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The scalar sector of $SU(2)$ gauge theory with $N_F = 2$ fundamental flavours

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We present a first non perturbative study of the flavour singlet scalar and pseudoscalar spectrum of SU(2) gauge theory with two fundamental Dirac flavours. This theory provides a minimal template for a wide class of Standard Model extensions featuring novel strong dynamics. After having discussed our computational method, we present our new results for the σ,η' and a_0 states. We evaluate the relevant disconnected contributions and obtain benchmark results that are crucial input for model building. This work summarises our recent contribution[1].

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1. Introduction

The understanding of the flavour singlet scalar and pseudoscalar states of gauge theories with fermions is crucial to design and constrain models of new physics based on underlying strong dynamics such as composite Higgs models. It is thus of pivotal importance to develop tools to explore it using non perturbative methods in various theories.

In this work, we consider an $SU(2)$ gauge field theory with two fermions in the fundamental representation. Because of the pseudo-realness of the fundamental representation of $SU(2)$, the model exhibits an $SU(4)$ flavour symmetry in the massless limit which spontaneoulsy breaks down to the $Sp(4)$ group thus leading to 5 Goldstone bosons. As proposed in [2], the theory can be embedded into the Standard Model in such a way that it interpolates between composite Goldstone Higgs and Technicolor models. In this scenario, the physical Higgs boson is then a mixture of one Goldstone boson and of the lightest scalar excitation, analogue to the σ meson in QCD, of the underlying strongly coupled theory. With the currently available constraints, such a model is compatible with experiments[3]. This model is thus relevant for the experimental search of new physics.

It is important to recall that the properties of the scalar resonance of the strong theory in isolation are not preserved in the full Beyond SM model, due to the many corrections from interactions with the SM gauge bosons and heavy fermions. The mass of the scalar resonance can, for example, become lighter due to the SM interactions [4] and consequently narrower for kinematical reasons. The result obtained from the lattice simulations, which are performed in isolation from the Standard Model, must then be interpreted with care.

The theory investigated here has previously been studied on the lattice and, in particular, it has been shown that the expected pattern of spontaneous chiral symmetry breaking is realized [5]. An estimate of the masses of the vector and axial-vector mesons in unit of the pseudoscalar meson decay constant have been obtained in [6]. The scattering properties of the Goldstone bosons of the theory have also been considered in [7] and more recently in [8] where a first attempt to compute resonnance parameters has been reported. The model has also been investigated in the context of possible DM candidates [9, 10, 11, 12].

Computing the lightest scalar state is notoriously difficult because of the large disconnected contributions to the relevant two point functions, which are extremely challenging to estimate accurately, and because such a state is expected to be a broad resonance decaying into two Goldstone bosons in the chiral limit. Note, that calculation in QCD made significant progress [13].

The scalar sector of strongly interacting theories have been studied in others gauge theories [14, 15, 16], but our benchmark results constitute a primer for the important theory investigated here. We also provide results on the mass of the pseudoscalar singlet state, the η' meson, which is not a Goldstone boson due to the axial anomaly.

This work is summarized a recent contribution[1], to which we refer the reader interested in more details.

2. Techniques & Results

We simulate the $SU(2)$ gauge theory with two Dirac fermions in the fundamental representation discretized using the Wilson action for two mass-degenerate fermions u, d and the Wilson plaquette action for the gauge fields. The numerical simulations have been performed using an improved version of the HiRep code first described in Ref. [17]. We chose the scale observable w_0^χ as obtained in [20] and defined at non zero Wilson Flow time and in the chiral limit in [18, 19].

Even if the flavour symmetry group is $Sp(4)$, we will use the $SU(2)_V$ terminology and thus use the notion of isospin throughout this work. In this work, we will focus on fermionic interpolating field operators.

