

## Strange and charmed baryons using $N_f = 2$ twisted mass QCD

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**Mauro Papinutto\*,Jaume Carbonell**

*Laboratoire de Physique Subatomique et de Cosmologie,  
UJF/CNRS-IN2P3/INPG, 53 rue des Martyrs, F-38026 Grenoble, France  
E-mail: mauro.papinutto@lpsc.in2p3.fr, carbonel@lpsc.in2p3.fr*

**Vincent Drach**

*NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany  
E-mail: vincent.drach@desy.de*

**Constantia Alexandrou**

*Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus  
Computer-based Science and Technology Research Center, 20 Kavafi Str., 2121 Nicosia, Cyprus  
E-mail: alexand@ucy.ac.cy*



We compute the mass spectrum for strange/charmed baryons in the partially quenched approach using  $N_f = 2$  twisted mass QCD configurations. We investigate two main issues: the size of lattice artefacts using three values of the lattice spacing (the smallest of which is approximately 0.05 fm) and the dependence of baryon masses on meson (or quark) masses. We thus perform a global fit in order to extrapolate simultaneously to the continuum limit and to the physical point. We estimate the masses of  $\Omega_{sss}$ ,  $\Xi_{dss}$ ,  $\Lambda_{uds}$ ,  $\Omega_{ccc}$ ,  $\Xi_{dcc}$ ,  $\Lambda_{udc}$ .

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\*Speaker.

## 1. Introduction

Simulations with two light degenerate sea quarks ( $N_f = 2$ ) and including also the strange sea quark ( $N_f = 2 + 1$ ) are nowadays standard. The ETM Collaboration has generated a substantial sample of  $N_f = 2$  ensembles at four values of the lattice spacing (ranging from 0.1 to 0.05 fm), several values of the light sea quark mass and several physical volumes. Using these ensembles one can study the cut-off effects on observables and the insight gained provides valuable input for the choice of parameters for the  $N_f = 2 + 1 + 1$  (i.e. including both the strange and the charm sea quarks) simulations under production. Preliminary results using  $N_f = 2 + 1 + 1$  simulations have been presented at this conference [1].

In the present study we therefore use  $N_f = 2$  ensembles with a partially quenched setup in which the strange and charm quarks are added only as valence quarks. For heavy quarks the compton wavelength of the associated heavy-light meson is small compared to present attainable lattice spacings which means that cut-off effects can be large. The charm quark mass is at the upper boundary of the range of masses that can be simulated at present for the coarsest lattice spacing used in the continuum limit extrapolation ( $a \sim 0.1 fm$  for which  $m_c a \lesssim 1$ ). In order to safely control this extrapolation it is thus important to assess the size of lattice artefacts affecting the observables of interest.

Our goal is to extend the study of Ref. [2] by including a finer lattice spacing  $a \simeq 0.051$  fm. We would like, in addition, to compute the low-lying spectrum of strange and charmed baryons. In this contribution we present preliminary results for the masses of the strange baryons  $\Omega_{sss}$ ,  $\Xi_{dss}$ ,  $\Lambda_{uds}$  and the corresponding charmed baryon obtained by substituting the strange quark with the charm quark ( $\Omega_{ccc}$ ,  $\Xi_{dcc}$ ,  $\Lambda_{udc}$ ). Preliminary results for the low-lying strange baryon spectrum with  $N_f = 2 + 1 + 1$  gauge configurations [3] and for the spectrum of static-light baryons with  $N_f = 2$  configurations [4] have also been presented at this conference.

## 2. Setup

The lattice discretization used for the doublet of degenerate light quarks is Wilson twisted mass QCD at maximal twist [5] whose action reads (in the twisted basis)

$$S_{\text{light}}^{\text{tmQCD}} = a^4 \sum_x \bar{\chi}_l(x) \left( \frac{\gamma_\mu}{2} (\nabla_\mu + \nabla_\mu^*) - \frac{a}{2} \nabla_\mu^* \nabla_\mu + M_{cr} + i\gamma_5 \tau_3 \mu_l \right) \chi_l(x) \quad (2.1)$$

where  $\nabla_\mu$ ,  $\nabla_\mu^*$  are forward and backward covariant derivatives,  $M_{cr}$  is the Wilson critical mass and  $\mu_l$  is the light quark mass.

