2018-01-31

Control law and state estimators design for multi-agent system with reduction of communications by event-triggered approach

Viel, CXS

http://hdl.handle.net/10026.1/10558

NNT : 2017SACLS274

Doctorat Paris-Saclay

École doctorale n^o 580 : STIC | Sciences et technologies de l'information et de la communication

Spécialité doctorale "Robotique"

par

Christophe VIEL

Control law and state estimators design for multi-agent system with reduction of communications by event-triggered approach

Loi de guidage coopérative et estimateurs d'état pour système multi-agent avec réduction des communications par méthode event-triggered

Thèse présentée et soutenue à l'Onera le 26 septembre 2017

Jury

Dimos DIMAROGONAS,	Professeur, KTH	Rapporteur
Sophie TARBOURIECH,	Chercheur, LAAS-CNRS	Rapporteur
Jacques BLANC-TALON,	Ingenieur, DGA	Examinateur
Antonio LORIA,	Professeur, L2S Centrale-Supélec	Examinateur
Luc JAULIN,	Professeur, ENSTA-Bretagne	Examinateur
Hélène PIET-LAHANIER,	Chercheur, ONERA	Directeur de thèse
Michel KIEFFER,	Professeur, L2S Centrale-Supélec	Directeur de thèse
Sylvain BERTRAND,	Chercheur, ONERA	Encadrant

ONERA, Chemin de la Hunière, 91123 Palaiseau, France

Acknowledgements

Je veux tout d'abord remercier Sophie Tarbouriech et Dimos Dimaragonas pour leur investissement en tant que rapporteurs de ma thèse. Ma reconnaissance va à Antonio Loria, Luc Jaulin et Jacques Blanc-Talon pour avoir accepté de participer au jury de soutenance. Merci également à Antonio Loria, Luc Jaulin, et Jacques Blanc-Talon pour leur investissement en tant que membre du jury à ma soutenance. Merci aussi à la DGA et à l'ONERA pour avoir financé cette thèse durant ces trois années.

Je remercie mes encadrants de thèse, Hélène Piet-Lahanier, Michel Kieffer et Sylvain Bertrand, pour leur support et leur patience, surtout dans les débuts où la méthodologie ne m'était pas encore acquise. Merci pour votre confiance et pour le temps que vous m'avez consacré, en partage de connaissance comme en relecture de travaux. J'ai pu développer mes connaissances personnelles et gagner en rigueur sous votre directive. J'espère que mes résultats sont à la hauteur de leurs attentes après ces trois années de travail en leur collaboration.

Merci également aux doctorants et les stagiaires du DCPS qui se sont succédés, les anciens comme les nouveaux, avec qui j'ai passé de bons moments et dont les conversations du midi m'ont permis de relâcher la pression ou de relativiser mes problèmes. Merci pour leurs conseils et leurs encouragements. Leurs retours positifs après mes présentations orales m'auront permis de prendre d'avantage confiance en moi et de progresser dans cet exercice qui me reste difficile.

A mes parents, qui m'ont toujours soutenu et permis de faire les études que je voulais faire. Merci d'avoir cru en moi.

A Mélissa, qui a été là durant les moments difficiles, m'a soutenu et m'a encouragé durant ces trois années.

Contents

Abbreviation

MAS: Multi-Agent System

UAV: Unmanned Aerial Vehicle

AUV: Autonomous Underwater Vehicle

CTC: Communication Triggering Condition

T–S: Takagi–Sugeno

ALOHA: Additive Links On-line Hawaii Area

CSMA: Carrier Sense Multiple Access

RTS: Ready to Send

CAN: Controller Area Network

ISS: Input-to-State Stability

ISpS: Input-to-State practically Stable

Notations

Chapter 2: Distributed event-triggered consensus of linear multi-agent systems with bounded perturbations

x_i	state of Agent i.
y_i^j	estimate of the state x_i performed by Agent j.
y^j	vector gathering the estimates performed by Agent j of the states of all agents.
y	vector $\left[\begin{array}{ccc} (y_1^1)^T & \dots & (y_N^N)^T \end{array}\right]^T$ of estimates performed by each agent of its own state
v^i	common estimate of y^i performed by all agents.
e_i^j	estimation error between x_i and y_i^j.
$t_{j,k}$	time instant at which the k-th message has been sent by Agent j.
$t_{j,k}^i$	the time at which the k-th message sent by Agent j has been received by Agent i.
t_ℓ^i	time of reception by Agent i of the ℓ-th message, whatever the sending agent.

Chapter 3: Distributed event-triggered control for multi-agent formation stabilization

q_i	vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$
q	vector $\left[\begin{array}{cccc} q_1^T & q_2^T & \dots & q_N^T \end{array}\right]^T \in \mathbb{R}^{N.n}$, *configuration* of the MAS
x_i	state vector $\left[q_i^T, \dot{q}_i^T\right]^T$ of Agent i
$\hat{q}_i^j$	estimate of q_i performed by Agent j.
$\hat{q}^j$	estimate of q performed by Agent j.
e_i^j	estimation error between q_i and $\hat{q}_i^j$.
r_{ij}	relative coordinate vector $r_{ij} = q_i - q_j$ between agents i and j.
r_{ij}^*	desired value for r_{ij}.
$t_{j,k}$	time at which the k-th message is sent by Agent j.
$t_{j,k}^i$	time at which the k-th message sent by Agent j is received by Agent i.
t_ℓ^i	time of reception by Agent i of the ℓ-th message, whatever the sending agent.

Chapter 4: Distributed event-triggered for multi-agent formation stabilization and tracking control

q_i	vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$
q	vector $\left[\begin{array}{cccc} q_1^T & q_2^T & \dots & q_N^T \end{array}\right]^T \in \mathbb{R}^{N.n}$, *configuration* of the MAS
x_i	state vector $\left[q_i^T, \dot{q}_i^T\right]^T$ of Agent i
$\hat{q}_i^j$	estimate of q_i performed by Agent j.
$\hat{q}^j$	estimate of q performed by Agent j.
$\hat{x}_i^j$	estimate of x_i performed by Agent j.
e_i^j	estimation error between q_i and $\hat{q}_i^j$.
r_{ij}	relative coordinate vector $r_{ij} = q_i - q_j$ between agents i and j.
r_{ij}^*	desired value for r_{ij}.
q_0	reference trajectory
q_i^*	reference trajectory for Agent i, $q_i^* = q_0 + r_{i1}^*$
r_i	trajectory error for Agent i, $r_i = q_i - q_i^*$
$t_{j,k}$	time at which the k-th message is sent by Agent j.
$t_{j,k}^i$	time at which the k-th message sent by Agent j is received by Agent i.
v_i	collision avoidance term of Agent i.
r_c	collision radius
r_a	avoidance radius

Chapter 5: Packet dropout in distributed event-triggered for multi-agent formation stabilization

q_i	vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$
q	vector $\left[\begin{array}{cccc} q_1^T & q_2^T & \dots & q_N^T \end{array}\right]^T \in \mathbb{R}^{N.n}$, *configuration* of the MAS
x_i	state vector $\left[q_i^T, \dot{q}_i^T\right]^T$ of Agent i
$\hat{q}_i^j$	estimate of q_i performed by Agent j.
$\hat{q}^j$	estimate of q performed by Agent j.
$\hat{x}_i^j$	estimate of x_i performed by Agent j.
$\breve{q}_i^j$	additional estimate of $\hat{q}_i^j$ performed by Agent i.
$\breve{e}_i^j$	additional estimate of e_i^j performed by Agent i.
e_i^j	estimation error between q_i and $\hat{q}_i^j$.
r_{ij}	relative coordinate vector $r_{ij} = q_i - q_j$ between agents i and j.
r_{ij}^*	desired value for r_{ij}.
q_0	reference trajectory
q_i^*	reference trajectory for Agent i, $q_i^* = q_0 + r_{i1}^*$
r_i	trajectory error for Agent i, $r_i = q_i - q_i^*$
$t_{j,k}$	time at which the k-th message is sent by Agent j.
$t_{j,k}^i$	time at which the k-th message sent by Agent j is received by Agent i.
$\tilde{\alpha}_{j,k}^i$	stochastic variable of the k-th message sent by Agent j is transmit to an Agent i

Chapter 6: Communication delay in distributed event-triggered for multi-agent formation stabilization

q_i	vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$
q	vector $\begin{bmatrix} q_1^T & q_2^T & \dots & q_N^T \end{bmatrix}^T \in \mathbb{R}^{N.n}$, *configuration* of the MAS
x_i	state vector $\left[q_i^T, \dot{q}_i^T\right]^T$ of Agent i
$\hat{q}_i^j$	estimate of q_i performed by Agent j.
$\hat{q}^j$	estimate of q performed by Agent j.
$\hat{x}_i^j$	estimate of x_i performed by Agent j.
e_i^j	estimation error between q_i and $\hat{q}_i^j$.
r_{ij}	relative coordinate vector $r_{ij} = q_i - q_j$ between agents i and j.
r_{ij}^*	desired value for r_{ij}.
q_0	reference trajectory
q_i^*	reference trajectory for Agent i, $q_i^* = q_0 + r_{i1}^*$
r_i	trajectory error for Agent i, $r_i = q_i - q_i^*$
$t_{j,k}$	time at which the k-th message is sent by Agent j.
$t_{j,k}^i$	time at which the k-th message sent by Agent j is received by Agent i.

Chapter 1

Introduction and state-of-art

1.1 Introduction

Multi-Agent System (MAS) has been an important subject of research this last decade with applications to mobile robots, like unmanned air vehicles (UAVs), autonomous underwater vehicles (AUVs), satellite constellations, spacecraft, aircraft, and automated highway systems. They can be used in domain like exploration of unknown zones, surveillance or maintenance for difficult-access areas. Cooperation between agents in a same fleet, where vehicles can be identical or different, can take many forms: exchange of information, allocation of objectives, synchronization of speed, execution of several jobs simultaneously, avoidance of collision... For cooperative estimation, the cooperation drives agents to share their measurements so that they can improve the accuracy or the reliability of the resulting estimate. This can be applied to enhance the global vision of the environment.

However, cooperation between agent induces that each agent can gather information from others agents and/or an exchange between them. This results in new problems to consider and choices to make: communication can be centralized or distributed, information on other agents can be provided by sensors or broadcast messages, which in turn requires to define how and when an agent needs to broadcast a message. When vehicle are very spaced or in presence of obstacles, using sensors for relative location or sharing visual information can become difficult or even intractable, thus requiring transfer of communication between agents to obtain these information. However, other issues arise from communication: network saturation, energy to broadcast messages, limited broadband, conflict in communication protocol made by a large number of agents. All these problems induce the necessity to limit the number of communication between agents in order to manage them better. Therefore, the limitation of the amount of communication requires the agents' control law, estimators and protocols of communication to be adapted.

This last decade, a large number of methods have been developed to reduce the number of communication between agents in problem of consensus, and more recently in the general frame of formation problem. Consensus is an important problem in cooperative control, see *e.g.* [77, 113, 13, 37, 36]. In such problems, the state components of several agents have to converge to the same value (example of applications: all vehicles reach the same position and speed). Formation control consists in driving and maintaining all agents of a MAS to some reference, possibly time-varying configuration, defining, *e.g.*, their relative positions, orientation, and speed. Various approaches have been considered for that purpose as described in [112, 87, 82, 72, 26, 14, 15].

Consensus and formation control usually require significant exchange of information between neighboring agents so that each agent can properly evaluate its control law accounting for the control inputs depending on its neighbor states. However, for distributed systems, the state values of the other agents are not permanently available. This is why many authors, see *e.g.* [77], consider permanent communication between agent, or periodically updated as in [36], which is more practical. When discrete time communication is used, , each agent usually builds estimators of the states of its neighbors to enable proper computation of their control laws. However, in absence of permanent communication, the convergence of the system to the consensus or the desired formation shape depends on the quality of the state estimates and the time between two updates of information. Thus, specific methods have been developed to guarantee the convergence of the system while reducing the number of broadcast information, such as intermittent communication [117] and more recently event-triggered communication.

In this approach, a communication is broadcast when a condition, based on chosen parameters and some threshold, is fulfilled. It is well-suited to applications where number of communications should be minimized, *e.g.*, to improve furtivity, to reduce energy consumption, or limit interference between transmitted data packets. Application examples with such constraints are presented in [59, 60] for the case of a fleet of vehicles, or in [5] where agents aim at merging local feature-based maps.The main difficulty consists in determining the communication triggering condition (CTC) that will ensure the completion of the task assigned to the MAS, *e.g.*, reaching some consensus, maintaining a formation, *etc.* In a distributed strategy, each agent maintains an estimate of its own state only using the information it has shared with its neighbors. It then evaluates the quality of the estimate of its state made by its neighbors. When the discrepancy between this own state estimate and its actual state reaches some threshold, the agent triggers a communication to update estimation made by its neighbors.

Using this triggering condition [23, 24] considers agents whose dynamic is modeled as a single integrator and considers that the threshold decreases with time while the fleet reaches the consensus. The decreasing threshold allows to obtain higher frequency of communication when thus system converges, and thus when agents need to obtain a more accurate control value to reach the consensus. However, communication frequency may become so high that the communication is almost permanent when consensus is close, and the simplicity of the dynamical model of agents may not be appropriate for description of complex vehicles. Thus, an event-triggered method for a double integrator dynamical system was developed by [94, 57]. Threshold is exponentially decreasing with time, reaching higher frequency of communication when system converges, but is bounded by a periodic communication when consensus is reached. The method is still limited by its dynamics representation, which may prove too simple for real cases. This motivates new developments by [136, 37, 35] who considers a multi-agent system with general linear dynamics. State-dependent thresholds are then considered to ensure suitable convergence property for the system. Problems of communication delay are studied in [39, 135, 79], providing a communication model closer to physical phenomena.

All these studies proposed an event-triggered approach allowing to obtain a consensus without requiring to permanent communication. However, in all these methods, state perturbation had been not considered and the number of communications remains large. Moreover, these previous approaches are sensitive to perturbations. This issue has been partly addressed by [45, 19] who proposes an event-triggered method to mitigate the impact of perturbations in the case of dynamics described by simple integrator.

Some recent works combine event-triggered approaches with distance-based or displacement-based formation control [61, 98, 99]. In these approaches, the dynamics of the agents are described by a simple integrator, and control input is considered constant between two communications. The proposed CTCs are all centralized, considering different threshold formulations. A constant threshold is considered in [98] and a time-varying threshold in [61, 99]. CTC depends then on the relative positions between agents and the relative discrepancy between actual and estimated agent states. They allows to reduce the number of triggered communications when the system converges to the desired formation. A minimal time between two communications, named inter-event time, is also defined. Nevertheless, no perturbations are considered in all these works. Finally, Logic-Based Communication (LBC) techniques have been introduced in [84, 3] to reduce the number of communications in decoupled nonlinear MAS to reach a desired formation. Agents have to follow parameterized paths, designed in a centralized way. CTC introduced by LBC lead all agents to follow the paths in a synchronized way to set up a desired formation. Communication delays, as well as packet losses are considered.

The objective of this thesis is to develop distributed controls and estimators for multi-agent system to reduce the number of communication by using event-triggered strategy and taking state perturbation into account. The study is dedicated to two main topics: first the problem of consensus for a system with a general linear dynamics, second the formation control and tracking problem for a system modeled by Euler-Lagrange dynamics.

Chapter 1 presents the state-of-art of the different approaches and the lists of notations and acronyms needed for this thesis. More precisely, concepts of distributed control, graph theory and communication protocol are presented. State-of-art of consensus approach, formation control and event-triggered strategy are also described.

In Chapter 2, reduction of number of communication for consensus problem is studied. Taking as reference the work of [37], this study first recalls the main trigger parameters used in [37]. The triggering condition depends mostly on the discrepancy between estimate state and current state. The objective of the work presented in this chapter is to limit this discrepancy so that the number of communications is decreased.

As the model proposed by [37] did not consider perturbations, the introduction of additive perturbations requires to enlarge the study of the stability of the global system. Various models of perturbations have been tested to observe their impact on the global stability conditions and the variations on the number of communication. The stability is analyzed via a Lyapunov function where the perturbations are introduced. Influence of the perturbation on the estimate discrepancy has also been studied.

In this context, a new estimator has been developed to reduce the estimate discrepancy, and so the number of triggering. The main idea is to take into account the control input of the agents in the estimate model to lower the bounds on the difference between the actual state and the estimated one. The CTC and communication protocols are adapted to the new features.

Finally, performances obtained by the new and the reference methods are compared. Extension to this work to non-linear dynamics and time-varying topology is discussed.

In Chapter 3, the problem of formation control for a multi-agent systems (MAS) with an event-triggered strategy is considered. Agent dynamics are described by an Euler-Lagrange system including perturbations. A control input is studied to lead agents to an unique oriented desired target formation and to maintain the formation despite the presence of perturbations. The control law is derived from [82]. Two estimates for computing distributed controls are proposed, providing a trade-off between computation time and amount of triggered communications.

Convergence to the desired formation and stability of the global system are studied using Lyapunov analysis. A distributed CTC is designed guarantying the convergence. Performances obtained by the proposed approach are illustrated with simulations focusing on the reduction in communications obtained.

Chapter 4 presents an extension of the previous approach to the case when the model of the agents depends on some unknown parameters. The control input and estimator model are modified to include terms of compensation of the errors due to the unknown parameters. Furthermore, problem of tracking a desired trajectory while maintaining the desired formation and reducing the number of broadcast information is considered. Contrary to LBC techniques, a single *a priori* trajectory is determined to follow the desired path. Some conditions on the reference trajectory to allow tracking by agents with state limitations are defined. Convergence of the system to the desired formation and the desired trajectory is studied using CTC. Finally, performances obtained on simulations are presented to illustrate formation error, tracking error and reduction in communications obtained. The problem of collisions avoidance between agents and communication delays are also discussed.

Chapter 5 considers the problem of loss of information due to packet dropout. Previous estimator is adapted to tackle this issue using a distributed CTC, based on the expectation of the estimate error made by the neighbor of the agent. This condition is derived from stability analysis using stochastic Lyapunov function as in [25, 95]. A communication protocol is developed that insure absence of Zeno behavior. The expectation of the estimate error required for CTC computation, is evaluated with an new estimator. Proof of convergence of the system with the proposed CTC is presented.

Chapter 6 tackles the issue of communication delay, without packet dropout. To avoid broadcasting of outdated information, the message content is modified to transmit a prediction of agent state to update estimation made by neighbors. Moreover, agent updates the estimation of its own state with the broadcast value to guarantee the synchronization of estimations made by all agents. The CTC is modified using the prediction state, allowing to take into account the delay in message reception. Two prediction models are proposed to offer a trade off between the accuracy of the prediction and the computation time.

A concluding chapter synthesizes the results presented in this thesis and describes some potential directions for future works.

1.2 Cooperation among the agents

Cooperative approaches are often inspired by biology considering schools of fish, flocks of birds, groups of bees, and swarms of social bacteria. Earliest methods were based on simple local individual coordination rules to define the global group behavior [89, 103]. In MAS, cooperation among the agents of the system is used to accomplish tedious and complex missions, such as surveillance or area exploration. In these methods, measurements collected from different agents are associated to improve the cooperative detection and localization. A MAS is justified when the global efficiency of the agents is larger than the sum of the efficiency of each agent. Another advantage of MAS over a single agent system is its robustness to vehicle loss due to failures. However, the cooperative movements of several vehicles imply to tackle the problem of collision avoidance and limitation of distances of communication.

Three important characteristics should be sought for the agents in a MAS: autonomy, local information and decentralized/distributed control and communication. Autonomy is to be taken in the sense that agents can control their own trajectories to reach their objectives. Local information translates the fact that the agents have limited access to the state of the other agents. Local information can be obtained with different means like sensors or radio communications. For example, state information of other vehicles can be obtained by the measurements delivered by relative position sensors [105, 22, 55, 6], or be communicated between vehicles using a wireless network, or by using visual devices like color LED [101, 33]. Finally, in decentralized/distributed control and communication each agent computes its own control input and takes its own decision on when it is needed to broadcast messages, as described in Section 1.3.

1.3 Centralized, decentralized and distributed control

In a multi-agent system, control and communications can be designed to be centralized, decentralized or distributed. In centralized control, all information on agents states is broadcast and used by a central controller which evaluates the control inputs of all agents in the network before broadcasting it to them. This central controller can be an agent in the network or a separate station. In a centralized control, agents don't need to communicate between them but only with the central controller. This kind of control input have been mostly studied during the last decade [97, 127, 75, 110]. However, they present some major drawbacks. First, agents are dependent of the central controller: if the central controller cannot broadcast message during a short time or breaks down, agents cannot take any further decision. They can either stop to move, or continue their trajectories at the risk of collidingwith an obstacle. Same problem occurs when an agent cannot receive message from the central controller due to a radio failure, interferences, obstacles on the road, or a distance between agent and the central controller that is beyond the reception range.

Another disadvantage is that the number of broadcast messages between the controller and agents increases with the number of agents: a large number of agents induces an important time calculation and a risk of network saturation. Furthermore, in practical implementation, a sampling interval is required between transmissions of agents measurements to the central controller, the computation time of each agent control inputs by the central controller, and the transmission of these values to all agents in the network: in some case, computation of the control variables might become unytractable.

To overcome these issues, decentralized and distributed controls have been developed. In decentralized systems, agents compute their own control inputs independently on the system. However, absence of communication between controllers limits the achievable performances and the possible cooperative missions. For example, for formation fleet, each agent follows a desired trajectory insuring the maintain of the formation. Discrepancy between desired and current relative locations of agents cannot be corrected by the agent neighbors due to the absence of communication.

In distributed control, agents compute their own control inputs using local information similarly to decentralized control, but agents are also able to exchange information between them, which enhances the cooperation. This type of methods is intended to fill the gap between centralized and decentralized schemes. Recently, the potential advantages of distributed control have attracted many researchers. Approaches have been developed for applications in many areas including cooperative control consensus [77, 10, 13], formation control [82, 72, 105, 99] or flocking [89, 103, 91, 7]. Note that the presence of a leader in the network can also be considered for centralizing some information and thus increasing system performance [15, 90, 62]. Similarly, an hierarchy in sub-groups can also be integrated to centralize information within the neighborhood and, select useful information before transmission to other groups, avoiding problem of network saturation. Leaders can also be used to allocate objectives within the fleet. In these methods, even if some agents are considered as leader and other as followers, agents are still in charge of computing their own control.

1.4 Stability of dynamic systems

Classical notions introduced in what follows are taken from [11, 53].

1.4.1 Autonomous system

The autonomous system is described by

$$\dot{x}(t) = f(x(t)) \tag{1.1}$$

where $f : \mathcal{D} \to \mathbb{R}^n$ is a locally Lipschitz map from a domain $\mathcal{D} \subset \mathbb{R}^n$ into $\mathbb{R}^n$. Assume that $x_e \in \mathbb{R}^n$ is an equilibrium point of (1.1), *i.e* $f(x_e) = 0$. The characterization and the study of the stability of (1.1) is performed relatively to x_e . To simplify the problem, all definitions and results are stated considering $x_e = 0$ as the equilibrium point. There is no loss of generality as any equilibrium point can be brought back to the origin using the change of variable $\bar{x} = x - x_e$ and the resulting system $\dot{\bar{x}} = f(\bar{x})$, where $\bar{x}_e = 0$.

1.4.2 Lyapunov stability

Stability of systems (1.1) can be demonstrated using Lyapunov stability definition. It is formulated as follows

Definition 1. The equilibrium point $x_e = 0$ of system (1.1) is defined as

1. **Stable** if for each $\epsilon > 0$, there exists a scalar $\eta > 0$ such that

$$\|x(0)\| \leq \eta \Rightarrow \|x(t)\| \leq \epsilon, \forall t \geq 0$$

   where $x(t, x(0))$ is the solution of (1.1) with the initial condition $x(0)$.

2. **attractive** if there exists η such as

$$\|x(0)\| \leq \eta \Rightarrow \lim_{t\to\infty} x(t) = 0$$

3. **asymptotically stable** if stable and attractive.

4. **unstable** if it is not stable.

Stability is defined in a neighborhood of the origin. The neighborhood $S(x, \epsilon)$ of a point x is a set characterized by the parameter ϵ and containing x. Then, the proof of stability of the origin is established if a neighborhood $S(0, \epsilon)$ can be found such that every trajectory starting from $S(0, \eta)$ will remain within $S(0, \epsilon)$. A system is *asymptotically stable* if any trajectory inside $S(0, \epsilon)$ goes towards the origin $x = 0$. A system is called unstable if no trajectory can remain within $S(0, \epsilon)$ for any η (Examples in Figure 1.1).

Definition 2. Define the set $\mathcal{D}$ consisting of all initial conditions $x(0)$ from which any trajectory of the system (1.1) converges to the origin. $\mathcal{D}$ is named the domain of attraction of the origin. Then, the origin is defined as

1. **locally** asymptotically stable if the domain of attraction is strictly induced in $\mathbb{R}^n$, *i.e.* $\mathcal{D} \subset \mathbb{R}^n$.

2. **globally** asymptotically stable if the domain of attraction is $\mathbb{R}^n$, *i.e.* $\mathcal{D} = \mathbb{R}^n$.

From Definition 1, the knowledge of the trajectory of x is needed to study and conclude to the stability of the system, which is, in general, difficult or impossible to obtain. Thus, another formulation of the stability has been formulated. Based on Lyapunov function, this method is called the Lyapunov's second method.

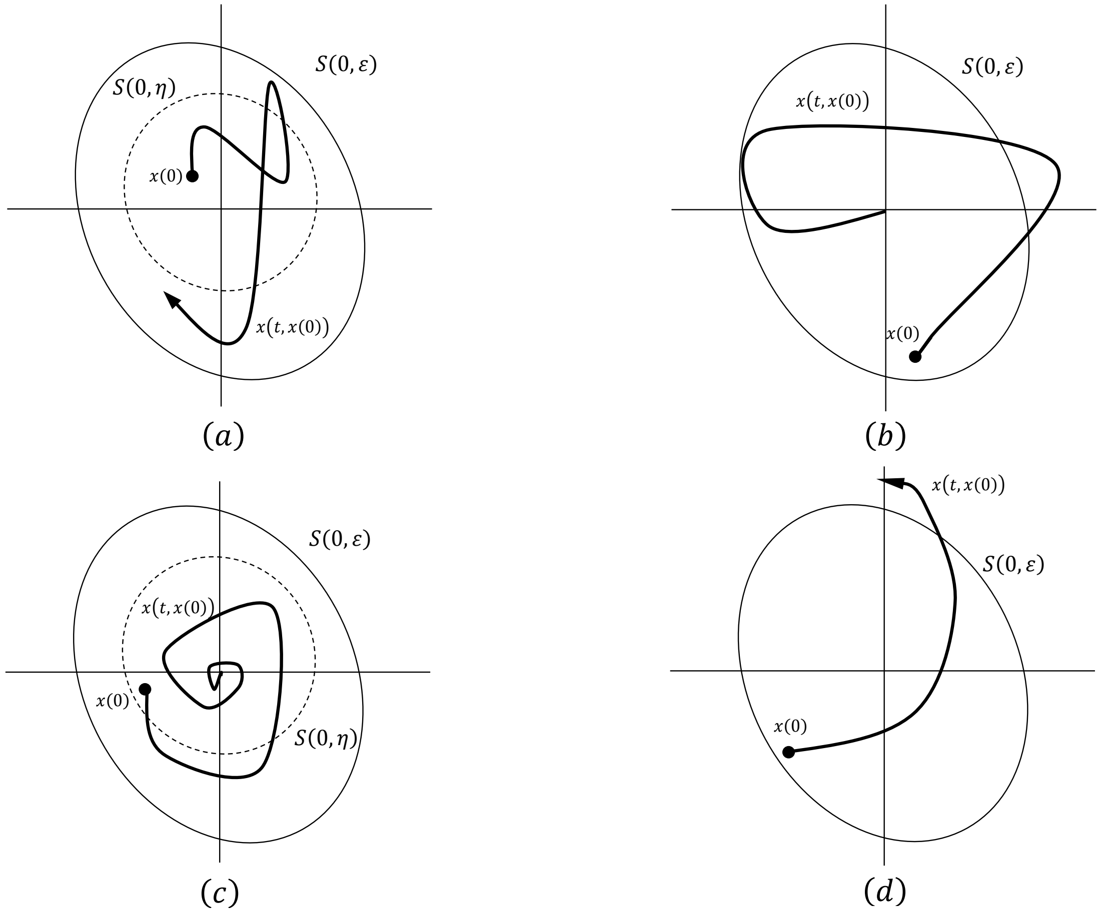


Figure 1.1: Example of system(a) Stable; (b) Attractive; (c) Asymptotically stable; (d) Unstable

1.4.3 Lyapunov's second Method

This method aims at analysing the stability of the equilibrium point without relying on the trajectory of the system. This method is based on the use of the Lyapunov function. The following theorem gives sufficient conditions for the stability of system (1.1).

Theorem 1. *Let $x_e = 0$ be an equilibrium point for (1.1) and $\mathcal{D} \subset \mathbb{R}$ be a domain contaning $x_e = 0$. Let the candidate Lyapunov function $V : \mathcal{D} \rightarrow \mathbb{R}$ be a continuously differentiable function such that*

$$V(0) = 0 \text{ and } V(x) > 0 \text{ in } \mathcal{D} - \{0\} \tag{1.2}$$

$$\dot{V}(x) \leq 0 \text{ in } \mathcal{D} \text{ with } \dot{V}(x) = \frac{\partial V}{\partial x} f(x). \tag{1.3}$$

Then, $x = x_e$ is stable Moreover, if

$$\dot{V}(x) < 0 \, in \, \mathcal{D} - \{0\} \tag{1.4}$$

then $x = x_e$ is asymptotically stable.

Definition 3. A continuously differentiable fonction $V(x)$ satisfying (1.2)-(1.3) is called a Lyapunov function.

Note that Theorem 1 gives only sufficient conditions for stability. Thus, no conclusion can be drawn if a Lyapunov function can not be found. Note also that the choice of the candidate Lyapunov function is an important issue in the study of the stability and there is no general procedure for finding it. A commonly used Lyapunov function is of the form $V = x^T P x$ where P is a symmetric positive definite matrix.

1.4.4 LaSalle Invariance Principe

Let first define some definitions

Definition 4. The set $\mathcal{M}$ is said to be

1. Invariant if $x_0 \in \mathcal{M}$ implies that $x(t, x_0) \in \mathcal{M}\ \forall t \in R$.
2. Positively invariant if $x_0 \in \mathcal{M}$ implies that $x(t, x_0) \in \mathcal{M}\ \forall t \geq 0$.

In previous section, Theorem 1 provides an efficient way to ensure the stability of a dynamic system. However, no conclusion on the asymptotic stabilization can be obtained if the chosen Lyapunov only verifies $\dot{V}(x) \leq 0$. Although there are other points different from the origin where $\dot{V}(x) = 0$, if one can prove that any trajectory should no be attracted to these points, apart from $x = 0$, then the trajectory must converge to zero. This is the LaSalle's Theorem or LaSalle Invariance Principle.

Theorem 2. *Let*

1. *$\mathcal{M}$ be a positively invariant set with respect to the system (1.1)*
2. *$V : \mathcal{M} \to \mathbb{R}$ be a Lyapunov function such that $\dot{V}(x) \leq 0$, $\forall x \in \mathcal{M}$.*
3. *$E = \left\{x \in \mathcal{M};\ \dot{V}(x) = 0\right\}$*
4. *L be the largest invariant set contained in E.*

Then every trajectory starting in $\mathcal{M}$ converges to L.

Since proof of asymptotic stability implies proof that $x(t) \to 0$ as $t \to \infty$, Theorem 2 requires to demonstrate that the largest invariant set in E reduces to the origin. In that case, the following corollary allows to conclude to the asymptotic stability of the origin.

Corollary 1. Let

1. $x = 0$ be an equilibrium point of the system (1.1)
2. $V : \mathcal{M} \to \mathbb{R}$, respectively $V : \mathbb{R}^n \to \mathbb{R}$, be a Lyapunov function such that $\dot{V}(x) \leq 0$, $\forall x \in \mathcal{M}$.
3. $E = \left\{x \in \mathcal{M};\ \dot{V}(x) = 0\right\}$ and assume that no trajectory can stay in E, other than the origin.

Then $x = 0$ is asymptotically stable, respectively globally asymptotically stable.

1.4.5 Input-to-State Stability (ISS) and input-to-state practically stable (ISpS)

Let first define some definitions

Definition 5. (From [53]) Let define the following Kappa-class:

Class $\mathcal{K}$: a continuous function $\beta(.) : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\beta(0) = 0$.

Class $\mathcal{K}_\infty$: a continuous function $\beta(.) : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}_\infty$ if it is $\mathcal{K}$, $a = \infty$ and $\lim_{r\to\infty} \beta(r) = \infty$.

Class $\mathcal{KL}$: a continuous function $\beta(r, s) : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if for each fixed s, the function $\beta(., s)$ belongs to $\mathcal{K}$, and for each fixed r, the function $\beta(r, .)$ is decreasing and $\lim_{s\to\infty} \beta(r, s) = 0$.

Consider the system

$$\dot{x}(t) = f(t, x(t), u(t)) \tag{1.5}$$

where $u(t)$ is a continuous, bounded function of t for all $t \geq 0$. The system (1.5) is call input-to-state. Suppose the unforced system $\dot{x}(t) = f(t, x(t), 0)$ has a globally uniformly stable equilibrium point x_e, what can we say about the behavior of the system in presence of a bounded input $u(t)$? Due to the boundedness of u, it is possible to show that, in some case, the system converges to a ball of radium r, where r depend on $\sup(\|u\|)$.

Definition 6. (Definition 2.1 from [48]) The system (1.5) is said to be input-to-state practically stable (ISpS) if there exists a function β of class $\mathcal{KL}$, a function γ of class $\mathcal{K}$ and a non-negative constant d such that, for each initial condition $x(0)$ and each measurable essentially bounded control $u(.)$ defined on $[0, \infty)$, the solution $x(.)$ of the system (1.5) exists on $[0, \infty)$ and satisfies

$$\|x(t)\| \leq \beta(\|x(0)\|, t) + \gamma(\|u(t)\|) + d. \tag{1.6}$$

When (1.6) is satisfied with $d = 0$, then system (1.5) is said to be to be input-to-state stable (ISS).

Then, the following theorem gives sufficient conditions for the stability of system (1.5).

Theorem 3. *A smooth (i.e. $\mathcal{C}^\infty$) function V is said to be an ISpS-Lyapunov function for the system (1.5) if there exist class $\mathcal{K}_\infty$ function $\underline{\alpha}(.)$, $\bar{\alpha}(.)$, $\alpha(.)$, a function γ of class $\mathcal{K}$ and a non-negative constant d such that it can be found*

$$\underline{\alpha}(\|x(t)\|) \leq V(x,t) \leq \bar{\alpha}(\|x(t)\|), \quad \forall x \in \mathbb{R}^n$$

and

$$\frac{\partial V}{\partial x} f(x,t) \leq -\alpha(\|x(t)\|) + \gamma(\|u(t)\|) + d, \quad \forall x \in \mathbb{R}^n$$

or

$$\|x(t)\| \geq \gamma(\|u(t)\|) + d \Rightarrow \frac{\partial V}{\partial x} f(x,t) \leq -\alpha(\|x(t)\|), \quad \forall x \in \mathbb{R}^n.$$

1.4.6 Comparison Method

Quite often when we study the state equation $\dot{x} = f(x,t)$, we need to compute bounds on the solution $x(t)$ without computing the solution itself. A useful tool is the comparison lemma. The comparison lemma compares the solution of the scalar differential inequality $\dot{v}(t) \leq f(t, v(t))$ with the solution of another scalar differential equation $\dot{w}(t) = f(t, w(t))$. The lemma applies even when $v(t)$ is not differential, but has an upper right-hand derivative $D^+v(t)$which satisfies a differential inequality.

Lemma 1. (Comparison Lemma, from [53]) Consider the scalar differential equation

$$\begin{aligned} \dot{w} &= f(t,w) \\ w(t_0) &= w_0 \end{aligned}$$

where $f(t,w)$ is continuous in t and locally Lipschitz in w, for all $t \geq 0$ and all $w \in \mathcal{D} \subset \mathbb{R}$. Let $[t_0, T)(T$ could be infinity) be the maximal interval of existence of the solution $w(t)$, and suppose $w(t) \in \mathcal{D}$ for all $t \in [t_0, T)$. Let $v(t)$ be a continuous function whose upper right-hand derivative $D^+v(t)$ satisfies the differential inequality

$$\begin{aligned} D^+v(t) &\leq f(t,v) \\ v(t_0) &\leq w_0 \end{aligned}$$

with $v(t) \in \mathcal{D}$ for all $t \in [t_0, T)$. Then, $v(t) \leq w(t)$ for all $t \in [t_0, T)$.

Then, consider the perturbed systems

$$\dot{x}(t) = f(x,t) + g(x,t) \tag{1.7}$$

where $g(x,t)$ is considered as a perturbation of the system $\dot{x}(t) = f(x,t)$. Let $V(x,t)$ be a Lyapunov function for the nominal system (1.7) and suppose the derivative of V along the trajectories of $\dot{x}(t) = f(x,t)$ satisfies the differential inequality

$$\dot{V} \leq h(t,V)$$

By the Lemma 1,

$$V(t, x(t)) \leq y(t)$$

where

$$\begin{aligned} \dot{y}(t) &= h(t,y) \\ y(0) &= V(x(0), 0). \end{aligned}$$

This approach is particularly useful when the Lyapunov function can not respect all conditions needs to use other theorems like LaSalle Theorem or ISpS Lyapunov.

1.5 Graph theory

This section recalls some classical notions related to graph theory used in this thesis. The notations introduced in what follows are taken from [21].

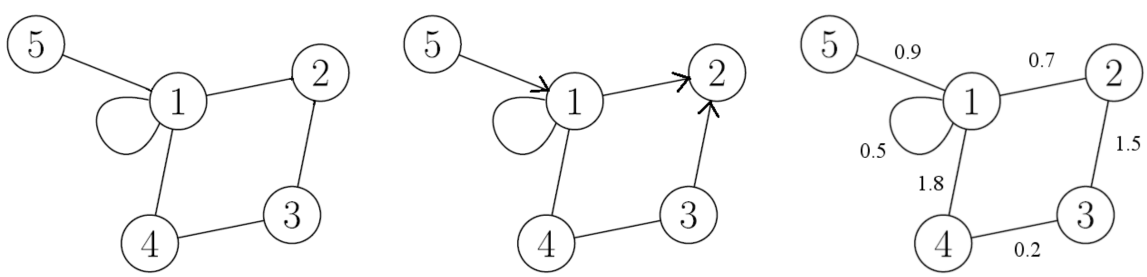
1.5.1 Connectivity notions

Graph theory is research area shared by mathematics and computer science. It is the study of graphs, which mathematical structures used to model relations between objects. A graph $\mathcal{G}$ is defined a set of nodes (or vertices), interconnected by edges. A node represents an object (place, computer, person, cell, vehicle, agent) and every edge represents a relation between two objects (a distance, a connection, a logical link, a speed, a communication). In our study, a node is a member of our fleet, name it an agent, and edges indicate possible communications between agents.

A graph is denoted $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, ..., N\}$ is the set of the N nodes and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ the set of edges. A subgraph of a graph $\mathcal{G}$ is another graph formed from a subset of the vertices and edges of $\mathcal{G}$. Two nodes directly linked by an edge in the graph are said to be neighbors. The set of neighbors of an node i is $\mathcal{N}_i = \{j \in \mathcal{N} | \, (i, j) \in \mathcal{E}, \, i \neq j\}$. N_i, the cardinal number of $\mathcal{N}_i$, is the number of neighbors of node i. $\mathcal{E}_{ij}$ is the edge between the node i and node j.

Edges can be directed, indicating a direction from a node to the other one, as illustrated in Figure 1.2 (b). A directed edge represents a one-way interaction from a node to an other node, for example a one-way communication. A graph with directed edges is called a *directed graph*, and edges are called arcs. An *undirected graph* is equivalent to a directed graph where all arcs are doubled, as illustrated in Figure 1.2 (a). An edge $\mathcal{E}_{ij}$ can be weighted to give importance of the connection between nodes i and j compared to others edge. A graph where edges are weighted is called a *weighted graph*, as illustrated in Figure 1.2 (c). A unweighted graph is equivalent to a weighted graph where every edges possess a unit weight.

Finally, a graph where edges do not change with time is called a graph with *fixed topology.* By opposition, a graph where edges change with the time is called a graph with *time-varying topology.* In practice, the topology of a graph can be time-varying because connections can appear/disappear due to the influence of the distance between nodes, to interference, to material imperfections, or simply to the choice of the communication strategy.



(a) Undirected graph $\mathcal{G}_1$ (b) directed graph $\mathcal{G}_2$ (c) Undirected weight graph $\mathcal{G}_3$

Figure 1.2: Communication graph

1.5.2 Path and complex graph

In a graph, a *path* is a set of adjacent edges, *i.e.,* edges sharing one vertex, which allow to link two nodes, which are not necessarily neighbors. The number of edges defines the length of the path. In a directed graph, a path is a set of adjacent edges which allows one to link two nodes by respecting the direction of arcs. A directed path is called a *chain.* A directed graph is call *connected* if for all nodes, there exist paths which can connect it to all other nodes. A graph is said to be *strongly connected* if every node is reachable from every other node. A graph is call *fully-connected* or *complete* if every nodes is connected by an unique edge (one in each direction) with all other nodes.

A graph contains a *cycle* if there is a non-trivial path that starts and ends at the same node. In opposite, a graph is call *acyclic* if it contains no cycles. A connected acyclic graph with an unique root is called a *tree*. A directed *spanning tree*, or simply a spanning tree, is a graph where every subgraph is a directed tree. A graph is called a *ring* if it is cyclic and composed by only one path.

Examples of previous particular graphs are illustrated in Figure 1.3.

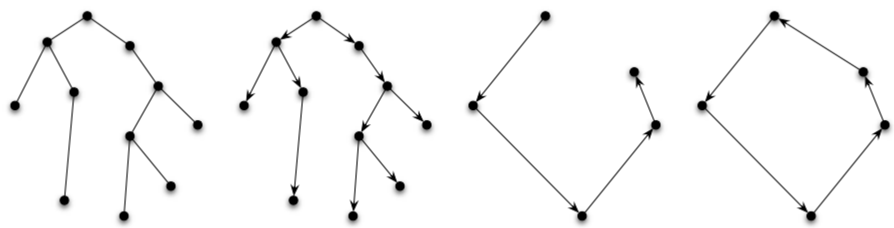


Figure 1.3: From left to right, tree, directed tree, chain, and ring graphs

1.5.3 Adjacency matrix A

The adjacency matrix $A = [a_{ij}]_{N\times N}$ associated to a graph $\mathcal{G}$ is a square matrix of size $N \times N$, where N is the number of nodes. The value of each non-negative element a_{ij} of the matrix is the weight of the edge $\mathcal{E}_{ij}$. Thus if there is no connection between two nodes i and j, $a_{ij} = 0$. If the graph is undirected, one has $a_{ij} = a_{ji}$ for all (i, j). Moreover, if the graph is unweighted, one has $a_{ij} = 1$ or $a_{ij} = 0$ for all (i, j). The adjacency matrix associated to graph on Figure 1.2 can be expressed as

$$A(\mathcal{G}_1) = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(\mathcal{G}_2) = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(\mathcal{G}_3) = \begin{bmatrix} 0.5 & 0.7 & 0 & 1.8 & 0.9 \\ 0.7 & 0 & 1.5 & 0 & 0 \\ 0 & 1.5 & 0 & 0.2 & 0 \\ 1.8 & 0 & 0.2 & 0 & 0 \\ 0.9 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

1.5.4 Degree matrix D

The in-degree matrix $D_{\text{in}} = [d_{\text{in},ij}]_{N\times N}$ and out-degree matrix $D_{\text{out}} = [d_{\text{out},ij}]_{N\times N}$ associated to a graph $\mathcal{G}$ is a diagonal square matrix of size $N \times N$ defined by

$$D_{\text{out}} = \text{diag}(A1_N)$$
$$D_{\text{in}} = \text{diag}\left(A^T 1_N\right)$$

where $1_N \in \mathbb{R}^{N\times 1}$ is the all-one vector. The graph is *weight-balanced* if $D_{\text{out}} = D_{\text{in}}$.

1.5.5 Laplacian matrix L

The Laplacian matrix is $L = [l_{ij}]_{N\times N}$ associated to a graph $\mathcal{G}$ is a square matrix of size $N \times N$ defined by

$$L = D_{\text{out}} - A$$

L is symmetric iff $\mathcal{G}$ is undirected. Moreover, in that case, every row and every column of L sums to zero, which mean L satisfies $L1_N = 0$. L has only one null eigenvalue $\lambda_1(L)$, and all its other non-zero eigenvalues $\lambda_2(L) \leq \lambda_3(L) \leq \ldots \leq \lambda_N(L)$ are strictly positive. If the graph is fully-connected, $\lambda_i(L) = N$ for all $i \in [2 \ldots N]$.

1.6 Communication protocol

A communication protocol is a system of rules that allows two or more entities of a communication system to transmit information via any kind of variation of a physical quantity. These rules can define message syntax, synchronization of communication approach, detection of collision between transmitted packets of data, bandwidth assignment, communication architecture, or error recovery. A protocol often describes different case of a communications between two entities.

The study of communication protocol allows to understand a part of material constraints link communications and take it in account in the developpement of communication approach. Its offers an openning on methods to transmit information, message content, or number of agents receiving a message from one transmission or afte after several hops: all theses attributs will be used to develop approachs reducing the number of broadcast messages in next chapters of this thesis. They can also be used to help to solve technical problem as the packet dropout.

Initially, communication protocols were proposed for wired network. In September 1968, the University of Hawaii began a research program to use radio communications for computer-to-computer and console-to-computer links. The main idea was to use a unique radio channel for all messages instead of assigning a subchannel for each message as done previously, which limited the number of simultaneous computers connected. However, this unique channel leads to collisions between broadcast data packets. Then, protocols were developed to solve the data collision problem. Protocols for message re-transmissions were proposed to obtain a reliable network with an efficient data transmission. The first one was the ALOHAnet protocol.

1.6.1 ALOHAnet protocol

The goal of the Additive Links On-line Hawaii Area (ALOHA) system [1] is to provide a radio communication alternative to conventional wired communications. It can be used when all nodes send and receive on the same channel. ALOHA protocols describe rules to solve collision problems between broadcast data packets.

The first version of the protocol, named ”Pure ALOHA”, can be expressed from each station by the following step.

- A station sends data when needed.
- If, while it is transmitting data, the station receives any data from another station, there is a message collision. Then, the station finishes to broadcast the message and defines a random waiting time. The message is broadcast again after this time. Note that all stations associated to the collision will need to broadcast their message again later.

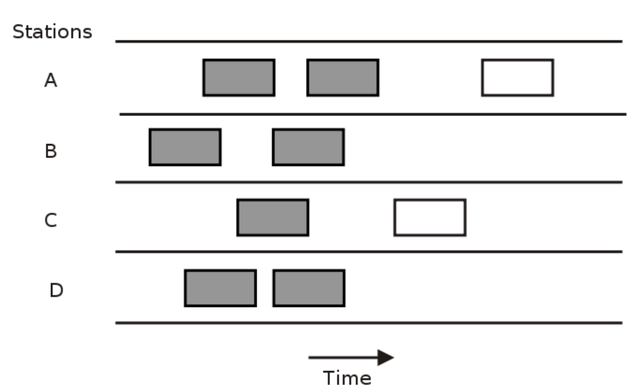


Figure 1.4: Pure ALOHA protocol. Boxes indicate message frames. Black boxes indicate frames which have collided. White boxes indicates messages broadcast successfully.

Figure 1.4 proposes an example of Pure ALOHA. In this example, station A transmits its message successfully after two colliding communications, and station B after one.

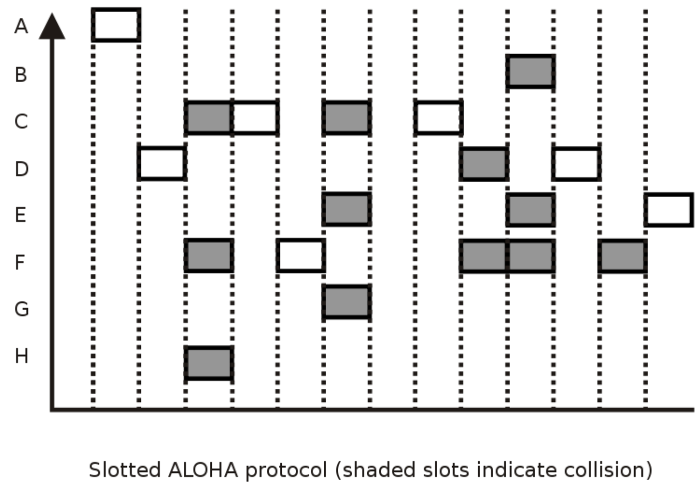


Figure 1.5: Slotted ALOHA protocol. Boxes indicate message frames. Black boxes indicate frames which have collided indicate when frames are in the same slots. White boxes indicates messages broadcast successfully.

Pure ALOHA does not check if the channel is busy before transmitting. Then, ALOHA can not use the full capacity of the communications channel since collisions can occur and data may have to be sent again. The list of messages waiting for transmission can be large if the number of stations is important, which makes no-collision frames difficult to obtain if the concept of ”transmit later” is not specified.

Then, an improvement to the original ALOHA protocol is the Slotted ALOHA, which introduced discrete time-slots and increased the maximum throughput. Consider the following assumptions

- All time-slot frames have the same length.
- A station can send message only at the beginning of a time-slot.
- Stations cannot generate a message while transmitting or trying to transmit.
- If a collision is detected, a station waits a random time before trying again to transmit.
- The population of stations attempts to transmit according to a Poisson distribution.

Figure 1.5 proposes an example of Slotted ALOHA.

Since a station can send only at the beginning of a time slot, collisions can only occur during this time-slot and their number is reduced. However, stations still do not try to detect if another station is transmitting before it transmits its own message, which still leads to collision at the beginning of a time slot.

This problem leads to the development of CSMA, a ”listen before send” random-access protocol described in the following section.

1.6.2 CSMA

Based on ALOHAnet, the Carrier Cense Multiple Access (CSMA) is a ”listen before send” random-access protocol. A station verifies the absence of an other message broadcast by an another station on the transmission medium before transmitting itself. If a message is detected, the station waits for the end of the neighbor transmission before initiating its own transmission. Then, stations using CSMA can send and receive information on the same medium and in the same bandwidth while avoiding more collisions than with the ALOHA protocol. The first implementation of CSMA was Ethernet. With this kind of communication “someone speaks, others wait” is also one of the weakness of CSMA and Ethernet protocol.

Indeed, while one station is transmitting, others machines receive and must wait even if they also need to transmit data. Thus, a communication with a large throughput can lead to a saturation of the network.

Variations on basic CSMA include addition of collision avoidance, collision detection, and collision resolution approach. Transmissions by one station are generally received by all other stations connected to the medium in most of these strategies.

CSMA/CD : Collision Detection

The CSMA/CD is used to improve the performance of CSMA by terminating transmission as soon as a collision is detected, thus shortening the time required before a retry can be attempted.

The CSMA/CD is based on the ALOHA protocol, where each station decides when it needs to transmit a message. When a station needs to send data, it checks if it receives any data from another station. If it is the case, the station waits for the end of the neighbor transmission, and begins to transmit after. If the station detects an other station broadcasting a message at the beginning or during the transmission (collision), it stops sending data immediately. A random waiting time is defined before trying to transmit again the message to avoid collision with the same machine. Note that a message is erased if it is not transmitted successfully after 16 trials to avoid network saturation. Then, a large number of stations can also lead to a problem of network saturation.

The CSMA/CD can be assimilated to a group of person where everybody can speak when he wants. If two persons speak at the same time, they stop a try again after a short time. It can be noticed CSMA/CD are mostly for wired network or fully-connected network, because stations need to be able to detect collisions with other stations. When a station receives messages from two neighbors which are not themselves neighbors, both stations cannot detect the collision and messages cannot be received. CSMA/CA was created to solve this problem.

CSMA/CA : Collision Avoidance

CSMA/CA [9] is mostly used for networks where two stations can transmit to a third one without detecting each other (distance between the two stations too large). A station is defined as transmission leader and authorize or not the communication when station asks it. To implement CSMA/CA, a station broadcasts a short RTS (Ready to Send) frame with few information to ask the communication. If the communication is accepted, the leader station broadcasts a CTS (Clear To Send) frame, and the station transmits its message. In opposite, if the transmission station is busy, the transmission is deferred for a random time interval.

This method is used in the WIFI network, where the leader station are named Access Points (AP). The CSMA/CA can be assimilated to a classroom where pupils ask to answer a question and where the teacher decides who speaks first.

CSMA/CR : Collision Resolution

The CSMA/CR is used to improve CSMA/CD performance. The difference is if many stations transmit at the same time, a station continues to transmit message since the broadcast signal is identical. Station stops to transmit data when signals begin differents. This protocol allows to finish a communication from one station without waiting delay or re-transmission.

This method is used in CAN network. The CSMA/CA can be assimilated to singers who can continue to sing together since they are reading the same score.

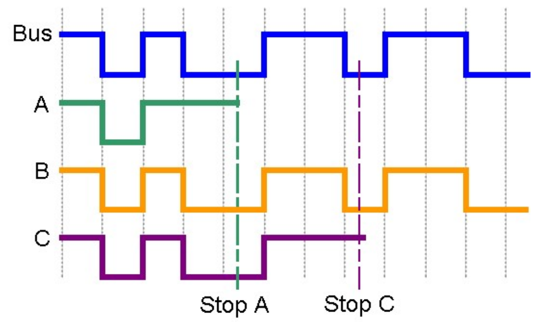


Figure 1.6: Example of system with the CSMA/CR protocol

CSMA access modes

The access modes are transmission process. Strategies exposed here are added rules on CSMA protocols previously exposed: they define if a station can transmit information immediately or later when it needs and the transmission medium is idle. Transmission medium can be managed by only one access modes strategy respected by all users. Else, rules are not harmonized between stations and transmissions become unbalanced or impossible. Note that some communication protocols can work only with one defined access mode. Access modes are summarized in Figure 1.7.

CSMA 1-persistent 1-persistent CSMA is an aggressive transmission strategy. When the transmitting station is ready to transmit and the transmission medium is idle, the station transmits immediately. If it is busy, it waits until the transmission medium becomes idle by listening the medium continuously. When the medium becomes idle, the station transmits immediately without trying to know if an other station wants to transmit also. In case of a collision, the station waits for a random period of time before following the same procedure again.

1-persistent CSMA is used in ALOHA systems and CSMA/CD systems including Ethernet.

CSMA Non-persistent Non persistent CSMA is a non aggressive transmission strategy. When the transmitting station is ready to transmit and the transmission medium is idle, the station transmits immediately. If it is busy, it waits for a random period of time without listening the medium. After this time, it checks the transmission medium and tries to transmit again by following the same steps.

This approach reduces the risk of collision, but can induce a longer delay before transmission compared to 1–persistent.

CSMA 0-persistent In this approach, a supervisory station assigns transmission order to each station. Slots of fixed time are managed, and each station transmits during the time-slot assigned to it while other stations wait.

0-persistent CSMA is used by CobraNet and LonWorks.

CSMA P-persistent This is an approach between 1-persistent and non-persistent CSMA access modes. When the transmitting station is ready to transmit and the transmission medium is idle, the station transmits a message with a probability p. If the station does not transmit (the probability of this event is $1 - p$), it waits until the next available time-slot. If the medium is busy, it waits until the transmission medium becomes idle by listening the medium continuously. When the medium become idle, the station transmits again a message with probability p.

CSMA 1-persistent is a CSMA p-persistent with a probability $p = 1$. p-persistent CSMA is used in CSMA/CA

1.6.3 Token Ring

Token ring is a communications protocol for local area networks. Messages travel around a ring graph and circulate in a unique direction (direct ring graph topology), which allows one to avoid message collisions

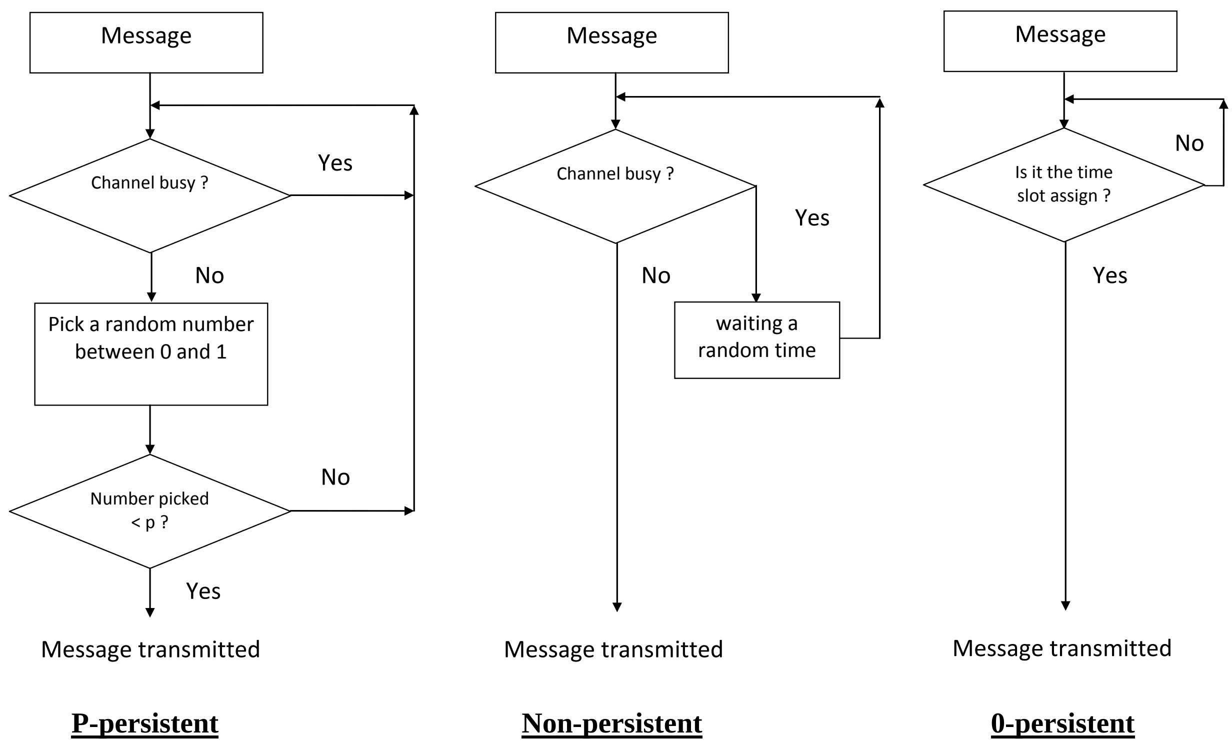


Figure 1.7: CSMA access modes

as in contention-based access protocol like CSMA or Ethernet. Messages are contained in frames, and a limited number of frames is employed inside the network.

The protocol uses a three-byte frame called a ”token” which is added at the head of the message. The token contains the destination of the message and a special byte which takes the value 1 if the frame is busy and 0 else. A station can transmit a message only if the token is idle. As the communication network is a direct ring, each station receives the message broadcast by the previous station and reads its token. If the station is the message recipient, it changes the token back to 0 and transmits the message to the next station. When the message gets back to the originator, it sees that the token has been changed to 0 and thus the message has been received successfully. Message is removed from the frame, which can be used by an other agent to transmit information.

Token Ring is more deterministic compared to Ethernet or CSMA/CD protocol, and collision between frames are impossible. Thus, Token Ring keeps its performance constant with a large number of station, when Ethernet degrades with an higher number of collisions. It can also manage a “priority access”, in which some station can have priority over the token, which is not possible on Ethernet where stations have equal influence on the network. However, Ethernet has less topology constraints than Token Ring which needs a direct ring topology, sometime difficult to obtain with wired network and even more difficult in wireless network. Moreover, Token Ring is more complex to implement than Ethernet and requires specialized processor.

Later versions of Ethernet has gradually eclipsed Token ring by its lower cost, a higher throughput and lower structure.

Token Ring access modes The data transmission process can be summarized as follows:

- Empty information frames are continuously circulated on the ring.
- When a station needs to transmit a message, it modifies the token of a idle frames and changes the token to 1. Frame is sent in the network.
- The frame is examined by each successive station. The station that identifies itself as the message destination copies it from the frame and changes the token back to 0.
- When the frame gets back to the originator, it sees that the token has been changed to 0 and that the message has been copied and received. It removes the message from the frame.
- The frame continues to circulate as an ”empty” frame, ready to be taken by a station when a message needs to be sent.

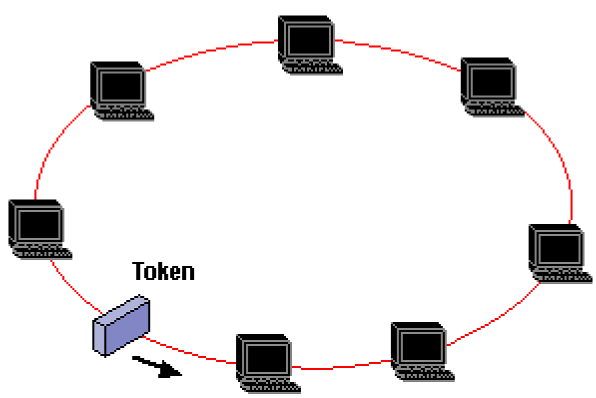


Figure 1.8: System with CSMA/CR protocol

1.7 State-of-art : Consensus problem

Consensus is an important problem in cooperative control, see [77, 113, 13, 37, 36]. In such problem, several agents have to be synchronized to a common value: this problem is called the consensus or agreement problem in the literature. Applications of consensus in cooperative control of multi-agent systems are numerous.

In the case of multi-vehicle systems, it can be interesting for the vehicles to achieve a consensus on positions, like in rendez-vous problem to reach an objective at the same instant [30, 96]. It can also be used to achieve a cooperative motion in formation [32, 58] or flock [90, 89], or to synchronize the orientations of the vehicles [75, 100, 8]. In this case, the consensus can be defined in terms of (relative) positions of the vehicles, velocities and / or orientations. A consensus on the agents' speed can also to help avoiding collisions between vehicles [114, 80].

In the case of power grids, consensus approach can also be used to control distributed energy sources in order to ensure the production of a predetermined amount of active or reactive power. Each source can be considered as a node of a network with communication capabilities to neighbor sources [42, 27].

Consensus can also be used like in [93] to couple oscillators and kinematic models of groups of self-propelled particles.

In addition to control approaches, it is also worth mentioning that consensus is also used for distributed estimation. State estimation from noisy measurements provided by multiple sensors can indeed be addressed by a distributed consensus approach. Local information fusion is performed in the multi sensor network taking into account communication links between its nodes and using a consensus protocol as in [123, 78, 92].

Although consensus problems have a history in computer science, we will focus here on their applications to cooperative control of multi-agent systems. The main purpose of this section is to summarize the recent progress of consensus methods proposed by the cooperative control community.

1.7.1 Consensus definition

Consider a multi-agent system composed of N agents, which communication topology is described by an graph $\mathcal{G}$, as exposed in Section 1.5. It is assumed that neighbor agents, *i.e.* agents between which a communication link exists as defined in$\mathcal{G}$, can exchange information. Let's define by x_i the state of the i th agent. It is assumed that a consensus is desired on this state vector.

Two type of consensus can be defined: asymptotic consensus and bounded consensus. An asymptotic consensus is obtained when the state of all agents converge asymptotically to the same value (see Definition 7) . A bounded consensus is obtained if the discrepancy between the states of the agents converge within a bounded domain (see Definition 8).

Definition 7. The multi-agent system reaches an *asymptotic consensus* iff

$$\lim_{t\to\infty} \|x_j(t) - x_i(t)\|^2 = 0, \tag{1.8}$$

for all pairs(i, j) of agents.

Definition 8. The multi-agent system reaches a *bounded consensus* iff there exists some $\varepsilon > 0$ such that

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| \leqslant \varepsilon \tag{1.9}$$

for all pairs(i, j) of agents

1.7.2 Continuous-time and discrete-time consensus

As described in [77, 113, 74, 76], a continuous-time consensus protocol can be obtained by the dynamic equation

$$\dot{x}_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(t)\left(x_i(t) - x_j(t)\right) \tag{1.10}$$

where $\mathcal{N}_i$ is the set of the Agent i's neighbors, $a_{ij}(t)$ the (i,j)-th element of the adjacency matrix A associated to the communication graph $\mathcal{G}$, and t the time instant . Remark that (1.10) can be written in matrix form as $\dot{x} = -Lx$, where L is the Laplacian matrix associate to the communication graph $\mathcal{G}$ and $x = \begin{bmatrix} x_1^T, & \dots, & x_N^T \end{bmatrix}^T$. If consensus is reached, the state of each agent remains constant with respect to time. Continuous-time consensus requires permanent communications between the agents. Thus a significant exchange of information is required which makes its practical implementation almost impossible.

As presented in [77, 42, 67, 63, 27] a discrete-time consensus protocol can be obtained by considering for Agent i the difference equation

$$x_i(k+1) = x_i(k) - \epsilon \sum_{j \in \mathcal{N}_i} a_{ij}(k)(x_i(k) - x_j(k)). \tag{1.11}$$

where $\epsilon > 0$ is a constant coefficient that can be used for example to account for a discretization period and where k denotes the time index. Again, if consensus is reached, the state of each agent remains constant with respect to time. Communications between the agents are required at each time step k which is more easy to implement in practice than continuous-time consensus. Nevertheless a huge number of communications is still required, which will motivate the work of communication reductions

In both cases, if $\sum_{j \in \mathcal{N}_i} a_{ij} = 1$ the state of each agent is updated using a weighted mean of the differences between its own state and the states of its neighbor. In continous-time consensus approaches considered in [63, 102, 96, 42], the coefficients $a_{ij}(t)$ of (1.10) are not defined as the coefficients of the adjacency matrix but as time-varying weights $a_{ij}(t) = m_{ij}(t)$ that follow specific rules.

In [63], these coefficients are defined by $m_{ij}(t) = \frac{1}{N_i}$, where N_i is the cardinal number of $\mathcal{N}_i$, if Agent i and j are neighbors, $m_{ij}(t) = 0$ else, such as to obtain an average of the states of the neighbors. Then, each agents converges to a barycenter of these states , without favoring a specific agent. If a distance $d_{ij}(t)$ can be defined between the states of any Agent i and Agent j, [102] proposes to define $m_{ij}(t) = \frac{1}{2d_{ij}}$ to favor the convergence of the Agent i to its closest neighbors. This methods tends to scatter the agents into "smaller groups" when they are initially too distant from each other. Thus [96] proposes a method where $m_{ij}(t)$ is weighted to preserve the "weakest links". Longer distances make thus increase the value of $m_{ij}(t)$, making the agents converges to an unique group.

As previously mentionned, coefficients used in the consensus equation (1.10)-(1.11) can be chosen as elements of the adjacency matrix or using other rules that may also depend on the communication topology and account for its changes. In [42, 27], the coefficients m_{ij} are chosen as $m_{ij}(k) = \frac{1}{1+D_{\text{out,j}}}$ in the case of a discrete-time consensus to manage distributed sources in power grids. In this way, topology switches are taken into account to adapt the number of contributing energy sources, allowing to account for the power demand (Example Figure 1.9).

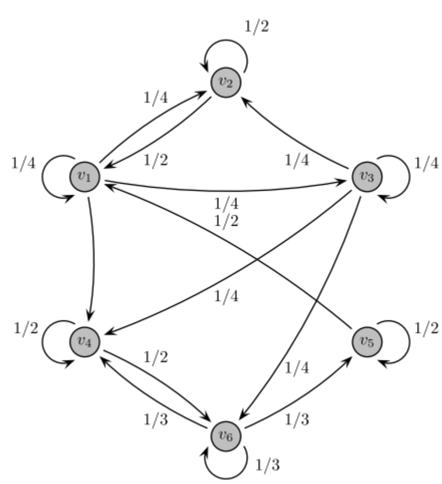


Figure 1.9: Example of weight $m_{ij}(t)$ with strategy proposed by [42].

1.7.3 Influence of the communication topology

Many works in cooperative control assumes a static fully connected communication topology, where each agent can communicate with all the other agents. However, real-world communication topologies are usually not fully connected. In many cases they would depend on communication ranges of the agents. In case of mobile agents, the communication topology hence changes with respect to time and communication links may appear or disappear between the agents.

Nevertheless, some conditions on the communication graph must be satisfied to obtain a consensus. It is shown in [46] that the consensus can be achieved if the union over the time of all the undirected communication graphs is connected *frequently enough*. As an extension to this work, [70] shows that a consensus can be achieved asymptotically in the case of time varying topologies if the union of the directed communication graphs contains a spanning tree *frequently enough*. In [78, 76], consensus is shown to be obtained in case of a directed communication graph if it is also strongly connected and balanced. A solution to ensure this condition can be the introduction of a virtual leader to guarantee a connected graph between agents as in pinning methods [124, 125].

Therefore, properties of the communication graph related to its connectivity have a strong impact on the convergence to a consensus and on the speed of convergence. [77] shows that a consensus can be reached between all the agents only if a connected graph (time-invariant or time-varying) is used. Moreover, the greater the number of connections is, the faster the converge to the consensus will be (Example on Figure 1.10).

1.7.4 Communication strategies for consensus

When the communication graph is not fully connected (e.g. agents with limited communication range) theoretical and practical issues related to communications arise. Most consensus control strategies are based on two types of communications : information relay or local information.

In a communication strategy based on "information relays", an information received by one agent is transmitted to all other agents to which a communication link exists. This process may allow each agent to obtain information on every other agent of the system, provided that the communication topology is connected. This strategy allows each agent to get better global information on the MAS and helps the convergence to a unique consensus. However, it induces a large number of communications between the agents.

In a "local information" based method, information are exchanged between direct neighbor agents only and are therfeore not relayed anymore to other non-direct neighbor agents. With this communication strategy less data are transmitted between the agents helping to reduce computational and communication burden. However, relying on more local information may tend to scatter the agents into several groups, also increasing the risk to break communication links between them.

1.7.5 Consensus with communication delay

In multi-agent systems, communication delays naturally arise because of the bandwidth limitation or saturation of the communication channels, the possible asymmetry of the communication graph, the time required to compress and extract data in the broadcast message, or the limited transmission speed due to the physical characteristics of the transmission medium (*e.g.* acoustic wave communications between underwater vehicles).

Since consensus approaches require communications, especially directly between the agents in the distributed case, communication delays may impact on the convergence of the agents to a consensus.

Let $\tau_{ij}(t)$ denote the time delay associated to the information transmission from Agent j to Agent i. $\tau_{ij}(t)$ can be assumed to be constant and identical for all agents [78] ($\tau_{ij}(t) = \tau \ \forall (i,j)$ and $\forall t$), time-varying [10] or distance-varying [96]. In most of the works like [78, 77, 10, 13], communication delay is assumed to be upper-bounded such that $\tau_{ij}(t) < \tau_{\max} \ \forall (i,j), \forall t$. The estimation or knowledge of τ_{ij} is of a huge importance to decide if it can be compensated (*e.g.* [134]) or if robustness in the convergence analysis has to be addressed [78, 77, 13].

The continuous-time consensus approach proposed in [78, 77, 10, 13] in presence of known time-delays

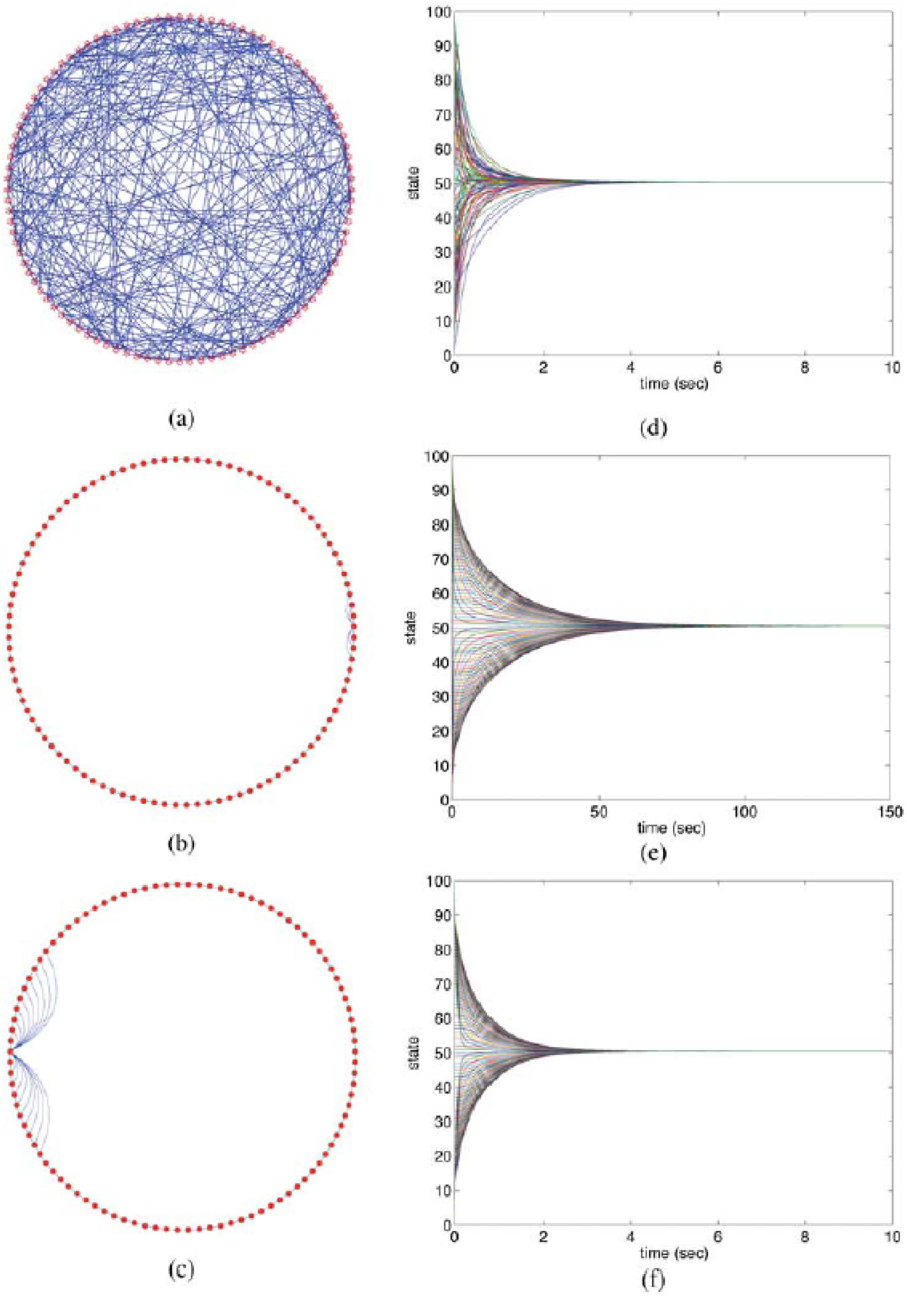


Figure 1.10: Consensus with (a) A small-world with 300 links, (b) a regular lattice with interconnections to three nearest neighbors and 300 links, (c) a regular lattice with interconnections to the 10 nearest neighbors and 1000 links. [77]

is defined by

$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} a_{ij} \left[x_i \left(t - \tau_{ij} \left(t \right) \right) - x_j \left(t - \tau_{ij} \left(t \right) \right) \right]$$

where the value of $x_i\left(t - \tau_{ij}\left(t\right)\right)$ is used to be synchronized with the received information x_j. Note that system converges faster to the consensus as the delay is smaller. In [78], a simple case with a constant known communication delay, *i.e.* $\tau_{ij} = \tau \ \forall \left(i, j\right)$, is studied. In the case of an undirected, connected communication graph with fixed topology, the consensus is proven to be reached iff

$$\tau = \left[0,\ \frac{\pi}{2\lambda_{\max}(L)}\right)$$

Another consensus approach can be used by considering that the time delay only affects the broadcast information and not the own state of the agent. In this case, a continuous-time consensus protocol can be formulated as

$$\dot{x}_i = -\sum_{j\in\mathcal{N}_i} a_{ij}\left[x_i(t) - x_j(t-\tau_{ij})\right].$$

In this framework, some methods exist to compensate the communication delay and improve the speed of convergence and its accuracy. In [134], a distributed protocol is proposed to actively compensate for communication delays, based on a prediction of the agent behavior. Agents with linear dynamics, a fixed topology and a fixed communication delay τ are considered and the consensus equations are defined as

$$\begin{aligned}
\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) \\
u_i(t) &= F\left(e^{A\tau}z_i(t) + \sum_{j\in\mathcal{N}_i} a_{ij}\int_{-\tau}^{0} Bu_j(t+s)\,ds\right) \\
z_i(t) &= \sum_{j\in\mathcal{N}_i} a_{ij}\left(x_i(t-\tau) - x_j(t-\tau)\right)
\end{aligned}$$

where F is a gain matrix such that $A + BF$ is semi-definite negative. If the delays are exactly known and bounded, the work of [134] proves that this consensus problem can be solved. Furthermore, the delays are allowed to be time-varying and unknown in the particular case of agents with open-loop dynamics containing only one zero eigenvalue.

1.7.6 Dynamic models of the agents

As introduced above, there exist works in the literature where the agents are considered to be governed by models more complex than first-order dynamics. In [131, 94, 19], a second-order consensus problem is studied, which can be expressed as

$$\begin{aligned}
\dot{x}_i(t) &= v_i \\
\dot{v}_i(t) &= \alpha\sum_{j\in\mathcal{N}_i} l_{ij}(t)\left[x_i(t) - x_j(t)\right] + \beta\sum_{j\in\mathcal{N}_i} l_{ij}(t)\left[v_i(t) - v_j(t)\right]
\end{aligned}$$

where α and β are positive constants and l_{ij} is an element of the Laplacian matrix L.

In contrast to the first-order consensus problem, it has been shown in [85] that agents with second-order dynamics may not converge to an asymptotic consensus even if the network topology contains a direct spanning tree. Some other works like [131] propose conditions on α and β to guarantee convergence in this case.

In [20, 115, 67, 37, 38, 35, 36, 39, 34, 134], a general linear model is considered and the consensus protocol is defined in the case of time-invariant topologies by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{1.12}$$

$$u_i(t) = F\sum_{j=1}^{N} a_{ij}\left(x_i(t) - x_j(t)\right) \tag{1.13}$$

where A and B are respectively the state and control matrices, and where F a matrix designed such that $A + BF$ is semi-definite negative to ensure closed-loop stability of the system. F can be found such that $F = B^T P$ where P is the solution of the Riccati equation

$$PA + A^T P - 2PBB^T P + 2\alpha P < 0, \tag{1.14}$$

where $\alpha > 0$ is a positive design parameter. Remark that the general linear dynamics consensus protocol (1.12) can be written in matrix form as

$$\dot{x}(t) = \bar{A}x(t)$$

where $x = \begin{bmatrix} x_1^T & \dots & x_i^T & \dots & x_N^T \end{bmatrix}^T$ and $\bar{A} = I_N \otimes A + L \otimes (BF)$.

1.7.7 Consensus and estimators

In previous sections, communication between agents have been considered either permanent or periodic depending on the time representation that is used (continuous-time or discrete-time). In the more general case of limited communications (periodic, event-based, etc.), current information on the state of neighbor agents may not be available for the evaluation of the control imput. This information can be replaced by estimates of the states of neighbor agents as proposed in [37, 38, 35, 36, 107, 106, 109, 128].

This can be expressed for simple integrator dynamics and time-invariant topologies by

$$\begin{aligned} \dot{x}_i(t) &= u_i(t) \\ u_i(t) &= \sum_{j \in \mathcal{N}_i} a_{ij} \left(y_i^i(t) - y_j^i(t) \right) \end{aligned}$$

or for general linear dynamics by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{1.15}$$

$$u_i(t) = F \sum_{j \in \mathcal{N}_i} a_{ij} \left(y_i^i(t) - y_j^i(t) \right) \tag{1.16}$$

where y_j^i denotes the estimate of Agent j's state made by Agent i. Some classical example of estimators are:

(i) Zero-hold estimator: between two reception times, the estimate $y_j^i(t)$ performed by Agent i is considered to be equal to the last received state value send by Agent j, *i.e.* $y_j^i(t) = x_j(t_{j,k})$. In that case, the control input is also maintained constant between two instants of reception of messages sent by neighbors of Agent i. This is the approach adopted in [94, 23, 57].

(ii) Dynamic estimator based on a prediction model: the estimate is computed as the solution of a differential system which imitates the dynamics (or a part of it) of the agent's. For example, in [35, 37, 38], $y_j^i(t)$ is computed by considering the dynamic equation

$$\begin{aligned} \dot{y}^i(t) &= \mathrm{A}y^i(t) \\ y_j^i(t) &= x_j(t_{j,k}) \end{aligned}$$

where $y^i(t) = \begin{bmatrix} y_1^{iT}(t) & \dots & y_i^{iT}(t) & \dots & y_N^{iT}(t) \end{bmatrix}^T$. The choice of the estimator dynamics, and hence the possible introduction of estimation errors in the consensus protocol, has a direct impact on the properties of the obtained consensus (asymptotic or bounded)

1.7.8 Virtual agent, leader and Pinning consensus

In many consensus approaches, the common value to which the state of the agents converge mainly depends on the chosen consensus protocol and the initial states of the agents. If one would like to influence the transient behavior of the agents or the obtained consensus value, different methods can be used.

A first approach consists in introducing virtual agents without modifying the consensus protocol. These virtual agents are considered as other standard agents in the consensus protocol, but their control input can be defined in a different way to influence the global behavior of the MAS. Virtual agents can be used to drive the consensus to a desired setpoint, to avoid obstacles in the case of multi-vehicles systems as in [90, 89].

Another approach consists in introducing one or several agent(s) that will be considered by others as leader(s). A leader can be real or virtual and is used to impose a reference to the MAS by making all the other agents, named *followers*, to track the leader and thus converge to a same objective. This approach is often reffered to as *consensus tracking [113, 13, 118]*.

Addition of a virtual leader may also help improving the connectivity in the communication graph (*e.g.* by ensuring the existence of a spanning tree): when agents belong to different communication subgraphs, a virtual leader that becomes a common reference between these agents hence connects these subgraphs togeteher. This strategy can be helpful in cases where the agents may split in different subgroups (presence of obstacles for multi-vehicles systems, packet dropouts, etc.). When consensus is used to obtain a formation, a leader can also be used to help to maintain the desired geometrical configuration.

An example of method using leaders in consensus problems is pinning control theory for synchronization of dynamical networks [124, 125]. A virtual leader is added to the network as neighbor of one or very few agents and defines a desired reference. Assuming a connected communication graph, all the agents of the network will therefore be synchronized with this virtual leader even if they have no direct communication link to it.

1.8 State-of-art: Formation control

Within the recent decades, formation control constitutes an important research topic in multi-agent control domain. Formation control refers to the problem of controlling the relative positions, velocity and orientations of agents to maintain them as a fleet while allowing the group to move towards a common objective. It aims at accomplishing complex missions such as mapping, exploration, monitoring environments or data collection while insuring security and autonomous navigation.

Formation control design requires first to select a feasible formation pattern, given the constraints on the variations of agent states and thus determine the series of actions on each agent so as to maintain the formation shape, drive it to the objective and potentially enable switching between formation patterns. Formation control may be separated into formation tracking control and formation regulation control. Formation tracking control is defined a potentially distributed control law that each agent applies to reach a desired formation. The formation regulation control is applied once the agents have reached the required positions on the desired pattern and aims at correcting any variation of the agent states in a neighborhood of the desired state values. The formation may be either rigid or flexible depending whether the formation structure remain fixed (rigid formation) or can evolve for a short duration of time, *e.g.* to avoid obstacles, before rejoining the initial desired structure (flexible formation).

Given the numerous results on formation control and their varieties, different attempts have been made to classify the approaches as presented in Figure 1.11. In this section, only the main classes of formation control are recalled: formation tracking control, virtual structure and flocking control.

1.8.1 Classification of Formation tracking control

In [73, 18], a first classification of formation control schemes is defined relying on the function of the agent state components on which the control law depends. The classes consist in Position-based control, Displacement-based control and Distance-based control. They can be compared in terms of requirements of sensing abilities and interaction topology. In order to present the different approaches, the following example, a single-integrator $\dot{p}_i(t) = u_i$ where p_i is the Agent i current position will be used. Denote $y_i = g_i(p_1, \ldots, p_N)$ the measurement and $z_i = h_i(p_i)$the output of Agent i. The desired formation is expressed as $F(z) = F(z^*)$.

- **Position-based control** [84, 3, 130, 121]
  The agents control their own positions defined in a global coordinate frame. The desired formation is prescribed also in this global frame. In such case, the measurement consists in the absolute position of the agent in this frame. Interaction between agents is not an absolute requirement as the resulting formation can be reached by individual control as in [121]. The resulting control expresses as $u_i = -k_p(p_i(t) - p_i^*(t))$ where the denomination p stands for position. This control law can be modified by accounting for the interaction between agents and adding control input of the form $\sum_{j\in N_i} w_{ij}(p_j - p_i)$ with N_i the set of neighbors of Agent i and w_{ij} the components of the Laplacian matrix describing the connection graph. In position-based control, the control law is most often centralized and handled by a global administrator. Few distributed approaches have been proposed

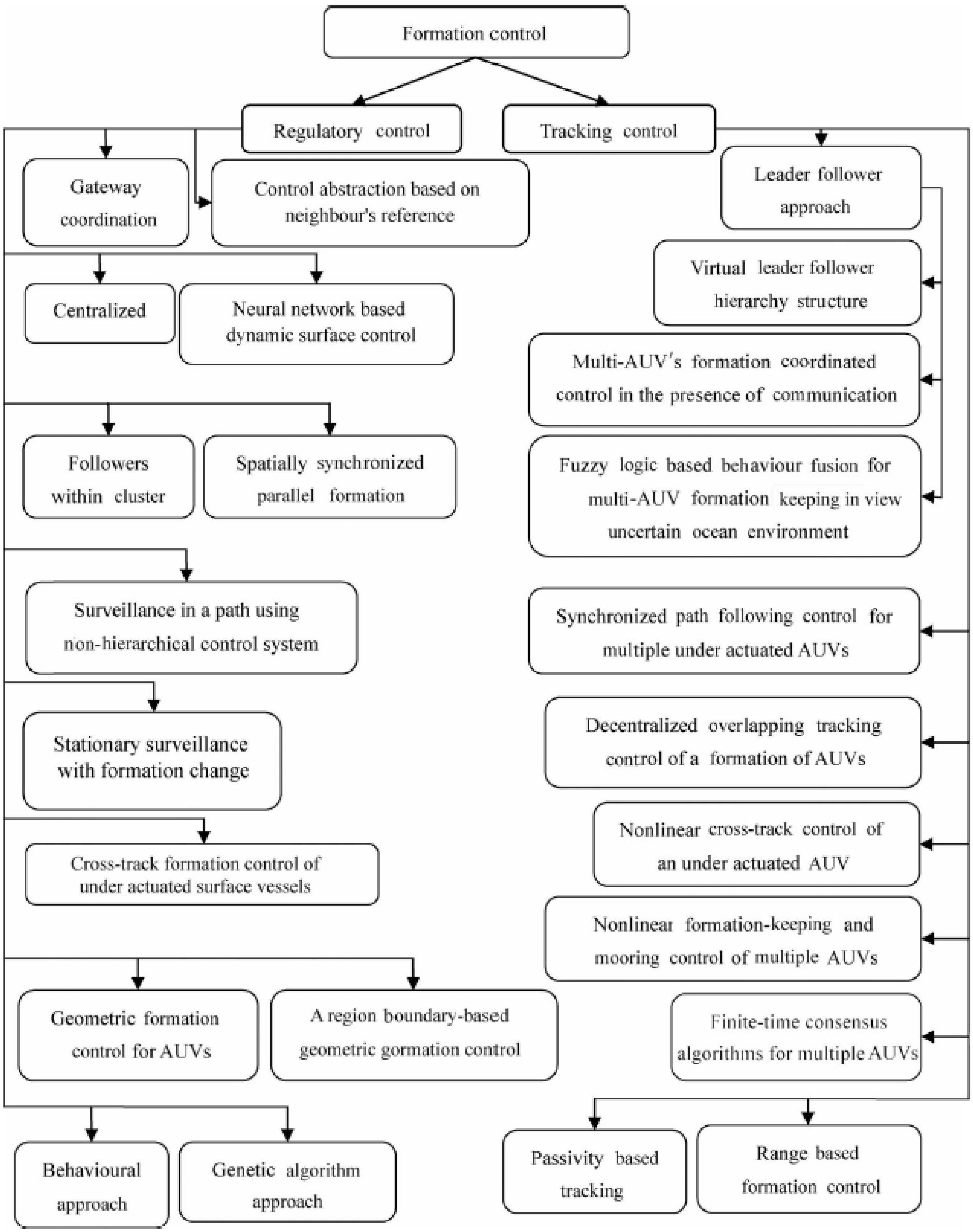


Figure 1.11: A proposed classification of formation control of MAS [22].

in this scheme

- **Displacement-based control** [82, 72, 105, 104, 99]
  The agents control their relative positions with respect to their neighbors defined in a global coordinate frame. The desired formation is no longer defined as a set of positions but as desired displacements. In such case, the measurements are performed in each agent local frame and consist of the coordinates of its neighbors in this frame. This implies that an agent needs to know its local frame orientation in the global coordinate system. Interaction between agents is an higher requirement, the graph must be connected. The resulting control expresses as $u_i = k_p \sum_{j \in N_i} w_{ij} \left(p_j - p_i - p_j^* + p_i^*\right)$ which stresses the need for interaction. Then, an unique formation shape with an unique orientation in the global coordinate system is obtained. Connectivity preservation has been studied by [47]. The interconnection makes it possible to enhance robustness against failures as studied in [21].