We define the following interpolating operators:

$$\mathscr{O}_{\bar{q}q}^{(\Gamma,\pm)}(x) \,=\, \overline{u}(x)\Gamma u(x) \pm \overline{d}(x)\Gamma d(x)\,, \tag{2.1}$$

where $\Gamma = \{\gamma_5, \mathbb{1}\}$. We extract the meson masses from zero-momentum two-point correlation functions:

$$C_{\Gamma,\pm}^{(t_0)}(t) = -\frac{1}{N_f L^3}\sum_{\vec{x},\vec{x}_0}\left\langle \mathscr{O}_{\bar{q}q}^{(\Gamma,\pm)\dagger}(t,\vec{x})\mathscr{O}_{\bar{q}q}^{(\Gamma,\pm)}(t_0,x_0)\right\rangle. \tag{2.2}$$

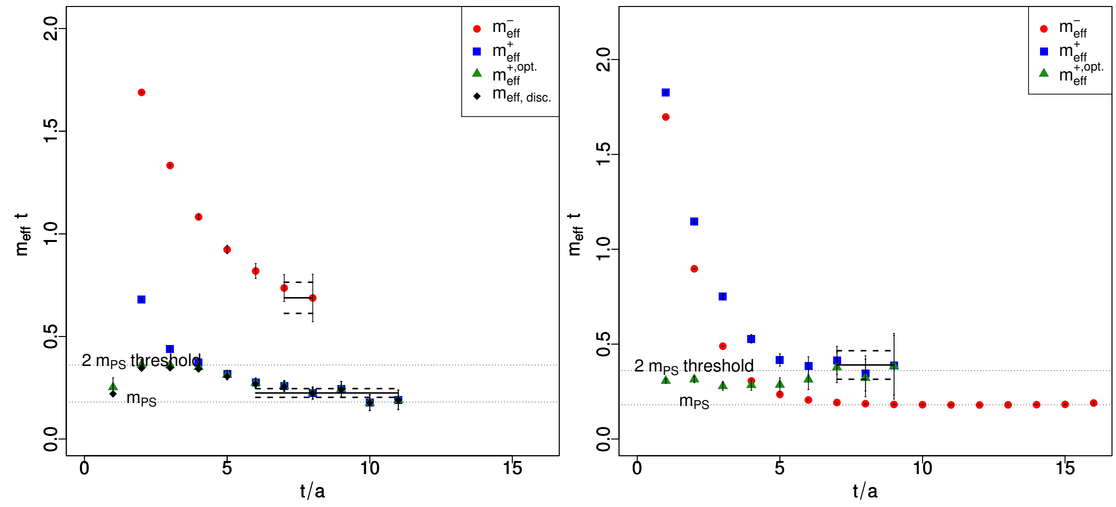
The evaluation of the two-point functions involve a so-called disconnected contribution in the case iso-scalar channel, corresponding to the operator $\mathscr{O}_{\bar{q}q}^{(\Gamma,+)}(x)$. In order to improve the signal over noise ratio, we perform vaccum substraction and average over the timeslice of the source position t_0. We use $Z_2 \times Z_2$ single time slice stochastic sources [21] to estimate the connected part of meson 2-point correlators. Assuming that no accidental symmetry forces the operator $\mathscr{O}_{\bar{q}q}^{(\Gamma,+)}$ to have a vanishing overlap with the ground state, the two-points correlator decays exponentially at large Euclidean time with the decay rate inversely proportional to the mass of the σ or η' bound states depending on the matrix Γ chosen when no decay channels are opened.

2.1 Effective masses

We define an effective mass $m_{\rm eff}^{(\Gamma,\pm)}(t)$ as in [22, 23] by the solution of the implicit equation:

$$\frac{C_{\Gamma,\pm}^{(t_0)}(t-1)}{C_{\Gamma,\pm}^{(t_0)}(t)} = \frac{e^{-m_{\rm eff}^{(\Gamma,\pm)}(t)(T-(t-1))} + e^{-m_{\rm eff}^{(\Gamma,\pm)}(t)(t-1)}}{e^{-m_{\rm eff}^{(\Gamma,\pm)}(t)(T-t)} + e^{-m_{\rm eff}^{(\Gamma,\pm)}(t)t}}\,, \tag{2.3}$$

