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## A Non-Minimum Phase Robust Non-linear Neuro-Wavelet Predictive Control Strategy for a Quadruple Tank Process

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**Abstract:** In process industries model-plant mismatch is a significant problem. Quadruple Tank Process (QTP) can be configured both in minimum phase and non-minimum phase (NMP). However, in NMP, the control of QTP poses a challenge. This paper addresses that and presents a novel robust wavelet based non-minimum phase control (NMPC) strategy for the challenging QTP using genetic algorithm to find the optimised value of the manipulated variables in NMPC at every sampling time. The QTP is modelled based on wavelet neural network. The simulation results indicate that significant improvements have been achieved both in modelling and control strategies for a QTP system compare to conventional approaches such as the Levenberg-Marquardt

**Keywords:** Wavelet Neural Network (WNN); right hand plane zero (RHPZ); non-minimum-phase (NMP), Non-linear Model Predictive Control (NMPC); quadruple-tank process (QTP) Genetic Algorithms (GA); Multi Input Multi Output (MIMO); model-plant mismatch (MPM); Non-linear Optimisation; Coupled Tank System (CTS).

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## 1 INTRODUCTION

The design of control strategies for plants with non-minimum phase (NMP) characteristics presents several difficulties owing to the presence of right hand plane zeros (RHPZ). A shortcoming of linearising the open loop behaviour of non-linear plants using state feedback is the inability to deal with the NMP characteristics of the system. This becomes more problematic where a multiple-input multiple-out (MIMO) system is involved owing to the loop interactions. Moreover, model-plant mismatch (MPM) is a significant problem in the process industries. MPM and parameter discrepancies result due to wear and tear, equipment degradation and many years of usage. The control of a quadruple tank process (QTP) is a very interesting and challenging topic in control engineering. QTP belongs to systems that are complex in nature and exhibit nonlinearities. The design of such a MIMO non-linear plant is an extremely arduous task. This is essentially as a result of their loop interactions which invariably causes difficulties in feedback control design (Srinivasarao & Subbaiah 2013; Owa et al. 2013). The QTP is commonly employed in the literature to describe the concept of problems associated with multi-variable control. The problems include internal instabilities, high sensitivity, time delay in the system (Shneiderman & Palmor 2010), imposition of a maximum bandwidth restrictions, phase drop in the frequency response and the performance limitations owing to the presence of right hand plane zeroes (RHPZ) (Malar & Thyagarajan 2009; Srinivasarao & Subbaiah 2013; Johansson 2000; Shneiderman & Palmor 2010; Gatzke et al. 2000). The QTP is important in that it has the easy features to position one of its multi-variable zeroes on each side of the 's' plane (Malar & Thyagarajan 2009). Therefore, QTP can be configured to exhibit either a non-minimum phase (NMP) or minimum phase (MP) system as shown in Figure 1. However, the presence of the NMP system poses a major challenge in the control field as there are difficulties to achieve required control performances (Shneiderman & Palmor 2010).

The response of a NMP system to a step input has an initial undershoot. Moreover, the NMP nature of a plant imposes hindrances for the linear feedback designs (Johansson 2000). Sliding mode controller based on coupling feedback linearization for quadruple tank process is presented by Biswas (2009) which operates in a limited zone. The limitation of sliding mode controller is that it is not continuous. Therefore, the motivation behind this work was to use techniques from intelligent control to model and control the challenging problem of QTP.

Most efficient industrial process plants and their operational activities today require operating systems very close to the boundary of the acceptable operating domain (Findeisen & Allgower 2002) and hence linear models are mostly inadequate to represent sufficiently the plant dynamics (Tricaud 2008). As a result of this, most researchers employ the use of soft computing and intelligent approaches such as the wavelet neural network (WNN) and a genetic algorithm (GA) as tools in the modelling of complex systems by utilising input-output data sets (Malar & Thyagarajan 2009) and more recently a hybrid biogeography-based particle swarm optimisation technique (Sangapillai, 2016). In view of this, not much work has been undertaken in using soft computing techniques with a non-linear model predictive control (NMPC) strategy for the QTP. Authors' previous work (Owa et al 2013) proposed a NMPC strategy for a coupled tank system.

In this paper the scheme is extended to QTP and analysis of the impact of both MP and NMP is presented. A novel approach is employed in this work using a wavelet as the activation function in the neural network to model the QTP due to its inherent capability of both time and frequency signal localisation, which ultimately helps in achieving a global minimum solution. Wavelets are one of the most exciting research areas in signal processing today and researchers have increasingly seized the opportunity to employ wavelet functions with its choice of different mother wavelet in various modelling disciplines (Maalla et al. 2008; Huang et al. 2002; Lin et al. 2003; Jajangiri et al. 2010; Lu et al. 2005; Meng & Sun 2008; Kuraz 2006; Lilong et al. 2011; Coca & Billings 2012; Oussar et al. 1998).

Conventional approaches of training the NN structures such as the back propagation methods have been extensively used. However, there is a problem of the solution becoming trapped in a local minimum point thereby results in model inaccuracies. For this reason, a GA is employed for the training of the wavelet neural network (WNN) structure. In previous work (Owa et al 2013), a GA was found to be superior to an approach such as the gradient descent. Moreover, classical controllers like the PID have been very promising for years but finds it hard to tackle difficult systems like the NMP.

This paper employs the use of a soft computing approach for the design of a more effective and better non-linear WNN of the QTP. This removes the inaccuracies and various assumptions in mathematical models of the plant responsible for a degraded performance of the control strategy.

Little work appears to have been recorded in the area of robust WNN-NMPC for the NMP QTP. The focus of this work is to use time-frequency localisation feature of the wavelet to design an efficient non-linear MIMO model for the QTP that can operate not just in one region of operation. Moreover, the first principle QTP model was built using SIMULINK design and this represents the real plant during the simulation study. The WNN (black box model) were applied to model QTP because this paper/approach aims at exploring and using trained input-output data for system identification. This black box model could be useful in some process industries where the dynamic equation cannot be obtained or are not readily available. This further leads to design an optimised robust WNN-NMPC for a QTP based on a WNN model. The results are benchmarked against the cited literature in the paper.

The remaining sections of this paper are described as follows: section 2 describes the QTP system while section 3 contains the details of the system identification and modelling using a GA. Section 4 presents the NMPC strategies with a GA online optimisation process. Finally, the concluding part of this paper is presented in section 5.

## 2 DESCRIPTION OF QUADRUPLE TANK PROCESS

Johansson (2000) in his work showed that the QTP system always has two transmission zeroes as indicated in Figure 1. The values  $\gamma_1$  and  $\gamma_2$  are inflow ratio into the Tanks 1 and 2 respectively. When  $(0 < \gamma_1 + \gamma_2 < 1)$ , one of the transmission zeroes is located in RHP whereas there are no RHPZ for  $(1 < \gamma_1 + \gamma_2 < 2)$  case. Moreover, when  $\gamma_1 + \gamma_2 = 1$ , the system has a transmission zero at the origin while the quotient  $\gamma_1/\gamma_2$  gives the zero direction.

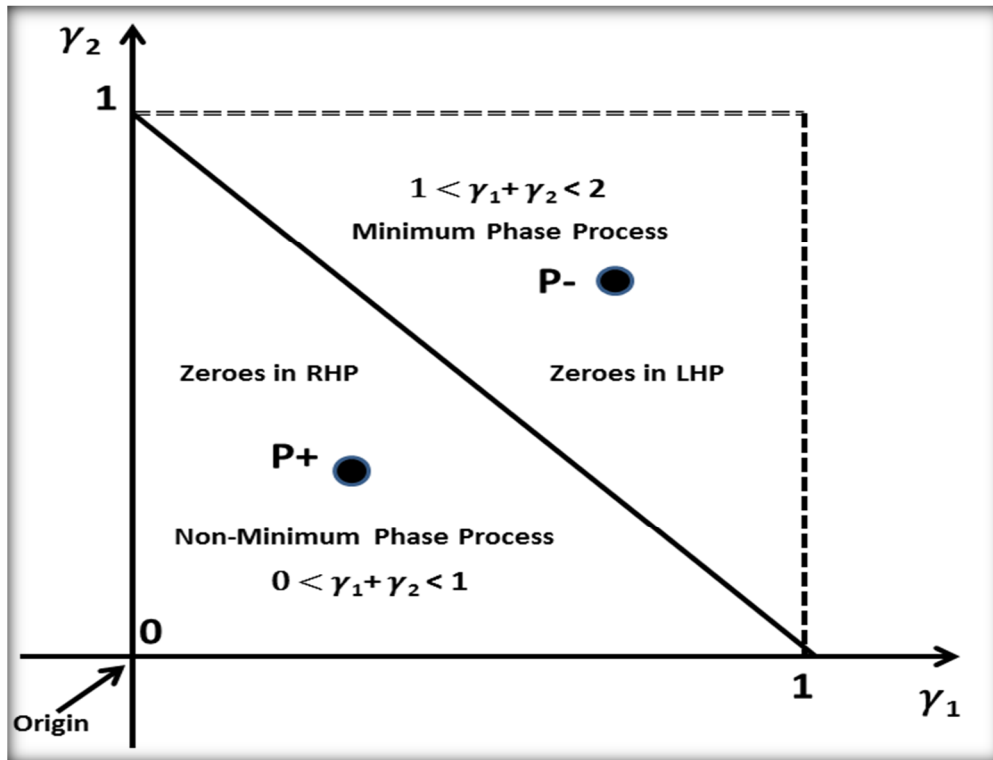


Figure 1 Minimum and Non-minimum-phase regions

The schematic diagram of a QTP is shown in Figure 2 and the main aim here is to control the fluid level in the two lower tanks with the two pumps. The QTP consists of two pumps and four interconnected water tanks. Its manipulated variables or process inputs are voltages ( $u_1$  and  $u_2$ ) to the pumps and the controlled variables (outputs) are the heights of the fluids in the two lower tanks (Tank 1 and Tank 2). From Figure 2, the flow from pump 1 is split into both Tank 1 and Tank 4 whilst the flow from pump 2 is split into both Tank 2 and Tank 3. Here, fluids enter indirectly into the lower tanks via the upper tanks.