The strange and charm (which in the following are referred to as ‘‘heavy’’) quarks are added here only as valence quarks *à la* Osterwalder-Seiler and their action reads

$$S_{\text{heavy}}^{\text{OS}} = a^4 \sum_x \sum_{h=s}^c \bar{\chi}_h(x) \left( \frac{\gamma_\mu}{2} (\nabla_\mu + \nabla_\mu^*) - \frac{a}{2} \nabla_\mu^* \nabla_\mu + M_{cr} + i\gamma_5 \mu_h \right) \chi_h(x) \quad (2.2)$$

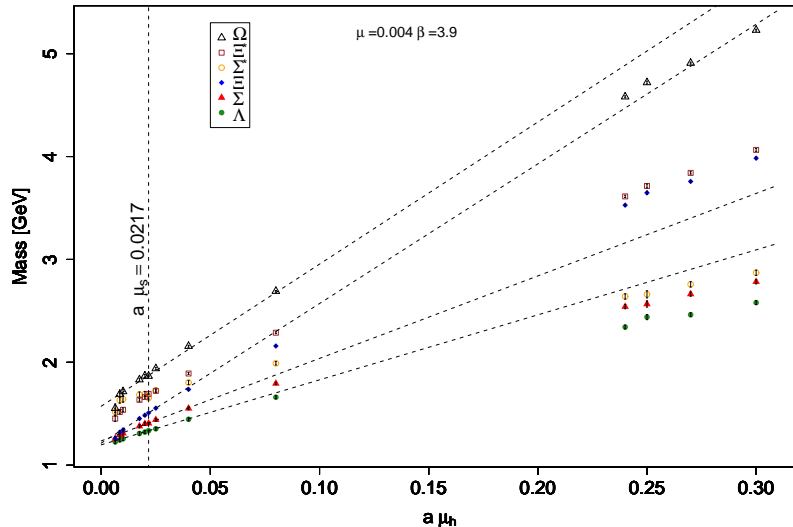
where  $\mu_s$  and  $\mu_c$  are the strange and charm (valence) quark masses. In order to remove the determinant of the strange and charm quarks, ghosts have to be added correspondingly. Concerning the gauge actions, ETMC uses the tree-level Symanzik improved gauge action.

The main advantage of this regularization with respect to the standard Wilson one is that the spectrum and the matrix elements extracted from correlation functions are automatically  $O(a)$  improved [6]. The drawback is that parity and isospin are explicitly broken by  $O(a^2)$  lattice artefacts

and are recovered only in the continuum limit. Here we use ETMC configurations generated at three values of the lattice spacing  $a \in \{0.051, 0.064, 0.080\}$  fm and physical volumes  $L \sim 2.0 \div 2.4$  fm (the scale has been set through  $f_\pi$  in Ref. [7]). Light sea quark masses correspond to pion masses  $M_\pi \in [290, 520]$  MeV while partially quenched valence strange/charm quarks correspond to heavy-light meson masses  $M_K \in [520, 710]$  MeV and  $M_D \in [1.80, 2.40]$  GeV. In all we have 40 different combinations  $(M_\pi, M_{hl})$ . In order to combine data at different lattice spacings we express the value of the masses in units of  $r_0$  [8]. For the three lattice spacings considered here the values  $r_0/a \in \{8.36, 6.73, 5.36\}$  are taken from Ref. [7]

### 3. Numerical results

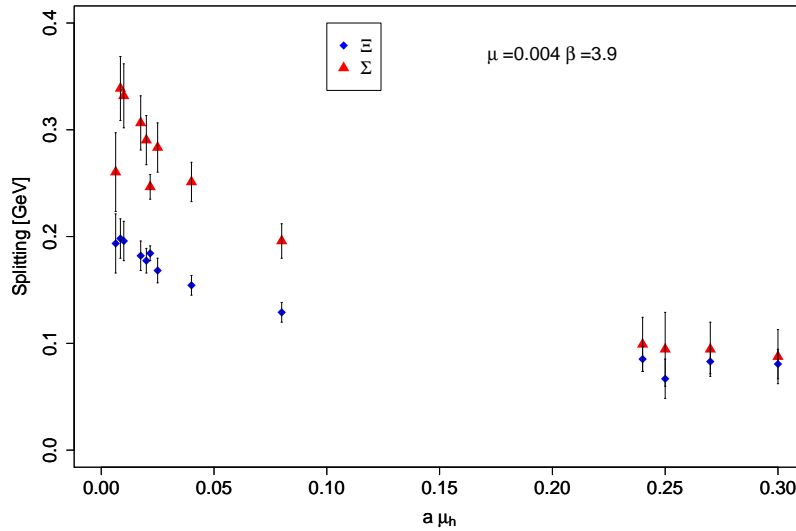
An important issue in our study is the dependence of the baryon masses upon the “heavy” quark mass  $\mu_h$  in the strange and in the charm region. At  $a = 0.080$  fm and  $M_\pi \simeq 340$  MeV this dependence is shown in Fig. 1.



