2016-12

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James, MN

http://hdl.handle.net/10026.1/10512

10.4028/www.scientific.net/SSP.258.117
Solid State Phenomena
Trans Tech Publications, Ltd.

Interpretation of Plasticity Effects using the CJP Crack Tip Field Model

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Keywords: Crack tip displacement field; Crack tip stress field; Stress intensity factors; T-stress; Plasticity-induced shielding; Plastic zone size; Plastic zone shape.

Abstract. This paper will outline the development of a model of crack tip fields that represents an innovation in incorporate the influences on crack tip displacement and stress fields of the zone of local plasticity that envelops a growing fatigue crack. The model uses assumed distributions of elastic stresses induced at the elastic-plastic boundary via wake contact and compatibility requirements, and defines a set of modified elastic stress intensity factors to characterise the crack stress or displacement tip field. In particular, recent work will be presented that compares the interpretation of plasticity-induced shielding obtained from trends observed in K_R and K_F with values of so-called 'crack closure' obtained via traditional strain gauge determination.

Introduction

This paper will very briefly outline the development of a model of crack tip fields that represents an attempt to incorporate the influences on crack tip displacement and stress fields of the zone of local plasticity that envelops a growing fatigue crack. In a fashion somewhat analogous to the original formulation of the elastic stress intensity factor by Irwin, the model uses assumed distributions of elastic stresses induced at the elastic-plastic boundary via wake contact and compatibility requirements, and defines a set of modified elastic stress intensity factors [1] to characterise the crack tip field. These stress intensity factors (SIFs) reflect a combination of applied stress and any plasticity-induced *elastic* stresses (that characterise the so-called plasticity-induced crack tip shielding) and the model therefore leads to a stress intensity factor that drives crack growth (called K_F which, in the absence of plasticity-induced shielding, is identical to K_I) and a retarding stress intensity factor (K_R) that includes influences from crack wake contact (so-called closure) as well as stresses induced by compatibility requirements at the elastic-plastic interface, and which have an effect on the elastic stress field ahead of the crack. It also calculates a value for the T-stress.

The original development was performed on a birefringent material (polycarbonate) using a stress-based approach that allowed direct comparison with full-field photoelastic fringe patterns [1], and this was then extended to include a displacement-based solution that allowed comparison between the analytical full field solution and full-field experimental data acquired from digital image correlation (DIC) techniques. This allowed the model to be applied to metallic compact tension or other standard fracture mechanics specimens [2]. The model was referred to by its originators as the Christopher-James-Patterson (CJP) model of crack tip fields, recognising the distinct multidisciplinary contributions necessary to its development (applied mechanics, fatigue/fracture mechanics and experimental mechanics). The model was further extended from uniaxial (K_I) loading to include biaxial (K_I and K_{II}) loading [3].

Recent work on the CJP model has been aimed at investigating how it might characterise the growth of inclined cracks subject to biaxial loading [4], comparing its ability to characterise plastic zone size and shape with that of the Williams and Westergaard models of elastic crack tip stresses [5], and with a comparison of the interpretation of plasticity-induced closure obtained from trends observed in K_R and K_F with values obtained via traditional strain gauge determination. This paper will briefly review some of this work, in particular the ability of the CJP model to characterise plasticity-induced fatigue crack closure.

CJP Model

The CJP model is a novel mathematical model developed by Christopher, James and Patterson based on Muskhelishvili complex potentials. The authors postulated that the plastic enclave which exists around the tip of a fatigue crack and along its flanks will shield the crack from the full influence of the applied elastic stress field and that crack tip shielding includes the effect of crack flank contact forces (so-called crack closure) as well as compatibility-induced interfacial shear stress at the elastic-plastic boundary.