- **Distance-based control** [12, 32, 54]
  The agents control their inter distance to achieve the desired formation. This formation is defined as desired distances between agents and is no longer unique in terms of orientation and translation with respect to the global frame. In such case, the measurements are performed in each agent local frame and consists of the coordinates of its neighbors in this frame. Knowledge of their own frame orientation in the global frame is no longer needed. As the desired formation expresses in terms of desired distances, the fleet can be described as a rigid body. Therefore, the interaction graph requires to be rigid or persistent. Various control laws have been defined in this scheme. They can either be obtained as a gradient function of the sum of weighted relative distances between agents. Another expression of the control law studied in [54] is $u_i = -k_p \nabla(\sum_{j \in N_i} (\|p_j - p_i\|^2 - \|p_j^* - p_i^*\|^2)$. In any case, the control law is always a non-linear function of the state.

Position-based control is particularly beneficial in terms of the interaction topology, but it requires agents more advanced sensors to measure their own position accurately, much more than the other approaches. Moreover, an *a priori* trajectory has to be evaluated for each agent, which induces problems of path following generation.

In opposite, distance-based control is more advantageous in terms of the sensing capability because only need to know the relative distance between agents. However, it requires more interactions between agents to obtain a stable formation shape where displacement control can obtain a formation with only $N - 1$ interactions. It also requires short inter-agent distance if the distance is measured by sensors. Thus, displacement-based control is moderate in terms of both sensing capability and interaction topology compared to the other approaches. A trade-off between the amount of interactions among agents and the requirement on the sensing capability of each agents is required to choose the formation approach.
Although the classification of formation control in terms of position, displacement and distance based control is of interest as it allows to evaluate some trade-off between the complexity of measurement versus the level of information exchange and connectivity, other classifications exist that focus on other aspects of the problems. The formation problem can be seen as either a specific shape that the agents must achieve or a set of distributed trajectories that the agents track with prescribed synchronicity [13]. These two features have been referred respectively as formation producing and formation tracking. These problems have been studied through matrix theory based approach [32], potential function based approach [75] and Lyapunov based approach [62] to name a few.

Another potential classification of formation control problems can rely on how the desired formations are expressed. If an explicit description is provided by desired positions or desired inter-agent displacement, the approaches would be defined as morphous formation control while if the formations are described as expected behaviors, *e.g.* collision avoidance and cohesion, they belong to amorphous formation control techniques. Leader-follower and virtual structure approaches are widely used examples of morphous formation control. Flocking and behavioral formation are amorphous formation types. Description of these techniques is presented in the following paragraphs.

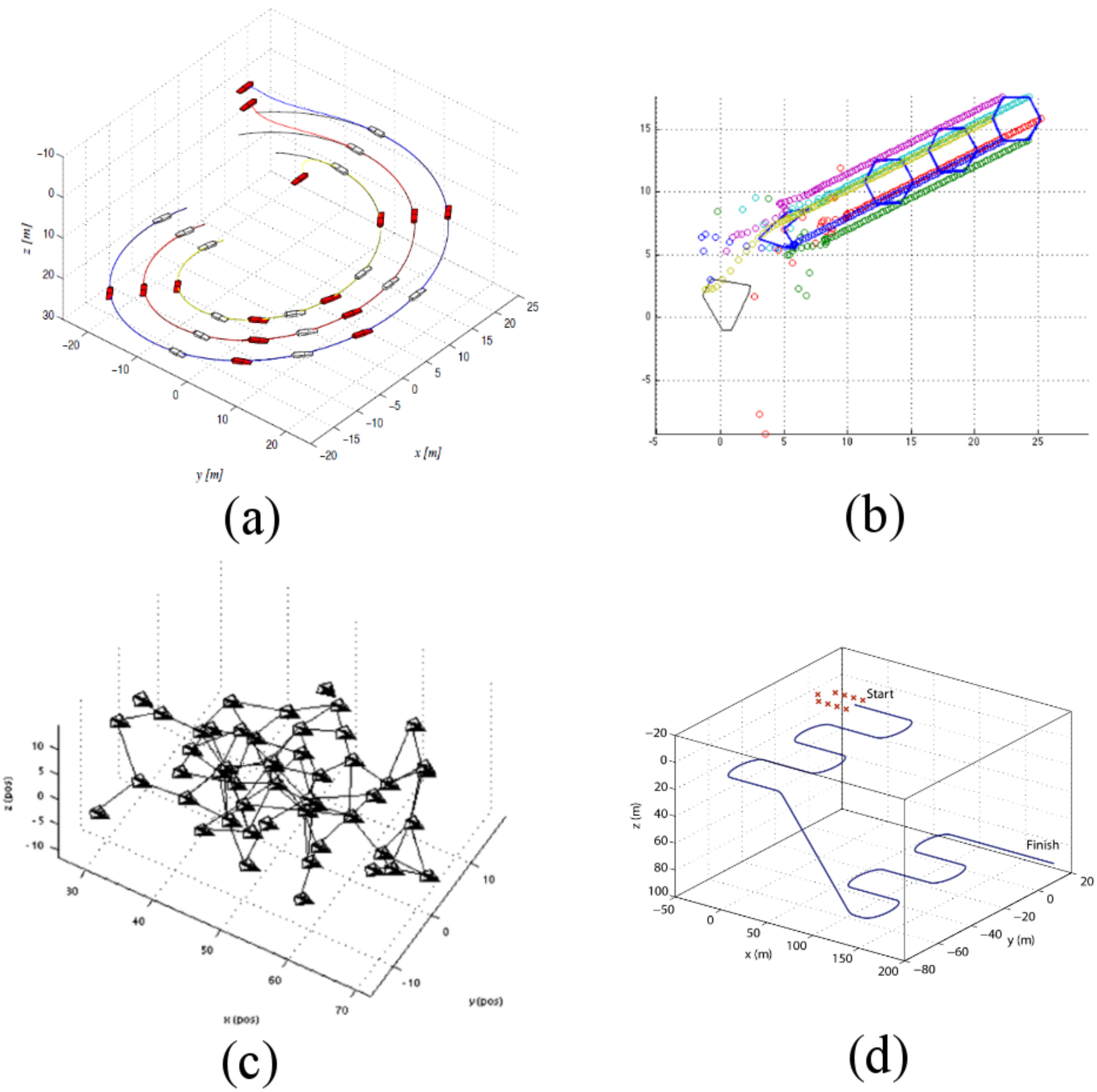


Figure 1.12: Tracking control formation. (a) Position-based approach [3]. (b) Displacement-based approach [82]. (c) Virtual structure: distance-based approach [75]. (d) Leader-follower approach [65].

1.8.2 Leader-follower method

In leader-follower techniques [26, 15, 6, 65], a trajectory is designed only for some leader agent based on mission goals. The other follower agents, aim at tracking the leader as well as maintaining some target formation defined with respect to the leader. A virtual leader has been considered in [14, 15, 90] to gain robustness to leader failure. This requires a good synchronization among agents of the state of the virtual leader. In the way idea, hierarchy structure can be used to select several leader to lead more efficiently the fleet, improved stability of the system and avoid collisions. However, permanent communication between agents and the leader is assumed: in most works, agents evaluate their control input with value based on information received from the leader. Note data on the environment can be collected by all agents, centralized by leader before be transmitted to others agents. Agent's control can also be combined with some behavioral rules to manage obstacle avoidance or described agent behavior if it loses its leader.

Different leader-follower methods have been considered. In [69], agents positions around the leader are defined but not allocate to a specific agent. A protocol assign a position to an agent if it is the closer to it location and if this position is free. This attribution is distributed and so not require intervention of the leader to attribute position. Messages are periodically broadcast from each agent to check their position. In case of loss of agent, its former positions is assign to an other one to fill the gap such as obtain the most compact formation possible.

[16, 15] proposed an optimal control for formation approach for non-holonomic vehicle with obstacles avoidance. A moving reference point represents an agent following a predefined reference trajectory. The real-time movement of the reference point can be known in advance or in-flight through wireless communication by each agent. Each agent must try to keep the prescribed relative distance and angle from this reference point. An obstacle avoidance cost function, evaluated using agents coordinate and a prediction of agents coordinate, guarantees obstacle avoidance while fleet doesn't deviate too far away from the desired trajectory. Inter-vehicle collision avoidance is also ensured by collision cost function.

[65] proposed a distributed coordinated tracking for multiple networked Euler–Lagrange systems where only a subset of the followers has access to the leader. A distributed adaptive control, using the information of both the neighbors and the neighbor' neighbors, is defined to account for parametric uncertainties. A distributed continuous estimator is designed to estimate the leader's coordinate when agent has not access to its current coordinate permanently. The control is robust to bounded perturbation.

Using only sensors information, [6] develops a model to identify V-formations or circular formations of an existing MAS formation. The model considers the location of entities to determine formation shape and adapt its control to insert agent inside the existing formation. For example, a least squares is used to define the median of the V-formation, and select the branch with the smallest number of agents to choose this future new agent position. Leader, which can be virtual or real, is located at the head of the V-formation and at the center of the circular formation. Other agents take position around it and track its trajectory.

Synchronous formation control approach using distributed control is proposed in [62, 97]. Virtual leader is used as reference of the formation. Consensus on agents velocity and integration of the tracking error allow a synchronization of the network to the desire formation. In [62], sufficient conditions are proposed to guarantee the converge of the system in presence of sampled-data and communication delays.

1.8.3 Virtual structure

Virtual structure [87, 112] is an alternative way to address the problem of leader failure. In a virtual structure, no agent has a predominant position regarding the others agent, but the entire formation is treated as a single entity where the agent control is designed to satisfy constraints between neighbors. A variety of virtual structure method can be formulated. For example, cooperative path following (CPF), distance-based control, and displacement-based control are virtual structure methods. In distance-based control, the constraints are distances between agents. In displacement-based control, relative coordinate or speed vectors between agents are imposed. Recently, [82, 72] propose a stable formation control law by using tensegrity structure, following the convention in [4]. In architecture, a tensegrity structure is a structural principle based on the use of isolated components in compression inside a net of continuous tension. Components are linked by compressed members (usually bars or struts) which prevent component to touch each other, and by tensioned members (usually cables) which prevent components to be separated from a fixed distance. The whole draws the system spatially and allows to obtain a stable structure.

Then, this specific structure, close to the displacement-based control, allows to introduce flexibility in the structure by not only using fixed distance, but also rejections and attractions between agents. [82] proposes also a method to define weights in the control input.

Consensus strategies have also been mostly applied to achieve vehicle formations (Cf. Section 1.7). In [32], information exchanged are studied to improve system stability and the vehicle formation energy. Consensus is reach on the only equilibrium point, defined as the desired relative position of the vehicles. The effectiveness of the method is shown by a Nyquist stability criterion on the formation stabilization. Fixed time communication delay is also considerate. Still inspired from distributed consensus approach, [112] proposes a nonlinear formation control method for non-holonomic mobile robots. Since the reference trajectory is not know continuously, a finite-time distributed observer of the desired trajectory is proposed. Thus, the formation control design can be done with limited information of the desired trajectory. Method allows a flexible, intermittent, and time varying communication topology among vehicles.

An other kind of virtual structure approach is presented in [17]. A control law is proposed to create formations of different choosing shapes. Areas are defined and the agents move and spread homogeneously inside. When the fleet move in the global system, agents inside the formation moves while preserving its shape. Gradient attracts agents inside the area while keeping them within the delimited area and the most spread as possible. Note to spread the agents over a desired area, some authors like [21] use the Voronoi repartition to optimize the distribution of the agents without allocate them a specific position.

In [87, 86], a virtual structure method for spacecraft formation is proposed. The proposed decentralized control input allows agents to track a desired position in the formation and to maintain it. [104, 105] address the problem of decentralized state estimation in fixed topology formations of vehicles. Displacement-based control is used to guarantee a fixed distance and orientation between agents, and so no changing of topology can happen. Presence of perturbations are considerate. Another method developed by [12] proposes an adaptive control input for formation stabilization for and heterogeneous Euler-Lagrange system. Control law is based on distance-based approach and ensuring asymptotic convergence of the inter-vehicle distance errors to zero.

[97] presents a centralized synchronization approach to trajectory tracking of Euler-Lagrange vehicles while maintaining time-varying formations. Each vehicle tracks its desired trajectory while synchronizing its position with others agents to keep relative displacement with them, as required by the formation. Then, control is based on the formation error, measured by the position error and the synchronization error. A synchronous controller for each vehicle's translation is defined to guarantee that both position and synchronization errors approach zero asymptotically. Moreover, a rotary controller is also designed to ensure that the robot is always oriented toward its desired position. Both translation and rotary controls are supported by a centralized high-level control for task monitoring and vehicle global localization.

The virtual structure method is often combined with leader-following method to drive fleet to a desired target or trajectory. Derived from leader approach, a sub-category of virtual structure is virtual leader approach [14, 15]. Instead of a real vehicle used as the leader of the formation, a virtual vehicle described by its current state is used to place other agents around it and drive the formation. Thus, a synchronization of the virtual leader current state must be managed between agent.

1.8.4 Flocking methods and Behavior-based control

In behavior-based controls, also named flocking control or swarm control [89, 103, 75, 91, 7, 33, 101, 90, 52], are inspired by birds, fish or bacterium behavior. They impose several behavior rules (attraction, repulsion, imitation) to each agent. Their combination leads the MAS to follow some desired behavior. Such approach requires the availability to each agent of observations of the state of its neighbors. These observations may be deduced from measurements provided by sensors embedded in each agent or from information communicated by its neighbors. In all cases, these observations are assumed permanently available. In addition, if a satisfying global behavior may be obtained by the MAS, Behavior-based flocking cannot impose a precise configuration between agents. Moreover, in opposite with formation controls where the distances desired among agents are always fixed, flocking control defines no constraints on distance among AUVs.

In the first works made by [88], agents follows three rules : flock centering, which attempt to stay close to nearby neighbors, collision avoidance, to avoid collisions with nearby agents, and velocity matching, to match velocity with nearby agents. These rules can be expressed an attraction force between agents, repulsion force if agents too close, and velocity synchronization. By following these three rules, the MAS converge to a compact group without collision which moves in a same direction with the same velocity, like a fleet of bird (cf Figure 1.13).

In [75, 90, 91, 51], the fleet keeps the formation in a predefined geometric shape. Studying [89] works, it defines the same three behavior rules. However, so some cost functions have been create to modulate the attractive and repulsive forces between agents and improve the global behavior of the fleet. Moreover, virtual agents are associated to obstacles, which allows agents to avoid its without introduce new rules. In phase of obstacle avoidance, the formation can split itself in two part to bypass an obstacle before go back to its initial shape. Collective potentials are defined to penalize deviation from the optimal compact formation, even during phase of obstacle avoidance. The repulsive potential function between agent and obstacle/agent is developed such as be inversely proportional to the norm of the distance between them. All these rules are summary in three distributed flocking algorithms to lead to a self-organizing flocking behavior.

Collision-free formation control for second-order vehicles is considered in [64]. The proposed control approach uses Laplacian graph and potential function to achieve a desired formation and collision avoidance. Security distance is defined such as guarantee the repulsive force start its action enough early to avoid the collision without disturb system convergence.

Finally, [40] proposed a distributed method of coordination without communication, using only agent' sensors. Distributed control allows to form a certain pattern and following a designated vehicle referred as leader, without a priori knowledge of the path leader is following. Agents use sensor to detect leader and closer neighbors, before move to obtain a desire distance and orientation with its. A symmetric axis center on the leader is used to help the organization of agent's destination point.

Figure 1.13: Example of flocking control

1.8.5 Issues in formation control

While fulfilling basic requirements such as trajectory tracking, following path generation, formation shape generation, switching between formation shapes, designing efficient formation control must also be performed by accounting for the environmental effects and the limitations and constraints of the vehicles and the communication network they realize. Among environmental effects, external perturbations (*e.g.* wind effects for aerial vehicles) and presence of obstacles can affect the convergence to the desired formation. Uncertainty on the relative positions of agents may lead to collision. Limitation of speed and acceleration play a major role in the efficient realization of the formation.

Environmental disturbance To obtain a high degree of reliability, disturbances, noises measurement and their effects on the agents must be modeled with a suitable degree of accuracy to evaluate their effects on the stability of the system. In UAV and AUV for example, the main sources of the dynamic disturbances are wind and wave, which scatter agents in the formation and create discrepancy between an agent current state and its estimate made by one of these neighbors. Thus, model of these effects must be generated and be considered in the control law and stabilization of the global system.

Communication constraints According to most control and coordination strategies studied, a wireless network is necessary in the fleet to gather information on others agents when information cannot be obtained using sensors. Note in some cases, formation shape is designed using communication topology constraints such as keeping distance between agents lower than communication range. The resulting formation thus always enables exchange between agents. In case of AUV, disturbances of sea layers, small bandwidth, and strong attenuation of signal in underwater medium induce chance of data packet loss and/or dropout due to attenuation in the environment. In a minimal case and/or a large number of problems, communication delays may appear and can disturb or make obsolete received information. Thus delay compensation must be employed while designing formation control strategies. Finally, in case of low data rate due to specific problem like in acoustic communication or limited data bandwidth, compact control language or linear quadratic optimal control must be employed. The control algorithm must also be based on on board computing power.

Some works try to join the formation control with the event-triggered communication (cf Section 1.9). [61, 98, 99, 112] model each agent with a simple integrator and proposed an event based on the error measurement. Command control is keeping constant between two broadcast messages.

Collision and obstacle When a group of multiple vehicles moves in formation, it is necessary to avoid collision between agents as well as avoid collision with obstacles intersecting the formation path. Thus, obstacle avoidance is highly essential in formation control. Remark the obstacle may be static or dynamic. Collision and obstacle avoidance can be managed in real-time obstacle avoidance using sensor and potential function based approach like in flocking methods, or offline by defining a tracking trajectory which avoid obstacles.

1.9 State-of-art: Event-triggered method

Distributed cooperative control usually requires significant exchange of information between neighboring agents so that each agent can properly evaluate its control law. In a UAV fleet for example, computation of the cooperative control law of an agent requires that values of speed and position of other vehicles must be updated regularly. That's why controlling a network with limited communication resources is a challenging task. Indeed, in absence of direct measurements, delivery of a message may induce delays, potential loss of information and additional expenses in terms of energy. Others issues are network bandwidth saturation and loss of stealth for military applications. Sensible selection of the information content and of the time when it is required may prove an efficient way to tackle these issues.

Many approaches for consensus or flocking in multi agent networks assume permanent communication, as in [74, 77, 113]. They often rely on continuous updating of neighbor state values, which involves continuous communication with all agents in the fleet. This results in heavy communication load and high frequency bandwidth. It also involves a large quantity of information processing at each time instant especially for a large number of agents. Therefore, practical implementation of the methods in multi-agent systems becomes soon intractable. In order to decrease the amount of processed information and the associated communication burden, methods have been proposed that limit their requirement to discrete time information publishing.

In periodic communication strategy, or discrete communication, as in [36, 42, 63], agents update their command with new information broadcast and received at a constant period T. To avoid saturation of message reception, the instants of communication of all agents may be shifted in time, but with the same constant period T. This method requires a synchronization of clocks between agents in the fleet. Although numerous developments have been performed in the field of periodic sampling methods, they still present flaws in terms of heavy communication loads and large amount of information to be processed.

Another approach suggested to overcome this drawback is intermittent communication [114, 116, 117, 119]. In intermittent communication strategy, the periods of communication broadcast alternate with periods of absence of communication. When communication are broadcast, it can be effected by means of permanent or periodic communication strategy. However, the absence of communication periods result in a global decrease of the amount of transmission required. However, the duration of broadcasting is often longer than the silent period and during broadcast, there is no restriction on information sent.

Instead of a priori planning of communication time, it seems more efficient to consider broadcasting only when it is required and thus to define a condition that will trigger communication, if fulfiled.

Event-triggered communication, or event-based control communication, is a promising approach to limit communication. The main difficulty consists in determining the communication triggering condition (CTC) that will ensure the quality of the completion of the task assigned to the MAS, *e.g.*, reaching some consensus, maintaining a formation, *etc.* As the state values of the other agents are not permanently available, it becomes mandatory for each agent to dispose of estimates of the state values of its neighbors to compute its cooperative control law. However, it is difficult to assess the quality of the state estimates. Therefore, to dispose of a suitable reference, each agent estimates its own state only using the information it has shared with its neighbors. When the discrepancy between its own state estimate and its actual state reaches some threshold, the agent triggers a communication. This type of approach has been considered, *e.g.*, in [136, 35, 94, 39, 107, 23, 106]. The methods developed in this context mainly differ from each other by the complexity of the dynamic model of the agents [136, 35, 94, 45, 28], by the structure of the state estimator [23, 39, 107, 106], and by the determination of the threshold of the CTC [94, 110, 31, 135, 28].

In this thesis, the event-trigger methods presented are used to decrease the amount of broadcast information between agents of a fleet. However, similar approaches have also been developed to save energy in static sensor' networks [50, 68, 95, 25]. Note that, event-triggered strategies described here are dedicated to consensus problem at the exception of the approaches exposed in Section 1.9.9.

1.9.1 Notations

In this study, agents broadcast messages at specific instant. If τ_{ij} is the communication time between two Agents i and j, broadcast messages are not received at the same instant that it has been sent. The following notation are introduced to underline the difference between the time when a message has been sent and when it has been received:

- $t^i_{j,k}$ denotes the time at which the k-th message sent by Agent j has been received by Agent i, and $t^i_{j,k+1}$ denotes the next.
- $t_{j,k}$ denotes the time at which the last message has been sent by Agent j, and $t_{j,k+1}$ denotes the next.
- t^i_k denotes the time of reception by Agent i of the last sent message, whatever the sending agent.

Remark if a communication delay τ_{ij} exists between Agent i and Agent j, the instant when Agent i receives the k-th message broadcast by Agent j is equal to $t^i_{j,k} = t_{j,k} + \tau_{ij}$. Else if $\tau_{ij} = 0$, one obtains $t^i_{j,k} = t_{j,k}$.

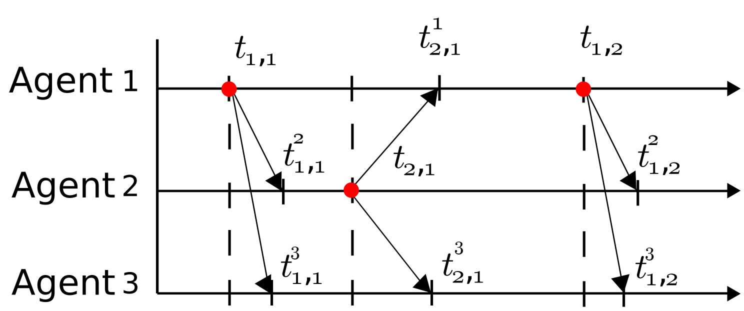


Figure 1.14: Examples of communication instants $t_{i,k}$ and receiption time $t^j_{i,k}$ for agents i and j where $(i, j) \in [1, 2, 3]$.

1.9.2 Communication triggering condition

The idea of event-triggered communication is that information are not updated continuously, but only at specific instants which are not necessarily periodic. Then, information are broadcast at appropriate time and only when agents need it. This appropriate time is derived from the state of a condition, named communication triggering condition (CTC), which will lead to broadcast a message when it is satisfied. As said before, the main difficulty consists in determining a CTC which constitutes a good trade-off between limited information and good performances of completion of the task assigned to the MAS.

The CTC is of the general form

$$f_i\left(.\right) > 0 \tag{1.17}$$

where f_i, called an event, depends on chosen parameters. Most of the times, the condition f_i associated with an Agent i depends of the discrepancy between Agent i state estimate and its current state.

In centralized system, decisions are taken by a leader, and so (1.17) is evaluated using information from all agents. It decides when an agent has to share an information with an other one. In case of distributed system, every agent decides itself when it has to transmit information to its neighbors. Thus, f_i is expressed using local information and local estimate values. Agent i broadcasts a message to its neighbors when its own condition (1.17) is satisfied. Note that as condition f_i mostly depends on the error between agent current state and its estimate, CTC may prove very sensitive to state perturbation.

1.9.3 Distributed event-triggered and estimators

The triggering condition depends on the discrepancy between its own state estimate and its actual value. This estimate is updated using the information it has shared with its neighbors and a dynamical model of evolution. Hence the error is due to potential time tag differences between agents and difference in estimate model structure and real agent dynamics. It is thus of major importance that there exists a synchronization of the update of estimators especially in the presence of communication delay. It is also necessary that the estimators of all agents are structurally identical. Some examples of estimators are presented in the following section.

One of the simplest estimator structure used e. g. in [94, 135, 129, 30, 43, 112, 136] is

$$y_i^j\left(t\right) = x_i\left(t_{i,k}^j\right) \tag{1.18}$$

where x_i is Agent i state and y_i^j is the estimate of the state of Agent i performed by Agent j. The estimate remains constant until the next triggering condition. In presence of a communication delay τ_{ij} like in [84], estimators are updates at time $t = t_{i,k} + T$, where $T > \tau_{ij}$ for all (i, j). This protocol allows all agents to receive the message from Agent i before updating y_i^j, and to do it in a synchronized way. However, drawback of (1.18) is its low accuracy, which leads to frequent triggered communication.

Another estimator presented in [37, 38, 35] is

$$\dot{y}_i^j\left(t\right) = Ay_i^j\left(t\right), \tag{1.19}$$

$$y_i^j\left(t_{i,k}^j\right) = x_i\left(t_{i,k}^j\right) \tag{1.20}$$

where A is the agent dynamic matrix. The state evolution due to the control inputs is not taken into account. For consensus, this estimator reflects agent state evolution when the consensus is reached, corresponding to a zero control input. In [38], a compensating term is added of the form

$$y_i^j\left(t_{i,k}^j\right) = e^{A\tau_{ij}}x_i\left(t_{i,k}^j\right). \tag{1.21}$$

to account for the communication delay when estimator is updated. (1.21) thus tolerate absence of synchronization between agents. However, even if (1.20) is more accurate than (1.18), it still doesn't take into account the control inputs.

1.9.4 Selection of an event-triggered condition

The design and selection of an event triggered condition is of major importance. Although systematic procedure to define a CTC doesn't exist, two schemes can be distinguished.

The first scheme consists in defining CTC by seeking for a function f_i translating some mission requirements. For example, in case where agents need to not exceed some security distance in a formation, the function f_i depends on the error between agent's current coordinate and its coordinate estimation and a threshold to compare this error with the security distance bounds. Thresholds may be constant or time-varying, as in [94, 23, 31, 28] where the threshold is time decreasing exponentially. Advantages of these methods are that they often guarantee a limited number of communication and can be easily distributed. However, the convergence and the stability of the global controlled system are often difficult to prove. Thus this scheme to construct CTC is mostly limited to simple and double integrator dynamics.

The second scheme to elaborate suitable CTC consists in deriving it from the conditions of global stability of the system. Usually, this is performed using a Lyapunov function which integrates a function of estimation errors. Bounds on this function that insure satisfaction of Lyapunov theorem conditions provides the expression of CTC . This scheme is used for system with complex dynamics to guarantee global stability and convergence [37, 35, 39, 110, 23, 81, 107, 106, 122]. However, this approach presents some drawbacks . First the resulting CTC guarantees convergence to a stable system but doesn't ensure decrease of the number of communications. In order to avoid continuous communication, proof of absence of Zeno behavior (cf. Section 1.9.5) must be established. An other disadvantage is the difficulty to obtain a distributed CTC from considerations on the global system stability. It is potentially feasible to transform the designed CTC so that it only requires only local information, but it is usually at the expense of the number of trigger. Finally, it is often difficult to provide a physical interpretation in terms of mission requirement of the resulting function.

The following example [94] is presented to illustrate the determination of a CTC. The problem to be addressed is consensus in a network. The dynamics of each agent is described by

$$
\begin{aligned}
\dot{x}_i(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(t) \\
u_i(t) &= \sum_{j \in \mathcal{N}_i} \left(y_i^i(t) - y_j^i(t) \right) \\
y_j^i(t) &= x_i\left(t_{j,k}^i\right)
\end{aligned}
$$

where $x_i \in \mathbb{R}^n$ is the state of Agent i, $u_i \in \mathbb{R}^m$ is its control input evaluated using $y_j^i \in \mathbb{R}^n$, the estimate of the state of Agent j performed by Agent i. Assume that the communication graph is connected and there is no communication delay. Thus all Agent i's neighbors have the same estimate of Agent i state, $(m, j) \in \mathcal{N}_i$, $y_i^i = y_i^j = y_i^m$.

The CTC is defined by $f_i > 0$ where

$$
f_i\left(t, e_i^i(t)\right) = |e_i^i(t)| - \left(c_0 + c_1 e^{-\alpha t}\right) \tag{1.22}
$$

with $e_i^i(t) = x_i(t) - y_i^i(t)$ is the error between Agent i state estimate and its current state, c_0, c_1 and α are constant design parameters.

The CTC associated with (1.22) translates the requirements for reaching a consensus into f_i becomes positive when the error e_i^i between the current state x_i and its own estimation y_i^i is larger than the adaptive threshold defined by c_0, c_1 and α. It indicates that Agent i trajectory diverges from the trajectory estimated by other agents. The consensus makes it necessary to correct this error by broadcasting a message containing Agent i state values to others agents. Thus, when $f_i > 0$, a communication is broadcast, e_i^i is reset to zero and f_i becomes negative. As $c_1 e^{-\alpha t}$ is time decreasing, less triggers occur at the beginning of the mission, when agents are remotely located and the error is growing faster. When agents become closer, consensus requires increased accuracy on the state estimate. The decrease of the threshold reflects this need and the CTC increases the number of communications. The constant term c_0 guarantees there is no continuous triggering event by keeping f_i negative when e_i^i is reset to zero.

Figure E.2 compares results obtained with periodic communication and event-trigger communication developed by [94]. Less broadcast messages are needed using event-triggered method than periodic communication method. Moreover, both systems converge at the same time.

Most often, the CTC is considered to be computed continuously. Since MAS are generally sampled-data systems, event-triggered methods based on discrete sampling characteristics appears to be more practical. Thus, by combining event-triggered control and periodic sampled-data control, some methods, [135, 57, 66, 79], check the event condition periodically. The control inputs are also only computed at the same sampling time and hold to this value over the periodic time interval. This simplifies the prediction of the error evolution.

This sampling period guarantees the minimum inter-event time studied in Section 1.9.5. The CTC must guarantee that the system converges during the sampling period, using only local information without

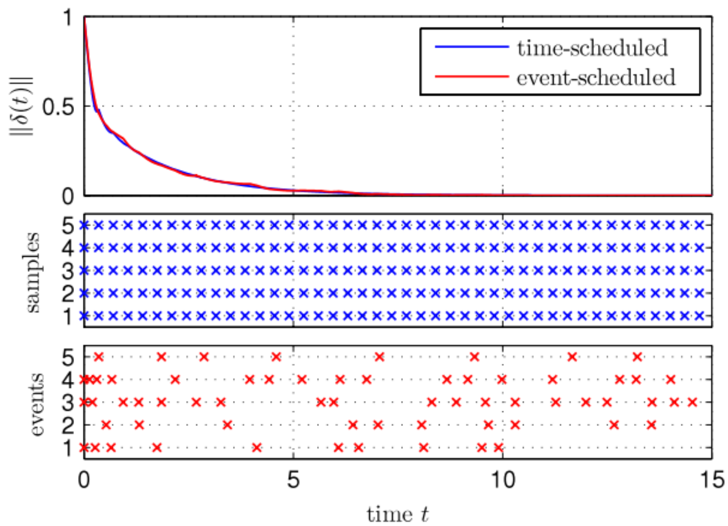


Figure 1.15: Comparison between event-triggered method proposed by [94] and basic periodic communication method.

broadcasting message.

1.9.5 Zeno behavior and inter-event time

The Zeno behavior corresponds to cases when an infinite number of discrete transitions is made in a finite time interval. A CTC which can not ensure the non-existence of Zeno behavior may potentially be continuously triggering. The absence of Zeno behavior may be established by proving the existence of a positive minimal time $\tau_i = t_{i,k+1} - t_{i,k}$ between two triggering events, called inter-event time. The maximal number of communications made by the CTC can be upper-bounded by a periodic communication with period T equal to the inter-event time τ_i.

Definition of minimal inter-event time has been presented in [20, 24, 23, 29, 31, 37, 38, 35, 34, 39, 60, 98, 30, 136, 133, 49]. It can be derived by determining an upper-bound of the estimation error e_i obtained at the time after the CTC has been satisfied. If it is strictly positive, absence of Zeno behavior is proved. The resulting inter-event times are often very small and may prove difficult to obtain when the agent dynamics is complex.

Guaranteing the absence of Zeno behavior can be directly done using a CTC evaluated at periodic instant [135, 57, 66]. However, this requires to prove system convergence with this periodic evaluation.

1.9.6 Event-triggered with communication delay

In most practical applications, there exists a delay between the instant when a message is broadcast and the time of reception. In [38, 39, 79, 135], this communication delay is considered and accounted for in a condition that guarantees system convergence.

[135] study a periodic event-triggered consensus problem for a MAS with simple integrator dynamics. Two event-triggered strategies combined with sampled-data control are proposed: the first one is based on an exponential delay function and the second on a quadratic Lyapunov function. The time delay is assumed to be upper-bounded by the sampling period. Then, a sufficient condition on the value of the sampling period and time delay is obtained to guarantee the asymptotic stability of the system.

[38] develops a distributed event-based consensus protocol for linear systems with limited communication and transmission delays. Event is classically designed as a function of discrepancy between current and estimate states, which triggers when it overcomes a specified threshold. The estimator of agent state takes in account the communication delay with which agents received information, to improve model accuracy. Lower-bounds on the inter-event times are computed and shown to be positive to prove absence

of Zeno behavior. [39] extends this previous work by considering that agents are connected by directed graphs. Event condition structure is deeply modified due to loss of symmetry in the system, but is still a distributed function and absence of Zeno behavior can also be proved.

[79] proposes a discrete event-triggered communication approach for a class of networked Takagi–Sugeno (T–S) fuzzy systems with communication delay and state perturbation. Delay is assumed to be upper bounded by a parameter δ. Thus, the sampling period between two events calculation is chosen larger than δ, so that broadcast messages are received before the CTC is re evaluated . Parameter δ constitutes a trade-off between the sampling period and the allowable network-induced delay. The control law is defined by logic Zero-Order-Hold (ZOH) . Note that in this case, the sampling period of the control law can be chosen lower than the sampling period between two events,contrary to [135] where there must be identical. The last transmitted state values are used as estimation of agent state for computing the control law. With the appropriate selection of the sampling period, the discrete event-triggered communication guarantees absence of Zeno behavior.

1.9.7 Package dropout, perturbations, switching topology and input saturation.

Communication delay is not the only disturbing factor that can affect the performances of event-triggered system. Other factors that can potentially counteract the stabilizability of the system are packet dropouts and variable topology.

The packet dropouts (also called "missing measurements") are known to be one of the most frequently observed phenomena in networked systems. In networks of large size, the packet dropouts may result in severe failures, especially when communication are constrained. Since event-triggered approach is designed to transmit a message when it is absolutely required, loss of part of these information can lead to unstability of the system. Moreover, agents cannot know if their broadcast messages have been received when the system is decentralized. As packet dropouts are not predictable, evaluating its effect must rely on stochastic model of occurrence of such a loss of data. In [25, 95], discrete-time stochastic non-linear systems with packet dropouts are considerate. The events are built using estimation errors like classical CTC, but are weighted with stochastic value variables. The number of triggering communication rises to counterbalance the potential loss information. A stochastic Lyapunov is used to prove the stability and the convergence of the system.

Another problem is the case of direct communication graphs. Sometime, communication between to agent can be only in one way, because of material problem like a broken receptor. Thus, the asymmetry of the communication graph makes it difficult to flock the agents when an agent cannot received information and only broadcast it. In Pinning method [124, 125], the presence of a virtual leader helps to converge even with a direct graph and a switching topology.

As CTC is evaluated using local information for distributed control, a change in the connection graph modifies the expected information sent by the neighboring agents. Event-triggered control schemes with time-varying topology have been proposed by [57, 66, 56] . Events are modified whenever a change occurs in the time-varying communication graphs.

1.9.8 Dynamic model and Event-triggered

In event-based centralized [111, 28, 126] or decentralized approaches [24, 31, 132, 23, 29, 34], simple integrator is the most often used dynamic model. It allows to define or build simple CTCs as in [29, 45, 31, 132, 30, 66, 99, 34, 24, 23, 135] where it is obtained by comparing error between agent state and its own estimation or between communicated and estimated average of relative positions of neighbors. In [23, 135, 24, 31], estimates of agents state correspond to the last received information. Time-varying topology is addressed in [66].

Double integrator model is treated by [94, 59, 60], where triggering conditions are associated with a state-independent and exponentially decreasing threshold. [49] studies problem of the centralized event-triggered based on a leader-following second-order consensus. Event condition is evaluated using the norm of agent error and its neighbor errors. More recently, [57] proposed an asynchronous sampling distributed event-triggered method in leader-follower formation with switching topology. To guarantee the tracking convergence, the CTC depends on the error of the estimate of agent position and velocity.

The impact of the switching topology on Zeno behavior is studied. It shows absence of Zeno behavior is guaranteed during intervals where topology is fixed, but witching instants can induce trigger of the CTC. Consequently, the topology switching can lead to a permanent communication if there is no lower bound on the time between two topology switches. However, the existence of this lower bound is sufficient to prove that there is not accumulation message in the sequences of inter-event times.

In [136, 37, 38, 35, 34, 39, 110, 41, 20, 43, 50, 79, 95, 120, 126, 128, 107, 106] a general linear model is considered

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{1.23}$$

$$u_i(t) = F \sum_{j \in \mathcal{N}_i} a_{ij}\left(y_i^i(t) - y_j^i(t)\right), \tag{1.24}$$

as presented in Section 1.7.6. As the proof of convergence of the system becomes more difficult, CTC are derived from the stability analysis of the global system, guarantying the stability and the convergence of it.

In [2, 28, 133, 129, 109, 111, 71, 25] nonlinear dynamical systems are represented as

$$\dot{x}_i = f(t, x_i) + g(x_i)\, u_i \tag{1.25}$$

where f is supposed to be globally Lipschitz.

In this case, the proof of absence of Zeno behavior is complex due to difficulty in predicting the error behavior. The assumption of f being globally Lipschitz provides a framework to treat this issue and demonstrate the global convergence.

In [71], the consensus problem of multi-agent systems is studied using Euler-Lagrange dynamics model. A centralized event-triggered strategy based on agent' position and velocity is proposed to limit the number of messages and to guarantee the global convergence. A non linear system modeled as a Takagi–Sugeno fuzzy system is presented in [111] using a discrete event-triggered communication method with communication delay. The estimator consists in a logic zero order holder. Finally, [2, 133] study the problem of event-triggered pinning control for the synchronization of complex networks of nonlinear dynamical systems. Conditions on parameters of the control inputs are presented that guarantee the global convergence of the system with the absence of Zeno behavior and exponential decrease of the norm error.

In most of the works presented in this section, state perturbations are not tackled. In [50, 95, 111, 120, 68], the event-triggered approach is studied in presence of noise in a static sensor networks without control inputs. These methods introduce in the event a time-varying coefficient which corrects partly the noise influence. A Kalman filter is used for enhancing the quality of estimation and thus reduce the influence of perturbations on the number of broadcast messages. However, most of these method are designed for centralized system. For distributed control methodsthe perturbations are not considered during the design of the control law and estimator which makes them very sensitive to perturbations. This results usually in a large increase of the number of broadcast messages. [79, 45, 111] propose an event-triggered method to mitigate the impact of perturbations in the case of vehicle dynamics described by simple integrator.

Some other issues can also be considered, as, for example, input saturation. [129] proposes a distributed event-triggered adaptive consensus control for nonlinear MAS subject to input saturation. Input saturation affects mostly the stability of the system but has not an direct impact on the creation or evaluation of the CTC.

1.9.9 Event-triggered and formation control

Most event-triggered approaches have been applied so that a MAS can reach a consensus. Recent works combine event-triggered communication approaches with distance-based or displacement-based formation control [61, 98, 99]. In these approaches, the dynamics of the agents are described by a simple integrator, and the control input is assumed constant between two communications. The proposed CTCs are centralized, and differ by the threshold formulations. A constant threshold is considered in [98]. [61, 99] define CTC where the thresholds are time-varying, depending on the relative positions between agents and the relative discrepancy between actual agent state and estimated state. It allows to reduce the number of communications when the system converges to the desired formation. A minimal inter-event time is also defined and no perturbations are considered.

Logic-Based Communication (LBC) techniques have been introduced in [84, 127, 3, 130] to decrease the number of communications. They bear common features to event-triggered method, but the triggering condition is based on formal logic. MAS with decoupled nonlinear agent dynamics are considered in [84, 3]. Agents have to follow parameterized paths, designed in a centralized way. CTC introduced by LBC lead all agents to follow the paths in a synchronized way to set up a desired formation. Communication delays, as well as packet losses are considered. Input-to-state stability conditions are established but absence of Zeno behavior is not analyzed.

1.10 Thesis outline

The subject of this thesis is the determination of distributed cooperative control of a multi-agent system with limited communications. The agents are mobile autonomous vehicles moving in an unknown environment. They dispose of their own means of measurements to measure their own state values and rely on communication link to obtain information on the state values or processed data of their neighboring agents. The communication links are summarized via a connection graph. The objective of this work is to decrease the amount of information to be transmitted between agents while managing the fleet.

This manuscript is organized in four chapters.

Chapter 1 has been dedicated to present the definitions and tools required in this study. Concepts of distributed control, graph theory and communication protocol have been presented. State-of-art of consensus approach, formation control and event-triggered strategy have been described.

Chapter 2 addresses the problem of distributed event-triggered communications for consensus of a multi-agent system with general linear dynamics and state perturbations. A control law and estimators of other agent's states are designed. An event-triggered communication strategy is defined to decrease the number of broadcast messages while insuring convergence to a stable consensus. Simulations illustrate performances obtained and comparisons with the reference method [37] of the quality of results on various cases are presented.

Chapter 3 is dedicated to the problem of formation control in multi-agent systems and presents an event-triggered strategy to reduce the number of communications between agents. Agents are assumed to have full-knowledge of the parameters of their dynamic Euler-Lagrange models. A control law and two estimators of other agent states are proposed. An event-triggered communication strategy is defined to reduce the number of broadcast messages while converging to a stable formation. State perturbations are considered. Simulations illustrates the performances obtained with both estimators.

Chapter 4 extends the problem studied in Chapter 3 to formation tracking control. Furthermore, the parameters of the agent's dynamical models are now supposed unknown. An adaptive control law is proposed, guaranteeing convergence. The previous estimator structures are re-designed to account for the uncertainty on the model parameters. An event-triggered communication strategy is defined to decrease the number of broadcast messages while converging to a stable formation and tracking the reference trajectory.

Chapter 5 extends the results presented in Chapter 4 to tackle the issue of packet dropouts. A new estimation model is defined, and the event-triggered communication strategy takes in account the expectation of the estimate error due to the loss of messages. A protocol is introduced to solve the problem of Zeno behavior.

In Chapter 6, communication delays are now considered. A prediction model is introduced to predict the triggering instant and adapt accordingly the time for broadcasting the message. Two structures of prediction models are proposed.

The final chapter presents a general conclusion and some perspectives of work. The proofs of the results presented in Chapters 2 to 6 are presented in the Appendix.

1.11 Author publications

- Viel, C., Bertrand, S., Piet-Lahanier, H., and Kieffer, M. (2016). New state estimator for decentralized event-triggered consensus for multi-agent systems. in IFAC ICONS 2016, IFAC-PapersOnLine, 49(5), 365-370.
- Viel, C., Bertrand, S., Kieffer, M and Piet-Lahanier, H. (2017). New state estimators and communication protocol for distributed event-triggered consensus of linear multi-agent systems with bounded perturbations. IET Control Theory & Applications, 11(11), 1736–1748.
- Viel, C., Bertrand, S., Piet-Lahanier, H., and Kieffer, M, (2017). Distributed event-triggered consensus for multi-agent formation stabilization, to appear in Proc. IFAC World Conference, Toulouse.
- Viel, C., Bertrand, S., Piet-Lahanier, H., and Kieffer, M, (2017). Distributed event-triggered control strategies formulti-agent formation stabilization and tracking, submitted to Automatica.

Chapter 2

Distributed event-triggered consensus of linear multi-agent systems with bounded perturbations

2.1 Introduction

This chapter addresses the problem of distributed event-triggered communications for consensus of a multi-agent system (MAS) with both general linear dynamics and state perturbations. This work extends results presented in [37, 35] by analyzing the effect of state perturbations on the consensus and on the communication requirements. Moreover, to reduce communications, this chapter proposes an improved estimator of the agent states, derives an estimator of the estimation error, and introduces an adapted communication protocol. By taking into account the control input of the agents, the proposed estimator allows the MAS to obtain a consensus with much less communications than with the approach in [37, 35]. The proposed technique is thus well-suited to applications where communications should be minimized, *e.g.*, to improve furtivity, reduce energy consumption, or limit collisions between transmitted data packets. Application examples with such constraints are exposed in [59, 60] for the case of a fleet of vehicles, or in [5] where agents aim at merging local feature-based maps.

With this approach, estimates of the states of all the agents (not only neighboring ones) are required to evaluate all control laws. More estimates are performed, but this reduces the communication frequency. A convergence analysis is achieved while considering state perturbations composed of two components: one common to all agents, and one agent-specific. Absence of Zeno behavior is shown. The case of a time-varying topology is also discussed.

This chapter starts in Section 2.2 with the problem formulation, and in Section 2.2.2, with a detailed description of the reference method which inspired this work.

The communication triggering condition (CTC), presented in Section 2.3, requires a new state estimator, described in Section 2.4, along with an adapted communication protocol.

A second estimator is exposed in Section 2.4.4 to obtain an implementable distributed event-triggering strategy presented in Section 2.5.

Section 2.6 compares the performance of the proposed approach to state-of-the-art results from [37, 35].

Finally, Sections 2.7 and 2.8 extend the previous results to non-linear dynamical systems and propose some solutions to address the case of time-varying topologies.

2.2 Problem formulation and reference solution

The dynamics of the agents of the MAS are first described. They incorporate state perturbation and control inputs. A CTC condition, which can be evaluated for any form of estimator, is also defined in Section 2.3 to let the choice of the estimator in Section 2.4.

2.2.1 Dynamical model with state perturbations

As in [37], one considers first a fixed, undirected, and connected communication graph $\mathcal{G}$ with adjacency matrix A_c. Time-varying topologies are studied later in Section 2.8.

In this section, the dynamics of a generic agent i is modeled as

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + d_i(t) \qquad (2.1)\\ u_i(t) &= c_1 F \sum_{j\in\mathcal{N}_i} \left(y_i^i(t) - y_j^i(t)\right). \qquad (2.2) \end{aligned}$$

In (2.1), $x_i \in \mathbb{R}^n$ is the state of Agent i, $u_i \in \mathbb{R}^m$ is its control input evaluated using $y_j^i \in \mathbb{R}^n$, the estimate of the state of Agent j performed by Agent i as described in Section 2.4. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. One has $c_1 = c + c_2$ with $c = 1/\lambda_2(L)$ and $c_2 \geq 0$ a design parameter. $F = -B^T P$ where P is a symmetric positive semi-definite matrix, solution of the Riccati equation

$$PA + A^T P - 2PBB^T P + 2\alpha P < 0, \qquad (2.3)$$

with $\alpha > 0$.

Remark 1. The parameter c, and so $\lambda_2(L)$, is related to the communication graph Laplacian matrix L. Its knowledge is required by each agent to evaluate its control input (2.2). Since the structure of the communication graph $\mathcal{G}$ is fixed, it can be assumed that the communication graph is initially known by all agents, or that a flooding method like that exposed in Section 2.4.3 can be initiated at $t = 0$ to deduce it. Thus, L and c can be computed by each agent and the control input u_i can be evaluated in a fully distributed way.

Contrary to [37], one considers in (2.1) an additive perturbation $d_i \in \mathbb{R}^n$. This perturbation is assumed to be such that

$$d_i(t) = m(t) + s_i(t), \qquad (2.4)$$

where $m(t) \in \mathbb{R}^n$ is a bounded time-varying perturbation with $\|m(t)\| \leq M_{\max}$ identical for all agents and $s_i(t) \in \mathbb{R}^n$ is a bounded agent-specific perturbation with, for all $i = 1, \ldots, N$, $\|s_i(t)\| \leq S_{\max}$, where $M_{\max} \geq 0$ and $S_{\max} \geq 0$ are known bounds. This two-parts additive perturbation model can be used, *e.g.*, to represent the combined effect of a uniform wind field on a fleet of drones and specific attitude-dependent turbulence affecting differently each drone.

The vector of all state perturbations is then

$$d(t) = 1_N \otimes m(t) + s(t) \qquad (2.5)$$

with $s(t) = \left[s_1(t)^T \ldots s_N(t)^T\right]^T$.

Splitting $d_i(t)$ in two parts allows taking into account the effect of two types of perturbations on the consensus. The perturbation $m(t)$ affects identically all agents, but has no effect on the convergence to the consensus contrary to the agent-specific perturbation $s_i(t)$. Indeed, if one considers the situation where the states of all agents have converged to the same value, the control inputs (2.2) become equal to zero. Thus, $m(t)$ affects identically the dynamics (2.1) of all agents. The agents move, but their state will still be identical. This is not the case with $s_i(t)$ which is specific for each agent. Nevertheless, as will be seen later, both perturbations have an impact on the CTC.

The problem considered here consists in designing a distributed control scheme, robust to perturbations, to drive the agents to a bounded consensus, while limiting the communications between agents. For that purpose, communication time instants are chosen locally by each agent using an event-triggered approach introduced in Section 2.3.

In this chapter, as in [37], we suppose that there is no communication delay and agents know perfectly their own state.

2.2.2 Reference solution

The problem considered in [37], agents are described with the simplified dynamics where $d_i(t) = 0$. The estimate $y_j^i(t)$ is obtained as follows

$$\begin{aligned} \dot{y}_j^i(t) &= Ay_j^i(t), \ \forall t \in \left]t_{j,k}^i, t_{j,k+1}^i\right[, \qquad (2.6)\\ y_j^i\left(t_{j,k}^i\right) &= x_j\left(t_{j,k}^i\right) \qquad (2.7) \end{aligned}$$

with $t_{j,k}$ the time instant at which the k-th message has been sent by Agent j and $t^i_{j,k}$ the time at which this message has been received by Agent i. In [37], it is assumed that there is no communication delay between agents, so $t^i_{j,k} = t_{j,k}$ for all $i \in \mathcal{N}_j$. The time of reception by Agent i of the ℓ-th message is t^i_ℓ, whatever the sending agent. The time at which the last message has been sent by agent j is denoted $t_{j,k}$. and $t_{j,k+1}$ denote the next one.

Let

$$y^i = \left[y_1^{iT}, y_2^{iT}, \dots y_N^{iT}\right]^T \in \mathbb{R}^{N.n}$$

be the vector gathering the estimates of the states of all agents performed by Agent i. The vector

$$y = \left[\begin{array}{ccc} \left(y_1^1\right)^T & \dots & \left(y_N^N\right)^T \end{array} \right]^T \in \mathbb{R}^{N^2.n}$$

gathers the estimates performed by each agent of its own state. Similarly, let

$$e = \left[\begin{array}{ccc} \left(e_1^1\right)^T & \dots & \left(e_N^N\right)^T \end{array} \right]^T \in \mathbb{R}^{N^2.n}$$

where $e_i = y_i^i - x_i$ is the estimation error between x_i and y_i^j.

It can be noticed that Agent i only uses the estimates y_j^i of the states of its neighbors $j \in \mathcal{N}_i$ to evaluate its control input (2.2). Moreover, since there is no communication delay or losses, (2.6)-(2.7) guarantee that $y_j^i = y_j^\ell$, $\forall i \in \mathcal{N}_j$ and $\forall \ell \in \mathcal{N}_j$. As a consequence, the estimate of the state of Agent j is the same for all its neighbors, thus each agent knows the estimates available at its neighbors. However, the influence of the control input (2.2) is not considered in the estimator (2.6). This induces an estimation error $e_i = y_i^i - x_i$ growing with time.

For that purpose, the communication time instants $t_{i,k}$ are chosen locally by Agent i using an event-triggered approach considering a threshold δ_i calculated from the state estimation error e_i, see Theorem 4. In addition, the delay between two successive communications (inter-event time) is shown in [37] to be lower-bounded, ensuring the absence of Zeno behavior.

Let $\widehat{L} = L \otimes P$, $\overline{L} = \widehat{L} A_c + A_c^T \widehat{L}$, $A_c = \overline{A} + \overline{B}_1$, $\overline{A} = I_N \otimes A$, $\overline{B}_1 = c_1 L \otimes (BF)$, $M = PBB^TP$ and

$$\beta = \frac{\lambda_{\min>0}(-\overline{L})}{\lambda_{\max}(\widehat{L})}. \tag{2.8}$$

It is proven in [37] that $\bar{L}$ is semi-definite negative. In the following theorem, the initial states are considered to be known by all agents.

Theorem 4 ([37], Th. 3)**.** *Assume that (A, B) is controllable and that the communication graph is connected and undirected. Then agents which dynamic described by (2.1)-(2.2) and $d_i = 0$ achieve a bounded asymptotic consensus with*

$$\lim_{t\to\infty} \|x_j(t) - x_i(t)\|^2 \leq \frac{N\eta}{\beta\lambda_{\min}(P)} \tag{2.9}$$

for all $(i, j) \in \mathcal{N}$, if the communications are triggered when

$$\delta_i > \sigma z_i^T \Theta_i z_i + \eta \tag{2.10}$$

where $z_i = \sum_{i=1}^N \left(y_i^i - y_j^i\right)$, $0 < \sigma \leq 1$, $\eta > 0$ is some design parameter and

$$\begin{aligned} \delta_i \quad = \quad & 2\left(c_2 - c\right) N_i z_i^T PBB^T P e_i + \left[2cN_i^2\left(1 + b_i\right) + \frac{c_2 - c}{b_i} N_i \right. \\ & \left. + cN_i\left(N - 1\right)\left(b_i + \frac{3}{b_i}\right)\right] e_i^T PBB^T P e_i \end{aligned} \tag{2.11}$$

$$\Theta_i = \left(2c_2 - b_i N_i\left(c_2 - c\right)\right) PBB^T P. \tag{2.12}$$

Moreover, the inter-event time can be lower-bounded $t_{i,k+1} - t_{i,k} \geq \tau_i$ where

$$\tau_i = \frac{\ln\left(\left(\frac{\eta}{k_2} + g\right)^2 - g + 1\right)}{\|A\|} \tag{2.13}$$

$$\begin{aligned} k_2 &= \left|2cN_i^2\left(1+b_i\right)+\frac{c_2-c}{b_i}N_i+cN_i\left(N-1\right)\left(b_i+\frac{3}{b_i}\right)\right| \qquad (2.14)\\ &\quad\times\|PBB^TP\|\left(\frac{z_{imax}\|cBF\|}{\|A\|}\right)^2 \\ g &= \frac{|2(c_2-c)N_i|\|A\|/\|cBF\|}{\left|2cN_i^2\left(1+b_i\right)+\frac{c_2-c}{b_i}N_i+cN_i\left(N-1\right)\left(b_i+\frac{3}{b_i}\right)\right|} \qquad (2.15)\end{aligned}$$

where z_{imax} satisfies $\forall t$ $\|z_i(t)\| \leq z_{imax}$ and $\|A\| = \max_{1\leq i\leq n}\sum_{j=1}^{n}|a_{ij}|$.

The CTC (2.10) guarantees the stability and the convergence of the global system only if agents used the estimator dynamics (2.6): the proof presented in [37] does not allow the use of an other estimator.

The CTC (2.10) mainly depends on e_i^i. Thus, a communication is triggered by Agent i when e_i^i becomes large. Since our aim is to reduce the number of communications triggered, keeping δ_i as small as possible, so keeping e_i^i small, is a way for improvement. Furthermore, the presence of state perturbations is not considered in the dynamics (2.1). Such perturbations may be an important source of discrepancy between the current state x_i and the state estimate y_i.

Then, in the next part of the chapter, our presentation focuses on the design of a more accurate estimator, and on the definition of a new CTC adapted to this new estimator. Moreover, the presence of perturbations is considered and their influence on the stability of the MAS is studied.

2.3 Event-triggered consensus

The introduction of the perturbations modifies the stability of the global system, and the solution proposed in [37] does not apply anymore. Moreover, the event-triggered method developed in [37] is closely dependent on the estimator (2.6). Our aim is to develop an event-triggered method which can guarantee the global stability of the MAS system 1) in presence of state perturbation; 2) without a specific condition on the estimator dynamics. This last point allows one to choose the state estimator in a second time and to choose the one that reduces the number of broadcast message. We choose here a "deducted" event to solve our problem.

This section introduces an event-triggered strategy to reduce the number of communications in Theorem 5. For that purpose, we assume that the estimates y_i^j for all i and j, are perfectly known by all agents in the network. This imposes strong constraints on the estimators embedded in each agent and on the communication protocol. These constraints will be relaxed in Section 2.5 to allow a practical implementation of the proposed technique. In the following theorem, the initial states are considered to be known by all agents.

Theorem 5. *Assume that (A, B) is controllable and that the communication graph is connected and undirected with a fixed topology described by the Laplacian matrix L. Consider some design parameter $\eta > 0$. Agents with dynamics (2.1)-(2.1) achieve a bounded consensus with*

$$\forall(i,j) \quad \lim_{t\to\infty}\|x_i-x_j\|^2 \leq \frac{N^3\eta}{\beta\lambda_{\min}(P)} \qquad (2.16)$$

if the bound on the perturbation satisfies

$$S_{\max} \leq \sqrt{\frac{\alpha\left\|c_2\lambda_2(L)M\right\|}{\lambda_{\max}(P)}}\sqrt{\frac{N\eta}{\lambda_{\min}(P)\beta}} \qquad (2.17)$$

and if communications are triggered when

$$\bar{\delta}_i \geq \rho z_i^T\Theta z_i + \eta \qquad (2.18)$$

with $\Theta_i = (2c_2 - b_iN_i(c_2-c))M$, $1 \geq \rho > 0$ a design parameter and

$$\begin{aligned}\bar{\delta}_i &= c_1\left[\left(z_i - N_ie_i^i\right)^T M\sum_{j\in\mathcal{N}_i}\Delta_{ij} + \frac{N_i}{2b_i}e_i^{iT}Me_i^i + \left(1+\frac{b_i}{2}\right)N_i\sum_{j\in\mathcal{N}_i}\left(\Delta_{ij}^TM\Delta_{ij}\right)\right]\\ &\quad+2\left(c_2-c\right)N_iz_i^TMe_i^i + \left[2c\left(N_i\right)^2\left(1+b_i\right)+\frac{c_2-c}{b_i}N_i+cN_i\left(N-1\right)\left(b_i+\frac{3}{b_i}\right)\right]e_i^{iT}Me_i^i \qquad (2.19)\end{aligned}$$

and $z_i = \sum_{j\in\mathcal{N}_i}\left(y_i^i - y_j^i\right)$, $\Delta_{ij} = y_i^j - y_i^i$, $0 < b_i < \frac{2c_2}{(c-c_2)N_i}$ *if* $c_2 > c$, $b_i > 0$ *otherwise.*

Note that the variables P, α, c_2 and c are derived from the control definition. N_i is the cardinal number of the set of the neighbors $\mathcal{N}_i$ of Agent i and β is defined in (2.8). The decision variables of Theorem 5 are η, b_i and ρ.

The proof of Theorem 5 is in Appendix A.1.

From (2.17) and (2.19), one sees that η can be used to adjust the trade-off between the bound on the consensus error and the amount of triggered communications. If $\eta = 0$ and if there is no perturbation, the system achieves an asymptotic consensus.

The CTC (2.19) mainly depends on e_i^i and Δ_{ij}. A communication is triggered by Agent i when the estimate y_i^i of its own state x_i is not satisfying, *i.e.*, when e_i^i becomes large. It is also triggered when the discrepancy Δ_{ij} between this estimate and that made by other agents y_i^j with their available information is large.

The two perturbations have a direct impact on e_i^i and thus on the frequency of communications. The sufficient condition (2.17) on $S_{\max}$ to have a consensus depends on η and on the measure of connectivity $\lambda_2(L)$ of the graph. Systems with more connected graphs are more robust to perturbations. $M_{\max}$ does neither influence the quality of the consensus, nor its convergence.

To reduce the number of communications triggered, one has to keep $\bar{\delta}_i$ as small as possible. This is done by keeping e_i^i and Δ_{ij} small, which is achieved by building accurate estimates y_i^i and y_i^j, as described in Section 2.4. Then, since in a distributed context, the y_i^js cannot be easily made available to all agents, the CTC introduced in Theorem 5 is difficult to implement. This issue is addressed in Section 2.4.4.

2.4 Agents state estimation and communication protocol

2.4.1 Agents state estimation

As exposed in the Section 2.2.2, the estimate $y_j^i(t)$ proposed by [37] is evaluated without considering control input. As already noticed, the absence for estimation of the control input in (2.6) induces a growing gap of the estimation error e_i^i. To reduce the number of messages broadcast by each agent, a new dynamic is considerate. It represents the agent behaviors by accounting for the control input evaluated by each agent and its dynamic behavior. It allows to be more accurate and so stay close to the current state, so keeping e_i^i small. Thus, the estimate $y_j^i(t)$ is evaluated as

$$\begin{aligned}
\dot{y}_j^i(t) &= Ay_j^i(t) + B\tilde{u}_j^i(t), \qquad t_{j,k}^i \leq t < t_{j,k+1}^i && (2.20)\\
\tilde{u}_j^i(t) &= c_1F\sum_{p\in\mathcal{N}_j}\left(y_j^i(t) - y_p^i(t)\right) && (2.21)\\
y_j^i\left(t_{j,k}^i\right) &= x_j\left(t_{j,k}^i\right), && (2.22)
\end{aligned}$$

where (2.20) takes into account the control input of the agents. Considering all the agents, (2.20)-(2.22) can be rewritten as

$$\begin{aligned}
\dot{y}^i(t) &= A_cy^i(t) && (2.23)\\
y_j^i\left(t_{j,k}^i\right) &= x_j\left(t_{j,k}^i\right), && (2.24)
\end{aligned}$$

where $A_c = \overline{A} + \overline{B}_1$, $\overline{A} = I_N \otimes A$, and $\overline{B}_1 = c_1L\otimes(BF)$.

To determine the control inputs applied by Agent j, Agent i needs to perform an estimate of the state of all the neighbors of Agent j. However, to perform an estimate of the state of an Agent ℓ neighbor of Agent j, an evaluation of all neighbors of Agent ℓ is required. As the communication graph is connected, Agent i will have to evaluate the state of all agents in the network to determine the control inputs applied by all other agents.

Remark 2. If there is no perturbation, *i.e.*, $M_{\max} = 0$ and $S_{\max} = 0$, the estimate error e_i^i vanishes. Moreover, in absence of perturbation, if for some time instant t_k, $y^i(t_k) = y^j(t_k)$ for all $(i,j)\in\mathcal{N}$, then $y^i(t) = y^j(t)$ for all (i,j) for all $t > t_k$. As a consequence, $\Delta_{ij}(t) = 0$ and $e_i^i(t) = 0$ for all (i,j) for all $t > t_k$. No communication will be triggered for $t > t_k$.

Note in the first works, an estimator where the control input $\tilde{u}_j^i(t) = \tilde{u}_j^i\left(t_{j,k}^i\right)$ was constant has been proposed to avoid to evaluate the state of all agents in the network. However, this estimator obtains worst performance in terms of CTC compared to (2.7)-(2.6). Thus, this method has been abandon for (2.20)-(2.22).

2.4.2 Communication protocol: fully-connected graph

In this section, the communication graph is assumed as fully connected. As in [37], the message broadcast by an Agent i at $t_{i,k}$ contains the state $x_i(t_{i,k})$ of Agent i. Agent $j \in \mathcal{N}_i = \mathcal{N}$ uses it to update its estimate y_i^j according to (2.22).

With a fully connected graph, the information transmitted by some agent is received without delay by all other agents in the network. As a consequence, one has $y_i^i(t) = y_i^j(t)$ and $\Delta_{ij} = 0$ for all $(i, j) \in \mathcal{N}^2$.

In this case, the CTC in Theorem 5 can be evaluated. Communications are triggered mainly due to the state perturbations.

2.4.3 Communication protocol: not fully-connected graph

From now, the communication graph is no more fully connected. Assume first that a message broadcast by Agent i at $t_{i,k}$ contains only its state $x_i(t_{i,k})$. Only neighboring agents receive the message and use $x_i(t_{i,k})$ to update their estimates y_i^j, $j \in \mathcal{N}_i$, according to (2.22).

A relaying is necessary to allow other agents updating y_i^j, $j \notin \mathcal{N}_i$. Two strategies are discussed in what follows.

Flooding Method

With the first strategy, a message received by an agent is immediately retransmitted to its neighbors.

When an Agent broadcasts a message at $t_{i,k}$, this message contains $t_{i,k}$ and the state $x_i(t_{i,k})$ of Agent i. When some Agent j, neighbor of Agent i, receives this message, it broadcasts $t_{i,k}$ and $x_i(t_{i,k})$ to its own neighbors if it has not done it previously. This message is further broadcast by the neighbors. This is a typical flooding strategy [44, 83], which enables all the network receiving the message.

Since there is no communication delay, one has $y_i^i(t) = y_i^j(t)$ and $\Delta_{ij} = 0$ for all $(i, j) \in \mathcal{N}^2$ as in Section 2.4.2.

With this method, each time a communication is triggered for a given agent, the same message is broadcast up to N times, depending on the topology. This technique is not competitive compared to that presented in [37].

Delayed flooding method

With the proposed alternative strategy, when a message is received by some agent, this agent waits until its CTC is satisfied to broadcast its own state as well as updated estimates of the states of all agents in the network evaluated from information in the messages received from its neighbors. This requires to store and broadcast a vector containing the time instants at which the communication has been triggered for each agent.

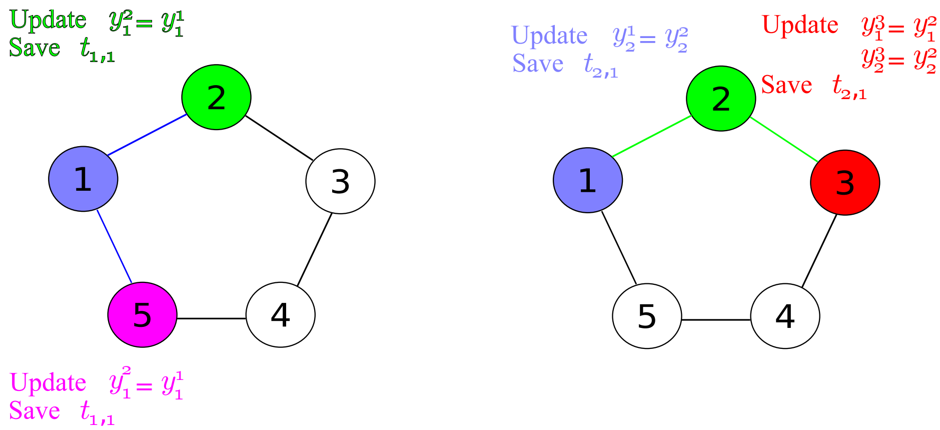
Thus, when a communication is triggered at $t_{i,k}$, Agent i first updates $y_i^i(t_{i,k}) = x_i(t_{i,k})$. Then, instead of transmitting only $t_{i,k}$ and $x_i(t_{i,k})$, it broadcasts the vector y^i and a vector

$$T^i = \left[t_{1,k_1}, \ldots, t_{i-1,k_{i-1}}, t_{i,k}, t_{i+1,k_{i+1}} \ldots t_{N,k_N}\right]$$

of time instants, where each t_{j,k_j} represents the time at which the triggering condition of Agent j has been satisfied.

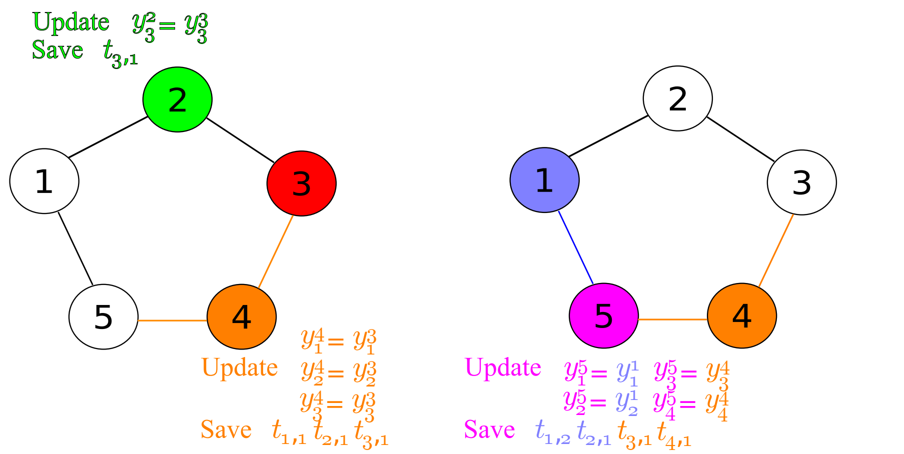
When some Agent ℓ receives the message from Agent i, it compares the time instants in T^i with those of its own T^ℓ. Each components of y^ℓ such that $t_{i,k} > t_{\ell,k}$, *i.e.*, corresponding to a more recent triggering instant, are replaced by those of y^i. The vector T^ℓ is updated accordingly.

Example 1 illustrates this information diffusion strategy.



(a) CTC satisfied for Agent 1.

(b) CTC satisfied for Agent 2.



(c) CTC satisfied for Agent 3.

(d) CTC satisfied simultaneously for Agents 1 and 4.

Figure 2.1: Delayed flooding protocol

Example 1. In Figure 2.1(a), the CTC is satisfied at $t_{1,1}$ for Agent 1. It updates its own estimate $y_1^1 = x_1$ and the first component of T^1 with $t_{1,1}$. Then it broadcasts T^1 and y^1. Its neighbors, Agents 2 and 5, receive this message. Agent 2, since the first component $t_{1,1}$ of T^1 is more recent than that of T^2, updates y_1^2 as $y_1^2 = y_1^1$. The first component of T^2 is now $t_{1,1}$. Agent 5 performs the same updates.

In Figure 2.1(b), the CTC is satisfied for Agent 2 which performed the update $y_2^2 = x_2$ and sets the second component of T^2 to $t_{2,1}$. It broadcasts then T^2 and y^2. Agent 3, once it receives this message, using T^2, knows that its estimates of the states of Agents 1 and 2 are outdated and performs the updates $y_1^3 = y_1^2$ and $y_2^3 = y_2^2$. The two first components of T^3 are now $t_{1,1}$ and $t_{2,1}$. Agent 1 updates only $y_2^1 = y_2^2$ and the second component of T^1 to $t_{2,1}$. A similar behavior is observed in Figure 2.1(c). In Figure 2.1(d), the CTC is satisfied simultaneously for Agents 1 and 4. Since the first components of T^1 is larger than that of T^4,*i.e.*, $t_{1,2} > t_{1,1}$, Agent 5 uses y_1^1 coming from Agent 1 to update y_1^5. It uses y_4^4 coming from Agent 4 to update y_4^5.

Example 2. An other example of the communication exchanges is proposed in Figure 2.2.

The proposed communication protocol has been designed so that once a message has been sent, (i) the estimation error e_i^i and discrepancies Δ_{ij} are reset to zero, and (ii) the CTC in Theorem 5, is no longer satisfied.

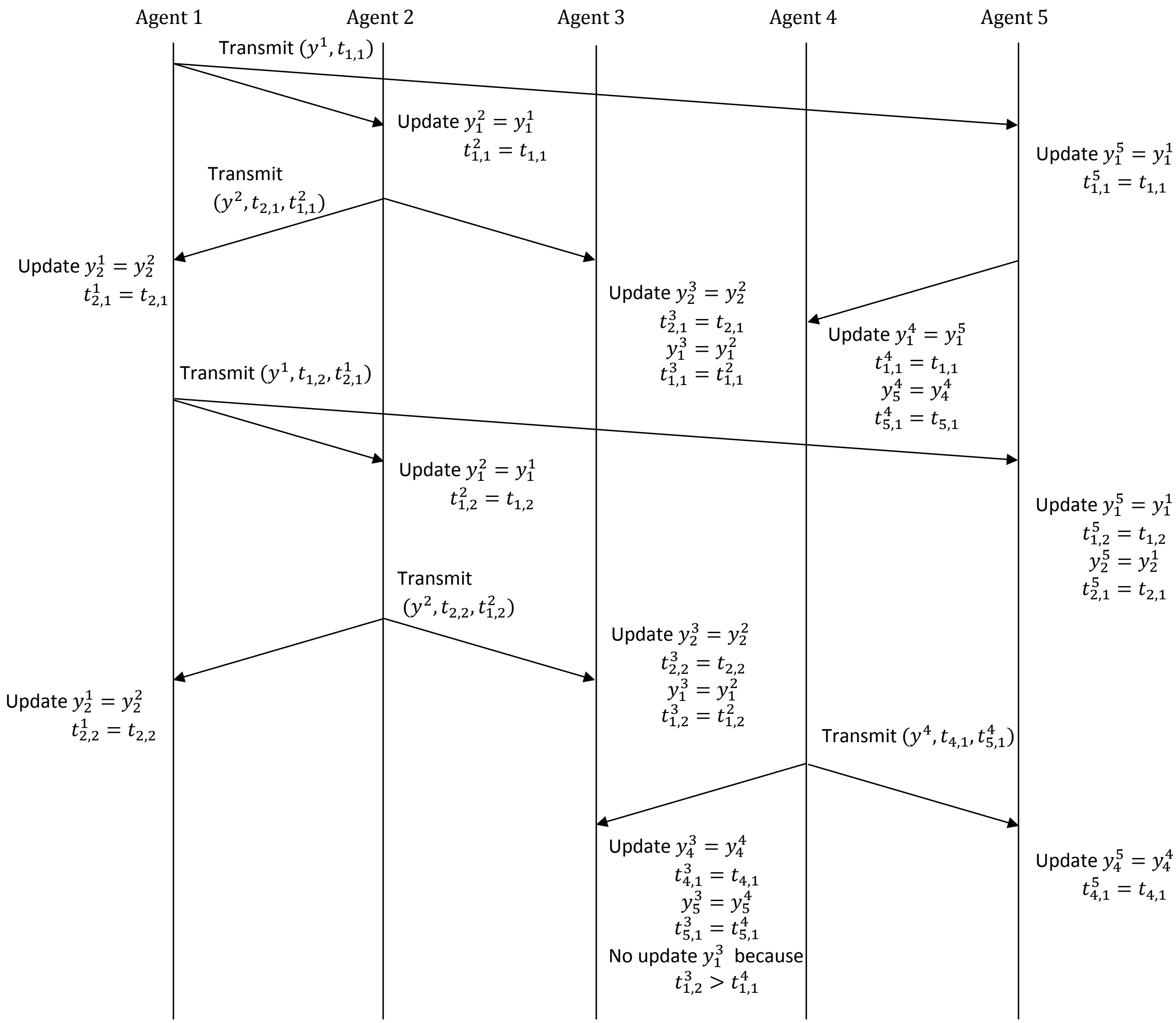
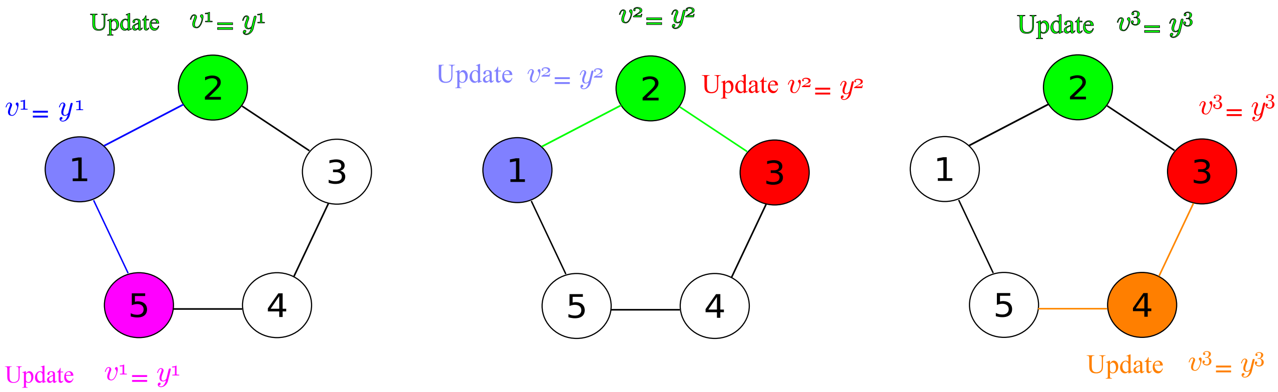


Figure 2.2: Example of delayed flooding protocol with the communication exposed in Figure 2.1.



(a) (b) (c)

Figure 2.3: Update estimator v^i

2.4.4 Estimation v^i of estimate y^i by Agent j

The delayed flooding protocol of Section 2.4.3 allows each Agent i having access to y_j^i, for all $j \in \mathcal{N}$. Nevertheless, Agent i is not able to access y_i^j, which is required to evaluate its CTC in Theorem 5.

In a first time, an idea was to introduce an estimation of the discrepancy Δ_{ij}. Indeed, it can be shown that the global vector $\Delta = \left[\begin{array}{cccc} \Delta_{11}^T & \Delta_{12}^T & \dots & \Delta_{NN}^T \end{array} \right]^T$ can be expressed as $\Delta = \exp\left(Z\left(t - t_k\right)\right) \Delta\left(t_k\right)$ where $Z \in \mathbb{R}^{\left(N^2 n\right) \times \left(N^2 n\right)}$ and t_k the time at the last instant where an agent in the network has broadcast a message. However, as see in Section 2.4 with the update of y_i^j, $\Delta\left(t_k\right)$ can not be updated at each instant t_k because an Agent i can only measured discrepancies Δ_{ij} of its neighbors $j \in \mathcal{N}_i$. Thus, this method was abandoned to the following ones.

To address this issue, each Agent i evaluates an *additional* estimates $v^j = \left[v_1^{jT} \dots v_N^{jT}\right]^T \in \mathbb{R}^{N.n}$ of y^j for all $j \in \mathcal{N}_i \cup \{i\}$, with the constraint that the estimates v^i performed by Agents i and $j \in \mathcal{N}_i$ have to be identical. For that purpose, the estimate v^i performed by Agent i and all its neighbors $j \in \mathcal{N}_i$ is updated only when the CTC is satisfied for Agent i and when it broadcasts a message. The v^js are thus less frequently updated than the y^is and are less accurate. Both estimators are evaluated simultaneously by each agent. Introducing v^j does not require any modification of the delayed flooding protocol. Agent i uses the v^is to check the CTC and y^i to evaluate the control inputs.

The dynamics of the additional estimate v^i is

$$\dot{v}_j^i\left(t\right) = Av_j^i\left(t\right) + B\bar{u}_j^i\left(t\right), \quad t_k^i \leq t < t_{k+1}^i \tag{2.25}$$

$$\bar{u}_j^i\left(t\right) = c_1 F \sum_{p \in \mathcal{N}_\ell} \left(v_j^i\left(t\right) - v_p^i\left(t\right)\right) \tag{2.26}$$

$$v^i\left(t_{i,k}\right) = y^i\left(t_{i,k}\right) \tag{2.27}$$

$$v_j^i\left(t_{j,k}\right) = y_j^j\left(t_{j,k}\right), \quad j \in \mathcal{N}_i. \tag{2.28}$$

Considering all the agents, (2.25)-(2.28) can be rewritten as

$$\dot{v}^i\left(t\right) = A_c v^i\left(t\right) \tag{2.29}$$

$$v^i\left(t_{i,k}\right) = y^i\left(t_{i,k}\right) \tag{2.30}$$

$$v_j^i\left(t_{j,k}\right) = y_j^j\left(t_{j,k}\right), \quad j \in \mathcal{N}_i. \tag{2.31}$$

Example 3. In Figure 2.3(a), the CTC is satisfied at $t_{1,1}$ for Agent 1. It updates its own estimate $y_1^1 = x_1$, the first component of T^1 with $t_{1,1}$, and its own additional estimate $v^1 = y^1$. Then, Agent 1 broadcasts T^1 and y^1. Its neighbors, Agents 2 and 5, receive this message as seen in Figure 2.3(a). Agent 2, since the first component $t_{1,1}$ of T^1 is more recent than that of T^2, updates y_1^2 as $y_1^2 = y_1^1$ and updates the additional estimate of Agent 1 $v^1 = y^1$. The first component of T^2 is now $t_{1,1}$. Agent 5 performs the same updates. Since there is no communication delay, the additional estimates v^1 evaluated by Agent 1, 2, and 5 are identical.

2.5 Distributed event-triggered consensus

Using the additional estimate v^i introduced in Section 2.4.4, Theorem 6 in Section 2.5.1 introduces a CTC that can be evaluated by each agent in a distributed way. Section 2.5.2 introduces then an implementable distributed event-triggered consensus algorithm.

2.5.1 CTC in distributed context

As in Theorem 5, the initial states are considered to be known by all agents. For practical implementation as well as in the following illustration examples, this condition can be relaxed by using two possible methods. The first one consists in making each agent trigger communications at $t = 0$ using the delayed flooding method presented in Section 2.4.3 : each agent will receive information from all other agents and initialize their estimators. The second method consists in making each Agent i initialize the state estimators for all other agents with its own value of the state, and trigger a communication at time $t = 0$, in order to update the estimates of its neighbors. Thus, additional estimators v^i are updated with the first communication. Moreover, each Agent j which is not a neighbor of Agent i and the estimate y_j^i of which is not updated by the first communication will have no impact on the control inputs (2.2) and (2.21) because $\forall j \notin \mathcal{N}_i \; y_j^i - y_i^i = 0$. Estimators will then be updated with more accurate values by next triggered communications.

Theorem 6. *Assume that (A, B) is controllable and that the communication graph is connected and undirected with a fixed topology described by the Laplacian matrix L. Consider some design parameter $\eta > 0$. Agents which dynamics is (2.1)-(2.1) achieve a bounded consensus with*

$$\forall (i,j) \quad \lim_{t\to\infty} \|x_i - x_j\|^2 \le \frac{N^3\eta}{\beta\lambda_{\min}(P)} \tag{2.32}$$

if the following condition on the perturbation bound is satisfied:

$$S_{\max} \le \sqrt{\frac{\alpha \|c_2\lambda_2(L)M\|}{\lambda_{\max}(P)}}\sqrt{\frac{N\eta}{\lambda_{\min}(P)\beta}} \tag{2.33}$$

and if communications are triggered when

$$\tilde{\delta}_i \ge \rho z_i^T \Theta z_i + \eta \tag{2.34}$$

with $\Theta_i = (2c_2 - b_i N_i (c_2 - c)) M$, $1 \ge \rho > 0$ a design parameter,

$$\begin{aligned}
\tilde{\delta}_i \;=\;& c_1 \left[\frac{1}{2b_{i2}} \left(z_i - N_i e_i^i\right)^T M \left(z_i - N_i e_i^i\right) + \frac{b_{i2}}{2} \sum_{j\in\mathcal{N}_i} N_j \left(y_j^i - v_j^i\right)^T M \left(y_j^i - v_j^i\right) \right.\\
& + \left(z_i - N_i e_i^i\right)^T M \sum_{j\in\mathcal{N}_i} \left(v_i^j - y_i^i\right) + \frac{N_i}{2b_i} e_i^{iT} M e_i^i + 2\left(1 + \frac{b_i}{2}\right) N_i \sum_{j\in\mathcal{N}_i} \left[\left(v_i^j - y_i^i\right)^T M \left(v_i^j - y_i^i\right) \right.\\
& \left.\left. + \left(y_j^i - v_j^i\right)^T M \left(y_j^i - v_j^i\right) \right] \right] + 2\left(c_2 - c\right) N_i z_i^T M e_i^i \\
& + \left[2c\left(N_i\right)^2 \left(1 + b_i\right) + \frac{c_2 - c}{b_i} N_i + c N_i (N-1) \left(b_i + \frac{3}{b_i}\right) \right] e_i^{iT} M e_i^i
\end{aligned} \tag{2.35}$$

and $z_i = \sum_{j\in\mathcal{N}_i} \left(y_i^i - y_j^i\right)$, $M = PBB^TP$, $0 < b_i < \frac{2c_2}{(c-c_2)N_i}$ if $c_2 > c$, $b_i > 0$ otherwise.

Note that the variables P, α, c_2 and c are derived from the control definition. N_i is the cardinal number of the set of the neighbors $\mathcal{N}_i$ of Agent i and β is defined in (2.8). The decision variables of Theorem 6 are η, b_i, b_{i2} and ρ. Note that the value of $\beta\lambda_2(L)$ can be derived from remark 1.

The proof of Theorem 6 is in Appendix A.2 and the proof of absence of Zeno behavior in Appendix A.3.

The difference between Theorems 5 and 6 lies in the evaluation of the CTC. The term $\bar{\delta}_i$ in (2.18) has been replaced by $\tilde{\delta}_i$ in (2.34), which mainly depends on the discrepancy between the state estimates y_j^i and the estimates of these state estimates v_j^i .

When an Agent i broadcasts a message, the estimation error e_i^i and the discrepancies $y_j^i - v_j^i$ and $v_i^j - y_i^i$ are reset according to (2.22), (2.28), and (2.27). As a consequence, the CTC (2.34) in Theorem 6 is no more satisfied.

2.5.2 Summary of the distributed event-triggered consensus algorithm

Results of Section 2.4 to 2.5 describing the proposed distributed event-triggered consensus approach are summarized in Algorithm 1 for some Agent i. This description is generic in the sense that all agents are controlled and trigger communications in the same way. The main loop of this algorithm is repeated until it is stopped by some external event (end of the mission, end of simulation time, etc.).

```
%% Initialization
T^i = 0_n.
if x(0) is known then
  y^i(0) ← x(0)
  for j = 1...N do
    if j ∈ N_i then
      v^j(0) ← x(0)
    end if
  end for
else
  for j = 1...N do
    y^i_j(0) ← x_i(0)
  end for
  v^i(0) ← y^i(0)
  Broadcast a message
  % Message received?
  for j = 1...N, j ≠ i do
    if a message is received from Agent j then
      Update y^i and T^i as presented in Section 2.4.3,
      v^j(t^i_{j,k}) ← y^j(t^i_{j,k}).
    end if
  end for
end if

%% Main loop
% Message received?
for j = 1...N, j ≠ i do
  if a message is received from Agent j then
    Update y^i and T^i as presented in Section 2.4.3,
    v^j(t^i_{j,k}) ← y^j(t^i_{j,k}).
  end if
end for
```

Algorithm 1: Control algorithm for Agent i

2.6 Example

Consider a network of $N = 5$ agents with unstable dynamics taken from [37] described by the following state and control matrices

$$A = \begin{bmatrix} 0.48 & 0.29 & -0.3 \\ 0.13 & 0.23 & 0 \\ 0 & -1.2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ -1.5 & 1 \\ 0 & 1 \end{bmatrix}.$$

Solving (2.3) with $\alpha = 1$, one obtains

$$P = \begin{bmatrix} 4.8436 & 5.4783 & -1.1082 \\ 5.4783 & 7.0514 & -1.4299 \\ -1.1082 & -1.4299 & 0.3778 \end{bmatrix}.$$

The network topology is linear with Laplacian matrix

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

with $\lambda_2(L) = 0.382$.