where T is lattice temporal extent. At large euclidean time t, $m_{\rm eff}^{(\Gamma,\pm)}(t)$ approaches the value of the mass of the lightest state with the same quantum numbers as the operator $\mathscr{O}_{\bar{q}q}^{(\Gamma),\pm}$. We also have up to two alternative definition of the effective mass referred to as $m_{\rm eff}^{(\Gamma,{\rm opt.})}(t)$, $m_{\rm eff,disc}^{(\Gamma,+)}(t)$. They are used only as cross check, we therefore refer the interested reader to [1], for precise definitions. We observe clear plateaus for most of our ensembles. However, in our most chiral ensemble we observe short plateaux before the signal becomes dominated by noise. For all our ensembles, the masses are always extracted by performing single exponential fits to the full correlators on a range $[t_1/a, t_2/a]$ where t_2/a is set by the last timeslice where the effective mass is well-defined. We then choose by inspection the value of t_1/a for each ensemble. We show in Fig. 1 the mass, its error, and the fitting range obtained from this fit as horizontal lines for the most chiral run used in this analysis ($\beta = 2.0$, $m_0 = 0.958$, $L = 32$). A careful study of the excited state contamination in that regime would certainly help to extract more reliably the ground of state but is beyond the scope of this benchmark study.



(a) *iso-vector and iso-scalar scalar operator.* (b) *iso-vector and iso-scalar pseudoscalar operator.*

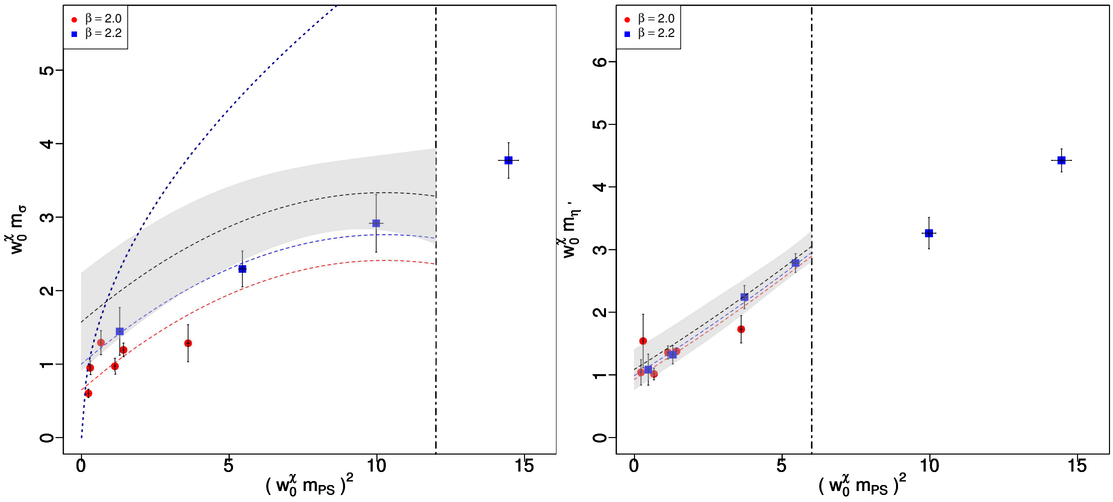
Figure 1: *Effective masses in the scalar and pseudoscalar case ($\beta = 2.0$, $m_0 = 0.958$, $L = 32$). The disconnected part has been measured on 2200 configurations.*

2.2 Extrapolations

Once the masses are extracted for each ensemble they are expressed in units of the scale w_0^χ, as determined in [20], and expressed as functions of $(w_0^\chi m_{\rm PS})^2$. The chiral and continuum extrapolations are then carried out by using the same strategy as in [20]. For each quantity we perform a global fit, including all the available data, to the following fit ansätz:

$$w_0^\chi m_X = w_0^\chi m_X^\chi + A(w_0^\chi m_{\rm PS})^2 + B(w_0^\chi m_{\rm PS})^4 + C\frac{a}{w_0}\,. \tag{2.4}$$

