space did make the task more difficult.

In addition to the difficulty of writing in limited area of the page, the teachers also said that they had a lot of trouble keeping track of what the question was asking. One of the goals that I had for this task was to emulate the difficulty that a dyslexic student may have when reading the question. Whilst the actual process of reading the question cannot be directly emulated, I felt if the difficulty in reading was emulated, it was a good way to show the teachers how difficult it is for the students to read and interpret a question, even if they had previously seen the question as several of the teachers did, and even if they had successfully completed before. In fact, several teachers, including me, actually answered parts of the question incorrectly even though we had seen and correctly answered the question before.
Finally, almost all of the teacher also stated that doing just this one question was very tiring. One teacher, Zara, stated that she was “exhausted” after doing this question. This was a feeling that re-emerged for her after she completed the second part of this activity.

Figure 71: Upside Down Question #36 from the August 2015 NYS Geometry Regents (Common Core)

Figure 72, shows the question that the second part of the professional development was based on. In this activity, the word problem was presented right
side up. The teachers however had to write their workings and solution upside down.

![Upside Down Writing Question](image)

**Figure 72: An Upside Down Writing Question that is Based on the NYS Regents Algebra I (Common Core) Curriculum**

This activity was more impactful in helping teachers understand the difficulties that the students have faced. Several teachers have reported that they had difficulty determining where to start writing, and also had difficulty keeping track of their work. In addition, the teachers also stated that it was difficult to read what they had written in the previous line just seconds before they had to re-write some of the workings on the next line. One of the first-year assistant teachers, Yanna, reiterated how tiring this experience was, and how much more
compassion she would have for students. Rebecca, a ninth year lead teacher, said that “this was eye opening” despite her extensive experience. Her comment was very revealing to me because I felt that she had a solid understanding of this particular issue since we discussed it several times during her time as a teacher at my specialist school, including her first two years at the school where she worked with me as an assistant teacher. After completing this activity she additionally said that it was a good reminder, and “reminds me of the problems they [the students] have”.

An interesting side note about this activity is that most teachers who completed this did not write their names on either side of the paper. Students not writing their names on assignments is a common source of frustration for teachers in all of the departments in my school. When I presented this task to my fellow PhD candidates and supervisors, they all openly discussed how difficult it would be to write their names on the papers. To my surprise, when I quickly examined their papers, no one wrote his or her name on either side of the paper. After the discussion that they had, I fully expected that all of them would write their names on the paper.

9.3.3 Helping Teachers Develop Strategies to Teach Students with Visual-Spatial Difficulties

This professional development session was about twenty minutes in length. It was attended by two lead teachers, and four assistant teachers. Most of the same teachers who attended the professional development session about writing difficulties took part, in addition to an assistant teacher who was absent
on the day that the writing difficulties session was held. This assistant teacher was a second year teacher, and was an assistant science teacher the previous school year. One lead teacher who was present at the writing difficulties meeting could not attend this professional development presentation. In addition, my Director of Studies also attended this session.

I started this session by reviewing some of the visual-spatial misconceptions that the students had in two and three dimensions. A lot of the images that I displayed came from my pilot study where I interviewed students. In addition, I showed several problems that students commonly had difficulties with, most notably questions with parallel lines that were cut by at least one transversal. Whilst my intention was to discuss these types of questions in detail, the open nature of the meetings encouraged teachers to discuss any issues that have come up in their own classes recently.

Rebecca asked that I change the topic from parallel lines cut by a transversal to the Midsegment Theorem. She said that some of her students in one of her eleventh grade classes (which includes Diana) had difficulty distinguishing between the midsegment of the triangle and the base that it is parallel to. Rebecca noted that this was primarily a visual-spatial issue because the students were able to correctly solve these types of questions such as the one shown in Figure 73 as long as the students were able to identify the base and midsegment correctly.
Figure 73: Question about the Lengths of the Midsegment and Parallel Base of Triangle ABC

Whilst this was not a topic that was preplanned, as a department, we have developed a way of thinking about such problems. I suggested a simple method that the students would be able to learn relatively quickly, and also practise and remember with greater success. If a student identifies the peak of the triangle (assuming the parallel segments were horizontal), he or she can simply move his or her finger down to the first parallel segment. Rebecca said that her students were generally able to identify this as the middle side; they were instructed to label this as “mid.” As they moved their finger down to the other parallel segment that was actually a side of the triangle, they would label this as “LONGER.” Again, Rebecca’s students were generally able to identify this as a segment that was longer than the midsegment. Once this was done, Rebecca’s students were able to write a correct equation such as: 

\[ 2 \text{ (mid) } = \text{LONGER}, \quad 2 \ (4x - 2) = 7x + 1, \]

correctly find the value of \( x \), and then find the length of the midsegment and the measure of the side of the triangle that was parallel to it.

Writing “mid” and “LONGER” was important too because it reminded students that the word “LONGER” had twice as many letters as “mid,” so twice
the number of letters in “mid” equals the number of letters in “LONGER.” It helps to remind students of the relationship between the lengths of the midsegment and the base of the triangle. Also writing “mid” in lowercase and “LONGER” in uppercase helps to remind students of the sizes of each of these segments as well.

A few days after the meeting, I spoke to Rebecca and Yanna, the assistant teacher who worked with Rebecca, to determine the effectiveness of this strategy. Both of them reported that the students who were struggling were able to use this strategy correctly, and after just a short time within the 40 minute class, were able to do it independently. This was also somewhat true for Diana, who I have been working with for this research.

I also spoke to the other teachers who attended the meeting. Whilst they did were not teaching this specific topic, they did state that they could use this specific strategy, or develop their own strategies that is based on this model for other topics.

9.3.4 Understanding Difficulties with Mental Mathematics

Teachers who are new to my school, or new to teaching in a specialist school exclusively for students with learning disabilities, often have difficulty understanding why high school students have difficulty with basic calculations. They are often frustrated by the inability that students have in performing the calculations accurately, or with speed. Even questions that many teachers believe are simple, such as “100 x 1” may be difficult for students because of their
memory issues, impulsivity, or poor number sense. This may be a result of difficulties that are similar to those students who have verbal dyscalculia.