**Figure 1:** The dependence of the octet and decuplet baryon masses on  $\mu_h$ . Dashed lines correspond to linear fits performed in the strange region.

From Fig. 1 we observe that baryon masses depend linearly on  $\mu_h$  both in the strange and in the charm region but with two different slopes. This behavior will be further discussed in what follows. For what concern meson masses, in the case of the Kaon we observe a dependence  $M_K^2 \propto \mu_h$ , in agreement with the fact that the Kaon can still be considered a pseudo Goldstone boson. For the  $D$  meson instead we observe a dependence  $M_D \propto \mu_h$  as predicted by heavy quark effective theory (HQET), with no evidence of  $1/\mu_h$  term. In the following we will consider the functional dependence of baryon masses upon  $M_\pi$  and  $M_{hl}$  because this allows to extrapolate to the physical point without knowing the values of the renormalized quark masses. The observations above imply that baryon masses depend quadratically on  $M_K$  in the strange region while depend linearly on  $M_D$  in the charm region.

From Fig. 1 and Fig. 2 it is also evident that the splitting between  $J = 1/2$  and  $J = 3/2$  states ( $\Sigma/\Sigma^*$  and  $\Xi/\Xi^*$ ) clearly diminishes with the increase of  $\mu_h$ . In quark models, this observation



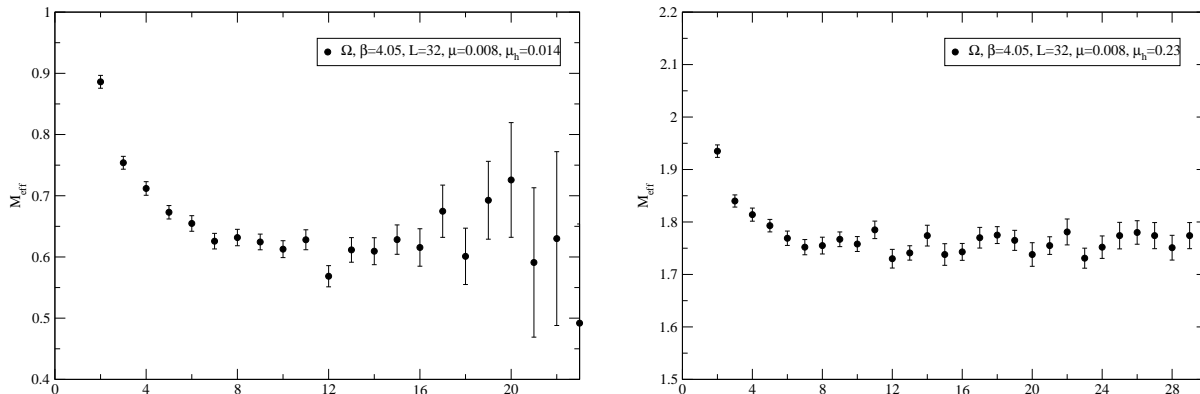
**Figure 2:**  $\Sigma/\Sigma^*$  and  $\Xi/\Xi^*$  splittings as function of  $\mu_h$ .

is explained thanks to the fact that the spin-spin coupling part of the  $q - q$  potential is inversely proportional to the masses of the two quarks  $\frac{s_i \cdot s_j}{\mu_i \mu_j}$ . In HQET, the splitting of baryons containing one heavy quark (e.g. the  $\Sigma_{udh}/\Sigma_{udh}^*$ ) is proportional to  $1/\mu_h$ .