Each agent is assumed to have access only to its own state. The vector of initial states is

$$\begin{aligned} x(0) &= \left[\begin{bmatrix} 8.5067 \\ -0.6568 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1.7367 \\ -0.1855 \\ 0 \end{bmatrix}^T \begin{bmatrix} -0.0340 \\ -0.4651 \\ 0 \end{bmatrix}^T \ldots \right. \\ & \qquad \left. \ldots \begin{bmatrix} -0.7768 \\ -0.3803 \\ 0 \end{bmatrix}^T \begin{bmatrix} -0.6568 \\ 1.5076 \\ 0 \end{bmatrix}^T \right]^T. \end{aligned}$$

The simulation duration is T = 5 s. Euler method is used to integrate agents dynamics over intervals of the form $[kdt, (k+1)\,dt[$ with a step $dt = 0.01$ s. As the system has been discretised, the CTC is evaluated every instant kdt, inducing a minimum period between the transmission of two messages by the same agent set to dt. The perturbation $d(t)$ is assumed of constant value over each interval of the form $[kdt, (k+1)\,dt[$. The agent-specific component of $d(t)$ is $s_i(t) = [0, s_{i,2}(t), 0]^T$ where $s_{i,2}(t)$ is a zero-mean Gaussian noise with standard deviation σ_m, truncated at $S_{\max} = \sigma_m$ with $|s_{i,2}| = \|s_i\| \leq S_{\max}$, such as to satisfy condition (2.34) in Theorem 6. The component of the perturbation common to all the agents is $m(t) = [0, m_2(t), 0]^T$. Two cases are considered: a constant value $m_2(t) = M_{\max}$ (see, *e.g.*, Figure 2.7 (a)) or a zero-mean Gaussian noise truncated at the standard deviation σ_m, such that $|m_2| = \|m\| < M_{\max}$ (Figure 2.7 (b)).

The parameters of the CTC are set as follows $\eta = 0.1$, $c = \frac{1}{\lambda_2(L)}$, $c_2 = 0.1$, $b_i = 1.36$, $b_{2i} = 1$ and $\rho = 0.5$. The value of c is imposed, that of c_2 is taken from [37]. The other values are chosen to reduce the number of required communications.

The proposed approach is compared to that of [37], evaluating in both cases the total number of messages broadcast $N_{\mathrm{m}} \leqslant \overline{N}_{\mathrm{m}} = N\mathrm{T}/dt$. The residual communication ratio

$$R_{\mathrm{com}} = 100\frac{N_{\mathrm{m}}}{\overline{N}_{\mathrm{m}}} \tag{2.36}$$

of the number of broadcast messages is expressed in %. R_{com} indicates the proportions of time slots during which a communication has been triggered. It should be as small as possible.

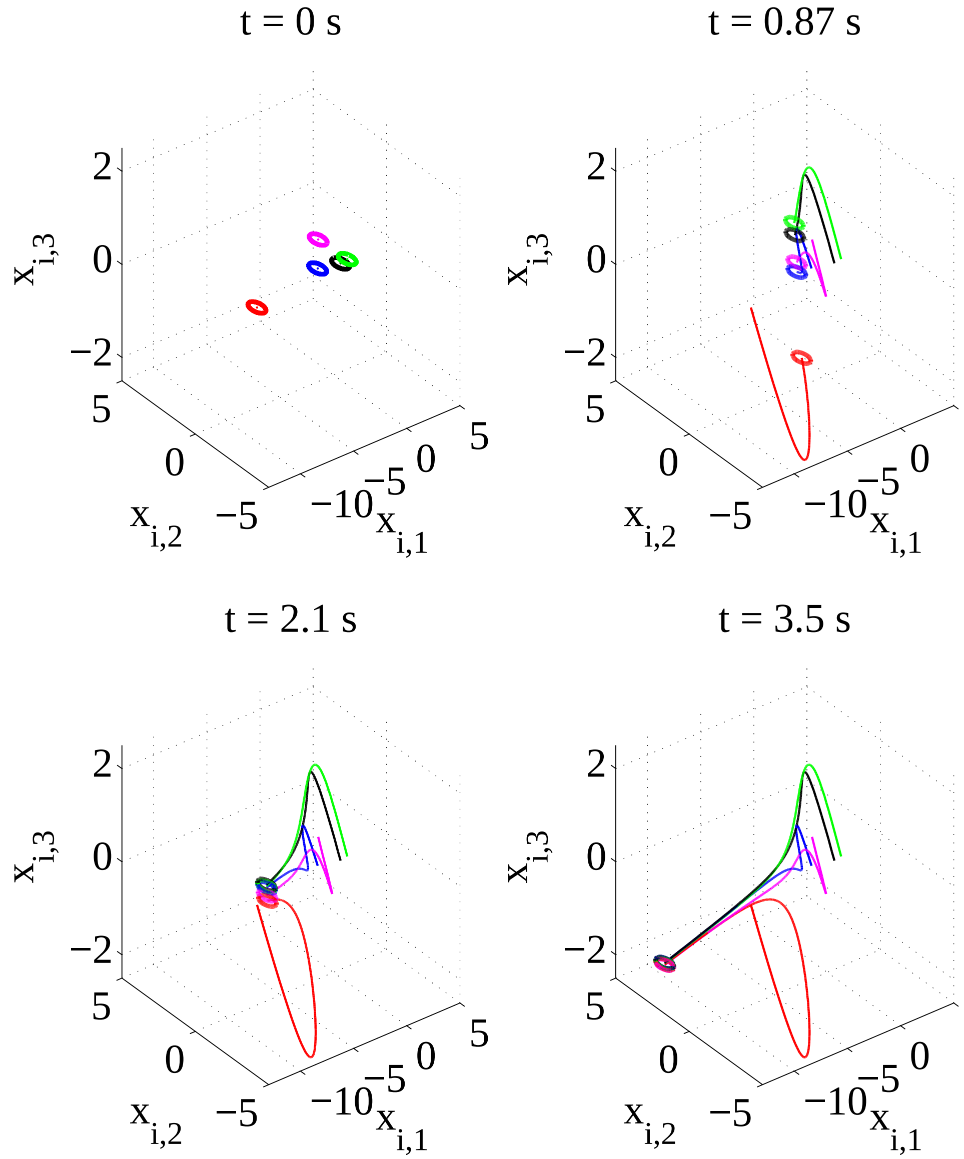
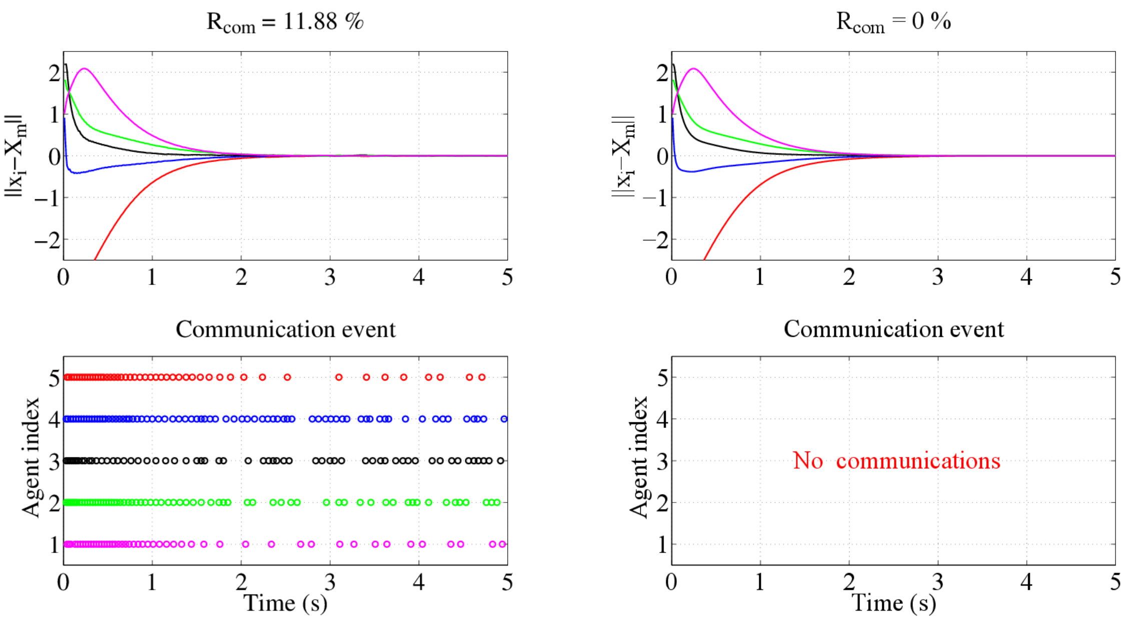


Figure 2.4: Convergence of agents to a consensus with control (2.2) and Theorem 6. Corresponding mapping between agent index and curve color: magenta: 1, green: 2, black: 3, blue: 4, red: 5.



(a) Error with respect to consensus and time instants of the broadcast messages when considering the reference estimator (2.6) from [37].

(b) Error with respect to consensus and time instants of the broadcast messages when considering the proposed estimator(2.20).

Figure 2.5: Comparison between estimator (2.6) and new estimator (2.20) without perturbation. $X_m = \frac{1}{N}\sum_{i=1}^{N} x_i$. Initial state is known. Corresponding mapping between agent index and curve color: magenta: 1, green: 2, black: 3, blue: 4, red: 5.

2.6.1 Without perturbation

Figure 2.5 compares the performance in terms of consensus error and number of broadcast messages for each agent, considering both estimators (2.20) and (2.6).

The figures show the time instant when each agent transmits a message. It can be seen that agents reduce communication and they converge to the same trajectories even in presence of reduce information.

When the initial conditions are perfectly known by all the agents and there is no perturbation, no communications are required when using the proposed estimator (2.20).

Figure 2.6 shows the results when each agent only knows its own initial state. With the estimator (2.6) from [37], a first communication is enough to initialize the estimates of all agents, since each agent only estimates the states of its neighbours. With the proposed estimator (2.20), estimators are initialized using the second protocol described in Section 2.5. The delayed flooding method allows then an update of the estimates of all agents. After a short transient period, only few communications are required.

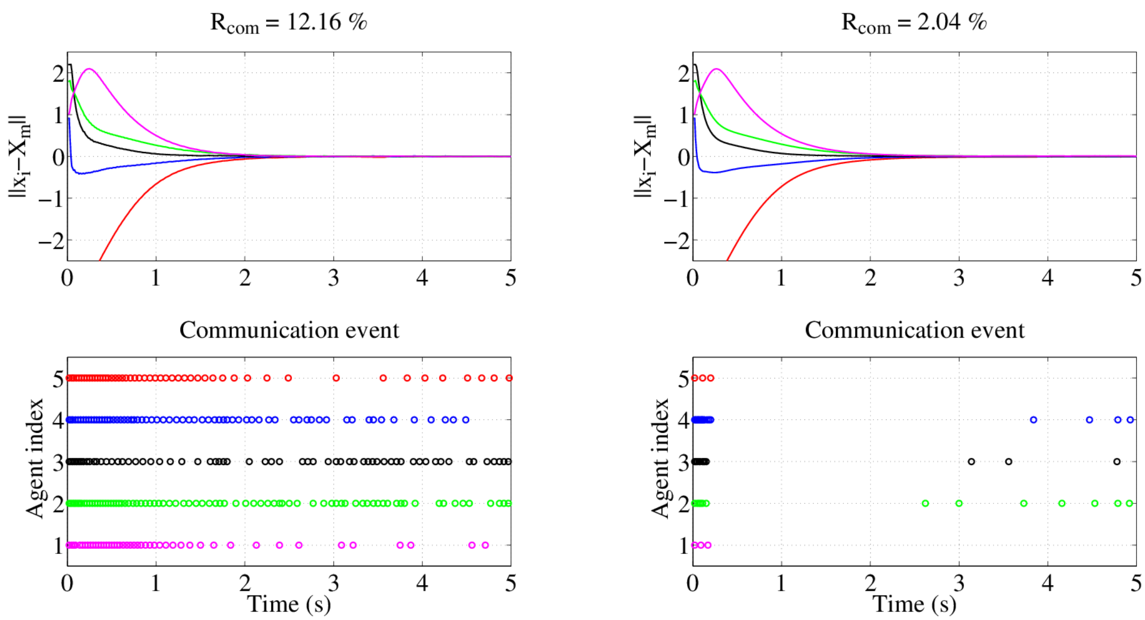
2.6.2 With perturbations

Figure 2.7 shows the evolution of R_{com} as a function of $S_{\max}$ for different values of $M_{\max}$, when $m(t)$ is constant (Figure 2.7(a)) and when it is described by a truncated Gaussian distribution (Figure 2.7(b)).

As could be expected, all types of perturbations, e.g. identical or agent-specific, increase the number of communications required. For high level of perturbations, keeping $\tilde{\delta}_i$ small can become impossible, resulting in the need for a permanent communication between agents.

With the considered parameters, the value of the upper-bound on $S_{\max}$ introduced in Theorem 2 is $S_{\max} = 16.17$. With this value, the sufficient condition (2.34) is satisfied for the following simulations, and one observes that a consensus is always reached.

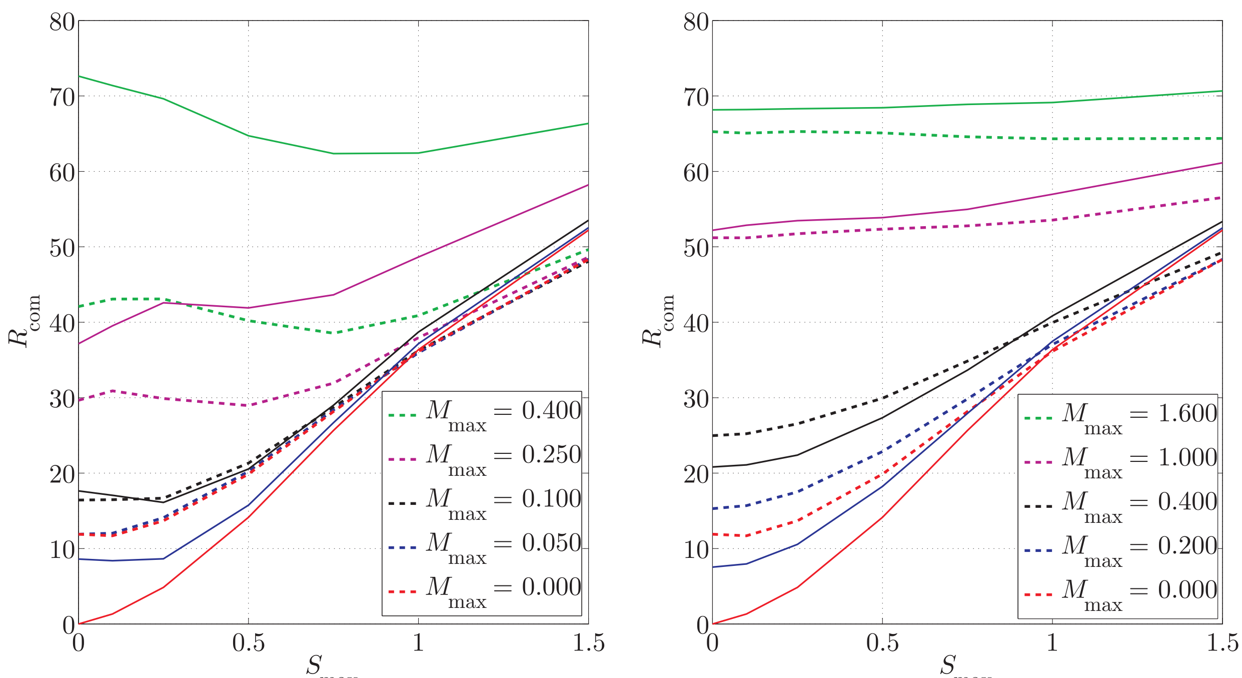
When the level of perturbation is low, the number of communications triggered by each agent is less with the proposed estimator (2.20) than with estimator (2.6). When $S_{\max}$ or $M_{\max}$ are large, the estimator (2.20) provides equivalent or even worse performance in terms of number of triggered communications compared to (2.6). This is mainly due to the additional terms introduced in the CTC (2.34).



(a) Error with respect to consensus and time instants of the broadcast messages when considering the reference estimator (2.6) from [37].

(b) Error with respect to consensus and time instants of the broadcast messages when considering the proposed estimator (2.20)

Figure 2.6: Comparison between estimator (2.6) and new estimator (2.20) without perturbation. $X_m = \frac{1}{N}\sum_{i=1}^{N} x_i$. Initial state unknown. Corresponding mapping between agent index and curve color: magenta: 1, green: 2, black: 3, blue: 4, red: 5.



(a) $m(t) = M_{\max}$ is constant

(b) $m(t)$ is truncated Gaussian

Figure 2.7: Evolution of R_{com} as a function of $S_{\max}$ for different values of $M_{\max}$ when considering the reference estimator (2.6) (dashed) and the proposed estimator (2.20) (plain).

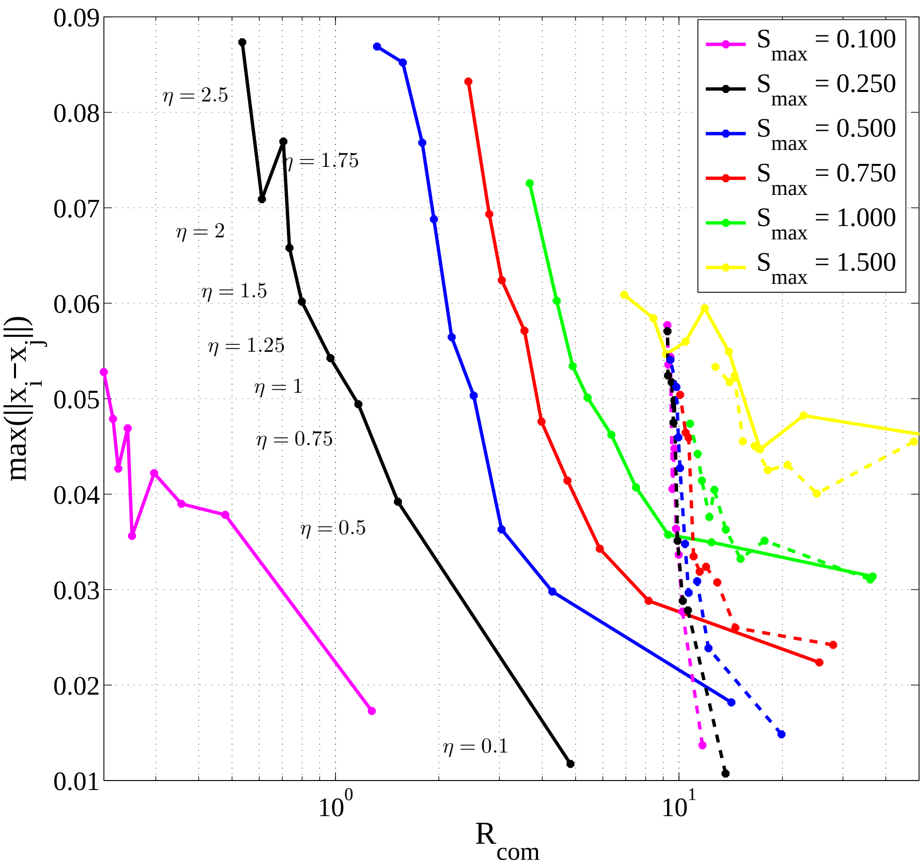


Figure 2.8: Evolution of R_{com} and $\max_{i,j\in\mathcal{N}}(\|x_i - x_j\|)$ for different values of $\eta = \{\ 0.1,\ \ 0.5,\ \ 0.75,\ \ 1,\ \ 1.25,\ \ 1.5,\ \ 1.75,\ \ 2,\ \ 2.5\ \}$ with the reference estimator (2.6) (dashed) and the proposed estimator (2.20) (plain) for different values of $S_{\max}$. Each point is the mean value over 50 simulations.

2.6.3 Choosing η in the CTC

The parameter η in the CTC allows to reach a trade-off between the disagreement with respect to consensus and the reduction in the number of triggered communications. Figure 2.8 illustrates this trade-off for

$$\eta \in \{\ 0.1,\ \ 0.5,\ \ 0.75,\ \ 1,\ \ 1.25,\ \ 1.5,\ \ 1.75,\ \ 2,\ \ 2.5\ \}$$

and different values of $S_{\max}$ for the proposed estimator and that proposed by [37].

It can be seen that the proposed estimator outperforms that proposed by Garcia in terms of number of communications, while the order of magnitude of the consensus disagreement remains relatively close. Using the proposed estimator, R_{com} can be significantly reduced which is not the case using the reference estimator of Garcia up to $S_{\max} = 1.5$, with which R_{com} cannot be reduced below the value of 10.

Figure 2.8 provides some guidelines to select the value of η when communications constraints or when some bound on the disagreement with respect to consensus have to be satisfied.

2.7 Extension to linear time-varying systems

In the previous section, the CTC has been developed for multi-agent systems with linear time invariant (LTI) dynamics. In practice, the dynamics of many systems may not be time-invariant. This is for example the case when one looks to approximate the dynamics of a nonlinear systems by performing linearizations or using a T-S fuzzy representation. In this section, we extend results obtained in Theorem 6 to the case of LTV systems. Proof in Appendices A.1, A.2 and A.3 remain valid without modifications.

Assume that the dynamics of an Agent i can be represented by the following LTV system:

$$\dot{x}_i(t) = A(t)\,x_i(t) + B(t)\,u_i(t) \quad (2.37)$$

$$u_i(t) = c_1 F(t) \sum_{j\in\mathcal{N}_i} \left(y_i^i(t) - y_j^i(t)\right) \quad (2.38)$$

Theorem 6 is still valid with the new matrices $A(t)$ and $B(t)$ if a constant matrix $P(t)$ can be found such as

$$\forall t, \quad P(t)\,A(t) + A(t)^T P(t) - 2P(t)\,B(t)\,B(t)^T P(t) + 2\alpha P(t) < 0.$$

Estimators y^i and v^i are also rewritten using $A(t)$ and $B(t)$. Theorem 6 and proof in Appendix A are still valid with these matrices $A(t)$ and $B(t)$.

An example of LTV representation can be obtained from the Takagi–Sugeno (T–S) fuzzy-model-based approach that can be used to represent or approximate nonlinear systems [79] by a T-S fuzzy system of the form

$$\dot{x}_i(t) = \sum_{m=1}^{r} \mu_m(\theta(t))\left[\mathcal{A}_m x_i(t) + \mathcal{B}_m u_i(t)\right] \quad (2.39)$$

$$u_i(t) = c_1 F \sum_{j\in\mathcal{N}_i} \left(y_i^i(t) - y_j^i(t)\right)$$

with $r \in \mathbb{N}^*$ and where $\mathcal{A}_m$ and $\mathcal{B}_m$ are constant known matrices of appropriate dimensions. The weighting functions $\mu_m(\theta(t))$ depend on variables $\theta(t) \in \mathbb{R}^p$, $p \in \mathbb{N}^*$and satisfy the following property for $m = 1, 2, \ldots, r$

$$\forall t, \quad \mu_m(\theta(t)) \geq 0$$

$$\sum_{m=1}^{r} \mu_m(\theta(t)) = 1.$$

This parametric representation allows for example to perform a linearization of nonlinear dynamics at r different linearization points instead of only one, hence resulting in a wider domain of validity of the system dynamics approximation.

Let us now define the matrices $A(t)$ and $B(t)$ as

$$A(t) = \sum_{m=1}^{r} \mu_m(\theta(t))\,\mathcal{A}_m \quad (2.40)$$

$$B(t) = \sum_{m=1}^{r} \mu_m(\theta(t))\,\mathcal{B}_m, \quad (2.41)$$

and rewrite the dynamical system (2.39) as

$$\dot{x}_i(t) = A(t)\,x_i(t) + B(t)\,u_i(t)$$

$$u_i(t) = c_1 F(t) \sum_{j\in\mathcal{N}_i} \left(y_i^i(t) - y_j^i(t)\right)$$

with $F(t) = -B(t)^T P(t)$ and $P(t)$ a symmetric positive semi-definite matrix

$$P(t) = \sum_{m=1}^{r} \mu_m(\theta(t))\,\mathcal{P}_m \quad (2.42)$$

where $\mathcal{P}_m$ is the solution of the Riccati equation

$$\forall m \in [1 \ldots r], \quad \mathcal{P}_m\mathcal{A}_m + \mathcal{A}_m^T\mathcal{P}_m - 2\mathcal{P}_m\mathcal{B}_m\mathcal{B}_m^T\mathcal{P}_m + 2\alpha\mathcal{P}_m < 0$$

In the same way, estimators y^i and v^i are rewritten using $A(t)$, $B(t)$ and $P(t)$.

Theorem (6) and proof in Appendix A are still valid with the new matrices (2.40) and (2.41), which allows to extend results of Theorem 6 to T-S fuzzy system (2.39).

2.8 Time varying topology

There are many cases where the communication graph $\mathcal{G}$ may change with respect to time: communication links may indeed appear or disappear between agents depending on their inter-distances (limitations of the communication range) or due to the presence of obstacles or failures of the communication equipments, etc. The communication topology must therefore be considered as time-varying, and the instants when the communication graph changes are called topology switches. Thus, the convergence of the global system should be analyzed when dealing with such time-varying topology. Some works in literature like [124, 125, 78] have shown that a consensus can be guaranteed if there exists a direct spanning tree in the communication graph.

Before studying the stability of the global system in presence of time-varying topology for our proposed method, let us first arise two questions related to time-varying topologies: (i) How a given Agent can detect that there is a switch in the topology (ii) How a given Agent can inform other agents that it has updated its list of neighbors ?

2.8.1 Detecting topology switches

Define t_h as the time instant at which the topology switches. A new connection (or new edge) between two Agents i and j is detected by Agent i if it receives a message from Agent j at $t \geq t_h$ and if Agent j was not a neighbor of Agent i on the interval $t_{h-1} \leq t < t_h$ ($j \notin \mathcal{N}_i$). Thus, discovering a new neighbor is pretty easy. However, when Agent i does not receive any message from Agent j, it is more complicated in event-triggered approaches to distinguish between the case of a non triggered communication (CTC of Agent j not satisfied) and the case where the communication link between Agents i and j is broken.

To overcome this problem, a simple strategy may consist in making all the agents to broadcast a message at fixed periodic time instants. If a message from Agent j, neighbor of Agent i, is missing, Agent i considers that the communication link is broken and Agent j is not one of its neighbors anymore. On the opposite, if an Agent k receives a message from Agent j, it adds it to its neighbors. Inconvenient of this strategy is that it results in adding many communications to the communication protocol defined in Section 2.5.2, which is contrary to our objective to limit the number of broadcast messages between agents.

2.8.2 Influence of topology switches on control input

Assume that the problem exposed in the previous paragraph has been solved. Estimators (2.20) and (2.25) introduced in previous sections assume that for any Agent i of the network $\mathcal{N}$ its set $\mathcal{N}_i$ of neighbors is known by all the agents. Indeed, the knowledge of the whole network $\mathcal{N}$ is required to evaluate the estimated controls (2.21) and (2.26). Thus, when an agent detects a topology switch, it needs to inform others agents of its new set of neighbors which cannot systemically detect this modifications instantaneously. Note that this problem does not exist in methods like the one of [37] since estimators only require the states of the neighbor agents.

Some methods to solve this problem or design control laws independent of the topology are proposed in the following paragraphs.

Informing other agents that an agent has updated its list of neighbours

Proposition 1: Flooding delay strategy A first idea is to use the flooding method to transmit the new set $\mathcal{N}$ to all agents. With this method, each time a topology switch is detected by a given agent, a communication is triggered and the same message is broadcast up to N times, depending on the new topology. This technique is not efficient to reduce the number of broadcast messages. Moreover, Theorem 6 guarantees system convergence on the interval $t \in [t_h, t_{h+1}[$. A proof of the system convergence with discontinuities between intervals due to topology switches has not been obtained.

Proposition 2: Flooding delay strategy and estimated set of neighbors The second idea is to use the delayed flooding method to make each Agent i to transmit the set $\mathcal{N}_i$ along with y^i and T^i like exposed in Section 2.4.3 each time the agent needs to broadcast a message, *i.e.* when the CTC is satisfied.

Let us introduce the vector

$$\hat{\mathcal{N}}^i(t) = \left\{\hat{\mathcal{N}}_1^i(t), \ldots, \hat{\mathcal{N}}_N^i(t)\right\},$$

where $\hat{\mathcal{N}}_j^i(t)$ is the estimate of $\mathcal{N}_j$ performed by Agent i, with $\hat{\mathcal{N}}_i^i(t) = \mathcal{N}_i(t)$ and $\hat{\mathcal{N}}_j^i(t) = \hat{\mathcal{N}}_j^i\left(t_{j,k}^i\right)$. $\hat{\mathcal{N}}_i^i(t)$ is updated by Agent i at every instant $t = t_h$ of change detection in the topology. Then, let us rewrite the control input u_i in (2.2), $\tilde{u}_i$ in (2.21) and $\bar{u}_i$ in (2.26) as

$$u_i(t) = c_1 F \sum_{j \in \hat{\mathcal{N}}_i^i} \left(y_i^i(t) - y_j^i(t)\right) \tag{2.43}$$

$$\tilde{u}_j^i(t) = c_1 F \sum_{p \in \hat{\mathcal{N}}_j^i} \left(y_j^i(t) - y_p^i(t)\right) \tag{2.44}$$

$$\bar{u}_j^i(t) = c_1 F \sum_{p \in \hat{\mathcal{N}}_j^i} \left(v_j^i(t) - v_p^i(t)\right). \tag{2.45}$$

Note that these modifications affect the calculation of y^i but not v^i since Agent i has not broadcast a message. Thus, this discrepancy leads to satisfy the CTC more quickly and so broadcast a message to update missing information.

Then, instead of transmitting only T^i and y^i, Agent i broadcasts now also the vector $\hat{\mathcal{N}}^i$ of its estimate of $\mathcal{N}$. When some Agent ℓ receives the message from Agent i and compares the time instants in T^i with those of its own T^ℓ, as described in Section 2.4.3, each components of $\hat{\mathcal{N}}^\ell$ such that $t_{i,k} > t_{\ell,k}$, *i.e.*, corresponding to a more recent triggering instant, are replaced by those of $\hat{\mathcal{N}}^i$. Inconvenient of this method is that the discrepancy between estimators y_i and v_i leads to a communication each time the topology switches.

The problem of this method is that Theorem 6 and its proof presented in Appendix A do not guarantee anymore the stability of the global system. Indeed, the matrix $\tilde{B} = T\left(I_N \otimes \overline{B}_1\right)$ used in the Lyapunov function has to be rewritten such as $\tilde{B} = f\left(BF, \ \hat{\mathcal{N}}_1^1, \ \hat{\mathcal{N}}_2^1, \ \ldots, \ \hat{\mathcal{N}}_N^N \right)$, where f is a function which depends on the estimates of the set $\mathcal{N}$ made by all agents. In this more complex case a proof has not been obtained.

Control input independent of the topology switches

Proposition 3: Control input associated to the fully-connected graph A third idea is to introduce a control with less dependence on the topology. This one consists in using the control input that would be associated to the "fully-connected graph", regardless of the current real topology. It can be expressed as

$$\begin{aligned} u_i(t) &= c_1 F \sum_{j=1}^{N} \left(y_i^i(t) - y_j^i(t)\right) \\ \tilde{u}_j^i(t) &= c_1 F \sum_{p=1}^{N} \left(y_j^i(t) - y_p^i(t)\right) \\ \bar{u}_j^i(t) &= c_1 F \sum_{p=1}^{N} \left(v_j^i(t) - v_p^i(t)\right). \end{aligned}$$

All estimates of other agents are used to evaluate the control input. In opposite to the previous proposition 2 (flooding delay strategy and estimated set of neighbors), the stability of the global system with this control can be shown by following the steps of the proof in Appendix A and using as Lyapunov function $V_N = x^T L_N x$, where $L_N = N I_N - 1_{N\times N}$ is the Laplacian associate to the fully-connected graph. Then, the centralized CTC proposed in Theorem 5 with the following expressions for z_i, Θ_i and $\overline{\delta}_i$ guarantees the convergence to a bounded consensus and reduces the number of broadcast messages

$$\begin{aligned}\bar{\delta}_i &= c_1\left[\left(z_i - Ne_i^i\right)^T M\sum_{j=1}^{N}\Delta_{ij} + \frac{N}{2b_i}e_i^{iT}Me_i^i + \left(1+\frac{b_i}{2}\right)N\sum_{j=1}^{N}\left(\Delta_{ij}^T M\Delta_{ij}\right)\right. \\ &\left. +2\left(c_2-c\right)N_i z_i^T Me_i^i + \left[2cN^2\left(1+b_i\right) + \frac{c_2-c}{b_i}N + cN\left(N-1\right)\left(b_i+\frac{3}{b_i}\right)\right]e_i^{iT}Me_i^i\right. \quad (2.46)\\ \Theta_i &= \left(2c_2 - b_i N\left(c_2-c\right)\right)M \quad (2.47)\\ z_i &= \sum_{j=1}^{N}\left(y_i^i - y_j^i\right). \quad (2.48)\end{aligned}$$

However, these modifications are not adapted for the Theorem 6. Indeed, the evaluation of the CTC requires an additional estimation v^i for all agents in the network. Since the additional estimator v^i cannot be updated by an Agent j if Agent i is not one of its neighbors, this method is impossible to implement in a distributed way, excepted by using the flooding protocol described in Section 2.4.3. However, remind that this protocol induces a large number of broadcast messages.

Proposition 4: control input associated to the minimum connected subgraph With a similar idea of reducing the dependence to the topology, another control law is proposed to solve the problem. Let us first assume that there exists a minimum subgraph $\mathcal{G}_{\min}\left(\mathcal{N}_{\min},\mathcal{E}_{\min}\right)$ such as all agents are connected, identical for all time-varying topologies $\mathcal{G}\left(t\right)$, $\forall t>0$. Then, Agent i control input is evaluated by using the time-constant set of neighbors $\mathcal{N}_{\min,i}$, associated to $\mathcal{G}_{\min}$, which is independent of the current topology. The expressions of the control inputs are given by

$$\begin{aligned}u_i\left(t\right) &= c_1 F\sum_{j\in\mathcal{N}_{\min,i}}\left(y_i^i\left(t\right) - y_j^i\left(t\right)\right) \quad (2.49)\\ \tilde{u}_j^i\left(t\right) &= c_1 F\sum_{p\in\mathcal{N}_{\min,j}}\left(y_j^i\left(t\right) - y_p^i\left(t\right)\right) \quad (2.50)\\ \bar{u}_j^i\left(t\right) &= c_1 F\sum_{p\in\mathcal{N}_{\min,j}}\left(v_j^i\left(t\right) - v_p^i\left(t\right)\right). \quad (2.51)\end{aligned}$$

Information received from Agent j, neighbour of Agent i, such as $j\in\mathcal{N}_i\left(t\right)$ and $j\notin\mathcal{N}_{\min,i}$ are still used to update estimates made by Agent i following the delayed flooding protocol, but without being used in the control inputs defined in (2.49)-(2.51).

Using the Lyapunov function $V_{\min} = x^T L_{\min} x$ where $L_{\min}$ is the Laplacian matrix associated to the graph $\mathcal{G}_{\min}$, Theorems 5 and 6 are still valid on the interval $[t_h, t_{h+1}[$ with $z_i = \sum_{j\in\mathcal{N}_{\min,i}}\left(y_i^i - y_j^i\right)$, $\Theta_i = \left(2c_2 - b_i N_{\min,i}\left(c_2-c\right)\right)M$, where $N_{\min,i}$ is the cardinal number of $\mathcal{N}_{\min,i}$, and the expression (2.35) of $\tilde{\delta}_i$ using $\mathcal{N}_{\min,i}$ instead of $\mathcal{N}_i$. Moreover, as $V_{\min}$ is independent of the time-varying topology, Lyapunov function $V_{\min}$ allows also to prove the stability of the global system on all intervals. It has been observed by simulations that using such a control law considering a fewer number of agents in $\mathcal{N}_i$ allows to reduce the number of triggered, as it would be expected, but reduces the speed of convergence.

2.8.3 Time-varying strategy

The approach based on using the minimum connected subgraph proposed in Section 2.8.2 allows to obtain a MAS consensus in presence of a time-varying topology. Using this method also solves the problem of detection of topology switches exposed in Section 2.8.1 since the control law (2.49) and the estimated control inputs (2.50) and (2.51) are independent of variations in the topology. The main issue of this method is to find a minimum connected subgraph $\mathcal{G}_{\min}$ existing for all the time-variations of the topology.

2.9 Conclusion

In this chapter, a distributed event-triggered communication strategy has been proposed to reach consensus in multi-agent systems with a reduced need for communication compared to state-of-the-art techniques.

To obtain this result, each agent has to manage simultaneously two estimators of the states of the other agents of the network. The first provides an accurate agent state estimate of all agents, which does not necessarily coincide among all agents. The second estimator considers only the neighbors of each agent and is less accurate but its value is constrained to coincide when two agents are neighbors. Both estimators are used to trigger communications.

A flooding delay communication protocol has been developed to guarantee the reset of estimation errors without adding broadcast messages to the initial strategy.

The proposed distributed event-triggered communication technique enables to obtain a reduced number of communications while enabling the agents to reach a bounded consensus in presence of state perturbations. Convergence to consensus has been studied and absence of Zeno behavior proved.

Simulations have shown the effectiveness of the proposed estimators in presence of state perturbations when their level is moderate. A guideline to select some design parameter to obtain a trade-off between communications constraints and bound on the consensus disagreement has been proposed.

Finally, extensions of this results to time-varying linear systems (including T-S fuzzy representations) and to the case of a time-varying topology have been presented.

Extensions of this work will focus on the case of influence of packet drops during transmission of messages, and handling time delays in communications. Using stochastic Lyapunov functions like in [25, 95] will allow to find an adapted CTC to make the system converge even in presence of packet dropouts. These methods use an expectation of the estimation error to take into account the lost of information in the communications.

In case of communication delay, the CTC would need to be satisfied more frequently so as to compensate the effect of the transmission delay. Moreover, communication delay should be taken into account in the state estimators managed by the agents as proposed in [38].

Packet dropout will be studied in Chapter 5 and communication delays in Chapter 6, in the case of formation control.

Chapter 3

Distributed event-triggered control for multi-agent formation stabilization

In chapter 2, the problem of communication reduction in consensus of multi-agents systems with linear dynamics has been studied and an event-triggered method has been proposed. In this chapter, the problem considered is formation control of multi-agent systems with Euler-Lagrange dynamics. Reduction of communications is still considered by proposing an event-triggered strategy.

More precisely, displacement-based formation control where agent dynamics are described by Euler-Lagrange dynamics including a state perturbation is considered. This work extends results presented in [82] by introducing an event-triggered strategy, and results of [61, 98, 99] by addressing systems with more complex dynamics than a simple integrator. This is the first approach to distributed event-triggered control of multiple Euler-Lagrange systems found in the literature. To obtain distributed control laws, estimators of other agents' states are introduced. The proposed distributed CTC involves the relative discrepancy between the actual and estimated agent states: a communication is triggered when the discrepancy between the actual state of an agent and its estimate reaches some threshold. The impact of state perturbations on the formation and on the communications is analyzed. A condition for the convergence of the MAS to a stable formation is also studied.

Hypotheses and some notations are introduced in Section 3.1.

The considered formation parametrization is presented in Section 3.2 and the new decentralized control law, based on estimates of the agents' states described Section 3.2.3, is proposed in Section 3.2.2.

The CTC is presented in Section 3.3.

A simulation example is considered in Section 3.4 to illustrate the reduction in communications obtained. Finally, conclusions are drawn in Section 3.5.

3.1 Notations and hypotheses

Let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$ and let $q = \left[\begin{array}{cccc} q_1^T & q_2^T & \dots & q_N^T \end{array}\right]^T \in \mathbb{R}^{N.n}$ be the *configuration* of the MAS. The dynamics of each agent is described by the Euler-Lagrange system

$$M_i\left(q_i\right)\ddot{q}_i + C_i\left(q_i,\, \dot{q}_i\right)\dot{q}_i = \tau_i + d_i \tag{3.1}$$

where $\tau_i \in \mathbb{R}^n$ is some control input described in Section 3.2.2, $M_i\left(q_i\right) \in \mathbb{R}^{n\times n}$ is the inertia matrix of Agent i, $C_i\left(q_i, \dot{q}_i\right) \in \mathbb{R}^{n\times n}$ is the matrix of the Coriolis and centripetal term on Agent i, and d_i is the additive external state perturbation satisfying $\|d_i\| < D_{\max}$. The state vector of Agent i is $x_i^T = \left[q_i^T, \dot{q}_i^T\right]$. Assume that the dynamics satisfy the following assumptions:

A1) $M_i\left(q_i\right)$ is symmetric positive and there exists $k_M > 0$ satisfying $\forall x,\, x^T M_i\left(q_i\right) x \leq k_M x^T x$.

A2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric or negative definite and there exists $k_C > 0$ satisfying $\forall x$, $x^T C_i(q_i, \dot{q}_i)\, x \leq k_C \|\dot{q}_i\|\, x^T x$.

A3) For all agent pairs $(i, j) \in \mathcal{N}$, if Agent j knows q_i and $\dot{q}_i$, it can evaluate $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

In what follows, the notations M_i and C_i are used to replace $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$. In this chapter, one assumes that each Agent i is able to measure without error its own state x_i. Moreover, it is assumed that there is no communication delay between agents.

3.2 Formation control problem

This section aims at designing a decentralized control strategy to drive the MAS to a desired target formation in some global reference frame $\mathcal{R}$, while reducing as much as possible the communications between agents. The target formation is first described in Section 3.2.1. The potential energy of the MAS is introduced to quantify the discrepancy between the target and current formations. The distributed control introduced in Section 3.2.2 tries to minimize this potential energy. To evaluate the control input of each agent despite the communications at discrete time instants only, estimators of the coordinate vectors of all agents are managed by each agent, as presented in Section 3.2.3. A CTC is designed to limit this discrepancy by updating the estimators as described in Section 3.3.

3.2.1 Formation parametrization

Consider the relative coordinate vector $r_{ij} = q_i - q_j$ between two agents i and j and the target relative coordinate vector r^*_{ij} for all $(i,\, j) \in \mathcal{N} \times \mathcal{N}$. A target formation is defined by the set $\left\{r^*_{ij}, (i,\, j) \in \mathcal{N} \times \mathcal{N}\right\}$. Consider, without loss of generality, the first agent as a reference agent and introduce the target relative configuration vector $r^* = \begin{bmatrix} r^{*T}_{11} & \dots & r^{*T}_{1N} \end{bmatrix}^T$. Any target relative configuration vector r^*_{ij} can be expressed as $r^*_{ij} = r^*_{1i} - r^*_{1j}$. In this chapter, the target configuration is considered to be time-invariant, *i.e.* $\dot{r}^*_{ij} = 0$. Extension to time-varying formations will be considered in Chapter 4 along with the tracking of a reference trajectory.

The *potential energy* $P(q,\, t)$ of the formation represents the disagreement between r_{ij} and $r^*_{ij}$and is defined by

$$P(q,\, t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_{ij} - r^*_{ij} \right\|^2 \tag{3.2}$$

where the $k_{ij} = k_{ji}$ are some *spring coefficients*, which can be positive or null, and $k_{ii} = 0$. $P(q,\, t)$ has been introduced for tensegrety formations in [72, 82]. The minimum number of non-zero coefficients k_{ij} $i, j \in \mathcal{N}$ to properly define a target formation is $N-1$. Indeed, for a given r^*, all target relative coordinate vectors r^*_{ij} between any pair of agents i and j can be expressed from components of r^*. Nevertheless, a number of non-zero k_{ij} larger than $N-1$ introduces robustness in the formation, in particular with respect to the loss of an agent. The values of the k_{ij}s that make a given r^* an equilibrium formation may be chosen using the method developed in [82]. (Cf. Appendix B.1).

Definition 9. The MAS asymptotically converges to the target formation with a bounded error iff there exists some $\varepsilon_1 > 0$ such as

$$\lim_{t \to \infty} P(q,\, t) \leqslant \varepsilon_1. \tag{3.3}$$

A control law designed to reduce the potential energy $P(q,\, t)$ allows a bounded convergence of the MAS. To describe the evolution of $P(q,\, t)$, one introduces as in [82]

$$g_i = \frac{\partial P(q,\, t)}{\partial q_i} = \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r^*_{ij} \right) \tag{3.4}$$

$$\dot{g}_i = \sum_{j=1}^{N} k_{ij} \left(\dot{r}_{ij} - \dot{r}^*_{ij} \right) \tag{3.5}$$

$$s_i = \dot{q}_i + k_p g_i \tag{3.6}$$

where g_i and $\dot{g}_i$ characterize the evolution of the discrepancy between the current and target formations and k_p is a positive scalar design parameter.

3.2.2 Distributed control

The control law proposed in [82] is defined as $\tau_i = \tau_i(q_i, \dot{q}_i, q)$ and aims at reducing $P(q, t)$, thus making the MAS converge to the target formation in case of permanent communication. In this approach, each agent evaluates its control input using the state vectors of its neighbors obtained via permanent communication. Here, in a distributed context with limited communications between agents, agents cannot have permanent access to q. Thus, one introduces the estimate $\hat{q}_j^i$ of q_j performed by Agent i to replace the missing information in the control law. The MAS configuration estimated by Agent i is denoted as $\hat{q}^i = \left[\begin{array}{ccc} \hat{q}_1^{iT} & \dots & \hat{q}_N^{iT} \end{array}\right]^T \in \mathbb{R}^{N.n}$. The way $\hat{q}_j^i$ is evaluated is described in Section 3.2.3.

In a distributed context with limited communications, with the help of $\hat{q}^i$, Agent i is able to evaluate

$$\begin{aligned}
\bar{g}_i &= \sum_{j=1}^{N} k_{ij}\left(\bar{r}_{ij} - r_{ij}^*\right) && (3.7)\\
\dot{\bar{g}}_i &= \sum_{j=1}^{N} k_{ij}\left(\dot{\bar{r}}_{ij} - \dot{r}_{ij}^*\right) && (3.8)\\
\bar{s}_i &= \dot{q}_i + k_p \bar{g}_i && (3.9)
\end{aligned}$$

with $\bar{r}_{ij} = q_i - \hat{q}_j^i$ and $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$. Using $\bar{g}_i$, $\dot{\bar{g}}_i$ and $\bar{s}_i$, Agent i is able to evaluate the following distributed control input to be used in (3.1)

$$\tau_i\left(q_i, \dot{q}_i, \hat{q}^i, \dot{\hat{q}}^i\right) = -k_s\bar{s}_i - k_g\bar{g}_i - k_p\left(M_i\left(q_i\right)\dot{\bar{g}}_i + C_i\left(q_i, \dot{q}_i\right)\bar{g}_i\right). \quad (3.10)$$

for some $k_g > 0$ and $k_s \geq 1 + k_p\left(k_M + 1\right)$ a design parameter.

Section 3.2.3 introduces the estimator $\hat{q}_j^i$ of q_j needed in the control 3.10.

3.2.3 Estimator dynamics and control law

In what follows, the time instant at which the k-th message is sent by Agent j is denoted $t_{j,k}$. Let $t_{j,k}^i$ be the time at which the k-th message sent by Agent j is received by Agent i. In this chapter, we assume that there is no communication delay between agents. Therefore, $t_{j,k}^i = t_{j,k}$ for all $i \in \mathcal{N}_j$. When a communication is triggered at $t_{i,k}$ for Agent i, it broadcasts a message containing $t_{i,k}$, $q_i\left(t_{i,k}\right)$ and $\dot{q}_i\left(t_{i,k}\right)$. Once a message is received by neighbors of Agent i, its content is used to update their estimate of the state of Agent i as presented in this section.

To get accurate estimates, the dynamics of the estimator are chosen so as to imitate as much as possible the agents' dynamics. Following the idea of chapter 2, the estimate $\hat{q}_j^i$ of q_j evaluated by Agent i is therefore evaluated considering

$$\begin{aligned}
M_j\left(\hat{q}_j^i\right)\ddot{\hat{q}}_j^i + C_j\left(\hat{q}_j^i, \dot{\hat{q}}_j^i\right)\dot{\hat{q}}_j^i &= \hat{\tau}_j^i,\ \forall t \in \left[t_{j,k}^i, t_{j,k+1}^i\right[&& (3.11)\\
\hat{q}_j^i\left(t_{j,k}^i\right) &= q_j\left(t_{j,k}^i\right) && (3.12)\\
\dot{\hat{q}}_j^i\left(t_{j,k}^i\right) &= \dot{q}_j\left(t_{j,k}^i\right), && (3.13)
\end{aligned}$$

This estimator (3.11) managed by Agent i requires an estimate $\hat{\tau}_j^i$ of τ_j evaluated by Agent j. This estimated control input $\hat{\tau}_j^i$ can be evaluated with one of the two following proposed methods.

Basic control:

$$\hat{\tau}_j^i = -k_s\dot{\hat{q}}_j^i \quad (3.14)$$

In this case, for all i, j such that $k_{ij} \neq 0$, Agents i and j must be connected in the communication graph. The main advantage of this control input is that the estimates of other agent state are not required.

Accurate control:

$$\hat{\tau}_j^i = -k_s\hat{s}_j^i - k_g\hat{g}_j^i - k_p\left(M_j\left(\hat{q}_j^i\right)\dot{\hat{g}}_j^i + C_j\left(\hat{q}_j^i, \dot{\hat{q}}_j^i\right)\hat{g}_j^i\right) \quad (3.15)$$

where $\hat{s}_j^i = \dot{\hat{q}}_j^i + k_p \hat{g}_j^i$, $\hat{g}_j^i = \sum_{k=1}^N k_{jk}\left(\hat{r}_{jk}^i - r_{jk}^*\right)$, $\dot{\hat{g}}_j^i = \sum_{k=1}^N k_{jk}\left(\dot{\hat{r}}_{jk}^i - \dot{r}_{jk}^*\right)$, and $\hat{r}_{jk}^i = \hat{q}_j^i - \hat{q}_k^i$. This expression of the control input makes the estimator more accurate than (3.14) and so helps the estimate $\hat{q}_j^i$ to remain closer to q_j. Note that if there is no perturbation, *i.e.*, $D_{\max} = 0$, the discrepency between $\hat{q}_j^i$ and q_j vanishes. The price to be paid for this method is that every agent needs to maintain an estimator of the state of all other agents, and a fully-connected communication graph is hence required to update it.

Errors appear between q_i and its estimate $\hat{q}_i^j$ obtained by an other Agent j due to the presence of state perturbations and the non-permanent communication. The errors for the estimates performed by Agent j are expressed as

$$e_i^j = \hat{q}_i^j - q_i, \; j \in \mathcal{N} \quad (3.16)$$
$$e^j = \hat{q}^j - q. \quad (3.17)$$

These errors are used in Section 3.3 to trigger communications when e_i^i and $\dot{e}_i^i$ become too large.

Since one assumed that there is no communication delay, these estimators satisfy $\hat{q}_j^i = \hat{q}_j^j$, $\forall (i,j) \in \mathcal{N}$. Estimates are used in the evaluation of the agents control law, but are also used in the evaluation of the CTC presented in what follows.

3.3 Event-triggered communications

Theorem 7 introduces a CTC used to trigger communications to ensure a bounded convergence of the MAS to the target formation. A message broadcast by an Agent i contains the state x_i. The initial value of the state vectors are considered to be known by all agents. In practice, this condition can be satisfied by triggering a communication from all agents at time $t = 0$ to initialize the estimates of its neighbors.

Let $k_{\max} = \max_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} (k_{\ell j})$ and $k_{\min} = \min_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} (k_{\ell j} \neq 0)$, $\alpha_i = \sum_{j=1}^N k_{ij}$, $\alpha_{\min} = \min_{i=1,\dots,N} \alpha_i$ and $\alpha_{\mathrm{M}} = \max_{i=1,\dots,N} \alpha_i$.

Theorem 7. *Consider a MAS with agent dynamics given by (3.1) and the control law (3.10). Consider some design parameters* $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$, $c_3 = \frac{\min\left\{k_1, k_p, \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right\}}{\max\{1, k_M\}}$ *and* $k_1 = k_s - (1 + k_p (k_M + 1))$. *In absence of communication delays, the system (3.1) is input-to-state practically stable (ISpS) and the agents can be driven to some target formation such that*

$$\lim_{t \to \infty} P(q,t) \leq \xi \quad (3.18)$$

where ξ *satisfies*

$$\xi = \frac{2N}{k_g c_3}\left[D_{\max}^2 + \eta\right] \quad (3.19)$$

if the communications are triggered when one of the following conditions is satisfied

$$\begin{aligned} k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta \leq{}& \alpha_M^2 \left(k_e e_i^{iT} e_i^i + k_p k_M \dot{e}_i^{iT} \dot{e}_i^i\right) \\ &+ \alpha_M k_C^2 k_p \left\|e_i^i\right\|^2 \sum_{j=1}^N k_{ji}\left[\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right]^2 + k_g b_i \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 \end{aligned} \quad (3.20)$$

$$\left\|\dot{q}_i\right\| \geq \left\|\dot{\hat{q}}_i^i\right\| + \eta_2 \quad (3.21)$$

with $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$.

The proof of Theorem 7 is given in Appendix B.2

Corollary 2. Consider a MAS with agent dynamics given by (3.1) and the control law (3.10). For any Agent i, let $t_{i,k}$ and $t_{i,k+1}$ be two consecutive communication instants at which the CTC of Theorem 7 have been satisfied. Then $t_{i,k+1} - t_{i,k} > 0$.

The proof of Corollary 2 is provided in Appendix B.3.

The CTCs proposed in Theorem 7 are analyzed assuming that the estimators of the state of the agents and the communication protocol is such that $\forall (i,j) \in \mathcal{N} \times \mathcal{N}$,

$$\hat{x}_i^i(t) = \hat{x}_i^j(t) \tag{3.22}$$
$$\hat{x}_i^i(t_{i,k}) = x_i^i(t_{i,k}), \tag{3.23}$$

where (3.22) is called the *estimate synchronization condition* and (3.23) the *estimator reset condition*. Theorem 7 is valid independently of the way the estimate $\hat{x}_i^i$ of x_i is evaluated provided that (3.22) and (3.23) are satisfied. Hence, the control law can be estimated using the models described in (3.14) or (3.15) in Theorem 7.

From (3.18) and (3.20), one sees that η can be used to adjust the trade-off between the bound ξ on the formation error and the amount of triggered communications. If $\eta = 0$ and if there is no perturbation, the system converges asymptotically.

The CTC (3.21) is related to the discrepancy between $\dot{q}_i$ and $\dot{\hat{q}}_i^i$. Choosing a small value of η_2 may lead to frequent communications. On the contrary, when η_2 is large, (3.20) is more likely to be satisfied. A value of η_2 that corresponds to a trade-off between the two CTCs (3.20) and (3.21) has thus to be found to minimize the amount of communications.

The CTCs (3.20) and (3.21) mainly depend on e_i^i and $\dot{e}_i^i$. A communication is triggered by Agent i when the estimate state $\hat{x}_i^i$ of its own state vector x_i is not satisfying, *i.e.*, when e_i^i and $\dot{e}_i^i$ becomes large. To reduce the number of triggered communications, one has to keep e_i^i and $\dot{e}_i^i$ as small as possible. This may be achieved by increasing the accuracy of the estimator, as proposed in Section 3.2.3, but possibly at the price of a more complex structure for the estimator.

The perturbations have a direct impact on e_i^i and $\dot{e}_i^i$, and, as a consequence, on the frequency of communications. The bound (3.19) on the potential energy, and hence on formation errors, is also affected by the perturbation through $D_{\max}$.

Parameters k_p, k_g and k_s in the control law are chosen to ensure stability. They can be tuned to adjust the speed of convergence. The condition $k_s \geq 1 + k_p(k_M + 1)$ has to be verified. In the performed simulations k_s has been chosen close to $1 + k_p(k_M + 1)$ and $k_g > k_p$ leading to good performances in terms of speed of convergence and damping.

The choice of the parameter α_{M} determines the number of broadcast messages. Taking the spring coefficients k_{ij} such that $\alpha_{\mathrm{M}} = \max_{i=1\dots N}\left(\sum_{j=1}^N k_{ij}\right) < 1$ leads to a reduction in the number of triggered communication since the CTC (3.20) is less frequently verified. Nevertheless choosing small values for k_{ij} impacts the speed of convergence since these coefficient appear in the control law. This influence can be counter balanced by choosing $k_g > \frac{1}{\min_{i=1\dots N}(k_{ij})}$.

The number of broadcast messages is also influenced by the parameter b_i. Choosing b_i close to $\frac{1}{k_g}$ reduces the influence of the term $k_g b_i \left\|\dot{q}_i - \dot{q}_i^*\right\|^2$ in the CTC, which is not reset when a message is broadcast and hence have a direct impact on the number of communications.

3.4 Example

Consider a set of $N = 6$ agents with coordinate vector $q_i \in \mathbb{R}^2$. The performance of the proposed algorithm will be evaluated considering the following two dynamical models, assumed identical for all the agents. For Model 1, one has

$$M_i^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} C_i^1(\dot{q}_i) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \|\dot{q}_i\|,$$

with $k_g = 15$, $k_p = 1$, $k_M = \left\|M_i^1\right\| = 1$, $k_C = \left\|\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right\| = 0.1$ and $k_s = 3$. The norm $\|M\|$ returns the largest singular value of the matrix M. For Model 2, one considers

$$M_i^2 = \begin{bmatrix} 0.56 & -2.23 \\ -2.23 & 9.28 \end{bmatrix} C_i^2(\dot{q}_i) = \begin{bmatrix} 1.40 & -1.76 \\ -1.76 & 2.99 \end{bmatrix} \|\dot{q}_i\|,$$

with $k_g = 15$, $k_p = 0.185$, $k_M = \|M_i^2\| = 9.81$, $k_C = \left\| \begin{bmatrix} 1.40 & -1.76 \\ -1.76 & 2.99 \end{bmatrix} \right\| = 6.33$, and $k_s = 3$. The initial vector state $x(0)$ is such that

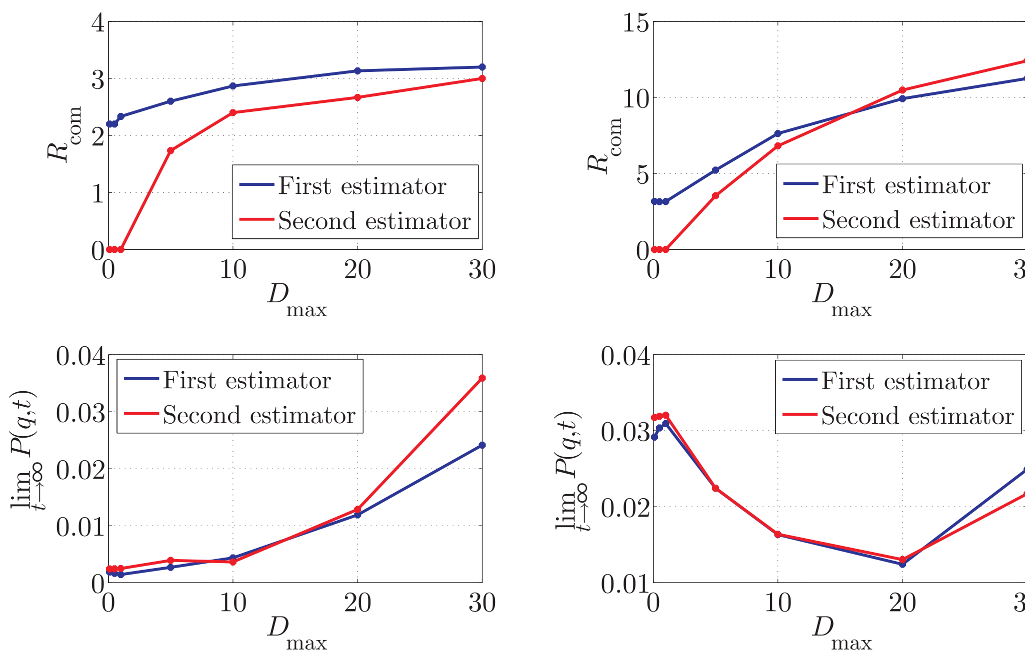
$$q(0) = \left[\begin{bmatrix} -3.98 \\ -3.01 \end{bmatrix}^T \begin{bmatrix} -0.19 \\ 0.83 \end{bmatrix}^T \begin{bmatrix} 8.76 \\ -0.50 \end{bmatrix}^T \begin{bmatrix} 3.51 \\ -4.16 \end{bmatrix}^T \begin{bmatrix} 1.75 \\ -2.42 \end{bmatrix}^T \begin{bmatrix} 2.01 \\ 0.24 \end{bmatrix}^T \right]^T$$

and $\dot{q}(0) = 0_{2N}$. The vector of relative configurations representing a hexagon

$$r^* = \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 3 \\ \sqrt{3} \end{bmatrix}^T \begin{bmatrix} 2 \\ 2\sqrt{3} \end{bmatrix}^T \begin{bmatrix} 0 \\ 2\sqrt{3} \end{bmatrix}^T \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}^T \right]^T.$$

A stress matrix has been computed using the approach in [82]. Its components are such that $k_{i(i+1)} = k_{i(i-1)} = 0.3$, $k_{ii} = 0$ and $k_{ij} = 0.1$ for all (i,j) such that $|i-j| > 1$. One obtains $\alpha_{\mathrm{M}} = 0.9$.

A full-connected communication graph is considered. The simulation duration is $T = 2.5\,s$ for Model 1 and $T = 6\ s$ for Model 2. Matlab's ode45 integrator is used with a step size $\Delta t = 0.01$ s. Since time has been discretized, the minimum period between the transmission of two messages by the same agent is set to Δt. The perturbation $d(t)$ is assumed constant over each interval of the form $[k\Delta t, (k+1)\Delta t[$, $k \in \mathbb{N}$. The components of $d(t)$ are chosen to be independent realizations of a zero-mean uniformly distributed noise $U\left(-\frac{D_{\max}}{\sqrt{2}}, \frac{D_{\max}}{\sqrt{2}}\right)$ and are thus such that $\|d\|^2 \leq D_{\max}$. Let N_{m} be the total number of messages broadcast during a simulation. Performance are evaluated by comparing N_m to the maximum number of messages that can be broadcast $\overline{N}_{\mathrm{m}} = NT/\Delta t \geq N_m$. The percentage of residual communications is defined as $R_{\mathrm{com}} = 100\frac{N_{\mathrm{m}}}{\overline{N}_{\mathrm{m}}}$ and expressed in %. R_{com} indicates the proportions of time slots during which a communication has been triggered.



(a) Results for Model 1. (b) Results for Model 2.

Figure 3.1: Evolution of R_{com} and $P(q,t)$ for different values of $D_{\max} \in \{0.1, 0.5, 1, 5, 10, 20, 30\}$."Estimator 1" reffers to the estimator with the basic input (3.14) and "Estimator 2" with accurate input (3.15).

Figure 3.2 shows the trajectories of the agents when the control (3.10) is applied along with the CTC defined in Theorem 7. It can be seen that agents converge to the desired formation with a limited number of communications. Figure 3.1 shows the evolution of the communication ratio R_{com} and of the

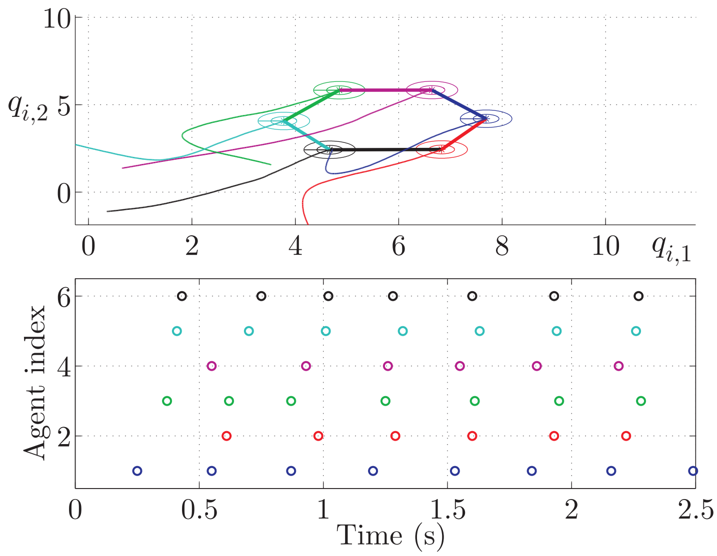


Figure 3.2: Hexagonal formation with the accurate estimator input (3.15). $D_{\max} = 20$. Top: Agent trajectory to an hexagonal formation. Agents are represented by circles. Bottom: communications time instant. Corresponding mapping between agent index and curve color: blue: 1, red: 2, green: 3, magenta: 4, light blue: 5, black: 6.

potential energy once the system has converged, for different values of $D_{\max}$. When $D_{\max}$ is small, the accurate estimator (3.15) provides better performance in terms of communication reduction than the basic estimator (3.14). As expected, the potential energy obtained once the system has converged increases for both estimators with the level of perturbations.

In the case of Model 2, the *basic estimator* (i.e. estimator with basic input (3.14)) makes the system converge to a smaller and smaller asymptotic potential energy when perturbations increase from $D_{\max} = 0$ to $D_{\max} = 20$. After this value, the asymptotic potential energy increases with the level of perturbations as for the *accurate estimator*. It can be explained by the high number of triggered communications inducing more frequent resets of the estimators, which allows obtaining more accurate estimates of the state of the agents, and thus a more accurate formation.

When $D_{\max}$ gets large, the performance of both estimators gets closer. In that case, the simplest estimator should be preferred.

3.5 Conclusion

This chapter presents an event-triggered communication strategy to reach a static target formation for MAS with perturbed Euler-Lagrange dynamics. Two estimators of different complexity and accuracy have been considered to provide the missing information required by the control, allowing a trade-off between computation time and amount of triggered communications. A distributed event-triggered condition have been proposed to reduce the number of communications while guaranteing a convergence to the target formation with a bounded error. Convergence to a desired formation and influence of state perturbations on the convergence and on the amount of required communications have been studied. Moreover, the time interval between consecutive communications has been shown to be strictly positive. Simulations have shown the effectiveness of the proposed method in presence of state perturbations when their level remains moderate. Chapter of this work has been presented in the 2017 IFAC World Congress paper [108].

In this chapter, the inertia matrix and the matrix of the Coriolis and centripetal term are considered to be known by the agents, which may not be always the case. Thus, next chapter will present an adaptive control to overcome problems due to parametric uncertainties on the inertia matrix and the matrix of the Coriolis and centripetal term. Moreover, the considered problem will be extended to time-varying formations and reference trajectory tracking.

Chapter 4

Distributed event-triggered for multi-agent formation stabilization and tracking control

This chapter proposes a strategy to reduce the number of communications for displacement-based formation control while following a desired reference trajectory. Agent dynamics are described by Euler-Lagrange models and include perturbations. Contrary to Chapter 3, the inertia matrix as well as the Coriolis/centripetal matrix are considered to be unknown by agents. Thus, estimates of these quantities are introduced to evaluate the control input of each agent. Moreover, to obtain efficient distributed control laws, each agent uses an estimator of the state of the other agents. As in Chapter 3, the proposed distributed CTC involves the inter-agent displacements and the relative discrepancy between the actual and estimated agent states. A single *a priori* trajectory has to be evaluated to follow the desired path. The effect of the state perturbations on the formation and on the communications are analyzed. Conditions for the Lyapunov stability of the MAS have been introduced and the time interval between consecutive communications has been shown to be strictly positive.

Some hypotheses are introduced in Section 4.1 and the formation parametrization is described in Section 4.2. Since the problem considered here is to drive a formation of agents along a desired reference trajectory, the designed distributed control law consists of two parts. The first part (already studied in Section 3.2.2) drives the agents to some target formation and maintains the formation, despite the presence of perturbations. In this chapter, this control is rewritten to become adaptive and robust to uncertainties in the inertia matrix and in the Coriolis/centripetal matrix. It is also based on estimates of the states of the agents described Section 4.2.4. The second part (see Section 4.2.3) is dedicated to the tracking of the desired trajectory. Communication instants are chosen locally by Agent i using an event-triggered approach introduced in Section 4.3.A simulation example is considered in Section 4.5 to illustrate the reduction of the communications obtained by the proposed approach. Finally, conclusions are drawn in Section 4.6.

4.1 Notations and hypotheses

Consider a MAS forming a network of N agents. For some vector $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T \in \mathbb{R}^n$, we define $|x| = \begin{bmatrix} |x_1| & |x_2| & \dots & |x_n| \end{bmatrix}^T$ where $|x_i|$ is the absolute value of the i-th component of x. Similarly, the notation $x \geq 0$ will be used to indicate that each component x_i of x is non negative, *i.e.*, $x_i \geq 0 \ \forall i \in \{1 \dots n\}$.

Let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i in some global fixed reference frame $\mathcal{R}$ and let $q = \begin{bmatrix} q_1^T & q_2^T & \dots & q_N^T \end{bmatrix}^T \in \mathbb{R}^{N.n}$ be the *configuration* of the MAS. The dynamics of each agent is described by the Euler-Lagrange model

$$M_i(q_i)\,\ddot{q}_i + C_i(q_i,\,\dot{q}_i)\,\dot{q}_i + G = \tau_i + d_i, \tag{4.1}$$

where $\tau_i \in \mathbb{R}^n$ is some control input described in Section 4.2.3, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix of Agent i, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis/centripetal matrix of Agent i, G accounts for the acceleration due to gravity supposed to be known and constant, and d_i is a time-varying state perturbation satisfying $\|d_i(t)\| < D_{\max}$. The state vector of Agent i is $x_i^T = \left[q_i^T, \dot{q}_i^T\right]$. Assume that the dynamics satisfy the following assumptions:

A1) $M_i(q_i)$ is symmetric positive and there exists $k_M > 0$ satisfying $\forall x$, $x^T M_i(q_i) x \leq k_M x^T x$.

A2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric or negative definite and there exists $k_C > 0$ satisfying $\forall x$, $x^T C_i(q_i, \dot{q}_i) x \leq k_C \|\dot{q}_i\| x^T x$.

A3) There exists $\dot{q}_{\max} \in \mathbb{R}_+^n$ and $\ddot{q}_{\max} \in \mathbb{R}_+^n$ such that $|\ddot{q}_i| \leq \ddot{q}_{\max}$ and $|\dot{q}_i| \leq \dot{q}_{\max}$.

A4) The left-hand side of (4.1) can be linearly parametrized as

$$M_i(q_i) x_1 + C_i(q_i, \dot{q}_i) x_2 = Y_i(q_i, \dot{q}_i, x_1, x_2) \theta_i \tag{4.2}$$

for all vectors $x_1, x_2 \in \mathbb{R}^n$, where $Y_i(q_i, \dot{q}_i, x_1, x_2)$ is a regressor matrix with known structure and θ_i is a vector of unknown but constant parameters associated with the i-th agent.

A5) For each $i = 1, \ldots, N$, θ_i is such that $\theta_{\min,i} < \theta_i < \theta_{\max,i}$, with known $\theta_{\min,i}$ and $\theta_{\max,i}$.

Assumptions A1, A2, and A4 have been previously considered, *e.g.*, in [65, 62].

Moreover, one assumes that

A6) each Agent i is able to measure without error its own state x_i,

A7) there is no packet losses or communication delay between agents.

In what follows, the notations M_i and C_i are used to replace $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

4.2 Control problem

This section aims at designing a decentralized control strategy to drive a MAS to a desired target formation in some global reference frame $\mathcal{R}$, while reducing as much as possible the communications between agents. The target formation is described in Section 4.2.1. As in Chapter 3, the potential energy $P(q, t)$ of the MAS is introduced to quantify the discrepancy between the current and target formations. Moreover, the problem of tracking a desired trajectory is formulated in Section 4.2.2. The proposed adaptive distributed control, introduced in Section 4.2.3, tries to minimize the potential energy and distance between the reference trajectory and agents. Estimators of the coordinate vectors of all agents and an estimate of the matrices M_i and C_i, are presented in Section 4.2.4. These estimators are used when evaluating the control input.

4.2.1 Formation parametrization

The parametrization described in Section 3.2.1is briefly recalled. The relative coordinate vector between two agents i and j is $r_{ij}(t) = q_i(t) - q_j(t)$ and the target relative coordinate vector is denoted r_{ij}^* for all $(i, j) \in \mathcal{N}$. A target formation is defined by the set $\left\{r_{ij}^*(t), (i, j) \in \mathcal{N}\right\}$ and without loss of generality, the first agent as a reference. The control law in Section 4.2.3 is designed to reduce the potential energy $P(q, t)$ of the formation. This potential energy involves the difference between r_{ij} and r_{ij}^* as follows

$$P(q, t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_{ij}(t) - r_{ij}^*(t) \right\|^2 \tag{4.3}$$

where the terms $k_{ij} = k_{ji}$ are some spring coefficients, which can be positive or null.

4.2.2 Time-varying formation and tracking trajectory

In this section, the MAS has to follow some reference trajectory $q_1^*(t)$, while remaining in a desired formation. Agent 1, taken as the reference agent, aims at following $q_1^*(t)$. It is assumed that all agents have access to $q_1^*(t)$. Moreover, assume that the target formation can be time-varying and is represented by the relative configuration vector $r^*(t)$. Therefore the reference trajectory of each agent can be expressed as $q_i^*(t) = q_1^*(t) + r_{i1}^*(t)$.

To guarantee that individual reference trajectories can be tracked by each agent, it is assumed that for $i = 1, \ldots, N$,

$$|\dot{q}_i^*| < \dot{q}_{\max} \tag{4.4}$$
$$|\ddot{q}_i^*| < \ddot{q}_{\max}. \tag{4.5}$$

Definition 10. The MAS reaches its tracking objective iff there exists $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that (3.3) is satisfied and

$$\lim_{t\to\infty} \|q_1(t) - q_1^*(t)\| \leqslant \varepsilon_2, \tag{4.6}$$

i.e., iff the reference agent asymptotically converges to the reference trajectory, and the MAS asymptotically converges to the target formation with bounded errors.

A distributed control law is designed to satisfy this target. Introduce the error terms

$$\begin{aligned} r_i &= q_i - q_i^* \\ \hat{r}_i^j &= \hat{q}_i^j - q_i^*. \end{aligned}$$

The terms g_i, $\bar{g}_i$, $\hat{g}_i^j$, $\bar{s}_i$ and $\hat{s}_i^j$, introduced in Chapter 3, are now redefined as follows to address the trajectory tracking problem

$$g_i = \sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right) + k_0 r_i \tag{4.7}$$
$$\bar{g}_i = \sum_{j=1}^{N} k_{ij}\left(\bar{r}_{ij} - r_{ij}^*\right) + k_0 r_i \tag{4.8}$$
$$\hat{g}_i^j = \sum_{j=1}^{N} k_{ij}\left(\hat{r}_{ij}^j - r_{ij}^*\right) + k_0 \hat{r}_i^j \tag{4.9}$$
$$s_i = \dot{q}_i - \dot{q}_i^* + k_p g_i \tag{4.10}$$
$$\bar{s}_i = \dot{q}_i - \dot{q}_i^* + k_p \bar{g}_i \tag{4.11}$$
$$\hat{s}_i^j = \dot{\hat{q}}_i^j - \dot{q}_i^* + k_p \hat{g}_i^j \tag{4.12}$$

with $\bar{r}_{ij} = q_i - \hat{q}_j^i$ and $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$ and where $k_0 \geq 0$ is a positive design parameter which may be used to control the tracking error with respect to the reference trajectory. When no reference trajectory is considered, $k_0 = 0$.

4.2.3 Distributed control with tracking term

The control law proposed in [82] is defined as $\tau_i = \tau_i(q_i, \dot{q}_i, q)$ and aims at reducing $P(q, t)$, thus making the MAS converge to the target formation in case of permanent communication. In this approach, each agent evaluates its control input using the state vectors of its neighbors obtained via permanent communication. In Chapter 3, a distributed control with limited communications between agents has been studied, but the inertia matrix and the Coriolis/centripetal matrix are consider to be known by all agents, which is difficult in practice. Here, agents cannot have permanent access to q and have no access to the vector θ_i. Thus, one introduces the estimate $\hat{q}_j^i$ of q_j performed by Agent i to replace the missing information in the control law, and the estimate $\bar{\theta}_i$ of θ_i is used with the regressor matrix Y_i to replace the unknown matrices M_i and C_i. The MAS configuration estimated by Agent i is denoted as $\hat{q}^i = \left[\begin{array}{ccc} \hat{q}_1^{iT} & \ldots & \hat{q}_N^{iT} \end{array}\right]^T \in \mathbb{R}^{N.n}$. The evaluation of $\hat{q}_j^i$ is described in Section 4.2.4.

In a distributed context with limited communications, using $\bar{g}_i$ and $\bar{s}_i$, Agent i is able to evaluate the following adaptive distributed control input to be used in (4.1)

$$\begin{aligned} \tau_i\left(q_i,\,\dot{q}_i,\,\hat{q}^i,\,\dot{\hat{q}}^i\right) &= -k_s\bar{s}_i - k_g\bar{g}_i + G - Y_i\left(q_i,\,\dot{q}_i,\,\dot{\bar{p}}_i,\,\bar{p}_i\right)\bar{\theta}_i, && (4.13)\\ \dot{\bar{\theta}}_i &= \Gamma_i Y_i\left(q_i,\,\dot{q}_i,\,\dot{\bar{p}}_i,\,\bar{p}_i\right)^T\bar{s}_i && (4.14)\end{aligned}$$

where $\bar{p}_i = k_p\bar{g}_i - \dot{q}_i^*$ and $\dot{\bar{p}}_i = k_p\dot{\bar{g}}_i - \ddot{q}_i^*$ with the design parameters $k_g > 0$, $k_s \geq 1 + k_p\left(k_M + 1\right)$ and Γ_i an arbitrary symmetric positive definite matrix.

Section 4.2.4 introduces the estimator $\hat{q}_j^i$ of q_j needed in the control (4.13).

4.2.4 Communication protocol and estimator dynamics

In what follows, the time instant at which the k-th message is sent by Agent j is denoted $t_{j,k}$. Let $t_{j,k}^i$ be the time at which the k-th message sent by Agent j is received by Agent i. In this chapter, one assumes again that there is no communication delay between agents. Therefore, $t_{j,k}^i = t_{j,k}$ for all $i \in \mathcal{N}_j$. When a communication is triggered at $t_{i,k}$ by Agent i, it broadcasts a message containing $t_{i,k}$, $q_i\left(t_{i,k}\right)$, $\dot{q}_i\left(t_{i,k}\right)$ and its estimated matrix $\bar{\theta}_i\left(t_{i,k}\right)$. Once a message is received by neighbors of Agent i, its content is used to update their estimate of the state of Agent i as presented in the next section.

Estimator dynamics

Following the idea of [107, 106], the estimate $\hat{q}_j^i$ of q_j made by Agent i is evaluated considering

$$\begin{aligned} \hat{M}_j^i\left(\hat{q}_j^i\right)\ddot{\hat{q}}_j^i + \hat{C}_j^i\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i\right)\dot{\hat{q}}_j^i + G &= \hat{\tau}_j^i,\ \forall t \in \left[t_{j,k}^i,\,t_{j,k+1}^i\right[&& (4.15)\\ \hat{q}_j^i\left(t_{j,k}^i\right) &= q_j\left(t_{j,k}^i\right) && (4.16)\\ \dot{\hat{q}}_j^i\left(t_{j,k}^i\right) &= \dot{q}_j\left(t_{j,k}^i\right), && (4.17)\end{aligned}$$

where $\hat{M}_j^i\left(\hat{q}_j^i\right)$ and $\hat{C}_j^i\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i\right)$ are estimates of M_j and C_j computed from $Y_j\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i,\,x,\,y\right)$ and $\bar{\theta}_j\left(t_{j,k}^i\right)$ using

$$\hat{M}_j^i\left(\hat{q}_j^i\right)x + \hat{C}_j^i\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i\right)y = Y_j\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i,\,x,\,y\right)\bar{\theta}_j\left(t_{j,k}^i\right).$$

The estimator (E.34) managed by Agent i requires an estimate $\hat{\tau}_j^i$ of τ_j evaluated by Agent j. This estimate, used by Agent i, is evaluated as

$$\begin{aligned} \hat{\tau}_j^i &= -k_s\hat{s}_j^i - k_g\hat{g}_j^i + G - Y_j\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i,\,\dot{\hat{p}}_j^i,\,\hat{p}_j^i\right)\hat{\theta}_j^i && (4.18)\\ \dot{\hat{\theta}}_j^i &= \Gamma_j Y_j\left(\hat{q}_j^i,\,\dot{\hat{q}}_j^i,\,\dot{\hat{p}}_j^i,\,\hat{p}_j^i\right)^T\hat{s}_j^i && (4.19)\\ \hat{\theta}_j^i\left(t_{j,k}^i\right) &= \bar{\theta}_j\left(t_{j,k}^i\right) && (4.20)\end{aligned}$$

where $\hat{p}_j^i = k_p\hat{g}_j^i - \dot{q}_j^*$, $\dot{\hat{p}}_j^i = k_p\dot{\hat{g}}_j^i - \ddot{q}_j^*$, $\hat{s}_j^i = \dot{\hat{q}}_j^i - \dot{q}_j^* + k_p\hat{g}_j^i$, $\hat{g}_j^i = \sum_{k=1}^N k_{jk}\left(\hat{r}_{jk}^i - r_{jk}^*\right)$, $\dot{\hat{g}}_j^i = \sum_{k=1}^N k_{jk}\left(\dot{\hat{r}}_{jk}^i - \dot{r}_{jk}^*\right)$, $\hat{r}_{jk}^i = \hat{q}_j^i - \hat{q}_k^i$, and $\hat{\theta}_j^i$ is the estimate of $\bar{\theta}_j$.

Errors appear between q_i and its estimate $\hat{q}_i^j$ obtained by an other Agent j due to the presence of state perturbations, the non-permanent communication, and the mismatch between θ_i, $\bar{\theta}_i$, and $\hat{\theta}_i$. The errors for the estimates performed by Agent j are expressed as

$$\begin{aligned} e_i^j &= \hat{q}_i^j - q_i,\ j \in \mathcal{N} && (4.21)\\ e^j &= \hat{q}^j - q. && (4.22)\end{aligned}$$

These errors are used in Section 4.3 to trigger communications when e_i^i and $\dot{e}_i^i$ become too large. Figure 4.1 summarizes the overall structure of the estimator and of the controller.

Remark 3. The structure of the estimator for $\hat{\tau}_j^i$ is chosen so as to get an accurate estimate for q in order to keep the e_i^is and $\dot{e}_i^i$s small. In absence of perturbations, *i.e.,* when $D_{\max} = 0$ and if θ_i is perfectly known, *i.e.,* $\bar{\theta}_i = \hat{\theta}_i^i = \theta_i$, the estimation error e_i^i introduced in (E.39) vanishes. The price to be paid for the use of this estimator structure for $\hat{\tau}_j^i$ is that every agent needs to maintain an estimator of the state of all other agents.

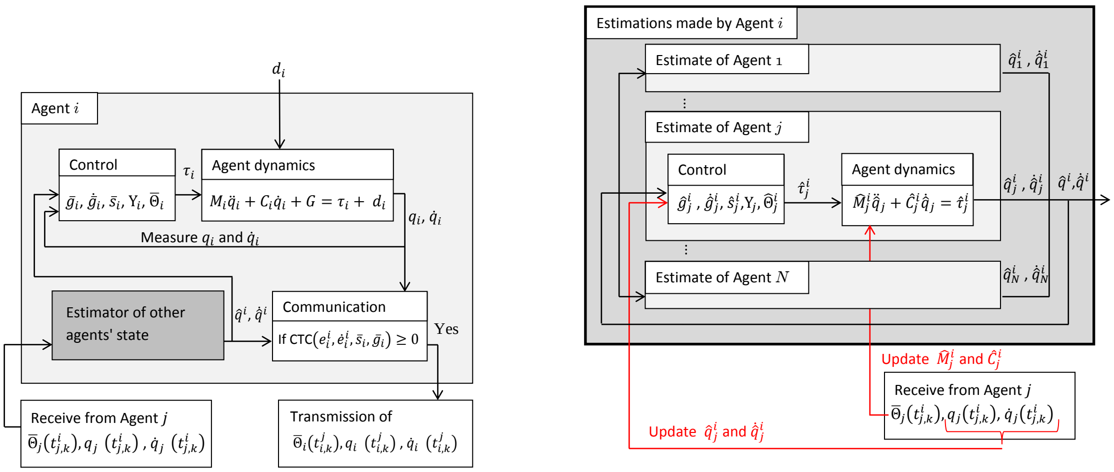


Figure 4.1: Formation control system architecture

Communication protocol

When a communication is triggered at $t_{i,k}$ by Agent i, it broadcasts a message containing $t_{i,k}$, $q_i\left(t_{i,k}\right)$, $\dot{q}_i\left(t_{i,k}\right)$ and its estimated $\bar{\theta}_i\left(t_{i,k}\right)$. We assume that this message is received by all other agents, either directly when the network is fully connected, or after several hops when the network is only connected. The latter case requires the use of a flooding protocol [44, 83]. Since communications have been assumed without delay, one has $\hat{q}_i^i\left(t\right) = \hat{q}_i^j\left(t\right)$ for all $\left(i,\, j\right) \in \mathcal{N}^2$. This simplifies the stability study in Appendix C.1.

4.3 Event-triggered communications

Theorem 8 introduces a CTC used to trigger communications to ensure a bounded asymptotic convergence of the MAS to the target formation. The initial value of the state vectors are considered to be known by all agents. In practice, this condition can be satisfied by triggering a communication from all agents at time $t = 0$ to initialize the estimates of the state of the neighbors of all agents.

Let $k_{\max} = \max_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} \left(k_{\ell j}\right)$ and $k_{\min} = \min_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} \left(k_{\ell j} \neq 0\right)$, $\alpha_i = \sum_{j=1}^{N} k_{ij}$, $\alpha_{\min} = \min_{i=1,\dots,N} \alpha_i$ and $\alpha_{\mathrm{M}} = \max_{i=1,\dots,N} \alpha_i$. Define also for $\bar{\theta}_i \in \mathbb{R}^p$ and $\bar{\theta}_i = \left[\bar{\theta}_{i,1},\, \dots,\, \bar{\theta}_{i,p}\right]^T$

$$\Delta\theta_{i,\max} = \begin{bmatrix} \max\left\{\left|\bar{\theta}_{i,1} - \theta_{\min,i,1}\right|, \left|\bar{\theta}_{i,1} - \theta_{\max,i,1}\right|\right\} \\ \vdots \\ \max\left\{\left|\bar{\theta}_{i,p} - \theta_{\min,i,p}\right|, \left|\bar{\theta}_{i,p} - \theta_{\max,i,p}\right|\right\} \end{bmatrix} \tag{4.23}$$

and $\Delta\theta_i = \bar{\theta}_i - \theta_i$.

