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A multi-envelope vertical coordinate system for numerical ocean modelling

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Abstract A Multi-Envelope generalised coordinate system for numerical ocean modelling is introduced. In this system, computational levels are curved and adjusted to multiple ‘virtual bottoms’ (aka envelopes) rather than following geopotential levels or the actual bathymetry. This allows defining computational levels which are optimised to best represent different physical processes in different sub-domains of the model. In particular, we show how it can be used to improve the representation of tracer advection in the ocean interior. The new vertical system is compared with a widely used z-partial step scheme. The modelling skill of the models is assessed by comparison with the analytical solutions or results produced by a model with a very high resolution z-level grid. Three idealised process-oriented numerical experiments are carried out. Experiments show that numerical errors produced by the new scheme are much smaller than those produced by the standard z-partial step scheme at a comparable vertical resolution. In particular, the new scheme shows superiority in simulating the formation of a cold intermediate

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layer in the ocean interior and in representing dense water cascading down a steep topography.

Keywords Ocean modelling · Vertical coordinate · Oceanic transport

1 Introduction

When designing an ocean model, the choice of the vertical coordinate system must be pursued very carefully (Griffies, 2004), especially in regional areas where local smaller-scale processes interact with large-scale oceanographic features (Kantha and Clayson, 2000; Gangopadhyay and Robinson, 2002). Numerical discretisation introduces truncation errors specific to the chosen vertical coordinate system, and hence influences the representation of physical processes (Haidvogel and Beckmann, 1999). Currently, three main vertical coordinates are typically used in ocean modelling, namely z -level (geopotential), terrain-following and isopycnic, but all of them have deficiencies (see e.g. Chassignet et al. 2006).

The z -level coordinates are a natural framework for describing horizontal pressure gradients. However, the z -level system generates an unnatural step-like representation of bottom topography and consequently introduces an error in simulating near-bottom processes, including dense water overflows (e.g. Ezer and Mellor 2004; Ivanov et al. 2004). Gerdes (1993a) concluded that the crude approximation to the actual topography used in z -level models results in large errors in the simulated mass transport in regions where planetary and topographic beta-effects are of comparable magnitudes. Horizontal overshoots of dense water due to step-like representation of bottom topography lead to spurious convective mixing. Recently, Ezer (2016) showed that the unrealistic representation of topographic slopes in z -ocean models has a negative impact on the simulation of the dynamics of western boundary currents and consequently of large-scale circulation.

The disadvantages of z -level grids initiated intensive development of terrain-following grids for ocean modelling (Blumberg and Mellor, 1987; Haidvogel et al., 1991; Ezer and Mellor, 1992). The terrain-following coordinate (σ -coordinate system) offers a smooth representation of bottom topography and a natural parametrisation of the bottom boundary layer (Mellor et al., 2002). However, it introduces a pressure gradient error, in particular on steep slopes (Haney, 1991; Mellor et al., 1994, 1998).

The use of computational surfaces that are not aligned with isopycnals (which is generally the case of both z - and σ -coordinate systems) in simulating tracer transport introduces the contamination of slow diapycnal processes by fast isopycnal exchanges (e.g. Roberts and Marshall 1998; Griffies et al. 2000b). As a consequence, spurious diapycnic mixing poses a major problem in non-isopycnal models (see Holt et al. 2017 and references therein). Such deficiencies are not present in vertical grids where computational levels follow isopycnals (so-called isopycnic grids), e.g. used in the MICOM ocean model, see Bleck (1998). However, isopycnic models experience difficulties in weakly stratified areas, such as over the continental shelf or in the upper or bottom mixed layers (Griffies et al., 2000a). Legg et al. (2006) compared the performance of isopycnal and z -models in representing dense cascades while Legg et al. (2009) pointed out the importance of a correct simulation of oceanic overflows in numerical climate models.

43 In order to minimise the disadvantages of the various vertical coordinate sys-
44 tems, further modifications were introduced either to the vertical grids themselves
45 or to the numerical representation of the governing equations. For example, the
46 introduction of shaved (Adcroft et al., 1997) or partial (Pacanowski et al., 1998)
47 cells which slightly change the shape of ‘pure’ z -coordinate grids was proposed to
48 improve the representation of bottom topography in z -models. The z -partial steps
49 approach is now widely used for global (Barnier et al., 2006) and regional (e.g.,
50 Oddo et al. 2009; Trotta et al. 2016) ocean models. A stretched terrain-following s -
51 coordinate system (Song and Haidvogel, 1994) and its several variants (e.g. Madec
52 et al. 1996; Siddorn and Furner 2013) as well as advanced methods in calculation
53 of pressure gradients (Shchepetkin, 2003) were developed to improve σ -coordinates
54 flexibility and accuracy.

55 The concept of a generalised vertical coordinate system (see for example Kasa-
56 hara 1974 or Mellor et al. 2002) allowed in principle the development of vertical
57 grids of various complexity, as for example the hybrid vertical schemes where dif-
58 ferent ‘pure’ grids were applied to different sub-domains of the ocean. The aim of
59 this was to better represent the differing physical processes which might prevail in
60 different sub-domains, by using one specific grid rather than another. Examples
61 of those methods are the HYCOM model (Bleck and Boudra, 1981; Bleck, 2002),
62 the vertical grids by Gerdes (1993a,b), Madec et al. (1996), Shapiro et al. (2013)
63 or the Song and Hou (2006) parametric vertical coordinate system.

64 The idea of Arbitrary Lagrangian-Eulerian (ALE) vertical coordinates (Hirt
65 et al., 1974) permitted the development of z^* - (Adcroft and Campin, 2004) and
66 \tilde{z} -coordinates (Leclair and Madec, 2011) and the adaptive σ -based coordinate
67 (Hofmeister et al., 2010).

68 A significant improvement in terrain-following schemes was achieved by in-
69 troducing the idea of the ‘enveloping’ bathymetry, where computational surfaces
70 follow a ‘virtual bottom’ (aka envelope) rather than the real bathymetry (Enriquez
71 et al., 2005; Dukhovskoy et al., 2009; Shapiro et al., 2013). This solution allows
72 the reduction of slopes of computational surfaces and the reduction of pressure
73 gradient errors to an acceptable level.

74 In this study, we introduce the ‘Multi-Envelope s -coordinate’ (hereinafter MEs-
75 coordinate). It extends the classical concept of terrain-following coordinates by
76 defining s -levels which follow multiple envelopes rather than a single one as is the
77 case in existing models. This approach allows to combine the ideas behind the
78 hybrid schemes (best representation of different physics in different sub-domains
79 of the model) and numerical improvements (e.g. enveloping) developed for ‘pure’
80 vertical discretisation grids. The new vertical system represents a generalised co-
81 ordinate system, since all non-isopycnal vertical grids (both ‘pure’ and hybrid) can
82 be considered a special case of MEs-coordinates.

83 The paper is organized as follows. Section 2 defines the MEs-coordinate, detail-
84 ing its features. Section 3 describes the idealised model domain, the design of the
85 three different vertical grids and the set up of the three numerical experiments. In
86 Section 4, the results are presented, analysed and discussed. Section 5 summarises
87 our main conclusions.

2 The Multi-Envelope s -coordinate

In this paper we show how the MEs system can be used to improve the representation of the oceanic transport in a non-isopycnal coordinate model. The MEs-coordinate combines the s -coordinate concept and the idea of ‘enveloping’ the bottom topography.

Let us consider a local Cartesian x, y, z coordinate system with a downward vertical unit vector \hat{z} . A stretched envelope-following s -coordinate can be defined as

$$z = S(\sigma, \eta, H_e) \quad (1)$$

where $\eta(x, y, t)$ is the deviation of the sea surface from its unperturbed position, $H_e(x, y)$ is a smoothed version of the actual bottom topography (aka bathymetry envelope) and $-1 \leq \sigma \leq 0$, with $\sigma = 0$ at $z = \eta$ and $\sigma = -1$ at $z = H_e$. A general stretching function is represented by $S(\sigma, \eta, H_e)$. It can be, for example, the one by Song and Haidvogel (1994), Shchepetkin and McWilliams (2005) or Siddorn and Furner (2013).

The MEs vertical system defines n arbitrary reference surfaces (hereafter called envelopes) $H_e^k(x, y, t)$, with $0 \leq k \leq n$ and $n \in \{2m + 1\}$ with m a positive integer such that

$$\eta = H_e^0 < H_e^1 < \dots < H_e^{n-1} < H_e^n \quad (2)$$

Each envelope $H_e^k(x, y, t)$ moves with the free-surface according to the following equation:

$$H_e^k = h_e^k + \eta \left(1 - \frac{h_e^k}{h} \right) \quad (3)$$

where $h_e^k(x, y)$ is the depth of the k^{th} envelope when the ocean free-surface is unperturbed ($\eta = 0$) and $h = h_e^n$.

The envelopes divide the ocean model vertical domain into n sub-zones D_i , with $1 \leq i \leq n$. Each sub-zone D_i is bounded by envelopes H_e^{i-1} at the top and H_e^i at the bottom.

The non-dimensional σ_i -coordinate is defined for each sub-zone D_i as

$$\sigma_i = - \frac{z - H_e^{i-1}}{H_e^i - H_e^{i-1}} \quad (4)$$

with $\sigma_i(H_e^{i-1}) = 0$ and $\sigma_i(H_e^i) = -1$. Then, the MEs-coordinate is defined as a piecewise function

$$\begin{cases} z|_{D_i} = S_i(\sigma_i, H_e^{i-1}, H_e^i), & \text{if } i \in \{2m + 1\} \\ z(x, y, \sigma_i, t)|_{D_i} = P_{x,y,i}^3(\sigma_i), & \text{if } i \in \{2m\} \end{cases} \quad (5a)$$

$$(5b)$$

The function $S_i(\sigma_i, H_e^{i-1}, H_e^i)$ in Equation 5a represents a general stretching function. For example, in the case of the classical Song and Haidvogel (1994) stretching function, MEs coordinates are defined as

$$z|_{D_i} = H_e^{i-1} + h_c^i \sigma_i - C_i(\sigma_i)(H_e^i - h_c^i - H_e^{i-1}) \quad (6)$$

where h_c^i is the critical depth at which transition from pure σ to the stretched s -coordinate occurs and $C_i(\sigma_i)$ is the hyperbolic function of Song and Haidvogel (1994) (their $C(s)$).

The function $P_{x,y,i}^3(\sigma_i)$ in Equation 5b is a complete cubic spline whose coefficients are determined by the following three constraints:

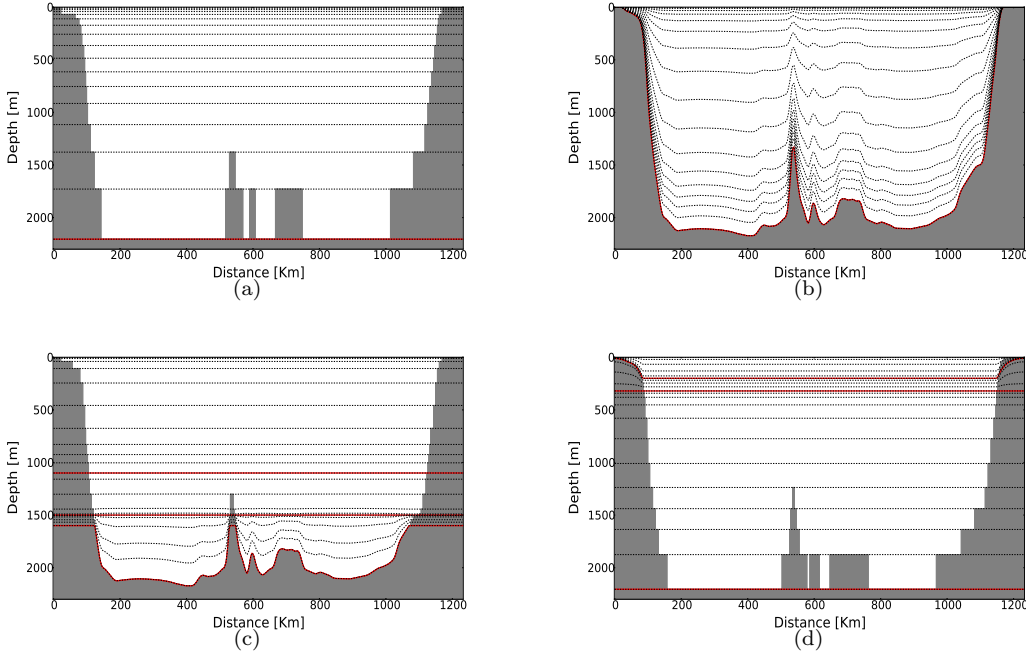


Fig. 1 Sketches depicting ‘pure’ z - (a) and σ - (b) grids and hybrid Madec et al. (1996) z -on-top-of- s (c) and Shapiro et al. (2013) s -on-top-of- z (d) approaches as retrieved with the MEs-coordinate system. Envelopes H_e^i used to define each specific configuration are shown in red.

