# Integrated Numerical Modelling System for Extreme Wave Events at the Wave Hub Site

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## **Abstract**

This paper examines an extreme wave event which occurred during a storm at the Wave Hub site in 2012. The extreme wave of **9.57** m height was identified from a time series of the heave data collected by an Oceanor Seawatch Mini II Buoy deployed at the site. An energy density spectrum was derived from this time series and then used to drive a physical model, which represents the extreme wave at 1:20 scale in Plymouth University's new COAST Lab. The NewWave technique was used to define the input to the physical model. The experiment is reproduced in a numerical wave tank using the fully nonlinear CFD library OpenFOAM® and the wave generation toolbox waves2Foam. Results are evaluated, and issues regarding the predictions of a numerical model that is driven by the NewWave input signal are discussed. This study sets the basis for further research in coupling field data, physical modelling and numerical modelling in a more efficient and balanced way. This will lead to the new approach of composite modelling that will be implemented in future work.

Keywords: Rogue wave, Storm event, Wave Hub, OpenFOAM®, waves2Foam, NewWave, Integrated modelling

#### Introduction

One of the most promising renewable energy resources, yet immature in development, is ocean wave energy. The available wave energy density is greatest offshore, however this is also where wave energy converters (WEC) are exposed to the harshest environmental conditions. When trying to ensure the survivability of offshore structures, attention frequently focuses on extreme wave events, referred to as freak or rogue waves. These waves are single waves that are larger and steeper than expected for a given sea state. They also are characterized by a high crest between two deep troughs. Numerous mechanisms have been proposed for the generation of these extreme waves, including: wave-wave interaction, wave-current interaction, bathymetry, wind effect, self-focusing instabilities and directional effects (Zhao, 2010). However, a consensus on generation mechanisms has yet to be reached (Dysthe et al., 2009).

Extreme waves are usually considered to be rare events as they depend on synchronisation of many factors. However, despite their low probability and uncertainty of occurrence, they can have a catastrophic effect on ships, offshore platforms and wave energy converters and have become a real threat to the ocean industry, indicating a requirement for further and more intensive study (Zhao, 2010).

In this work, an extreme wave event was identified from a storm which occurred at the Wave Hub site, located 16 kilometres off the north coast of Cornwall. The site has an average depth of 55m, and covers an 8 km² area in one of the world's most energetic wave climates. The Wave Hub project provides offshore grid-connected infrastructure for the demonstration of full scale WEC technologies in real sea conditions (Wave Hub, 2013).

The need for more profound understanding of the formation and the hydrodynamics of rogue waves is of vital importance. The complexity of modelling extreme waves, either physically or numerically, arises due to the highly non-linear transient character of such an event within an irregular or multi-frequency sea-state. Effective numerical modelling (NM) of extreme waves requires random wave simulations that demand very long computational time in order to capture near-extreme events. Concurrently, physical modelling (PM) is limited by the wave tank length, which restricts the generation of a full-scale extreme wave with long time evolution that allows all the desired nonlinear interactions of random events to take place. An alternative, easy way to generate extreme waves is by making use of energy focusing (Zhao, 2010). This can be achieved by adjusting the wave phases of the spectrum components of the wave group, so that all linear wave components are superposed to focus simultaneously at a given location and time (Westphalen et al., 2008). Applying NewWave theory (Tromans et al., 1991) the focused wave represents the average shape of an extreme wave profile consistent with a random process with a specified energy spectrum. Focusing is considered to occur when the wave group becomes most compact in space and the local energy density highest (Ning et al., 2009).

Reproducing the event using physical modelling (PM) and then numerical modelling (NM) has to take into account all constraints and advantages linked with each technique. One can say that the greatest advantage of the PM is that it reproduces the natural interactions taking place, including all the nonlinear effects. On the other hand, PM is expensive, has inherent scale effects, and provides a small data set relatively to NM. On the contrary, NM provides high density data across the whole domain, such as pressures and velocities, which are not possible to extract at the same density from a physical tank test. A detailed SWOT analysis of PM and NM proves that both are developing and improving, NM still relies on physical models for its development, and yet their strengths are often complementary (Sutherland and Barfuss, 2012). A new approach in modelling has recently emerged: composite modelling (CM) is defined as "the integrated and balanced use of physical and numerical models" (Deliverable, 2009). The principle of composite modelling consists in subdividing a very complex and complicated problem into several simple and more easily tractable processes which can be described by the most appropriate methods in order to get the most reliable process models, including physical and validated numerical and analytical models (Oumeraci, 1999). CM seems to be a very promising technique and provides a unique opportunity for researchers and engineers to understand the uncertainties and limitations of both the PM and the NM, however it is still in its infancy (Sutherland and Barfuss, 2012).