Mode I Solution. In the original formulation of this model, crack tip stress fields were characterised as [1]:

$$\begin{aligned}
\sigma_x &= -\frac{1}{2}(A+4B+8F)r^{-\frac{1}{2}}\cos\frac{\theta}{2} - \frac{1}{2}Br^{-\frac{1}{2}}\cos\frac{5\theta}{2} - C \\
&\quad -\frac{1}{2}Fr^{-\frac{1}{2}}\left[\ln(r)\left(\cos\frac{5\theta}{2}+3\cos\frac{\theta}{2}\right)+\theta\left(\sin\frac{5\theta}{2}+3\sin\frac{\theta}{2}\right)\right]+O\left(r^{\frac{1}{2}}\right) \\
\sigma_y &= \frac{1}{2}(A-4B-8F)r^{-\frac{1}{2}}\cos\frac{\theta}{2} + \frac{1}{2}Br^{-\frac{1}{2}}\cos\frac{5\theta}{2} + H \\
&\quad +\frac{1}{2}Fr^{-\frac{1}{2}}\left[\ln(r)\left(\cos\frac{5\theta}{2}-5\cos\frac{\theta}{2}\right)+\theta\left(\sin\frac{5\theta}{2}-5\sin\frac{\theta}{2}\right)\right]+O\left(r^{\frac{1}{2}}\right) \\
\sigma_{xy} &= -\frac{1}{2}r^{-\frac{1}{2}}\left(A\sin\frac{\theta}{2}+B\sin\frac{5\theta}{2}\right) \\
&\quad -Fr^{-\frac{1}{2}}\sin\theta\left[\ln(r)\cos\frac{3\theta}{2}+\theta\sin\frac{3\theta}{2}\right]+O\left(r^{\frac{1}{2}}\right)
\end{aligned} \tag{1}$$

Five coefficients (*A*, *B*, *C*, *F* and *H*) are therefore used to define the stress fields around the crack tip. This model can be also solved in terms of displacement [2]:

$$\begin{aligned}
2G(u+iv) &= \kappa\left[-2(B+2F)z^{\frac{1}{2}}+4Fz^{\frac{1}{2}}-2Fz^{\frac{1}{2}}\ln(z)-\frac{C-H}{4}z\right] \\
&\quad -z\left[-(B+2F)\bar{z}^{-\frac{1}{2}}-F\bar{z}^{-\frac{1}{2}}\overline{\ln(z)}-\frac{C-H}{4}\right] \\
&\quad -\left[A\bar{z}^{\frac{1}{2}}+D\bar{z}^{\frac{1}{2}}\overline{\ln(z)}-2D\bar{z}^{\frac{1}{2}}+\frac{C+H}{2}\bar{z}\right]
\end{aligned} \tag{2}$$

In the mathematical analysis, the assumption $D + F = 0$ must be made in order to give an appropriate asymptotic behaviour of the stress along the crack flank. Therefore, crack tip displacement fields are defined from the five coefficients: *A*, *B*, *C*, *F* and *H*.

The CJP model provides three stress intensity factors to characterise the stress and displacement fields around the crack tip; an opening mode stress intensity factor K_F, a retardation stress intensity factor K_R, and a shear stress intensity factor K_S, and it also finds a value for the *T*-stress. The opening mode stress intensity factor K_F is defined using the applied remote load traditionally characterised by K_I but which is modified by force components derived from the stresses acting across the elastic-plastic boundary and which therefore influence the driving force for crack growth. Thus, unlike the classical K_I, K_F includes the effect of plasticity-induced crack shielding and it is linear with the load as long as there is no shielding effect. K_F is defined from the asymptotic limit of σ_y as $x \rightarrow +0$, along $y = 0$, i.e. towards the crack tip on the crack plane ahead of the crack tip:

$$K_F = \lim_{r \to 0}\left[\sqrt{2\pi r}\left(\sigma_y + 2Fr^{-1/2}\ln r\right)\right] = \sqrt{\frac{\pi}{2}}\left(A - 3B - 8F\right) \tag{3}$$