1. Monotonicity:

$$\partial_{\sigma_i} z|_{D_i} > 0, \text{ with}$$

$$\begin{cases} -1 \leq \sigma_i \leq 0, & \text{if } i = n \\ -1 < \sigma_i \leq 0, & \text{if } i < n \end{cases}$$

2. Continuity:

$$z|_{D_i}(\sigma_i = -1) = z|_{D_{i+1}}(\sigma_{i+1} = 0)$$

3. Continuity of the first derivative:

$$\partial_{\sigma_i} z|_{D_i}(\sigma_i = -1) = \partial_{\sigma_{i+1}} z|_{D_{i+1}}(\sigma_{i+1} = 0)$$

121 A description of the method used to determine the coefficients of complete cubic
 122 splines $P_{x,y,i}^3(\sigma_i)$ is given in Appendix 1. Under these conditions, the Jacobian of
 123 the transformation from z to σ is continuous, ensuring one of the requirements of
 124 improved accuracy formulated by Marti et al. (1992) and Treguier et al. (1996).

125 The new MEs represents a generalised coordinate system, in the sense that
 126 ‘pure’ and hybrid non-isopycnal vertical coordinates can be considered a special
 127 case of MEs-coordinate. For example, z -grids are simply generated by defining
 128 a single horizontal envelope $H_e^1 = \max(H_B)$, where $H_B(x, y)$ is the actual
 129 bathymetry (see Figure 1(a)). Similarly, terrain-following σ -coordinates can be

130 generated by choosing $H_e^1 = H_B$, see Figure 1(b)). Figures 1(c) and 1(d) show
 131 how hybrid ‘z-on-top-of-s’ (Madec et al., 1996) and ‘s-on-top-of-z’ (Shapiro et al.,
 132 2013) grids, respectively, can be easily generated with the MEs vertical system.
 133 In MEs all grid cells are full, both near the bottom and in the interior, and their
 134 shape is dictated by the corresponding envelope.

135 An important feature of the MEs system is that envelopes H_e^i can be arbitrary
 136 chosen surfaces. This implies that they can be designed to optimise the
 137 representation of those physical processes that are prioritised, allowing the mod-
 138 eller to manage and control the design of model levels with enhanced flexibility.
 139 Figure 2 shows an example of MEs design by using five reference surfaces H_e^i .

140 In this configuration, sub-zone D_5 has model levels which follow envelope H_e^5 ,
 141 a smooth version of the actual bottom topography up to 1500 m. This enables
 142 realistic simulations of dense water overflows over the ocean bottom while reducing
 143 pressure gradient errors. In sub-zone D_3 , model levels are horizontal. Zones D_2 and
 144 D_4 work as transition zones which gradually reduce the slope of s -levels towards
 145 geopotential surfaces in D_3 .

146 The upper envelope H_e^1 follows the ‘main pycnocline’ (i.e. long-term mean pyc-
 147 nocline) in open ocean areas but it follows the topography in coastal regions.
 148 Such an envelope allows to obtain realistic simulations of both dense water cas-
 149 cades in shelf areas and the formation of a cold intermediate layer in the open sea.
 150 The pycnocline-shaped envelope reduces the angle between the computational sur-
 151 faces and the isopycnals, and hence reduces the spurious diapycnal mixing, thus
 152 performing similar to isopycnal coordinate systems.

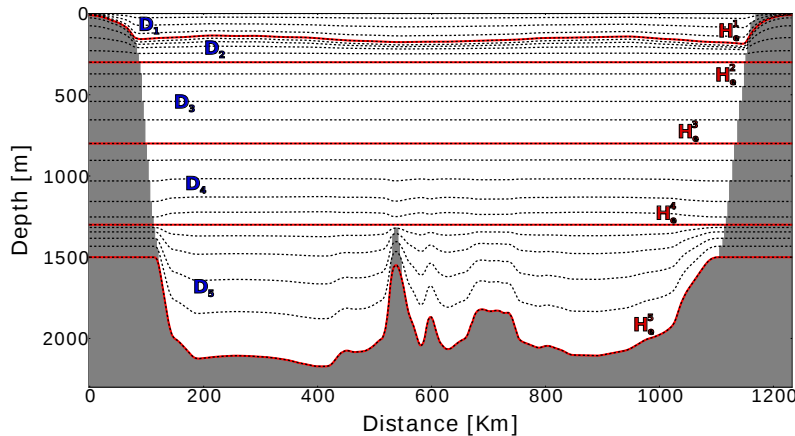


Fig. 2 Conceptual sketch of the MEs vertical coordinate system. In this example, five envelopes H_e^i are used to define MEs-levels.

153 To clarify this effect, let us consider the idealised case of a two-layer immiscible
 154 fluid depicted in Figure 3.

155 In this case, tracer advection and diffusion occurs exclusively along the isopyc-
 156 nal surface, as represented by black and green arrows in Figure 3(a), and there is

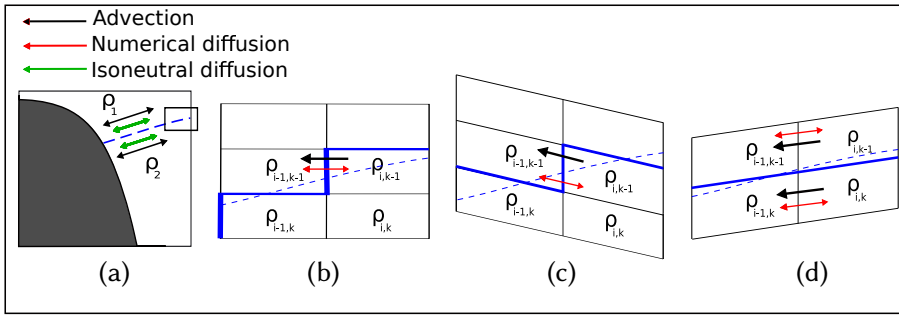


Fig. 3 Idealised two density layers baroclinic ocean (a) and its representation with geopotential z -levels (b), terrain-following s -levels (c) and the MEs vertical system with the upper envelope H_e^1 designed to follow the main pycnocline in open ocean areas (d). The real pycnocline is represented by the dashed blue lines, while the pycnocline simulated by the models is shown with the solid blue lines. See the text for more detailed explanations.

157 no diapycnal mixing. Figures 3(b), 3(c) and 3(d) illustrate how the real isopycnal
 158 surface is represented with z -level, s -level and MEs grids, respectively.

159 Black arrows in Figures 3(b) and 3(c) show how advection is simulated in z -
 160 and s -models, resulting in the spurious mixing across different densities due to
 161 much stronger ‘along-computational-level’ numerical diffusion (see the red arrows)
 162 , which transfers mass and momentum between the density layers.

163 The rotation of the diffusion operator to align the lateral diffusion with isopyc-
 164 nals (Redi, 1982) would have reduced this undesirable effect. However, s -models
 165 typically use geopotentially oriented diffusion, because of the difficulties in com-
 166 puting isoneutral diffusion (Barnier et al., 1998; Marchesiello et al., 2009; Furner,
 167 2012; Lemarié et al., 2012). Another approach (widely used in regional models)
 168 could be the subtraction of climatological temperature and salinity fields before
 169 the lateral diffusion fluxes are calculated, hence diffusing only tracer anomalies,
 170 following Mellor and Blumberg (1985).

171 If model levels mimic the pycnocline as in the MEs model, the angle between
 172 the isopycnals and computational surfaces is small, see Figure 3(c), and the spu-
 173 rious diapycnal mixing arising from numerical errors of the advective schemes is
 174 significantly reduced.

175 3 Experiments to assess model skill

176 In this section we assess the modelling skills of the MEs scheme in comparison to
 177 the widely used z -level with partial steps scheme by performing a set of idealised
 178 numerical experiments with an axisymmetric ocean basin.

179 The model domain is a bowl-shape basin with a diameter of 500 km, maximum
 180 depth of 1000 m and the downward positive topography H_B defined by

$$H_B = \max\left\{h_0 \exp\left(\frac{x^2}{2\sigma^2} + \frac{y^2}{2\sigma^2}\right), 1000\right\} \quad (7)$$

181 with $h_0 = 25000$ m, $\sigma = 8$, and $x, y \in [-40$ km , 40 km] (see Figure 4(a)). The
 182 slope at the 200 m isobath of the idealised basin is $\approx 1.5\%$.

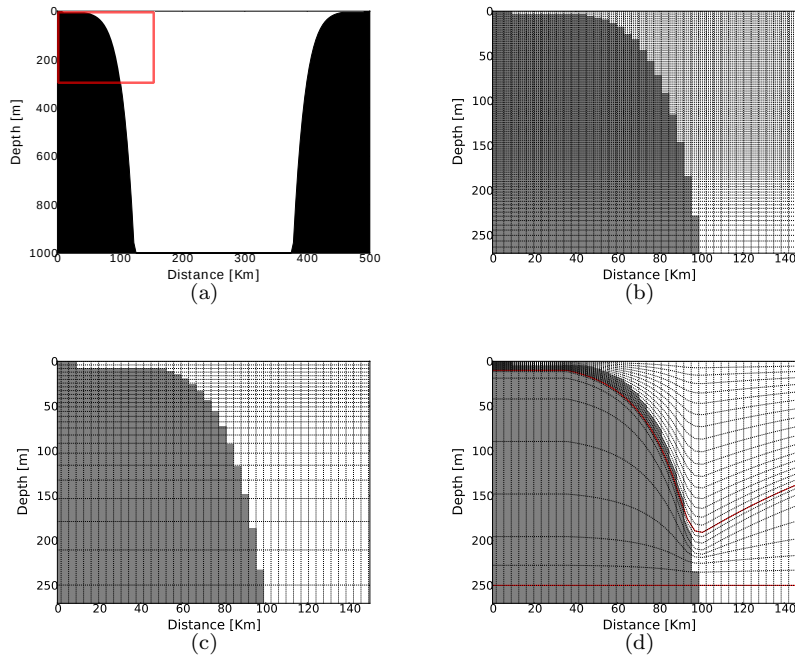


Fig. 4 Cross sections of the topography H_B of the idealised domain (a) and zfs -150 (b), zps -34 (c) and MEs-34 vertical (d) grids configured for this study. For the numerical grids, only the portion of the domain highlighted with the red square in panel (a) is shown for clarity.

183 In order to use the MEs grid for our computations, we modified the Nucleus
 184 for European Modelling of the Ocean (NEMO) Ocean General Circulation model
 185 code accordingly. The NEMO hydrodynamic component is a three dimensional, fi-
 186 nite differences, free-surface primitive equation ocean model suitable for modelling
 187 ocean circulation at regional and global scales. It solves the incompressible, hydro-
 188 static, Boussinesq approximated primitive equations along with a non-linear equa-
 189 tion of state. NEMO provides a selection of various turbulence closure schemes. In
 190 this study we use the NEMO 3.6-stable code, see Madec (2008).

191 3.1 Model grids

192 All the numerical experiments are carried out by using two models which have
 193 the same horizontal mesh but two different vertical grids: one uses the common
 194 z -level with partial steps (hereafter called zps) while the second uses the new MEs
 195 scheme. In the horizontal, the mesh has 140 grid points in both the zonal and the
 196 meridional directions and a uniform grid spacing $\Delta x = \Delta y \approx 3.57 \text{ km}$. For better
 197 comparison between the MEs against the zps vertical grid, both models have the
 198 same number of 34 numerical levels and hereafter they are called MEs-34 and
 199 zps -34, respectively. For the same reason, the computational level n° 26 is placed
 200 at the same depth of 250m for both grids.