In this work, effort was put into utilizing the strengths of each technique (PM and NM) in order to get the best outcome with relatively less uncertainty. The input signal for the physical wave paddles is derived from the energy spectrum produced by the field buoy data using the NewWave formulation. In the NM, instead of simulating the whole physical tank, local surface elevation measurements were used from the physical tank to drive the numerical solution to a point closer to the focusing position. This means that only a small region of the tank has to be simulated, making the computation highly efficient and saving considerable

computational time (Ning et al., 2009). Furthermore, the passive absorbing mechanism of the relaxation zone at the end of the numerical wave tank (NWT) ensured that no reflections are taking place in the NM, something unlikely to be achieved in PM.

The scope of this study is to present a robust technique which includes analysis of the field data, and their reliable replication using both physical and numerical modelling. The aim is to prepare reliable tools for further studies using the principles of CM, considering the current work as a first but vital step on the way to CM. Future research of WECs' survivability in the offshore environment will be carried out, especially for the Wave Hub site, as well as for other areas of possible instalment of WECs. These include the regions of Cornwall and Finistère, the island communities of le Parc Naturel Marin d'Iroise and the Isles of Scilly, that are suggested by the MERiFIC Interreg European project for future adoption of marine energy.

# **Field Data and Offshore Wave Climate**

The use of field data is of great importance as it gives a realistic view of the magnitude and the probability of occurrence of extreme waves. Using the spectrum from real sea data in PM and NM, ensures the generation of an event which is consistent in its characteristics with a sea state at the Wave Hub site.

Wave data used in the present study are recorded by an Oceanor Seawatch Mini II Buoy deployed in close proximity to the Wave Hub with coordinates: 50.205380N, -5.363430E. The buoy is set-up to store real-time displacement in the heave mode, as well as north and east displacement, and provides wave parameters determined over 30 minutes. For this study, outputs were considered and used to determine extreme wave conditions over the period from March 2012 to September 2012;  $H_s$ , the significant wave height determined in the frequency domain and  $H_{max}$ , defined as the maximum wave height in the record.

Dysthe et al. (2009) describe a criterion for identifying rogue waves using the condition,  $H_{max}/H_s > 2$ . Applying this to the Wave Hub data returned a total of 128 occurrences over the period between March and September 2012. However the majority of these are deemed not to be of sufficient energy to be of threat to the survivability of typical WECs. Thus, further criteria are next applied to pick out extraordinarily large wave events during periods of persistent storminess:

- 1. A storm threshold is set using the methods described by Hovland et al. (2010), given by  $H_s=1.5*H_s$  mean=2.51m, where  $H_s$  mean is the yearly average of  $H_s$  at Wave Hub.
- 2. The storm duration has to persist for a minimum of 12 hours.

These criteria are still found to be insufficiently rigorous in the identification of extreme events. Through an analysis of the distribution of  $H_s$  versus  $H_{max}/H_s$  it is found that occurrences where  $H_s>4m$ , stand out, therefore a further criteria for the identification of extreme events is set at  $H_s>4m$ .

Through this selection process, five storm events are identified during the period, as shown in Figure 1. The storm event taking place on the 15<sup>th</sup> of August 2012 at 6:30 pm is chosen as the reference case for experimental implementation in the Ocean basin and numerical testing in the NWT. The time series of the buoy heave data is reported in Figure 2. Spectral energy density is calculated from this heave time series using the Welch method (Welch, 1967). This spectrum is then used to physically model the sea state at a 1:20 scale.

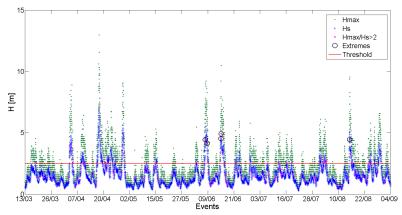


Figure 1: Selection of the storm events

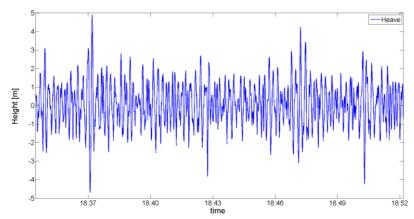


Figure 2: Heave data from the buoy for the storm event on 15/08/12

# **Physical Experiments**

An hour long sea state is generated in Plymouth University's 35m long by 15.6m wide Ocean Basin to demonstrate the ability to recreate the measured Wave Hub spectrum at 1:20 scale. Wave generation is possible over a frequency band between 0.1 to 2.0 Hz. Frequencies which give an integer number of waves within the run time of one hour are used, with the amplitude of each frequency component  $(a_n)$  determined by equation (1):