The retardation stress intensity factor K_R characterises shielding forces applied in the plane of the crack and which provide a retarding effect on fatigue crack growth. Thus, K_R is evaluated from σ_x in the limit as $x \rightarrow -0$, along $y = 0$, i.e. towards the crack tip along the crack flank:

$$K_R = \lim_{r \to 0}\left[\sqrt{2\pi r}\,\sigma_x\right] = -(2\pi)^{3/2} F \tag{4}$$

It is proposed in the CJP model that a shear term arises from the requirement of compatibility of displacements at the elastic-plastic boundary of the plastically deformed crack wake, as plastic deformation is a constant volume effect, (equivalent to Poisson ratio, $\nu = 0.5$), while elastic deformation occurs with Poisson ratio, $\nu = 0.3$. The mathematical analysis produces a *ln* term, which has the same form as the stress field terms associated with dislocations. In this respect, it is interesting to note that Riemelmoser and Pippan [6] proposed a dislocation model for plasticity-induced closure in plane strain and that their model led to a shear stress along the crack wake from elastic rotation of the lattice in the plastic wake. The net stress effect is essentially the same as that arising from the compatibility concept that the CJP model assumes in generating the stress terms. A shear stress intensity factor K_S is therefore defined in the CJP model that characterises this compatibility-induced shear stress along the plane of the crack at the interface between the plastic enclave and the surrounding elastic field and is derived from the asymptotic limit of σ_{xy} as $x \rightarrow -0$, along $y = 0$, i.e. towards the crack tip along the crack wake:

$$K_S = \lim_{r \to 0}\left[\sqrt{2\pi r}\,\sigma_{xy}\right] = \mp\sqrt{\frac{\pi}{2}}\left(A + B\right) \tag{5}$$

A positive sign indicates y > 0, and a negative sign that y < 0. The *T*-stress, which is found as components T_x in the *x*-direction and T_y in the *y*-direction is given by:

$$\begin{aligned} T_x &= -C \\ T_y &= -H \end{aligned} \tag{6}$$

Mode I and II Solution. Extending this model to include both Mode I and Mode II loading requires an additional force parameter representing an anti-symmetrical shear force on either side of the crack. The Mode I CJP Model is defined as:

$$\left|\sigma_y - \sigma_x + 2i\sigma_{xy}\right| = Az^{-\frac{1}{2}} + Bz^{-\frac{3}{2}}\bar{z} + Cz^0 + Dz^{-\frac{1}{2}}\ln(z) + Ez^{-\frac{3}{2}}\bar{z}\ln(z) \tag{7}$$

The extension to the model was obtained by making the coefficients A and B complex and making the assumptions $A = A_r + i3B_i$, $B = B_r + iB_i$, $D + E = 0$, to give the Mode I and Mode II version as [3]:

$$\left|\sigma_y - \sigma_x + 2i\sigma_{xy}\right| = (A_r + i3B_i)z^{-\frac{1}{2}} + (B_r + iB_i)z^{-\frac{3}{2}}\overline{z} + Cz^0 + Dz^{-\frac{1}{2}}\ln(z) + Ez^{-\frac{3}{2}}\overline{z}\ln(z) \tag{8}$$

Reference [3] gives full details of the solution of this equation for cases of crack tip stresses and crack tip displacements along with the relevant expressions for the three stress intensity parameters and the T-stress.

Characterising Plasticity-Induced Closure

The original intention when this work was started was to develop a full-field model of crack tip stresses that could be fitted to full-field photoelastic fringe patterns, and that would then allow a single point wake contact stress (pressure) to be identified explicitly [7]. Accepting that the model was approximate in its manner of dealing with wake contact stress (single point rather than a power law distribution), and that the fitting between experimental data and analytical results was complex, requiring global optimisation via a genetic algorithm and local optimisation using the downhill Simplex method, the results were rather interesting and are repeated here in Fig. 1. Fig. 1 shows the behaviour of the wake contact pressure (black diamonds) throughout two load cycles applied to a standard polycarbonate compact tension (CT) specimen tested at R = 0.1. Polycarbonate is a useful model material to study plasticity-induced closure, as it is known to show this phenomenon [8] and is also birefringent. It is clear that the values of the wake contact pressure extracted from this relatively complex approach are random and do not follow a consistent pattern. This is in contrast to the values of K_I and K_{II} produced by the model which do follow a logical pattern and which could be interpreted as demonstrating the existence of plasticity-induced closure and crack tip blunting.