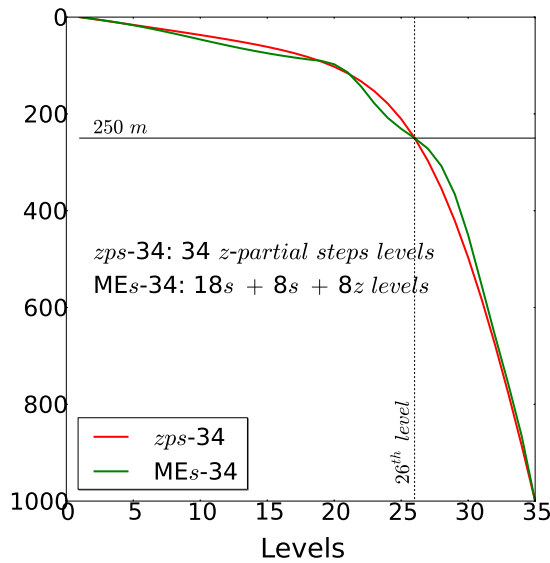


Fig. 5 Vertical distribution of *zps*-34 (red) and *MEs*-34 (green) model levels in the middle of the computational domain. The depth of the 26th level (250 m) is also shown.

201 One experiment is also carried out with a *z*-full step vertical grid at a very
 202 high vertical resolution of 150 levels (hereafter *zfs*-150). This simulation is used
 203 as a reference where analytical solutions are not available (see Section 3.2).

204 The *zps*-34 grid uses a standard NEMO-3.6 *z*-partial steps scheme (Figure
 205 4(c)) with a minimum layer thickness of 4 m. The partial step parameters are
 206 tuned in such a way that the topography represented with 34 levels is close to the
 207 one discretised with 150 geopotential levels.

208 The *MEs*-34 grid is configured by using three *envelopes* (see Figure 4(d)). The
 209 middle H_e^2 and the deep H_e^3 envelopes are horizontal and located at 250 m and
 210 1000 m respectively. Therefore, the deeper D_3 zone of the *MEs* grid is effectively
 211 discretised with a *z*-coordinate grid. The upper envelope H_e^1 of the *MEs*-34 grid
 212 is dome-shaped in the ocean interior, following a typical shape of the thermocline
 213 in a sea with a cyclonic circulation, but it follows an ‘enveloping’-bathymetry over
 214 the continental slope and shelf.

215 The ‘enveloping’-bathymetry is a smoothed version of the actual bathymetry
 216 with a maximum depth of 200 m and a minimum depth of 10 m. It is obtained
 217 by applying the Martinho and Batteen (2006) smoothing algorithm to the actual
 218 topography, which reduces the maximum value of the slope parameter (Mellor
 219 et al., 1998) defined as

$$r \equiv \frac{|H_b - H_a|}{H_b + H_a} \quad (8)$$

220 where H_a and H_b are the depths of adjacent grid cells. With the H_e^1 envelope, the
 221 value of r is reduced from $r = 0.13$ (actual bathymetry) to 0.09 (H_e^1 envelope),
 222 allowing the reduction of pressure gradients errors.

Exp. Name	Oceanic process	Ideal test process	Initial ocean setup	Perturb.	Assess. of models' skills
HPGE (Sec. 3.2.1)	Ocean circulat.	Evolution of a stably stratified ocean at rest	Horiz. uniform vert. stable stratification, no motion	-	Comparison with analytical solution
CASC (Sec. 3.2.2)	Dense water cascading upon the shelf	Gravity current over steep topography	No stratific., no motion	Dense ring upon the shelf and the shelf-break	Comparison with analytical solution
CILF (Sec. 3.2.3)	CIL formation in the ocean interior	Sinking and spreading of a dense cold patch	Cyclonic ocean with 2 density layers	Cylindrical dense water patch in the upper layer	Comparison with high vert. resol. model solut.

Table 1 Oceanic processes tested in this study together with the associated experiment setup and the method used to evaluate models skills.

223 The uppermost envelope H_e^1 has a parabolic shape in deep areas ($H_B(x, y) \geq$
224 200) given by equation

$$H_e^1 = A + B(x^2 + y^2) \quad (9)$$

225 where $A = 87.22$ and $B = 273.33$. The MEs-34 configuration uses 18 levels in the
226 upper (D_1) zone, 8 levels in the central (D_2) zone and 8 levels in the deeper (D_3)
227 zone. The configurations of the two 34 levels vertical grids are presented in Figure
228 5, where the vertical distributions of *zps*-34 (red) and MEs-34 (green) model levels
229 in the middle of the computational domain are compared.

230 The *zfs*-150 model uses a standard NEMO-3.6 *z*-full step grid (Madec, 2008)
231 with the stretched function tuned in such a way that layers thickness up to 200 m
232 depth is almost constant with a value of ≈ 2 m (Figure 4(b)).

233 3.2 Experiment set-up

234 We carry out three idealised process-oriented numerical experiments which mimic
235 three typical oceanic conditions. The first experiment (hereinafter called HPGE)
236 is designed to assess the generation of spurious currents due to horizontal pressure
237 gradient errors (see Section 3.2.1). The second experiment (hereinafter called CASC)
238 represents dense water cascading from the continental shelf (Ivanov et al., 2004),
239 see Section 3.2.2. The third experiment (hereinafter CILF) simulates the formation
240 of a cold intermediate layer over a permanent thermocline, a process observed in
241 many subarctic seas (Chubarenko and Demchenko, 2010; Cyr et al., 2011). The
242 latter process is monitored in our simulations by using a passive tracer (see Section
243 3.2.3). The inventory of the experiments is given in Table 1.

244 The skills of MEs-34 and *zps*-34 models are assessed by comparison with known
245 analytical solutions for the first and the second experiments. In the third experi-
246 ment the analytical solution is not available and the comparison is made against
247 a reference numerical simulation which uses *zfs*-150.

248 In all the numerical experiments, the time-splitting formulation for the non-
249 linear free surface is applied, with the baroclinic and barotropic time-steps equal

Physical and Comput. NEMO specific setup	HPGE EXP.	CASC EXP.	CILF EXP.
EOS	non-linear (TEOS10)	non-linear (TEOS10)	linear (Roquet et al., 2015) $\lambda_1 = \lambda_2 = 0.0$ $\mu_1 = \mu_2 = \nu = 0.0$
Lateral diffusivity	$8 [m^2 s^{-1}]$	$8 [m^2 s^{-1}]$	$10^{-7} [m^2 s^{-1}]$
Vertical diffusivity	$10^{-7} [m^2 s^{-1}]$	GLS	$10^{-7} [m^2 s^{-1}]$
Vertical viscosity	$10^{-5} [m^2 s^{-1}]$	GLS	$10^{-5} [m^2 s^{-1}]$

Table 2 Physical and computational NEMO setup specific of the three experiments. If not specified, NEMO standard values are used (see Madec 2008).

250 to 150 s and 7.5 s, respectively. The Asselin time filter parameter is 0.1. We use
 251 the pressure Jacobian scheme together with a leapfrog time scheme for calculation
 252 of the hydrostatic pressure gradient term. The Total Variance Dissipation (TVD)
 253 and Energy and ENstrophy (EEN) conservative schemes are used for tracer and
 254 momentum advection, respectively. All the simulations are performed using the
 255 *f-plane* approximation ($f \approx 10^{-4}$). For the lateral diffusion of momentum, we use
 256 a second order operator aligned with horizontal levels together with a fourth order
 257 operator discretised along model levels (O’Dea et al., 2012). The Laplacian and
 258 bi-laplacian viscosity coefficients are constant with values equal to $10^2 [m^2 s^{-1}]$
 259 and $-2 \cdot 10^9 [m^4 s^{-1}]$, respectively. The lateral diffusion is simulated by using a
 260 horizontal harmonic operator with constant diffusivity (see Table 2 for the values
 261 used in each experiment). The vertical diffusivity and viscosity coefficients are con-
 262 stant in the HPGE and CILF experiments while are computed using the Generic
 263 Length Scale (GLS) turbulent closure scheme (Umlauf and Burchard, 2003, 2005)
 264 tuned following Wobus et al. (2013) in the CASC experiment (see Table 2). In
 265 the HPGE and CILF experiments we reduce the explicit vertical diffusivity to the
 266 minimum value allowed by model stability ($10^{-7} [m^2 s^{-1}]$), in order to isolate the
 267 effect of spurious numerical diffusion linked to the vertical discretisation scheme.
 268 All the models use no-slip lateral boundary conditions and a log-layer enhanced
 269 quadratic bottom friction parametrisation with minimum and maximum bottom
 270 drag coefficient values equal to $2.5 \cdot 10^{-3}$ and 10^{-1} , respectively. Convection is
 271 parameterised by applying enhanced vertical diffusion on tracers in regions where
 272 the stratification is unstable. The enhanced vertical mixing coefficient is set equal
 273 to $10 m^2 s^{-1}$.

274 3.2.1 Generation of spurious currents

275 In this experiments we assess the accuracy of the *zps* and *MEs* vertical schemes
 276 in representing horizontal pressure gradients. In *zps* models, the near bottom grid
 277 points within a vertical level are not necessarily at the same depth as the grid points
 278 in the interior, resulting in problems with pressure gradient errors and spurious
 279 diapycnal diffusion (Pacanowski et al., 1998).

280 The initial condition for each run is obtained by horizontally spreading the
 281 temperature and salinity profiles showed in Figure 6, so that there are no horizontal
 282 pressure gradients, there is no initial circulation and the sea surface is flat. There is
 283 no meteorological forcing or river discharge. In the absence of any external forcing,
 284 the analytical solution for current velocities and horizontal density gradients is
 285 zero. However, numerical errors due to the vertical discretisation may lead to

286 errors in the pressure gradient computation, generating spurious current velocities
 287 (see for example Berntsen 2002).

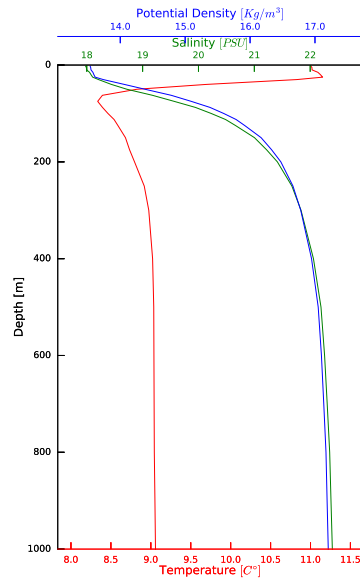


Fig. 6 Vertical profiles of temperature, salinity and potential density anomaly used as initial condition for the HPGR experiment. They are basin averaged mean annual climatologies computed from MyOcean Black Sea Reanalysis from 1992 to 2012 (MyOcean2, 2014).

288 The HPGE experiments consist of two prognostic simulations, one for each
 289 vertical grid, where the NEMO model is run for 30 days without any external
 290 forcing. The computational and physical NEMO settings are listed in Table 2
 291 (HPGE experiments).

292 3.2.2 Dense water cascading on the shelf

293 In the second experiment we investigate the ability of the two 34 levels models to
 294 properly represent the flow of dense water down a steep topographic slope.

295 We consider an initial axisymmetric, three-dimensional density ring of dense
 296 water with a homogeneous density $\rho + \Delta\rho$, situated upon the shelf and an ambient
 297 ocean with constant density ρ . The initial velocity is zero everywhere.

298 The initial condition used for the numerical simulations is shown in Figure 7.
 299 The axisymmetrical dense ring is confined in coastal areas, has a maximum depth
 300 of 50 m and temperature, salinity and potential density anomaly σ_r of 10°C , 21
 301 PSU and 16.00 kg m^{-3} , respectively. Ambient water temperature is 12°C and
 302 salinity is 20 PSU , yielding a potential density anomaly of $\sigma_o = 14.94 \text{ kg m}^{-3}$.

