This copy of the thesis has been supplied on condition that anyone who consults it is understood to recognise that its copyright rests with its author and that no quotation from the thesis and no information derived from it may be published without the author's prior consent.
Novel Particle Model for the Prediction of Stability and Episodic Collapse of Coastal Cliffs and Levees

by

Johan Richard Vandamme

A thesis submitted to the University of Plymouth
in partial fulfilment for the degree of

Doctor of Philosophy

School of Marine Science and Engineering

Faculty of Science and Technology

November 2011
Novel Particle Model for the Prediction of Stability and Episodic Collapse of Coastal Cliffs and Levees by Johan Richard Vandamme

Abstract

This thesis investigates the WCSPH model by considering fluid entry and exit, and integrates the WCSPH method into a new, novel, particle-based Bluff Morphology Model (BMM). Using the BMM, this thesis investigates the stability, collapse and equilibrium position of soft coastal bluffs (cliffs).

Fluid and floating object interaction using a novel adaptation of the WCSPH method is investigated by incorporating a floating object model. In particular, this thesis examines the water impact, hydrodynamic forces, fluid motions, and movement of objects in the conventional case studies of object entry and exit from still water. A two-dimensional wedge drop analysis was examined, and the hydrodynamic forces show acceptable agreement with published experimental and numerical results. Simulations for water entry and exit of a buoyant and neutral density cylinder compares well with the previous experimental, numerical and empirical studies. These results provide a good foundation to evaluate the accuracy and stability of WCSPH for modelling complex flows, and therefore offers a platform for the use of WCSPH in a Bluff Morphology Model.

The BMM combines a multiple wedge displacement method with an adapted Weakly Compressible Smoothed Particle Hydrodynamics (WCSPH) method. At first the wedge method is applied to compute the stability of the bluff. Once the critical failure mechanism of the bluff slope has been identified, if the Factor of Safety for the mechanism is less than 1, the adapted WCSPH method is used to predict the failure movement and residual shape of the slope. The model is validated against benchmark test cases of bluff stability for purely frictional, purely cohesive, and mixed strength bluff materials including 2D static water tables. The model predictions give a good correlation with the expected values, with medium resolution models producing errors of typically less than 2.0%. In addition, the prediction of lateral movement of a surveyed cliff and the dynamic collapse of a vertical bluff are computed, and compare well with published literature.

This model is further extended to then investigate the effect of two dimensional seepage on the stability and collapse of soil slopes and levees. To incorporate the seepage in the model, Darcy’s Law is applied to the interactions among neighbouring soil particles and ghost particles are introduced along the enclosed soil boundary to ensure that no fluid crosses the boundary. The contribution of partially saturated soils and matric suction, as well as the change in hydraulic conductivity due to seepage, are predicted well by this model. The predicted time evolution of slope stability and seepage induced collapse are in reasonable agreement with the experimental results for homogeneous frictional sand and multiple layered cohesive soils. Rapid drawdown over a sand soil is also investigated, and the location and time of the levee collapse occurrence are captured well. A toe erosion model is incorporated within the numerical model, and the location and quantity of erosion caused by lateral seepage is well predicted. The interplay of erosion, seepage and slope instability is examined.
Contents

Chapter 1 – Introduction .................................................................................................1

  1.1 Background ..............................................................................................................1

  1.2 Coastal Cliff Failures .............................................................................................3

    1.2.1 Short Term Coastal Erosion ............................................................................3

    1.2.2 Short Term Cliff Collapse ............................................................................5

    1.2.3 Causes of Collapse .......................................................................................8

    1.2.4 Soil Properties ...............................................................................................9

  1.3 Aims and Objectives .............................................................................................10

Chapter 2 – Methodology ..............................................................................................13

  2.1 Potential Models ..................................................................................................13

    2.1.1 Soil Profile and Stability Models ....................................................................13

    2.1.2 Hydraulic Erosion Models ............................................................................20

    2.1.3 Seepage Models ............................................................................................21

    2.1.4 Novelty of SPH approach .............................................................................23

  2.2 Previous SPH models .........................................................................................25

    2.2.1 Smoothed Particle Hydrodynamics applied to Fluids ....................................25

    2.2.2 SPH applied to Solids and Fluid-Solid Interactions ........................................35

    2.2.3 SPH applied to Soil Movements ....................................................................39

  2.3 Present SPH Model Improvements .....................................................................41
4.3 Results ........................................................................................................109
4.3.1 Bluff Slope Stability Analysis ...............................................................109
4.3.2 Bluff Slope Failures .............................................................................121
4.4 Conclusions .............................................................................................125

Chapter 5 – Investigation of Levee Instability Caused by Seepage and Erosion
Using a Particle Method ..................................................................................128
5.1 Introduction ..............................................................................................128
5.2 Bluff Morphology Model ..........................................................................130
5.2.1 Stability Evaluation .............................................................................131
5.2.2 Seepage Analysis ................................................................................132
5.2.3 Erosion & Undercutting Effects ............................................................135
5.2.4 Soil Parameters ....................................................................................136
5.3 Previous Experimental and Numerical Model Results ............................137
5.3.1 Lysimeter Experiments of Homogeneous Cohesionless Soil of Fox et
al. (2007a) ....................................................................................................137
5.3.2 Lysimeter Experiments of Inhomogeneous Soil of Chu-Agor et al.
(2008) .........................................................................................................138
5.3.3 Draw Down Failure Experiments of Budhu and Gobin (1996) ..........139
5.4 Model Validation ......................................................................................139
5.4.1 Seepage Through Homogeneous Soil ...............................................139
5.4.2 Seepage Through Non-Homogeneous Soil .......................................145
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Angle of Wedge Base</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Angle of Interwedge Boundary</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Unit Weight</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Angle Between Particles and the Vertical</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Total Soil Stress</td>
</tr>
<tr>
<td>(\sigma')</td>
<td>Effective Soil Stress</td>
</tr>
<tr>
<td>(\phi')</td>
<td>Effective Angle of Internal Friction</td>
</tr>
<tr>
<td>(\phi_{mob}')</td>
<td>Mobilised Angle of Internal Friction</td>
</tr>
<tr>
<td>(\Pi)</td>
<td>Empirical viscosity approximation</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>Balance of Lateral Forces on a Particle</td>
</tr>
<tr>
<td>(a)</td>
<td>Acceleration</td>
</tr>
<tr>
<td>(A)</td>
<td>Area</td>
</tr>
<tr>
<td>(c')</td>
<td>Effective Cohesion of Soil</td>
</tr>
<tr>
<td>(c_{mob}')</td>
<td>Mobilised Cohesion</td>
</tr>
<tr>
<td>(C_u)</td>
<td>Maximum undrained shear strength</td>
</tr>
<tr>
<td>(D)</td>
<td>Deposition Flux</td>
</tr>
<tr>
<td>(E)</td>
<td>Erosion Flux</td>
</tr>
</tbody>
</table>
$FoS$  Factor of Safety  

$F$  Force  

$F_{out}$  Out of Balance Force  

$h$  WCPSH Kernel Smoothing Length  

$k$  Soil Permeability  

$K$  Rankines Earth Pressure Coefficient  

$K_a,K_p$  Rankines Active and Passive Earth Pressure Coefficients  

$l$  Length  

$m$  Mass  

$n$  Porosity  

$N_x,N_z$  Normal Components (in the $x$ and $z$ direction, respectively)  

$N'$  Effective Normal Force  

$N$  Normal Force  

$p$  Pressure  

$Q$  Volumetric Flow Rate  

$r_u$  Pore Pressure Coefficient  

$t$  Time  

$T$  Shear Stress  

$x$
\[ U, U_w \] Uplift Force Caused by Water Pressure

\[ V_j \] Volume of Particle \( j \)

\[ W(x_a - x_j), W_{ij} \] WCPSH Kernel Function

\[ W \] Weight of Soil Wedge

\[ z_b \] Bed Elevation

---

**List of Abbreviations**

**BMM** Bluff Morphology Model

**CCCL** Coastal Construction Control Line

**CIP** Constrained Interpolation Profile

**FOS** Factor of Safety

**ISPH** Incompressible Smoothed Particle Hydrodynamics

**RMSE** Root Mean Squared Error

**SPH** Smoothed Particle Hydrodynamics

**SPS** Sub-Particle Scale

**WCSPH** Weakly Compressible Smoothed Particle Hydrodynamics
List of Figures

Fig. 1.1 Coastal Erosion seen threatening a listed WWII coastal structure in Cornwall................................................................................................................................................2

Fig. 1.2 Example of partial cliff collapse on a soft, unprotected cliff [Source Bristol University]........................................................................................................................................6

Fig. 2.1 Representation of mobilisation of angle of shearing resistance $\phi'_{mob}$ and strain for two soil types...........................................................................................................................45

Fig. 2.2 Diagrammatic representation of Wedge numbering and Boundary Labelling..........................................................................................................................................................48

Figure 2.3 Flow Chart of model components, transfer of variables, and model outputs........................................................................................................................................................58

Fig. 3.1 Comparison of WCSPH to the experimental results of Greenhow (1987) for plunging wedges of 30° (a) and 45° (b) deadrise angles at a constant velocity into still water (images courtesy of the Massachusetts Institute of Technology)..................................................................................................................69

Fig. 3.2 The velocity vector plots of the fluid domain under impact of a 30° wedge; the jets described in Greenhow (1987) are clearly shown: (a) 0.004 s; (b) 0.016 s; (c) 0.02 s.........................................................................................................................................................71
Fig 3.3 (a) the falling velocity of a 30° deadrise angle wedge, timed from the moment of entry; (b) the vertical force exerted on the wedge by the fluid body..........................................................................................................................73

Fig. 3.4 Pressure under the impact of a 30° wedge with WCSPH on the left hand side and the ISPH results of Shao (2009) on the right.........................................................75

Fig. 3.5 Slamming coefficient \[p/(0.5\rho v^2)\] plotted against dimensionless depth for 2D wedges plotted over the predicted values from Greenhow (1987); multiple lines for the 60° and 45° wedges show the theories of Borg (1959) and Wagner (1932)...........................................................................................................................................77

Fig. 3.6 Free surface deformation during water entry of a cylinder of density 500kgm\(^{-3}\); WCSPH results (left) compared against the model results using CIP by Zhu et al. (2007) (centre) and experimental results of Greenhow and Lin (1983) (right) at times after impact: (a) 0.005 s; (b) 0.030 s; (c) 0.085 s; (d) 0.120 s........79

Fig. 3.7 Depth penetration of a cylinder of density 500 kgm\(^{-3}\): WCSPH results, model results using CIP by Zhu et al. (2007), and experimental results of Greenhow and Lin (1983).................................................................................................................................80

Fig. 3.8 Free surface deformation during water entry of a cylinder of density 1,000kgm\(^{-3}\); WCSPH results (left) compared against the model results using CIP by Zhu et al. (2007) (centre) and experimental results of Greenhow and Lin (1983) (right); images shown at time after impact: (a) 0.015 s; (b) 0.110 s; (c) 0.200 s; (d) 0.450 s.................................................................................................................................82
Fig. 3.9 Depth penetration of a cylinder of density 1,000 kgm$^{-3}$: WCSPH results, model results using CIP by Zhu et al. (2007), and experimental results of Greenhow and Lin (1983).................................................................................................................................84

Fig. 3.10 Free surface deformation attributable to a forced movement cylinder rising through the free surface for dimensionless time ($t = Ut/d$) (a) 0.4; (b) 0.6; red dots show the numerical results presented in Greenhow and Moyo (1997).........................86

Fig. 3.11 Velocity plots of the fluid domain as a free movement of a cylinder of density 250 kgm$^{-3}$ as it (a) rises through the fluid; (b) aligns with the free surface; (c) reaches maximum elevation.................................................................................................................88

Fig. 3.12 Comparison of different resolution runs for a cylinder rise, showing the expected reduction in the asymmetry around the cylinder: (a) particle diameter of 0.025 m; (b) $d = 0.016$ m.......................................................................................................................89

Fig. 4.1 Assumed relationship between mobilised angle of shearing resistance ($\phi_{mob}'$) and strain along the assumed failure surface.................................................................99

Fig. 4.2 Conceptualised cycle of Bluff Morphology Model (BMM) mechanism to find failure surfaces..................................................................................................................101

Fig. 4.3 Wedge boundaries of an arbitrary slip surface (also showing wedge boundary suffixes)...................................................................................................................102

Fig. 4.4 Root Mean Square Error of the model when predicting mobilised angle of shearing resistance ($\phi_{mob}'$) for a linear slip in a dry, non-cohesive soil.................110
Fig. 4.5 Predicted mobilised angle of shearing resistance ($\phi'_\text{mob}$) for a linear slip in a wet, non-cohesive soil using 8 wedges.................................................................111

Fig. 4.6 Root Mean Square Error of the Bluff Morphology Model (BMM) when predicting the mobilised angle of shearing resistance ($\phi'_\text{mob}$) for a linear slip in a wet, non-cohesive soil, where $\gamma_{\text{soil}}/\gamma_{\text{water}} = 1.91$.................................................................112

Fig. 4.7 Schematic of the slope domain (a). Taylor’s stability coefficients for a non-frictional soil where the solid lines show D=1 and the dotted lines D=2. The points are the computed slope stability values (b).................................................................114

Fig. 4.8 Root Mean Square Error of the Bluff Morphology Model (BMM) when predicting Taylor’s stability coefficient for a non-frictional soil, for various numbers of wedges within each slip mechanism, where $H$ is the cliff height in metres, and $G$ is the density of the soil relative to the density of water.........................................................116

Fig. 4.9 Factor of Safety of a cohesive wet slope with the Cliff Morphology Model and the Slope/W (Finite Element Method) (Durrani, 2007). Analysis at the toe (a), midpoint (b) and the crest (c) of the slope.................................................................117

Fig. 4.10 Factor of Safety for a typical glacial till deposit cliff, showing surveyed position of July 2007 (solid) and February 2008 (dashed) by Quinn et al. (2010)........................................................................................................118

Fig. 4.11 Horizontal displacement of glacial till failure. The Fast Lagrangian Analysis of continuum (FLAC) model (Quinn et al., 2010) (a) compared to the present Bluff Morphology Model (BMM) (b).................................................................120
Fig. 4.12 Particle positions in a catastrophic collapse of a vertical cliff face. The present Bluff Morphology Model (BMM) (a) compared to Discrete Element Method (Iwashita and Hakuno, 1990) (b)...........................................................................................................123

Fig. 4.13 Particle velocity vectors in a catastrophic collapse of a vertical cliff face. The present Bluff Morphology Model (BMM) (a+c) compared to Discrete Element Method (Iwashita and Hakuno, 1990) (b+d) at two different stages of failure..................................................................................................................................................124

Fig. 5.1 Diagrammatic representation of particle tessellation, where neighbours are $\delta l$ distance apart, and seepage flows over a boundary of area $A$........................................132

Fig. 5.2 The relationship of (a) Volumetric water content and (b) hydraulic conductivity to Pore Water Pressure properties used by the Bluff Morphology Model (dashed line) compared to that of Ng & Shi (1998).................................................................134

Fig. 5.3 Typical set up of the lysimeter experiments of Fox et al. (2007). The water inflow reservoir is at a controlled level and the bank angle is set at 60°.................137

Fig. 5.4 The propagation of the wetting front across a sand bank at t=110s, 240s, and 310s. (a-c): experimental photos from (Fox et al. 2007b); (d-f): BMM model (saturated particles red, unsaturated black)........................................................................................................140

Fig. 5.5 Model predictions of Cumulative Seepage using a spatial resolution between 0.006m and 0.014m......................................................................................141
Fig. 5.6 BMM predictions of the change in the critical Factor of Safety over the course of 1500s of seepage for a sand bank during the lysimeter experiment of Fox et al. [18]. Time of observed experimental failure given by line at t=1000s........142

Fig. 5.7 – Soil Profiles of the Factor of Safety before seepage (a) and after 1500s of seepage (b) for the case $\phi_b = 0^\circ$. Black particles are those with no Factor of Safety found by the BMM, with observed failure of Fox, Chu-Agor et al. (2007) (black dashed line).................................................................................................................................144

Fig. 5.8 Soil Profile of lysimeter experiments by Chu-Agor et al. (2008). The water level is kept at a constant height H, and the depth of the Silt Loam Layer, BH, varies with different experiments...............................................................146

Fig 5.9 –BMM predictions of the change in the critical Factor of Safety over the 600s of seepage for a non-homogeneous layered cohesive lysimeter experiment of Chu-Agor, Wilson et al. (2008). Time of observed experimental failure given by line at t=600s............................................................................................................146

Fig. 5.10 Maximum undercut of the Bluff Morphology Model, compared to the conditions used by Chu-Agor et al. (2008).............................................................................148

Fig. 5.11 The change in critical Factor of Safety of the soil profile compared to the maximum undercutting depth (a) and volume of erosion (b) for the three different values of $\phi_b$ as modelled by the BMM.................................................................149

Fig. 5.12 Pore water pressure profile of a sand bank after an initially saturated profile is exposed to 405 seconds of rapid draw down.................................150
Fig. 5.13 Soil Profile of the Factor of Safety at failure. Black particles are those with no Factor of Safety found by the BMM, and the observed failure of Budhu et al. (1996) is shown in black dashed line.

Figure 6.1 Diagram illustrating Input and Output Variables of the BMM.

List of Tables

Table 5.1 Summary of soil parameters from field tests of Chu-Agor et al. (2008).
Acknowledgement

Firstly I would like offer my deepest gratitude to my Director of Studies, Qingping Zou, for her patience, encouragement, drive and commitment to my research. Her tenacity and endurance has spurred me on and developed me to be able to write this thesis. My heartfelt thanks are also extended to Dominic Reeve and Ed Ellis for their consideration and insight towards my research.

I am grateful for the insight and example of many who have seen, commented on, and spurred my work, in particular Robert Dalrymple of Johns Hopkins University, Alan Trenhaile of the University of Windsor, John Loveless of the University of Bristol, Jim Griffiths of Plymouth University, and John Vines and Zhong Peng, formerly of Plymouth University.

I would also like to thank the support of Plymouth University for funding this study and research. In addition, I am grateful to Plymouth University and the Royal Academy of Engineering for the financial support of attending conferences. I am also grateful to the support staff of Plymouth University, in particular Lucy Cheetham, Barbara Fuller and Cathy Hodson.

Finally, I offer my heartfelt thanks to my wife, Kathryn Amy, for her dedication, support and love throughout the duration of my research. Her kindness and encouragement was the bedrock I built upon.
Author’s Declaration

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award without prior agreement of the Graduate Committee.

This study was financed with the aid of the University of Plymouth postgraduate research scholarship. Additional support was provided by the Faculty of Technology.

The author presented an internal seminar to members of the academic staff of the School of Marine Sciences and Engineering, and also attended a number of training seminars run by the Graduate Committee.

Parts of this thesis have been accepted for publications to the Journal of Waterway, Port, Coastal and Ocean Engineering, and the Journal of Geomorphology, and submitted to the Journal of Computers and Geotechnics. In addition, three international conferences were attended and work from this thesis was presented.

Conferences Attended:

- International Conference on Coastal Dynamics, Tokyo, Japan, Sep 7th – 12th, 2009 (Presenting Paper)

Word count of main body of thesis: 37,072

Signed: Date:
Relevant Publications

Journal Papers:


Conference Papers:


Chapter 1 – Introduction

1.1 Background

With significant financial, social and political assets, the coastlines of countries around the world are of critical importance to manage, monitor, and understand. As a direct response to climate change and anticipated sea-level rise, research is constantly striving to improve the predictions of coastal behaviour in years to come. Within this broad stroke of research, it is imperative that the concepts of wave climates and coastal erosion, and the stability and collapse of coastal cliffs and bluffs are well understood.

Into this framework we must project the typical coastal profile, frequently backed by a steep slope of soil or rock. These cliffs, or bluffs, form the bulk of the defence in many parts of the world against the destructive forces of the coastal climate. By better modelling the processes of erosion and collapse of these cliffs and bluffs, we can improve our understanding of the dynamics of this vital asset.

The fundamental process of cliff erosion is driven by the ongoing morphology of the environment. Within the shoreline, different earth materials will respond in wildly different ways to the combined forces of marine erosion, tidal wetting, groundwater flux, and sediment movement. Human interaction into this environment adds another order of complexity in addition. Appending to these primary drivers of coastal morphology, the secondary factors of wind, geological
movement and chemical or biological weathering complete a complicated environment.

Figure 1.1 Coastal Erosion seen threatening a listed WWII coastal structure in Cornwall
Combining the critical nature of these factors into a reasonable and accurate prediction of coastal erosion is of real significance, particularly to protect assets such as the Pillbox seen in Fig. 1.1. As a result of the complexity of the macroscale, some of the models applied to coastal erosion can be grouped together in terms of statistical extrapolation, using historical maps or photographs as a way to gauge the rate of recession. However, this is problematic, not least in the scarcity and resolution of data, and as such severely constricts the usefulness of any such approach (Lee et al. 2001).
An alternative approach then, is to consider an episodic collapse of a cliff to be a probabilistic event, as considered by Hall et al. (2002) in an independent gamma process, or as considered by Lee et al. (2001) where episodic collapse was considered using the discrete method. However, any such probabilistic model relies on data from previous collapses, and provides only a generic rate of failure as a solution. Such a model is incapable of predicting with any accuracy the location, magnitude and cause of any given failure.

In terms of physical process modelling, the research is wide and varied, notably due to the different outcomes and desired predictions. The following section details the physical models and groups them systematically depending on their model outcomes.

**1.2 Coastal Cliff Failures**

When considering the episodic and sudden, catastrophic collapse of a coastal cliff, dune, levee, or breakwater, there is a significant division in research between the experimental data, the models of coastal erosion, and the data of coastal collapse. The physical processes of coastal morphology can be modelled with software to predict rate, change and progression of the coastal profile.

**1.2.1 Short Term Coastal Erosion**

Models of short term beach erosion are either closed loop or open loop models, with the closed loop models assuming that a profile will, if exposed to the same conditions for a long enough period of time, achieve equilibrium. Several of these models have been built and tested, including the following.
SBEACH (Janbu 1957; Morgenstern and Price 1965) is a two dimensional cross-shore model based on the principle that the dissipation of wave energy per unit volume of water is the significant destructive force. It incorporates wave run-up and set-up and gives an accurate (although marginally under-predicted (Sarma 1973)) dune profile. It is similar in both results and methods to the model developed by Kriebel (1982; 1999) called EDUNE. This model allows for constant wave run-up but no wave set-up and cannot simulate variable sediment size or properties.

Another model called Coastal Construction Control Line (CCCL) developed by Chiu and Dean (1984; 1986) uses an exponential folding time scale of 13 hours and includes longshore variability. CCCL does not simulate wave run-up and it is incapable of processing variable sediment properties. Although consistent, CCCL tends to over-predict results (Sarma 1973).

However, these models consider the longer term erosion predictions, whereas storm events offer a new dynamic of coastal morphology, with a combination of wind, rain, and marine elements forcing erosion. Indeed, it is clear from the research that there is investigation into both marine induced erosion (Carter 1988) and erosion caused by rainfall infiltrating the cliff surface, and exfiltrating at the toe of the slope, or simply through runoff (Wainwright 1996). Any attempt to model the whole picture of coastal morphology within storm-events would do well to incorporate both physical elements.
1.2.2 Short Term Cliff Collapse

With regard to the experimental and observational data on short term cliff collapse, there is a breadth of data across a number of geographical, geological, and periodical areas. The most influential of these are summarised below.

Brian Collins and his associates have authored two key papers studying the landslides in the weakly lithified sand cliffs in California. They have collected data over 5 years for a 1.5km stretch of coastline just south of the San Andreas Fault. The first paper (Collins et al. 2006) looks at an empirical prediction of the quaternary marine terrace deposits. They identified that there were two of the main failures present, those were wave-induced, and rain induced failures.

The second of these papers (Collins and Sitar 2007) is based at the same site as his previous studies, examines in more depth the spatial and temporal processes that cause the landsliding in the cliffs. They particularly impress the importance of effective geotechnical mapping; both the horizontal and vertical exposed lithology can be used to determine the prevalent failure mechanism. They also separate the cliff failures depending on the cementation strength within the sands. As there is an approximate correlation between cementation strength and slope angle at failure, a simple on site empirical relationship can be used for landsliding predictions.
To the north of the site used by Collins, a report commissioned by the Oregon Department of Land Conservation and Development looked at the current methods of predicting short term stability of the cliffs. Komar (1987) has concluded that the most significant improvement to this process would be improving “models to specifically assess the susceptibility to erosion [and subsequent failing]”. An example of such a short term stability failure can be seen in Figure 1.2.

Again on the east US coast, a feasibility study of allowing the erosion of marine bluffs to take place in order to bulk up beach size for environmental reasons allowed the investigation of unprotected coastal bluffs, although this is published without any results for the scheme (Johannessen et al. 2007).

Chalk cliffs, inherently weak by nature, are also well researched in terms of periodic landslide activity. Busby and Jackson (2006) studied the cliffs around
Beachy Head, SE England. They studied the electrical anisotropy of the cliffs and observed that before a cliff fall took place there was a large temporal change in the coefficient of anisotropy, which they interpreted to be a reduction in fracture dilatancy. This may allow instrumentation to be set up in order to check the stability and potential impending collapse of the cliff.

In France, Larson et al. (1989) studied the immediate seismic activity of a chalk cliff before its collapse. While the topic itself is not directly related to the research material, the documentation of the collapses are nonetheless appropriate, as is the attempt to predict the failure of the cliff face.