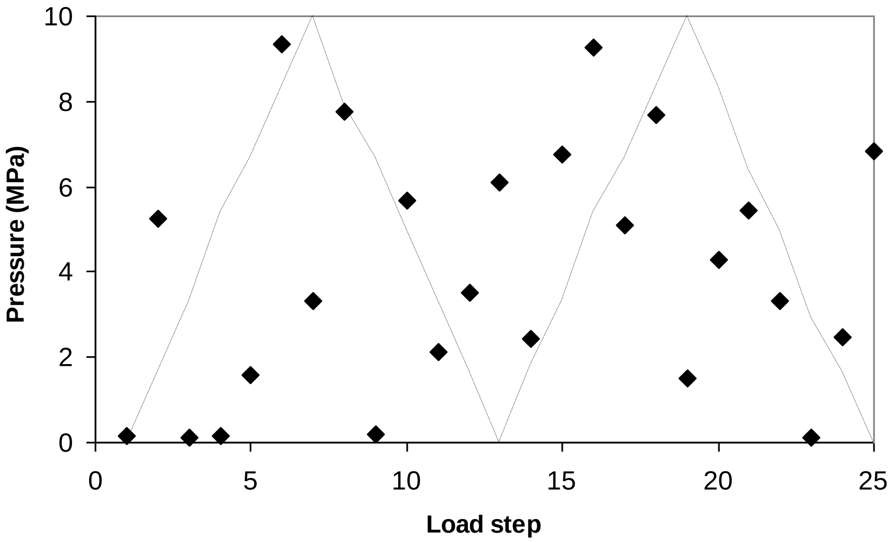

Fig. 1a Wake contact pressure evaluated at 25 steps through two load cycles. The dashed lines indicate the applied fatigue cycles in relation to the load step positions.

These results led to a re-appraisal of whether crack wake contact (the original idea behind crack closure) could explain plasticity-induced shielding, or whether other influences, e.g. compatibility of displacements at the elastic-plastic boundary also needed to be captured. This was motivated partly by knowledge of the model put forward by Riemelmoser and Pippan [6] and partly by other known problems in rationalising crack growth data using the supposed closure-free value of stress intensity ΔK_{eff} where the opening value of the stress intensity factor K_{op} was evaluated using standard techniques [9]. The model proposed in references [1, 2] incorporates both a power law distribution of wake contact pressure behind the crack tip and the elastic shear stresses induced at the elastic-plastic boundary through compatibility requirements. It does not give explicit values of wake contact pressure but instead builds the shielding influences into the definitions of the new stress intensity factors defined in the model, i.e. K_F and K_R. Fig. 2 shows data acquired through a half cycle of fatigue loading, from fitting the CJP model to full-field photoelastic fringe patterns obtained from a polycarbonate CT specimen using phase-stepping. The crack was 35.0 mm long and was tested at R = 0.1, 0.5 Hz and P_{max} = 120 N in the load cycle. Fig. 3 gives equivalent data for a crack 30.3 mm long tested at R = 0.3.