Theorem 8. *Consider a MAS with agent dynamics given by (4.1) and the control law (4.13). Consider some design parameters* $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$,

$$c_3 = \frac{\min\left\{1, k_1, k_p, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)\right\}}{\max\left\{1, k_M\right\}}$$

and $k_1 = k_s - \left(1 + k_p\left(k_M + 1\right)\right)$. *The system (4.1) is input-to-state practically stable (ISpS) and the agents can be driven to some target formation such that*

$$\lim_{t\to\infty} \sum_{i=1}^{N} k_0 \left\|r_i\right\|^2 + \frac{1}{2} P\left(q, t\right) \leq \xi \tag{4.24}$$

with

$$\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \tag{4.25}$$

where $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T\Gamma_i^{-1}\Delta\theta_i\right)\right)$, *if the communications are triggered when one of the following conditions is satisfied*

$$\begin{aligned} k_s\bar{s}_i^T\bar{s}_i + k_pk_g\bar{g}_i^T\bar{g}_i + \eta \le{}& \alpha_M^2\left(k_e e_i^{iT}e_i^i + k_pk_M\dot{e}_i^{iT}\dot{e}_i^i\right) \\ &+ \alpha_M k_C^2 k_p \left\|e_i^i\right\|^2 \sum_{j=1}^N k_{ji}\left[\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right]^2 + k_g b_i \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 \\ &+ k_p\left\|e_i^i\right\|\left[\alpha_M^2\left(1 + \left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right) + \frac{\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2}{\left(1+\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)}\right] \end{aligned} \tag{4.26}$$

$$\left\|\dot{q}_i\right\| \quad \ge \quad \left\|\dot{\hat{q}}_i^i\right\| + \eta_2 \tag{4.27}$$

with $k_e = k_sk_p^2 + k_gk_p + \frac{k_g}{b_i}$, *and* $Y_i = Y_i\left(q_i,\, \dot{q}_i,\, \dot{\bar{p}}_i,\, \bar{p}_i\right)$.

The proof of Theorem 8 is given in Appendix C.1.

Corollary 3. Consider a MAS with agent dynamics given by (4.1) and the control law (4.13). For any Agent i, let $t_{i,k}$ and $t_{i,k+1}$ be two consecutive communication instants at which the CTC of Theorem 8 have been satisfied. Then $t_{i,k+1} - t_{i,k} > 0$.

The proof of Corollary 3 is provided in Appendix C.2

The CTCs proposed in Theorem 8 are analyzed assuming that the estimators of the state of the agents and the communication protocol is such that $\forall\,(i,j) \in \mathcal{N}\times\mathcal{N}$,

$$\hat{x}_i^i(t) = \hat{x}_i^j(t) \tag{4.28}$$

$$\hat{x}_i^i(t_{i,k}) = x_i^i(t_{i,k}), \tag{4.29}$$

where (4.28) is called the *estimate synchronization condition* and (4.29) the *estimator reset condition.* Theorem 8 is valid independently of the way the estimate $\hat{x}_i^i$ of x_i is evaluated provided that (4.28) and (4.29) are satisfied.

From (E.40) and (4.26), one sees that η can be used to adjust the trade-off between the bound ξ on the formation and tracking errors and the amount of triggered communications. If $\eta = 0$, there is no perturbation and θ_i is perfectly known as in Chapter 3, the system converges asymptotically

The CTC (4.27) is related to the discrepancy between $\dot{q}_i$ and $\dot{\hat{q}}_i^i$. Choosing a small value of η_2 may lead to frequent communications. On the contrary, when η_2 is large, (4.26) is more likely to be satisfied. A value of η_2 that corresponds to a trade-off between the two CTCs (4.26) and (4.27) has thus to be found to minimize the amount of communications.

The CTCs (4.26) and (4.27) mainly depend on e_i^i and $\dot{e}_i^i$. A communication is triggered by Agent i when the state estimate $\hat{x}_i^i$ of its own state vector x_i is not satisfying, *i.e.*, when e_i^i and $\dot{e}_i^i$ becomes large. To reduce the number of triggered communications, one has to keep e_i^i and $\dot{e}_i^i$ as small as possible. This may be achieved by increasing the accuracy of the estimator, as proposed in Section 4.2.4, but possibly at the price of a more complex structure for the estimator.

The perturbations have a direct impact on e_i^i and $\dot{e}_i^i$, and, as a consequence, on the frequency of communications. (4.25) shows the impact of $D_{\max}$ and η on the formation and tracking errors: in presence of perturbations, the formation and tracking errors cannot reach a value below a minimum value due to the perturbations. At the cost of a larger formation and tracking errors, η can reduce the number of triggered communications and so can reduce the influence of perturbations on the CTC (4.26).

The discrepancy between the actual values of M_i and C_i and of their estimates $\hat{M}_i^i$ and $\hat{C}_i^i$ determines the accuracy of $\bar{\theta}_i$, so $\Delta\theta_{i,\max}$, and the estimation errors. Even in absence of state perturbations, due to the linear parametrization, it is likely that $\hat{M}_i^i \neq M_i$, $\hat{C}_i^i \neq C_i$ and $\Delta\theta_{i,\max} > 0$, which leads to the satisfaction of the CTCs at some time instants. Thus, the CTC (4.26) leads to more communications when the model of the agent dynamics is not accurate, requiring thus more frequent updates of the estimate of the states of agents.

Guidelines to choose parameters k_p, k_g, k_s, b_i and α_{M} are described in Section 3.3

4.4 Inter-agent collision avoidance

In this section, an inter-agent collision avoidance mechanism is proposed, inspired from [114, 90].

Let r_a be the avoidance distance and r_c be the collision distance, with $r_a > r_c$. If $\|q_i - q_j\| < r_a$ for some $i \neq j$, Agents i and j have to start a collision avoidance procedure, to avoid collision when $\|q_i - q_j\| < r_c$. For that purpose, one introduces a collision avoidance term in (4.13), expressed as

$$v_i\left(\bar{r}_{i1}, \ldots, \bar{r}_{iN}\right) = k_{\mathrm{a}} \sum_{j \neq i} \operatorname{sgn}\left(\bar{r}_{ij}\right) \left(\frac{r_a - r_c}{\left| \|\bar{r}_{ij}\| - r_c \right|} \right)^2 z\left(\bar{r}_{ij}\right) \tag{4.30}$$

$$z\left(\bar{r}_{ij}\right) = \frac{\left(\|\bar{r}_{ij}\| - r_a - \left| \|\bar{r}_{ij}\| - r_a \right| \right)}{2 \left| \|\bar{r}_{ij}\| - r_a \right|} \tag{4.31}$$

where $\bar{r}_{ij} = q_i - \hat{q}_j^i$ and $k_{\mathrm{a}} > 0$ is some design parameter. Note that r_a must be such as $\forall (i,\, j) \;\; r_a < \left\| r_{ij}^* \right\|$. The agent control input (4.13) and the estimator control inputs (4.18) accounting for the collision avoidance then become

$$\tau_i = \tau_i + v_i\left(\bar{r}_{i1}, \ldots, \bar{r}_{iN}\right) \tag{4.32}$$

$$\hat{\tau}_i^j = \hat{\tau}_i^j + v_i\left(\hat{r}_{i1}^i, \ldots, \hat{r}_{iN}^i\right). \tag{4.33}$$

The terms v_i are functions of $\bar{r}_{ij}$, thus of the state estimate $\hat{q}_j^i$. The efficiency of v_i depends on e_i^i. If e_i^i is too large, it may be difficult to anticipate a collision. Therefore, an additional CTC is introduced in Theorem 8 to ensure that agents are able to start a collision avoidance mechanism before a collision.

Theorem 9. *Consider a MAS with agent dynamics given by (4.1) and the control law (4.32). Consider some design parameters $\eta \geq 0$, $\xi \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$, and $\sigma \in \left]0, 1\right]$. The agents can be driven to some target formation such that*

$$\lim_{t \to \infty} P\left(q, t\right) + \sum_{i=1}^{N} k_0^2 \left\| r_i \right\|^2 \leq \xi \tag{4.34}$$

if a communication is triggered when either(4.27), (4.26) or the following inequality

$$\frac{\sigma}{2} \left(\left\| \hat{r}_{ij}^i \right\| - r_c \right) < \left\| e_i^i \right\| \text{ for some } i \neq j \tag{4.35}$$

are satisfied. Agents will then be able to start a collision avoidance mechanism before collision occurs.

See Appendix C.3 for a proof of the second part of Theorem 9.

4.5 Example

The performance of the proposed algorithm is evaluated considering a set of $N = 6$ agents. Two models will be considered to describe the dynamics of the agents.

4.5.1 Models of the agent dynamics and estimator

Double integrator (DI)

The first model consists of the dynamical system

$$M_i\left(q_i\right) \ddot{q}_i + C_i\left(q_i,\, \dot{q}_i\right) \dot{q}_i \quad = \quad \tau_i + d_i$$

with $q_i \in \mathbb{R}^2$ and where

$$M_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} C_i\left(\dot{q}_i\right) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \left\| \dot{q}_i \right\|. \tag{4.36}$$

Then the vectors $\bar{\theta}_i(0) = \hat{\theta}_i^j(0)$, $i = 1, \dots, N$ are obtained using (4.2). In place of the estimator in Section 4.2.4 a first less accurate estimate of x_j made by Agent i, is evaluated as

$$\hat{q}_j^i(t) = q_j\left(t_{j,k}^i\right) \tag{4.37}$$

$$\dot{\hat{q}}_j^i(t) = \dot{q}_j\left(t_{j,k}^i\right). \tag{4.38}$$

This estimator allows one to better observe the tradeoff between the potential energy of the formation and the communication requirements.

Finally, choose $k_M = \|M_i\| = 1$, $k_C = \|C_i\| = 0.1$, $k_p = 1$, $k_g = 15$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$, and $k_0 = 2$.

Surface ship (SS)

The second model considers surface ships with coordinate vectors $q_i = \begin{bmatrix} x_i & y_i & \psi_i \end{bmatrix}^T \in \mathbb{R}^3$, $i = 1 \dots N$, in a local earth-fixed frame. For Agent i, (x_i, y_i) represents its position and ψ_i its heading angle. The dynamics of the agents is described by the surface ship dynamical model taken from [55], assumed identical for all agents, and expressed in the body frame as

$$M_{\mathrm{b},i}\dot{\mathrm{v}}_i + C_{\mathrm{b},i}(\mathrm{v}_i)\mathrm{v}_i + D_{\mathrm{b},i}\mathrm{v}_i = \tau_{\mathrm{b},i} + d_{\mathrm{b},i}, \tag{4.39}$$

where $\mathrm{v}_i = \begin{bmatrix} u_i & v_i & r_i \end{bmatrix}^T$ is the velocity vector in the body frame, $\tau_{\mathrm{b},i}$ is the control input, $d_{\mathrm{b},i}$ is the perturbation, and

$$\begin{aligned}
M_{\mathrm{b},i} &= \begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix} \\
C_{\mathrm{b},i}(\mathrm{v}_i) &= \begin{bmatrix} 0 & 0 & -33.8v_i - 1.0115r_i \\ 0 & 0 & 25.8u_i \\ 33.8v_i + 1.0115r_i & -25.8u_i & 0 \end{bmatrix} \\
D_{\mathrm{b},i} &= \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.86 & -0.11 \\ 0 & -0.11 & -0.5 \end{bmatrix}.
\end{aligned}$$

At $t = 0$, one assumes that Agent i has access to estimates $\hat{M}_{\mathrm{b},i}^i$ of $M_{\mathrm{b},i}$, $\hat{C}_{\mathrm{b},i}^i$ of $C_{\mathrm{b},i}$, and $\hat{D}_{\mathrm{b},i}^i$ of $D_{\mathrm{b},i}$ described as

$$\begin{aligned}
\hat{M}_{\mathrm{b},i}^i &= \left(1_{3\times 3} + 0.1\Xi_i^{\mathrm{M}}\right) \odot M_{\mathrm{b},i} \\
\hat{C}_{\mathrm{b},i}^i &= \left(1_{3\times 3} + 0.1\Xi_i^{\mathrm{C}}\right) \odot C_{\mathrm{b},i} \\
\hat{D}_{\mathrm{b},i}^i &= \left(1_{3\times 3} + 0.1\Xi_i^{\mathrm{D}}\right) \odot D_{\mathrm{b},i},
\end{aligned}$$

where $1_{3\times 3}$ is the 3×3 matrix of ones, Ξ_i^{M}, Ξ_i^{C}, and Ξ_i^{D} are matrices which components are independent and identically Bernoulli random variables with values in $\{-1, 1\}$, and $\odot$ is the Hadamard product. These estimates are transmitted at $t = 0$ to all other agents. As a consequence, the estimates of $M_{\mathrm{b},i}$ and $C_{\mathrm{b},i}$ made by all agents at $t = 0$ are all identical.

The model (4.39) is expressed with the coordinate vectors q_i in the local earth-fixed frame using the transform

$$\begin{aligned}
\dot{q}_i &= J_i(\psi_i)\mathrm{v_i} \\
J_i(\psi_i) &= \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

where $J_i(\psi_i)$ is a simple rotation around the z-axis in the earth-fixed coordinate. Define $J_i^{-T} = \left(J_i^{-1}\right)^T$. Then, (4.39) can be rewritten as

$$J_i^{-T}M_{\mathrm{b},i}J_i^{-1}\ddot{q}_i + J_i^{-T}\left[C_{\mathrm{b,i}}(\mathrm{v}) - M_{\mathrm{b,i}}J_i^{-1}\dot{J}_i + D_{\mathrm{b},i}\right]J_i^{-1}\dot{q}_i = J_i^{-T}\tau_{\mathrm{b}} + J_i^{-T}d_{\mathrm{b},i}$$

and so

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = \tau_i + d_i$$

where

$$M_i(q_i) = J^{-T} M_{\mathrm{b}} J^{-1},$$
$$C_i(q_i, \dot{q}_i) = J_i^{-T}\left[C_{\mathrm{b},i}\left(J_i^{-1}\dot{q}_i\right) - M_{\mathrm{b},i}J_i^{-1}\dot{J}_i + D_{\mathrm{b},i}\right]J^{-1},$$

and τ_i is the control input in earth-fixed coordinates (4.13).

Then the vectors $\bar{\theta}_i(0) = \hat{\theta}_i^j(0)$, $i = 1, \ldots, N$ are obtained using (4.2). The estimator described in Section 4.2.4 is employed.

Finally, choose $k_M = \|M_i\| = 33.8$, $k_C = \|C_v(1_N)\| = 43.96$, $k_p = 6$, $k_g = 20$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$, and $k_0 = 1.5$.

Simulation parameters

One chooses the components of the initial value $x(0)$ of the state vector as

$$q(0) = \left[\begin{bmatrix} -0.35 \\ -1.11 \\ 0 \end{bmatrix}^T \begin{bmatrix} 4.59 \\ -4.59 \\ 0 \end{bmatrix}^T \begin{bmatrix} 4.72 \\ 2.42 \\ 0 \end{bmatrix}^T \ldots \right.$$
$$\left. \ldots \begin{bmatrix} 0.64 \\ 1.36 \\ 0 \end{bmatrix}^T \begin{bmatrix} 3.53 \\ 1.56 \\ 0 \end{bmatrix}^T \begin{bmatrix} -1.26 \\ 3.36 \\ 0 \end{bmatrix}^T \right]^T,$$

and $\dot{q}(0) = 0_{Nn\times 1}$. The vector of relative target configurations corresponds to a hexagon formation

$$r^* = \left[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} 3 \\ \sqrt{3} \\ 0 \end{bmatrix}^T \ldots \right.$$
$$\left. \ldots \begin{bmatrix} 2 \\ 2\sqrt{3} \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 2\sqrt{3} \\ 0 \end{bmatrix}^T \begin{bmatrix} -1 \\ \sqrt{3} \\ 0 \end{bmatrix}^T \right]^T.$$

Using the approach developed in [82], the following stress matrix can be computed from r^*

$$K = 0.1\begin{bmatrix} 0 & 1.85 & 0 & 0.926 & 0 & 1.85 \\ 1.85 & 0 & 1.85 & 0 & 0.926 & 0 \\ 0 & 1.85 & 0 & 1.85 & 0 & 0.926 \\ 0.926 & 0 & 1.85 & 0 & 1.85 & 0 \\ 0 & 0.926 & 0 & 1.85 & 0 & 1.85 \\ 1.85 & 0 & 0.926 & 0 & 1.85 & 0 \end{bmatrix}$$

and $\alpha_i = \sum_{j=1}^{N} k_{ij} = 0.463$, for all $i = 1, \ldots, N$ and $\alpha_{\mathrm{M}} = 0.463$.

A fully-connected communication graph is considered. The simulation duration is $T = 2\ s$. Matlab's ode45 integrator is used with a step size $\Delta t = 0.01$ s. Since time has been discretized, the minimum delay between the transmission of two messages by the same agent is set to Δt. The perturbation $d_i(t)$ is assumed of constant value over each interval of the form $[k\Delta t, (k+1)\Delta t[$. The components of $d_i(t)$ are independent realizations of zero-mean uniformly distributed noise $U\left(-\frac{D_{\max}}{\sqrt{3}}, \frac{D_{\max}}{\sqrt{3}}\right)$ and are thus such that $\|d_i(t)\| \leq D_{\max}$. Let N_{m} be the total number of messages broadcast during a simulation. The performance of the proposed approach is evaluated comparing N_{m} to the maximum number of messages that can be broadcast $\overline{N}_{\mathrm{m}} = NT/\Delta t \geq N_{\mathrm{m}}$. The percentage of residual communications is defined as $R_{\mathrm{com}} = 100\frac{N_{\mathrm{m}}}{\overline{N}_{\mathrm{m}}}$. R_{com} indicates the percentage of time slots during which a communication has been triggered.

When a tracking has to be performed, one considers the target trajectory of the first agent

$$\dot{q}_1^* (t) = \begin{bmatrix} 4\sin(0.4t) \\ 4\cos(0.4t) \\ 0.4t \end{bmatrix},$$

the other agents having to remain in formation. Define the tracking error $\varepsilon_0 = q_1 - q_1^*$.

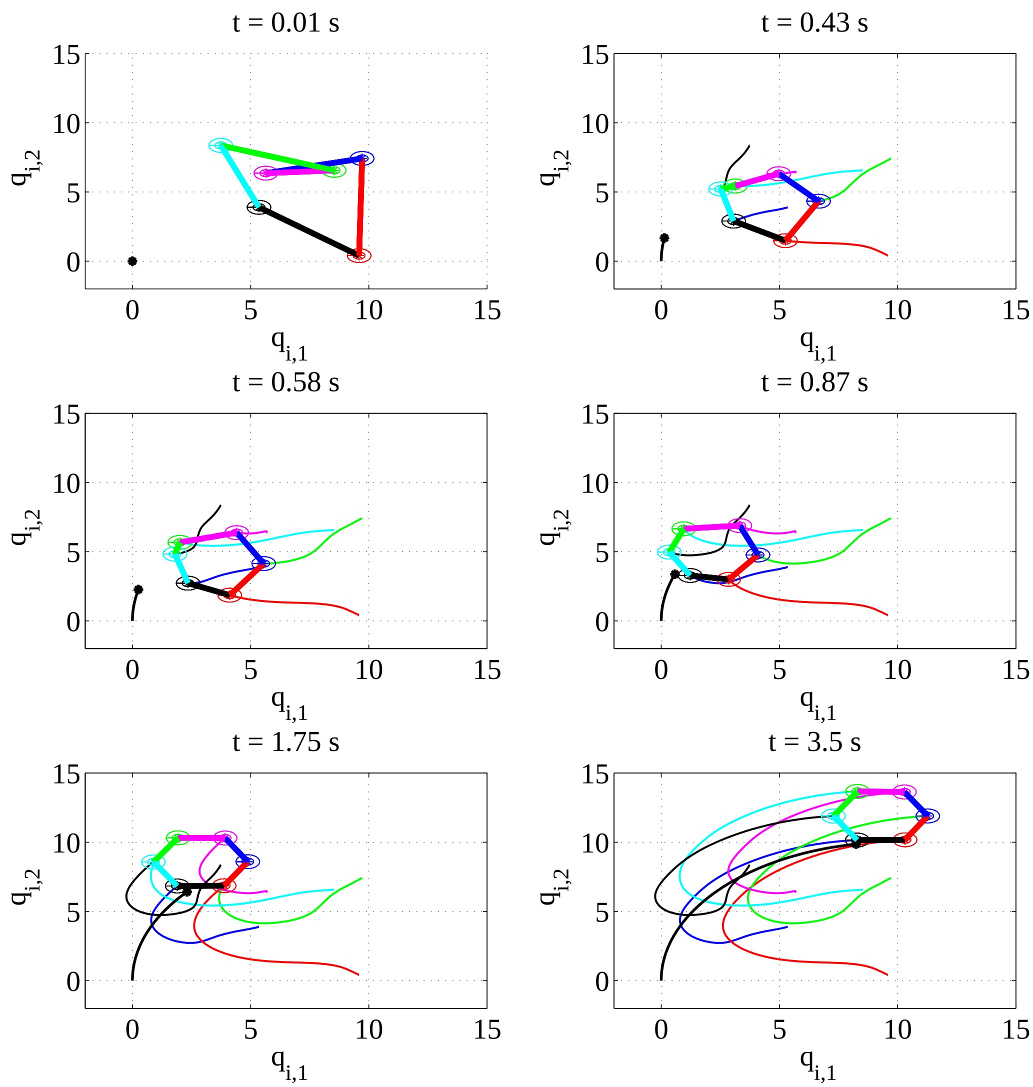
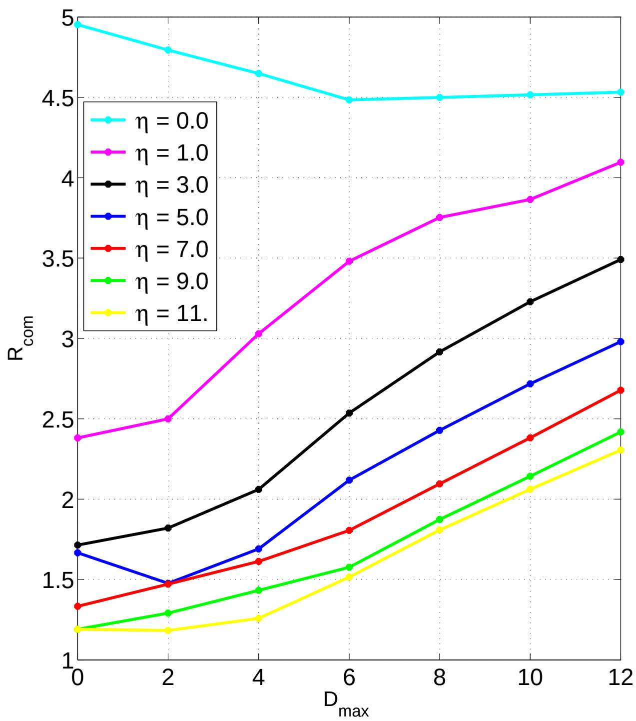


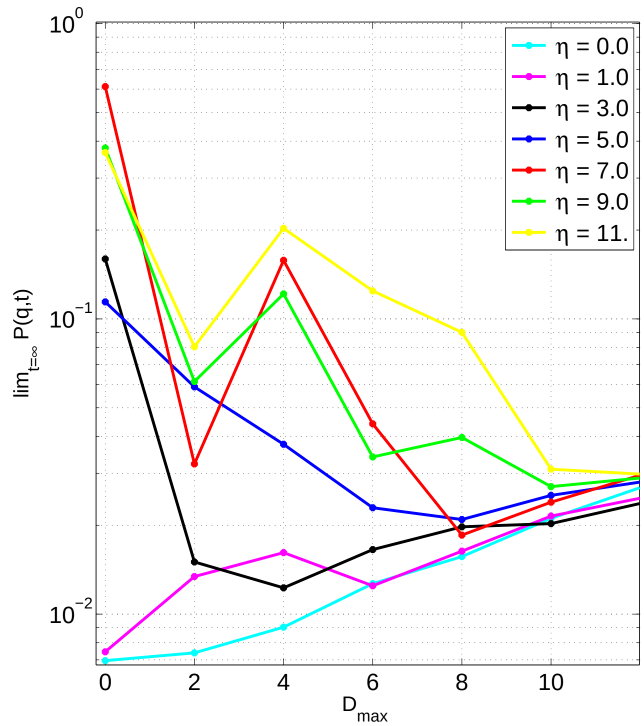
Figure 4.2: Convergence of agents to a formation with tracking control (4.13) and Theorem (8). Agents are represented by circles. Corresponding mapping between agent index and curve color: magenta: 1, green: 2, black: 3, blue: 4, red: 5, cyan: 6. Large black line: reference trajectory q_1^*

4.5.2 Formation control with DI

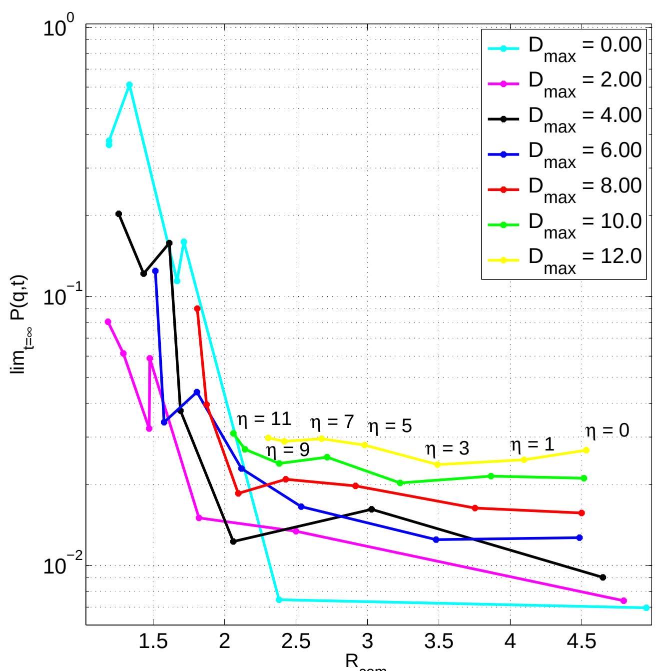
Figure 4.3 shows the evolution of the communication ratio R_{com} and of the potential energy at $t = T$. For all simulations, one has $P(q, T) \leq \xi$ for the different values of $D_{\max}$ and η.



(a)



(b)

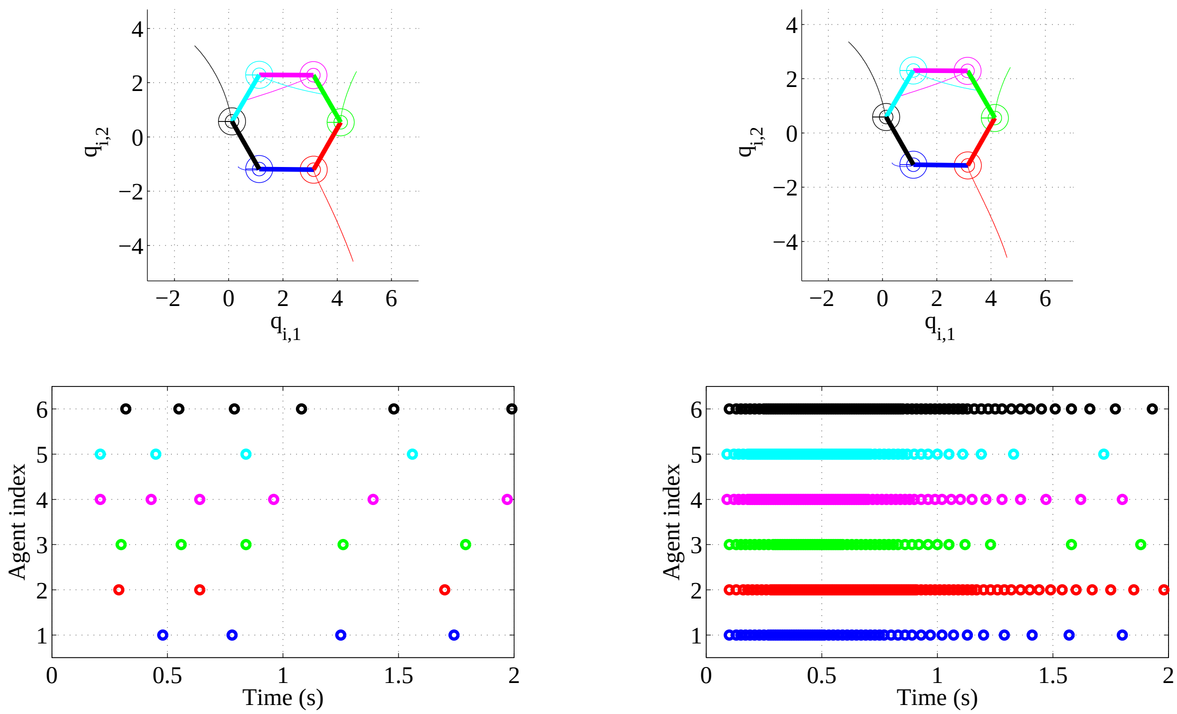


(c)

Figure 4.3: Evolution of R_{com} and $P(q,t)$ for different values of $D_{\max} \in \{ 0, \; 2, \; 4, \; 6, \; 8, \; 10, \; 12 \}$, $\eta \in \{ 0, \; 1, \; 3, \; 5, \; 7, \; 9, \; 11 \}$, and $\eta_2 = 7.5$. The DI model (4.36) and the constant estimator (4.37)-(4.38) are considered.

In Figure 4.3 (a), the number of communications obtained once the system has converged increases as the level of perturbations becomes more important, as expected. Increasing η in the CTC 4.26 helps reducing R_{com} . Nevertheless, increasing η also increases the potential energy $P(q,T)$ of the formation, as can be seen in Figure 4.3 (b). In Figure 4.3 (b), when $\eta \geq 3$, one observes that the potential energy starts to decrease with the level of perturbation $D_{\max}$ to increase again when $D_{\max}$ gets large. To explain this surprising behavior, Figure 4.3 (c) shows that there exists a threshold $\overline{R}_{\text{com}} = 2.25$ below which the potential energy significantly increases to ensure proper convergence. Therefore η should be chosen such that R_{com} remains above this threshold. Even large values of $D_{\max}$ can be tolerated provided that η is chosen large enough to provide a sufficient amount of communications.

4.5.3 Formation control with ship dynamical model



(a) Accurate estimator (E.34).

(b) Constant estimator (4.37).

Figure 4.4: Hexagonal formation with $D_{\max} = 20$, $\eta = 20$ and $\eta_2 = 7.5$. Agents are represented by circles. In (a), $R_{\text{com}} = 2.61\%$ and $P(q,T) = 0.001$. In (b) $R_{\text{com}} = 18.25\%$ and $P(q,T) = 0.001$.

Figure 4.4 shows the trajectories of the agents when the control (4.13) is applied and the communications are triggered according to the CTC of Theorem 8. Figure 4.4 (a) illustrates the results obtained using the accurate estimator (E.34), Figure 4.4(b) illustrates results obtained using the simple estimator (4.37). The agents converge to the desired formation with a limited number of communications, even in presence of perturbations.

Figure 4.5 shows the evolution of R_{com} and of $P(q,T)$ parametrized by η for different values of $D_{\max}$. For all simulations, one has $P(q,T) \leq \xi$ for the different values of $D_{\max}$ and η. As expected and shown in Section 4.5.2, the potential energy obtained once the system has converged increases with $D_{\max}$. It can also be observed that increasing η reduces the number of messages broadcast, without a significant impact on $P(q,T)$, contrary to what was observed with the DI with simple estimator.

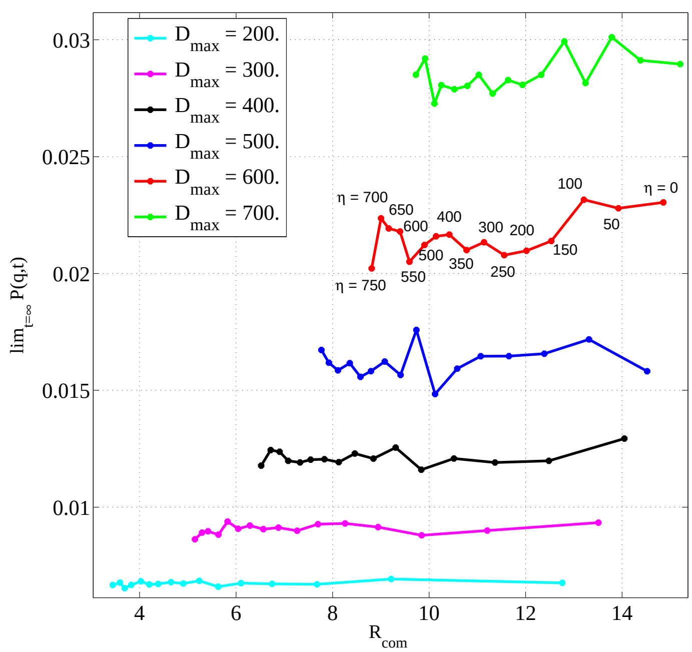


Figure 4.5: Evolution of R_{com} and $P(q,t)$ for different values of $D_{\max} \in \{\ 200,\ \ 300,\ \ \ldots,\ \ 700\ \}$, $\eta \in \{\ 0,\ \ 50,\ \ 100,\ \ \ldots\ \ 750\ \}$ and $\eta_2 = 7.5$. Model (4.39) and accurate estimator (E.34) are considerate.

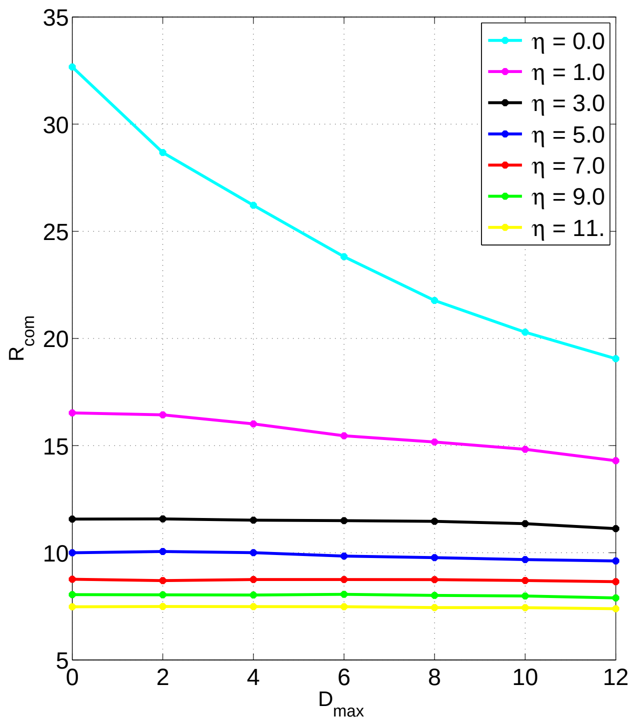
4.5.4 Tracking control with DI

The simulation duration is $T = 3.5\,s$. Figures 4.6 and 4.7 show the evolution of the communication ratio R_{com}, the potential energy and the tracking error at $t = T$.

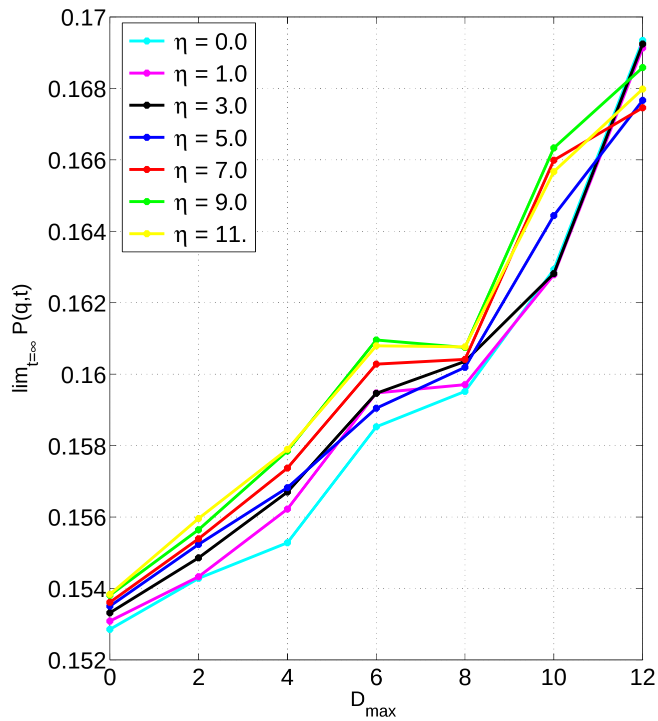
In Figure 4.6 (a), the number of communications obtained once the system has converged decreases as the level of perturbation becomes more important, especially when η is small, which was not excepted. Such behavior is not observed with the accurate estimator (E.34), where R_{com} increases when the perturbations become more important, as illustrated in Figure 4.10 (a) with the ship model. This behavior can be explained by the fact that a large $D_{\max}$ makes $\|\bar{g}_i\|$ and $\|\bar{s}_i\|$ larger, which reduces the number of times the CTC (4.26) is satisfied, even if the error $\left\|e_i^i\right\|$ is also affected. Difference with accurate estimator is the error e_i^i is keeping small by the estimator, so the influence of perturbations is more significant on e_i^i than on $\|\bar{g}_i\|$ or $\|\bar{s}_i\|$, which leads to a larger number of communications triggered.

Figure 4.6 (a) illustrates that the parameter η in the CTC (4.26) can help reducing R_{com} . It can be seen that there exists for R_{com} a threshold ($R_{\text{com}} = 7$) which R_{com} cannot reach : we can deduce a minimal number of communications is required for system converge with the constant estimator (4.37)-(4.38).

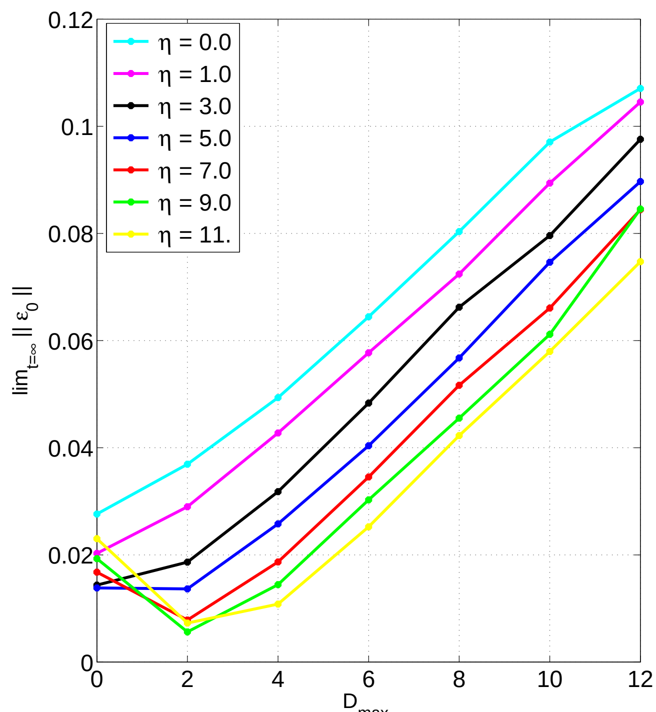
Figures 4.6 (b) and (c) show that the potential energy of the formation $P(q,t)$ and the tracking error ε_0 increase when the perturbation level increases. The influence of parameter η is also illustrated: Figure 4.6 (b) shows that a larger value of η leads to an increase of $P(q,t)$, but reduces ε_0. Indeed, the less communications, the more difficult it is for some Agent i to be synchronized with the others agents to reach the target formation. However, be less synchronized with the other agents allows Agent i to be more synchronized with its target trajectory q_i^* , inducing a small tracking error ε_0. Thus, a trade off between the $P(q,t)$ and ε_0 has to be reached, shown Figure 4.7.



(a)

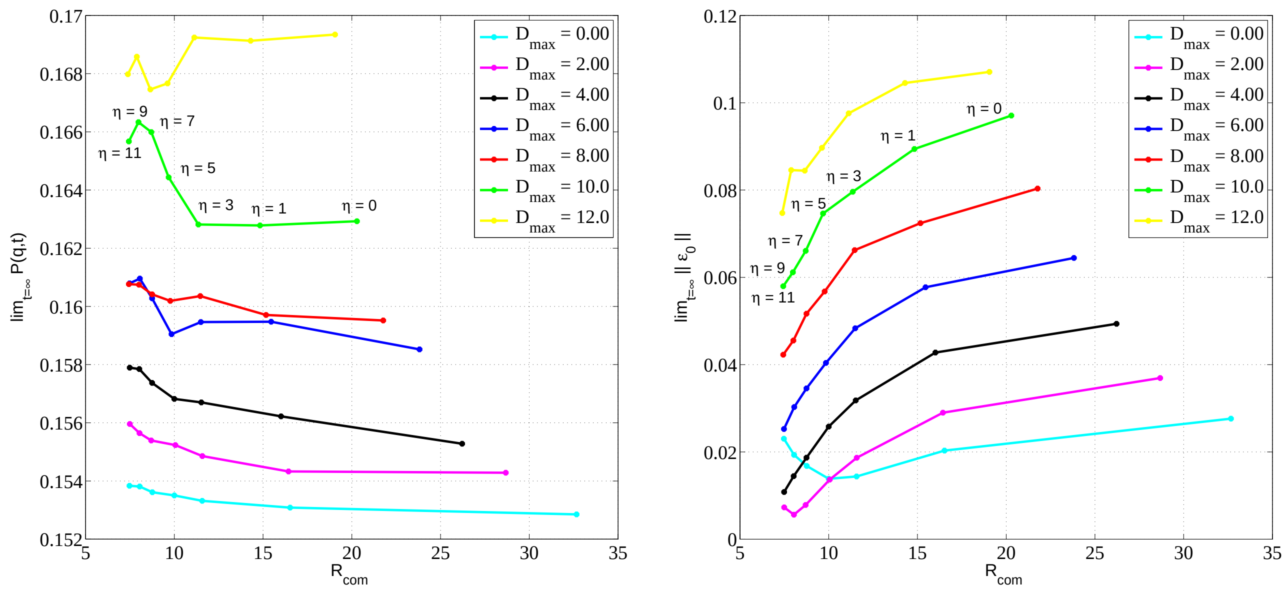


(b)



(c)

Figure 4.6: Evolution of R_{com}, $P(q,t)$ and ε_0 for different values of $D_{\max} \in \{\, 0,\ 2,\ 4,\ 6,\ 8,\ 10,\ 12 \,\}$, $\eta \in \{\, 0,\ 1,\ 3,\ 5,\ 7,\ 9,\ 11 \,\}$ and $\eta_2 = 7.5$. Model (4.36) and constant estimator (4.37)-(4.38) are considerate.



(a) (b)

Figure 4.7: Evolution of R_{com}, $P(q,t)$ and ε_0 for different values of $D_{\max} \in \left\{ 0, \;\; 2, \;\; 4, \;\; 6, \;\; 8, \;\; 10, \;\; 12 \right\}$, $\eta \in \left\{ 0, \;\; 1, \;\; 3, \;\; 5, \;\; 7, \;\; 9, \;\; 11 \right\}$ and $\eta_2 = 7.5$. DI model (4.36) and constant estimator (4.37)-(4.38) are considerate.

4.5.5 Tracking with surface ship model

The simulation duration is $T = 2.5\,s$.

Figures 4.10 and 4.9 show the evolution of the communication ratio R_{com}, the potential energy and the tracking error at $t = T$.

In Figure 4.10 (a), the number of communications obtained once the system has converged increases as the level of perturbations becomes more important. The parameter η in the CTC 4.26 can help to reduce R_{com} . Figure 4.10 (b) and (c) show that the potential energy of the formation $P(q,t)$ and the tracking error ε_0 also increase when the perturbation level increases. Influence of parameter η is also illustrated : Figure 4.10 (c) shows that increasing η results in make ε_0 decrease when $D_{\max} > 200$. Influence of η on $P(q,t)$ is less clearly detectable than in the case of the DI model.

In Figure 4.9, it can be observed that R_{com} cannot be reduced below the value of 1: a minimum number of communications is indeed required to converge with the accurate estimator (E.34).

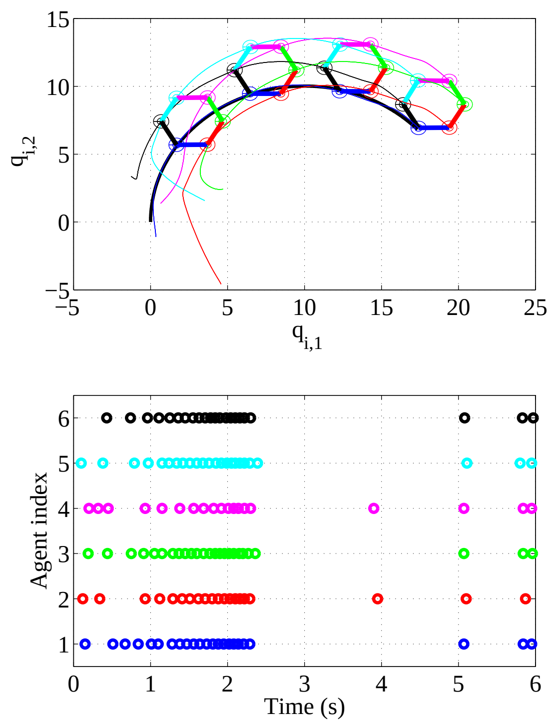
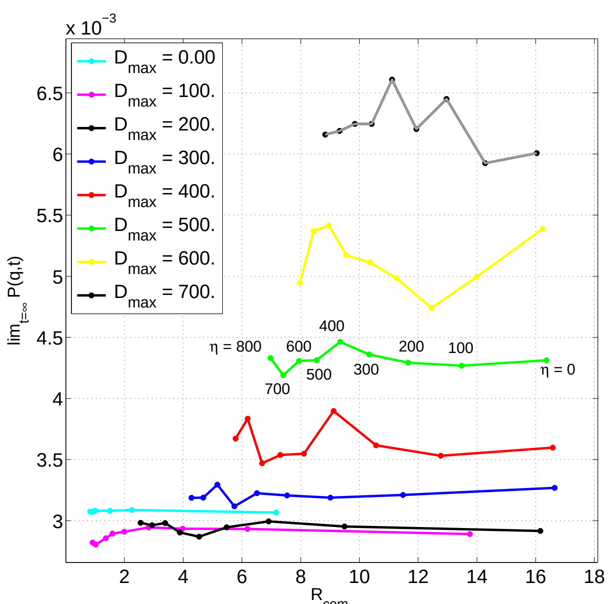
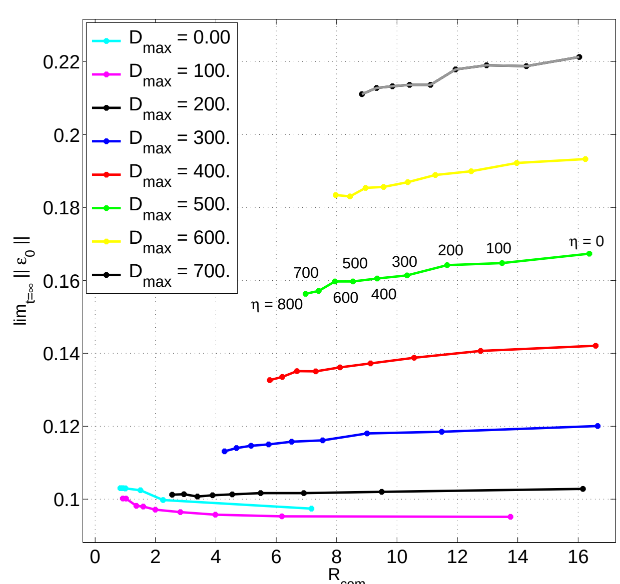


Figure 4.8: Hexagonal formation and tracking problem with $D_{\max} = 50$, $\eta = 50$, and $\eta_2 = 7.5$. Circles represents agents (top figure) and communication events (bottom figure). $R_{\text{com}} = 5\%$, $P(q,T) = 0.001$ and $\|\varepsilon_0\| = 0.1$. $T = 6\,\text{s}$.

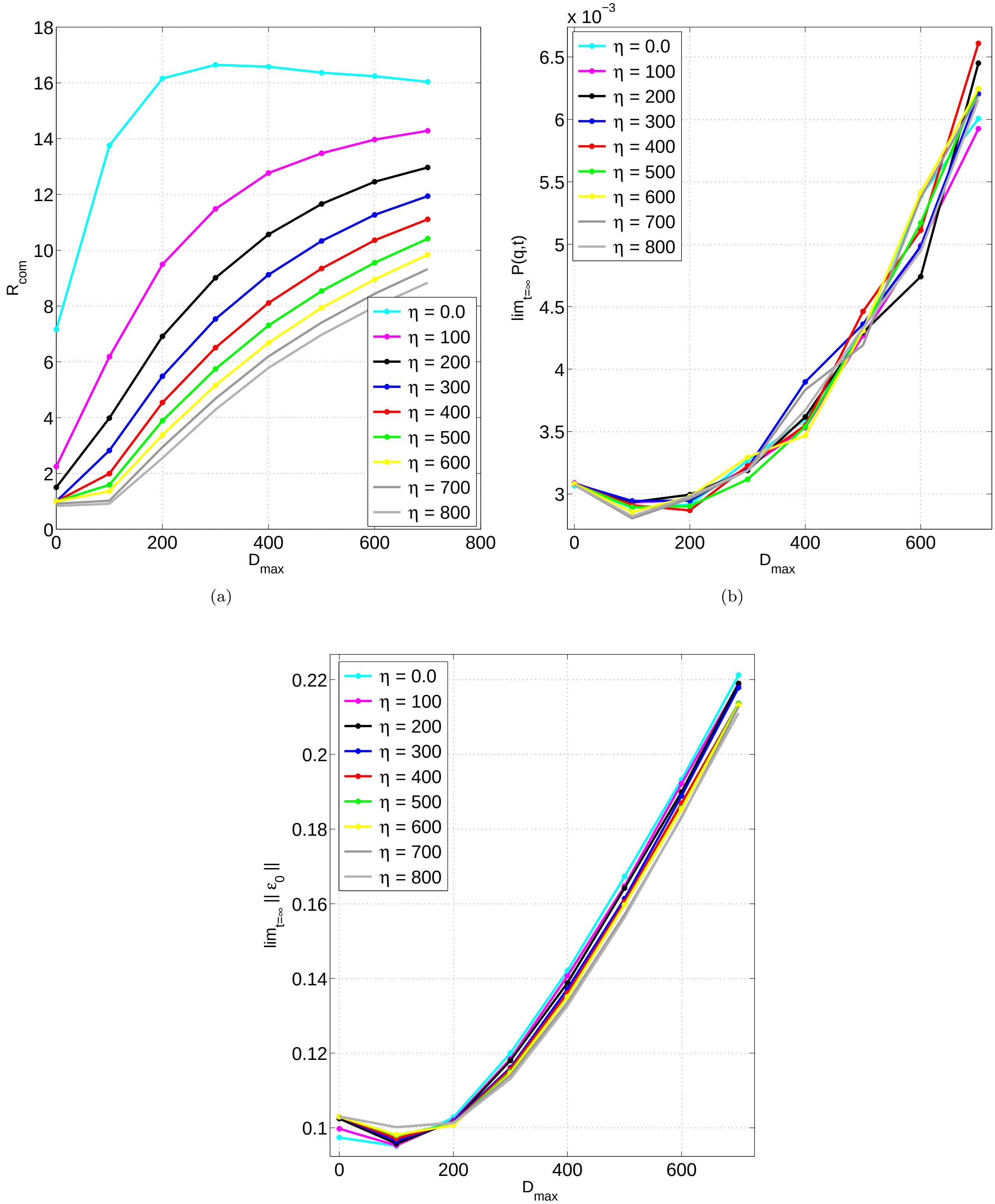


(a)



(b)

Figure 4.9: Evolution of R_{com}, $P(q,t)$ and ε_0 for different values of $D_{\max} \in \{\ 0,\quad 100,\quad 200,\quad \ldots\quad 700\ \}$, $\eta \in \{\ 0,\quad 100,\quad 200,\quad \ldots\quad 800\ \}$ and $\eta_2 = 7.5$. The SS model (4.39) and accurate estimator (E.34) are considered.



(a) (b) (c)

Figure 4.10: Evolution of R_{com}, $P(q,t)$ and ε_0 for different values of $D_{\max} \in \{0, 100, 200, \ldots 700\}$, $\eta \in \{0, 100, 200, \ldots 800\}$and $\eta_2 = 7.5$. The SS model (4.39) and accurate estimator (E.34) are considered.

4.6 Conclusion

This Part presents an adaptive control and event-triggered communication strategy for formation stabilization and tracking of multi-agent systems with perturbed Euler-Lagrange dynamics. Uncertainties are considered on the inertia matrix and the matrix of the Coriolis and centripetal term in the agents' dynamics, so that they cannot be considered known by the agents. An estimator has been proposed to provide the missing information required by the control laws. Convergence to a desired formation and influence of state perturbations on the convergence and on the amount of required communications have been studied. Tracking control to follow a desired trajectory has also been considered and added to the formation control. A distributed event-triggered condition to converge to the desired formation and follow the reference trajectory while reducing the number of communications has been proposed. Moreover, the time interval between consecutive communications has been shown to be strictly positive. Simulations have shown the effectiveness of the proposed method in presence of state perturbations when their level remains moderate. Two dynamics models have been considered for the agents: a double integrator mode to illustrate the main performances, and a surface ship vessel model to illustrate the capacity of the algorithm to handle more complex nonlinear dynamics.

In the next Parts, the considered problem will be extended to communication delays and package dropouts.

Chapter 5

Packet dropout in distributed event-triggered for multi-agent formation stabilization

This chapter tackles the issue of the influence of packet dropouts for event-triggered formation tracking. As in Chapter 4, Agent dynamics are described by Euler-Lagrange models including perturbations, and the inertia matrix and the matrix of the Coriolis/centripetal parameter are considered to be unknown.

Packet dropout is a frequent phenomenon in networked systems and may be a severe cause of failure, especially in the case of event-triggered communications. Since event-triggered approaches are based on the idea that a message is transmitted only when required, a loss of information may have a critical impact on the system and its stability. Moreover, detection of a missing transmission can be very difficult especially when the system is distributed. Thus, this work adapts the method presented in Chapter 4 to account for the influence of packet dropouts during transmission of messages.

Model of packet dropouts is exposed in Section 5.1. A centralized CTC is presented in Section 5.2. A new state estimator is then proposed in Section 5.4.1.

Adaptation to distributed estimation is described in Section 5.4.3. Communication instants are chosen locally by Agent i as described in Section 5.4.4.

A simulation example is presented in Section 5.5 to illustrate the reduction of the communications obtained by the proposed approach. Finally, conclusions are drawn in Section 5.6.

5.1 Model of packet dropouts

Due, for example, to the limited communication bandwidth, a message broadcast between two agents could be subject to packet dropout. To model this phenomenon in the transmission of a message from Agent j to Agent i, the update of the state estimation of Agent j performed by Agent i is described as in [25] by

$$\hat{q}_j^i\left(t_{j,k}^{i+}\right) = \tilde{\alpha}_{j,k}^i q_j\left(t_{j,k}\right) + \left(1 - \tilde{\alpha}_{j,k}^i\right)\hat{q}_j^i\left(t_{j,k}^{i-}\right) \tag{5.1}$$

where $\tilde{\alpha}_{j,k}^i$ is a random variable used to represent a stochastic occurrence of packet dropout in the transmission of the k-th message sent by Agent j to an Agent i. The $\tilde{\alpha}_{j,k}^i$'s, $k \in \mathbb{N}$, are assumed to be modeled by a Bernouilli stochastic process with the following probabilities

$$P\left(\tilde{\alpha}_{j,k}^i = 1\right) = \bar{\alpha}$$
$$P\left(\tilde{\alpha}_{j,k}^i = 0\right) = 1 - \bar{\alpha}$$

with $0 \leq \bar{\alpha} \leq 1$. With this model, the k-th message is successfully received by Agent i if $\tilde{\alpha}_{j,k}^i = 1$. The message is lost if $\tilde{\alpha}_{j,k}^i = 0$. Remark that $\tilde{\alpha}_{j,k}^j$ is always equal to 1 as there is *no communication between Agent j and itself.*

Let $\hat{q}_j^i\left(t\left|q_j\left(t_{j,k}^i\right)\right.\right)$ be the estimate of $q_j(t)$ made by Agent i updated by $q_j\left(t_{j,k}^i\right)$. Then, since the broadcast message can be subject to packet dropouts, it can be expressed

$$\begin{aligned}\hat{q}_j^i\left(t_{j,k}^{i+}\right) &= \tilde{\alpha}_{j,k}^i q_j\left(t_{j,k}\right) + \left(1-\tilde{\alpha}_{j,k}^i\right)\hat{q}_j^i\left(t_{j,k}^{i-}\right)\\ &= \tilde{\alpha}_{j,k}^i q_j\left(t_{j,k}\right) + \left(1-\tilde{\alpha}_{j,k}^i\right)\left[\tilde{\alpha}_{j,k-1}^i\hat{q}_j^i\left(t_{j,k}^{i+}\left|q_j\left(t_{j,k-1}^i\right)\right.\right) + \left(1-\tilde{\alpha}_{j,k-1}^i\right)\hat{q}_j^i\left(t_{j,k}^{i+}\left|\hat{q}_j^i\left(t_{j,k-1}^i\right)\right.\right)\right]\end{aligned}$$

Note that, if Agent i has received the k-th message broadcast by Agent j, one has $\hat{q}_j^i\left(t_{j,k}^{i+}\right) = \hat{q}_j^j\left(t_{j,k}^{i+}\right)$.

The above equations are easily extended to the case of $\dot{\hat{q}}_j^i\left(t_{j,k}^{i+}\right)$which is also part of the broadcast message.

For the sake of simplicity, the notations $\hat{q}_j^i(t)$ are used to replace $\hat{q}_j^i\left(t\left|q_j\left(t_{j,k}^i\right)\right.\right)$.

5.2 Centralized event-triggered communications with packet dropouts

In the following sections, we first study the case where the system is centralized and agents have access *to the estimates performed by all the agents*. The distributed case will be studied in the Section 5.4.

Consider first the case when the expectations $\mathbb{E}\left(e_i^j(t)\right)$ of estimation errors $e_i^j(t)$ for all i and j, are perfectly known by all the agents of the network even though there are packet dropouts. The following CTC is designed to trigger communications to ensure a bounded asymptotic convergence of the MAS to a target formation.

Assume that each agent knows the initial state vector of all the other agents (see Section 4.3 of Chapter 4 for more details). Introduce $k_{\max} = \max_{\substack{\ell=1\dots N\\ j=1\dots N}}(k_{\ell j})$ and $k_{\min} = \min_{\substack{\ell=1\dots N\\ j=1\dots N}}(k_{\ell j}\neq 0)$, $\alpha_i = \sum_{j=1}^N k_{ij}$, $\alpha_{\min} = \min_{i=1,\dots,N}\alpha_i$ and $\alpha_{\max} = \max_{i=1,\dots,N}\alpha_i$. Let $\bar{\theta}_i = \left[\bar{\theta}_{i,1},\dots,\bar{\theta}_{i,p}\right]^T \in \mathbb{R}^p$ and $\Delta\theta_{i,\max}$ be the same as defined in (4.23).

Theorem 10. *Consider a MAS with agent dynamics given by (4.1) and the control law (4.13). Consider some design parameters* $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$, $c_3 = \frac{\min\left\{1,k_1,k_p,k_0,2k_0\left(2k_0+\frac{\alpha_{\min}k_{\min}}{k_{\max}}\right)\right\}}{\max\{1,k_M\}}$ *and* $k_1 = k_s - (1 + k_p(k_M+1))$. *In absence of communication delays, the system (4.1) is input-to-state practically stable (ISpS) and the agents can be driven to some target formation such that*

$$\lim_{t\to\infty}\mathbb{E}\left(\sum_{i=1}^N k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right) \leq \xi \tag{5.2}$$

where ξ *satisfies*

$$\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \tag{5.3}$$

where $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T\Gamma_i^{-1}\Delta\theta_i\right)\right)$, *if the communications are triggered when one of the following conditions is satisfied*

$$\begin{aligned}k_s\bar{s}_i^T\bar{s}_i + k_p k_g\bar{g}_i^T\bar{g}_i + \eta \leq \alpha_M &\left[\sum_{j=1}^N k_{ij}\left(k_e\mathbb{E}\left(\left\|e_i^j\right\|^2\right) + k_p k_M\mathbb{E}\left(\left\|\dot{e}_i^j\right\|^2\right)\right)\right.\\ &\left.+k_p k_C^2\sum_{j=1}^N k_{ij}\mathbb{E}\left(\left\|e_i^j\right\|^2\right)\left[\left\|\dot{\hat{q}}_j^i\right\|+\eta_2\right]^2\right] + k_g b_i\left\|\dot{q}_i-\dot{q}_i^*\right\|^2\\ &+k_p\sqrt{\sum_{j=1}^N k_{ij}\mathbb{E}\left(\left\|e_i^j\right\|^2\right)}\left[\alpha_M\left(1+\left\|\left|Y_i\right|\Delta\theta_{i,\max}\right\|^2\right) + \frac{\left\|\left|Y_i\right|\Delta\theta_{i,\max}\right\|^2}{\left(1+\left\|\left|Y_i\right|\Delta\theta_{i,\max}\right\|^2\right)}\right]\end{aligned} \tag{5.4}$$

$$\|\dot{q}_i\| \quad \geq \quad \left\|\dot{\hat{q}}_i^i\right\| + \eta_2 \tag{5.5}$$

with $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$ *and* $Y_i = Y_i\left(q_i,\, \dot{q}_i,\, \dot{\bar{p}}_i,\, \bar{p}_i\right)$.

The proof of Theorem 10 is given in Appendix D.1. Contrary to the case without packet-dropout, as in Chapter 3 or Chapter 4, a proof of absence of Zeno behavior is more difficult to obtain because $\mathbb{E}\left(e_i^j\left(t\right)\right)$ is not reset to zero at $t = t_{i,k}$ if the message has been lost. This point will be discussed in Section 5.3.2.

Note that without packet dropout, Theorem 10 becomes equivalent to Theorem 8.

5.3 Communication protocol

5.3.1 Message content

When a communication is triggered at $t_{i,k}$ by Agent i, it broadcasts a message containing $t_{i,k}$, $x_i\left(t_{i,k}\right)$, $\bar{\theta}_i\left(t_{i,k}\right)$ and $\hat{x}^i = \left[\hat{x}_1^{iT}, \ldots, \hat{x}_N^{iT}\right]^T$. We assume that this message is transmitted to all agents j if $k_{ij} \neq 0$. The message is received directly if Agent i and j are neighbor or after several hops. The latter case requires the use of a flooding protocol [44, 83]. Due to packet dropout problem, the first method and/or a fully-connected communication graph is recommended. Remind there is no communication delay in this chapter.

5.3.2 Strategy to solve Zeno behavior

With the CTC proposed in Theorem 10 the absence of Zeno behavior cannot be proven. Indeed, if the message has not been received by Agent j, the error e_i^j is not reset to zero at $t = t_{i,k}$ and the CTC (5.4) is still satisfied after the message has been sent. To address this issue, the following strategy is proposed.

A minimum delay $\tau_{\min} \leq t_{i,k+1} - t_{i,k}$ is imposed before performing a new evaluation of the CTC (5.4) once it has been satisfied. Note that $\tau_{\min}$ hence corresponds to the minimum time between two communications transmitted by a same agent. Nevertheless, due to the possible error on e_i^j, this time constraint does not guarantee that the CTC will not remain satisfied at each next instants it will be evaluated.

Consider the case where the CTC (5.4) remains satisfied at the instant $t = t_{i,k} + \tau_{\min}$ and possibly at other next instants $t_{i,K} = t_{i,K-1} + \tau_{\min}$ with $K > k$. A message is broadcast at each of these instants where the CTC is still verified. The probability of a successful reception of one of theses messages (no packet dropout) by neighbor agents of Agent i increases. Thus the expectation $\mathbb{E}\left(\left\|e_i^j\right\|^2\right)$ is decreasing and the CTC (5.4) is prone to be not satisfied anymore. Moreover, if $\tau_{\min}$ is chosen enough small, *i.e.* such that the evolution of agents' states can be neglected over this period of time ($x_i\left(t_{i,k+1}\right) \simeq x_i\left(t_{i,k}\right)$), it can be assumed that all agents will receive the same information to update their estimators and $\mathbb{E}\left(\left\|e_i^j\right\|^2\right)$ can be expressed as proposed in Lemma 3.

Lemma 2. Let define the constant $\epsilon > 0$ chosen such that $\forall t \in I_{k,\epsilon} = \left[t_{i,k},\, t_{i,k} + \epsilon\right]$, $x_i\left(t\right) \simeq x_i\left(t_{i,k}\right)$ and $\hat{x}_i\left(t\right) \simeq \hat{x}_i\left(t_{i,k}\right)$. Consider that $\tau_{\min}$ is chosen such that $\tau_{\min} \leq \epsilon/K$ with $K \geq 2$. For all $t \in I_{k,\epsilon}$ and $\forall \ell \in \left[k, \ldots k + K - 1\right]$ such $t_{i,\ell+1} - t_{i,\ell} = \tau_{\min}$, *i.e.* the CTC (5.4) triggers every $\tau_{\min}$ since the instant $t = t_{i,k}$, one has

$$\mathbb{E}\left(\left\|e_i^j\left(t_{i,\ell}\right)\right\|^2\right) \simeq \left(1 - \left(1 - \bar{\alpha}\right)^{\ell-k}\right)\left\|\hat{q}_i^i\left(t_{i,\ell}\right) - q_i\left(t_{i,\ell}\right)\right\|^2 + \left(1 - \bar{\alpha}\right)^{\ell-k}\left\|\hat{q}_i^j\left(t_{i,\ell}\right) - q_i\left(t_{i,\ell}\right)\right\|^2.$$

Thus if $\tau_{\min}$ is taken enough small, *i.e.* K be enough large,

$$\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right) \to 0 \quad \text{when } t \to t_{i,k} + \tau_{\min} K$$

and, as shown in proof of AppendixD.4, the CTC (5.4) will stop to be satisfied at $t = t_{i,k} + \epsilon$.

Proof of Lemma 3 is proposed in Appendix D.2 and in AppendixD.4.

Note that this protocol guarantees the convergence and the absence of Zeno behavior, but can induce a large number of triggered communications over the intervals $I_{k,\epsilon}$.

Remark 4. Proof of no Zeno behavior can be obtained by Lemma 3 only for independent Bernoulli-distributed probabilities. In practice where hardware failures may be more suitably modeled by Markov processes, messages broadcast after a lost message have a lot of chance to be also lost, and so there is no guarantee that the CTC would stop to be satisfied.

5.4 Distributed problem

The CTC proposed in Theorem 10 assumes that Agent i knows $\mathbb{E}\left(e_i^j(t)\right)$ for all j. However, in a distributed context, the estimation $\hat{q}_i^j$ is no longer available to all agents. It is then necessary to define a new estimator of Agent i's state as proposed in Section 5.4.1 and introduce an additional estimator. The first estimator is used to evaluate the control input without problem of packet dropout, and is also the most optimistic estimation of other agent estimation. The second estimator in Section 5.4.2 considers the worst case of estimation, where agents never receive information from others agents. Both estimators are used to evaluate the expected value of the estimation error of $\mathbb{E}\left(e_i^j(t)\right)$.

5.4.1 New estimator

Let first define a new estimator model as

$$\hat{M}_j^i\left(\hat{q}_j^i\right)\ddot{\hat{q}}_j^i + \hat{C}_j^i\left(\hat{q}_j^i, \dot{\hat{q}}_j^i\right)\dot{\hat{q}}_j^i + G = \hat{\tau}_j^i, \ \forall t \in \left[t_{j,k}^i, t_{j,k+1}^i\right[\quad (5.6)$$

$$\hat{q}_j^i\left(t_{j,k}^i\right) = q_j\left(t_{j,k}^i\right) \quad \text{if } \tilde{\alpha}_{j,k}^i = 1 \quad (5.7)$$

$$\dot{\hat{q}}_j^i\left(t_{j,k}^i\right) = \dot{q}_j\left(t_{j,k}^i\right) \quad \text{if } \tilde{\alpha}_{j,k}^i = 1 \quad (5.8)$$

where

$$\hat{\tau}_j^i = -k_s\left(\dot{\hat{r}}_j^i + k_p k_0 \hat{r}_j^i\right) - k_g k_0 \hat{r}_j^i + G - Y_j\left(\hat{q}_j^i, \dot{\hat{q}}_j^i, \dot{\hat{m}}_j^i, \hat{m}_j^i\right)\hat{\theta}_j^i. \quad (5.9)$$

$$\dot{\hat{\theta}}_j^i = \Gamma_j Y_j\left(\hat{q}_j^i, \dot{\hat{q}}_j^i, \dot{\hat{m}}_j^i, \hat{m}_j^i\right)^T\left(\dot{\hat{r}}_j^i + k_p k_0 \hat{r}_j^i\right) \quad (5.10)$$

with $\hat{r}_j^i = \hat{q}_j^i - q_j^*$, and $\hat{m}_j^i = k_p k_0 \hat{r}_j^i - \dot{q}_j^*$ if $k_0 > 0$, *i.e.* in the case of a reference trajectory to be tracked, $\hat{m}_j^i = 0$ else. Remark if $k_0 = 0$, $\dot{q}_j^* = 0$.

Remark 5. When the formation converges to the target configuration, it can be observed that $\hat{g}_j^i = k_0 \hat{r}_j^i$ and $\hat{s}_j^i = \dot{\hat{r}}_j^i + k_p k_0 \hat{r}_j^i$.

Contrary to the control laws defined in (4.18) which required estimation of all agent states, the control (5.9) only requires Agent j information to update $\hat{q}_j^i$. This makes it less dependent of the communications and so limits issues due to packet loss. Moreover, Agent i needs to perform only estimation of its own state and of those of agents j such that $k_{ij} \neq 0$. However, (5.6) is less accurate than (4.18).

When considering packet loss, if there exists an instant $t = t_{j,k}^i$ such that $\hat{x}_j^i\left(t_{j,k}^i\right) = x_j\left(t_{j,k}^i\right)$ for $x_i = \left[q_i^T, \dot{q}_i^T\right]^T$, *i.e.* $\tilde{\alpha}_{j,k}^i = 1$, then $\hat{x}_j^i(t) = \hat{x}_j^j(t)$ $\forall t \in \left[t_{j,k}^i, t_{j,k+1}^i\right[$ which corresponds to the synchronization assumption presented in Chapter 4.

5.4.2 Additional estimator

In Theorem 10, Agent i is assumed to know $\mathbb{E}\left(\left\|e_i^j\right\|\right)$, *i.e.* $\hat{q}_i^j$. However, in a distributed context, this is no longer true. To address this issue, each Agent i evaluates an additional estimates $\breve{q}_i^j$, which is an estimate of $\hat{q}_i^j$ made by the Agent i, for all Agent j such that $k_{ij} \neq 0$. The estimate $\breve{q}_i^j$ is updated only when Agent i receives a message from Agent j, *i.e.* when $t = t_{j,k}^i$, $\breve{q}_i^j\left(t_{j,k}^i\right) = \hat{q}_i^j\left(t_{j,k}^i\right)$. This guarantees that $\breve{q}_i^j(t) = \hat{q}_i^j(t)$ for $t \in \left[t_{j,k}^i,\, t_{i,k+1}\right[$, *i.e.* the time interval during which Agent i doesn't broadcast a message.

The dynamics of $\breve{q}_i^j$ is described as

$$\begin{aligned} \breve{M}_i^j\left(\breve{q}_i^j\right)\ddot{\breve{q}}_i^j + \breve{C}_i^j\left(\breve{q}_i^j,\, \dot{\breve{q}}_i^j\right)\dot{\breve{q}}_i^j + G &= \breve{\tau}_i^j & (5.11)\\ \breve{q}_i^j\left(t_{j,k}^i\right) &= \hat{q}_i^j\left(t_{j,k}^i\right) & (5.12)\\ \dot{\breve{q}}_i^j\left(t_{j,k}^i\right) &= \dot{\hat{q}}_i^j\left(t_{j,k}^i\right) & (5.13) \end{aligned}$$

where

$$\breve{\tau}_i^j = -k_s\left(\dot{\breve{r}}_i^j + k_p k_0 \breve{r}_i^j\right) - k_g k_0 \breve{r}_i^j + G - Y_j\left(\breve{q}_i^j,\, \dot{\breve{q}}_i^j,\, \dot{\breve{m}}_i^j,\, \breve{m}_i^j\right)\breve{\theta}_i^j. \quad (5.14)$$

$$\dot{\breve{\theta}}_i^j = \Gamma_i Y_i\left(\breve{q}_i^j,\, \dot{\breve{q}}_i^j,\, \dot{\breve{m}}_i^j,\, \breve{m}_i^j\right)^T\left(\dot{\breve{r}}_i^j + k_p k_0 \breve{r}_i^j\right) \quad (5.15)$$

with $\breve{r}_i^j = \breve{q}_i^j - q_i^*$and $\breve{m}_i^j = k_p k_0 \breve{r}_i^j - \dot{q}_i^*$ if $k_0 > 0$, $\breve{m}_i^j = 0$ else.

The estimate $\breve{x}_i^j$ represents the worst possible estimation of $\hat{x}_i^j$, because it considers that Agent j never receives information from Agent i to update its estimation. Similarly, the estimation $\hat{x}_i^i$ represents the most optimistic estimation of $\hat{x}_i^j$, because it considers Agent j receives all messages from Agent i.

5.4.3 Expectation of the estimation error

Since $\breve{q}_i^j$ is updated less frequently than $\hat{q}_i^j$, and $\hat{q}_i^j = \hat{q}_i^i$ if $\hat{q}_i^j$ is updated using the last message broadcast by Agent i, $\left\|e_i^j\right\|$ can be upper-bounded by the *worst case* error $\left\|\breve{e}_i^j\right\|$, described in this section.

First, let study the evaluation of e_i^j. Define the instant $t_{j,h}^i$ when the h-message is broadcast by the Agent j and is assumed to be received successfully by Agent i. In the following section, let t satisfies $t \geq t_{j,h}^i$.

$\forall t \in \left[t_{j,h}^i,\, t_{i,k}^j\right[$, one has $\hat{q}_i^j(t) = \breve{q}_i^j(t)$ and so the estimation error can be evaluated as

$$\begin{aligned} e_i^j(t) &= \hat{q}_i^j(t) - q_i(t)\\ &= \breve{q}_i^j(t) - q_i(t) \quad \forall t \in \left[t_{j,h}^i,\, t_{i,k}^j\right[\end{aligned} \quad (5.16)$$

At the instant $t = t_{i,k}$, Agent i broadcasts a message. If the message is received, *i.e.* $\tilde{\alpha}_{i,k}^j = 1$, one gets $\hat{q}_i^j(t) = \hat{q}_i^i(t)\ \forall t \in [t_{i,k},\, t_{i,k+1}[$. Else $\hat{q}_i^j(t) = \breve{q}_i^j(t)\ \forall t \in [t_{i,k},\, t_{i,k+1}[$. The estimation error becomes

$$\begin{aligned} e_i^j\left(t_{i,k}^+\right) &= \left[\tilde{\alpha}_{i,k}^j \hat{q}_i^j(t_{i,k}) + \left(1 - \tilde{\alpha}_{i,k}^j\right)\hat{q}_i^j\left(t_{i,k}^-\right)\right] - q_i(t_{i,k})\\ &= \left[\tilde{\alpha}_{i,k}^j \hat{q}_i^i(t_{i,k}) + \left(1 - \tilde{\alpha}_{i,k}^j\right)\breve{q}_i^j\left(t_{i,k}^-\right)\right] - q_i(t_{i,k}) \end{aligned} \quad (5.17)$$

and then

$$e_i^j(t) = \left[\tilde{\alpha}_{i,k}^j \hat{q}_i^i(t) + \left(1 - \tilde{\alpha}_{i,k}^j\right)\breve{q}_i^j(t)\right] - q_i(t) \quad \forall t \in \left[t_{i,k},\, \min\left\{t_{i,k+1}, t_{j,h+1}^i\right\}\right[.$$

Since $\breve{q}_i^j$ is updated less frequently than $\hat{q}_i^j$, let the additional estimation error $\breve{e}_i^j(t)$ be defined as

$$\breve{e}_i^j(t) = \breve{q}_i^j(t) - q_i(t) \quad \forall t \in \left[t_{j,h}^i,\, t_{i,k}\right[\quad (5.18)$$

$$\breve{e}_i^j(t) = \left[\tilde{\alpha}_{i,k}^j \hat{q}_i^i(t) + \left(1 - \tilde{\alpha}_{i,k}^j\right)\breve{q}_i^j(t)\right] - q_i(t) \quad \forall t \in [t_{i,k},\, t_{i,k+1}[\quad (5.19)$$

Thus, using previous study of e_i^j and the communication protocol described in Section 5.3.2 for $\breve{x}$ instead of $\hat{x}$ one obtains

- If $\forall t \in \left[t^i_{j,h},\, t_{i,k}\right[$

$$\mathbb{E}\left(\left\|\breve{e}^j_i(t)\right\|^2\right) = \left\|\breve{q}^j_i(t) - q_i(t)\right\|^2 \tag{5.20}$$

- If $t > t_{i,k}$,

$$\mathbb{E}\left(\left\|\breve{e}^j_i(t)\right\|^2\right) = \bar{\alpha}\left\|\hat{q}^i_i(t) - q_i(t)\right\|^2 + (1-\bar{\alpha})\left\|\breve{q}^j_i(t) - q_i(t)\right\|^2 \tag{5.21}$$

- In the case of Section 5.3.2, if $t > t_{i,k+K}$ where $\exists K \in \mathbb{N}$, $K \geq 2$ and $t_{i,k+K} - t_{i,k} \leq \epsilon$,

$$\mathbb{E}\left(\left\|\breve{e}^j_i(t)\right\|^2\right) = \left(1-(1-\bar{\alpha})^K\right)\left\|\hat{q}^i_i(t) - q_i(t)\right\|^2 + (1-\bar{\alpha})^K\left\|\breve{q}^j_i(t) - q_i(t)\right\|^2 \tag{5.22}$$

and $\mathbb{E}\left(\left\|\breve{e}^j_i(t)\right\|^2\right) \geq \mathbb{E}\left(\left\|e^j_i(t)\right\|^2\right)$. Proof of (5.20)-(5.21)-(5.22) is presented in Appendix D.3.

Similar expressions can be obtained for $\mathbb{E}\left(\left\|\dot{\breve{e}}^j_i(t)\right\|^2\right)$.

Remark 6. $\forall t \in \left[t^i_{j,h},\, t_{i,k}\right] \cup [t_{i,k},\, t_{i,k+1}[$, (5.16)-(5.18) and (5.17)-(5.19) lead to $\mathbb{E}\left(\left\|\breve{e}^j_i(t)\right\|^2\right) = \mathbb{E}\left(\left\|e^j_i(t)\right\|^2\right)$.

Evaluation of $\mathbb{E}\left(\left\|\breve{e}^j_i(t)\right\|^2\right)$ and $\mathbb{E}\left(\left\|\dot{\breve{e}}^j_i(t)\right\|^2\right)$ will be used in the Section 5.4.4 to evaluate the proposed distributed CTC.

5.4.4 Distributed event-triggered communications with packet dropout

Using the additional estimate $\breve{q}^j_i$ and the additional estimation error $\breve{e}^j_i$ introduced in Section 5.4, Theorem 10 of Section 5.2 can be evaluated by each agent in a distributed way as proposed in Theorem 11.

As in Theorem 10, the initial values of the state vectors are considered to be known by all agents. In practice, this condition can be satisfied by triggering a communication from all agents at time $t = 0$ to initialize the estimates of the state of the neighbors of all agents.

Theorem 11. *Consider a MAS with agent dynamics given by (4.1) and the control law (4.13). Consider some design parameters $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$, $c_3 = \frac{\min\left\{1, k_1, k_p, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)\right\}}{\max\{1, k_M\}}$ and $k_1 = k_s - (1 + k_p(k_M + 1))$. In absence of communication delays, the system (4.1) is input-to-state practically stable (ISpS) and the agents can be driven to some target formation such that*

$$\lim_{t\to\infty} \mathbb{E}\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right) \leq \xi \tag{5.23}$$

where ξ satisfies

$$\xi = \frac{N}{k_g c_3}\left[D^2_{\max} + \eta + c_3\Delta_{\max}\right] \tag{5.24}$$

where $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T \Gamma_i^{-1}\Delta\theta_i\right)\right)$, if the communications are triggered when one of the following conditions is satisfied

$$\begin{aligned}
k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta \leq \alpha_M &\left[\sum_{j=1}^{N} k_{ij}\left(k_e \mathbb{E}\left(\left\|\breve{e}^j_i\right\|^2\right) + k_p k_M \mathbb{E}\left(\left\|\dot{\breve{e}}^j_i\right\|^2\right)\right)\right.\\
&\left. + k_p k_C^2 \sum_{j=1}^{N} k_{ij}\mathbb{E}\left(\left\|\breve{e}^j_i\right\|^2\right)\left[\|\dot{\hat{q}}^i_j\| + \eta_3\right]^2\right] + k_g b_i\left\|\dot{q}_i - \dot{q}^*_i\right\|^2\\
&+ k_p\sqrt{\sum_{j=1}^{N} k_{ij}\mathbb{E}\left(\left\|\breve{e}^j_i\right\|^2\right)}\left[\alpha_M\left(1 + \left\|\left|Y_i\right|\Delta\theta_{i,\max}\right\|^2\right) + \frac{\left\|\left|Y_i\right|\Delta\theta_{i,\max}\right\|^2}{\left(1 + \left\|\left|Y_i\right|\Delta\theta_{i,\max}\right\|^2\right)}\right]
\end{aligned} \tag{5.25}$$

$$\|\dot{q}_i\| \quad \geq \quad \left\|\dot{\hat{q}}_i^i\right\| + \eta_2 \tag{5.26}$$

with $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, $Y_i = Y_i\left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i\right)$.

The proof of Theorem 11 is given in Appendix D.1. Contrary to the case without packet-dropout as in Chapter 3 or Chapter 4, proof of absence of Zeno behavior is more difficult to obtain because $\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)$ is not reset to zero at $t = t_{i,k}$ if a message has not been received. A solution has been proposed in the Section 5.3.2 and 5.4.3, and an optional complementary method is proposed in Section 5.4.5.

The CTCs proposed in Theorem 11 are analyzed assuming that the estimators of the states of the agents and the communication protocol are such that $\forall (i,j) \in \mathcal{N} \times \mathcal{N}$, $\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)$ can be evaluated by Agent i: other estimators can be proposed for $\hat{q}_i^j$ or $\breve{q}_i^j$, provided that this condition is satisfied.

Remark 7. If there is no packet dropout, *i.e.* $\bar{\alpha} = 1$, Theorem 11 becomes equivalent to the Theorem 8.

5.4.5 Optional additional communication protocol

Contrary to Theorem 10 where e_i^j is assumed to be known and so can be reset at $t_{i,k}$ if message has been successfully received, $\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)$ is not reset to zero in Theorem 11. Protocols introduced in Section 5.3.2 and 5.4.3 guarantee the convergence of the formation and the absence of Zeno behavior, but it can induce a large number of trigger.

To reduce the number of trigger, an optional method based on a idea similar to message RTS and CTS used CSMA/CA protocol can be employed. In this strategy, when Agent j received a message from Agent i, it broadcasts a short frame MR (Message received) to inform Agent i that its message has been received. If Agent i receives the MR frame from Agent j, it can update its additional estimation as $\breve{x}_i^j\left(t_{i,k}\right) = x_i^i\left(t_{i,k}\right)$ and so reset $\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)$ to zero. Else, as the MR frame can also be subject of packet drop, no conclusion can be settled.

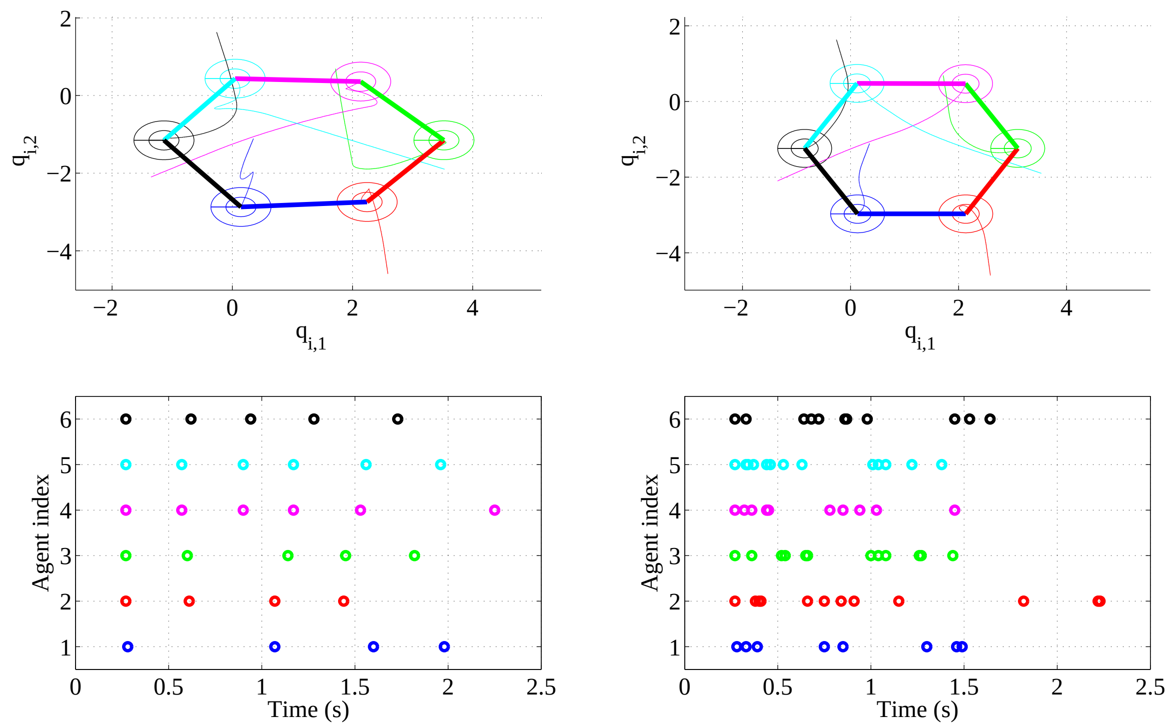
The additional protocol can require in practice a second transmission station embedded on the agent to send the MR message using another frequency. If an unique frequency is employed to transmit usual message and RM message, the following improvement can be made : when Agent j needs to broadcast a MR to Agent i, it transmits $t_{j,k}$, $x_j\left(t_{j,k}\right)$, $\bar{\theta}_j\left(t_{j,k}\right)$ and $\hat{x}^j = \left[\hat{x}_1^{jT}, \dots, \hat{x}_N^{jT}\right]^T$ to update its neighbors estimates and use the communication. Then, $\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)$ is reset to zero, $\breve{x}_i^j\left(t_{i,k}\right) = x_i^i\left(t_{i,k}\right)$ and $\hat{x}_j^i$ is updated with the current value. The only difference with a classic message is that no RM is asked.

Finally, note this protocol induces an important additional number of communications: it must be used only when the number of continuous trigger by CTCs is important. Thus, a RM can be asked by Agent i only when its CTC (5.25) triggers two times in a row, *i.e.* if $t_{i,k+2} - t_{i,k} = 2\tau_{\min}$.

5.5 Example

Consider the same dynamics, coefficients and simulation parameters that used in Section 4.5 in Chapter 4. No reference trajectory is considered. The simulation duration is $T = 2.s$. Matlab's ode45 integrator is used with a step size $\Delta t = 0.01$ s. To implement the communication protocol described in Section 5.3.2, the minimum delay between the transmission of two messages by the same agent is set to $\tau_{\min} = 0.0025$ s, and Agent states are considered constant over each interval of the form $[k\Delta t, (k+1)\Delta t[$. Let N_{m} be the total number of messages broadcast during a simulation. The performance of the proposed approach is evaluated comparing N_{m} to the maximum number of messages that can be broadcast $\overline{N}_{\mathrm{m}} = NT/\tau_{\min} \geq N_{\mathrm{m}}$. The percentage of residual communications is defined as $R_{\mathrm{com}} = 100\frac{N_{\mathrm{m}}}{\overline{N}_{\mathrm{m}}}$. R_{com} indicates the percentage of time slots during which a communication has been triggered. Note that as $\tau_{\min} < \Delta t$, the number of time slots in simulations presented in this section is larger than the one in Section 4.5 in Chapter 4.

The CTC (4.26) in Theorem 8 from Chapter 4 and the new CTC (5.25) from Theorem 11 are compared in presence of packet dropout where $\bar{\alpha} = 0.5$. Remind Theorem 8 has not been studied for case with packet dropout.



(a) Convergence with CTC (4.26) from Chapter 4.

(b) Convergence with CTC (5.25) and using expectation calculation (5.20)-(5.21)-(5.22).