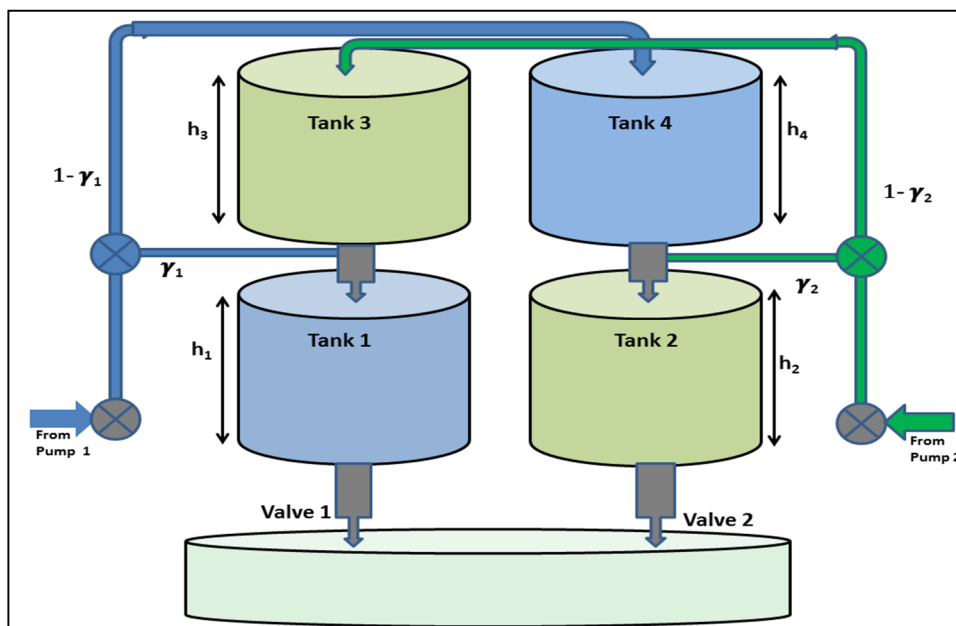


Figure 2 Schematic Diagram of a Four-Tank system/Quadruple Tank Process

The outputs are level voltage measurement from sensor device. QTP can easily be built by the use of a two double-tank processes. The output of each of the two pumps is split into two ratios using a three-way valve as shown in Figure 2.

The control strategy is to control the fluid levels in the two lower tanks. If both flow ratios  $\gamma_1$  and  $\gamma_2$  are big ( $1 < \gamma_1 + \gamma_2 < 2$ ) most of the fluid is going directly into the lower tanks (1 and 2). Secondly, if both flow ratios  $\gamma_1$  and  $\gamma_2$  are small ( $0 < \gamma_1 + \gamma_2 < 1$ ), fluid will first enter the upper tanks (3 and 4) before draining into the lower tanks. Therefore, pump 1 indirectly fills tank 2 and pump 2 indirectly fills tank 1 as explained earlier.

The physical parameters of the QTP used in (Johansson 2000; Suja Mani Malar & Thyagarajan 2009; Srinivasarao & Subbaiah 2013) are given in Table 1.

Table 1 Physical parameters of the QTP in (Johansson 2000; Malar & Thyagarajan 2009; Srinivasarao & Subbaiah 2013)

Symbol	Description	Value
<b>Tank 1 &amp; Tank 3</b>	Area of Tanks $A_1$ and $A_3$	$2.8 \cdot 10^{-4} \text{m}^2$
<b>Tank 2 &amp; Tank 4</b>	Area of Tanks $A_2$ and $A_4$	$3.2 \cdot 10^{-4} \text{m}^2$
<b>Tank 1-4</b>	Maximum height of the tanks	0.2m
	Area of valve orifice $a_1$ and $a_3$	$7.1 \cdot 10^{-6} \text{m}^2$
	Area of valve orifice $a_2$ and $a_4$	$5.7 \cdot 10^{-6} \text{m}^2$
<b>g</b>	Gravitational constant	$9.80 \text{ ms}^{-2}$

The derivation of complex mathematical models of non-linear systems is a challenging topic in many engineering disciplines (Malar et al, 2009). The non-linear dynamic equations for the QTP shown in Figure 2 are determined by relating the flow  $Q_i$  into the tank to the flow  $Q_o$  leaving through the tanks valves. Equation 1 is obtained by applying mass balance flow equation on a tank (Laubwald 2005).

$$Q_i - Q_o = A \frac{dh}{dt} \quad (1)$$

where A is the cross-sectional area of the tank and h is the height of the fluid in the tank. The unit of Equation 1 is expressed in  $\text{m}^3 \text{s}^{-1}$ . The flow through the valve can also be expressed as shown in Equation 2 (Laubwald 2005).

$$Q_o = \delta_x \beta_x \alpha_x \sqrt{2gh_x} \quad (2)$$

where  $\alpha_x$  is the cross sectional area of the orifice and  $\delta_x$  is the discharge coefficient of the valve. The parameter  $\delta_x$  takes into account all the fluid characteristics which includes losses and irregularities in the systems in such a way that the two sides of the equation balance and cancel out. Also,  $\beta_x$  is the valve opening and it is expressed as a ratio value.

At any given time, the heights of fluid in the four tanks relate to the fluid inlet rates and fluid outlet rates. Therefore, equations 1 and 2 can be combined together using the mass balances and Bernoulli's law for the QTP and apply to the four tanks in order to derive the four equations given in Equation 3, which represents the tanks 1 to 4 respectively.

$$\begin{aligned}
\frac{dh_1}{dt} &= \frac{1}{A_1} * \{-a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + (\gamma_1 k_1 u_1)\} \\
\frac{dh_2}{dt} &= \frac{1}{A_2} * \{-a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + (\gamma_2 k_2 u_2)\} \\
\frac{dh_3}{dt} &= \frac{1}{A_3} * \{-a_3\sqrt{2gh_3} + (1 - \gamma_2) * k_2 u_2\} \\
\frac{dh_4}{dt} &= \frac{1}{A_4} * \{-a_4\sqrt{2gh_4} + (1 - \gamma_1) * k_1 u_1\}
\end{aligned} \tag{3}$$

where,

$A_i$  is the cross sectional area of the  $i^{\text{th}}$  tank ( $\text{m}^2$ )

$a_i$  is the cross sectional area of small outlet valve orifice the  $i^{\text{th}}$  tanks ( $\text{m}^2$ )

$h_i$  is the water level in the  $i^{\text{th}}$  tank (m)

$\gamma_1$  and  $\gamma_2$  are the ratios in which the outputs of the two pumps 1 and 2 are split.

$u_1$  and  $u_2$  are the voltages applied to pumps 1 and 2 respectively.

$k_1$  and  $k_2$  are respective constants of the two pumps and the units are expressed in  $\text{m}^3 \text{s}^{-1} \text{v}^{-1}$ .

The subscripts 1 to 4 refer to Tanks 1 to 4 respectively. The rest of the parameters are given in Table 1. Figure 3 shows the SIMULINK representation of the MIMO coupled tank equations where the input 1 is  $u_1$ , input 2 is  $u_2$ , output 1 is  $h_1$  and output 2 is  $h_2$ . This representation of the plant will be used in simulation to test the NMPC algorithm.

The operating parameters of minimum-phase and NMP system in (Johansson 2000; Malar & Thyagarajan 2009; Srinivasarao & Subbaiah 2013) are provided in Table 2.

Table 2 Operating parameters of non-minimum-phase and minimum-phase system (Johansson 2000; Malar & Thyagarajan 2009; Srinivasarao & Subbaiah 2013)

Parameters	Minimum Phase	Non-minimum Phase
$h_1^0, h_2^0$ (m)	0.124, 0.127	0.126, 0.13
$h_3^0, h_4^0$ (m)	0.018, 0.014	0.048, 0.049
$u_1^0, u_2^0$ (v)	3.00, 3.00	3.15, 3.15
$k_1, k_2$ ( $\text{m}^3 \text{s}^{-1} \text{v}^{-1}$ )	$3.33 * 10^{-6}$ , $3.35 * 10^{-6}$	$3.14 * 10^{-6}$ , $3.29 * 10^{-6}$
$\gamma_1, \gamma_2$	0.70, 0.60	0.43, 0.34

The valve positions settings for QTP are provided in Table 3.

Table 3 Valve positions for the Quadruple Tank Process

Valves settings	Zero Location	Description
If $1 < \gamma_1 + \gamma_2 < 2$	Zero in LHP	Minimum phase (P-)
If $0 < \gamma_1 + \gamma_2 < 1$	Zero in RHP	Non-minimum phase (P+)
If $\gamma_1 + \gamma_2 = 1$	Zero at Origin	
$\gamma_1/\gamma_2$		Gives zero direction