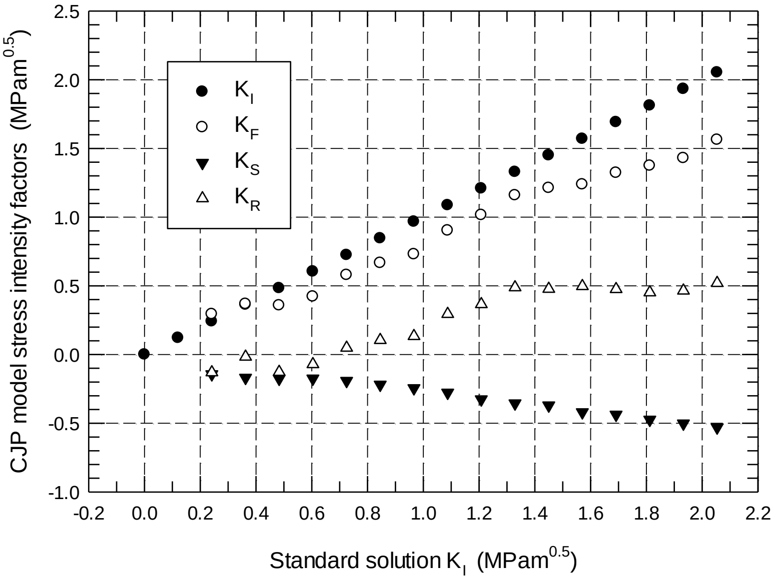

Fig. 2 Values of the stress intensity factors defined in the CJP model through a single half-cycle of loading at R = 0.1. The values of K_I obtained from the standard wide range solution for a CT specimen are also shown.

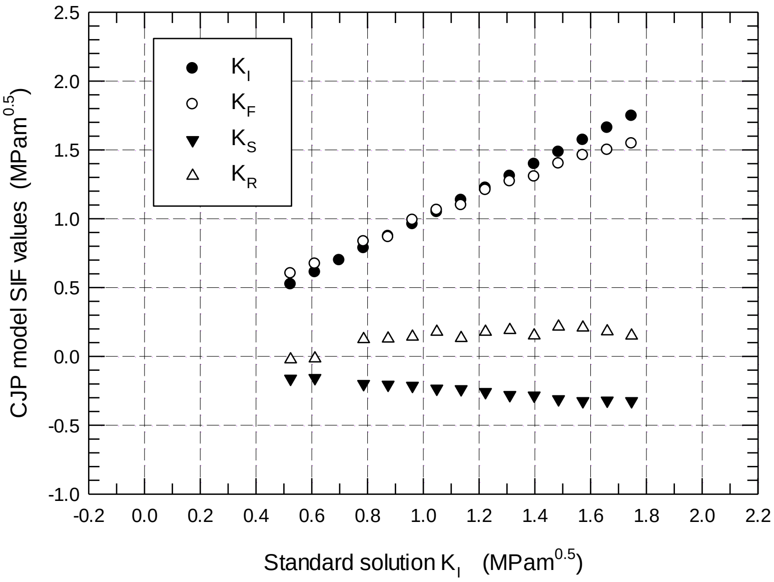

Fig. 3 Values of the stress intensity factors defined in the CJP model through a single half-cycle of loading at R = 0.3. The values of K_I obtained from the standard wide range solution for a CT specimen are also shown.

The K_F and K_R data at R = 0.1 are interpreted as showing that crack tip shielding is occurring during the lower part of the fatigue cycle, as it is clear that the value of K_F remains constant, within the limits of experimental error, up to a value of the nominal K_I of about 0.5 MPa√m. The same observation is true for the value of K_R which is slightly negative up a similar value of K_I. Standard wisdom states that a negative SIF has no physical meaning, but in terms of modelling closure and crack retardation arising from plasticity-induced shielding, negative SIF values are well-established, e.g. [10]. It therefore seems that in this polycarbonate material at R = 0.1 plasticity-induced shielding is present over perhaps the bottom 30% of the fatigue cycle in nominal SIF terms. Consideration of the trends in K_F and K_R at R = 0.1 gives an interpretation that shielding is present up to the point in the load cycle where the value of K_F starts to increase monotonically and K_R becomes positive. The other interesting point is that the increase in K_F changes slope in the upper part of the fatigue cycle, and the increase in K_R also levels out (above a value of $K_I \approx 1.35$ MPa√m). This is interpreted as reflecting the influence of crack tip blunting in the upper part of the fatigue cycle. Note that using the standard definition of SIF, $K_C < 3$ MPa√m in this polycarbonate material and the peak value of nominal SIF applied in this fatigue loading is 2.05 MPa√m which would be expected to lead to significant craze-induced blunting of the crack tip [11].