Figure 5.1: Hexagonal formation with the DI (4.36) and new estimator (5.1) without perturbations. The CTC (4.26) in Chapter 4 and new CTC (5.25) are compared. $\bar{\alpha} = 0.5$, $D_{\max} = 0$, $\eta = 1$ and $\eta_2 = 7.5$. Agents are represented by circles. In (a), $R_{\text{com}} = 0.5\%$ and $P(q) = 0.142$. In (b) $R_{\text{com}} = 2.15\%$ and $P(q) = 0.001$.

In Figure 5.1 (a), the new estimator (5.1) allows to obtain a reduced number of communication without perturbations. Compare to estimators proposed in Chapter 4, the reduction of communication is better than the one of the constant estimator (4.37) but worst than the one of the accurate estimator (4.18). However, the CTC (4.26) used in Figure 5.1 (a) doesn't allow to converge with a small potential energy $P(q,T)$ in presence of packet dropout. In opposite, the CTC (5.25) used in Figure (b) allows to converge with a small potential energy $P(q,T)$ even in presence of packet dropout. However, the cost is a larger number of communications.

In Figure 5.2, performance of CTC (5.25) are compared for the DI model and different value of $\bar{\alpha}$. The number of communications obtained once the system has converged increases as the level of $\bar{\alpha}$ becomes more important, as expected. Nevertheless, since there is communication lost, *i.e.* $\bar{\alpha} < 1$, increasing $\bar{\alpha}$ does not make important modification on the potential energy $P(q,T)$ of the formation : CTC (5.25) guarantees the same accuracy on the potential energy at the cost of the communication ratio R_{com}. This one can become very important when the probability of success of message transmission $\bar{\alpha}$ is low.

As for the case without packet dropout in Section 4.5.2 in Chapter 4, the number of communications obtained once the system has converged decreases as the level of perturbations becomes more important, which was not excepted. Again, it can be note this behavior is not observed with the Surface ship, where R_{com} increases when perturbations becomes more important, as illustrate in Figure 5.3.

Figure 5.3 shows the evolution of R_{com} and of $P(q,T)$ for different values of $D_{\max}$ and $\bar{\alpha}$. For all simulations, one has $P(q,T) \leq \xi$ for the different values of $D_{\max}$ and $\bar{\alpha}$. As expected, the number of communications increases with $\bar{\alpha}$ and $D_{\max}$. Nevertheless, increasing $\bar{\alpha}$ does not make important modifications on the potential energy $P(q,T)$ of the formation, as it was observed with the DI with

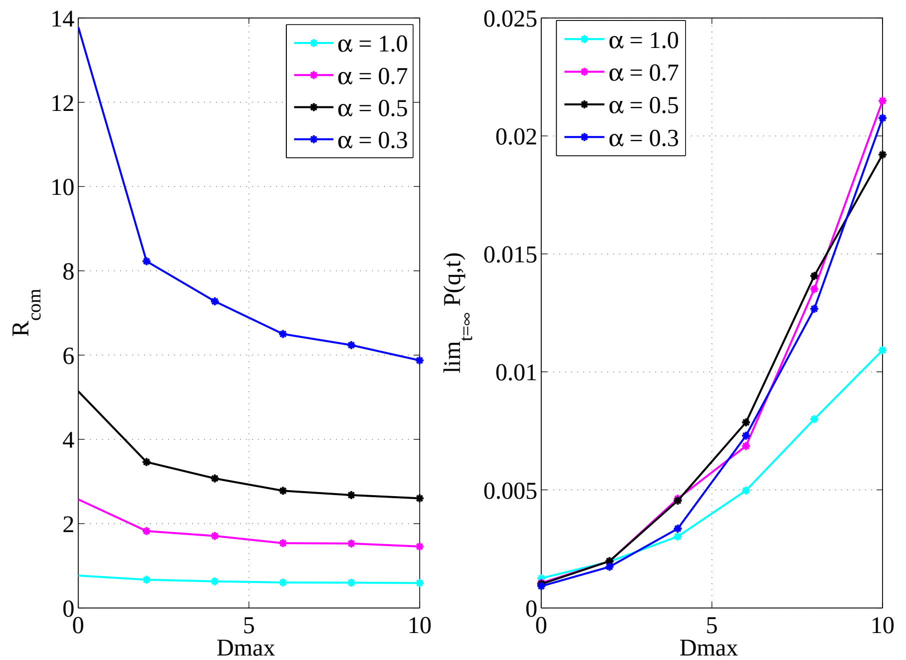


Figure 5.2: Evolution of R_{com} and $P(q,t)$ for different values of $D_{\max} = \{\ 0\ \ 2\ \ 4\ \ 6\ \ 8\ \ 10\ \}$, $\eta = 0$ and $\eta_2 = 7.5$. Model (4.36) and new estimator (5.1) are considerate.

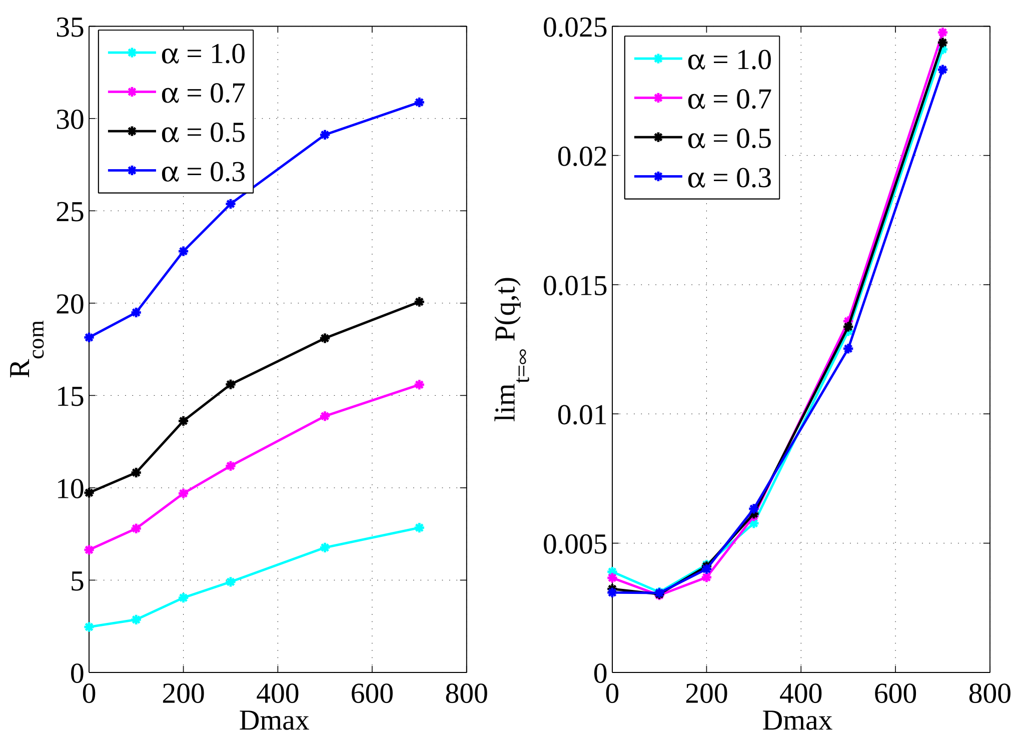


Figure 5.3: Evolution of R_{com} and $P(q,t)$ for different values of $D_{\max} = \{\ 0\ \ 100\ \ 200\ \ 300\ \ 500\ \ 700\ \}$, $\eta = 200$ and $\eta_2 = 7.5$. Model (4.39) and new estimator (5.1) are considerate.

simple estimator : this one is more sensible to the state perturbations.

5.6 Conclusion

This chapter addresses the problem of packet dropout by adapting methods proposed in the previous chapter. Influence of packet dropout on the estimators has been studied. Due to the loss information, estimators cannot be synchronized all the time, making the CTC impossible to be evaluated in a distributed way. Thus, estimator has been modified and an additional estimator has been introduced to address this issue. The first estimator is used to evaluate the control input without problem of packet dropout, and is also the most optimistic estimation of other agent estimation. The second estimator considers the worst case of estimation, where agents never receive information from others agents. Both estimators are used to evaluate the expected value of the estimation error, used in a distributed formulation of the CTC to trigger communications. Convergence to the target formation and reference trajectory has been studied and absence of Zeno behavior has been solved using a particular communication protocol.

In future work, communication delay will also be considered along with packet dropout. Moreover, Markov chains will also be considered instead of Bernouilli process to obtain a more realistic model of lost information during communications.

Chapter 6

Communication delay in distributed event-triggered for multi-agent formation stabilization

This chapter proposes a strategy to reduce the number of communications for displacement-based formation control while following a desired reference trajectory, in presence of bounded communication delays. As in Chapter 4, agent dynamics are described by Euler-Lagrange models and include perturbations. The inertia matrix and the Coriolis/centripetal matrix are consider to be unknown by the agents. Packet losses are not considered here. Work exposed in this chapter is a preliminary study which has to be improved: some conditions in the new CTC has to be precised, and more simulations have to be performed. Moreover, absence of Zeno behavior has not been shown.

In multi-agent systems, time-varying delays may arise naturally due to the distance between agents, to the temporary unavailability of the channel, to the time to encode and decode data in the messages broadcast, to the limited capacity of the channel, which incurs a nonzero transmission duration. This communication delay has to be taken into account by the agents, to avoid updating their state estimators with outdated information. The proposed approach takes into account various sources of communication delays. One assumes that the communication delay may be upper bounded and that this upper bound is known and is the same for all agents. The CTCs studied in previous chapters guarantee the convergence of the MAS provided that all state estimators are updated instantaneously after the transmission, which is also assumed without delay. To account for a bounded delay, communication has to be anticipated compared to the delay-free case. Moreover, in order to keep all state estimators synchronized, since the actual communication delays of different agents may differ, the state estimators have to be updated only once all agents have received the transmitted packet.

The proposed technique is inspired from [84], which describes a logic-based approach for path-following while holding a formation pattern of a network of robotic vehicles in presence of bounded communication delays. The CTC proposed in Theorem 12 has been adapted to account for bounded communication delays. A communication protocol and a prediction of the state of all agents are described to allow a practical implementation of the proposed technique.

The problems induced by communication delays and their impact in the broadcast packet content is exposed in Section 6.1. The estimation and prediction of the states of the agents is exposed in Section 6.3. The adapted CTC, which aims is to mitigate the communication delay, is presented in Section 6.2. A simulation example is considered in Section 6.4 to illustrate the reduction of the communications obtained by the proposed approach. Finally, conclusions are drawn in Section 6.5.

Notations exposed in Chapter 4 are used in this chapter.

6.1 Transmission delays and broadcast packet contents

One assumes in what follows that the clock of all agents are perfectly synchronized.

Let $\tau_{ij} = t^j_{i,k} - t_{i,k}$ be the delay between the time $t = t_{i,k}$, at which Agent i has broadcast a message and the instant $t = t_{i,k}^{j}$ at which Agent j has received it. Assume that for all pair of agents (i, j), τ_{ij} can be upper-bounded by a known constant delay τ_d.

In Theorems 8 and 11, conditions ensure the convergence and stability of the MAS if a CTC is satisfied at time $t_{i,k}$, the estimated state $\hat{x}_i^j\left(t_{i,k}^j\right)$ is reset at time $t_{i,k}^j$ using $x_i\left(t_{i,k}^j\right)$. Thus, to satisfy this condition in presence of communication delay, a message containing $x_i\left(t_{i,k}^j\right)$ must be broadcast at time $t = t_{i,k} \leq t_{i,k}^j - \tau_{ij}$. Nevertheless, $x_i\left(t_{i,k}^j\right)$ cannot be known at $t = t_{i,k}$. To address this issue, a prediction $\tilde{x}_i^i\left(t_{i,k}^j\right)$ of the state $x_i\left(t_{i,k}^j\right)$ made by Agent i must be evaluated and transmitted.

Let $\tilde{x}_i^i\left(t + \tau_d\right) \in \mathbb{R}^n$ be the prediction at time $t + \tau_d$ of the state $x_i\left(t + \tau_d\right)$ made by Agent i at time t. The prediction model will be studied in Section 6.1.

6.1.1 Communication protocol

When a communication is triggered at $t_{i,k}$ by Agent i, it broadcasts a message containing $t_{\text{up},\,i} = t_{i,k} + \tau_d$, $\tilde{q}_i^i\left(t_{i,k} + \tau_d\right)$, $\dot{\tilde{q}}_i^i\left(t_{i,k} + \tau_d\right)$ and its prediction $\tilde{\theta}_i^i\left(t_{i,k} + \tau_d\right)$, which is the prediction of its estimated $\bar{\theta}_i$. We assume that this message is received by all other agents, either directly when the network is fully connected, or after several hops when the network is only connected.

6.1.2 Estimators update and synchronization

In Theorems 8 and 11, agents have also to have synchronized state estimators satisfying $\hat{x}_i^i(t) = \hat{x}_i^j(t)$ $\forall (i, j) \in \mathcal{N}$. Since $\tau_{ij}(t)$ is unknown, may be time varying, but is such that $\tau_{ij}(t) < \tau_d$, agents have to update their estimate of x_i at time $t_{i,k} + \tau_d$, when all agents have received the message. Thus, using the value $t_{\text{up},\,i}$ transmitted in Agent i message, all state estimators are synchronized at the same instant, see Figure 6.1. One obtains if $\tilde{\alpha}_{i,k}^j = 0$

$$
\begin{aligned}
\hat{x}_i^j\left(t_{\text{up},\,i}\right) &= \tilde{x}_i^i\left(t_{i,k} + \tau_d\right) \quad \forall j \in \mathcal{N}, && (6.1)\\
\hat{x}_i^i\left(t_{\text{up},\,i}\right) &= \tilde{x}_i^i\left(t_{i,k} + \tau_d\right). && (6.2)
\end{aligned}
$$

The main drawback of this approach lies in the fact that estimators are updated using a prediction of the state x_i and not with its actual value. Since there always exists a discrepancy between the prediction and the current state value, the estimation error $e_i^i\left(t_{i,k} + \tau_d\right)$ does not vanish at $t = t_{i,k} + \tau_d$. Using the prediction model, one should be able to upper-bound the estimation error such as $\left\|e_i^i\left(t_{i,k} + \tau_d\right)\right\| \leq \epsilon$. The absence of Zeno behavior must be proved despite the presence of ϵ.

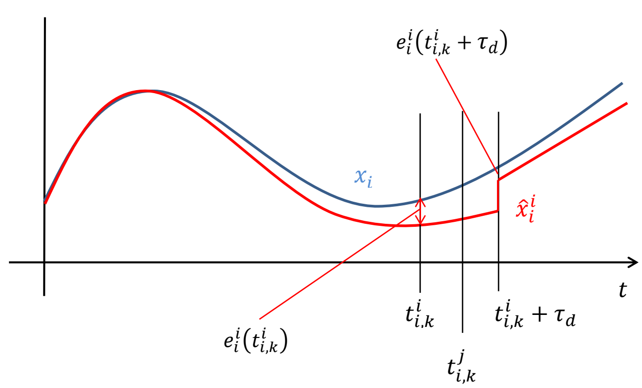
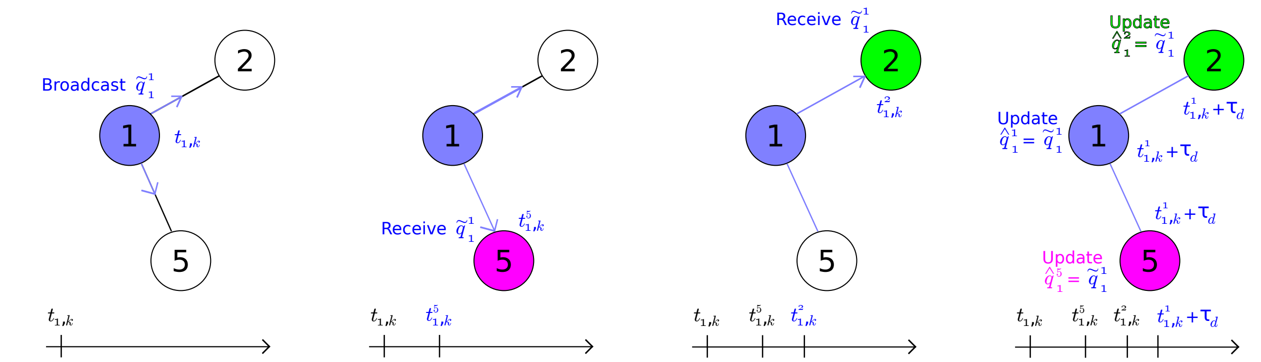


Figure 6.1: Comparison between x_i and $\hat{x}_i^i$ in presence of communication delay



(a) CTC satisfied for Agent 1 at $t = t_{1,k}$. (b) Message received by Agent 5 (c) Message reveived by Agent 2. (d) Update of estimators $\hat{q}_i^j$ for $j \in \mathcal{N}$.

Figure 6.2: Communication protocol with communication delay

6.2 Distributed event-triggered strategy with communication delay

As explained in Section 6.1, a message has to be broadcast earlier to account for communication delays. Ideally, each agent should be able to detect not later than at t if the CTC (6.6) will be satisfied at time $t + \tau_d$. A new CTC is defined using the predictions of $\bar{s}_i(t+\tau_d)$, $\bar{g}_i(t+\tau_d)$, $e_i^i(t+\tau_d)$, $\dot{e}_i^i(t+\tau_d)$, $\dot{q}_i^i(t+\tau_d)$, $\dot{\hat{q}}_i^i(t+\tau_d)$, $\bar{p}_i(t+\tau_d)$, and $\dot{\bar{p}}_i(t+\tau_d)$.

Theorem 12. *Consider a MAS with agent dynamics given by (4.1) and the control input (4.13). Consider some positive design parameters η, η_2, β_e, $\beta_{\dot{e}}$, β_g, β_s, β_q, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$,*

$$c_3 = \frac{\min\left\{1, k_1, k_p, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)\right\}}{\max\left\{1, k_M\right\}} \tag{6.3}$$

and $k_1 = k_s - (1 + k_p(k_M + 1))$. In presence of communication delays, the system (4.1) is input-to-state practically stable (ISpS) and the agents can be driven to some target formation such that

$$\lim_{t\to\infty} \sum_{i=1}^{N} k_0 \left\|r_i\right\|^2 + \frac{1}{2} P(q,t) \leq \xi \tag{6.4}$$

with

$$\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right] \tag{6.5}$$

where $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i\right)\right)$, if the communications are triggered when one of the following conditions is satisfied

$$\begin{aligned}
k_s \left\|\tilde{s}_i^i(t+\tau_d)\right\|^2 + k_p k_g \left\|\tilde{g}_i^i(t+\tau_d)\right\|^2 + \eta \leq \alpha_M^2 &\left(k_e \left\|\tilde{e}_i^i(t+\tau_d)\right\|^2 + k_p k_M \left\|\dot{\tilde{e}}_i^i(t+\tau_d)\right\|^2\right) \\
&+ \alpha_M k_C^2 k_p \left\|\tilde{e}_i^i(t+\tau_d)\right\|^2 \sum_{j=1}^{N} k_{ji}\left[\left\|\dot{\tilde{q}}_j^i(t+\tau_d)\right\| + \eta_2\right]^2 \\
&+ k_g b_i \left\|\dot{\tilde{q}}_i^i(t+\tau_d) - \dot{q}_i^*(t+\tau_d)\right\|^2 \\
&+ k_p \left\|\tilde{e}_i^i(t+\tau_d)\right\|^2 \left[\alpha_M^2\left(1 + \left\||Y_i|\,\Delta\tilde{\theta}_{i,\max}\right\|^2\right) + \frac{\left\||Y_i|\,\Delta\tilde{\theta}_{i,\max}\right\|^2}{\left(1 + \left\||Y_i|\,\Delta\tilde{\theta}_{i,\max}\right\|^2\right)}\right]
\end{aligned} \tag{6.6}$$

$$\left\|\dot{\tilde{q}}_i^i\right\| \geq \left\|\dot{\tilde{\tilde{q}}}_i^i\right\| + \eta_2 \tag{6.7}$$

with $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, $Y_i = Y_i\left(\tilde{q}_i\left(t+\tau_d\right), \dot{\tilde{q}}_i\left(t+\tau_d\right), \dot{\tilde{p}}_i\left(t+\tau_d\right), \tilde{p}_i\left(t+\tau_d\right)\right)$, *and*

$$\Delta\tilde{\theta}_{i,\max} = \begin{bmatrix} \max\left\{\left|\tilde{\theta}_{i,1}^i\left(t+\tau_d\right) - \theta_{\min,i1}\right|, \left|\tilde{\theta}_i^i\left(t+\tau_d\right) - \theta_{\max,i1}\right|\right\} \\ \vdots \\ \max\left\{\left|\tilde{\theta}_{i,p}^i\left(t+\tau_d\right) - \theta_{\min,ip}\right|, \left|\tilde{\theta}_p^i\left(t+\tau_d\right) - \theta_{\max,ip}\right|\right\} \end{bmatrix}. \tag{6.8}$$

and if $\forall t \geq 0$ *the following conditions are satisfied*

$$\left\|\bar{s}_i\left(t\right)\right\|^2 + \beta_s \geq \left\|\tilde{s}_i^j\left(t\right)\right\|^2 \tag{6.9}$$

$$\left\|\bar{g}_i\left(t\right)\right\|^2 + \beta_g \geq \left\|\tilde{g}_i^j\left(t\right)\right\|^2 \tag{6.10}$$

$$\left\|\dot{\tilde{q}}_i^i\left(t\right) - \dot{q}_i^*\left(t\right)\right\|^2 \geq \left\|\dot{q}_i\left(t\right) - \dot{q}_i^*\left(t\right)\right\|^2 - \beta_q \tag{6.11}$$

$$\left\|\tilde{e}_i^i\left(t\right)\right\|^2 \geq \left\|e_i^i\left(t\right)\right\|^2 - \beta_e \tag{6.12}$$

$$\left\|\dot{\tilde{e}}_i^i\left(t\right)\right\|^2 \geq \left\|\dot{e}_i^i\left(t\right)\right\|^2 - \beta_{\dot{e}}. \tag{6.13}$$

Proof. If the conditions (6.9)-(6.13) are satisfied for all $t \geq 0$, thus the triggering conditions (6.6)-(6.7) are satisfied when the triggering condition (4.26)-(4.27) of Theorem 8 evaluated at the time $t+\tau_d$ are satisfied. Thus, since the communication are triggered at time $t_{i,k}$ using the conditions of Theorem 12, conditions of Theorem 11 will be satisfied at time $t_{i,k}+\tau_d$. Thus, the system is ISpS and the agents can be driven to some target formation such that $\lim_{t\to\infty}\sum_{i=1}^N k_0\left\|r_i\right\|^2 + \frac{1}{2}P\left(q,t\right) \leq \xi$ with $\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]$. □

The values of $\beta_e, \beta_{\dot{e}}, \beta_g, \beta_s$, and β_q must be chosen enough small to avoid useless communications due to the CTC (6.6) while Conditions (6.9)-(6.13) are satisfied. These values mostly depend of the accuracy of the prediction model evaluating $\tilde{s}_i^i$, $\tilde{g}_i^i$, $\dot{\tilde{q}}_i$, $\tilde{e}_i^i$, and $\dot{\tilde{e}}_i^i$.

The prediction $\tilde{e}_i^i$ and $\dot{\tilde{e}}_i^i$ have to take in account the reset of estimator at $t_{i,k}+\tau_d$, else the CTC (6.6) will be continuously triggered over the interval $[t_{i,k}, t_{i,k}+\tau_d[$. However, absence of Zeno behavior has not been shown, and conditions $\beta_e, \beta_{\dot{e}}, \beta_g, \beta_s$ has to be precised.

6.3 Prediction model

6.3.1 Prediction via Euler integration

Using a basic Euler integration, one easily obtains a prediction model with the following form $y\left(t+\tau_d\right) = y\left(t\right) + \tau_d\dot{y}\left(t\right)$. Let's study it.

Prediction of $x_i\left(t+\tau_d\right)$, $\hat{x}_j^i\left(t+\tau_d\right)$ $\forall j \neq i$ $\forall t \geq 0$, and $\breve{x}_j^i\left(t+\tau_d\right)$, $\forall j \in \mathcal{N}$ With Euler integration, the agent dynamics and their control inputs are not taken into account in the prediction model. Thus, the prediction of the state of other agents evaluated by Agent i is expressed for all t as

$$\tilde{x}_i^i\left(t+\tau_d\right) = x_i\left(t\right) + \dot{x}_i\left(t\right)\tau_d. \tag{6.14}$$

Similarly, the prediction of $\hat{x}_j^i$ is expressed as

$$\tilde{\hat{x}}_j^i\left(t+\tau_d\right) = \hat{x}_j^i\left(t\right) + \dot{\hat{x}}_j^i\left(t\right)\tau_d. \tag{6.15}$$

With these basic prediction method, the updates performed on the estimates $\hat{x}_j^i$ between time t and time $t+\tau_d$ are not known by Agent i, and thus cannot be taken into account, leading to discrepancies between the predictions and the actual value of the predicted variable.

Nevertheless, information on the future updates of $\hat{x}_i^i$ are known by Agent i. It may be taken into account to obtain a more accurate prediction $\tilde{\hat{x}}_i^i$, described in the following section.

Prediction of $\tilde{\hat{x}}_i^i(t+\tau_d)$ When $t \geq t_{i,k}+\tau_d$, all messages broadcast by Agent i has been received and estimators for all agents have been synchronized. Thus, the prediction model can be expressed as

$$\tilde{\hat{x}}_i^i(t+\tau_d) = \hat{x}_i^i(t)+\dot{\hat{x}}_i^i(t)\tau_d \quad t\in[t_{i,k}+\tau_d,\, t_{i,k+1}[\tag{6.16}$$

However, the synchronization (6.1)-(6.2) induces that all agents update their estimate of Agent i state at the time $t = t_{i,k}+\tau_d$ using the prediction $\tilde{x}_i^i(t_{i,k}+\tau_d)$, *i.e.* $\forall j \in \mathcal{N}\ \hat{x}_i^j(t_{i,k}+\tau_d) = \tilde{x}_i^i(t_{i,k}+\tau_d)$. Thus, since Agent i will update $\hat{x}_i^i$ at the instant $t = t_{i,k}+\tau_d$,$\tilde{\hat{x}}_i^i(t+\tau_d)$ can be expressed as

$$\begin{aligned}\tilde{\hat{x}}_i^i(t+\tau_d) &= \tilde{x}_i^i(t_{i,k}+\tau_d)+\dot{\tilde{x}}_i^i(t_{i,k}+\tau_d)\left((t+\tau_d)-(t_{i,k}+\tau_d)\right)\\ &= \left[x_i(t_{i,k})+\dot{x}_i(t_{i,k})\tau_d\right]+\left[\dot{x}_i(t_{i,k})+\ddot{x}_i(t_{i,k})\tau_d\right](t-t_{i,k})\end{aligned} \tag{6.17}$$

where $\ddot{x}_i(t_{i,k}) = \left[\ddot{q}_i(t_{i,k})^T,\, 0_n^T\right]$. Thus, prediction of $\tilde{\hat{x}}_i^i$ can be rewritten for all t as

$$\tilde{\hat{x}}_i^i(t+\tau_d) = x_i(t_{i,k})+\dot{x}_i(t_{i,k})(\tau_d+t-t_{i,k})+\ddot{x}_i(t_{i,k})\tau_d(t-t_{i,k}). \tag{6.18}$$

Note that $\tilde{\hat{x}}_i^i(t+\tau_d)$ is only used by Agent i to evaluate its CTCs (6.6) and (6.7), and has not to be performed by others agents.

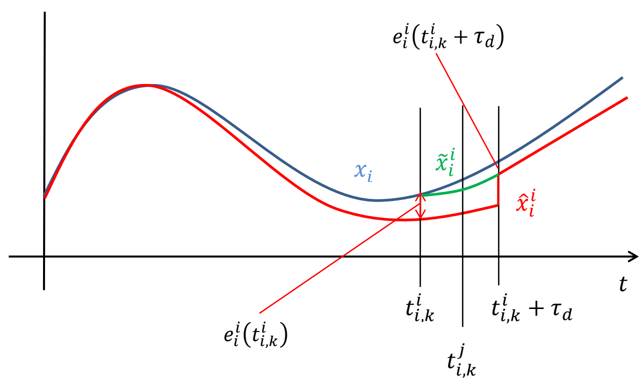


Figure 6.3: Comparison between x_i, $\hat{x}_i^i$ and $\tilde{x}_i^i$ in presence of communication delay. $\tilde{x}_i^i$ is evaluate off-line at time $t_{i,k}^i$ and its processing time τ_p must be managed during the time-slot τ_d such as $\tau_d > \tau_p+\tau_{ij}(t)$.

Other prediction The predicted quantities $\tilde{e}_i^i(t+\tau_d)$, $\dot{\tilde{e}}_i^i(t+\tau_d)$, $\dot{\tilde{q}}_j^i(t+\tau_d)$, $\dot{\tilde{\hat{q}}}_j^i(t+\tau_d)$, $\tilde{g}_i^i(t+\tau_d)$, and $\tilde{s}_i^i(t+\tau_d)$ are then deduced from $\tilde{x}_i^i(t+\tau_d)$, $\tilde{\hat{x}}_i^i(t+\tau_d)$, and $\tilde{\hat{x}}_j^i(t+\tau_d)$.

The main advantage of this approach is its limited processing requirements. Nevertheless, the maximum tolerable delay τ_d has to be small enough compared to have a prediction $\tilde{x}_i^i$ close to the actual state value x_i.

Section 6.3.2 proposes a more accurate predictor and analyzes its advantages and its drawbacks.

6.3.2 Accurate prediction

To stay close to the agent behavior, the dynamic of the prediction state $\tilde{x}_i^i$ can be expressed as

$$\hat{M}_i^i(\tilde{q}_i^i)\ddot{\tilde{q}}_i^i+\hat{C}_i^i(\tilde{q}_i^i,\dot{\tilde{q}}_i^i)\dot{\tilde{q}}_i^i = \tilde{\tau}_i^i+\tilde{d}_i \tag{6.19}$$

$$\tilde{x}_j^i(t_{ini}) = \hat{x}_j^i(t_{ini}) \text{ if } j\neq i, \tag{6.20}$$

$$\tilde{x}_i^i(t_{ini}) = x_i(t_{ini}) \tag{6.21}$$

where t_{ini} is the time at which the prediction is evaluated, $\hat{x}_j^i = \left[\hat{q}_j^{iT}, \dot{\hat{q}}_j^{iT}\right]^T$, $\tilde{\tau}_i^i$ is the predicted control input, and $\tilde{d}_i$ an prediction of the state perturbation, equal, *e.g.*, to its mean value. As the matrices M_i and C_i are unknown by agents, taking $\hat{M}_i^i$ and $\hat{C}_i^i$ induce a discrepancy between the prediction and the actual state value.

Focus now on the predicted control input $\tilde{\tau}_i^i$. A simple way to evaluate $\tilde{\tau}_i^i$ can be to choose it as constant: $\tilde{\tau}_i^i(t) = \tau_i^i(t_{i,k})$. This predicted control input is easy to implement but leads to a discrepancy between τ_i^i and $\tilde{\tau}_i^i$, and so between $\tilde{x}_i^i$ and x_i.

An alternative way is to choose the predicted control $\tilde{\tau}_i^i$ as in (5.9) where $\hat{x}_i^i$ is replaced by $\tilde{x}_i^i$. To evaluate it, a prediction of $\hat{q}^i$ and $\dot{\hat{q}}^i$ is required. Consider the prediction of the state estimate $\tilde{\hat{x}}_j^i = \left[\tilde{\hat{q}}_j^{iT}, \dot{\tilde{\hat{q}}}_j^{iT}\right]^T$ evaluated starting from $t = t_{ini}$ using the dynamical model

$$\hat{M}_j^i\left(\tilde{\hat{q}}_j^i\right)\ddot{\tilde{\hat{q}}}_j^i + \hat{C}_j^i\left(\tilde{\hat{q}}_j^i, \dot{\tilde{\hat{q}}}_j^i\right)\dot{\tilde{\hat{q}}}_i^i = \tilde{\hat{\mathfrak{q}}}_j^i \tag{6.22}$$

$$\tilde{\hat{x}}_j^i(t_{ini}) = \hat{x}_j^i(t_{ini}) \quad \forall j \in \mathcal{N} \tag{6.23}$$

$$\tilde{\hat{x}}_i^i(t_{i,k} + \tau_d) = \tilde{x}_i^i(t_{i,k} + \tau_d) \tag{6.24}$$

where $\tilde{\hat{\mathfrak{f}}}_j$ is the prediction of the estimate control performed using (4.18) where the vector $\hat{q}^i$ is replaced by $\tilde{\hat{q}}^i$. Here, (6.24) expresses the fact that $\hat{x}_i^i$ is updated by Agent i at time $t = t_{i,k} + \tau_d$, as in Section 6.3.1.

Then, the predicted control $\tilde{\tau}_i^i$ is performed as in (4.18) where x_i^i is replaced by $\tilde{x}_i^i$ and $\hat{x}^i$ by $\tilde{\hat{x}}^i$.

Using the previous predictions, the predicted errors $\tilde{e}_i^i$ and $\dot{\tilde{e}}_i^i$ may be evaluated as

$$\tilde{e}_i^i(t+\tau_d) = \tilde{\hat{q}}_i^i(t+\tau_d) - \tilde{q}_i^i(t+\tau_d) \tag{6.25}$$

$$\dot{\tilde{e}}_i^i(t+\tau_d) = \dot{\tilde{\hat{q}}}_i^i(t+\tau_d) - \dot{\tilde{q}}_i^i(t+\tau_d). \tag{6.26}$$

In the same way, the predicted quantities $\tilde{g}_i^i$, $\tilde{s}_i^i$ used in Theorem 12 can be expressed as

$$\tilde{g}_i^i = \sum_{i=1}^{N} k_{ij}\left(\tilde{q}_i^i - \tilde{\hat{q}}_i^i - r_{ij}^*\right) \tag{6.27}$$

and

$$\tilde{s}_i^i = \dot{\tilde{q}}_i^i - q_i^* + k_p\tilde{g}_i^i. \tag{6.28}$$

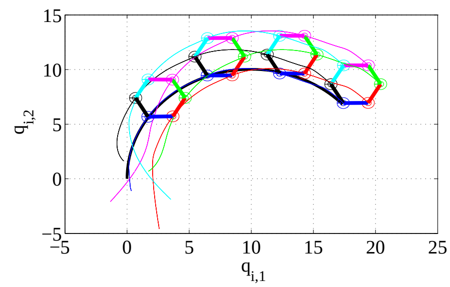
This approach leads to an accurate prediction of x_i. The main drawback for this method is that a prediction of all agents state is needed to evaluate $\tilde{\tau}_i^i(t)$ for all $t \in \left]t_{j,k}^i, t_{j,k+1}^i\right]$, which involves significant processing efforts. Moreover, the processing time τ_p must be managed during the time-slot τ_d such as $\tau_d > \tau_p + \tau_{ij}(t)$. This makes such approach more difficult to implement in practice the technique presented in Section 6.3.1.

6.4 Example with communication delay

Consider the same dynamics, coefficients, simulation parameters, and tracking trajectory than those considered in Section 4.5. A constant communication delay for all agents as $\tau_{ij} = 0.03\ \forall(i,j)$ and τ_d is taken such $\tau_d = 0.03$.

Figure 6.4 shows the trajectories of the agents when the control input (4.13) is applied to obtain a desired formation and tracking a reference trajectory. The CTCs defined in Theorem 12 and the prediction model exposed in Section 6.3.1 are used. It can be seen that agents converge to the desired formation and reference trajectory with a limited number of communications, even in presence of perturbation and communication delay. The communication ratio R_{com} is larger in presence of communication delay than without communication as exposed in Section 4.5. This is due to the CTC defined in Theorem 12 which is more restrictive than the CTC introduced in Theorem 8. Moreover, the predicted values used in the CTC induce a larger discrepancy between the estimated and the actual state values, which lead to a a CTC which is more likely to be satisfied.

Clearly, additional simulations would be required to finish this study.



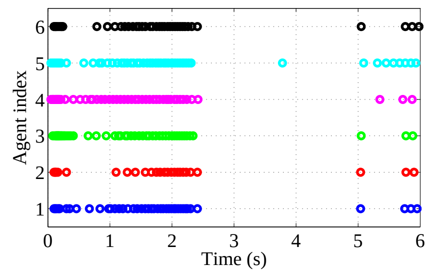


Figure 6.4: Hexagonal formation and tracking problem with $D_{\max} = 20$, $\eta = 20$, $\eta_3 = 10$ and $\tau_d = 0.03$. Agents are represented by circles. Model (4.39) and estimator (4.18) are considerate. $R_{\text{com}} = 7.41\%$, $P(q) = 0.001$ and $\|\varepsilon_0\| = 0.14$.

6.5 Conclusion

This chapter presents problem of communication delay, adapted to methods proposed in previous parts. Influence of communication delay on the message content has been studied. To balance effect of communication delay, a prediction value of agent state is transmit to others agents to update their estimators in a synchronized way. The CTC has been adapted to take in account the communication delay and trigger earlier such that compensate it. Two prediction models of different complexity and accuracy have been considered have been proposed. Convergence to the target formation and target position has been studied. However, absence of Zeno behavior have not been shown.

In future work, the considered problem will be extended to problem of time-varying topology and control saturation. Combine method with packet dropout effect studied in previous chapter is also considerate. Moreover, some conditions in the CTC has to be precised.

Chapter 7

Conclusions and Perspectives

Conclusions

In this thesis, event-triggered communication techniques have been proposed to decrease the number of communication to be transmitted in a multi-agent system driven by distributed cooperative control laws. The agents are mobile autonomous vehicles moving in an unknown environment. They dispose of their own means of measurements to measure their own state values and rely on communication link to obtain information on the state values or processed data of their neighboring agents. The communication links are summarized via a connection graph.
The main contributions of this thesis are twofold:

- First, a distributed event-triggered communication technique to reach consensus with a reduced number of communication using a general linear dynamic model with state perturbations has been developed and the results obtained compared with those of other approaches.
- Second, an event-triggered strategy has been developed to manage flocking and trajectory tracking for a fleet of vehicles modeled with Euler-Lagrange dynamics equations with state perturbation. The problem complexity is regularly increased to take into account uncertainty on the model dynamics, communication delay and packet dropout.

The first approach considers agents with general linear dynamic model, state perturbations and a fixed communication graph without communication delay. The method relies on the simultaneous use of two estimators of the states of the other agents in the network. The first provides an accurate state estimate of all agents in the fleet by introducing a dynamical observer of the states including the control inputs. The second estimator considers only the agents in the neighborhood of each agent and is less accurate because updated less frequently than the first estimator. However, its value is constrained to coincide when two agents are neighbors. The output errors of both estimators are used in the expression of the triggering condition. Flooding delay communication protocol has been developed to guarantee the reset of estimators error without adding broadcast message to the initial strategy. Conditions between the perturbation level and the consensus error are defined. Convergence to consensus has been studied and absence of Zeno behavior proved. Simulations have shown the effectiveness of the proposed estimators in presence of state perturbations with moderate level and enabled comparisons with the results obtained using the state-of-art method. Influence of the knowledge of the initial conditions has been exposed. A guideline to select some design parameters to obtain a trade-off between communication constraints and bound on the consensus disagreement has been proposed. Finally, extensions of this results to time-varying linear dynamics model and the case of a time-varying topology have been discussed.
The second method is dedicated to the development of an event-triggered communication strategy to reach a target formation for MAS with Euler-Lagrange dynamics and state perturbations. Two estimators of different complexity and accuracy, inspired from the previous technique, have been considered to provide the missing information required by the control, allowing a trade-off between computation time and amount of triggered communications. A distributed event-triggered condition has been proposed to limit the number of communication while guaranteeing convergence to the target formation with a bounded error. Convergence to a desired formation and influence of state perturbations on the convergence and on

the amount of required communications have been studied. Moreover, the time interval between consecutive communications has been shown to be strictly positive. Simulations have shown the effectiveness of the proposed method in presence of state perturbations when their level remains moderate.
Extension to the initial method to cases with uncertainty on the parameters of the dynamical model has been developed. On-line identification of the model parameters has been proposed to provide the missing information required by the control law. Tracking control to follow an reference trajectory has been considered and added to the formation control input. A distributed event-triggered condition to converge to a desired formation and follow the reference trajectory while reducing the number of communications is presented. Simulations illustrated the effectiveness of the proposed method in presence of state perturbations. A guideline to select some design parameters to obtain a trade-off between communications constraints and the bounded error of the target formation and tracking trajectory is proposed. The time interval between consecutive communications has been shown to be strictly positive.
Communication delay and packet dropouts have also been studied. For the first case, two prediction models of different complexity and accuracy have been considered. Convergence to the target formation and target position has been studied. To account for potential packet dropouts, adaptations of the estimator structure and of the triggering conditions to the stochastic characteristics of the occurrence of loss of information has been performed by considering the expected value of the estimate error due to loss of information. To guarantee absence of Zeno behavior, a specific communication protocol has been developed.

Perspectives

Several mid-term and long term directions are proposed below.

Modeling of packet dropout is based on assumption of loss of information to be mutually independent Bernoulli-distributed, and solutions have been designed using these characteristics. In practice, packet dropout can also be represented by Markov chain: a lost message can be due to the presence of an obstacle or a receiver failure, which results in the fact that the events are not independent. Adaptations of triggering strategies to more realistic loss probabilities could be of interest in order to increase the robustness of the approach to this issue. Introduction of time delays leads to modification of the communication protocols but some conditions in the CTC need to be relaxed to obtain a less pessimistic updating frequency. Moreover, accounting for communication delays and packet dropouts in a joint manner would constitue an important improvement. A potential way of handling both could be to generalize the probabilistic description of the packet dropout to the time of arrival of a message.

In all event-triggered communication methods proposed in this thesis, it is assumed the CTCs are continuously evaluated. Since MAS are generally sampled-data systems, event-triggered methods based on discrete sampling characteristics are more practical. Combining event-triggered techniques and periodic sampled-data control will allow to be closer to a real system where condition is evaluated at discrete time instant.

Finally, each agent has been assumed to measure its own state values without error which constitutes a very unrealistic condition. State observer has to be introduced and impact of a noise measurement needs to be studied. Modelling the measurement uncertainty using bounded error context could be a way to integrate this additional perturbation in the global triggering condition but may lead to pessimistic decisions and increase of the number of trigger. Extension of the works presented in [50, 95, 111, 120, 68], to the case of dynamical models of agents could proved to be an interesting direction.

Bibliography

[1] N. Abramson. The aloha system: another alternative for computer communications. In *Proceedings of the November 17-19, fall joint computer conference*, pages 281–285. ACM, 1970.

[2] A. Adaldo, F. Alderisio, D. Liuzza, G. Shi, D.V. Dimarogonas, M. Di Bernardo, and K.H. Johansson. Event-triggered pinning control of switching networks. In *IEEE Transactions on Control of Network Systems*, volume 2, pages 204–213, 2015.

[3] A. P. Aguiar and A. M. Pascoal. Coordinated path-following control for nonlinear systems with logic-based communication. In *IEEE Decision and Control Conference*, pages 1473–1479, 2007.

[4] A.Y. Alfakih and V.H. Nguyen. On affine motions and universal rigidity of tensegrity frameworks. In *Linear Algebra and its Applications*, volume 439, pages 3134–3147, 2013.

[5] R. Aragues, J. Cortes, and C. Sagues. Distributed consensus algorithms for merging feature-based maps with limited communication. In *Robotics and Autonomous Systems*, volume 59, pages 163–180, 2011.

[6] F.J.M Arboleda, J. Alberto G. Luna, S.A.G. Arias, et al. Identification of v-formations, circular, and doughnut formations in a set of moving entities with outliers. In *Abstract and Applied Analysis*. Hindawi Publishing Corporation, 2013.

[7] M Arboleda, F. Javier, G. Luna, J. Alberto, G. Arias, and S. Alonso. Identification of v-formations and circular and doughnut formations in a set of moving entities with outliers. In *Abstract and Applied Analysis*, volume 1. Hindawi Publishing Corporation, 2014.

[8] D. Bauso, L. Giarré, and R. Pesenti. Attitude alignment of a team of uavs under decentralized information structure. In *IEEE Conference on Control Applications (CCA)*, volume 1, pages 486–491, 2003.

[9] G. Bianchi, L. Fratta, and M. Oliveri. Performance evaluation and enhancement of the csma/ca mac protocol for 802.11 wireless lans. In *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, volume 2, pages 392–396, 1996.

[10] P-A. Bliman and G. Ferrari-Trecate. Average consensus problems in networks of agents with delayed communications. In *Automatica*, volume 44, pages 1985–1995, 2008.

[11] J. Boada. *Satellite control with saturating inputs.* PhD thesis, ISAE, 2010.

[12] X. Cai and M. de Queiroz. Adaptive rigidity-based formation control for multirobotic vehicles with dynamics. In *IEEE Transactions on Control Systems Technology*, volume 23, pages 389–396, 2015.

[13] Y. Cao and W. Ren. Distributed coordinated tracking with reduced interaction via a variable structure approach. In *IEEE Transactions on Automatic Control*, volume 57, pages 33–48, 2012.

[14] Z. Chao, L. Ming, Z.. Shaolei, and Z. Wenguang. Collision-free uav formation flight control based on nonlinear mpc. In *Proc. IEEE Electronics, Communications and Control (ICECC)*, pages 1951–1956, 2011.

[15] Z. Chao, S.-L. Zhou, L. Ming, and W-G Zhang. Uav formation flight based on nonlinear model predictive control. In *Mathematical Problems in Engineering*, volume 1, pages 1–15. Hindawi Publishing Corporation, 2012.

[16] Zhou Chao, Lei Ming, Zhou Shaolei, and Zhang Wenguang. Collision-free uav formation flight control based on nonlinear mpc. In *International Conference on Electronics, Communications and Control (ICECC)*, pages 1951–1956, 2011.

[17] C. Chern Cheah, S.P. Hou, and J.J.E. Slotine. Region-based shape control for a swarm of robots. In *Automatica*, volume 45, pages 2406–2411, 2009.

[18] Y.Q. Chen and Z. Wang. Formation control: a review and a new consideration. In *IEEE International Conference on Intelligent Robots and Systems (IROS)*, pages 3181–3186, 2005.

[19] L. Cheng, Z. Hou, and M. Tan. A mean square consensus protocol for linear multi-agent systems with communication noises and fixed topologies. In *IEEE Transactions on Automatic Control*, volume 59, pages 261–267, 2014.

[20] Teng-Hu Cheng, Zhen Kan, Justin R Klotz, John M Shea, and Warren E Dixon. Decentralized event-triggered control of networked systems-part 1: Leader-follower consensus under switching topologies. In *American Control Conference (ACC)*, volume 13 of *3*, pages 5438–5443, 2015.

[21] J. Cortes and S. Martinez. *Distributed Control of Robotic Networks.* Princeton University Press, 2009.

[22] B. Das, B. Subudhi, and B.B. Pati. Cooperative formation control of autonomous underwater vehicles: An overview. In *International booktitle of Automation and Computing*, volume 13, pages 199–225. Springer, 2016.

[23] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson. Distributed event-triggered control for multi-agent systems. In *IEEE Transactions on Automatic Control*, volume 57, pages 1291 –1297, 2012.

[24] D. V. Dimarogonas and K. H. Johansson. Event-triggered control for multi-agent systems. In *Proceedings of IEEE Conference on Decision and Control, CDC/CCC*, pages 7131–7136, 2009.

[25] Derui Ding, Zidong Wang, Bo Shen, and Hongli Dong. Event-triggered distributed h state estimation with packet dropouts through sensor networks. In *IET Control Theory & Applications*, volume 9, pages 1948–1955, 2015.

[26] K-D Do. Formation tracking control of unicycle-type mobile robots with limited sensing ranges. In *IEEE Transactions on Control Systems Technology*, volume 16, pages 527–538, 2008.

[27] A. D. Dominguez-Garcia and C. N. Hadjicostis. Coordination and control of distributed energy resources for provision of ancillary services. In *First IEEE International Conference on Smart Grid Communications (SmartGridComm)*, pages 537–542, 2010.

[28] L. Dong, X. Zhong, C. Sun, and H. He. Predictive event-triggered control based on heuristic dynamic programming for nonlinear continuous-time systems. In *International Joint Conference on Neural Networks (IJCNN)*, pages 1–7, 2015.

[29] Y. Fan and G. Feng. Event-triggered control of multi-agent systems with suboptimal triggering. In *Control Conference (ASCC)*, pages 1–6, 2013.

[30] Y. Fan, G. Feng, and Y. Wang. Event-triggered control of multi-agent systems by combined measurement approach. In *Chinese Control Conference (CCC)*, pages 6250–6255, 2012.

[31] Y. Fan, G. Feng, and Y. Wang. Distributed event-triggered control of multi-agent systems with combinational measurements. In *Automatica*, volume 49, pages 671 – 675, 2013.

[32] A. J. Fax and R. M. Murray. Information flow and cooperative control of vehicle formations. In *IEEE Transactions on Automatic Control,*, volume 49, pages 1465–1476, 2004.

[33] E. Ferrante, A.E. Turgut, A. Stranieri, C. Pinciroli, M. Birattari, and M. Dorigo. A self-adaptive communication strategy for flocking in stationary and non-stationary environments. In *Natural Computing*, volume 13, pages 225–245. Springer, 2014.

[34] E. Garcia, Y. Cao, and D.W. Casbeer. Model-based event-triggered multi-vehicle coordinated tracking control using reduced order models. In *booktitle of the Franklin Institute*, volume 351, pages 4271–4286, 2014.

[35] E. Garcia, Y. Cao, X. Wang, and D. W. Casbeer. Cooperative control with general linear dynamics and limited communication: Centralized and decentralized event-triggered control strategies. In *American Control Conference (ACC)*, volume 1, pages 159–164, 2014.

[36] E Garcia, Y Cao, X Wang, and D. W. Casbeer. Cooperative control with general linear dynamics and limited communication: Periodic updates. In *American Control Conference (ACC)*, volume 1, pages 3195–3200, 2014.

[37] E. Garcia, Y. Cao, X. Wang, and D. W. Casbeer. Decentralized event-triggered consensus with general linear dynamics. In *Automatica*, volume 50, pages 2633–2640, 2014.

[38] E. Garcia, Y. Cao, X. Wang, and D. W. Casbeer. Event-triggered cooperative control with general linear dynamics and communication delays. In *IEEE Annual Conference on Decision and Control (CDC)*, volume 1, pages 2914–2919, 2014.

[39] E. Garcia, Y. Cao, X. Wang, and D. W. Casbeer. Decentralized event-triggered consensus of linear multi-agent systems under directed graphs. In *American Control Conference (ACC)*, volume 1, pages 5764–5769, 2015.

[40] V. Gervasi and G. Prencipe. Coordination without communication: the case of the flocking problem. In *Discrete Applied Mathematics*, volume 144, pages 324–344, 2004.

[41] G. Guo, L. Ding, and Q-L. Han. A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems. In *Automatica*, volume 50, pages 1489–1496, 2014.

[42] C.N. Hadjicostis and T. Charalambous. Average consensus in the presence of delays and dynamically changing directed graph topologies. abs/1210.4778, 2012.

[43] W. Heemels, K. H. Johansson, and P. Tabuada. An introduction to event-triggered and self-triggered control. In *IEEE Conference on Decision and Control (CDC)*, pages 3270–3285, 2012.

[44] W. R. Heinzelman, J. Kulik, and H. Balakrishnan. Adaptive protocols for information dissemination in wireless sensor networks. In *Proceedings of the 5th Annual ACM/IEEE International Conference on Mobile Computing and Networking*, pages 174–185, 1999.

[45] A. Hu, J. Cao, M. Hu, and L. Guo. Event-triggered consensus of multi-agent systems with noises. In *booktitle of The Franklin Institute*, volume 352, pages 3489–3503, 2014.

[46] A. Jadbabaie, J. Lin, and A.S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. In *IEEE Transactions on automatic control*, volume 48, pages 988–1001, 2003.

[47] M. Ji and M. Egerstedt. Distributed coordination control of multiagent systems while preserving connectedness. volume 23, pages 693–703, 2007.

[48] Z.-P. Jiang, I. MY Mareels, and Y. Wang. A lyapunov formulation of the nonlinear small-gain theorem for interconnected iss systems. volume 32, pages 1211–1215, 1996.

[49] H. Jiangping, C. Guanrong, and L. Han-Xiong. Distributed event-triggered tracking control of second-order leader-follower multi-agent systems. In *Proceedings 30th Chinese Control Conference (CCC)*, volume 1, pages 4819–4824, 2011.

[50] Z. Jin, T. Hu, C. Sun, and L. Zhang. Event-triggered state fusion estimation for wireless sensor networks with feedback. In *Proceedings 34th Chinese Control Conference (CCC)*, volume 1, pages 4610–4614, 2015.

[51] A. Kahn, J. Marzat, and H. Piet-Lahanier. Formation flying control via elliptical virtual structure. In *IEEE International Conference on Networking, Sensing and Control (ICNSC)*, pages 158–163, 2013.

[52] X. Kang, H. Xu, and X. Feng. Fuzzy logic based behavior fusion for multi-auv formation keeping in uncertain ocean environment. In *OCEANS*, pages 1–7, 2009.

[53] H. K. Khalil. *Nonlinear Systems, 3rd ed.* Upper Saddle River,NJ: Prentice-Hall, 2002.

[54] L. Krick, M. E. Broucke, and B. A. Francis. Stabilisation of infinitesimally rigid formations of multi-robot networks. volume 82, pages 423–439, 2009.

[55] E Kyrkjebø, K.Y. Pettersen, M. Wondergem, and H. Nijmeijer. Output synchronization control of ship replenishment operations: Theory and experiments. In *Control Engineering Practice*, volume 15, pages 741 – 755, 2007.

[56] H. Li, Z. Chen, L. Wu, H. K. Lam, and H. Du. Event-triggered fault detection of nonlinear networked systems. In *IEEE Transactions on Cybernetics*, volume PP, pages 1–12, 2016.

[57] H. Li, X. Liao, T. Huang, and W. Zhu. Event-triggering sampling based leader-following consensus in second-order multi-agent systems. In *IEEE Transactions on Automatic Control*, volume 60, pages 1998–2003, 2015.

[58] Z. Lin, B. Francis, and M. Maggiore. Necessary and sufficient graphical conditions for formation control of unicycles. In *IEEE Transactions on automatic control*, volume 50, pages 121–127, 2005.

[59] S. Linsenmayer. *Event-triggered control of multi-agent systems with double-integrator dynamics: Application to vehicle platooning and flocking algorithms.* PhD thesis, KTH Royal Institute of Technology, Sweden, 2014.

[60] S. Linsenmayer and D. V. Dimarogonas. Event-triggered control for vehicle platooning. In *American Control Conference (ACC)*, pages 3101–3106, 2015.

[61] Q. Liu, Z. Sun, J. Qin, and C. Yu. Distance-based formation shape stabilisation via event-triggered control. In *Proceedings Chinese Control Conference (CCC)*, pages 6948–6953, 2015.

[62] Z Liu, W Chen, J Lu, H Wang, and J Wang. Formation control of mobile robots using distributed controller with sampled-data and communication delays. In *IEEE Transactions on Control Systems Technology*, volume 24, pages 2125–2132, 2016.

[63] J. Lunze. Synchronisation of autonomous agents with minimum communication. In *IEEE 52nd Annual Conference on Decision and Control (CDC)*, pages 5415–5420, 2013.

[64] A. Mahmood and Y. Kim. Collision-free vehicle formation control using graph laplacian and edge-tension function. In *World Congress*, volume 19, pages 1808–1812, 2014.

[65] J Mei, W Ren, and G Ma. Distributed coordinated tracking with a dynamic leader for multiple euler-lagrange systems. In *IEEE Transactions on Automatic Control*, volume 56, pages 1415–1421, 2011.

[66] X. Meng and T. Chen. Event based agreement protocols for multi-agent networks. In *Automatica*, volume 49, pages 2125–2132, 2013.

[67] Z. Meng, C. Yongcan, and W. Ren. Stability and convergence analysis of multi-agent consensus with information reuse. In *International booktitle of Control*, pages 1081–1092, March 2009.

[68] A. Molin, H. Sandberg, and K. H. Johansson. Consistency-preserving event-triggered estimation in sensor networks. In *IEEE Conference on Decision and Control (CDC)*, pages 7494–7501, 2015.

[69] Sérgio Monteiro and Estela Bicho. Robot formations: Robots allocation and leader-follower pairs. In *Proc. IEEE Robotics and Automation, ICRA*, pages 3769–3775, 2008.

[70] L. Moreau. Stability of multiagent systems with time-dependent communication links. In *IEEE Transactions on automatic control*, volume 50, pages 169–182, 2005.

[71] B. Mu, H. Li, W. Li, and Y. Shi. Consensus for multiple euler-lagrange dynamics with arbitrary sampling periods and event-triggered strategy. In *Intelligent Control and Automation (WCICA), 2014 11th World Congress on*, pages 2596–2601. IEEE, 2014.

[72] B. Nabet and N-E Leonard. Tensegrity models and shape control of vehicle formations. volume 1, pages 1–31, 2009.

[73] K-K Oh, M-C Park, and H-S Ahn. A survey of multi-agent formation control. In *Automatica*, volume 53, pages 424 – 440, 2014.

[74] R. Olfati-Saber. Consensus protocols for networks of dynamic agents. In *American Control Conference*, pages 1–6, 2003.

[75] R. Olfati-Saber. Flocking for multi-agent dynamic systems: Algorithms and theory. In *Proc. IEEE Transactions on Automatic Control*, volume 51, pages 401–420. IEEE, 2006.

[76] R. Olfati-Saber, W. B. Dunbar, and R. M. Murray. Cooperative control of multi-vehicle systems using cost graphs and optimization. In *Proceedings of the American Control Conference*, volume 1. Citeseer, 2003.

[77] R. Olfati-Saber, A. J. Fax, and R. M. Murray. Consensus and cooperation in networked multi-agent systems. In *Proceedings of the IEEE*, volume 95, pages 215–233, 2007.

[78] R. Olfati-Saber and R. M. Murray. Consensus problems in networks of agents with switching topology and time-delays. In *Automatic Control, IEEE Transactions on*, volume 49, pages 1520–1533, 2004.

[79] C. Peng, Q.-L. Han, and D. Yue. To transmit or not to transmit: a discrete event-triggered communication scheme for networked takagi–sugeno fuzzy systems. In *IEEE Transactions on Fuzzy Systems*, volume 21, pages 164–170. IEEE, 2013.

[80] L. Peng, Y. Zhao, B. Tian, J. Zhang, B-H Wang, H-T Zhang, and T. Zhou. Consensus of self-driven agents with avoidance of collisions. In *Phys. Rev. E*, volume 79, pages 026113,1–6. American Physical Society, Feb 2009.

[81] Wen Qin, Zhong-xin Liu, and Zeng-qiang Chen. Observer-based consensus for nonlinear multi-agent systems with intermittent communication. In *Neurocomputing*, volume 154, pages 230–238, 2015.

[82] Y. Qingkai, C Ming, F. Hao, C. Jie, and H. Jie. Distributed formation stabilization for mobile agents using virtual tensegrity structures. In *Proceedings Chinese Control Conference (CCC)*, pages 447–452, 2015.

[83] A Rahman, W Olesinski, and P Gburzynski. Controlled flooding in wireless ad-hoc networks. In *Wireless Ad-Hoc Networks, 2004 International Workshop on*, pages 73–78, 2004.

[84] F. C. Rego, A. P. Aguiar, and A. M. Pascoal. A packet loss compliant logic-based communication algorithm for cooperative path-following control. In *IFAC Proceedings Volumes*, volume 46, pages 262–267, 2013.

[85] W. Ren and E. Atkins. Second-order consensus protocols in multiple vehicle systems with local interactions. In *AIAA Guidance, Navigation, and Control Conference and Exhibit*, page 6238, 2005.

[86] W. Ren and R. Beard. Virtual structure based spacecraft formation control with formation feedback. In *AIAA Guidance, Navigation, and control conference and exhibit*, page 4963, 2002.

[87] W. Ren and R. Beard. Decentralized scheme for spacecraft formation flying via the virtual structure approach. In *Guidance, Control, and Dynamics*, volume 27, pages 73–82, 2004.

[88] Craig W. Reynolds. Flocks, herds and schools: A distributed behavioral model. In *ACM Siggraph Computer Graphics*, volume 21, pages 25–34. ACM, 1987.

[89] Craig W Reynolds. Steering behaviors for autonomous characters. In *Game developers conference*, pages 763–782, 1999.

[90] Y. Rochefort. *Méthodes pour le guidage coopératif*. PhD thesis, STITS, 2013.

[91] Y. Rochefort, H. Piet-Lahanier, S. Bertrand, D. Beauvois, and D. Dumur. Guidance of flocks of vehicles using virtual signposts. In *IFAC Proceedings Volumes*, volume 44, pages 5999–6004, 2011.

[92] M. Rosencrantz, G. Gordon, and S. Thrun. Decentralized sensor fusion with distributed particle filters. In *Proceedings of the Nineteenth conference on Uncertainty in Artificial Intelligence*, pages 493–500. Morgan Kaufmann Publishers Inc., 2002.

[93] R. Sepulchre, D. Paley, and N. Leonard. Collective motion and oscillator synchronization. In *Cooperative control*, pages 189–205. Springer, 2005.

[94] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson. Event-based broadcasting for multi-agent average consensus. In *Automatica*, volume 49, pages 245–252, 2013.

[95] D. Shi, T. Chen, and L. Shi. Event-triggered maximum likelihood state estimation. In *Automatica*, volume 50, pages 247–254, 2014.

[96] N. Sorensen and W. Ren. Rendezvous problem in multi-vehicle systems: Information relay and local information based strategies. In *Adaptive and Learning Systems, 2006 IEEE Mountain Workshop on*, pages 183–188, 2006.

[97] D. Sun, C. Wang, W. Shang, and G. Feng. A synchronization approach to trajectory tracking of multiple mobile robots while maintaining time-varying formations. In *Proc. IEEE Robotics*, volume 25, pages 1074–1086. IEEE, 2009.

[98] Z. Sun, Q. Liu, C. Yu, and B.D.O. Anderson. Generalized controllers for rigid formation stabilization with application to event-based controller design. In *European Control Conference (ECC)*, pages 217–222, 2015.

[99] T. Tang, Z. Liu, and Z. Chen. Event-triggered formation control of multi-agent systems. In *Proceedings 30th Chinese Control Conference (CCC)*, pages 4783–4786, 2011.

[100] H. G. Tanner, A. Jadbabaie, and G. J. Pappas. Stable flocking of mobile agents, part i: Fixed topology. In *IEEE 42nd Conference on Decision and Control*, volume 2, pages 2010–2015, 2003.

[101] A.E. Turgut, H. Çelikkanat, F. Gökçe, and E. Şahin. Self-organized flocking in mobile robot swarms. In *Swarm Intelligence*, volume 2, pages 97–120. Springer, 2008.

[102] S. Vanka, M. Haenggi, and V. Gupta. Convergence speed of the consensus algorithm with interference and sparse long-range connectivity. In *IEEE booktitle of Selected Topics in Signal Processing*, volume 5, pages 855–865, 2011.

[103] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet. Novel type of phase transition in a system of self-driven pinproceedingss. In *Phys. Rev. Lett.*, volume 75, pages 1226–1229. American Physical Society, Aug 1995.

[104] D. Viegas, P. Batista, P. Oliveira, and C. Silvestre. Decentralized h2 observers for position and velocity estimation in vehicle formations with fixed topologies. In *Systems & Control Letters*, volume 61, pages 443–453, 2012.

[105] D. Viegas, P. Batista, P. Oliveira, C. Silvestre, and C.L.P. Chen. Distributed state estimation for linear multi-agent systems with time-varying measurement topology. In *Automatica*, volume 54, pages 72–79, 2015.

[106] C. Viel, S. Bertrand, M. Kieffer, and H. Piet-Lahanier. New state estimators and communication protocol for distributed event-triggered consensus of linear multi-agent systems with bounded perturbations. volume 11, page 1736 – 1748, 2017.

[107] C Viel, S Bertrand, H Piet-Lahanier, and M Kieffer. New state estimator for decentralized event-triggered consensus for multi-agent systems. In *Proceedings IFAC ICONS 2016*, volume 49, pages 365–370, 2016.

[108] C. Viel, S. Bertrand, H. Piet-Lahanier, and M Kieffer. Distributed event-triggered consensus for multi-agent formation stabilization, to appear in proc. ifac world conference, toulouse. 2017.

[109] A. Wang, T. Dong, and X. Liao. Event-triggered synchronization strategy for complex dynamical networks with the markovian switching topologies. In *Neural Networks*, volume 74, pages 52 – 57, 2016.

[110] D. Wang, Z. Wang, W. Wang, and J. Lian. Event-triggered based containment control of multi-agent systems with general linear dynamics. In *Information and Automation, 2015 IEEE International Conference on*, pages 1064–1069, 2015.

[111] H. Wang, P. Shi, and J. Zhang. Event-triggered fuzzy filtering for a class of nonlinear networked control systems. In *Signal Processing*, volume 113, pages 159–168, 2015.

[112] J. Wang, M. Obeng, T. Yang, G. Staskevich, and B. Abbe. Formation control of multiple non-holonomic mobile robots with limited information of a desired trajectory. In *IEEE International Conference on Electro/Information Technology*, pages 550–555, 2014.

[113] R. Wei. Consensus tracking under directed interaction topologies: Algorithms and experiments. In *American Control Conference (ACC)*, volume 1, pages 742–747, 2008.

[114] G. Wen, Z. Duan, Z. Li, and G. Chen. Flocking of multi-agent dynamical systems with intermittent nonlinear velocity measurements. In *International Journal of Robust and Nonlinear Control*, volume 22, pages 1790 –1805, 2012.

[115] G. Wen, Z. Duan, W. Ren, and G. Chen. Distributed consensus of multi-agent systems with general linear node dynamics and intermittent communications. In *International booktitle of Robust and Nonlinear Control*, volume 24, pages 2438–2457, 2014.

[116] G. Wen, Z. Duan, W. Yu, and G. Chen. Consensus in multi-agent systems with communication constraints. In *International booktitle of Robust and Nonlinear Control*, volume 22, pages 170–182. John Wiley & Sons, Ltd, 2012.

[117] G. Wen, Z. Duan, W. Yu, and G. Chen. Consensus of second-order multi-agent systems with delayed nonlinear dynamics and intermittent communications. In *International Journal of Control*, volume 86, pages 322–331, February 2013.

[118] G. Wen, G. Hu, W. Yu, J. Cao, and G. Chen. Consensus tracking for higher-order multi-agent systems with switching directed topologies and occasionally missing control inputs. In *Systems & Control Letters*, volume 62, pages 1151–1158, 2013.

[119] Guanghui Wen, Zhisheng Duan, and Guanrong Chen. Distributed consensus of multi-agent systems with general linear node dynamics through intermittent communications. In *Control and Decision Conference (CCDC)*, pages 1–5, May 2012.

[120] J. Wu, Q.-S. Jia, K. H. Johansson, and L. Shi. Event-based sensor data scheduling: Trade-off between communication rate and estimation quality. In *IEEE Transactions on automatic control*, volume 58, pages 1041–1046, 2013.

[121] J. Wu and M. Qi. Research on formation handling of serial swarm robots. In *Third Global Congress on Intelligent Systems (GCIS)*, pages 338–341. IEEE, 2012.

[122] J. Wu, V. Ugrinovskii, and F. Allgöwer. Cooperative estimation for synchronization of heterogeneous multi-agent systems using relative information. In *World Congress*, volume 19, pages 4662–4667, 2014.

[123] L. Xiao and S. Boyd. Fast linear iterations for distributed averaging. In *Systems & Control Letters*, volume 53, pages 65–78, 2004.

[124] W. Xiong, D.W.C. Ho, and C. Huang. Pinning synchronization of time-varying polytopic directed stochastic networks. In *Physics Letters A*, volume 374, pages 439–447, 2010.

[125] W. Xiong, D.W.C. Ho, and Z. Wang. Consensus analysis of multiagent networks via aggregated and pinning approaches. In *IEEE Transactions on Neural Networks*, volume 22, pages 1231–1240, 2011.

[126] W. Xu, D. W. C. Ho, L. Li, and J. Cao. Leader-following consensus of general linear multi-agent systems: Event-triggered schemes. In *Asian Control Conference (ASCC)*, pages 1–6, 2015.

[127] Y. Xu and J.P. Hespanha. Communication logic design and analysis for networked control systems. In *Current trends in nonlinear systems and control*, pages 495–514. Springer, 2006.

[128] D. Yang, W. Ren, and X. Liu. Decentralized consensus for linear multi-agent systems under general directed graphs based on event-triggered/self-triggered strategy. In *Proc. IEEE International Conference on Decision and Control (CDC)*, pages 1983–1988, 2014.

[129] X. Yin, D. Yue, and S. Hu. Adaptive periodic event-triggered consensus for multi-agent systems subject to input saturation. In *International booktitle of Control*, volume 89, pages 653–667. Taylor & Francis, 2016.

[130] J.K. Yook, D.M. Tilbury, and N.R. Soparkar. Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators. In *IEEE transactions on control systems technology*, volume 10, pages 503–518, 2002.

[131] W. Yu, G. Chen, and M. Cao. Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. In *Automatica*, volume 46, pages 1089–1095, 2010.

[132] H. Zhang, R. Yang, H. Yan, and Q. Chen. Distributed event-triggered control for consensus of multi-agent systems. In *booktitle of the Franklin Institute*, volume 352, pages 3476–3488, 2015.

[133] B. Zhou, X. Liao, T. Huang, and G. Chen. Pinning exponential synchronization of complex networks via event-triggered communication with combinational measurements. In *Neurocomputing*, volume 157, pages 199–207, 2015.

[134] B. Zhou and Z. Lin. Consensus of high-order multi-agent systems with large input and communication delays. In *Automatica*, volume 50, pages 452–464, 2014.

[135] F. Zhou, Z. Huang, W. Liu, L. Li, and K. Gao. Periodic event-triggered condition design for the consensus of multiagent systems with communication delays. In *Mathematical Problems in Engineering*, volume 1, pages 1–9, 2016.

[136] W. Zhu, Z.P. Jiang, and G. Feng. Event-based consensus of multi-agent systems with general linear models. In *Automatica*, volume 50, pages 552–558, 2014.

Appendix A

Appendix of Chapter 2

A.1 Consensus convergence

The system gathering the dynamics of all the agents is

$$\dot{x}(t) = \overline{A}x(t) + \tilde{B}\tilde{y}(t) + d(t)$$

where $x = \left[x_1^T \dots x_N^T\right]^T$, $\overline{A} = 1_N \otimes A$, $\tilde{B} = T\left(I_N \otimes \overline{B}_1\right)$, $\overline{B}_1 = c_1 L \otimes BF$, $\tilde{y} = \left[y^{1T}\, y^{2T} \dots y^{NT}\right]^T \in \mathbb{R}^{N^2 n}$ is the vector gathering the estimates of the states of Agents $1, \dots, N$ performed by all agents. Define $e = y - x \in \mathbb{R}^{Nn}$.

A matrix $T \in \mathbb{R}^{Nn \times N^2 n}$ is also defined to extract, from vector $\tilde{y}$, all terms y_k^k, $k = 1 \dots N$:

$$\begin{aligned} T\tilde{y} &= T\left[y^{1T}\, y^{2T} \dots y^{NT}\right]^T \\ &= \left[y_1^{1T}\, y_2^{2T} \dots y_N^{NT}\right]^T \\ &= y. \end{aligned}$$

This matrix can be expressed as $T = \left(\left(I_N \otimes 1_N^T\right) \circ \left(1_N^T \otimes I_N\right)\right) \otimes 1_n$, with $\circ$ the entrywise matrix product. One may easily show that $T\tilde{y}\left(1_N \otimes y\right) = y$.

Define the candidate Lyapunov function : $V = x^T \widehat{L} x$, with $\widehat{L} = L \otimes P$. Since the graph is undirected and P is symmetric, L and $\widehat{L}$ are symmetric and

$$\dot{V} = 2\left(x^T \widehat{L}\left(\overline{A}x + \tilde{B}\tilde{y}\right) + d^T(t)\,\widehat{L}x\right) \tag{A.1}$$

Define $\dot{V}_1 = 2x^T \widehat{L}\left(\overline{A}x + \tilde{B}\tilde{y}\right)$. The next section will show that $\dot{V}_1$ is upper bounded by $x^T \overline{L} x$. Then introduce $\dot{V}_2 = x^T \overline{L} x + 2d^T(t)\,\widehat{L}x$, where one reminds that $\overline{L} = \widehat{L}A_c + A_c^T \widehat{L}$ and $A_c = \overline{A} + \overline{B}_1$. An upper bounds for $\dot{V}_2$, also evaluated in what follows, is then used to upper bound $\dot{V}$.

A.1.1 Upper bound for $\dot{V}_1$

Let $\Delta_{ij} = y_i^j - y_i^i$ and define $\Delta(t) = \left[\Delta_{11}^T(t)\ \Delta_{12}^T(t) \dots \Delta_{N,N-1}^T(t)\ \Delta_{NN}^T(t)\right]^T \in \mathbb{R}^{N^2 n}$. Note first that $\tilde{y} = 1_N \otimes y + \Delta$ and $\tilde{e} = 1_N \otimes e + \Delta$.

$$\begin{aligned} \dot{V}_1 &= 2x^T \widehat{L}\left(\overline{A}x + \tilde{B}\tilde{y}\right) \\ &= 2x^T \widehat{L}\left(\overline{A}x + \tilde{B}\left(1_N \otimes y + \Delta\right)\right) \end{aligned} \tag{A.2}$$

Since $\tilde{B} = T\left(I_N \otimes \overline{B}_1\right)$, $\overline{B}_1 = c_1 L \otimes (BF)$, and $T\left(1_N \otimes y\right) = y$, one obtains

$$\begin{aligned} \tilde{B}\left(1_N \otimes y\right) &= T\left(I_N \otimes \overline{B}_1\right)\left(1_N \otimes y\right) \\ &= T\left(I_N \otimes \left(\overline{B}_1 y\right)\right) \\ &= \overline{B}_1 y \end{aligned}$$

and

$$\dot{V}_1 = 2x^T\widehat{L}\left(\overline{A}x+\overline{B}_1 y\right)+2x^T\widehat{L}\tilde{B}\Delta. \tag{A.3}$$

Consider

$$\dot{V}_{11} = 2x^T\widehat{L}\left(\overline{A}x+\overline{B}_1 y\right) \tag{A.4}$$

and

$$\dot{V}_{12} = 2x^T\widehat{L}\tilde{B}\Delta. \tag{A.5}$$

The expression of $\dot{V}_{11}$ can be found in [37], where it is shown that $\dot{V}_{11} = x^T\overline{L}x+\sum_{i=1}^N\left(\delta_i - z_i^T\Theta_i z_i\right)$ with

$$\begin{aligned}\delta_i = \; & 2\left(c_2-c\right)N_i z_i^T PBB^T Pe_i^i + \\ & \Bigg[2cN_i^2\left(1+b_i\right)+\frac{c_2-c}{b_i}N_i \\ & +cN_i\left(N-1\right)\left(b_i+\frac{3}{b_i}\right)\Bigg]e_i^{iT}PBB^TPe_i^i.\end{aligned} \tag{A.6}$$

Using the expression of $\tilde{B}$ and $\overline{B}_1$, and the fact that $e=y-x$, $\dot{V}_{12}$ may be rewritten as

$$\dot{V}_{12} = 2\left(\widehat{L}\left(y-e\right)\right)^T T\left(I_N\otimes\left(c_1L\otimes\left(BF\right)\right)\right)\Delta.$$

Using the property of T,

$$T\left(I_N\otimes\left(c_1L\otimes\left(BF\right)\right)\right)\Delta = \begin{bmatrix} c_1BF\sum_{k\in\mathcal{N}_1}\left(\Delta_{11}-\Delta_{1k}\right) \\ \vdots \\ c_1BF\sum_{k\in\mathcal{N}_N}\left(\Delta_{NN}-\Delta_{Nk}\right)\end{bmatrix}.$$

Since $\Delta_{ii}=0$, $\widehat{L}=L\otimes P$, and $F=-B^TP$ one may rewrite $\dot{V}_{12}$ as

$$\begin{aligned}\dot{V}_{12} = \; & c_1\sum_{i=1}^N\Bigg[\sum_{j\in\mathcal{N}_i}\left(y_i^i-y_j^j\right)^T\left(-PBB^TP\right)\sum_{k\in\mathcal{N}_i}\left(-\Delta_{ik}\right) \\ & -\sum_{j\in\mathcal{N}_i}\left(e_i^i-e_j^j\right)^T\left(-PBB^TP\right)\sum_{k\in\mathcal{N}_i}\left(-\Delta_{ik}\right)\Bigg]\end{aligned} \tag{A.7}$$

One may rewrite $\sum_{j\in\mathcal{N}_i}\left(y_i^i-y_j^j\right)^T$ as

$$\begin{aligned}\sum_{j\in\mathcal{N}_i}\left(y_i^i-y_j^j\right)^T & = \sum_{j\in\mathcal{N}_i}\left(y_i^i-y_j^i+\left(y_j^i-y_j^j\right)\right)^T \\ & = z_i^T+\sum_{j\in\mathcal{N}_i}\Delta_{ji}^T\end{aligned}$$

Inserting this expression in (A.7) and defining $M=PBB^TP$, one gets

$$\begin{aligned}\dot{V}_{12} = \; & c_1\sum_{i=1}^N\Bigg[z_i^TM\sum_{k\in\mathcal{N}_i}\Delta_{ik}+\sum_{j\in\mathcal{N}_i}\left(\Delta_{ji}\right)^TM\sum_{k\in\mathcal{N}_i}\Delta_{ik} \\ & -N_ie_i^{iT}M\sum_{k\in\mathcal{N}_i}\Delta_{ik}+\sum_{j\in\mathcal{N}_i}e_j^{jT}M\sum_{k\in\mathcal{N}_i}\Delta_{ik}\Bigg].\end{aligned} \tag{A.8}$$

Using $x^T y \leq \frac{1}{2b_i} x^T x + \frac{b_i}{2} y^T y$ for any $b_i > 0$, one obtains

$$\begin{aligned}
\sum_{j \in \mathcal{N}_i} (\Delta_{ji})^T M \sum_{k \in \mathcal{N}_i} \Delta_{ik} &= \sum_{j \in \mathcal{N}_i} \sum_{k \in \mathcal{N}_i} (\Delta_{ji})^T M \Delta_{ik} \\
&\leq \sum_{j \in \mathcal{N}_i} \sum_{k \in \mathcal{N}_i} \left[\frac{1}{2b_{i1}} \Delta_{ij}^T M \Delta_{ij} + \frac{b_{i1}}{2} \Delta_{ik}^T M \Delta_{ik} \right] \\
&\leq \sum_{j \in \mathcal{N}_i} N_i \left[\left(\frac{1}{2b_{i1}} + \frac{b_{i1}}{2} \right) \Delta_{ij}^T M \Delta_{ij} \right]
\end{aligned}$$

and

$$\begin{aligned}
\sum_{j \in \mathcal{N}_i} e_j^{jT} M \sum_{k \in \mathcal{N}_i} \Delta_{ik} &= \sum_{j \in \mathcal{N}_i} \sum_{k \in \mathcal{N}_i} e_j^{jT} M \Delta_{ik} \\
&\leq \sum_{j \in \mathcal{N}_i} \sum_{k \in \mathcal{N}_i} \left[\frac{1}{2b_i} e_j^{jT} M e_j^j + \frac{b_i}{2} \Delta_{ik}^T M \Delta_{ik} \right] \\
&\leq \sum_{j \in \mathcal{N}_i} N_i \left[\frac{1}{2b_i} e_j^{jT} M e_j^j + \frac{b_i}{2} \Delta_{ij}^T M \Delta_{ij} \right]
\end{aligned}$$

Using these upper bounds in (A.8), one gets

$$\begin{aligned}
\dot{V}_{12} \leq\ & c_1 \sum_{i=1}^{N} \left[\left(z_i - N_i e_i^i \right)^T M \sum_{k \in \mathcal{N}_i} \Delta_{ik} + N_i \sum_{j \in \mathcal{N}_i} \left(\frac{1}{2b_{i1}} + \frac{b_{i1}}{2} \right) \right. \\
& \left. \times \Delta_{ij}^T M \Delta_{ij} + \sum_{j \in \mathcal{N}_i} N_i \left[\frac{1}{2b_i} e_j^{jT} M e_j^j + \frac{b_i}{2} \Delta_{ij}^T M \Delta_{ij} \right] \right].
\end{aligned}$$

Choosing arbitrarily $b_{i1} = 1$, one gets

$$\begin{aligned}
\dot{V}_{12} \leq\ & c_1 \sum_{i=1}^{N} \left[\left(z_i - N_i e_i^i \right)^T M \sum_{k \in \mathcal{N}_i} \Delta_{ik} + \frac{N_i}{2b_i} e_i^{iT} M e_i^i \right. \\
& \left. + \sum_{j \in \mathcal{N}_i} N_i \left(1 + \frac{b_i}{2} \right) \Delta_{ij}^T M \Delta_{ij} \right].
\end{aligned} \tag{A.9}$$

Inserting (A.9) in $\dot{V}_1$ and one obtains

$$\dot{V}_1 \leq x^T \overline{L} x + \sum_{i=1}^{N} \left(\bar{\delta}_i - \sigma z_i^T \Theta_i z_i \right),$$

where

$$\begin{aligned}
\bar{\delta}_i =\ & c_1 \left[\left(z_i - N_i e_i^i \right)^T M \sum_{j \in \mathcal{N}_i} \left(y_i^j - y_i^i \right) + \frac{N_i}{2b_i} e_i^{iT} M e_i^i \right. \\
& \left. + \left(1 + \frac{b_i}{2} \right) N_i \sum_{j \in \mathcal{N}_i} \left(\left(y_i^j - y_i^i \right)^T M \left(y_i^j - y_i^i \right) \right) \right] + \delta_i.
\end{aligned} \tag{A.10}$$

Since $\overline{L}$ is semi-definite negative, $\dot{V}_1 \leq 0$ if, for $i, j = 1 \ldots N$, the communication events are triggered when $\bar{\delta}_i \geq \rho z_i^T \Theta z_i$ with $0 < \rho \leq 1$.

Remark 8. With $\delta_i > \rho z_i^T \Theta z_i$ and no perturbation, $V_1(t)$ converges asymptotically to zero. In order to reduce the number of broadcast communications, a threshold η can be introduced so that $\bar{\delta}_i \geq \rho z_i^T \Theta z_i + \eta$.

A.1.2 Upper bound for V

Assuming that there is no perturbation, one is now interested in bounding $\|x_i - x_j\|$ when the CTC (2.34) is satisfied.

First note that $x^T\widehat{L}x \geq 0$, so

$$x^T\widehat{L}x \leq \lambda_{\max}\left(\widehat{L}\right)x^Tx$$

and that $x^T\overline{L}x \leq 0$, so

$$-x^T\overline{L}x \geq \lambda_{\min>0}\left(-\overline{L}\right)x^Tx.$$

Combining these results, one obtains

$$x^T\widehat{L}x\frac{1}{\lambda_{\max}\left(\widehat{L}\right)} \leq x^Tx \leq -x^T\overline{L}x\frac{1}{\lambda_{\min>0}\left(-\overline{L}\right)}$$

and thus

$$x^T\overline{L}x \quad \leq \quad -\beta x^T\widehat{L}x,$$

where $\beta = \frac{\lambda_{\min>0}\left(-\overline{L}\right)}{\lambda_{\max}\left(\widehat{L}\right)}$.

With the triggering condition defined in Theorem 5, one obtains

$$\begin{aligned}\dot{V}(t) &\leq x^T\overline{L}x + \sum_{i=1}^{N}\left(\delta_i - \rho z_i^T\Theta_i z_i\right) \\ &\leq -\beta V(t) + N\eta \end{aligned} \tag{A.11}$$

from which one deduces that $V(t) \leq V(0)e^{-\beta t} + \frac{N\eta}{\beta}$. Consequently,

$$\lim_{t\to\infty} V(t) \leq \frac{N\eta}{\beta}. \tag{A.12}$$

$V(t)$ may be rewritten as

$$\begin{aligned} V(t) &= x^T\widehat{L}x \\ &= \sum_{i=1}^{N}\left(x_i^TP\sum_{k\in\mathcal{N}_i}(x_i - x_k)\right) \\ &= \sum_{i=1}^{N}\sum_{k\in\mathcal{N}_i}\left(x_i^TP(x_i - x_k)\right) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{k\in\mathcal{N}_i}2\left(x_i^TPx_i - x_i^TPx_k\right) \end{aligned}$$

As the graph is undirected, $V(t)$ becomes

$$\begin{aligned} V(t) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{k\in\mathcal{N}_i}\left(x_i^TPx_i - 2x_i^TPx_k + x_k^TPx_k\right) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{k\in\mathcal{N}_i}(x_i - x_k)^TP(x_i - x_k). \end{aligned} \tag{A.13}$$

Since the graph is connected, each term $(x_i - x_k)^TP(x_i - x_k)$ appears twice in (A.13). Thus, from (A.12) and (A.13), one has

$$\begin{aligned}
\lim_{t\to\infty} V(t) &\leq \frac{N\eta}{\beta} \\
\lim_{t\to\infty} \sum_{i=1}^{N} \sum_{k\in\mathcal{N}_i} \|x_i - x_k\|^2 \lambda_{\min}(P) &\leq \frac{N\eta}{\beta} \\
\forall k \in \mathcal{N}_i, \quad \lim_{t\to\infty} \|x_i - x_k\|^2 &\leq \frac{N\eta}{\beta\lambda_{\min}(P)} \qquad \text{(A.14)}
\end{aligned}$$

The graph is connected, thus for any pair of agents (i,j), there exists a path between them linking neighboring agents, the indexes of these agents are k_1, k_2,...,k_m and

$$\begin{aligned}
\|x_i - x_j\| &\leq \|x_i - x_{k_1}\| + \|x_{k_1} - x_{k_2}\| + \ldots + \|x_{k_m} - x_j\| \\
&\leq N \max_{k\in\mathcal{N},\, \ell\in\mathcal{N}_k} \|x_k - x_\ell\|. \qquad \text{(A.15)}
\end{aligned}$$

Combining (A.14) and (A.15), one gets

$$\forall (i,j) \in \mathcal{N}, \quad \lim_{t\to\infty} \|x_i - x_j\|^2 \leq \frac{N^3\eta}{\beta\lambda_{\min}(P)}.$$

The perturbations terms do not appear in $\bar{\delta}_i$ and Θ_i, but they impact the estimation error and the communication triggering frequency.

A.1.3 Upper bound for $\dot{V}_2$

Since $L1_N = 0$ one has

$$(L \otimes P)(1_N \otimes m) = ((L1_N) \otimes (Pm)) = 0$$

and one deduces

$$\begin{aligned}
\dot{V}_2 &= 2x^T \widehat{L} d + x^T \overline{L} x \\
&= 2x^T (L \otimes P)(1_N \otimes m + s) + x^T \overline{L} x \\
&= 2x^T \widehat{L} s + x^T \overline{L} x.
\end{aligned}$$

Let $\dot{V}_{21} = 2x^T \widehat{L} s$ and $\dot{V}_{22} = x^T \overline{L} x$. Then, considering a sequence of $b_i > 0$, $i = 1, \ldots, N$, one has

$$\begin{aligned}
\dot{V}_{21} &= 2 \sum_{i=1}^{N} \left(\sum_{j\in\mathcal{N}_i} (x_i - x_j)^T P s_i \right) \\
&\leq 2 \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \left(\frac{b_i}{2} (x_i - x_j)^T P (x_i - x_j) + \frac{1}{2b_i} s_i^T P s_i \right) \\
&\leq \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \left(b_i (x_i - x_j)^T P (x_i - x_j) + \frac{1}{b_i} \lambda_{\max}(P) S_{\max}^2 \right). \qquad \text{(A.16)}
\end{aligned}$$

To bound $\dot{V}_{22}$, using the expression of $\overline{L}$, one gets

$$\dot{V}_{22} = x^T \left[L \otimes \left(PA + A^T P \right) - (LL) \otimes \left(2c_1 PBB^T P \right) \right] x$$

Since $M = PBB^T P$ and L are symmetric semi-define positive matrices, one obtains

$$\begin{aligned}
x^T (L \otimes M) x &= x^T \left(I_N \otimes \left(B^T P \right) \right)^T (L \otimes I_n) \left(I_N \otimes \left(B^T P \right) \right) x \\
&= x^T \left(I_N \otimes \left(B^T P \right) \right)^T U^T \Lambda U \left(I_N \otimes \left(B^T P \right) \right) x
\end{aligned}$$

where Λ is a diagonal matrix with elements $\Lambda_i = \lambda_i (L \otimes I_n)$, $i = 1 \dots Nn$, and U is the matrix of corresponding eigenvectors.