$$a_n = \sqrt{2S(f_n)\Delta f},\tag{1}$$

where  $S(f_n)$  is the spectral energy density at frequency  $f_n$  and  $\Delta f$  is the interval between the frequency components. A random phase modifier is used to generate a random sea state.

The measured surface elevation, 14m from the paddle, is shown in Figure 3 (left). All results are presented at full scale. Figure 3 (right) shows the energy spectral density calculated from this measurement, compared against the Wave Hub spectrum. The wave hub spectrum has been reproduced relatively well by the physical experiments, with a peak frequency within 5% of the required value. The total energy within the tank spectrum is less however, probably due to energy dissipation caused by wave breaking.

Although the spectral content is well reproduced, rogue wave events are less so. The largest wave in Figure 2 had a 9.57m wave height, compared to a maximum height of 7.43m observed in the tank (corresponding to a Relative Error for  $H_{max}$  of 22.3%).

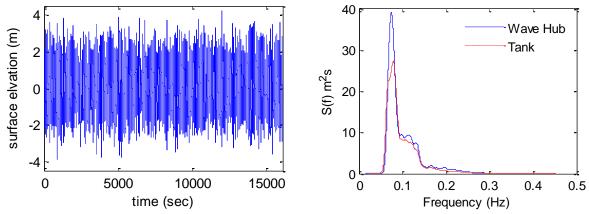


Figure 3: Measured surface elevation 14m from the wave paddles (left). Comparison between the energy spectral density measured at Wave Hub and that measured during the physical experiments (right).

A comparison between the wave height distributions (Figure 4) also confirms that reproduction of the statistical extremes is sub optimal. This is likely to be influenced by the smoothing inherent in the spectral estimation and by the phase randomisation. However, without smoothing, the uncertainty in the estimation of the spectral content is large. A further reason for this is that within the basin there is insufficient time for the nonlinear wave-wave interactions to develop.

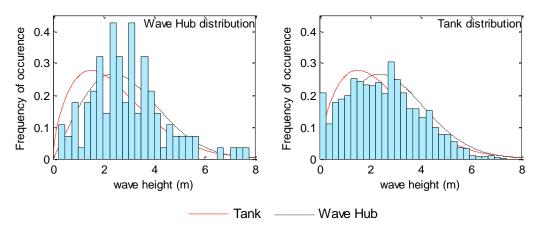


Figure 4: Comparison between the wave height distributions measure at Wave Hub and those measured during the physical experiments.

An approach commonly used is to embed a focused wave group into a random sea state (Clauss et al., 2007). This has the advantage of guaranteeing the generation of an extreme event while not requiring long duration simulations. As the first step in this process a NewWave wave group is generated based on the measured spectrum.

### NewWave theory

The NewWave formulation was introduced by Tromans et al. (1991) and is used frequently by the offshore industry as a design wave. The concept of the NewWave theory is to generate an extreme wave, at a known position and time, through the superposition of small amplitude linear waves of different frequencies. NewWave is a compact wave packet with a local time history identical to the scaled inverse Fourier transform of the spectrum of the sea state (Hunt-

Raby et al., 2011). For large crests, the most probable values of water elevation around the crest can be generated in a reproducible way.

The measured wave hub spectrum is used as the input for the NewWave group generated in the tank. Measurement of the resulting wave is conducted at 19.65m. This is set as the theoretical focus point. A wave group is defined as being focused when all components have the same phase at the same point in time and space. The method proposed by Chaplin (1996) is used to try and achieve this, where the phase differences at the focus point are measured and used to correct the wave paddle input.

Due to the interaction of waves of different frequencies new components are generated which do not satisfy the linear dispersion relation. The surface elevation predicted by theory is therefore not achieved and the focus is poor. One approach to improve the efficiency of the focus is to adjust the theoretical focus point relative to the measurement point (Ransley et al., 2013). This however requires a process of trial and error and is very time intensive. The empirical adjustment of phases used here results in a similar improvement in focus but requires fewer wave iterations.