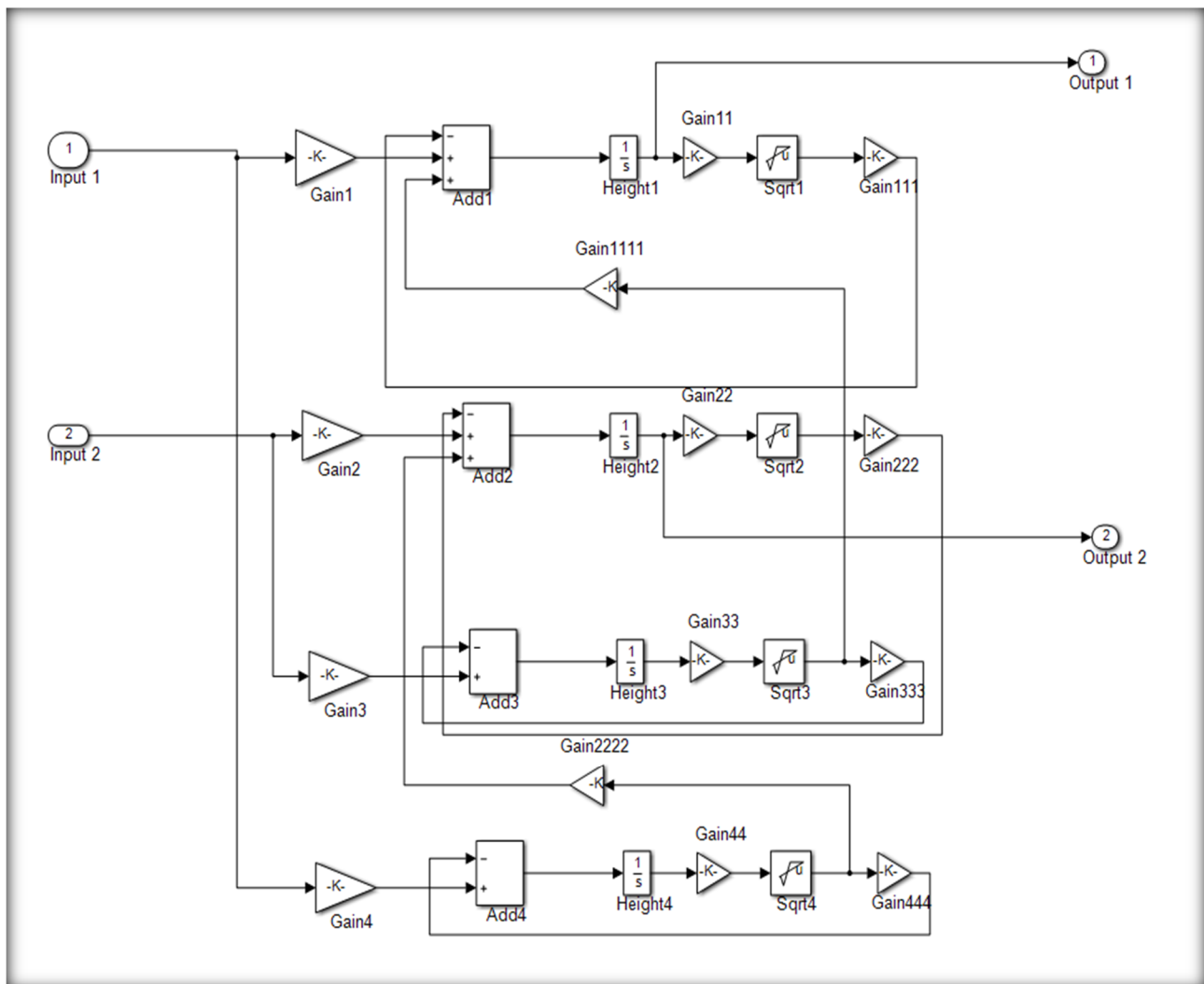


Figure 3 The SIMULINK® design of MP and NMP QTP



### 3 SYSTEM IDENTIFICATION AND MODELLING OF QTP

System identification techniques are applied in order to model the QTP. The model will be used to predict the behaviour of the QTP. The process involves applying the combinations of pseudo-random binary signal (PRBS) and uniformly distributed noise input signals to excite the SIMULINK® design model (Figure 3) for the QTP in order to generate input-output data. Before the commencement of data collection, all the four tanks have zero initial conditions, as there are no initial fluids when the open loop excitation is carried out.

Three sets of different input-output data of 2000 sample each were collected from open loop responses for both MP and NMP of the QTP with a sampling rate  $T_s$  of one second. Figures 4 (a-c) show the three data sets for the NMP open loop response data collected for analysis while Figures 5 (a-c) show the three data sets for the MP open loop response data collected for analysis. The entire open loop datasets exhibit expected behaviour since control strategy was not employed in the data gathering stage. The maximum height limits of 20cm were not exceeded and hence there are no tank overflows. Also, both tanks did not run out of fluid throughout the whole process even though both valves 1 & 2 in Figure 2 are in the open state.

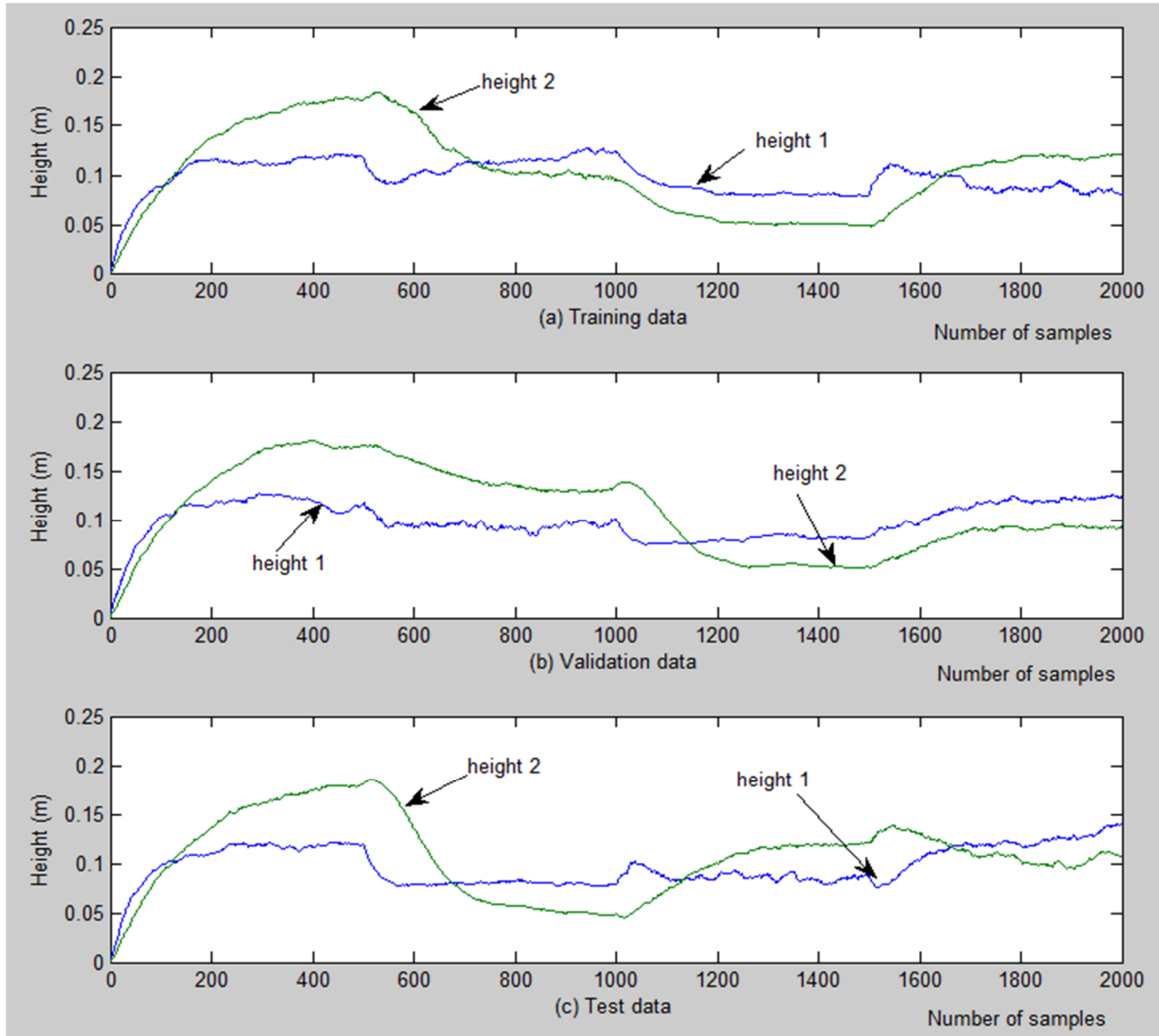


Figure 4 Non-minimum phase open loop response data collected for analysis

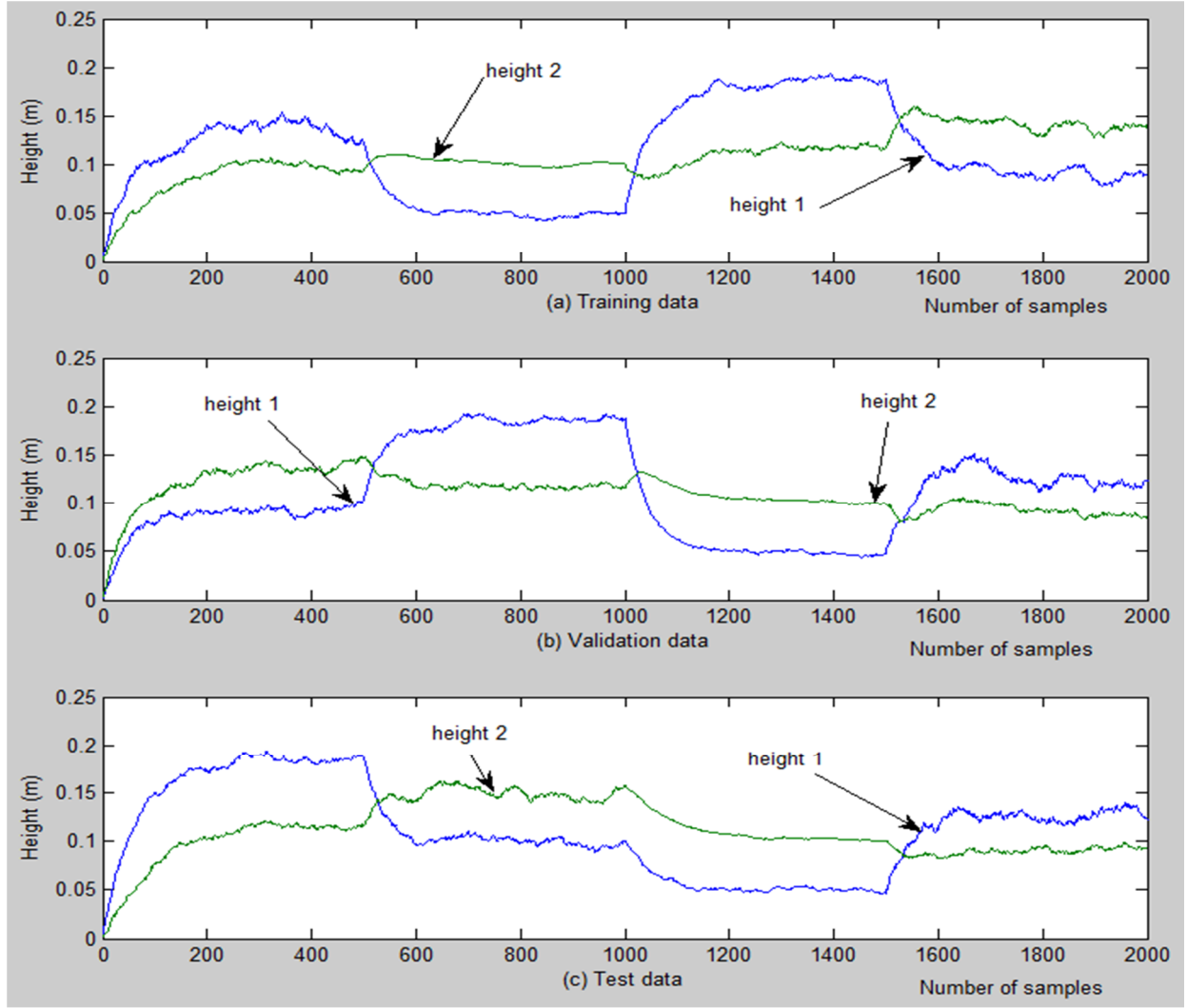


Figure 5 Minimum phase open loop response data collected for analysis

### 3.1 WNN Training with a Genetic algorithm

The WNN has wavelets functions in the hidden layer, which is also referred to as a wavelet layer. Training of a WNN involves finding the unknown weights between input to hidden layer ( $WI_{ji}$ ), hidden to output layer ( $WO_{kj}$ ), translation factor ( $b_j$ ) and dilation (expansion) factor ( $a_j$ ). For further details of WNN, the interested reader is referred to (Leavey et al. 2003).