Introducing $q = U\left(I_N \otimes \left(B^T P\right)\right) x$, one obtains

$$\begin{aligned} x^T (L \otimes M) x &= q^T \Lambda q \\ &= \sum_{i=1}^{Nn} q_i^2 \lambda_i (L \otimes I_n) \\ &\leq \frac{1}{\lambda_{\min>0} (L \otimes I_n)} \sum_{i=1}^{Nn} q_i^2 \lambda_i (L \otimes I_n)^2 . \end{aligned}$$

Since $\lambda_{\min>0} (L \otimes I_n) = \lambda_{\min>0} (L) \lambda_{\min>0} (I_n) = \lambda_2 (L)$, one obtains

$$\begin{aligned} x^T (L \otimes M) x &\leq \frac{1}{\lambda_2 (L)} q^T \Lambda^2 q \\ &\leq \frac{1}{\lambda_2 (L)} x^T \left(I_N \otimes \left(B^T P\right)\right)^T ((LL) \otimes I_n) \left(I_N \otimes \left(B^T P\right)\right) x \\ &\leq \frac{1}{\lambda_2 (L)} x^T ((LL) \otimes M) x \end{aligned}$$

and thus

$$-x^T ((LL) \otimes M) x \leq -\lambda_2 (L) x^T (L \otimes M) x. \tag{A.17}$$

Injecting (A.17) in $\dot{V}_{22}$, one gets

$$\begin{aligned} \dot{V}_{22} &= x^T \bar{L} x \\ &= x^T \left[L \otimes \left(PA + A^T P\right) - 2c_1 (LL) \otimes M\right] x \\ &\leq x^T \left[L \otimes \left(PA + A^T P - 2c_1 M \lambda_2 (L)\right)\right] x \end{aligned}$$

Reminding that $c_1 = c + c_2$ and $c = \frac{1}{\lambda_2(L)}$, one gets

$$\begin{aligned} \dot{V}_{22} &\leq x^T \left[L \otimes \left(PA + A^T P - 2\left(\frac{1}{\lambda_2 (L)} + c_2\right) M \lambda_2 (L)\right)\right] x \\ &\leq x^T \left[L \otimes \left(PA + A^T P - 2M - 2c_2 M \lambda_2 (L)\right)\right] x. \end{aligned}$$

Using (2.3), one obtains

$$\dot{V}_{22} \leq x^T \left[L \otimes (-2\alpha P - 2c_2 \lambda_2 (L) M)\right] x$$

and using (A.13), $\dot{V}_{22}$ becomes

$$\dot{V}_{22} \leq \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \left[(x_i - x_j)^T (-2\alpha P - 2c_2 \lambda_2 (L) M) (x_i - x_j)\right] \tag{A.18}$$

Since $\dot{V}_2 = \dot{V}_{21} + \dot{V}_{22}$, combining (A.16) and (A.18), one gets

$$\begin{aligned} \dot{V}_2 \leq & \sum_{i=1}^{N} \left(\sum_{j \in \mathcal{N}_i} \left[(x_i - x_j)^T ((b_i - \alpha) P - c_2 \lambda_2 (L) M) (x_i - x_j)\right] \right. \\ & \left. + \frac{N_i}{b_i} \lambda_{\max} (P) S_{\max}^2 \right). \end{aligned} \tag{A.19}$$

One now searches a condition on $S_{\max}$ to ensure that V_2 is decreasing. Having $\dot{V}_2 \leq 0$ is equivalent to

$$\sum_{i=1}^{N} \left(\frac{N_i}{b_i} \lambda_{\max} (P) S_{\max}^2\right) \leq \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \left[(x_i - x_j)^T ((\alpha - b_i) P + c_2 \lambda_2 (L) M) (x_i - x_j)\right]. \tag{A.20}$$

Sufficient conditions to satisfy (A.20) are for $i = 1, \ldots, N$

$$\frac{N_i}{b_i}\lambda_{\max}(P)\,S_{\max}^2 \le \sum_{j\in\mathcal{N}_i}\left[(x_i - x_j)^T\left((\alpha - b_i)\,P + c_2\lambda_2(L)\,M\right)(x_i - x_j)\right]. \tag{A.21}$$

Each inequality (A.21) is satisfied if the following condition holds

$$\frac{N_i}{b_i}\lambda_{\max}(P)\,S_{\max}^2 \quad \le \quad \sum_{j\in\mathcal{N}_i}\left[(x_i - x_j)^T(x_i - x_j)\,\|(\alpha - b_i)\,P + c_2\lambda_2(L)\,M\|\right]$$

with $\|M\| = \max_{i=1:n}(|\lambda_i(M)|)$. This provides an upper bound for $S_{\max}$

$$S_{\max}^2 \le \sum_{j\in\mathcal{N}_i}\|x_i - x_j\|^2\,\frac{b_i\,\|(\alpha - b_i)\,P + c_2\lambda_2(L)\,M\|}{\lambda_{\max}(P)\,N_i}. \tag{A.22}$$

Using (A.14) in (A.22), one gets

$$\begin{aligned} S_{\max}^2 &\le \frac{b_i\,\|(\alpha - b_i)\,P + c_2\lambda_2(L)\,M\|}{\lambda_{\max}(P)\,N_i}\,\frac{N_i N\eta}{\beta\lambda_{\min}(P)} \\ S_{\max} &\le \sqrt{\frac{b_i\,\|(\alpha - b_i)\,P + c_2\lambda_2(L)\,M\|}{\lambda_{\max}(P)}}\sqrt{\frac{N\eta}{\lambda_{\min}(P)\,\beta}}. \end{aligned}$$

Choosing $b_i = \alpha$, one obtains

$$S_{\max} \quad \le \quad \sqrt{\frac{\alpha\,\|c_2\lambda_2(L)\,M\|}{\lambda_{\max}(P)}}\sqrt{\frac{N\eta}{\lambda_{\min}(P)\,\beta}} \tag{A.23}$$

Using (A.23) in (A.19), one finally gets $\dot{V}_2 \le 0$, which leads to $\dot{V} \le 0$. The system converges thus to a bounded consensus.

A.2 Proof of Theorem 6

Starting from (A.9) in Appendix A.1.1, one has

$$\begin{aligned} \dot{V}_{12} \quad \le \quad & c_1\sum_{i=1}^{N}\left[\left(z_i - N_i e_i^i\right)^T M\sum_{j\in\mathcal{N}_i}\left(y_i^j - y_i^i\right) + \frac{N_i}{2b_i}e_i^{iT}Me_i^i\right. \\ & \left. + \left(1 + \frac{b_i}{2}\right)N_i\sum_{j\in\mathcal{N}_i}\left(\Delta_{ij}^T M\Delta_{ij}\right)\right]. \end{aligned} \tag{A.24}$$

Expressing Δ_{ij} as $\Delta_{ij} = y_i^j - v_i^j + v_i^j - y_i^i$ in (A.24), one gets

$$\begin{aligned} \dot{V}_{12} \quad \le \quad & c_1\sum_{i=1}^{N}\left[\left(z_i - N_i e_i^i\right)^T M\sum_{j\in\mathcal{N}_i}\left(y_i^j - v_i^j\right)\right. \\ & + \left(z_i - N_i e_i^i\right)^T M\sum_{j\in\mathcal{N}_i}\left(v_i^j - y_i^i\right) \\ & \left. + \frac{N_i}{2b_i}e_i^{iT}Me_i^i + \left(1 + \frac{b_i}{2}\right)N_i\sum_{j\in\mathcal{N}_i}\left(\Delta_{ij}^T M\Delta_{ij}\right)\right] \end{aligned} \tag{A.25}$$

Using $xy \leq \frac{1}{2b_{i2}}x^Tx + \frac{b_{i2}}{2}y^Ty$, with $b_{i2} > 0$,

$$\begin{aligned}
\dot{V}_{12} \leq\ & c_1 \sum_{i=1}^{N} \left[\frac{1}{2b_{i2}} \left(z_i - N_i e_i^i\right)^T M \left(z_i - N_i e_i^i\right) \right. \\
& + \frac{b_{i2}}{2} \sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right)^T M \sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right) \\
& + \left(z_i - N_i e_i^i\right)^T M \sum_{j\in\mathcal{N}_i} \left(v_i^j - y_i^i\right) + \frac{N_i}{2b_i} e_i^{iT} M e_i^i \\
& \left. + \left(1 + \frac{b_i}{2}\right) N_i \sum_{j\in\mathcal{N}_i} \left(\Delta_{ij}^T M \Delta_{ij}\right) \right] \qquad \text{(A.26)}
\end{aligned}$$

Let

$$\begin{aligned}
\dot{V}_{12a} &= \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right)^T M \sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right) \\
\dot{V}_{12b} &= \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \left(\Delta_{ij}^T M \Delta_{ij}\right)
\end{aligned}$$

Both terms are upper-bounded in what follows.

Upper-bound for $\dot{V}_{12a}$ Using $xy \leq \frac{1}{2}x^Tx + \frac{1}{2}y^Ty$, $\dot{V}_{12a}$ can be upper bounding

$$\dot{V}_{12a} \leq \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} N_i \left(y_i^j - v_i^j\right)^T M \left(y_i^j - v_i^j\right) \qquad \text{(A.27)}$$

As the communication graph is undirected, one gets

$$\dot{V}_{12a} \leq \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} N_j \left(y_j^i - v_j^i\right)^T M \left(y_j^i - v_j^i\right). \qquad \text{(A.28)}$$

Upper-bound for $\dot{V}_{12b}$ Introducing v_i^j in $\dot{V}_{12b}$,

$$\begin{aligned}
\dot{V}_{12b} = & \sum_{i=1}^{N} \left(\sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right)^T M \left(y_i^j - v_i^j\right) \right. \\
& + \sum_{j\in\mathcal{N}_i} \left(v_i^j - y_i^i\right)^T M \left(v_i^j - y_i^i\right) \\
& \left. + 2 \sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right)^T M \left(v_i^j - y_i^i\right) \right) \qquad \text{(A.29)}
\end{aligned}$$

Using again $xy \leq \frac{1}{2}x^Tx + \frac{1}{2}y^Ty$ one has

$$\begin{aligned}
& 2 \sum_{j\in\mathcal{N}_i} \left(y_i^j - v_i^j\right)^T M \left(v_i^j - y_i^i\right) \\
\leq & \sum_{j\in\mathcal{N}_i} \left(y_j^i - v_j^i\right)^T M \left(y_j^i - v_j^i\right) \\
& + \sum_{j\in\mathcal{N}_i} \left(v_i^j - y_i^i\right)^T M \left(v_i^j - y_i^i\right)
\end{aligned}$$

Injecting this expression in (A.29) leads to

$$\begin{aligned}\dot{V}_{12b} &\leq \sum_{i=1}^{N}\left(2\sum_{j\in\mathcal{N}_i}\left(y_i^j - v_i^j\right)^T M\left(y_i^j - v_i^j\right)\right.\\ &\left.+2\sum_{j\in\mathcal{N}_i}\left(v_i^j - y_i^i\right)^T M\left(v_i^j - y_i^i\right)\right)\end{aligned}$$

As the communication graph is undirected, one gets

$$\begin{aligned}\dot{V}_{12b} &\leq \sum_{i=1}^{N}\left(2\sum_{j\in\mathcal{N}_i}\left(y_j^i - v_j^i\right)^T M\left(y_j^i - v_j^i\right)\right.\\ &\left.+2\sum_{j\in\mathcal{N}_i}\left(v_i^j - y_i^i\right)^T M\left(v_i^j - y_i^i\right)\right)\end{aligned} \tag{A.30}$$

Upper bound for $\dot{V}_{12}$ Finally, combining (A.28) and (A.30) in (A.26), one obtains

$$\begin{aligned}\dot{V}_{12} &\leq c_1\sum_{i=1}^{N}\left[\frac{1}{2b_{i2}}\left(z_i - N_i e_i^i\right)^T M\left(z_i - N_i e_i^i\right)\right.\\ &+\frac{b_{i2}}{2}\sum_{j\in\mathcal{N}_i} N_j\left(y_j^i - v_j^i\right)^T M\left(y_j^i - v_j^i\right)\\ &+\left(z_i - N_i e_i^i\right)^T M\sum_{j\in\mathcal{N}_i}\left(v_i^j - y_i^i\right) + \frac{N_i}{2b_i} e_i^{iT} M e_i^i\\ &+2\left(1+\frac{b_i}{2}\right) N_i\sum_{j\in\mathcal{N}_i}\left[\left(v_i^j - y_i^i\right)^T M\left(v_i^j - y_i^i\right)\right.\\ &\left.\left.+\left(y_j^i - v_j^i\right)^T M\left(y_j^i - v_j^i\right)\right]\right].\end{aligned} \tag{A.31}$$

The upper bound for $\dot{V}_1$ becomes

$$\dot{V}_1 \leq x^T \overline{L} x + \sum_{i=1}^{N}\left(\tilde{\delta}_i - \rho z_i^T \Theta_i z_i\right)$$

with $\tilde{\delta}_i$, ρ and Θ_i as expressed in Theorem 6.

As a consequence, if, for $i, j = 1 \ldots N$, the communications are triggered when $\tilde{\delta}_i > \rho z_i^T \Theta z_i$, then $\dot{V}_1 \leq 0$. The rest of the proof is identical to the one of Theorem 6.

A.3 Proof of absence of Zeno behavior

Two cases are considered: $D_{\max} = 0$ and $D_{\max} > 0$. Consider first the case with no perturbation ($D_{\max} = 0$). In this case, the estimate error e_i^i vanishes. Moreover, since the initial states are assumed to be known by all agents, $y^i(t) = y^j(t) = v^i(t)$ for all (i, j) and for all $t > 0$. As a consequence, the discrepancies $y_j^i - v_j^i = 0$ and $v_i^j - y_i^i = 0$ for all (i, j) and for all $t > 0$. No communication will be triggered, which excludes the possibility of a Zeno behavior.

Consider now the case with $D_{\max} > 0$ and let us proove the absence of Zeno behavior. To do so, let us show that the inter-event time $t_{i,k+1} - t_{i,k}$ is strictly positive.

As the CTC (2.34) mainly depends on e_i^i, we begin by studying the time derivative of this error. From the definition of e_i^i and by remarking that $u_i^i(t) = \tilde{u}_i^i(t)$, it can be expressed as

$$\begin{aligned}\dot{e}_i^i &= \dot{y}_i^i - \dot{x}_i\\ &= \left(Ay_i^i + B\tilde{u}_i^i(t)\right) - \left(Ax_i^i + Bu_i^i + d_i\right)\\ &= Ae_i^i - d_i.\end{aligned} \tag{A.32}$$

Then, it can observed that the derivative of $\left\|e_i^i\right\|$ satisfies

$$\begin{aligned}
\frac{d}{dt}\left\|e_i^i\right\| &= \frac{e_i^{iT}\dot{e}_i^i}{\left\|e_i^i\right\|} \\
\frac{d}{dt}\left\|e_i^i\right\| &= \frac{1}{\left\|e_i^i\right\|}e_i^{iT}\left(Ae_i^i - d_i\right) \\
\frac{d}{dt}\left\|e_i^i\right\| &\leq \frac{1}{\left\|e_i^i\right\|}\left(\|A\|\left\|e_i^i\right\|^2 + \left\|e_i^i\right\| D_{\max}\right) \\
\frac{d}{dt}\left\|e_i^i\right\| &\leq \|A\|\left\|e_i^i\right\| + D_{\max}, \qquad \text{(A.33)}
\end{aligned}$$

Solving the differential equation (A.33) leads to

$$\left\|e_i^i\right\| \leq e^{\|A\|(t-t_{i,k})}\alpha - \frac{D_{\max}}{\|A\|} \qquad \text{(A.34)}$$

for $t \geq t_{i,k}$with α a constant. Remind that the error e_i^i is reset to zero when a message is broadcast by Agent i, so$\left\|e_i^i\left(t_{i,k}\right)\right\| = 0$. This is used to identify the value of $\alpha = \frac{D_{\max}}{\|A\|}$, and to obtain then the general solution of (A.33) for $t \geq t_{i,k}$:

$$\left\|e_i^i\right\| \leq \left(e^{\|A\|(t-t_{i,k})} - 1\right)\frac{D_{\max}}{\|A\|}. \qquad \text{(A.35)}$$

From the CTC (2.34) a new communication will be triggered when $\tilde{\delta}_i = \rho z_i^T \Theta z_i + \eta$. Introducing

$$\check{\delta}_i = \tilde{\delta}_i - e_i^{iT} M e_i^i \bar{b}_i. \qquad \text{(A.36)}$$

A new communication is hence triggrered when

$$\check{\delta}_i + e_i^{iT} M e_i^i \bar{b}_i = \rho z_i^T \Theta z_i + \eta \qquad \text{(A.37)}$$

and one has

$$\begin{aligned}
\check{\delta}_i + \lambda_{\max}(M)\left\|e_i^i\right\|^2 \bar{b}_i &\geq \rho z_i^T \Theta z_i + \eta \\
\left\|e_i^i\right\|^2 &\geq \frac{1}{\lambda_{\max}(M)\,\bar{b}_i}\left(\rho z_i^T \Theta z_i - \check{\delta}_i + \eta\right) \qquad \text{(A.38)}
\end{aligned}$$

since $\lambda_{max}(M) > 0$ and $\bar{b}_i > 0$. Using (A.38) along with (A.35) evaluated at time $t_{i,k+1}$, where the CTC is satisfied, allows to obtain

$$\left(e^{\|A\|(t_{i,k+1}-t_{i,k})} - 1\right)^2\left(\frac{D_{\max}}{\|A\|}\right)^2 \geq \frac{1}{\lambda_{\max}(M)\,\bar{b}_i}\left(\rho z_i^T \Theta z_i - \check{\delta}_i + \eta\right). \qquad \text{(A.39)}$$

Considering the following two assumptions.

Assumption 1: $t_{i,k+1} = t_{i,k}$ According to (2.22), (2.28), and (2.27), $\tilde{\delta}_i = 0$ at $t = t_{i,k}$. As a consequence, the CTC (2.34) in Theorem 6 cannot be satisfied, which contradicts the considered assumption.

Assumption 2: $t_{i,k+1} > t_{i,k}$ According to (A.32), for all $t \in]t_{i,k}, t_{i,k+1}[$ one has $\left\|e_i^i(t)\right\| > 0$. Since $e_i^{iT} M e_i^i \geq \left\|e_i^i\right\| \lambda_{\min}(M)$ and using the fact that$\lambda_{\min}(M) > 0$ since $M = PBB^TP$ is symmetric positive, one deduces that $e_i^{iT} M e_i^i > 0$ for all $t \in]t_{i,k}, t_{i,k+1}[$. This expression and (A.36) imply $\check{\delta}_i < \tilde{\delta}_i$ and

$$\left(\rho z_i^T \Theta z_i - \check{\delta}_i + \eta\right) > 0 \qquad \text{(A.40)}$$

Then using (A.40) and $\left(e^{\|A\|(t_{i,k+1}-t_{i,k})}-1\right)>0$ in (A.39), one gets

$$\left(e^{\|A\|(t_{i,k+1}-t_{i,k})}-1\right)\frac{D_{\max}}{\|A\|} \quad \geq \quad \sqrt{\frac{1}{\lambda_{\max}\left(M\right)\bar{b}_i}\left(\rho z_i^T\Theta z_i-\check{\delta}_i+\eta\right)}.$$

In presence of perturbations, $D_{\max}>0$, thus

$$e^{\|A\|(t_{i,k+1}-t_{i,k})} \geq 1+\sqrt{\frac{\|A\|^2}{\lambda_{\max}\left(M\right)\bar{b}_i D_{\max}^2}\left(\rho z_i^T\Theta z_i-\check{\delta}_i+\eta\right)}$$

$$t_{i,k+1}-t_{i,k} \geq \frac{1}{\|A\|}\ln\left[1+\sqrt{\frac{\|A\|^2}{\lambda_{\max}\left(M\right)\bar{b}_i D_{\max}^2}\left(\rho z_i^T\Theta z_i-\check{\delta}_i+\eta\right)}\right]. \tag{A.41}$$

Since $\left(\rho z_i^T\Theta z_i-\check{\delta}_i+\eta\right)>0$, from (A.41), one deduces that $t_{i,k+1}-t_{i,k}>0$, which excludes the possibility of a Zeno behavior.

Appendix B

Appendix of Chapter 3

B.1 Calculation of spring coefficients k_{ij}

In [82], the spring coefficients $k_{ij} > 0$ are expressed as

$$k_{ij} = \begin{cases} \frac{\omega_{ij}}{1-\beta_{ij}^c} & \text{, if } \omega_{ij} > 0 \\ \frac{\omega_{ij}}{1-\beta_{ij}^s} & \text{, if } \omega_{ij} < 0 \end{cases}$$

where $\beta_{ij}^c \in (0, 1)$ and $\beta_{ij}^s \in (1, +\infty)$ are constants and where ω_{ij} are elements of the *stress matrix* $\Omega = [\omega_{ij}]_{N\times N}$ associated to the desired formation shape. ω_{ij} is positive or negative using analogy to structures defined using incompressible *struts* or inextensible *cable*s, with $\omega_{ij} = \omega_{ji}$. In tensegrety structures, an equilibrium structure is obtained only if for all $i \in \mathcal{N}$

$$\sum_{j\in\mathcal{N}_i} \omega_{ij} r_{ij}^* = 0.$$

[82] proposes a method to generate a stress matrix Ω such as the associated tensegrity structure corresponds to a stable formation. Main steps are described in the following paragraph.

The stress matrix Ω is expressed as

$$\Omega = DD^T$$

where D is computed as solution of the following equality

$$\bar{N}^T D = 0_{(n+1)\times(N-n-1)} \tag{B.1}$$

with $D \neq 0_{N\times(N-n-1)}$ and $\bar{N} = \left[[q_1^* \dots q_N^*]\ 1_N^T\right] \in \mathbb{R}^{N\times(n+1)}$.

More details on the calculation of D is presented in [82].

B.2 Proof of formation convergence

Considering a given value of $D_{\max}$and η, one shows first that the MAS converges asymptotically to some bounded region. Then one evaluates the impact of $D_{\max}$ and η on the size of this region.

B.2.1 Proof of the ISpS of system

Consider the candidate Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{N} s_i^T M_i s_i + \frac{k_g}{4} P(q, t) \tag{B.2}$$

Taking the time derivative of V leads to

$$\dot{V} = \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T\dot{M}_i s_i + s_i^T M_i\dot{s}_i\right] + \frac{k_g}{4}\frac{d}{dt}P\left(q,\, t\right) \tag{B.3}$$

where $\dot{s}_i = \ddot{q}_i + k_p\dot{g}_i$. It can be shown that

$$\begin{aligned}
& \frac{1}{4}\frac{d}{dt}P\left(q,t\right) \\
= & \frac{1}{4}\frac{d}{dt}\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\left\|r_{ij}-r_{ij}^*\right\|^2 \\
= & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left(\dot{r}_{ij}-\dot{r}_{ij}^*\right)^T\left(r_{ij}-r_{ij}^*\right)\right] \\
= & \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\left[\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(r_{ij}-r_{ij}^*\right)-\left(\dot{q}_j-\dot{q}_j^*\right)^T\left(r_{ij}-r_{ij}^*\right)\right] \\
= & \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\left[\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(r_{ij}-r_{ij}^*\right)-\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(r_{ji}-r_{ji}^*\right)\right]
\end{aligned} \tag{B.4}$$

and since $r_{ji} = -r_{ij}$

$$\begin{aligned}
\frac{1}{4}\frac{d}{dt}P\left(q,t\right) &= \sum_{i=1}^{N}\left(\dot{q}_i-\dot{q}_i^*\right)^T\left[\sum_{j=1}^{N}k_{ij}\left(r_{ij}-r_{ij}^*\right)+k_0 r_i\right] \\
&= \sum_{i=1}^{N}\left(\dot{q}_i-\dot{q}_i^*\right)^T g_i
\end{aligned} \tag{B.5}$$

Thus

$$\dot{V} = \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T\dot{M}_i s_i + s_i^T M_i\dot{s}_i + k_g\left(\dot{q}_i-\dot{q}_i^*\right)^T g_i\right] \tag{B.6}$$

One focuses now on the term $M_i\dot{s}_i$ and tries to find an equivalent expression. One may write

$$\begin{aligned}
M_i\dot{s}_i + C_i s_i &= M_i\left[\ddot{q}_i + k_p\dot{g}_i\right] + C_i\left[\dot{q}_i + k_p g_i\right] \\
&= \tau_i + k_p\left(M_i\dot{g}_i + C_i g_i\right) + d_i
\end{aligned} \tag{B.7}$$

Using (3.10), one gets

$$M_i\dot{s}_i + C_i s_i = -k_s\bar{s}_i - k_g\bar{g}_i - k_p\left(M_i\left(\dot{\bar{g}}_i - \dot{g}_i\right) + C_i\left(\bar{g}_i - g_i\right)\right) + d_i \tag{B.8}$$

Now, introducing (3.4) in (3.6), one may write

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p\left[\sum_{i=1}^{N}k_{ij}\left(q_i - q_j - r_{ij}^*\right)\right]. \tag{B.9}$$

Since $e_j^i = \hat{q}_j^i - q_j$, one gets

$$\begin{aligned}
s_i &= \dot{q}_i - \dot{q}_i^* + k_p\left[\sum_{i=1}^{N}k_{ij}\left(q_i - \hat{q}_j^i + e_j^i - r_{ij}^*\right)\right] \\
&= \dot{q}_i - \dot{q}_i^* + k_p\left[\sum_{i=1}^{N}k_{ij}\left(\bar{r}_{ij} - r_{ij}^*\right)\right] + k_p\sum_{\substack{j=1 \\ j\neq i}}^{N}k_{ij}e_j^i \\
&= \bar{s}_i + k_p E_j^i
\end{aligned} \tag{B.10}$$

with since $k_{ii} = 0$

$$E_j^i = \sum_{i=1}^{N} k_{ij} e_j^i. \tag{B.11}$$

In the same way, it can be obtained $g_i = \bar{g}_i + E_j^i$. One gets

$$M_i \dot{s}_i + C_i s_i = -k_s \bar{s}_i - k_g \bar{g}_i + k_p \left(M_i \dot{E}_j^i + C_i E_j^i \right) + d_i. \tag{B.12}$$

Let $\dot{V}_1 = \sum_{i=1}^{N} 2k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i \right)$. Using (B.12) in (B.3), one obtains

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} \left[s_i^T \left[\frac{1}{2} \dot{M}_i - C_i \right] s_i - k_s s_i^T \bar{s}_i - k_g \left(\dot{q}_i + k_p g_i \right)^T \bar{g}_i \right. \\ &\quad \left. + k_g \dot{q}_i^T g_i + s_i^T d_i \right] + \frac{1}{2} \dot{V}_1 \end{aligned} \tag{B.13}$$

Remind than $\frac{1}{2}\dot{M}_i - C_i$ is skew symmetric or definite negative, so $s_i^T \left[\frac{1}{2}\dot{M}_i - C_i \right] s_i \leq 0$. For all $b > 0$ and all vectors x and y of similar size, one has

$$x^T y \leq \frac{1}{2} \left(b x^T x + \frac{1}{b} y^T y \right). \tag{B.14}$$

Using (B.14) with $b = 1$, one deduces that $d_i^T s_i \leq \frac{1}{2} \left(D_{\max}^2 + s_i^T s_i \right)$ and that

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^{N} \left[-k_s s_i^T \bar{s}_i - g_i^T \bar{g}_i k_g k_p + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \right. \\ &\quad \left. + k_g \dot{q}_i^T \left(g_i - \bar{g}_i \right) \right] + \frac{1}{2} \dot{V}_1 \end{aligned} \tag{B.15}$$

One notices that $r_{ij} = q_i - q_j = q_i - \hat{q}_j^i + e_j^i = \bar{r}_{ij} + e_j^i$, thus

$$\begin{aligned} \left\| s_i - \bar{s}_i \right\|^2 &= s_i^T s_i - 2 s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\ \left\| k_p E_j^i \right\|^2 &= s_i^T s_i - 2 s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\ s_i^T \bar{s}_i &= -\frac{1}{2} \left\| k_p E_j^i \right\|^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2} \bar{s}_i^T \bar{s}_i \end{aligned} \tag{B.16}$$

In the same way, one may obtain $g_i^T \bar{g}_i = -\frac{1}{2} \left\| E_j^i \right\|^2 + \frac{1}{2} g_i^T g_i + \frac{1}{2} \bar{g}_i^T \bar{g}_i$. Define $\left\| E_j^i \right\|_L^2 = E_j^{iT} L E_j^i$ for any matrix L. Injecting it and using (B.16) in (B.15) leads to

$$\begin{aligned} \dot{V} &\leq \frac{1}{2} \sum_{i=1}^{N} \left[-\left(k_s - 1 \right) s_i^T s_i - \frac{k_s}{2} \bar{s}_i^T \bar{s}_i + \left(k_s k_p^2 + k_g k_p \right) \left\| E_j^i \right\|^2 \right. \\ &\quad \left. -2 k_p k_g \frac{1}{2} g_i^T g_i - 2 k_p k_g \frac{1}{2} \bar{g}_i^T \bar{g}_i + D_{\max}^2 + 2 k_g \dot{q}_i^T E_j^i \right] + \frac{1}{2} \dot{V}_1 \end{aligned} \tag{B.17}$$

Using (B.14) with $b = b_i > 0$, one shows that $2\dot{q}_i^T \left(g_i - \bar{g}_i \right) \leq \left(b_i \left\| \dot{q}_i \right\|^2 + \frac{1}{b_i} \left\| E_j^i \right\|^2 \right)$. Using this result in (B.17) and using $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$ one gets

$$\begin{aligned} \dot{V} &\leq \frac{1}{2} \sum_{i=1}^{N} \left[-\left(k_s - 1 \right) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + k_e \left\| E_j^i \right\|^2 + b_i k_g \left\| \dot{q}_i \right\|^2 \right. \\ &\quad \left. - k_p k_g g_i^T g_i - k_p k_g \bar{g}_i^T \bar{g}_i + D_{\max}^2 \right] + \frac{1}{2} \dot{V}_1 \end{aligned} \tag{B.18}$$

Consider now $\dot{V}_1$. Using (B.14) with $b = 1$, the fact that M_i is symmetric positive definite, and that $x^T M_i x < k_M x^T x$, one obtains

$$\dot{V}_1 \leq \sum_{i=1}^{N} k_p \left(\left(k_M + 1 \right) s_i^T s_i + \left[k_M \dot{E}_j^{iT} \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i \right] \right) \tag{B.19}$$

$$\begin{aligned}\sum_{i=1}^{N} 2k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i\right) &\leq \sum_{i=1}^{N} k_p \left(s_i^T M_i s_i + s_i^T s_i + \left[\dot{E}_j^{iT} M_i \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i\right]\right) \\ &\leq \sum_{i=1}^{N} k_p \left(\left(k_M + 1\right) s_i^T s_i + \left[k_M \dot{E}_j^{iT} \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i\right]\right) \qquad \text{(B.20)}\end{aligned}$$

Focus on the terms $E_j^{iT} C_i^T C_i E_j^i$

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &= \sum_{i=1}^{N} \left(\sum_{j=1}^{N} k_{ij} e_j^i\right)^T C_i^T C_i \left(\sum_{\ell=1}^{N} k_{i\ell} e_\ell^i\right) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} k_{ij} e_j^{iT} \left\|C_i\right\|^2 e_\ell^i \qquad \text{(B.21)}\end{aligned}$$

Using (B.14) with $b = 1$, one gets

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &\leq \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} k_{ij} \left(e_j^{iT} \left\|C_i\right\|^2 e_j^i + e_\ell^{iT} \left\|C_i\right\|^2 e_\ell^i\right) \\ &\leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} k_{ij} \left(e_j^{iT} \left\|C_i\right\|^2 e_j^i\right) \\ &\leq \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} k_{ij} \left(e_j^{iT} \left\|C_i\right\|^2 e_j^i\right) \qquad \text{(B.22)}\end{aligned}$$

Since one has assumed that (3.23) and (3.22) are satisfied, one has $\hat{q}_j^i = \hat{q}_j^j$ and $e_j^i = e_j^j$. As a consequence,

$$\sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\|e_j^i\right\|^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\|e_j^j\right\|^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ji} \left\|e_i^i\right\|^2. \qquad \text{(B.23)}$$

and since $k_{ij} = k_{ji}$

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &\leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} \sum_{j=1}^{N} \left[k_{ij} \left\|e_i^i\right\|^2 \left\|C_j\right\|^2\right]\right) \\ &\leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} \sum_{j=1}^{N} \left[k_{ij} \left\|e_i^i\right\|^2 k_C^2 \left\|\dot{q}_j\right\|^2\right]\right). \qquad \text{(B.24)}\end{aligned}$$

The second CTC (3.21) leads to

$$\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i \leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} k_C^2 \left\|e_i^i\right\|^2 \sum_{j=1}^{N} k_{ij} \left(\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right)^2\right). \qquad \text{(B.25)}$$

In the same way, one shows that $\sum_{i=1}^{N} E_j^{iT} E_j^i \leq \sum_{i=1}^{N} \alpha_{\mathrm{M}}^2 \left\|e_i^i\right\|^2$ and $\sum_{i=1}^{N} \dot{E}_j^{iT} \dot{E}_j^i \leq \sum_{i=1}^{N} \alpha_{\mathrm{M}}^2 \dot{e}_i^{iT} \dot{e}_i^i$. Injecting it in (B.18), one gets

$$\begin{aligned}\dot{V} \leq\ & \frac{1}{2} \sum_{i=1}^{N} \left[-\left(k_s - 1 - k_p \left(k_M + 1\right)\right) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + D_{\max}^2\right. \\ & -k_p k_g g_i^T g_i - k_p k_g \bar{g}_i^T \bar{g}_i + k_g b_i \left\|\dot{q}_i\right\|^2 + k_p k_M \alpha_{\mathrm{M}}^2 \left\|\dot{e}_i^i\right\|^2 \\ & +\alpha_{\mathrm{M}}^2 \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}\right) \left\|e_i^i\right\|^2 + \alpha_{\mathrm{M}} k_p k_C^2 \left\|e_i^i\right\|^2 \sum_{j=1}^{N} k_{ij} \left[\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right]^2 \qquad \text{(B.26)}\end{aligned}$$

The CTC (3.20)leads to

$$\begin{aligned}\dot{V} &\leq \frac{1}{2}\sum_{i=1}^{N}\left[-\left(k_s-1-k_p\left(k_M+1\right)\right)s_i^T s_i - k_g k_p g_i^T g_i + D_{\max}^2+\eta\right] && \text{(B.27)}\\ &\leq \frac{1}{2}\sum_{i=1}^{N}\left[-k_1 s_i^T s_i - k_g k_p g_i^T g_i + D_{\max}^2+\eta\right] && \text{(B.28)}\end{aligned}$$

where $k_1 = \left(k_s - 1 - k_p\left(k_M+1\right)\right)$.

Following the steps given in Appendix B.4.1 from (B.47) to (B.50), one shows that

$$\dot{V} \leq -c_3 V + \frac{N}{2}\left[D_{\max}^2+\eta\right] \tag{B.29}$$

where $c_3 > 0$ is a positive constant. Define the function W such that $W(0) = V(0)$ and

$$\dot{W} = -c_3 W + \frac{N}{2}\left[D_{\max}^2+\eta\right]. \tag{B.30}$$

Using the initial condition $W(0) = V(0)$, the solution of (B.30) is

$$W(t) = \exp\left(-c_3 t\right)V(0) + \left(1-\exp\left(-c_3 t\right)\right)\frac{N}{2c_3}\left[D_{\max}^2+\eta\right]. \tag{B.31}$$

Then, using the Comparison Lemma (Lemma 3.4 [53]), one has $V(t) \leq W(t)$ and so

$$V(t) \leq \exp\left(-c_3 t\right)V(0) + \left(1-\exp\left(-c_3 t\right)\right)\frac{N}{2c_3}\left[D_{\max}^2+\eta\right] \tag{B.32}$$

Since M_i is symmetric, there exists a matrix S_{M_i} such that $M_i = S_{M_i}^T S_{M_i}$. Introduce now

$$\begin{aligned}y_M &= \begin{bmatrix} \left(S_{M_1}s_1\right)^T & \ldots & \left(S_{M_i}s_i\right)^T & \ldots & \left(S_{M_N}s_N\right)^T \end{bmatrix}^T\\ z &= \begin{bmatrix} y_M^T & \sqrt{\frac{k_g}{2}P\left(x,t\right)} \end{bmatrix}^T\end{aligned}$$

Then, $V(t)$ can be rewritten as

$$V(z) = \frac{1}{2}z^T z. \tag{B.33}$$

Using (B.33) in (B.32), one has $\forall t \geq 0$

$$\begin{aligned}\left\|z(t)\right\|^2 &\leq \exp\left(-c_3 t\right)\left\|z(0)\right\|^2 + \left(1-\exp\left(-c_3 t\right)\right)\frac{N}{c_3}\left[D_{\max}^2+\eta\right]\\ \left\|z(t)\right\| &\leq \sqrt{\exp\left(-c_3 t\right)\left\|z(0)\right\|^2 + \left(1-\exp\left(-c_3 t\right)\right)\frac{N}{c_3}\left[D_{\max}^2+\eta\right]}\\ \left\|z(t)\right\| &\leq \sqrt{\exp\left(-c_3 t\right)\left\|z(0)\right\|^2} + \sqrt{\left(1-\exp\left(-c_3 t\right)\right)\frac{N}{c_3}\left[D_{\max}^2+\eta\right]}\\ \left\|z(t)\right\| &\leq \exp\left(-\frac{c_3}{2}t\right)\left\|z(0)\right\| + \sqrt{\frac{N}{c_3}\left[D_{\max}^2+\eta\right]} && \text{(B.34)}\end{aligned}$$

and therefore

$$\left\|z(t)\right\| \leq \beta\left(\left\|z(0)\right\|, t\right) + \rho \tag{B.35}$$

with $\rho = \sqrt{\frac{N}{c_3}\left[D_{\max}^2+\eta\right]}$, $\beta\left(\left\|z(0)\right\|, t\right) = \exp\left(-\frac{c_3}{2}t\right)\left\|z(0)\right\|$ and $\beta \in \mathcal{KL}$ (see definition of class $\mathcal{KL}$ functions in Section 1.4.5). Using Definition 2.1 from [48], (B.35) implies that the system is input-to-state practically stable (ISpS).

B.2.2 Convergence of V

From (B.35), we know that the system is ISpS. Moreover, from (B.29), one has

$$\dot{V} \leq -c_3 V + \frac{N}{2}\left[D_{\max}^2 + \eta\right] \tag{B.36}$$

Then, if initially

$$-c_3 V\left(0\right) + \frac{N}{2}\left[D_{\max}^2 + \eta\right] < 0 \tag{B.37}$$

one has $\dot{V} \leq 0$ and V is decreasing. Then, one has from (B.32)

$$\begin{aligned}
\lim_{t\to\infty} V\left(t\right) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \\
\lim_{t\to\infty} \frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i\right) + \frac{k_g}{4} P\left(q,t\right) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \\
\lim_{t\to\infty} \frac{k_g}{2} P\left(q,t\right) &\leq \frac{N}{c_3}\left[D_{\max}^2 + \eta\right] - \lim_{t\to\infty}\sum_{i=1}^{N}\left(s_i^T M_i s_i\right) \\
\lim_{t\to\infty} P\left(q,t\right) &\leq \frac{2N}{k_g c_3}\left[D_{\max}^2 + \eta\right]
\end{aligned} \tag{B.38}$$

Asymptotically, the formation and tracking error are bounded.

B.3 Proof of $t_{i,k+1} - t_{i,k} > 0$

From the CTC (7), a communication is triggered at $t = t_{i,k}^-$ when

$$\begin{aligned}
k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta = \alpha_{\mathrm{M}}^2 &\left(k_e \left\|e_i^i\right\|^2 + k_p k_M \left\|\dot{e}_i^i\right\|^2\right) \\
&+ \alpha_{\mathrm{M}} k_C^2 k_p \left\|e_i^i\right\|^2 \sum_{j=1}^{N} k_{ji}\left[\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right]^2 + k_g b_i \left\|\dot{q}_i\right\|^2
\end{aligned} \tag{B.39}$$

with $k_e = \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}\right)$. Then, the estimation errors e_i^i and $\dot{e}_i^i$ are reset and one has $e_i^i\left(t_{i,k}^+\right) = 0$ and $\dot{e}_i^i\left(t_{i,k}^+\right) = 0$. As a consequence, the CTC (3.20) in Theorem 7 is not satisfied at $t = t_{i,k}^+$ iff

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta > k_g b_i \left\|\dot{q}_i - \dot{q}_i^*\right\|^2. \tag{B.40}$$

To prove $t_{i,k+1} > t_{i,k}$, one has to show that (B.40) is satisfied.

Using the property $x^T y \geq -\frac{1}{2}\left(b_{i2} x^T x + \frac{1}{b_{i2}} y^T y\right)$ for some $b_{i2} > 0$, one deduces that

$$\begin{aligned}
\bar{s}_i^T \bar{s}_i &= k_p^2 \bar{g}_i^T \bar{g}_i + \left\|\dot{q}_i\right\|^2 + 2k_p \bar{g}_i^T \dot{q}_i \\
&\geq \left(k_p^2 - k_p b_{i2}\right)\bar{g}_i^T \bar{g}_i + \left(1 - \frac{k_p}{b_{i2}}\right)\left\|\dot{q}_i\right\|^2.
\end{aligned} \tag{B.41}$$

Using (B.41), a sufficient condition for (B.40) to be satisfied is

$$\begin{aligned}
k_s\left(k_p^2 - k_p b_{i2}\right)\bar{g}_i^T \bar{g}_i + k_s\left(1 - \frac{k_p}{b_{i2}}\right)\left\|\dot{q}_i\right\|^2 + k_p k_g \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \left\|\dot{q}_i\right\|^2 \\
k_s\left(1 - \frac{k_p}{b_{i2}}\right)\left\|\dot{q}_i\right\|^2 + \left[k_p k_g + k_s\left(k_p^2 - k_p b_{i2}\right)\right]\bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \left\|\dot{q}_i\right\|^2 \\
k_1 \bar{g}_i^T \bar{g}_i + \eta &> k_2 \left\|\dot{q}_i\right\|^2
\end{aligned} \tag{B.42}$$

where $k_1 = \left[k_p k_g + k_s \left(k_p^2 - k_p b_{i2}\right)\right]$ and $k_2 = \left[k_g b_i - k_s \left(1 - \frac{k_p}{b_{i2}}\right)\right]$. To ensure that the inequality (B.42) is satisfied independently of the values of $\bar{g}_i$ and $\dot{q}_i$, it is sufficient to find b_i and b_{i2} such that $k_1 > 0$ and $k_2 < 0$. Consider first k_1.

$$\begin{aligned} k_p k_g + k_s \left(k_p^2 - k_p b_{i2}\right) &> 0 \\ \frac{k_g}{k_s} &> \left(-k_p + b_{i2}\right) \\ \frac{k_s k_p + k_g}{k_s} &> b_{i2}. \end{aligned} \tag{B.43}$$

Focus now on k_2

$$\begin{aligned} k_g b_i - k_s \left(1 - \frac{k_p}{b_{i2}}\right) &< 0 \\ \frac{k_g b_i}{k_s} &< 1 - \frac{k_p}{b_{i2}} \\ \frac{1}{b_{i2}} &< \frac{1}{k_p}\left(1 - \frac{k_g b_i}{k_s}\right). \end{aligned} \tag{B.44}$$

Remind that $b_{i2} > 0$, which implies $\frac{k_g b_i}{k_s} < 1$ that is $b_i < \frac{k_s}{k_g}$. Therefore one gets

$$\frac{k_s k_p}{k_s - k_g b_i} < b_{i2}.$$

Finally, one has to find a condition on b_i such that (B.43) and (B.44) can be satisfied simultaneously

$$\frac{k_s k_p + k_g}{k_s} > b_{i2} > \frac{k_s k_p}{k_s - k_g b_i}. \tag{B.45}$$

One may find such a b_{i2} if

$$\begin{aligned} k_s - k_g b_i &> \frac{k_s^2 k_p}{k_s k_p + k_g} \\ \frac{1}{k_g}\left(k_s - \frac{k_s^2 k_p}{k_s k_p + k_g}\right) &> b_i \\ b_i &< \frac{k_s}{k_s k_p + k_g}. \end{aligned} \tag{B.46}$$

which also ensures that $b_i < \frac{k_s}{k_g}$. Thus, once $b_i < \frac{k_s}{k_s k_p + k_g}$, there exists some b_{i2} such that (B.45) is satisfied. As a consequence $t_{i,k+1} - t_{i,k} > 0$.

B.4 Complementary proof elements

B.4.1 Differential equation of V

From (B.27), one gets

$$\dot{V} \leq \frac{1}{2} \sum_{i=1}^{N} \left[-k_{\mathrm{m}} \left(s_i^T s_i - k_g g_i^T g_i\right) + D_{\max}^2 + \eta\right] \tag{B.47}$$

where $k_{\mathrm{m}} = \min\{k_1, k_p\}$. Using (B.57) from Appendix B.4.2, one may write

$$\sum_{i=1}^{N} g_i^T g_i \geq \frac{\alpha_{\min} k_{\min}}{k_{\max}} P(q,t) \tag{B.48}$$

Define $k_3 = \frac{\alpha_{\min} k_{\min}}{k_{\max}}$. Then

$$\begin{aligned}
\dot{V} &\leq -\frac{1}{2}\sum_{i=1}^{N}\left(k_{\mathrm{m}} s_i^T s_i\right) - \frac{k_3 k_g}{4} P\left(q,t\right) + \frac{N}{2}\left(D_{\max}^2 + \eta\right) \\
&\leq -\frac{1}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N} k_{\mathrm{m}}\left(k_{\mathrm{M}} s_i^T s_i\right) + \frac{k_3 k_g}{4} P\left(q,t\right)\right] + \frac{N}{2}\left(D_{\max}^2 + \eta\right) \\
&\leq -\frac{k_4}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N}\left(k_{\mathrm{M}} s_i^T s_i\right) + \frac{k_g}{4} P\left(q,t\right)\right] + \frac{N}{2}\left(D_{\max}^2 + \eta\right)
\end{aligned} \tag{B.49}$$

with $k_M^* = 1$ if $k_M < 1$ and$k_M^* = k_M$ else, and $k_4 = \min\left(k_{\mathrm{m}}, k_3\right)$. Let $c_3 = \frac{k_4}{k_M^*}$ and one gets

$$\begin{aligned}
\dot{V} &\leq -c_3\left[\frac{1}{2}\sum_{i=1}^{N} s_i^T M_i s_i + \frac{k_g}{4} P\left(q,t\right)\right] + \frac{N}{2}\left[D_{\max}^2 + \eta\right] \\
\dot{V} &\leq -c_3 V + \frac{N}{2}\left[D_{\max}^2 + \eta\right]
\end{aligned} \tag{B.50}$$

The evaluation of c_3 is described in Section B.4.3.

B.4.2 Upper-bound on $\sum_{i=1}^{N} g_i^T g_i$

From (3.4), one has

$$\sum_{i=1}^{N} g_i^T g_i = \sum_{i=1}^{N}\left\|\sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right)\right\|^2 \tag{B.51}$$

One may write

$$\begin{aligned}
\sum_{i=1}^{N}\left\|\sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right)\right\|^2 &= \sum_{i=1}^{N}\left(\sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right)\right)^T\left(\sum_{\ell=1}^{N} k_{i\ell}\left(r_{i\ell} - r_{i\ell}^*\right)\right) \\
&= \sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} k_{i\ell} k_{ij}\left(r_{ij} - r_{ij}^*\right)^T\left(r_{i\ell} - r_{i\ell}^*\right)
\end{aligned} \tag{B.52}$$

Using the fact that, for any vectors a and b, $2a^T b = a^T a + b^T b - (a-b)^T (a-b)$, one deduces

$$\sum_{i=1}^{N}\left\|\sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right)\right\|^2 = \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N} k_{i\ell} k_{ij}\left[\left\|r_{ij} - r_{ij}^*\right\|^2 + \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 - \left\|\left(r_{ij} - r_{ij}^*\right) - \left(r_{i\ell} - r_{i\ell}^*\right)\right\|^2\right]\right] \tag{B.53}$$

We remark that $\left(r_{ij} - r_{ij}^*\right) - \left(r_{i\ell} - r_{i\ell}^*\right) = \left(r_{ij} - r_{i\ell}\right) - \left(r_{ij}^* - r_{i\ell}^*\right) = r_{\ell j} - r_{\ell j}^*$. Injecting it into (B.53) yields

$$\sum_{i=1}^{N}\left\|\sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right)\right\|^2 = \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N} k_{i\ell} k_{ij}\left[\left\|r_{ij} - r_{ij}^*\right\|^2 + \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 - \left\|r_{\ell j} - r_{\ell j}^*\right\|^2\right]\right]$$

with $k_{\max} = \max_{\substack{\ell = 1 \ldots N \\ j = 1 \ldots N}} (k_{\ell j})$

$$k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 \geq \sum_{i=1}^{N} \left[\frac{1}{2} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left[\left\| r_{ij} - r_{ij}^* \right\|^2 + \left\| r_{i\ell} - r_{i\ell}^* \right\|^2 - \left\| r_{\ell j} - r_{\ell j}^* \right\|^2 \right] \right]$$

$$k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 \geq \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{i\ell} - r_{i\ell}^* \right\|^2 - \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{\ell j} - r_{\ell j}^* \right\|^2$$

$$k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 \geq \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2 - \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2$$

$$k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 \geq \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2 \tag{B.54}$$

Let $k_{\min} = \min_{\substack{\ell = 1 \ldots N \\ j = 1 \ldots N}} (k_{\ell j} \neq 0)$ and $\alpha_{\min} = \min_{i=1,\ldots,N} \alpha_i$ with $\alpha_i = \sum_{j=1}^{N} k_{ij}$. One may write

$$\begin{aligned} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2 &= \sum_{i=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} \sum_{j=1}^{N} k_{ij} k_{\ell j} \left\| r_{ij} - r_{ij}^* \right\|^2 \\ &\geq \sum_{i=1}^{N} \sum_{\ell=1}^{N} k_{i\ell} k_{\min} \sum_{j=1}^{N} k_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2 \\ &\geq \sum_{i=1}^{N} \alpha_i k_{\min} \sum_{j=1}^{N} k_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2 \\ &\geq \alpha_{\min} k_{\min} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2 \\ &\geq 2 \alpha_{\min} k_{\min} P(q,t) \end{aligned} \tag{B.55}$$

Injecting (B.55) in (B.54) to get

$$k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 \geq \alpha_{\min} k_{\min} P(q,t)$$

$$\sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 \geq \frac{\alpha_{\min} k_{\min}}{k_{\max}} P(q,t) \tag{B.56}$$

and, using (B.51), one obtains:

$$\sum_{i=1}^{N} g_i^T g_i \geq \frac{\alpha_{\min} k_{\min}}{k_{\max}} P(q,t) \tag{B.57}$$

B.4.3 Evaluation of c_3

Let evaluate c_3:

$$
\begin{aligned}
c_3 &= \frac{k_4}{k_M^*} \\
&= \frac{\min\left(k_{\mathrm{m}}, k_3\right)}{\max\left\{1, k_M\right\}} \\
&= \frac{\min\left\{\min\left\{k_1, k_p\right\}, \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right\}}{\max\left\{1, k_M\right\}} \\
&= \frac{\min\left\{k_1, k_p, \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right\}}{\max\left\{1, k_M\right\}}
\end{aligned}
$$

where $k_1 = k_s - 1 - k_p\left(k_M + 1\right)$, $\alpha_{\min} = \min_{i=1,\ldots,N} \alpha_i$, $k_{\max} = \max_{\substack{\ell = 1 \ldots N \\ j = 1 \ldots N}} \left(k_{\ell j}\right)$ and $k_{\min} = \min_{\substack{\ell = 1 \ldots N \\ j = 1 \ldots N}} \left(k_{\ell j} \neq 0\right)$.

Appendix C

Appendix of Chapter 4

C.1 Proof of Theorem 8

Consider a given value of $D_{\max}$ and η, one shows first that the MAS is input-to-state practically stable. One then evaluates the influence of $D_{\max}$ and η on the behavior of the MAS.

C.1.1 Proof of the input-to-state practical stability of the MAS

Consider the continuous positive-definite candidate Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i + \Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i\right) + \frac{k_g}{2}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N} k_0 \left\|q_i - q_i^*\right\|^2\right] \tag{C.1}$$

where $\Delta\theta_i = \bar{\theta}_i - \theta_i$. The time derivative of V is

$$\dot{V} = \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i + \Delta\theta_i^T \Gamma_i^{-1} \dot{\bar{\theta}}_i\right] + \frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N} k_0 \left\|q_i - q_i^*\right\|^2\right] \tag{C.2}$$

where, from (4.10), one has $\dot{s}_i = \ddot{q}_i - \ddot{q}_i^* + k_p \dot{g}_i$. Injecting (4.14) in (C.2) one obtains

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i + \Delta\theta_i^T Y_i\left(q_i,\, \dot{q}_i,\, \dot{\bar{p}}_i,\, \bar{p}_i\right)\bar{s}_i\right] \\ &\quad + \frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N} k_0 \left\|q_i - q_i^*\right\|^2\right].\end{aligned} \tag{C.3}$$

The last term in (C.3) may be written as

$$\begin{aligned}
& \frac{1}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\,t\right)+\sum_{i=1}^{N}k_{0}\left\Vert q_{i}-q_{i}^{*}\right\Vert ^{2}\right] \\
= & \frac{1}{4}\frac{d}{dt}\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\left\Vert r_{ij}-r_{ij}^{*}\right\Vert ^{2}+\frac{1}{2}\frac{d}{dt}\sum_{i=1}^{N}k_{0}\left\Vert q_{i}-q_{i}^{*}\right\Vert ^{2} \\
= & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left(\dot{r}_{ij}-\dot{r}_{ij}^{*}\right)^{T}\left(r_{ij}-r_{ij}^{*}\right)+k_{0}\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}\left(q_{i}-q_{i}^{*}\right)\right] \\
= & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left[\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}\left(r_{ij}-r_{ij}^{*}\right)-\left(\dot{q}_{j}-\dot{q}_{j}^{*}\right)^{T}\left(r_{ij}-r_{ij}^{*}\right)\right]\right. \\
& \left.+k_{0}\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}\left(q_{i}-q_{i}^{*}\right)\right] \\
= & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left[\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}\left(r_{ij}-r_{ij}^{*}\right)-\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}\left(r_{ji}-r_{ji}^{*}\right)\right]\right. \\
& \left.+k_{0}\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}r_{i}\right]. \qquad (C.4)
\end{aligned}$$

Since $r_{ji}=-r_{ij}$, one gets

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\,t\right)+\sum_{i=1}^{N}k_{0}\left\Vert q_{i}-q_{i}^{*}\right\Vert ^{2}\right] & = \sum_{i=1}^{N}\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}\left[\sum_{j=1}^{N}k_{ij}\left(r_{ij}-r_{ij}^{*}\right)+k_{0}r_{i}\right] \\
& = \sum_{i=1}^{N}\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}g_{i}. \qquad (C.5)
\end{aligned}$$

Combining (C.3) and (C.5), one obtains

$$\dot{V}=\sum_{i=1}^{N}\left[\frac{1}{2}s_{i}^{T}\dot{M}_{i}s_{i}+s_{i}^{T}M_{i}\dot{s}_{i}+\Delta\theta_{i}^{T}Y_{i}\left(q_{i},\,\dot{q}_{i},\,\dot{\bar{p}}_{i},\,\bar{p}_{i}\right)\bar{s}_{i}+k_{g}\left(\dot{q}_{i}-\dot{q}_{i}^{*}\right)^{T}g_{i}\right]. \qquad (C.6)$$

One focuses now on the term $M_{i}\dot{s}_{i}$. Using again (4.10), one may write

$$M_{i}\dot{s}_{i}+C_{i}s_{i}=M_{i}\left(\ddot{q}_{i}-\ddot{q}_{i}^{*}+k_{p}\dot{g}_{i}\right)+C_{i}\left(\dot{q}_{i}-\dot{q}_{i}^{*}+k_{p}g_{i}\right) \qquad (C.7)$$

Using (4.1), one gets

$$M_{i}\dot{s}_{i}+C_{i}s_{i} = \tau_{i}+d_{i}-G+M_{i}\left(k_{p}\dot{g}_{i}-\ddot{q}_{i}^{*}\right)+C_{i}\left(k_{p}g_{i}-\dot{q}_{i}^{*}\right), \qquad (C.8)$$

where one used (4.1). Now, introducing (4.13), one gets

$$\begin{aligned}
M_{i}\dot{s}_{i}+C_{i}s_{i} = & -k_{s}\bar{s}_{i}-k_{g}\bar{g}_{i}-Y_{i}\left(q_{i},\,\dot{q}_{i},\,k_{p}\dot{\bar{g}}_{i}-\ddot{q}_{i}^{*},k_{p}\bar{g}_{i}-\dot{q}_{i}^{*}\right)\bar{\theta}_{i} \\
& +M_{i}\left(k_{p}\dot{g}_{i}-\ddot{q}_{i}^{*}\right)+C_{i}\left(k_{p}g_{i}-\dot{q}_{i}^{*}\right)+d_{i} \qquad (C.9)
\end{aligned}$$

In what follows, one uses Y_{i} in place of $Y_{i}\left(q_{i},\,\dot{q}_{i},\,k_{p}\dot{\bar{g}}_{i}-\ddot{q}_{i}^{*},k_{p}\bar{g}_{i}-\dot{q}_{i}^{*}\right)$ to lighten notations. Since $\Delta\theta_{i}=\bar{\theta}_{i}-\theta_{i}$, one obtains

$$\begin{aligned}
s_{i}^{T}M_{i}\dot{s}_{i} = & -k_{s}s_{i}^{T}\bar{s}_{i}-k_{g}s_{i}^{T}\bar{g}_{i}-s_{i}^{T}C_{i}s_{i}+s_{i}^{T}\left(M_{i}\left(k_{p}\dot{g}_{i}-\ddot{q}_{i}^{*}\right)+C_{i}\left(k_{p}g_{i}-\dot{q}_{i}^{*}\right)\right) \\
& -s_{i}^{T}Y_{i}\left(\theta_{i}+\Delta\theta_{i}\right)+s_{i}^{T}d_{i}. \qquad (C.10)
\end{aligned}$$

Using (4.2) in (C.10) leads to

$$-s_{i}^{T}Y_{i}\left(\theta_{i}+\Delta\theta_{i}\right) = -s_{i}^{T}Y_{i}\Delta\theta_{i}-s_{i}^{T}\left(M_{i}\left(k_{p}\dot{\bar{g}}_{i}-\ddot{q}_{i}^{*}\right)+C_{i}\left(k_{p}\bar{g}_{i}-\dot{q}_{i}^{*}\right)\right). \qquad (C.11)$$

Considering (4.2) and (C.10) in (C.6), one gets

$$\dot{V} = \sum_{i=1}^{N} \left[\frac{1}{2} s_i^T \dot{M}_i s_i - k_s s_i^T \bar{s}_i - k_g s_i^T \bar{g}_i - s_i^T C_i s_i + s_i^T \left(M_i \left(k_p \dot{g}_i - \ddot{q}_i^* \right) + C_i \left(k_p g_i - \dot{q}_i^* \right) \right) \right.$$
$$- s_i^T \left(M_i \left(k_p \dot{\bar{g}}_i - \ddot{q}_i^* \right) + C_i \left(k_p \bar{g}_i - \dot{q}_i^* \right) \right) - s_i^T Y_i \Delta\theta_i + \bar{s}_i^T Y_i \Delta\theta_i$$
$$\left. + k_g \left(\dot{q}_i - \dot{q}_i^* \right)^T g_i + s_i^T d_i \right]. \tag{C.12}$$

Now, introduce (4.7) in (4.10) to get

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p \left[\sum_{i=1}^{N} k_{ij} \left(q_i - q_j - r_{ij}^* \right) + k_0 r_i \right]. \tag{C.13}$$

Since $e_j^i = \hat{q}_j^i - q_j$, one gets

$$\begin{aligned} s_i &= \dot{q}_i - \dot{q}_i^* + k_p \left[\sum_{i=1}^{N} k_{ij} \left(q_i - \hat{q}_j^i + e_j^i - r_{ij}^* \right) + k_0 r_i \right] \\ &= \dot{q}_i - \dot{q}_i^* + k_p \left[\sum_{i=1}^{N} k_{ij} \left(\bar{r}_{ij} - r_{ij}^* \right) + k_0 r_i \right] + k_p \sum_{\substack{j=1 \\ j \neq i}}^{N} k_{ij} e_j^i \\ &= \bar{s}_i + k_p E_j^i, \end{aligned} \tag{C.14}$$

with since $k_{ii} = 0$

$$E_j^i = \sum_{i=1}^{N} k_{ij} e_j^i. \tag{C.15}$$

Using similar derivations, one may show that

$$g_i = \bar{g}_i + E_j^i. \tag{C.16}$$

Replacing (C.14) and (C.16) in (C.12), one gets

$$\begin{aligned} \dot{V} = & \sum_{i=1}^{N} \left[s_i^T \left[\frac{1}{2} \dot{M}_i - C_i \right] s_i - k_s s_i^T \bar{s}_i - k_g \left(\dot{q}_i - \dot{q}_i^* + k_p g_i \right)^T \bar{g}_i \right. \\ & \left. + k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i \right) + k_p E_j^{iT} Y_i \Delta\theta_i + k_g \left(\dot{q}_i - \dot{q}_i^* \right)^T g_i + s_i^T d_i \right]. \end{aligned} \tag{C.17}$$

Let

$$\dot{V}_1 = \sum_{i=1}^{N} 2 k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i \right)$$

and

$$\dot{V}_2 = 2 k_p \sum_{i=1}^{N} E_j^{iT} Y_i \Delta\theta_i.$$

Since $\frac{1}{2}\dot{M}_i - C_i$ is skew symmetric or definite negative, $s_i^T \left[\frac{1}{2}\dot{M}_i - C_i \right] s_i \leq 0$. For all $b > 0$ and all vectors x and y of similar size, one has

$$x^T y \leq \frac{1}{2} \left(b x^T x + \frac{1}{b} y^T y \right). \tag{C.18}$$

Using (C.18) with $b = 1$, one deduces that $d_i^T s_i \leq \frac{1}{2}\left(D_{\max}^2 + s_i^T s_i\right)$ and that

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^{N}\Big[-k_s s_i^T \bar{s}_i - k_g k_p g_i^T \bar{g}_i + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \\ & + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T \left(g_i - \bar{g}_i\right)\Big] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right) \end{aligned} \quad \text{(C.19)}$$

One notices that $r_{ij} = q_i - q_j = q_i - \hat{q}_j^i + e_j^i = \bar{r}_{ij} + e_j^i$, thus

$$\begin{aligned} \|s_i - \bar{s}_i\|^2 &= s_i^T s_i - 2 s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\ \left\|k_p E_j^i\right\|^2 &= s_i^T s_i - 2 s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\ s_i^T \bar{s}_i &= -\frac{1}{2}\left\|k_p E_j^i\right\|^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2} \bar{s}_i^T \bar{s}_i \end{aligned} \quad \text{(C.20)}$$

In the same way, from (C.20), one shows that

$$g_i^T \bar{g}_i = -\frac{1}{2}\left\|E_j^i\right\|^2 + \frac{1}{2} g_i^T g_i + \frac{1}{2} \bar{g}_i^T \bar{g}_i. \quad \text{(C.21)}$$

Injecting (C.21) in (C.19),

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^{N}\Big[\frac{k_s}{2}\left(k_p^2\left\|E_j^i\right\|^2 - s_i^T s_i - \bar{s}_i^T \bar{s}_i\right) + k_p k_g \frac{1}{2}\left(\left\|E_j^i\right\|^2 - g_i^T g_i - \bar{g}_i^T \bar{g}_i\right) + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \\ & + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T \left(g_i - \bar{g}_i\right)\Big] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right) \\ \leq & \sum_{i=1}^{N}\Big[-\frac{(k_s - 1)}{2} s_i^T s_i - \frac{k_s}{2} \bar{s}_i^T \bar{s}_i + \frac{k_s k_p^2 + k_g k_p}{2}\left\|E_j^i\right\|^2 - \frac{1}{2} k_p k_g\left(g_i^T g_i + \bar{g}_i^T \bar{g}_i\right) + \frac{1}{2} D_{\max}^2 \\ & + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T \left(g_i - \bar{g}_i\right)\Big] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right). \end{aligned} \quad \text{(C.22)}$$

Using (C.18) with $b = b_i > 0$, one shows that $2\dot{q}_i^T\left(g_i - \bar{g}_i\right) \leq \left(b_i \|\dot{q}_i\|^2 + \frac{1}{b_i}\left\|E_j^i\right\|^2\right)$. Using this result in (C.22), one gets

$$\begin{aligned} \dot{V} \leq & \frac{1}{2}\sum_{i=1}^{N}\Big[-\left(k_s - 1\right) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}\right)\left\|E_j^i\right\|^2 + b_i k_g \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 \\ & - k_p k_g\left(g_i^T g_i + \bar{g}_i^T \bar{g}_i\right) + D_{\max}^2\Big] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right). \end{aligned} \quad \text{(C.23)}$$

Consider now $\dot{V}_1$. Using (C.18) with $b = 1$, the fact that M_i is symmetric positive definite, and that $x^T M_i x < k_M x^T x$, one obtains

$$\begin{aligned} \sum_{i=1}^{N} 2 k_p s_i^T\left(M_i \dot{E}_j^i + C_i E_j^i\right) & \leq \sum_{i=1}^{N} k_p\left(s_i^T M_i s_i + s_i^T s_i + \left[\dot{E}_j^{iT} M_i \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i\right]\right) \\ & \leq \sum_{i=1}^{N} k_p\left(\left(k_M + 1\right) s_i^T s_i + \left[k_M \dot{E}_j^{iT} \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i\right]\right) \end{aligned} \quad \text{(C.24)}$$

Focus now on the terms $E_j^{iT} C_i^T C_i E_j^i$

$$\begin{aligned} \sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i & = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} k_{ij} e_j^i\right)^T C_i^T C_i\left(\sum_{\ell=1}^{N} k_{i\ell} e_\ell^i\right) \\ & \leq \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} k_{i\ell} k_{ij}\left\|C_i\right\|^2 e_j^{iT} e_\ell^i. \end{aligned} \quad \text{(C.25)}$$

Using (C.18) with $b=1$, one gets

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &\leq \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} k_{i\ell}k_{ij}\left\|C_i\right\|^2\left(e_j^{iT}e_j^i + e_\ell^{iT}e_\ell^i\right)\\ &\leq \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} k_{i\ell}k_{ij}\left\|C_i\right\|^2\left(e_j^{iT}e_j^i\right)\\ &\leq \sum_{i=1}^{N}\alpha_i\sum_{j=1}^{N} k_{ij}\left\|C_i\right\|^2\left(e_j^{iT}e_j^i\right).\end{aligned} \tag{C.26}$$

Since one has assumed that (4.29) and (4.28) are satisfied, one has $\hat{q}_j^i = \hat{q}_j^j$, $e_j^i = e_j^j$. As a consequence,

$$\sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij}\left\|e_j^i\right\|^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij}\left\|e_j^j\right\|^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} k_{ji}\left\|e_i^i\right\|^2. \tag{C.27}$$

and since $k_{ij} = k_{ji}$,

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &\leq \sum_{i=1}^{N}\left(\alpha_{\mathrm{M}}\sum_{j=1}^{N}\left[k_{ij}\left\|e_i^i\right\|^2\left\|C_j\right\|^2\right]\right)\\ &\leq \sum_{i=1}^{N}\left(\alpha_{\mathrm{M}}\sum_{j=1}^{N}\left[k_{ij}\left\|e_i^i\right\|^2 k_C^2\left\|\dot{q}_j\right\|^2\right]\right).\end{aligned} \tag{C.28}$$

Then, the second CTC (4.27) leads to

$$\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i \leq \sum_{i=1}^{N}\left(\alpha_{\mathrm{M}} k_C^2\left\|e_i^i\right\|^2\sum_{j=1}^{N} k_{ij}\left(\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right)^2\right). \tag{C.29}$$

Similarly, one shows that

$$\sum_{i=1}^{N} E_j^{iT} E_j^i \leq \sum_{i=1}^{N}\alpha_{\mathrm{M}}^2\left\|e_i^i\right\|^2$$

and

$$\sum_{i=1}^{N} \dot{E}_j^{iT} \dot{E}_j^i \leq \sum_{i=1}^{N}\alpha_{\mathrm{M}}^2\left\|\dot{e}_i^i\right\|^2.$$

Consider now $\dot{V}_2$

$$\begin{aligned}\dot{V}_2 &= 2k_p\sum_{i=1}^{N} E_j^{iT} Y_i \Delta\theta_i\\ &= 2k_p\sum_{i=1}^{N}\left(\sum_{j=1}^{N} k_{ij}e_j^i\right)^T Y_i\Delta\theta_i.\end{aligned} \tag{C.30}$$

Since $e_j^i = e_j^j$, one gets

$$\begin{aligned}\dot{V}_2 &= 2k_p\sum_{i=1}^{N}\left(\sum_{j=1}^{N} k_{ij}e_j^j\right)^T Y_i\Delta\theta_i\\ &= 2k_p\sum_{j=1}^{N}\sum_{i=1}^{N}\left(k_{ji}e_i^i\right)^T Y_j\Delta\theta_j\\ &= 2k_p\sum_{i=1}^{N} e_i^{iT}\sum_{j=1}^{N} k_{ji}Y_j\Delta\theta_j.\end{aligned} \tag{C.31}$$

Let $0_n = [0, \ldots 0]^T \in \mathbb{R}^n$ be the all-zero vector. If $e_i^i = 0_n$, one has $2k_p e_i^{iT} \sum_{j=1}^N k_{ji} Y_j \Delta\theta_j = 0$. Considering now the case $e_i^i \neq 0_n$. Using (C.18) with $b = b_{i2} > 0$, one obtains

$$\dot{V}_2 = 2k_p \sum_{i=1}^N E_j^{iT} Y_i \Delta\theta_i \tag{C.32}$$

$$\leq k_p \sum_{i=1}^N \left(b_{i2} E_j^{iT} E_j^i + \frac{1}{b_{i2}} \left\| Y_i \Delta\theta_i \right\|^2 \right). \tag{C.33}$$

Since $\sum_{i=1}^N E_j^{iT} E_j^i \leq \sum_{i=1}^N \alpha_{\mathrm{M}}^2 \left\| e_i^i \right\|^2$, one gets

$$\dot{V}_2 \leq \sum_{i=1}^N k_p \left(\alpha_{\mathrm{M}}^2 b_{i2} \left\| e_i^i \right\|^2 + \frac{1}{b_{i2}} \left\| \left| Y_i \right| \left| \Delta\theta_i \right| \right\|^2 \right)$$

$$\leq \sum_{i=1}^N k_p \left(\alpha_{\mathrm{M}}^2 b_{i2} \left\| e_i^i \right\|^2 + \frac{1}{b_{i2}} \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2 \right), \tag{C.34}$$

where $\Delta\theta_{i,\max}$ is given by (4.23).

Since $e_i^i \neq 0_n$, choosing $b_{i2} = \frac{1 + \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2}{\left\| e_i^i \right\|}$, one obtains $\dot{V}_2 \leq \dot{V}_3$ with

$$\dot{V}_3 = \sum_{i=1}^N k_p \left[\alpha_{\mathrm{M}}^2 \left(\frac{1 + \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2}{\left\| e_i^i \right\|} \right) \left\| e_i^i \right\|^2 + \frac{\left\| e_i^i \right\| \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2}{\left(1 + \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2 \right)} \right]$$

$$= \sum_{i=1}^N k_p \left\| e_i^i \right\| \left[\alpha_{\mathrm{M}}^2 \left(1 + \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2 \right) + \frac{\left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2}{\left(1 + \left\| \left| Y_i \right| \Delta\theta_{i,\max} \right\|^2 \right)} \right]. \tag{C.35}$$

Injecting (C.24), (C.29), and (C.35) in (C.23), one gets

$$\begin{aligned} \dot{V} \quad \leq \quad & \frac{1}{2} \sum_{i=1}^N \Big[- \left(k_s - 1 - k_p \left(k_M + 1 \right) \right) s_i^T s_i - k_s \bar{s}_i^T \bar{s}_i + D_{\max}^2 \\ & - k_p k_g g_i^T g_i - k_p k_g \bar{g}_i^T \bar{g}_i + k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 + k_p k_M \alpha_{\mathrm{M}}^2 \left\| \dot{e}_i^i \right\|^2 \\ & + \alpha_{\mathrm{M}}^2 \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right) \left\| e_i^i \right\|^2 + \alpha_{\mathrm{M}} k_p k_C^2 \left\| e_i^i \right\|^2 \sum_{j=1}^N k_{ij} \left[\left\| \dot{\hat{q}}_j^i \right\| + \eta_2 \right]^2 \Big] + \frac{1}{2} \dot{V}_3. \end{aligned} \tag{C.36}$$

The CTC (4.26) leads to

$$\dot{V} \leq \frac{1}{2} \sum_{i=1}^N \left[- \left(k_s - 1 - k_p \left(k_M + 1 \right) \right) s_i^T s_i - k_g k_p g_i^T g_i + D_{\max}^2 + \eta \right]$$

$$\dot{V} \leq \frac{1}{2} \sum_{i=1}^N \left[-k_1 s_i^T s_i - k_g k_p g_i^T g_i + D_{\max}^2 + \eta \right] \tag{C.37}$$

with $k_1 = k_s - 1 - k_p \left(k_M + 1 \right)$.

Following the steps given in Appendix C.1.3 from (C.53) to (C.57), one shows that

$$\dot{V} \leq -c_3 V + \frac{N}{2} \left[D_{\max}^2 + \eta \right] + \frac{c_3}{2} \sum_{i=1}^N \left(\Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i \right), \tag{C.38}$$

where $c_3 > 0$ is a positive constant. Introducing $\Delta_{\max} = \max_{i=1:N} \left(\sup_{t>0} \left(\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i \right) \right)$, one has

$$\dot{V} \leq -c_3 V + \frac{N}{2} \left[c_3 \Delta_{\max} + D_{\max}^2 + \eta \right]. \tag{C.39}$$

Define the function W such that $W(0) = V(0)$ and

$$\dot{W} = -c_3 W + \frac{N}{2}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]. \tag{C.40}$$

Using the initial condition $W(0) = V(0)$, the solution of (C.40) is

$$W(t) = \exp(-c_3 t) V(0) + (1 - \exp(-c_3 t)) \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]. \tag{C.41}$$

Then, using the Lemma 3.4 in [53] (Comparison lemma), one has $V(t) \le W(t)$ and so

$$V(t) \le \exp(-c_3 t) V(0) + (1 - \exp(-c_3 t)) \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right] \tag{C.42}$$

Since M_i and Γ_i are symmetric, there exists matrices S_{M_i} and S_{Γ_i} such that $M_i = S_{M_i}^T S_{M_i}$ and $\Gamma_i = S_{\Gamma_i}^T S_{\Gamma_i}$. Introduce now

$$y_M = \begin{bmatrix} (S_{M_1} s_1)^T & \dots & (S_{M_i} s_i)^T & \dots & (S_{M_N} s_N)^T \end{bmatrix}^T \tag{C.43}$$

$$y_\Gamma = \begin{bmatrix} \left(S_{\Gamma_1}^{-1} \Delta\theta_1\right)^T & \dots & \left(S_{\Gamma_i}^{-1} \Delta\theta_i\right)^T & \dots & \left(S_{\Gamma_N}^{-1} \Delta\theta_N\right)^T \end{bmatrix}^T \tag{C.44}$$

$$y_q = \begin{bmatrix} (q_1 - q_1^*)^T & \dots & (q_i - q_i^*)^T & \dots & (q_N - q_N^*)^T \end{bmatrix}^T \tag{C.45}$$

$$z = \begin{bmatrix} y_M^T & y_\Gamma^T & \sqrt{k_g k_0} y_q^T & \sqrt{\frac{k_g}{2} P(x, t)} \end{bmatrix}^T \tag{C.46}$$

Then, $V(t)$ can be rewritten as

$$V(z) = \frac{1}{2} z^T z. \tag{C.47}$$

Using (C.47) in (C.42), one has $\forall t \ge 0$

$$\begin{aligned}
\|z(t)\|^2 &\le \exp(-c_3 t) \|z(0)\|^2 + (1 - \exp(-c_3 t)) \frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right] \\
\|z(t)\| &\le \sqrt{\exp(-c_3 t) \|z(0)\|^2 + (1 - \exp(-c_3 t)) \frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]} \\
\|z(t)\| &\le \sqrt{\exp(-c_3 t) \|z(0)\|^2} + \sqrt{(1 - \exp(-c_3 t)) \frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]} \\
\|z(t)\| &\le \exp\left(-\frac{c_3}{2} t\right) \|z(0)\| + \sqrt{\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]}
\end{aligned} \tag{C.48}$$

and so

$$\|z(t)\| \le \beta(\|z(0)\|, t) + \rho \tag{C.49}$$

with $\rho = \sqrt{\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]}$, $\beta(\|z(0)\|, t) = \exp\left(-\frac{c_3}{2} t\right) \|z(0)\|$, and $\beta \in \mathcal{KL}$ (see Section 1.4.5). Using Definition 2.1 from [48], (C.49) implies that the MAS is input-to-state practically stable.

C.1.2 Convergence of V

From (C.49), we know the system is ISpS. Moreover, from (C.38), one has

$$\dot{V} \le -c_3 V + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2} \sum_{i=1}^{N} \left(\Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i\right) \tag{C.50}$$

Then, if initially

$$-c_3 V(0) + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2} \sum_{i=1}^{N} \left(\Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i\right) < 0 \tag{C.51}$$

one has $\dot{V} \leq 0$ and V is decreasing. Then, one has from (C.42)

$$\begin{aligned}
\lim_{t\to\infty} V(t) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
\lim_{t\to\infty} \frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i + \Delta\theta_i{}^T \Gamma_i^{-1}\Delta\theta_i\right) + \frac{k_g}{2}\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
\lim_{t\to\infty} \frac{k_g}{2}\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
&\quad - \lim_{t\to\infty} \frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i + \Delta\theta_i{}^T \Gamma_i^{-1}\Delta\theta_i\right) \\
\lim_{t\to\infty} \sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t) &\leq \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right].
\end{aligned} \tag{C.52}$$

Asymptotically, the formation and tracking error are bounded.

C.1.3 Complementary proof elements

Differential equation satisfied by V

From (C.37), one gets

$$\dot{V} \leq \frac{1}{2}\sum_{i=1}^{N}\left[-k_{\mathrm{m}}\left(s_i^T s_i - k_g g_i^T g_i\right) + D_{\max}^2 + \eta\right] \tag{C.53}$$

where $k_{\mathrm{m}} = \min\{k_1, k_p\}$. Using (C.64), one may write

$$\begin{aligned}
\sum_{i=1}^{N} g_i^T g_i &\geq \sum_{i=1}^{N} k_0^2\left\|r_i\right\|^2 + \left(2k_0 + \frac{\alpha_{\min}k_{\min}}{k_{\max}}\right)P(q,t) \\
&\geq k_2\left(\sum_{i=1}^{N} k_0^2\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right)
\end{aligned} \tag{C.54}$$

where

$$k_2 = \begin{cases} 2\left(2k_0 + \frac{\alpha_{\min}k_{\min}}{k_{\max}}\right) & \text{if } \left(2k_0 + \frac{\alpha_{\min}k_{\min}}{k_{\max}}\right) < \frac{1}{2} \\ 1 & \text{else.} \end{cases}$$

Then

$$\begin{aligned}
\sum_{i=1}^{N} g_i^T g_i &\geq k_2\left(\sum_{i=1}^{N} k_0^2\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right) \\
&\geq k_3\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right)
\end{aligned} \tag{C.55}$$

where $k_3 = k_2k_0$ if $k_0 < 1$, $k_3 = 1$ else. Then

$$\begin{aligned}
\dot{V} &\leq -\frac{1}{2}\sum_{i=1}^{N}\left(k_{\mathrm{m}}s_i^T s_i\right) - \frac{k_3k_g}{2}\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right) + \frac{N}{2}\left(D_{\max}^2 + \eta\right) \\
&\leq -\frac{1}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N} k_{\mathrm{m}}\left(k_{\mathrm{M}}s_i^T s_i\right) + \frac{k_3k_g}{2}\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right)\right] + \frac{N}{2}\left(D_{\max}^2 + \eta\right) \\
&\leq -\frac{k_4}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N}\left(k_{\mathrm{M}}s_i^T s_i\right) + \frac{k_g}{2}\left(\sum_{i=1}^{N} k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t)\right)\right] + \frac{N}{2}\left(D_{\max}^2 + \eta\right)
\end{aligned} \tag{C.56}$$

with $k_M^* = 1$ if $k_M < 1$ and $k_M^* = k_M$ else, and $k_4 = \min(k_\mathrm{m}, k_3)$. Let $c_3 = \frac{k_4}{k_M^*}$ and one gets

$$\dot{V} \leq -c_3 \left[\frac{1}{2} \sum_{i=1}^{N} \left[s_i^T M_i s_i + \Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i \right] + \frac{k_g}{2} \left(\sum_{i=1}^{N} k_0 \left\| r_i \right\|^2 + \frac{1}{2} P(q,t) \right) \right]$$
$$+ \frac{N}{2} \left[D_{\max}^2 + \eta \right] + \frac{c_3}{2} \sum_{i=1}^{N} \Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i$$
$$\dot{V} \leq -c_3 V + \frac{N}{2} \left[D_{\max}^2 + \eta \right] + \frac{c_3}{2} \sum_{i=1}^{N} \left(\Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i \right). \tag{C.57}$$

The evaluation of c_3 is described in Appendix C.1.3.

Upper-bound on $\sum_{i=1}^N g_i^T g_i$

From (4.7), one may write

$$\sum_{i=1}^{N} g_i^T g_i = \sum_{i=1}^{N} \left[\sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) + k_0 r_i \right]^T \left[\sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) + k_0 r_i \right]$$
$$= \sum_{i=1}^{N} \left[\left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 + \left\| k_0 r_i \right\|^2 + 2 \left(k_0 r_i \right)^T \left(\sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right) \right]. \tag{C.58}$$

Let

$$P_1 = \sum_{i=1}^{N} r_i^T \left(\sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right). \tag{C.59}$$

Since $r_{ij} - r_{ij}^* = q_i - q_j - \left(q_i^* - q_j^* \right) = r_i - r_j$,

$$P_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} r_i^T \left(r_i - r_j \right)$$
$$= \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left(r_i^T r_i - r_i^T r_j \right). \tag{C.60}$$

Using the fact that $2a^T b = a^T a + b^T b - (a-b)^T (a-b)$, one gets

$$P_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left(\left\| r_i \right\|^2 - \frac{1}{2} \left(\left\| r_i \right\|^2 + \left\| r_j \right\|^2 - \left\| r_i - r_j \right\|^2 \right) \right). \tag{C.61}$$

Since $k_{ij} = k_{ji}$ and $r_i - r_j = r_{ij} - r_{ij}^*$

$$P_1 = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_i \right\|^2 - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ji} \left\| r_j \right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2$$
$$= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_i \right\|^2 - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_i \right\|^2 + P(q,t)$$
$$= P(q,t). \tag{C.62}$$

Injecting P_1 in (C.58), one gets

$$\sum_{i=1}^{N} g_i^T g_i = \sum_{i=1}^{N} \left[\left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^* \right) \right\|^2 + \left\| k_0 r_i \right\|^2 \right] + 2 k_0 P(q,t) \tag{C.63}$$

and using (C.70), one gets

$$\sum_{i=1}^{N} g_i^T g_i \geq \sum_{i=1}^{N} k_0^2 \|r_i\|^2 + \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) P\left(q,t\right) \tag{C.64}$$

Upper-bound on $\sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2$

One may write

$$\begin{aligned}\sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 &= \sum_{i=1}^{N} \left(\sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right)^T \left(\sum_{\ell=1}^{N} k_{i\ell} \left(r_{i\ell} - r_{i\ell}^*\right) \right) \\ &= \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} \left(r_{ij} - r_{ij}^*\right)^T \left(r_{i\ell} - r_{i\ell}^*\right) \end{aligned} \tag{C.65}$$

Using the fact $2a^T b = a^T a + b^T b - (a-b)^T (a-b)$

$$\sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 = \sum_{i=1}^{N} \left[\frac{1}{2} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} \left[\left\|r_{ij} - r_{ij}^*\right\|^2 + \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 - \left\|r_{ij} - r_{ij}^* - \left(r_{i\ell} - r_{i\ell}^*\right)\right\|^2 \right] \right] \tag{C.66}$$

One has

$$\begin{aligned}\left(r_{ij} - r_{ij}^*\right) - \left(r_{i\ell} - r_{i\ell}^*\right) &= \left(r_{ij} - r_{i\ell}\right) - \left(r_{ij}^* - r_{i\ell}^*\right) \\ &= r_{\ell j} - r_{\ell j}^*\end{aligned}$$

Injecting this result in (C.66) leads to

$$\sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 = \sum_{i=1}^{N} \left[\frac{1}{2} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} \left[\left\|r_{ij} - r_{ij}^*\right\|^2 + \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 - \left\|r_{\ell j} - r_{\ell j}^*\right\|^2 \right] \right] \tag{C.67}$$

with $k_{\max} = \max_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} \left(k_{\ell j}\right)$

$$\begin{aligned}
k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 &\geq \sum_{i=1}^{N} \left[\frac{1}{2} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left[\left\|r_{ij} - r_{ij}^*\right\|^2 + \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 - \left\|r_{\ell j} - r_{\ell j}^*\right\|^2 \right] \right] \\
k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 &\geq \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{ij} - r_{ij}^*\right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 \\
&\quad - \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{\ell j} - r_{\ell j}^*\right\|^2 \\
k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 &\geq \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{ij} - r_{ij}^*\right\|^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{ij} - r_{ij}^*\right\|^2 \\
&\quad - \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{ij} - r_{ij}^*\right\|^2
\end{aligned}$$

$$k_{\max} \sum_{i=1}^{N} \left\| \sum_{j=1}^{N} k_{ij} \left(r_{ij} - r_{ij}^*\right) \right\|^2 \geq \frac{1}{2} \sum_{i=1}^{N} \sum_{\ell=1}^{N} \sum_{j=1}^{N} k_{i\ell} k_{ij} k_{\ell j} \left\|r_{ij} - r_{ij}^*\right\|^2. \tag{C.68}$$

Let $k_{\min} = \min_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} (k_{\ell j} \neq 0)$ and $\alpha_{\min} = \min_{i=1,\dots,N} \alpha_i$. One may write

$$\begin{aligned}
\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} k_{i\ell}k_{ij}k_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^{2} &= \sum_{i=1}^{N}\sum_{\ell=1}^{N} k_{i\ell}\sum_{j=1}^{N} k_{ij}k_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^{2} \\
&\geq \sum_{i=1}^{N}\sum_{\ell=1}^{N} k_{i\ell}k_{\min}\sum_{j=1}^{N} k_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^{2} \\
&\geq \sum_{i=1}^{N} \alpha_i k_{\min}\sum_{j=1}^{N} k_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^{2} \\
&\geq \alpha_{\min}k_{\min}\sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^{2} \\
&\geq 2\alpha_{\min}k_{\min}P\left(q,t\right)
\end{aligned} \tag{C.69}$$

Injecting (C.69) in (C.68) one gets

$$\begin{aligned}
k_{\max}\sum_{i=1}^{N}\left\|\sum_{j=1}^{N} k_{ij}\left(r_{ij}-r_{ij}^{*}\right)\right\|^{2} &\geq \alpha_{\min}k_{\min}P\left(q,t\right) \\
\sum_{i=1}^{N}\left\|\sum_{j=1}^{N} k_{ij}\left(r_{ij}-r_{ij}^{*}\right)\right\|^{2} &\geq \frac{\alpha_{\min}k_{\min}}{k_{\max}}P\left(q,t\right).
\end{aligned} \tag{C.70}$$

Evaluation of c_3

One has

$$\begin{aligned}
c_3 &= \frac{k_4}{k_M^*} \\
&= \frac{\min\left(k_{\mathrm{m}}, k_3\right)}{\max\left\{1, k_M\right\}} \\
&= \frac{\min\left\{\min\left\{k_1, k_p\right\}, \min\left\{k_2 k_0, 1\right\}\right\}}{\max\left\{1, k_M\right\}} \\
&= \frac{\min\left\{k_1, k_p, 1, k_2 k_0\right\}}{\max\left\{1, k_M\right\}} \\
&= \frac{\min\left\{k_1, k_p, 1, k_0 \min\left\{2\left(2k_0 + \frac{\alpha_{\min}k_{\min}}{k_{\max}}\right), 1\right\}\right\}}{\max\left\{1, k_M\right\}} \\
&= \frac{\min\left\{k_1, k_p, 1, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min}k_{\min}}{k_{\max}}\right)\right\}}{\max\left\{1, k_M\right\}}
\end{aligned} \tag{C.71}$$

where $k_1 = k_s - 1 - k_p\left(k_M + 1\right)$, $\alpha_{\min} = \min_{i=1,\dots,N} \alpha_i$, $k_{\max} = \max_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} (k_{\ell j})$ and $k_{\min} = \min_{\substack{\ell = 1 \dots N \\ j = 1 \dots N}} (k_{\ell j} \neq 0)$.