Because of this frustration that these teachers face, I felt that it was important to address this issue in a professional development meeting. This meeting was attended by three lead teachers, and four assistant teachers. All of the teachers in the high school mathematics department meeting attended this meeting. The meeting was about twenty minutes in length. In addition, my Director of Studies also attended this session.

Figures 74 and 75 show the two sides of the sheet that was given to the teachers. Teachers had to evaluate each question by referring to the “Key for Number and Letter Correspondence” on the back page of the handout. They were not permitted to fold the page so that they could see the questions and the key at the same time. Instead, they had to flip back and forth. This was done to show teachers what it was like for students who had memory deficiencies to try to retrieve information that they know, even if they have seen it many times before, but simply have difficulty retrieving the information.
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**Figure 74:** The Front Page of the Handout on Decoding and Dyscalculia

**Key for Number and Letter Correspondence**

- One = p
- Two = k
- Three = L
- Four = g
- Five = m
- Six = Z
- Seven = M
- Eight = G
- Nine = Y
- Ten = c
- Zero = W

**Figure 75:** The back page of the Handout on Decoding and Dyscalculia
For the first three questions, the teachers were permitted to change the letters to numbers, solve the problem, and then covert the final answer back into letters. For the final question, however, the teachers had to only write letters for their workings, and also the final answer. This question was the most difficult not just for this reason, but it was also the question with the most steps. In addition, this question involved not only division, but also subtraction, and multiplication.

There were several important issues that emerged in the discussion after the activity. Firstly, whilst the teachers started the problem, I kept giving directions. This is an unconscious habit that I have seen many teachers develop when giving work, especially new work, to their students. Whilst they may feel that this is helpful to students, it actually distracts them from learning the new work. In fact, the Director of Studies remarked that this was very “annoying.” This was a point that I wanted to drive home to the teachers in their work.

Secondly, the exhaustion issue re-emerged in this activity. Several teachers noted that it was very tiring to flip back and forth an apply the information from one page to another. They also noted that it must be very difficult to do many questions during a class, if completing only four questions was so challenging.

Thirdly, the teachers who were very competent in arithmetic said that they could not easily rely on their number sense because they had to change letters to numbers and then back again. This is essentially a variation of decoding, which is often a prominent difficulty for students with dyslexia.

Finally, the question has “p,” and not “P.” Whilst “P” was on the Key for Number and Letter Correspondence, “p” was not. Originally, I made a typo, but
it was serendipitous as it led to a good discussion point. As American mathematics teachers, we sometimes use multiple notations to represent the same thing. Whilst this is probably as not much the case in the U.K., it reminded teachers that we must be very consistent with our notation, and also directly review the various notations that we use.

9.3.5 Psychoeducational Evaluations for Mathematics Teachers

A lot of the professional development sessions that various professionals in my school have presented on understanding psychoeducational evaluations has been at an introductory level, and these presenters have never discussed MLDs in any substantial detail. This is partly due to the typical profile of our students’ learning disabilities and partly due to the dearth of information on this subject. This session sought to give the mathematics teachers in my department a better understanding of what information to look for when a student is diagnosed with an MLD, and also even if they are not.

As mentioned in Chapter 2 of this thesis, Mathematics Disorder is no longer listed in DSM-V. Mathematics Disorder, dyslexia, and other commonly diagnosed learning disabilities, are classified under a more general term, SLD.

This session had multiple parts that were split up into the different types of tests that are part of a standard psychoeducational evaluation including memory, executive functioning, and visual-spatial tests and subtests. All of the tests that were described in this session were part of the WISC-IV or the WJ-III since the psychologists and psychiatrists who write evaluations for the students in my school usually use these tests.
9.3.5.1 Digit Span Tests

The first memory test that I described to the teachers was the *Digit Span Test* which is a subtest of the WISC-IV. It requires students to say a series of numbers in the same order in which the evaluator presented them. The evaluator starts by reading two digits aloud and then proceeds to more digits if the student who is being evaluated is able to say the digits correctly.

Rosenthal (2006, p.131-132) wrote that this test “*is a task of short-term auditory memory, sequencing, and simple verbal expression.*” Deficiencies in short-term memory can help teachers to understand that students may have difficulties that will make it a challenge for them to memorise not just arithmetic facts, but also memorising the steps in an algorithm that a student uses to solve a problem.

Jones (2009, p.310) wrote that the *Numbers Reversed Test* which is part of the WJ-III, and the *Digits Backwards* test, a subsection of the *Digit Span Test* in WISC-IV have similar procedures and are designed to determine if a student’s active working memory is intact. In addition, Jones (2009, p.310-311) wrote that these tests may reveal if a student’s has attentional deficits.

The importance of the test was readily apparent to the teachers. Callie, a second year assistant teacher said “*it’s really good to know this ... it makes it easier to talk to the parents [about their child’s learning disabilities].*”

9.3.5.2 Letter Number Sequencing Test

The *Letter Number Sequencing Test* which is part of WISC-IV is another test that aims to measure a student’s working memory. Murtagh (2009, p.102)
wrote that this test is used to determine not only a student’s short term memory, but also a student’s working memory. Along with the different digit span tests, this also helps to establish if a student has memory and attentional issues.

9.3.5.3 Arithmetic Tests

The \textit{Arithmetic Test} and the \textit{Calculation, Math Fluency, Applied Problems, and Quantitative Concepts} test which are part of the WISC-IV and WJ-III respectively are used to determine if a student has difficulty completing arithmetic questions.

9.3.5.4 Visual-Spatial Tests

The \textit{Block Design Test} and the \textit{Block Design, and Object Assembly} test which are part of the WISC-IV and WJ-III respectively, are subtests that require students to re-create block designs by using different coloured blocks. In addition to determining if students have visual-spatial deficits, these timed tests may also serve as a measure of problem solving and perceptual reasoning skills.