The journal papers concerning dune sites are also very wide reaching in their breadth of subject matter. Although there are many concerning the movement of sand dunes, from Aagaard (2007) studying dunes in Denmark to Anfuso (2007) studying them in SW Spain, the long term movement of the dunes themselves are not deemed close enough to the research topic to be discussed intensely, save to note in passing that these lightly or non-consolidated deposits are liable to large scale movement and morphology due to wind forces as well as those of a more hydrodynamic nature, requiring any model to be three phase. However, under storm conditions, the dune movement caused by the wave action is significantly intensified, and the dunes are often saturated due to elevated waves or rainfall, the result of which is that they behave as coastal bluffs for a short time scale.

Another short term model was developed by Kogure et al. (1997) who looked at modelling the falling of limestone blocks after the development of notches and tension cracks. The analysis correlates well with the results, showing the blocks
are predominantly failing due to toppling after the undercut at the base of the cliff initiates instability.

1.2.3 Causes of Collapse

In the papers above, cliff collapse has been considered as a result of an unstable surface, or a failure plane, within the cliff. In many cases, this collapse is seen as a predominantly two dimensional feature, the failures acting in plane perpendicular to the shoreline, with the edges of any failure mass caused by spatial variability or unpredictable anomalies in the land mass. The instabilities, when considered in a two dimensional framework, capture the critical features of the morphology of the coastline. In order maximise the use of any model created, the cause of these instabilities should be drawn out and considered in turn.

Firstly, instability can be considered prominently as a result of erosion, either by sea, seepage, air, or by human cause. This is primarily due to the undercutting, or notching, of a cliff as this removes areas of resisting soil mass and presents a more unstable slope. Erosion must be considered in any model that deals with morphology of coastal areas, although predictions of erosion may not necessarily need to be through models of the wave climate, as erosion of the cliff itself appears to happen at a stable rate, especially for soils with stronger characteristics (Belov et al. 1999). It is also important to consider that erosion rates are often predicted to be a gradual change of level over time, however, storm-based scour can reduce beach levels on an intensive scale such that a time period of four to five years is required before restoration (Morton et al. 1994).
Secondly, instability is considered as a result of changes in the water table, either through coastal inundation or rainfall events. As a result of this, a two dimensional seepage analysis has a significant impact on the overall stability of a slope. The pore water pressures should include both the saturated soils positive water pressures, and the unsaturated soils negative water pressures, also known as matric suction (Fredlund et al. 1978; Fredlund and Rahardjo 1993).

As a result of these first two considerations, it is important to consider the inclusion of a wave climate model, as the coastal areas are under attack from the hydraulic forces of the sea. This would also better enable the modelling and predictions of the change in stability caused by the rising in sea levels, changes in wave profiles in storms, or the installation of breakwaters, floating or otherwise.

Finally, a failure event can ultimately only be caused by a mobilising force, most commonly of gravitational acceleration, overpowering the resisting force of the soil shear strength, suction and any inertia forces that may be stabilising the slope. As a result of this, the key consideration in any numerical model is an accurate soil model, which can perform stability analysis of single homogeneous, and multiple heterogeneous soils with a variety of strength conditions.

1.2.4 Soil Properties

Throughout this thesis, the concept of a soil is that of a three phase material, with a constant solid particles phase, and a series of interconnected voids. These may be filled with either air or water, or a combination of both. The particles of the model represent areas (two dimensionally) or volumes (three dimensionally) of soils, and are not created to reproduce the individual soil particles themselves.
Each soil used within the model is created using a set of scalar parameters applied to the model particles. As a result of this, each model particle may only represent one soil type, and increasing the heterogeneity of the model will require increasing the resolution of the model.

The scalar parameters applied to each of the model particles represent the mass of the particle, the proportion of particle filled with soil, water, and air, the maximum cohesion and angle of internal friction available for that soil type, the earth pressure, pore water pressure, and permeability. Other factors, including the resolution dimensions, value of water density and gravitational acceleration, are universal constants applied to the entire model.

The shear strength of the soil, a critical function in the analysis of slope stability, is computed through a displacement stepping algorithm, where the soils mobilise the cohesive and frictional strength through small increases in the strain along a path of interest. This allows multiple soils to reach peak strengths at different stages, without compromising the accuracy of the model predictions of the Factor of Safety, that is, the ratio of the maximum possible shear strength, to the shear strength required to stabilise the slope on any particular failure plane.

1.3 Aims and Objectives

The main aim of this thesis is to introduce a numerical, particle based model that can consider the morphology and stability of soft coastal cliffs (bluffs), dunes, and backwaters. This will be achieved through the following objectives:
• To create an accurate model of soil slope stability, through a particle method that is able to consider multiple soils, pore water pressure effects, and 2D water table variations.

• To import a modified version of Smoothed Particle Hydrodynamics into the soil stability model to analyse numerically and reproduce the collapse and equilibrium position of an episodic failure.

• To explore the feasibility and importance of a particle based wave climate model, with a focus on the erosion and engineering of nearshore wave activities.

• To explore the feasibility and importance of a two dimensional seepage model, including negative pore water pressures and its effect on hydraulic conductivity.

• To integrate a successful combination of soil stability, collapse, erosion and seepage into a working model of slope stability.

• To use WCSPH to simulate the pressures and movement of a forced and free object entering and exiting the fluid domain.

The organisation of this thesis is set out as follows. Chapter 2 presents a comparison of potential models in simulating the slope morphology of a coastal zone. It also considers the methodology of the model created for this present work, and expands on the description and methods used. Chapter 3 explores the feasibility of using Weakly Compressible Smoothed Particle Hydrodynamics by considering situations of solid and fluid interaction under high pressure scenarios. Chapter 4 validates and explores the Bluff Morphology Model using the WCSPH module for typical benchmark tests of static water tables and
without considering erosion. Chapter 5 validates the seepage and erosion module within the model, and identifies the critical nature of slope instability caused by a two dimensional seepage and exfiltration erosion. Finally, Chapter 6 concludes the work presented and makes a case for the model in light of the objectives listed above. In addition, it highlights the additional work planned to further improve on the current model.
Chapter 2 – Methodology

2.1 Potential Models

Due to the significant range of models afforded the scientific community, it was important to select the correct type of model in order to maximise the potential gains of the research. This section examines model types in more detail, drawing on the relevant examples, and evaluating their likely effectiveness in the subject area.

2.1.1 Soil Profile and Stability Models

Due to the continuous nature of most intact soils, it is most common for soil profiles and stability models to be analysed using a continuous element based method. Limit Equilibrium (LE) and Finite Element Method (FEM) are the two most common forms of numerical model, and can be traced back as far as 1967 (Duncan 1996).

Limit Equilibrium methods require assumptions of the slip surface locations, which traditionally required a number of iterations. The ever increasing computational power of standard CPUs allows for such iterations to be computed in a very small timescale, and as such Limit Equilibrium methods are still standard practice (Wei et al. 2009).

Using traditional computational methods, Limit Equilibrium split the assumed failure plane into slices, usually vertical, and one of a number of mathematical methods, each with its own assumptions regarding the interaction between the slices is used to compute the stability of the assumed failure plane. An excellent
review of traditional Limit Equilibrium Methods can be found in Nash (1987). These methods are used by numerical models due to the rapid iteration through potential failure planes.

Limit Equilibrium can be extended to a three dimensional model by considering columns, as opposed to slices, as in Lam and Fredlund (1993). The shape of inter-column forces significantly alter the computation depending on the assumptions derived of their direction and/or magnitude. However, Limit Equilibrium Models remain a comprehensive analysis tool to examine the Factor of Safety of soil slopes, assuming a fully elastic soil surface.

Finite Element Methods, in contrast to Limit Equilibrium, require no such assumptions over the location of the slip surface locations, allowing for a more “natural” failure as the mesh distorts under self loading (Griffiths and Lane 1999). However, this does not speed up computational time, as the analysis of every mesh through the slope requires significant computational resources.

Using Finite Element Analysis typically requires a soil failure to deform under perfectly plastic conditions, where the material remains structurally intact, and strains across the failure plane. This requires that only stress-strain states that are stable are reached within the soil mass, which then requires the plastic limit condition to be significantly higher than other models would require (Laouafa and Darve 2002).

In order to improve the velocity field of Finite Element Models, Chen et al. (2003) considered a rigid finite element model, which allows for discontinuities in the velocity field at all inter-element boundaries. This method then allows for
an approach similar to the particle methods discussed below. In addition, the use of rigid elements mitigates the straining of the mesh and the typical continuous mass assumptions of typical Finite Element Methods.

Finite Element Methods are often packaged within many industrial standard Slope Stability models, for example SLOPE/W (Krahn 2004c). A key strength of the FEM models is that they are able to apply multiple soil strengths and water tables to different elements, and thus solve complex soil analysis problems when there is a significant quantity of spatial variability (Karpurapu and Bathurst 1995).

Another key issue regarding the performance of Finite Element Analysis is the Collapse Mechanism. When a soil is shown to fail, there is a numerical instability within the FE model, as the movement between the mobilised, shearing soil mass slides across a small, but finite, thickness of material known as a shear band, or failure plane. This thin band and highly differential movement causes significant problems for Eulerian computational analysis (Pamin and De Borst 1995), and mesh regeneration is a computationally intensive method to avoid numerical diffusion or total model failure (Markauskas et al. 2005).

Finite Element Models have been successfully integrated with other methods of modelling, as hybrid models. These offer a significant advantage over pure Finite Element Models, as they can mitigate some of the drawback of FEM by using another approach. Coupling with a Boundary Element Method (Kamalian et al. 2006) allows for the soil to be meshed only using the boundary elements.
This reduces computational resources, and thus allows a significantly higher magnitude of detail to be analysed at the soil surface.

FEM has also been coupled with a Shear Strength Reduction technique by Nian et al. (2010) to discover the effect of seepage on a two dimensional slope stability. This method, whilst capable of considering the evolution of seepage and stability over time still suffers from the restriction of applying the strength reduction through a mesh based analysis. This leads to unphysical collapse predictions, and irregular phreatic surface calculations.

Another method of modelling slope failure is the Distinct Element Method (DEM), after Cundall and Strack (1979). Distinct Element Method is a discrete, Lagrangian method that offers prediction of collapse of soil mass. This is achieved through modelling the soil mass as an assembly of contacting discs, or particles. This offers a freedom from the restraints of a mesh, and theoretically allows for a fully collapsible soil domain.

Distinct Element Method has been used to model the deformation of a soil under extreme failure, or differential loading successfully. It is known to model with accuracy the discontinuous nature of the soil mass, which is of particular significance as it is a dominant factor in the behaviour of failed soils (Tanaka et al. 2000).

Due to the nature of the distinct elements, DEM is significantly more accurate when applied to modelling movement or deformation within a granular, or non-cohesive soil profiles (Boettcher 1999) than when it models the cohesive soils such as clay and silt. This is due to the interaction between the elements
confined to modelling the normal forces only, without the shearing resistance required to accurately model a cohesive soil.

Having considered two distinct element methods, it can be summarised that the continuous Finite Element Method offers a significant advantage when considering the overall stability of a soil slope. This method can offer a fairly comprehensive description of the stability distribution across a slope, and can be used to consider the stability of a variety of soil strengths and non-homogeneous soil profiles.

However, Finite Element Method fails to fully model the collapse and plastic strain of the soil model, without unphysical movement across the failure plane. The Distinct Element Method offers a significantly better reproduction of a soil collapse, especially for a non-cohesive frictional soil, however this fails to offer the stability prediction of the Finite Element Method.

Although other Eulerian methods of soil stability and collapse exist, many of them fail to improve on the issues presented by the research on Finite Element Methods, resulting in unphysical predictions of collapse. In addition to this, history dependent materials are highly problematic to model successfully within a Eulerian method. As a result of this, there is a clear need to incorporate a Lagrangian method within any model developed for this research.

A number of other Lagrangian methods have been tested in the research literature. Finite Difference based Fast Lagrangian Analysis of Continua (FLAC) is capable of resolving soil domains under large strain using low-order elements
(Cundall 1988). However, using this method requires a high density resolution, as strain or distortion within an element is not accurately computed.

Finite Difference Method has also been developed into a Lagrangian Particle Finite Difference Method (LPFDM), which projects the Fast Lagrangian Analysis of Continua formulations onto a Lagrangian Particle Method (LPM). This offers the consistency of a fixed Eulerian mesh, which is used to solve the equations of motion of the material, and the Lagrangian Particle Method, which maintains the material history and thus any strength requirements. This is a highly computationally intensive method that allows for significant strain and distortion within the modelled domain. However, this can result in some unphysical voids appearing on the planes of movement (Konagai and Johansson 2001).

Using an entirely meshless approach to modelling fluid movement, Smoothed Particle Hydrodynamics was adapted to model fluid flow by Monaghan (1994), and has since been used to study a wide range of solid and fluid flow. The mesh-free method of numerical modelling allows objects of high shear to be modelled with little numerical instability. When applied to a soil model, SPH allows for an accurate and stable method to model large deformations (Bui et al. 2008). This method uses the SPH framework to model the collapse and movement of unstable soil material. However, in this case, there is no attempt to map the stability or potential failure planes within the profile. Aside from the work of these authors, there is not a significant quantity of work applying the use of SPH to a soil domain; however it has been used to successfully model a wide range of other phenomena.
Many traditional slope stability models are based on a traditional mathematical model of circular arc or parallel linear slip analyses. These models are usually based on a homogenous soil profile and use basic analysis of soil geometries using a continuous mathematical representation of the slope on a spreadsheet, or similar. These mathematical models are usually based on the Swedish (Fellenius) method of slices, or a derivative of this method (Donald and Chen 1997) to examine the stability of the slope in question. Error margins of this method typically range from 5-20\% (Craig 1974) depending on the bluff conditions. This method consistently over predicts the disturbing force, thus producing a conservative estimate of slope stability. Non-circular failure slopes are conventionally analysed using a multiple wedge method (Donald and Chen 1999; McCombie 2009). These mathematical models of slope stability can be applied into a particle or element based model, if programmed correctly.

A number of models also look to predict the movement of failed soil sections when they can be modelled as debris flows. These models tend to consider the flow of soil to be dominantly a Non-Newtonian fluid behaviour, and can translate shear resistance of the solid particles into a viscosity within the models. In order to account for the stability of these soils, a yield stress is required before flow begins, and these are described as Bingham fluid. Models of these include that developed by Tang et al. (2010) and can be used to model some earth flows including mudslides (Abadie et al. 2010).

The Bingham fluid model has been expanded to consider the effect of some coarse particles held within the fluid mixture, and thus considering a two phase frictional material (Shao et al. 2002). These models can be applied to the
analysis of impact of debris flows (Roth et al. 2010), however they are constrained by the properties of the flow behaviour, and cannot be applied to all soil material failures.

There are also a number of short term coastal morphology models available that are based on equilibrium profiles of the beach. These include SBEACH (Larson and Kraus 1989; Larson et al. 1989), EDUNE (Kriebel and Dean 1985; Kriebel 1986) and CCCL (Chiu and Dean 1984; Chiu and Dean 1986). These models tend to consider either an equilibrium profile or an avalanching failure, where slopes steeper than the angle of internal friction redistribute, and as a result of this, are not considered suitable to meet the objectives of this thesis.

2.1.2 Hydraulic Erosion Models

When considering the total coastal environment, a model must be capable of predicting the erosion by wave climate. There are a number of potential models that are currently used for these, and these are discussed here in brief.

Wave induced erosion, when considering a specific area of interest, has been examined by Ruggiero et al. (2001) for the Oregon coast. In the paper, the erosion over a specific height is examined for extreme tidal conditions. Although not strictly a numerical model, the approach used gives a valid empirical model data set for extreme tidal conditions.

Analytically, a wave impact model has been developed by Larson et al. (2004). This model is used to predict erosion and recession of dunes under impact of large waves during severe storms. The model variables are run-up height and an empirical transport coefficient. The data shows adequate two dimensional
prediction of storm erosion, but is limited to cases of erosion dominated by wave impact, and ignores other hydrological factors.

XBeach (eXtreme Beach behaviour model) has been designed as a Lagrangian model of sediment movement which models the shallow water equations into a Generalized Lagrangian Mean (GLM) formulation (Walstra et al. 2000). This results in a robust physics-based model that is specifically designed for inundation and hurricane modelling (Roelvink et al. 2006).

In addition to this, the WCSPH model has been extended to include modelling of erosion within a breaking wave environment (Zou 2007). This allows for soil erosion and suspension in the fluid domain, and is achieved by applying scalar parameters to the fluid domains of soil suspension, as the soil particles are at this stage very small in size compared to the size of the domain.

Sediment transport within this model is divided into bed and suspended load. Bed load is limited to within a distance of twice the sediment diameter to the bed itself, and is driven mainly by skin friction or by bed shear stress. Suspended load occurs throughout the fluid domain and is driven by the turbulence within the fluid body. This method also applies a constant fall velocity over all the sediment, and in doing so can solve sediment diffusion problems (Zou 2007).

2.1.3 Seepage Models

Modelling two dimensional seepage within a soil stability framework has received much attention over recent years. Flow through a porous media has been considered by Zhu et al. (1999) where SPH has been modified to flow
through porous media by using solid particles within a unit cell to obstruct the fluid flow.

Using FEM, Gasmo et al. (2000) investigated the effect and location of infiltration on slope stability. This model included unsaturated flow and the respective permeability changes within the soil domain. However, these functions did not represent field conditions accurately, and thus the model could not quantify infiltration quantities or rates, which was further diffused by using a Eulerian method of modelling.

Also using FEM, SEEP/W (Krahn 2004a) was built as an add-on to the SLOPE/W model. This model also considers variable permeability, or hydraulic conductivity, to vary with the negative pore water pressure, or matric suction, within the soil. This model applies an iterative solver to Darcy’s Law of seepage, and does not consider moisture leaving the soil domain as vapour. It is also Eulerian in its approach, and as such is sensitive to the meshing and boundary conditions applied to the numerical model (Krahn 2004a).

Recently, a mixed Lagrangian-Eulerian model has been developed to model the movement of seepage in mixed soil domains (Jeyisanker and Gunaratne 2009). This models a three dimensional non-homogeneous soil profile by a particulate model, using discrete elements in a staggered, structured grid, and solving the Navier-Stokes equations for particle motion. This allows for rapid changes in properties, in this case the compaction of the soil, and seepage forces are well predicted. At this stage, however, there is no prediction of erosion forces or a model of erosion in place within the model. In addition, due to the choice of
equation used, the accuracy of the model is somewhat compromised when considering very low permeability soils.

2.1.4 Novelty of SPH approach

Due to the order of magnitude of the differential movement within a soil domain that can occur at the time of a cliff collapse, it is believed that using a mesh-based numerical model will cause significant straining and distortion of the mesh in the event of failure. Using a mesh-free method allows the user to model the complete cycle of coastal bluff dynamics, from instability to collapse.

At this stage in computational development, the use of a particle method places a significant demand on the computation of the model runs. As a result, care must be taken to both minimise the demand on the processor by way of simplification and efficiency savings, as well as a careful control of the potential scope of the project. It is for these reasons that a two dimensional model is chosen for this research. This allows for an order of magnitude savings on the intensity of the model without a significant reduction in the modelling accuracy, as the coastal morphology is often considered a constant in the direction of the shore, as discussed in Section 1.2.3.

Considering the expressed need for a mesh free model, there is consensus in the research discussion in section 2.1.1 that a particle method offers the best method of producing a Lagrangian analysis of the soil domain of the coastal model domain. This allows for extreme collapse to be predicted and measured, as well as an accurate history tracking of the soil types and particle history.
Considering the choice of particle domain, it is clear from the literature that the Smoothed Particle Hydrodynamics model offers a comprehensive method of solving fluid flows, and due to its adaptation to solid mechanics modelling, it is envisaged that a soil adaptation would be achievable and offer considerable benefits. The source code for Weakly Compressible Smoothed Particle Hydrodynamics (WCSPH) is also available under a GNU General Public Licence, ensuring easy access and availability to this model.

However, considering the numerical intensity of this model type, it is not deemed appropriate to consider an entire soil domain using the WCSPH technique. An additional technique is required to ascertain the stability of the model, before any instabilities can be modelled using WCSPH. Due to the potential failure shapes within a two dimensional coastal model, a multiple wedge analysis method has been selected as the best means of determining the potential instability within the model. Applying this technique to a particle-based model should allow the optimum compromise between the detail of results and the resources required for any analysis.

In addition to this, the model must include a computational analysis of seepage through a two dimensional application of Darcy’s Law. This could be achieved through a fixed neighbour interaction, or a function based on the proximity of particles. This function choice will depend on the computational requirements and availabilities for each method.

A final part of the model will be to include erosion as a numerical model. This should be calculated both through inherent instability and pore water pressure
forcing within the soil, but also as a wave driven erosion, or as input from a standalone wave erosion model. This will be accessible using a particle domain to model the soil, as eroded particles can simply be removed from the model with no need to remesh. Incorporating an SPH erosion model into the soil stability model will be pursued if viable.

2.2 Previous SPH models

Within this section the current art state of the models in question is discussed, and the history of the model development expanded. This is taken for the three phases of the potential model, the fluid movement, the solid object its interactions with the fluid domain, and finally the soil phase.

2.2.1 Smoothed Particle Hydrodynamics applied to Fluids

Smoothed Particle Hydrodynamics was initially developed for modelling and studying astrophysics within vacuums (Gingold and Monaghan 1977; Lucy 1977), a comprehensive discussion of which can be found in Monaghan (1992), showing the diverse applications and robust details of the model type. This model has since been adapted to model and reproduce free surface flows by Monaghan (1994). At the time, this revolutionary approach of grid free Lagrangian modelling of a fluid allowed a significant advancement of the study into fluid concepts. The results of this paper presented the case for the ease of tracking a free surface using this technique compared with other grid based Eulerian methods. The mathematics of this model is complex and highly varied since its inception, and so an overview is presented here.
Smoothed Particle Hydrodynamics is based on the principle of the Navier-Stokes equations being applied to the particle interactions as they flow across the fluid domain. This free movement across the domain is controlled by a smoothing kernel that applies a weighted averaging to the collisions of particles based on their proximity to one another. This allows for particle movement to continue freely across the computational domain.

Each particle within the numerical model is assigned a series of scalar parameters, including mass, pressure, velocity components, location, and so on. This scalar data set \( f(x) \) is then interpreted at each timestep after Dalrymple and Rogers (2006).

\[
f(x_a) = \sum_j f_j W(x_a - x_j)V_j \tag{2.2.1}
\]

In this case, \( V_j \) is the volume of the particle \( j \), which is a function of its density. The mass of any particle within the SPH method stays constant, ensuring conservation of mass provided no particle is removed from the computational domain. The smoothing function, or kernel, here is denoted by \( W(x_a - x_j) \), and can be chosen from a number of different shapes including quadratic and cubic. This works as a weighted smoothing function, allowing particles close in proximity to have a greater affect on the value of \( f(x) \). Universal properties, such as gravitational acceleration, are applied following this summation.

Due to the sum across all particles in Equation 2.2.1, it is noted that the particles across the entire domain theoretically interact with every other. In order to increase the computational efficiency of the model, this interaction is restricted to particles within a distance of \( 2h \) from each other, where \( h \) is a user-specified
value relative to the initial particle resolution. Due to the diminishing value of the smoothing function, this curtailment of interactions allows for a more efficient model without significantly affecting the accuracy (Monaghan 1994).

Ensuring conservation of mass and momentum, the changes in the velocity and density of a particle are computed using the following formulas applied to particle $a$ after Monaghan (1994):

\[
\frac{dv_a}{dt} = -\sum_j m_j \left( \frac{p_a}{\rho_a^2} + \frac{p_j}{\rho_j^2} + \Pi_{aj} \right) \nabla_a W_{aj} + g
\]  

(2.2.2)

\[
\frac{d\rho_a}{dt} = \sum_j m_j (v_a - v_j) \cdot \nabla_a W_{aj}
\]  

(2.2.3)

Where $j$ is all other particles within the radius of $2h$ from the particle $a$. The parameters of particle $a$ and $j$ are denoted by the suffix on the variable parameters, of which $p$ is the pressure, $v$ the velocity, $m$ the mass and $\rho$ the density. $\Pi_{aj}$ is an empirical approximation of the viscosity effects (Monaghan 1994), and $W_{aj}$ is the kernel function chosen.

Given that the mass of any particle remains constant through the simulation, the conservation of mass is ensured if no particles enter or leave the computational domain. There are two distinct SPH adaptations when considering the density of the fluid. In Weakly Compressible SPH, discussed here, the density of all particles is considered a variable, and the volume therefore changes within the fluid domain. This departure from normal practice prevents the density of particles near the free surface tending towards zero and inducing inaccuracies into the calculation. In addition to this, the variation within the fluid density is small and typically is 1% or less (Dalrymple and Rogers 2006).
Using the equation (2.2.3) allows the model to compute all time dependent variables in one time step, using only one particle interaction sweep, whereas without a weakly compressible fluid whose density is approximated, there is a single iteration to calculate the density, then an additional iteration is required to calculate the change of velocity of the particles (Monaghan 1994).

Due to the mesh-free nature of the computational method, there is a possibility that the particles could end up overlapping and occupying a similar space. In order to prevent this from occurring, Monaghan (1989) developed the XSPH correction which acts as a velocity averaging profile over particles in close proximity. This ensures their velocity is similar and prevents particles from becoming separated by a distance so short that the kernel function applies an unphysical weighting to the particles movement calculations. Although this method is not strictly physical, the viscosity of a fluid acts in a similar manner, and inaccuracies are negligible (Monaghan 1989).