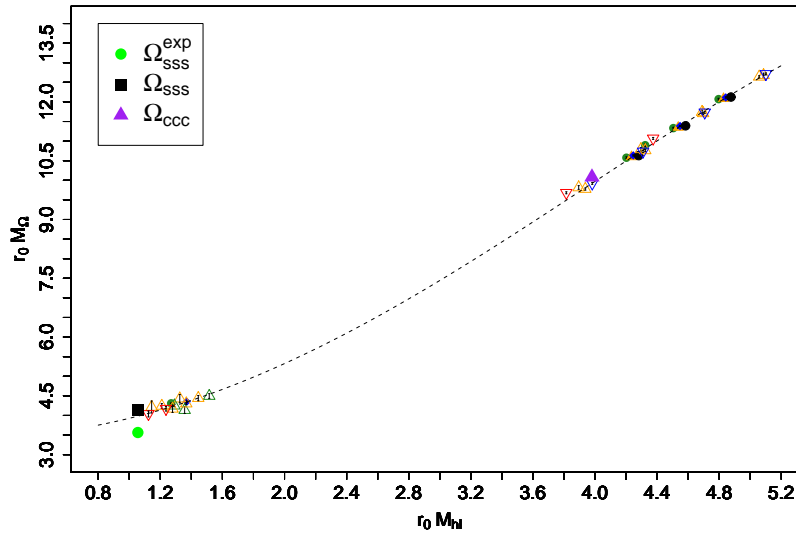
Hadron masses  $M_H$  are extracted from the two point correlators  $C_H(t) = \sum_{\mathbf{x}} \langle H(t, \mathbf{x}) H^\dagger(0, \mathbf{0}) \rangle$  of the corresponding interpolating operators  $H$  at large time distances. The interpolating operators  $H$  are those of Ref. [2] and to improve their overlap with the ground state we apply Gaussian smearing and use APE smearing for the links that enter the hopping function. At large Euclidean time separation the value of the hadron mass can be extracted by fitting the effective mass defined by  $M_H^{\text{eff}}(t) = \frac{1}{a} \ln \frac{C_H(t)}{C_H(t+a)}$  to a constant.

It turns out that the statistical error on  $M_H^{\text{eff}}(t)$  for the strange baryons grows faster in time than in the case of the charmed baryons. In the case of the  $\Omega_{sss}$  and  $\Omega_{ccc}$  this is illustrated in Fig. 3. It is easy to show that the statistical error on  $M_{\Omega_{hhh}}^{\text{eff}}(t)$  is

$$\Delta M_{\Omega_{hhh}}^{\text{eff}}(t) \propto \exp(M_{\Omega_{hhh}} - \frac{3}{2}M_{\bar{h}h})t \quad (3.1)$$



**Figure 3:** Left:  $M_{\Omega_{sss}}^{\text{eff}}(t)$ . Right:  $M_{\Omega_{ccc}}^{\text{eff}}(t)$ .



**Figure 4:**  $M_\Omega$  as function of  $M_{hl}$ . The dashed line is an interpolating form between the  $s$  and  $c$  region.

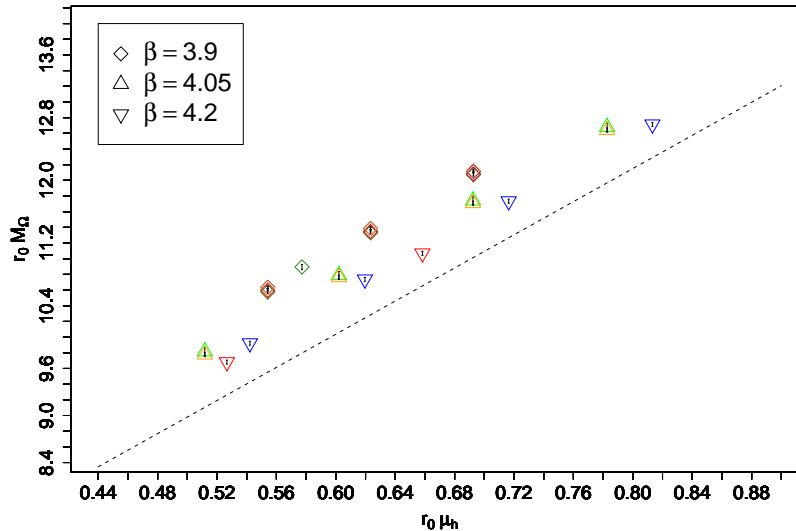
where  $M_{\bar{h}h}$  is the mass of the  $\bar{h}h$  meson made of an heavy and an anti-heavy quark. This phenomenon is then probably explained by the fact that the gap  $\Delta_{\Omega_{ccc}} \equiv M_{\Omega_{ccc}} - \frac{3}{2}M_{\bar{c}c}$  has a smaller value than  $\Delta_{\Omega_{sss}} \equiv M_{\Omega_{sss}} - \frac{3}{2}M_{\bar{s}s}$ . At the physical point  $M_{\Omega_{sss}} = 1672$  MeV while the unphysical  $\bar{s}s \equiv \eta_s$  meson would have a mass  $M_{\bar{s}s} \approx \sqrt{2M_K^2 - M_\pi^2} \approx 690$  MeV [9] and the gap  $\Delta_{\Omega_{sss}} \approx 640$  MeV. In the charm case instead, the preliminary prediction from the present work gives  $M_{\Omega_{ccc}} \approx 4730$  MeV while the  $\bar{c}c$  meson can be identified with the  $\eta_c$  meson which has a mass  $M_{\eta_c} = 2980$  MeV. The gap  $\Delta_{\Omega_{ccc}} \approx 260$  MeV is therefore sensibly smaller than in the strange case. Presumably, this fact remains true for the values of the meson masses we have in our simulations but we still need to check numerically this conjecture.