9.3.5.5 Executive Functioning Tests

The \textit{Wisconsin Card Sorting Test} (WCST) and \textit{Delis-Kaplan Executive Functioning System} (D-KEFS) test are not directly part of the WISC-IV or WJ-III but are still used by many of the psychologists and psychiatrists who write the psychoeducational evaluations for the students in my school. The former test is a subtest of the \textit{PEBL Psychological Test Battery}. Pearson (n.d.), the company that owns the rights to the D-KEFS describes it as the “\textit{the first nationally \ldots}
standardized set of tests to evaluate higher level cognitive functions in both children and adults.” The D-KEFS has appeared a lot more in the more recent set of psychoeducational evaluations that students in my specialist school have.

Explaining how the executive functioning tests are given, and their consequences was especially useful for a lot of the new teachers, and helped them to understand the difficulties that our students have had. In particular, Yanna said “this makes a lot more sense,” referring to a pair of students that she worked extensively with who had significant executive functioning issues. It seemed that she had developed a deeper understanding of executive functioning difficulties after this part of the session.

9.4 Individual Sessions with Teachers

In addition to the group sessions, I met with individual teachers, or a teaching team which was made up of the lead teacher and assistant teacher who worked together on a daily basis. Usually the teachers approached me with a topic that they found difficult to teach, or were trying to find an alternative method for teaching a particular lesson.

In these shorter sessions I usually showed the teachers one or two strategies for approaching a particular lesson. The lessons that they asked about were not always dealing with visual-spatial aspects, but did focus on not just one or two topics.

The sessions were also helpful because I was able to see the types of difficulties that the teachers had when they were teaching their classes.
allowed me to think about future topics of discussion for the intervention sessions with the teachers.

9.5 The Importance of these Activities

It became apparent that these sessions were very important. In addition to providing help to teachers develop strategies in teaching their classes, it also gave the entire department a chance to work closely together. These sessions transformed the meetings that we held into a much more useful and dynamic discussions.

As previously mentioned, the topics for the individual sessions with the teachers were almost exclusively set by an individual teacher or a teaching team. Since I did not usually choose the topics for these sessions, the teachers who requested these sessions had a vested interest in gaining the most out of each of the sessions. Susan, a fourth year Lead Teacher, who worked with me during her first year at my school, seemed to gain a lot from these sessions. She may have felt more comfortable seeking my help since she and I worked together from the start of her tenure at the school. Susan had sought help for a variety of topics ranging from introductory algebra to geometry and trigonometry. In each session, she and I spoke not just about different teaching strategies, but also the students that she was teaching. This seemed to be just as important as the strategies since I was able to provide information about the students that I taught in previous years, or from information that I gleaned from a particular student’s psychoeducational evaluation.
These meetings gave her some confidence that she did not have before, and it seemed to help her to reduce her anxiety, which Susan had acknowledged. Beilock (2009, p.952) who had conducted research on the role that a female teacher’s anxiety plays in the classroom had noted that female students, sometimes internalise the teacher’s anxiety themselves. If this was the only beneficial outcome from the individual sessions, that achievement alone was well worth the time that was taken to hold the meetings.

Overall, several teachers have said individual sessions gave them a sense of confidence especially when working with academically or behaviourally challenging students. The insights that they gained helped them additional valuable tools to educate their students.
Chapter 10: Analysis of Interventions with the Teachers in my Department

This chapter will focus on the analysis of the intervention sessions that I conducted with the teachers in my department. The primary goals of this chapter are to determine if each intervention worked, and why they may or may not have worked. Whilst a variety of methods may be used to determine if the interventions were successful, I have chosen to mainly focus on teacher interviews to determine if they felt that the interventions were effective. In particular, I looked for evidence to suggest that the teachers changed their teaching style, or were able to gain a better understanding of the struggles that their students faced.

The analysis of the interviews will be separated into two parts: an analysis of the group intervention sessions that were shown in Table 4, and an analysis of the intervention sessions that I conducted with an individual teacher or teaching pair.

10.1 An Analysis of the Group Intervention Sessions

This section of this chapter will focus on the efficacy of the five group intervention sessions that I conducted with the teachers in my department. These five sessions were held throughout the course of my degree program.

10.1.1 An Analysis of the Introduction to MLDs Session

This was the first group session that I conducted with the teachers in my department. In a way, it was the springboard to this strand of my research. Several teachers, especially the veteran mathematics lead teachers who attended this workshop, reported that this was important not just for the content that they...
learned, but it also made them feel more confident in asking me for help. Eric, who had been teaching at the school for nine years at the time when he attended this session, said “This was the first time [that] I heard most of the names,” referring to the different types of dyscalculia and the other MLDs that were presented. Rebecca, who was in her eighth year at the school, also attended this session and said that it was important to have “language to talk to the parents and students with.” Both Eric, Rebecca, and another second year lead teacher, who has since left the school for another position, all remarked that they would be keen to learn more about MLDs during our regularly scheduled department meetings which are shorter in length. The middle school principal who also attended this session said of the workshop “it was good to see what the students are seeing” because “as teachers we don’t expect to see this.” She continued to say that it was a good reminder for her and other teachers.

In addition to the efficacy of the lessons, as mentioned in the previous chapter both Eric and Rebecca said that they felt more confident in asking for my help from me after this session. Eric, who has taught at the school for more than ten years said that this was the first meaningful professional development session about mathematics that was held at the school. They further said that they wanted to learn more about how the learning disabilities appeared for specific students in their classes.

10.1.2 An Analysis of the Session on Difficulties with Writing

The motivation for this session came from the teachers in my department. Many of the teachers, in particular the newer assistant teachers, have said that they had difficulty understanding why students had so much trouble with the
difficulty with writing. This session which is made up of two parts was aimed to help the teachers understand the difficulties that the students have with writing.

It appears that Yanna, a first year assistant teacher, may have been the person who was the most affected by this session. She said that she finally understood “how worried they [the students] must feel” when she had to do the two writing activities. Just as Rebecca, the lead teacher who works with Yanna said, she too said that the activities were exhausting.