In this work, a Morlet wavelet  $\varphi(x)$  is selected as a mother wavelet. The wavelet  $\varphi(x)$  is expressed in Equation 4 and is used as the activation function for the neurons in the hidden layers of the WNN layer instead of the sigmoidal activation function used in (Malar & Thyagarajan 2009).

$$\varphi(x) = \cos(1.75x) \exp\left(-\frac{x^2}{2}\right) \quad (4)$$

A wavelet transform allows exceptional localisation in the time domain via translation (a shifting process) and in the frequency domain via dilation (a scaling process) of the mother wavelet. The effect of this shifting and scaling process is to produce a time-frequency representation of the signal. The wavelet basis functions are shifted in time domain to maintain the same number of oscillations and its frequency scaled in amplitude to maintain energy. Owing to their capability to localise in time, wavelet transforms readily

lend themselves to non-stationary signal analysis. The architecture of a MIMO WNN of a QTP is shown in Figure 6 where  $y_1$  and  $y_2$  are the respective heights of tank 1 and 2.

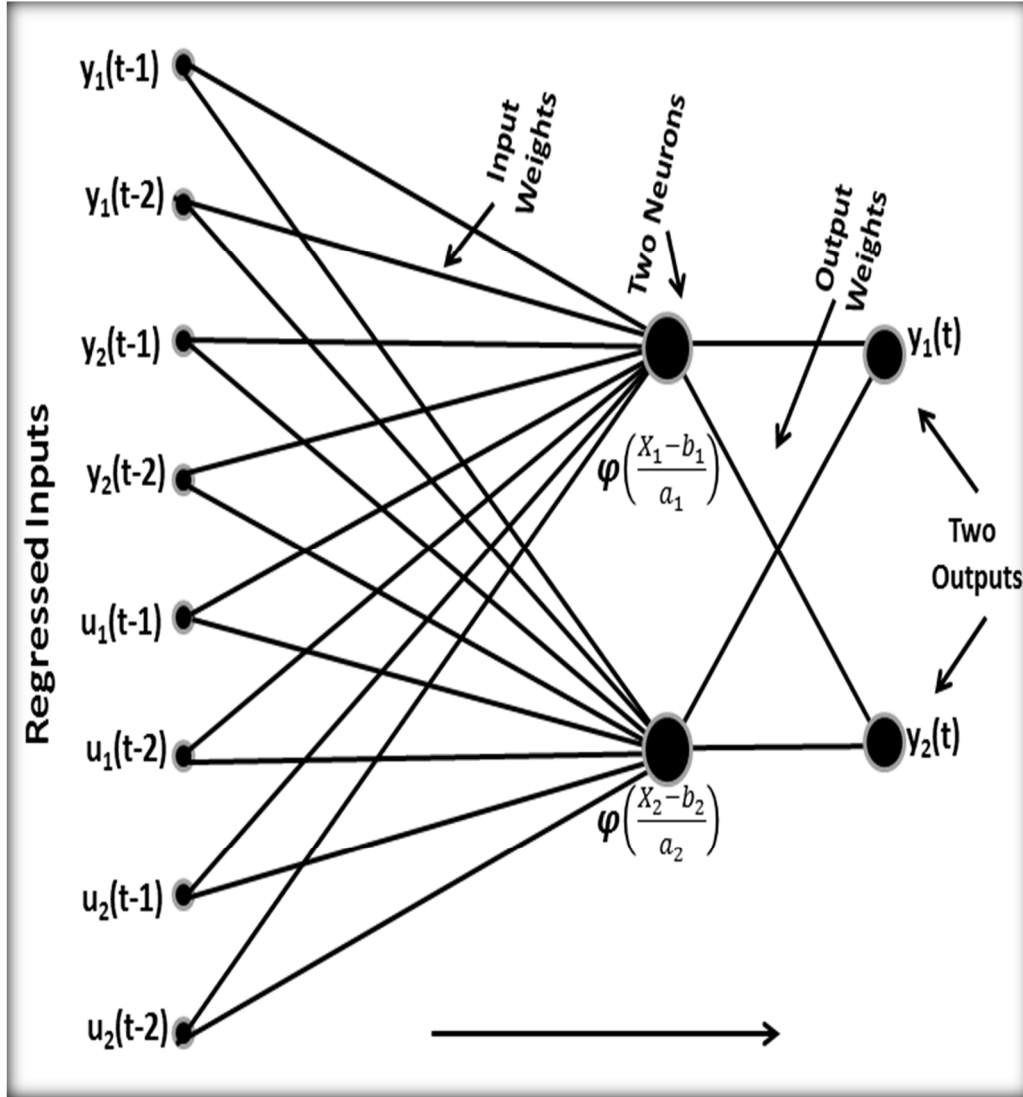


Figure 6 Structure of a proposed MIMO WNN model

GA is a stochastic global search and standard optimisation method that operates on population of potential solutions by applying the principle of survival of the fittest to evolve a better candidate for a particular solution. Here a GA is used to obtain the best population that WNN weights that minimises the objective function. In this paper, real-valued genes are used to represent population chromosomes as it provides faster optimisation as chromosomes conversion to its phenotypes prior to their function implementation is not necessary. The parameters used and the working principle of a GA are based on available literature (Pham & Karaboga 2000; Zbigniew Michalewicz 1996). The weight optimisation uses procedures called selection, crossover, and mutations. The optimisation parameters are given in Table 4 are derived through some heuristic trials and expert knowledge.

Table 4 WNN and GA Parameters

Parameter Description	Parameter Values	
	Cited Literature (Malar & Thyagarajan 2009)	Proposed work
Crossover Ratio Value	n/a	0.65
Mutation Ratio Value	n/a	0.05
Population size	n/a	200
Number of generations	10000	1000
Training Algorithm for WNN	Levenberg-Marquardt	Genetic Algorithm
Activation Function for NN	Sigmoidal	Morlet Wavelet
Number of Input Nodes	6	8 (Figure 6)
Number of Output Nodes	2	2 (Figure 6)
Number of Hidden layers	1	1 (Figure 6)
Number of Hidden layers nodes	7	2 (Figure 6)
Input delay, Output delay		2, 2 (Figure 6)
Architecture		Multi Layer Perceptron (MLP)

The population is applied to the objective function and the individual performances obtained are ranked based on the performance of how the errors between the obtained model and the training data (target) are reduced using Equation 5.

$$mse = \sum_{n=1}^N \sum_{k=1}^S \frac{(y_{model_k}^n - y_{target_k}^n)^2}{2 * N} = \sum_{n=1}^N \frac{e^{n^2}}{2 * N} \quad (5)$$

where

$y_{model}$  is the output of the model being trained

$y_{target}$  is the real output of the training data

$N$  is the number of samples to be trained

$S$  is the number of outputs

$L$  is the number of regressed inputs in the NN structure, and

$e$  is error difference between the model and the target.

The fitness value is calculated using equation 6

$$fitness(x_i) = \frac{1}{x_i + 1} \quad (6)$$

where  $x_i$  is the value of the performance in the objective function

### 3.2 WNN Modelling Results

The results of the WNN modelling are given in Table 5. The MSE for both MP and NMP are calculated using equation 5. The MSEs were calculated for the training, validation and the test data. However, MP gave lower MSEs as compared to NMP in all the three data collected. The low MSE values show that both designed Black box models can conveniently fit the three data sets. The mean and the variance of the two-input signals for the three data samples used to excite the plant are also given in Table 5. In order to further test the effectiveness of the design WNN model, the various input data are cross –correlated with the

prediction errors. The autocorrelation is the cross-correlation of the prediction errors with itself at different points in time; this corresponds to finding the similarities between the prediction errors as functions of the time lags between them.

Table 5 Modelling results for both MP and NMP

Performance Function (Outputs)	Data One Training		Data Two Validation		Data Three Testing	
NMP - MSE (m <sup>2</sup> )	2.02*10 <sup>-4</sup>		2.22*10 <sup>-4</sup>		2.15*10 <sup>-4</sup>	
MP - MSE (m <sup>2</sup> )	9.80*10 <sup>-7</sup>		1.00*10 <sup>-6</sup>		9.97*10 <sup>-7</sup>	
INPUTS	Input 1	Input 2	Input 1	Input 2	Input 1	Input 2
NMP-Mean(volts)	2.8679	2.7890	2.8504	2.8667	2.8808	2.8242
NMP-Variance(volts)	3.3891	2.0024	3.5958	1.8829	3.3988	2.0923
MP-Mean(volts)	2.8383	2.8349	2.8305	2.8261	2.8486	2.8122
MP-Variance(volts)	3.4487	1.9684	3.4263	2.0065	3.4111	2.0242

For a perfect prediction model, there should only be one non-zero value of the autocorrelation function, and it should occur at the zero lag. This would mean that the prediction errors were completely uncorrelated with each other. The aim is to make sure that the model is good enough so that there would be no correlation within the prediction error. In this work, it is however, admissible if the error correlation falls within the 10% confidence interval limit around zero.

Figures (7 & 8) are the autocorrelation and correlation plots for both outputs (1 & 2) for the NMP and MP respectively and fall within accepted confidence interval.