## **Numerical Method**

When modelling extreme free surface problems with Computational Fluid Dynamics (CFD) methods, it is essential to employ advanced fully nonlinear models to take into account viscous and green-water effects in multiphase flows. Accurate simulation of a moving fluid interface, particularly one which is highly distorted, is extremely challenging. Arguably the most appropriate strategy in this case is the volume of fluid (VoF) method (Hirt and Nichols, 1981). This is a surface capturing Navier-Stokes (NS) solver which has been proven to accommodate highly distorted, multivalued free-surface and topological changes like wave breaking and recombination (Greaves, 2004). The VoF method provides the required design tool for the modelling of extreme wave loading on marine structures which, once properly calibrated and validated, offers a high density of test data that is both accurate and cost effective.

In this work OpenFOAM is utilised as a robust and advanced open source CFD code widely used in the industry. OpenFOAM can be a useful tool for coastal engineering applications as it solves 3D domains and considers two-phase flows with several turbulence models, as well as appropriate boundary conditions for wave generation and absorption (Higuera et al., 2012). The numerical model used is the Navier-Stokes solver, interFoam, supplied with the open source CFD toolbox OpenFOAM®. This solver simultaneously solves the Reynolds averaged Navier-Stokes (RANS) equations and the incompressible continuity equation for the combined flow of the two immiscible fluids using the finite volume method of discretization and a VoF approach similar to the formulation of Hirt and Nichols (1981).

The pressure-velocity coupling is achieved through the PISO algorithm. The interface is captured using the volume fraction scalar field,  $\alpha$  (which equals 0 for air and 1 for water), and a compression term is added to limit the interface smearing. This freely available toolbox, released by OpenCFD Ltd®, is gaining popularity in coastal engineering studies where large computational demands, requiring parallel processing, are common place, and typically expensive to run on commercial platforms (Jacobsen et al., 2012). Further to this, the wave generation toolbox developed by Jacobsen et al. (2012), waves2Foam, which offers generic wave generation and an absorption scheme termed 'wave relaxation zones', has been adopted.

It has been shown that waves2Foam coupled with OpenFOAM® can accurately model the propagation and breaking of water waves (Jacobsen et al., 2012).

In order to assess the ability of this numerical method to model a focussed wave event, a two-dimensional numerical wave tank (NWT) is constructed to compare results with the physical experiment described above. The computational domain is discretised with a grid of 80,500 cells in 3 blocks. A global coordinate system is defined with the origin at the still water level and the x-axis pointing in the direction of wave propagation. The vertical side of grid cells is chosen to be relatively coarse near the sea bed, but then becomes finer towards the free surface. The region from z = -0.2 m to z = 0.3m (which contains the free-surface) has a uniform, square-celled mesh at a resolution of  $0.02 \times 0.02$ cm. In the horizontal direction the cell size is uniform across the whole domain. Care is taken not to exceed the suggested size ratio of 1.1 between adjacent cells (OpenCFD®, 2012). Convergence tests have shown this discretisation is sufficient. A no-slip boundary condition is applied at the sea bed and an outlet condition that allows air to both enter and leave the computational domain is applied on the top boundary.

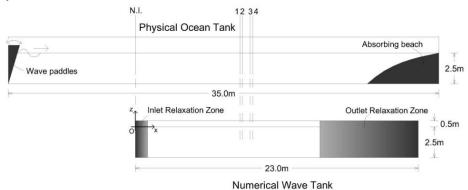


Figure 7: Schematic of Physical Ocean Tank and the NWT used in OpenFOAM. The focusing point is located at wave gauge 3.

The computational domain is 23m long and 2.5m deep and is designed to match the central part of the physical domain. The inlet boundary is located at the position of the upstream wave gauge thus allowing the physical result recorded there to be used as the inlet boundary condition in the computational domain. Moreover, using this technique of the truncated domain, the complexity of the transporting functions of the wave paddles is not taken into account and the computational domain is shorter, saving significant computational time. The amplitudes and phase angles of 243 wave components, with frequencies evenly spaced between 7.8x10<sup>-3</sup> and 1.89Hz, are derived by means of a fast Fourier transform (FFT) of the time series recorded at wave gauge 1 during the physical experiments. Figure 6 shows that the linear combination of these components recreates the measured time series precisely.