Overall, it seemed that the teachers were able to better understand the difficulties that their students face when writing. Their responses indicated that they may be more likely to be empathetic to their students’ challenges.

10.1.3 An Analysis of Helping Teachers Develop Strategies to Teach Students with Visual-Spatial Difficulties

The original aim of this session was to show several strategies for teachers in the department to teach about the graphs of perpendicular lines. However, the topic quickly changed when Rebecca sought assistance on how to better teach the Midsegment Theorem.

Before analysing the efficacy of this session, I feel that it is important to note that the teachers felt that they were welcome to change the topics of the meetings, and critique any information that I presented throughout our meetings. In fact, Eric, Rebecca, and Susan, who have been the lead teachers in the department for the last three years all said this in their annual reviews at the end of the year. This openness to change topics and hear new ideas is in my opinion perhaps just as important as any information or strategies that I presented.
Rebecca and Yanna, who was working with Rebecca as an assistant teacher, both reported that the strategies that I presented in this session were very helpful. They were teaching this topic to a class that included Diana and other students who had typically struggled with problems that have visual-spatial elements to them. Rebecca said that when she used this strategy that the students were “able to figure out where to write mid,” referring to the identification of the midsegment, and then correctly identify the base. She further said that writing “LONGER” for the side of the triangle that is parallel to the midsegment was better than writing base because the side is not always at the bottom.

The approach that I presented in this session again showed the importance of centring. When Rebecca’s students were able to focus on and recognise the centre of the diagram, that is the most important aspects of the problem, they were able to solve the problems more easily. It seemed that her students were able to overcome the difficulty of starting the problem, which for some students, and especially for Diana, is sometimes the most difficult part of completing her mathematics work independently.

10.1.4 An Analysis of Understanding Difficulties with Mental Mathematics

This session seemed to be very eye opening not only for the newer teachers like Yanna, but also for veteran teachers like Eric and Susan. The main goal of this session was to help teachers to understand how difficult it is for students to do mental calculations. Whilst the activity that was presented in this lesson was not focused on addressing visual-spatial deficits, it was nevertheless important since it was a relevant topic and was suggested by several teachers.
Yanna who was circumspect about the broad use of calculators by our students when she first started to teach at the school said “I can see how hard it is for them [the students].” In particular, she noted that much like the writing activity the questions were very difficult to complete, and it was “tiring to do all of them [problems].”

The latter sentiment was echoed by Eric who said that “it would be good for new people [teachers] to see this.” He went on to say that it was a good way for teachers to see what our students face when doing what may be simple for their non-learning disabled peers, or even for teachers who find arithmetic to be simple.

Susan said that the typo that I made was good because “we [the teachers in the department] might not think about it,” referring to the multiple notations that we use for a given term. This may have been especially relevant to Susan since she was teaching geometry which has multiple, albeit similar notations for the most fundamental concepts such as angles.

This session also seemed to have helped the teachers understand, or at the very least empathise with the students and better understand the difficulties that students with MLDs face. It may ultimately help them to improve their ability to teach these students.

10.2 An Analysis of the Individual Sessions with the Teachers

The sessions that I conducted with individual teachers focused primarily on developing strategies for addressing visual-spatial deficits. Whilst this was not the only topic for the sessions, it is the topic that I will discuss in this section of this chapter.
10.2.1 An Analysis of Intervention Sessions with Susan

Susan and I met several times to discuss how to teach certain topics. I will focus on four of the sessions in this section. One session will focus on discussing MLDs and learning disabilities in general. The subsequent three sessions that I will describe will be centred around teaching topics that students find difficult because of their visual-spatial deficits.

Susan and I met for these four sessions and at least seven other sessions at her request. She worked with me as an assistant teacher during her first year at the school. She said that this was one of the reasons why she felt comfortable seeking help from me. I suspect that even if Susan had not worked directly with me in a shared classroom, she still would have sought help from me because of the open and collegial nature of our department. Several teachers including Susan have said this on various occasions.

10.2.2 An Analysis of Speaking to a Parent about a Student’s Learning Disabilities

During one of our meetings, Susan approached me about how to speak to the mother of a child about her daughter’s MLDs and LDs in general. In order to give her a better answer to her questions, I asked her to reschedule our meeting so I could read the student’s psychoeducational evaluation. When I did meet with her two days later, I provided Susan a list of talking points for her conversation with the parent.

When I spoke to Susan after her meeting with the parent, she said that it was helpful to have the talking points. Even though the parent was reluctant to
accept some aspects of her child’s MLDs and LDs, Susan said it “*would’ve been better than not having them [the talking points].*” In particular, Susan said it was also helpful to point out specific parts of the child’s psychoeducational evaluation when explaining how they related to the student’s difficulties.

10.2.3 An Analysis of Teaching Students about the Angles Formed by Parallel Lines that are Cut by a Transversal

The results of this session were similar to the results of the intervention session that I did with Carol as described in section 7.4.4 of this thesis. I showed Susan the same techniques that I showed to Carol. Susan said that writing the locations of the angles made it easier for her students to identify the corresponding angles. Additionally she stated “*when they [the students in her class] saw [the] upper left angles or the upper right angles, it made it easier for them to see what angles were equal [in measure].*” She also said that most of the students were able to remember how to use these techniques independently.

10.2.4 An Analysis of Teaching Students how to Plot Points on a Rectangular Coordinate Plane

During this intervention session, I showed Susan a technique that I have used over the years in the introductory algebra classes that I have taught. The labelling technique for the $x$ and $y$-values for the points, and the arrows that were drawn for each seemed to help some of her students. Whilst one of the three students who had a lot of difficulty graphing points found this useful, the other two students that Susan felt needed additional assistance did not. Susan seemed to believe that this technique worked for one of the students because this student
likes clear steps that can be followed. Susan said that this student had a lot of anxiety and this “helped to take away the unknown [for her].”