### 3.1 $\Omega_{sss}$ and $\Omega_{ccc}$

In Fig. 4 we present all the 40 data points for the  $\Omega$  mass fitted to a functional form which interpolates between the strange and the charm region. This plot already shows the smallness of lattice artefacts. The functional form reduces, in the strange region, to the form  $M_\Omega = M_0 + AM_\pi^2 + BM_{hl}^2$ . In the charm region it reduces instead to  $M_\Omega = D + EM_\pi^2 + FM_{hl}$ . The two forms fit well the data: using 13 data points in the strange region we obtain  $\chi_{d.o.f.}^2 = 1.56$ ; using 27 data points in the charm region we find  $\chi_{d.o.f.}^2 = 1.15$ . Lattice artefacts are visible in the strange region and the inclusion of a term  $A_0a^2$  to the functional form above lower the  $\chi_{d.o.f.}^2$  from 1.56 to 0.92. For charmed baryons we do not see any cut-off effect.

This is due however to the choice of studying the behaviour of baryon masses as function of meson masses. Had we chosen to study their dependence upon the renormalized quark masses  $\mu_l$  and  $\mu_h$  (obtained from the bare masses by multiplying them by  $Z_\mu$  taken from Ref. [10]) we would have immediately remarked the presence of lattice artefacts, at least in the charm region. In this region, a fit to the form  $M_\Omega = D + E\mu_l + F\mu_h$  (i.e. not including lattice artefacts) is not sufficient and gives a huge  $\chi_{d.o.f.}^2$ . In order to obtain a reasonable  $\chi_{d.o.f.}^2 = 1.18$  one needs to add lattice artefacts (both  $\mu_h$ -independent and  $\mu_h$ -dependent). Fig. 5 shows the data points together with the curve obtained by plotting the fitting function after setting  $a = 0$ .

Lattice artefacts are instead hardly visible in the strange region. Here, a fit to the form  $M_\Omega = A + B\mu_l + B\mu_h$  gives a reasonable  $\chi_{d.o.f.}^2 = 1.39$  and adding lattice artefacts (dependent or



**Figure 5:**  $M_\Omega$  as function of  $\mu_h$  in the charm region. The dashed line is the continuum limit obtained by plotting the fitting function described in the text after extrapolating to  $a = 0$ .

independent on the quark masses) does not improve the fit.

We remark that  $M_\Omega$  depends very mildly on  $M_\pi$  and therefore the extrapolation to the physical  $M_\pi$  seems not to pose any problem. By interpolating also to the physical value of  $M_K$  we get the result  $M_{\Omega_{sss}} = 1.86(20)$  GeV which is consistent with the analysis in Ref. [2] but still 10% larger than the experimental value. Due to the previous considerations and the analysis performed, this discrepancy seems not to be related to the continuum limit extrapolation or to the extrapolation in the light quark mass. Extrapolation to the physical  $(M_\pi, M_D)$  point gives the prediction  $M_{\Omega_{ccc}} = 4.73(40)$  GeV (the experimental value is not known) and the ratio  $M_{\Omega_{sss}}/M_{\Omega_{ccc}} = 0.393(54)$