A number of authors (Dyka and Ingel 1995; Dyka et al. 1997; Monaghan 2000) have considered approaches to correct the instability in the SPH method under tensile forces. This happens as a result of a tensile force occurring between particles that are stretched apart and the particles end up attracting each other. Although more significant in problems of solid mechanics, tensile instability can be seen in highly turbulent fluid and gas flows (Monaghan 2000). A small repulsive term considered between SPH particles is introduced to resist the particle clumping that forms through tensile instability. This term has now been incorporated into the kernel function and allows for modelling of turbulent flows with significantly less physical and numerical fragmentation between particles.
Containing a fluid domain of mesh free particles is a set of particles acting as a boundary condition for the fluids. There are a number of competing boundary methods used within the WCSPH model. Monaghan (1994) applies a purely repulsive force, the magnitude of which is set by the proximity of the fluid particle to the boundary wall, and is based on the initial spacing between the particles. This method therefore ensures that particles at the boundary have a zero normal component of velocity, and pressure forces when they move within \( r \), the original spacing of the model. A similar method is to use solid particles fixed in space whose response is similar to the fluid particles within the domain (Gómez-Gesteira and Dalrymple 2004). This also exerts a repulsive force against particles that move within \( 2h \) of the boundary particle.

Another method of computing the boundary conditions is to use image particles that are created to mirror the physical fluid particles exactly, so that solid boundary conditions are strictly satisfied (Cummins and Rudman 1999).

In order to model the turbulence and complexities smaller than the size of the particle, the Sub-Particle Scaling (SPS) for compressible fluids was introduced by Gotoh et al. (2001) and used, for example, by Dalrymple and Rogers (2006). This is achieved using Favre-averaging as this does not introduce extra terms into the conservation of mass equation, and is achieved using a flat-top spatial filter.

Another key development within WSPH is that of the Shephard Filter. This is used as the application of the Large Eddy Simulation (LES) description of viscosity effects can lead to unphysical undulating surfaces. This is caused by the density variations between particles being magnified through a slightly
positive feedback loop in the equation of state. Therefore, a filtering is applied across the particles every 40 time steps such that:

\[
\rho_i = \frac{\sum_j \rho_j w_{ij} V_j}{\sum_j W_{ij} V_j}
\]  

(2.2.4)

This method is applied to particles within the smoothing length \((2h)\) of particle \(j\).

This application still allows the WCSPH model to capture the highly non-linear hydrodynamic processes such as wave impact, overturning wave fronts, and jet formation.

Although often computed as a purely single phase model, using SPH particles to also model the air within an experimental domain has shown significant improvements on the free surface and pressure predictions. Colagrossi and Landrini (2003) considered air as another fluid within the numerical domain, and developed a method of modelling interfacial flows within the SPH method. This allows predictions of movement of air bubbles within the fluid, as well as a more robust and well defined free surface prediction. However, this additional accuracy requires significantly greater computational resources, and also the additional number of particles for the same resolution decreases model stability.

Smoothed Particle Hydrodynamics has also been proven to work for non uniform model resolutions. This method, as published by (Oger et al. 2006), allows for a smaller particle size in the areas required for a larger level of detail, with larger particles being used for areas where less interest, or less complex hydraulic features, are needed. For the cases published, of wedge drop into fluid, the key areas of study are easily identified and thus the spatial resolution function can be defined before the simulation begins. Although this method provides a high level
of detail and minimises the computational requirements of the model, it is significantly harder to implement when areas interest are not known before, or change during the simulation.

As a result of this, and to combat the ever increasing complexity of the models being used, SPH has been developed as a parallel source code, suitable for multi-core processing (Zou 2007; Ferrari et al. 2009). The communication times between cores are low enough if the domain is shared well between processors. This parallel SPH has been adapted further to consider a massively parallel SPH code that can be run on the GPU (Graphics Processing Unit) at significantly faster ratios than even parallel SPH (Hérault et al. 2010). At the time of writing, this model is in beta-testing but is showing significant possibility in reducing the numerical burden of SPH modelling.

Weakly Compressible SPH has been chosen as the subject of this model over the similar model Incompressible Smoothed Particle Hydrodynamics (ISPH) (Cummins and Rudman 1999; Shao and Lo 2003). Although ISPH is not diluting the pressure component of the model through compressibility, this typically only accounts for a 1% change in density (Dalrymple and Rogers 2006), and the two methods offer comparable results with similar run times. Although ISPH allows longer time steps across the model, the computation takes longer for each time step. WCSPH allows a higher resolution than ISPH for any given memory size (Colagrossi and Landrini 2003). ISPH also can suffer from instabilities and random noise disturbance (Xu et al. 2009).
The application of WCSPH as a mesh free method of modelling fluid flows has allowed for a set of numerical results that explore a number of features aided by the Lagrangian nature of the approach. Wave breaking, as explored by Dalrymple and Knio (2001) initiated a number of in depth studies of the mechanics of wave overturning, breaking, mixing, and other key wave dominated fluid processes (Rogers and Dalrymple 2004; Dalrymple and Rogers 2006; Dalrymple 2007). The analysis of particle position and ability of the method to consider divergent, convergent, and separating flows allows for a detailed consideration of wave dynamics.

Considering this, WCSPH has also been applied to the case of green water overtopping of a structure (Gómez-Gesteira et al. 2005). The separation and merging of flow is considered well using a particle model and the “rooster tail” jets are well predicted. Wave overtopping has also been considered using the ISPH method (Shao et al. 2006), where in both cases the Lagrangian method offers a significant improvement of capture of the free surface for these cases.

Some consideration has already been given the erosion caused by breaking waves using the WCSPH method (Zou 2007). This model could be used with the aims of the thesis in the prediction of the erosion of the bluff, dune, or cliff front. The application of a particle method to erosion calculations makes the boundary movement computationally easier to define and thus model.

The sediment transport model used by Zou (2007) consists of a sediment concentration term being given to each particle, with a constant fall velocity applied to all sediment. This suspended load then occurs throughout the fluid.
domain and is driven by the turbulence within the fluid body. Change of bed elevation is calculated using the following equation:

\[
(1 - n) \frac{\partial z_b}{\partial t} = D - E \quad (2.2.5)
\]

Where \( z_b \) is the bed elevation, \( n \) the porosity of the sediment, and \( D \) and \( E \) are the deposition and erosion fluxes respectively at the fluid-sediment boundary. Therefore this model allows erosion and deposition of the soil by elevating or lowering the boundary particles to accommodate the flux of sediment.

Although often considered a weaker facet of the WCSPH models, the pressure prediction for impact of fluids on solid structures (Gómez-Gesteira and Dalrymple 2004), as well as the impact of a solid movement on the fluid (Monaghan et al. 2003) has been considered with Smoothed Particle Hydrodynamics. In both these cases, the compressibility within the fluid and associated changed in density are not significant enough to alter the predictions of the fluid response and the pressure field within the fluid. Since these papers, more developments on the pressure evaluation within SPH have been considered and published (Molteni and Colagrossi 2009).

The interaction of the fluid with other objects has been considered in terms of the fluid flowing across a stationary object in a number of cases, for example (Colagrossi et al. 2007; Lee et al. 2008). In these cases, aspects of fluid flow including vortex shedding, pressure contours, and free surface tracking are considered within the domain of fluid interaction with fixed objects.
SPH has previously been considered as part of a hybrid model to consider phenomena that could be too computationally intensive to predict as a single type model. It has been coupled with wave generation-propagation models, such as SWAN (Crespo et al. 2008a) in order to consider the detailed nearshore fluid state as a result of wave predictions from a larger geographical area. It has also been coupled with an FEM model to consider violent interactions between fluid and solid but deformable domains (Fourey et al. 2010). These examples show the potential for SPH models to be included within other numerical methods and models.

Considering storm surge on a beach profile has not been extensively modelled using an SPH technique. However, similar fluid properties are examined when modelling dam break behaviour over dry and wet beds, as examined by (Crespo et al. 2008b). This paper shows the dissipation mechanisms of wave breaking and bottom friction are reproduced well, and the predicted fluid velocities fit well with the experimental results.

This section has considered the basic equations and mechanisms of Weakly Compressible Smoothed Particle Hydrodynamics, as well as the current art state. The main developments of the Lagrangian method have been summarised along with key equations including conservation of momentum and mass, and the application of the kernel, or smoothing function. The lack of diffusion of densities at the free surface caused by the artificial compression of the fluid allows for accurate tracking of the free surface with only minor discrepancies across the density of the fluid.
The key considerations of WCSPH have also been considered, including the different boundary conditions, Sub-Particle Scaling (SPS), the Shephard Filter, and spatial velocity averaging (XSPH). These features are all included into the WCSPH source code (v1.0) used for the duration of this PhD work, which was accessed from the SPHysics website (SPHYSICS 2007).

This section has also given a brief overview of the art state of SPH modelling applied to the fluid domain and fluid properties. The key applications of fluid SPH modelling that are relevant to the aims and objectives of the thesis are stated and summarised.

### 2.2.2 SPH applied to Solids and Fluid-Solid Interactions

SPH has also been used for a wide range of numerical models for the application of solid objects in terms of fracture and fluid response. A number of these are described in this section. Using SPH for the modelling of a brittle solid, for example, allows for accurate modelling of propagation of cracks, impacts and cratering (Benz and Asphaug 1995). The lack of mesh or continuous fabric used within the modelling of a material allows for breakup of the material to be modelled without any diffusion at the edge boundaries, allowing for accurate modelling of material cutting, for example in machine milling (Limido et al. 2007). In order to do this, the SPH method is adapted to add a shear strength component within the momentum equation that allows for the breakdown of material once the peak shear stress is reached, and hence the fracture propagation across the solid domain. In addition to this, the use of a particle model allows each particle history to be uniquely tracked and stored, which optimised the processes of finding material that is beyond peak strength. This particle history
process is also used in the modelling of injection moulding, where fluid movement is non-Newtonian and solidifies based on temperature. The adaptation of SPH to such a property has been documented by Fan et al. (2010), where the mesh free nature of SPH optimises the modelling of complex solid to fluid interactions.

When considering the interface between the fluid and the solid domain, SPH has been used to simulate the erosion of non-cohesive bed sediments. Manenti et al. (2009) considered the erosion of a cohesionless soil through a multiphase SPH model. These particles, when suspended in the water column are considered as a viscosity increase of the original fluid. Diffusion within suspended particles of soil is alluded to as two phase flow problem which is considered through Navier Stokes and continuity equations.

Soil particles that are not suspended are excluded from the SPH calculations of the fluid domain, and a pressure is applied from a lithostatic condition with the pressure of the soil particles above applied additionally. The sediment layer is assumed saturated, and the net vertical pressure, or effective stress, on the soil particles is considered as the difference of the total pressure and the uplifting fluid pressure. The movement of a solid soil particle is applied once the lateral hydrodynamic frictional stresses overcome the friction of the soil surface. In this model, once a particle is moved from its position on the soil bed, it is considered suspended in the fluid domain (Manenti et al. 2009). This model achieves reasonable convergence with experimental data, however suffers from a number of assumptions including the pressure distribution within the soil bed, the
application of internal friction angle over a particle length, and the lack of lateral pressure forcing on soil erosion.

A similar approach of scour was made by Ulrich and Rung (2010). In this method, the interaction between the fluid layer and the soil layer is confined to a viscous fluid layer between these two regions. This layer varies in thickness and location depending on the concentration of soil in fluid suspension. The soil here is modelled as a highly viscous fluid whose viscosity depends on the cohesion and internal frictional strength of the soil. As a result of this methodology of modelling, the soil will creep and continue to move after the equilibrium position has been reached. This method therefore requires high values of viscosity, and therefore is significantly less suitable for soils of low strength. However, the interaction between fluid and soil domain, namely the erosion of soil due to high velocity fluid flows, are well modelled provided the choice of particle size is adequate for the scenario in question.

When considering the fluid response on solid objects, there have been a number of studies considering the SPH response to a fixed velocity solid object entry. Gong et al. (2009) consider a two dimensional wedge entering a calm water at a variety of constant speeds. The pressure response to the impact is evaluated and a non-reflective boundary layer is applied to suppress the reflection of the sound wave. This offers some accuracy improvements to the pressure distribution within the fluid domain but does not change the response of the free surface to the impact.
These authors also considered the case of a circular disc entering the calm fluid, and in this case considered a cylindrical co-ordinate set to better match the deformation of the fluid under impact (Gong and Liu 2009). The response of the fluid including the closure of the air cavity is examined and reproduced, though the three dimensional modelling requires a significant increase in computation and the subsequent drop of resolution.

Wedge entry modelling with SPH using complex initial particle spacing with a varying resolution has been considered by Oger et al. (2006) which considered at different angles of attack of the wedge and the resulting jet formation and fluid pressure response.

A variable resolution was also applied to the case of a heaving cone in initially still water was considered by Omidvar et al. (2011). Here the peak pressure on the cone was replicated well using Weakly Compressible SPH code with lateral non-reflective boundary layers but no conditions applied to the bottom boundary.

Despite the research on forced object entry, there is far less research considering the object exit from a fluid. Considering the rise of a fluid of smaller density (Grenier et al. 2009) applies many of the principles of object exit from a fluid, but is highly deformable and applies different physical processes to the surface piercing section of the model. Aside from this, there is little published material on objects rising through fluids with Weakly Compressible SPH.

Modelling floating objects using the Incompressible SPH has been documented by Shao (2009). In this thesis, the concept of free-falling objects is modelled to predict slamming forces and pressure distribution of the fluid and the response of
the object to this fluid force. Different model resolutions were used to consider the application of these ISPH to these complex scenarios.

This section has considered the state of the art SPH considering the application and interaction of fluid and solid particles. Scour, erosion and floating objects have all been considered and typical published results considered and evaluated.

2.2.3 SPH applied to Soil Movements

Using Smoothed Particle Hydrodynamics to model soils is a convenient application of Lagrangian modelling techniques, allowing for storage of particle history, strength, and other scalar parameters to each model. However, it is a highly computationally intensive model and in some cases previously discussed, the application modelling of soil using SPH as a viscous fluid results in more equilibrium position predictions due to creep and material flow. The tensile instability (Monaghan 2000), as discussed previous sections, is a more significant problem with a soil due to the internal friction and shear strength at zero strain (apparent cohesion). As a result of this material property, the final equilibrium position of the modelled material may not be a horizontal level, which can result in numerical in instability in certain cases.

The catastrophic collapse of a soil profile using SPH has been considered by Bui et al. (2008). This approach uses standard WCSPH and applies the Drucker-Prager model, where the strain conditions are always plane and the material functions as either perfectly elastic or perfectly plastic. This methodology differs from the alternative Mohr-Coulomb model particularly concerning the application of the tensile stresses.
The SPH model used was applied only to cases of catastrophic collapses, where the movement of the soil was entirely considered through the adapted SPH equations. As a result of this, a significant adaptation to the artificial stress method was required to overcome the tensile instability, particularly for a non-cohesive soil of high strength. When considering a cohesive soil, the tensile instability caused computational failure and was overcome by using an artificial stress method that is significantly different to typical physical simulations with SPH. As a result of this, the results of the computations of cohesive strength soils were less representative of the experimental data than that of the non-cohesive soils models.

This model also considered the strain through the soil, allowing a significantly detailed consideration of strain and shear bands. This application of SPH allows for detailed analysis over areas of high relative displacement, however the strain within the particles was not stored and therefore the soil did not behave as a perfectly elastic-plastic material over an entire model duration.

SPH has also been considered when modelling the soil deformation under high deformation scenarios, such as the Cone Penetrometer Test (CPT). Kulak and Bojanowski (2011) considered a hybrid SPH model where the SPH particles were placed under the cone. This was found to predict the resistance force of the soil on the cone well, with soil density being the most sensitive parameter to the prediction of force. In this case, the SPH was confined to a very small radius under the cone in order to conserve computational resources, and the interface between SPH and the hexagonal solid Lagrangian model presented some distortion of strain.
It is clear from this section that although Smoothed Particle Hydrodynamics offers a robust method of modelling soil movements, strain and related phenomena, one of the reasons for the lack of published research in this area is due to the computational intensity of the soils, as well as the diffusion of results when a hybrid model is used. Therefore, a fully particle model which considers the stability of the soil slope separately from the post failure movement and equilibrium position should be an optimised method to pursue this phenomena and meet the aims and objectives of this PhD thesis.

2.3 Present SPH Model Improvements

In this section the summaries of the improvements to the computational model are listed and explained. This section showcases the contribution to knowledge of this PhD project and allows the reader to grasp in more detail the work done to produce the figures within chapters 3, 4 and 5.

2.3.1 Water Entry and Exit of Floating Object

This section explains the adaptations to the WCSPH source code that enabled the work in this PhD thesis Chapter 3, which was used to further the understanding and application of WCSPH before applying it to the desired subject area of coastal morphology.

Initially, a development was made to the source code to allow the generation of fluid and boundary particles at more specific and user defined co-ordinates. This allowed for the creation of fixed solid objects of more complex geometry than the options available to the source code release of the data generator program.
Having achieved this particle placement of fluid and boundary particles, a model was written for the behaviour of a floating object. This allowed the modelling of fluid response to a forced obstacle movement – e.g. the plunging of an object at a constant velocity. In addition to this, the model allowed for the feedback from the fluid domain to influence the movement of the obstacle, a freely floating object. This was achieved by the following set of equations:

\[ F_x = \sum_j p_j a_j * N x_j * W_{aj} \]  \hspace{1cm} (2.3.1)

\[ F_z = \sum_j (p_j a_j * N z_j * W_{aj}) + g * m_{obs} \]  \hspace{1cm} (2.3.2)

In effect, the resultant force in either of the two orthogonal directions can be computed through an integration of the pressures of fluid particles around the object, which are resolved with respect to the surface normals of the object. This method considers the fluid pressures using the same kernel function \( W_{aj} \) as the fluid domain, where the distance between particles is taken as the distance between the fluid particle and the nearest obstacle particle. For the vertical direction, using the mass of the object as a constant allowed the floating object to be modelled as a hollow set of particles, reducing the array size needed for any given simulation.

Rotation can be computed by applying each pressure component as a moment around the centroid of the object, a user defined co-ordinate set. This moment is then applied to the object about the centroid. However, this degree of freedom can be restrained within the model if necessary, or, in the case of circular objects, to reduce the computational requirements of the model.
When considering the movement of a solid object within the fluid domain, the fluid movement is significantly more dynamic and variable with time than the movement of a solid object, particularly when the object is high in mass and subsequently inertia. As a result of this, it was found that minimal adverse effects on the outcome of the model resulted from considering the object on a different time step than the fluid domain. Therefore, the movement of the obstacle is expressed in its own time step:

\[
\frac{dv_{obs}}{dt_{obs}} = -\frac{F_{obs}}{m_{obs}}
\]  

(2.3.3)

Depending on the mass of the obstacle, the model was found to stay consistent and accurate with time steps of up to twenty times the fluid time step. This allowed some computational efficiency to be made and also appeared to minimise some resonance between the two phases.

The soil erosion method of (Zou 2007), as discussed in Section 2.2.1, has been adapted to allow for large values of erosion. As the erosion or deposition is displayed as a change in the elevation of the boundary particles, a large differential erosion condition can produce a large gap in the boundary conditions, as the x co-ordinate of the particle remains fixed throughout the model duration.

In order to combat this cause of potential failure within the model, the boundary particles are re-staggered every 100 time steps, where necessary. This involves ensuring that no two boundary particles are further than the initial particle spacing away from each other, and therefore creates boundary particles when the gaps are too large.
Unfortunately this subsequently adds to the computational demand of the model, as this effectively operates as a re-meshing of the boundary. In addition to this, it can produce a numerically ‘smoothed’ profile of the bed. However, for the duration of model runs that are realistic within the scope of this thesis, these are not significant sources of computational error.

2.3.2 Soil Stability

Due to the investigation into current literature and the reasons stated in Section 2.1 and 2.2, it was decided to use a particle method to model the coastal slope environment. Using only WCSPH for modelling the soil erosion and movement would be computationally intensive and potentially unstable when considering cohesive soils and tension within the soil environment, as discussed in Section 2.2.3. As a result of this, a new particle stability model was built, based on many similar features to Weakly Compressible Smoothed Particle Hydrodynamics, to allow stability to be modelled coherently, efficiently and robustly, and then to import this data to an WCSPH model within the program in order to model the dynamics of collapse through the WCSPH method.

As a result of this methodology, the bluff, dune or levee is constructed of a number of discrete particles, each of which is assigned a number of specific scalar parameters including mass, pore water pressure, and soil material type. These particles represent a unit area of soil material, and although the soil type itself can be variable within the domain, each particle must consist of only one soil type.
Each soil type used within the computation is described in terms of its density, and its shear strength capacities, which are given in terms of cohesion $c$ and angle of internal shearing resistance $\phi$. When considered in isolation from the effects of water pressure, these values are notated with the dash symbol to show the effective angle of internal shearing resistance $\phi'$ and the effective cohesion $c'$. These values are maximum strength parameters for each soil type, and each soil may reach peak values of shear strength under different strains, as shown in Fig. 2.1. This is handled by the stability method through a displacement stepping algorithm used within the Multiple Wedge Method (McCombie 2009).

![Fig. 2.1 Representation of mobilisation of angle of shearing resistance $\phi_{mob}'$ and strain for two soil types](image)

Given the strain along a slip surface must be constant to ensure continuity of the material within the soil profile, the values of internal friction, and similarly, cohesion, are found for various soils at specified values of strain, and are thus
considered the mobilised values of internal friction and cohesion, and denoted $\phi_{mob}'$ and $c_{mob}'$ respectively. As can be seen in Figure 2.1, it is entirely possible for one soil to have strained beyond peak strength while another soil is still strengthening, and as such this method can process more complex soil problems than those that consider purely elastic-plastic soil behaviours. At this stage however, the model can only consider the relationship between strain and $\phi'$ as a set of linear relationships.

The stability method used is a displacement stepping multiple wedge method, after McCombie (2009). This method searches for potential failure planes that can be drawn by a convex set. This allows the potential failure surfaces to be analysed from linear translational to circular rotational, and any combination therein. Once this surface is assumed, it is then broken down into a number of wedges for the stability analysis to be considered, which is a user defined parameter and offers increased accuracy with increased wedges, depending on the resolution being fine enough (Section 4.3).

The model algorithm for finding potential failure surfaces is described in Section 4.2.1 and shown in Figure 4.2. Despite the number of failure surfaces that are discounted due to lack of particles within each wedge or daylighting through an uneven terrain, the model can calculate the stability of a standard soil domain with 70,000 particles at a rate of approximately 3,000 potential failure planes per minute on a single core of a 2.4GHz processor in a standard desktop PC.

Splitting the failure surfaces into a discrete number of wedges is achieved by putting chords of the curved failure surface between each wedge intersection.
Assuming a negligible amount of soil dilation, the interwedge boundaries are constructed by bisecting the angle on the failure plane. This ensures that each wedge strains linearly along the failure plane and the displacement between wedges, and therefore the magnitude of the interwedge forces are compatible with the general straining of the soil. During the stability analysis of the slope, the soil inside each wedge is assumed maintain its original shape, and thus suffer no strain except along the boundaries.

Should the interwedge boundaries intersect within the soil domain, the wedge boundaries are rotated the minimum amount to ensure that particles are not considered to be within wedge overlaps. Although this eliminates particles being counted in two wedges, which is a source of significant error, the interwedge forces are now mobilising at a slightly different angle and as such some error has been brought into the method. As a result of this, increasing wedge numbers particularly for rotational failures can increase the inaccuracy of the model, and the number of wedges modelled must be selected carefully.

Once the slip surface has been assumed and split into the number of wedges required, the mass of the particles within each wedge are summed, the pore water pressure along the wedge boundaries, and the proportion of the various soil types along the wedge boundaries are calculated. The use of only the boundary particles to compute the soil strength is a direct result of the assumption discussed above, that each wedge maintains a zero strain state inside the wedge area. Therefore, the strength is directly dependent on the shear strength induced by the strain within the wedge boundaries. The method used in calculating the stability of the model works on an iterative calculation based on statics.
Starting with the upslope wedge (the wedge at the highest elevation), the mass and pore water pressures are resolved through static geometry to calculate the resultant forces on the failure plane and the forces acting on the wedge directly below. This is denoted as the normal force \((N)\) and is considered to act perpendicular to the interwedge boundary. These forces act equal and opposite to each other over the wedge boundary, whose suffixes are detailed in the Fig. 2.2.

![Fig. 2.2 Diagrammatic representation of Wedge numbering and Boundary Labelling](image)

Once the normal forces have been computed, finding the effective normal force \((N')\) is simply a question of subtracting the integrated pore water pressure \((U_w)\) over the boundary for each wedge boundary.

\[
N' = N - U_w
\]  

(2.3.4)

In the case of this method, it should be noted that the stability analysis happens at a specific point in time, such that the value of the water pressure \(U_w\) does not
change over the course of the analysis, despite any seepage that may be occurring during the overarching model timeframe.

Given the displacement method of the model first calculates a value of \( N' \) from a position of zero strain, the values of shear stress along the wedge boundaries will be zero. After the first ‘step’ of the model, increasing the strain across the failure plane, the values of shear stress are calculated using one of the following formulas:

\[
T_{i,2} = c'_{i,2} + N'_{i,2} \tan(\phi'_{i,2}) \quad (2.3.5)
\]

\[
T_{i,2} = c'_{i,2} + (N - U_a) \tan(\phi'_{i,2}) + (U_a - U_{w(i,2)}) \tan(\phi_b) \quad (2.3.6)
\]

Equation 2.3.5 is used in the case of fully saturated or fully dry soil, and equation 2.3.6 used in the case of partially saturated soil. In these cases, the shear force \( T \) is found using the effective cohesion \( c' \), and the angle of internal friction \( \phi' \). The unsaturated soil has a shear strength component of the suction within the soil grains known as matric suction. This depends on negative (relative to atmospheric) pore water pressure, as well as the angle of matric suction \( \phi_b \) which typically falls between ten and twenty degrees (Fredlund et al. 1978). In equation 2.3.6 the matric suction value is measured as the absolute water pressure \( U_w \) which must be lower than the air pressure which is taken as a model constant, \( U_a \).