10.2.5 An Analysis of Teaching Students how to Graph the Solution to Linear Inequalities on a Number Line

Susan sought help to find a more effective way to teach her students how to graph the solution to a linear inequality on a number line. The method that I showed her had students draw tails for less than, less than or equal to, greater than, and greater than or equal to inequality symbols. This made each of these symbols appear to be arrows and I thought would make it easy for students to see which way they should shade on the number line. It is a successful technique that I have used with students in the past.

Susan mentioned that this topic was somewhat difficult for the students in one of her tenth grade classes and especially for one of her students in this class. Susan said that whilst the technique worked for three or four students, the other five students in her class did not find it helpful. One student in particular, who is highly anxious, and has a variety learning and emotional issues ranging from dyslexia to anxiety to Obsessive Compulsive Disorder (OCD), according to her psychologist, told Susan that she was “confused” and wanted to know “where they [the symbols and arrows] for the inequalities came from.” As this student’s ninth grade teacher, I was not surprised by her comments or her confusion. It appeared that she coped with her anxiety by asking a lot of questions, which is something that other teachers and I have experienced with this, and to some extent other students with similar difficulties. As this student’s ninth grade teacher, I was not surprised by her comments or her confusion. It appeared that
she coped with her anxiety by asking a lot of questions, which is something that other teachers and I have experienced with this, and to some extent other students with similar difficulties.

Other students in Susan’s class who had difficulties with this technique said that they just could not see how to shade the line. Susan said that a different student said that “I just don’t get it.” Susan did not ask what he meant by this, and moved on to a different technique to teach him.

10.3 An Intervention with a Middle School Class

A middle school teacher who I have worked on the same floor with for several years approached me about a problem that she was facing when she was teaching her class how to solve linear equations. When she was describing some of the problems that some of her students had, it seemed that she was describing a difficulty that both Carol and Diana had. She said that several students in her eighth grade class, including one student who I subsequently taught in ninth grade, had difficulty starting to solve linear equations.

When I introduced her to the technique of centring the problem by drawing lines around the equal sign, and circling the variable and its corresponding coefficient, and then solving the problem. The middle school teachers said that she found this method to be successful. When I asked her what helped her students, she said that she thought it helped because the students “didn’t have to figure out how to start it [the problem].” Though she did not ask her students what made it easier, she seemed to think that doing this helped the students focus on the question. This is a theme re-emerged from my work with
Carol and Diane. The centring process seemed to have been successful and made it easier for students to complete their work.

In my work with the teachers in my department specifically, and my school in general, the centring theme emerged not only in geometry, but also in algebra and trigonometry. Whilst this principle is fairly simple, its power and utility has been quite substantial and is something that I will explore in future research.

Overall, the intervention sessions with the teachers seemed to have worked. Some of the strategies that I have used with Carol and Diana have worked for other students in both the middle school and high school. The major themes of the successful strategies involve educating teachers about MLDs and how they may appear in the classroom. Additionally, educating teachers to speak to parents about their children’s MLDs and LDs in general have helped to empower the teachers and give them more confidence in teaching their students. Finally, centring was a major theme that seemed to be very impactful not just for Carol and Diana, but also other students in the school. Perhaps the greatest impact of centring is that it helped the teachers in my school to utilise a technique they did not necessarily consider before, and provided them with a powerful set of tools that they could use when they taught their lessons.
Chapter 11: Conclusion

This chapter will summarise the findings of my research, state its implications, and offer future questions for research in this field. In addition to stating the findings, I will enumerate the implications for the students, teachers, and parents in my school. The implications for constituents outside of my school will also be considered.

11.1 A Restatement of the Research Questions

The research questions that were originally stated in section 3.1 of my thesis were:

- What are the types of visual-spatial deficits that the students I have interviewed have? Also, how can these deficits be classified by extending Karagiannakis’ model?

- What components will form an effective intervention plan for students with visual-spatial deficits?

- What types of interventions will work with teachers who are teaching students who are diagnosed with MLDs and other LDs, and especially those with visual-spatial deficits?

I believe that my research has helped me to answer these questions. Whilst my research cannot answer these questions in an all-encompassing or unqualified manner, it does provide evidence to support my answers to these questions.

11.1.1 Identification and Categorisation of the Visual-Spatial Deficits of Students in my School

The results of the mathematical interviews that I conducted in the pilot study and the additional mathematical interviews have helped me to determine the types of visual-spatial deficits that the students I interviewed appeared to
have. The interviews revealed that some students had two-dimensional, some students had three-dimensional, and some students had both two and three-dimensional visual-spatial deficits.

The questions that I asked the students when they answered questions incorrectly sought to gain an insight into their thought process. Whilst some of their responses, although wrong, seemed to be something that teachers may not be surprised by, some of the responses were very unexpected. As a teacher with almost seventeen years of experience in my specialist school, even I was sometimes incredibly surprised by some of their explanations which gave a glimpse into their interesting but ultimately flawed thought process.

The revised version of Karagiannakis’ model that I proposed was especially useful in categorising the deficits and misconceptions since it separated Karagiannakis’ visual-spatial category into two-dimensional and three-dimensional visual-spatial deficits. This provided an excellent and simplified method to categorise these visual-spatial deficits. I hope that in the future it can be used by educators to inform their teaching, and also provides a basis for speaking to parents about their children’s visual-spatial deficits.

11.1.2 Addressing the Visual-Spatial Deficits with Students

In my interventions with Carol and Diana two themes emerged from my intervention sessions with them. First, direct instruction played a very important role in helping them to start overcoming their deficits. Both of these students said this explicitly in their intervention sessions and follow up interviews. They additionally said that when their teachers did not provide this type of instruction, they found it more difficult to learn.
Matthaei’s research seemed to corroborate the benefits of direct instruction to some extent as well. Some of the most successful strategies that she used in her interventions used elements of direct instruction. In addition, recent literature (Gurganus, 2017, p.201) on teaching students with LDs tends to support this claim.

Another theme that emerged from my work with Carol and Diana was the effectiveness of pre-teaching certain skills that are required for class lessons that have visual-spatial aspects. Diana seemed to have benefitted from learning the skills ahead of time. Diana said that she was not just aided by seeing some of the work ahead of time, but it also helped to diminish her anxiety because she realised that she had known some of the work very well whilst she was in the lesson.