303 If such initial condition is allowed to evolve freely, the dense water will tend
 304 to descend downslope driven by the gravitational force while the Coriolis force
 305 will deflect such motion toward the right (Northern hemisphere). In the absence

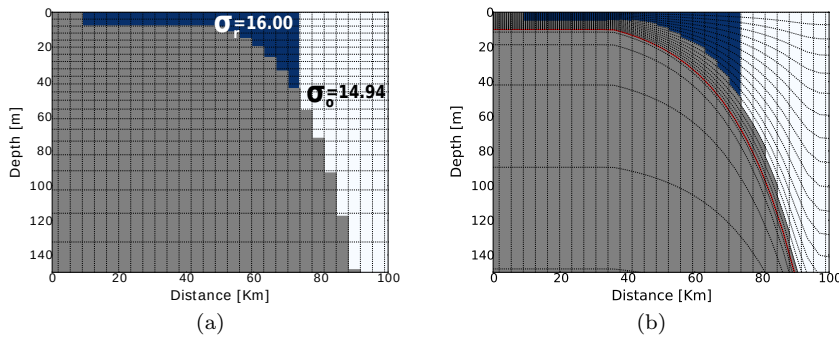


Fig. 7 Meridional cross-sections in the middle of the domain of the potential density initial condition for CASC experiments.

of friction an equilibrium eventually will be reached. For a constant bottom slope angle θ , the geostrophic current velocity u_g is given by $u_g = \frac{g'}{f} \tan \theta$ (Nof, 1983), where g' is the reduced gravity $g' = \frac{g\Delta\rho}{\rho_0}$ and f is the Coriolis parameter. In the presence of friction, a tongue of dense water of approximately 2 Ekman depths will continue to descend.

In the case of a fully developed cascading without entrainment and ambient current, there is an analytical solution for the downslope velocity (Shapiro and Hill, 1997) given by

$$u_{SH97} = 0.2u_g \quad (10)$$

where u_g is the along slope geostrophic velocity (Nof, 1983). While the formula by Shapiro and Hill (1997) was derived for constant slopes, we compare our model results against this formula as the horizontal curvature of our domain is negligible as compared to the topographic slope, and the change of the slope over the length of the tongue is small. A similar approach was used in Wobus et al. (2011, 2013).

One month-long NEMO runs are performed with the computational and physical settings listed in Table 2 (CASC experiments). The GLS turbulence closure scheme is configured according to Wobus et al. (2013). The convective adjustment parameterization is used following Laanaia et al. (2010). The experiment is conducted with 2 vertical grids, the *zps*-34 and the *MEs*-34.

3.2.3 Formation of Cold Intermediate Layer

In the third experiment, we assess the ability of the *zps*-34 and *MEs*-34 vertical grids to represent the formation of a Cold Intermediate Layer (CIL) over a permanent pycnocline by monitoring the advection of a passive tracer in the ocean interior. The experiment simulates the sinking and spreading of a dense (cold) patch of water in an idealised cyclonic ocean with a doming pycnocline.

The initial condition is axisymmetric and represents a two-layer fluid with a cold cylindrical patch at the centre of the basin (see Figure 8). The main pycnocline is defined by Equation 9 with $A = 92.92$ and $B = 193.33$ (note: the pycnocline does not coincide with the H_e^1 envelope). Salinity is equal to 35 PSU and is constant everywhere. The initial velocity is zero. We use a linear equation of state with

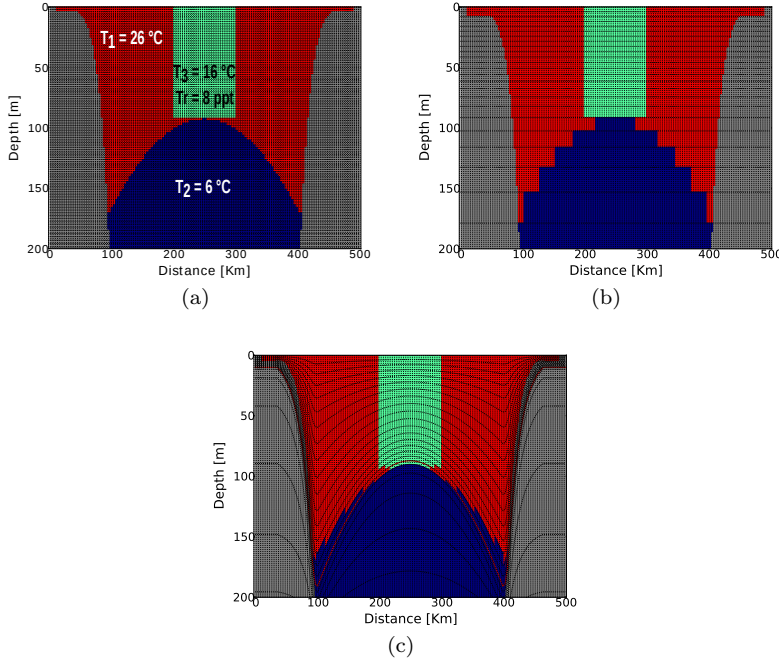


Fig. 8 Meridional cross-sections in the middle of the domain of temperature and salinity initial condition fields defined on *zfs-150* (a), *zps-34* (b) and *MEs-34* (c) models' grids for the CILF experiment.

335 coefficients shown in Tab. 2 (CILF experiment). Temperature and density anomaly
 336 above (i.e. in layer 1) and below (layer 2) the pycnocline are $T_1 = 26^\circ\text{C}$, $\sigma_1 =$
 337 23.4 kg m^{-3} and $T_2 = 6^\circ\text{C}$, $\sigma_2 = 26.7 \text{ kg m}^{-3}$, respectively. The cylindrical
 338 dense convective patch has a radius of 50 km , a maximum depth of 92.94 m and
 339 temperature, density anomaly and passive tracer concentration equal to $T_3 =$
 340 16°C , $\sigma_3 = 25.0 \text{ kg m}^{-3}$ and $C = 8 \text{ ppt}$, respectively. The ratio between the
 341 volume of the cold dense patch (green slug in Figure 8) and the volume of the
 342 domed denser layer (blue fluid portion in Figure 8) is 0.011 in all the models.

343 Explicit tracer diffusion is negligibly small in order to isolate the numerical
 344 diffusion linked to advection schemes. However, we use a standard high value (10
 345 m^2s^{-1}) of vertical diffusivity for convective adjustment. The computational and
 346 physical settings are listed in Table 2 (CILF experiment). We use the numerical
 347 solution of the very high vertical resolution *zfs-150* model as a reference to evaluate
 348 the performance of both *zps-34* and *MEs-34* vertical schemes.

349 The numerical simulations are performed for 60 days. When the lateral ex-
 350 change and spreading of an oceanic cold water patch occurs, baroclinic instabilities
 351 break up the mixed patch and homogeneous water sinks and spreads out at its
 352 neutrally buoyant level (see fig. 3 in Marshall and Schott 1999).

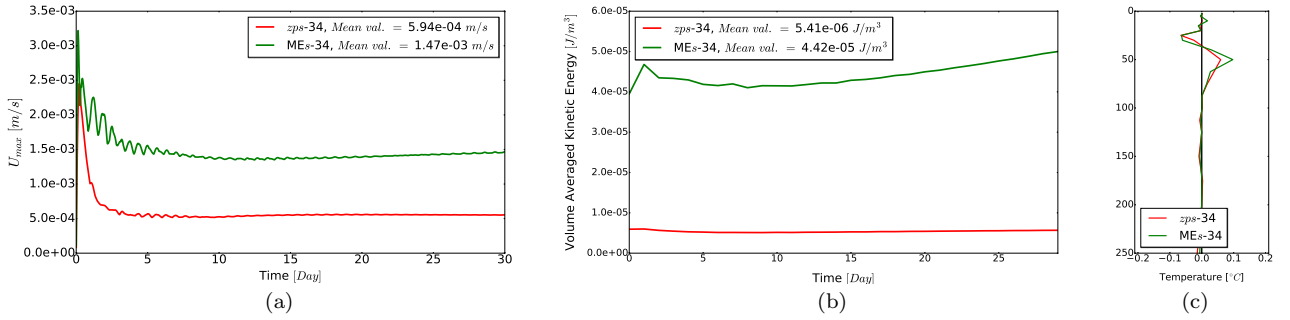


Fig. 9 (a) Time series of spurious currents maximum values, (b) time series of basin averaged Kinetic Energy and (c) differences between temperature profiles extracted in the middle of the domain after 30 days of simulation and the initial condition of zps -34 (red) and MEs -34 (green) models.

353 4 Results and Discussion

354 4.1 Horizontal pressure gradients errors

355 The numerical results of this experiment demonstrate that horizontal pressure gra-
 356 dient errors appear in both MEs and zps models. After 31 days, spurious currents
 357 develop in both models, however their absolute values are small in both cases. In
 358 the zps -34 model they are localized only in proximity of the sloping sea-floor while
 359 in the MEs -34 model they affect all the domain.

360 Time series of the maximum value of spurious currents computed over the whole
 361 domain (Figure 9(a)) show that u_{max} values are less than $5 \cdot 10^{-3} \text{ ms}^{-1}$, i.e. well
 362 within the acceptable margin of error and are comparable with the accuracy of
 363 high-precision instruments (Valeport, 2017). The averaged over the length of the
 364 simulation maximum error for the zps -34 model is $0.59 \cdot 10^{-3} \text{ ms}^{-1}$, which is
 365 slightly better than the one of the MEs model, where the average maximum value
 366 is $1.47 \cdot 10^{-3} \text{ ms}^{-1}$.

367 The time series of the basin averaged Kinetic Energy (KE) due to spurious
 368 currents are compared in Figure 9(b). The zps -34 model has a time averaged KE
 369 of $5.41 \cdot 10^{-6} \text{ Jm}^{-3}$, which corresponds to an average speed of $1.02 \cdot 10^{-4} \text{ ms}^{-1}$.
 370 The MEs -34 model shows slightly higher but still very low values: basin averaged
 371 KE of $4.42 \cdot 10^{-5} \text{ Jm}^{-3}$ and average speed of $2.93 \cdot 10^{-4} \text{ ms}^{-1}$. After one month of
 372 simulation, the KE in the MEs model does not reach an equilibrium. In the case of
 373 σ -coordinates, this behaviour has been classified as *sigma error of the second kind*
 374 (SESK) (Mellor et al., 1998) and it has been reported and studied in a number of
 375 publication (see for example Shchepetkin 2003 and references therein).

376 Figure 9(c) presents differences between the temperature profiles extracted in
 377 the middle of the domain of the two models after 30 days of simulation and the
 378 initial condition, showing that the same level of spurious mixing is obtained with
 379 both the models.

380 As discussed in Sec. 3.1, the doming of the computational levels in MEs -34 was
 381 introduced to deal with ocean domains characterized by a cyclonic circulation. In

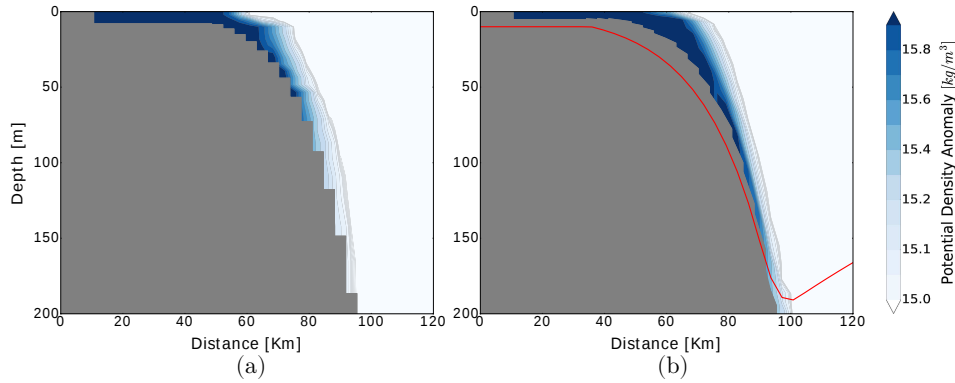


Fig. 10 Cross section in the middle of the domain showing the cascade simulated by the *zps-34* (a) and the *MEs-34* (b) models at day 6.