As a result of this method, the shear forces are calculated for a model step using the normal forces from the previous step. Although this causes a slight
inaccuracy due to the iterative nature of the calculation, as long as the model
displacement step is small, there is no significant error.

Using this iteration, the principles of statics are applied to the uppermost wedge
first, and the Normal forces are found using the following:

\[
N'_{i,2} = \frac{1}{\cos(\alpha_i) + \sin(\alpha_i) \cdot \tan(\beta_{i+1})}\left\{W_i - T_{i,1} (\sin(\beta_i) \cdot \tan(\beta_{i+1}) + \cos(\beta_i)) + \\
N'_{i,1} (\sin(\beta_i) - \cos(\beta_i) \cdot \tan(\beta_{i+1})) - T_{i,2} (\sin(\alpha_i) - \cos(\alpha_i) \cdot \tan(\beta_{i+1})) + T_{i,3} (\cos(\beta_{i+1}) + \sin(\beta_{i+1}) \cdot \tan(\beta_{i+1}))\right\} - U_{i,2}
\]

\[2.3.7\]

\[
N'_{i,3} = \frac{1}{\cos(\beta_{i+1})} \left\{T_{i,1} \sin(\beta_i) + N'_{i,1} \cos(\beta_i) + N'_{i,2} \sin(\alpha_i) - T_{i,2} \cos(\alpha_i) - T_{i,3} \sin(\beta_{i+1})\right\} - U_{i,3}
\]

\[2.3.8\]

Where $W_i$ is the weight of the wedge, $\alpha_i$ is the angle of the boundary $i, 2$ and $\beta_i$
is the angle of the upslope boundary $i, 1$. The lowermost wedge will only have
one Normal force $N'_{i,2}$ as the downslope side of the wedge will be the slope
surface. As a result, when considering the slope mechanism through statics,
there will be an external restraining force, or an “out of balance force”
maintaining equilibrium of the slope mechanism. This is expressed as:

\[
F_{out} = T_{i,1} \sin(\beta_i) + N'_{i,1} \cos(\beta_i) + N'_{i,2} \sin(\alpha_i) - T_{i,2} \cos(\alpha_i)
\]

\[2.3.9\]

While $F_{out}$ remains positive, the friction within the slope is insufficient to
maintain the equilibrium of the assumed failure plane. Therefore, the value of
strain along the failure plane is increased such that the value of $F_{out}$ reduces to
either zero, meaning the shear force is sufficient to maintain slope equilibrium, or
the value of $F_{out}$ will minimise to some positive value and fail to reduce beyond this point for increasing strain.

If the value of $F_{out}$ reduces to zero, the assumed failure plane is stable and the soil strength can resist against the failure computed. As a result of this, the Factor of Safety ($FoS$) against failing is calculated as:

$$FoS = \frac{c'_\text{max} + \sigma' \tan(\phi'_{\text{max}})}{c'_\text{mob} + \sigma' \tan(\phi'_{\text{mob}})} \quad (2.3.10)$$

As can be seen in this case, the minimum value of $FoS$ (Factor of Safety) for a stable soil is 1.0. Along the base of the soil slope, the minimum value of $FoS$ is stored for each particle across all potential slip surfaces computed. This allows a stability map to be drawn of the slope profile, allowing a swift quantitative examination of slope stability (for example Figure 4.10).

Should the value of $F_{out}$ fail to fall below 0, the slope as computed is physically unstable and will fail immediately. However, in some cases, more than one unstable failure plane may be observed, and as such the model continues cycling through the potential failure planes until the value of $F_{out}$ has been computed for all potential failure planes. Following this, and using Newton’s second law:

$$a = F_{out} / \sum_i W_i \quad (2.3.11)$$

The slope computed to have the largest acceleration $a$ is considered to be the dominant failure, and the corresponding failure plane is passed to the SPH adapted soil flow model which is discussed in the following section.
Should there be a small slip that does not jeopardise the structural integrity of the main slope, i.e. a slip surface that is entirely contained within the angled slope front, if the mass of the slope is deemed small enough, the soil particles contained within the slope can be removed, as a prediction of erosion. This is seen with cases of two dimensional seepage and tends to happen soon before the episodic collapse of the slope. Although removing the particles negated the consideration of the equilibrium position of the failed material, as the failed soil may slump against the slope toe, the effect on the computed outcome is minor as the slope size is sufficiently small.

The stability analysis of the soil slope as documented in this section allows for a significant computational saving when considering the analysis of a soil bluff, dune or levee over a period of dynamic interaction. This method also prevents the model from applying an unphysical creep of the solid, as seen in some models discussed in Section 2.2. The use of the multiple wedge analysis offers a robust and detailed analysis of the stability of a soil slope at any point, allowing the prediction of time of collapse, as well as the mechanism of failure.

2.3.3 Seepage

In order to fully model two dimensional seepage until failure of a slope, the particles were given scalar parameters to describe the proportion of saturation of voids. Using this, the soil mass could be calculated considering the mass of the soil with the additional mass of the fluid added on. This approach to three phase modelling is necessary as the water particles and soil particles are small enough, and the scale is large enough, to consider soil as a single homogeneous material,
with properties of permeability and shear strength that depend on the degree of saturation.

The importance of modelling seepage within the soil domain has been discussed within Section 1.2.3. In order to do this, the model has tessellated the original particle placement in a hexagonal placement. This allows a fixed area of fluid for each particle interaction, as using a SPH type approach would result in cross-sectional areas between the soil particles being unique for each particle placement, and therefore a more intense computation, with less control over the conservation of mass.

Seepage quantity considered across each particle interaction is taken in each time step. The movement of fluid across neighbouring particles is applied through a two dimensional formation of Darcy’s Law to calculate the flow rate $Q$.

$$Q = k \frac{\delta p}{\delta l} A$$  \hspace{1cm} (2.3.12)

Here $k$ is the permeability of the particle coupling, which in working practice within the model is taken as the lower permeability of the two particles considered. $\delta p$ is the pressure difference between the water pressure of the two particles, which is corrected for elevation, $\delta l$ is the distance between the two particles, and $A$ is the area of the intersection between the two particles, taken as the side of the hexagonal particle boundary with a unit depth.

The flow rate is converted to a dimensionless volume of water ($\Delta Vol$) that transfers between particles in the current time step as shown:

$$\Delta Vol = \frac{(Q \, dt)}{V_p}$$  \hspace{1cm} (2.3.13)
Where $V_p$ is the volume of the particle, and $dt$ is the time step. This dimensionless seepage volume is effectively a proportion of the particle that will be filling, or emptying, over this time step. The value of $dt$ stays constant throughout the model run, and is calculated at the start of the model to ensure the condition below.

$$\left| \sum_j \Delta Vol \right| \leq V_p, n$$ (2.3.14)

Where $n$ here represents the porosity of the soil, which typically falls between 25-50% for most engineering soils (Lambe and Whitman 1969). As long as the value of $dt$ is low enough that the sum over the six neighbours conforms to Equation 2.3.14, the simulation can control the distribution of pore water. Should this condition be violated, unphysical quantities of pore water pressure may remain in the particle, causing numerical diffusion, and potentially, model breakdowns.

Considering the problem of boundary particles, the BMM creates static ghost particles along the enclosed boundaries of the soil domain. These are similar in concept and procedure to those of Monaghan and Kajtar (2009) for the SPH method. Any vertical pore water movement is reflected so that the seepage velocity is tangential to the boundary conditions, and thus no fluid crosses the closed boundary.

When considering boundaries of the soil that are open to the air, the ghost particles are modelled as empty particles of zero pore water pressure, and any fluid that is discharged into these is immediately discarded. Although this causes a slight inaccuracy as the fluid running down the soil surface is discarded, the
quantity of fluid seeping out of the soil domain (exfiltration) is typically of such small magnitudes and velocities that any physical effects are negligible within the scenarios discussed in the aims and objectives of this thesis. The total exfiltration in each time step can be captured by summing the volume of fluid within the open boundary ghost particles, before this volume is reset at the start of the following time step.

A two dimensional model of seepage within the soil domain must also consider the effect of partially saturated soils. As can be seen from Ng and Shi (1998), the effect of partial saturation within a soil causes a matric suction, or a pore water pressure lower than atmospheric air pressure. This in turn effects the hydraulic conductivity of the soil, and hence its permeability. In this model, the effect on suction and hydraulic conductivity is applied based only on the partial saturation value, and does not consider direction of saturation, i.e. whether the soil is wetting or drying. These relationships are expressed as split domain relationship based on the constitutive equations of Ng and Shi (1998), and assume homogeneity and isotropic conditions within each soil type in the computational domain.

In addition to the consideration of seepage within the soil domain, there is also a model for the erosion effects of exfiltration, which typically is often ignored, yet has been shown to be important in the prediction of stability and collapse (Crosta and Di Prisco 2000).

As the method of modelling chosen is a particle model, the eroded particles can be removed from the computational domain without loss of model integrity. This
removal of particles assumes the eroded particles are held either in suspension, or driven through bed shear by the exiting fluid away from the slope. In reality, some particles may remain near the toe of the slope whilst others are moved away from the slope, and in order to accurately model this there would need to be a free surface fluid model in addition to the seepage, which is a significant undertaking.

Erosion caused by seepage can be considered as a force equilibrium approach. Under saturated conditions, the soil particle will experience lateral pressures caused by the differential hydraulic head on either side.

\[ \Psi = c' + \sigma' \tan(\phi') - [\Delta\sigma' + \Delta u] \]  

(2.3.15)

Here \( \Delta\sigma' \) represents the difference in horizontal earth stresses, and \( \Delta u \) represents the difference in horizontal pore water pressures on either side of the particle. Therefore, if \( \Psi \) is negative, the disturbing forces are greater than the shear strength of the soil, and the particle is considered eroded and removed from the soil domain.

**2.3.4 Prediction of Earth Movement**

SPH has also been adapted to predict the movement and equilibrium of a soil failure. This is done through including shear resistance and lateral earth pressure coefficients in the WCSPH method. Lateral earth pressures are varied and taken according to Rankine’s Earth Pressure coefficients \( K \), where:

\[ K = 1 - \sin(\phi') \]  

(2.3.16)
This equation is applied if the particles are not moving towards or away from one another. However, if there is a relative lateral compression or tension between the soil particles, on the plane perpendicularly between each particle, then:

\[ K_a = \frac{1 - \sin(\phi')}{1 + \sin(\phi')} \]  \hspace{1cm} (2.3.17)

\[ K_p = \frac{1 + \sin(\phi')}{1 - \sin(\phi')} \]  \hspace{1cm} (2.3.18)

Where \( K_a \) is used if the soil is in an active state, i.e. the particles are moving away from each other, and \( K_p \) is used if the particles are moving towards one another, and the soil is in a passive state. The relevant \( K \) factor is used in the following expression to calculate the pressure exerted between soil pairs (\( P_\theta \)):

\[ P_\theta = P_{vertical} \cdot \{ K + \sin(\phi') \cdot \cos(\theta) \} \]  \hspace{1cm} (2.3.19)

Where \( \theta \) is the angle of the vector between the two particles and the vertical, and \( P_{vertical} \) is the vertical soil pressure, which is the stored WCSPH pressure, for the soil particle.

The second addition to the WCSPH code to allow the modelling of soil movement was the introduction of shear resistance, which is applied within the kernel as a direct retardation of particles, perpendicular to the vector between the two particles, such that:

\[ \frac{\partial \mathbf{v}_i}{\partial t} = \sum_j (P_{\theta_i} + P_{\theta_j}) \cdot \sqrt{\tan(\phi')} \cdot \sin(\theta) / (2 \cdot p \cdot m_i) \]  \hspace{1cm} (2.3.20)

Where the change in velocity of particle \( i \) is considered through the sum of all particles \( j \) within the smoothing length. \( P_{\theta_i} \) is the pressure of particle \( i \) in the
direction of $j$, and $dx_c$ is the contact area between the two particles, found through the kernel function. This shear resistance is calculated using the residual angle of internal shearing resistance, $\phi_r'$.

Using these two adaptations, the fluid modelling of WCSPH can be adapted to model the flow of soils. This method is more suitable than simply considering soils to be a highly viscous flow, as due to the shear resistance, the equilibrium position can be reached and maintained.

An overview of the model created using the adaptations and developments described in the previous sections is shown diagrammatically in Figure 2.3. This flow chart shows the original Bluff Morphology Model, as documented in Chapter 4, and the extension discussed in Chapter 5, with the respective input parameters and outputs given by the numerical model.

![Flow Chart of model components, transfer of variables, and model outputs](image-url)

**Figure 2.3 Flow Chart of model components, transfer of variables, and model outputs**
This section has presented the adaptations to the existing models in order to specify the methodology and parameters of the work within this PhD thesis. The following chapters consider the applications of these models and their adaptations.
Chapter 3 – Modelling Floating Object Entry and Exit Using Smoothed Particle Hydrodynamics

Having explained the value in using Weakly Compressible Smoothed Particle Hydrodynamics as a model for this thesis, this chapter explores the use of WCSPH within scenarios of high differential movement, impact and fluid separation by introducing a model for fluid and floating object coupling within the WCSPH model. This allows the author to explore WCSPH in more detail and offers a suitable platform from which to model a coastal bluff.

3.1 Introduction

The study of interactions between fluid and floating object, although a challenging subject area in its own right, has become a more pressing practical problem with the increase in demand for wave energy extraction, and offshore explorations. Many of the objects designed will be placed in high sea states, and subsequently they must be designed to withstand the forces arising from extreme wave conditions. Often the greatest forces on these objects are incurred through object entry and exit during their movement. Numerical modelling of this type of situation could prove a cost effective method for simulating such conditions, not suffering from the scaling problems present in physical models. Due to the anticipated range of structures and wave conditions, it is important that the numerical technique is accurate and robust.

Although there are many methods of numerical simulation for wave dynamics, including the modelling of floating objects, (Greenhow and Moyo 1997; Zhao et
al. 1997; Yan and Ma 2007; Zhang et al. 2010) the traditional methods of modelling are grid based. These methods subsequently encounter a marked increase in computational difficulty when phenomena such as flow separation, vortex shedding, surface piercing, flow coherence or large differential movement are involved in the simulation. As a result, it becomes increasingly difficult to accurately capture the movement and fluid response to a floating object.

The increasing computational power that is available to researchers has meant that numerical methods have evolved beyond the Eulerian grid-based methods of modelling. The computational method presented in this chapter is particle based; creating a dexterous mesh-free numerical modelling technique.

Initially developed for the study of the particle motion in highly turbulent scenarios within astrophysics by Gingold and Monaghan (1977) and Lucy (1977), Smoothed Particle Hydrodynamics (SPH) has been adapted for free surface flows and other hydrodynamic problems; (Monaghan 1992; Monaghan 1994; Monaghan 2000; Gómez-Gesteira and Dalrymple 2004; Gómez-Gesteira et al. 2005; Dalrymple 2007; Rogers et al. 2008; Dalrymple et al. 2009; Ferrari 2010; Gómez-Gesteira et al. 2010).

Developments of the SPH method have recently diverged towards new formats and methods of with subsequent reduction in CPU demands, including a GPU (Hérault et al. 2010) and a parallelised version (Ferrari et al. 2009) of SPH.

Although many multi-phase and multi-resolution methods have been developed for SPH, the main mathematical processes of WCSPH, have not changed significantly, with only slight additions to the hydrodynamic pressure evaluation
(Molteni and Colagrossi 2009) and changes to the angular momentum conservation, (Ataie-Ashtiani and Mansour-Rezaei 2009).

With WCSPH, the pressure values of the fluid particles are dependent on the change in density and a state equation, which can cause large pressure fluctuations, as noted by Xu et al. (2009). This inaccuracy can be reduced by re-meshing the particles across a uniform grid as proposed by Chaniotis et al. (2002), but this subsequently contravenes the mesh-free nature of SPH and thus is not used in the method of this chapter.

In this study, an extra module was developed to simulate the movement of solid bodies within the SPH program. To achieve this, an object is considered to be composed of solid boundary particles, their local positions fixed relative to each other, and their global positioning dependent on the hydrodynamic forcing of the water particles which act normally to the obstacle surface. This approach is different from the work of Ataie-Ashtiani and Mansour-Rezaei (2009) where the SPH modelling of an object movement is pre-defined and does not respond to any hydrodynamic conditions.

This approach presents a convenient method with which to model many types of floating objects within many different situations. Within this chapter, the research of modelling floating objects with SPH is extended by examining the full range of object movements using a standard and continuous particle placement, using regular spacing of the fluid particles, in contrast to the complex initial radial particle spacing of Oger et al. (2006).
The objective of this present study is to combine the floating object movement prediction model with the SPH model to investigate the interaction of flow and floating objects. The model results are then compared with benchmark test cases of water entry and exit, or wedges and cylinders. Pressures and velocities are compared with published results.

3.2 SPH Modelling

The mathematical basis for the SPH method is modelling the fluid domain as a number of discrete particles whose interactions are based on the Navier-Stokes equations. The representation of the fluid domain as particles uses a Lagrangian approach that allows the detailed examination of fluid responses. As explored in the previous section, SPH has been used in a variety of free turbulent surface flows.

The main form of SPH is Weakly Compressible SPH, although fully incompressible methods (ISPH) have also been developed e.g. (Shao and Lo 2003; Shao et al. 2006). Although ISPH tends to predict pressure fluctuations more accurately, the overall results of both methods are often comparable. Run times for identical numerical models are similar, as the ISPH method takes longer per time step but will use larger time steps throughout the run. WCSPH allows for a smaller particle size, giving a higher resolution, for any given memory size (Colagrossi and Landrini 2003).

3.2.1 Hydrodynamic Model

Once an initial geometry is defined, the particles that have been created in the numerical domain are assigned scalar parameters that include mass, pressure,
velocity components and so on. The values of these properties for all the particles can then be interpolated to compute any one of the scalar quantities for any given particle, using a smoothing function which is known as the kernel function.

The conservation of momentum and mass, as shown by Monaghan in his (1994) paper, is applied to each particle as shown in equations 2.2.2 and 2.2.3, but reproduced here for convenience:

\[
\frac{dv_a}{dt} = -\sum_j m_j \left( \frac{p_a}{\rho_j} + \frac{p_j}{\rho_j} + \Pi_{aj} \right) \nabla W_{aj} + g \tag{3.2.1}
\]

\[
\frac{d\rho_a}{dt} = \sum_j m_j (v_a - v_j). \nabla W_{aj} \tag{3.2.2}
\]

where \( j \) is all other particles within the active kernel function radius of \( 2h \), \( p_j \) is the pressure; \( v_j \) is the velocity; \( m_j \) the mass and \( \rho_j \) the density of particle \( j \). \( \Pi_{aj} \) is an empirical approximation of the viscosity effects (Monaghan 1994) and \( W_{aj} \) is the kernel function.

The method used by the author is an extension of WCSPH, which has been further developed by the author from the open-source SPHYSICS code (v1.0.002) published on the University of Manchester website (SPHYSICS 2007) by developing a method to model the feedback between moving objects and fluid motion, as described in the subsequent sections. In this weakly compressible approach, the compressibility is simulated by altering the density of the particles, and subsequently calculating their volume. Conservation of mass is guaranteed therefore, providing that no particles enter or leave the numerical domain.
The development of SPH is well documented, for example, in (Dalrymple and Rogers 2006), and therefore not described in great detail here. The model version used here includes the XSPH correction (Monaghan 1989), and the kernel functions used have been integrated with artificial pressure to correct tensile instability as shown in Dyka and Ingel (1995), Dyka et al. (1997), and Monaghan (2000). Sub-particle turbulence is modelled after Gotoh et al. (2001) (Gotoh et al. 2004) and Dalrymple and Rogers (2006). Execution time of the modelling has been decreased by using the linked-list, as suggested by Viccione et al. (2008). WCSPH uses a self-correcting time step, and as such this is not listed in the parameters used for each case study in this chapter.

3.2.2 Solid Particle Modelling

The boundaries of the numerical domain in the SPH method are constructed from particles according to the domain geometry. These particles have their position defined, either as fixed in space or moving with a specified function in time, for example a paddle wave maker. These particles are then included within the kernel integration and will exert proportional repulsive forces in opposition to the movement of a fluid towards the boundary. The two main approaches of modelling the boundary particles are set out by Monaghan (1994), and by Gómez-Gesteira et al. (2005).

The mechanisms of floating object modelling within the SPH simulation has been achieved by using solid particles to construct the shape of the object in question. In contrast to the method employed by Campbell et al. (2008), this allows a homogeneous method of computation, and improves the efficiency of the model. The effect of the floating object particles on the fluid is included
within the kernel function, but the hydrodynamic forcing onto the floating object particles is computed independently, in the predictor-corrector mechanism of the marching time step method. The average pressure of the fluid particles on the object surface over the previous object time step is then translated to a force by considering the fluid particles within the kernel function and the obstacle geometry, as seen in equations 3.2.3 and 3.2.4. The gravitational force on the object is applied using the initial weight input, instead of the combined weight of the particles, so that a hollow obstacle can represent a solid one, and thus saving some computational time.

The net forces on the object are found through integrating the pressure field of the fluid surrounding the object particles, as shown in the equations below. The pressure component of each particle acts at the respective point on the object, through the surface normal. Thus rotation can be computed through the pressure moment about the centroid of the object.

\[ F_x = \sum_j p_j a_j * N x_j * W_{aj} \]  \hspace{1cm} (3.2.3)

\[ F_z = \sum_j (p_j a_j * N z_j * W_{aj}) + g * m_{obs} \]  \hspace{1cm} (3.2.4)

Where \( F_x \) and \( F_z \) are the components of the force on the object in the \( x \) and \( z \) directions respectively, the particles \( j \) are those that make up the object, \( m_{obs} \) is the total mass of the object, and \( p_j, a_j \) and \( N x_j, N z_j \) are the pressures, surface area and normal component in the \( x \) and \( z \) directions of the \( j \) particle respectively.
The model uses the same kernel function and smoothing length as the fluid, to ensure consistency between the two phases. In addition, the authors found that numerical stability increased when the boundary particles were placed within the initial domain “grid” spacing, as this prevented obstacle particle cluster. The normal vectors for the model were derived from the initial geometry to avoid errors in computing the normal vector of the floating object after the particles had been subject to slight movement caused by the discretization. However, this can result in an apparent visual inconsistency regarding the distance from the fluid to the obstacle, but this problem is minor, and subsequently reduces with increased resolution.

Due to the rapid change in parameters of the fluid domain, and the slower response of the object, to save computational time, the movement of the object is calculated over a longer time step than the fluid motion, in contrast to the method applied by the SPHYSICSv2.0 code, which updates at every time step. The forces and pressures on the obstacle are then considered over ten time steps, and the object movement is computed for the longer time period, as seen in equation 3.2.5, where $dt_{obs}$ is equal to ten times the timestep used for the fluid domain. This particular ratio of time steps was found to function well for the cases studied, however other test cases may require different ratios.

$$\frac{dv_{obs}}{dt_{obs}} = - \frac{F_{obs}}{m_{obs}}$$

(3.2.5)

### 3.3 Results

Results are presented for a variety of cases inherent to modelling the physical processes in the behaviour of floating objects within the developed WCSPH
code. Initially, results are shown for the wedge submersion into a still fluid, both at a forced constant velocity, and as a free falling velocity dependent on the hydrodynamic response. Secondly, the method is used to examine forced cylinder entry and exit, and finally free object movement when a buoyant cylinder is placed in still water.

3.3.1 Wedge Entry

The initial test is a case study of object entry into a still fluid. This case consists of a 2D wedge being immersed into a fluid at rest, and was first published by Greenhow (1987). Wedges with deadrise angles of 30° and 45° were plunged through the water surface at a constant speed of 2ms⁻¹, capturing the surface elevation. The cubic-spline kernel function was used, with a smoothing length of 0.92, the shepard filter, and a Laminar viscosity of 1x10⁻⁶ m²/s. No additional density correction technique has been used. The deadrise angle is the angle between the side of the wedge and the horizontal. Figure 3.1 shows the photographs of the experiment (Greenhow 1987) and the corresponding WCSPH numerical simulation.

Figure 3.1 serves to demonstrate the suitability of SPH for moving object interaction with fluid modelling. The size and direction of the jets are reproduced well considering the resolution of the solution. The jets and splash are underestimated due to the particle size and discretization, as variation of the viscosity showed little effect on the result. However, even with a particle diameter of 0.0025m, the computational time of the simulation still took 22 hours to run 0.1s of simulated time, on a single 2.4GHz processor, and thus higher resolutions are not feasible at this stage. The 30° wedge produces jets which are
angled much closer to horizontal than the 45° wedge, as expected. Another apparent advantage of using a particle method such as SPH in numerical simulations of cases such as this is the ability to allow for fluid body separation, such as the spray formation seen on the left-hand side of the 45° wedge. This phenomenon is much more complex for a computational method that uses grids or meshes in place of particles.

Fig. 3.1 Comparison of WCSPH to the experimental results of Greenhow (1987) for plunging wedges of 30° (a) and 45° (b) deadrise angles at a constant velocity into still water
(images courtesy of the Massachusetts Institute of Technology)

This test case shows the accuracy when considering a simplified case with a defined movement and a still fluid. In reality, any design for a floating object would be required to model the object response to the fluid as opposed to the case in Figure 3.1 which constrains the velocity to a constant throughout the test.
As such the following test shows the results of a wedge plunging into the fluid at a defined entry velocity, wherein the subsequent wedge movement is a result of the fluid forces on the intruding object. There are plenty of examples of physical and theoretical testing around the subject area of wedge slamming, which is important to not only the renewable energy industry but also the shipping and ocean transport industry. Aside from Greenhow (1987), extensive work has been carried out by Zhao and Faltinsen in (1993) and (1997) considering the entry of arbitrarily shaped 2D bodies, as well as the impact study of Cointe (1987) and the detailed vertical and oblique entry of wedges presented by Judge et al. in (2004).