My recommendations for working with students who have these deficits include:

- Pre-teaching the underlying skills that are needed for a lesson that has visual-spatial features
- Using direct instruction to teach these students
- Emphasise centring when possible, especially since it may help these students start a problem which is sometimes the most difficult step.

11.1.3 Addressing MLDs and Specifically Visual-Spatial Deficits with Teachers

The other major part of addressing visual-spatial deficits centred around educating teachers about MLDs and specifically the visual-spatial deficits that affect students who are learning mathematics in our school. This was primarily achieved through department meetings with the entire department, and also
through discussions with individual teachers or teaching pairs that were made up of the lead and assistant teachers who taught classes with each other. The latter approach may have ultimately been more effective than the former because of the individualised nature of these meetings.

The department meetings provided a forum for teachers to address specific difficulties or concerns that they had, even if they were not related to the agenda that was set for a specific meeting. The topics for discussion during the meetings were mainly suggested by the teachers in the department. Sometimes, I decided the topics for the meetings from the formal and informal discussions that I had with the teachers. All of these meetings provided specific strategies that helped teachers to teach specific topics, such as the meeting about the Midsegment Theorem, or gave the teachers some insight into how frustrated our students were when they could not do tasks that students who did not have LDs and the teachers themselves would find to be simple.

The sessions that focused on developing practical strategies for teaching students with visual-spatial deficits were generally well received. The teachers said that the strategies could generally be implemented in the short forty minute sessions that they had to teach each class. They also said that the strategies could be easily repeated, which is an essential part of effectively teaching students in our school.

My recommendations for intervention sessions with teachers include:

- Focusing on only one topic unless the session is intended to address a broad range of topics
- Giving teachers an ample amount of time to contribute to the meetings, and especially ask questions
• Having a willingness to change the original agenda in order to address the needs and concerns of the teachers

• Provide practical approaches that the teachers can implement in the time that is allotted for a class lesson.

11.1.4 Addressing MLDs and Specifically Visual-Spatial Deficits with Teachers Outside of my Specialist School

The presentations that I have done for teachers outside of my school were especially impactful. I was usually invited to give these presentations by my former colleagues, in particular, one who had previously worked as the high school psychologist in my specialist school for almost ten years. Whilst he had considerable knowledge about MLDs, he requested that I make several presentations to teachers in his new school that he was working in because of my practical experience in the classroom.

On numerous occasions, teachers have spoken to me after the presentation about their students, but most often about themselves. Several teachers have said that the students that they have taught had a specific type of MLD that they learned about for the first time in my presentation. They often went on to say that the wished that they had known this information earlier because they would have approached their students who potentially had MLDs with more patience and understanding.

11.2 Further Research Questions

The further research questions that I will propose here are intended to build on my research and enhance the knowledge in this field overall. The further research questions focus on visual-spatial deficits, and MLDs that address a
variety of the aspects of students who have LDs that are struggling to learn mathematics.

11.2.1 Prevalence of the Types of Visual-Spatial Deficits in Other Schools

One question that my research did not answer is whether mathematics teachers in other schools, especially non-specialist schools, have taught students who have visual-spatial deficits that are the same or similar to the visual-spatial deficits that the students that I interviewed during this research had. If students in other schools do have these visual-spatial deficits, my research could be more impactful to teachers and students to a much larger group of students and their teachers.

In addition to its impact potentially beneficial impact to students, this type of research could hopefully encourage other researchers to do more research in this field, and develop a more comprehensive set of strategies that could help students with visual-spatial deficits.

11.2.2 Awareness of MLDs by Primary, Secondary School, and Higher Education Mathematics Teachers

As far as I am aware, there is currently no research available that discusses the extent to which mathematics teachers at any level know about MLDs and their impact. From my discussions with teachers and researchers, including those who have advanced degrees in educating students with LDs, very few have had any education beyond learning very basic information about dyscalculia. Drew and Trott (2011) did a survey about dyscalculia in higher education through her work for the Eureka Centre for Mathematical Confidence
(Drew, 2015 p.337). Whilst the survey (Drew, 2015, p.337-343) sought to determine how higher education institutions in the U.K. screened for dyscalculics, determined the number of dyscalculics that each higher education institution had, the degree programs that these students were enrolled in, the comorbidity with dyslexia and perhaps other LDs, and supports that these institutions offered, it did not determine to what extent the professors who teach these students were familiar with MLDs. I firmly believe that determining if teachers at all levels are familiar with MLDs can help to highlight the lack of awareness and perhaps help to create more formal training programmes for these educators.

11.2.3 Developing Effective Teacher Training Programmes

Whilst this research addressed the interventions that seem to work in helping teachers to understand MLDs and more effectively teach students visual-spatial deficits, it does mainly focus on helping the teachers in my specialist school. Since non-specialist schools, or even other specialist schools can be quite different in student populations, teacher backgrounds, and educational goals, it may not be best to use the approaches that I have used in my work with teachers in other schools.

Based on my discussions with teachers and other colleagues in my specialist school, some of them did not fully accept that MLDs affect students to the same extent that language-based LDs do. Some even suggested that MLDs may not exist, though none of them have said this directly. I believe that any training programme for teachers and other professionals who work with students who have an MLD or even an LD that is not mathematical in nature, should
include a rigorous description and justification for MLDs by using the most recent research in neuroscience. The neuroscience has especially helped me to persuade some of my more reluctant colleagues into accepting MLDs have a big impact on how students in our school learn mathematics.

11.2.4 Developing Effective Education Programmes for Parents

In my discussions with the teachers in my school and other schools it was clear that the parents of MLD students and more generally LD students had greater exposure to language-based LDs such as dyslexia. There seemed to be greater awareness and perhaps acceptance of these types of disabilities by parents as well because their children’s teachers have been able to discuss and explain these disabilities to them in an informed manner. In my discussions with parents in my school, many of whom have learned quite a bit of language-based LDs, it seemed that they learned very little about MLDs.