382 this experiment we use *MEs-34* for an ocean with largely horizontal isopycnals and
 383 an absence of any background circulation. In order to evaluate a potential negative
 384 effect of curved computational levels in the ocean interior we also performed an
 385 additional simulation with the same grid set-up of the *MEs-34* grid but using a
 386 modified upper envelope H_e^1 which is horizontal in the ocean interior. Hereinafter
 387 we call this grid *SH13-34*, since it follows Shapiro et al. (2013), see Figure 1(d).
 388 Comparisons of numerical results obtained with the *MEs-34* and the *SH13-34* grids
 389 demonstrate that inclining the model levels in the ocean interior (used in *MEs-34*)
 390 does not increase the magnitude of spurious currents. The time-averaged maximum
 391 value of spurious currents in the *SH13-34* is $1.46 \cdot 10^{-3} \text{ m s}^{-1}$ as compared to
 392 $1.47 \cdot 10^{-3} \text{ m s}^{-1}$ in *MEs-34*. This result supports the use of *MEs-34* type models
 393 with the curved upper envelope even in areas without cyclonic circulations or
 394 where ocean fronts are weak or moderate.

395 4.2 Dense water cascading on the shelf

396 We evaluate the *zps-34* and *MEs-34* models' performance in representing dense
 397 water overflows down a steep topography by comparing the numerical results of
 398 the downslope velocity with theoretical values given by Shapiro and Hill (1997).

399 The downslope speed is defined as the speed of the plume head in an az-
 400 imuthally averaged sense. The plume is defined as a water mass with potential
 401 density $\geq 1014.99 \text{ kg m}^{-3}$. The speed is computed using the horizontal distance
 402 of each grid cell representing the plume head from the middle of the domain.

403 Time series of the plume edge depths show that both models reproduce a dense
 404 water cascading with nearly constant downward speed (Figure 10). The plume head
 405 reaches the deepest zone of the model topography (1000 m) after 11 days in the
 406 case of the *MEs-34* model and after 14 days with the *zps-34* grid.

407 In order to compare the numerical and analytical solutions, we compute the
 408 downslope velocity u_{model} of the simulated cascades only when the plume edge is
 409 located in areas where the topographic slope is between 0.006 and 0.020 and the
 410 depth is less than 800m (see Figure 11(a)). To compute the Nof's velocity we use
 411 a slope of 0.014, the mean value of the actual bottom slope.

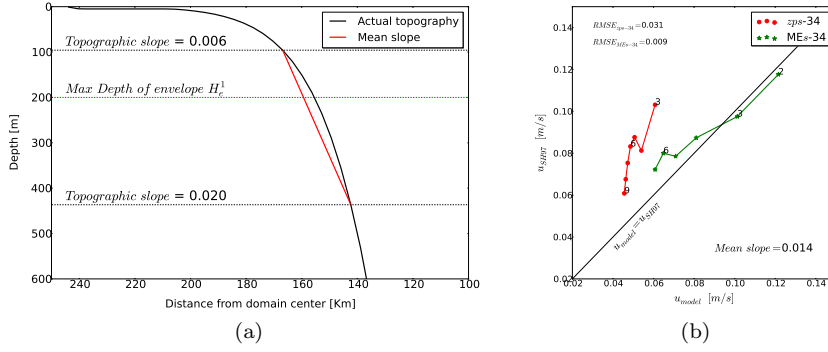


Fig. 11 (a) Actual topography of the model domain (black) and the slope used to compute the Nof (1983) velocity (red). The locations where the topographic slope is equal to 0.006 and 0.020 and the maximum depth of envelope H_e^1 of the MEs model are shown as well. (b) Comparison between the daily values of the downslope velocity predicted by the Shapiro and Hill (1997) theoretical model with the numerical ones obtained with the *zps-34* (red) and the MEs-34 (green) models. Numbers indicate the day.

412 In order to compute the reduced gravity g' , we consider a reference potential
 413 density ρ_0 given by the daily mean of azimuthally-averaged potential densities in
 414 model cells just above the model bathymetry. The ambient water density ρ_a is obtained
 415 by computing the daily mean of azimuthally-averaged potential densities in
 416 model bottom cells with values less than $1014.99 \text{ kg m}^{-3}$. Finally, the daily potential
 417 density ρ_c representative of the dense cascade of each model run is computed by
 418 daily averaging potential densities of bottom cells where the azimuthally-averaged
 419 potential density is between 1015.35 and $1014.99 \text{ kg m}^{-3}$.

420 Figure 11(b) shows the comparison between the daily values of the downslope
 421 velocity given by the analytical solution (Shapiro and Hill, 1997) and the numerical
 422 solutions obtained with the *zps-34* and the MEs-34 models.

423 Results show that the MEs-34 model performs significantly better than the *zps-34*
 424 model. In the *zps-34* model, the dense water cascade crosses the analysed zone
 425 (i.e. the area between the water depths of 90 and 450 m, see Figure11(a)) from
 426 day three to day 9. Throughout the entire period, the *zps-34* underestimates the
 427 downslope speed of cascading, especially in the beginning of the event (day 3).
 428 The RMS error of the *zps-34* model is 0.031 ms^{-1} , which is high (about 50%)
 429 compared to the average downslope speed of $0.05 - 0.07 \text{ ms}^{-1}$. On the other hand,
 430 in the MEs-34 model the plume descends faster, has lower loss of density due
 431 to entrainment, and crosses the analysed zone from day 2 to 7. The modelled
 432 downslope speeds are in the range of $0.06-0.12 \text{ ms}^{-1}$ and are almost equal to the
 433 analytical solution, with a RMS error of 0.009 ms^{-1} , or about 10% of the average
 434 speed. The fact that the downslope cascading in *zps-34* is slower than in MEs-34
 435 is probably due to the enhanced artificial mixing (reducing g') which characterises
 436 *z*-type models with step-like topography (see Figure 10). This agrees with other
 437 gravity current overflow experiments results (see for example fig. 2 in Ezer 2005).

438 Figure 11(b) shows that during the days 6 and 7 of the MEs-34 simulation,
 439 the plume reaches the lower computational zone D2, which has some horizontal
 440 (geopotential) levels. The accuracy of the simulation slightly decreases at this point

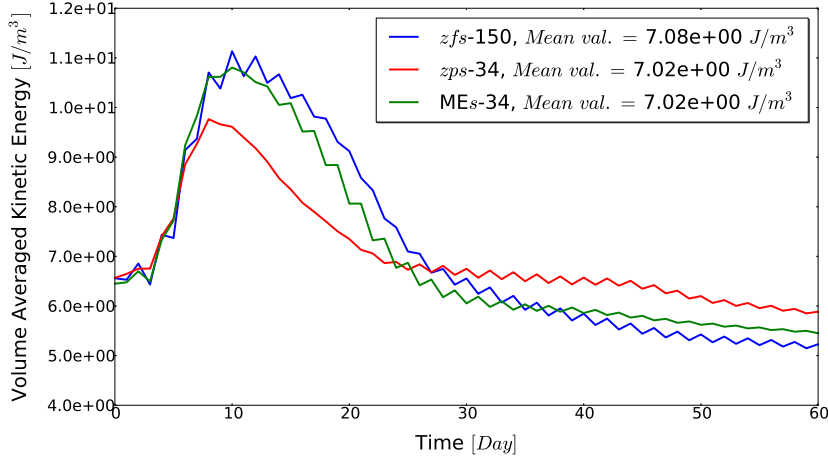


Fig. 12 Time series of the volume averaged KE for the *zfs-150* (blue), *zps-34* (red) and *MEs-34* (green) models.

441 in time as the cascade head reaches a point in the vertical coordinate system which
 442 begins to resemble a *z*-level grid.

443 4.3 Formation of Cold Intermediate Layer

444 For this experiment, an analytical solution is not available. Therefore, we compare
 445 the results of *zps-34* and *MEs-34* models with the reference solution produced by
 446 the high resolution *zfs-150* model.

447 A *zfs-150* simulation is significantly more expensive computationally than a
 448 simulation performed with the other two low resolution models. In this experiment
 449 for example, the duration of the *zfs-150* simulation on our HPC cluster was 70556
 450 *s* (≈ 19.6 *hr*), while *zps-34* and *MEs-34* numerical runs took 17579 *s* (≈ 4.9 *hr*)
 451 and 21646 *s* (≈ 6.0 *hr*), respectively.

452 We begin the analysis with the comparison of the 60 days long time series of
 453 the volume averaged Kinetic Energy (KE) of the three models (Figure 12).
 454 After a few days of spin-up, all the simulations seem to represent the same general
 455 dynamics: a first energetic stage where the dense cold patch sinks and spreads
 456 along the permanent pycnocline and a second less active regime where the CIL is
 457 at its neutrally buoyant level and geostrophy is the leading dynamics.

458 The time series of basin averaged KE produced with *MEs-34* and the reference
 459 *zfs-150* models are quite similar, with a RMS error equal to 0.15 Jm^{-3} (or ap-
 460 proximately 2% of the mean KE). Both models show a maximum of KE at day 10
 461 with values of 10.81 Jm^{-3} in the case of the *MEs-34* model and 11.13 Jm^{-3} for
 462 the reference *zfs-150* model.

463 On the other hand, the *zps-34* model simulates a shorter and less energetic first
 464 phase and a moderately more vigorous geostrophic stage, with a RMS error of 0.96
 465 Jm^{-3} (or 14% of the mean KE). The maximum of KE in the *zps-34* simulation is
 466 9.76 Jm^{-3} and is reached at day 8, i.e. 2 days earlier than the reference.

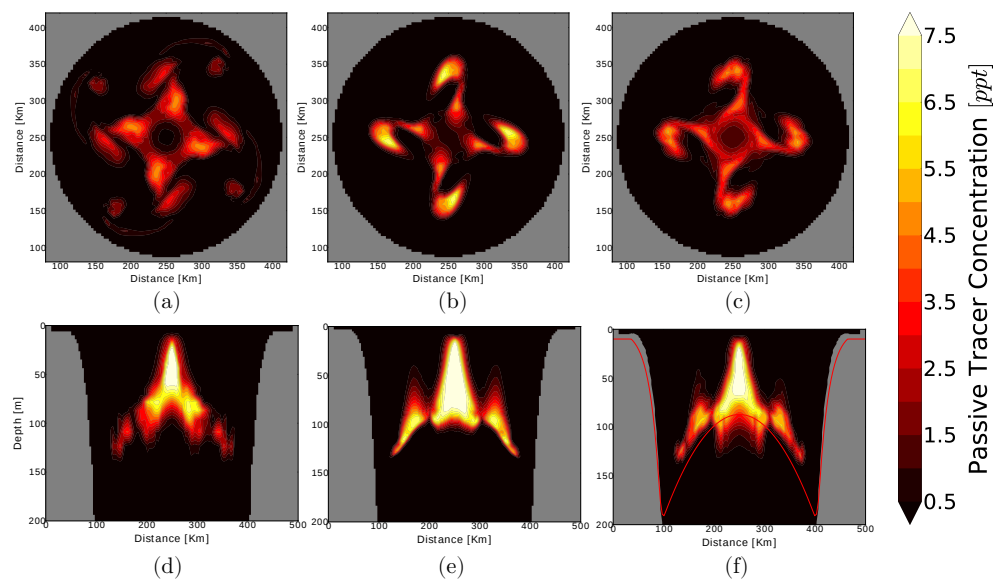


Fig. 13 Passive tracer concentration after 18 days. *First row*: horizontal distribution maps obtained at 105 m depth with the *zps-34* (a), the *zfs-150* (b) and the *MEs-34* (c) models. *Second row*: meridional cross sections obtained with the *zps-34* (d), the *zfs-150* (e) and the *MEs-34* (f) models.