Figure 3.2 shows the sequence of images with the wedge (with a 30° deadrise angle) plunging into the still water with an initial velocity of \(2\text{ms}^{-1}\). The initial penetration into the surface causes the fluid to move down and to the side of the incoming wedge. These jets are attached to the wedge surface and propagate further up as the wedge progresses deeper into the fluid domain. Eventually the jets detach from the wedge surface and shoot to the side as seen in the final image. The maximum velocity of the jets is \(17.1\text{ms}^{-1}\) in the final image, being \(15.8\text{ms}^{-1}\) in the second image. These values compare well with the values predicted numerically in previous research (Oger et al. 2006; Shao 2009).
Fig. 3.2 The velocity vector plots of the fluid domain under impact of a 30° wedge; the jets described in Greenhow (1987) are clearly shown: (a) 0.004 s; (b) 0.016 s; (c) 0.02 s
The second test is of a wedge with an entry velocity of 6.15ms$^{-1}$, and the results are compared with those published by Shao (2009) who used ISPH with an identical resolution. The cubic-spline kernel function was again used, and the viscosity was Laminar & SPS method, with a $\nu$ value of $8 \times 10^{-7}$ m$^2$/s. The run time for 0.4s of simulation time was 25 hours. The results show the water surface elevation, fluid domain velocity vectors and pressure contours for the time following the initial impact.

Panel (a) of Figure 3.3 shows the close correlation between the velocity predictions of weakly compressible SPH results and the existing ISPH results. The RMSE error of the predicted velocities to the measured value is 0.36%. The velocity of the WCSPH wedge closely follows the expected trend and gives a much smoother profile than the results shown by Kleefsman (2005), and the results sit comfortably within the data points measured by Zhao et al. (1997). Towards the maximum time values measured there is a slight inaccuracy where the simulated results decelerate more slowly than the other results, however this is still well within reasonable tolerance.

Panel (b) of Figure 3.3 shows the force prediction, a factor that is often considered the least accurate parameter of WCSPH. The pressure of the fluid is affected by the kernel function, but the smoothing length has a slight effect on the force, although the test case (with h=0.95) was nonetheless under predicted. Although the initial rise in upwards force seems languid, this appears to be the result of the pressure wave moving away from the obstacle resulting in a slight underestimation of the fluid pressure. The smoothing length had no real bearing on this effect. The peak and residual forces are predicted correctly and the
profiles of the results are well matched, with a RMSE between the predicted pressure and the measured ones of 12.4%.

Fig 3.3 (a) The falling velocity of a 30° deadrise angle wedge, timed from the moment of entry; (b) the vertical force exerted on the wedge by the fluid body
Pressure oscillations within WCSPH, as shown in Figure 3.3, are not uncommon (Gong et al. 2009; Gotoh 2009), and although they are not a source of significant error due to their high frequency and low amplitude, they can cause inconsistency within the computation and as such must be minimised. Research is still ongoing to correct the problem (Molteni and Colagrossi 2009; Antuono et al. 2010). The slight oscillations within the results are likely to be the result of a slight feedback resulting from the time step and sound wave speed used, causing a feedback between pressure waves, or could be caused by the different time steps used within the method.

Although these fluctuations can cause some unphysical results when left unchecked, the fluctuations experienced by the authors are fairly small. Nonetheless, these can significantly affect the prediction of the movement of a floating object. As a result of this, the longer time step used to model the floating object goes some way to dampen the effect of the oscillations on the obstacle. Hence, the deceleration of the obstacle is predicted with greater smoothness than the force, which is merely an instantaneous integration of the pressure over the projected horizontal face of the wedge. The force used to plot Figure 3.3 was extracted at specific time periods, without the averaging over the 10 time steps, which could have smoothed the profile significantly.

The decrease in force is significantly steeper than the other predictions, which is believed to be caused by the pressure wave radiating out at a higher velocity than the wedge, causing a more sudden drop in the pressure of the particles at a close proximity, which has a high impact on the pressure integration due to the shape of the cubic-spline kernel function that was used.
Fig. 3.4 Pressure under the impact of a 30° wedge with WCSPH on the left hand side and the ISPH results of Shao (2009) on the right at t=0.004s (a) and t=0.016s (b)
Although it is traditionally viewed as one of the weaknesses of weakly compressible SPH, the pressure induced in the entire fluid domain can be compared with the ISPH results, as shown in Figure 3.4. The two sets of data were computed using identical particle sizes (diameter = 0.01m).

Image (a) of Figure 3.4 clearly shows a bulb of high pressure under the initial impact of the wedge, with no disturbance to the fluid further afield. The ISPH has a faster response time for pressure variations due to the incompressible nature, which may be why the highest pressure is at the surface instead of below the nose of the wedge. The maximum pressure under the wedge is in the region of 100kPa. This area of high pressure diffuses as the water moves upwards and sideways along the wedge, as shown in the second figure where the maximum pressure is around 70kPa.

An obvious discrepancy between the two sets of results is displayed in the second time step shown in Figure 3.4, where the surface profile of the ISPH model already displays some splashing and a more significant jet formation than the authors’ results, which show a more uniform fluid surface which forms a jet later in the simulation, producing a more powerful jet than the ISPH. This is logical, considering the instantaneous pressure transfer in ISPH compared to the sound wave dependent WCSPH.

The discretized nature of the particle model presents difficulties when computing the prediction of jet volume, so it is hard to know which result is more accurate. However, the WCSPH result predicts a more powerful jet than the ISPH results, which are understood to be significantly weaker than those found in Oger et al.
(2006). The methodology used in the work by Oger et al. (2006) is based on the weakly compressible SPH method, however the initial conditions were altered to give a complex radial spacing and variable resolution, affording a very fine resolution at the surface to try to predict the jets accurately, and thus are not shown within this thesis.

Fig. 3.5 Slamming coefficient \([p/(0.5\rho u^2)]\) plotted against dimensionless depth for 2D wedges plotted over the predicted values from Greenhow (1987); multiple lines for the 60° and 45° wedges show the theories of Borg (1959) and Wagner (1932)

The results of the 30° wedge entry show the performance of a normally configured weakly compressible SPH in predicting the fluid forces upon an object and also the forces within the fluid domain itself. This is a crucial step
towards modelling a floating object over longer time periods, where there will be inevitable instances of exit and/or entrance into the fluid surface by the object.

The slamming coefficient \([p/0.5\rho v^2]\) for various wedge angles have also been examined, for which there are fewer results for comparison. Figure 3.5 shows some initial results of slamming coefficients for wedges of 30\(^\circ\), 45\(^\circ\), and 60\(^\circ\) deadrise angles, using the same computational conditions as for Figures 3.1 and 3.2. The results are compared against the theoretical predictions of Greenhow (1987), Borg (1959) and Wagner (1932).

The slamming coefficient comparison in Figure 3.5 shows that the WCSPH model tends towards the trend and peak values of slamming well. Although there is some noise within the results, this does not detract from the overall similarities.

### 3.3.2 Cylinder Entry

Also considered are the free surface profiles caused by a free velocity cylinder dropped into calm water. Cylinders of density 500kgm\(^{-3}\) and 1000kgm\(^{-3}\), with diameter of 0.11m are dropped through a distance of 0.5m, until they reach the water surface. The water depth is 0.30m. Both simulations were run with a cubic-spline kernel, and the smoothing length was 0.9. The Laminar & SPS viscosity method was used with a \(\nu\) value of \(1 \times 10^{-6} \text{ m}^2/\text{s}\), and the resolution used was 0.0065. Results are compared to the experiments of Greenhow and Lin (1993) and the CIP (Constrained Interpolation Profile) method of Zhu et al. (2007).
Fig. 3.6 Free surface deformation during water entry of a cylinder of density 500kgm$^{-3}$; WCSPH results (left) compared against the model results using CIP by Zhu et al. (2007) (centre) and experimental results of Greenhow and Lin (1983) (right) at times after impact: (a) 0.005 s; (b) 0.030 s; (c) 0.085 s; (d) 0.120 s

Figure 3.6 shows the surface movement resulting from the impact of the 500kgm$^{-3}$ cylinder on the water surface. It can be seen that the impact of the cylinder causes a jet on each side to be formed, and this jet appears to flow in a more vertical direction as the cylinder continues into the fluid. In addition to
this, the flow separation as the cylinder keeps plunging is also well predicted with the WCSPH method, with the water surfaces on either side being straighter, and closer to the experimental data than the CIP method (Zhu et al. 2007).

Due to the same discretization problem that was experienced in the wedge entry, the jets are not as accurately calculated as the experimental results show, however, simulating a run of 1s took over 6 hours, and a resolution high enough to accurately capture the jet phenomena would require excessive computational resources.

Fig. 3.7 Depth penetration of a cylinder of density 500 kgm$^{-3}$: WCSPH results, model results using CIP by Zhu et al. (2007), and experimental results of Greenhow and Lin (1983)

The penetration depth of the cylinder can be seen in Figure 3.7. This shows an excellent agreement with the experimental data for the first 0.4 seconds. The apparent over prediction of the penetration compared to the experimental data is also reflected in the CIP method, and at this stage it is not clear why this discrepancy occurs.
Figure 3.8 shows the images of a cylinder with density of \(1000 \text{kgm}^{-3}\) being dropped into the fluid. As the WCSPH model does not include an air phase, the speed of impact is identical to the previous test. This cylinder penetrates to a much deeper level, reaching the bottom of the tank at \(t=0.50\)s. The method used by Zhu et al. (2007) was to reflect the velocity of the cylinder upwards upon contact with the boundary particles, which is the same method used by the authors due to the reflective nature of the boundary particles.

The slight particle clumping that can be seen in Figure 3.8 is a longstanding problem in the WCSPH method. Measures to minimise particle clumping, such as XSPH (Monaghan 1989), artificial stress within the kernel function (Monaghan 2000) and recent research in viscous fluids (Fang et al. 2009) has gone some way to address the problem. However, the unphysical particle clumping is caused by the shape of the kernel function, and the gradient of the kernel function when the particles are in close proximity is too slight (Vaughan et al. 2008). As a result of this, reducing the smoothing length, and subsequently the kernel function minimises the effect.

The initial image shows the formation of the jets and the flow separation around the side of the cylinder. As the progression through the simulation continues, the free surface becomes more fragmented, as evidenced by the layers of yellow and green in the results of Zhu et al. (2007), and by the disordered scatter of the particles in the authors’ WCSPH method. Both methods seem to predict the angle of the jets at 0.110 seconds to be angling inwards more than the experiment shows, however the WCSPH method is closer to the experimental data.
Fig. 3.8 Free surface deformation during water entry of a cylinder of density $1,000\text{kgm}^{-3}$; WCSPH results (left) compared against the model results using CIP by Zhu et al. (2007) (centre) and experimental results of Greenhow and Lin (1983) (right); images shown at time after impact: (a) 0.015 s; (b) 0.110 s; (c) 0.200 s; (d) 0.450 s
It is evident from Figure 3.6 that the cylinder particles are within the initial “grid” spacing, and as such the representation is not entirely circular. This is because when a completely circular obstacle was created without using the regularly spaced particles, the model suffered regular instabilities creating excessive artificial pressures. As a result of this, creating the cylinder through regular spaced particles reduced these excessive artificial pressures, and the results from it are significantly improved. It is important to note, therefore, that although the particles are aligned to the initial grid, they retain the normal vector of the cylinder. This then contributes to a visual increase in the separation between the fluid and the location of the grid points where the particles are displayed. Although this effect is reduced substantially with higher resolutions, as seen in (Ataie-Ashtiani and Mansour-Rezaei 2009), it is still representative of an unphysical property and may thus have some effect on the density and pressure calculations. However, given the surface vector of the cylinder remains in continuous, rather than discrete form, the pressure values are well conserved, as can be seen through the velocity comparisons.

The final two images show the water surface closing over the submerged obstacle. The shape of the water surface is well predicted by the WCSPH method, including the near-vertical jets continuing at 0.200 seconds, and the height of the cumulating water column at 0.450s.

Figure 3.9 shows the penetration depth of the cylinder after the initial impact at 0.3 seconds. The WCSPH results are closely correlated to the CIP results, which sit in reasonable correlation between the experimental data points. The
experimental result at 0.34 seconds appears to be slightly anomalous, considering the latter points.

![Graph showing depth penetration over time for different methods.]

**Fig. 3.9 Depth penetration of a cylinder of density 1,000 kg m\(^{-3}\): WCSPH results, model results using CIP by Zhu et al. (2007), and experimental results of Greenhow and Lin (1983)**

### 3.3.3 Cylinder Exit

The exit of an object from the fluid domain is critical to the design of floating objects and wave energy converters. The behaviour of an obstacle exit, however, has significantly less published research available for comparison, but some test cases have been modelled and the results are shown below.

When a cylinder is submerged into the fluid domain and forced to rise through the surface of the fluid, the free surface deformation has been presented by Greenhow and Moyo (1997) and is compared to the SPH numerical results in Figure 3.10. This test case involved a cylinder of diameter 0.5m with a density of 1000kgm\(^{-3}\), and was submerged at a depth of 1m before being forced to rise at
a constant motion of 1ms\(^{-1}\) upwards. The particle size was 0.02m, and the simulation was run using a cubic-spline kernel of smoothing length 0.85, and a Laminar & SPS viscosity (\(\nu\)) value of \(1\times10^{-6}\) m\(^2\)/s. The smoothing length is as short as feasible to minimise clumping and discretization effects without increasing instability. Further runs with smaller smoothing lengths increased instability, and resulting in model crashes or unphysical voids appearing. The results are compared to the numerical results of Greenhow and Moyo (1997) at comparable time steps.

Figure 3.10 shows good correlation with the numerical results, predicting the fluid height over the rising cylinder correctly. The RMSE error of the fluid above the cylinder is 1.25% with a 0.65% error across the whole surface in the first panel, 0.32% above the cylinder and 0.54% over the whole surface in the second image. The general shape is well matched by both frames, and the error level reduces as the cylinder progresses further to the free surface.

When considering true motion of the cylinder through the fluid, it is important to consider a cylinder whose movement, resulting from the fluid forces, is unbounded. In the following case, a cylinder of the same diameter as the previous test was given a density of 250kgm\(^{-3}\), and initially set up with its centre 1.0m below the still water surface which was at 1.5m. The particle size was 0.0175m, providing a domain with a total number of particles slightly exceeding 14,500, and the simulation was run using a cubic-spline kernel of smoothing length 0.84, with a Laminar & SPS viscosity value of \(1\times10^{-6}\) m\(^2\)/s.
Fig. 3.10 Free surface deformation attributable to a forced movement cylinder rising through the free surface for dimensionless time ($t = Ut/d$) (a) 0.4; (b) 0.6; red dots show the numerical results presented in Greenhow and Moyo (1997)
A velocity vector plot of the test case is displayed in Figure 3.11. The images show the obstacle as it moves at a constant speed, as it breaches the free surface, and as it reaches its maximum elevation. When observing the images, it is important to note that a side-effect of the SPH method is that when a fluid particle becomes separated from the rest of the domain by a distance greater than the smoothing length $2h$, having no additional particle interactions will cause it to be only affected by gravity. In some cases, this can cause a particle rejoining the fluid to have a disproportionate effect on the domain at the point of impact.

The vector plot (a) within Figure 3.11 clearly shows the water flowing around the cylinder in a similar manner as is predicted for the test with a controlled cylinder movement. The fluid on top of the cylinder moves upwards at the same speed as the cylinder and outwards where there is no water forces to prevent this. The space left by the cylinder is quickly filled with water flowing down and inwards from the sides of the obstacle, creating the eddies that can be seen propagating through the sequence of images.

A slight asymmetry can be observed in the results, and is due to the discrete nature of SPH, whereby the particles are not aligned with the central axis, and subsequently the fluid response varies to a degree laterally. Decreasing the particle size would reduce the asymmetric response; however it also increases the potential instabilities within the model.
Fig. 3.11 Velocity plots of the fluid domain as a free movement of a cylinder of density 250 kgm$^3$ as it (a) rises through the fluid; (b) aligns with the free surface; (c) reaches maximum elevation.
Fig. 3.12 Comparison of different resolution runs for a cylinder rise, showing the expected reduction in the asymmetry around the cylinder: (a) particle diameter of 0.025 m; (b) particle diameter of 0.016 m
Plot (b) shows the beginning of a slight asymmetry to the numerical solution, showing a lower jet flow under the obstacle on the left hand side compared to the right. The water surface over the top of the obstacle is still symmetrical.

The final image shows the cylinder at its maximum elevation. The remaining particles of water are being shed over the obstacle, and the residual eddies below the cylinder can be clearly seen. These eventually cease and the fluid domain enters a state of equilibrium.

The asymmetry that is observed in Fig. 3.11 is caused by the discrete nature of water particles being forced to shed on one side or the other of the rising obstacle. As a result of this, the asymmetry reduces down for smaller particle sizes, as can be seen clearly in Figure 3.12.

### 3.4 Conclusions and Discussion

In this chapter the development of WCSPH has been described, to include the behaviour of modelling floating object movement as a response to hydrodynamic forces within a fluid domain. This was achieved with an adaption to the WCSPH model that allowed floating objects to be defined and modelled. Phenomena that are traditionally complex to simulate correctly, such as surface piercing and impact pressure of object entry and exit have been modelled successfully. In particular, this chapter has examined the water impact, hydrodynamic forces, fluid motions and movement of objects in the typical case studies of object entry and exit from still water. These case studies show that SPH is a viable method to study floating objects. This work forms a solid grounding for exploring the design and modelling of floating objects using SPH, including wave energy.
capture devices, and therefore offers a reasonable introduction into the use of particle models within this thesis.

The new object modelling code is based on a system of two different time steps for each method within the simulation. The water response to the movement is calculated within the original kernel in each time step, and the obstacle movement as a result of the fluid response is computed after this using a distinct kernel function. This has been validated using wedge and cylinder entry, as well as cylinder exit. Initially considering forced plunging of a wedge into the fluid domain, the fluid free surface and movement agrees well with the experimental results of Greenhow and Lin (1983).

Computed values of object velocities and fluid forces have been moderately well predicted for the case of a free wedge slamming into still fluid, and the pressure within the fluid domain also shows good agreement with the results of Shao (2009). Slamming coefficients of wedges of different geometries tend towards the expected theoretical values (Greenhow 1987).

Cylinder entry into still water has been investigated, and the fluid response as well as the dynamic response of the cylinder is well predicted, as compared to experimental and numerical techniques (Zhu et al. 2007). Investigation into cylinder rise has shown that the WCSPH method predicts free surface deformation which compares well to previous results (Greenhow and Moyo 1997; Zhang et al. 2010).

To fully explore the potential opportunities of the SPH method, more research of these phenomena could be pursued. Further work opportunities include more
detailed modelling of single or multiple objects within the fluid domain, and air-water interaction within SPH (Rogers et al. 2009). In addition, consideration of different turbulence models may offer a higher resolution capture of certain phenomena within this discipline.

SPH is a computationally intensive method of modelling, however all tests were completed on a single core of a 2.4GHz processor of a standard desktop computer, with run times less than 40 hours. Further developments currently underway are progressing towards a version of SPH which will run on the Graphics Processing Unit (GPU) of a computer, and will dramatically reduce the time of computations and should allow for significantly higher resolution modelling (Hérault et al. 2010).

When considering this research in the context of this thesis, it is important to note the computational time of the models compared to the model time. Although a 40 hour run time is acceptable, when the modelled timescale is little over a minute, this poses a significant problem for the adaptation of floating object modelling to the wave transformation with the idea of modelling erosion, the timescale becomes increasing implausible. In addition to this, as seen in Appendix 1, a research paper on the coupling of a Smoothed Particle Hydrodynamics wave climate and erosion model to the Bluff Morphology Model (Vandamme et al. 2009) results in model runs that cannot run in the same order of magnitude as real time. Although the conceptual modelling is there, it cannot be deemed an appropriate avenue of future exploration until a significantly faster method of particle modelling is available to the research community.
Chapter 4 – A Novel Particle Method for Modelling the Episodic Collapse of Coastal Bluffs

In this chapter, the overarching slope stability model is introduced and explained. It is used to investigate the stability, collapse and equilibrium position of soft coastal cliffs (bluffs). This model combines a multiple wedge displacement method with an adapted Weakly Compressible Smoothed Particle Hydrodynamics (WCSPH) method. The chapter considers benchmark cases of soil stability in order to consider the applicability of the model, and validates the results against purely frictional, purely cohesive, and mixed strength bluff materials including 2D static water tables.

4.1 Introduction

The development of climate change predictions, combined with the increasing risk and value of coastal and marine assets, has prompted a surge in interest in the modelling and predictions of coastal, shoreline and bluff movements. It is imperative that the mechanisms of coastal bluff collapse are fully understood in order to predict the location of a slope failure and improve the accuracy of quantified asset risk and implement appropriate stabilisation measures. This knowledge, however, requires a full assessment of factors affecting coastal bluff stability and collapse over short timescales.

Previous research in the field of coastal landsliding has focused on understanding why landslides form through a retrospective analysis, either by considering the ground profile following a slip, or by considering mechanisms and processes that
cause mass failures, allowing for empirical models (Walkden and Hall 2005; Trenhaile 2009), or by gaining a data set that allows probabilistic extrapolation (Lee et al. 2001; Furlan 2008). Although coastal landsliding typically consists of large episodic collapses, there has been a considerable amount of research that considers spatially and temporally diluted monitoring systems, which result in significant smoothing of the dramatic collapse episodes themselves. Quinn et al. (2010) considers this to be directly contributing to a poor understanding of coastal slope processes.

As a result of this, and the spatial variability of the coastlines studied, there is significant debate about the key factors that control failure events and occurrences associated with bluff failure. Some individual mechanisms are well documented, for example (Hutchinson 1970; Dixon and Bromhead 2002), yet there is little in the way of a numerical model with which coastal slope failures can be back-analysed and the failure mechanism best identified.

The authors believe that this is primarily due to the complexity of the issue at hand, where any such model must consider a wide range of factors in order to attempt to replicate a failure event with any accuracy. These factors include the geological layering (Bromhead and Ibsen 2004), rainfall and pore water pressures (Caine 1980; Iverson 2000), particle orientation and cementation (Martins et al. 2005) and toe erosion (Walkden and Dickson 2008).

Research on the modelling of short term coastal cliff and bluff erosion is still relatively unexplored. Although there are several models of beach profile change, most of these are closed loop models which, under constant boundary
conditions, tend towards an equilibrium beach profile. SBEACH (Larson and Kraus 1989; Larson et al. 1989) is a two dimensional cross-shore model which takes the significant destructive force to be that of the dissipation of wave energy per unit volume of water. It incorporates wave run-up and wave set-up and outputs an accurate, although marginally under-predicted (Zheng and Dean 1997) dune profile. SBEACH is similar in both method and results to EDUNE, (Kriebel and Dean 1985; Kriebel 1986). EDUNE models a constant wave run-up but neglects set-up and variable sediment properties.

Other equilibrium profile methods include the Coastal Construction Control Line (CCCL) (Chiu and Dean 1984; Chiu and Dean 1986). This method models a uniform sediment profile, and has a tendency to over-predict results (Zheng and Dean 1997), and has not been subsequently developed significantly in research literature.

Another popular model is XBEACH, (Roelvink et al. 2009). This is a 2D depth averaged model, which considers the height of the surface to be a scalar parameter that is designed to model nearshore and dune response during storm events. It is a grid based model that solves momentum balance and mass transfer on a staggered grid, such that mass balance is solved in cell centres, and momentum balance is solved at cell interfaces. Shallow water momentum equations are solved after Walstra et al., (2000). XBEACH has been shown to have a high sensitivity to storm surge inputs, and a general overestimation of erosion. This could be due, in part, to the avalanching algorithm used to predict dune collapse (McCall 2008), that is, the algorithm of the model calculates that the dune will collapse when the slope angle is greater than the angle of repose of
the dune material, but the dune remains constant if the slope angle is less than this.

Applying these types of medium term beach equilibrium models directly to collapse of coastal bluffs and dunes, however, can be problematic. Over a long enough time, all beach profiles will conform to the equilibrium position, but the episodic nature of bluff and cliff erosion is such that the discrete, sudden events are not well captured by transitions among different equilibrium positions (Quinn et al. 2010). The slope stability element is a critical factor in the accuracy of the short term predictions of these models.

When considering the slope stability, XBeach, along with many other nearshore models uses an avalanching algorithm to define the maximum dune slope, and considers only wave and marine erosion to be the cause of bluff collapse, ignoring the geotechnical, hydrological and geological parameters that would influence the type and size of slip. Slope stability programs, from a geotechnical field, consider rotational, as well as translational slip failures. Considering all possible types of failure is necessary to ensure the prediction is as accurate as possible.

The majority of slope stability models use the Swedish (Fellenius) method of slices, or a derivative of this method (Donald and Chen 1997) to examine the stability of the slope in question. Error margins of this method typically range from 5-20% (Craig 1974) depending on the bluff conditions. This method consistently over predicts the disturbing force, thus producing a conservative estimate of slope stability. Non-circular failure slopes are conventionally
analysed using a multiple wedge method (Donald and Chen 1999; McCombie 2009).

A common slope stability software package is SLOPE/W (Krahn 2004b) which analyses the stability of a user defined slope section with varying earth materials and engineering reinforcements, and under dynamic/seismic loading. Slope/W has a variety of slope analysis methods built in, including the Fellenius method, and derivatives of it, but can also use a Finite Element Method, inside a limit equilibrium framework. This affords it a more accurate stress distribution than through the alternative circular arc analyses (Krahn 2004b).