The final topic for future research that I am proposing is to determine what educational programmes could be developed for parents to help them understand the nature and impact of MLDs on their children’s mathematics education. In my discussions with some parents who have children who may have MLDs, there seemed to have been a start to a greater understanding, and also an inclination to have greater patience in working with their children on their mathematics homework. I suspect that having more formal programmes will help parents to a greater extent.
11.3 Impact on my Teaching

This research already has and certainly will continue to have a substantial impact on my teaching. From this research, I have developed a model to work with students who appeared to have visual-spatial deficits. I have already used the same strategies, with minor modifications to meet the individualised needs of different students, that I have used to help Carol and Diana. The results have largely been successful, and will be a focus for future research that I intend to do.

So far, I have not worked with a student where I have needed to make major modifications to the strategies that I have used with Carol and Diana. However, sometimes new strategies have had to be created. For example, I worked with a student who had difficulty with the traditional method for factoring quadratic polynomials. I showed him a method that made him rely on his visual-spatial abilities. Whilst this was actually a strength for this student, the method that I was able to develop to help this student had a basis in my work with Carol and Diana. Without this experience, I am not certain that I would have been able to develop a method that helped him this effectively.

11.4 Impact on my work as a Mathematics Department Chairman

My work as a leader in my department has already been transformed in very significant ways. In addition to the teacher training programmes that were mentioned in section 11.2.3 of this thesis, some of which have already been implemented, the mathematics teachers in the high school, and some in the middle school and elementary school have approached me to ask for help in teaching certain topics. This is especially true for topics that are difficult for students with visual-spatial challenges or deficits to learn.
In the research that I intend to do in the future, I will aim to formalise the process by which I work with these teachers who usually seek my assistance informally. One of my future goals in my research is to create a set of resources that will help teachers directly. In addition, I will aim to help them to develop their own strategies when they are working with their own students.

11.5 Summary of the Diagnostic Methods

The mathematical interviews that I conducted in my pilot study were essential to setting a strong foundation for the identification of students with potential visual-spatial deficits. This of course, led me to identify students who would be good candidates to receive specific interventions that addressed their visual-spatial deficits.

When I approached the students that I identified about working with me to receive further interventions, Carol and Diana eagerly agreed to work with me. Whilst the positive relationship that I developed with them helped me to convince them to work with me, their eagerness came from a desire to improve their own mathematical abilities as well as reduce their anxiety. Both of them stated this directly to me, and independently of each other.

11.6 Summary of the Intervention Strategies with Carol and Diana

The intervention strategies that I used were primarily based on my experience teaching students before I started this research, and the individual needs of Carol and Diana. I sought to develop strategies that both Carol and Diana could replicate independently in what they would consider to be high-stress situations, such as examinations.
The strategies that I used with them were purposely simplified as much as possible, whilst simultaneously aiming to help them to develop an understanding of the underlying concepts of the mathematics that they were learning. However, this was not always possible. Despite this, the strategies were for the most part successful, in helping them improve their performance.

The intervention sessions for Carol and Diana were almost always for a short period of time. They were rarely longer than 15 to 20 minutes in length. Since we often met during lunch or at the end of a school day, they seemed to be tired or not as attentive as they would be at other times in the day. This was especially true for Diana who was diagnosed with ADHD by her psychologist. The short sessions allowed them to sustain their attention more than they probably could have during longer sessions.

Finally, the intervention sessions almost always focused on one topic. This made the goal of each session very clear both for the students and for me. The focus on one topic also made it easier to keep the sessions short.

11.7 Summary of the Intervention Strategies with the Teachers in my Specialist School

The intervention sessions with the teachers were modelled on the sessions with Carol and Diana. Each session was usually 15 to 20 minutes in length, and almost always focused on one topic.

This was done for several reasons. Firstly, focusing on only one topic ensured that the teachers could discuss a topic in greater detail. It afforded them time to ask questions, and make meaningful contributions to the session. As
several teachers said, it also gave them time to think about the questions that they asked, and also gave them a chance to participate because they had known that there was not any other topic on the agenda. Ultimately, they did not feel rushed.

Secondly, focusing on one topic per session modelled how to teach a lesson to all of the teachers, especially the teachers who were new to my specialist school. Most lessons that are taught by the high school mathematics teachers, with the exception of revision lessons, primarily focus on one topic with questions of varying difficulty, and ample opportunity for questions where the teachers provide guided assistance to the students, and questions where the students can practise what they have learned independently. The benefits of focusing on one topic indirectly became apparent according many of the teachers who participated in this study.

11.8 Conclusion

The research in this field is still scarce and needs to be developed quite a bit in the future. My hope is that this research will motivate other researchers to develop a deeper understanding of MLDs and visual-spatial deficits at the secondary school level. I further hope that teachers, school administrators, psychologists, parents, and even students will gain a deeper understanding of how these disabilities impact children with LDs and more specifically MLDs. In addition, I hope that they start to understand and believe that these students can learn mathematics, but in a manner that is different than most students who do not have LDs.
APPENDIX A: Diagnostic Criteria for ADHD as Defined by DSM-V (BASC, 2013, p.1-2)

*Diagnostic Criteria for Inattention*

- Often fails to give close attention to details or makes careless mistakes in schoolwork, work, or other activities
- Often has difficulty sustaining attention in tasks or play activities
- Often does not seem to listen when spoken to directly
- Often does not follow through on instructions and fails to finish schoolwork, chores, or duties in the workplace (not due to oppositional behaviour or failure of comprehension)
- Often has difficulty organising tasks and activities
- Often avoids, dislikes, or is reluctant to engage in tasks that require sustained mental effort (such as schoolwork or homework)
- Often loses things necessary for tasks or activities at school or at home (e.g. toys, pencils, books, assignments)
- Often easily distracted by extraneous stimuli
- Often forgetful in daily activities

In order to be diagnosed with the inattentive subtype, at least six of the criteria must have persisted for at least six months.
Diagnostic Criteria for Hyperactivity