C.2 Proof of $t_{i,k+1} - t_{i,k} > 0$

From the CTC (4.26), a communication is triggered at $t = t_{i,k}^-$ when

$$\begin{aligned} k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta = {} & \alpha_{\mathrm{M}}^2 \left(k_e \left\| e_i^i \right\|^2 + k_p k_M \left\| \dot{e}_i^i \right\|^2 \right) \\ & + \alpha_{\mathrm{M}} k_C^2 k_p \left\| e_i^i \right\|^2 \sum_{j=1}^{N} k_{ji} \left[\left\| \dot{\hat{q}}_j^i \right\| + \eta_2 \right]^2 + k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 \\ & + k_p \left\| e_i^i \right\| \left[\alpha_{\mathrm{M}}^2 \left(1 + \left\| |Y_i| \, \Delta\theta_{i,\max} \right\|^2 \right) + \frac{\left\| |Y_i| \, \Delta\theta_{i,\max} \right\|^2}{\left(1 + \left\| |Y_i| \, \Delta\theta_{i,\max} \right\|^2 \right)} \right] \end{aligned} \tag{C.72}$$

with $k_e = \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right)$. Then, the estimation errors e_i^i and $\dot{e}_i^i$ are reset and one has $e_i^i \left(t_{i,k}^+ \right) = 0$ and $\dot{e}_i^i \left(t_{i,k}^+ \right) = 0$. As a consequence, the CTC (4.26) in Theorem 8 is not satisfied at $t = t_{i,k}^+$ iff

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta > k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 . \tag{C.73}$$

To prove that $t_{i,k+1} > t_{i,k}$, one has to show that (C.73) is satisfied.

Using the property $x^T y \geq -\frac{1}{2} \left(b_{i2} x^T x + \frac{1}{b_{i2}} y^T y \right)$ for some $b_{i2} > 0$, one deduces that

$$\begin{aligned} \bar{s}_i^T \bar{s}_i &= k_p^2 \bar{g}_i^T \bar{g}_i + \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 + 2 k_p \bar{g}_i^T \left(\dot{q}_i - \dot{q}_i^* \right) \\ &\geq \left(k_p^2 - k_p b_{i2} \right) \bar{g}_i^T \bar{g}_i + \left(1 - \frac{k_p}{b_{i2}} \right) \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 . \end{aligned} \tag{C.74}$$

Using (C.74), a sufficient condition for (C.73) to be satisfied is

$$\begin{aligned} k_s \left(k_p^2 - k_p b_{i2} \right) \bar{g}_i^T \bar{g}_i + k_s \left(1 - \frac{k_p}{b_{i2}} \right) \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 + k_p k_g \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 \\ k_s \left(1 - \frac{k_p}{b_{i2}} \right) \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 + \left[k_p k_g + k_s \left(k_p^2 - k_p b_{i2} \right) \right] \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 \\ k_1 \bar{g}_i^T \bar{g}_i + \eta &> k_2 \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 \end{aligned} \tag{C.75}$$

where $k_1 = \left[k_p k_g + k_s \left(k_p^2 - k_p b_{i2} \right) \right]$ and $k_2 = \left[k_g b_i - k_s \left(1 - \frac{k_p}{b_{i2}} \right) \right]$. To ensure that the inequality (C.75) is satisfied independently of the values of $\bar{g}_i$ and $\dot{q}_i$, it is sufficient to find b_i and b_{i2} such that $k_1 > 0$ and $k_2 < 0$. Consider first k_1.

$$\begin{aligned} k_p k_g + k_s \left(k_p^2 - k_p b_{i2} \right) &> 0 \\ \frac{k_g}{k_s} &> \left(-k_p + b_{i2} \right) \\ \frac{k_s k_p + k_g}{k_s} &> b_{i2}. \end{aligned} \tag{C.76}$$

Focus now on k_2

$$\begin{aligned} k_g b_i - k_s \left(1 - \frac{k_p}{b_{i2}} \right) &< 0 \\ \frac{k_g b_i}{k_s} &< 1 - \frac{k_p}{b_{i2}} \\ \frac{1}{b_{i2}} &< \frac{1}{k_p} \left(1 - \frac{k_g b_i}{k_s} \right). \end{aligned} \tag{C.77}$$

Since $b_{i2} > 0$, one has $\frac{k_g b_i}{k_s} < 1$ and so $b_i < \frac{k_s}{k_g}$. Then

$$\frac{k_s k_p}{k_s - k_g b_i} < b_{i2}. \tag{C.78}$$

Finally, one has to find a condition on b_i such that (C.76) and (C.77) can be satisfied simultaneously

$$\frac{k_s k_p + k_g}{k_s} > b_{i2} > \frac{k_s k_p}{k_s - k_g b_i}. \tag{C.79}$$

One may find b_{i2} if

$$\begin{aligned} k_s - k_g b_i &> \frac{k_s^2 k_p}{k_s k_p + k_g} \\ \frac{1}{k_g}\left(k_s - \frac{k_s^2 k_p}{k_s k_p + k_g}\right) &> b_i \\ b_i &< \frac{k_s}{k_s k_p + k_g}. \end{aligned} \tag{C.80}$$

which also ensures that $b_i < \frac{k_s}{k_g}$. Thus, once $b_i < \frac{k_s}{k_s k_p + k_g}$, there exists some b_{i2} such that (C.79) is satisfied. As a consequence $t_{i,k+1} - t_{i,k} > 0$.

C.3 Possibility to start a collision avoidance mechanism before collision

The CTC 4.35 ensures that $\forall (i,j) \in \mathcal{N}$

$$\begin{aligned} \frac{\sigma}{2}\left(\left\|\hat{r}_{ij}^i\right\| - r_c\right) &> \left\|e_i^i\right\| \\ \frac{1}{2}\left(\left\|\hat{r}_{ij}^j\right\| - r_c\right) + \frac{1}{2}\left(\left\|\hat{r}_{ij}^i\right\| - r_c\right) &> \left\|e_i^i\right\| + \left\|e_j^j\right\| \end{aligned} \tag{C.81}$$

Since there is no communication delay, the estimator (E.34)-(E.38) guarantees $\hat{r}_{ij}^i = \hat{r}_{ij}^j$. One gets

$$\begin{aligned} \left\|\hat{r}_{ij}^i\right\| - r_c &> \left\|e_i^i\right\| + \left\|e_j^j\right\| \\ \left\|\hat{r}_{ij}^i\right\| - \left\|e_i^i\right\| - \left\|e_j^j\right\| - r_c &> 0 \\ \left\|\hat{r}_{ij}^i - e_i^i + e_j^j\right\| - r_c &> 0 \\ \forall (i,j) \in \mathcal{N} \quad \left\|r_{ij}\right\| &> r_c. \end{aligned} \tag{C.82}$$

The CTC allows thus agents, which are close to collide, to update their state estimates. This update is performed before a collision has actually occured.

The candidate Lyapunov function V introduced in (C.1) with the new control input (4.32). Following the same steps from (C.1) to (C.10), one gets

$$\begin{aligned} s_i^T M_i \dot{s}_i = & -k_s s_i^T \bar{s}_i - k_g s_i^T \bar{g}_i - s_i^T C_i s_i + s_i^T \left(M_i \left(k_p \dot{g}_i - \ddot{q}_i^*\right) + C_i \left(k_p g_i - \dot{q}_i^*\right)\right) \\ & - s_i^T Y_i \left(\theta_i + \Delta\theta_i\right) + s_i^T d_i - s_i^T \bar{v}_i \end{aligned}$$

Using the property $x^T y \leq \frac{1}{2}\left(x^T x + y^T y\right)$, one deduces that $\bar{v}_i^T s_i \leq \frac{1}{2}\left(\bar{v}_i^T \bar{v}_i + s_i^T s_i\right)$. Then

$$\begin{aligned} \dot{V} \leq \sum_{i=1}^{N} \Bigg[& s_i^T \left[\frac{1}{2}\dot{M}_i - C_i\right] s_i - k_s s_i^T \bar{s}_i - k_g \left(\dot{q}_i - \dot{q}_i^* + k_p g_i\right)^T \bar{g}_i \\ & + k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i\right) + k_g \left(\dot{q}_i - \dot{q}_i^*\right)^T g_i + s_i^T d_i + \frac{1}{2}\left(\bar{v}_i^T \bar{v}_i + s_i^T s_i\right) \Bigg]. \end{aligned}$$

Following similar steps from (C.17) to (C.37), one gets

$$\dot{V} \leq \frac{1}{2}\sum_{i=1}^{N}\left[-\left(k_s - 2 - k_p\left(k_M + 1\right)\right)s_i^T s_i - 2k_g k_p g_i^T g_i + D_{\max}^2 + \eta + \frac{1}{2}\bar{v}_i^T \bar{v}_i\right].$$

When a collision avoidance mechanism is started, the derivative of the Lyapunov function may be temporarily positive due to the term $\frac{1}{2}\bar{v}_i^T \bar{v}_i$. In absence of such mechanism, $\bar{v}_i = 0$. This is in particular the case when r_{ij} is close to r_{ij}^*. The convergence of the MAS may be thus only temporarily affected by collision avoidance mechanisms.

Appendix D

Appendix of Chapter 5

D.1 Proof of convergence with packet dropout

Inspired by the proof developed in [25, 95], consider the continuous positive–definite candidate Lyapunov function

$$V = \mathbb{E}\left(\frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i + \Delta\theta_i^T \Gamma_i^{-1}\Delta\theta_i\right) + \frac{k_g}{2}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N} k_0 \left\|q_i - q_i^*\right\|^2\right]\right) \tag{D.1}$$

where $\Delta\theta_i = \bar{\theta}_i - \theta_i$.

Let define t_1 is the first message sent in the network, whatever the sending agent. If $t \in [0,\, t_1[$, one may write

$$V\left(t\right) = \frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i + \Delta\theta_i^T \Gamma_i^{-1}\Delta\theta_i\right) + \frac{k_g}{2}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N} k_0 \left\|q_i - q_i^*\right\|^2\right] \tag{D.2}$$

and the time derivative of V exists and can be evaluated as

$$\dot{V}\left(t\right) = \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i + \Delta\theta_i^T \Gamma_i^{-1}\dot{\bar{\theta}}_i\right] + \frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N} k_0 \left\|q_i - q_i^*\right\|^2\right].$$

where, from (4.10), one has $\dot{s}_i = \ddot{q}_i - \ddot{q}_i^* + k_p \dot{g}_i$.

Considering now the case $t \geq t_1$. Let $\tilde{\alpha}_k^i$ be the random variable used to represent a stochastic occurrence of packet dropout linked to the reception of the k-th message by Agent i, whatever the sending agent. Since $\tilde{\alpha}_k^i$ is assumed to be modeled by a Bernoulli stochastic process, independent of time and agents' state, one may write

$$\begin{aligned}
V\left(t\right) = {} & \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\tilde{\alpha}_k^i\left(s_i^T M_i s_i + \Delta\theta_i^T \Gamma_i^{-1}\Delta\theta_i\right) \\
& + \frac{k_g}{2}\left[\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\tilde{\alpha}_k^i\left(\left\|r_{ij} - r_{ij}^*\right\|^2 + k_0\left\|q_i - q_i^*\right\|^2\right)\right] \\
& + \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\left(1 - \tilde{\alpha}_k^i\right)\left(s_i^T M_i s_i + \Delta\theta_i^T \Gamma_i^{-1}\Delta\theta_i\right) \\
& + \frac{k_g}{2}\left[\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\left(1 - \tilde{\alpha}_k^i\right)\left(\left\|r_{ij} - r_{ij}^*\right\|^2 + k_0\left\|q_i - q_i^*\right\|^2\right)\right].
\end{aligned}$$

where $K \in \mathbb{N}$ is the number of all messages broadcast in the network at the instant t since $t = 0$. Thus, the time derivative of V exists and can be evaluated as

$$
\begin{aligned}
\dot{V}(t) = & \sum_{i=1}^{N}\sum_{k=1}^{K}\tilde{\alpha}_k^i\left[\frac{1}{2}s_i^T\dot{M}_is_i + s_i^TM_i\dot{s}_i + \Delta\theta_i^T\Gamma_i^{-1}\dot{\bar{\theta}}_i\right] \\
& + \frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\tilde{\alpha}_k^i\left(\left\|r_{ij}-r_{ij}^*\right\|^2 + k_0\left\|q_i-q_i^*\right\|^2\right)\right] \\
& + \sum_{i=1}^{N}\sum_{k=1}^{K}\left(1-\tilde{\alpha}_k^i\right)\left[\frac{1}{2}s_i^T\dot{M}_is_i + s_i^TM_i\dot{s}_i + \Delta\theta_i^T\Gamma_i^{-1}\dot{\bar{\theta}}_i\right] \\
& + \frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\left(1-\tilde{\alpha}_k^i\right)\left(\left\|r_{ij}-r_{ij}^*\right\|^2 + k_0\left\|q_i-q_i^*\right\|^2\right)\right]
\end{aligned}
$$

which is equal to

$$
\dot{V} \quad = \quad \mathbb{E}\left(\sum_{i=1}^{N}\left[\frac{1}{2}s_i^T\dot{M}_is_i + s_i^TM_i\dot{s}_i + \Delta\theta_i^T\Gamma_i^{-1}\dot{\bar{\theta}}_i\right] + \frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N}k_0\left\|q_i-q_i^*\right\|^2\right]\right) \tag{D.3}
$$

Injecting (4.14) in (D.3) one obtains

$$
\begin{aligned}
\dot{V} \quad = \quad & \mathbb{E}\sum_{i=1}^{N}\left[\frac{1}{2}s_i^T\dot{M}_is_i + s_i^TM_i\dot{s}_i + \Delta\theta_i^TY_i\left(q_i,\, \dot{q}_i,\, \dot{\bar{p}}_i,\, \bar{p}_i\right)\bar{s}_i\right] \\
& + \mathbb{E}\left(\frac{k_g}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N}k_0\left\|q_i-q_i^*\right\|^2\right]\right).
\end{aligned} \tag{D.4}
$$

In (D.4),

$$
\begin{aligned}
& \frac{1}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N}k_0\left\|q_i-q_i^*\right\|^2\right] \\
= \quad & \frac{1}{4}\frac{d}{dt}\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}\left\|r_{ij}-r_{ij}^*\right\|^2 + \frac{1}{2}\frac{d}{dt}\sum_{i=1}^{N}k_0\left\|q_i-q_i^*\right\|^2 \\
= \quad & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left(\dot{r}_{ij}-\dot{r}_{ij}^*\right)^T\left(r_{ij}-r_{ij}^*\right) + k_0\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(q_i-q_i^*\right)\right] \\
= \quad & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left[\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(r_{ij}-r_{ij}^*\right) - \left(\dot{q}_j-\dot{q}_j^*\right)^T\left(r_{ij}-r_{ij}^*\right)\right]\right. \\
& \left. + k_0\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(q_i-q_i^*\right)\right] \\
= \quad & \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{j=1}^{N}k_{ij}\left[\left(\dot{q}_i-\dot{q}_i^*\right)^T\left(r_{ij}-r_{ij}^*\right) - \left(\dot{q}_i-\dot{q}_i^*\right)^T\left(r_{ji}-r_{ji}^*\right)\right]\right. \\
& \left. + k_0\left(\dot{q}_i-\dot{q}_i^*\right)^Tr_i\right].
\end{aligned} \tag{D.5}
$$

Since $r_{ji} = -r_{ij}$, one gets

$$
\begin{aligned}
\frac{1}{2}\frac{d}{dt}\left[\frac{1}{2}P\left(q,\, t\right) + \sum_{i=1}^{N}k_0\left\|q_i-q_i^*\right\|^2\right] \quad & = \quad \sum_{i=1}^{N}\left(\dot{q}_i-\dot{q}_i^*\right)^T\left[\sum_{j=1}^{N}k_{ij}\left(r_{ij}-r_{ij}^*\right) + k_0r_i\right] \\
& = \quad \sum_{i=1}^{N}\left(\dot{q}_i-\dot{q}_i^*\right)^Tg_i.
\end{aligned} \tag{D.6}
$$

Combining (D.4) and (D.6), one obtains

$$\dot{V} = \mathbb{E}\left(\sum_{i=1}^{N}\left[\frac{1}{2}s_i^T\dot{M}_i s_i + s_i^T M_i \dot{s}_i + \Delta\theta_i^T Y_i\left(q_i,\, \dot{q}_i,\, \dot{\bar{p}}_i,\, \bar{p}_i\right)\bar{s}_i + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T g_i\right]\right) \tag{D.7}$$

One focuses now on the term $M_i\dot{s}_i$. Using again (4.10), one may write

$$M_i\dot{s}_i + C_i s_i = M_i\left(\ddot{q}_i - \ddot{q}_i^* + k_p\dot{g}_i\right) + C_i\left(\dot{q}_i - \dot{q}_i^* + k_p g_i\right)$$

and by using (4.1), one gets

$$M_i\dot{s}_i + C_i s_i \quad = \quad \tau_i + d_i - G + M_i\left(k_p\dot{g}_i - \ddot{q}_i^*\right) + C_i\left(k_p g_i - \dot{q}_i^*\right), \tag{D.8}$$

Now, introducing (4.13), one gets

$$\begin{aligned} M_i\dot{s}_i + C_i s_i \quad = \quad & -k_s\bar{s}_i - k_g\bar{g}_i - Y_i\left(q_i,\, \dot{q}_i,\, k_p\dot{\bar{g}}_i - \ddot{q}_i^*, k_p\bar{g}_i - \dot{q}_i^*\right)\bar{\theta}_i \\ & + M_i\left(k_p\dot{g}_i - \ddot{q}_i^*\right) + C_i\left(k_p g_i - \dot{q}_i^*\right) + d_i \end{aligned} \tag{D.9}$$

In what follows, one uses Y_i to represent $Y_i\left(q_i,\, \dot{q}_i,\, k_p\dot{\bar{g}}_i - \ddot{q}_i^*, k_p\bar{g}_i - \dot{q}_i^*\right)$. Since $\Delta\theta_i = \bar{\theta}_i - \theta_i$, one obtains

$$\begin{aligned} s_i^T M_i\dot{s}_i \quad = \quad & -k_s s_i^T\bar{s}_i - k_g s_i^T\bar{g}_i - s_i^T C_i s_i + s_i^T\left(M_i\left(k_p\dot{g}_i - \ddot{q}_i^*\right) + C_i\left(k_p g_i - \dot{q}_i^*\right)\right) \\ & - s_i^T Y_i\left(\theta_i + \Delta\theta_i\right) + s_i^T d_i \end{aligned} \tag{D.10}$$

Using (4.2) leads to

$$-s_i^T Y_i\left(\theta_i + \Delta\theta_i\right) \quad = \quad -s_i^T Y_i\Delta\theta_i - s_i^T\left(M_i\left(k_p\dot{\bar{g}}_i - \ddot{q}_i^*\right) + C_i\left(k_p\bar{g}_i - \dot{q}_i^*\right)\right). \tag{D.11}$$

Considering (4.2) and (D.10) in (D.7), one gets

$$\begin{aligned} \dot{V} = \mathbb{E}\Bigg(\sum_{i=1}^{N}\Bigg[&\frac{1}{2}s_i^T\dot{M}_i s_i - k_s s_i^T\bar{s}_i - k_g s_i^T\bar{g}_i - s_i^T C_i s_i + s_i^T\left(M_i\left(k_p\dot{g}_i - \ddot{q}_i^*\right) + C_i\left(k_p g_i - \dot{q}_i^*\right)\right) \\ & - s_i^T\left(M_i\left(k_p\dot{\bar{g}}_i - \ddot{q}_i^*\right) + C_i\left(k_p\bar{g}_i - \dot{q}_i^*\right)\right) - s_i^T Y_i\Delta\theta_i + \bar{s}_i^T Y_i\Delta\theta_i \\ & + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T g_i + s_i^T d_i\Bigg]\Bigg). \end{aligned} \tag{D.12}$$

Now, introduce (4.7) in (4.10) to get

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p\left[\sum_{i=1}^{N}k_{ij}\left(q_i - q_j - r_{ij}^*\right) + k_0 r_i\right]. \tag{D.13}$$

Since $e_j^i = \hat{q}_j^i - q_j$, one gets

$$\begin{aligned} s_i \quad & = \quad \dot{q}_i - \dot{q}_i^* + k_p\left[\sum_{i=1}^{N}k_{ij}\left(q_i - \hat{q}_j^i + e_j^i - r_{ij}^*\right) + k_0 r_i\right] \\ & = \quad \dot{q}_i - \dot{q}_i^* + k_p\left[\sum_{i=1}^{N}k_{ij}\left(\bar{r}_{ij} - r_{ij}^*\right) + k_0 r_i\right] + k_p\sum_{\substack{j=1\\ j\neq i}}^{N}k_{ij}e_j^i \\ & = \quad \bar{s}_i + k_p E_j^i \end{aligned} \tag{D.14}$$

with since $k_{ii} = 0$

$$E_j^i = \sum_{i=1}^{N}k_{ij}e_j^i. \tag{D.15}$$

Using similar derivations, one may show that

$$g_i = \bar{g}_i + E_j^i. \tag{D.16}$$

Replacing (D.14) and (D.16) in (D.12), one gets

$$\begin{aligned}\dot{V} &= \mathbb{E}\left(\sum_{i=1}^{N}\left[s_i^T\left[\frac{1}{2}\dot{M}_i - C_i\right]s_i - k_s s_i^T\bar{s}_i - k_g\left(\dot{q}_i - \dot{q}_i^* + k_p g_i\right)^T\bar{g}_i\right.\right.\\ &\left.\left. +k_p s_i^T\left(M_i\dot{E}_j^i + C_i E_j^i\right) + k_p E_j^{iT} Y_i \Delta\theta_i + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T g_i + s_i^T d_i\right]\right). \end{aligned} \tag{D.17}$$

Let $\dot{V}_1 = \sum_{i=1}^{N} 2k_p s_i^T\left(M_i\dot{E}_j^i + C_i E_j^i\right)$ and $\dot{V}_2 = 2k_p\sum_{i=1}^{N} E_j^{iT} Y_i\Delta\theta_i$. Remind than $\frac{1}{2}\dot{M}_i - C_i$ is skew symmetric or definite negative, so $s_i^T\left[\frac{1}{2}\dot{M}_i - C_i\right]s_i \leq 0$. For all $b > 0$ and all vectors x and y of similar size, one has

$$x^T y \leq \frac{1}{2}\left(b x^T x + \frac{1}{b} y^T y\right). \tag{D.18}$$

Using (D.18) with $b = 1$, one deduces that $d_i^T s_i \leq \frac{1}{2}\left(D_{\max}^2 + s_i^T s_i\right)$ and that

$$\begin{aligned}\dot{V} &\leq \mathbb{E}\left(\sum_{i=1}^{N}\left[-k_s s_i^T\bar{s}_i - k_g k_p g_i^T\bar{g}_i + \frac{1}{2}s_i^T s_i + \frac{1}{2}D_{\max}^2\right.\right.\\ &\left.\left. +k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T\left(g_i - \bar{g}_i\right)\right] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right)\right) \end{aligned} \tag{D.19}$$

One notices that $r_{ij} = q_i - q_j = q_i - \hat{q}_j^i + e_j^i = \bar{r}_{ij} + e_j^i$, thus

$$\begin{aligned}\left\|s_i - \bar{s}_i\right\|^2 &= s_i^T s_i - 2s_i^T\bar{s}_i + \bar{s}_i^T\bar{s}_i\\ \left\|k_p E_j^i\right\|^2 &= s_i^T s_i - 2s_i^T\bar{s}_i + \bar{s}_i^T\bar{s}_i\\ s_i^T\bar{s}_i &= -\frac{1}{2}\left\|k_p E_j^i\right\|^2 + \frac{1}{2}s_i^T s_i + \frac{1}{2}\bar{s}_i^T\bar{s}_i \end{aligned} \tag{D.20}$$

In the same way, from (D.20), one shows that $g_i^T\bar{g}_i = -\frac{1}{2}\left\|E_j^i\right\|^2 + \frac{1}{2}g_i^T g_i + \frac{1}{2}\bar{g}_i^T\bar{g}_i$. Injecting it in (D.19)

$$\begin{aligned}\dot{V} &\leq \mathbb{E}\left(\sum_{i=1}^{N}\left[\frac{k_s}{2}\left(k_p^2\left\|E_j^i\right\|^2 - s_i^T s_i - \bar{s}_i^T\bar{s}_i\right) + k_p k_g\frac{1}{2}\left(\left\|E_j^i\right\|^2 - g_i^T g_i - \bar{g}_i^T\bar{g}_i\right) + \frac{1}{2}s_i^T s_i + \frac{1}{2}D_{\max}^2\right.\right.\\ &\left.\left. +k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T\left(g_i - \bar{g}_i\right)\right] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right)\right)\\ &\leq \mathbb{E}\left(\sum_{i=1}^{N}\left[-\frac{(k_s - 1)}{2}s_i^T s_i - \frac{k_s}{2}\bar{s}_i^T\bar{s}_i + \frac{k_s k_p^2 + k_g k_p}{2}\left\|E_j^i\right\|^2 - \frac{1}{2}k_p k_g\left(g_i^T g_i + \bar{g}_i^T\bar{g}_i\right) + \frac{1}{2}D_{\max}^2\right.\right.\\ &\left.\left. +k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T\left(g_i - \bar{g}_i\right)\right] + \frac{1}{2}\left(\dot{V}_1 + \dot{V}_2\right)\right). \end{aligned} \tag{D.21}$$

Using (D.18) with $b = b_i > 0$, one shows that $2\dot{q}_i^T\left(g_i - \bar{g}_i\right) \leq \left(b_i\left\|\dot{q}_i\right\|^2 + \frac{1}{b_i}\left\|E_j^i\right\|^2\right)$. Using this result in (D.21), one gets

$$\begin{aligned}\dot{V} &\leq \frac{1}{2}\sum_{i=1}^{N}\left[-\left(k_s - 1\right)\mathbb{E}\left(s_i^T s_i\right) - k_s\mathbb{E}\left(\bar{s}_i^T\bar{s}_i\right) + \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}\right)\mathbb{E}\left(\left\|E_j^i\right\|^2\right) + b_i k_g\mathbb{E}\left(\left\|\dot{q}_i - \dot{q}_i^*\right\|^2\right)\right.\\ &\left. -k_p k_g\mathbb{E}\left(g_i^T g_i + \bar{g}_i^T\bar{g}_i\right) + D_{\max}^2\right] + \frac{1}{2}\mathbb{E}\left(\dot{V}_1 + \dot{V}_2\right) \end{aligned} \tag{D.22}$$

Consider now $\dot{V}_1$. Using (D.18) with $b = 1$, the fact that M_i is symmetric positive definite, and that $x^T M_i x < k_M x^T x$, one obtains

$$\begin{aligned}\sum_{i=1}^{N} 2k_p s_i^T \left(M_i \dot{E}_j^i + C_i E_j^i\right) &\leq \sum_{i=1}^{N} k_p \left(s_i^T M_i s_i + s_i^T s_i + \left[\dot{E}_j^{iT} M_i \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i\right]\right) \\ &\leq \sum_{i=1}^{N} k_p \left(\left(k_M + 1\right) s_i^T s_i + \left[k_M \dot{E}_j^{iT} \dot{E}_j^i + E_j^{iT} C_i^T C_i E_j^i\right]\right) \end{aligned} \tag{D.23}$$

Focus now on the terms $E_j^{iT} C_i^T C_i E_j^i$

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &= \sum_{i=1}^{N} \left(\sum_{j=1}^{N} k_{ij} e_j^i\right)^T C_i^T C_i \left(\sum_{\ell=1}^{N} k_{i\ell} e_\ell^i\right) \\ &\leq \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} k_{i\ell} k_{ij} \left\|C_i\right\|^2 e_j^{iT} e_\ell^i. \end{aligned} \tag{D.24}$$

Using (D.18) with $b = 1$, one gets

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &\leq \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} k_{i\ell} k_{ij} \left\|C_i\right\|^2 \left(e_j^{iT} e_j^i + e_\ell^{iT} e_\ell^i\right) \\ &\leq \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} k_{i\ell} k_{ij} \left\|C_i\right\|^2 \left(e_j^{iT} e_j^i\right) \\ &\leq \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} k_{ij} \left\|C_i\right\|^2 \left(e_j^{iT} e_j^i\right) \end{aligned} \tag{D.25}$$

Since $k_{ij} = k_{ji}$, one gets

$$\sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij} \left\|e_j^i\right\|^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} k_{ji} \left\|e_i^j\right\|^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} k_{ij} \left\|e_i^j\right\|^2 \tag{D.26}$$

and so

$$\begin{aligned}\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i &\leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} \sum_{j=1}^{N} \left[k_{ij} \left\|e_i^j\right\|^2 \left\|C_j\right\|^2\right]\right) \\ &\leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} \sum_{j=1}^{N} \left[k_{ij} \left\|e_i^j\right\|^2 k_C^2 \left\|\dot{q}_j\right\|^2\right]\right). \end{aligned} \tag{D.27}$$

Then, the second CTC (5.26) leads to

$$\sum_{i=1}^{N} E_j^{iT} C_i^T C_i E_j^i \leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} k_C^2 \sum_{j=1}^{N} k_{ij} \left\|e_i^j\right\|^2 \left(\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right)^2\right). \tag{D.28}$$

Similarly, one shows that $\sum_{i=1}^{N} E_j^{iT} E_j^i \leq \sum_{i=1}^{N} \alpha_{\mathrm{M}} \sum_{j=1}^{N} k_{ij} \left\|e_i^j\right\|^2$ and $\sum_{i=1}^{N} \dot{E}_j^{iT} \dot{E}_j^i \leq \sum_{i=1}^{N} \alpha_{\mathrm{M}} \sum_{j=1}^{N} k_{ij} \left\|\dot{e}_i^j\right\|^2$.

Consider now $\mathbb{E}\left(\dot{V}_2\right)$

$$\begin{aligned}\mathbb{E}\left(\dot{V}_2\right) &= 2k_p \mathbb{E}\left(\sum_{i=1}^{N} E_j^{iT} Y_i \Delta\theta_i\right) \\ &= 2k_p \mathbb{E}\left(\sum_{i=1}^{N} \left(\sum_{j=1}^{N} k_{ij} e_j^i\right)^T Y_i \Delta\theta_i\right). \end{aligned} \tag{D.29}$$

Let $0_n = [0, \ldots 0]^T \in \mathbb{R}^n$ be the all-zero vector. If $e^i_j = 0_n$, one has $e^{iT}_j Y_i \Delta\theta_i = 0$. Considering now the case $e^i_j \neq 0_n$. Using (D.18) with $b = b_{i2} > 0$, one obtains

$$\mathbb{E}\left(\dot{V}_2\right) = 2k_p \mathbb{E}\left(\sum_{i=1}^{N} E^{iT}_j Y_i \Delta\theta_i\right) \tag{D.30}$$

$$\leq k_p \mathbb{E}\left(\sum_{i=1}^{N}\left(b_{i2} E^{iT}_j E^i_j + \frac{1}{b_{i2}} \left\|Y_i \Delta\theta_i\right\|^2\right)\right). \tag{D.31}$$

Since $\sum_{i=1}^N \mathbb{E}\left(E^{iT}_j E^i_j\right) \leq \sum_{i=1}^N \alpha_{\mathrm{M}} \sum_{j=1}^N k_{ij} \mathbb{E}\left(\left\|e^j_i\right\|^2\right)$, one gets

$$\begin{aligned}\mathbb{E}\left(\dot{V}_2\right) &\leq k_p \sum_{i=1}^{N}\left(b_{i2}\alpha_{\mathrm{M}} \sum_{j=1}^{N} k_{ij} \mathbb{E}\left(\left\|e^j_i\right\|^2\right) + \frac{1}{b_{i2}} \left\||Y_i|\,|\Delta\theta_i|\right\|^2\right)\\ &\leq k_p \sum_{i=1}^{N}\left(b_{i2}\alpha_{\mathrm{M}} \sum_{j=1}^{N} k_{ij} \mathbb{E}\left(\left\|e^j_i\right\|^2\right) + \frac{1}{b_{i2}} \left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)\end{aligned} \tag{D.32}$$

where $\Delta\theta_{i,\max}$ is given by (4.23). Since $\exists j \in \mathcal{N}\; e^i_j \neq 0_n$, choose $b_{i2} = \frac{1+\left\||Y_i|\Delta\theta_{i,\max}\right\|^2}{\sqrt{\sum_{j=1}^N k_{ij}\mathbb{E}\left(\left\|e^j_i\right\|^2\right)}}$ and one obtains $\mathbb{E}\left(\dot{V}_2\right) \leq \mathbb{E}\left(\dot{V}_3\right)$ where

$$\begin{aligned}\mathbb{E}\left(\dot{V}_3\right) &= k_p \sum_{i=1}^{N}\left[\alpha_{\mathrm{M}}\left(\frac{1+\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2}{\sqrt{\sum_{j=1}^N k_{ij}\mathbb{E}\left(\left\|e^j_i\right\|^2\right)}}\right)\sum_{j=1}^{N} k_{ij}\mathbb{E}\left(\left\|e^j_i\right\|^2\right) + \frac{\sqrt{\sum_{j=1}^N k_{ij}\mathbb{E}\left(\left\|e^j_i\right\|^2\right)}\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2}{\left(1+\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)}\right]\\ &= k_p \sum_{i=1}^{N}\sqrt{\sum_{j=1}^{N} k_{ij}\mathbb{E}\left(\left\|e^j_i\right\|^2\right)}\left[\alpha_{\mathrm{M}}\left(1+\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right) + \frac{\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2}{\left(1+\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)}\right]\end{aligned} \tag{D.33}$$

Injecting (D.23), (D.28), and (D.33) in (D.22), one gets

$$\begin{aligned}\dot{V} \leq\; & \frac{1}{2}\sum_{i=1}^{N}\Big[-\left(k_s - 1 - k_p\left(k_M+1\right)\right)\mathbb{E}\left(s_i^T s_i\right) - k_s\mathbb{E}\left(\bar{s}_i^T\bar{s}_i\right) + D^2_{\max}\\ & -k_p k_g \mathbb{E}\left(g_i^T g_i\right) - k_p k_g \mathbb{E}\left(\bar{g}_i^T \bar{g}_i\right) + k_g b_i \mathbb{E}\left(\left\|\dot{q}_i - \dot{q}_i^*\right\|^2\right) + k_p k_M \sum_{j=1}^{N}\alpha_{\mathrm{M}} k_{ij}\mathbb{E}\left(\left\|\dot{e}^j_i\right\|^2\right)\\ & +\left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}\right)\sum_{j=1}^{N}\alpha_{\mathrm{M}} k_{ij}\mathbb{E}\left(\left\|e^j_i\right\|^2\right) + k_p k_C^2 \sum_{j=1}^{N} k_{ij}\alpha_{\mathrm{M}}\mathbb{E}\left(\left\|e^j_i\right\|^2\right)\left[\left\|\dot{q}^i_j\right\| + \eta_2\right]^2\Big] + \frac{1}{2}\dot{V}_3\end{aligned} \tag{D.34}$$

The CTC (5.4) or (5.25) lead to

$$\begin{aligned}\dot{V} &\leq \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}\left[-\left(k_s - 1 - k_p\left(k_M+1\right)\right)s_i^T s_i - k_g k_p g_i^T g_i + D^2_{\max} + \eta\right]\\ \dot{V} &\leq \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}\left[-k_1 s_i^T s_i - k_g k_p g_i^T g_i + D^2_{\max} + \eta\right]\end{aligned} \tag{D.35}$$

with $k_1 = k_s - 1 - k_p\left(k_M + 1\right)$.

Following the steps given in Appendix D.1.1 from (D.53) to (D.57), one shows that

$$\dot{V} \leq -c_3 V + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2}\sum_{i=1}^{N}\mathbb{E}\left(\Delta\theta_i{}^T\Gamma_i^{-1}\Delta\theta_i\right), \tag{D.36}$$

where $c_3 > 0$ is a positive constant. Introducing $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T\Gamma_i^{-1}\Delta\theta_i\right)\right)$, one has

$$\dot{V} \leq -c_3 V + \frac{N}{2}\left[c_3\Delta_{\max} + D_{\max}^2 + \eta\right]. \tag{D.37}$$

Define the function W such that $W(0) = V(0)$ and

$$\dot{W} = -c_3 W + \frac{N}{2}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]. \tag{D.38}$$

Using the initial condition $W(0) = V(0)$, the solution of (D.38) is

$$W(t) = \exp(-c_3 t)\, V(0) + (1 - \exp(-c_3 t))\,\frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]. \tag{D.39}$$

Then, using the Lemma 3.4 in [53] (Comparison lemma), one has $V(t) \leq W(t)$ and so

$$V(t) \leq \exp(-c_3 t)\, V(0) + (1 - \exp(-c_3 t))\,\frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \tag{D.40}$$

Since M_i and Γ_i are symmetric, there exists matrices S_{M_i} and S_{Γ_i} such that $M_i = S_{M_i}^T S_{M_i}$ and $\Gamma_i = S_{\Gamma_i}^T S_{\Gamma_i}$. Introduce now

$$y_M = \begin{bmatrix} (S_{M_1}s_1)^T & \dots & (S_{M_i}s_i)^T & \dots & (S_{M_N}s_N)^T \end{bmatrix}^T \tag{D.41}$$

$$y_\Gamma = \begin{bmatrix} \left(S_{\Gamma_1}^{-1}\Delta\theta_1\right)^T & \dots & \left(S_{\Gamma_i}^{-1}\Delta\theta_i\right)^T & \dots & \left(S_{\Gamma_N}^{-1}\Delta\theta_N\right)^T \end{bmatrix}^T \tag{D.42}$$

$$y_q = \begin{bmatrix} (q_1 - q_1^*)^T & \dots & (q_i - q_i^*)^T & \dots & (q_N - q_N^*)^T \end{bmatrix}^T \tag{D.43}$$

$$z = \begin{bmatrix} y_M^T & y_\Gamma^T & \sqrt{k_g k_0}\, y_q^T & \sqrt{\frac{k_g}{2}P(x, t)} \end{bmatrix}^T \tag{D.44}$$

Then, $V(t)$ can be rewritten as

$$V(z) = \frac{1}{2}\mathbb{E}\left(z^T z\right). \tag{D.45}$$

Using (D.45) in (D.40), one has $\forall t \geq 0$

$$\mathbb{E}\left(\|z(t)\|^2\right) \leq \exp(-c_3 t)\,\mathbb{E}\left(\|z(0)\|^2\right) + (1 - \exp(-c_3 t))\,\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]. \tag{D.46}$$

Since the variance is always definite semi-positive, *i.e.* $0 \leq Var(\|z(t)\|)$, one has

$$\begin{aligned} Var(\|z(t)\|) &\leq \mathbb{E}\left(\|z(t)\|^2\right) - \mathbb{E}(\|z(t)\|)^2 \\ 0 &\leq \mathbb{E}\left(\|z(t)\|^2\right) - \mathbb{E}(\|z(t)\|)^2 \\ \mathbb{E}(\|z(t)\|)^2 &\leq \mathbb{E}\left(\|z(t)\|^2\right). \end{aligned} \tag{D.47}$$

Using (D.47) and $\mathbb{E}\left(\|z(0)\|^2\right) = \|z(0)\|^2$ in (D.46), one gets

$$\begin{aligned}
\mathbb{E}\left(\|z(t)\|^2\right) &\le \exp(-c_3 t)\,\mathbb{E}\left(\|z(0)\|^2\right) + (1-\exp(-c_3 t))\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
\mathbb{E}(\|z(t)\|)^2 &\le \exp(-c_3 t)\,\mathbb{E}\left(\|z(0)\|^2\right) + (1-\exp(-c_3 t))\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
\mathbb{E}(\|z(t)\|) &\le \sqrt{\exp(-c_3 t)\,\mathbb{E}\left(\|z(0)\|^2\right) + (1-\exp(-c_3 t))\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]} \\
\mathbb{E}(\|z(t)\|) &\le \sqrt{\exp(-c_3 t)\,\mathbb{E}\left(\|z(0)\|^2\right)} + \sqrt{(1-\exp(-c_3 t))\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]} \\
\mathbb{E}(\|z(t)\|) &\le \exp\left(-\frac{c_3}{2}t\right)\|z(0)\| + \sqrt{\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]}
\end{aligned} \tag{D.48}$$

and so

$$\mathbb{E}(\|z(t)\|) \le \beta(\|z(0)\|, t) + \rho \tag{D.49}$$

with $\rho = \sqrt{\frac{N}{c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]}$, $\beta(\|z(0)\|, t) = \exp\left(-\frac{c_3}{2}t\right)\|z(0)\|$, and $\beta \in \mathcal{KL}$. Using Definition 2.1 from [48], (D.49) implies that the MAS is input-to-state practically stable.

Convergence of V

From (D.49), we know the system is ISpS. Moreover, from (D.36), one has

$$\dot{V} \le -c_3 V + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2}\sum_{i=1}^{N}\mathbb{E}\left(\Delta\theta_i{}^T\Gamma_i^{-1}\Delta\theta_i\right) \tag{D.50}$$

Then, if initially

$$-c_3 V(0) + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2}\sum_{i=1}^{N}\mathbb{E}\left(\Delta\theta_i{}^T\Gamma_i^{-1}\Delta\theta_i\right) < 0 \tag{D.51}$$

one has $\dot{V} \le 0$ and V is decreasing. Then, one has from (D.40)

$$\begin{aligned}
\lim_{t\to\infty} V(t) &\le \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
\lim_{t\to\infty}\mathbb{E}\left(\frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i + \Delta\theta_i{}^T\Gamma_i^{-1}\Delta\theta_i\right) + \frac{k_g}{2}\left(\sum_{i=1}^{N}k_0\|r_i\|^2 + \frac{1}{2}P(q,t)\right)\right) &\le \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
\lim_{t\to\infty}\frac{k_g}{2}\mathbb{E}\left(\sum_{i=1}^{N}k_0\|r_i\|^2 + \frac{1}{2}P(q,t)\right) &\le \frac{N}{2c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right] \\
&\quad - \lim_{t\to\infty}\frac{1}{2}\sum_{i=1}^{N}\mathbb{E}\left(s_i^T M_i s_i + \Delta\theta_i{}^T\Gamma_i^{-1}\Delta\theta_i\right) \\
\lim_{t\to\infty}\mathbb{E}\left(\sum_{i=1}^{N}k_0\|r_i\|^2 + \frac{1}{2}P(q,t)\right) &\le \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right].
\end{aligned} \tag{D.52}$$

Asymptotically, the formation and tracking error are bounded.

D.1.1 Complementary proof elements

Differential equation satisfied by V

From (D.35), one gets

$$\dot{V} \leq \frac{1}{2}\sum_{i=1}^{N} \mathbb{E}\left[-k_{\mathrm{m}}\left(s_i^T s_i - k_g g_i^T g_i\right) + D_{\max}^2 + \eta\right] \tag{D.53}$$

where $k_{\mathrm{m}} = \min\{k_1, k_p\}$. Using (C.64), one may write

$$\begin{aligned}\sum_{i=1}^{N} g_i^T g_i &\geq \sum_{i=1}^{N} k_0^2 \|r_i\|^2 + \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) P(q,t) \\ &\geq k_2 \left(\sum_{i=1}^{N} k_0^2 \|r_i\|^2 + \frac{1}{2} P(q,t)\right)\end{aligned} \tag{D.54}$$

where

$$k_2 = \begin{cases} 2\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) & \text{if } \left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right) < \frac{1}{2} \\ 1 & \text{else.}\end{cases}$$

Then

$$\begin{aligned}\sum_{i=1}^{N} g_i^T g_i &\geq k_2 \left(\sum_{i=1}^{N} k_0^2 \|r_i\|^2 + \frac{1}{2} P(q,t)\right) \\ &\geq k_3 \left(\sum_{i=1}^{N} k_0 \|r_i\|^2 + \frac{1}{2} P(q,t)\right)\end{aligned} \tag{D.55}$$

where $k_3 = k_2 k_0$ if $k_0 < 1$, $k_3 = 1$ else. Then

$$\begin{aligned}\dot{V} &\leq \mathbb{E}\left(-\frac{1}{2}\sum_{i=1}^{N}\left(k_{\mathrm{m}} s_i^T s_i\right) - \frac{k_3 k_g}{2}\left(\sum_{i=1}^{N} k_0 \|r_i\|^2 + \frac{1}{2}P(q,t)\right) + \frac{N}{2}\left(D_{\max}^2 + \eta\right)\right) \\ &\leq \mathbb{E}\left(-\frac{1}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N} k_{\mathrm{m}}\left(k_{\mathrm{M}} s_i^T s_i\right) + \frac{k_3 k_g}{2}\left(\sum_{i=1}^{N} k_0 \|r_i\|^2 + \frac{1}{2}P(q,t)\right)\right] + \frac{N}{2}\left(D_{\max}^2 + \eta\right)\right) \\ &\leq \mathbb{E}\left(-\frac{k_4}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N}\left(k_{\mathrm{M}} s_i^T s_i\right) + \frac{k_g}{2}\left(\sum_{i=1}^{N} k_0 \|r_i\|^2 + \frac{1}{2}P(q,t)\right)\right] + \frac{N}{2}\left(D_{\max}^2 + \eta\right)\right)\end{aligned} \tag{D.56}$$

with $k_M^* = 1$ if $k_M < 1$ and $k_M^* = k_M$ else, and $k_4 = \min(k_{\mathrm{m}}, k_3)$. Let $c_3 = \frac{k_4}{k_M^*}$ and one gets

$$\begin{aligned}\dot{V} &\leq \mathbb{E}\left(-c_3\left[\frac{1}{2}\sum_{i=1}^{N}\left[s_i^T M_i s_i + \Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i\right] + \frac{k_g}{2}\left(\sum_{i=1}^{N} k_0 \|r_i\|^2 + \frac{1}{2}P(q,t)\right)\right]\right. \\ &\quad \left. + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2}\sum_{i=1}^{N} \Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i\right) \\ \dot{V} &\leq -c_3 V + \frac{N}{2}\left[D_{\max}^2 + \eta\right] + \frac{c_3}{2}\sum_{i=1}^{N} \mathbb{E}\left(\Delta\theta_i{}^T \Gamma_i^{-1} \Delta\theta_i\right).\end{aligned} \tag{D.57}$$

The evaluation of c_3 is described in Appendix C.1.3.

D.2 Proof of Lemma 3

Let define the set $\{t_{i,k},\, t_{i,k+1},\, \ldots,\, t_{i,k+K}\}$ where $\forall \ell \in [k \ldots k+K]$, $t_{i,\ell+1} - t_{i,\ell} = \tau_{\min}$ and $t_{i,k+K} - t_{i,k} \leq \epsilon$.

At $t_{i,k+2}$, one has

$$\begin{aligned} e_i^j\left(t_{i,k+2}\right) &= \tilde{\alpha}_{i,k+2}^j q_i\left(t_{i,k+2}\right) + \left(1-\tilde{\alpha}_{i,k+2}^j\right)\hat{q}_i^j\left(t_{i,k+2}|\hat{q}_i^j\left(t_{i,k+2}^-\right)\right) - q_i\left(t_{i,k+2}\right) \\ &= \left(1-\tilde{\alpha}_{i,k+2}^j\right)\left[\hat{q}_i^j\left(t_{i,k+2}|\hat{q}_i^j\left(t_{i,k+2}^-\right)\right) - q_i\left(t_{i,k+2}\right)\right] \\ &= \left(1-\tilde{\alpha}_{i,k+2}^j\right)\left[\left[\tilde{\alpha}_{i,k+1}^j\hat{q}_i^j\left(t_{i,k+2}|\,q_i\left(t_{i,k+1}\right)\right) + \left(1-\tilde{\alpha}_{i,k+1}^j\right)\hat{q}_i^j\left(t_{i,k+2}|\,\hat{q}_i^j\left(t_{i,k+1}\right)\right)\right] - q_i\left(t_{i,k+2}\right)\right] \end{aligned} \tag{D.58}$$

If the message is received, *i.e.* $\tilde{\alpha}_{i,k+2}^j = 1$, one gets $\hat{q}_i^j\left(t\right) = \hat{q}_i^i\left(t\right)\ \forall t \in [t_{i,k+1},\, t_{i,k+2}[$. Thus, using $\hat{q}_i^j\left(t|\,q_i\left(t_{i,k+1}\right)\right) = \hat{q}_i^i\left(t|\,q_i\left(t_{i,k+1}\right)\right)$, one has

$$\begin{aligned} e_i^j\left(t_{i,k+2}\right) = \left(1-\tilde{\alpha}_{i,k+2}^j\right)&\left[\tilde{\alpha}_{i,k+1}^j\hat{q}_i^i\left(t_{i,k+2}|\,q_i\left(t_{i,k+1}\right)\right) + \left(1-\tilde{\alpha}_{i,k+1}^j\right)\hat{q}_i^j\left(t_{i,k+2}|\,\hat{q}_i^j\left(t_{i,k+1}\right)\right)\right. \\ &\left. -q_i\left(t_{i,k+2}\right)\right] \end{aligned} \tag{D.59}$$

Studying $\hat{q}_i^j\left(t_{i,k+2}|\,\hat{q}_i^j\left(t_{i,k-1}\right)\right)$, one has

$$\begin{aligned} e_i^j\left(t_{i,k+2}\right) = \left(1-\tilde{\alpha}_{i,k+2}^j\right)&\left[\tilde{\alpha}_{i,k+1}^j\hat{q}_i^i\left(t_{i,k+2}|\,q_i\left(t_{i,k+1}\right)\right) + \left(1-\tilde{\alpha}_{i,k+1}^j\right)\left[\tilde{\alpha}_{i,k}^j\hat{q}_i^j\left(t_{i,k+2}|\,q_i\left(t_{i,k}\right)\right)\right.\right. \\ &\left.\left. + \left(1-\tilde{\alpha}_{i,k}^j\right)\hat{q}_i^j\left(t_{i,k+2}|\,\hat{q}_i^j\left(t_{i,k}\right)\right)\right] - q_i\left(t_{i,k+2}\right)\right] \end{aligned} \tag{D.60}$$

and by using $\hat{q}_i^j\left(t|\,q_i\left(t_{i,k+1}\right)\right) = \hat{q}_i^i\left(t|\,q_i\left(t_{i,k+1}\right)\right)$, one gets

$$\begin{aligned} e_i^j\left(t_{i,k+2}\right) = \left(1-\tilde{\alpha}_{i,k+2}^j\right)&\left[\tilde{\alpha}_{i,k+1}^j\hat{q}_i^i\left(t_{i,k+2}|\,q_i\left(t_{i,k+1}\right)\right) + \left(1-\tilde{\alpha}_{i,k+1}^j\right)\left[\tilde{\alpha}_{i,k}^j\hat{q}_i^i\left(t_{i,k+2}|\,q_i\left(t_{i,k}\right)\right)\right.\right. \\ &\left.\left. + \left(1-\tilde{\alpha}_{i,k}^j\right)\hat{q}_i^j\left(t_{i,k+2}|\,\hat{q}_i^j\left(t_{i,k}\right)\right)\right] - q_i\left(t_{i,k+2}\right)\right]. \end{aligned} \tag{D.61}$$

For all $\forall t \in [t_{i,k},\, t_{i,k}+\epsilon]$, $x_i\left(t\right) \simeq x_i\left(t_{i,k}\right)$ and $\hat{x}_i^j\left(t\right) \simeq \hat{x}_i^j\left(t_{i,k}\right)$. In particular : $q_i\left(t_{i,k+2}\right) \simeq q_i\left(t_{i,k+1}\right)$ and $\hat{q}_i^j\left(t_{i,k+2}\right) \simeq \hat{q}_i^j\left(t_{i,k+1}\right) \simeq \hat{q}_i^j\left(t_{i,k}\right)$. Thus, it can be written

$$\begin{aligned} e_i^j\left(t_{i,k+2}\right) &\simeq \left(1-\tilde{\alpha}_{i,k+2}^j\right)\left[\tilde{\alpha}_{i,k+1}^j\hat{q}_i^i\left(t_{i,k+2}|\,q_i\left(t_{i,k+2}\right)\right) + \left(1-\tilde{\alpha}_{i,k+1}^j\right)\left[\tilde{\alpha}_{i,k}^j\hat{q}_i^i\left(t_{i,k+2}|\,q_i\left(t_{i,k+2}\right)\right)\right.\right. \\ &\quad\left.\left. + \left(1-\tilde{\alpha}_{i,k}^j\right)\hat{q}_i^j\left(t_{i,k+2}|\,\hat{q}_i^j\left(t_{i,k+2}\right)\right)\right] - q_i\left(t_{i,k+2}\right)\right] \\ &\simeq \left(1-\tilde{\alpha}_{i,k+2}^j\right)\left[\left[\tilde{\alpha}_{i,k+1}^j + \tilde{\alpha}_{i,k}^j\left(1-\tilde{\alpha}_{i,k+1}^j\right)\right]\hat{q}_i^i\left(t_{i,k+2}\right) + \left(1-\tilde{\alpha}_{i,k+1}^j\right)\left(1-\tilde{\alpha}_{i,k}^j\right)\hat{q}_i^j\left(t_{i,k+2}\right)\right. \\ &\quad\left. -q_i\left(t_{i,k+2}\right)\right] \end{aligned} \tag{D.62}$$

Thus, for any $t_{i,k+K}$

$$e_i^j\left(t_{i,k+K}\right) \simeq \left(1-\tilde{\alpha}_{i,k+K}^j\right)\left[\left(\sum_{p=k}^{k+K-1}\tilde{\alpha}_{i,p}^j\prod_{\ell=p+1}^{k+K-1}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\right)\hat{q}_i^i\left(t_{i,k+K}\right) + \prod_{\ell=k}^{k+K-1}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\hat{q}_i^j\left(t_{i,k+K}\right) - q_i\left(t_{i,k+K}\right)\right]. \tag{D.63}$$

Using (D.63) and $\forall t \in [t_{i,k},\, t_{i,k}+\epsilon]$, $x_i\left(t\right) \simeq x_i\left(t_{i,k}\right)$ and $\hat{x}_i^j\left(t\right) \simeq \hat{x}_i^j\left(t_{i,k}\right)$, one gets

$$\mathbb{E}\left(\left\|e_i^j\left(t_{i,k+K}\right)\right\|^2\right) \simeq \mathbb{E}\left(\left\|\left[\left(\sum_{p=k}^{k+K-1}\tilde{\alpha}_{i,p}^j\prod_{\ell=p+1}^{k+K}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\right)\hat{q}_i^i\left(t_{i,k}\right) + \prod_{\ell=k}^{k+K}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\hat{q}_i^j\left(t_{i,k}\right) - q_i\left(t_{i,k}\right)\right]\right\|^2\right) \tag{D.64}$$

Since variables $\tilde{\alpha}_{i,\ell}^j$ are independent Bernoulli-distributed, one gets

$$\mathbb{E}\left(\left\|e_i^j\left(t_{i,k+K}\right)\right\|^2\right) \simeq \left(\sum_{p=1}^{K}\bar{\alpha}\left(1-\bar{\alpha}\right)^p\right)\left\|\hat{q}_i^i\left(t_{i,k}\right) - q_i\left(t_{i,k}\right)\right\|^2 + \left(1-\bar{\alpha}\right)^K\left\|\hat{q}_i^j\left(t_{i,k}\right) - q_i\left(t_{i,k}\right)\right\|^2 \tag{D.65}$$

Since $\hat{q}_i^i(t_{i,k}) = q_i(t_{i,k})$, one has

$$\mathbb{E}\left(\left\|e_i^j(t_{i,k+K})\right\|^2\right) \simeq (1-\bar{\alpha})^K \left\|\hat{q}_i^j(t_{i,k}) - q_i(t_{i,k})\right\|^2 \tag{D.66}$$

Since $\bar{\alpha} \leq 1$, (D.66) is decreasing when K grows and converges to zero if $K \to \infty$. The proof Appendix D.4 shows absence of Zeno behavior if $\mathbb{E}\left(\left\|e_i^j(t_{i,k+K})\right\|^2\right) = 0$, so the CTC is no more satisfied.

D.3 Evaluation of additional expectation using the Lemma 3

Let study the following three cases for the expression of $\breve{e}_i^j$, depending on the considered time interval:

Case(5.20) If $\forall t \in \left[t_{j,h}^i,\, t_{i,k}\right[$, one has

$$\begin{aligned}\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right) &= \mathbb{E}\left(\left\|\breve{q}_i^j(t) - q_i(t)\right\|^2\right)\\ &= \left\|\breve{q}_i^j(t) - q_i(t)\right\|^2.\end{aligned} \tag{D.67}$$

Case(5.21) If $\forall t \in [t_{i,k},\, t_{i,k+1}[$, one has

$$\begin{aligned}\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right) &= \mathbb{E}\left(\tilde{\alpha}_{i,k}^j\left\|\hat{q}_i^i - q_i\right\|^2 + \left(1-\tilde{\alpha}_{i,k}^j\right)\left\|\breve{q}_i^j - q_i\right\|^2\right)\\ &= \bar{\alpha}\left\|\hat{q}_i^i - q_i\right\|^2 + (1-\bar{\alpha})\left\|\breve{q}_i^j - q_i\right\|^2.\end{aligned} \tag{D.68}$$

Case(5.22) If $t > t_{i,k+K}$ where $\exists K \in \mathbb{N}, K \geq 2$ and $t_{i,k+K} - t_{i,k} \leq \epsilon$, similarly to the proof of Appendix D.2, it can be obtained

$$e_i^j(t_{i,k+K}) \simeq \left(\sum_{p=k}^{k+K}\tilde{\alpha}_{i,p}^j\prod_{\ell=p+1}^{k+K}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\right)\hat{q}_i^i(t_{i,k+K}) + \prod_{\ell=k}^{k+K}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\hat{q}_i^j(t_{i,k+K}) - q_i(t_{i,k+K})$$

which can be upper-bounded by

$$\breve{e}_i^j(t_{i,k+K}) \simeq \left(\sum_{p=k}^{k+K}\tilde{\alpha}_{i,p}^j\prod_{\ell=p+1}^{k+K}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\right)\hat{q}_i^i(t_{i,k+K}) + \prod_{\ell=k}^{k+K}\left(1-\tilde{\alpha}_{i,\ell}^j\right)\breve{q}_i^j(t_{i,k+K}) - q_i(t_{i,k+K}).$$

One finally gets

$$\mathbb{E}\left(\left\|\breve{e}_i^j(t_{i,k+K})\right\|^2\right) = \left(1-(1-\bar{\alpha})^K\right)\left\|\hat{q}_i^i(t_{i,k}) - q_i(t_{i,k})\right\|^2 + (1-\bar{\alpha})^K\left\|\breve{q}_i^j(t_{i,k}) - q_i(t_{i,k})\right\|^2 \tag{D.69}$$

D.4 Absence of Zeno behavior

From the CTC (5.26), a communication is triggered at $t = t_{i,k}^-$ when

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta = \alpha_{\mathrm{M}} \left[\sum_{j=1}^{N} k_{ij} \left(k_e \mathbb{E}\left(\left\| e_i^j \right\|^2 \right) + k_p k_M \mathbb{E}\left(\left\| \dot{e}_i^j \right\|^2 \right) \right) + k_p k_C^2 \left\| e_i^i \right\|^2 \sum_{j=1}^{N} k_{ij} \mathbb{E}\left(\left\| e_i^j \right\|^2 \right) \left[\left\| \dot{\hat{q}}_j^i \right\| + \eta_2 \right]^2 \right] + k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 + k_p \sum_{j=1}^{N} k_{ij} \mathbb{E}\left(\left\| e_i^j \right\|^2 \right) \left[\alpha_{\mathrm{M}} \left(1 + \left\| \left| Y_i \right| \left| \Delta\theta_{i,\max} \right| \right\|^2 \right) + \frac{\left\| \left| Y_i \right| \left| \Delta\theta_{i,\max} \right| \right\|^2}{\left(1 + \left\| \left| Y_i \right| \left| \Delta\theta_{i,\max} \right| \right\|^2 \right)} \right] \tag{D.70}$$

with $k_e = \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right)$. Then, the expectation $\mathbb{E}\left(\left\| e_i^j \right\|^2 \right)$ and $\mathbb{E}\left(\left\| \dot{e}_i^j \right\|^2 \right)$ converges to zero since communication protocol described Section 5.3.2. As a consequence, the CTC (6.6) in Theorem 11 is not satisfied at $t = t_{i,k}^+$ iff

$$k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta > k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 . \tag{D.71}$$

Similarly to Appendix C.2, the absence of Zeno behavior can be proven.

Similar proof can be make by replacing $\mathbb{E}\left(\left\| e_i^j \right\|^2 \right)$ is replaced by $\mathbb{E}\left(\left\| \breve{e}_i^j \right\|^2 \right)$.

Appendix E

Résumé francais

E.1 Introduction

Les systèmes multi-agents (MAS) ont fait l'objet d'importantes recherches ces dernières décennies, avec des domaines d'application divers tels les véhicules autonomes aériens (UAVs), les véhicules autonomes sous-marins (AUVs), satellites. Ces systèmes sont utilisés pour différentes applications telles que l'exploration, la surveillance, ou la maintenance dans les zones difficiles d'accès. La coopération entre agents ne peut cependant avoir lieu que si les agents peuvent collecter ou recevoir des informations sur les autres membres de la flotte. Ces informations peuvent être obtenues à partir de mesures provenant de capteurs embarqués mais, de ce fait, sont limitées essentiellement à des informations de positions ou d'orientation relatives. L'échange d'informations ayant un contenu plus diversifié est envisagé par le biais de communications inter-agents. Cependant cet échange doit s'effectuer en évitant dans la mesure du possible la saturation du réseau. De plus, les performances attendues doivent être obtenues en présence de délais de communication et de perte de messages. Réduire du nombre de communications devient donc une nécessité afin de mieux gérer celles-ci. Néanmoins, réduire le nombre d'informations échangées entre agents implique que les lois de commandes, les estimateurs et les protocoles utilisés par les agents utilisés soient adaptés à la diminution .

Ces dernière décennies, de nombreux chercheurs ont développé des méthodes avec communication réduite permettant de réaliser un consensus multi-agents, et plus récemment une formation. Pour obtenir un consensus, [77, 113, 13, 37, 36], l'état des agents doit converger vers une même valeur (par exemple pour un ensemble de véhicules, même vitesse, même position...). L'obtention d'une formation consiste à diriger et maintenir une flotte de véhicules suivant une configuration désirée. De nombreuses approches ont été proposées à cet effet dans la littérature, voir [112, 87, 82, 72, 26, 14, 15].

L'obtention d'un consensus et la réalisation d'un vol en formation sont desproblèmes nécessitant d'ordinaire un nombre important d'échanges d'informations sur l'état des agents voisins, afin d'évaluer de façon distribuée les lois de commande des agents. Certains auteurs s'appuient sur une communication supposée disponible en permanence [77] ou une publication périodique des informations nécessaires [36] . D'autres méthodes ont été proposées qui permettent de réduire de manière plus importantes le nombre de communications nécessaires, en utilisant par exemple les communications par intermittence [117], ou encore les communications déclenchées par évènement , dites " event-triggered ".

Dans ce type d'approches, une communication est transmise lorsque une condition est remplie. Cette condition se traduit en général par la comparaison d'une expression incluant différentes composantes de l'état des agents avec un seuil. La principale difficulté réside dans la détermination de la condition de déclenchement des communication (CTC), permettant à la fois d'obtenir un nombre réduit de communication tout en assurant la convergence du système vers le consensus ou la formation désirée.

Dans le cas des commandes distribuées, chaque agent estime l'état de ses voisins pour évaluer sa propre commande [39]. Chaque agent fait également une estimation de son propre état basée sur les informations accessibles aux voisins. Cette estimation permet à l'agent de connaitre l'estimation de son état tel qu'il est calculé par ses voisins. Pour déclencher une communication, la condition s'effectue sur la comparaison entre l'erreur d'estimation de l'étatde l'agent et un seuil. Cette approche est considérée dans de nombreux travaux pour le problème de consensus, *e.g.* [136, 35, 94, 39, 107, 23, 106]. Ceux-ci diffèrent par la complexité du modèle dynamique des agents [136, 35, 94], la structure des estimateurs [23, 39, 107, 106], et la détermination du seuil de déclenchement de la CTC [94, 106].

Plusieurs travaux présentent les méthodes de commande pour créer une formation en utilisant des communication par évènement déclenchant [61, 98, 99]. Dans ces travaux, la dynamique des agents est principalement décrite par un simple intégrateur, avec une commande constante entre chaque communication. Les CTCs dépendent de variables en provenance de tous les agents, avec différents choix de seuils de déclenchement. Un seuil constant est défini dans [98]. [61, 99] considèrent un seuil variable avec le temps, où la CTC dépend également des positions relatives entre agents et de l'erreur effectuée par les estimateurs. Ces CTCs permettent ainsi de réduire le nombre de communication de manière variable en fonction de la précision exigée par le système, afin de converger vers la formation désirée. Les perturbations d'état ne sont pas considérées.

L'objectif de cette thèse est de développer des lois de commande et des estimateurs distribués pour un système multi-agents capables d'assurer la convergence vers un consensus ou une formation de structure donnée tout en limitant le nombre de communications. Les communications effectuées entre agents sont déclenchées par des évènements. Les modèles d'évolution incluent des perturbations d'état.

La Partie 1 décrit les notions et outils de bases utilisées dans ce document, ainsi que l'état de l'art sur les méthodes de consensus, le vol en formation et les méthodes d'évènement déclenchant.

La Partie 2 aborde le problème du consensus. Un méthode event-triggered pour obtenir un consensus borné avec un nombre d'information réduit, tout en prenant en compte la présence de perturbations. La CTC se basant sur l'erreur entre l'état réel et l'état estimé, un nouvel estimateur a été élaboré afin de réduire de manière efficace cette erreur. Un protocole de communication et un deuxième estimateur sont également présentés afin de permettre la mise en oeuvre pratique de la méthode de façon distribuée. L'analyse de la convergence est effectuée. Des extensions de ces approches sont finalement discutées.

La partie 3 traite le problème de formation et de poursuite de trajectoire pour un système dynamique Euler-Lagrange. On y définit une méthode event-triggered pour obtenir une formation, respectant des bornes sur les erreurs en présence de perturbations. Une structure de loi de commande adaptative est proposée afin de compenser les composantes inconnues de la dynamique du système. Un estimateur de structure similaire de la dynamique du système est également développé afin de réduire l'erreur d'estimation du système. Une CTC est proposée, basée sur l'erreur d'estimation et la distance inter agents. La flotte est supposée suivre une unique trajectoire de référence. L'étude de la stabilité du système et de la convergence vers la formation désirée ainsi que la poursuite a été effectuée.

La Partie 4 étend les résultats obtenus dans la Partie 3 aux problèmes de pertes de données. L'estimateur est adapté pour tenir compte de l'influence des pertes de communications. Une nouvelle CTC est introduite, basée sur l'espérance de l'erreur faite par les agents voisins. Un protocole de communication est développé afin de garantir l'absence de paradoxe de Zeno. Les conditions et la preuve de la convergence du système ont été établies.

La Partie 5 propose également une extension des résultats obtenus dans la Partie 3 en considérant les délais de communications. Afin d'éviter l'envoi d'informations obsolètes, le contenu du message envoyé est modifié afin de transmettre une prédiction de l'état de l'agent permettant de mettre à jour les estimateurs des voisins. De plus, l'agent met à jour sa propre estimation en utilisant le contenu du message afin de garantir la synchronisation des estimateurs. La CTC est modifiée en utilisant les états prédits, afin de tenir compte des délais de transmission. Deux modèles de prédiction sont proposés, proposant un compromis entre la précision de la prédiction et la complexité de calcul.

La dernière partie est constituée d'une conclusion générale sur les travaux présentés dans cette thèse et d'une description des perspectives de travaux futurs.

E.1.1 Système coopératif

Les recherches sur la collaboration entre agents se sont initialement inspirés des comportements biologiques, tel que le vol des oiseaux ou les essaims d'abeilles. Les premières méthodes se basent sur des règles individuelles permettant de définir un comportement global [89, 103]. Dans les systèmes multi-agents (MAS), la coopération entre agents permet de réaliser des missions tel que la surveillance, l'exploration ou manoeuvre dans les zones à risque pour des opérateurs humains. L'utilisation d'un MAS n'est justifié que si l'efficacité globale du système est supérieure à la somme des efficacités individuelles. Un autre avantage lié au MAS est la robustesse du système en cas de perte d'un de ses membres. Cependant, la coopération s'appuie en général sur des besoins d'échange. Il est de ce fait nécessaire de résoudre les problèmes liés aux communication entre agents.

E.1.2 Contrôle centralisé, décentralisé et distribué

Les lois de commande d'un MAS peuvent être conçues pour être centralisées, décentralisées ou distribuées. Dans les commandes centralisées, les informations des agents sont transmises à une unité centrale, qui évalue la commande pour tous les agents avant de leur renvoyer. Les agents ne communiquent pas entre eux. Bien que faisant l'objet de nombreuses études[97, 127, 75, 110], cette méthode possède l'inconvénient que les agents dépendent étroitement de l'unité centrale, et sont incapables d'effectuer des décisions seuls. La commande décentralisée permet à chaque agent de calculer sa propre loi de manière indépendante. Aucune communication entre les agents n'est effectuée à cet effet, ce qui peut limiter les performances de la mission à accomplir. Finalement, la commande distribuée permet à chaque agent d'évaluer sa commande de manière indépendante, tout en permettant aux agents d'échanger des informations entre eux. L'envoi des messages est décidé par les agents eux-même. Ces avantages en font des méthodes très prisées dans les problème d'obtention de consensus [77, 10, 13], le vol en formation [82, 72, 105, 99] ou le flocking [89, 103, 91, 7].

E.1.3 Notions de théorie des graphe

Une communication entre deux agents est nommée une liaison. Une liaison peut être à double sens ou à sens unique (ex : cas d'un émetteur/récepteur défaillant chez un agent). L'ensemble des agents d'un système et des liaisons entre eux est nommé graphe de liaison. Deux agents directement liés via un graphe de liaison sont dit "voisins", et on note $\mathcal{N}_i$ l'ensemble des voisins de l'Agent i. Un graphe possédant uniquement des liaisons bidirectionnelles est dit non-orienté. Autrement, il est dit orienté. Un graphe dont les liaisons ne changent pas avec le temps est dit à topologie fixe. Par opposition, un graphe dont les liaisons changent avec le temps est dit à topologie variable (des liaisons peuvent apparaitre/disparaitre en fonction de l'éloignement ou le rapprochement des agents entre eux, suivant la portée de leur récepteur par exemple). On dit qu'un graphe est entièrement connecté quand tous les agents sont directement liés les uns aux autres. Plusieurs formes de graphe particulières sont présentées en Figure E.1.

On définit la matrice d'adjacence $\mathrm{A}(t)$ la matrice dont les éléments a_{ij} sont différents de zéro, $a_{ij} \neq 0$, s'il existe une liaison entre l'Agent i et l'Agent j, soit une communication possible, et $a_{ij} = 0$ sinon. On définit également la matrice des degrés sortant $D_{\text{out}} = \text{diag}(\mathrm{A}1_N)$ et la matrice des degrés entrant $D_{\text{in}} = \text{diag}\left(\mathrm{A}^T 1_N\right)$, où N est le nombre d'agents et 1_N le vecteur de dimension N dont les composantes sont toutes égales à 1. Enfin, on définit la matrice de Laplace L telle que $L = D_{\text{out}} - \mathrm{A}$. Ces matrices permettent de modéliser le graphe de liaison, et seront utilisées dans la suite de ce document. On définit $\otimes$ comme étant le produit de Kronecker, ainsi que $\lambda_{\max}(M)$ et $\lambda_{\min}(M)$ les valeurs propres maximale et minimale de la matrice M.

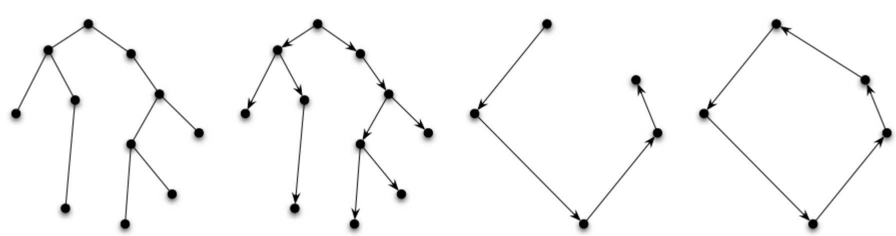


Figure E.1: Formes de graphe particulières. De gauche à droite, arbre, arbre orienté, chaine, anneau

E.1.4 Stabilité au sens de Lyapunov

La stabilité au sens de Lyapunov, et plus précisément la seconde méthode de Lyapunov, vise à caractériser la stabilité des systèmes autonomes autour d'un point d'équilibre, sans connaitre les trajectoires du système autour des points. Pour cela, on définit avant tout une fonction de Lyapunov candidate.

Définition 11. Une fonction de Lyapunov candidate V est une fonction de$\mathcal{X} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^+$ telle que V et ses dérivées partielles soient continues, et V est définie positive (*i.e.* $V > 0\ \forall x \neq 0$ et $V(0) = 0$).

Le théorème suivant donne des conditions suffisantes pour la stabilité des systèmes autonomes.

Théorème 13. *Définissons un ensemble $\mathcal{X} \subset \mathbb{R}^n$ qui contient l'origine et $V : \mathcal{X} \rightarrow \mathbb{R}^+$ une fonction Lyapunov candidate*

1. *Si $\dot{V}(x) \leq 0\ \forall x \in \mathcal{X}$, alors l'origine est localement stable.*
2. *Si $\dot{V}(x) < 0\ \forall x \in \mathcal{X}$, $x \neq 0$, alors l'origine est localement asymptotiquement stable.*

où $\dot{V}$ est la dérivée temporelle de V.

E.1.5 Protocole de communication

Un protocole de communication est un système de règles permettant à deux entités de communication de transmettre des informations. Ces règles définissent la syntaxe des messages, la synchronisation des communications, la détection des collisions entre messages, l'assignation des bandes passantes, des instants de communication ...

Initialement conçus pour les réseaux filaires, les protocoles de communication ont été repensés avec les radio fréquences. Une seule bande de fréquence est généralement utilisée pour envoyer tous les messages au lieu d'en assigner une à chacun. Cependant, cette bande de fréquence unique peut être sujette à des collisions entre les données échangées par les agents au même instant. Des protocoles ont donc été développés pour gérer ces problèmes et assurer une communication efficace.

Protocole ALOHAnet

Dans la première version du protocole, nommé Pure ALOHA, une station émet un message quand elle en a besoin. Si, durant la transmission, la station reçoit des données provenant d'une autre station, il y a collision. La station finit d'envoyer son message et définit un temps aléatoire d'attente. Le message est retransmis une nouvelle fois à la fin de ce temps. Le protocole est répété jusqu'à ce que le message est été envoyé avec succès. Afin d'améliorer le débit, une deuxième version nommée le Slotted ALOHA introduit des créneaux horaire à intervalles fixes : un agent ne peut communiquer qu'au début d'un créneau, limitant ainsi les collisions en assurant qu'aucune ne peut apparaitre durant la transmission du message.

On notera que la liste des messages en attente peut devenir importante du fait d'un grand nombre de stations, pouvant conduire à une saturation du réseau. De plus, on remarque que les stations ne cherchent pas à détecter si un message est en cours de transmission avant d'envoyer un message, ce qui a motivé l'élaboration du protocole CSMA, présenté dans la section suivante.

CSMA

Basé sur les travaux de l'ALOHAnet, le CSMA (Carrier Cense Multiple Access) est un protocole d'accès aléatoire "écoute avant envoie", où les stations vérifient l'absence de communication sur le canal avant de tenter de transmettre un message. Si une transmission est détectée, la station attend la fin de la transmission avant de tenter de transmettre à nouveau, permettant d'éviter un grand nombre de collisions. La première implémentation du CSMA fut l'Ethernet. On remarquera que "l'écoute avant l'envoi" est également un point faible du CSMA, car les stations sont obligées d'atteindre qu'une transmission soit terminée pour pouvoir envoyer un message à son tour. Plusieurs variations du CSMA existent.

Dans le CSMA/CD, les transmissions sont interrompues dès qu'une collision est détectée, réduisant le temps requis avant de retenter une communication. Dans le CSMA/CA, une station est définie comme coordinateur et autorise ou non la communication quand un agent la demande. Enfin, le CSMA/CR autorise les stations à transmettre simultanément tant que le message transmis est identique pour les deux stations.

E.1.6 Commande pour le consensus

Le consensus est un problème très étudié dans le domaine de la commande coopérative, [77, 113, 13, 37, 36]. Dans le problème de consensus, plusieurs agents doivent se synchroniser en une valeur commune. Dans le cas des systèmes multi-agents, cette valeur commune peut être une mesure de plusieurs capteurs, une vitesse de synchronisation pour éviter les collisions entre agents, ou un objectif à atteindre au même instant.

Définition

Un consensus peut être asymptotique ou borné. Un consensus est défini comme asymptotique si l'état de tous les agents convergent vers la même valeur. De même, un consensus est considéré comme borné si l'écart d'état entre les agents peut être borné par une constante quand le système global converge.

Définition 12. Un réseau d'agents atteint un consensus asymptotique ssi

$$\lim_{t\to\infty} \|x_j(t) - x_i(t)\|^2 = 0, \tag{E.1}$$

pour toutes les paires d'agents (i, j) dans le réseau. Un réseau d'agents atteint un consensus borné ssi il existe une constante $\varepsilon > 0$ telle que

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| \leqslant \varepsilon \tag{E.2}$$

pour toutes les paires d'agents (i, j) dans le réseau.

Cependant, plusieurs conditions sur le graphe de communication doivent être respectées pour obtenir un consensus. Il est montré dans [46] qu'un consensus ne peut être atteint que si l'union des graphe de communication dans le temps est connecté suffisamment souvent tant que le système évolue. Afin d'étendre les résultats précédents [70] montre qu'un consensus en présence d'une topologie variable peut être atteint asymptotiquement si l'union des graphes orientés forme suffisamment souvent un arbre orienté. L'étude du problème de consensus avec un graphe orienté présenté par [78, 76] montre qu'un graphe fortement connecté est nécessaire pour atteindre le consensus. Quelques solutions existent pour satisfaire ces conditions, notamment l'introduction d'un agent virtuel pour garantir un graphe connecté, comme dans les méthodes de pinning [124, 125].

Commande de consensus

Comme décrit dans [77, 113, 74, 76], un système de consensus peut être modélisé sous la forme

$$\dot{x}_i(t) = -\sum_{j\in\mathcal{N}_i} m_{ij}(t)\left(x_i(t) - x_j(t)\right) \tag{E.3}$$

où $\mathcal{N}_i$ est l'ensemble des voisins de l'Agent i, $m_{ij}(t)$ est une pondération entre les valeurs i et j. Cette pondération $m_{ij}(t)$ est souvent choisie égale à $a_i(t)$, élément de la matrice d'adjacence $\mathrm{A}(t)$ associé au graphe de communication $\mathcal{G}(t)$. Cependant, [63, 102, 96, 42] proposent d'autres valeurs pour m_{ij} afin d'optimiser la rapidité de convergence du système et s'assurer du rassemblement des agents en un groupe unique. De manière générale, on notera que plus le nombre de voisins d'un agent est important, plus la convergence vers le consensus sera rapide.

E.1.7 Commande pour la formation de flottes

Le contrôle de formation consiste à faire converger et maintenir les agents d'un flotte vers une formation désirée, possiblement variable dans le temps. Plusieurs approches ont été proposées dans la littérature regrouper en méthodes comportementales, nommée "behavior-based flocking" [89, 103, 75, 91, 7], ou méthode de suivi de formation, nommée "formation tracking" [26, 15, 6, 65, 87].

Méthodes comportementales (behavior-based flocking)

Fortement inspirées des comportements biologiques tel que les oiseaux ou les bactéries, les méthodes comportementales [89, 103, 75, 91, 7] imposent plusieurs règles de comportement (attraction, répulsion, imitation) pour chaque agent. Leur combinaison conduit le système MAS à suivre un comportement désiré. Ces approches requièrent que chaque agent puisse observer l'état de ses voisins, via des capteurs ou communication faite entre agents. Dans tous les cas, ces observations sont supposées accessibles de manière permanente. De plus, les méthodes comportementales ne peuvent aboutir à une configuration précise entre les agents.

Suivi de formation (formation tracking)

Différentes méthodes de suivi de formation peuvent être considérées. Dans les méthodes meneur-suiveurs, nommée "leader-follower" [26, 15, 6, 65], une trajectoire est définie pour un agent leader afin de remplir une mission. Les autres agents (followers) suivent le leader et tentent de maintenir une formation autour de ce leader de référence. On note que ces méthodes sont dépendantes du leader, dont la moindre panne peut suffire à mettre en péril toute la formation. Aussi, un leader virtuel peut être considéré [14, 15, 90] afin d'obtenir un système plus robuste aux défaillances. Les structures virtuelles, introduites dans [87, 112, 75], ne nécessitent pas la présence d'un leader en imposant des contraintes directement entre agents. Ces méthodes peuvent être divisées en contraintes imposant une position, une distance, ou un vecteur entre agents. Enfin, une dernière catégorie de méthodes consiste à imposer à chaque agent une trajectoire prédéfinie, conduisant à la formation souhaitée, comme dans [97, 3].

On notera que dans la plupart de ces méthodes, une communication permanente est exigée.

E.1.8 Gestion des communications à l'aide d'évènements déclenchant (event-triggered)

L'utilisation d'évènements déclenchant sont à l'origine d'approches prometteuses quand le nombre de communications est restreint ou pour limiter le nombre de collisions au sein d'un réseau. Dans ces méthodes, un message est envoyé quand une condition, nommée CTC (Communication Triggering Condition) ou condition d'évènement, est remplie. Elles permettent ainsi de ne communiquer qu'en cas de besoin. La plus grande difficulté de ces méthodes résident dans le fait de trouver une condition assurant la stabilité du système, la réussite de la mission et une réduction efficace du nombre de communication.

Dans les systèmes distribués, les états des autres agents n'étant pas accessibles en permanence, chaque agent maintient des estimateurs des états de ses voisins pour évaluer sa commande [39]. Cependant, en l'absence de communication permanente, la qualité de ces estimateurs, en terme de précision de la reconstruction, est difficile à évaluer. Aussi, chaque agent maintient également une estimation de son propre état, effectué à partir des informations partagées avec les autres agents. Dès que l'erreur entre cette estimation et l'état actuel de l'agent dépasse un certain seuil, une communication est effectuée (ou "déclenchée") pour mettre à jour l'estimateur des voisins.

Paradoxe de Zeno et Minimum intervalle de temps

Le paradoxe de Zeno décrit le phénomène correspondant à une infinité de déclenchement de la condition d'évènement durant un intervalle de temps fini, créant ainsi une communication permanente. Pour établir l'absence de paradoxe de Zeno, de nombreux chercheurs, *e.g.* [20, 24, 23, 29, 31, 37, 38, 35, 34, 39, 60, 98, 30, 136, 133, 49], démontrent systématiquement l'existence d'intervalle de temps minimum entre deux communications.

Event-triggered et consensus

La plupart des méthodes event-triggered ont été développées afin de limiter le nombre de communication au sein d'un consensus. Dans [23], la dynamique des agents est modélisée par un simple intégrateur. Un seuil décroissant avec le temps est utilisé, ce qui implique une augmentation de la fréquence de communication avec le temps. Dans [94], le modèle dynamique est un double intégrateur et la condition de déclenchement de communication CTC dépend d'un seuil exponentiellement décroissant avec le temps, indépendant de l'état des agents. Un terme constant est également considéré afin de maintenir un seuil minimum. Un modèle de dynamique linéaire généralisé est considéré dans [136, 37, 35] avec des seuils variables en fonction de l'état des agents, assurant la convergence du système. La présence de perturbations a été partiellement étudiée par [45, 19], qui proposent une méthode event-triggered réduisant l'impact des perturbations dans le cas de simples intégrateurs.

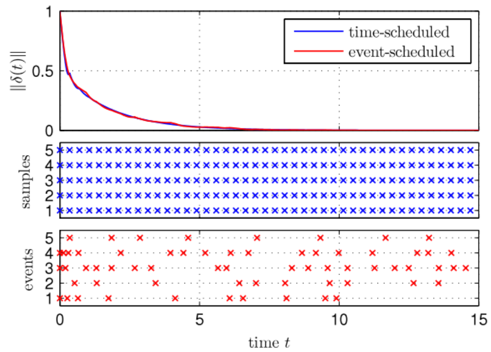


Figure E.2: Comparaison entre la méthode de Seyboth et une communication périodique classique.

Event-triggered et formation

Quelques travaux récents combinent les approches event-triggered avec des méthodes de formation [61, 98, 99]. Dans ces travaux, la dynamique des agents est décrite par un simple intégrateur, et la commande est considérée comme constante entre deux communications. Ces CTCs nécessitent des informations en provenance de tous les agents, avec cependant différents types de seuils de déclenchement: un seuil constant est utilisé dans [98], et des seuils variables avec le temps dans [61, 99]. Les CTC dépendent également des positions relatives entre agents et des erreurs d'estimation. Ainsi, les CTC permettent de réduire le nombre de communication quand le système converge vers la forme désirée. On note que l'absence de paradoxe de Zeno a été démontrée, et que les systèmes ne considèrent pas de perturbations d'état.

Les Logic-Based Communications (LBC), introduites dans [84, 127, 3, 130], semblent également être des approches intéressantes pour réduire les communications. Utilisant également une condition pour gérer les communications, le problème d'un système multi-agent non-linéaire est étudié. Dans ces méthodes, chaque agent suit un chemin paramétré prédéfini, calculé de manière centralisé. La CTC introduite permet aux agents de suivre leurs trajectoires de manière synchronisée, menant à la formation désirée. Les délais de communication et pertes de données sont considérées. Cependant, l'absence de paradoxe de Zeno n'a pas été analysée.

Perte de données, topologie variable et perturbations

Si la plupart des recherches sur les event-triggered se basent principalement sur l'étude de la dynamique des agents et de leur commande, d'autres contraintes doivent être considérées. Dans [25, 95], un système dynamique avec des pertes de données est considéré. La CTC est construite à l'aide d'un variable stochastique conduisant à des déclenchements supplémentaires afin de compenser les éventuelles pertes de données. Les méthodes proposées par [57, 66, 56] permettent d'adapter leurs conditions avec des topologies variables. Elles exigent cependant que l'ensemble des agents émettent une communication à chaque nouvelle configuration.