None of these existing models can fully resolve the location, type and equilibrium position of a slope collapse. This chapter presents the novel approach, using a particle method, for stability analysis and collapse predictions of a coastal bluff. Using controlled parameters, this method provides detailed slope stability analysis based on a multiple wedge method so that deep seated rotational failures are considered. This method also incorporates shallow translational “avalanching” failures. In addition to this, once identified, this model allows for the failure mechanism and collapse to be simulated in order to predict the final profile shape of the bluff in question. This is a useful addition to coastal bluff research since a generic slope model which predicts both stability and collapse in a mesh-free, non-distorting method has not yet been published. The applications of this model run beyond analysis and predictions, and this model could be used to design temporary or permanent works designed to maintain the stability of the bluff, as well as to further understand the nature and development of slip surfaces.
4.2 Bluff Morphology Model

The model discussed in this chapter is a particle based bluff morphology model, which can be used in hybrid with any 2D beach erosion model or data to predict bluff collapse without using an avalanching method.

This model is designed to consider the stability and movement of bluffs made of weak earth materials, which can be adequately modelled as a series of continuous materials where spatial variability such as joints or bedding planes are negligible. This allows it to be applied to both weak coastal bluffs and dunes. When considering a coastal bluff of complex geology, multiple different materials can be modelled by assigning the relevant properties and parameters to the corresponding numerical particles.

The model set-up represents areas of bluff material as computational particles, which are initially tessellated in a hexagonal pattern, and allows for each particle to be assigned specific scalar parameters including mass, pore water pressure, and bluff material type. The bluff material type of the particle indicates both the cohesion and maximum angle of shearing resistance ($\phi'$). These are both mobilised over a displacement stepping within the model, such that peak cohesion and peak $\phi'$ are mobilised at the same displacement, creating a mobilised cohesion ($c'_{mob}$) and a mobilised angle of shearing resistance ($\phi'_{mob}$), which are calculated using a user-specified relationship between $\phi'_{mob}$ and displacement. An example of the model visualisation of this relationship is shown in Figure 4.1.
Fig. 4.1 Assumed relationship between mobilised angle of shearing resistance ($\phi_{mob}$) and strain along the assumed failure surface

In this method, each particle can have a unique set of properties to represent the bluff material. However, this limits the accuracy of representing bluff materials numerically through two ways. Firstly, if the material is highly spatially variable, the accuracy will be limited by the accuracy of the sampling used to investigate the bluff itself. Secondly, a cliff or bluff material that has significant spatial strength variability, for example a rock with bedding planes and joints, would be too complex to model as the joints and bedding planes can only be modelled through a line of numerical particles, which in turn presents a resolution-based problem. Thus this chapter considers homogeneous bluffs to test the accuracy of the model.
The particle model is an advantageous method for predicting sudden collapse of a coastal bluff. With no mesh regeneration or straining under large relative distortion, the model allows for a fully Lagrangian model which can track the history of each particle, ensuring, for example, that bluff material particles on a failure surface will remain as material of a residual strength.

The model has been developed with intent to predict collapse under erosion events. As a result, the particle method can move or remove particles to ensure the bluff profile in the program correlates well to the profile data. These data can be supplied by any experimental or numerical modelling method. Initial work has been done using a Weakly Compressible Smoothed Particle Hydrodynamics model (Zou 2007; Vandamme et al. 2009), although the runtimes of the WCSPH model limit the usefulness of this particular partnership.

4.2.1 Stability Analysis

When modelling the stability of the bluff, it was decided to use a wedge analysis as opposed to a traditional circular arc analysis, as this allows a more exhaustive search through the potential slip planes. Using the method presented by McCombie (2009), the model can search throughout the bluff and calculate the mobilised angle of shearing resistance. This subsequently allows accurate predictions of collapse and offers a comprehensive insight into the sub-surface material.

For ease of comprehension, the method of cycling through the calculated failure surfaces is conceptualised in Figure 4.2. Initially, the method assumes an entry point of an arbitrary failure surface on the soil surface, and then an exit point
above this. Many potential slips that fit these two points are then analysed, from linear to a near-circular analysis where the back of the potential failure surface is vertical (steps 3 and 4 in the diagram). If the slip surface daylights (as shown on A in Figure 4.2), then the entry point is moved to the appropriate place. The exit point is then stepped across the domain repeating this loop, and then the entry point is moved and the loop begins again. The resolution of calculating the stability planes is determined by the model user, and the potential slip surfaces are cycled through at a rate of approximately 3,000 per minute on a standard desktop CPU with a domain of 70,000 particles, and thus it allows the model to compute a wide ranging stability analysis.

Fig. 4.2 Conceptualised cycle of Bluff Morphology Model (BMM) mechanism to find failure surfaces
Once the failure surface is defined, it is discretized into a number of wedges. These can be arbitrary in location along the plane, however wedge boundaries must coincide with any failure surface vertices in order to ensure compatible displacement along the boundary, such that each wedge displaces linearly. In order to achieve this, the failure surface is constructed with straight lines between the wedge boundaries, as shown by Figure 4.3. The number of wedges used is a user-defined variable, and sensitivity analysis for this is discussed, for example, with Figures 4.4 and 4.6.

![Wedge boundaries of an arbitrary slip surface](image)

Fig. 4.3 Wedge boundaries of an arbitrary slip surface (also showing wedge boundary suffixes)
Figure 4.3 shows an arbitrary failure surface with 4 wedges as selected by the model. The wedge boundaries are angled such that they bisect the internal angle of the failure surface equally, assuming that each soil wedge has no differential movement and that any dilation can be neglected McCombie (2009). However, this can cause wedge boundaries to intersect, creating a potential source of significant error. When the boundaries do intersect, the wedge boundaries are rotated equally to ensure no overlap within the bluff, however this reduces the accuracy of the force mobilisation on the interwedge boundaries and can subsequently reduce accuracy. Having defined the wedge boundaries, the weight of the wedges and the material properties along each vertex are found and computed.

Beginning with the uppermost wedge, the forces acting on each wedge are resolved in the horizontal and vertical direction, and solved to find the normal forces acting on the wedge from each boundary, i.e. the Normal Forces ($N$). The interwedge boundary force is equal and opposite and subsequently each wedge can be resolved until the final wedge, where a horizontal force that maintains equilibrium is applied and calculated.

Normal forces on the wedges ($N$) are calculated iteratively over all the wedges, with the force on the interwedge boundaries being reflected, such that

$$N_{i,1} = N_{i-1,3} \quad (4.2.1)$$

The effective force ($N'$) is defined as follows, where $N$ is the normal force, and $U$ the integrated pore water pressure ($u$) over the wedge boundary.
The shear force on the wedge boundaries, \( T \) is the combination of the cohesive force \((c')\) and the product of the angle of shearing resistance of the soil \((\phi')\) computed using the material profile and current displacement, and the effective force \((N')\) from the previous displacement step, such that

\[
T_{i,2} = \{C'_{i,2} + (N'_{i,2}) \tan(\phi'_{i,2})\} \tag{4.2.3}
\]

Due to the displacement stepping method, the value of \( T_i \) will vary with the displacement, and as a result of this, for any given displacement, the mobilised values of \( c' \) and \( \phi' \) will be referred to as \( c'_{mob} \) and \( \phi'_{mob} \) respectively.

Starting with the uppermost wedge, the normal forces are found through resolving the forces on the subsequent wedges, as shown in equations 2.3.7 and 2.3.8, but reproduced here for convenience.

\[
N'_{i,2} = \frac{1}{\cos(\alpha_i)+\sin(\alpha_i)\tan(\beta_{i+1})}\{W_i - T_{i,1}(\sin(\beta_i) \* \tan(\beta_{i+1}) + \cos(\beta_i)) + N'_{i,1}(\sin(\beta_i) - \cos(\beta_i) \* \tan(\beta_{i+1})) - T_{i,2}(\sin(\alpha_i) - \cos(\alpha_i) \* \tan(\beta_{i+1})) + T_{i,3}(\cos(\beta_{i+1}) + \sin(\beta_{i+1}) \* \tan(\beta_{i+1}))\} - U_{i,2} \tag{4.2.4}
\]

\[
N'_{i,3} = \frac{1}{\cos(\beta_{i+1})}\{T_{l,1} \sin(\beta_i) + N'_{i,1} \cos(\beta_i) + N'_{i,2} \sin(\alpha_i) - T_{i,2} \cos(\alpha_i) - T_{i,3} \sin(\beta_{i+1})\} - U_{i,3} \tag{4.2.5}
\]

Where \( W_i \) is the weight of the wedge. When the normal forces acting on the last wedge are known, the last wedge can be resolved in both directions. As this final wedge has no value for \( N'_{i,3} \), a balancing force \( F_{out} \) is introduced to
keep the mechanism in equilibrium. This horizontal “out of balance” force ($F_{out}$) can be described as a sum of the wedge forces, as shown in equation 4.2.6 where the subscript $i$ notates the final wedge.

$$F_{out} = T_{i,1} \sin(\beta_i) + N'_{i,1} \cos(\beta_i) + N'_{i,2} \sin(\alpha_i) - T_{i,2} \cos(\alpha_i)$$

(4.2.6)

If this force is positive, the slip mechanism is unstable with the computed shear forces, and the displacement is increased across the slip boundary. This increased displacement increases the strain along the boundary, and therefore the new values of $c_{mob}'$ and $\phi_{mob}'$ are found. These are used to find the new values of the shear strength, $T$, which are computed using the $N'$ values of the previous displacement step as per equation 4.2.3. This causes a slight inaccuracy, which can be minimised by using a very small displacement.

Eventually, either the out of balance force will reduce to zero, or reach a minimum and begin rising. If it reaches zero, then the mechanism is stable, and the values of $c_{mob}'$ and $\phi_{mob}'$ at this point are the equilibrium values. The particles at the base of all the different slip mechanisms store the highest values of $c_{mob}'$ and $\phi_{mob}'$, and these are used to build the safety maps, as shown in the results (e.g. Fig 4.10).

If, however, the out of balance force does not reduce to zero, then the model domain cannot be in equilibrium, and the slip mechanism with the highest failure rate, i.e. the one with the fastest acceleration, will be modelled with the failure mechanics method, as detailed in Section 4.2.2.
The nature of this technique and the particle method means that it is convenient to represent multiple materials within the computational domain, and some failure surfaces may be such that equilibrium is not found until one material is failed and in residual strength, while the other is still approaching the peak value of $\phi_{mob}'$. Any material that ends up in residual strength can have its particle history stored, so that the model will not falsely attribute peak strength criteria to these particles.

### 4.2.2 Failure Mechanics

Once the critical failure mechanism of the slope has been identified, the model uses a Smoothed Particle Hydrodynamics (SPH) type method to model the subsequent collapse of the bluff. The computational domain of the Bluff Morphology Model (BMM) applies scalar parameters to each of the particles, including mass, vertical earth pressure, pore water pressure, and velocity components. The values of these properties can be interpolated using the following equation, to compute any one of the scalar quantities $f(x)$ for any given particle:

$$ f(x) = \sum_j f_j W(x - x_j)V_j $$

(4.2.7)

The scalar interpolation here of the function $f(x)$ is scaled by the function $(f_j)$ of the particle $j$ and $V_j$, being the volume of the particle. Volumes are explicitly calculated using the density and the mass, the latter of which remains constant for the duration of the simulation, ensuring conservation of mass. The smoothing function $W(x - x_j)$ is known as the kernel function, and although many types exist in SPH simulations, the cubic kernel was used for this model. This function
acts as a weighted average for the summation of particles. This weighting is based on the proximity of the two particles, and although theoretically this should be applied to all particles irrespective of distance apart, the cubic kernel function is curtailed at a distance of $2h$, where $h$ is a user defined parameter, but typically related to the initial particle spacing. This significantly reduces the computational time needed.

Similarly to the SPH method, the interacting particles are stored in a ‘linked-list’, which is updated with each time step, allowing more efficient program runs.

Conservation of momentum and mass, as detailed by Monaghan (1994) is applied to the particle $a$ in the form shown in equations 2.2.2 and 2.2.3, but repeated here for convenience:

$$\frac{\partial v_a}{\partial t} = \sum_j m_j \left( \frac{v_a}{\rho_a} + \frac{p_j}{\rho_j^2} + \Pi_{aj} \right) \nabla_a W_{aj} + g \quad (4.2.8)$$

$$\frac{\partial \rho_a}{\partial t} = \sum_j m_j (v_a - v_j) \cdot \nabla_a W_{aj} \quad (4.2.9)$$

Where $j$ is all other particles within the radius of $2h$, $p_j$ is the pressure; $v_j$ the velocity; $m_j$ the mass and $\rho_j$ the density of the particle $j$. $\Pi_{aj}$ is an empirical approximation of the viscosity effects (Monaghan 1994), and $W_{aj}$ is the kernel function. The SPH-type method used has been adapted from the open-source SPHysics code published on the University of Manchester website (SPHYSICS 2007). This code already includes the XSPH correction (Monaghan 1989), and the tensile correction (Dyka and Ingel 1995; Dyka et al. 1997; Monaghan 2000) which has been integrated into the kernel function. This method uses a weakly compressible approach to fluid movement, and it has been adapted to study the
flow of solids by including shear resistance and lateral earth pressure coefficients based on Rankine soil properties. Although negative pore water pressures can be modelled, negative soil pressures are rarely seen in large volumes within the field of interest, and typically act as a stabilising force, ergo these are considered negligible. The equations used to model the variable lateral earth pressures are shown below:

\[ K_0 = 1 - \sin(\phi') \]  
\[ P_{\theta} = P_{\text{vertical}} \{ K_0 + \sin(\phi') \cos(\theta) \} \]

Where \( K_0 \) is the Rankine earth pressure coefficient, for a soil with low lateral strain, and is dependant only on the maximum, not the mobilised \( \phi' \), and is related to the angular earth pressure as shown, where \( \theta \) is the angle between the two particles and the vertical, and \( P_{\text{vertical}} \) is the vertical pressure exerted by the bluff material weight above the particle.

The shear resistance of the bluff material is applied to particles within the smoothing length of \( 2h \), as are all the properties within the SPH method. This is computed using the SPH kernel, and is applied directly as a retardation of the particles, perpendicular to the vector between the two particles, such that

\[ \frac{\partial \nu_s}{\partial t} = \sum_j (P_{\theta i} + P_{\theta j}) \, dx_c \, \tan(\phi') \sin(\theta) / (2 \, m_i) \]

Where \( \partial \nu_s \) is the change in the velocity of particle \( i \) caused by shear resistance. \( P_{\theta i} \) is the pressure of particle \( i \) exerted in the direction of \( j \), \( dx_c \) is the contact area between the two particles, which is calculated using the kernel function. \( m_i \)
is the mass of the model particle \( i \), and \( \phi'_r \) is the residual shear resistance angle of the particles.

### 4.3 Results

#### 4.3.1 Bluff Slope Stability Analysis

A simple validation of the model involves testing linear slip surfaces in a dry non-cohesive soil. Using this method, the linear failure surfaces are tested and the \( \phi'_\text{mob} \) values needed for stability of the slip surface are computed. In order to satisfy the laws of motion, the theoretical mobilised angle of shearing resistance (\( \phi'_\text{mob} \)) will be equal to the inclination above the horizontal of the linear slip surface (\( \alpha \)).

Figure 4.4 shows the RMSE for the linear failure surfaces, depending on the number of wedges used, where the error is defined by the difference between the value of \( \phi'_\text{mob} \) and the inclination of the slip surface (\( \alpha \)). As can be seen from Figure 4.4, the accuracy increases dramatically with the increasing number of wedges until approaching a minimum value of 0.664%. However, the number of potential slip surfaces that the model can analyse decreases as the number of wedges increases. This is because each wedge must contain multiple particles for the method to work, and increasing wedge numbers, for any given model at a set particle count, decreases the number of particles per wedge. When considering a high number of wedges, the number of potential slip surfaces that contain sufficient particle numbers decreases, hence the number of analysed cases decreases. In order to find the critical slope and achieve a high accuracy at
the same time, the optimum number of wedges has been found to be between five and twelve, depending on the scenario considered.

Consider a saturated ground, such that the pore water pressure is expressed by

\[ u = r_u * \gamma_{soil} * z \]  \hspace{1cm} (4.3.1)

Where \( u \) is the pore water pressure, \( \gamma_{soil} \) is the unit weight of the soil, \( r_u \) is the pore pressure coefficient, and \( z \) is the vertical depth below the free surface. Linear failure surfaces through a non-cohesive bluff material with an \( r_u \) value equal to the ratio of the unit weight of water to the unit weight of the material
would be expected to mobilise an angle of shear resistance as shown below, where $\beta_{slip}$ is the angle of inclination of the straight failure surface.

$$\tan \left( \phi'_{mob} \right) = \tan(\beta_{slip}) \left/ \left\{ 1 - \frac{r_u}{\cos^2(\beta_{slip})} \right\} \right.$$  

(4.3.2)

Figure 4.5 shows the results of linear failure surface analysis when considering a saturated soil with pore pressure defined in equation (4.3.1). The saturated unit weight of the soil is 19.4, and therefore the pore pressure coefficient ($r_u$) is 0.5056. The observed relationship between the angle of a linear failure surface and that of $\phi'_{mob}$ is linear, with a gradient very close to 2.0. It is apparent that there is some scatter in the results, with a tendency for slight over prediction of the value of $\phi'_{mob}$ in the middle of the graph.

![Graph showing predicted mobilised angle of shearing resistance for linear slip in a wet, non-cohesive soil using 8 wedges](image)

Fig. 4.5 Predicted mobilised angle of shearing resistance ($\phi'_{mob}$) for a linear slip in a wet, non-cohesive soil using 8 wedges
As the pore water pressure adds to the spatial variability of the model parameters, the error of this scenario is significantly larger than the previous benchmark case. However, this error decreases significantly with an increasing number of wedges. Figure 4.6 shows the RMSE error compared to the number of wedges in the analysis, showing a minimum error of 1.4% for 11 wedges.

Additional benchmark tests have been carried out to consider the ability of the program to model circular, non-circular and compound slips in addition to linear translational slips. Due to the methodology used, the wedge boundaries are always linear, and as such a truly circular slip surface will be approximated into a polygonal form.

Fig. 4.6 Root Mean Square Error of the Bluff Morphology Model (BMM) when predicting the mobilised angle of shearing resistance ($\phi_{mob}$) for a linear slip in a wet, non-cohesive soil, where $\gamma_{soil}/\gamma_{water} = 1.91$
When considering a bluff material with no internal frictional strength, the cohesive strength needed for the stability of any given bluff geometry can be predicted by Taylor’s Chart (Craig 1974) which has been calculated using the principle of geometric similarity. The failure of a non-frictional, homogenous slope will always be a wide circular failure mechanism, extending as deep as possible into the bluff. For this reason, the solution is dependent on the depth to a firm stratum, and this is expressed as a factor of the slope height, $D$.

Figure 4.7 shows a comparison of the Taylor’s stability coefficient with the values predicted by the BMM. Each slope geometry is analysed by the BMM, and the most critical slip mechanism, i.e. the one that gives the lowest Factor of Safety ($FoS$), is outputted by the model, and subsequently used to give a stability coefficient $N_s$, as shown below.

$$N_s = \frac{c_u}{FoS \cdot \gamma_{soil} \cdot H}$$ \hspace{1cm} (4.3.3)

Where $c_u$ is the maximum undrained shear strength, $FoS$ is the Factor of Safety, and $H$ is the height of the slope. This expected value of $N_s$ also depends on the distance between the top of the slope and firm stratum ($D \cdot H$), where $H$ is the slope height, and two cases have been considered, $D=1$ and $D=2$. The theoretical predictions converge rapidly after $D=2$, and as uniform undrained shear strength is extremely rare in practical applications with any real depth, greater values of $D$ have been ignored.
Fig. 4.7 Schematic of the slope domain (a). Taylor’s stability coefficients for a non-frictional soil where the solid lines show D=1 and the dotted lines D=2. The points are the computed slope stability values (b)
Figure 4.7 shows a good correlation between the theoretical and predicted values of $N_s$, although the values are slightly under predicted for low slope angles, they are within reasonable tolerance and are more accurate when considering the steeper slope. As Taylor’s stability coefficient is inversely proportional to the Factor of Safety, the under prediction implies that the $FoS$ of the model was too high, probably caused by the critical slip mechanism not being found, which may in part be due to the linear approximation of the wedge boundaries. There is no real variation in the results when different densities or slope heights are computed. Changing the number of wedges decreases the error initially as a smoother slip profile is modelled, however increasing errors in adjusting the wedge boundaries to avoid overlap cause an increase in the error past 10 wedges, as seen in Figure 4.8.

Having considered the cases of zero frictional strength, and zero cohesive strength, a further case study is now considered with a homogenous clay type soil. The slope angle is $20^\circ$, with a height of 6m. The soil considered has a cohesion intercept of 2.5kNm$^{-2}$, and a peak friction angle of $20^\circ$. The water table is at the ground surface, and has an $r_u$ value of 0.3.

The model tests slopes for all shapes from linear through arc and circular slips. The critical, i.e. the minimum value of $\phi'_mob$ of each soil particle is recorded and as such an overall picture can be seen of the total of $\phi'_mob$ at each particle within the slope. This allows every particle to retain a FoS, and as such a detailed cross sectional profile is visible.
Fig. 4.8 Root Mean Square Error of the Bluff Morphology Model (BMM) when predicting Taylor’s stability coefficient for a non-frictional soil, for various numbers of wedges within each slip mechanism, where $H$ is the cliff height in metres, and $G$ is the density of the soil relative to the density of water.

Figure 4.9 shows the validation of the geometry with a comparison to the analytical model of Slope/W (Durrani 2007). The present BMM model allows for a highly detailed breakdown of the slope surface compared to the Slope/W data, but this figure shows an excellent agreement between the two data sets.
Fig. 4.9 Factor of Safety of a cohesive wet slope with the Cliff Morphology Model and the Slope/W (Finite Element Method) (Durrani, 2007). Analysis at the toe (a), midpoint (b) and the crest (c) of the slope.
Due to the relationship between strain and strength mobilisation, it is possible to consider the strain of an essentially stable cliff surface, in addition to finding those that may be on the verge of a failure event. When considering a cliff constructed of a predominantly glacial deposit (Quinn et al. 2010), using generic soil properties for glacial till \( (c' = 10\, \text{kPa}, \phi' = 30^\circ) \), the stability analysis for the entire slope can be seen in Figure 4.10, suggesting a predominantly linear failure with a curvature towards the toe of the cliff. The slight anomalies in the form of the surface particles on the face of the cliff are due to the resolution of the model and should be discounted. It is likely that the stability of these particles is overrepresented. This stability map, agrees well with the description of failure (Quinn et al. 2010).

The lines on Figure 4.10 show the surveyed cliff position, and indicate a similar mode and magnitude of failure to the model predictions. It is likely, as seen from

---

**Fig. 4.10 Factor of Safety for a typical glacial till deposit cliff, showing surveyed position of July 2007 (solid) and February 2008 (dashed) by Quinn et al. (2010)**
the undulation between 5 and 10 metres on the x-axis, that the failure in question was more complex than a single collapse. Indeed, although it is possible that tension cracks and toe erosion played a significant part in the failure of the slope, the new cliff surface still can be traced through the areas of significant instability, where the Factor of Safety is 1.45 or lower. This may well be an over prediction of stability also due to the pore water, and rainfall around the time of the slip, which are not documented.

Having constructed the stability of the slope in question, it is easy to plot the lateral strain of the particles in this formation. Lateral strain over a measurable quantity can be converted into displacement, and Figure 4.11 shows a comparison between the particle method of this thesis, and the fast Lagrangian analysis of continuum (FLAC) code of Quinn et al. (2010). This method produces a plot of displacement and stress, not stability, however the concepts are related.

Figure 4.11 shows a distribution of the lateral (x-direction) movement of the slope from Figure 4.10, compared with retrospective analysis of the mass failure. This is based on the data presented by Quinn et al. (2010) in the Withernsea area of north-eastern England. The magnitude and distribution of the flow contours are similar, with a slightly more erratic distribution of the contours in the BMM, which is partially caused by the resolution clipping the boundaries on the base of the expected slip. An additional anomaly between the two results is the gradient at the top of the slip pattern. This is likely to be caused by tension cracking, or a significant weakness of the soil properties at the top of the slope that is not considered by the current BMM. However, the results show similarity in method
and magnitude of failure mode, which is critical to the model’s accuracy and applicability.

Fig. 4.11 Horizontal displacement of glacial till failure. The Fast Lagrangian Analysis of continuum (FLAC) model (Quinn et al., 2010) (a) compared to the present Bluff Morphology Model (BMM) (b)
4.3.2 Bluff Slope Failures

Having considered the slope stability methods the model can also predict the significant collapse, failure movement and equilibrium position of the bluff, based on the critical slope mechanism. Once the slope boundary enters the plastic deformation phase, the mobilised earth material collapses using the WCSPH adaption method previously outlined in section 4.2.2.

When considering the collapsed profile of a bluff, the final equilibrium shape will depend on how catastrophic collapse is, i.e. the quantity of mobilised material, and the orientation of the failure plane, are critical parameters that control the final position of the collapsed material. This is likely to depend on the rate and magnitude of change of the bluff parameter that induces collapse. Bluff collapse can be induced by fluxes in pore water pressure, erosion, or reduction of material strength. The nature of the failure and change in the bluff profile will depend entirely on the volume of earth material mobilised, as shallow slips in a non-cohesive earth material will result in small scale “avalanching” as opposed to a deep seated rotational failure that will cause a catastrophic change to the final equilibrium profile of the bluff.

To induce a catastrophic bluff collapse and test the accuracy of the model, a sudden change of bluff parameters is used. In this case, a dambreak type scenario is examined, where a vertical bluff of non cohesive dry material is allowed to fall at \( t=0 \). The initial bluff geometry and parameters are identical to those used in the Distinct Element Method by Iwashita and Hakuno (1990). The
initial geometry is a 3m high vertical slope, and the material is dry with a \( \phi'_{\text{max}} \) value of 11°, and a density of 2000kgm\(^{-3}\).