In contrast, at R = 0.3 the value of K_F increases monotonically from the start of the fatigue cycle and the value of K_R is ≥ 0 from the start of the fatigue cycle. This would be expected from the proportion of the cycle that experiences plasticity-induced shielding at R = 0.1.

Similar data have been obtained using the displacement solution of the CJP model and DIC techniques on 25 x 24 x 1 mm Grade 5 titanium CT specimens [5] (see Fig. 4).

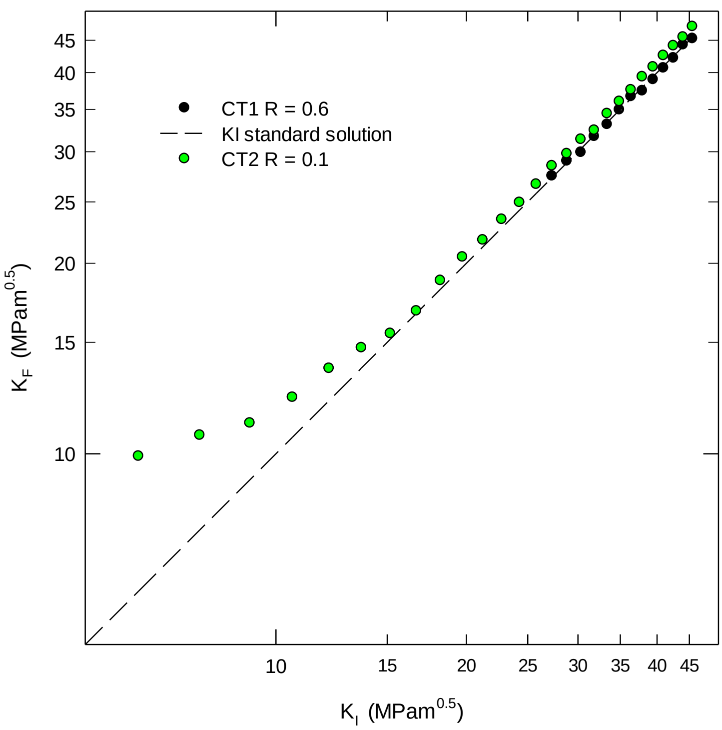

Fig. 4 Values of K_F through a loading half-cycle as a function of the applied nominal K_I from two titanium compact tension specimens. Data for CT1 was obtained at R = 0.6 and a = 9.20 mm, and for CT2 R = 0.1 and a = 9.40 mm.

Some additional work, using 2 mm thick compact tension specimens machined from 2024-T6 aluminium alloy, has considered the relationship between the opening load determined from back-face strain gauge measurements using the offset compliance technique and the trends in K_F and K_R. In this case CJP SIF data were acquired using DIC techniques at R = 0.05, 0.3 and 0.5. Fig. 5

shows typical offset compliance data recorded during a single fatigue cycle at R = 0.05 and the trace clearly shows an amplified change in slope of the curve at an approximate value of applied load of 180 N in a fatigue cycle where P_{max} = 746 N, i.e. a ratio of $K_{op}/K_{max} \approx 0.25$.