La présence de perturbations est également à prendre en compte : certaines méthodes telles [50, 95, 111, 120, 68] proposent des CTCs couplées à des filtre de Kalman et des coefficients variables pour atténuer l'influence des bruits de mesures. Ces méthodes nécessitent cependant de centraliser les informations. [79, 45, 111] étudient le problème de commande distribuée pour simple intégrateur avec des perturbations d'état.

E.2 Nouveaux estimateurs et protocole de communication pour event-triggered consensus appliqués à un système linéaire avec perturbations d'état bornées

Dans cette partie, on se propose de reprendre la méthode event-triggered développée par [37] pour obtenir un consensus borné avec un nombre d'informations réduit, tout en prenant en compte la présence de perturbations. On introduit pour cela un nouvel estimateur dans le but de réduire de manière plus efficace le nombre de communications échangées. Un protocole de communication est également présenté afin de permettre la mise en oeuvre pratique de la méthode. Avec cette approche, une estimation de l'état de tous les agents (et pas seulement des voisins) est requis pour évaluer toutes les commandes des estimateurs. Les estimations réalisées possèdent donc un niveau de complexité plus élevé, mais ceci permet de réduire la fréquence de communication. L'analyse de la convergence est effectuée en considérant des perturbations d'état séparées en deux composantes aléatoires : l'un commune à tous les agents, l'autre spécifique pour chacun.

En utilisant les notions introduites dans la Section E.1, on introduit la formulation du problème dans la Section E.2.1. La CTC, présentée dans la Section E.2.4, requis le nouvel estimateur décrit en Section E.2.2 couplé au protocole de communication présenté Section E.2.2. Un deuxième estimateur est également exposé en Section E.2.3 afin d'obtenir une implémentation décentralisée de la CTC.

E.2.1 Formulation du problème

Comme dans [37], le réseau est composé de N agents, avec un graphe de communication $\mathcal{G}$ non-orienté et une topologie fixe, donc une matrice d'adjacence A constante. On considère la dynamique et la commande distribuée suivante :

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + d_i(t) && \text{(E.4)} \\ u_i(t) &= c_1 F \sum_{j \in \mathcal{N}_i} \left(y_i^i(t) - y_j^i(t) \right). && \text{(E.5)} \end{aligned}$$

où $x_i \in \mathbb{R}^n$ est l'état de l'Agent i, y_j^i l'estimation de l'état de l'Agent j par l'Agent i décrite dans la Section E.2.2, et $u_i \in \mathbb{R}^m$ est la commande, $i = 1, \ldots, N$. $A \in \mathbb{R}^{n \times n}$ et $B \in \mathbb{R}^{n \times m}$. $c_1 = c + c_2$ avec $c = 1/\lambda_2(L)$ et $c_2 \geq 0$ sont des paramètres de réglage. $F = -B^T P$ où P est une matrice semi-définie symétrique, solution de l'équation de Riccati

$$PA + A^T P - 2PBB^T P + 2\alpha P < 0, \tag{E.6}$$

avec $\alpha > 0$. L'estimation de l'état y_j^i est décrite dans la section suivante.

Contrairement à [37], on ajoute des perturbations à la dynamique de l'agent définie par l'équation (E.4):

$$\begin{aligned} d_i(t) &= m(t) + s_i(t), && \text{(E.7)} \\ d(t) &= 1_N \otimes m(t) + s(t) && \text{(E.8)} \end{aligned}$$

avec $s(t) = \left[s_1(t)^T \ldots s_N(t)^T \right]^T$, $m(t) \in \mathbb{R}^n$ la composante variant avec le temps mais considérée identique pour tous les agents et bornée par $\|m(t)\| \leq M_{\max}$, et $s_i(t) \in \mathbb{R}^n$ la composante spécifique à chaque agent, également bornée par $\|s_i(t)\| \leq S_{\max}$ $\forall t$, $i = 1, \ldots, N$. Les deux composantes de $d(t)$ permettent de représenter l'effet combiné d'un vent uniforme sur une flotte d'agents et les turbulences affectant de manière différente chacun d'entre eux.

Le problème considéré ici consiste à créer une commande distribuée, robuste aux perturbations, permettant d'obtenir un consensus borné tout en limitant le nombre d'informations échangées. Pour cela, les instants de communications sont choisis localement par chaque agent, suivant la méthode d'event-triggered introduite dans la Section E.2.4.

Dans cette étude, on considèrera qu'il n'y a pas de délai de communication et que les agents connaissent parfaitement leur propre état.

E.2.2 Modèle d'estimation et protocole de communication

Modèle d'estimation

On définit l'estimation $y_j^i(t)$ de l'état de l'Agent j par l'Agent i. On peut définir ainsi l'erreur d'estimation $e_j^i = y_j^i - x_j$. Dans le Théorème 14 de la Section E.2.4, une communication étant déclenchée quand e_i^i dépasse un certain seuil. Aussi, le modèle de notre estimateur a été créé afin de représenter au mieux la dynamique réelle des agent et minimiser e_i^i. Chaque agent évalue sa propre estimation des états de tous les autres agents. L'évolution de $y_j^i(t)$ est modélisée par :

$$\dot{y}_j^i(t) = Ay_j^i(t) + B\tilde{u}_j^i(t), \ \forall t \in \left]t_{j,k}^i, t_{j,k+1}^i\right[\tag{E.9}$$

$$\tilde{u}_j^i(t) = c_1 F \sum_{p \in \mathcal{N}_j} \left(y_j^i(t) - y_p^i(t)\right) \tag{E.10}$$

$$y_j^i\left(t_{j,k}^i\right) = x_j\left(t_{j,k}^i\right), \tag{E.11}$$

avec $t_{j,k}^i$ le temps à l'instant où le k-ième message envoyé par l'Agent j a été reçu par l'Agent i. Le temps où le k-ième message a été envoyé par l'Agent j est noté $t_{j,k}$ et $t_{j,k+1}$ définit le temps où le$(k+1)$-ième message sera envoyé. On note $y^i = \left[y_1^{iT}, y_2^{iT}, \dots y_N^{iT}\right]^T$ avec $y^i \in \mathbb{R}^{Nn}$ le vecteur des estimations de l'état de tous les agents par l'Agent i.

Protocole de communication

Dans [37], un message envoyé par l'Agent i à $t = t_{i,k}$ contient son état $x_i(t_{i,k})$. Cette valeur est utilisée par ses voisins $j \in \mathcal{N}_i$ pour mettre à jour y_i^j. Cela n'est cependant pas possible si $j \notin \mathcal{N}_i$. Afin de contourner ce problème, un protocole de communication nommé "delayed flooding method" a été élaboré.

Quand il s'avère nécessaire qu'un agent envoie des informations, il transmet un message contenant le vecteur y^i , ainsi que la liste

$$T^i = \left[t_{1,k_1}, \dots, t_{i-1,k_{i-1}}, t_{i,k}, t_{i+1,k_{i+1}} \dots t_{N,k_N}\right]$$

où chaque t_{j,k_j} représente le temps à l'instant où la condition d'évènement l'Agent j a été satisfaite. Quand un Agent ℓ reçoit le message venant de l'Agent i, il compare sa propre liste de temps T^ℓ avec T^i. Pour chaque composant de y^ℓ tel que $t_{i,k} > t_{\ell,k}$, *i.e.*, l'élément de y^i a été mis à jour le plus récemment, celui-ci est remplacé par celui de y^i. Le vecteur T^ℓ est également mis à jour en conséquence.

Cette méthode permet de mettre à jour les estimateurs en relayant les informations d'un agent à un autre uniquement quand celui-ci doit communiquer.

E.2.3 Estimation v^i de l'état estimé y^i par l'Agent j

Le delayed flooding protocol présenté dans la Section E.2.2 permet à chaque Agent i d'avoir accès à y_j^i, pour tout $j \in \mathcal{N}$. Cependant, l'Agent i ne peut avoir accès à y_i^j, connu uniquement de l'Agent j, et requis pour l'évaluation de la CTC présentée dans le Théorème 14.

Pour résoudre ce problème, chaque Agent i possède un estimateur additionnel $v^j = \left[v_1^{jT} \dots v_N^{jT}\right]^T \in \mathbb{R}^{Nn}$ de y^j pour tout $j \in \mathcal{N}_i \cup \{i\}$, avec pour contrainte que les estimations v^i réalisées par les Agents i et $j \in \mathcal{N}_i$ soient identiques. v^i est réalisé par l'Agent i et tous ses voisins $j \in \mathcal{N}_i$, et mis à jour seulement quand la CTC est satisfaite par l'Agent i. Les v^js sont donc mis à jour moins fréquemment que les y^is et sont donc moins précis. Les deux estimateurs sont évalués simultanément par chaque agent. Introduire v^j ne requit aucune modification pour le delayed flooding protocol.

La dynamique de l'estimateur additionnel v^i est exprimée sous la forme

$$\dot{v}_j^i(t) = Av_j^i(t) + B\bar{u}_j^i(t), \quad t_k^i \leq t < t_{k+1}^i \tag{E.12}$$

$$\bar{u}_j^i(t) = c_1 F \sum_{p \in \mathcal{N}_\ell} \left(v_j^i(t) - v_p^i(t)\right) \tag{E.13}$$

$$v^i(t_{i,k}) = y^i(t_{i,k}) \tag{E.14}$$

$$v_j^i(t_{j,k}) = y_j^j(t_{j,k}), \quad j \in \mathcal{N}_i. \tag{E.15}$$

E.2.4 Méthode event-triggered

Dans le Théorème 14, l'état initial de tous les agents est supposé connu par tous les agents. Dans la partie expérimentale, cette condition sera relaxée : Agent i initialisera l'état des estimateurs de tous les autres agents avec la valeur de son propre état. Une communication est déclenchée à $t = 0$ pour mettre à jour les estimateurs des voisins de l'Agent i. Tous les autres agents agissent de même.

Introduisons $\widehat{L} = L \otimes P$, $\overline{L} = \widehat{L} A_c + A_c^T \widehat{L}$, $A_c = \overline{A} + \overline{B}_1$, $\overline{A} = I_N \otimes A$, $\overline{B}_1 = c_1 L \otimes (BF)$, $M = PBB^T P$ et $\beta = \frac{\lambda_{\min>0}(-\overline{L})}{\lambda_{\max}(\widehat{L})}$. Il est prouvé dans [37] que $\bar{L}$ est semi-définie positive.

Théorème 14. *Supposons que (A, B) est contrôlable et le graphe de communication est non-orienté et connecté avec une topologie fixe, décrite par la matrice de Laplace L. On considère le paramètre de réglage $\eta > 0$. Les agents dont la dynamique est décrite par (E.4) achève un consensus borné*

$$\forall (i, j) \quad \lim_{t \to \infty} \|x_i - x_j\|^2 \leq \frac{N^3 \eta}{\beta \lambda_{\min}(P)} \tag{E.16}$$

si la condition suivante sur les perturbations est satisfaite

$$S_{\max} \leq \sqrt{\frac{\alpha \|c_2 \lambda_2(L) M\|}{\lambda_{\max}(P)}} \sqrt{\frac{N\eta}{\lambda_{\min}(P)\beta}} \tag{E.17}$$

et si une communication est déclenchée quand la condition suivante est satisfaite

$$\tilde{\delta}_i \geq \rho z_i^T \Theta z_i + \eta \tag{E.18}$$

avec $\Theta_i = (2c_2 - b_i N_i (c_2 - c)) M$, $1 \geq \rho > 0$ un paramètre de réglage

$$\begin{aligned}
\tilde{\delta}_i \;=\; & c_1 \Bigg[\frac{1}{2b_{i2}} \left(z_i - N_i e_i^i\right)^T M \left(z_i - N_i e_i^i\right) + \frac{b_{i2}}{2} \sum_{j \in \mathcal{N}_i} N_j \left(y_j^i - v_j^i\right)^T M \left(y_j^i - v_j^i\right) \\
& + \left(z_i - N_i e_i^i\right)^T M \sum_{j \in \mathcal{N}_i} \left(v_i^j - y_i^i\right) + \frac{N_i}{2b_i} e_i^{iT} M e_i^i + 2\left(1 + \frac{b_i}{2}\right) N_i \sum_{j \in \mathcal{N}_i} \Bigg[\left(v_i^j - y_i^i\right)^T M \left(v_i^j - y_i^i\right) \\
& + \left(y_j^i - v_j^i\right)^T M \left(y_j^i - v_j^i\right) \Bigg] \Bigg] + 2(c_2 - c) N_i z_i^T M e_i^i \\
& + \left[2c (N_i)^2 (1 + b_i) + \frac{c_2 - c}{b_i} N_i + c N_i (N - 1) \left(b_i + \frac{3}{b_i}\right) \right] e_i^{iT} M e_i^i
\end{aligned} \tag{E.19}$$

et $z_i = \sum_{j \in \mathcal{N}_i} \left(y_i^i - y_j^i\right)$, $M = PBB^T P$, $0 < b_i < \frac{2c_2}{(c - c_2) N_i}$ si $c_2 > c$, $b_i > 0$ sinon.

La preuve du Théorème 14 se trouve en Annexe A.2 et la preuve de l'absence de paradoxe de Zeno dans l'Annexe A.3.

En observant (E.17) et (E.16), on remarque que η peut être utilisé pour obtenir un compromis entre la valeur de l'erreur bornée du consensus et le nombre de communications. Si $\eta = 0$ et s'il n'y a pas de perturbation, le système achève un consensus asymptotique.

La CTC (E.18) dépend majoritairement de e_i^i, de l'écart $y_j^i - v_j^i$ et de l'écart $v_i^j - y_i^i$. Une communication est donc envoyée par l'Agent i quand l'erreur d'estimation de y_i^i ou v_i^i devient trop importante. Les deux perturbations ont un impact direct sur e_i^i, et donc sur la fréquence de communications. On remarque également que $M_{\max}$ ne possède une influence que sur la qualité du consensus mais non sur sa convergence.

E.2.5 Extension aux systèmes linéaires variables dans le temps

Dans les sections précédentes, la CTC a été développée pour des systèmes à dynamique linéaire constante dans le temps. En pratique, la plupart des systèmes ne sont pas de structure invariante avec le temps. C'est par exemple le cas des approximations de système non-linéaire, ou des modèles Fuzzy Takagi–Sugeno (T–S) [79].

On peut ainsi définir les matrices $A(t)$ et $B(t)$ telles que

$$\dot{x}_i(t) = A(t)\, x_i(t) + B(t)\, u_i(t) \tag{E.20}$$

$$u_i(t) = c_1 F \sum_{j \in \mathcal{N}_i} \left(y_i^i(t) - y_j^i(t) \right) \tag{E.21}$$

Le Théorème (14) et la preuve dans l'Annexe A restent valides avec les nouvelles matrices $A(t)$ et $B(t)$, permettant d'élargir les résultats obtenus par le Théorème 14 aux systèmes linéaires variables au cours du temps. La matrice P est réévaluée au cours du temps en fonction des valeurs de $A(t)$ et $B(t)$.

E.2.6 Topologie variable avec le temps

Le graphe de communication $\mathcal{G}$ peut changer au cours du temps. En effet, des liens peuvent apparaitre/disparaitre à cause d'interférences ou de la distance entre agents. Il faut donc garantir que la convergence du système est toujours assurée même en présence de changement de topologie. Quelques travaux de la littérature comme [124, 125, 78] montrent qu'un consensus peut être atteint s'il existe un sous-graphe en arbre dans le graphe de communication. Étudions ce problème dans le cadre de notre méthode.

Détection du changement de topologie La première difficulté est de pouvoir détecter quand un changement de topologie a lieu. Si l'apparition d'une nouvelle connexion entre agents est facile à détecter (réception d'un message venant d'un agent non-voisin jusqu'à présent), la détection de la disparition d'une connexion est bien plus ardue dans un système à communication réduite. En effet, il est difficile de savoir si l'on ne reçoit plus de communication venant d'un voisin parce que celui-ci n'a pas émis de message ou parce que le lien entre les deux agents a disparu. Un méthode de détection pourrait être d'émettre systématiquement un message à intervalle fixe : la non réception de ce message indiquerait que la communication est brisée. Cela implique cependant un plus grand nombre de communications dans le système.

Adaptation de la commande à la topologie variable En supposant que le problème précédent ait été résolu, il s'agit par la suite d'avertir le reste de la flotte du changement de topologie. La connaissance de celle-ci est en effet nécessaire au calcul de la commande et des estimateurs des agents. Une méthode alternative est de rendre l'évaluation de la commande partiellement indépendante de la topologie. Pour cela, on définit un sous-graphe de communication minimum $\mathcal{G}_{\min}(\mathcal{N}_{\min}, \mathcal{E}_{\min})$ que l'on suppose toujours existant dans toutes les topologies rencontrées. On réécrit la commande de l'Agent i en utilisant l'ensemble des voisins minimum $\mathcal{N}_{\min,i}$, tel que $u_i(t) = c_1 F \sum_{j \in \mathcal{N}_{\min,i}} \left(y_i^i(t) - y_j^i(t) \right)$, de même pour $\tilde{u}_j^i(t)$ et $\bar{u}_j^i(t)$.

Les informations reçues par l'Agent i provenant de l'Agent j tel que $j \in \mathcal{N}_i(t)$ et $j \notin \mathcal{N}_{\min,i}$ sont utilisées pour l'estimation des états des agents, mais pas pour l'élaboration de la commande.

En utilisant la fonction de Lyapunov $V_{\min} = x^T L_{\min} x$ où $L_{\min}$ est la matrice de Laplace associée à $\mathcal{G}_{\min}$, le Théorème 14 est valide, quelque soit la topologie en utilisant $z_i = \sum_{j \in \mathcal{N}_{\min,i}} \left(y_i^i - y_j^i \right)$, $\Theta_i = (2c_2 - b_i N_{\min,i}(c_2 - c))\, M$, et réécrivant $\tilde{\delta}_i$ en utilisant $\mathcal{N}_{\min,i}$ à la place de $\mathcal{N}_i$.

E.2.7 Conclusion

Ce chapitre présente une méthode de communication event-triggered distribuée pour système multi-agents avec une réduction du nombre de communications comparé à l'état de l'art.

Pour obtenir ces résultats, chaque agent évalue simultanément deux estimateurs de l'état des agents dans le réseau.

Le premier fournit une estimation précise de l'état de tous les agents, mais dont la valeur des estimations diffère entre les agents. Le deuxième estimateur est moins précis, mais est construit de manière à fournir des valeurs identiques entre voisins. Les deux estimateurs sont utilisés pour déterminer les communications.

Un protocole de communication par relais a été développé pour garantir la mise à jour des estimateurs sans ajouter de communications supplémentaires à la stratégie initiale.

Une communication par event-triggered distribuée permettant d'obtenir un nombre réduit de communications tout en atteignant un consensus borné a été développée en tenant compte de perturbations d'état. La convergence du consensus a été étudiée et l'absence de paradoxe de Zeno a été prouvée.

Des simulations montrent l'efficacité des estimateurs proposés en présence de perturbations, quand celles-ci sont d'un niveau modéré. Quelques lignes directrices ont été données afin de choisir un compromis entre l'erreur de consensus et le nombre de communications.

Enfin, des extensions des résultats obtenus pour les dynamiques non-linéaires T-S fuzzy et le cas des topologies variables ont été proposées. Les prochains travaux sur cette étude devront se focaliser sur l'influence des pertes de données durant les transmissions, ainsi que sur les délais de communications.

E.3 Déplacement en formation avec poursuite d'un objectif pour un système multi-agent par méthode event-triggered distribuée

Cette partie propose une stratégie de commande distribuée permettant la réduction du nombre de communications au sein d'un système multi-agents devant évoluer en formation tout en poursuivant une trajectoire de référence. La dynamique des agents est décrite par un système de type Euler-Lagrange incluant des perturbations sur l'état. La matrice d'inertie et la matrice regroupant les effets centripètes et Coriolis sont supposées inconnues par les agents. Une loi de commande adaptative est proposée en se basant sur une estimation de ces paramètres ainsi que sur une estimation des états des agents voisins (non disponibles de manière continue). Une CTC distribuée basée sur l'erreur d'estimation d'état et les distances inter-agents garantit la réduction du nombre de communications. L'effet des perturbations sur la convergence du système est analysé, et l'absence de paradoxe de Zeno est démontrée.

Les notations et hypothèses utilisées sont introduites dans la Section E.3.1. La définition du problème de déplacement en formation est faite en Section E.3.2 et le problème de poursuite d'une trajectoire de référence est introduit dans la Section E.3.2. Une commande adaptative est définie en Section E.3.2 afin de conduire la flotte vers la formation désirée tout en suivant la trajectoire choisie. La communication est gérée via la méthode event-triggered introduite dans la Section E.3.4.

E.3.1 Notations et hypothèses

On considère le système composé de N agents, dont la topologie est décrite par un graphe non-orienté $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. L'ensemble des voisins de l'Agent i est $\mathcal{N}_i = \{j \in \mathcal{N} | (i,j) \in \mathcal{E},\ i \neq j\}$. N_i est le nombre d'élément de $\mathcal{N}_i$. Pour un vecteur quelconque $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T \in \mathbb{R}^n$, on définit $|x| = \begin{bmatrix} |x_1| & |x_2| & \dots & |x_n| \end{bmatrix}^T$ où $|x_i|$ est la valeur absolue de la i-ième composante de x. De la même manière, la notation $x \geq 0$ est utilisée pour indiquer que chaque composante de x est positive ou nulle, *i.e.* $\forall i \in \{1 \dots n\}\ x_i \geq 0$.

On définit $q_i \in \mathbb{R}^n$ le vecteur des coordonnées de l'Agent i dans un repère global quelconque $\mathcal{R}$ et $q = \begin{bmatrix} q_1^T & q_2^T & \dots & q_N^T \end{bmatrix}^T \in \mathbb{R}^{N.n}$ la *configuration* du MAS. La dynamique de chaque agent est décrite par un système Euler-Lagrange tel que

$$M_i(q_i)\,\ddot{q}_i + C_i(q_i,\, \dot{q}_i)\,\dot{q}_i + G = \tau_i + d_i(t)\,, \tag{E.22}$$

où $\tau_i \in \mathbb{R}^n$ est la commande, $M_i(q_i) \in \mathbb{R}^{n\times n}$ est la matrice d'inertie de l'Agent i, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ est la matrice des termes de Coriolis et centripètes de l'Agent i, G contient les effets de l'accélération gravitationnelle, supposés connus, et $d_i(t)$ est le vecteur perturbation vérifiant $\|d_i(t)\| < D_{\max}$. Le vecteur d'état de l'Agent i est $x_i^T = \left[q_i^T, \dot{q}_i^T\right]$. On suppose que les hypothèses suivantes sont respectées :

A1) $M_i(q_i)$ est définie symétrique positive et il existe $k_M > 0$ satisfaisant $\forall x,\ x^T M_i(q_i)\, x \leqslant k_M x^T x$,

A2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ est une matrice antisymétrique ou définie négative, et il existe $k_C > 0$ satisfaisant $\forall x,\ x^T C_i(q_i, \dot{q}_i)\, x \leqslant k_C \|\dot{q}_i\|\, x^T x$,

A3) Il existe $\dot{q}_{\max} \in \mathbb{R}_+^n$ et $\ddot{q}_{\max} \in \mathbb{R}_+^n$ tels que $|\ddot{q}_i| \leq \ddot{q}_{\max}$ et $|\dot{q}_i| \leq \dot{q}_{\max}$.

A4) Le terme gauche de (E.22) peut être représenté sous une forme linéaire en le vecteur θ_i des paramètres inconnus mais constants associés à l'Agent i :

$$M_i(q_i)\, x_1 + C_i(q_i, \dot{q}_i)\, x_2 = Y_i(q_i, \dot{q}_i, x_1, x_2)\, \theta_i \tag{E.23}$$

pour tous vecteurs $x_1, x_2 \in \mathbb{R}^n$, et où $Y_i(q_i, \dot{q}_i, x_1, x_2)$ est la matrice de régression.

A5) Les composantes du vecteur θ_i sont bornées :$\theta_{\min,i} < \theta_i < \theta_{\max,i}$.

Dans la suite du texte, les notations M_i et C_i sont utilisées pour remplacer $M_i(q_i)$ et $C_i(q_i, \dot{q}_i)$. Dans cette étude, on suppose que chaque Agent i est capable de mesurer sans erreur son propre état x_i. De plus, on considère qu'il n'y a pas de délai de communication.

E.3.2 Définition de la commande

Le but de cette section est de définir une loi de commande distribuée permettant de conduire un MAS vers une formation désirée dans le repère global $\mathcal{R}$, tout en suivant une trajectoire de référence et en réduisant autant que possible le nombre de communications entre les agents. La formation à atteindre est décrite dans la Section E.3.2 et la paramétrisation du problème de poursuite d'une trajectoire de référence est définie dans la Section E.3.2.

Paramétrisation du déplacement en formation

On considère le vecteur de coordonnés relatives $r_{ij} = q_i - q_j$ entre deux agents i et j, ainsi que le vecteur de coordonnées relatives désirées r_{ij}^* pour tout $(i, j) \in \mathcal{N}$. La formation désirée est définie par l'ensemble $\left\{r_{ij}^*, (i, j) \in \mathcal{N}\right\}$. On considère, sans perdre de généralité, que le premier agent est l'agent de référence et on introduit le vecteur de configuration relatif désiré $r^* = \left[\begin{array}{ccc} r_{11}^{*T} & \ldots & r_{1N}^{*T} \end{array}\right]^T$. Chaque vecteur relatif désiré r_{ij}^* peut ainsi être exprimé comme $r_{ij}^* = r_{1i}^* - r_{1j}^*$.

On définit l'énergie potentiel $P(q, t)$ de la formation tel qu'introduit dans [72, 82],

$$P(q, t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} k_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2 \tag{E.24}$$

où les $k_{ij} = k_{ji}$ sont des coefficients de pondération, pouvant être positifs ou nuls, avec $k_{ii} = 0$.

Définition 13. Le MAS converge asymptotiquement vers la formation désirée avec une erreur bornée ssi il existe $\varepsilon_1 > 0$ tel que

$$\lim_{t\to\infty} P(q, t) \leqslant \varepsilon_1. \tag{E.25}$$

Afin d'obtenir une convergence bornée pour le MAS, on cherchera à construire une loi de commande permettant de réduire $P(q, t)$ au cours du temps.

Formation variable avec le temps et trajectoire de référence

Dans cette section, le système multi-agent doit suivre une trajectoire de référence $q_1^*(t)$, tout en maintenant la formation désirée. L'Agent 1 est toujours considéré comme l'agent de référence cherchant à suivre $q_1^*(t)$. Cette trajectoire de référence $q_1^*(t)$ est la seule devant être définie et on supposera qu'elle est connue de tous les agents. De même, on suppose que le vecteur $r^*(t)$ peut évoluer au cours du temps afin de faire évoluer la formation désirée. On peut ainsi définir la trajectoire individuelle de chaque Agent i telle que $q_i^*(t) = q_1^*(t) + r_{i1}^*(t)$. Pour garantir que la trajectoire individuelle peut être suivie par chaque agent, on impose que $|\dot{q}_i^*| < \dot{q}$ et $|\ddot{q}_i^*| < \ddot{q}_{\max}$.

Définition 14. Le MAS atteint sa trajectoire de référence ssi il existe $\varepsilon_1 > 0$ et $\varepsilon_2 > 0$ tels que (E.25) est satisfait et

$$\lim_{t\to\infty} \|q_1(t) - q_1^*(t)\| \leqslant \varepsilon_2, \tag{E.26}$$

i.e., ssi l'agent de référence a convergé sur la trajectoire de référence et que le MAS a convergé vers sa formation désirée avec une erreur bornée.

Afin de pouvoir créer une commande distribuée permettant d'atteindre l'objectif souhaité, on introduit les erreurs de trajectoire $r_i = q_i - q_i^*$ et $\hat{r}_i^j = \hat{q}_i^j - q_i^*$.

Ainsi, une commande cherchant à réduire l'énergie potentielle $P(q, t)$ et l'erreur de trajectoire r_i permet d'obtenir une convergence bornée du MAS. Pour décrire l'évolution de $P(q, t)$ et r_i, on redéfinit les termes suivant

$$g_i = \frac{\partial P(q, t)}{\partial q_i} + k_0 r_i = \sum_{j=1}^{N} k_{ij}\left(r_{ij} - r_{ij}^*\right) + k_0 r_i \tag{E.27}$$

$$\dot{g}_i = \sum_{j=1}^{N} k_{ij}\left(\dot{r}_{ij} - \dot{r}_{ij}^*\right) + k_0 \dot{r}_i \tag{E.28}$$

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p g_i \tag{E.29}$$

où g_i et $\dot{g}_i$ caractérisent l'évolution de l'écart entre la formation actuelle et la formation désirée, $k_p > 0$ est un paramètre de réglage. Le paramètre de réglage $k_0 \geq 0$ peut être choisi nul s'il n'y a pas de trajectoire à suivre.

Commande distribuée

La commande proposée par [82] permet de réduire $P(q, t)$ pour converger vers la formation désirée dans le cas d'une communication permanente, supposant donc un accès continu de chaque agent à l'état de ses voisins. Dans notre cas, les agents n'ont pas accès aux informations de leurs voisins en permanence : on introduit donc une estimation $\hat{q}_j^i$ de q_j réalisée par l'Agent i pour remplacer les informations manquantes dans la commande. On note ainsi $\hat{q}^i = \left[\begin{array}{ccc} \hat{q}_1^{iT} & \ldots & \hat{q}_N^{iT} \end{array}\right]^T \in \mathbb{R}^{N.n}$ le vecteur des coordonnées estimées par l'Agent i. L'évolution de $\hat{q}_j^i$ est décrite dans la Section E.3.3. De plus, les agents n'ayant pas accès au vecteur θ_i, une estimation $\bar{\theta}_i$ de θ_i est donc également implémentée pour pouvoir définir la loi de commande.

Dans le contexte d'une communication distribuée et limitée, chaque Agent i peut évaluer les termes

$$\bar{g}_i = \sum_{j=1}^{N} k_{ij}\left(\bar{r}_{ij} - r_{ij}^*\right) + k_0 r_i \tag{E.30}$$

$$\bar{s}_i = \dot{q}_i - \dot{q}_i^* + k_p \bar{g}_i \tag{E.31}$$

avec $\bar{r}_{ij} = q_i - \hat{q}_j^i$ et $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$. A partir de ces termes, on peut ensuite évaluer la loi de commande de l'Agent i définie par

$$\tau_i\left(q_i, \dot{q}_i, \hat{q}^i, \dot{\hat{q}}^i\right) = -k_s \bar{s}_i - k_g \bar{g}_i + G - Y_i\left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i\right)\bar{\theta}_i \tag{E.32}$$

$$\dot{\bar{\theta}}_i^i = \Gamma_i Y_i\left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i\right)^T \bar{s}_i \tag{E.33}$$

où $\bar{p}_i = k_p \bar{g}_i - \dot{q}_i^*$, $\dot{\bar{p}}_i = k_p \dot{\bar{g}}_i - \ddot{q}_i^*$, $k_g > 0$, $k_s \geq 1 + k_p(k_M + 1)$ est un paramètre de réglage et Γ_i une matrice positive choisie arbitrairement.

La Section E.3.3 introduit l'estimateur $\hat{q}_j^i$ de q_j nécessaire à la définition de cette loi de commande (E.3.2).

E.3.3 Protocole de communication et estimateurs d'état

Dans la suite, on notera $t_{j,k}$ l'instant où le k-ième message est envoyé par l'Agent j. On note également $t_{j,k}^i$ l'instant où le k-ième message, envoyé par l'Agent j est reçu par l'Agent i. Quand une communication est déclenchée à $t_{i,k}$ par l'Agent i, celui-ci envoie un message contenant $q_i(t_{i,k})$, $\dot{q}_i(t_{i,k})$ et son vecteur $\bar{\theta}_i(t_{i,k})$. Quand le message est reçu par d'autres agents, son contenu est utilisé pour mettre à jour l'estimation de l'état de l'Agent i comme présenté dans la section suivante. On supposera que le message est reçu par tous les agents, voisins ou non de l'Agent i, qu'il soit transmis directement, comme dans le cas d'un graphe entièrement connecté, ou par plusieurs transmissions successives. Dans ce dernier cas, le protocole de flooding [44, 83] est utilisé. Comme il n'y a pas de délai de communication, on en déduit que $\hat{q}_i^i(t) = \hat{q}_i^j(t)$ pour tout $(i, j) \in \mathcal{N}^2$.

Dynamique des estimateurs

Suivant l'idée présentée dans la Partie 2, l'estimation $\hat{q}_j^i$ de q_j réalisée par l'Agent i est évalue en considérant

$$\hat{M}_j^i\left(\hat{q}_j^i\right)\ddot{\hat{q}}_j^i + \hat{C}_j^i\left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i\right)\dot{\hat{q}}_j^i + G = \hat{\tau}_j^i,\ \forall t \in \left[t_{j,k}^i,\, t_{j,k+1}^i\right[\tag{E.34}$$

$$\hat{x}_j^i\left(t_{j,k}^i\right) = x_j\left(t_{j,k}^i\right) \tag{E.35}$$

avec $\hat{x}_j^i = \left[\hat{q}_j^{iT},\, \dot{\hat{q}}_j^{iT}\right]$ où $\hat{M}_j^i\left(\hat{q}_j^i\right)$ et $\hat{C}_j^i\left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i\right)$ sont des estimations de M_j et C_j réalisées à partir des éléments de $\bar{\theta}_j\left(t_{j,k}^i\right)$ par la relation

$$\hat{M}_j^i\left(\hat{q}_j^i\right)x + \hat{C}_j^i\left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i\right)y = Y_j\left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i,\, x,\, y\right)\bar{\theta}_j\left(t_{j,k}^i\right).$$

L'estimateur (E.34) réalisé par l'Agent i requiert une estimation $\hat{\tau}_j^i$ de la commande τ_j employée par l'Agent j. Cette estimation est évaluée par

$$\hat{\tau}_j^i = -k_s\hat{s}_j^i - k_g\hat{g}_j^i + G - Y_j\left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i,\, \dot{\hat{p}}_j^i,\, \hat{p}_j^i\right)\hat{\theta}_j^i \tag{E.36}$$

$$\dot{\hat{\theta}}_j^i = \Gamma_j Y_j\left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i,\, \dot{\hat{p}}_j^i,\, \hat{p}_j^i\right)^T\hat{s}_j^i \tag{E.37}$$

$$\hat{\theta}_j^i\left(t_{j,k}^i\right) = \bar{\theta}_j\left(t_{j,k}^i\right) \tag{E.38}$$

où $\bar{p}_i = k_p\bar{g}_i - \dot{q}_i^*$, $\dot{\bar{p}}_i = k_p\dot{\bar{g}}_i - \ddot{q}_i^*$, $\hat{s}_j^i = \dot{\hat{q}}_j^i + k_p\hat{g}_j^i$, $\hat{g}_j^i = \sum_{k=1}^N k_{jk}\left(\hat{r}_{jk}^i - r_{jk}^*\right)$, $\dot{\hat{g}}_j^i = \sum_{k=1}^N k_{jk}\left(\dot{\hat{r}}_{jk}^i - \dot{r}_{jk}^*\right)$, $\hat{r}_{jk}^i = \hat{q}_j^i - \hat{q}_k^i$, et $\hat{\theta}_j^i$ est l'estimation de $\bar{\theta}_j$.

Il existe une erreur entre q_i et son estimation $\hat{q}_i^j$ réalisée par l'Agent j à cause de la présence de perturbations et de la communication non permanente, mais également de la différence entre θ_i, $\bar{\theta}_i$, et $\hat{\theta}_i$. Les erreurs d'estimation réalisées par l'Agent j peuvent sont définies par

$$e_i^j = \hat{q}_i^j - q_i,\ j \in \mathcal{N} \tag{E.39}$$

Ces erreurs sont utilisées dans la Section E.3.4 pour déclencher les communications quand e_i^i et $\dot{e}_i^i$ deviennent trop importantes.

E.3.4 Communication par Event-triggered

Le Théorème 15 introduit une condition de déclenchement des communications de manière à assurer une convergence asymptotique vers la formation désirée et la trajectoire de référence. La valeur initiale de l'état de tous les agents est supposée connue par tous. Cette condition peut être satisfaite en effectuant une première communication à $t = 0$. On définit $\alpha_i = \sum_{j=1}^N k_{ij}$ et $\alpha_{\mathrm{M}} = \max_{i=1,\ldots,N}\alpha_i$. On définit aussi pour $\bar{\theta}_i = \left[\bar{\theta}_{i,1},\, \ldots,\bar{\theta}_{i,p}\right]^T$

$$\Delta\theta_{i,\max} = \begin{bmatrix} \max\left\{\left|\bar{\theta}_{i,1} - \theta_{\min,i1}\right|,\left|\bar{\theta}_{i,1} - \theta_{\max,i1}\right|\right\} \\ \vdots \\ \max\left\{\left|\bar{\theta}_{i,p} - \theta_{\min,ip}\right|,\left|\bar{\theta}_{i,p} - \theta_{\max,ip}\right|\right\} \end{bmatrix}.$$

Théorème 15. *Considérons un MAS dont la dynamique des agents est décrite par (E.22) et de loi de commande (E.32). On considère les paramètres* $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$,

$$c_3 = \frac{\min\left\{1, k_1, k_p, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min}k_{\min}}{k_{\max}}\right)\right\}}{\max\left\{1, k_M\right\}}$$

et $k_1 = k_s - (1 + k_p(k_M + 1))$. *En l'absence de délai de communication, les agents convergent vers la formation et la trajectoire désirées en vérifiant*

$$\lim_{t\to\infty}\sum_{i=1}^N k_0\left\|r_i\right\|^2 + \frac{1}{2}P(q,t) \leq \xi \tag{E.40}$$

avec $\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3\Delta_{\max}\right]$ *où* $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T\Gamma_i^{-1}\Delta\theta_i\right)\right)$, *si les communications sont déclenchées quand l'une des conditions suivantes est respectée*

$$k_s\bar{s}_i^T\bar{s}_i + k_pk_g\bar{g}_i^T\bar{g}_i + \eta \leq \alpha_M^2\left(k_ee_i^{iT}e_i^i + k_pk_M\dot{e}_i^{iT}\dot{e}_i^i\right) + \alpha_Mk_C^2k_p\left\|e_i^i\right\|^2\sum_{j=1}^N k_{ji}\left[\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right]^2$$
$$+k_gb_i\left\|\dot{q}_i - \dot{q}_i^*\right\|^2 + k_p\left\|e_i^i\right\|\left[\alpha_M^2\left(1 + \left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right) + \frac{\left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2}{\left(1 + \left\||Y_i|\,\Delta\theta_{i,\max}\right\|^2\right)}\right] \tag{E.41}$$

$$\|\dot{q}_i\| \geq \left\|\dot{\hat{q}}_i^i\right\| + \eta_2 \tag{E.42}$$

avec $k_e = k_sk_p^2 + k_gk_p + \frac{k_g}{b_i}$ *et* $Y_i = Y_i\left(q_i,\, \dot{q}_i,\, \dot{\bar{p}}_i,\, \bar{p}_i\right)$.

La CTC proposée dans le Théorème 15 a été construite en supposant que les estimateurs de l'état des agents et que le protocole de communication garantissent que $\forall\,(i,j) \in \mathcal{N}\times\mathcal{N}$,

$$\hat{x}_i^i\,(t) = \hat{x}_i^j\,(t) \tag{E.43}$$
$$\hat{x}_i^i\,(t_{i,k}) = x_i^i\,(t_{i,k})\,. \tag{E.44}$$

Le Théorème 15 est valide indépendamment de la façon dont les estimateurs $\hat{x}_i^i$ de x_i respectent (E.43) et (E.44).

De (E.40) et (E.41), on peut observer que η permet un compromis entre la borne ξ de l'erreur de formation et de poursuite, et le nombre de communications.

On note qu'une petite valeur de η_2 amène à un grand nombre de déclenchements de (E.42), et, à l'opposé, une grande valeur amène à un grand nombre de déclenchements de (E.41). Le réglage de ce paramètre η_2 permet donc d'obtenir un compromis.

Les CTCs (E.41) et (E.42) dépendant principalement de e_i^i et $\dot{e}_i^i$, une communication est déclenchée quand les erreurs d'estimation deviennent trop importantes. Garder e_i^i et $\dot{e}_i^i$ petits via un estimateur précis est donc recommandé afin de diminuer le nombre de communications.

Les erreurs d'estimation e_i^i et $\dot{e}_i^i$ sont dues aux perturbations et à la différence entre M_i et C_i et leur estimations respectives $\hat{M}_i^i$ et $\hat{C}_i^i$, déterminée par la précision de $\bar{\theta}$. Ainsi, même en absence de perturbation, ces différences font que les CTCs seront toujours satisfaites au bout d'un certain temps.

Le choix des paramètres α_{M}, k_g, k_p et b_i déterminent aussi le nombre de communications réalisées. Choisir k_{ij} tel que α_i est petit permet de réduire le nombre de déclenchements de (E.41). La contre partie d'un α_i petit est la diminution de la robustesse aux perturbations. Ces effets peuvent être tempérés par les choix de k_p et k_g, qui ne doivent cependant pas être choisis trop grands pour limiter le nombre de déclenchements de (E.41).

E.3.5 Conclusion

Dans cette Section, une commande adaptative et une stratégie de communication de type event-triggered ont été élaborées pour système multi-agents, afin d'atteindre une formation désirée et de suivre une trajectoire de référence. La dynamique considéré pour les agents est de type Euler-Lagrange et prend en compte une perturbation sur l'état. Les matrices d'inertie et de forces de Coriolis étant supposées inconnues, des estimateurs des paramètres inconnus et des états des agents ont été proposés pour remplacer les informations manquantes. Une condition distribuée de déclenchement des communications a été proposée permettant la convergence du système vers la formation désirée et la trajectoire de référence tout en réduisant le nombre de communications. L'absence de paradoxe de Zeno a été démontrée.

Dans les sections suivantes, les résultats obtenus seront étendus au problème de pertes de donnée et aux délais de communications.

E.4 Déplacement en formation d'un système multi-agents avec pertes de données dans les communications

Cette partie porte sur le problème de pertes de données durant les transmissions de messages. Comme dans la partie précédente, on considère un système multi-agents avec une dynamique Euler Lagrange soumise à des perturbations, et l'on désire obtenir une formation prédéfinie tout en poursuivant une trajectoire de référence.

Les pertes de données sont des phénomènes courant dans les réseaux et sont la cause de nombreuses pannes, en particulier dans les systèmes ou les communications sont gérées par des méthodes de type event-triggered. En effet, la réduction des communications fait que l'information contenue dans chaque message est d'autant plus importante pour assurer la convergence du système. De plus, la détection d'une perte de donnée est difficile à réaliser quand le système est distribué.

Les hypothèses sont formulées dans la Section E.3.1 sont conservées. Les problèmes liés aux pertes de données sont exposés dans la Section E.4.1. Un nouvel estimateur adapté à ces problèmes est décrit dans la Section E.4.2. Le calcul de l'espérance de l'erreur d'estimation, nécessaire à l'évaluation de la nouvelle CTC décrite dans la Section E.4.3, est abordé dans la Section E.4.4 en utilisant un estimateur additionnel introduit en Section E.4.4.

Le problème des pertes de données est étudié dans la Section E.5. Le nouveau contenu des messages est décris dans la Section E.5.1 : il s'agit d'une prédiction de l'état des agents, définie en Section E.5.3. Une CTC, se déclenchant plus tôt afin de compenser les problèmes de pertes de communications, est proposée en Section E.4.3.

E.4.1 Problème des pertes de données

Du fait de la limitation des bandes passantes et des perturbations extérieurs (vagues, interférence, présence d'obstacles...), les messages envoyés peuvent être sujet à des pertes de données. Une modélisation de ce phénomène peut être faite au niveau de la mise à jour des estimateurs par la relation

$$\hat{q}_j^i\left(t_{j,k}^{i+}\right) = \tilde{\alpha}_{j,k}^i q_j\left(t_{j,k}\right) + \left(1 - \tilde{\alpha}_{j,k}^i\right)\hat{q}_j^i\left(t_{j,k}^{i-}\right) \tag{E.45}$$

où $\tilde{\alpha}_{j,k}^i$ est un processus stochastique traduisant la perte d'information quand le message k envoyé par l'Agent j est transmis à l'Agent i. Cette variables peut prendre la valeur de 1 ou 0 selon une distribution de Bernoulli suivant la probabilité $P\left(\tilde{\alpha}_{j,k}^i = 1\right) = \bar{\alpha}$ et $P\left(\tilde{\alpha}_{j,k}^i = 0\right) = 1 - \bar{\alpha}$ avec $0 \leq \bar{\alpha} \leq 1$. Le k-ième message est reçu avec succès par l'Agent i si $\tilde{\alpha}_{j,k}^i = 1$. On remarque que $\tilde{\alpha}_{j,k}^j$ est toujours égal à 1.

On définit $\hat{q}_j^i\left(t, q_j\left(t_{j,k}^i\right)\right)$ l'estimation de $q_j(t)$ effectuée par l'Agent i en utilisant la valeur $q_j\left(t_{j,k}^i\right)$ comme dernière valeur de mise à jour. Par défaut, $\hat{q}_j^i(t) = \hat{q}_j^i\left(t, q_j\left(t_{j,k}^i\right)\right)$. On note que si l'Agent i reçoit le k-ième message envoyé par l'Agent j, $\hat{q}_j^i\left(t_{j,k}^i\right) = \hat{q}_j^j\left(t_{j,k}^i\right)$.

Dans le Théorème 16, l'Agent j a besoin de la valeur de e_i^j, donc de $\hat{q}_i^j$. Cependant, $\hat{q}_i^j$ est inconnu par l'Agent i. Du fait des pertes de données, on ne peut être assuré de la synchronisation de $\hat{q}_i^j$ et $\hat{q}_i^i$ comme c'était le cas avec l'estimateur (E.46), car on ne peut pas savoir si les Agents i et j ont accès aux mêmes informations.

La Section E.4.2 proposera donc un nouveau modèle d'estimateur de $\hat{q}_i^i$ ne requérant pas les estimations des états des autres agents $\hat{q}_j^i$. De plus, un estimateur additionnel est introduit dans la Section E.4.4 afin de permettre d'évaluer e_i^j dans la Section E.4.4.

E.4.2 Nouvel estimateur

On définit le nouveau modèle d'estimation tel que

$$\hat{M}_j^i\left(\hat{q}_j^i\right)\ddot{\hat{q}}_j^i + \hat{C}_j^i\left(\hat{q}_j^i, \dot{\hat{q}}_j^i\right)\dot{\hat{q}}_j^i + G = \hat{\tau}_j^i, \ \forall t \in \left[t_{j,k}^i, t_{j,k+1}^i\right[\tag{E.46}$$

$$\hat{x}_j^i\left(t_{j,k}^i\right) = x_j\left(t_{j,k}^i\right) \quad \text{if } \tilde{\alpha}_{j,k}^i = 1 \tag{E.47}$$

avec

$$\hat{\tau}_j^i = -k_s\left(\dot{\hat{r}}_j^i + k_p k_0 \hat{r}_j^i\right) - k_g k_0 \hat{r}_j^i + G - Y_j\left(\hat{q}_j^i, \dot{\hat{q}}_j^i, \dot{\hat{m}}_j^i, \hat{m}_j^i\right)\hat{\theta}_j^i. \tag{E.48}$$

$$\dot{\hat{\theta}}_j^i = \Gamma_j Y_j\left(\hat{q}_j^i, \dot{\hat{q}}_j^i, \dot{\hat{m}}_j^i, \hat{m}_j^i\right)^T\left(\dot{\hat{r}}_j^i + k_p k_0 \hat{r}_j^i\right) \tag{E.49}$$

où $\hat{r}_j^i = \hat{q}_j^i - q_j^*$, et $\hat{m}_j^i = k_p k_0 \hat{r}_j^i - \dot{q}_j^*$si $k_0 > 0$, $\hat{m}_j^i = 0$ sinon. Note que si $k_0 = 0$, $\dot{q}_j^* = 0$.

Remarque 9. Quand la formation converge vers la formation désirée, on observe que $\hat{g}_j^i = k_0 \hat{r}_j^i$ et $\hat{s}_j^i = \dot{\hat{r}}_j^i + k_p k_0 \hat{r}_j^i$.

Le modèle d'estimation (E.46) et la commande (E.48) sont évalués en utilisant uniquement les informations de l'Agent i, permettant d'éviter une partie des problèmes de pertes d'informations. Ainsi, on n'a besoin d'effectuer uniquement une estimation de l'état des agents j tel que $k_{ij} \neq 0$ et de soi-même. On note que la commande E.48 est moins précise que E.36.

De même, s'il existe un instant $t = t_{j,k}^i$ tel que $\hat{x}_j^i \left(t_{j,k}^i\right) = x_j \left(t_{j,k}^i\right)$, *i.e* $\tilde{\alpha}_{j,k}^i = 1$, alors $\hat{x}_j^i(t) = \hat{x}_j^j(t)$ $\forall t \in \left[t_{j,k}^i, t_{j,k+1}^i\right[$ et $e_j^i(t) = e_j^j(t)$.

Le problème pour évaluer $e_j^i(t)$ quand $\tilde{\alpha}_{j,k}^i = 0$ est décrit dans la Section E.4.4.

E.4.3 Condition de déclenchement en présence de pertes de données

L'étude d'une nouvelle CTC prenant en compte les pertes de données a conduit à montrer que celle-ci était dépendante de e_i^j. Cependant, l'Agent i ne peut avoir accès à e_i^j sans une communication permanente. Aussi, un estimateur additionnel $\breve{q}_i^j$ a été introduit dans la Section E.4.4 afin d'obtenir l'erreur additionnelle $\breve{e}_i^j$ telle que $\left\|\breve{e}_i^j\right\|^2 \leq \left\|e_i^j\right\|^2$. Cette erreur est utilisée dans le Théorème 16.

Comme dans le Théorème 15, la valeur initiale des vecteurs d'état est considérée comme connue par tous les agents. En pratique, cette condition peut être satisfaite en effectuant une première communication par tous les agents à l'instant $t = 0$ permettant d'initialiser les estimateurs.

Théorème 16. *On considère le MAS avec des agents respectant la dynamique (E.22) et la commande (E.36). On considère les paramètres de réglages* $\eta \geq 0$, $\eta_2 > 0$, $0 < b_i < \frac{k_s}{k_s k_p + k_g}$, $c_3 = \frac{\min\left\{1, k_1, k_p, k_0, 2k_0\left(2k_0 + \frac{\alpha_{\min} k_{\min}}{k_{\max}}\right)\right\}}{\max\{1, k_M\}}$ *et* $k_1 = k_s - (1 + k_p(k_M + 1))$. *Les agents peuvent converger vers la formation désirée en vérifiant*

$$\lim_{t\to\infty} P(q,t) + \sum_{i=1}^{N} k_0^2 \|r_i\|^2 \leq \xi \tag{E.50}$$

avec $\xi = \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta + c_3 \Delta_{\max}\right]$ *où* $\Delta_{\max} = \max_{i=1:N}\left(\sup_{t>0}\left(\Delta\theta_i^T \Gamma_i^{-1} \Delta\theta_i\right)\right)$, *si les communications sont déclenchées quand l'une des conditions suivantes est satisfaite*

$$\begin{aligned} k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta \leq \alpha_M &\left[\sum_{j=1}^{N} k_{ij}\left(k_e \mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right) + k_p k_M \mathbb{E}\left(\left\|\dot{\breve{e}}_i^j\right\|^2\right)\right)\right. \\ &\left. + k_p k_C^2 \sum_{j=1}^{N} k_{ij} \mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)\left[\left\|\dot{\hat{q}}_j^i\right\| + \eta_2\right]^2\right] + k_g b_i \|\dot{q}_i - \dot{q}_i^*\|^2 \\ &+ k_p \sum_{j=1}^{N} k_{ij} \mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right)\left[\alpha_M\left(1 + \||Y_i|\,\Delta\theta_{i,\max}\|^2\right) + \frac{\||Y_i|\,\Delta\theta_{i,\max}\|^2}{\left(1 + \||Y_i|\,\Delta\theta_{i,\max}\|^2\right)}\right] \end{aligned} \tag{E.51}$$

$$\|\dot{q}_i\| \quad \geq \quad \left\|\dot{\hat{q}}_i^i\right\| + \eta_2 \tag{E.52}$$

avec $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, $Y_i = Y_i\left(q_i, \dot{q}_i, \dot{\bar{p}}_i, \bar{p}_i\right)$.

Le problème du paradoxe de Zeno est décrit dans la Section E.4.4.

Les CTCs proposées dans le Théorème 16 sont analysées en supposant que les estimateurs et le protocole de communication garantissent que $\forall (i,j) \in \mathcal{N} \times \mathcal{N}$, e_i^j ou $\breve{e}_i^j$ peuvent être évalués par l'Agent i.

Remarque 10. S'il n'y a pas de pertes de données, *i.e.* $\bar{\alpha} = 1$, on obtient $\mathbb{E}\left(\left\|\breve{e}_i^j\right\|^2\right) = \left\|\breve{e}_i^j\right\|^2 = \left\|e_i^j\right\|^2$. De plus, si les estimateurs sont synchronisés de manière à obtenir $e_i^j = e_i^i$, le Théorème 16 devient équivalent au Théorème 15.

E.4.4 Estimateur additionnel et erreur d'estimation

Protocole de communication

Quand une communication est déclenchée à $t_{i,k}$ par l'Agent i, il transmet un message contenant $t_{i,k}$, $x_i\left(t_{i,k}\right)$, $\bar{\theta}_i\left(t_{i,k}\right)$ et $\hat{x}^i = \left[\hat{x}_1^{iT}, \ldots, \hat{x}_N^{iT}\right]^T$. On suppose que le message est transmis à tous les voisins $j \in \mathcal{N}_i$ si les paramètres k_{ij} sont choisis tel que $k_{ij} = 0$ si $\forall j \notin \mathcal{N}_i$. Sinon, on suppose que le message est reçu par tous les agents j si $k_{ij} \neq 0$, que cela soit de manière directe quand le graphe est entièrement connecté, ou après plusieurs sauts si le graphe est connecté.

Paradoxe de Zeno

En présence de pertes de données, il n'est pas garanti que les estimations $\hat{q}_i^j$ soient mises à jour quand l'Agent i envoie un message à $t = t_{i,k}$, et donc que les erreurs e_i^j et $\breve{e}_i^j$ soient remises à zéro. Aussi, la CTC E.51 dans la Section E.4.4 peut être toujours satisfaite même après qu'une communication ait été déclenchée. Pour résoudre ce problème, on impose un délai minimum après satisfaction d'une CTC E.51 avant de l'évaluer à nouveau, et on définit le Lemme 3.

Lemme 3. On définit la constante $\epsilon > 0$ telle que $\forall t \in I_{k,\epsilon} = \left[t_{i,k}, t_{i,k} + \epsilon\right]$, $x_i\left(t\right) \simeq x_i\left(t_{i,k}\right)$, $\hat{x}_i\left(t\right) \simeq \hat{x}_i\left(t_{i,k}\right)$ et $\breve{x}_i\left(t\right) \simeq \breve{x}_i\left(t_{i,k}\right)$. On considère que $\tau_{\min}$ est choisi tel que $\tau_{\min} K \leq \epsilon$ où $\exists K \in \mathbb{N}$ et $K \geq 2$. $\forall t \in I_{k,\epsilon}$ et $\forall \ell \in [k, \ldots k+K]$ tels que $t_{i,\ell+1} - t_{i,\ell} = \tau_{\min}$, *i.e* la CTC (E.51) est satisfaite tous les $\tau_{\min}$ depuis l'instant $t = t_{i,k}$, on a

$$\mathbb{E}\left(\left\|e_i^j\left(t_{i,\ell}\right)\right\|^2\right) \simeq \left(1 - \left(1 - \bar{\alpha}\right)^{\ell-k}\right)\left\|\hat{q}_i^i\left(t_{i,\ell}\right) - q_i\left(t_{i,\ell}\right)\right\|^2 + \left(1 - \bar{\alpha}\right)^{\ell-k}\left\|\hat{q}_i^j\left(t_{i,\ell}\right) - q_i\left(t_{i,\ell}\right)\right\|^2.$$

Ainsi, si $\tau_{\min}$ est choisi suffisamment petit, *i.e.*choisi K suffisamment grand,

$$\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right)t \rightarrow 0 \quad \text{quand } t \rightarrow t_{i,k} + \tau_{\min} K$$

et la CTC (E.51) ne sera plus satisfaite quand $t = t_{i,k} + \epsilon$.

La preuve du Lemme 3 est décrite dans l'Annexe D.2. L'existence de $\tau_{\min}$ et du Lemme 3 garantissent l'absence de paradoxe de Zeno. Néanmoins, on peut obtenir un grand nombre de communication durant l'intervalle $I_{k,\epsilon}$.

Estimateur additionnel

Durant l'étude du Théorème 16, l'Agent j avait initialement besoin de e_j^i, donc de $\hat{q}_j^i$. Cependant, l'Agent j ne peut avoir accès à $\hat{q}_j^i$ tout le temps à cause des pertes de données. Pour résoudre ce problème, on a défini l'estimateur additionnel $\breve{q}_j^i$, estimation de $\hat{q}_j^i$ réalisée par l'Agent j. Chaque Agent j évalue $\breve{q}_j^i$, une estimation de $\hat{q}_j^i$ pour tous les Agent i tels que $k_{ji} \neq 0$. $\breve{q}_j^i$ est mis à jour quand l'Agent j reçoit un message de l'Agent i, *i.e.* quand $t = t_{i,k}^j$, $\breve{q}_j^i\left(t_{i,k}^j\right) = \hat{q}_j^i\left(t_{i,k}^j\right)$. On garantit ainsi que $\breve{q}_j^i\left(t\right) = \hat{q}_j^i\left(t\right)$ pour $t \in \left[t_{i,k}^j, t_{j,k+1}\right[$, *i.e* tant que l'Agent j n'envoie pas de message.

La dynamique de $\breve{q}_j^i$ est exprimée par

$$\hat{M}_j^i\left(\breve{q}_j^i\right)\ddot{\breve{q}}_j^i + \hat{C}_j^i\left(\breve{q}_j^i, \dot{\breve{q}}_j^i\right)\dot{\breve{q}}_j^i + G = \breve{\tau}_j^i, \ \forall t \in \left[t_{i,k}^j, t_{j,k+1}\right[\tag{E.53}$$

$$\breve{x}_j^i\left(t_{i,k}^j\right) = \hat{x}_j^i\left(t_{i,k}^j\right) \quad \text{if } \tilde{\alpha}_{i,k}^j = 1 \tag{E.54}$$

avec

$$\breve{\tau}_j^i = -k_s\left(\dot{\breve{r}}_j^i + k_p k_0 \breve{r}_j^i\right) - k_g k_0 \breve{r}_j^i + G - Y_j\left(\breve{q}_j^i, \dot{\breve{q}}_j^i, \dot{\breve{m}}_j^i, \breve{m}_j^i\right)\breve{\theta}_j^i. \tag{E.55}$$

$$\dot{\breve{\theta}}_j^i = \Gamma_j Y_j\left(\breve{q}_j^i, \dot{\breve{q}}_j^i, \dot{\breve{m}}_j^i, \breve{m}_j^i\right)^T\left(\dot{\breve{r}}_j^i + k_p k_0 \breve{r}_j^i\right) \tag{E.56}$$

où $\breve{r}_j^i = \breve{q}_j^i - q_j^*$et $\breve{m}_j^i = k_p k_0 \breve{r}_j^i - \dot{q}_j^*$si $k_0 > 0$, $\breve{m}_j^i = 0$ sinon.

L'estimation $\breve{x}_j^i$ représente la pire estimation possible de $\hat{x}_j^i$, car elle considère que l'Agent i n'a reçu aucune information venant de l'Agent j pour se mettre à jour. De la même manière, $\hat{x}_j^j$ représente l'estimation la plus optimiste de $\hat{x}_j^i$, dans le cas où l'Agent i a reçu tous les messages provenant de l'Agent j.

Espérance de l'erreur d'estimation

Comme $\breve{q}_i^j$ est mis à jour moins fréquemment que $\hat{q}_i^j$, et $\hat{q}_i^j = \hat{q}_i^i$ si $\hat{q}_i^j$ est mis à jour en utilisant le dernier message envoyé par l'Agent i, l'erreur $\left\|e_i^j\right\|$ peut être majorée par l'erreur $\left\|\breve{e}_i^j\right\|$ décrite dans cette section. En étudiant e_i^j et en utilisant le protocole de communication décrit Section E.4.4, on obtient l'évaluation suivante de $\mathbb{E}\left(\left\|\breve{e}_i^j(t)\right\|^2\right) \geq \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)$

- Si $\forall t \in \left[t_{j,h}^i,\, t_{i,k}\right[$
$$\mathbb{E}\left(\left\|\breve{e}_i^j(t)\right\|^2\right) = \left\|\breve{q}_i^j(t) - q_i(t)\right\|^2 \tag{E.57}$$

- Si $t > t_{i,k}$ et $t_{i,k} - t_{i,k-1} > \epsilon$,
$$\mathbb{E}\left(\left\|\breve{e}_i^j(t)\right\|^2\right) = \bar{\alpha}\left\|\hat{q}_i^i(t) - q_i(t)\right\|^2 + (1-\bar{\alpha})\left\|\breve{q}_i^j(t) - q_i(t)\right\|^2 \tag{E.58}$$

- Si $t > t_{i,k}$ et $\exists K \in \mathbb{N}^*$ tel que $t_{i,k} - t_{i,k-K} \leq \epsilon$,
$$\mathbb{E}\left(\left\|\breve{e}_i^j(t)\right\|^2\right) = \left(1 - (1-\bar{\alpha})^{K+1}\right)\left\|\hat{q}_i^i(t) - q_i(t)\right\|^2 + (1-\bar{\alpha})^{K+1}\left\|\breve{q}_i^j(t) - q_i(t)\right\|^2 \tag{E.59}$$

La preuve de (E.57)-(E.58)-(E.59) est décrite dans l'Annexe D.3.

Remarque 11. Sur l'intervalle $\forall t \in \left[t_{j,h}^i,\, t_{i,k}\right] \cup [t_{i,k},\, t_{i,k+1}[$, on a $\mathbb{E}\left(\left\|\breve{e}_i^j(t)\right\|^2\right) = \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)$.

E.4.5 Conclusion

Cette section a abordé le problème des pertes de données lors des communications entre agents. Les méthodes introduites précédemment ont été adaptées dans cette section pour traiter cette problématique. L'influence des pertes de données sur les estimateurs a été étudiée : celles-ci empêchent notamment les estimateurs de se synchroniser, imposant de créer une nouvelle CTC. Les estimateurs ont été modifiés afin d'être moins sensibles aux pertes d'informations, et un estimateur additionnel, représentant le pire cas où aucun message n'a été reçu, a été introduit. Ces deux estimateurs permettent une évaluation distribuée de la nouvelle CTC. La convergence du système vers la formation désirée et la trajectoire de référence a été étudiée. De plus, un protocole de communication a été mis en place pour résoudre les problèmes de paradoxe de Zeno.

Les perspectives sont d'étendre la méthode proposée aux problèmes de délais de communications. De plus, une modélisation des pertes de données par une chaine de Markov, plus réaliste, pourra être considérée.

E.5 Problème des délais de communication

Dans cette section, le problème des délais de communication bornés entre les agents est étudié et les résultats de la Section E.3 sont adaptés pour pouvoir le résoudre. Notamment, la condition du Théorème 15 a été adaptée, en s'inspirant de [84]. Le protocole de communication et des modèles de prédiction de l'état des agents sont décrits afin de permettre l'implémentation pratique de la méthode.

Le problème des délais de communications et leur influence sur la définition du contenu des messages à adopter sont décrits en Section E.5.1. Ces messages contiennent désormais une prédiction de l'état des agents, calculée en Section E.5.3. Une nouvelle CTC, déclenchant en avance pour contre-balancer les problèmes de délais, est proposée en Section E.5.2. Enfin, les conclusions sont présentés en Section E.5.4.

E.5.1 Contenu du message

On définit $\tau_{ij} = t_{i,k}^{j} - t_{i,k}$ le délai de communication entre l'instant où l'Agent i émet un message et l'instant où l'Agent j le reçoit. On suppose que τ_{ij} peut être majoré par une constant $T > \tau_{ij}(t)$ pour tout couple (i,j). Considérons l'influence des délais de communications : dans le Théorème 15, la condition (E.41) garantit la stabilité et la convergence du système global si à l'instant $t_{i,k}^{j}$, l'estimation $\hat{x}_i^j\left(t_{i,k}^{j}\right)$ est mise à jour avec la valeur $x_i\left(t_{i,k}^{j}\right)$. Ainsi, pour garantir la condition tout en prenant en compte les délais de communications, le message contenant $x_i\left(t_{i,k}^{j}\right)$ doit être envoyé à $t = t_{i,k} \leq t_{i,k}^{j} - \tau_{ij}$. Cependant, $x_i\left(t_{i,k}^{j}\right)$ ne peut être connu à $t = t_{i,k}$. Une prédiction $\tilde{x}_i^i\left(t_{i,k}^{j}\right)$ de $x_i\left(t_{i,k}^{j}\right)$ doit donc être effectuée.

Pour tout vecteur $y \in \mathbb{R}^n$, on définit $\tilde{y}_i^i(t+T) \in \mathbb{R}^n$ la prédiction de l'état $y_i(t+T)$ effectuée par l'Agent i . Le modèle de prédiction sera étudié dans la Section E.5.1.

Dans le Théorème 16, les agents ont besoin que les estimateurs soient synchronisés de manière à assurer que $\hat{x}_i^i(t) = \hat{x}_i^j(t)\ \forall (i,j) \in \mathcal{N}$. Tant que $\tau_{ij}(t)$ est inconnu par les agents et que $\tau_{ij}(t) < T$, les agents mettront à jour leur estimation de x_i à l'instant $t_{i,k}+T$, quand tous les agents auront reçu le message envoyé par l'Agent i. Ainsi, les valeurs estimées sont synchronisées.

$$\begin{aligned} \hat{x}_i^j(t_{i,k}+T) &= \tilde{x}_i^i(t_{i,k}+T) \quad \forall j \in \mathcal{N}_i, \\ \hat{x}_i^i(t_{i,k}+T) &= \tilde{x}_i^i(t_{i,k}+T). \end{aligned}$$

L'inconvénient de cette méthode est que les estimateurs sont mis à jour en utilisant une prédiction de x_i et non la véritable valeur, donc l'erreur $e_i^i(t_{i,k}+T)$ n'est pas remise complètement à zéro. De fait, l'absence de paradoxe de Zeno ne peut être prouvée.

Le problème de la synchronisation des estimateurs ayant été résolu, la prochaine section se concentrera sur la nouvelle CTC.

E.5.2 Condition de déclenchement en présence de délais de communication

Comme expliqué dans la section précédente, le message doit être envoyé en avance pour compenser le délai de communication. On cherche donc à savoir à l'instant t si la condition sera satisfaite à l'instant $t+T$ en utilisant les prédictions $\bar{s}_i(t+T)$,$\bar{g}_i(t+T)$, $e_i^i(t+T)$, $\dot{e}_i^i(t+T)$, $\dot{q}_i^i(t+T)$, $\dot{\hat{q}}_i^i(t+T)$, $\bar{p}_i(t+T)$ et $\dot{\bar{p}}_i(t+T)$.

Théorème 17. *On considère un MAS avec la dynamique (E.22) et la commande (E.32). On considère les paramètres de réglage η, ξ, η_2, β_e, $\beta_{\dot{e}}$, β_g, β_s, β_q et $0 < b_i < \frac{k_s}{k_s k_p + k_g}$. En présence de délai de communications $\tau_{ij} < T$, les agents peuvent converger vers la formation désirée en vérifiant*

$$\lim_{t\to\infty} P(q,t) + \sum_{i=1}^{N} k_0^2 \left\|r_i\right\|^2 \leq \xi \tag{E.60}$$

si les communications sont déclenchées quand l'une de ces conditions est satisfaite

$$\begin{aligned} k_s \left\|\tilde{s}_i^i(t+T)\right\|^2 + k_p k_g \left\|\tilde{g}_i^i(t+T)\right\|^2 + \eta \leq \alpha_M^2 \left(k_e \left\|\tilde{e}_i^i(t+T)\right\|^2 + k_p k_M \left\|\dot{\tilde{e}}_i^i(t+T)\right\|^2\right) \\ + \alpha_M k_C^2 k_p \left\|\tilde{e}_i^i(t+T)\right\|^2 \sum_{j=1}^{N} k_{ji} \left[\left\|\dot{\tilde{q}}_j^i(t+T)\right\| + \eta_2\right]^2 + k_g b_i \left\|\dot{\tilde{q}}_i^i(t+T) - \dot{q}_i^*(t+T)\right\|^2 \\ + k_p \left\|\tilde{e}_i^i(t+T)\right\|^2 \left[\alpha_M^2 \left(1 + \left\|\left|Y_i\right| \Delta\theta_{i,\max}\right\|^2\right) + \frac{\left\|\left|Y_i\right| \Delta\theta_{i,\max}\right\|^2}{\left(1 + \left\|\left|Y_i\right| \Delta\theta_{i,\max}\right\|^2\right)}\right] \end{aligned} \tag{E.61}$$

$$\left\|\dot{\tilde{q}}_i^i\right\| \geq \left\|\dot{\hat{\tilde{q}}}_i^i\right\| + \eta_2 \tag{E.62}$$

avec $k_e = k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}$, $Y_i = Y_i\left(\tilde{q}_i(t+T), \dot{\tilde{q}}_i(t+T), \dot{\tilde{p}}_i(t+T), \tilde{p}_i(t+T)\right)$,

$$\Delta\theta_{i,\max} = \begin{bmatrix} \max\left\{\left|\tilde{\theta}_{i,1}^i(t+T) - \theta_{\min,i1}\right|, \left|\tilde{\theta}_i^i(t+T) - \theta_{\max,i1}\right|\right\} \\ \vdots \\ \max\left\{\left|\tilde{\theta}_{i,p}^i(t+T) - \theta_{\min,ip}\right|, \left|\tilde{\theta}_p^i(t+T) - \theta_{\max,ip}\right|\right\} \end{bmatrix}.$$

et si les conditions suivantes sont respectées

$$\|\bar{s}_i(t)\|^2 + \beta_s \geq \left\|\tilde{s}_i^j(t)\right\|^2 \quad \text{(E.63)}$$

$$\|\bar{g}_i(t)\|^2 + \beta_g \geq \left\|\tilde{g}_i^j(t)\right\|^2 \quad \text{(E.64)}$$

$$\left\|\dot{\tilde{q}}_i^i(t) - \dot{q}_i^*(t)\right\|^2 \geq \|\dot{q}_i(t) - \dot{q}_i^*(t)\|^2 - \beta_q \quad \text{(E.65)}$$

$$\left\|\tilde{e}_i^i(t)\right\|^2 \geq \left\|e_i^i(t)\right\|^2 - \beta_e \quad \text{(E.66)}$$

$$\left\|\dot{\tilde{e}}_i^i(t)\right\|^2 \geq \left\|\dot{e}_i^i(t)\right\|^2 - \beta_{\dot{e}}, \quad \text{(E.67)}$$

Les valeurs de $\beta_e, \beta_{\dot{e}}, \beta_g, \beta_s, \beta_q$ doivent être choisies suffisamment petite pour éviter des déclenchements inutiles de la condition (E.61) tout en respectant les conditions (E.63)-(E.67).

E.5.3 Modèle de prédiction

Modélèe de prédiction simple par discrétisation d'Euler

En utilisant la méthode d'Euler, on obtient un modèle de prédiction de la forme $y(t+T) = y(t) + T\dot{y}(t)$.

Prédiction de $x_i(t+T)$ et $\hat{x}_j^i(t+T)$, $\forall j \neq i\ \forall t \geq 0$ Dans cette méthode, la dynamique des agents et leur commande ne sont pas prises en compte. Ainsi, la prédiction de l'état de l'Agent i peut s'exprimer $\forall t \geq 0$ par

$$\tilde{x}_i^i(t+T) = x_i(t) + \dot{x}_i(t)\,T$$
$$\tilde{\hat{x}}_j^i(t+T) = \hat{x}_j^i(t) + \dot{\hat{x}}_j^i(t)\,T$$

On remarque que les futures mise à jour de $\hat{x}_j^i$ ne peuvent être connues par l'Agent i, créant une différence avec la véritable valeur. Cependant, l'Agent i connait les futures mises à jour de $\hat{x}_i^i$, permettant d'obtenir une prédiction plus précise.

Prédiction de $\tilde{\hat{x}}_i^i(t+T)$ Quand $t \geq t_{i,k} + T$, tous les messages envoyés par l'Agent i ont été reçus par les autres agents et leurs estimateurs ont été synchronisés. Le modèle de prédiction peut donc être écrit sous la forme $\tilde{x}_i^i(t+T) = x_i(t) + \dot{x}_i(t)\,T$. Cependant, quand $t \in [t_{i,k},\, t_{i,k}+T[$, l'Agent i mettra à jour $\hat{x}_i^i$ à l'instant $t = t_{i,k} + T$ en utilisant la prédiction $\tilde{x}_i^i(t_{i,k}+T)$ réalisée à l'instant $t_{i,k}$. Ainsi, $\tilde{\hat{x}}_i^i(t+T)$ peut être exprimé par $\forall t \geq 0$

$$\tilde{\hat{x}}_i^i(t+T) = x_i(t_{i,k}) + \dot{x}_i(t_{i,k})(T + t - t_{i,k}) + \ddot{x}_i(t_{i,k})\,T\,(t - t_{i,k})$$

Autres prédictions En utilisant les prédictions $\tilde{x}_i^i(t+T)$, $\tilde{\hat{x}}_i^i(t+T)$ et $\tilde{\hat{x}}_j^i(t+T)$, on peut évaluer les termes $\tilde{e}_i^i(t+T)$, $\dot{\tilde{e}}_i^i(t+T)$, $\dot{\tilde{q}}_j^i(t+T)$, $\dot{\tilde{\hat{q}}}_j^i(t+T)$, $\tilde{g}_i^i(t+T)$ et $\tilde{s}_i^i(t+T)$.

L'avantage de cette méthode est sa simplicité et son faible temps de calcul. Cependant, le délai T doit être suffisamment petit pour garantir que la prédiction $\tilde{x}_i^i$ soit suffisamment proche de la valeur réelle x_i.

Modèle de prédiction précis

Afin d'essayer d'être plus proche du comportement réel des agents, la dynamique de la prédiction de $\tilde{x}_i^i$ peut être modélisée par

$$\begin{aligned} \hat{M}_i^i(\tilde{q}_i^i)\,\ddot{\tilde{q}}_i^i + \hat{C}_i^i(\tilde{q}_i^i, \dot{\tilde{q}}_i^i)\,\dot{\tilde{q}}_i^i &= \tilde{\tau}_i^i + \tilde{d}_i \\ \tilde{x}_j^i(t_{i,k}) &= \hat{x}_j^i(t_{i,k}) \text{ si } j \neq i, \\ \tilde{x}_i^i(t_{i,k}) &= x_i(t_{i,k}) \end{aligned} \quad \text{(E.68)}$$

où t_{ini} est l'instant où la prédiction est évaluée, $\hat{x}_j^i = \left[\hat{q}_j^{iT}, \dot{\hat{q}}_j^{iT}\right]^T$, $\tilde{\tau}_i^i$ est la prédiction de la commande, et $\tilde{d}_i$ une prédiction des perturbations si cela est possible (on choisira généralement $\tilde{d}_i = \mathbb{E}\left(d_i\right)$). On note que la différence entre (M_i, C_i) et $\left(\hat{M}_i^i, \hat{C}_i^i\right)$ induit une erreur de prédiction.

On s'intéresse maintenant à la commande $\tilde{\tau}_i^i$. On peut choisir$\tilde{\tau}_i^i\left(t\right) = \tau_i^i\left(t_{i,k}\right)$, sous la forme (E.48) où $\hat{x}_i^i$ est remplacé par $\tilde{x}_i^i$. Si l'on souhaite faire une prédiction de la commande (E.36), une prédiction des estimés $\hat{q}_j^i$ est également nécessaire. On utilise alors le modèle

$$\begin{aligned}\hat{M}_j^i\left(\tilde{\hat{q}}_j^i\right)\ddot{\tilde{\hat{q}}}_j^i + \hat{C}_j^i\left(\tilde{\hat{q}}_j^i, \dot{\tilde{\hat{q}}}_j^i\right)\dot{\tilde{\hat{q}}}_i^i &= \tilde{\hat{\mathfrak{q}}}_j^i \\ \tilde{\hat{x}}_j^i\left(t_{ini}\right) &= \hat{x}_j^i\left(t_{ini}\right) \quad \forall j \in \mathcal{N} \\ \tilde{\hat{x}}_i^i\left(t_{i,k}+T\right) &= \tilde{x}_i^i\left(t_{i,k}+T\right)\end{aligned} \tag{E.69}$$

où$\tilde{\hat{\mathfrak{J}}}$ et $\tilde{\tau}_i^i$ sont les prédictions de la commande (E.36) et où $\hat{q}^i$ est remplacé par $\tilde{\hat{q}}^i$. (E.69) exprime la mise à jour de $\hat{x}_i^i$ à l'instant $t = t_{i,k} + T$ comme dans la Section E.5.3.

En utilisant les prédictions $\tilde{x}_i^i\left(t+T\right)$, $\tilde{\hat{x}}_i^i\left(t+T\right)$ et $\tilde{\hat{x}}_j^i\left(t+T\right)$, on peut évaluer les termes $\tilde{e}_i^i\left(t+T\right)$, $\dot{\tilde{e}}_i^i\left(t+T\right)$, $\dot{\tilde{q}}_j^i\left(t+T\right)$, $\dot{\tilde{\hat{q}}}_j^i\left(t+T\right)$, $\tilde{g}_i^i\left(t+T\right)$ et $\tilde{s}_i^i\left(t+T\right)$. Cette approche permet une meilleure prédiction de x_i, mais son principal inconvénient est son cout en temps de calcul τ_p, qui doit respecter la contrainte $T > \tau_p + \tau_{ij}\left(t\right)$. Cela en fait une méthode beaucoup plus difficilement implémentable que celle proposée dans E.5.3.

E.5.4 Conclusion

Cette section présente une adaptation de la méthode initialement proposée en Section E.3 pour le déplacement en formation et le suivi d'une trajectoire de référence, en présence de délais de communication. L'influence des délais sur le contenu des messages a été étudiée. Pour contre-balancer les effets des délais de communications, une prédiction de la valeur de l'état est transmise aux autres agents afin de mettre à jour leurs estimateurs de manière synchronisée. La CTC a été adaptée pour prendre en compte les délais de communications et est déclenchée en avance afin de les compenser. Deux modèles de prédiction de différentes complexités et précisions ont été proposés. La convergence du système vers la formation désirée et la trajectoire de référence a été étudiée, ainsi que l'absence de paradoxe de Zeno.

Dans de futurs travaux, le problème considéré sera étendu à ceux de topologie variable et de saturation de la commande. La combinaison des délais de communications avec les pertes d'informations sera également traitée.

E.6 Conclusions et perspectives

Conclusion

Dans cette thèse, des techniques de communication de type event-triggered et des lois de commandes distribuées ont été proposées, pour réduire le nombre de communications transmises au sein d'un système multi-agent coopératif. Les principales contributions de cette thèse sont les suivantes :

- Premièrement, une méthode de réduction de communications par event-triggered distribuée a été proposée pour le problème d'obtention d'un consensus entre agents possédant une dynamique linéaire généralisé, incluant des perturbations sur l'état. Les résultats obtenus ont été comparés avec d'autres approches.

- Deuxièmement, une stratégie event-triggered distribuée a été développée pour le problème de déplacement en formation et de suivi d'une trajectoire de référence par un système multi-agents. La dynamique considérée pour les agents est de type Euler-Lagrange et inclue également une perturbation sur l'état. Des extensions ont été proposées dans les cas avec incertitudes sur le modèle dynamique, présence de délais de communications et pertes de données.

Dans la première approche, le graphe communications et considéré comme fixe et les communications et sans délais. La méthode utilise simultanément deux estimateurs de l'état des agents : le premier permet une mesure précise de l'état de tous les agents de la flotte en prenant en compte une estimation de la dynamique et de la commande des agents. Le second considère seulement les voisins de chaque agent et est moins précis car mis à jour moins fréquemment que le premier. Cependant, sa valeur coïncide entre deux voisins. L'erreur d'estimation des deux estimateurs est utilisée pour évaluer la condition de déclenchement. Un protocole de communication nommé Flooding delay a été développé afin de mettre à jour les estimateurs sans ajouter de communication supplémentaire. Le lien entre les perturbations et l'erreur de consensus a été mis en évidence, et la convergence vers le consensus et l'absence de paradoxe de Zeno a été prouvé. Enfin, des extensions de ces résultats aux modèles linéaires variables dans le temps et au cas de topologies variables ont été discutées.

Dans la seconde approche, des incertitudes sur le modèle dynamique étant considérées : une estimation de ces paramètres ainsi que des états des agents de la flotte a été introduite pour remplacer les informations manquantes. Ces estimations sont utilisées afin d'obtenir une loi de commande distribuée. Une CTC distribuée, basée sur les erreurs d'estimation, a été définie pour réduire le nombre de communications tout en garantissant la convergence du système vers la formation et la trajectoire voulues avec une erreur bornée. L'absence de paradoxe de Zeno a également été prouvée.

Les délais de communication ainsi que les pertes de données ont aussi été étudiés. Dans le premier cas, deux modèles de prédictions de différente complexité et précision ont été considéré. La convergence vers la formation et la trajectoire de référence choisie a été étudiée et l'absence de paradoxe de Zeno a été prouvée. Pour prendre en compte les délais de communications, un estimateur additionnel et une CTC basée sur des paramètres stochastiques ont été développés en considérant l'espérance de l'erreur l'estimation due aux pertes d'informations. Pour garantir l'absence de paradoxe de Zeno, un protocole de communication spécifique a été proposé.

Perspectives

Plusieurs objectifs à moyen et long termes sont proposés ci-dessous.

Les pertes de données modélisées dans cette étude sont considérées comme mutuellement indépendantes avec une distribution de Bernoulli, et les solutions proposées sont basées sur ces caractéristiques. En pratique, les pertes de données peuvent ne pas être indépendantes (présence d'obstacle masquant des agents, matériel défectueux), et être représentées par une chaine de Markov. L'adaptation des stratégies proposées à ces nouvelles contraintes serait une amélioration intéressante pour augmenter la robustesse du système global. Les délais de communication ont été introduits, mais les conditions sur la CTC ont besoin d'être relaxées afin d'obtenir une communication moins importante car trop pessimiste. De plus, considérer de manière simultanée les problèmes de délais de communication et de pertes de données constituerait une amélioration importante.

Dans les méthodes event-triggered étudiées dans cette thèse, il est supposé que les CTCs sont évaluées en permanence. Sachant que les MAS sont des systèmes généralement discrétisés, évaluer les CTCs de manière périodique permettrait une implémentation plus pratique et plus proche d'un système réel.

Enfin, chaque agent est supposé mesurer son propose état sans erreur, ce qui constitue une condition peu réaliste. Un observateur d'état doit être introduit et l'impact d'un bruit de mesure doit alors être étudié. Modéliser l'incertitude de mesure en utilisant une erreur bornée pourrait être une manière d'intégrer ces erreurs de mesures dans la CTC, mais conduirait à une prise de décision plus pessimiste et donc une augmentation du nombre de déclenchements. L'extension aux travaux présentés dans [50, 95, 111, 120, 68] pourrait être une direction intéressante.

Control law and state estimators design for multi-agent system with reduction of communications by event-triggered approach

A large amount of research work has been recently dedicated to the study of Multi-Agent System and cooperative control. Applications to mobile robots, like unmanned air vehicles (UAVs), satellites, or aircraft have been tackled to insure complex mission such as exploration or surveillance. However, cooperative tasking requires communication between agents, and for a large number of agents, the number of communication exchanges may lead to network saturation, increased delays or loss of transferred packets, from the interest in reducing them. In event-triggered strategy, a communication is broadcast when a condition, based on chosen parameters and some threshold, is fulfilled. The main difficulty consists in determining the communication triggering condition (CTC) that will ensure the completion of the task assigned to the MAS. In a distributed strategy, each agent maintains an estimate value of others agents state to replace missing information due to limited communication.

This thesis focuses on the development of distributed control laws and estimators for multi-agent system to limit the number of communication by using event-triggered strategy in the presence of perturbation with two main topics, i.e. consensus and formation control. The first part addresses the problem of distributed event-triggered communications for consensus of a multi-agent system with both general linear dynamics and state perturbations. To decrease the amount of required communications, an accurate estimator of the agent states is introduced , coupled with an estimator of the estimation error, and adaptation of communication protocol. By taking into account the control input of the agents, the proposed estimator allows to obtain a consensus with fewer communications than those obtained by a reference method. The second part proposes a strategy to reduce the number of communications for displacement-based formation control while following a desired reference trajectory. Agent dynamics are described by Euler-Lagrange models with perturbations and uncertainties on the model parameters. Several estimator structures are proposed to rebuilt missing information. The proposed distributed communication triggering condition accounts for inter-agent displacements and the relative discrepancy between actual and estimated agent states. A single a priori trajectory has to be evaluated to follow the desired path. Effect of state perturbations on the formation and on the communications are analyzed. Finally, the proposed methods have been adapted to consider packet dropouts and communication delays. For both types of problems, Lyapunov stability of the MAS has been developed and absence of Zeno behavior is studied.

Loi de guidage coopérative et estimateurs d'état pour système multi-agent avec réduction des communications par méthode event-triggered

Les systèmes multi-agents (MAS) et la commande coopérative ont fait l'objet de nombreuses recherches ces dernières années. Les domaines d'application sont très diverses et dans le cas des systèmes multi-véhicules, des approches ont été développées pour des unmanned air vehicles (UAVs), satellites, avions... Le type de missions envisagées sont des missions complexes telles l'exploration ou la surveillance de zones, la recherche et le suivi de cibles d'intérêt. Cependant, la coopération requière des échanges de communication entre les agents. Lorsque ceux-ci sont nombreux, cet échange peut conduire à des saturations du réseau, à l'augmentation des délais de transmission ou l'occurrence de pertes de paquets, d'où l'intérêt de réduire le nombre de communication. Dans les méthodes event-triggered, une communication est envoyée quand une condition, basée sur des paramètres choisis et un seuil prédéfini, est remplie. La principale difficulté est de définir une condition qui permettra de limiter les échanges sans dégrader l'exécution de la mission choisie. Dans le cas d'un système distribué, chaque agent doit maintenir une estimation de la valeur de l'état des autres agents afin de remplacer l'absence d'informations due à la communication réduite.

L'objectif de cette thèse est de développer des lois de commandes et des estimateurs distribuées pour un système multi-agent afin de réduire le nombre de communication par méthode event-triggered, tout en prenant en compte la présence de perturbations. L'étude est divisée en deux grandes parties. La première décrit une méthode de communication event-triggered permettant de converger vers un consensus pour un système multi-agents de modèle d'évolution dynamique linéaire généralisée et en présence de perturbations d'état. Pour réduire les communications, un estimateur précis de l'état des agents est proposé, couplé à un estimateur de l'estimation de l'erreur, ainsi qu'un protocole de communication adapté. En prenant en compte la commande appliquée à chaque agent, l'estimateur proposé permet d'obtenir un consensus avec un nombre bien inférieur de communication que de la méthode de référence dans l'état de l'art. La seconde partie propose une stratégie de réduction de communication pour une commande de vol en formation permettant de suivre une trajectoire de référence. La dynamique des agents est décrite par un système Euler-Lagrange incluant des perturbations et des méconnaissances sur les paramètres du modèle. Différentes structures d'estimateurs sont proposées pour reconstruire les informations manquantes. La condition d'event-triggered distribuée proposée est basée sur l'écart relatif entre les positions et vitesses réelles et désirées des agents, ainsi que l'erreur relative entre la valeur estimée de l'état de l'agent et la valeur réelle. Une trajectoire de référence unique est déterminée pour guider la flotte. L'effet des perturbations sur la formation et la communication a été analysé. Enfin, les méthodes proposées ont été adaptées pour tenir compte des dégradations de performances dues aux pertes de données et aux délais de communication. Pour les deux types d'approches présentées les conditions de la stabilité du MAS ont été obtenues par l'intermédiaire de fonctions de Lyapunov et l'absence de paradoxe de Zeno a été étudiée.