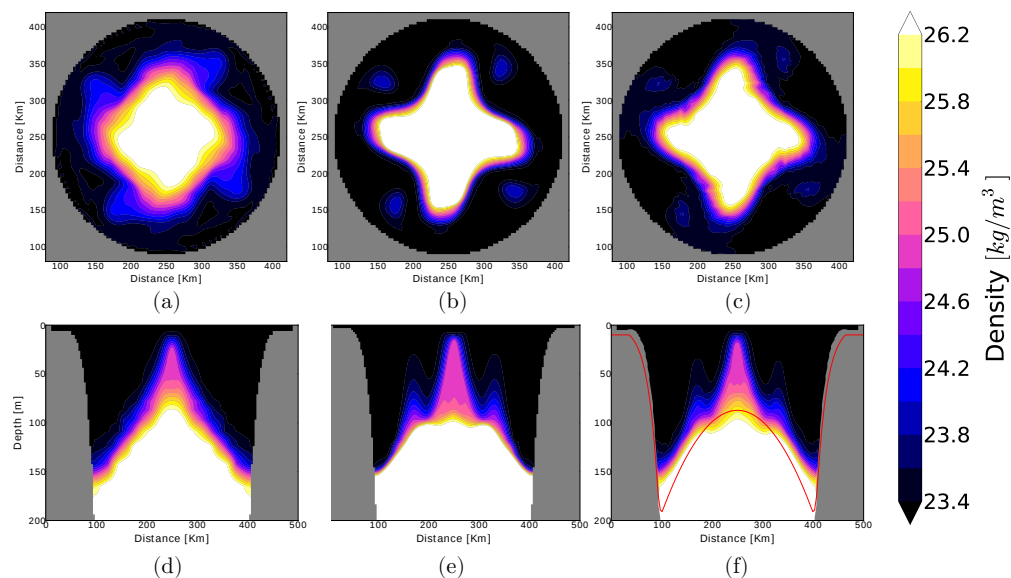


Fig. 14 Density anomaly distribution after 18 days. *First row*: horizontal maps at a depth of 120 m obtained with the *zps-34* (a), the *zfs-150* (b) and the *MEs-34* (c) models. *Second row*: meridional cross sections obtained with the *zps-34* (d), the *zfs-150* (e) and the *MEs-34* (f) models.

467 Daily averaged horizontal distribution maps and vertical cross sections of den-
 468 sity anomaly and passive tracer concentration after 18 and 50 days illustrate how
 469 the more energetic (day 18) and the less dynamical (day 50) stages of the CIL
 470 formation are represented by the three models.

471 After 18 days, the *zfs*-150 and *MEs*-34 models represent similar mesoscale
 472 baroclinic structures (see Figure 13(b)-(c) and Figure 14(b)-(e)-(d)-(f)). As ex-
 473 pected, the high resolution reference model *zfs*-150 is able to maintain the sharp
 474 pycnocline, both in the lateral and in the vertical directions (Figure 14(b)-(e)). The
 475 *MEs*-34 model demonstrates a similar capability, especially for horizontal gradi-
 476 ents (Figure 14(c)-(f)). On the other hand, Figure 13(a) and Figure 14(a)-(d) show
 477 that the *zps*-34 model generates stronger diapycnal diffusion and entrainment than
 478 *MEs*-34.

479 The transport of the passive tracer along the pycnocline after 18 days is simi-
 480 larly represented by both the *zfs*-150 and *MEs*-34 models (Figure 13(e)-(f)). To
 481 the contrary, the *zps*-34 model generates spurious mixed patches of tracer con-
 482 centration shown in blue in Figure 13(d).

483 This effect is probably due to the fact that the horizontal computational levels
 484 create a staggered representation of the pycnocline, and hence are subject to the
 485 same spurious mixing as when *z*-levels hit the sloping bottom.

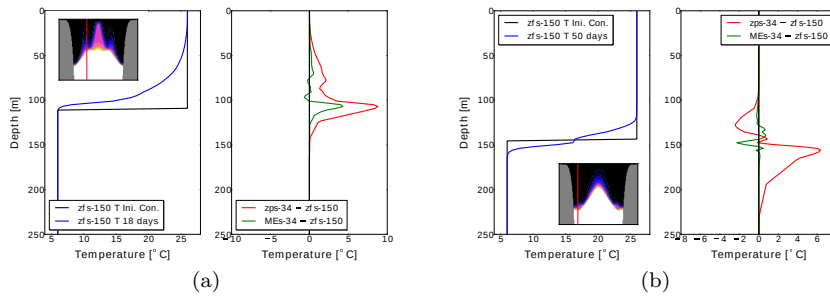


Fig. 15 Temperature vertical profiles simulated with the reference *zfs*-150 model (blue) and errors relative to the reference produced by the *zps*-34 (red) and the *MEs*-34 (green) models. The location of the profiles is shown in each cross section. (a) After 18 days and (b) after 50 days of simulations. The initial condition is shown in black.

486 Figure 15(a) shows a vertical profile of temperature simulated by the reference
 487 *zfs*-150 model in the proximity of the head of spreading dense water (blue profile,
 488 left sub-panel). It also shows the errors relative to the reference produced by the
 489 *zps*-34 (red) and *MEs*-34 (green) models, demonstrating that the *MEs*-34 grid has
 490 a significantly smaller error than the *zps*-34 model.

491 The vertical profiles of errors in horizontally averaged density and passive tracer
 492 concentration relative to the reference *zfs*-150 numerical solution after 18 days
 493 are presented in Figure 16(a)-(b). They show that the error generated by *MEs*-34
 494 model is approximately 50% smaller in comparison to the *zps*-34 model.

495 At day 50, all three models simulate a less active dynamics, where the lateral
 496 exchange and spreading of the dense cold water to its neutrally buoyant level is

497 terminated and geostrophic adjustment is the driving process, see Figures 17 and
 498 18.

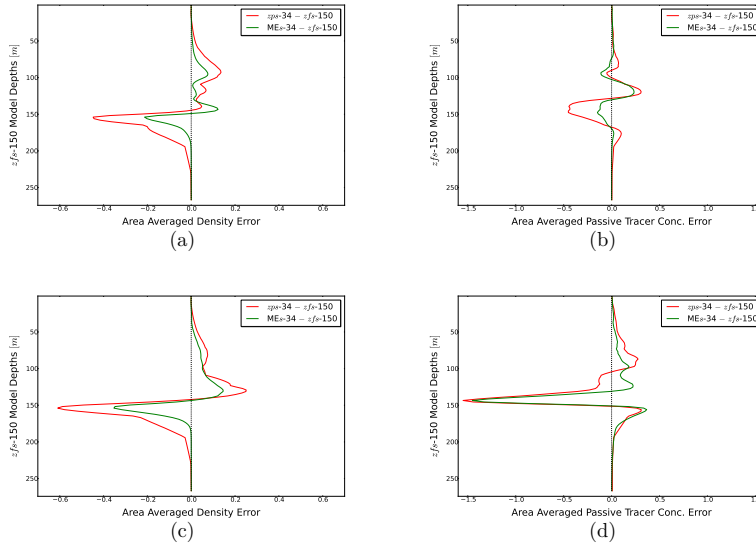


Fig. 16 Area averaged density and passive tracer concentration difference between the *zps*-34 (red) and the *MEs*-34 solutions and the reference *zfs*-150 one after 18 days (a-b) and 50 days (c-d) of simulation.

499 The reference *zfs*-150 solution shows that after 50 days the initial dense cold
 500 patch has formed a nearly uniform well-defined density layer with sharp fronts
 501 above the main pycnocline (see Figure 18(b)-(e)). The passive tracer is advected
 502 with low numerical diffusion, reaching depths of around 150 m at almost the origi-
 503 nal concentration (Figure 17(b)-(e)).

504 Figures 17(a)-(d) show the impact of the higher numerical diffusion of the *zps*-
 505 34 model in the transport of the passive tracer: the nearly uniform distribution
 506 along the pycnocline of the reference solution is lost and the passive tracer is mostly
 507 confined at depths shallower than 120 m. The maximum of tracer concentration
 508 is located at depths around 80-90 m. Figure 16(d) confirms that this is the case
 509 for the whole domain: at day 50, the *zps*-34 model simulates moderately higher
 510 tracer concentrations than the *zfs*-150 model at depths between 90-110 m and
 511 importantly lower values between 110-150 m.

512 After 50 days, the *MEs*-34 model represents a nearly uniform tracer distribu-
 513 tion along the main pycnocline up to 120-130 m (see Figure 17(f)). The *MEs*-34
 514 model simulates a horizontal passive tracer ring-shaped distribution at a depth
 515 of 120 m (Figure 17(c)) which is very similar to the reference *zfs*-150 solution
 516 (Figure 17(b)). This proves a lower artificial diffusion of the *MEs*-34 model in
 517 comparison to the *zps*-34 one. Figure 16(d) shows that at day 50 both *zps*-34
 518 and *MEs*-34 models generate slightly higher values than the reference solution. At
 519 depths around 150 m, both *zps*-34 and *MEs*-34 simulate lower values than the
 520 reference.

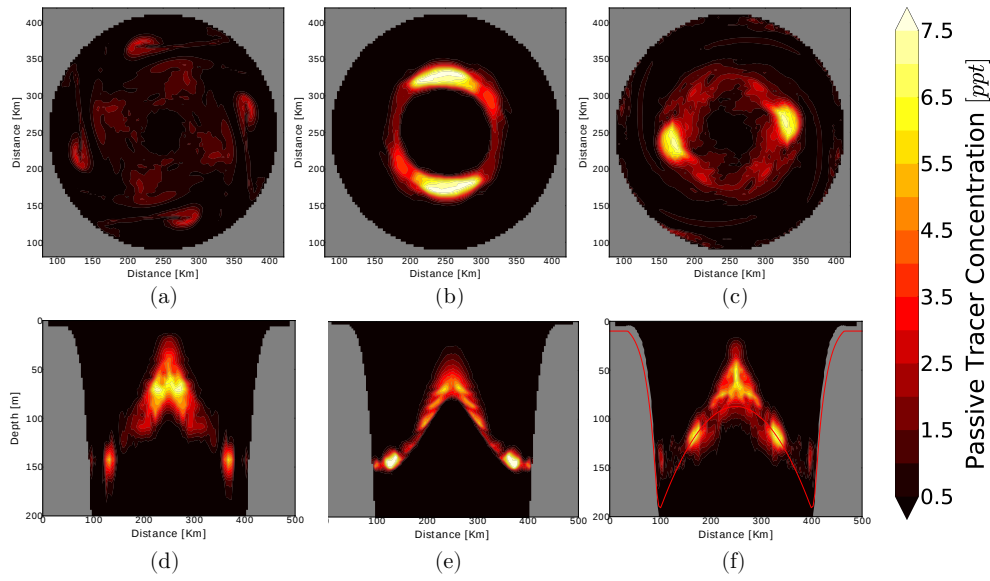


Fig. 17 Passive tracer concentration after 50 days. *First row*: horizontal distribution maps obtained at 120 m depth with the *zps-34* (a), the *zfs-150* (b) and the *MEs-34* (c) models. *Second row*: zonal cross sections obtained with the *zps-34* (d), the *zfs-150* (e) and the *MEs-34* (f) models.

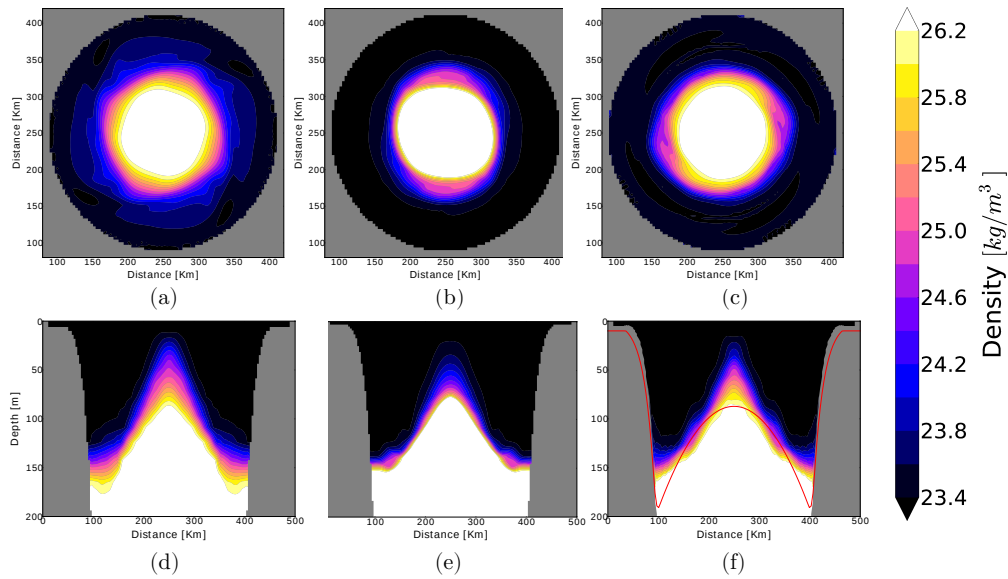


Fig. 18 Density anomaly distribution after 50 days. *First row*: horizontal maps at a depth of 120 m obtained with the *zps-34* (a), the *zfs-150* (b) and the *MEs-34* (c) models. *Second row*: zonal cross sections obtained with the *zps-34* (d), the *zfs-150* (e) and the *MEs-34* (f) models.