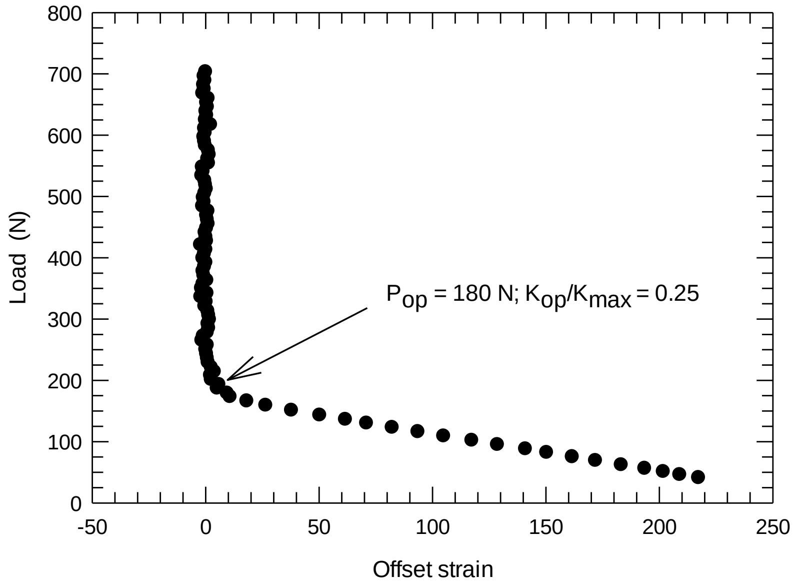

Fig. 5 Plot of offset compliance of a crack 44.2 mm long obtained using a back-face strain gauge on a 2 mm thick 2024-T6 aluminium CT specimen tested at R = 0.05. The opening point can be deduced to occur at ≈ 0.25 K_{max}.

Fig. 6 presents the data obtained for the average value of K_F (i.e. the average of the loading and unloading values at equivalent load steps) through a complete loading cycle using DIC and the CJP model. It is clear that there is a good correlation between the load at which a change in slope occurs here (180 N) and the offset compliance data.

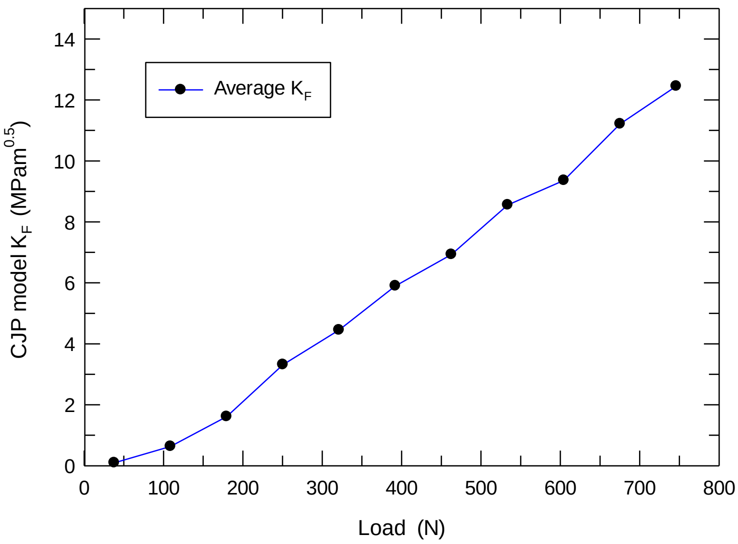

Fig. 6 Average value of K_F measured during a single loading-unloading cycle for a crack 30.0 mm long tested at R = 0.05 with K_{max} = 10.8 MPa√m.

Conclusions

This short paper has presented a summary of ideas underlying the innovative CJP model, the equations describing it in terms of crack tip stress or displacement fields, and some of the work aimed at identifying the potential value of K_F and K_R in characterising the presence of plasticity-induced shielding. The work presented in this paper demonstrates that the model offers potential advantages in characterising the influences of such shielding on the elastic stress field that drives crack growth. Work reported in reference [5] also shows that the CJP model better characterises the plastic zone size and shape in both plane stress and plane strain conditions than either of the two commonly used descriptions of the elastic stress field at a crack tip, i.e. the Williams and Westergaard models.

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