The Distinct Element Method by Iwashita and Hakuno (1990) is used as a comparison in Figures 4.12 and 4.13. It models the earth material as a mass-and-spring mechanism with non-uniform resolution, which can give rise to some unexpected behaviour when blocks of mass remain attached to their neighbours. This is especially noticeable in the overhang at 0.6 seconds in Figure 4.12. The BMM model, by contrast, achieves a much smoother failure as a direct result of the kernel function, which is far closer to what is expected when considering such a large slip of a non-cohesive material. Whilst this smoother function is a more physical and expected result when considering non-cohesive material, the nature of the SPH collapse method in a highly cohesive material such as clay collapse remains an ongoing research question.

Figure 4.12 clearly shows the collapse of a vertical bluff as a sudden mass movement, followed by some residual movement along the top of the failed earth material until equilibrium is reached. The two models predict similar initial failure patterns, with greater movement at the toe of the failed soil, creating an increasingly reclined front face, and smaller angle at the displaced bluff “edge”. By 1.0 seconds, the front face of the bluff has become almost linear, and the methods diverge slightly as the BMM method shows the failed material bunching up at the leading edge, as opposed to the smooth linear residual plane of the Distinct Element Method by Iwashita and Hakuno (1990).
Fig. 4.12 Particle positions in a catastrophic collapse of a vertical cliff face. The present Bluff Morphology Model (BMM) (a) compared to Discrete Element Method (Iwashita and Hakuno, 1990) (b)
Fig. 4.13 Particle velocity vectors in a catastrophic collapse of a vertical cliff face. The present Bluff Morphology Model (BMM) (a+c) compared to Discrete Element Method (Iwashita and Hakuno, 1990) (b+d) at two different stages of failure.

In Figure 4.13 it can be clearly seen that both models predict the highest velocities at the surface of the slump, a phenomenon echoed by experimental results (Lube et al. 2005). The present BMM does not push the debris as far along the lower boundary level, which may in part be due to the separately applied boundary conditions of the Distinct Element Method by Iwashita and
Hakuno (1990), as opposed to the method used by the authors, where the same properties of the earth material were used to define all boundary conditions.

4.4 Conclusions

A novel particle method of bluff morphology model has been constructed. It combines a multiple wedge displacement method with an adapted Weakly Compressible Smoothed Particle Hydrodynamics (WCSPH) method to compute the stability of the entire domain before modelling the failure and the subsequent equilibrium profile of a coastal bluff. The model shows good accuracy for predicting a wide range of stability scenarios and bluff material parameters. Model results of dry, non-cohesive earth material proved to be highly accurate, with root mean square errors of less than 2% for both dry and wet cases.

Tests on cohesive, non-frictional bluff materials have also been accomplished, with the relationship of Taylor’s stability coefficients well predicted, especially at higher slope angles. In addition, the profile of the bluff is well predicted and compared well with an industry standard package, Slope/W. It was also found that a more detailed cross section profile can be achieved with high accuracy using the novel BMM model.

The lateral movement of a cliff can be extrapolated from the lateral strain produced in a typical cliff makeup. When compared to a surveyed cliff environment, the results show similarity in method and magnitude of failure mode, which is critical to the model’s accuracy and applicability.

The particle method allows for accurate material tracking, and can model the collapse or failure of a bluff without complex mesh regeneration or straining.
The equilibrium profile of the bluff failure is highly dependent on how unstable the pre-collapse conditions are, which is usually dependent on the rate of change of the destabilising parameter. Results have shown the ability of the model to simulate catastrophic failures under a sudden change of slope conditions with good accuracy.

Sudden and catastrophic failures are very difficult to predict. Our model results indicate that although the final equilibrium position is relatively easier to correlate, the mechanism and subsequent movement of the mobilised material itself is highly variable. However, it is likely that a bluff section will fail first along a slip mechanism that has the highest “out-of-balance” force relative to the mobilised mass, i.e., the largest acceleration. In the case of multiple failure mechanisms existing in the bluff, which cannot reach equilibrium, the model used these parameters to identify the critical failure surface.

Overall, the new particle Bluff Morphology Model presented satisfies many of the conditions found in the modelling of coastal bluff collapse. At present the types of material modelled range from the purely cohesive to the purely frictional. The best model results can be achieved with a combination of these material parameters. The model is currently limited to bluffs with a fair percentage of homogeneity.

This model is capable of predicting the distribution of stability, and location and magnitude of failure mechanisms, and has the significant advantage of stability of a particle method. The model will be improved by including the variability of pore water pressures, two dimensional seepage and infiltration, and more
considerable variation in the geotechnical and geological properties of the bluff material. These extensions are critical to extend the functionality of the model allowing for a greater breadth of real-world predictions, including the likelihood of collapse and collapse rates with sea level rise, as well as the changes in the Factor of Safety of a bluff with complex geology on the exposed face. The model presented offers a suitable platform with which to investigate episodic bluff collapse events themselves, and the detailed expositions from this model may serve to improve the accuracy and the forecasts of longer term models.
Chapter 5 – Investigation of Levee Instability

Caused by Seepage and Erosion Using a Particle Method

This chapter considers the effect of two dimensional seepage on the stability of a slope. The Bluff Morphology Model (BMM) as discussed in Chapter 4 is extended in this chapter to include the effect of two dimensional seepage, partial saturation, and erosion on the stability of slopes. Static and dynamic external water tables are considered within this chapter, and the effect of seepage on the slope stability is discussed for different soil types.

5.1 Introduction

The effect of water on soil stability has been a long debated and complicated issue for geotechnical scientists and engineers. Seepage in particular, is often poorly represented in numerical models and the erosive power of seepage is often neglected entirely. Levees, rear sides of breakwaters and riverbanks are all subject to the problem of prediction of instability and collapse, and their structural integrity is critical for the protection of assets. Riverbank erosion alone has been recognized as a problem of global significance (Darby et al. 2000). It is therefore crucial to establish an accurate model prediction of soil slope stability under seepage conditions.

The stability of river banks, levees and other such soil slopes has received much attention in the past. Since the advent of computer software such as Slope/W (Krahn 2004c), General Limit Equilibrium and Finite Element Methods of
modelling slope stability have been used widely as engineering design tools to predict the safety of these slopes. The majority of these models compute the factor of safety (\( \text{FoS} \)) of a soil slope based on the limit equilibrium of rotational or translational failures of the soil body, by dividing the mobilised slope up into slices or wedges (Donald and Chen 1997; Donald and Chen 1999; McCombie 2009).

The slope stability problem is further complicated by the presence of seepage, erosion and undercutting caused by surface water. A partner program to Slope/W, SEEP/W has been developed by Krahn (2004a). This addition incorporates the movement in the water table, as well as dynamic pore water pressures, in the analysis of slope stability. This and other slope stability models rely on a computational mesh fixed in space. The model becomes unstable and inaccurate when high strain caused by large relative movement, or significant erosion, is present. The presence of two dimensional seepage often results in erosion and undercutting caused by surfacing water. This has been investigated by Chu-Agor et al. (2008) where a null region without soil strength is artificially created to replace the effect of removed soil properties.

The effect of seepage is important to identify for a number of reasons. Seepage is often the cause of liquefaction of surface soil particles, leading to erosion and episodic collapse of the bank (Fox et al. 2007b). The seepage affects the bank stability through the combination of toe erosion and loss of soil suction that reduces the restraining forces on the soil slope, or an increase in soil weight and pore water pressures that further increases the mobilising forces on the soil slope.
In some cases, falling external water tables cause a loss of containing pressure when the water level recedes (Simon et al. 2000).

Negative pore water pressure, known as matric suction, causes an apparent cohesion that significantly increases the slope stability. The loss of this matric suction when the soil within a bank is saturated, significantly reduces bank stability, and thus may trigger bank failures or episodic collapse (Simon and Curini 1998).

Inundation of the soil by rainwater or lateral seepage has been identified as an important factor in the stabilities of banks, slopes (Crosta and Di Prisco 2000), bluffs, breakwaters (Dentale et al. 2009) and levees. This chapter presents a novel approach, using a particle method for stability analysis of seepage driven collapse of slopes. The particle model allows for the distinct change of soil types, water pressures, and removal of earth material without any re-meshing, straining, or distortion of the numerical set-up.

In this chapter, a particle method for slope stability will be constructed such that it incorporates the significant soil features described above. The model results are compared to the laboratory experiments and the outputs of the Finite Element Model Slope/W. The aim of this chapter is to analyse the changes of slope stability, and the predictions of the mechanisms and times of collapse for soils under two dimensional seepage.

5.2 Bluff Morphology Model

The Bluff Morphology Model was originally designed as a stability and collapse prediction tool for slopes and coastal bluffs. The model has recently been used
successfully as a stability analysis tool to investigate the Episodic Collapse of Soft Coastal Bluffs by Vandamme et al. (2011). This model is extended to incorporate the seepage, erosion and undercutting in this chapter.

5.2.1 Stability Evaluation

The model set-up uses a hexagonal particle tessellation when considering a static earth profile. These particles are arranged in order to best represent the soil profile, and then assigned a set of scalar parameters including mass, pore water pressure, position, and soil properties. It would be theoretically possible to assign an individual set of soil material strength properties to each particle within the model, although this would significantly slow the analysis down. In this chapter, however, multiple soil types are considered, but each soil type is taken as homogenous and uniform within its boundaries.

In order to assess the stability of a soil slope, the model uses a displacement-stepping wedge method based on the concept of McCombie (2009), as explored in Chapter 4. This allows for a variety of potential slip surfaces to be analysed, in particular, from large rotational failures to linear translational failures. Once the analysis of each slope is completed, the Factor of Safety along the slope boundary is computed. This in turn can be assigned to the particles on the boundary of the slope, and each particle retains the lowest Factor of Safety assigned to it over many potential slip surfaces. This procedure provides the spatial distribution of factor of safety of the entire slope at any given time.

Where multiple soil types exist in the same wedge boundary, the model calculates the shear strength pro-rata to the proportion of each soil types. These
soil types do not have to reach peak strength at the same time for the computation to work. Due to the numerical averaging of the soil types, increasing the number of wedges in each potential slip surface allows for greater accuracy of the shear strength, however it also increases the computational time required. In this chapter, analyses were performed using 6-12 wedges per slip which was found to be sufficient for the present study.

5.2.2 Seepage Analysis

The particle method solves two dimensional seepage using Darcy’s Law across neighbouring particles, as seen in Figure 5.1. Given the initial hexagonal placement of particles, each particle must be analysed for seepage in six directions at each time step. The time steps are small enough to avoid unphysical emptying of voids during any single time step.

Fig. 5.1 Diagrammatic representation of particle tessellation, where neighbours are $\delta l$ distance apart, and seepage flows over a boundary of area $A$. 

132
Darcy’s Law is thus applied:

\[ Q = k \frac{\delta p}{\delta l} A \]

\[ \Delta Vol = \frac{Q \, dt}{v_p} \]

(5.2.1)

(5.2.2)

Where \( k \) is the minimum permeability of the two particles considered, \( \delta p \) is the pressure differential over the distance (\( \delta l \)) between particles, and \( A \) is the cross sectional area between particles. This flow rate is converted to a dimensionless volume of water by multiplying by the time step, \( dt \), and dividing by the volume of each particle. As a two dimensional model, we consider all variables in term of a unit thickness in the third dimension (depth).

In order to ensure accurate boundary conditions are satisfied, the model creates ghost particles along the enclosed boundaries of the earth material. These ghost particles play a similar role to the Smoothed Particle Hydrodynamics ghost particles (for example, see (Monaghan and Kajtar 2009)). They reflect the pore water pressures of the soil particles along the boundary and ensure that no fluid crosses the constrained fluid boundary. Open boundaries are modelled as air particles that the soil can interact with. Any fluid that seeps into these air particles is immediately discharged, and a sum of the fluid in these particles indicates the total quantity of fluid exiting the soil.

The application of a particle model to seepage also considers the effect of Matric Suction on partially saturated soil particles. The contribution of suction are included based on the proportion of water within each soil particle, and as such disregards whether the soil is wetting, or drying following (Ng and Shi 1998).
Fig. 5.2 The relationship of (a) Volumetric water content and (b) hydraulic conductivity to Pore Water Pressure properties used by the Bluff Morphology Model (dashed line) compared to that of Ng & Shi (1998)
This suction reduces the hydraulic conductivity, as shown in Figure 5.2. The functions relating the hydraulic conductivity and pore water content used within the model are a simplified version of the constitutive equations of Ng and Shi (1998), assuming homogeneity and isotropic conditions within the soil samples, and are shown in Figure 5.2.

Including matric suction in the strength formula of (4.2.3) gives the formula of Fredlund et al. (1978)

\[ \tau = c' + (\sigma - u_a)\tan(\phi') + (u_a - u_w)\tan(\phi_b) \]  \hspace{1cm} (5.2.3)

where \( \sigma \) is the total stress on the soil, \( u_a \) is the air pressure, \( u_w \) the water pressure, and \( \phi_b \) the angle of matric suction, which is typically between ten and twenty degrees (Fox et al. 2007a). It should be noted that for a saturated soil, the equation reduces to (4.2.3), as \( \phi_b \) attains a maximum value of \( \phi' \) under saturated conditions (Fredlund and Rahardjo 1993). In this model, \( \phi_b \) is taken as a constant and the strength of a saturated soil is computed using (4.2.3).

5.2.3 Erosion & Undercutting Effects

The Bluff Morphology Model incorporates the prediction of erosion in three different ways. Firstly, the model can represent predetermined erosion using a profile function as a function of time. This ensures that the development of the stability within the model can be checked for a predetermined erosion condition. Eroded particles are removed from the model to suit the new earth profile. As a result of this, only soil areas as large as the resolution are removed, and this prevents complications with varying sized particles.
Secondly, in addition to this method of following pre-predicted erosion, the model also checks for particles stability with an inbuilt liquefaction model, using a modified shear strength formula.

\[ \Psi = c' + \sigma' \tan(\phi') - \{\Delta \sigma' + \Delta u\} \]  

(5.2.4)

Where \( \Delta \sigma' \) represents the difference in horizontal earth stresses, and \( \Delta u \) the difference in horizontal water pressures on either side of the particle. If \( \Psi \) is negative, the disturbing forces on the particle are greater than the restoring forces, and the particle is unstable, and hence it is removed.

Thirdly, the Bluff Morphology Model predicts earth material erosion through the stability method. Once seepage erosion happens, small failure events often start to develop within the bank. These failures are usually small enough to be classed as undercutting, as opposed to episodic collapse, and occur before the catastrophic failure event of the bank. The developments to the BMM can identify these small failures and subsequently remove the associated particles. These three methods can be used simultaneously, but it is more physical to use either the first method, or one or both of the second and third methods. The developed BMM used the second method of erosion to produce the results in the rest of this chapter, as the third method, even when applied, was not found to occur. This is probably due to the resolution of the models; however the impact of small scale slips remains an open question.

### 5.2.4 Soil Parameters

This chapter seeks to reproduce well documented laboratory tests of slope failure as a consequence of seepage. As a result of this, the soil parameters are
considered to be uniform within the individual soil boundaries and the soil properties of the published results are used.

The model simulation requires the cohesion, angle of internal friction, unit weight and permeability of the soils as input parameters. The hydraulic conductivity, as seen in Figure 5.2, is scaled to the saturated permeability of the soil, and the void ratio of the soils is taken as a constant 60%.

5.3 Previous Experimental and Numerical Model Results

5.3.1 Lysimeter Experiments of Homogeneous Cohesionless Soil of Fox et al. (2007a)

The set up of these experiments is illustrated in Figure 5.3.

![Fig. 5.3 Typical set up of the lysimeter experiments of Fox et al. (2007b). The water inflow reservoir is at a controlled level and the bank angle is set at 60°](image-url)
The inflow reservoir is kept at a constant height of 30cm, allowing lateral seepage to propagate under constant head boundary conditions. The soil used was a uniform sieved sand, with shear box tests results of \( c' \) at 0.25kPa and \( \phi' \) of 35°. The unit weight was approximately 19kN/m\(^3\). Laboratory tests were performed with slope angles of 26°, 36°, 45°, 60° and 90°.

**5.3.2 Lysimeter Experiments of Inhomogeneous Soil of Chu-Agor et al. (2008)**

Another set of lysimeter experiments were carried out by Chu-Agor et al. (2008), which in concept are similar to the experiment of Fox et al. (2007a), but using three different soil layers within the lysimeter. The soil profile is shown essentially three vertical soil layers, and these are composed of Silt Loam, Loamy Sand, and Clay Loam. The parameters of these soils are given in Table 5.1 below.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Apparent Cohesion (KPa)</th>
<th>Angle of internal friction</th>
<th>Unit Weight kNm(^3)</th>
<th>Permeability (ms(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt Loam</td>
<td>7.5</td>
<td>30°</td>
<td>16</td>
<td>1.7x10(^{-4})</td>
</tr>
<tr>
<td>Loamy Sand</td>
<td>1</td>
<td>25.5°</td>
<td>19</td>
<td>4x10(^{-3})</td>
</tr>
<tr>
<td>Clay Loam</td>
<td>15</td>
<td>35°</td>
<td>21</td>
<td>1.5x10(^{-5})</td>
</tr>
</tbody>
</table>

*Table 5.1 – summary of soil parameters from field tests of Chu-Agor et al. (Chu-Agor et al. 2008)*

Laboratory tests in this case were performed with varying water heights and slopes of the lysimeter. These tests have been chosen to evaluate the model’s
ability to account for seepage and erosion through multiple soil beds, and to account for the change in stability of a cohesive sediment.

5.3.3 Draw Down Failure Experiments of Budhu and Gobin (1996)

Finally the model is verified against the draw down failure experiment. This test involved a slope constructed of cohesionless sand in a tank. The tank is then filled with water to the top of the slope, and the water level is maintained to allow equilibrium pore water pressure throughout the slope. After this point, the water is drained at a steady rate until the slope failure occurs. The soil parameters in this experiment are an angle of internal friction of 32°, permeability coefficient of 5x10^{-3} ms^{-1}, and a unit weight of 19 kN/m^3. This test evaluates the model’s capability to predict the effect of inclined seepage flow with variable boundary conditions, instead of lateral seepage flow under steady state boundary conditions of the previous cases.

5.4 Model Validation

5.4.1 Seepage Through Homogeneous Soil

Figure 5.4 shows the progression of the wetted front across a uniform, homogenous sandy bank. The model uses the same parameters and dimensions as the lysimeter experiments of Fox et al. (2007a). The saturated particles of the model, shown in red, are considered saturated as their pore pressure is positive, and thus there is no effect of matric suction and no significant volume of air within the soil.

The predicted profile of the wetting front (panels d-f) that divides the saturated and unsaturated soil, progresses with time in the same fashion as the experiment
(panels a-c). These wetted profiles are a curved, near linear profile with a slope that decreases with time.

![Imagen de la página del documento]

Fig. 5.4 The propagation of the wetting front across a sand bank at t=110s, 240s, and 310s. (a-c): experimental photos from (Fox et al. 2007b); (d-f): BMM model (saturated particles red, unsaturated black)

The difference between the model and the experiment grows with time. The predicted profile by the Bluff Morphology Model is slightly more linear than the experimental data. However, the distance travelled along the base and the top of
the sand bank, are identical. The cause for the discrepancy will be discussed in Section 5.5.

Figure 5.5 shows the convergence of total seepage into the bank with respect to the resolution. Due to the particle based method of the model, the water content of model particles is responsible for the resultant water pressure, as shown in Figure 5.2. As a result of this, it is important to know the sensitivity of the resolution to the total seepage volume, which is controlled by the water pressure of neighbouring particles.

![Graph showing the convergence of total seepage into the bank with respect to resolution.](image)

Fig. 5.5 Model predictions of Cumulative Seepage using a spatial resolution between 0.006m and 0.014m
As can be seen from Figure 5.5, the initial and final water content of the entire soil bank are very similar for different resolutions, with the variation of the final volume less than 1.1%. As the resolution decreases, the rate of water absorption appears to slow, producing a smoother and more refined curve for the 0.006m resolution (where each particle occupies an area of 31mm²).

The consistency in Figure 5.5 for a range of particle sizes indicates the reliability of the prediction of the seepage rate and the pore water pressure distribution for a dynamic two dimensional seepage case. The result also suggests that a steady state seepage is predicted.

![Figure 5.6 BMM predictions of the change in the critical Factor of Safety over the course of 1500s of seepage for a sand bank during the lysimeter experiment of Fox et al. (2007b).](image)

Time of observed experimental failure given by line at t=1000s.
Figure 5.6 shows the variation of the critical Factor of Safety over the course of the lysimeter experiment with a homogenous sandy soil (Fox et al. 2007a). The soil parameters are given by the shear box test, and results for three values of $\phi_b$, with and without erosion are presented.

Figure 5.6 shows the expected downward progression of the slope stability as the water seeps through the sand. This is due to the additional water pressures, reducing the maximum resisting shear force (Equation 4.2.3), and the increased weight of the weight soil. The large variation in initial Factor of Safety is caused by the value of $\phi_b$ chosen, and the respective increase in matric suction (Equation 5.2.4).

The Factor of Safety is slightly over predicted by the BMM, as the experimental results recorded a slope failure just before $t = 1000s$. The authors believe the over prediction is due to the use of the shear box test results of soil strength parameter, where the soil was possibly not as well compacted, or as strong as in the Lysimeter apparatus. It is also possible that the cohesion recorded in the shear box test was an apparent cohesion caused by the soil suction instead of a genuine soil property. The removal of cohesion strength significantly lowers the Factor of Safety.

Figure 5.7 shows the profile of the bank and the Factor of Safety before (a) and after the seepage (b) for the case with erosion but without matric suction ($\phi_b=0$). It is evident that the particles are removed by the erosion module in the lower corner of the bank. The depth of the erosion compares well with the experiment.
results Fox et al. (2007a), although the location of peak erosion (maximum undercut) is lower in elevation than the experimental results.

Figure 5.7 shows the increase of instability due to seepage across the entire bank, when no matric suction of the soil is considered. The critical slip mechanism is towards the front of the slope (indicated by the red colour), buried beneath the exposed surface by the cohesion of the soil. Increasing the matric suction of the
unsaturated soil tends to decrease the predicted Factor of Safety, but has little effect on the shape and distribution of instability.

Figure 5.7 identifies the critical slip mechanism as a 10cm deep translational failure (indicated by the centre of the red contour), which is similar to the observed failure depth. The experimental results, however, showed an initial failure section breaking away leaving an overhanging crest intact, which must have failed soon after, although this secondary failure is not stated within the literature.

5.4.2 Seepage Through Non-Homogeneous Soil

The model is used to study the evolution of the critical factor of safety with time, in a non-homogenous, highly cohesive soil for two cases; with and without undercutting. The results are compared with the lysimeter experiment of Chu-Agor et al. (2008) for three soil layers of different permeabilities and different strengths, the numerical set up of which is shown in Figure 5.8.

The initial critical stability is shown to be highly dependent on the value of the matric suction ($\phi_b$), however the variance is not as pronounced as in the frictional soil case in Figure 5.7. This is because the predominant strength of the soil comes from the cohesion and the pore water pressure has a lesser effect on the stability of highly cohesive soils.
Fig. 5.8 Soil Profile of lysimeter experiments by Chu-Agor et al. (2008). The water level is kept at a constant height $H$, and the depth of the Silt Loam Layer, $BH$, varies with different experiments.

Fig 5.9 –BMM predictions of the change in the critical Factor of Safety over the 600s of seepage for a non-homogeneous layered cohesive lysimeter experiment of Chu-Agor et al. (2008). Time of observed experimental failure given by line at $t=600s$
The BMM predicts that the stability is significantly altered by the seepage of the water through the soil, in contrast to the Slope/W results of Chu-Agor et al. (2008). This progression is shown in Figure 5.9 for the six different cases used. This is likely to be due to a combination of the frictional component of the shear strength being reduced by the pore water pressure, and also the increased weight of the saturated soil. In all cases, the variation of the critical Factor of Safety for the slope converges over time, regardless of the value of matric suction. In Figure 5.9, we can see that the effect of erosion and undercutting plays a significant role on slope stability. The predictions without undercutting appear to maintain a stable soil profile, compared to those with undercutting, where the soil is predicted to transition towards an unstable slope at approximately 610s. This is in agreement with the experimental results where a slope failure was recorded at 600s.

A comparison of the modelled erosion predicted by the Bluff Morphology Model to the predetermined erosion used in Chu-Agor et al. (2008) can be seen in Figure 5.10. The predetermined erosion was chosen to match the observed experimental results at five points during the experiment. Figure 5.10 shows a smoother profile predicted by the BMM, where the majority of the erosion starts slightly later, but the two models approach identical final values.

The erosion predicted by the model consists of a triangular shaped void exclusively in the Loamy Sand layer, similar to that found by the experiments, although the leading edge of the undercut is more angular in the BMM, compared to the smoother profile of the Slope/W predictions. The depth of
erosion agrees well with the published data by Chu-Agor et al. (2008) (Figure 5.10).

**Fig. 5.10 Maximum undercut of the Bluff Morphology Model, compared to the conditions used by Chu-Agor et al. (2008)**

Figure 5.11 shows the critical FoS for the three cases of erosion. The exponential best fit lines have $R^2$ values of 0.950 and 0.906 respectively, and indicate the expected progression of the critical slope stability over the course of erosion due to the seepage. These results cannot be quantitatively compared to the Slope/W predictions of Chu-Agor et al. (2008) where critical FoS ranges from 1.0 to 0.5, which are therefore significantly different from those in Figure 5.11, but the shape of the curves and distribution of the values are similar in these two studies.
Fig. 5.11 The change in critical Factor of Safety of the soil profile compared to the maximum undercutting depth (a) and volume of erosion (b) for the three different values of $\phi_b$ as modelled by the BMM.
5.4.3 Rapid Draw Down

The model is used to simulate the case of rapid draw down. In this example, the soil slope was constructed at $32^\circ$, the tank flooded and a constant water level maintained until the soil is fully saturated and the pore water pressures are hydrostatic. After this, the external water level was lowered by 1.667 mm/s, until either the tank emptied, or failure occurred.