- Often fidgets with hands or feet or squirms in seat
- Often leaves seat in classroom or in other situations in which remaining seated is expected
- Often runs about or climbs excessively in situations in which it is inappropriate (in adolescents or adults, may be limited to subjective feelings of restlessness)
- Often has difficulty playing or engaging in leisure activities quietly
- Often talks excessively
- Is often 'on the go' or often acts as if 'driven by a motor'

Diagnostic Criteria for Impulsivity

- Often has difficulty awaiting turn in games or group situations
- Often blurts out answers to questions before they have been completed
- Often interrupts or intrudes on others, e.g. butts into other children’s games

In order to be diagnosed with the hyperactive-impulsive subtype, at least six of the criteria must have persisted for at least six months. In addition, the behaviour must be inconsistent with the child’s developmental level. In order to be diagnosed with the combined subtype, both sets of criteria must be met for the inattentive and hyperactive-impulsive types (LDAWE, n.d.).
The Diagnostic & Statistical Manual of Mental Disorders – Fifth Edition (DSM-V) (Reynolds and Kamphaus, 2013, p.1-2) defined ADHD as:

A. A persistent pattern of inattention and/or hyperactivity-impulsivity that interferes with functioning or development, as characterized by (1) and/or (2):

1. Inattention: Six (or more) of the following symptoms have persisted for at least 6 months to a degree that is inconsistent with developmental level and that negatively impacts directly on social and academic/occupational activities:

   Note: The symptoms are not solely a manifestation of oppositional behavior, defiance, hostility, or failure to understand tasks or instructions.

   For older adolescents and adults (age 17 and older), at least five symptoms are required.

   a. Often fails to give close attention to details or makes careless mistakes in schoolwork, at work, or during other activities (e.g., overlooks or misses details, work is inaccurate).
b. Often has difficulty sustaining attention in tasks or play activities (e.g., has difficulty remaining focused during lectures, conversations, or lengthy reading).

c. Often does not seem to listen when spoken to directly (e.g., mind seems elsewhere, even in the absence of any obvious distraction).

d. Often does not follow through on instructions and fails to finish schoolwork, chores, or duties in the workplace (e.g., starts tasks but quickly loses focus and is easily sidetracked).

e. Often has difficulty organizing tasks and activities (e.g., difficulty managing sequential tasks; difficulty keeping materials and belongings in order; messy, disorganized work; has poor time management; fails to meet deadlines).

f. Often avoids, dislikes, or is reluctant to engage in tasks that require sustained mental effort (e.g., schoolwork or homework; for older adolescents and adults, preparing reports, completing forms, reviewing lengthy papers).
g. Often loses things necessary for tasks or activities (e.g., school materials, pencils, books, tools, wallets, keys, paperwork, eyeglasses, mobile telephones).

h. Is often easily distracted by extraneous stimuli (for older adolescents and adults, may include unrelated thoughts).

i. Is often forgetful in daily activities (e.g., doing chores, running errands; for older adolescents and adults, returning calls, paying bills, keeping appointments).

2. Hyperactivity and impulsivity: Six (or more) of the following symptoms have persisted for at least 6 months to a degree that is inconsistent with developmental level and that negatively impacts directly on social and academic/occupational activities.

Note: The symptoms are not solely a manifestation of oppositional behavior, defiance, hostility, or a failure to understand tasks or instructions.

For older adolescents and adults (age 17 and older), at least five symptoms are required.

a. Often fidgets with or taps hands or feet or squirms in seat.
b. Often leaves seat in situations when remaining seated is expected (e.g., leaves his or her place in the classroom, in the office or other workplace, or in other situations that require remaining in place).

c. Often runs about or climbs in situations where it is inappropriate.

(Note: In adolescents or adults, may be limited to feeling restless.)

d. Often unable to play or engage in leisure activities quietly.

e. Is often “on the go,” acting as if “driven by a motor” (e.g., is unable to be or uncomfortable being still for extended time, as in restaurants, meetings; may be experienced by others as being restless or difficult to keep up with).

f. Often talks excessively.

g. Often blurts out an answer before a question has been completed (e.g., completes people’s sentences; cannot wait for turn in conversation).

h. Often has difficulty waiting his or her turn (e.g., while waiting in line).

i. Often interrupts or intrudes on others (e.g., butts into conversations, games, or activities; may start using other people’s things without asking or receiving permission; for adolescents and adults, may intrude into or take over what others are doing).

B. Several inattentive or hyperactive-impulsive symptoms were present prior to age 12 years.
C. Several inattentive or hyperactive-impulsive symptoms are present in two or more settings (e.g., at home, school, or work; with friends or relatives; in other activities).

D. There is clear evidence that the symptoms interfere with, or reduce the quality of, social, academic, or occupational functioning.

E. The symptoms do not occur exclusively during the course of schizophrenia or another psychotic disorder and are not better explained by another mental disorder (e.g., mood disorder, anxiety disorder, dissociative disorder, personality disorder, substance intoxication or withdrawal).

Specify whether:

314.01 (F90.2) Combined presentation: If both Criterion A1 (inattention) and Criterion A2 (hyperactivity-impulsivity) are met for the past 6 months.

314.00 (F90.0) Predominantly inattentive presentation: If Criterion A1 (inattention) is met but Criterion A2 (hyperactivity-impulsivity) is not met for the past 6 months.

314.01 (F90.1) Predominantly hyperactive/impulsive presentation: If Criterion A2 (hyperactivity-impulsivity) is met but Criterion A1 (inattention) is not met over the past 6 months.
Specify if:

In partial remission: When full criteria were previously met, fewer than the full criteria have been met for the past 6 months, and the symptoms still result in impairment in social, academic, or occupational functioning.

Specify current severity:

Mild: Few, if any, symptoms in excess of those required to make the diagnosis are present, and symptoms result in only minor functional impairments.

Moderate: Symptoms or functional impairment between “mild” and “severe” are present.

Severe: Many symptoms in excess of those required to make the diagnosis, or several symptoms that are particularly severe, are present, or the symptoms result in marked impairment in social or occupational functioning.
References