### 3.2 $\Lambda_{uds}$ and $\Lambda_{udc}$

In the case of the  $\Lambda$  baryon, the dependence on  $M_\pi$  is much stronger than in the previous case and the inclusion of the term proportional to  $M_\pi^3$  is crucial and reduces the  $\chi^2_{\text{d.o.f.}}$  of a factor  $\sim 0.5$  in both the strange and the charm region. Lattice artefacts are hardly visible and the functional forms we have used to fit are  $M_\Lambda = M_0 + AM_\pi^2 + BM_{hl}^2 + CM_\pi^3$  (in the strange region) and  $M_\Lambda = D + EM_\pi^2 + FM_{hl} + GM_\pi^3$  (in the charm region). Of course the inclusion of chiral logarithms would affect the extrapolation to the physical point. For these preliminary results we have however neglected them and performed only a rough fit using the forms written above.

By extrapolating to the physical  $(M_\pi, M_K)$  point we obtain  $M_{\Lambda_{uds}} = 1.20(10)$  GeV which has to be compared with the experimental value  $M_{\Lambda_{uds}}^{\text{exp}} = 1.116$  GeV. By extrapolating to the physical  $(M_\pi, M_D)$  point we have  $M_{\Lambda_{udc}} = 2.24(18)$  GeV which is in good agreement with the experimental value  $M_{\Lambda_{udc}}^{\text{exp}} = 2.286$  GeV.

### 3.3 $\Xi_{dss}$ and $\Xi_{dcc}$

Twisted mass QCD breaks explicitly isospin symmetry and thus  $\Xi_{uss}^0$  and  $\Xi_{dss}^-$  (or equivalently  $\Xi_{ucc}^{++}$  and  $\Xi_{dcc}^+$ ) are not degenerate. We thus perform a combined fit of both  $\Xi_{uss}^0$  and  $\Xi_{dss}^-$  data with the form  $M_{\Xi_{\{0,-\}}} = M_0 + AM_\pi^2 + BM_{hl}^2 + CM_\pi^3 + A_{\{0,-\}}a^2$  where the coefficients  $A_{\{0,-\}}$  are different for the two sets of data. Analogously we perform a combined fit of both  $\Xi_{ucc}^{++}$  and  $\Xi_{dcc}^+$  data with

the form  $M_{\Xi\{++,+}} = D + EM_\pi^2 + FM_{hl} + GM_\pi^3 + D_{\{++,+}}a^2$ . The dependence on  $M_\pi$  of the mass of the doubly charmed  $\Xi$  turns out to be considerably less pronounced than for the standard (strange)  $\Xi$ . The coefficient  $A_-$  is substantially smaller than  $A_0$  and in the charm case  $D_+$  is compatible with zero and can be removed from the fit function. Fits work well and in the continuum limit, at the physical point, we get  $M_{\Xi_{dss}} = 1.37(12)$  GeV (to be compared with  $M_{\Xi_{dss}}^{\text{exp}} = 1.32$  GeV) and  $M_{\Xi_{dcc}} = 3.52(25)$  GeV (in perfect agreement with  $M_{\Xi_{dcc}}^{\text{exp}} = 3.52$  GeV).

#### 4. Conclusions

In this preliminary study we have shown that, when baryon masses are analyzed as function of meson masses, lattice artefacts are always small and in some cases (notably in the charm region) hardly visible. They are instead clearly visible when baryon masses are analyzed as function of quark masses. As expected lattice artefacts are larger in the charm region, where they increase proportionally to  $\mu_h$ . The chiral extrapolation in the light quarks confirms to be critical and a term of order  $M_\pi^3$  is needed for both  $\Xi$  and  $\Lambda$  (it is particularly evident in this last case).  $M_{\Omega_{sss}}$  is still 10% larger than the experimental value and the source of this discrepancy seems not to be related to the continuum limit extrapolation or to the extrapolation in the light quark mass. Further investigations are needed to clarify this issue. Results for  $M_{\Xi_{dss}}$ ,  $M_{\Lambda_{uds}}$ ,  $M_{\Xi_{dcc}}$  and  $M_{\Lambda_{udc}}$  nicely agree with the experimental values. We have moreover obtained a prediction for  $M_{\Omega_{ccc}} = 4.73(40)$  GeV. We are computing all the correlation functions needed to extract the whole low-lying spectrum of strange/charmed baryons. A complete analysis, including a more careful assessment of both statistical and systematic errors, will be performed in the near future.

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