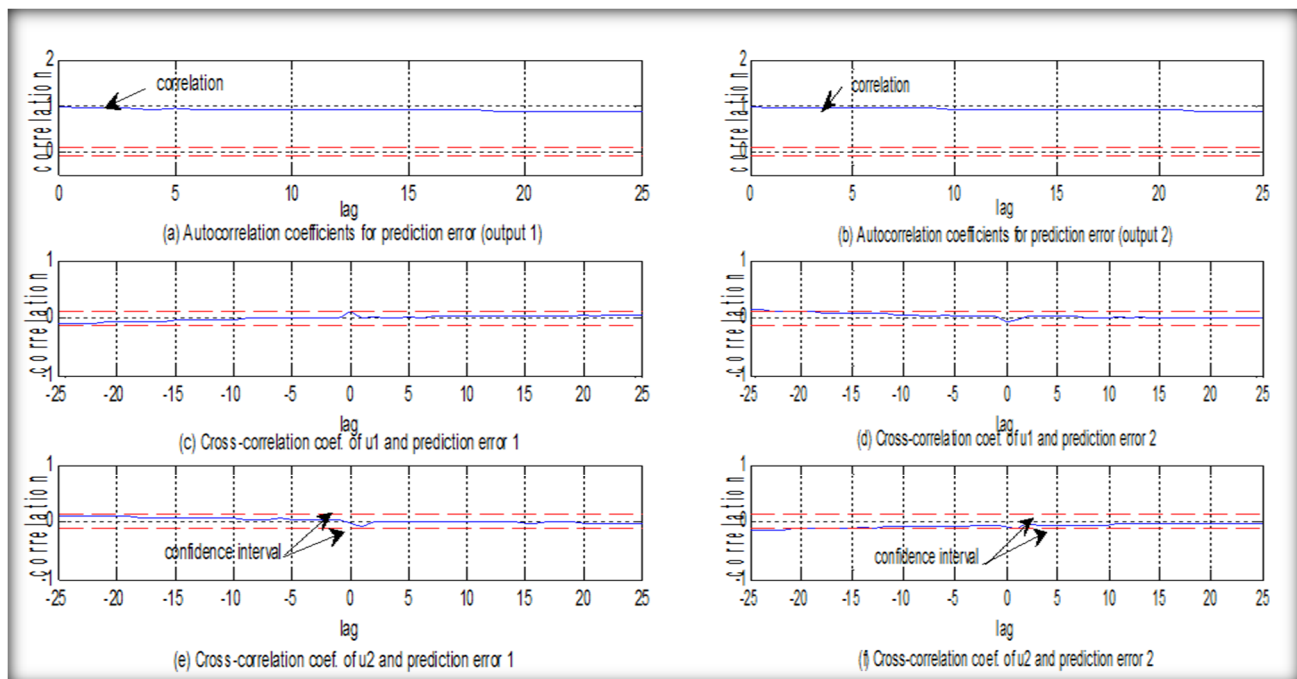


Figure 7 WNN Non-Minimum Phase system Correlation results

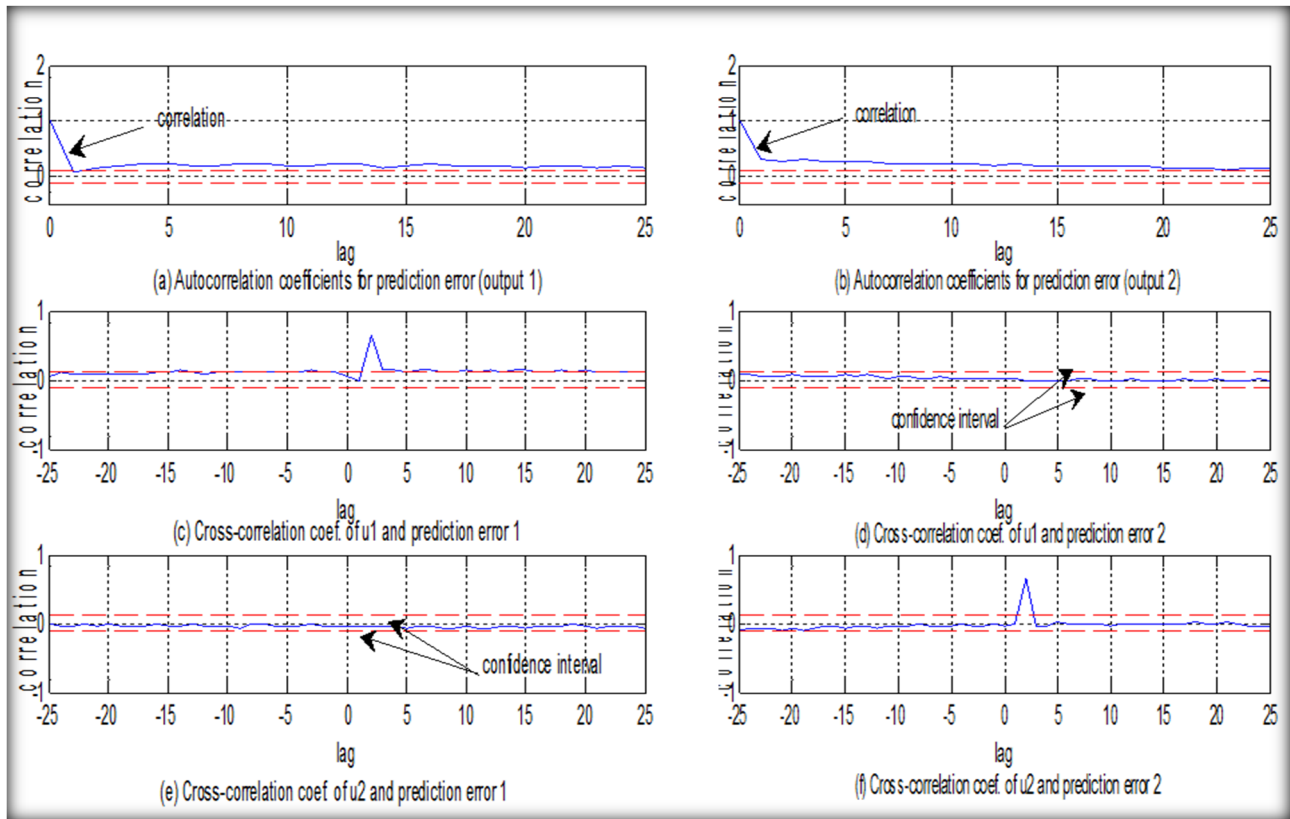


Figure 8 WNN Minimum Phase system Correlation results

The independent (test) data was used with the designed models to test the output height for both Tank 1 and Tank 2 respectively. The graphical results for the predicted outputs and the real plant outputs for the NMP are shown in Figure 9 while Figure 10 shows the results for the MP system. The noise exhibited in Figure 9 can be attributed to the high sensitivity and inherent internal instabilities in the problematic NMP whereas the MP system shows a better result.