The second order wave definition given by Sharma and Dean (1981) is used for each component to avoid the generation of undesired free waves. These waves appear when a linear wave-making inlet boundary condition is enforced (Bredmose and Jacobsen, 2011, Hunt et al., 2004, Zhao et al., 2009). The incident waves are generated within a 1m long relaxation zone, in which the wave field is enforced after every time step, using the update formula given in (Jacobsen et al., 2012, Bredmose and Jacobsen, 2011). Finally, another relaxation zone with a target solution of still water is located between x=15 m and x=23 m in order to absorb the waves. It has been found that an outlet relaxation zone of 8m sufficiently absorbs all incident waves in the case examined. Figure 7 shows a schematic of the computational domain.

In order to save computational resources the numerical simulation is initiated using the second order inlet values at  $t_{num} = 0$ s expanded across the entire domain, to replicate the physical conditions at  $t_{phys} = 60$ s, where  $t_{num} = t_{phys}$  - 60s. Due to the amplitudes are relatively small it is believed that this approach does not affect the outcome significantly.

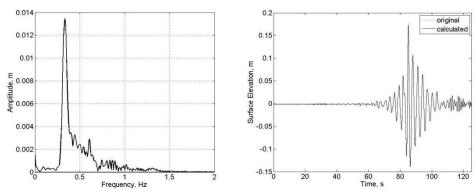


Figure 6: FFT of the time series measured at N.I. during the physical experiments (left). Comparison between the original and the reconstructed time series using 243 wave components derived from the FFT (right).

During the simulation the water and air have densities of  $1000 \text{kgm}^{-3}$  and  $1 \text{kgm}^{-3}$ ; and, kinematic viscosities of  $1 \times 10^{-6}$  kg m<sup>-1</sup> s<sup>-1</sup> and  $1.48 \times 10^{-5}$  kg m<sup>-1</sup> s<sup>-1</sup> respectively. A no turbulence model is applied. Implicit Euler time stepping is used and the adjustable TimeStep function within OpenFOAM® utilized to satisfy the Courant-Friedrichs-Lewy (CFL) condition to be equal or less than 0.25.

#### **Results and Discussion**

It is fairly clear from Figure 8 that there is a very good agreement between numerical and physical measurements up to 27s. After 32s important differences between the NM and PM results occur. It is believed that the discrepancies observed are caused by undesired reflections in the physical tank that affected the input signal for the NWT. That has also probably resulted to the loss of symmetry of the focusing event in the NWT, suggesting that focusing is not located at the same place in PM and NM. This is depicted in Figure 9, where the highest free surface elevation takes place further downstream in the NWT, at location 9.7m instead of 9.3m.

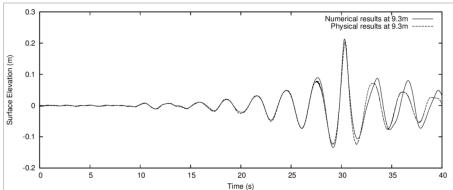


Figure 8: Comparison between the physical and numerical time series measured at the observed physical focus location.

The greatest achievement of the numerical simulation is that it replicates accurately the height and the spike shape of the crest of the extreme wave and matches the deepest troughs well, that is shown in Figure 8 between 27s and 32s. These two elements are the most important when considering wave loading on structures. Noteworthy is the effectiveness of the passive

absorbing mechanism of the relaxation zone, located between 15 and 23m. As shown in Figure 9, the incident waves are completely absorbed, something that is very difficult to achieve in a physical tank.

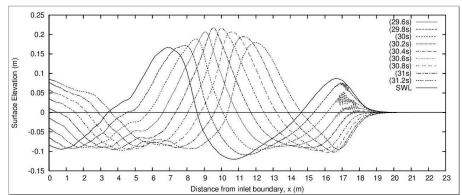


Figure 9: Snapshots of the free surface across the entire NWT at different times spanning the focus event.

#### **Conclusions**

Rogue waves pose a significant threat to the offshore industries. This work has demonstrated how extreme events measured in the field can be simulated numerically and in the laboratory. At the same time, it suggests that a procedure combining physical and numerical modelling of such real events is possible, and has great potential for testing survivability of WEC's. The target now is to improve the integration of physical and computational resources, in line with the concept of Composite Modelling.

There are several recommendations for future studies. Firstly, the wavelet analysis has been shown to be more effective than FFT for extreme wave studies (Zhao et al., 2009, Lin and Liu, 2004). Also, the use of an active absorption mechanism in the wave generating boundary may overcome issues surrounding the slight effect that the relaxation zone seemed to have on the wave propagation. Moreover, one can suggest the use of turbulence models to improve numerical simulation.

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