For this soil, the failure occurred at 411s after the first water level drop. The BMM predicted slope failure at 405s, and the positive pore water pressure distribution can be seen in Figure 5.12. The highly permeable sand is draining the pore water out through the front face, and the inclined pressure distribution indicates the rate of change of pore water pressure decreases with increasing distance to the open boundary.

![Pore water pressure profile of a sand bank after an initially saturated profile is exposed to 405 seconds of rapid draw down](image)

*Fig. 5.12 Pore water pressure profile of a sand bank after an initially saturated profile is exposed to 405 seconds of rapid draw down*
It is evident in this figure that the boundary particles below 0.08m in elevation are still saturated. This is due to the linearly decreasing water pressure on the boundary particles, modelled to a minimum value of zero.

Figure 5.13 shows the stability distribution at the same point. The particles in red are those which become unstable, but the exact location of the slip surface is unknown. However, given the fact that the entire profile of the observed failure is enveloped within the red section, the model predicts the size, timing and location of the failure well.

![Soil Profile of the Factor of Safety at failure.](image)

**Fig. 5.13** Soil Profile of the Factor of Safety at failure. Black particles are those with no Factor of Safety found by the BMM, and the observed failure of Budhu et al. (1996) is shown in black dashed line

The over predicted curvature of the failure plane is likely to be due to the matric suction of the partially saturated soil particles. However, it is also evident that there is a probable tension crack in the observations, leading to the vertical face...
at the back of the slip surface, which fails following a second draw down on the residual profile (Budhu and Gobin 1996).

5.5 Discussion

In this section, the results described previously are analysed and considered with regard to both the experimental data and sensitivity analysis.

5.5.1 Seepage through Homogenous Sand

The effect of two dimensional seepage and erosion on stability is examined by the model in a simple homogenous soil at a constant slope. This experiment provides the basic tests of the model capability due to its relative simplicity.

The propagation of the wetting front, as seen in Figure 5.4, is a clear indication of the models reliability of the prediction of the quantity of seepage, given the significant similarity between the distances the wetting front travelled during the various time steps. In this case the saturated particles were those deemed to have a positive water pressure – i.e. a water content of above 40% of the volume of voids.

The resolution of the model has a minimal effect on the seepage quantity (Figure 5.5). This result is likely to be due to the controlled interaction of each particle with its six neighbours, and the time step determined through consideration of the maximum volume of water movement under the highest pressure head differentials. This in turn leads to a time step that adjusts to the increasing resolution, and thus does not cause truncation errors over the volume of seepage.
The change in the critical Factor of safety in Figure 5.6 shows erosion has a significant effect on slope stability. In the case of a soil with low or zero cohesive strength, erosion effect comes into play through the removal of the retaining “toe” of the slope in as seen in Figure 5.7. In the experiment of Fox et al. (2007a), the erosion maintains a higher elevation on the slope than in the BMM. This may be caused by the non-physical removal of eroded particles by the BMM. In a laboratory experiment, eroded particles form a ‘scree slope’ of material at the toe of the slope, preventing the erosion at the very base. Given the model does not perform this function, the higher water pressures and eroding forces at the toe of the slope result in erosion that is more localised at the toe.

Considering the values of $\phi_b$ within the model shows a distinct contrast between the model results with and without erosion, as seen in Figure 5.6. The intact models, with no undercutting, show a convergence of the Critical Factor of Safety as the soil becomes more and more saturated, and the quantity of unsaturated particles, and thus the effect of matric suction decreases with time in the experiment. However, when the erosion of the particles is considered, this convergence of the Critical Factor of Safety for the different values of $\phi_b$ is less pronounced. This is due, in part, to the shift in the location of the critical failure plane. The experiments with large matric suction show an expected deeper, and more rotational failure slip, compared to a shallower and more translational slip predicted for small matric suction. This is consistent with the concept of matric suction acting in a similar way to cohesion, and the erosion provides a near-vertical toe of the slope that affords a variety of shapes of potential failure plane.
The experiment by Fox et al. (2007a) produced a failure just before 1000s, with a significant volume of material breaking away from the slope between the height of 50 and 250mm, approximately. This left a significant overhang of soil that is likely to have failed shortly after. The critical Factor of Safety of the model does not drop below 1 at this time frame, and is between 1.1 and 1.4 instead, depending on the value of \( \phi_b \). This is because the BMM is using soil parameters taken from the shearbox test of the soil – which is likely to be an overestimate of the soil in the lysimeter due to packing, matric suction, or experimental errors. In addition to this, the BMM does not consider tension cracking, which may cause the over prediction of stability and is likely to affect the angle at the top of the failure surface, as a tension crack reduces shear strength on near vertical sections of the slip surface to zero.

The volume of the potential failure within the slope surface ranges from 0.012\( \text{m}^3/\text{m} \) for \( \phi_b=0 \), 0.18\( \text{m}^3/\text{m} \) for \( \phi_b=10 \), and 0.021\( \text{m}^3/\text{m} \) for \( \phi_b=20 \). This compares reasonably well with the recorded experimental mobilised volume of 0.010\( \text{m}^3/\text{m} \), which rises to 0.017\( \text{m}^3/\text{m} \) when the overhanging soil is included.

Error between the model and the experiment is typically less than 10%. Considering the variability in soil parameters, this is a good model accuracy. This result suggests the importance of including tension cracks within the BMM to further replicate the physical conditions.

5.5.2 Seepage through Non-Homogenous Soil

The experiment of Chu-Agor et al. (2008) provides verification of the BMM against a non-homogeneous soil, with highly variable permeability and soil
strength parameters. This lysimeter experiment was created for the purpose of analysing the effect of seepage and erosion on cohesive earth material, and as a result of this, the failure mode and initial configuration is distinct from the test by Fox et al. (2007a).

The highly permeable layer of sand is sandwiched between much less permeable clay loam and silt loam (Fig. 5.8), therefore, the water first surfaced at the exposed face of the soil after only 190s. The saturated loamy sand layer also produces a downwards seepage into the lower clay layer, leaving only the upper silt layer with a significant portion of unsaturated material. This results in a highly varied pore water profile across the domain, a scenario closer to the real world applications of this model.

Due to the low permeability in the sand layer, the erosion predicted by the model is entirely contained within this soil domain. The cohesion of the upper silt layer keeps the overhang stable, and the lower clay layer has lower water pressures due to the low permeability. The depth of the undercut predicted by the model is compared to the void region that was programmed into the model comparison of Chu-Agor et al. (2008) in Figure 5.10. The Bluff Morphology Model shows a much smoother profile, indicating a more continuous erosion of the sand particles. The void regions in the Chu-Agor et al. (2008) were programmed in as there was no erosion calculation within their model, and so a coarse time step was chosen to reduce computational time. The similarity in the rate and the magnitude of the erosion by their and this study indicates the viability of the erosion prediction in the BMM.
The decrease of the stability in the soil slope is predicted over the course of the seepage by the BMM. In this case, the convergence of the critical FoS for various values of $\phi_b$ is due to the saturation of the lower soil layers, and hence the alleviated matric suction effect for the highly cohesive soil material. This applies equally to the model runs with and without undercutting. The expected reason for the divergence of the critical FoS for various values of $\phi_b$ at between $t=300s$ and $t=450s$ is caused by the erosion of soil allowing a greater variation of failure plane. As a result of this, the case with a low $\phi_b$ will have lower FoS for a steep linear failure surface, and the case of a high value of $\phi_b$ will have a lower FoS for larger, rotational slips. Without considering the erosion, there is significantly fewer places a failure plane can cross the downslope boundary, and therefore less potential variation in failure planes.

The change in the critical Factor of Safety with time for cases with erosion is much less uniform in the non-cohesive experiment. This is caused by the vertical face of the slope, as erosion is no longer physically more likely to be found at the toe of the slope and now particles are removed above the critical failure plane as well as below it. Eroding particles above the critical failure plane decreases the active weight within the failing soil mass, and therefore reduces the mobilising forces in the soil domain. Eventually, the reduction of cohesive strength as a result of the removed particles, reduces the critical Factor of Safety to a value that is lower than the case without erosion.

Although the reduction of the critical Factor of Safety is not uniform, this reduction can be related to the depth and volume of undercutting with an
exponential function as in Chu-Agor et al. (2008) (Fig. 5.11). The regression analysis of these two variables shows that the maximum undercutting depth is a more significant factor in the reduction of stability \( R^2=0.950 \) than the volume of removed soil \( R^2=0.906 \). Again, this is because the volume of soil is spread both above and below the critical failure surface and therefore decreases both the mobilising and restoring forces in the analysis, and a larger depth of undercut results in a larger number of potential failure surfaces.

The experimental results saw a failure occur at 600s after the initiation of seepage. The BMM at this instant predicts a critical Factor of Safety of only 1.020, a 2% over prediction (Fig. 5.9). This is possibly caused by the fact that the continuous soil nature was used in the model, while a more erratic and variable profile with potential slip surfaces and scars, as can be found in cohesive soils, may have been used in the experiment.

The episodic failure of the earth material was predicted by the BMM to be a predominantly linear failure at a rate of 0.07\( m^3/m \), which is close to the observed failure of 0.12\( m^3/m \). The BMM prediction of the failure surface on the upslope and downslope locations matches the experimental results. However, the BMM predicts a near linear slip between these points, whereas the experimental results show a curved and undulating failure plane. This is possibly a fault of the algorithm to detect potential failure planes in the BMM, and possibly an inaccuracy caused by a discontinuity in the experimental soil layers.
The model-data comparisons indicate the viability of the BMM. However, the failure planes within the soil mass that are predicted are too uniform, and spatial variability (Cho 2007) needs to be better represented in future studies.

5.5.3 Rapid Draw Down

Cases of rapid draw down are often numerically difficult to be modelled successfully. It is straightforward to set constant drainage as the boundary conditions in this model, as the surface particles act as the initiator of seepage through pressure difference between particle neighbours. Due to the particle nature of this model, it is easy to capture the sharp divides between the slope area above and under the water level.

One of the key features from this test run is the stability in the pore water pressures on the right hand side of the slope (Figure 5.12). These are not on the open boundary, and thus should remain near constant pressures. Over time, the water will drain across the entire soil domain so that the right hand boundary has reduced pressure. This can be seen in Figure 5.12 as a very slight deviation from the surface of the bank. However, the right hand boundary maintains the highest pressure across the domain, and this is indicative of an accurate seepage model.

Despite the highly permeable soil, this test run predicted no erosion from the bank face. Although this may, in part, be due to the resolution of the model, this result is also supported by the way the water surface decreased, allowing the shallow depth of soil to equalise under low differential pressure head.

The prediction of the slope failure at 405 seconds after the initiation of draw down is in excellent agreement with the experiment. In addition, the location of
the soil particles of critical instability shows the importance of slope failures below the water level.

The shape of the stability profile of Figure 5.13 again indicates the importance of considering matric suction in soils of partial saturation. The critical Factor of Safety is buried below the surface, and the residual angle of the BMM prediction is close to the residual angle of the experimental results, at 10° lower than the angle of internal friction.

It is very likely that the experimental results have a tension crack in the top face of the slip surface. Although this vertical slope later failed (Budhu and Gobin 1996), vertical tension cracks are a logical future subject for the model in order to improve results.

5.6 Conclusions

A novel particle method has been constructed to examine the stability of a soil bank under a variety of two dimensional seepage conditions. The model reproduces the change in stability and the soil morphology with acceptable accuracy. The seepage affects slope stability and profile for cohesive and frictional soils differently for different seepage and erosive conditions. The model predictions are typically within 10% accuracy.

Matric suction has been introduced into the Bluff Morphology Model by including the effect of partially saturated soils on the shear strength. A range of $\phi_b$ values have been used in the present study, and show an expected effect on the stability of partially saturated soils. The predicted contribution of hydraulic conductivity to stability is in good agreement with published data.
A sand slope has been modelled with a water reservoir behind it, at a constant head. The BMM predicted the seepage into the slope and the reduction in slope stability with acceptable agreement with the experiment, although stability is slightly over predicted. For a low cohesive soil, the choice of $\phi_b$ significantly affects the stability of a bank when seepage takes place. This is in contrast to highly cohesive materials, which are significantly affected by the erosion caused by the seepage, and less affected by the value of $\phi_b$.

Erosion at the toe of the slope is simulated by comparing the lateral hydraulic force and the available shear strength. Any unstable particles are removed automatically, which causes an inaccuracy due to the lack of a ‘scree’ pile. This can occasionally result in an over prediction of slope stability as the loose materials often form a retaining pressure against the remaining slope. The erosion causes a significant change to the stability of the slope and should be included in the modelling of this problem, as the inaccuracy induced by failing to consider a ‘scree slope’ is minimal. However the model would be more accurate if the mobilised particles remain in the computational domain so that the downward side of the slope has a potential build up of eroded particles, or scree. In the cases presented in this chapter, when the downward slope of the experiment is freely draining, and the soil particles are small, scree build-up is not a significant factor on the slope stability.

Considering a multi-soil domain with highly variable permeability, the model predicts the seepage, rate of erosion and time of collapse well. The stability of soil with high cohesive strength is affected significantly by erosion but not by the
angle of matric suction $\phi_b$. This result is in agreement with the experiment and numerical model of Chu-Agor et al. (2008).

It is clear that the effect of soil erosion on the critical factor of safety is more dependent on the maximum depth of the erosion than the volume of eroded material. Similar conclusions are drawn by previous studies of levees, banks and cliffs of high cohesive strength where notching is the predominant cause of collapse (Kogure et al. 2006).

Finally, the magnitude, location and time of failure are predicted well for a rapid draw down case. The associated pore water pressure reduction is modelled successfully, with exiting water on the front face and near-constant pore water pressures on the right hand of the soil sample.

The BMM is capable of predicting the distribution of stability, and location and magnitude of failure mechanisms caused by the erosion or change in pore water pressures by including a two dimensional seepage model. This capability will be improved by extending the soil strength domain to consider the effect of tension cracking at the top of slopes, which is often a precursor of failure.

In addition the increasing computational power allows for a more detailed resolution to accommodate the spatial variability within bluff material. This will enable the model to predict real-world scenarios by including the effect of sea level rise, and changes in water tables of banks and bluffs that are in danger of episodic failure events.
Chapter 6 – Conclusions and Discussion

6.1 Contribution to Knowledge

6.1.1 Short Term Coastal Morphology
The creation of a novel particle model has been used to examine the stability and collapse mechanisms of soil profiles. The model allows for a comprehensive analysis of stability and Factors of Safety within the soil profile, giving opportunity for a high level of coverage and analysis of a soil at rest. An overview of the input and output variables can be found in Figure 6.1.

The prediction of collapse, by finding locations of unstable failure planes, and then mechanising them using an SPH inspired model allows for a dynamic prediction of collapse velocity and equilibrium position of the slope profile. The resultant force that remains unopposed is used to drive the collapse of any residual material.

<table>
<thead>
<tr>
<th>Input Variables</th>
<th>Output Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil Properties</td>
<td>Stability Map</td>
</tr>
<tr>
<td></td>
<td>Erosion Location</td>
</tr>
<tr>
<td></td>
<td>Magnitude</td>
</tr>
<tr>
<td></td>
<td>Seepage and</td>
</tr>
<tr>
<td></td>
<td>Pore Water</td>
</tr>
<tr>
<td></td>
<td>Pressure</td>
</tr>
<tr>
<td></td>
<td>Time of Collapse</td>
</tr>
<tr>
<td></td>
<td>Mechanism of</td>
</tr>
<tr>
<td></td>
<td>Collapse</td>
</tr>
<tr>
<td></td>
<td>Equilibrium</td>
</tr>
<tr>
<td></td>
<td>Position</td>
</tr>
</tbody>
</table>

Fig. 6.1 Diagram illustrating Input and Output Variables of the BMM
The influence on seepage through an otherwise stable slope of earth material has been shown to be highly significant. For this reason, a two dimensional seepage analysis model has been developed and added into the Bluff Morphology Model. This has accurately modelled seepage through homogenous and non homogeneous soils, with appropriate steady state solutions and changes in pore water pressures.

Using this seepage model, predictions of erosion due to lateral seepage have been shown to be of significant importance when predicting the stability and timescale of collapse for an earth material under unsteady pore water conditions. Lateral erosion has been shown to be well reproduced when considered through the liquefaction and balance of lateral earth and hydraulic forces.

Unsaturated soils and the effect of matric suction on the stability of slopes is shown to be of critical importance. The impact of water content on pore water pressure and hydraulic conductivity has been modelled, and the resultant failures in two dimensional seepage conditions are examined for multiple soil types.

6.1.2 Floating Objects

The modelling of surface effects and interactions with floating objects has been investigated using a particle model with regular particle sizing and spacing. Complex surface effects of piercing and reformation, in addition to hydrodynamic forces on the obstacle have been shown to be predicted well with a Weakly Compressible fluid model. Using a longer time step to model the object movement has increased computational efficiency.
6.2 Conclusions

6.2.1 SPH Modelling of Slope Instability and Collapse

This PhD work has successfully taken the particle model concept of Smoothed Particle Hydrodynamics, and applied to a new model of slope stability and collapse. In addition to this, the application of WCSPH has been tested on object entry and exit.

The first section of this work introduced a novel particle model built to predict the stability and collapse mechanisms of a homogenous soil slope. This was done using the SPH concepts of applying scalar parameters to the soil particles, using the soil profile condensed down to the fewest possible parameters. Expressions of shear strength were derived using the Mohr-Coulomb failure relationship, and the normal earth stress was calculated through the soil profile, density and the water table.

The benefits of a particle model allowed for a comprehensive ability to model any intricate slope geometry, without the need of a complex mesh generation. In addition to this, the failure mechanics allowed for a physical shear along the failure plane with no distortion and unphysical effects. It also allows for a simple particle tracking analysis and a comprehensive stability profile across the slope. A simple stability check of ten thousand possible failure planes is completed in less than three minutes.

The hypothetical soils of entirely frictional or entirely cohesive soils have a stability analysis that is within 2% of the expected theoretical values. Comparisons to other methods indicate the model has similar, if not better
accuracy when predicting stability of soils of mixed cohesive and frictional
strength.

A two dimensional seepage model has been incorporated within the soil stability
model to allow for an accurate tracking of a wetting front, and changes in pore
water pressure under seepage. There is a scalar parameter applied to each
particle to allow for unsaturated water content, and this relates to the hydraulic
conductivity and the pore water pressure. This seepage is optimised through a
fixed neighbour list and a variable time step to ensure rapid convergence and
accuracy at large resolutions.

Seepage analysis is solved through a two dimensional form of Darcy’s Law, and
boundary conditions are typically resolved as open or closed. It is also possible
to pre-program boundary conditions to have linear or sinusoidal changes in water
pressure to represent a dynamic water table.

6.2.2 Instability and Collapse of Coastal Bluffs

Having created and validated a model to test soil stability of single and multiple
soil types, using the Mohr-Coulomb shear strength and including unsaturated soil
particles and matric suction, the particle model was extended to include
prediction of the collapse of an unstable slope.

Using a method that assumes failure planes allows for an optimised consideration
of soils with multiple unstable failure planes, for example, a vertical soil bluff, as
these can be considered and the soil mass with the largest acceleration used as
the mode of collapse to be modelled. This also allows for a significantly smaller
soil domain when modelling the collapse, as only the particles located within the unstable soil mass are considered by the collapse mechanism.

Modelling the catastrophic collapse of an unstable soil bank using this method is a very novel research outcome, and there was difficulty finding appropriate experimental validation data. The model has therefore been validated against a spring-mass computational model and shows excellent prediction of movement and equilibrium position.

Lateral movement of a cliff can be extrapolated from the strain values predicted with the stability map of the soil slope. The results of this method show highly comparable results to surveyed cliff environments.

Using a particle model means that the differential displacement within the soil can be monitored closely without any undue straining or unphysical results, and using the initial soil properties, any residual failure planes can be modelled by applying residual strength to the particles within the planes.

6.2.3 Seepage & Erosion Effects on Slope Instability

Having identified the necessity of including erosion within any coastal morphological model in Chapter 1, three different potential erosion models have been included, as discussed in Chapter 5.

Using a 2D seepage model within the extended BMM, predictions of time of collapse caused by seepage and exfiltration caused erosion are within 10% error of the experimental studies.
Using a variety of experimental data, the BMM has modelled the change of stability through to the time and position of collapse of a number of different soil slopes. In the case of low cohesive soils, it has been found that the choice of $\phi_b$ significantly affects the stability of a slope. In the case of highly cohesive materials, however, the erosion is the dominant cause of decreasing stability within a dynamic seepage environment.

Using lateral seepage forces to predict erosion within the soil slope compares well to experimental and model conditions. This erosion is a vital component to the prediction of stability of cohesive and non-homogeneous soils.

In addition, the magnitude, location and time of failure are predicted well for a soil slope under rapid draw down. This has demonstrated the ability of the model to predict slope collapse under both infiltration, and exfiltration conditions.

### 6.2.4 Water Entry and Exit of Floating Object

This work has also applied the Smoothed Particle Hydrodynamic model to predict the fluid response to object entry and exit. This was a combined model using solid obstacle particles under a different time step to investigate hydrodynamic forces, fluid response and object movement. The analysis of this data showed a strong correlation to empirical, numerical and experimental data.

This model, although too computationally intensive to be coupled with the Bluff Morphology Model, provided a platform to investigate phenomena traditionally hard to capture with non particle models, such as surface piercing, jets and impact pressure.
Modelling of free object entry showed a good prediction of hydrodynamic forces and the effect on object velocity and acceleration. Surface penetration and reformation was well predicted for cylinder entry. In addition, surface response to cylinder rise showed excellent production of water shedding and eddy formation. Asymmetry within the model caused by the lack of alignment with particles along the line of symmetry of the experiment is minimised through a reduction of the particle size, as any imbalance due to particle shedding is minimised through an increase in the total number of particles.

6.3 Comparison to Aims & Objectives

In Chapter 1 of this thesis, the aims and objectives of the work were set out. Following the conclusion of this work, a critical evaluation relating to these is presented below.

- A particle method which computes slope stability and a comprehensive exposition of Factor of Safety for multiple soil, pore water pressure effects and 2D water table variations has been created and validated against multiple sources.
- Using a Weakly Compressible Smoothed Particle Hydrodynamics based model to predict the movement of unstable soil particles, reproduction of the collapse and equilibrium position of an unstable cliff has been modelled and validated against published empirical comparators.
- Some coupling between a wave climate and erosion model has been explored within the context of a particle model. At this stage, the model was deemed too computationally intensive to pursue to any depth.
However, the Bluff Morphology Model maintains the capability of coupling with an erosion model by removing particles to maintain a profile. In addition, the ability of a particle model to capture the surface effects of object interaction have been proven successfully.

- Seepage has been successfully added into the Bluff Morphology Model, and considered through the medium of a fixed-neighbour interpretation of Darcy’s Law. This includes the concept of partial saturation, and models the change in hydraulic conductivity as well as negative pore water pressures. This has been investigated and shown that there is a significant relationship between two dimensional seepage and soil stability, particularly with modelling the inundation of partially saturated soil and the resultant lack of matric suction.

- These previous investigations have been compiled to produce a working model of slope stability under conditions of seepage caused by static or dynamic boundary conditions, which can be used, should failure be found, to model the collapse and equilibrium position of a failed cliff.

- Forced and free object movement within the fluid domain has been considered using the WCSPH model approach. Validation and comparison to the experimental and numerical data shows an accurate portrayal of complex flow phenomena. The use of composite time steps offered a computational saving without a compromise of accuracy.
6.4 Future Work

The research presented here is part of an ongoing set of objectives. As a result of this, there are a number of further aims that are considered to further develop and improve the working models.

With regard to the simulation of water entry and exit of free bodies, an investigation into the turbulence modelling of the fluid and the dependency of the object behaviour should be considered to broaden the research focus.

With regard to the Bluff Morphology Model, the prediction of tension cracking within the soil domain should be considered. This is likely to be a cause of the discrepancy between the numerical and experimental results of soils with (real or apparent) cohesive strength. This lack of tension cracking results in a slight overestimation of the stability of the soil, and also a slight misdirection of the prediction of the soil failure plane, as referenced in the text where a vertical, or overhang, of the original slope material remains.

In addition to this, the concept of spatial variability of data should be introduced. In experimental data, there is a significant degree of variability, as there is in the real world, of changes in compaction, pore water content, and residual failure planes. This lack of consideration of variability of soil parameters results in uniform and consistent numerical results, however, there is cause to consider a random distribution of error when considering a uniform soil slope.

Also, a suitable wave climate and erosion model needs to be found and coupled within the Bluff Morphology Model. This would allow a significant array of coastal parameters to be explored and considered, including concepts of wave
induced collapse, the change of stability under rising sea levels, and effect of offshore wave management on the stability of nearshore cliffs and levees.

It is expected that the addition of these components to the Bluff Morphology Model will allow for a significant improvement in the scope and accuracy of the numerical model, allowing for better understanding of episodic failure and transient stability problems. This increase in modelling accuracy of these features should then translate into a better understanding of long term coastal cliff, bluff and levee movement, and inform the social, political and environmental response to the threat of the sea.


Roth, A., Wendeler, C. and Amend, F. (2010). Use of Properly Designed Flexible Barriers to Mitigate Debris Flow Natural Hazards, ASCE.


Appendix 1 – Published Conference Papers