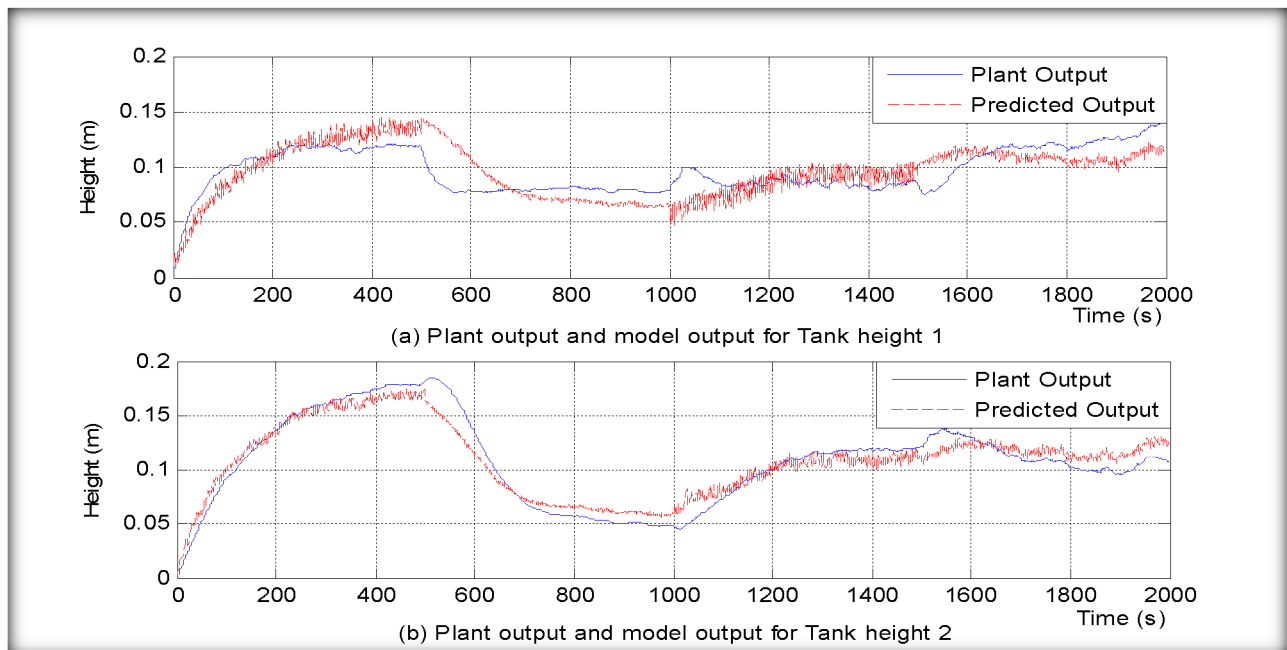


Figure 9 NMP WNN Model prediction for the independent Test data

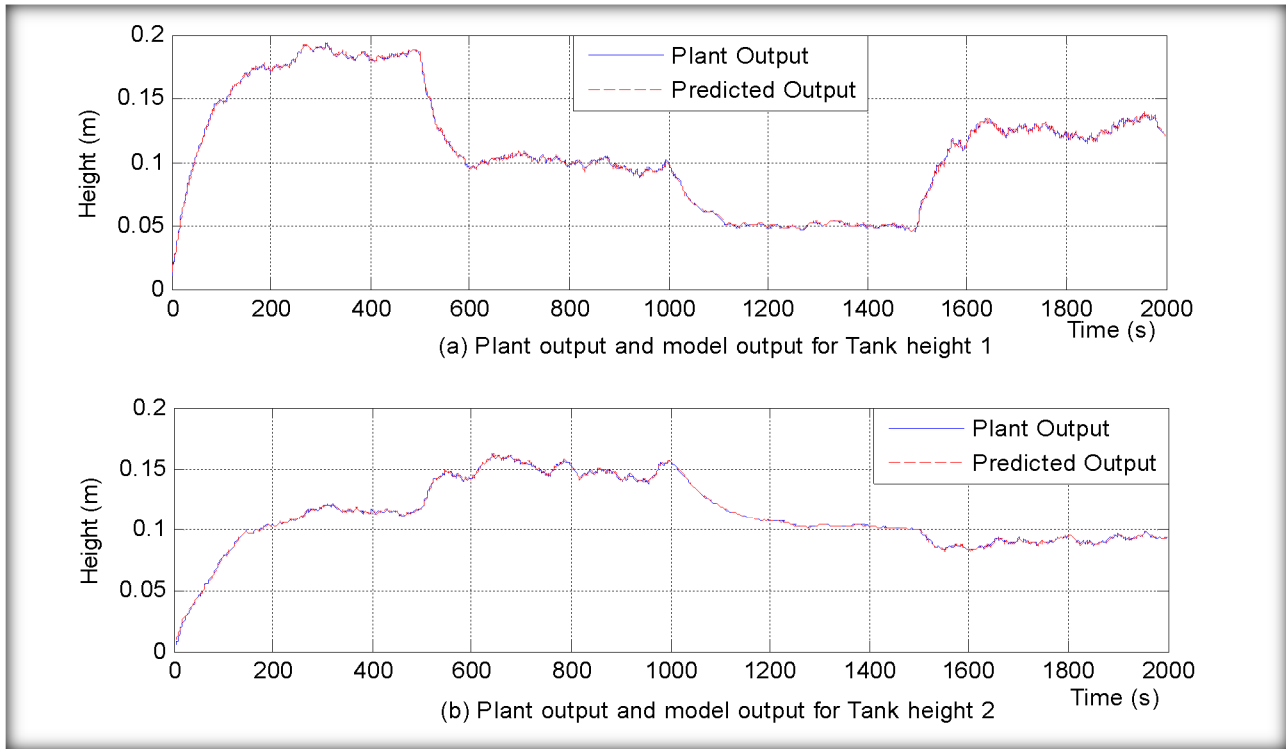


Figure 10 MP WNN Model prediction for the independent Test data

Furthermore, the results of WNN modelling is compared with the three other results (NN, Fuzzy and Neuro-Fuzzy modelling techniques) from Malar & Thyagarajan (2009) as shown in Table 6. The work of Malar & Thyagarajan (2009) deals on modelling of the QTP using soft computing approach and there is no control strategy involved. However, the work of Johansson (2000) uses both linearised model and physical equation for the PI control strategy. The results of the PI control strategy for QTP (Johansson 2000) will be compared against the WNN-NMPC strategy results in Section 4.1 It is concluded from the table that WNN has the least MSE values with more than 50% reduced error in modelling as compared with the best result in the published literature. WNN is therefore a better modelling approach than the other listed soft computing modelling techniques. The WNN will now be used in the NMPC strategy in next section.

Table 6 Modelling results comparison with cited literatures (Malar & Thyagarajan 2009)

Modelling Approach	Method of Acquisition	MSE (Training data) (m <sup>2</sup> )		MSE (Validation data) (m <sup>2</sup> )	
		MP	NMP	MP	NMP
<b>ANN modelling (Malar &amp; Thyagarajan 2009)</b>	Optimisation learning	2.00*10 <sup>-3</sup>	2.30*10 <sup>-3</sup>	8.60*10 <sup>-3</sup>	9.00*10 <sup>-3</sup>
<b>Fuzzy modelling (Malar &amp; Thyagarajan 2009)</b>	Knowledge based	2.80*10 <sup>-3</sup>	3.30*10 <sup>-3</sup>	7.30*10 <sup>-3</sup>	7.70*10 <sup>-3</sup>
<b>Neuro-Fuzzy Modelling (Malar &amp; Thyagarajan 2009)</b>	Knowledge based + learning	1.20*10 <sup>-3</sup>	1.10*10 <sup>-3</sup>	2.30*10 <sup>-3</sup>	5.50*10 <sup>-3</sup>

<b>WNN Modelling [Proposed]</b>	Genetic algorithm	$9.80 \times 10^{-7}$	$2.02 \times 10^{-4}$	$1.00 \times 10^{-6}$	$2.22 \times 10^{-4}$
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#### 4 NON-LINEAR CONTROL STRATEGY FOR THE QTP

This paper applies the non-linear control strategy proposed by the authors (Owa et al 2013) to a QTP both with NMP and MP. NMPC is an advanced control strategy in which the current and optimised manipulated control input is applied to the real plant. A finite prediction horizon open-loop optimal control problem is derived by obtaining a real time non-convex solution online at each sampling instant using a non-linear MIMO model for prediction. The optimisation produces an optimal control sequence where the first control action is applied to the plant. The schematic picture of the whole control strategy is shown in Figure 11. The predictor's task is to predict the plant's outputs based on the regressed input-output at every instant. This is carried out for different and random control moves within a prediction horizon range. The value of the prediction horizon should always be more than the control horizon. In this paper, NMPC strategy was implemented by using a GA to solve and minimise the complex non-linear optimisation cost function (see Equation 7) at every sampling instant.

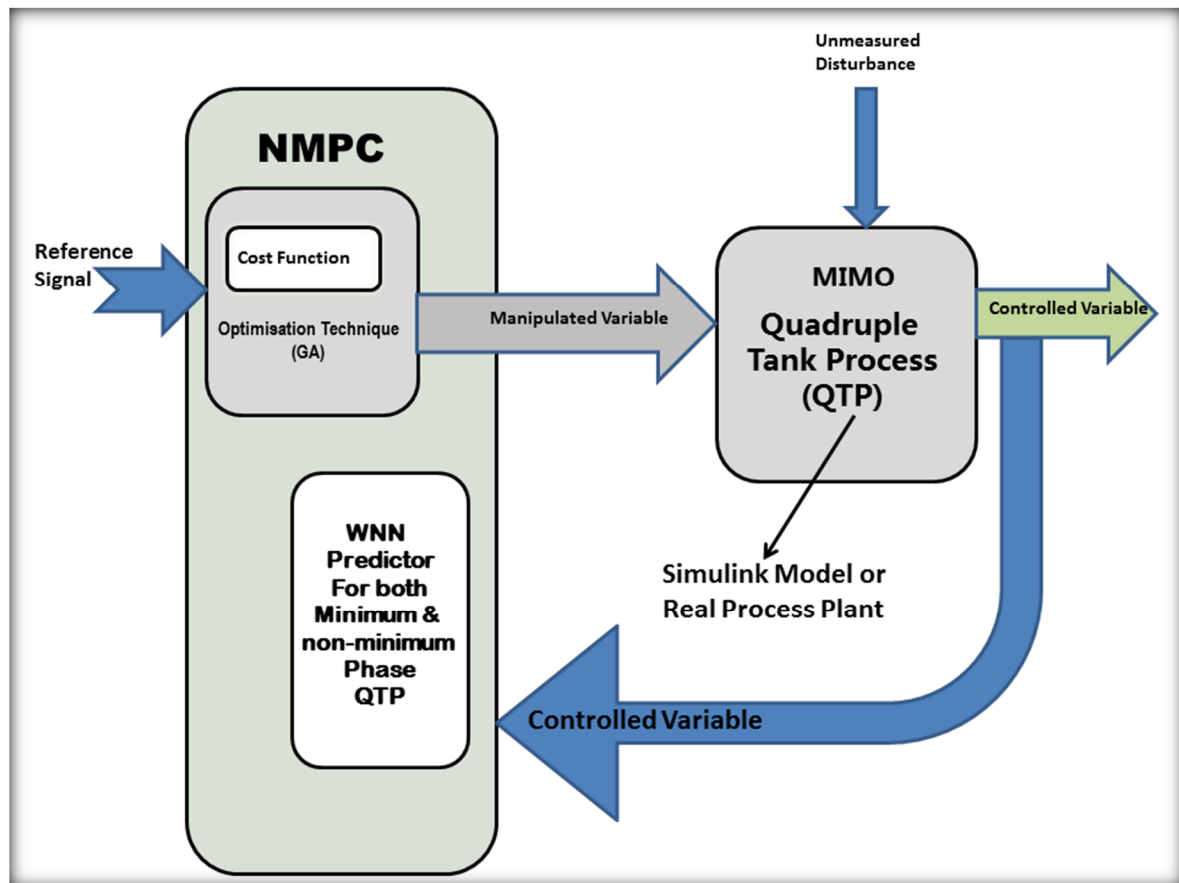


Figure 11 The structure of NMPC Strategy with GA Optimisation

This is used to obtain the optimised optimum manipulated control inputs vector that give the least error between the trajectories reference signals and the predicted output and the overall controller efforts used in terms of the voltage supplied to the pump.



$$J(\theta) = \left\{ \sum_{i=1}^p \left( \sum_{j=1}^{n_y} |w_{i+1,j}^y (y_j(k+i+1|k)) - r_j(k+i+1)|^2 + \sum_{j=1}^{n_u} |w_{i,j}^{\Delta u} \Delta u_j(k+i|k)|^2 \right) \right\} \quad (7)$$

The first summation on the RHS of Equation 7 represents the error in the reference valve and the prediction value while the second summation terms denote the rate of change in the controller actions which are the previous and current manipulated variables ( $\mathbf{u}_{j+1} - \mathbf{u}_j$ ). These are then calculated from the GA optimised manipulated variables. These parameters are the bounded random generated population in the GA. The values  $y_j$  and  $r_j$  stand for plant output and the trajectory reference value respectively while  $w^i$  denotes the weighting value. The parameters in Equation 7 that represent the random generated population in the GA are the manipulated variables in vector form, which represents the controller actions from pump 1, and pump 2 respectively. Termination measures were incorporated to abort the optimisation process once a defined sampling time is exceeded. This might sometimes lead to convergence to some optimal/sub optimal solution within the sampling time range. The NMPC strategy is designed in such a manner that the rate of change of the manipulated control input is controlled in small steps to prevent a substantial fluctuation in controller activities.

The population of GA is created such that the difference between consecutive control horizons is not more than a prescribed value of 1.5 volts within the voltage range between 0 and 10 volts. The parameters used in the NMPC strategy were selected after trials and errors as follows: 0.5 for crossover rate, 0.05 for mutation rate, control horizon 2, prediction horizon 5, population size 100 and maximum number of generations equal to 25 (Pham & Karaboga 2000; Zbigniew Michalewicz 1996).

#### 4.1 Control Strategies Results

The results of both NMPC and PID strategies are implemented to show the effectiveness of the proposed approach compare to conventional one. Two different ranges (upper and lower) are considered for control strategy for both the NMP and the MP system. In the MP upper range, the reference level of tank 1 and tank 2 are set in steps between 0.2m and 0.16m while the lower range are set to between 0.18m and 0.0125m. Similarly, the reference level of tank 1 and tank 2 for the NMP upper range are set between 0.2m and 0.05m while the lower range are set to range between 0.18m and 0.025m. There are two performance indices considered here to evaluate the performance of the control strategies: the MSE in Equation 8 and the ACE in Equation 9. The MSE is derived by adding all the squares of the error differences between the plant output and reference trajectory values for the two outputs and then divides the result by the total number of samples involved.

$$MSE = \frac{\sum_{j=1}^N (y1_j^r - y1_j^p)^2 + \sum_{j=1}^N (y2_j^r - y2_j^p)^2}{N} \quad (8)$$

In Equation 8, superscripts  $p$  and  $r$  stand for plant output and the reference value respectively while  $N$  represents the total number of samples. The ACE is defined as the summation of the squares of all the manipulated control variables inputs to the plant divided by the total number of samples and this is expressed as:

$$ACE = \frac{\sum_{j=1}^N u1_j^2 + \sum_{j=1}^N u2_j^2}{N} \quad (9)$$

The performance of the control strategies are analysed in simulation by utilising the MIMO QTP SIMULINK® model (Figure 3).