521 The *zps-34* model shows the formation of a more extended and diffusive CIL,
 522 with weaker horizontal and vertical gradients (Figure 18(a)-(d)). These artefacts
 523 are the result of the low vertical resolution combined with the step-like represen-
 524 tations of both pycnocline and advection.

525 A spurious downwelling event is produced with the *zps-34* model, while it is
 526 not present either in the reference *zfs-150* or *MEs-34* solutions (see Figure 18(d)-
 527 (e)-(f) and Figure 15(b)).

528 Figures 15(b), 16(c) and 18(c)-(f) show that the *MEs-34* model simulates the
 529 formation of a CIL closer to the reference *zfs-150* model, with lower diapycnal
 530 diffusion and sharper density fronts than the *zps-34* model.

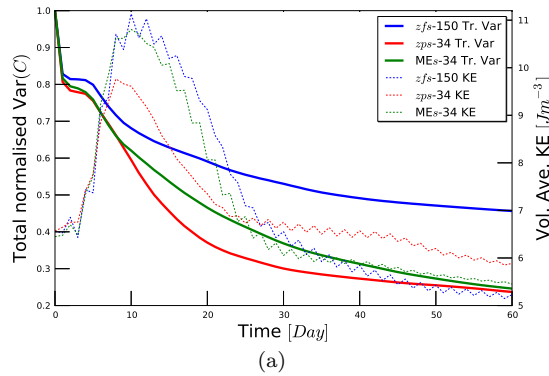


Fig. 19 Time series of the normalised passive tracer total variance (bold lines) and volume averaged KE (dashed lines) for the *zfs-150* (blue), *zps-34* (red) and *MEs-34* (green) models. Normalisation is done with respect to the total variance of the initial condition.

531 The numerical mixing due to discretisation errors of tracer advection schemes
 532 causes decay in time of the passive tracer total variance (Maqueda and Holloway,
 533 2006; Burchard and Rennau, 2008; Klingbeil et al., 2014), which is defined for a
 534 Boussinesq fluid as

$$Var(C) = \langle C^2 \rangle_V - \langle C \rangle_V^2 \quad (11)$$

535 where C is the concentration of the passive tracer and $\langle \bullet \rangle_V = V^{-1} \int \bullet dV$ repre-
 536 sents a global averaging operator in a ocean with volume $V = \int dV$.

537 Following James (1996), in Figure 19 we compare the cumulative loss with time
 538 of the discrete passive tracer total variance in the three models in terms of the
 539 ratio $Var(C)^n / Var(C)^0$, where n indicates the discrete time level and $n = 0$ is the
 540 initial condition. Numerical results show that, as expected, the reference *zfs-150*
 541 model has the lowest loss of variance with time, and hence the smallest numerical
 542 diffusion. The *MEs-34* model performs generally better than *zps-34*, especially
 543 during the more dynamic phase of the simulation. Both models give similar results
 544 after the end of the active phase.

545 5 Conclusions

546 In this study we present and assess the skills of a new vertical discretisation scheme
 547 which we call the ‘Multi-Envelope s -coordinate system’ or ‘MEs’. Our new system
 548 further develops the earlier concept of ‘enveloped bathymetry’, where model lev-
 549 els followed a ‘virtual bottom’ (aka envelope) rather than the actual bathymetry.
 550 Such ‘single-envelope’ system could be classed as an extreme case of the new ‘multi-
 551 envelope’ system. The multi-enveloping method allows the definition of computa-
 552 tional surfaces which are optimised to best represent the physical processes in
 553 question. This method provides greater flexibility in the designing of a vertical
 554 grid than currently available geopotential level or terrain-following systems. All of
 555 these systems can be obtained as specific implementations of MEs.

556 An assessment of the MEs model skill for a number of idealized process studies
 557 shows that MEs generates a small pressure gradient error, gives a better repre-
 558 sentation of dense water cascades down the continental slope and provides a more
 559 accurate simulation of formation of a cold intermediate layer, than a comparable
 560 z -partial steps system.

561 The MEs systems allows achieving a quality of simulation similar to a standard
 562 geopotential grid which has a much higher number of levels, and hence the MEs
 563 system is more computationally efficient.

564 The algorithm of creating MEs was implemented in NEMO for this study, but
 565 can easily adapted for any 3D ocean model.

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571 Appendix 1

572 For each (x, y) of the horizontal domain the complete cubic spline $P_{x,y,i}^3(\sigma_i)$ of the
 573 vertical sub-zone D_i can be written as

$$P_{x,y,i}^3(\sigma_i) = a_{x,y,i} + b_{x,y,i}(-\sigma_i) + c_{x,y,i}(-\sigma_i)^2 + d_{x,y,i}(-\sigma_i)^3 \quad (12)$$

574 where σ_i is given by Equation 4 and $-1 < \sigma_i \leq 0$.

575 Applying the three constraints defined in Section 2 leads to a tridiagonal linear
 576 system of four equations for the four unknowns $a_{x,y,i}$, $b_{x,y,i}$, $c_{x,y,i}$ and $d_{x,y,i}$
 577 (de Boor, 1978).

578 A modified version of the Fortran90 numerical library PPPACK (de Boor, 1978)
 579 has been introduced in the NEMO code to compute the four coefficients of the
 580 complete cubic spline $P_{x,y,i}^3(\sigma_i)$.

581 **References**

- 582 Adcroft, A. and Campin, J.-M. (2004). Rescaled height coordinates for accurate
583 representation of free-surface flows in ocean circulation models. *Ocean Modelling*,
584 7(3-4):269–284.
- 585 Adcroft, A., Hill, C., and Marshall, J. (1997). Representation of Topography by
586 Shaved Cells in a Height Coordinate Ocean Model. *Monthly Weather Review*,
587 125(9):2293–2315.
- 588 Barnier, B., Madec, G., Penduff, T., Molines, J.-M., Treguier, A.-M., Le Sommer,
589 J., Beckmann, A., Biastoch, A., Böning, C., Dengg, J., Derval, C., Durand, E.,
590 Gulev, S., Remy, E., Talandier, C., Theetten, S., Maltrud, M., McClean, J.,
591 and De Cuevas, B. (2006). Impact of partial steps and momentum advection
592 schemes in a global ocean circulation model at eddy-permitting resolution. *Ocean*
593 *Dynamics*, 56(5-6):543–567.
- 594 Barnier, B., Marchesiello, P., De Miranda, A. P., Molines, J.-M., and Coulibaly, M.
595 (1998). A sigma-coordinate primitive equation model for studying the circulation
596 in the South Atlantic. Part I: Model configuration with error estimates. *Deep*
597 *Sea Research I*, 45:543–572.
- 598 Berntsen, J. (2002). Internal pressure errors in sigma coordinate ocean models.
599 *Journal of Atmospheric and Oceanic Technology*, 21(1994):1403–1413.
- 600 Bleck, R. (1998). Ocean modeling in isopycnic coordinates. In Chassignet, E. P. and
601 Verron, J., editors, *Ocean Modeling and Parameterization, NATO ASI Mathe-*
602 *matical and Physical Sciences Series, Vol. 516*, pages 423–448. Kluwer Academic
603 Publishers, Dordrecht.
- 604 Bleck, R. (2002). An oceanic general circulation model framed in hybrid isopycnic-
605 Cartesian coordinates. *Ocean Modelling*, 4(1):55–88.
- 606 Bleck, R. and Boudra, D. (1981). Initial Testing of a Numerical Ocean Circulation
607 Model Using a Hybrid (Quasi-Isopycnic) Vertical Coordinate. *Journal of*
608 *Physical Oceanography*, 11(6):755–770.
- 609 Blumberg, A. F. and Mellor, G. L. (1987). A description of a three-dimensional
610 coastal ocean circulation model. In *Three-Dimensional Coastal Ocean Models*,
611 pages 1–16.
- 612 Burchard, H. and Rennau, H. (2008). Comparative quantification of physically and
613 numerically induced mixing in ocean models. *Ocean Modelling*, 20(3):293–311.
- 614 Chassignet, E. P., Hurlburt, H. E., Smedstad, O. M., Halliwell, G. R., Hogan, P. J.,
615 Wallcraft, A. J., and Bleck, R. (2006). Ocean Prediction with the Hybrid Co-
616 ordinate Ocean Model (HYCOM). In Chassignet, E. P. and Verron, J., editors,
617 *Ocean Weather Forecasting*, pages 413–426. Springer-Verlag, Berlin/Heidelberg.
- 618 Chubarenko, I. and Demchenko, N. (2010). On contribution of horizontal and
619 intra-layer convection to the formation of the Baltic Sea cold intermediate layer.
620 *Ocean Science*, 6(1):285–299.
- 621 Cyr, F., Bourgault, D., and Galbraith, P. S. (2011). Interior versus bound-
622 ary mixing of a cold intermediate layer. *Journal of Geophysical Research*,
623 116(C12):C12029.
- 624 de Boor, C. (1978). *A Practical Guide to Splines*. Springer-Verlag New York, New
625 York.
- 626 Dukhovskoy, D. S., Morey, S. L., Martin, P. J., O’Brien, J. J., and Cooper, C.
627 (2009). Application of a vanishing, quasi-sigma, vertical coordinate for simu-
628 lation of high-speed, deep currents over the Sigsbee Escarpment in the Gulf of

- 629 Mexico. *Ocean Modelling*, 28(4):250–265.
- 630 Enriquez, C. E., Shapiro, G. I., Souza, A. J., and Zatsepin, A. G. (2005). Hy-
631 drodynamic modelling of mesoscale eddies in the Black Sea. *Ocean Dynamics*,
632 55(5-6):476–489.
- 633 Ezer, T. (2005). Entrainment, diapycnal mixing and transport in three-dimensional
634 bottom gravity current simulations using the Mellor–Yamada turbulence
635 scheme. *Ocean Modelling*, 9(2):151–168.
- 636 Ezer, T. (2016). Revisiting the problem of the Gulf Stream separation: on the
637 representation of topography in ocean models with different types of vertical
638 grids. *Ocean Modelling*, 104:15–27.
- 639 Ezer, T. and Mellor, G. L. (1992). A Numerical Study of the Variability and the
640 Separation of the Gulf Stream, Induced by Surface Atmospheric Forcing and
641 Lateral Boundary Flows. *Journal of Physical Oceanography*, 22(6):660–682.
- 642 Ezer, T. and Mellor, G. L. (2004). A generalized coordinate ocean model and a
643 comparison of the bottom boundary layer dynamics in terrain-following and in
644 z-level grids. *Ocean Modelling*, 6(3-4):379–403.
- 645 Furner, R. (2012). A review of lateral diffusion for Met Office ocean shelf modelling.
646 Technical report, Met Office.
- 647 Gangopadhyay, A. and Robinson, A. R. (2002). Feature-oriented regional modeling
648 of oceanic fronts. *Dynamics of Atmospheres and Oceans*, 36(1-3):201–232.
- 649 Gerdes, R. (1993a). A primitive equation ocean circulation model using a gener-
650 al vertical coordinate transformation: 1. Description and testing of the model.
651 *Journal of Geophysical Research*, 98(C8):14683.
- 652 Gerdes, R. (1993b). A primitive equation ocean circulation model using a gener-
653 al vertical coordinate transformation: 2. Application to an overflow problem.
654 *Journal of Geophysical Research*, 98(C8):14703.
- 655 Griffies, S. M. (2004). *Fundamentals of Ocean Climate Models*. Princeton Univer-
656 sity Press, Princeton, NJ.
- 657 Griffies, S. M., Böning, C., Bryan, F. O., Chassignet, E. P., Gerdes, R., Hasumi,
658 H., Hirst, A., Treguier, A.-M., and Webb, D. (2000a). Developments in ocean
659 climate modelling. *Journal of Computational Physics*, 2(3-4):123–192.
- 660 Griffies, S. M., Pacanowski, R. C., and Hallberg, R. W. (2000b). Spurious Diapyc-
661 nal Mixing Associated with Advection in a z -Coordinate Ocean Model. *Monthly*
662 *Weather Review*, 128(3):538–564.
- 663 Haidvogel, D. and Beckmann, A. (1999). *Numerical Ocean Circulation Modeling*.
664 Imperial College Press.
- 665 Haidvogel, D. B., Beckmann, A., and Hedström, K. S. (1991). Dynamical sim-
666 ulations of filament formation and evolution in the Coastal Transition Zone.
667 *Journal of Geophysical Research*, 96(C8):15017.
- 668 Haney, R. (1991). On the pressure gradient force over steep topography in sigma
669 coordinate ocean models. *Journal of Physical Oceanography*, 21:610–619.
- 670 Hirt, C., Amsden, A., and Cook, J. (1974). An arbitrary Lagrangian–Eule-
671 rian computing method for all flow speeds. *Journal of Computational Physics*,
672 14(3):227–253.
- 673 Hofmeister, R., Burchard, H., and Beckers, J. M. (2010). Non-uniform adaptive
674 vertical grids for 3D numerical ocean models. *Ocean Modelling*, 33(1-2):70–86.
- 675 Holt, J., Hyder, P., Ashworth, M., Harle, J., Hewitt, H. T., Liu, H., New, A. L.,
676 Pickles, S., Porter, A., Popova, E., Allen, J. I., Siddorn, J., and Wood, R. (2017).
677 Prospects for improving the representation of coastal and shelf seas in global