The results of the fits for the σ and η' mesons are shown in Fig. 2. In the plots, the gray band indicates the 1σ confidence region for the continuum prediction, obtained by setting $a = 0$ with our best fit parameters. To give an idea of the fit quality we also plot the best fit curves at finite lattice spacing using the same color code as for the data points. The upper limit of the fitting range for each channel is shown by the vertical dashed-dotted line in the plots. In the scalar channels, we draw the threshold of the decay into Goldstone bosons by a blue dashed line. In the case of the σ meson we thus show that all our results lie below the two Goldstone boson mass threshold.



(a) σ (b) η'

Figure 2: *Combined chiral and continuum extrapolation of the scalar and pseudoscalar iso-scalar meson masses. Two pion threshold is depicted by a blue dotted line in the case of the σ. The black dottted curve is the best fit prediction for $a = 0$, while the red and blue curves are the best fit curve at finite lattice. The vertical dashed line indicates the largest value of $(w_0^\chi m_{\mathrm{PS}})^2$ included in the fit.*

In the case of the a_0, we have checked that finite volume effects are not significant on three different volumes ($L/a = 16, 24, 32$) at $\beta = 2.2$ and $m_0 = -0.75$. Since the estimate of the mass of the a_0 does not require the estimate of any disconnected loop contributions, we are able to obtain a signal on all data sets and thus include four lattice spacings in the extrapolation. However in the case, we observe that some of our data points lie above the 3 Goldstone boson mass threshold. For this reason, we also consider a fit which excludes the data points above threshold. In order to estimate the systematic effects introduced by the choice of the fitting ansatz of Eq. (2.4), we also performed fits imposing $B = 0$. The interested reader is refered to [1] for more details. Our final results have a relative error of the order 30% for the three states considered in this work. Our results clearly suggest that discretization errors are significant in the scalar sector as can be seen in Fig. 2. Investigating the chiral regime and reducing the statistical errors would clearly help to control the extrapolation.

3. Conclusion

We have presented a determination of the spectrum of the low lying scalar mesons (iso-triplet and iso-singlet) as well as of the η' for the SU(2) gauge theory with $N_f = 2$ fundamental Dirac fermions.

The results are obtained via numerical lattice simulations by using fermionic interpolating operators for the extraction of mass spectrum and include contributions from the disconnected diagrams. As expected, the results for the σ and η' channels receive large contribution from the disconnected part, have an exponentially decreasing signal over noise ratio at large euclidean separations and we observe short plateaux. It is not excluded that we lose the signal of the two point function before the asymptotic regime and this has to be kept in mind.

Assuming we indeed observe reach the asymptotic regime, our calculation clearly shows that the σ and a_0 are stable for most of our ensembles, which provides an *a posteriori* justification of method used to extract the masses of these states in our current setup. Excited states contamination is starting to be a problem on our most chiral ensemble. We have shown that discretization errors are significant in the scalar sector. At lower quark mass, it will become necessary to consider the two pion scattering process.

We find after a combined chiral and continuum extrapolation that $w_0^\chi m_{a_0} = 1.3(3)$, $w_0^\chi m_\sigma = 1.5(6)$ and $w_0^\chi m_{\eta'} = 1.0(3)$. In units of $F_{\rm PS}$ the results then read: $m_{a_0}/F_{\rm PS} = 16.7(4.9)$, $m_\sigma/F_{\rm PS} = 19.2(10.8)$ and $m_{\eta'}/F_{\rm PS} = 12.8(4.7)$ using that $w_0^\chi F_{\rm PS} = 0.078(13)$. For comparison we find $m_V/F_{\rm PS} = 13.1(2.2)$ and $m_A/F_{\rm PS} = 14.5(3.6)$.

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