Figures 12 & 13 show the MP control strategy results for WNN-NMPC and PID controllers respectively. Equation 10 is used to determine the response  $u(t)$  of the PID control strategy where  $e(t)$  = error,  $KI_p = K2_p$

$=100$ ,  $KI_i = K2_i = 0.5$  and  $KI_d = K2_d = 5$  are the respective gains of the proportional, integral and derivative actions of the strategy.

$$u(t) = \overbrace{K_p e(t)}^{\text{Proportional}} + \overbrace{K_i \int_0^t e(\tau) d\tau}^{\text{Integral}} + \overbrace{K_d \frac{d}{dt} e(t)}^{\text{Derivative}} \quad (10)$$

The two figures show that both control strategies are able to track the specified set points reference trajectories efficiently for a period of 250 seconds. For the upper range control scenarios (a & b in the figures), the WNN-NMPC strategy has MSE value of  $2.5 \times 10^{-3} \text{ m}^2$  and ACE value of  $49.72 \text{ v}^2$  whereas PID strategy has a MSE value of  $2.6 \times 10^{-3} \text{ m}^2$  and ACE value of  $51.97 \text{ v}^2$  (refer to Table 7). The combinations of both MSE and ACE are used in this work in order to further validate and reinforce the results obtained. MSE helps to determine how effective and efficient is the tracking of the set reference trajectory while the ACE is utilised to determine how much energy is expended or the effort involved in making the trajectory tracking possible; therefore, the lower the values of both MSE and ACE, the better the control strategy. The results for the lower range control (c & d in the figures) are also similar to the upper range. The MSE and the ACE show lower error results from WNN-NMPC strategy and is better than the PID control strategy that possesses more overshoot than the WNN-NMPC strategy (refer to Figures 12 & 13).

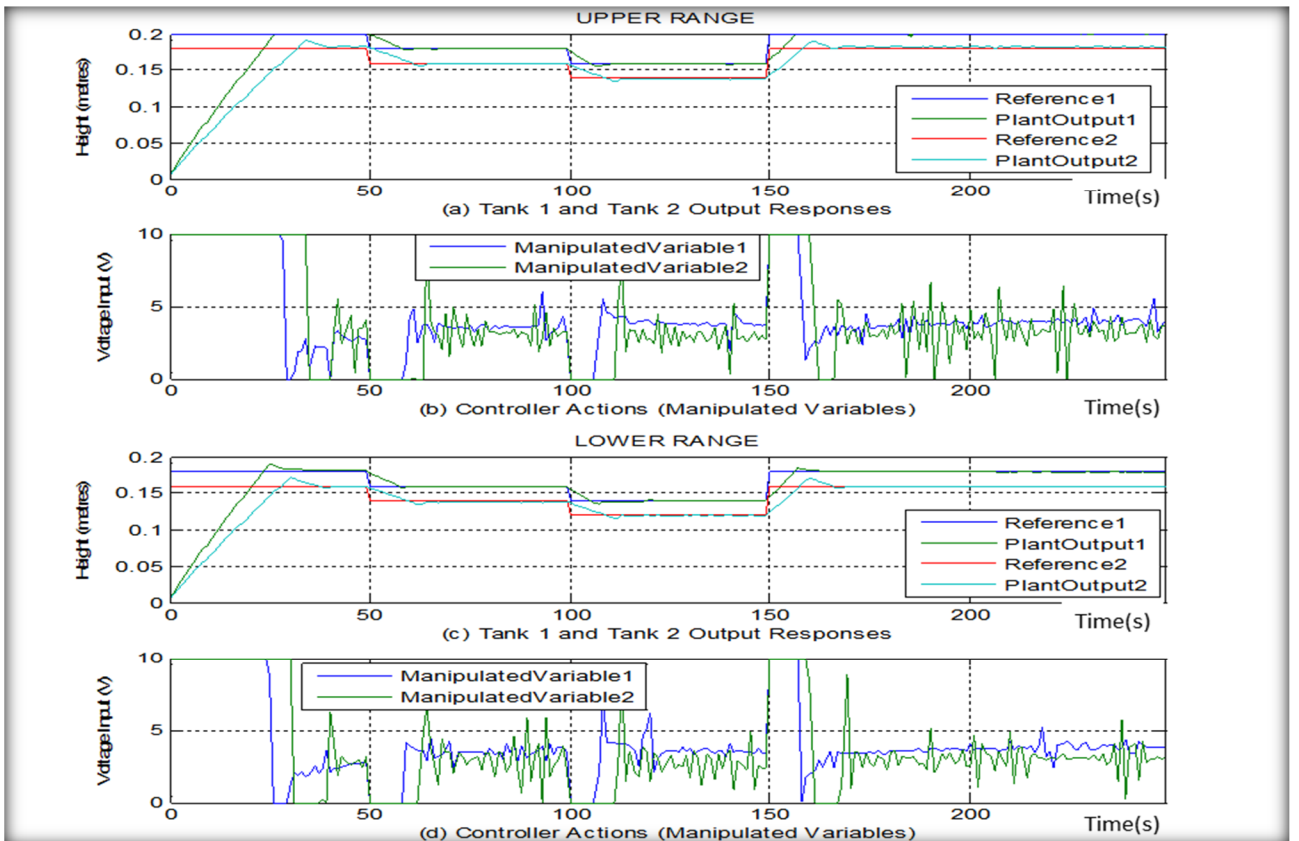


Figure 12 Different level controls for MP WNN-NMPC strategy

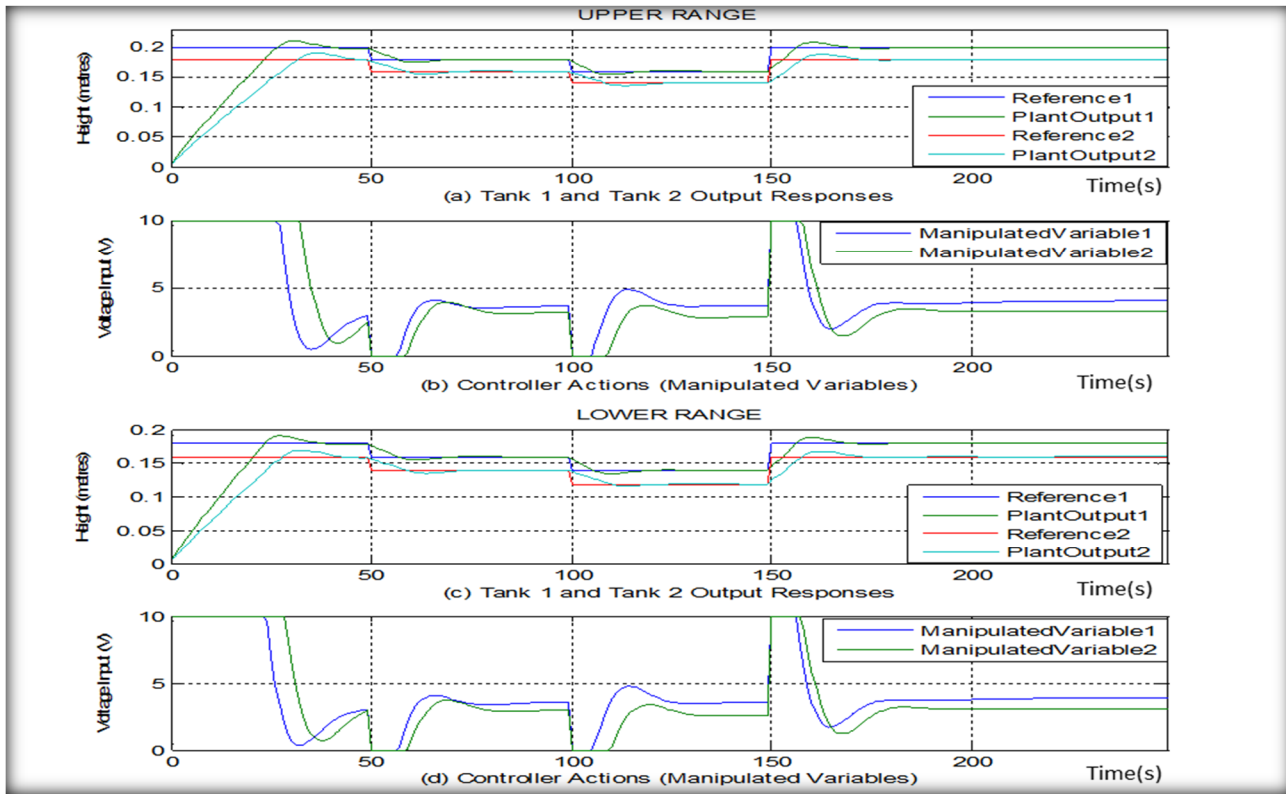


Figure 13 Different level controls for MP PID strategy

Furthermore, Figures 14 & 15 show the NMP control strategy results for WNN-NMPC and PID strategy respectively. This is a more challenging situation because it is harder to control NMP systems than the MP system (Johansson 2000). The two figures show that the proposed WNN-NMPC strategy could track the specified set points reference trajectory efficiently. For the upper range control scenarios, the WNN-NMPC strategy has MSE value of  $4.3 \times 10^{-3} \text{ m}^2$  and ACE value of  $40.58 \text{ v}^2$  while PID strategy has a MSE value of  $8.5 \times 10^{-3} \text{ m}^2$  and ACE value of  $43.64 \text{ v}^2$  (refer to Table 7). Both the MSE and the ACE obtained here show the superiority of the WNN-NMPC over the PID strategy. The complex nature of the NMP makes it difficult for the PID control strategy to perform effectively. Moreover, the PID control strategy possesses pronounced overshoots than the WNN-NMPC strategy (refer to Figures 14 & 15).