- 678 ocean models. *Geoscientific Model Development*, 10(1):499–523.
- 679 Ivanov, V. V., Shapiro, G. I., Huthnance, J. M., Aleynik, D. L., and Golovin,
680 P. N. (2004). Cascades of dense water around the world ocean. *Progress in*
681 *Oceanography*, 60(1):47–98.
- 682 James, I. (1996). Advection schemes for shelf sea models. *Journal of Marine*
683 *Systems*, 8(3-4):237–254.
- 684 Kantha, L. H. and Clayson, C. A. (2000). *Numerical Models of Oceans and Oceanic*
685 *Processes*. Academic Press.
- 686 Kasahara, A. (1974). Various Vertical Coordinate Systems Used for Numerical
687 Weather Prediction. *Monthly Weather Review*, 102(7):509–522.
- 688 Klingbeil, K., Mohammadi-Aragh, M., Gräwe, U., and Burchard, H. (2014). Quan-
689 tification of spurious dissipation and mixing – Discrete variance decay in a
690 Finite-Volume framework. *Ocean Modelling*, 81:49–64.
- 691 Laanaia, N., Wirth, A., Molines, J. M., Barnier, B., and Verron, J. (2010). On
692 the numerical resolution of the bottom layer in simulations of oceanic gravity
693 currents. *Ocean Science*, 6(2):563–572.
- 694 Leclair, M. and Madec, G. (2011). z-Coordinate, an Arbitrary Lagrangian-Eulerian
695 coordinate separating high and low frequency motions. *Ocean Modelling*, 37(3-
696 4):139–152.
- 697 Legg, S., Briegleb, B., Chang, Y., Chassignet, E. P., Danabasoglu, G., Ezer, T.,
698 Gordon, A. L., Griffies, S., Hallberg, R., Jackson, L., Large, W., Özgökmen,
699 T. M., Peters, H., Price, J., Riemenschneider, U., Wu, W., Xu, X., and Yang,
700 J. (2009). Improving Oceanic Overflow Representation in Climate Models: The
701 Gravity Current Entrainment Climate Process Team. *Bulletin of the American*
702 *Meteorological Society*, 90(5):657–670.
- 703 Legg, S., Hallberg, R. W., and Girton, J. B. (2006). Comparison of entrainment
704 in overflows simulated by z-coordinate, isopycnal and non-hydrostatic models.
705 *Ocean Modelling*, 11(1-2):69–97.
- 706 Lemarié, F., Kurian, J., Shchepetkin, A. F., Jeroen Molemaker, M., Colas, F., and
707 McWilliams, J. C. (2012). Are there inescapable issues prohibiting the use of
708 terrain-following coordinates in climate models? *Ocean Modelling*, 42:57–79.
- 709 Madec, G. (2008). NEMO ocean engine. *Note du Pôle de modélisation, Institut*
710 *Pierre-Simon Laplace (IPSL)*, (27):357pp.
- 711 Madec, G., Delecluse, P., Crépon, M., and Lott, F. (1996). Large-Scale Precon-
712 ditioning of Deep-Water Formation in the Northwestern Mediterranean Sea.
713 *Journal of Physical Oceanography*, 26(8):1393–1408.
- 714 Maqueda, M. M. and Holloway, G. (2006). Second-order moment advection scheme
715 applied to Arctic Ocean simulation. *Ocean Modelling*, 14(3-4):197–221.
- 716 Marchesiello, P., Debreu, L., and Couvelard, X. (2009). Spurious diapycnal mixing
717 in terrain-following coordinate models: The problem and a solution. *Ocean*
718 *Modelling*, 26(3-4):156–169.
- 719 Marshall, J. and Schott, F. (1999). Open-ocean convection: Observations, theory,
720 and models. *Reviews of Geophysics*, 37(1):1–64.
- 721 Marti, O., Madec, G., and Delecluse, P. (1992). Comment on 'Net diffusivity in
722 ocean general circulation models with nonuniform grids' by F. L. Yin and I. Y.
723 Fung. *Journal of Geophysical Research*, 97(C8):12763.
- 724 Martinho, A. S. and Batteen, M. L. (2006). On reducing the slope parameter in
725 terrain-following numerical ocean models. *Ocean Modelling*, 13(2):166–175.

- 726 Mellor, G. L. and Blumberg, A. F. (1985). Modeling Vertical and Horizontal
727 Diffusivities with the Sigma Coordinate System. *Monthly Weather Review*,
728 113(8):1379–1383.
- 729 Mellor, G. L., Ezer, T., and Oey, L.-Y. (1994). The Pressure Gradient Conun-
730 drum of Sigma Coordinate Ocean Models. *Journal of Atmospheric and Oceanic*
731 *Technology*, 11(4):1126–1134.
- 732 Mellor, G. L., Häkkinen, S. M., Ezer, T., and Patchen, R. C. (2002). A General-
733 ization of a Sigma Coordinate Ocean Model and an Intercomparison of Model
734 Vertical Grids. In Pinardi, N. and Woods, J., editors, *Ocean Forecasting: Con-*
735 *ceptual Basis and Applications*, pages 55–72. Springer Berlin Heidelberg, Berlin,
736 Heidelberg.
- 737 Mellor, G. L., Oey, L. Y., and Ezer, T. (1998). Sigma coordinate pressure gra-
738 dient errors and the seamount Problem. *Journal of Atmospheric and Oceanic*
739 *Technology*, 15(5):1122–1131.
- 740 MyOcean2 (2014). MyOcean catalogue of PRODUCTS, V4.0.
- 741 Nof, D. (1983). The translation of isolated cold eddies on a sloping bottom. *Deep*
742 *Sea Research Part A, Oceanographic Research Papers*, 30(2):171–182.
- 743 Oddo, P., Adani, M., Pinardi, N., Fratianni, C., Tonani, M., and Pettenuzzo, D.
744 (2009). A nested Atlantic-Mediterranean Sea general circulation model for op-
745 erational forecasting. *Ocean Science*, 5:461–473.
- 746 O’Dea, E. J., Arnold, A. K., Edwards, K. P., Furner, R., Hyder, P., Martin, M. J.,
747 Siddorn, J. R., Storkey, D., While, J., Holt, J. T., and Liu, H. (2012). An
748 operational ocean forecast system incorporating NEMO and SST data assimila-
749 tion for the tidally driven European North-West shelf. *Journal of Operational*
750 *Oceanography*, 5(1):3–17.
- 751 Pacanowski, R. C., Gnanadesikan, A., and Olume, V. (1998). Transient Re-
752 sponse in a Z-Level Ocean Model That Resolves Topography with Partial Cells.
753 *Monthly Weather Review*, 126(12):3248–3270.
- 754 Redi, M. H. (1982). Oceanic Isopycnal Mixing by Coordinate Rotation. *Journal*
755 *of Physical Oceanography*, 12(10):1154–1158.
- 756 Roberts, M. and Marshall, D. (1998). Do We Require Adiabatic Dissipation
757 Schemes in Eddy-Resolving Ocean Models? *Journal of Physical Oceanography*,
758 28(10):2050–2063.
- 759 Roquet, F., Madec, G., Brodeau, L., and Nycander, J. (2015). Defining a Simplified
760 Yet Realistic Equation of State for Seawater. *Journal of Physical Oceanography*,
761 45(10):2564–2579.
- 762 Shapiro, G. I. and Hill, A. E. (1997). Dynamics of Dense Water Cascades at the
763 Shelf Edge. *Journal of Physical Oceanography*, 27(11):2381–2394.
- 764 Shapiro, G. I., Luneva, M., Pickering, J., and Storkey, D. (2013). The effect of
765 various vertical discretization schemes and horizontal diffusion parameterization
766 on the performance of a 3-D ocean model: the Black Sea case study. *Ocean*
767 *Science*, 9(2):377–390.
- 768 Shchepetkin, A. F. (2003). A method for computing horizontal pressure-gradient
769 force in an oceanic model with a nonaligned vertical coordinate. *Journal of*
770 *Geophysical Research*, 108(C3):3090.
- 771 Shchepetkin, A. F. and McWilliams, J. C. (2005). The regional oceanic modeling
772 system (ROMS): a split-explicit, free-surface, topography-following-coordinate
773 oceanic model. *Ocean Modelling*, 9(4):347–404.

- 774 Siddorn, J. R. and Furner, R. (2013). An analytical stretching function that com-
775 bines the best attributes of geopotential and terrain-following vertical coordi-
776 nates. *Ocean Modelling*, 66:1–13.
- 777 Song, Y. and Haidvogel, D. (1994). A Semi-implicit Ocean Circulation Model Using
778 a Generalized Topography-Following Coordinate System. *Journal of Computa-*
779 *tional Physics*, 115(1):228–244.
- 780 Song, Y. T. and Hou, T. Y. (2006). Parametric vertical coordinate formulation for
781 multiscale, Boussinesq, and non-Boussinesq ocean modeling. *Ocean Modelling*,
782 11(3-4):298–332.
- 783 Treguier, A. M., Dukowicz, J. K., and Bryan, K. (1996). Properties of nonuniform
784 grids used in ocean general circulation models. *Journal of Geophysical Research:*
785 *Oceans*, 101(C9):20877–20881.
- 786 Trotta, F., Fenu, E., Pinardi, N., Bruciaferri, D., Giacomelli, L., Federico, I., and
787 Coppini, G. (2016). A Structured and Unstructured grid Relocatable ocean
788 platform for Forecasting (SURF). *Deep-Sea Research Part II: Topical Studies*
789 *in Oceanography*, 133:54–75.
- 790 Umlauf, L. and Burchard, H. (2003). A generic length-scale equation for geophys-
791 ical turbulence models. *Journal of Marine Research*, 61(2):235–265.
- 792 Umlauf, L. and Burchard, H. (2005). Second-order turbulence closure models
793 for geophysical boundary layers. A review of recent work. *Continental Shelf*
794 *Research*, 25(7-8):795–827.
- 795 Valeport (2017). Model 106 Current Meter.
796 [http://www.valeport.co.uk/Portals/0/Docs/Datasheets/Valeport-Model-](http://www.valeport.co.uk/Portals/0/Docs/Datasheets/Valeport-Model-106.pdf)
797 [106.pdf](http://www.valeport.co.uk/Portals/0/Docs/Datasheets/Valeport-Model-106.pdf). Accessed 12 November 2017.
- 798 Wobus, F., Shapiro, G. I., Huthnance, J. M., and Maqueda, M. A. M. (2013). The
799 piercing of the Atlantic Layer by an Arctic shelf water cascade in an idealised
800 study inspired by the Storfjorden overflow in Svalbard. *Ocean Modelling*, 71:54–
801 65.
- 802 Wobus, F., Shapiro, G. I., Maqueda, M., and Huthnance, J. (2011). Numerical sim-
803 ulations of dense water cascading on a steep slope. *Journal of Marine Research*,
804 69(2):391–415.