Table 7 NMPC Strategy results for both MP and NMP

<u>Scenarios</u>	<u>Minimum Phase (MP)</u>		<u>Non-Minimum Phase (NMP)</u>	
	<u>MSE (<math>\text{m}^2</math>)</u>	<u>ACE (<math>\text{v}^2</math>)</u>	<u>MSE (<math>\text{m}^2</math>)</u>	<u>ACE (<math>\text{v}^2</math>)</u>
<b>UPPER RANGE</b>				
<b>WNN-NMPC</b>	$2.5 \times 10^{-3}$	49.72	$4.3 \times 10^{-3}$	40.58
<b>PID</b>	$2.6 \times 10^{-3}$	51.97	$8.5 \times 10^{-3}$	43.64
<b>LOWER RANGE</b>				
<b>WNN-NMPC</b>	$1.8 \times 10^{-3}$	45.07	$3.2 \times 10^{-3}$	37.15
<b>PID</b>	$1.9 \times 10^{-3}$	47.06	$5.7 \times 10^{-3}$	38.19

The MP results for the NMPC and PID are provided in Figure 12 and 13 respectively while NMP results for the NMPC and PID are provided in Figure 14 and 15 respectively. In all the figures, there are different ranges of control. However, the focus is on the challenging NMP (figures 14 and 15) controls with range from 20cm (Maximum height of the tank) at the upper range to 2.5cm at the lower range. Hence, the

difference in the results can be more appreciated by comparing figure 14 (NMPC) and figure 15 (PID). The major highlights and differences of the better performances of the NMPC over PID strategies for the NMP system are as follows:

- (i) A much reduced MSE and ACE were obtained for both upper and lower ranges as shown in Table 7.
- (ii) The PID strategy finds it difficult to follow the trajectory tracking (Figure 15) unlike the NMPC strategy (Figure 14).
- (iii) The characteristic of the NMP system makes the PID controller tends to instabilities; figures 15 (b and d) shows the voltage input (manipulated variable) of tank 2. The responses of the voltage input in the NMPC are more stable and better.
- (iv) The rise time of PID is higher (between 80 and 100 seconds), the overshoot is out of range and it hardly settled in time. Conversely, the NMPC has a lower rise time of about 50 seconds.
- (v) The PID strategy has a steady state error whereas the NMPC has zero steady state error. In addition, the transient state of the NMPC strategy is more stable than that of the PID strategy.

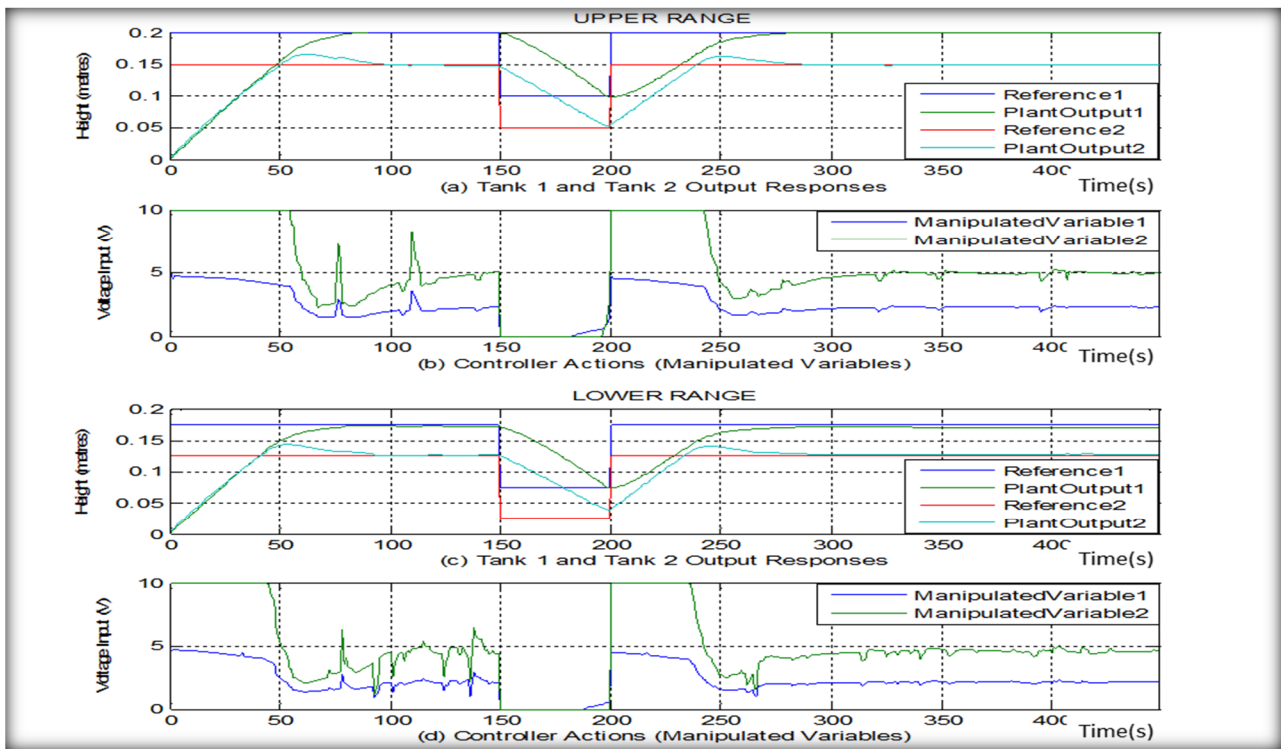


Figure 14 Different level controls for NMP WNN-NMPC strategy

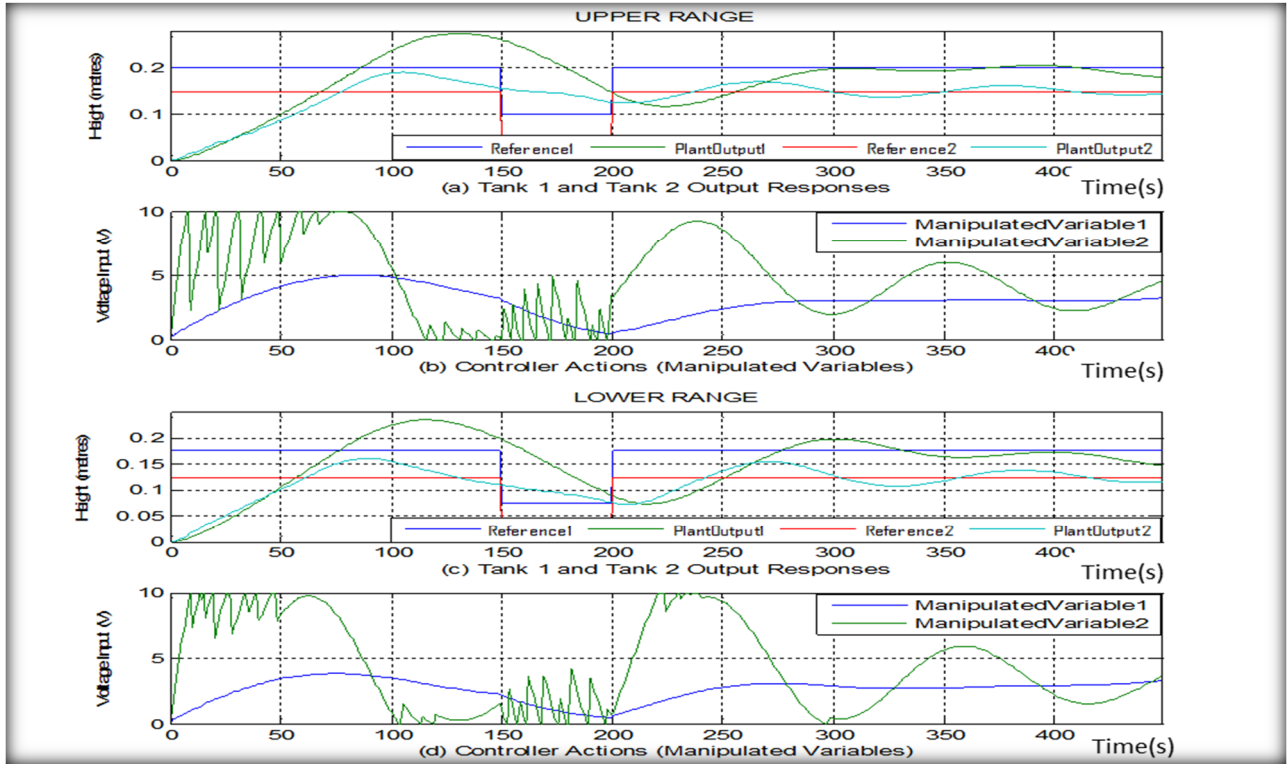


Figure 15 Different level controls for NMP PID strategy

Furthermore, the results indicate that the proposed WNN-NMPC strategy is more efficient than the PI control strategy results shown in the cited literature (Johansson 2000) for the NMP cases. For the MP, the proposed approach has zero overshoots while Johansson (2000) have equal overshoots of 10% in both outputs. Moreover, Johansson (2000) has twice the settling time than the proposed scheme.

In the NMP case, the proposed scheme also have zero per cent overshoots for both outputs while the second output of Johansson (2000) have more than 50% overshoot. Here, the settling time of Johansson (2000) is more than ten times longer than the proposed scheme for both outputs of the QTP. The PI control strategy parameters for Johansson (2000) are as follows: MP ( $K_1$ ;  $T_{i1}$ ) = (3.0; 30) and ( $K_2$ ;  $T_{i2}$ ) = (2.7; 40) whilst the NMP ( $K_1$ ;  $T_{i1}$ ) = (1.5; 110) and ( $K_2$ ;  $T_{i2}$ ) = (0.12; 220). The proposed scheme is also able to cover wider region of control operations than the PI control strategy reported in Johansson (2000).

## 5 CONCLUSIONS

Model-Plant mismatch (MPM) is a significant problem in the process industries. MPM and parameter discrepancies may be as a result of wear and tear, equipment degradation and many years of usage. Rather than an absolute reliance on the available mathematical dynamic QTP equations which are prone to uncertainties in parameter assumptions, a WNN nonlinear model is designed with the input-output data using a system identification approach. The first principle QTP model was built using SIMULINK design and this represents the QTP plant during the simulation study. In order to handle the difficulties in network training, a GA is employed in order to avoid being trapped in local minimum solutions. This paper proposes a novel model based WNN-NMPC strategy for MP and challenging problems of NMP of the MIMO QTP. The proposed method uses WNN for prediction at every sampling time which is very essential for plants with uncertain behaviours. Controllers for NMP systems are always difficult to design. There is an advantage of WNN-NMPC over PID controller because the latter does not have the predictive ability. Moreover, wavelets have good predictive ability since they can be localised in both time and frequency

domain. Simulation results have shown that the proposed WNN-NMPC strategy is able to control efficiently the difficult NMP system of the QTP for longer period of time than the results published in the literature. Furthermore, the GA is efficient in online non-convex optimisation and set point tracking for both upper and lower range of NMPC control strategies as compared with the results in cited literatures. The whole strategy is well suited for more complex and advanced MIMO chemical processes with varying interaction rates.

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